STAT406- Methods of Statistical Learning Lecture 1

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About me

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Prerequisites

- STAT306 or CPSC340
- You are comfortable working independently
- You are motivated and enjoy being challenged
- You want to be here and are interested in learning the material

Philosophy of the class

- We're here to help you learn (vs. teaching you)
- We'll encourage engagement, curiosity and generosity
- We'll have zero tolerance for plagiarism
- We favour steady work through the Term (vs. sleeping until finals)

Lectures



- Bring your laptop
- Prepare for class
- Ask, doubt, question, discuss

Lectures / Labs / Office hours

- Two weekly lectures, one weekly lab
- Ongoing evaluation you are expected to attend all course meetings
- Pre-lecture readings and activities
- Office hour: TBA, check Canvas next week
- It is a 4th year course expectations are high

Grades

- Quizzes (Webwork) (\approx 9): 10%
- Lab activities (\approx 9): 10%
- Four in-class midterms: 40%
- Final exam: 40%.
- There will be no make-up activities, quizzes, labs, or exams. Anything you miss (with official documentation) will be assigned to your final exam weight.

Textbook?

- No mandatory textbook, BUT:
- Several reference books all available on-line @ UBC Library or publisher
- Lecture slides + verbose scripts
- Most used: [JWHT13] An Introduction to Statistical Learning, James, Witten, Hastie, Tibshirani, R., 2013, Springer, New York.

Computer



- We will use R
 - Open source and free
 - "Standard" in Statistics community, some industry
 - Webwork quizzes rely on R

Computer

- YOU ARE RESPONSIBLE for learning
- We CAN HELP with R
- We WON'T teach all of R
- There are tons of on-line resources

Other resources

- Canvas: Webwork, grades
- GitHub: Slides, notes, notebooks
- PIAZZA: Discussions, announcements

piazza.com/ubc.ca/winterterm12018/stat406

Code



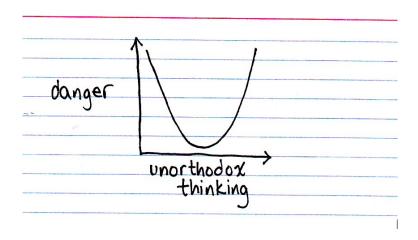
Thoughtlessly copying from the Prof

'It never ends well'

More about scripts

- You can read them from GitHub
- You can cut-and-paste block-by-block
- You can run / play with **notebooks**:
 - On your computer
 - On https://ubc.syzygy.ca/

Lectures?



thisisindexed.com

Discussion

Statistical learning

Discussion

Models versus "predictive algorithms"

- Y is the response variable
- X is a vector of auxiliary variables

$$Y = f(\mathbf{X}) + \varepsilon$$

- $f: \mathbb{R}^p \to \mathbb{R}$, unknown
- If $E[\varepsilon] = 0$

$$E[Y|X] = f(X)$$

• In a linear model, f is a linear function

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p + \varepsilon$$

• If $E[\varepsilon] = 0$

$$E[Y|X_1, X_2, \dots, X_p] = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p$$

- Why would we want to estimate the coefficients of the linear model?
- What's the connection with prediction?

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- What's the connection with prediction?
- "Best predictor"

$$\arg\min_{\mathbf{h}} E\left[(Y - \mathbf{h}(\mathbf{X}))^{2} \right] = E[Y|\mathbf{X}]$$

- Best predictor is the regression function
- We need to estimate E[Y|X]
- We propose a model (e.g. linear) for E[Y|X] and estimate it (e.g. estimate its parameters)
 - For example in a linear model, to estimate $f(\mathbf{X})$ we need to estimate $\beta_0, \beta_1, \ldots, \beta_p$

- Data (Y_1, \mathbf{X}_1) , (Y_2, \mathbf{X}_2) , ..., (Y_n, \mathbf{X}_n) ,
- Least squares estimator

$$\hat{\boldsymbol{\beta}} = \arg\min_{\beta_0,\beta} \sum_{i=1}^{n} (Y_i - \beta_0 - \boldsymbol{\beta}' \mathbf{X}_i)^2$$

ullet There is a closed form for \hat{eta}

$$\hat{eta}=(X'X)^{-1}X'\mathbf{Y}$$
 where $\mathbf{Y}=(Y_1,Y_2,\ldots,Y_n)'$ and $(1\ \cdots\ \mathbf{X}_1\ \cdots$

$$X = \begin{pmatrix} 1 & \cdots & \mathbf{X}_1 & \cdots \\ 1 & \cdots & \mathbf{X}_2 & \cdots \\ 1 & \cdots & \mathbf{X}_3 & \cdots \\ \vdots & \cdots & \cdots & \cdots \\ 1 & \cdots & \mathbf{X}_n & \cdots \end{pmatrix}$$

• As long as $E[\varepsilon] = E[\varepsilon | \mathbf{X}] = 0$ we have

$$E\left[\hat{\beta}\right] = \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{pmatrix}$$

- The LS estimator is consistent and unbiased
- Do we need any other assumption?

- Consider the air pollution data
- n = 60 observations
- p = 16, response variable: MORT
- A linear model:

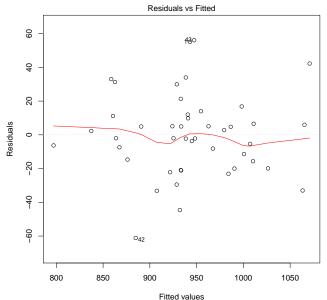
MORT =
$$\beta_0 + \beta_1$$
 PREC + β_2 JANT + . . . + ϵ or equivalently

$$E\left(\mathsf{MORT} \middle| \mathsf{PREC}, \mathsf{JANT}, \ldots\right)$$

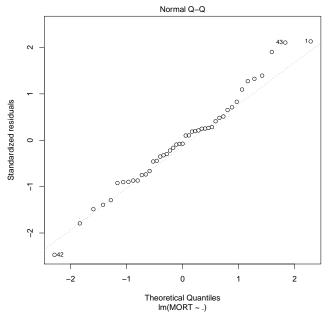
= $\beta_0 + \beta_1 \, \mathsf{PREC} + \beta_2 \, \mathsf{JANT} + \ldots$

- Randomly split into a training (n=45) and a test set (n=15)
- Use training set to fit a model
- Read data into object x.tr
- Fit the "full" model
- "Look" at the fit

Diagnostics



Diagnostics



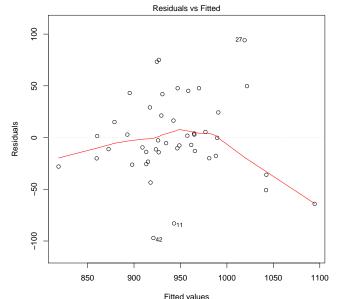
Diagnostics

```
> full <- lm(MORT ~ ., data=x.tr)
> summary(full)
> sum( resid(full)^2 )
[1] 25898.8
```

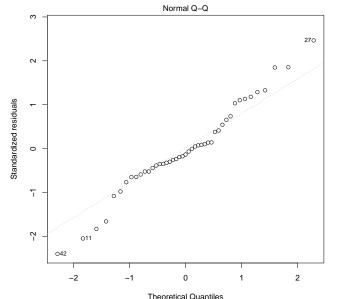
Fit a reduced model

```
> reduced <- lm(MORT ~ POOR + HC +
NOX + HOUS + NONW, data=x.tr)
> sum( resid(reduced)^2 )
[1] 66135.29
```

Diagnostics for reduced model



Diagnostics for reduced model



Discussion

Goodness of fit versus prediction power

Predictions

- Read the test set
- Use both models to predict MORT
- Compare both sets of predictions

Predictions

```
> x.te <- read.table('pollution-test.dat',...</pre>
>
> x.te$pr.full <- predict(full, newdata=x.te)
> x.te$pr.reduced <- predict(reduced,</pre>
       newdata=x.te)
>
> with (x.te, mean ( (MORT - pr.full) ^2 ))
[1] 4677.45
>
> with(x.te, mean( (MORT - pr.reduced)^2 ))
[1] 1401.571
```

Discuss

Ready, set, go!

- Quiz 0 (Review quizz) on Webwork
- Practice Quiz on Canvas
- Check canvas.ubc.ca often
- Visit github.com/msalibian/STAT406
- Use PIAZZA
- Read before class
- Attend the lab