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StarNet: Learning Schwarzschild Geodesics via Physics-Informed Neural Networks

"Somewhere, something incredible is waiting to be known." — Carl Sagan

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Abstract

This work introduces StarNet, a Physics-Informed Neural Network (PINN) designed to simulate geodesic motion in the curved spacetime of a Schwarzschild black hole without relying on traditional numerical integration. By directly embedding the geodesic equation and Christoffel symbols into the loss function, the network learns the geometry of spacetime as a continuous, differentiable function of proper time. The results demonstrate that StarNet successfully captures non-Newtonian relativistic phenomena, including perihelion precession and horizon crossing, while maintaining orbital stability over long integration times. This approach offers a novel, data-free method for modeling general relativity, with potential applications for inverse problems in astrophysics.

Introduction

RQ 1: Can a PINN accurately reconstruct particle trajectories (geodesics) in a curved vacuum spacetime without using traditional numerical integrators?

RQ 2: Can the network capture complex relativistic effects, such as perihelion precession and horizon crossing, solely by minimizing a physics-informed loss?

Methodology: The Physics

We model a vacuum spacetime defined by the Schwarzschild Metric $g_{\mu\nu}$ for a non-rotating black hole of mass M. The network minimizes the Geodesic Residual rather than training on data.

- **Metric:** $ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2 d\Omega^2$
- **Geodesic Equation:** $\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$

Methodology: The Network

- **Architecture:** Fully connected PINN with Softplus activation for higher-order differentiability.
- Inputs: Proper time τ.
- Hard Ansatz: We strictly enforce initial conditions (x_0, v_0) by defining the output as: $x_{net}(\tau) = x_0 + v_0\tau + \tau^2 \cdot \mathcal{N}(\tau;\theta)$

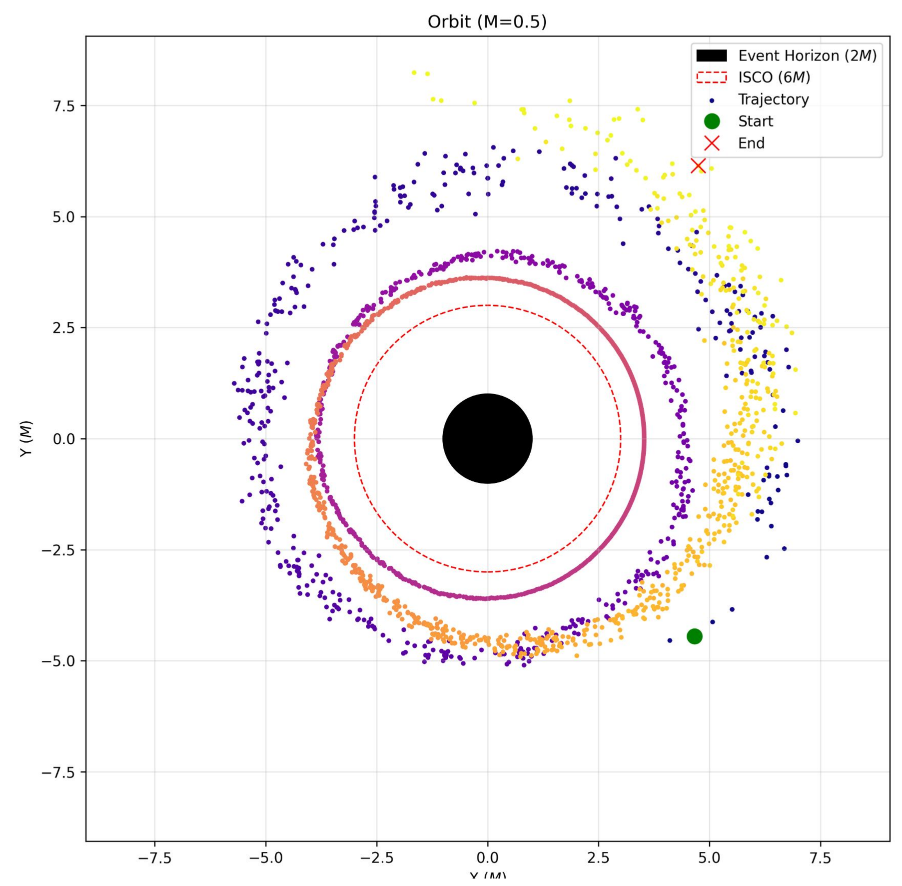

Figure 1: A bound orbit simulation over τ = 100. The PINN successfully captures non-Newtonian perihelion precession, resulting in the classic relativistic "rosette" pattern rather than a closed Newtonian ellipse.

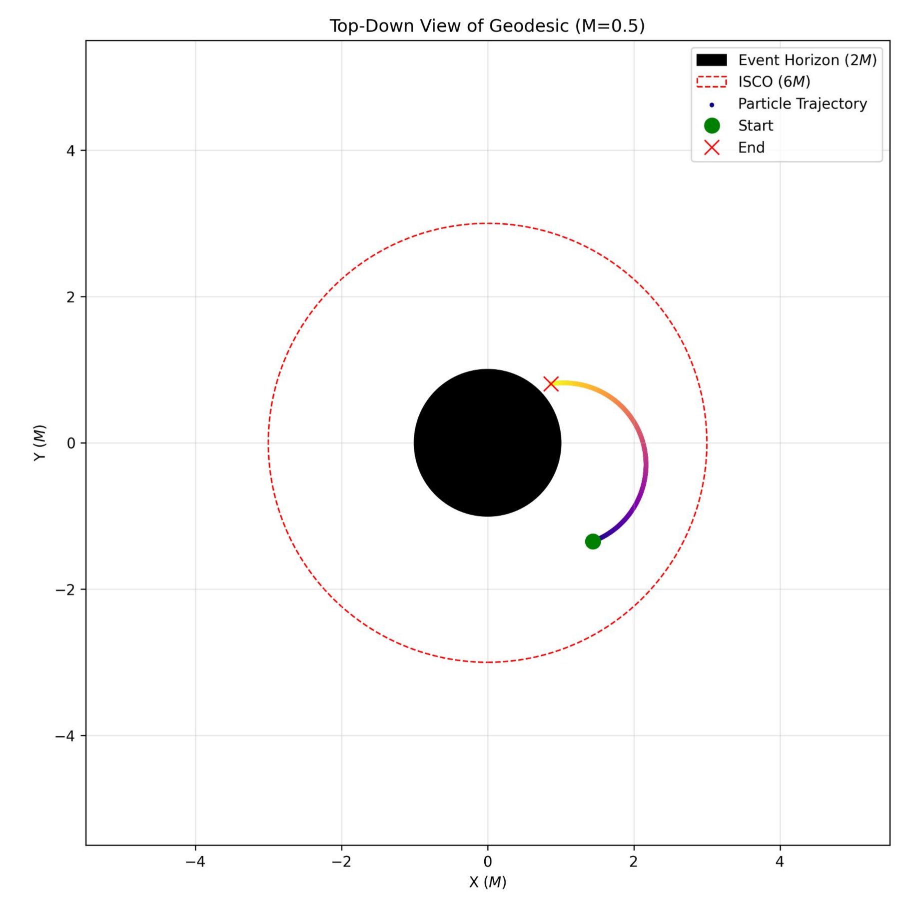

Figure 2: Top-down view of a scattering geodesic. The particle approaches the Event Horizon (2M), interacts with the potential barrier near the ISCO (6M), and escapes. The red dashed line marks the Innermost Stable Circular Orbit.

Comparison of Physical Models

Feature	Newtonian Gravity	StarNet PINN (GR)
Orbit Shape	Closed Ellipses	Precessing Rosettes (Figure 1)
Time Flow	Absolute (t)	Dilation (Proper time τ)
Event Horizon	None	Modeled at r = 2M
Integration Method	Discrete Steps (e.g. RK4)	Continuous Differentiable Function
ISCO Behavior	Stable at any radius	Unstable orbits below 6M

Discussion

- **Accuracy:** The model maintains orbital stability over long proper times $\tau = 100$ without the energy drift often seen in symplectic integrators.
- **Geometry Learning:** By embedding the Christoffel symbols $(\Gamma^\mu_{\alpha\beta})$ into the loss function, the network learns the "force" of gravity purely as geometric curvature.
- **Boundary Handling:** The Hard Ansatz ensures that even if the physics loss fluctuates, the trajectory origin remains exact.

Conclusion

StarNet successfully acts as a differentiable simulator for General Relativity, accurately reproducing key tests such as perihelion precession and horizon crossing without requiring labeled training data. By embedding the Christoffel symbols directly into the loss function, the network effectively "learns" the geometry of spacetime rather than just memorizing paths. This approach offers a powerful alternative to discrete numerical integrators, providing a continuous, analytic solution that maintains stability over long timescales.

Future Work

- Extend the metric to Kerr-Newman Spacetime to model rotating black holes and frame-dragging.
- Implement Inverse Problems: Use observed stellar orbits (e.g., S2 star) to regress the mass M and spin a of a black hole.

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