Discrete Mathematics

Homework 3

Deadline : 5/27

Exercise 4.1

18. Consider the following four equations:

1) $1 = 1$

2) $2 + 3 + 4 = 1 + 8$

3) $5 + 6 + 7 + 8 + 9 = 8 + 27$

4) $10 + 11 + 12 + 13 + 14 + 15 + 16 = 27 + 64$

Conjecture the general formula suggested by these four equations, and prove your conjecture.

24. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1 \qquad a_2 = 2 \qquad a_n = a_{n-1} + a_{n-2},\ n \geq 3.$$

a) Determine the values of a_3, a_4, a_5, a_6, and a_7.

b) Prove that for all $n \geq 1$, $a_n < (7/4)^n$.

Exercise 4.2

12. For $n \geq 0$ let F_n denote the nth Fibonacci number. Prove that

$$F_0 + F_1 + F_2 + \cdots + F_n = \sum_{i=0}^{n} F_i = F_{n+2} - 1.$$

16. Give a recursive definition for the set of all

a) positive even integers

b) nonnegative even integers

Exercise 4.3

10. If $n \in \mathbf{Z}^+$, and n is odd, prove that $8 \mid (n^2 - 1)$.

12. Determine the quotient q and remainder r for each of the following, where a is the dividend and b is the divisor.

a) $a = 23, \quad b = 7$ **b)** $a = -115, \quad b = 12$

c) $a = 0, \quad b = 42$ **d)** $a = 434, \quad b = 31$

18. For what base do we find that $251 + 445 = 1026$?

Exercise 4.4

12. Let $a, b \in \mathbf{Z}^+$ where $a \geq b$. Prove that $\gcd(a, b) = \gcd(a - b, b)$.

16. Let $a, b \in \mathbf{Z}^+$. Prove that there exist $c, d \in \mathbf{Z}^+$ such that $cd = a$ and $\gcd(c, d) = b$ if and only if $b^2 \mid a$.

Exercise 4.5

8. a) How many positive divisors are there for

$$n = 2^{14}3^9 5^8 7^{10} 11^3 13^5 37^{10}?$$

b) For the divisors in part (a), how many are

i) divisible by $2^3 3^4 5^7 11^2 37^2$?
ii) divisible by 1,166,400,000?
iii) perfect squares?
iv) perfect squares that are divisible by $2^2 3^4 5^2 11^2$?
v) perfect cubes?
vi) perfect cubes that are multiples of $2^{10}3^9 5^2 7^5 11^2 13^2 37^2$?
vii) perfect squares and perfect cubes?