1.

(1) False

$[p \vee (p \wedge q) \vee (p \wedge q \wedge r)] \wedge [(p \wedge r \wedge t) \vee t] \cancel{\Leftrightarrow} p \vee t$

$\Leftrightarrow p \wedge t$

(2) False

Let $n = 4$, $k = 6$; $\gcd(n, n+k) = \gcd(4, 10) = 2$

(3) True

$pqr \Rightarrow (1+1)(1+1)(1+1) = 8$; $p^7 = 7+1 = 8$

(4) True

$-4(2a+3b) + 17(a+b) = 17 \mid 9a+5b$

(5) False

"$\neg \forall x \forall y L(x,y)$" $\Rightarrow \exists x \neg [\forall y L(x,y)] \Rightarrow \exists x \exists y \neg L(x,y)$

Nobody loves everybody $\Rightarrow \forall x \neg [\exists y L(x,y)] \Rightarrow \forall x \forall y \neg L(x,y)$

So it means different thin

(6) True

10 的 Probability $= \frac{2^4}{2^9} = \frac{1}{32}$

12 的 Probability $= \frac{2^5}{2^{11}} = \frac{1}{64}$

$\Rightarrow \frac{P_{10}}{P_{12}} = \frac{\frac{1}{32}}{\frac{1}{64}} = \frac{1}{32} \times \frac{64}{1} = 2$

(7) False

$f(1) = f(-1) = (1, -1) \Rightarrow$ many to one

(8) False

Select n people $= \binom{2n}{n}$ and permutate other n people $= n!$

But AB = BA when they in the same group, so we need add $\frac{1}{(2!)^n}$ for n groups.

2.

(a) $60abcd^2$

$$\frac{5!}{1!1!1!2!} \times a \times b \times c \times d^2$$

(b) 56

$$C^8_3 = \frac{8!}{5!3!} = 56$$

(c) 36

$$\left.\begin{array}{l} z^0 = x^0 \sim x^{10} \Rightarrow 11 \\ z^1 = x^0 \sim x^8 \Rightarrow 9 \\ z^2 = x^0 \sim x^6 \Rightarrow 7 \\ z^3 = x^0 \sim x^4 \Rightarrow 5 \\ z^4 = x^0 \sim x^2 \Rightarrow 3 \\ z^5 = x^0 \Rightarrow 1 \end{array}\right\} 36$$

3. If $n \in \mathbb{Z}^+$, prove that $43 \mid 6^{n+2} + 7^{2n+1}$

when $n=1$, $43 \mid (6^3 + 7^3) \Rightarrow 43 \mid 559$ $\therefore$ True

Assume $n=k$, $43 \mid 6^{k+2} + 7^{2k+1}$ $\therefore$ True

Verify $n=k+1$, $43 \mid 6^{(k+1)+2} + 7^{2(k+1)+1}$

$$\Rightarrow 43 \mid 6 \cdot 6^{k+2} + 49 \cdot 7^{2k+1}$$

$$\Rightarrow 43 \mid 6(6^{k+2} + 7^{2k+1}) + 43 \cdot 7^{2k+1} \quad \therefore \text{True}$$

(since $43 \mid 6^{k+2} + 7^{2k+1}$ and $43 \mid 43 \cdot 7^{2k+1}$)

$\therefore$ By Mathematical Induction,

prove that $43 \mid 6^{n+2} + 7^{2n+1}$ for $n \in \mathbb{Z}^+$

4.

$\neg(p \wedge r)$

$\neg[p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)]$

$\Leftrightarrow \neg p \vee \neg(q \vee r) \vee \neg(\neg p \vee \neg q \vee r)$

$\Leftrightarrow \neg p \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$

$\Leftrightarrow [(\neg p \vee \neg q) \wedge (\neg p \vee \neg r)] \vee (p \wedge q \wedge \neg r)$

$\Leftrightarrow \neg(p \wedge r)$

5.

(a) $\emptyset$

$\emptyset \cap$ any set is $\emptyset$

(b) $\{a, \{\emptyset\}\}$

$\{\emptyset\} \Delta \{a, \emptyset, \{\emptyset\}\}$

$= \{\emptyset\} \cup \{a, \emptyset, \{\emptyset\}\} - \{\emptyset\} \cap \{a, \emptyset, \{\emptyset\}\}$

$= \{a, \emptyset, \{\emptyset\}\} - \{\emptyset\}$

$= \{a, \{\emptyset\}\}$

6. Both are knaves

If A lies, then B is knaves, so they won't be opposites type.

If A doesn't lie, B say they're opposite, it's a contradiction.

7. $3^n + 2^n + 2^{n-1}$

$$\sum_{k=\text{even}} (2^k + 2) \cdot C_k^n + \sum_{k=\text{odd}} (2^k + 1) \cdot C_k^n$$

$$= \sum_k 2^k C_k^n + \sum_k C_k^n + \sum_{\text{even}} C_k^n$$

$$= 3^n + 2^n + 2^{n-1}$$

8.

(a) 10

$x_1 + x_2 + x_3 < 8$, $x_1 > 0$, $x_2 > 0$, $x_3 > 2$

$\Rightarrow x_1 + x_2 + x_3 \le 7$, $x_1 > 0$, $x_2 > 0$, $x_3 > 2$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 7$, $x_1 > 0$, $x_2 > 0$, $x_3 > 2$, $x_4 \ge 0$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 7$, $x_1 \ge 1$, $x_2 \ge 1$, $x_3 \ge 3$, $x_4 \ge 0$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 7-1-1-3 = 2$, $x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0$

$\Rightarrow \binom{4+2-1}{2} = C^5_2 = 10$

(b) 6

x_1+x_2	x_3+x_4	
X 1	8	$\Rightarrow x_1, x_2 > 0$
2	4	
4	2	
X 8	1	$\Rightarrow x_3, x_4 > 0$

① $x_1 + x_2 = 2 \Rightarrow y_1 + y_2 = 0$ $(y_1, y_2 \ge 0) \Rightarrow H^2_0 = 1$
$x_3 + x_4 = 4 \Rightarrow y_3 + y_4 = 2$ $(y_3, y_4 \ge 0) \Rightarrow H^2_2 = 3$
$\Rightarrow 1 \times 3 = 3$

② $x_1 + x_2 = 4 \Rightarrow y_1 + y_2 = 2$ $(y_1, y_2 \ge 0) \Rightarrow H^2_2 = 3$
$x_3 + x_4 = 2 \Rightarrow y_3 + y_4 = 0$ $(y_3, y_4 \ge 0) \Rightarrow H^2_0 = 1$
$\Rightarrow 1 \times 3 = 3$

$3 + 3 = 6$

9.

Pick 2 objects from 4n objects, divides into 4 groups with n objects in each two cases

(1) two objects we pick are in same group $\Rightarrow C^4_1 C^n_2$

(2) two objects we pick are in different group $\Rightarrow C^4_2 C^n_1 C^n_1$

$\therefore 4\binom{n}{2} + 6n^2$