Discrete Mathematics (2016 Spring) Midterm II

1. (**40** points) For each of the following statements, determine and explain (required) whether it is correct or not.
   (a). Let R be a symmetric and transitive relation on a set A. If for every a in A there exists b in A such that (a, b) is in R, then R is an equivalence relation.
   (b). Let R be a relation on a set A. If R is reflexive and transitive, then $R^n=R$ for all positive integer n.
   (c). Let A be a set with |A| = n, and let R be an equivalence relation on A with |R| = r, r - n is even.
   (d). The number of different equivalence relations on a set of 4 elements is 15.
   (e). Only one the relations $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$, which represented by the zero-one matrices are partial orders.
   (f). Only two of (Z, =), (Z, $\neq$), (Z, $\geq$), (Z, $\nmid$) are posets.
   (g). If A is a language, the $(A^*)^+=A^*$.
   (h). The number of different set $A=\{a,b,c\} \subseteq Z^+, where\ a,b,c \geq 1$, satisfy the property $a \cdot b \cdot c = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$ is 121.

2. (**15** points)
   (a) A course gives a single choice quiz that has 4 questions, each with 4 possible responses. What is the minimum number of students to guarantee that at least 4 answer sheets must be identical?
   (b) How many selections from the set {1, 3, 5, 7, …, 33, 35} to guarantee that at least exist two integers x, y from our selection that $\gcd(x,y) \geq 2$?
   (c) How many times must we roll a single 4-face die in order to get the same score at least n times?

3. (**15** points) Determine the following relations are reflexive, symmetric, antisymmetric, or transitive.
   (a) Let A be a nonempty set and let $\mathcal{P}$ be the power set of A. A binary relation R on $A \times \mathcal{P}$ is define as the set $\{((a,T),(b,U)) | \{a,b\} is\ a\ subset\ of\ T \cap U\ for\ a,b \in A\ and\ T,U \in \mathcal{P}\}$.
   (b) $a, b \in \boldsymbol{Z}, and\ aRb\ if\ and\ only\ if\ a-b\ is\ prime.$
   (c) a, b are strings of 0s and 1s, and aRb if and only if the length of a is equal to or greater than b.

4. (**12**: 2,2,4,4 points) Let A={a, b, c, d}, B={1, 2, 3, 4, 5, 6}.
   (a) How many functions from A to B have f(a) $\neq$1?
   (b) How many one-to-one functions from A to B?
   (c) How many functions from A to B are nondecreasing?
   (d) How many onto functions from B to A satisfying f(1)=a?

5. (**13**: 2,3,4,4 points) Let A={a, b, c, d, e, f}.
   (a) How many closed binary operations f on A satisfy f(a, b) $\neq$c?
   (b) How many closed binary operations f on A have an identity and f(a, b)=c?
   (c) How many f in (b) are commutative?
   (d) How many equivalence relations on A that determine more than three (include three) equivalence classes and $a \in [b]$?

6. (**15** points) Design a problem that can be solved by two different FSMs that their different of the number of states is 2. Use the minimization process to reduce the bigger one.

(Stirling number of the second kind: S(4, 2)=7, S(4, 3)=6, S(5, 2)=15, S(5, 3)=25, S(5, 4)=10, S(6, 2)=31, S(6, 3)=90)