

Problem:

In this exam, you are given two 2D boxes consisting of full and empty cells. The goal is to convert the first box into the second with the minimum cost of operations (the operations are defined below). The boxes are represented as 2D char arrays. In the arguments, both boxes will be defined to include the same number of rows, yet the number of their columns may be different. For instance, Box-1 can be an array of the size of 10x12 whereas Box-2 can be an array of the size of 10x15. In order to represent empty cells, '-' character is used and for the full cells a letter is used. In the figure below, a few input box illustrations are given:

<div>(a)</div> <div><div>BOX-1</div><div><div>M</div><div>R</div><div>V</div><div>-</div></div></div> <div>A box defined as an array of 4x1</div>	<div><div>BOX-2</div><div><div><div>-</div><div>M</div><div>V</div></div><div><div>M</div><div>-</div><div>M</div></div><div><div>R</div><div>-</div><div>R</div></div><div><div>V</div><div>R</div><div>-</div></div></div></div> <div>A box defined as an array of 4x3</div>	<div><div>Resulting mem</div><div><div><div>0</div><div>3</div><div>5</div><div>8</div></div><div><div>3</div><div>4</div><div>6</div><div>8</div></div></div></div>
<div>(b)</div> <div><div>BOX-1</div><div><div><div>-</div><div>B</div><div>C</div></div><div><div>B</div><div>-</div><div>C</div></div></div></div> <div>A box defined as an array of 2x3</div>	<div><div>BOX-2</div><div><div><div>A</div><div>-</div><div>C</div><div>A</div><div>B</div></div><div><div>-</div><div>C</div><div>C</div><div>A</div><div>-</div></div></div></div> <div>A box defined as an array of 2x5</div>	<div><div>Resulting mem</div><div><div><div>0</div><div>1</div><div>2</div><div>4</div><div>6</div><div>7</div></div><div><div>1</div><div>2</div><div>2</div><div>4</div><div>6</div><div>7</div></div><div><div>2</div><div>2</div><div>3</div><div>5</div><div>7</div><div>6</div></div><div><div>4</div><div>4</div><div>4</div><div>3</div><div>5</div><div>6</div></div></div></div>
<div>(c)</div> <div><div>BOX-1</div><div><div><div>A</div><div>A</div><div>C</div><div>B</div><div>-</div></div><div><div>B</div><div>C</div><div>-</div><div>-</div><div>A</div></div><div><div>A</div><div>C</div><div>-</div><div>A</div><div>-</div></div><div><div>-</div><div>C</div><div>-</div><div>-</div><div>C</div></div></div></div> <div>A box defined as an array of 4x5</div>	<div><div>BOX-2</div><div><div><div>A</div><div>C</div><div>B</div><div>-</div><div>A</div><div>B</div><div>A</div></div><div><div>B</div><div>A</div><div>B</div><div>-</div><div>C</div><div>-</div><div>C</div></div><div><div>B</div><div>-</div><div>-</div><div>A</div><div>C</div><div>A</div><div>B</div></div><div><div>C</div><div>-</div><div>B</div><div>-</div><div>B</div><div>A</div><div>B</div></div></div></div> <div>A box defined as an array of 4x7</div>	<div><div>Resulting mem</div><div><div><div>0</div><div>4</div><div>6</div><div>9</div><div>10</div><div>14</div><div>17</div><div>21</div></div><div><div>3</div><div>3</div><div>5</div><div>8</div><div>9</div><div>13</div><div>16</div><div>20</div></div><div><div>7</div><div>5</div><div>7</div><div>10</div><div>11</div><div>10</div><div>13</div><div>17</div></div><div><div>8</div><div>6</div><div>7</div><div>10</div><div>11</div><div>11</div><div>14</div><div>18</div></div><div><div>10</div><div>8</div><div>9</div><div>12</div><div>12</div><div>13</div><div>13</div><div>17</div></div><div><div>12</div><div>10</div><div>10</div><div>13</div><div>14</div><div>15</div><div>15</div><div>19</div></div></div></div>

(d)	<p>BOX-1</p> <table><tr><td>-</td><td>Y</td><td>W</td><td>Y</td><td>-</td><td>-</td></tr><tr><td>-</td><td>Z</td><td>Y</td><td>Y</td><td>-</td><td>W</td></tr><tr><td>W</td><td>Y</td><td>Z</td><td>Y</td><td>-</td><td>-</td></tr><tr><td>W</td><td>W</td><td>-</td><td>Y</td><td>-</td><td>Y</td></tr><tr><td>-</td><td>Z</td><td>-</td><td>-</td><td>Z</td><td>Z</td></tr></table> <p>A box defined as an array of 5x6</p>	-	Y	W	Y	-	-	-	Z	Y	Y	-	W	W	Y	Z	Y	-	-	W	W	-	Y	-	Y	-	Z	-	-	Z	Z	<p>BOX-2</p> <table><tr><td>Z</td><td>W</td><td>-</td></tr><tr><td>W</td><td>Y</td><td>-</td></tr><tr><td>Y</td><td>Z</td><td>Y</td></tr><tr><td>-</td><td>Z</td><td>-</td></tr><tr><td>-</td><td>-</td><td>-</td></tr></table> <p>A box defined as an array of 5x3</p>	Z	W	-	W	Y	-	Y	Z	Y	-	Z	-	-	-	-	<p>Resulting mem</p> <table><tr><td>0</td><td>3</td><td>7</td><td>8</td></tr><tr><td>2</td><td>5</td><td>9</td><td>10</td></tr><tr><td>7</td><td>8</td><td>11</td><td>12</td></tr><tr><td>10</td><td>10</td><td>10</td><td>11</td></tr><tr><td>14</td><td>14</td><td>13</td><td>14</td></tr><tr><td>15</td><td>15</td><td>14</td><td>15</td></tr><tr><td>18</td><td>18</td><td>17</td><td>18</td></tr></table>	0	3	7	8	2	5	9	10	7	8	11	12	10	10	10	11	14	14	13	14	15	15	14	15	18	18	17	18
	-	Y	W	Y	-	-																																																																						
-	Z	Y	Y	-	W																																																																							
W	Y	Z	Y	-	-																																																																							
W	W	-	Y	-	Y																																																																							
-	Z	-	-	Z	Z																																																																							
Z	W	-																																																																										
W	Y	-																																																																										
Y	Z	Y																																																																										
-	Z	-																																																																										
-	-	-																																																																										
0	3	7	8																																																																									
2	5	9	10																																																																									
7	8	11	12																																																																									
10	10	10	11																																																																									
14	14	13	14																																																																									
15	15	14	15																																																																									
18	18	17	18																																																																									
(e)	<p>BOX-1</p> <table><tr><td>A</td><td>B</td><td>C</td><td>A</td><td>B</td><td>C</td></tr><tr><td>B</td><td>C</td><td>C</td><td>A</td><td>-</td><td>A</td></tr></table> <p>A box defined as an array of 2x6</p>	A	B	C	A	B	C	B	C	C	A	-	A	<p>BOX-2</p> <table><tr><td>A</td><td>B</td><td>B</td><td>A</td></tr><tr><td>B</td><td>A</td><td>-</td><td>C</td></tr></table> <p>A box defined as an array of 2x4</p>	A	B	B	A	B	A	-	C	<p>Resulting mem</p> <table><tr><td>0</td><td>2</td><td>4</td><td>5</td><td>7</td></tr><tr><td>2</td><td>0</td><td>2</td><td>3</td><td>5</td></tr><tr><td>4</td><td>2</td><td>1</td><td>2</td><td>4</td></tr><tr><td>6</td><td>4</td><td>3</td><td>4</td><td>3</td></tr><tr><td>8</td><td>6</td><td>5</td><td>6</td><td>5</td></tr><tr><td>9</td><td>7</td><td>6</td><td>5</td><td>6</td></tr><tr><td>11</td><td>9</td><td>8</td><td>7</td><td>7</td></tr></table>	0	2	4	5	7	2	0	2	3	5	4	2	1	2	4	6	4	3	4	3	8	6	5	6	5	9	7	6	5	6	11	9	8	7	7																		
	A	B	C	A	B	C																																																																						
B	C	C	A	-	A																																																																							
A	B	B	A																																																																									
B	A	-	C																																																																									
0	2	4	5	7																																																																								
2	0	2	3	5																																																																								
4	2	1	2	4																																																																								
6	4	3	4	3																																																																								
8	6	5	6	5																																																																								
9	7	6	5	6																																																																								
11	9	8	7	7																																																																								
(f)	<p>BOX-1</p> <table><tr><td>-</td><td>M</td><td>V</td><td>R</td></tr><tr><td>M</td><td>-</td><td>R</td><td>-</td></tr><tr><td>R</td><td>M</td><td>M</td><td>-</td></tr><tr><td>V</td><td>M</td><td>-</td><td>R</td></tr></table> <p>A box defined as an array of 4x4</p>	-	M	V	R	M	-	R	-	R	M	M	-	V	M	-	R	<p>BOX-2</p> <table><tr><td>M</td><td>-</td><td>-</td><td>V</td></tr><tr><td>V</td><td>R</td><td>R</td><td>M</td></tr><tr><td>-</td><td>-</td><td>-</td><td>-</td></tr><tr><td>-</td><td>M</td><td>-</td><td>R</td></tr></table> <p>A box defined as an array of 4x4</p>	M	-	-	V	V	R	R	M	-	-	-	-	-	M	-	R	<p>Resulting mem</p> <table><tr><td>0</td><td>2</td><td>4</td><td>5</td><td>8</td></tr><tr><td>3</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>6</td><td>8</td><td>9</td><td>10</td><td>11</td></tr><tr><td>9</td><td>10</td><td>12</td><td>13</td><td>13</td></tr><tr><td>11</td><td>12</td><td>14</td><td>15</td><td>15</td></tr></table>	0	2	4	5	8	3	5	6	7	8	6	8	9	10	11	9	10	12	13	13	11	12	14	15	15																
	-	M	V	R																																																																								
M	-	R	-																																																																									
R	M	M	-																																																																									
V	M	-	R																																																																									
M	-	-	V																																																																									
V	R	R	M																																																																									
-	-	-	-																																																																									
-	M	-	R																																																																									
0	2	4	5	8																																																																								
3	5	6	7	8																																																																								
6	8	9	10	11																																																																								
9	10	12	13	13																																																																								
11	12	14	15	15																																																																								

Your task is to convert the first box into the second box by using some operations resulting in the minimum cost. The conversion rules and operations are defined as follows:

- You should compare the boxes column by column. Each conversion operation is column-wise.
- A column could be deleted completely. The **deletion operation** costs as much as the number of full cells in the column. For instance; if the column consists of 5 cells where 3 of them full and 2 of them are empty, then deleting that column costs 3 units.
- For a column of Box-2, a new corresponding column could be inserted into Box-1 at any location (between two columns or as the initial column or as the final column). The **insertion operation** costs as much as the number of full cells inside the new column. For instance; if the newly inserted column consists of 5 cells where 3 of them full and 2 of them are empty, then inserting that column costs 3 units.
- A column could be converted into a new column by reordering its cells. For instance, if a column consists of 5 cells including ['X', 'A', '-', 'B', '-'], it can be reordered as ['A', '-', '-', 'B', 'X']. The **reordering operation** costs as much as the number of cells whose locations are changed. For the example given, since the

locations of the cells including 'A', 'X' and '-' changed only, it costs 3 units.

- A column could be converted into a new column by replacing its cells with some other cells. For the **replacement operation**, if a full cell is replaced with some other full cell, then it costs 1 unit. However, if an empty cell is replaced with a full cell, or vice versa, then it costs 2 units. For instance, if a column consists of 5 cells including ['X', 'A', '-', 'B', '-'], its cells can be replaced as ['X', 'C', 'D', '-', '-'], it costs $\langle \text{change from 'A' to 'C'} \rangle + \langle \text{change from '-' to 'D'} \rangle + \langle \text{change from 'B' to '-'} \rangle = 1 + 2 + 2 = 5$ units.
- Each operation is independent from each other. At each transition, apply only one of them.
- **HINT:** You should implement the dynamic programming column-wise. That is, for each column of Box-2, consider a corresponding column inside Box-1 which has been obtained by the operations above. The way of how to apply memoization is explained in the following parts.