Discrete Mathematics (2018 Spring) Midterm II

1. (**30** points) For each of the following statements, determine and explain (required) whether it is correct or not.
   (a). Let R and S be relations on X. If R and S are transitive, then $R \cap S$ is transitive.
   (b). The number of different set $A = \{a, b, c\} \subseteq Z^+, where\ a, b \geq 1, c > 1$, satisfy the property $a \cdot b \cdot c = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$ is 35.
   (c). If A is a language, then $(A^*)^+ = A^+$.
   (d). Define a relation R, if $(a, b) \in R \Leftrightarrow 2|(a - b)$. R is an equivalent relation.
   (e). The number of different equivalence relation on a set of 4 elements is 15.

2. (**12** points)
   (a) Let S be a set of seven positive integers whose maximum is at most 24. Show that the sums of the elements in all the nonempty subsets of S cannot all be distinct.
   (b) Show that in a group of **five** persons, there are at least two of them have the same number of friends in this group.
   (c) n+1 integers are chosen from 2n distinct integers. Show that among the integers chosen there are two integers a, b such that $a/b=2^k$ for some positive integer k.

3. (**8** points) If A = $\{a, b, c, d, e, f, g\}$, determine the number of relations on A that are
   (a) reflexive and symmetric but not transitive,
   (b) antisymmetric but not reflexive

4. (**5, 2, 3** points) Let p, q be two distinct primes. We denote relation $x\mathbf{R}y$ if x divides y. Under this relation **R**, (a) please draw the Hasse diagram of all positive divisors of p^2q^2 that are smaller than p^2q^2. (b) Please answer the maximum element(s), the greatest element(s), and (c) glb $\{p, pq\}$ and lub $\{pq, p^2, p^2q\}$.

5. (**3, 4, 3** points) Let A={a, b, c, d, e, f, g} (a) how many closed binary operations f on A satisfy f(a, b) $\neq$c? (b) How many closed binary operations f on A have an identity and f(a, b)=c? (c) How many f in (b) are commutative?

6. (**10** points) Suppose that R_1 and R_2 are equivalent relations on the set S. Determine whether each of the following combinations of R_1 and R_2 must be an equivalent relation. (a) $R_1 \cup R_2$, (b) $R_1 \cap R_2$, (c) $R_1 \oplus R_2$.

7. (**8** points) Find 1, 2, … k-equivalent state groups in the following FSM.

State	v		w	
	0	1	0	1
s_1	s_6	s_3	0	0
s_2	s_3	s_1	0	0
s_3	s_2	s_4	0	0
s_4	s_7	s_4	0	0
s_5	s_6	s_7	0	0
s_6	s_5	s_2	1	0
s_7	s_4	s_1	1	1

8. (**12** points) Let A be a set with |A|=n, and let R be a relation on A. (a) If R is antisymmetric. What is the maximum value for |R|? (b) If n=36 and R is an equivalent relation and partition A into disjoint equivalence classes A_1, A_2, A_3, where $|A_1|=|A_2|=|A_3|$. What is |R|? (c) If n=9, how many equivalence relations on A that have exactly one equivalence class of size 4?

(Stirling number of the second kind: S(4,2)=7, S(4, 3)=6, S(5, 2)=15, S(5, 3)=25, S(5, 4)=10, S(6, 2)=31, S(6, 3)=90, S(6, 4)=65, S(6,5)=15)