Discrete Mathematics

HW2

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Exercise 3.1

4. Which of the following statements are true?

a) $\emptyset \in \emptyset$ **b)** $\emptyset \subset \emptyset$ **c)** $\emptyset \subseteq \emptyset$

d) $\emptyset \in \{\emptyset\}$ **e)** $\emptyset \subset \{\emptyset\}$ **f)** $\emptyset \subseteq \{\emptyset\}$

(a) False

(b) False

(c) True

(d) True

(e) True

(f) True

12. Let $A = \{1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18\}$.

a) How many subsets of A contain six elements?

b) How many six-element subsets of A contain four even integers and two odd integers?

c) How many subsets of A contain only odd integers?

(a) $\binom{12}{6}=924$

(b) $\binom{6}{4}\binom{6}{2}=225$

(c) 63

26. For positive integers n, r show that

$$\begin{aligned}\binom{n+r+1}{r} &= \binom{n+r}{r} + \binom{n+r-1}{r-1} + \cdots \\ &\quad + \binom{n+2}{2} + \binom{n+1}{1} + \binom{n}{0} \\ &= \binom{n+r}{n} + \binom{n+r-1}{n} + \cdots \\ &\quad + \binom{n+2}{n} + \binom{n+1}{n} + \binom{n}{n}.\end{aligned}$$

$=\binom{n+r}{r}+\binom{n+r}{r-1}$

$=\binom{n+r}{r}+\binom{n+r-1}{r-1}+\binom{n+r-1}{r-2}$

$=\binom{n+r}{r}+\binom{n+r-1}{r-1}+\binom{n+r-1}{r-2}+\cdots\cdots\cdots+\binom{n+1}{1}+\binom{n}{0}$

又$\binom{n+r}{r}=\binom{n+r}{n}$ 得證

Exercise 3.2

2. If $A = [0, 3]$, $B = [2, 7)$, with $\mathcal{U} = \mathbf{R}$, determine each of the following:

a) $A \cap B$ **b)** $A \cup B$

c) $\overline{A}$ **d)** $A \triangle B$

e) $A - B$ **f)** $B - A$

(a) [2,3]
(b) [0,7)
(c) (-∞,0)∪(3,+∞)
(d) [0,1] ∪ [4, 6]
(e) [0,1]
(f) (3,7)

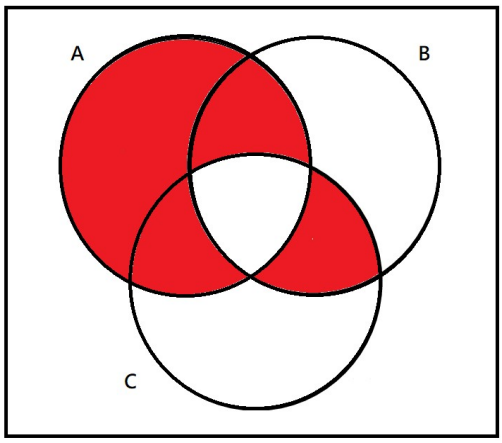
8. Using Venn diagrams, investigate the truth or falsity of each of the following, for sets $A, B, C \subseteq \mathcal{U}$.

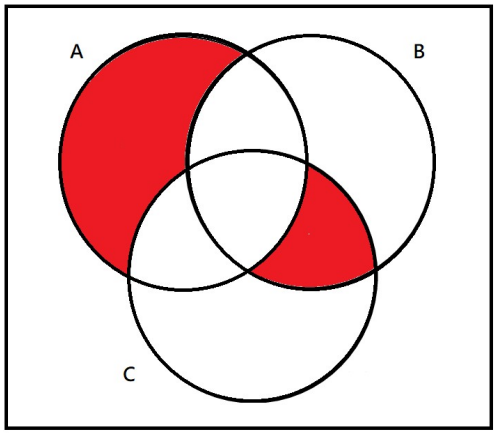
a) $A \triangle (B \cap C) = (A \triangle B) \cap (A \triangle C)$

b) $A - (B \cup C) = (A - B) \cap (A - C)$

c) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$

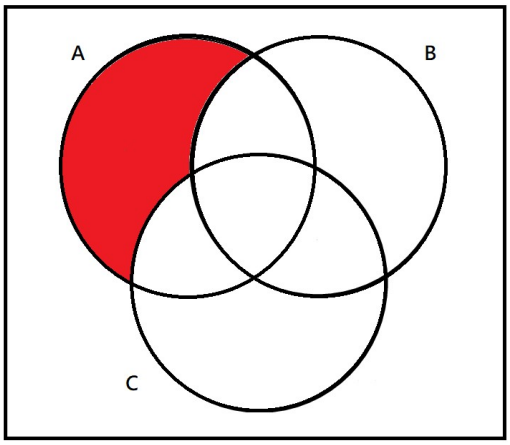
(a) False

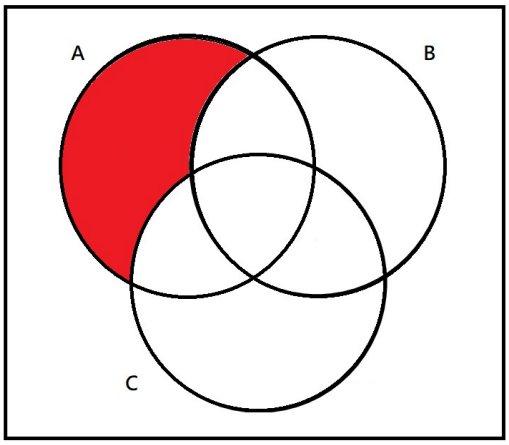




$A \triangle (B \cap C)$ $(A \triangle B) \cap (A \triangle C)$

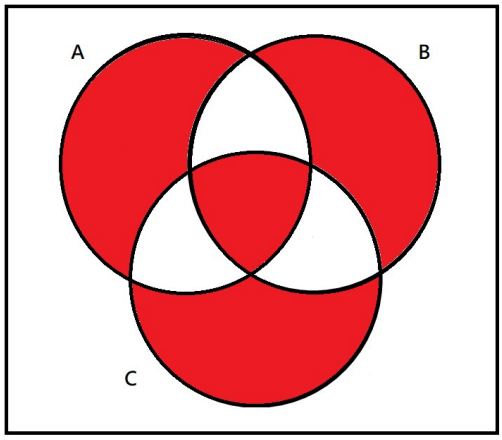
(b) True



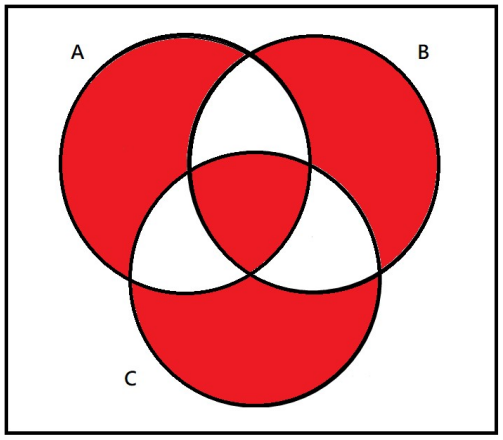


$A - (B \cup C)$ $(A - B) \cap (A - C)$

(c) True



$A \triangle (B \triangle C)$



$(A \triangle B) \triangle C$

18. For each $n \in \mathbf{Z}^+$ let $A_n = \{1, 2, 3, \ldots, n-1, n\}$. (Here $\mathcal{U} = \mathbf{Z}^+$ and the index set $I = \mathbf{Z}^+$.) Determine

$$\bigcup_{n=1}^{7} A_n, \quad \bigcap_{n=1}^{11} A_n, \quad \bigcup_{n=1}^{m} A_n, \quad \text{and} \quad \bigcap_{n=1}^{m} A_n,$$

where m is a fixed positive integer.

$\bigcup_{n=1}^{7} A_n = \{1, 2, 3, 4, 5, 6, 7\} = A_7$

$\bigcap_{n=1}^{11} A_n = \{1\} = A_1$

$\bigcup_{n=1}^{m} A_n = \{1, 2, 3, \ldots, m-1, m\} = A_m$

$\bigcap_{n=1}^{m} A_n = \{1\} = A_1$

Exercise 3.3

10. How many arrangements of the letters in CHEMIST have H before E, or E before T, or T before M? (Here "before" means anywhere before, not just immediately before.)

Total $-$!{H before E, or E before T, or T before M}

$=$ Total $-$ {H after E, and E after T, and T after M}

$= 7! - 5 \times 6 \times 7$

$= 5040 - 210$

$= 4830$

Exercise 3.4

5. The Tuesday night dance club is made up of six married couples and two of these twelve members must be chosen to find a dance hall for an upcoming fund raiser. (a) If the two members are selected at random, what is the probability they are both women? (b) If Joan and Douglas are one of the couples in the club, what is the probability at least one of them is among the two who are chosen?

(a) $\frac{\binom{6}{2}}{\binom{12}{2}} = \frac{5}{22}$

(b) $\frac{\binom{2}{1}\binom{10}{1}+\binom{2}{2}}{\binom{12}{2}} = \frac{7}{22}$

8. If three integers are selected, at random and without replacement, from {1, 2, 3, . . . , 99, 100}, what is the probability their sum is even?

3 偶數+2 奇數一偶數

$$\frac{\binom{50}{3} + \binom{50}{2}\binom{50}{1}}{\binom{100}{3}} = \frac{1}{2}$$

14. The freshman class of a private engineering college has 300 students. It is known that 180 can program in Java, 120 in Visual BASIC†, 30 in C++, 12 in Java and C++, 18 in Visual BASIC and C++, 12 in Java and Visual BASIC, and 6 in all three languages.

a) A student is selected at random. What is the probability that she can program in exactly two languages?

b) Two students are selected at random. What is the probability that they can (i) both program in Java? (ii) both program only in Java?

(a) $\frac{6+6+1}{300} = \frac{2}{25}$

(b) (i) $\frac{\binom{180}{2}}{\binom{300}{2}} = 0.36$

(ii) $\frac{\binom{162}{2}}{\binom{300}{2}} = 0.29$