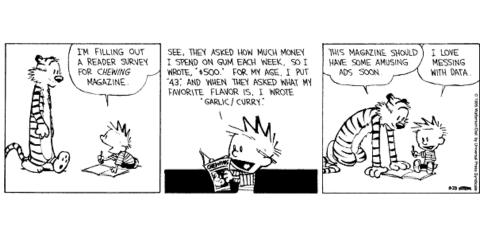
STAT406- Methods of Statistical Learning Lecture 2

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Mean squared prediction error (MSPE)

$$Y \longleftrightarrow \hat{f}_{n}(\mathbf{X})$$

$$\left(Y - \hat{f}_{n}(\mathbf{X})\right)^{2} ?$$

$$E\left[\left(Y - \hat{f}_{n}(\mathbf{X})\right)^{2}\right] ?$$

What are we "averaging" over? What is random?

What we want, typically, is

$$E_{(Y^*,\mathbf{X}^*)}\left[\left(Y^*-\hat{f}_n\left(\mathbf{X}^*\right)\right)^2\right]$$

where (Y^*, \mathbf{X}^*) are new, future observations, not used when computing ("training") \hat{f}_n .

If we assume that $Y = f(X) + \epsilon$, then

$$E_{(Y^*,\mathbf{X}^*)}\left[\left(Y^*-\hat{f}_n\left(\mathbf{X}^*\right)\right)^2\right]=$$

$$E_{(Y^*,\mathbf{X}^*)}\left[\left(f\left(\mathbf{X}^*\right)-\hat{f}_n\left(\mathbf{X}^*\right)\right)^2\right]+V(\epsilon)$$

- what assumptions are needed for this to be true?
- is it still true if I look at predictions for a single & fixed X₀?

What we want

$$E_{(Y^*,\mathbf{X}^*)}\left[\left(Y^*-\hat{f}_n\left(\mathbf{X}^*\right)\right)^2\right]$$

is very difficult to estimate

• Something similar:

$$E_{\left\{ \left(Y^{*},\mathbf{X}^{*}\right),\mathsf{data}\right\} }\left[\left(Y^{*}-\hat{f}_{n}\left(\mathbf{X}^{*}\right)\right)^{2}\right]$$

is easier to estimate (what is the difference, exactly?)

- Goodness of fit vs. prediction power
- How do we estimate prediction MSE?

$$E_{(Y^*,\mathbf{X}^*)}\left[\left(Y^*-\hat{f}\left(\mathbf{X}^*\right)\right)^2\right]$$

Can it be done without a test set?

Mean squared prediction error

How do we estimate the MSPE

$$E_{(Y^*,\mathbf{X}^*)}\left[\left(Y^*-\hat{f}(\mathbf{X}^*)\right)^2\right]$$
 ?

- (a): "Training / Testing" data sets
- (b): "Recycling" a single data set
- (c): "Direct estimates"

Test set approach

- Randomly split the data into a training set and a test set
- "Train" ("estimate", "fit") your model(s) using only the training set
- Use the estimated model(s) to obtain predictions for the test set (only)

Test set approach

• Check how well your model(s) did

$$\widehat{\mathsf{MSPE}} = \frac{1}{n_T} \sum_{j \in \mathcal{T}} (y_j - \hat{y}_j)^2$$

where:

 \mathcal{T} is the **test set**, $n_{\mathcal{T}}$ is the size of \mathcal{T} , and \hat{y}_{i} are the **predicted values**

- Back to the Pollution example
- Read the training set
- Train the full and the reduced models
- Read the test set
- Use both models to predict MORT
- Compare both sets of predictions

```
> x.te <- read.table('pollution-test.dat',...</pre>
>
> x.te$pr.full <- predict(full, newdata=x.te)
> x.te$pr.reduced <- predict(reduced,</pre>
       newdata=x.te)
>
> with (x.te, mean ( (MORT - pr.full) ^2 ))
[1] 4677.45
>
> with(x.te, mean( (MORT - pr.reduced)^2 ))
[1] 1401.571
```

Discuss

- Back to the Pollution example
- Repeat with a different training / test split
- Compare conclusions

Test set approach

- Pros:
- Estimates what we (generally) want to estimate
- It is computationally very fast

Test set approach

Cons:

- Estimated MSPE depends on the training set
- Estimated MSPE may be quite variablewhy is this a problem?
- One does not use all the available information for training the model(s)

Cross validation

- Leave-one-out
- Description
- Picture

CV - Pollution data

Run it on the pollution data

CV - leave-one-out

- Pros:
- It is not random!
- It uses almost all of the data for training
- Cons:
- It can be numerically very expensive (although "shortcuts" for linear models exist).

Cross validation

- K-fold
- Description
- Picture

CV - Revisit example data

• Run it on the pollution example

K-fold CV

- Pros:
- Faster than leave-one-out
- Less variable estimator for MSPE than leave-one-out (explain)
- Training set is larger than "training/test" method - less biased MSPE estimate (but more biased than leave-one-out)

K-fold CV

Cons:

- Choosing K is not trivial (K = 5 or K = 10 have been shown empirically to work well)
- Training set is smaller than leave-one-out's
- It is a random estimate of MSPE (it is a good idea to run it several times)

 Why compare models? Why not choose the "largest" (best fit to the data)?

- Correlated covariates:
- May reduce prediction accuracy
- Mask each other when included simultaneously in a model

- Correlated covariates have become prevalent
- Researchers can (and do) collect data "blindly"
- Data are collected without a specific question in mind