# STAT406- Methods of Statistical Learning Lecture 1

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#### About me

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# Prerequisites

- STAT306 or CPSC340
- You are comfortable working independently
- You are motivated and enjoy being challenged
- You want to be here and are interested in learning the material

# Philosophy of the class

- We're here to help you learn (vs. teaching you)
- We'll encourage engagement, curiosity and generosity
- We'll have zero tolerance for plagiarism
- We favour steady work through the Term (vs. sleeping until finals)

#### Lectures



- Bring your laptop
- Prepare for class
- Ask, doubt, question, discuss

#### Lectures / Labs / Office hours

- Two weekly lectures, one weekly lab
- Ongoing evaluation you are expected to attend all course meetings
- Pre-lecture readings and activities
- Office hour: TBA, check Canvas next week
- It is a 4th year course expectations are high

#### Grades

- Quizzes (Webwork) (pprox 9): 10%
- Lab activities ( $\approx$  9): 10%
- Four in-class midterms: 40%
- Final exam: 40%.
- There will be no make-up activities, quizzes, labs, or exams. Anything you miss (with official documentation) will be assigned to your final exam weight.

#### Textbook?

- No mandatory textbook, BUT:
- Several reference books all available on-line @ UBC Library or publisher
- Lecture slides + verbose scripts
- Most used: [JWHT13] An Introduction to Statistical Learning, James, Witten, Hastie, Tibshirani, R., 2013, Springer, New York.

### Computer



- We will use R
  - Open source and free
  - "Standard" in Statistics community, some industry
  - Webwork quizzes rely on R

## Computer

- YOU ARE RESPONSIBLE for learning
- We CAN HELP with R
- We WON'T teach all of R
- There are tons of on-line resources

#### Other resources

- Canvas: Webwork, grades
- GitHub: Slides, notes, notebooks
- PIAZZA: Discussions, announcements

piazza.com/ubc.ca/winterterm12018/stat406

#### Code



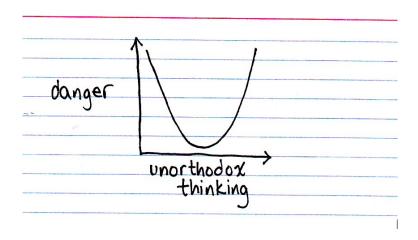
Thoughtlessly copying from the Prof

'It never ends well'

## More about scripts

- You can read them from GitHub
- You can cut-and-paste block-by-block
- You can run / play with notebooks:
  - On your computer
  - On https://ubc.syzygy.ca/

#### Lectures?



thisisindexed.com

#### Discussion

# Statistical learning

#### Discussion

# Models versus "predictive algorithms"

- Y is the response variable
- X is a vector of auxiliary variables

$$Y = f(\mathbf{X}) + \varepsilon$$

- $f: \mathbb{R}^p \to \mathbb{R}$ , unknown
- If  $E[\varepsilon] = 0$

$$E[Y|X] = f(X)$$

• In a linear model, f is a linear function

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p + \varepsilon$$

• If  $E[\varepsilon] = 0$ 

$$E[Y|X_1, X_2, \dots, X_p] = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p$$

- Why would we want to estimate the coefficients of the linear model?
- What's the connection with prediction?

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- What's the connection with prediction?
- "Best predictor"

$$\arg\min_{\mathbf{h}} E\left[ (Y - \mathbf{h}(\mathbf{X}))^2 \right] = E[Y|\mathbf{X}]$$

- Best predictor is the regression function
- We need to estimate E[Y|X]
- We propose a model (e.g. linear) for E[Y|X] and estimate it (e.g. estimate its parameters)
  - For example in a linear model, to estimate  $f(\mathbf{X})$  we need to estimate  $\beta_0, \beta_1, \ldots, \beta_p$

- Data  $(Y_1, \mathbf{X}_1)$ ,  $(Y_2, \mathbf{X}_2)$ , ...,  $(Y_n, \mathbf{X}_n)$ ,
- Least squares estimator

$$\hat{\boldsymbol{\beta}} = \arg\min_{\beta_0,\beta} \sum_{i=1}^{n} (Y_i - \beta_0 - \beta' \mathbf{X}_i)^2$$

• There is a closed form for  $\hat{\beta}$ 

$$\hat{\boldsymbol{\beta}} = (X'X)^{-1}X'\mathbf{Y}$$
 where  $\mathbf{Y} = (Y_1, Y_2, \dots, Y_n)'$  and 
$$X = \begin{pmatrix} 1 & \cdots & \mathbf{X}_1 & \cdots \\ 1 & \cdots & \mathbf{X}_2 & \cdots \\ 1 & \cdots & \mathbf{X}_3 & \cdots \\ \vdots & \cdots & \cdots & \ddots \\ 1 & \cdots & \mathbf{X}_n & \cdots \end{pmatrix}$$

• As long as  $E[\varepsilon] = E[\varepsilon | \mathbf{X}] = 0$  we have

$$E\left[\hat{\beta}\right] = \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{pmatrix}$$

- The LS estimator is consistent and unbiased
- Do we need any other assumption?

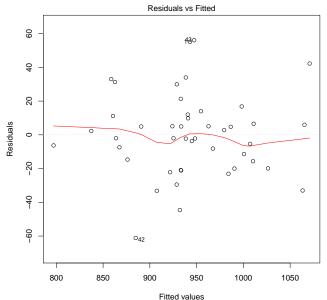
- Consider the air pollution data
- n = 60 observations
- p = 16, response variable: MORT
- A linear model:

MORT = 
$$\beta_0 + \beta_1$$
 PREC +  $\beta_2$  JANT + . . . +  $\epsilon$  or equivalently

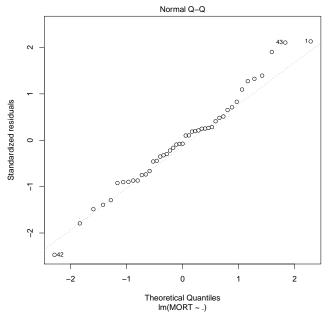
$$E\left(\mathsf{MORT} \middle| \mathsf{PREC}, \mathsf{JANT}, \ldots\right)$$
  
=  $\beta_0 + \beta_1 \, \mathsf{PREC} + \beta_2 \, \mathsf{JANT} + \ldots$ 

- Randomly split into a training (n=45) and a test set (n=15)
- Use training set to fit a model
- Read data into object x.tr
- Fit the "full" model
- "Look" at the fit

# **Diagnostics**



# **Diagnostics**



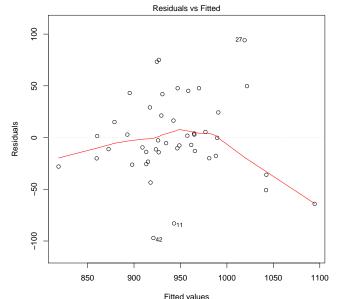
# Diagnostics

```
> full <- lm(MORT ~ ., data=x.tr)
> summary(full)
> sum( resid(full)^2 )
[1] 25898.8
```

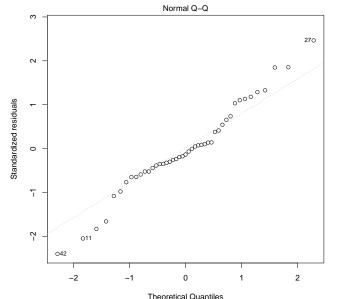
#### Fit a reduced model

```
> reduced <- lm(MORT ~ POOR + HC +
NOX + HOUS + NONW, data=x.tr)
> sum( resid(reduced)^2 )
[1] 66135.29
```

# Diagnostics for reduced model



# Diagnostics for reduced model



#### Discussion

# Goodness of fit versus prediction power

#### **Predictions**

- Read the test set
- Use both models to predict MORT
- Compare both sets of predictions

#### **Predictions**

```
> x.te <- read.table('pollution-test.dat',...</pre>
>
> x.te$pr.full <- predict(full, newdata=x.te)
> x.te$pr.reduced <- predict(reduced,</pre>
       newdata=x.te)
>
> with (x.te, mean ( (MORT - pr.full) ^2 ))
[1] 4677.45
>
> with(x.te, mean( (MORT - pr.reduced)^2 ))
[1] 1401.571
```

#### **Discuss**

# Ready, set, go!

- Quiz 0 (Review quizz) on Webwork
- Practice Quiz on Canvas
- Check canvas.ubc.ca often
- Visit github.com/msalibian/STAT406
- Use PIAZZA
- Read before class
- Attend the lab