4.29

(3.39)

$$f(x,y)=\frac{\binom{3}{x}\binom{2}{y}\binom{3}{4-x-y}}{\binom{8}{4}}\qquad \begin{array}{l}1\le x+y\le 4\\ x=0,1,2,3\\ y=0,1,2\end{array}$$

(a) $E(X^2Y-2XY)=\sum_{x=0}^{3}\sum_{y=0}^{2}(x^2y-2xy)f(x,y)$

$=(1-2)f(1,1)+(2-4)f(1,2)+(4-4)f(2,1)+$

$(8-8)f(2,2)+(9-6)f(3,1)+(0-0)f(0,0)+$

$(0-0)f(0,1)+(0-0)f(0,2)$

$=-\frac{18}{70}+-2\times\frac{9}{70}+3\times\frac{2}{70}=-\frac{3}{7}$

(b) $\mu_x-\mu_y$

x	0	1	2	3
$g(x)$	$\frac{5}{70}$	$\frac{30}{70}$	$\frac{30}{70}$	$\frac{5}{70}$

$g(x)=\sum_{y=0}^{2}f(x,y)$

$g(0)=f(0,0)+f(0,1)+f(0,2)$

$=0+\frac{2}{70}+\frac{3}{70}$

$g(1)=f(1,0)+f(1,1)+f(1,2)=\frac{3}{70}+\frac{18}{70}+\frac{9}{70}$

$g(2)=f(2,0)+f(2,1)+f(2,2)=\frac{9}{70}+\frac{18}{70}+\frac{3}{70}$

$g(3)=f(3,0)+f(3,1)=\frac{3}{70}+\frac{2}{70}$

y	0	1	2
$h(y)$	$\frac{15}{70}$	$\frac{40}{70}$	$\frac{15}{70}$

$h(y)=\sum_{x=0}^{3}f(x,y)$

$h(0)=f(0,0)+f(1,0)+f(2,0)+f(3,0)$

$=0+\frac{3}{70}+\frac{9}{70}+\frac{3}{70}=\frac{15}{70}$

$h(1)=f(0,1)+f(1,1)+f(2,1)+f(3,1)$

$=\frac{2}{70}+\frac{18}{70}+\frac{18}{70}+\frac{2}{70}=\frac{40}{70}$

$h(2)=f(0,2)+f(1,2)+f(2,2)+\cancel{f(3,2)}$

$=\frac{3}{70}+\frac{9}{70}+\frac{3}{70}=\frac{15}{70}$

$\mu_x-\mu_y=E(X)-E(Y)=(0\times\frac{5}{70}+1\times\frac{30}{70}+2\times\frac{30}{70}$

$+3\times\frac{5}{70})-(0\times\frac{15}{70}+1\times\frac{40}{70}+2\times\frac{15}{70})=\frac{35}{70}=\frac{1}{2}$

4.44

Find σ_{XY} on 3.39

$$\sigma_{XY} = E((X-\mu_X)(Y-\mu_Y)) = E(XY) - \mu_X\mu_Y$$

$$E(XY) = 1\times1\times\frac{18}{70} + 1\times2\times\frac{9}{70} + 2\times1\times\frac{18}{70} + 2\times2\times\frac{3}{70} + 3\times1\times\frac{2}{70} = \frac{90}{70}$$

$$\sigma_{XY} = \frac{9}{7} - \left(\frac{3}{2}\times1\right) = \frac{-3}{14}$$

4.60

y \ x	2	4
1	0.15	0.10
3	0.25	0.25
5	0.15	0.10

x	2	4
$g(x)$	0.55	0.45

y	1	3	5
$h(y)$	0.25	0.5	0.25

(a) $E(2X-3Y) = 2E(X) - 3E(Y)$

$E(X) = 2\times0.55 + 4\times0.45 = 2.9$

$E(Y) = 1\times0.25 + 3\times0.5 + 5\times0.25 = 3$

$E(2X-3Y) = 5.8 - 9 = -3.2$

(b) since X and Y are independent

$E(XY) = E(X)E(Y) = 8.7$

4.78 $P(\mu-2\sigma < X < \mu+2\sigma)$

$$f(x) = \begin{cases} 30x^2(1-x)^2 & 0<x<1 \\ 0 & \text{else} \end{cases}$$

$$\mu = \int_0^1 x f(x)\,dx = \int_0^1 30x^3(1-2x+x^2)\,dx$$

$$= \int_0^1 30x^3 - 60x^4 + 30x^5\,dx = \frac{30}{4}x^4 - 12x^5 + 5x^6 \Big|_0^1 = \frac{1}{2}$$

$$E(X^2) = \int_0^1 x^2 f(x)\,dx = \int_0^1 30x^4(1-2x+x^2)\,dx$$

$$= \int_0^1 30x^4 - 60x^5 + 30x^6\,dx = 6x^5 - 10x^6 + \frac{30}{7}x^7 \Big|_0^1$$

$$= \frac{2}{7}$$

$$\sigma^2 = E(X^2) - (E(X))^2 = \frac{2}{7} - \frac{1}{4} = \frac{1}{28} \quad \sigma = 0.189$$

$$\mu - 2\sigma = 0.122 \quad \mu + 2\sigma = 0.878$$

$$P(\mu-2\sigma < X < \mu+2\sigma) = \int_{0.122}^{0.878} 30x^2(1-2x+x^2)\,dx$$

$$= 10x^3 - 15x^4 + 6x^5 \Big|_{0.122}^{0.878} = 0.9700$$

4.98

	y		
x	0	1	2
0	0.12	0.04	0.04
1	0.08	0.19	0.05
2	0.06	0.12	0.30

$$g(k) = \sum_{y=0}^{2} f(k,y)$$

$$h(k) = \sum_{x=0}^{2} f(x,k)$$

(a)

x	0	1	2
g(x)	0.2	0.32	0.48

y	0	1	2
h(y)	0.26	0.35	0.39

The probability distribution of X given Y=2

x	0	1	2
$f_{X\mid Y=2}$	$\frac{4}{39}$	$\frac{5}{39}$	$\frac{30}{39}$

$$f_{X\mid Y=2}(x\mid 2) = \frac{f(x,2)}{h(2)}$$

(b) $E(X) = 0\times 0.2 + 1\times 0.32 + 2\times 0.48$

$= 1.28$

4.98

(b) $Var(X)$

$E(X^2) = 0^2 \times 0.2 + 1^2 \times 0.32 + 2^2 \times 0.48 = 2.24$

$Var(X) = E(X^2) - (E(X))^2 = 2.24 - (1.28)^2$

$= 0.602$

(c) $E(X|Y=2) = 1 \times \frac{5}{39} + 2 \times \frac{30}{39} = \frac{65}{39}$

$E(X^2|Y=2) = 1 \times \frac{5}{39} + 2^2 \times \frac{30}{39} = \frac{125}{39}$

$Var(X) = \frac{125}{39} - (\frac{65}{39})^2 = \frac{50}{117}$