

Note: (1) You **NEED** write down detail calculation process, otherwise scored 0. (2) The reverse side of this test paper can be used.

### **NCKU CSIE Discrete Mathematics (2019 Spring) Homework Quiz 1**

1. **(15 pts)** Find the coefficient of  $w^2x^2y^2z^2$  in the expansion of (a)  $(w^2 + x + y + z + 1)^9$ , (b)  $(2w - x + 3y + z - 2)^8$ , (c)  $(2w - x + 3y + z^{0.5} - 2)^{12}$
2. **(10 pts)** (a) How many nonnegative integer solutions are there to the pair of equations  $x_1 + x_2 + x_3 + \dots + x_7 = 36$ ,  $x_1 + x_2 + x_3 = 11$ . (b) How many solutions in (a) have  $x_1, x_2 > 0$ .
3. **(10 pts)** (a) How many compositions of 20 that have each summand a multiple of 3? (b) Let  $n, m, k$  be positive integers with  $n = mk$ . How many compositions of  $n$  have each summand a multiple of  $k$ ?
4. **(10 pts)** In how many ways can 16 be written as a sum of 2's and 3's if the order of the summands is (a) not relevant? (b) relevant?
5. **(15 pts)** Negate each of the following and simplify the resulting statement. (a)  $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$  (b)  $p \rightarrow (\neg q \wedge r)$  (c)  $\exists x[(p(x) \vee q(x)) \rightarrow r(x)]$
6. **(5 pts)** How many distinct four-digit integers can one make from the digits 3, 3, 3, 7, 7, and 8?
7. **(16 pts)** Define the connective "Nor" by  $(p \downarrow q) \Leftrightarrow \neg(p \vee q)$ , for any statements  $p, q$ . Represent the following using only this connective. (a)  $\neg p$  (b)  $p \vee q$ , (c)  $p \wedge q$  (d)  $p \rightarrow q$ .
8. **(14 pts)** (a) How many arrangements of the letters in MISCELLANEOUS have no pair of consecutive identical letters. (b) How many arrangements of the letters in CHEMIST have H before E, or E before T, or T before M? (Here "before" means anywhere before, not just immediately before.)