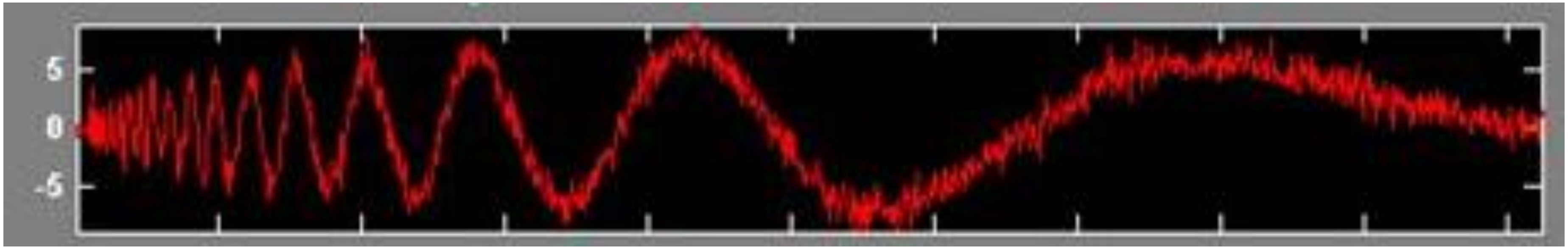
The Wavelet Transform

Motivation



Some signals obviously have spectral characteristics that vary with time

STATIONARITY OF SIGNAL

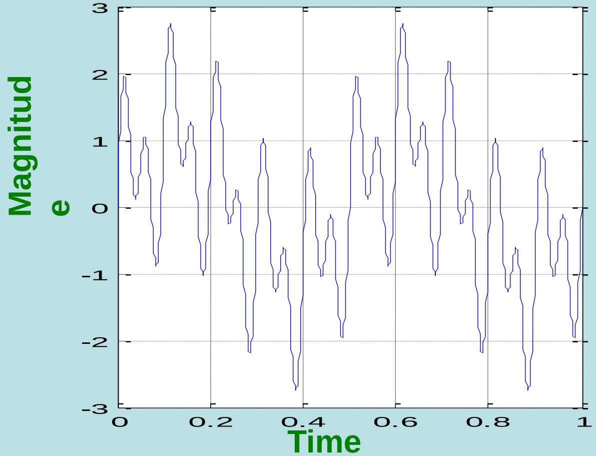
- Stationary Signal
  - Signals with frequency content unchanged in time
  - All frequency components exist at all times

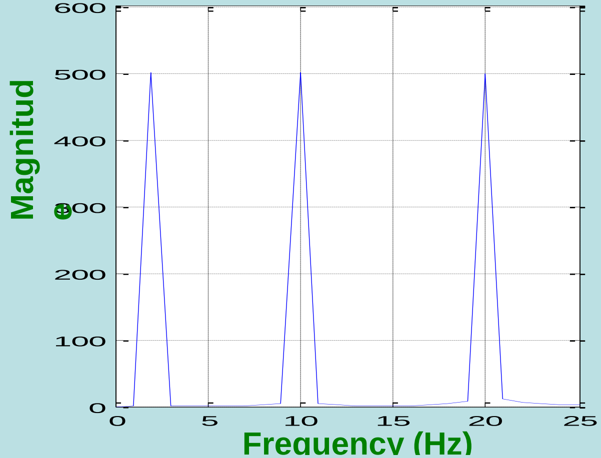
- Non-stationary Signal
  - Frequency changes in time
  - One example: the "Chirp Signal"

STATIONARITY OF SIGNAL

2 Hz + 10 Hz + 20Hz

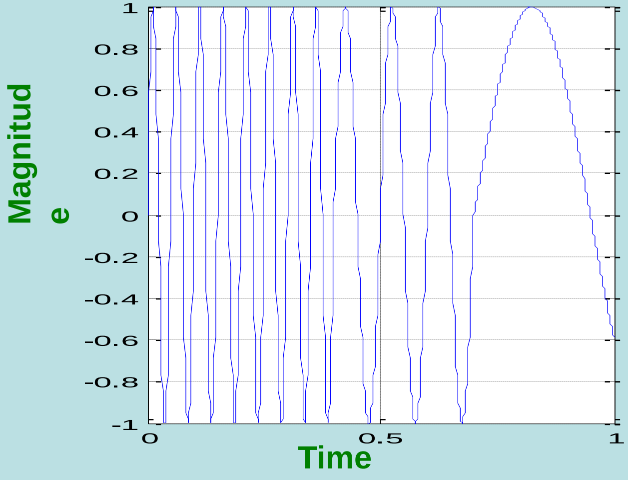
Stationary

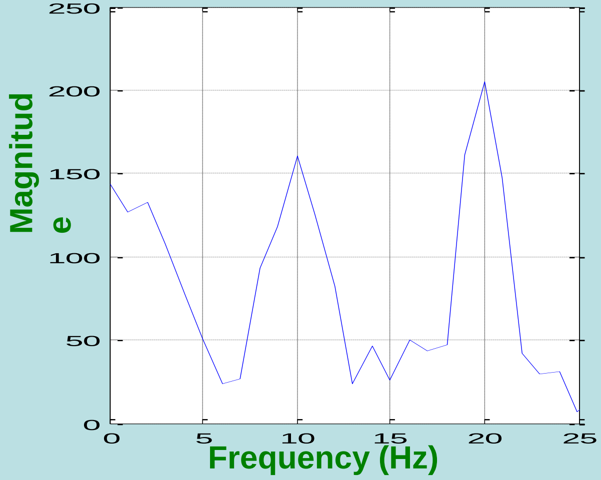




0.0-0.4: 2 Hz +
0.4-0.7: 10 Hz +
0.7-1.0: 20Hz

Non-Stationary



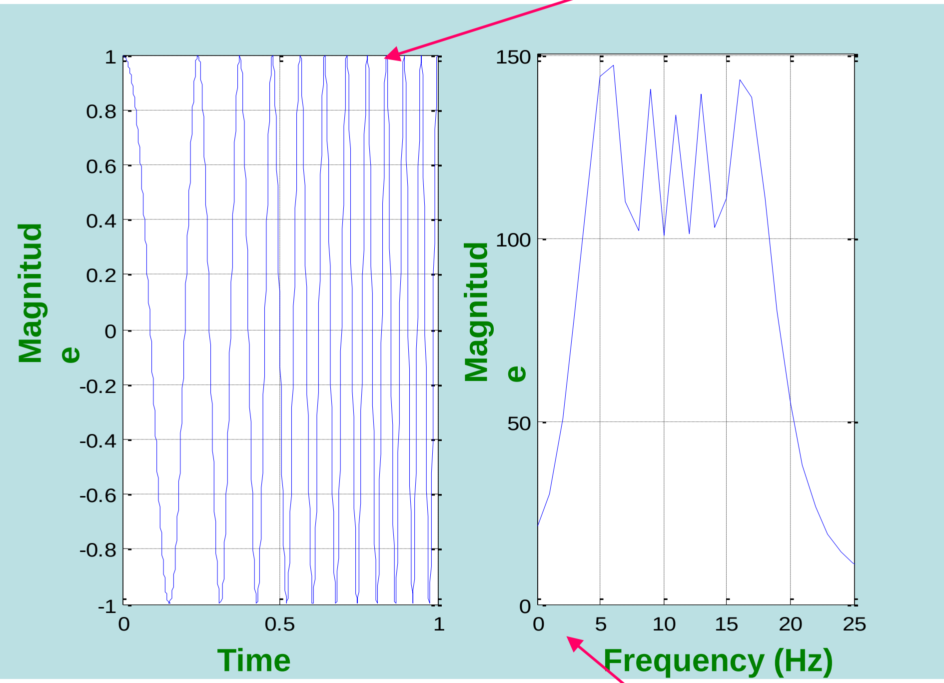


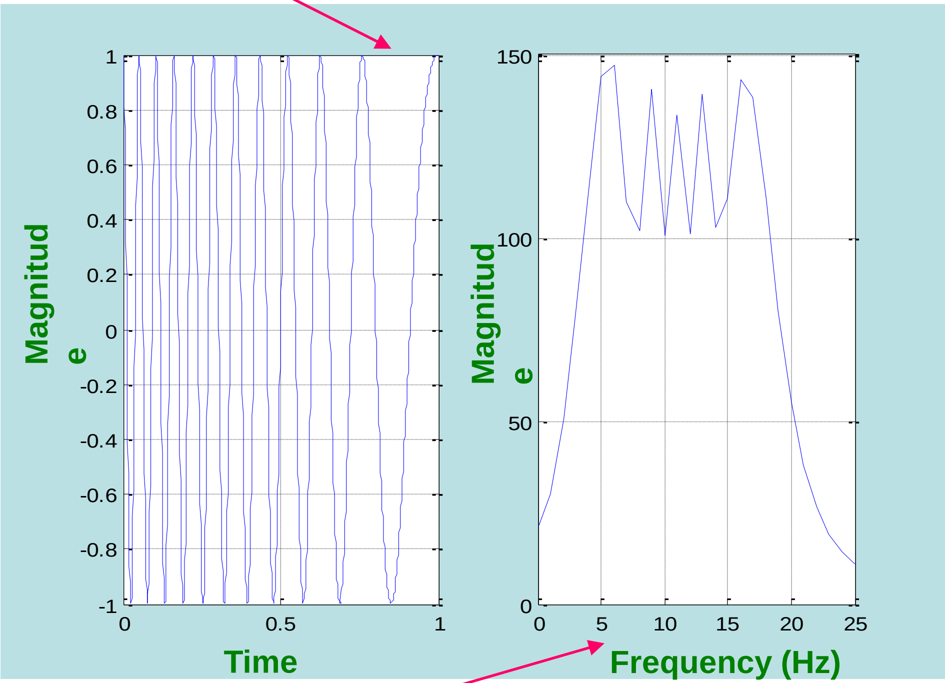
CHIRP SIGNALS

Frequency: 2 Hz to 20 Hz

Different in Time Domain

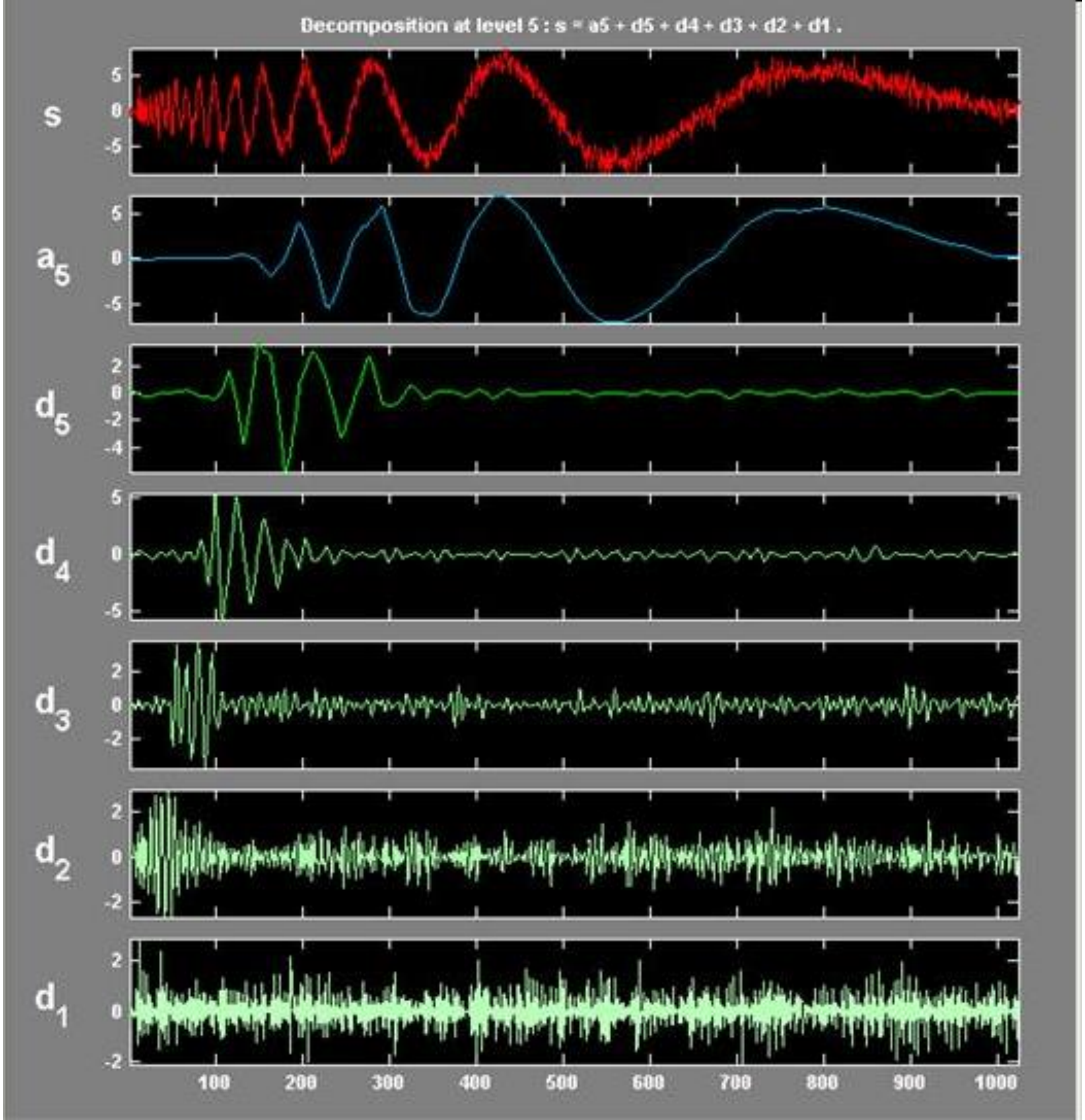
Frequency: 20 Hz to 2 Hz





Same in Frequency Domain

At what time the frequency components occur? FT can not tell!



Decomposition at level 5 : s = a5 + d5 + d4 + d3 + d2 + d1 .
s
a_5
d_5
d_4
d_3
d_2
d_1
100 200 300 400 500 600 700 800 900 1000

Wavelet Transform

$$\gamma(s,\tau) = \int f(t)\psi^*_{s,\tau}(t)dt$$

Inverse Wavelet Transform

$$f(t) = \int\int \gamma(s,\tau)\psi_{s,\tau}(t)d\tau ds$$

All wavelet derived from *mother wavelet*

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

Inverse Wavelet Transform

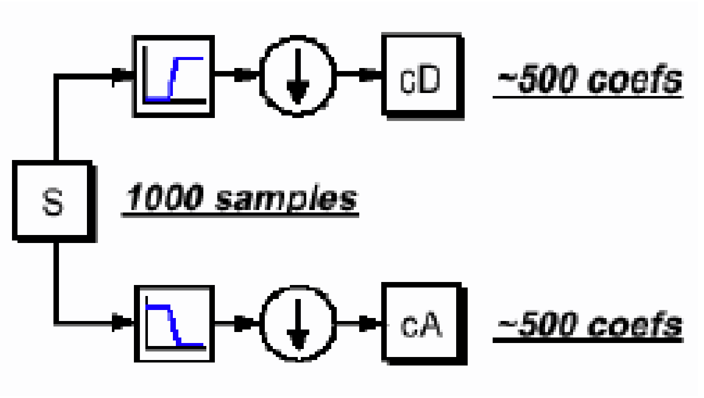
$$f(t) = \int\int \gamma(s,\tau)\psi_{s,\tau}(t)\,d\tau\,ds$$

time-series

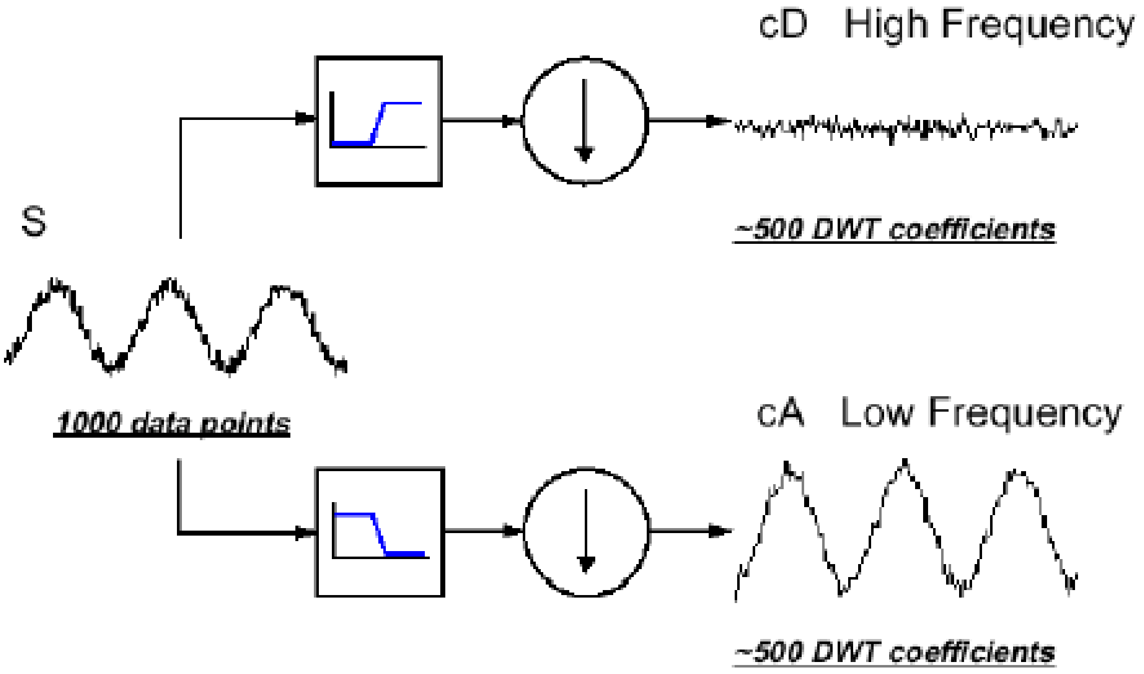
coefficients
of wavelets

wavelet with
scale, s and time, τ

build up a time-series as sum of wavelets of different scales, s, and positions, τ


cD
~500 coefs
S
1000 samples
cA
~500 coefs

An example:


cD High Frequency
~500 DWT coefficients
S
1000 data points
cA Low Frequency
~500 DWT coefficients