



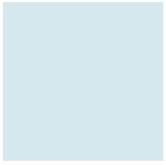
COMPILER CONSTRUCTION

Scanning

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Chapter 3

Theory and Practice of Scanning



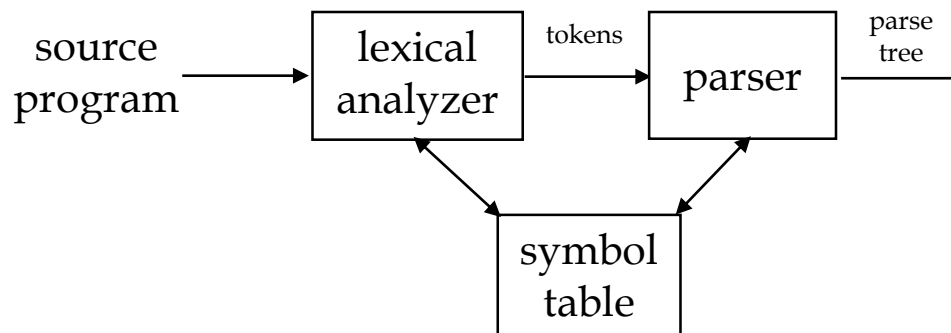
Scanner Generator

- All scanners perform much the same function
 - except for the **tokens** to be recognized
- A **scanner generator** helps eliminate the redundant effort of building a scanner
 - One can simply specifying which tokens the scanner is to recognize, and leaves the rest to the generator
 - E.g., Lex, Flex
- Programming a scanner generator is an example of **declarative programming**
 - we do not tell a scanner generator **how** (procedural programming) to scan but simply **what** to scan



Overview of a Scanner

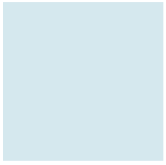
- A scanner transforms a **character stream** of source program into a **token stream**
 - Also called a **lexical analyzer** or **lexer**
- Reasons to separate lexical analysis
 - Simpler design is the most important consideration
 - Compiler efficiency is improved
 - Compiler portability is enhanced





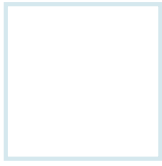
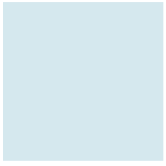
Definition of Tokens

- **Formal definitions of tokens** helps:
 - avoid writing lexical analysers (scanners) by hand; possible errors
 - simplify **specification** and implementation
 - understand the underlying techniques and technologies
 - For example, virtually all languages specify certain kinds of **rational constants**, which are often specified using decimal numerals such as 0.1 and 10.01
 - Can .1 or 10. be allowed?
- C, C++, Java say **YES**
- But, Pascal and Ada say **NO**
- Because 1..10 should be interpreted as a **range specifier** (1 to 10) in Pascal and Ada
- If .1 and 10. are allowed, then we'll have **two constants** (i.e., 1. and .10), which will lead to immediate **grammar error** after the ``two constants``



Outline

- In the following pages, we introduces:
 - Regular Expression
 - Finite Automata
 - Deterministic Finite Automata
 - Nondeterministic Finite Automata
- The above things are the key for a scanner to recognize the given regular expression (of a token)



Regular Expression (RE)

- RE
 - Is a way to specify various sets of strings
 - Can specify the structure of the tokens used in a programming language
- In this course, we simply walk through the basic RE concepts required to build a scanner
 - as it should be covered thoroughly in Computing Theory



Regular Expression (Cont'd)

- Regular set
 - A set of strings defined by a regular expression
- Token class
 - A regular set whose structure is defined by a RE
- Lexeme
 - A particular instance of a token class; or, instance of the token

Token class	Sample Lexemes	Informal Description of Pattern
const	const	const
if	if	if
relop	<, <=, =, <>, >, >=	< or <= or = or <> or > or >=
id	pi, count, D2	letter followed by letters and digits
num	0, 3.14, 6.02E23	any number constant



Regular Expression (Cont.)

- **Vocabulary**, denoted Σ (Sigma)
 - A finite character set
 - Normally the character set used by a computer, e.g., the **ASCII character set**, which contains **128 characters**
 - Java, however, uses the Unicode character set, including ASCII characters and a wide variety of other characters
- **Empty (or null) string**, denoted λ (Lambda)
 - represents an empty buffer in which no characters have yet been matched
 - represents also an optional part of a token
 - E.g., an integer literal may begin with a plus or minus; or, if it is unsigned, it may begin with λ



Regular Expression (Meta-character)

- **Meta-character**

- Any punctuation character or regular expression operator
- When used as an ordinary character, **a meta-character must be quoted to avoid ambiguity**

- The six symbols are meta-characters

() ' * + |

- NOTE: the symbol ' is the Apostrophe character as used in *don't*

- The four single-character tokens are often used in a programming language

- '(' | ')' | ';' | ','
(left parenthesis, right parenthesis, semicolon, and comma)
- Parentheses are used to delimit expressions



Regular Expression (Operations)

- Large (or infinite) sets are conveniently represented by **operations** on finite sets of characters and strings

- The three essential operations for RE

Let r_1, r_2 be REs

1. **Catenation.** $r_1 r_2$

2. **Alternation.** $r_1 \mid r_2$ denotes either r_1 or r_2

3. **Kleene closure.** r_1^* denotes zero or more occurrences of r_1

\mid and * are the operators for Alternation and Kleene Closure, respectively



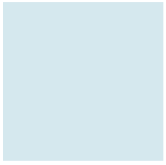
Regular Expression (Catenation)

- **Catenation**, an operation
 - Strings are built from characters in the character set Σ via catenation
- As **characters** are catenated to a string, it grows in length
 - For example, the string *'do'* is built by first catenating *d* to λ and then catenating *o* to the string *d*
 - The null string λ , when catenated with any string *s*, yields *s*
 - That is, $s\lambda \equiv \lambda s \equiv s$



Regular Expression (Catenation) (Cont'd)

- Catenation can be extended to **sets of strings**
 - Small finite sets are conveniently represented by listing their elements, which can be individual characters or strings of characters
- Let P and Q be sets of strings, and the symbol \in represents **set membership**
 - E.g., if $s1 \in P$ and $s2 \in Q$, then string $s1s2 \in (P Q)$



Regular Expression (Alternation)

- **Alternation** “ | ” can be extended to sets of strings

- Examples

1. Let P and Q be sets of strings

The string $s \in (P \mid Q)$ if, and only if, $s \in P$ **or** $s \in Q$

2. Let D be the set of the ten single digits

D is defined as $D = (0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9)$

- Note that the textbook often uses abbreviations such as $(0 \mid \dots \mid 9)$ rather than enumerate a complete list of alternatives
- The \dots symbol is not part of the regular expression notation



Regular Expression (Kleene Closure)

- Kleene closure

- ``*'' postfix operator, representing zero or more occurrences of the *decorated* character/string

- Examples

1. Let P be a set of strings

P^* represents all strings formed by the catenation of zero or more selections (possibly repeated) from P

Zero selections are represented by λ

2. LC^* is the set of all words composed only of **lowercase letters** and of **any length** (including the zero-length word, λ)



Regular Expression (More Examples)

- Let $\Sigma = \{a, b\}$
 - The regular expression $a \mid b$ denotes $\{a, b\}$
 - $(a \mid b)(a \mid b)$ denotes $\{aa, ab, ba, bb\}$
 - i.e., the set of all 2-letter strings of a's and b's
 - Another form is $aa \mid ab \mid ba \mid bb$
 - a^* denotes the set of all strings of a's
 - i.e. $\{\lambda, a, aa, aaa, \dots\}$
 - $(a \mid b)^*$ denotes the set of all strings of a's and b's
 - Another form is $(a^*b^*)^*$
 - $a \mid a^*b$ denotes the set containing the string a and all strings consisting of zero or more a's and followed by b's



Formal Definition

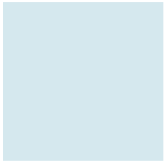
- \emptyset is a regular expression denoting the **empty set** (the set containing no strings)
- λ is a regular expression denoting the set that contains only the **empty string**
- The symbol s ($s \in \Sigma$) is a regular expression denoting $\{s\}$,
 - which refers to a **set** containing the single symbol
- If A and B are regular expressions then $A \mid B$, AB , and A^* are also regular expressions
They denote **3 operators**:
 - 1) **alternation**, 2) **catenation**, and 3) **Kleene closure** of the corresponding regular sets, respectively



Formal Definition (Cont'd)

- **Additional operators:**

- P^+ , sometimes called **positive closure** denotes all strings consisting of **one or more strings** in P catenated together: $P^* = (P^+ \mid \lambda)$ and $P^+ = PP^*$
- If A is a set of characters, **Not(A)**, denotes $(\Sigma - A)$, all characters in Σ , but not in A
- If k is a constant, then the set A^k represents all strings formed by catenating k (possibly different) strings from A , i.e., $A^k = (AAA \dots)$, k copies of A



Regular Expression (Another Example)

- A Java or C++ single-line comment that begins with `//` and ends with **Eol**
 - `Comment = // (Not(Eol))*Eol`
 - The above RE represents that a comment **begins with two slashes** and **ends at the first end-of-line**
 - Within the comment, any sequence of characters is allowed that does not contain an end-of-line
 - This guarantees that the first end-of-line we see ends the comment



Regular Expression (Extra Example)

- A fixed-decimal literal (for example, 23.456) can be defined as
 - $\text{Lit} = D+.D+$
 - **One or more digits** must be on both sides of the decimal point, so .12 and 35. are excluded
- Note: All **finite** sets are regular
 - However, some (but not all) infinite sets are regular



Beyond Regular Expression

- All regular sets can be defined by Context-Free Grammars (CFGs)
- CFGs are more powerful descriptive mechanism than regular expressions
 - REs are quite adequate for specifying **token-level** syntax
- For every regular expression we can create an efficient device, **finite automaton**,
 - which recognizes exactly those strings that match the regular expression's pattern



Finite Automata and Scanners

- A **finite automation** (FA) can be used to help **recognize** the tokens specified by a regular expression
 - To see if the given strings are belonging to the regular sets
- An FA consists of:
 - A finite set of *states*
 - A finite *vocabulary*, denoted Σ
 - A set of *transitions* (or moves) from one state to another, labeled with characters in Σ
 - A special state called the *start* state
 - A subset of the states called the *accepting*, or *final*, states
- An FA can also be represented graphically using a transition diagram, composed of the components shown in Fig. 3.1



A Finite Automaton

- An FA also can be represented graphically using a **transition diagram**
 - In practice, we first build the corresponding FA based on the given RE
 - Then, the built FA is used to determine if the given string is a valid token
- We now build the FA for the RE: $(abc^+)^+$

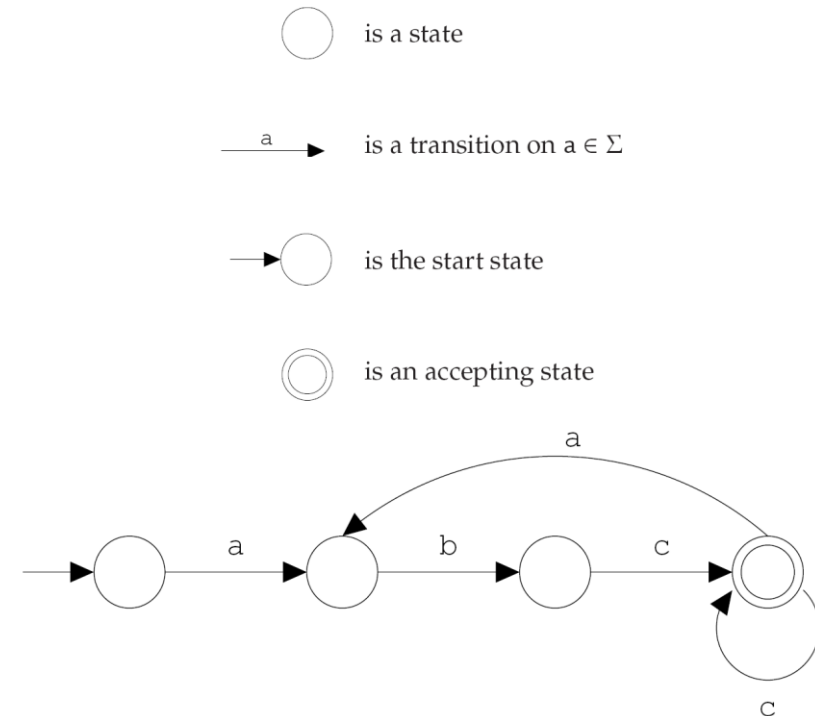
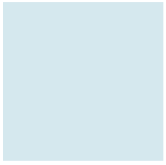
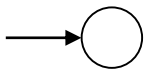


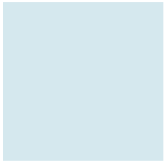
Figure 3.1: Components of a finite automaton drawing and their use to construct an automaton that recognizes $(abc^+)^+$.



Example: Build A Finite Automaton (Cont'd)

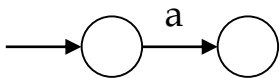
- Regular expression: $(abc^+)^+$
- The character to be read:
 - None
- Graph:
 - Add the starting state

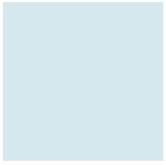




Example: Build A Finite Automaton (Cont'd)

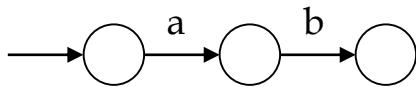
- Regular expression: $(abc^+)^+$
- The character to be read:
 - (a
- Graph:





Example: Build A Finite Automaton (Cont'd)

- Regular expression: $(abc^+)^+$
- The character to be read:
 - (ab
- Graph:



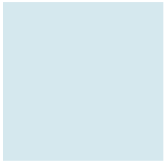


Example: Build A Finite Automaton (Cont'd)

- Regular expression: $(abc)^+$
- The character to be read:
 - (abc
- Graph:

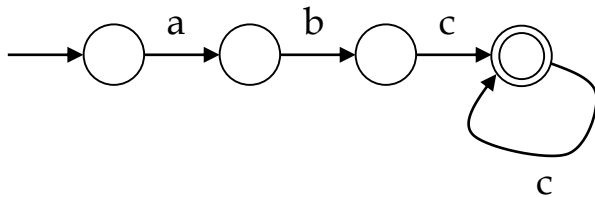


→ It looks like the string, ``**abc**'', is accepted by the regular expression, so we mark the current state as the **accepting state**



Example: Build A Finite Automaton (Cont'd)

- Regular expression: $(abc^+)^+$
- The character to be read:
 - $(abc^+$
- Graph:

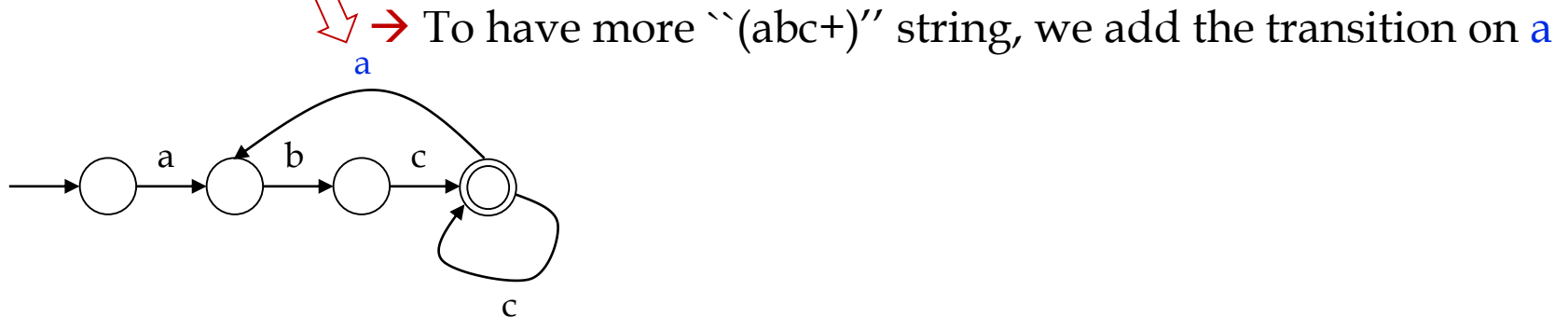


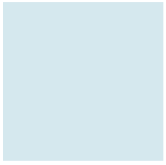


Example: Build A Finite Automaton (Cont'd)

- Regular expression: $(abc^+)^+$
- The character to be read:
 - $(abc^+)^+$

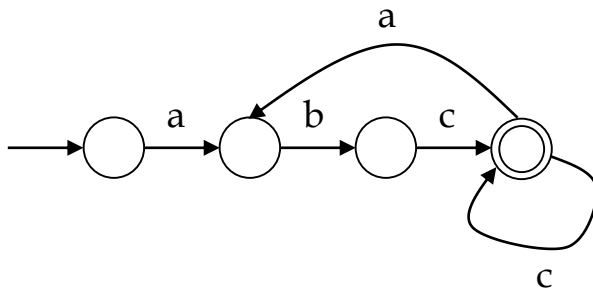
- Graph:





Example: Build A Finite Automaton (Cont'd)

- Regular expression: $(abc^+)^+$
- The character to be read:
 - $(abc^+)^+$
- Graph:





Example: Recognize the Token

- Given an input string and a RE: $(abc^+)^+$
 1. It starts from the **start state**
 2. Does the **next input character match the label on a transition from the current state?**

Yes, we move to the state pointed by the next character and set the next character as current character; otherwise, we stop

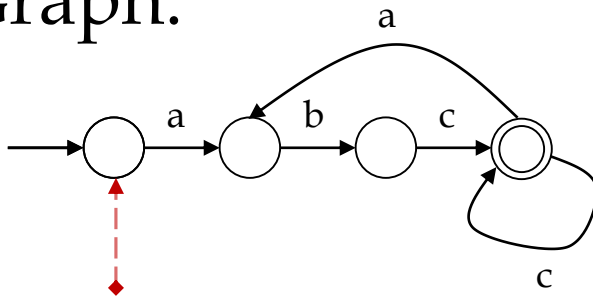
This step runs iteratively until no next character or no suitable transition
 3. If we finish (or stop) in an **accepting state**, the sequence of the characters forms a valid token; otherwise, a valid token has not been seen



Example: Recognize the Token (Cont'd)

- Regular expression: $(abc^+)^+$
- The given string: **abc**
- The character to be read:
 - **None** (at starting state)

- Graph:

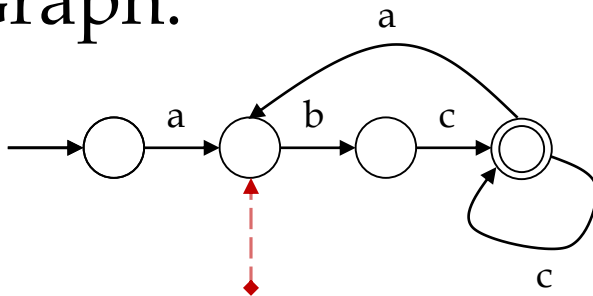




Example: Recognize the Token (Cont'd)

- Regular expression: $(abc^+)^+$
- The given string: **abc**
- The character to be read:
 - a

- Graph:

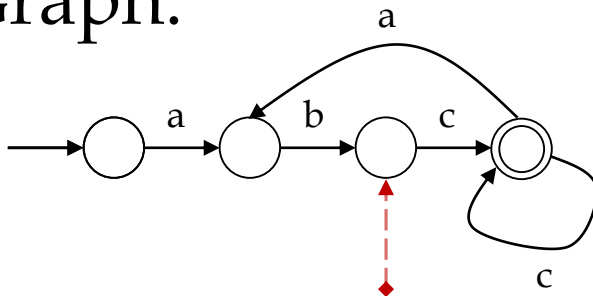




Example: Recognize the Token (Cont'd)

- Regular expression: $(abc^+)^+$
- The given string: **abc**
- The character to be read:
 - ab

- Graph:





Example: Recognize the Token (Cont'd)

- Regular expression: $(abc^+)^+$

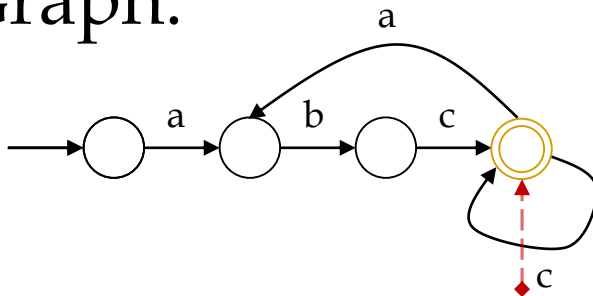
- The given string: **abc**

- The character to be read:

– abc

→ We have read out the characters and are at the **accepting state**, so we say the given string is a valid token.

- Graph:





Deterministic Finite Automata (DFA)

- A DFA is an FA that always allows **a unique transition for a given state and character**
- DFA is simple to program and are often used to drive a scanner
- DFA is conveniently represented in a computer by a **transition table**



Transition Table for DFA

- A transition table, T , is a two-dimensional array
 - indexed by a **DFA state** and a **vocabulary symbol**
 - Entries are either a **DFA state** or an **error flag** (often represented as a blank table entry)

Example

- If we are in state s and read character c ,
 - then $T[s, c]$ will be **the next state** we visit,
 - or $T[s, c]$ will contain **an error flag** indicating that c cannot extend the current token



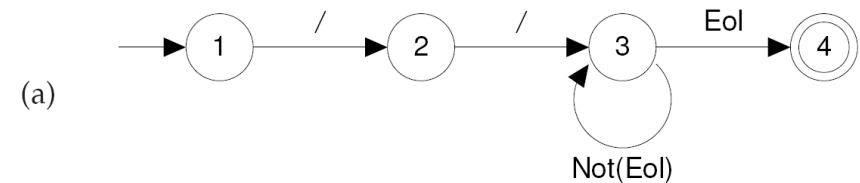
Example: DFA for the Single-Line Comment

- The single-line comment for Java or C++ can be represented by the regular expression

$\rightarrow // (\text{Not (Eol)})^* \text{Eol}$

- Fig. 3.2 is the corresponding DFA. (a) transition graph, & (b) transition table

- Initial state: 1
- Accepting state: 4
- Empty entries indicate *error flag*



(b)

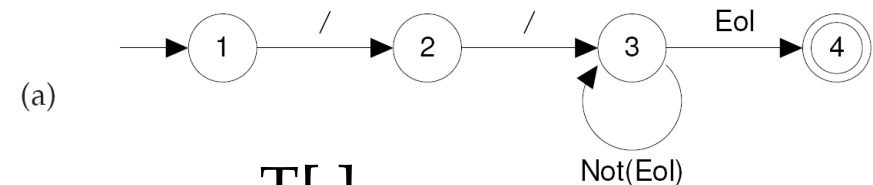
State	Character				
	/	Eol	a	b	...
1	2				
2	3				
3	3	4	3	3	3
4					

Figure 3.2: DFA for recognizing a single-line comment. (a) transition diagram; (b) corresponding transition table.



Example: DFA for the Single-Line Comment (Cont'd)

- $T[1, /]$
 - Initially, we are at the **state 1** and
 - if we have the next input character ``/``
 - The next state is **state 2**
 $\rightarrow T[1, /] = 2$
- $T[3, d]$
 - If we are at the **state 3** and
 - have the next input character ``d``
 - The next state is **state 3**
 $\rightarrow T[3, d] = 3$
- $T[3, \text{Eol}]$
 - If we are at the **state 3** and
 - have the next input character **Eol**
 - The next state is **state 4**
 $\rightarrow T[3, \text{Eol}] = 4$



(b)

$T[,]$

State	Character				
	/	Eol	a	b	...
1	2				
2	3				
3	3	4	3	3	3
4					

Figure 3.2: DFA for recognizing a single-line comment. (a) transition diagram; (b) corresponding transition table.



Coding the DFA

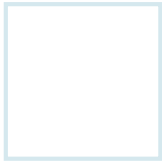
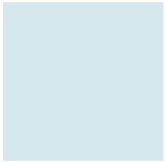
- A DFA can be coded in one of two forms:
 - Table-driven
 - Explicit control
 - A scanner could be built with either one of the two forms
- The *table-driven* based scanner is shown in Fig. 3.3
 - the transition table that defines the **transitions** in a DFA are **explicitly represented** in a runtime table that is “interpreted” by a driver program

```

/★ Assume CurrentChar contains the first character to be scanned ★/
State ← StartState
while true do
    NextState ← T[State, CurrentChar]
    if NextState = error
        then break
    State ← NextState
    CurrentChar ← READ()
if State ∈ AcceptingStates
then /★ Return or process the valid token ★/
else /★ Signal a lexical error ★/

```

Figure 3.3: Scanner driver interpreting a transition table.



Coding the DFA (Explicit Control)

- The *explicit control* based scanner is shown in Fig. 3.4
 - the transition table that defines a DFA's **actions appears implicitly as the control logic** of the program

```
/* Assume CurrentChar contains the first character to be scanned */  
if CurrentChar = '/'  
then  
    CurrentChar ← READ()  
    if CurrentChar = '/'  
    then  
        repeat  
            CurrentChar ← READ()  
        until CurrentChar ∈ { Eol, Eof }  
    else /* Signal a lexical error */  
else /* Signal a lexical error */  
if CurrentChar = Eol  
then /* Finished recognizing a comment */  
else /* Signal a lexical error */
```

Figure 3.4: Explicit control scanner.



About Lex Scanner Implementation

- **Sec. 3.5 Lex Scanner Generator**

- will be covered when we announce the programming homework #1
- You could read it first by yourself

- **Sec. 3.6 Other Scanner Generators**

- Is optional

- **Sec. 3.7 Practical Considerations of Building Scanners**

- should be read if you want to implement a more complex and efficient scanner



Regular Expression and Finite Automata

- REs are equivalent to FAs
- The main job of **scanner** is to
 - **transform a RE into an equivalent FA**
- To make an FA from a regular expression proceeds in two steps:
 1. It transforms the regular expression into an NFA
 2. It transforms the NFA into a DFA

RE → NFA → DFA



Nondeterministic Finite Automaton (NFA)

- An NFA **needs not** make **a unique** (deterministic) **choice** of which state to visit
 - when reading a particular input
- In other words
 - FAs that contain no λ transitions and that always have unique successor states for any symbol are **deterministic**



Example: NFAs

Fig. 3.17

- The NFA below is allowed to have a state that has two transitions coming out of it, label by the same symbol **a**

Fig. 3.18

- The NFA may also have transitions labeled with **λ**

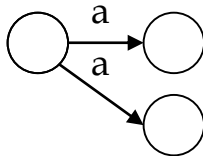


Figure 3.17: An NFA with two a transitions.

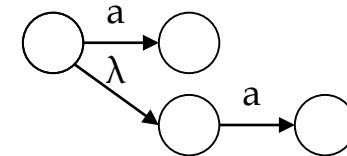


Figure 3.18: An NFA with a λ transition.



Key Elements of REs

- Recall that a regular expression is built of:
 - the *atomic* regular expressions
 a (a character in Σ) and λ (shown in Fig. 3.19)
 - with the **three operations**:
 AB , $A \mid B$, and A^*
 - Other operations, such as A^+ , are just abbreviation for the combinations for the above operations

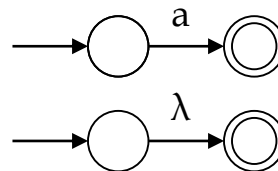


Figure 3.19: NFAs for a and λ .



Transforming RE to NFA

- Suppose we have FAs for A and B
 - The NFA for $A | B$ (shown in Fig. 3.20)
 - The states labeled A and B were the accepting states of the automata for A and B
 - we create a new accepting state for the combined FA
 - The NFA for AB (shown in Fig. 3.21)
 - The accepting state of the combined FA is the same as the accepting state of B
 - The NFA for A^* is (shown in Fig. 3.22)
 - The start state is an accepting state, so λ is accepted

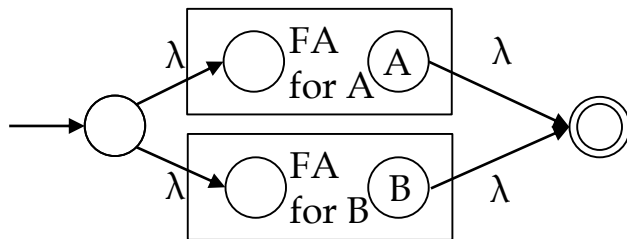


Figure 3.20: An NFA for $A | B$.

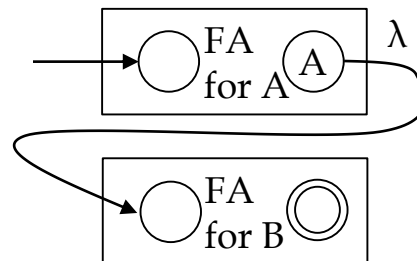


Figure 3.21: An NFA for AB .

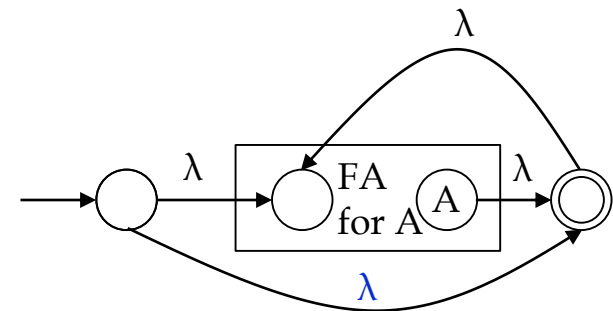


Figure 3.22: An NFA for A^* .



Example: RE to NFA

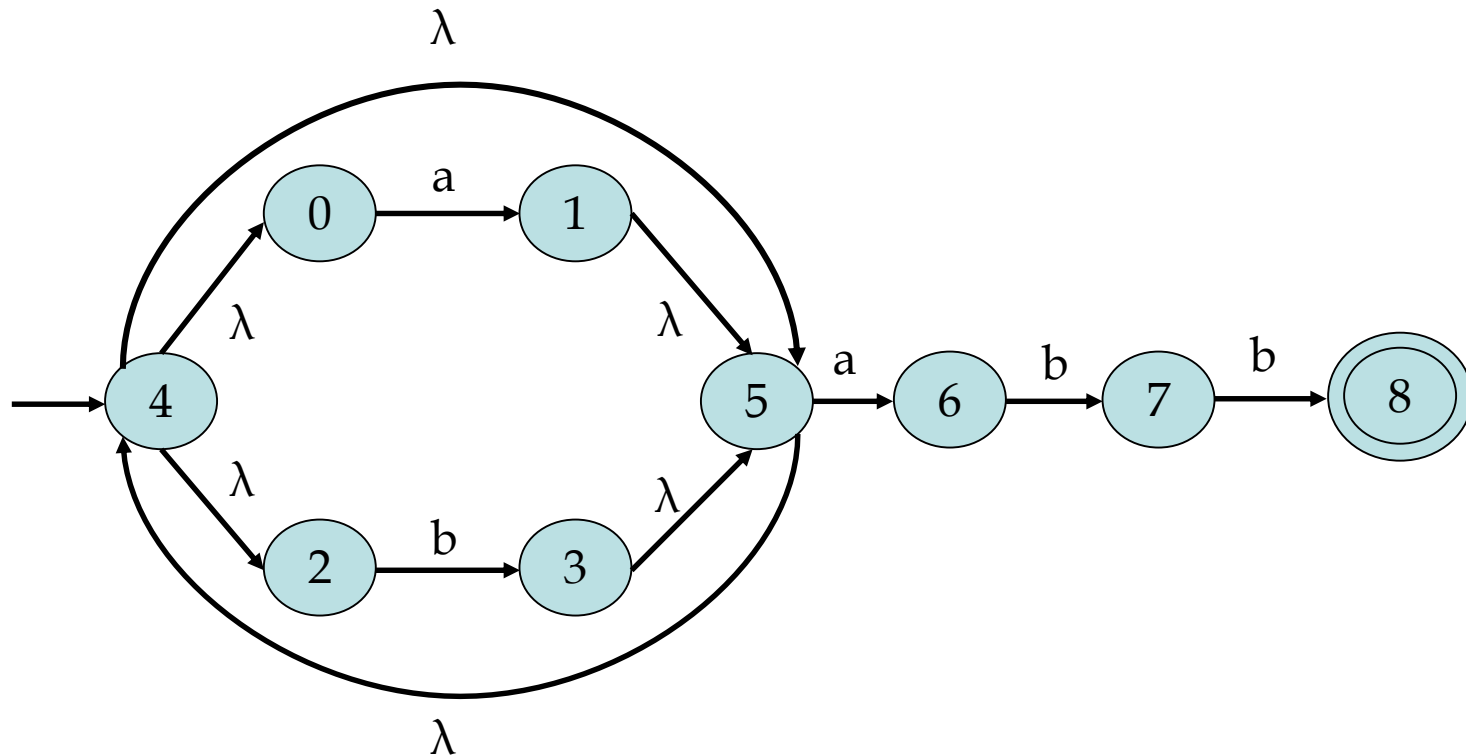
- Two steps for converting the regular expression, $(a \mid b)^*abb$, to the corresponding NFA:
 1. We create the NFA for $a, b, a \mid b, (a \mid b)^*$
 2. We create NFA for “abb”



Example: RE to NFA (Cont'd)

- Regular expression: $(a | b)^*abb$

1. NFA for $a, b, a | b, (a | b)^*$
2. NFA for "abb"
3. NFA for $(a | b)^*abb$

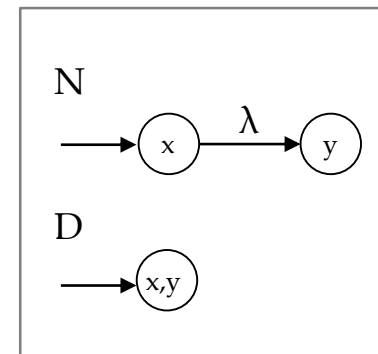




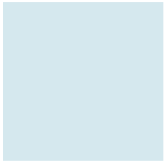
Transforming NFA to DFA

- **Subset construction algorithm**
 - transforming an NFA N to an equivalent DFA D
 - given in Fig. 3.23

Example of N and D



- **Key idea**
 - To construct **each state of D** with a **subset of states of N**
 - D will be in **the state $\{x,y,z\}$** after reading a given input character, if and only if N could be in **any of the states x,y , or z**



Notion

N: NFA (non-deterministic finite automata)

D: DFA (deterministic finite automata)

c
 $s \xrightarrow{c} t$: In N under char c , state s transits to t

c
 $S \xrightarrow{c} T$: In D, under char c , state S transits to T

S is a subset of $\{s \mid s \text{ in } N\}$



Start State

- The **start state of D**
 - is the set of all states to which N can transition without reading any input characters
 - that is, **the set of states reachable from the start state of N following only λ transitions**
- Algorithm **Close(S,T)**
 - computes those **states** that can be reached after only λ transitions, also known as **λ -successors**
 - shown in Fig. 3.23
- Once the start state of D is built, we begin to create successor states



Successor States of the Start State

- Place each state S of D on a **work list** when it is created
 - For each state S on the work list and each character c in the vocabulary,
 - we compute **S 's successor under c** ($S \xrightarrow{c} T$)
 - S is identified with some set of N 's states $\{n_1, n_2, \dots\}$
 1. We find all of the possible successor states to $\{n_1, n_2, \dots\}$ under c and obtain a set $\{m_1, m_2, \dots\}$
 2. We include the λ -successors of $\{m_1, m_2, \dots\}$
 - The resulting set of NFA states is included as a state T in D , and a transition from S to T , labeled with c , is added to D
 - We continue adding states and transitions to D until all possible successors to existing states are added
 - Because each state corresponds to a finite subset of N 's states, the process of adding new states to D must eventually terminate

General tips:
 Step 1: Find char successors
 Step 2: Find λ successors



Accepting State

- An accepting state of D is any set that **contains an accepting state of N**
- This reflects the convention that N accepts if there is any way it could get to its accepting state by choosing the “right” transitions



Example: NFA to DFA

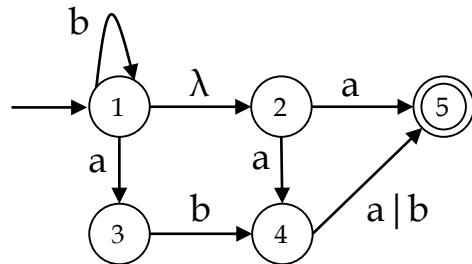


Figure 3.24: An NFA showing how subset construction operates.

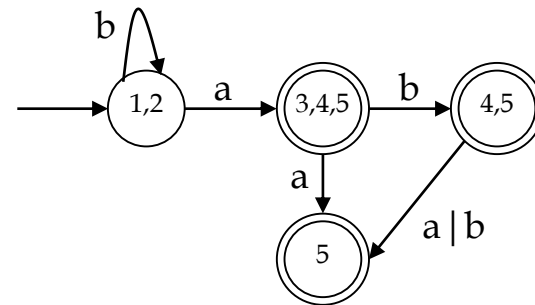
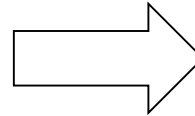


Figure 3.25: DFA created for NFA of Figure 3.24.

function makeDeterministic(N) **returns** DFA

① $D.StartState \leftarrow recordState(\{N.StartState\})$ {1}

② **foreach** $S \in WorkList$ **do**

$WorkList \leftarrow WorkList - \{S\}$

③ **foreach** $c \in \Sigma$ **do** $D.T(S, c) \leftarrow recordState(T \leftarrow \bigcup_{s \in S} t(s, c))$

④ $D.AcceptStates \leftarrow \{S \in D.States \mid S \cap N.AcceptStates \neq \emptyset\}$

end

λ-successors

function close(S, T) **return** Set

$ans \leftarrow S$

repeat

$changed \leftarrow false$

⑤ **foreach** $s \in ans$ **do**

⑥ **foreach** $t \in T(s, \lambda)$ **do**

if $t \notin ans$

then

⑦ $ans \leftarrow ans \cup \{t\}$

$changed \leftarrow true$

until not $changed$

return (ans)

end

function recordState(S) **return** Set

⑧ $S \leftarrow close(S, T)$ {1}

⑨ **If** $S \notin D.States$

then

$D.States \leftarrow D.States \cup \{S\}$

$WorkList \leftarrow WorkList \cup \{S\}$

return (S)

end

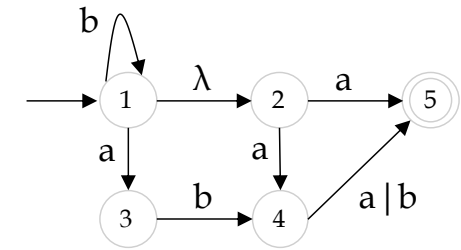


Figure 3.24: An NFA showing how subset construction operates.

To construct the start state of DFA:

→ **To find the set of states reachable from the start state of N following only λ transitions.**

- Start with state 1, the start state of N , and call RecordState(state 1) to find its λ -successors (Marker 1)
- RecordState() calls Close(state1, T). T includes states 2 and 3 (Marker 8), where T represents the following states of state 1 in N via any symbol.
- In Close(), set ans to state 1 (S).
And then for state 1 in ans (Marker 5) find the successor of state 1 via λ -transition, $t \in T(state\ 1, \lambda)$ (Marker 6).
Add t to ans , which is state 2 (Marker 7).
After that, return the ans set, states {1,2}, to RecordState() (Marker 8).
- Then, RecordState() will determine whether the set is in $D.States$ (Marker 9).
It is not, so it will be stored into $D.States$ and $WorkList$ (Marker 9).
- Now, we have constructed DFA's **start state as states {1,2}**.

function makeDeterministic(N) **returns** DFA

① $D.StartState \leftarrow recordState(\{N.StartState\})$

② **foreach** $S \in WorkList$ **do**

$WorkList \leftarrow WorkList - \{S\}$

③ **foreach** $c \in \Sigma$ **do** $D.T(S, c) \leftarrow recordState(T \cup_{s \in S} t(s, c))$

④ $D.AcceptStates \leftarrow \{S \in D.States \mid S \cap N.AcceptStates \neq \emptyset\}$

end

function close(S, T) **return** Set

$ans \leftarrow S$

repeat

$changed \leftarrow false$

⑤ **foreach** $s \in ans$ **do**

⑥ **foreach** $t \in T(s, \lambda)$ **do**

if $t \notin ans$

then

⑦ $ans \leftarrow ans \cup \{t\}$

$changed \leftarrow true$

until not $changed$

return (ans)

end

function recordState(S) **return** Set

⑧ $S \leftarrow close(S, T)$

⑨ **If** $S \notin D.States$

then

$D.States \leftarrow D.States \cup \{S\}$

$WorkList \leftarrow WorkList \cup \{S\}$

return (S)

end

Find all of the possible
successor states under c of
 N with the given states,
including λ states.

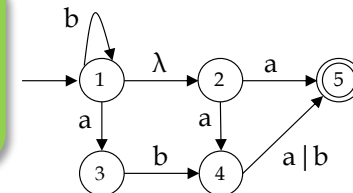


Figure 3.24: An NFA showing how subset construction operates.

Construct the successors of the start state $S = \{1, 2\}$ of DFA:

for each S in $WorkList$ ($S = \{1, 2\}$) do (Marker 2)

under char "a"

find S 's successor $D.T(S, c)$, where S is $\{1, 2\}$ and c is a (Marker 3):

state 1 transits to 3,

state 2 transits to 4,

state 2 transits to 5, we got $T = \{3, 4, 5\}$

RecordStates ($\{3, 4, 5\}$)

add $\{3, 4, 5\}$ to $D.States$ and $WorkList$

under char "b"

find S 's successor $D.T(S, c)$, where S is $\{1, 2\}$ and c is b):

state 1 transits to 1, we got $T = \{1\}$

RecordStates ($\{1\}$) calls close(), we got $T = \{1, 2\}$

$\{1, 2\}$ is already in $D.States$, so do not add it to $D.States$ and $WorkList$

function makeDeterministic(N) **returns** DFA

1 $D.StartState \leftarrow recordState(\{N.StartState\})$

2 **foreach** $S \in WorkList$ **do**

$WorkList \leftarrow WorkList - \{S\}$

3 **foreach** $c \in \Sigma$ **do** $D.T(S, c) \leftarrow recordState(T \leftarrow \bigcup_{s \in S} t(s, c))$

4 $D.AcceptStates \leftarrow \{S \in D.States \mid S \cap N.AcceptStates \neq \emptyset\}$

end

function close(S, T) **return** Set

$ans \leftarrow S$

repeat

$changed \leftarrow false$

5 **foreach** $s \in ans$ **do**

6 **foreach** $t \in T(s, \lambda)$ **do**

if $t \notin ans$

then

7 $ans \leftarrow ans \cup \{t\}$

$changed \leftarrow true$

until not $changed$

return (ans)

end

function recordState(S) **return** Set

8 $S \leftarrow close(S, T)$

9 **If** $S \notin D.States$

then

$D.States \leftarrow D.States \cup \{S\}$

$WorkList \leftarrow WorkList \cup \{S\}$

return (S)

end

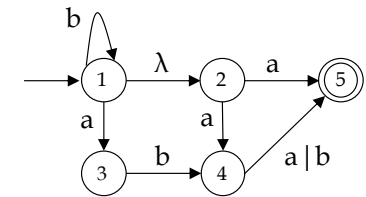
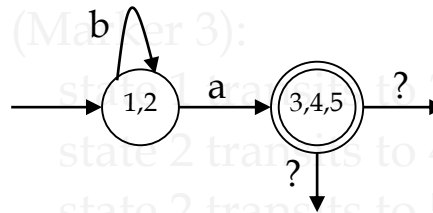


Figure 3.24: An NFA showing how subset construction operates.

Construct the successors of the start state $S = \{1, 2\}$ of DFA:

for each S in $WorkList$ ($S = \{1, 2\}$) do (Marker 2)

under char "a"



What we have established so far.

$\{3,4,5\}$

$\{1,2\} \xrightarrow{a} \{3,4,5\}$

$D.T(\{1,2\}, a) \leftarrow \{3,4,5\}$

under char "b"

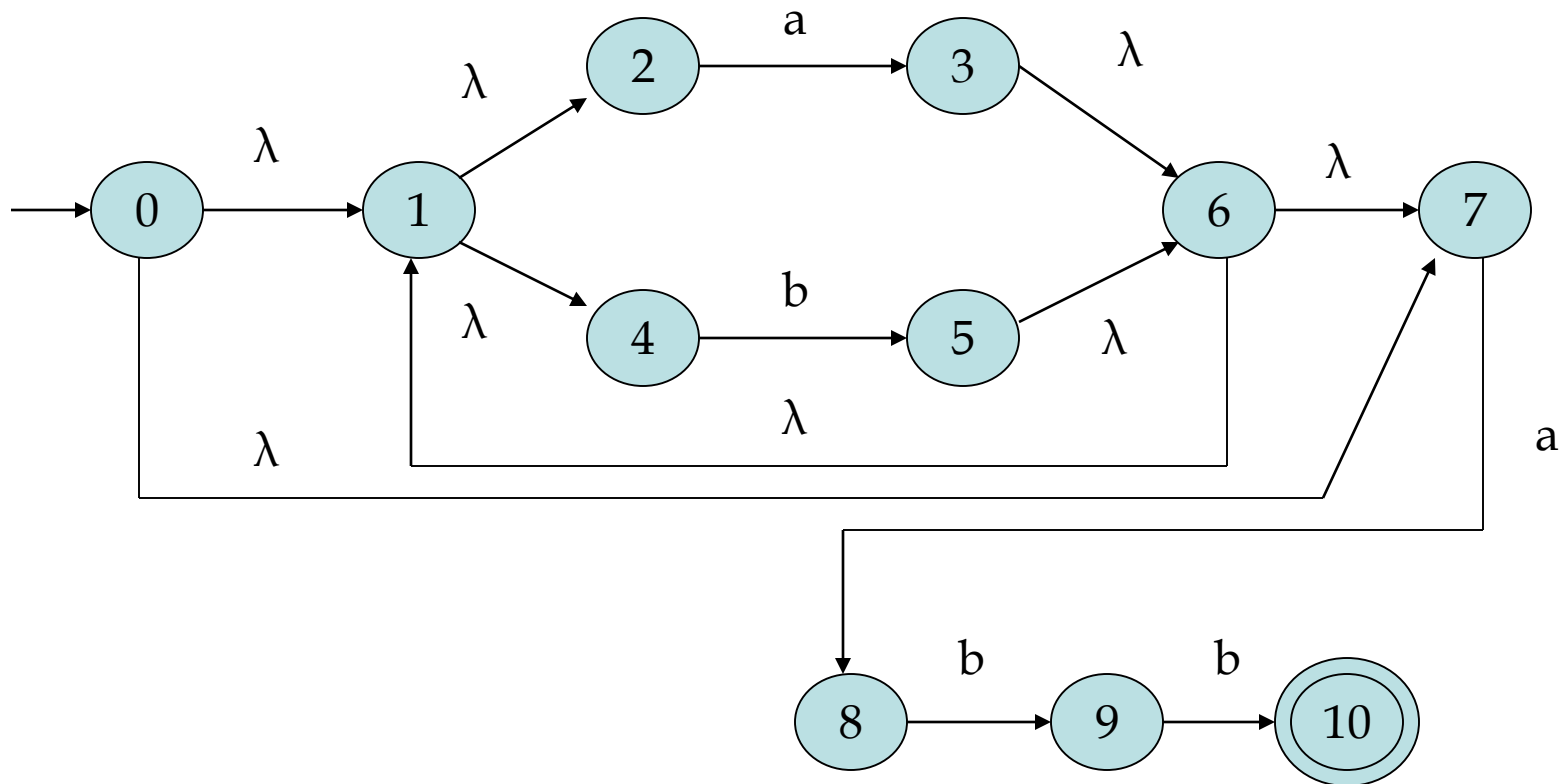
$\{1,2\} \xrightarrow{b} \{1,2\}$

$D.T(\{1,2\}, b) \leftarrow \{1,2\}$



Your Exercise

Given the NFA below, find its DFA.





Your Exercise (Cont'd)

The resulting DFA is:

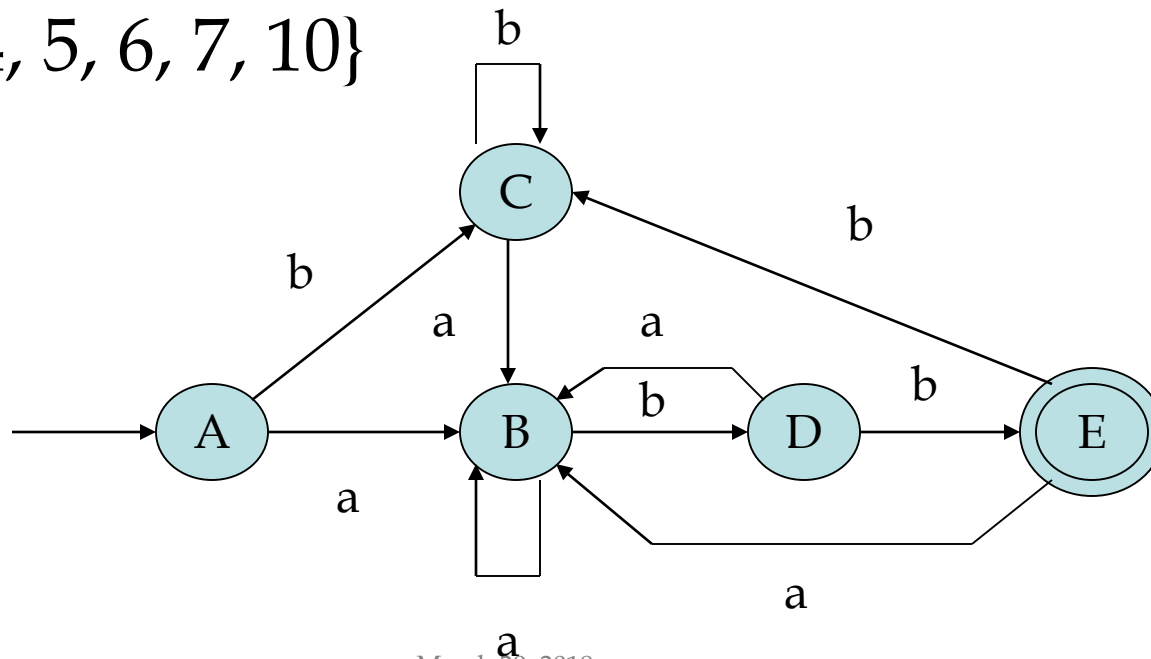
A {0, 1, 2, 4, 7}

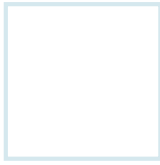
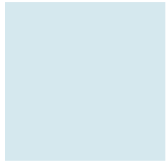
B {1, 2, 3, 4, 6, 7, 8}

C {1, 2, 4, 5, 6, 7}

D {1, 2, 4, 5, 6, 7, 9}

E {1, 2, 4, 5, 6, 7, 10}





QUESTIONS?