Programming Language

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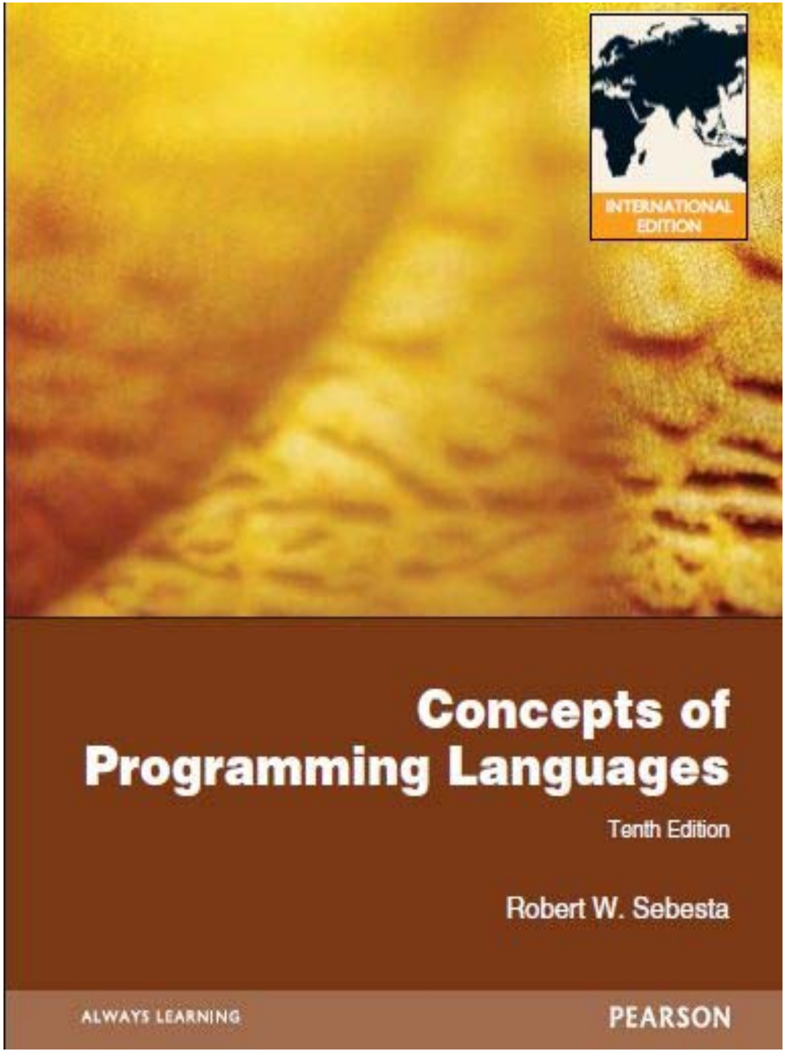


Lecture 5
LISP

Functional Programming: LISP

- LISt Processing language
  - Designed at MIT by McCarthy in 1960s.
- AI research needed a language to
  - Process data in lists (rather than arrays)
  - Symbolic computation (rather than numeric)
- Only two data types: atoms and lists

LISP variants

- LISP is a functional language. Variants are
  - Scheme
  - Emacs LISP
  - AutoLISP
  - Common Lisp
- Common Lisp and Scheme are the most popular
  - Scheme is for educational purpose
  - Common Lisp is the most popular functional languages

Objects

- Atoms
  - Numbers: 4 3.14 ½ #x16 #o22
  - Constants: pi t nil lambda-list-keywords
  - Characters: #\a #\Q #\space #\tab 預設的keywords
  - Strings: "foo" "Hello Hi" "@%!?#"
    - a sequence of characters bounded by double quotes
  - Booleans: T for true, NIL for false
  - Symbols: Dave num123 miles->km !_^_! 2nd-place *foo* 一串字元
    - Two special symbols: T for true and NIL for false

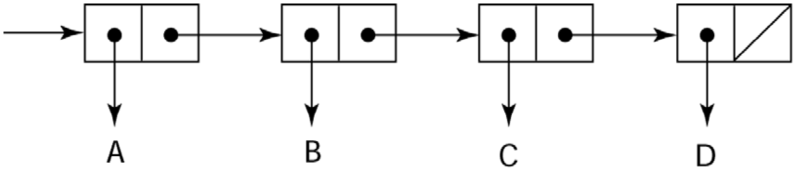
Objects (cont.)

- **Lists**: a list of atoms and/or lists, bounded by "(" and ")“,

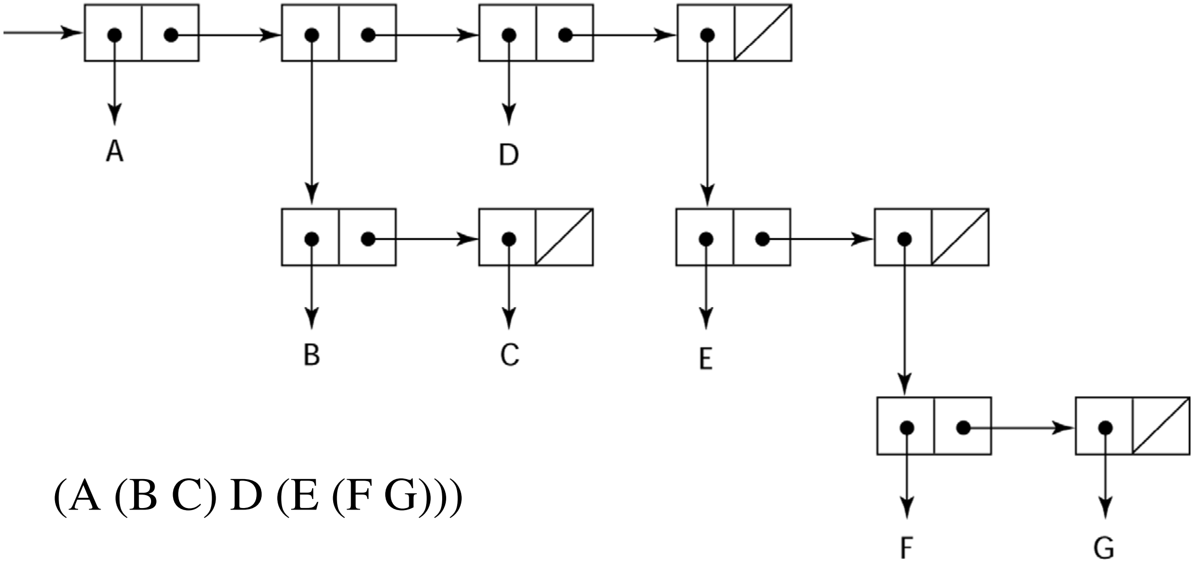
```
e.g.:
(a b c), (a (b c))
() 空的list
(a)  is equal to ( a () ) 含有a的list
(a b c d)
((a b) c (d e)) sub list
(((((a)))))
```

- Top elements of a list
  - example: top elements of list (a b c) are a, b, and c
  - top elements of list (a (b c)) are a and (b c)
  - nil: empty list, same as ().

Representation of Two LISP Lists



(A B C D)



(A (B C) D (E (F G)))

Example of comments

- Examples of comments 註解 ; and “ ”

```
;;; A comment formatted as a block of text
;;; outside of any function definition

(defun fib (n)
  ;; A comment on a line by itself
  (if (< n 3)
      1                   ; A comment on the same line as some code
      (+ (fib (- n 1))
         (fib (- n 2)))))

(setq *global-variable* 10)
(let (local-variable)
  (setq local-variable 15))
```

Substitution Model

- The basic rule: 前序
  - To evaluate a Scheme expression:
  - 1. **Evaluate** its **operands**
  - 2. **Evaluate** the **operator**
  - 3. **Apply** the operator to the evaluated operands

```
(operator operand1 operand2 ...)
```

```
(+ 3 (* 4 5))
```

Example expression evaluate

- Example: (+ 3(* 45))
  - evaluate 3
  - evaluate (* 45)
    - evaluate 4
    - evaluate 5
    - evaluate *
  - apply * to 4, 5→ 20
  - evaluate +
  - apply + to 3, 20 →23

Quote

- Use quote or ‘ to avoid procedure application

```
>(+ 1 2)      => 3
>(1 2 3)      => error
>‘(1 2 3)     => (1 2 3)
>(quote (1 2 3)) => (1 2 3)
```

不對list做運算

Primitive Numeric Function

- Primitive Numeric Function:
  - function only deal with numeric atoms
  - + – * /, sqrt

```
>42              => 42
>(* 3 7)         => 21
>(+ 5 7 8)        => 20
>(- 24 (* 4 3)   =>12
>(sqrt 4)          => 2
```

(+) => 0

Numerical Calculation Function

- There are some function
  - +, -, *, /
  - (sqrt 16) => 4
  - (expt 10 2) => 100
  - (max 1 2 3 4 5) => 5
  - (min 1 2 3 4 5) => 1
  - (mod (abs -27) 5) => 2
    絕對值

Example

- setq is a special form of function (with two arguments); Assign一個變數

```
>(setq x 3.0)
3.0
```
x = 3.0

```
>x
3.0
```

```
>(setq y x)
3.0
; the value of y is assigned as  the value of x
```
y = x

```
>y
3.0
```

```
>(+ x y)
6.0
```

Let evaluation

在let中使用的variable，出了let則無法使用

- Let function

Scoping：變數有效使用範圍

```
>(let ((x 5) (y 8)) (* x y )
40
```

```
>x
Error: Unbound variable
```

```
>(setq w 77)
77
>(let ((w 8) (x w)) (+ w x))
85
```

```
>w
77
```

```
>x
Error: Unbound variable
```

Essential Multiple Values

(values :this :that)   value：return 後面的parameter（可輸出多個值）

> :this ;

> :that

values

(values 4 (value))

> 4 ;

> nil

value assign到a b c

(multiple-value-bind (a b c) (values 2 3 5) (+ a b c))

>10

(multiple-value-bind (a b c) (values 2 3 6 5) (+ a b c))

> 11

Lambda expression

- Create a function by a lambda expression:
- (lambda (id1 id2 ...) exp1 exp2 ...)
  - id1 id2 ... – formal parameters
  - exp1 exp2 ... – body of the function

lambda：不代表特定function

➢ (lambda (x) (* x x))

function name 引數

No function names

=> nameless function

- Call a function by applying the evaluated lambda expression on its actual parameters

((lambda (x) (* x x)) 3) => 9

the function the actual parameter

Function Definition

- Lambda expression

  (lambda (x) (* x x))

  Formal parameter : x
  Function body: * x x

- Function application

  (lambda (x) (* x x))
  > error

  >((lambda (x) (* x x)) 7)
  49

Lambda function

- How can you reuse the function?
  - You can't!   (since it is nameless)
- What if you REALLY want to reuse it?
  - "defun"

Function Definition

- How to define a function? Function可以reuse

  (defun <function-name> (<formal-parameters>) <expression>)

- Bind a name to a function:

  ```
  >(defun square (x) (* x x) )
  ```

- Now call the function

  ```
  > (square 5)
  > 25
  ```

Example

- Design a function to find aX^2 + bX + c =0 answer.

```
[1]> (defun quadratic-roots (a b c)
"Returns the roots of a quadratic equation aX^2 + bX + c = 0"
(let ((discriminant (- (* b b) (* 4 a c))))
  (values (/ (+ (- b) (sqrt discriminant)) (* 2 a))
          (/ (- (- b) (sqrt discriminant)) (* 2 a)))))
QUADRATIC-ROOTS
[2]> (quadratic-roots 1 2 1)
-1 ;
-1
```

Example

- Rewrite a function to find <u>aX^2 + bX + c =0</u> answer.

If-else

```
(defun quadratic-roots-2 (a b c)
  (let ((num (- (* b b) (* 4 a c))))   ; zero if one root
   (cond ((zerop num) (/ (+ (- b) (sqrt num)) (* 2 a)))
         (t  (values (/ (+ (- b) (sqrt num)) (* 2 a))
               (/ (- (- b) (sqrt num)) (* 2 a)))))))
```

```
[6]> (quadratic-roots-2 1 -14 49)
7
[7]> (quadratic-roots-2 1 4 -5)
1 ;
-5
```

Conditions

- Boolean value true is T and false is NIL
- Predicates are expressions that evaluate to true or false
- Numberp 數語
  - test whether argument is a number
    - ➢ (numberp 9)
      T
    - ➢ (numberp ‘a)
      NIL

Some useful predicates

- `(= x y)` returns true if `x` and `y` are identical.
  - Or you can write `(equal x y)` or `(eq x y)`
  - Represents the same number or symbol or the same list.
- For example:
  - `(equal 6 6)      => true`

```
>(numberp x)          ; is x a number
>(stringp x)          ; is x a string?
>(listp x)            ; is x a list?
>(symbolp x)          ; is x a symbol?
>(atom x)             ; is x a atom?
>(null x)             ; is x nil?
```

Conditional Expressions: IF

- Conditional expressions come in two forms: condition 一定要其中一個符合

  (1) (if P E_1 E_2) ; if P then E_1 else E_2 ? :

  (2) (cond (P_1 E_1) ; if P_1 then E_1

  …… ; …

  (P_k E_k) ; else if P_k then E_k

  (t E_{k+1}) ; else E_{k+1}

Conditional Expression: IF

- Example:
  - (if a b c) => b if a is true, else c
  - (if (< 5 6) 1 2) → 1
  - (if (< 4 3) 1 2) → 2
- Anything other than NIL is treated as true:
  - (if 3 4 5) → 4 非nil皆為true
- if is a special form – evaluates its arguments only when needed:
  - (if (= 3 4) 1 (2)) → error
  - (if (= 3 4) 1 ‘(2)) → (2)
  - (if (= 3 4) 1 (if 5 2 3)) → 2

Conditional Expression: COND

- Compare x and y

```
(defun compare (x y)
 (cond ((equal x y) 'numbers-are-the-same)
       ((< x y) 'first-is-smaller)
       ((> x y) 'first-is-bigger)))
```

➢ (compare 3 4)
FIRST-IS-SMALLER

Let & defun & cond

- Example: “guess number”

```
(defun secret-number (number)
        (let ((secret 37))
                (cond ((= number secret) ‘True)
                ((< number secret) ‘too-low)
                ((> number secret) ‘too-high))))
```

Syntax (C vs. LISP)

C 中序	LISP 前序
1 + 2 + 3	(+ 1 2 3)
3 + 4 * 5	(+ 3 (* 4 5))
factorial (9)	(factorial 9)
(a == b) && (c != 0)	(and (= a b) (not (= c 0)))
(low < x) && (x < high)	(< low x high)
f (g(2,-1), 7)	(f (g 2 -1) 7)

定義好的function當operator使用

Example

C

```
if(a == 0)
        return f(x,y);
else
        return g(x,y);
```

Common LISP

```
(if (= a 0)
    (f x y)
    (g x y))
```

List manipulation in Lisp

- Three primitives and one constant
  - get head of list: car 取list中第一個element
  - get rest of list: cdr 取list中除了第一個element的其餘element
  - add an element to a list: cons
  - null list: nil or ()

List Operations

- cons – returns a list built from head and tail
  - (cons 'a '(b c d)) → (a b c d)
  - (cons 'a '()) → (a)
  - (cons '(a b) '(c d)) → ((a b) c d)
  - (cons 'a (cons 'b '())) → (a b)

List Operations

- car – returns first member of a list (head)
  - (car '(a b c d)) → a
  - (car '(a)) → a
  - (car '((a b) c d)) → (a b)
  - (car '(this (is no) more difficult)) → this
- cdr – returns the list without its first member (tail)
  - (cdr '(a b c d)) => (b c d)
  - (cdr '(a b)) => (b)
  - (cdr '(a)) => NIL
  - (cdr '(a (b c))) =>((b c))

List Operations

- null – returns T if the list is empty,
  returns NIL if not empty
  - ➢ (null ‘())
    T
- list – returns a list built from its arguments
  - (list 'a 'b 'c) → (a b c)
  - (list '(a b c)) → ((a b c))
  - (list '(a b) '(c d)) → ((a b) (c d))

List Operations

- length – returns the length of a list
  - (length '(1 3 5 7)) → 4
  - (length '((a b) c)) → 2
- reverse – returns the list reversed
  - (reverse '(1 3 5 7)) → (7 5 3 1)
  - (reverse '((a b) c)) → (c (a b))

Recursion

- How do you THINK recursively?
- Example: define factorial

$$\text{factorial}(n) = \underbrace{1 * 2 * 3 * \ldots(n-1)}_{\text{factorial }(n-1)} * n$$

$$\text{factorial}(n) = \begin{cases} 1 & \text{if } n=1 & \text{(the base case)} \\ n * \text{factorial}(n-1) & \text{otherwise} & \text{(inductive step)} \end{cases}$$

Version 1

- Example: define a factorial function

```
(defun  fact  (n)
    ( if ( = n 0 )
          1
          (* n    (fact(- n 1 )))))
```

Version 2: Tail Recursion

- a tail-recursive function is one in which the recursive call occurs last

```
(defun tailfact (n result)
   (if (= n 0)
     result
    (tailfact (- n 1 ) (* n result))))

(defun fact2 (n)
(tailfact n 1))
```

Steps of Version 1

Stack (space)

Time

```
(fact 5)
(* 5 (fact 4))
(* 5 (* 4 (fact 3)))
(* 5 (* 4 (* 3 (fact 2))))
(* 5 (* 4 (* 3 (* 2 (fact 1)))))
(* 5 (* 4 (* 3 (* 2 (* 1 (fact 0))))))
.
.
.
.
120
```

```
(defun  fact  (n)
   ( if ( = n 0 )
        1
        (* n    (fact(- n 1 )))))
```

Steps of Version 2

Stack

```
(fact 5)
(tailfact 5 1)
(tailfact 4 5)
(tailfact 3 20)
(tailfact 2 60)
(tailfact 1 120)
(tailfact 0 120)
120
```

Time

```
(defun tailfact (n result)
(if (= n 0)
result
(tailfact (- n 1 ) (* n result))))
```

Trace in Ver.1 and Ver.2

- Version 1:

```
[4]> (trace fact)
;; Tracing function FACT.
(FACT)
```

```
[6]> (fact 5)
1. Trace: (FACT '5)
2. Trace: (FACT '4)
3. Trace: (FACT '3)
4. Trace: (FACT '2)
5. Trace: (FACT '1)
6. Trace: (FACT '0)
6. Trace: FACT ==> 1
5. Trace: FACT ==> 1
4. Trace: FACT ==> 2
3. Trace: FACT ==> 6
2. Trace: FACT ==> 24
1. Trace: FACT ==> 120
120
```

Version 2:

```
[5]> (trace fact2)
;; Tracing function FACT2.
(FACT2)
```

```
[7]> (fact2 5)
1. Trace: (FACT2 '5)
1. Trace: FACT2 ==> 120
120
```

How does it work?

- Tail recursion requires two elements(more than two)
  - The tail recursive module must terminate with a recursive call that leaves no work on the stack to finish up.
  - Any storage must be done in the parameter list as opposed to the stack
  - The interpreter or compiler must be designed to recognize tail recursion and handle it appropriately

Let's have a look at the three programs

$$\sum_{k=1}^{100} k$$ = (sum-integers 1 100)

```
(define (sum-integers k  n)
   (if  (>  k  n)
        0
        (+  k
             (sum-integers (+ 1 k)  n))))
```

$$\sum_{k=1}^{100} k^2$$ = (sum-squares 1 100)

```
(define (sum-squares  k  n)
   (if  (>  k  n)
        0
        (+  (square K)
             (sum-squares  (+ 1 k)  n))))
```

$$\sum_{k=1,odd}^{101} k^{-2}$$ = (pi-sum 1 101)

```
(define (pi-sum  k  n)
   (if  (>  k  n)
        0
        (+  (/  1  (square  k))
             (pi-sum   (+  k 2)  n))))
```

Recursion

- Fibonacci:

$$
fib(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ fib(n-1) + fib(n-2) & \text{otherwise} \end{cases}
$$

```
(defun fib (n)
"Simple recursive Fibonacci number function"
(if (< n 2)
   n
 (+ (fib (- n 1)) (fib (- n 2)))))
```