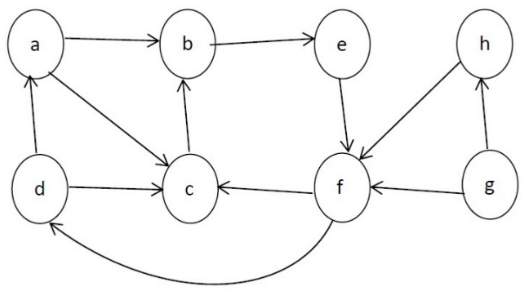
Algorithm Quiz2 solution

1. (25%)

G：



Given above graph $G = (V, E)$, please answer the following questions:

(a) Draw the adjacency matrix and adjacency lists for graph G.

(b) Use DFS (Depth first search) to find all vertices by starting from vertex a. If there are at least 2 edges to go, please choose the vertex in alphabet order. For instance, vertex a can go to vertex b or vertex d, choose vertex b instead of vertex d. (You need to write discovery and finish time for each vertex, and mark T (tree edge), B (back edge), F (forward edge) and C (cross edge) for all edges.)

(c) Analyze and give the time complexity (θ) for DFS algorithm.

Ans:

(a) Adjacency matrix:

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Adjacency lists: $a \to b \to c \to nil$

$b \to e \to nil$

$c \to b \to nil$

$d \to a \to c \to nil$

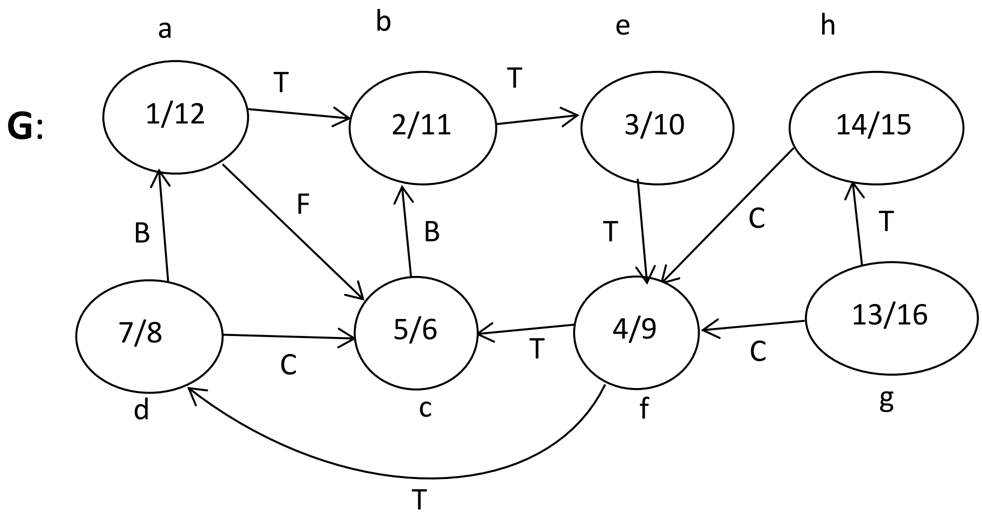
$e \to f \to nil$

$f \to c \to d \to nil$

$g \to f \to h \to nil$

$h \to f \to nil$

(b)

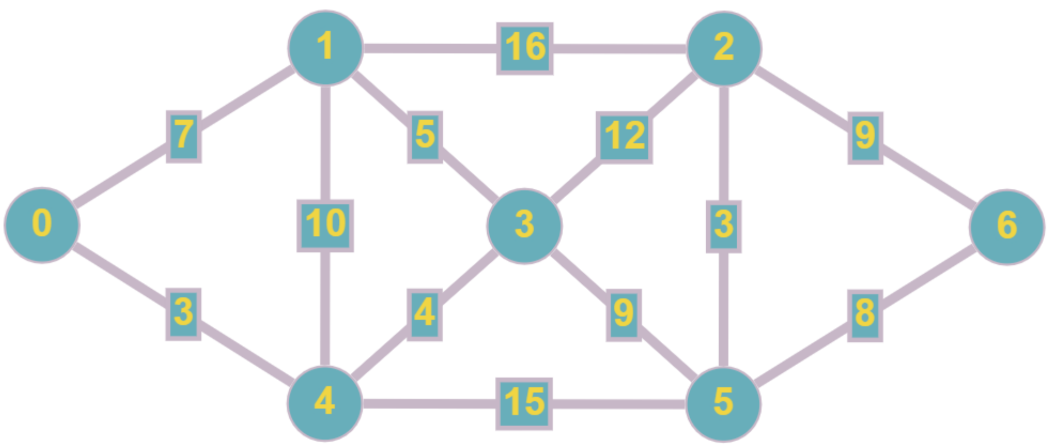


(c)

We only look once at most for all edges and vertices by using adjacency list.

Therefore, the time complexity is $\theta(V) + \theta(E) = \theta(V + E)$.

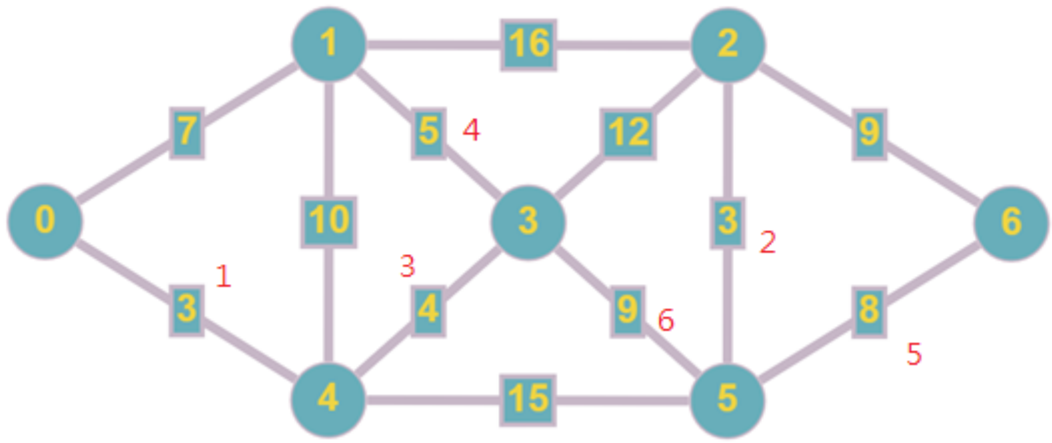
2. (25%)



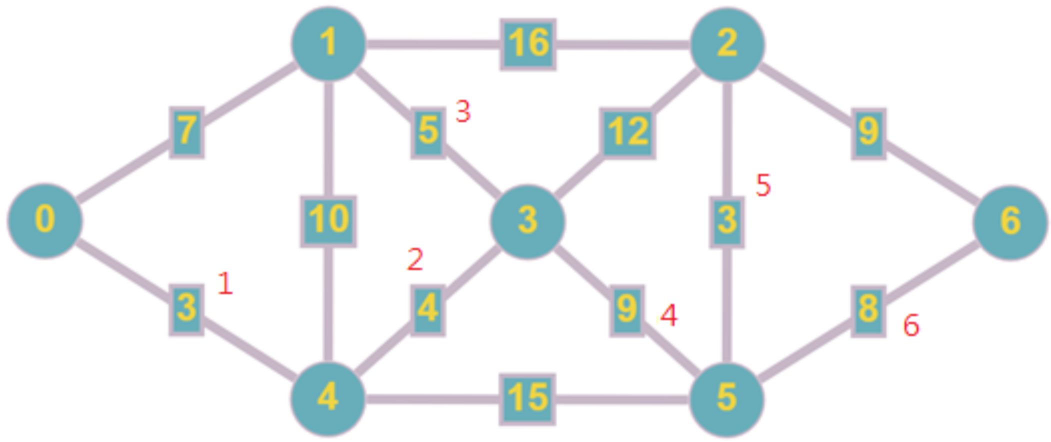
Find Minimum Spanning Trees by

(Write down the process)

(a) Kruskal's algorithm

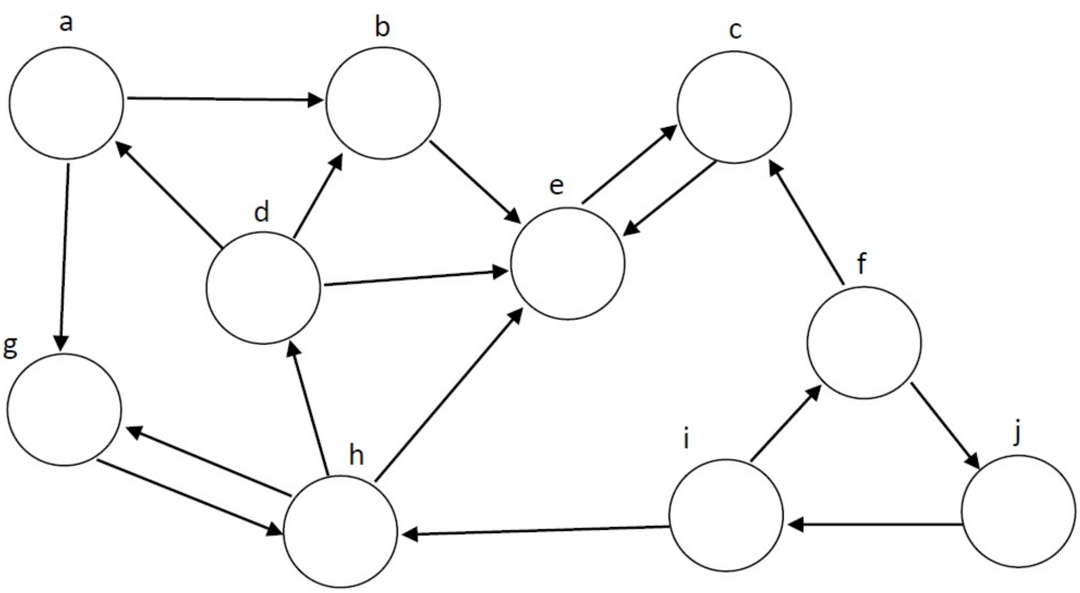


(b) Prim's algorithm (start at 0)



3. (25%)

(a) Find the strongly connected components.



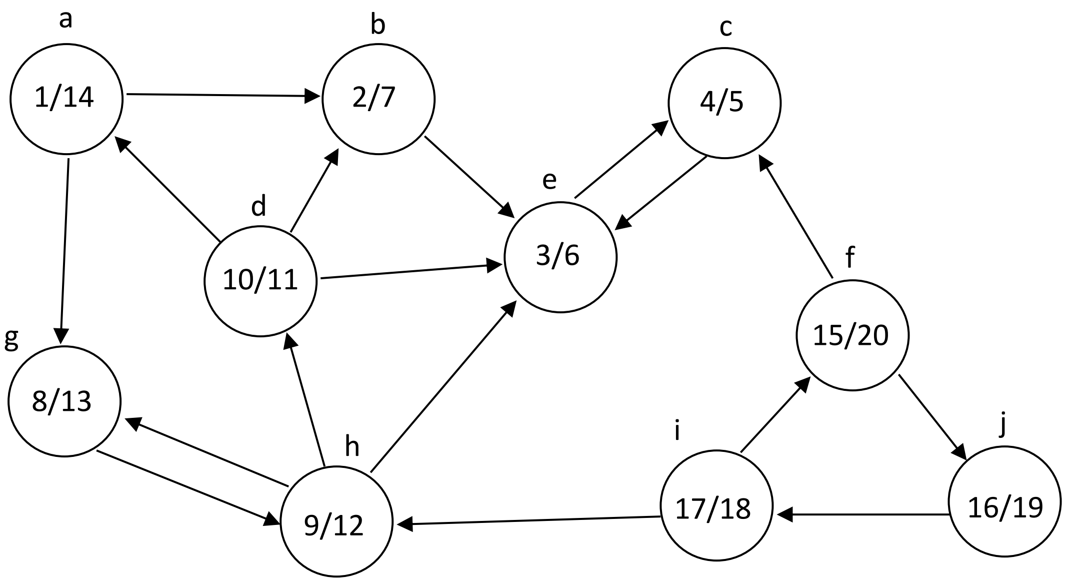
(b) Show the component graph of the previous graph.

(c) Explain why the component graph must be a DAG (directed acyclic graph).

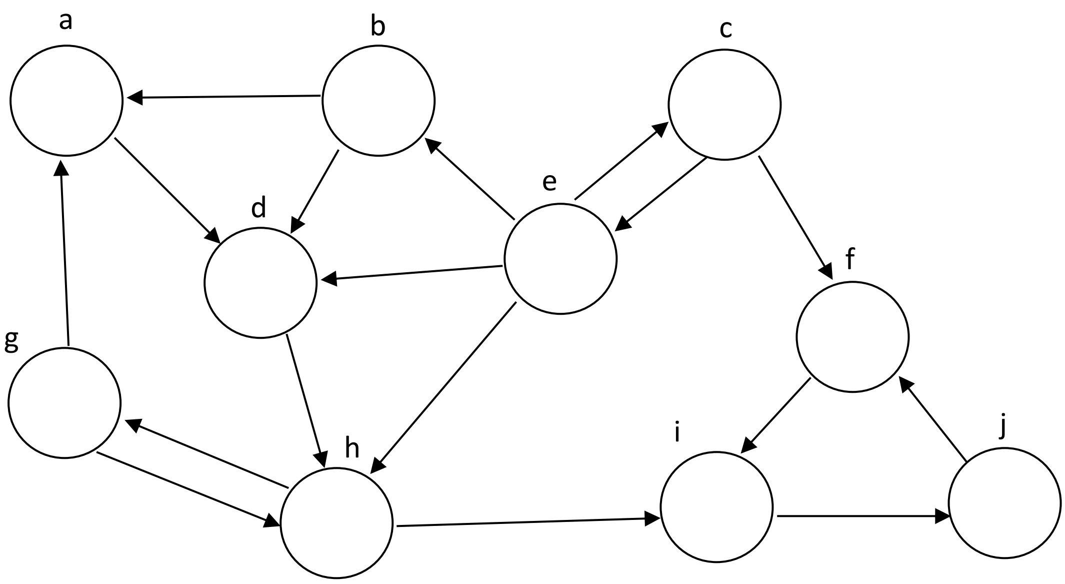
Ans:

(a)

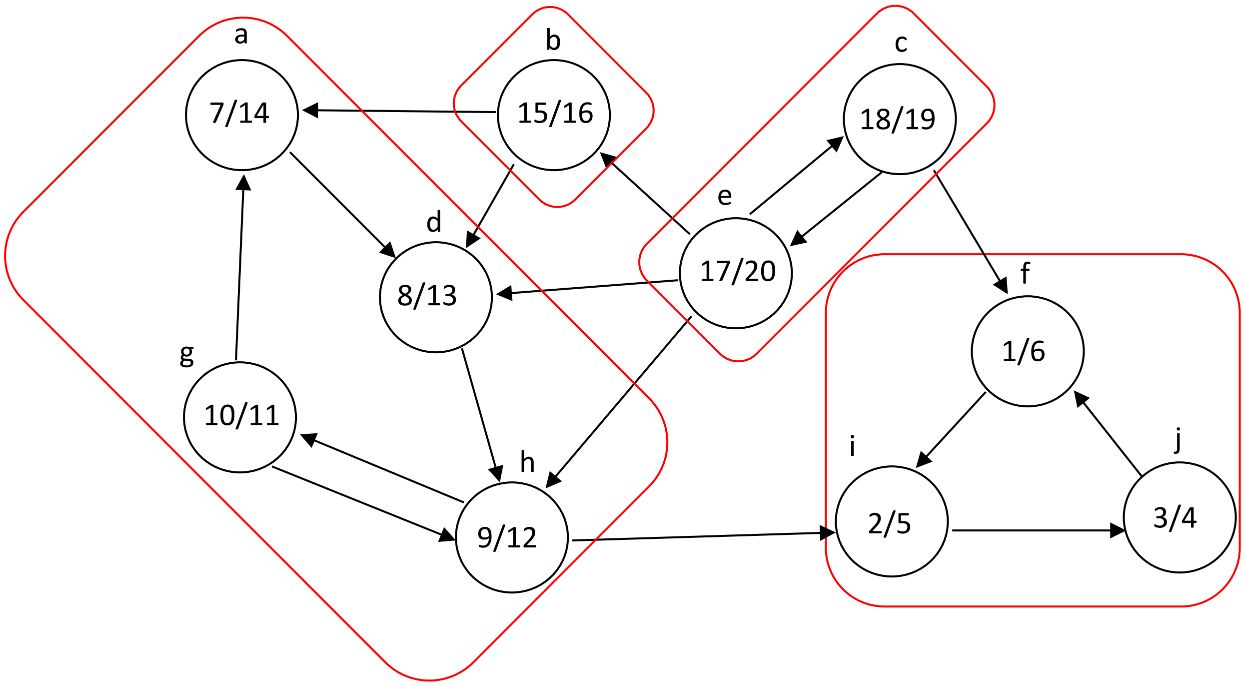
DFS(G)



G^T

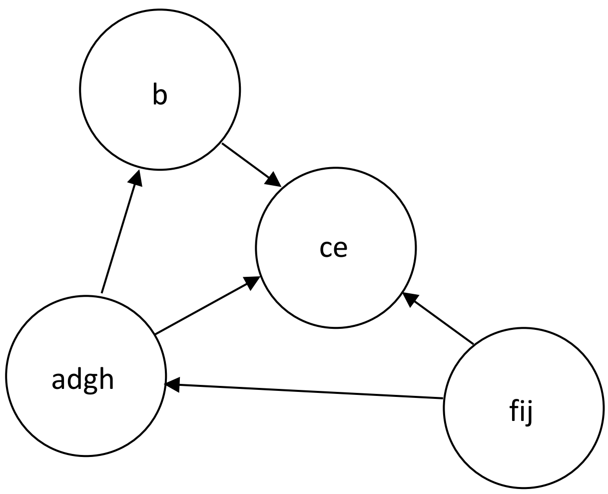


DFS(G^T), but in the main loop of DFS, consider the vertices in order of decreasing u.f (f>j>i>a>g>h>d>b>e>c)



(b)

G^{SCC}



(c)

Let C and C' be distinct strongly connected components in directed graph G=(V,E), let u, v ∈ C, let u', v' ∈ C', and suppose that G contains a path $u \rightsquigarrow u'$.

If G contains a path $v' \rightsquigarrow v$, then it contains paths $u \rightsquigarrow u' \rightsquigarrow v'$ and $v' \rightsquigarrow v \rightsquigarrow u$. Thus, u and v' are reachable from each other, thereby contradicting the assumption that C and C' are distinct strongly connected components.

4. (25%)

You are given a set S of n overlapping arcs of the unit circle. The arcs can be of different lengths. Please build an algorithm to find a largest subset P of these arcs such that no two arcs in P overlap (largest in terms of total number of elements, not in terms of total length of these arcs). Prove that your solution is optimal.

Ans:

<u>Greedy Algorithm</u>

Greedy choice strategy: Pick the one that ends first.

Step1:

For each arc, we separately assume it is in the solution, then repeat clockwise along the arc just as the **"activity-selection problem"**.

Step2:

Take the maximum among all these solutions.

<u>Prove correctness & optimum:</u>

We must choose some arc to be a part of our set. Once this arc is chosen, we cannot take any arc intersecting with this one, so we "delete" the arc from the circle. This leaves a line, and we can prove the correctness and optimum just as the **"activity-selection problem"**