Discrete Mathematics

Deadline: 2021/04/08

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1. Determine the number of paths in the xy-plane from (-3,-2) to (5,4), and must pass (0,0) on the way where each such path is made up of individual steps going one unit to the right (R) or one unit upward (U).

2. How many bit string of length 10 contain
a)exactly seven 1s
b)at most nine 1s
c)exactly three 1s and no consecutive 1s

3. How many arrangements of the letters in MISSISSIPPI have no consecutive S's ?

4. In how many ways can the symbols a, b, c, d, e, e, e, e, e be arranged so that no e is adjacent to another e?

5. How many triangles are determined by the vertices of a regular polygon of n sides? How many if no side of the polygon is to be a side of any triangle?

Exercise 2.1

4. Let p, q, r, s denote the following statements:

p: I finish writing my computer program before lunch.
q: I shall play tennis in the afternoon.
r: The sun is shining.
s: The humidity is low.

Write the following in symbolic form.

a) If the sun is shining, I shall play tennis this afternoon.

b) Finishing the writing of my computer program before lunch is necessary for my playing tennis this afternoon.

c) Low humidity and sunshine are sufficient for me to play tennis this afternoon.

6. Determine the truth value of each of the following implications.

a) If $3 + 4 = 12$, then $3 + 2 = 6$.

b) If $3 + 3 = 6$, then $3 + 4 = 9$.

c) If Thomas Jefferson was the third president of the United States, then $2 + 3 = 5$.

Exercise 2.2

4. For primitive statements p, q, r, and s, simplify the compound statement

$$[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]] \vee \neg q] \rightarrow s.$$

8. Write the dual for (a) $q \rightarrow p$, (b) $p \rightarrow (q \wedge r)$, (c) $p \leftrightarrow q$, and (d) $p \veebar q$, where p, q, and r are primitive statements.

10. Determine whether each of the following is true or false. Here p, q are arbitrary statements.

a) An equivalent way to express the converse of "p is sufficient for q" is "p is necessary for q."

b) An equivalent way to express the inverse of "p is necessary for q" is "$\neg q$ is sufficient for $\neg p$."

c) An equivalent way to express the contrapositive of "p is necessary for q" is "$\neg q$ is necessary for $\neg p$."

Exercise 2.3

4. For each of the following pairs of statements, use Modus Ponens or Modus Tollens to fill in the blank line so that a valid argument is presented.

a) If Janice has trouble starting her car, then her daughter Angela will check Janice's spark plugs.
Janice had trouble starting her car.
$\therefore$ _____

b) If Brady solved the first problem correctly, then the answer he obtained is 137.
Brady's answer to the first problem is not 137.
$\therefore$ _____

c) If this is a **repeat-until** loop, then the body of this loop is executed at least once.

_____

$\therefore$ The body of the loop is executed at least once.

d) If Tim plays basketball in the afternoon, then he will not watch television in the evening.

_____

$\therefore$ Tim didn't play basketball in the afternoon.

6. For primitive statements p, q, and r, let P denote the statement

$$[p \wedge (q \wedge r)] \vee \neg[p \vee (q \wedge r)],$$

while P_1 denotes the statement

$$[p \wedge (q \vee r)] \vee \neg[p \vee (q \vee r)].$$

a) Use the rules of inference to show that

$$q \wedge r \Rightarrow q \vee r.$$

b) Is it true that $P \Rightarrow P_1$?

8. Give the reasons for the steps verifying the following argument.

$$\begin{array}{l} (\neg p \vee q) \rightarrow r \\ r \rightarrow (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \rightarrow \neg t \\ \hline \therefore p \end{array}$$

Steps	**Reasons**
1) $\neg s \wedge \neg u$	
2) $\neg u$	
3) $\neg u \rightarrow \neg t$	
4) $\neg t$	
5) $\neg s$	
6) $\neg s \wedge \neg t$	
7) $r \rightarrow (s \vee t)$	
8) $\neg(s \vee t) \rightarrow \neg r$	
9) $(\neg s \wedge \neg t) \rightarrow \neg r$	
10) $\neg r$	
11) $(\neg p \vee q) \rightarrow r$	
12) $\neg r \rightarrow \neg(\neg p \vee q)$	
13) $\neg r \rightarrow (p \wedge \neg q)$	
14) $p \wedge \neg q$	
15) $\therefore p$	

Exercise 2.4

18. Negate and simplify each of the following.

a) $\exists x\,[p(x) \vee q(x)]$ **b)** $\forall x\,[p(x) \wedge \neg q(x)]$

c) $\forall x\,[p(x) \rightarrow q(x)]$

d) $\exists x\,[(p(x) \vee q(x)) \rightarrow r(x)]$

Exercise 2.5

10. Provide the missing reasons for the steps verifying the following argument:

$$\begin{array}{l} \forall x\,[p(x) \vee q(x)] \\ \exists x\,\neg p(x) \\ \forall x\,[\neg q(x) \vee r(x)] \\ \forall x\,[s(x) \rightarrow \neg r(x)] \\ \hline \therefore \exists x\,\neg s(x) \end{array}$$

Steps	Reasons
1) $\forall x\,[p(x) \vee q(x)]$	Premise
2) $\exists x\,\neg p(x)$	Premise
3) $\neg p(a)$	Step (2) and the definition of the truth for $\exists x\,\neg p(x)$. [Here a is an element (replacement) from the universe for which $\neg p(x)$ is true.] The reason for this step is also referred to as the *Rule of Existential Specification.*
4) $p(a) \vee q(a)$	
5) $q(a)$	
6) $\forall x\,[\neg q(x) \vee r(x)]$	
7) $\neg q(a) \vee r(a)$	
8) $q(a) \rightarrow r(a)$	
9) $r(a)$	
10) $\forall x\,[s(x) \rightarrow \neg r(x)]$	
11) $s(a) \rightarrow \neg r(a)$	
12) $r(a) \rightarrow \neg s(a)$	
13) $\neg s(a)$	
14) $\therefore \exists x\,\neg s(x)$	Step (13) and the definition of the truth for $\exists x\,\neg s(x)$. The reason for this step is also referred to as the *Rule of Existential Generalization.*