Example 4.1 Cellular system capacity (Example 5.1 of the printed notes)

Consider a cellular system in which there are a total of 1001 radio channels available for handling traffic. Suppose the area of a cell is 6 km^2 and the area of the entire system is 2100 km^2.

a) Calculate the system capacity if the cluster size is 7.

b) How many times would the cluster of size 4 have to be replicated in order to approximately cover the entire cellular area?

c) Calculate the system capacity if the cluster size is 4.

d) Does decreasing the cluster size increase the system capacity? Explain.

Solution:

Given:
The total number of available channels, $K = 1001$.
Cluster size, $N = 7$.
Area of cell, $A_{cell} = 6\ \mathrm{km}^2$.
Area of cellular system, $A_{sys} = 2100\ \mathrm{km}^2$.

a) Since the number of channels per cell is $J = K/N$, then $J = 1001/7 = 143$ channels/cell.
The coverage area of a cluster is

$$A_{cluster} = N \times A_{cell} = 7 \times 6 = 42\ \mathrm{km}^2.$$

The number of times the cluster has to be replicated to cover the entire cellular system is $M = A_{sys}/A_{cluster} = 2100/42 = 50.$
Therefore,

$$C = MJN = 50 \times 143 \times 7 = 50,050 \text{ channels}.$$

b) For $N = 4$, $A_{cluster} = 4 \times 6 = 24\ \mathrm{km}^2$. Therefore,

$$M = A_{sys}/A_{cluster} = 2100/24 = 87.5 \simeq 87.$$

c) With $N = 4$, $J = 1001/4 = 250$ channels/cell. The system capacity is then

$$C = 87 \times 250 \times 4 = 87,000 \ \text{ channels}.$$

d) From a) and c), it is seen that a decrease in N from 7 to 4 is accompanied by an increase in M from 50 to 87, and the system capacity is increased from 50,050 channels to 87,000 channels. Therefore, decreasing the cluster size does increase the system capacity.

$\square$

Example 4.2 Number of frequency channels (Example 5.2 in the printed notes)
Consider a cellular system with a total bandwidth of 30 MHz which uses two 25 kHz simplex channels to provide full duplex voice and control channels. Assuming that the system uses a 9-cell reuse pattern and 1 MHz of the total bandwidth is allocated for control channels,

a) calculate the total available channels,

b) determine the number of control channels,

c) determine the number of voice channels per cell,

d) determine an equitable distribution of control channels and voice channels in each cell.

Solution:

Given:

Total bandwidth	=	30 MHz
Channel bandwidth	=	25 kHz $\times$ 2 = 50 kHz/duplex channels

a) The total number of available channels $= \frac{30000}{50} = 600$.

b) The number of control channels $= \frac{1000}{50} = 20$.

c) The number of voice channels per cell $= \frac{600-20}{9} \simeq 64$.

d) Since only a maximum of 20 channels can be used as control channels, for $N = 9$, one way to allocate is 7 cells with 2 control channels and 64 voice channels each, and 2 cells with 3 control channels and 64 voice channels each.

Note that the channel allocation performed in part d) is not unique. Also, in practice one control channel per cell should be sufficient.

$\square$

Example 4.3 Number of cells in a cluster (Example 5.3 in the printed notes)
Verify that the cell cluster size is $N = i^2 + ij + j^2$, where i and j are the integer parameters determining the co-channel cells.

Solution:

A candidate cell has 6 nearest co-channel cells. By joining the centers of the 6 nearest neighboring co-channel cells, we form a large hexagon, as shown in Figure 11. This large hexagon has radius equal to D, which is also the co-channel separation. With the cell radius R, we have

$$D = \sqrt{3}RD_{norm} = \sqrt{3(i^2 + ij + j^2)}R.$$

In general, the area of a hexagon is proportional to the square of its radius. Let β (≈ 2.598) be the

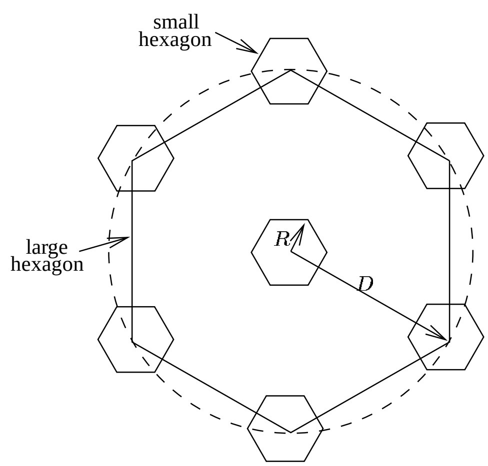

Figure 11: First tier co-channel interfering cells

proportional constant. Then the area of the large hexagon with radius D is

$$A_{\text{large}} = \beta D^2 = \beta[3(i^2 + ij + j^2)R^2]$$

and the area of a cell (the small hexagon) with radius R is

$$A_{\text{small}} = \beta R^2.$$

The number of cells in the large hexagon is then

$$\frac{A_{\text{large}}}{A_{\text{small}}} = 3(i^2 + ij + j^2) \tag{11}$$

On the other hand, from geometry it can be seen that, in general, the large hexagon encloses the center cluster of N cells plus 1/3 the number of the cells associated with six other peripheral large hexagons. Hence, the total number of cells enclosed by the large hexagon is

$$N + 6(\frac{1}{3}N) = 3N. \tag{12}$$

From Eq. (11) and Eq. (12), we have

$$N = i^2 + ij + j^2.$$

□

Example 4.8 Cell splitting and capacity increase (Example 5.7 of the printed notes)
Consider the cellular system shown in Figure 12, where the original cells have radius R. These cells are split into smaller cells, each with radius $R/2$. Suppose each base station is allocated 60 channels regardless

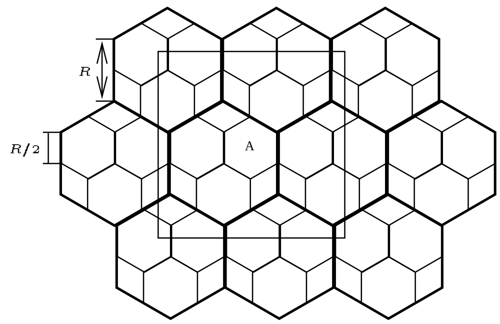

Figure 12: A cell splitting example with $R = 1$ km

of the cell size. There are obviously more small cells than original cells in the same coverage area. Since the number of channels allocated in a small cell is the same as in a large cell, it is obvious that cell splitting increases the number of channels within the same coverage area. Find the number of channels contained in a $3 \times 3\ \mathrm{km}^2$ area centered around (small) cell A for the following cases:

a) without cell splitting, i.e., just the original large cells, and

b) with cell splitting, i.e., using the small cells (microcells).

Solution:

To cover an area of $3 \times 3\ \mathrm{km}^2$ centered around cell A, we need to cover 1.5 km to the right, left, top and bottom of base station A, as shown in Figure 12. From Figure 12, it is observed that the $3 \times 3\ \mathrm{km}^2$ square centered around cell A contains more small cells than large cells. However, because of edge effect, the number of either type of cells contained within the square can only be an estimate. A reasonable estimate is that there are approximately 4 large cells (from visual observation). With a 1/2 radius split, the number of small cells within the square would be at most

$$(\frac{R}{R/2})^2 \times \text{ the number of large cells} = 4 \times 4 = 16 \text{ small cells.}$$

The above calculation would be correct if the enclosed area is infinitely large. With a finite area, it is necessary to take edge effect into consideration, so that the number of small cells contained within the $3 \times 3\ \mathrm{km}^2$ area would be less than 16. A reasonable estimate would be 15 small cells.

a) With an estimate of 4 base stations contained within the square, the number of channels equals $4 \times 60 = 240$.

b) With an estimate of 15 small cells, the number of channels contained in the square, with cell-splitting, is $15 \times 60 = 900$ channels, which is 3.75 times more channels than the unsplitting case.

Note that the upper bound is a 4 fold increase.

□