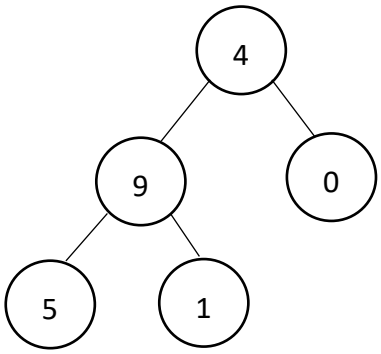
Algorithm Spring 2021 Final Exam

1. (10%) Considered a binary tree which contains digits from $0-9$ only, each root-to-leaf path would represent a number. If the leaf is the left-child or has no sibling, the number should be positive; otherwise, it should be negative. For instance, for a binary tree below, a root-to-leaf path $4 \to 9 \to 5$ which represents the number 495, and a root-to-leaf path $4 \to 9 \to 1$ which represents the number -491. Please answer the following questions.
   Input: $[4, 9, 0, 5, 1]$ (Suppose that the first index is 0)



(a) (5%) Given pseudo code for the problem that finding the total sum of all root-to-leaf numbers. Please fill in blanks. (Assume that input array is T.)

```
1.  dfs(Tree, i, num) {
2.      // Calculate value to Tree[i]
3.      val = ____(1)____
4.      // If there is no child, add the value of root-to-leaf to global variable "sum"
5.      if(Tree[i*2+1] == null && Tree[i*2+2] == null){
6.          // If leaf is right child, then multiply the value of root-to-leaf with (-1)
7.          if(____(2)____)
8.              val = val*(-1)
9.          sum = sum + val
10.         return
11.     }
12.     // search for left child
13.     if(Tree[i*2+1] != null)
14.         ____(3)____
15.     // search for right child
16.     if(Tree[i*2+2] != null)
17.         ____(4)____
18. }
```

```
19.
20.  sum = 0 // global variable
21.  sumNumbers(Tree, root) {
22.      if(Tree[root] == null)
23.          return 0
24.      dfs(Tree, root, 0)
25.      return sum
26.  }
27.
28. main() {
29.     print(sumNumbers(T, 0))
30. }
```

(b) (5%) What output will be printed if T is $[2, 7, 8, 0, 1, 3]$?

2. (10%)

(a) (5%) Why doesn't Dijkstra's algorithm work for the graph that has negative path?

(b) (5%) Given Dijkstra's algorithm below.

INITIALIZE-SINGLE-SOURCE(G, s)

```
1 for each vertex v ∈ G.V do
2     v.d = ∞
3     v.π = NIL
4 s.d = 0
```

RELAX(u, v, w)

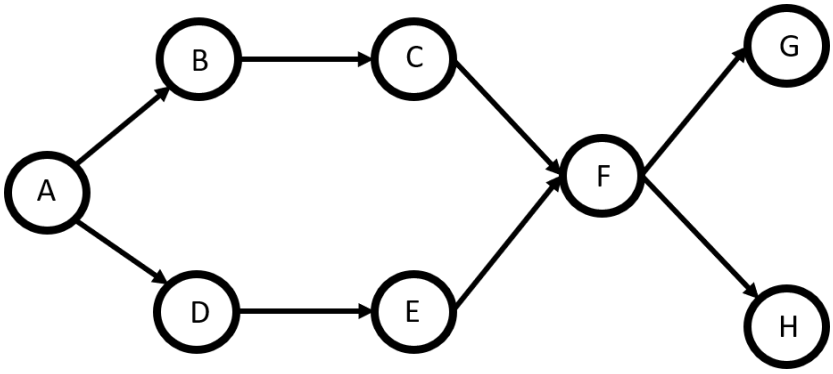
```
1 if v.d > u.d + w(u, v) then
2     v.d = u.d + w(u, v)
3     v.π = u
```

DIJKSTRA(G, w, r)

```
1 INITIALIZE-SINGLE-SOURCE(G, s)
2 S = ∅
3 Q = G.V
4 while Q ≠ ∅ do
5     u =EXTRACT-MIN(Q)
6     S = S ∪ {u}
7     for each vertex v ∈ G.Adj[u] do
8         RELAX(u, v, w)
```

Analyze and give time complexity for Dijkstra's algorithm while using binary heap. Suppose the input graph G is dense, which means that the graph can be assumed as complete graph. (You need to show your reason.)

3. (10%)



Consider the directed acyclic graph G above. How many topological orderings does it have?

4. (10%)

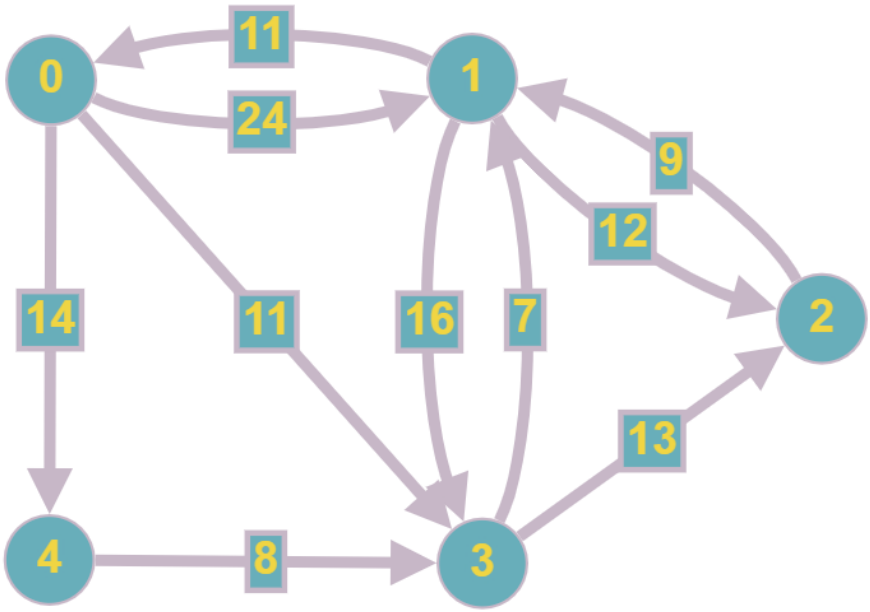
(a) (5%) Given 10 difference characters and their frequency, please build a Huffman tree.

Character	"C"	"O"	"S"	"M"	"_"	"P"	"A"	"X"	"Y"	"L"
Frequency	0.01	0.05	0.02	0.06	0.27	0.09	0.11	0.18	0.14	0.07

(Note 1: All edges along the path to a character contain a code digit. If they are on the left side of the tree, they will be a 0 (zero). If on the right, they well be a 1 (one).)
(Note 2: the frequency of left child < frequency of right child at the same height)

(b) (5%) Following the previous question (a), given a string:
0100111011110111101011001111001
Please decode the string by your Huffman tree.

5. (10%)



Find all pairs shortest path using Floyd-Warshall.
(Please write down process)

6. (10%) There are five cities in a network. The cost of building a road directly between i and j is the entry $a_{i,j}$ in the matrix below. Determine the least cost of making all the cities reachable from each other.

	0	1	2	3	4
0	0	12	17	13	14
1	12	0	8	7	9
2	17	8	0	3	13
3	13	7	3	0	10
4	14	9	13	10	0

7. (10%) Given a directed graph $G = (V, E)$, please design an algorithm to determine whether it contains a (directed) cycle.

8. (10%) Given a directed graph $G = (V,\ E)$, please design an algorithm to determine whether it contains a (directed) cycle of odd length.

9. (10%) Given a set of tasks, each task requests one unit of time to finish work. Moreover, a task has a deadline and reward. If the task is finished before the deadline, we get the reward. We cannot schedule multiple tasks at same time. (Hint: the deadline is at least one.)

   (a) (5%) Please design an algorithm to maximize total reward.
   (b) (5%) Consider any nonempty subproblem S_k, and let t_m be a task in S_k that is according to your choice. Please show that t_m is included in some maximum total reward subset of mutually compatible tasks of S_k.

10. (10%) Consider the following linear-programming system of difference constraints:

$$x_3 - x_1 \leq 4$$
$$x_2 - x_1 \leq 2$$
$$x_1 - x_4 \leq 10$$
$$x_5 - x_1 = 7$$
$$x_3 - x_2 \leq 5$$
$$x_2 - x_4 \leq -8$$
$$x_4 - x_5 \leq -4$$
$$x_5 - x_3 \leq 6$$
$$x_2 - x_5 \leq 9$$

   (a) (5%) Draw the constraint graph for these constraints.
   (b) (5%) Find a feasible solution or explain why no feasible solution exists.