STAT406- Methods of Statistical Learning Lecture 25

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- Response variable Y, vector of explanatory variables
 X = (X₁, X₂,..., X_p)'
- Interest in predicting Y or understanding the relationship
- Optimal predictor for Y based on X using a squared loss function?

The regression function

$$f(\mathbf{X}) = E[Y|\mathbf{X}]$$

- Model-based estimates (e.g. linear or polynomial regression)
- Feature selection can be important –
 prediction power negatively affected
 by highly correlated explanatory
 variables; sometimes the interest is in
 identifying all important features.

Estimate future prediction error:

$$E_{(Y,\mathbf{X})}\left[\left(Y-\hat{f}_{n}\left(\mathbf{X}\right)\right)^{2}\middle|\mathcal{D}_{n}\right]$$

- "Prediction error on the training set" (aka "residual sum of squares")?
- Cross-validation?
- AIC?, Mallow's C_p ?

- Regularization
- Ridge regression, LASSO, Elastic Net
- All have tuning parameters that need to be chosen
- CV? AIC?
- How many "parameters" does a regularized estimator have?
- EDF

What if the regression function

$$f(\mathbf{X}) = E[Y|\mathbf{X}]$$

is not linear in X?

- Model-based: non-linear regression
- Polynomial, splines approximations; kernel (local) regression estimators
- Regression trees
- Why trees if we have kernel regression or splines?

- Classification another prediction problem
- Optimal classifier using a 0-1 loss function?
- Need to estimate

$$P(Y=g|\mathbf{X})$$

for each class label g and each X

- Model-based estimators
- Given a model for the distribution of $\mathbf{X}|Y=g$ we can obtain

$$P(Y=g|\mathbf{X}) \propto f(\mathbf{X}|Y=g) P(Y=g)$$

- Model-free estimators
- Nearest neighbours
- Logistic / multinomial
- Trees

- Ensembles work better than individual predictors / classifiers
- Bagging is one way to build an ensemble, but variability of the combined prediction can be a concern
- Random Forests
- Out-of-bag prediction error estimates, feature importances
- Boosting

- Unsupervised learning, "no response"
- K-means, Dissimilarities (divisive)
- Model-based clustering, EM-algorithm (divisive)
- Hierarchical clustering (agglomerative)
- Principal Components, dimension reduction, "best" lower dimensional approximations