

# COMPILER CONSTRUCTION

# Code Analysis & Optimizations II

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# Chapter X

# Code Analysis and Optimizations



# Outline

- Using Control Flow Graphs
- Data Flow Analysis Example with Live Variable Analysis

- Note:
  - The contents of Code Analysis and Optimizations do not follow the chapter order in the textbook
  - You can find related materials online to know more about the contents, or refer to the books:
  - Some of the contents are available in the Ch. 14 of the **textbook**
    - Ch. 14 ~ 14.2.1, 14.3.2 (Live Variables), 14.4, and 14.5

  (The information of the following books is listed in **Reference** slide at the end of the file)
  - You may refer to **Advanced Compiler Design & Implementation**
    - Sec. 7.1, 8.1, 8.3, 8.4, and 14.1.3 (Live Variables Analysis)
  - You may refer to the reference book (**Dragon Book**)
    - Sec. 8.4 and 8.5
    - Sec. 9.1, 9.2, 9.3 (optional), and 9.4



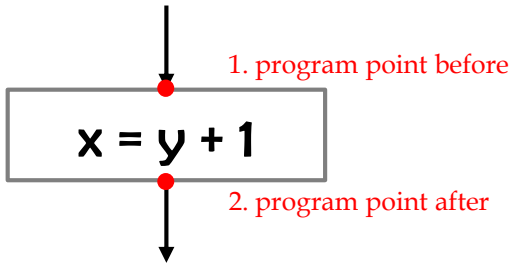
# Using CFGs

- Use CFG representation to **statically** extract information about the program
  - Reason at compile-time
  - About the run-time values of variables and expressions in all program executions
- Key ideas
  - Define **program points** in the CFG
  - Reason statically about how the ***information flows*** **between these program points**
- Extracted information examples
  - **Live variables**
  - **Copy propagation analysis**



# Program Points

- **Two program points** for each instruction:
  1. a program point before each instruction
  2. a program point after each instruction

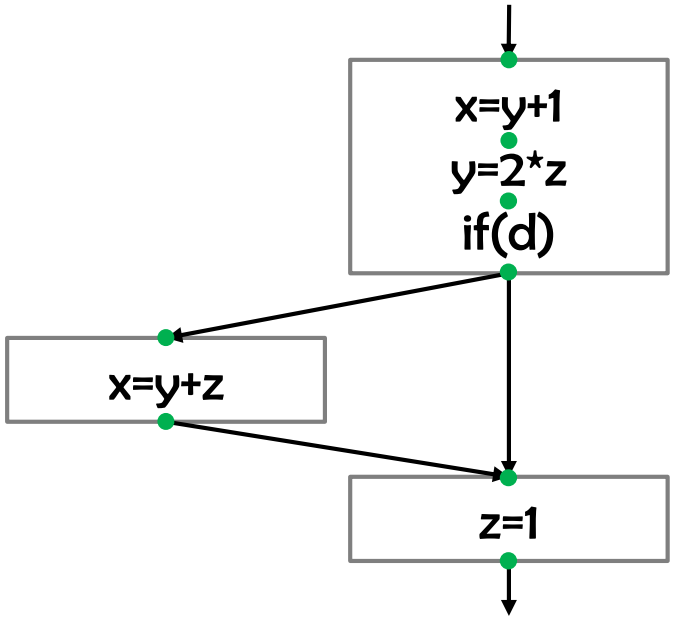



- In a basic block (BB):
  - The program point after an instruction is the

  →program point before the successor instruction

# Program Point: Example

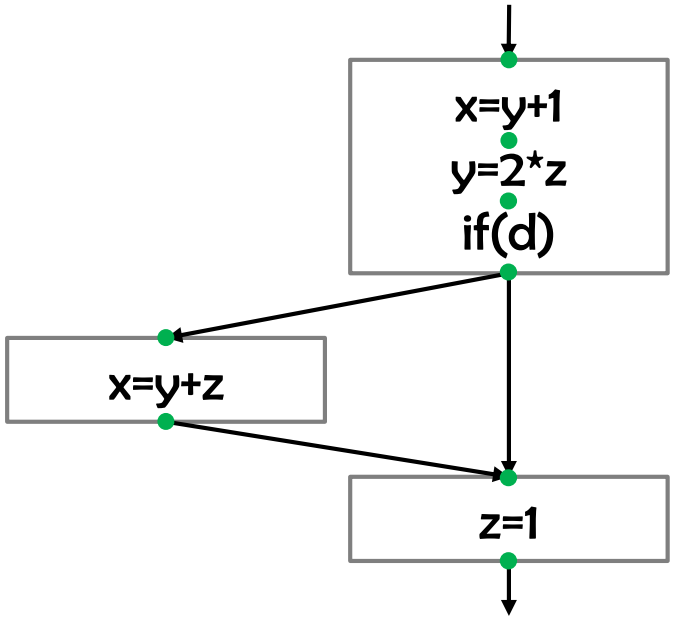
- Multiple successor BBs means that
  - point at the end of a block has multiple successor program points
- Depending on the execution, control flows
  - from a program point to one of its successors, and
  - from a program point to its multiple predecessors (of the BB)
- **How does *information* propagate between program points?**





# Flow of Extracted Information

- Question 1:
  - How does **information flow between the program points** before and after an **instruction**?
- Question 2:
  - How does information flow between successor and predecessor **basic blocks**?




- In plaintexts …
  - Q1: what is the effect of instructions?
  - Q2: what is the effect of control flow?



# Using CFGs

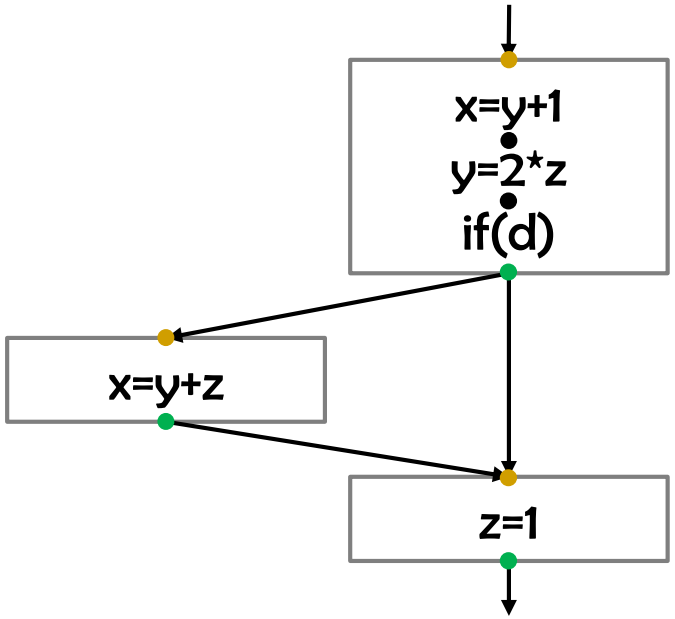
- To extract information
  - reason about how it propagates between program points
- Two typical program analyses are presented
  - Live variable analysis, which computes live variables are live at each program point
  - Copy propagation analysis, which computes the variable copies available at each program point
- We show how to use CFGs to compute the information at each program point for the analysis



# Live Variable Analysis

- Computes live variables at each program point
  - I.e., variables holding values which may be used later (in some execution of the program)

- For an **instruction I**, consider:
  - **in[I]**: live variables at program point before **I**
  - **out[I]**: live variables at program point after **I**
- For a **basic block B**, consider:
  - **in[B]**: live variables at beginning of **B**
  - **out[B]**: live variables at end of **B**

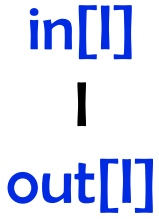



- If **I** is the first instruction in **B**, then **in[B]= in[I]**
- If **I'** is the last instruction in **B**, then **out[B]= out[I']**

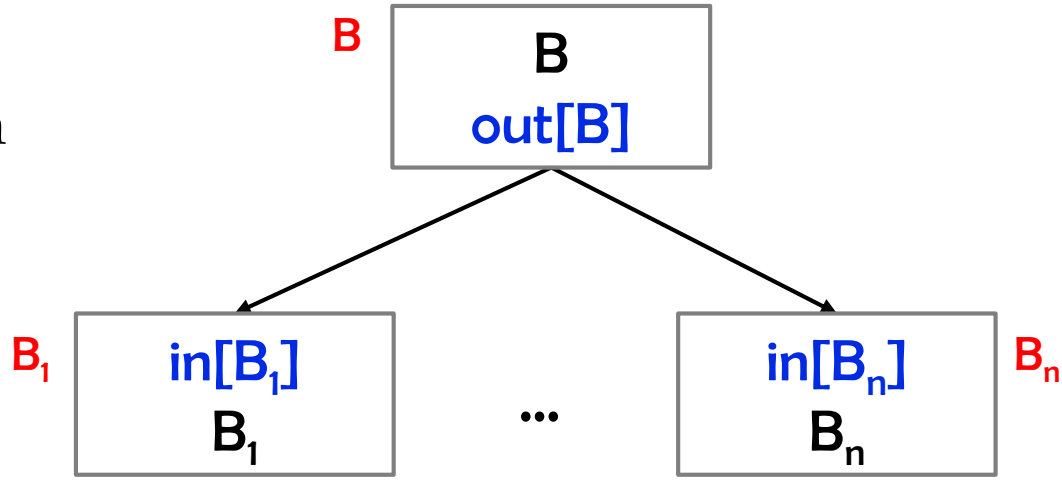


# How to Compute Liveness?

- Rephrase Question 1:
  - For each instruction **I**, what is the relation between **in[I]** and **out[I]**




- Rephrase Question 2:
  - For each basic block **B** with successor blocks $B_1$, $B_2$, …, $B_n$, what is the relation between **out[B]** and **in[$B_1$], in[$B_2$], … in[$B_n$]**





# Part 1: Analyze Instructions

- Question:
  - What is the relation between sets of live variables before and after an instruction?

in[I]
I
out[I]

- Examples of the appearances of in/out sets:

in[I] = {x, y, z}
x = y + z
out[I] = {y, z}

in[I] = {y, z, t}
x = y + z
out[I] = {y, z, t}

- … is there a general rule? Yes…



# Analyze Instructions

- Knowing variables live *after* **I**, can compute variables live *before* **I**
  - All variables live after **I** are also live before **I**, unless defines (writes) them
  - All variables that **I** uses (reads) are also live before instruction **I**

**in[I]**
**I**
**out[I]**

- Formal definition:
  - **in[I] = ( out[I] – def[I] ) ∪ use[I]**

  where:
  - **def[I]**: variables defined (written) by instruction **I**
  - **use[I]**: variables used (read) by instruction **I**
  - We learn how to compute **def[I]** and **use[I]** first



# Example: Computing Use/Def

- Compute **use[I]** and **def[I]** for each instruction **I**:

| Instruction I | use[I] | def[I] |
|---|---|---|
| x = y OP z | {y, z} | {x} |
| x = OP y | {y} | {x} |
| x = y | {y} | {x} |
| if (x) | {x} | {} |
| return x | {x} | {} |
| $x = f(y_1, ..., y_n)$ | $\{y_1, \ldots, y_n\}$ | {x} |

We ignore load and store instructions for now



# Example: Relationships between in/out Sets of Consecutive Instructions

- Property of the **in/out** sets for three instructions, I1, I2, and I3, in **B**

  Live1 = in[B] = in[$I_1$]

  Live2 = out[$I_1$] = in[$I_2$]

  Live3 = out[$I_2$] = in[$I_3$]

  Live4 = out[$I_3$] = out[B]

**Basic Block B**

| | |
|---|---|
| | Live1 |
| $I_1$ | x=y+1 |
| | Live2 |
| $I_2$ | y=2*z |
| | Live3 |
| $I_3$ | if(d) |
| | Live4 |

- Calculate live sets:

  Live1 = (Live2 - {x}) ∪ {y}

  Live2 = (Live3 - {y}) ∪ {z}

  Live3 = (Live4 - {}) ∪ {d}

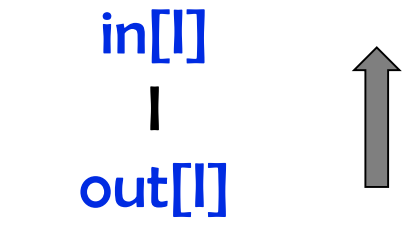
in[I] = ( out[I] – def[I] ) ∪ use[I]



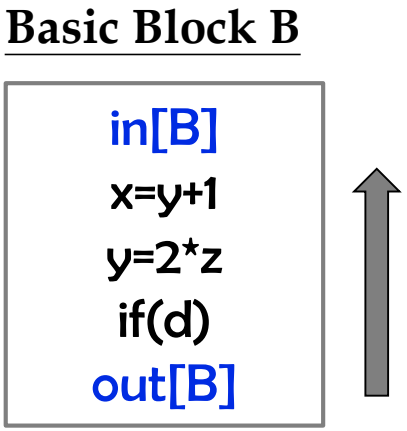
# Backward Flow for Liveness Analysis

- Relation:
  - in[I] = ( out[I] – def[I] ) ∪ use[I]

- **The information flows backward!!!**



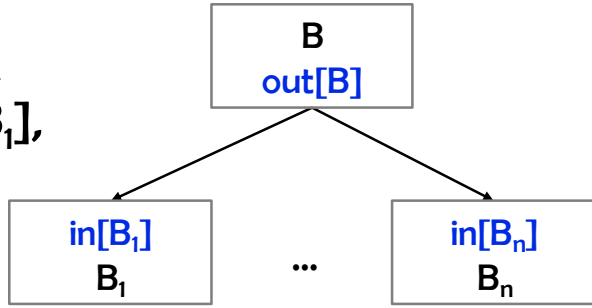

- **Information is calculated at different levels:**
  - **Instructions:** can compute **in[I]** if we know **out[I]**
  - **Basic Blocks:** information about live variable flows from **out[B]** to **in[B]**



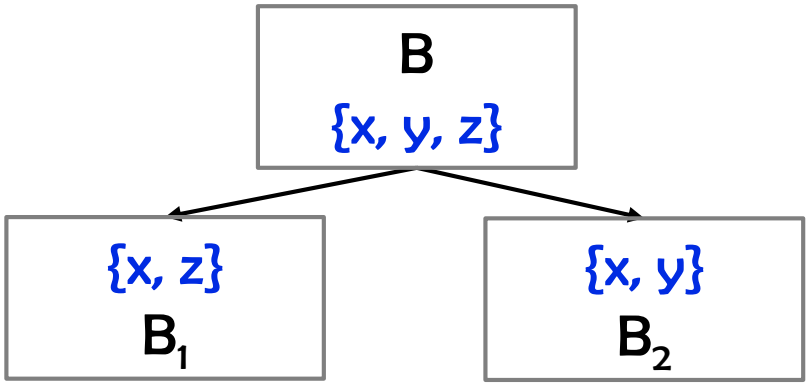


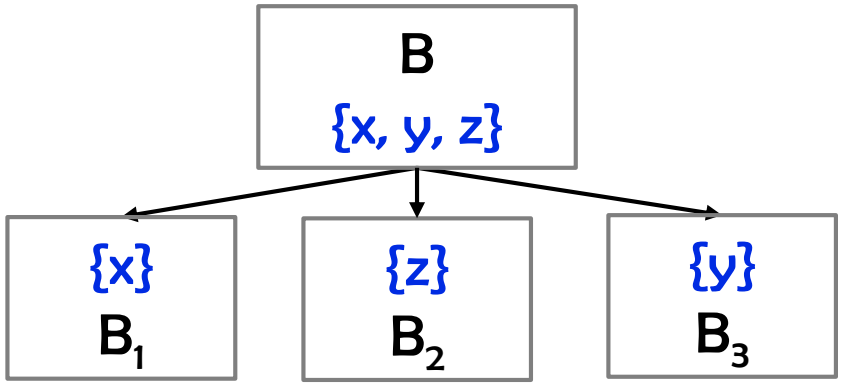
# Part 2: Analyze Control Flow

- Question:
  - For each basic block **B** with successor blocks $B_1$, $B_2$, …, $B_n$, what is the relation between **out[B]** and **in[$B_1$], in[$B_2$], … in[$B_n$]**




- Examples of the appearances of in/out sets:



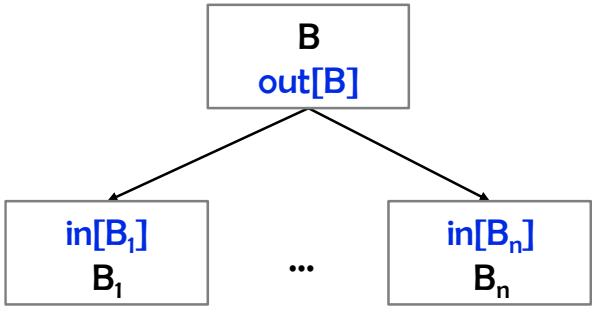




  - Information flows **backward** in the above examples
- What is the general rule?



# Analyze Control Flow (Backward)

- Rule:
  - A variables list live at **end** of basic block B if it is live at the beginning of one **successor** block
- **Characterizes all possible program executions!**
- Formal definition:
  - $out[B] = \bigcup_{B' \in successor(B)} in[B']$




- Again, information flows backward: from successors **B'** of **B** to basic block **B**



# Constraint System (Backward)

- Put **Part I and Part II** together
  - Start with CFG and derive a system of constraints between live variable sets

$$in[I] = ( out[I] - def[I] ) \cup use[I]$$ For each instruction I (Part I)

$$out[B] = \bigcup_{B' \in successor(B)} in[B']$$ For each basic block B (Part II)

- Solve constraints:
  1. Start with **empty sets** of live variables
  2. **Iteratively** apply constraints for instructions & BBs
  3. Stop when reaching a **fixed point**
  - Refer to the algorithm given in the next page



# Constraint Solving Algorithm

for all instructions in **in[I] = out[I] = ∅**

repeat

for each instruction **I**

$$in[I] = ( out[I] - def[I] ) \cup use[I]$$

for each basic block **B**

$$out[B] = \bigcup_{B' \in successor(B)} in[B']$$

until no change in live sets

Apply the constrains on the instructions for each BB

Apply the constrains on the BBs (Refer to the property in this page)

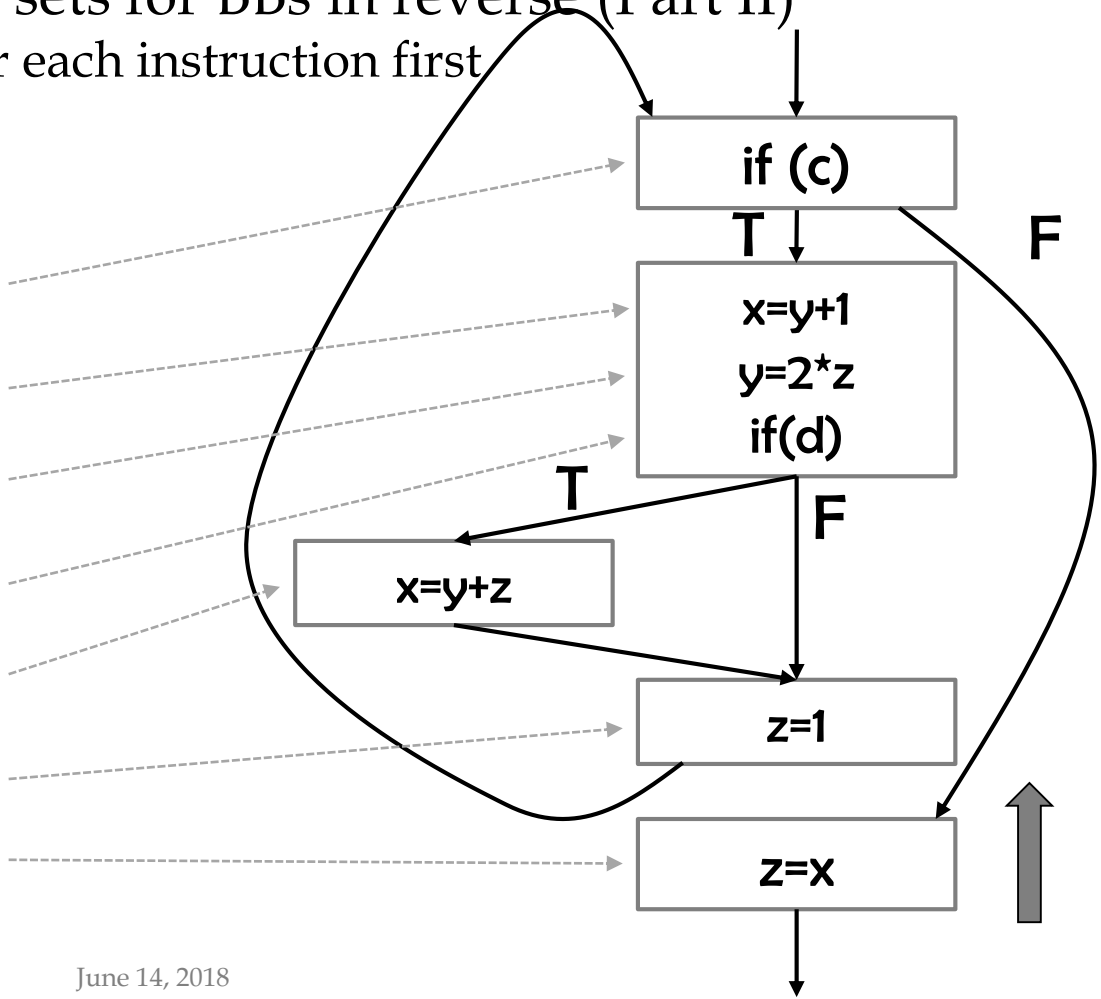
- It is important for the backward analysis to compute **in** with **out** (for a instruction and a BB)
- The order, for which instruction/BB applies the constrains, does not matter. Why?



# An Example

- Give the use and def sets for each instruction (Part I)
- You can compute **in/out** sets for BBs in reverse (Part II)
  - by deriving the **in/out** for each instruction first

| use[I] | def[I] |
|---|---|
| {c} | {} |
| {y} | {x} |
| {z} | {y} |
| {d} | {} |
| {y, z} | {x} |
| {} | {x} |
| {x} | {z} |





# Copy Propagation

- Determine copies available at each program point

- For an **instruction I**, consider:
  - **in[I]**: copies available at program point before **I**
  - **out[I]**: copies available at program point after **I**
- For a **basic block B**, consider:
  - **in[B]**: copies available at beginning of **B**
  - **out[B]**: copies available at end of **B**

- If **I** is the first instruction in **B**, then **in[B]= in[I]**
- If **I'** is the last instruction in **B**, then **out[B]= out[I']**



# Constraint System for Copy Propagation

- **Same *methodology*** with live variable analysis

1. Express flow of information (i.e., available copies)
   - For points before and after each instruction (**in[I], out[I]**)
   - For points at entry and exit of BBs (**in[B], out[B]**)

2. Build constraint system
   - using the relations between available copies

3. Solve constraints
   - to determine available copies at each point in the program



# Analyze Instructions for Available Copies

- Compute **out[I]** with **in[I]**
  - Remove from **in[I]** all copies of <u=v>, if variable u or v is written by **I**, and Keep all other copies from **in[I]**
  - If **I** is of the form x=y, add it to **out[I]**

**in[I]**
**I**
**out[I]**

- Formal definition:
  - **out[I] = ( in[I] – kill[I] ) ∪ gen[I]**

  where:
  - **kill[I]**: copies ***killed*** by instruction **I**
  - **gen[I]**: copies ***generated*** by instruction **I**



# Computing Kill/Gen

- Compute **gen[I]** and **kill[I]** for each instruction **I**

| Instruction I | gen[I] | kill[I] |
|---|---|---|
| x = y OP z | {} | {remove tuple contains x} |
| x = OP y | {} | {remove tuple contains x} |
| x = y | {x=y} | {remove tuple contains x} |
| if (x) | {} | {} |
| return x | {} | {} |
| x = f($y_1$, ..., $y_n$) | {} | {remove tuple contains x} |

We ignore load and store instructions for now

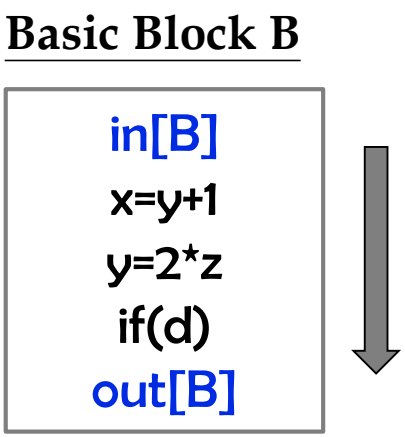


# Forward Flow for Copy Propagation

- Relation:
  - out[I] = ( in[I] – kill[I] ) ∪ gen[I]


- **The information flows forward!!!**

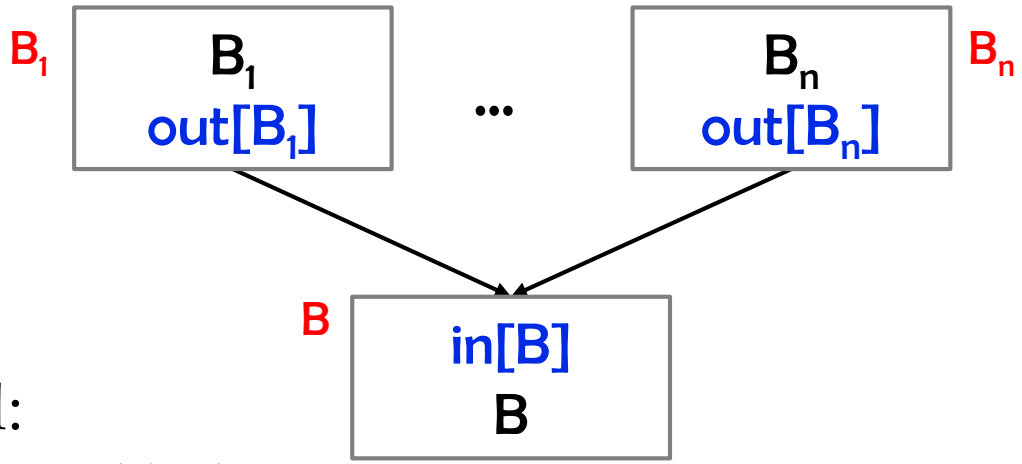
- **Instructions:** can compute **out[I]** if we know **in[I]**
- **Basic Blocks:** information about available copy flows from **in[I]** to **out[I]**





# Analyze Control Flow (Forward)

- Rule:
  - A copy assignment is available at **end** of basic block **B** if it is either in gen[B] of block **B** or it is available on entry of block **B** and not in kill[B]
  - A copy assignment is available on **entry** of block **B** if it is available on exit from all predecessors of block **B**
- **Characterizes all possible program executions!**
- Formal definition:
  - $in[B] = \bigcap_{B' \in predecessor(B)} out[B']$
- Information flows forward:
  - from predecessors **B'** of **B** to basic block **B**





# Constraint System (Forward)

- Build constraints
  - Start with CFG and derive a system of constraints between sets of available copies:=

$$out[I] = ( in[I] - kill[I] ) \cup gen[I]$$ For each instruction I

$$in[B] = \bigcap_{B' \in predecessor(B)} out[B']$$ For each basic block B

- Solve constraints:
  1. Start with empty sets of available copies
  2. Iteratively apply constraints
  3. Stop when we reach a fixed point



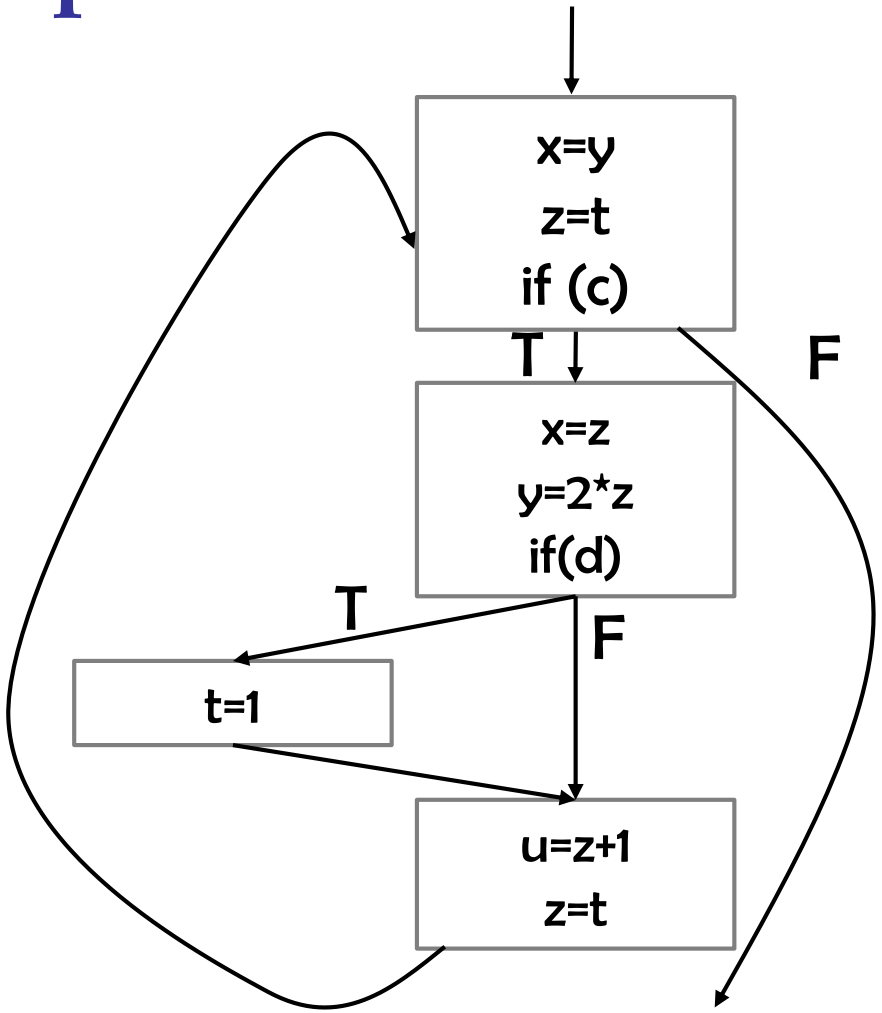
# The while-loop Example

- What are the available copies at the end of the program?

  x=y?

  z=t?

  x=z?





# Summary

- Extracting information about live variables and available copies is similar
  - Define the required information
    - Define information before/after instructions
    - Define information at entry/exit of basic blocks
  - Build constraints for instructions/control flow
  - Solve constraints to get needed information
- … is there a general framework?
  - Yes!!! Dataflow analysis framework
- We give the simplified definition below
  - You should find online materials to get familiar with the ideas introduced in this file



# Data Flow Analysis (Generalization)

- Live variable analysis and detection of available copies are similar
  - Define some information that they need to compute
  - Build constraints for the information
  - Solve constraints iteratively:
    - The information always *increases* during iteration
    - Eventually, it reaches a fixed point
- A general framework is necessary
  - The framework is applicable to many other analyses
  - I.e., live variable/copy propagation are the instances of the framework



# Dataflow Analysis Framework

- A common framework for many compiler analyses
  - Compute some information at each program point
  - The computed information characterizes all possible executions of the program

- **Basic methodology:**
  - Describe information about the program using an algebraic structure called ***lattice,***
    - which **formally defines** the representation of the information (e.g., set) , the related operations (e.g., union and join), the **transfer function** for **I** and **B**, etc.
  - Build constraints which show how instructions and control flow modify the information in the ***lattice***
  - Iteratively solve constraints



# Transfer Functions (Instruction)

- Dataflow analysis defines a **transfer function**,
  - **F** for each **instruction** in the program,
  - which describes how the instruction modifies the information in the lattice
- Consider
  - **in[I]** is information before **I**
  - **out[I]** is information after **I**
- The transfer function of a instruction **I**
  - for forward analysis: **out[I] = F(in[I])**
  - for backward analysis: **in[I] = F(out[I])**

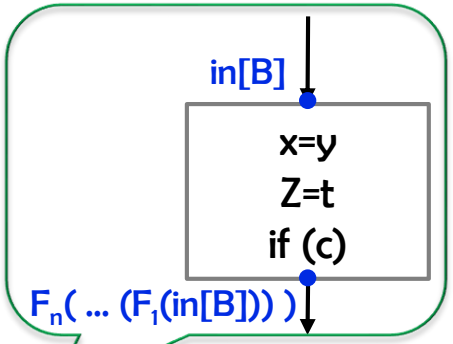


# Transfer Functions (Basic Block)

- Extend the concept of transfer function to BBs using function composition

Summarize the effect of instructions within a BB

- Consider
  - a basic block **B** consists of instructions $I_1$, ..., $I_n$ with transfer functions $F_1$, ..., $F_n$, respectively
  - **in[B]** is information before **B**
  - **out[B]** is information after **B**
- The transfer function of a basic block **B**
  - for forward analysis: $out[B] = F_n( \ldots (F_1(in[B])) )$
  - for backward analysis: $in[B] = F_1( \ldots (F_n(out[B])) )$

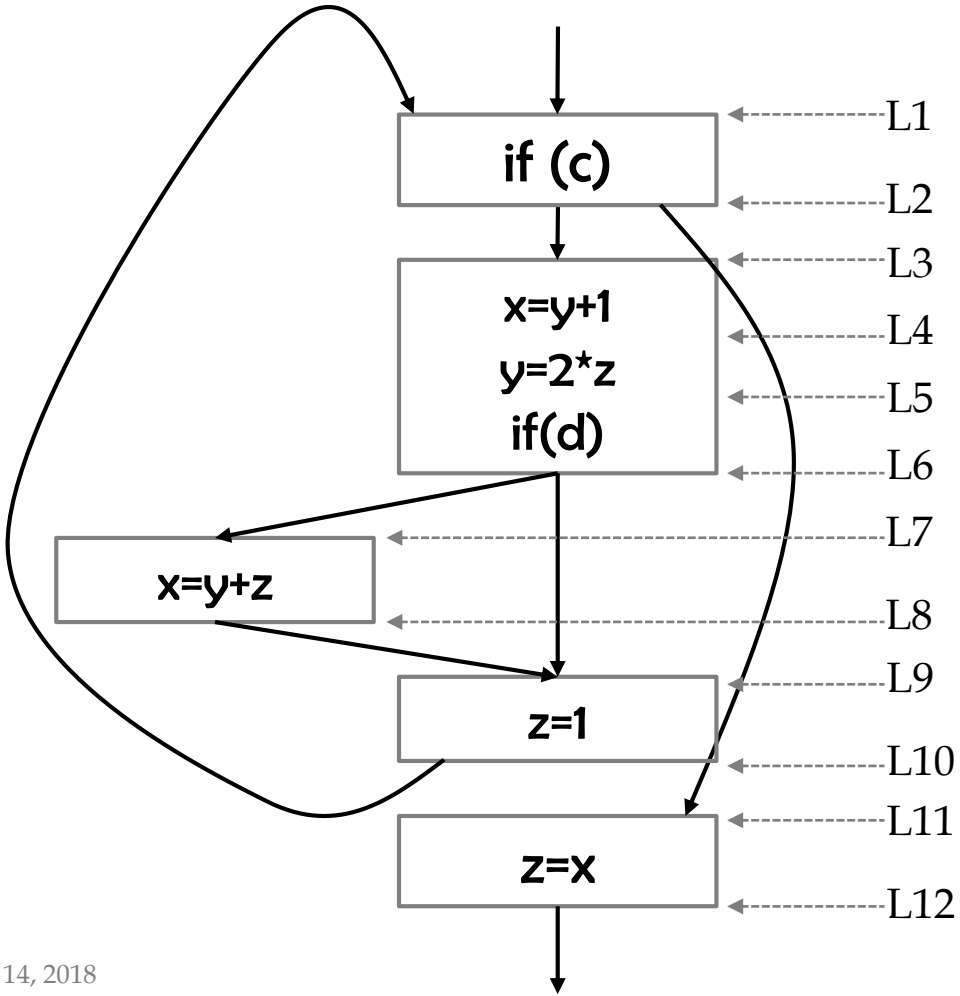


# Live Variable Analysis (Revisit)

- What are the **live variables** at each program point?

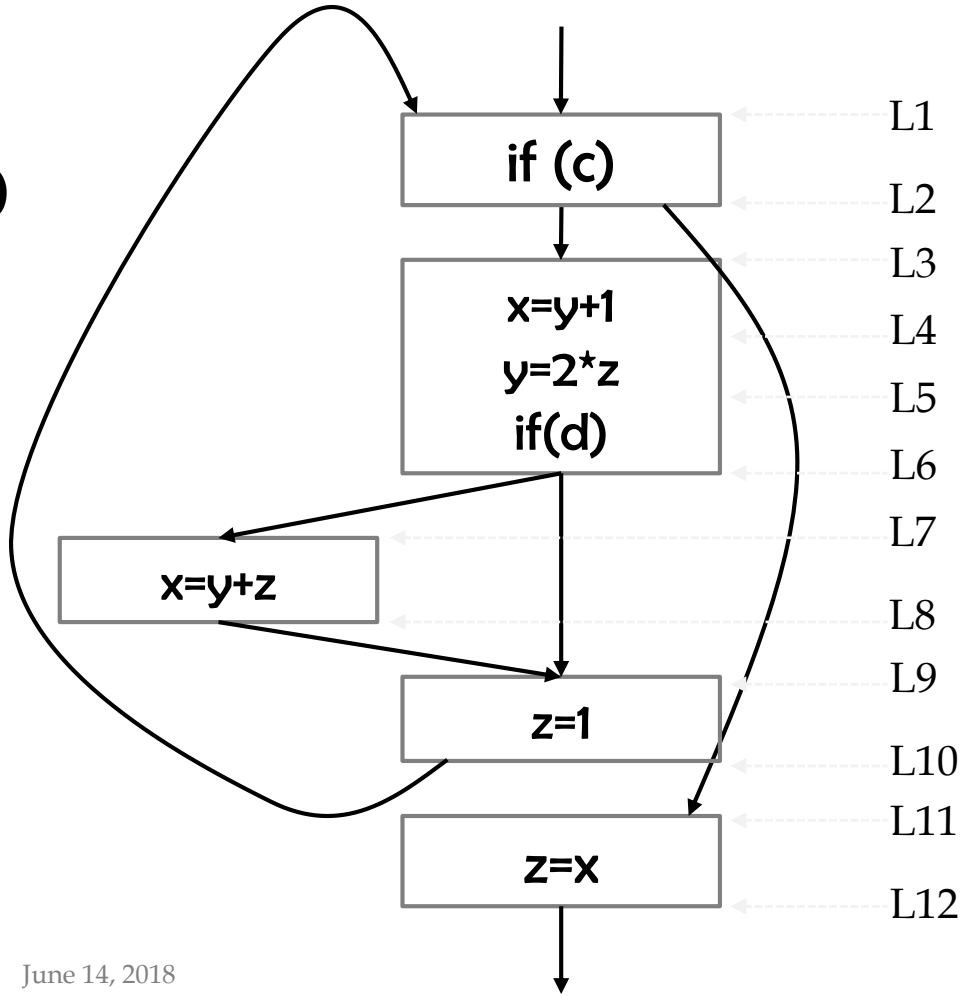
- Method:
  1. Define sets of live variables
  2. Build constrains
  3. Solve constraints

- We use **the analysis** as an example to concrete illustrate
  - how to perform Data Flow Analysis
  - by iteratively visiting the established graph and
  - applying the rules
  - until the fixed-point is reached!



# Derive Constraints

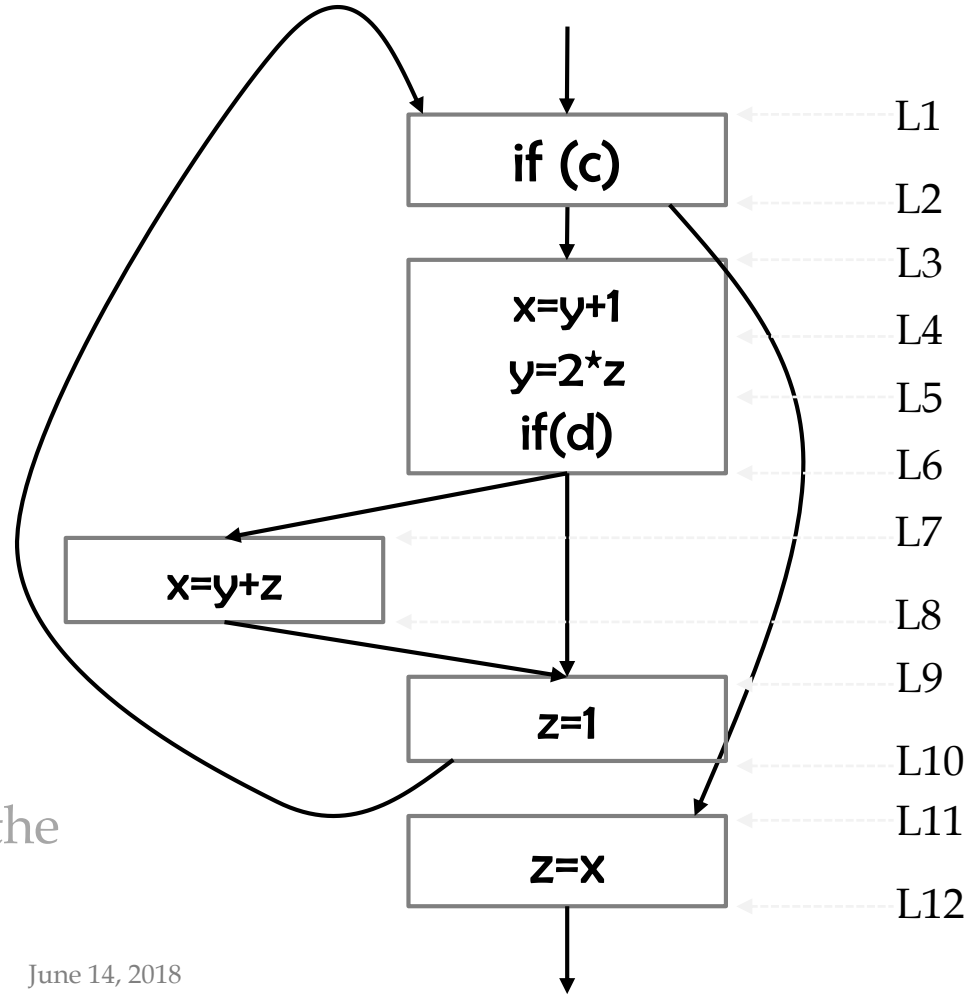
- Constraints for each instruction:
  - in[I] = ( out[I] – def[I] ) ∪ use[I]
- Constraints for control flow:
  - out[B] = $\bigcup_{B' \in \text{successor}(B)}$ in[B']





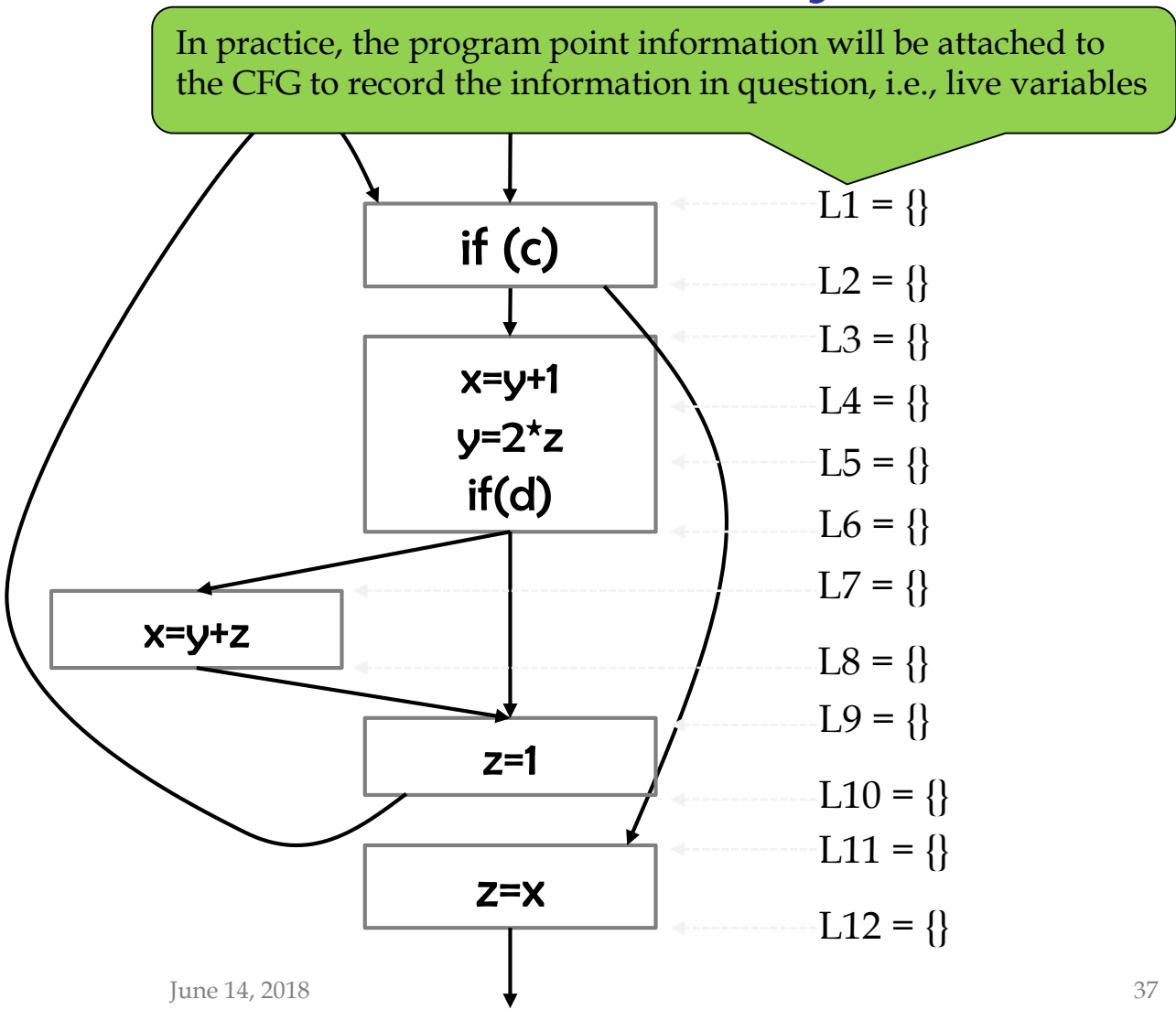
# Derive Constraints (Cont'd)

- L1 = L2 ∪ {c}
- L2 = L3 ∪ L11
- L3 = (L4 – {x} ) ∪ {y}
- L4 = (L5 – {y} ) ∪ {z}
- L5 = L6 ∪ {d}
- L6 = L7 ∪ L9
- L7 = (L8 – {x} ) ∪ {y, z}
- L8 = L9
- L9 = L10 – {z}
- L10= L1
- L11= (L12 – {z} ) ∪ {x}
- L12= … (? we do not know the successor inst. of L12)



# Initialization for Data Flow Analysis

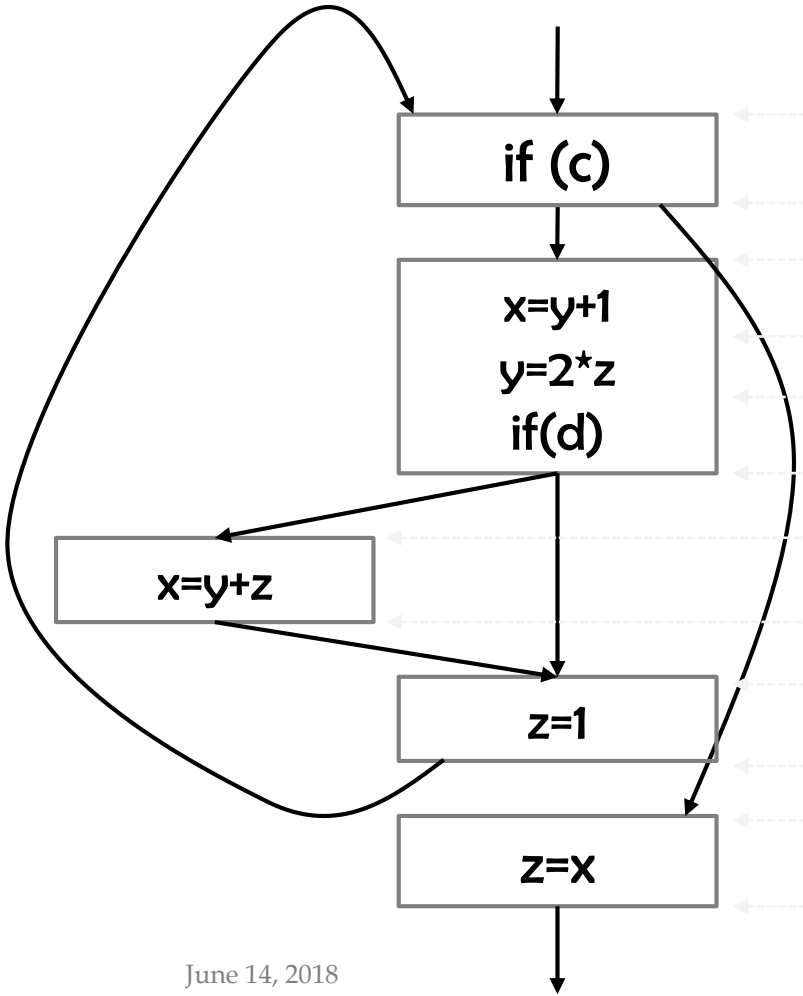
- L1 = L2 ∪ {c}
- L2 = L3 ∪ L11
- L3 = (L4 – {x} ) ∪ {y}
- L4 = (L5 – {y} ) ∪ {z}
- L5 = L6 ∪ {d}
- L6 = L7 ∪ L9
- L7 = (L8 – {x} ) ∪ {y, z}
- L8 = L9
- L9 = L10 – {z}
- L10= L1
- L11= (L12 – {z} ) ∪ {x}



# Iteration 1 of Data Flow Analysis

- L1 = L2 ∪ {c}
- L2 = L3 ∪ L11
- L3 = (L4 – {x} ) ∪ {y}
- L4 = (L5 – {y} ) ∪ {z}
- L5 = L6 ∪ {d}
- L6 = L7 ∪ L9
- L7 = (L8 – {x} ) ∪ {y, z}
- L8 = L9
- L9 = L10 – {z}
- L10= L1
- L11= (L12 – {z} ) ∪ {x}

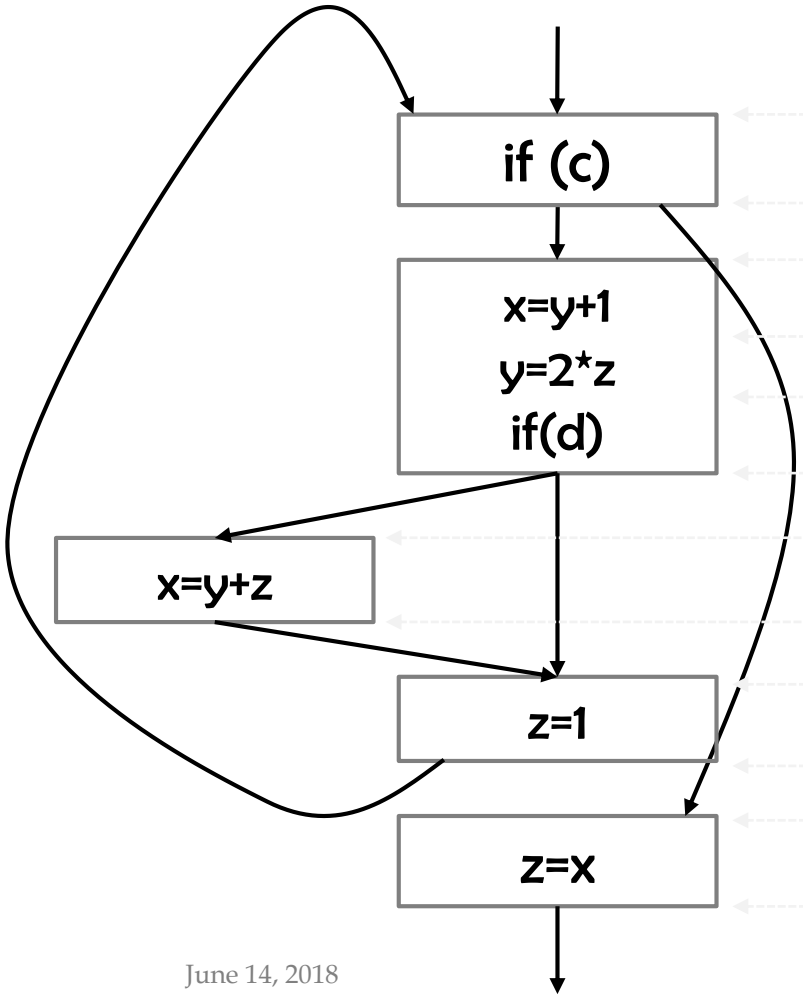
Apply the above constrains to the graph iteratively backward




L1 = {x, y, z, c, d}
L2 = {x, y, z, d}
L3 = {y, z, d}
L4 = {z, d}
L5 = {y, z, d}
L6 = {y, z}
L7 = {y, z}
L8 = {}
L9 = {}
L10 = {}
L11 = {x}
L12 = {}

# Iteration 2 of Data Flow Analysis

- L1 = L2 ∪ {c}
- L2 = L3 ∪ L11
- L3 = (L4 – {x} ) ∪ {y}
- L4 = (L5 – {y} ) ∪ {z}
- L5 = L6 ∪ {d}
- L6 = L7 ∪ L9
- L7 = (L8 – {x} ) ∪ {y, z}
- L8 = L9
- L9 = L10 – {z}
- L10= L1
- L11= (L12 – {z} ) ∪ {x}

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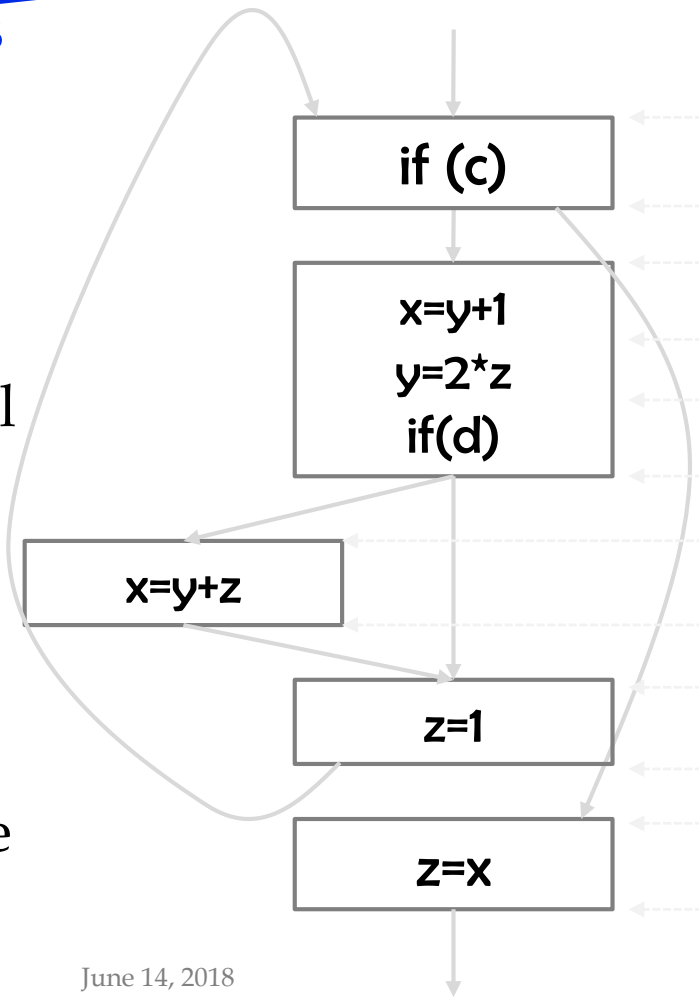
L2 = {x, y, z, c, d}
L3 = {y, z, c, d}
L4 = {x, z, c, d}
L5 = {x, y, z, c, d}
L6 = {x, y, z, c, d}
L7 = {y, z, c, d}
L8 = {x, y, c, d}
L9 = {x, y, c, d}
L10 = {x, y, z, c, d}
L11 = {x}
L12 = {}

We visit through the program points again!

# Reached Fixed-point at Iteration 3!!!

- Fixed-point means
  - the **live variable sets** of program points are the same with those we got in the previous iteration
  - I.e., **the sets** of the iteration 3 is identical to those listed in iteration 2 (previous slide)
- This implies that
  - compiler should keep the live variable sets of the previous iteration

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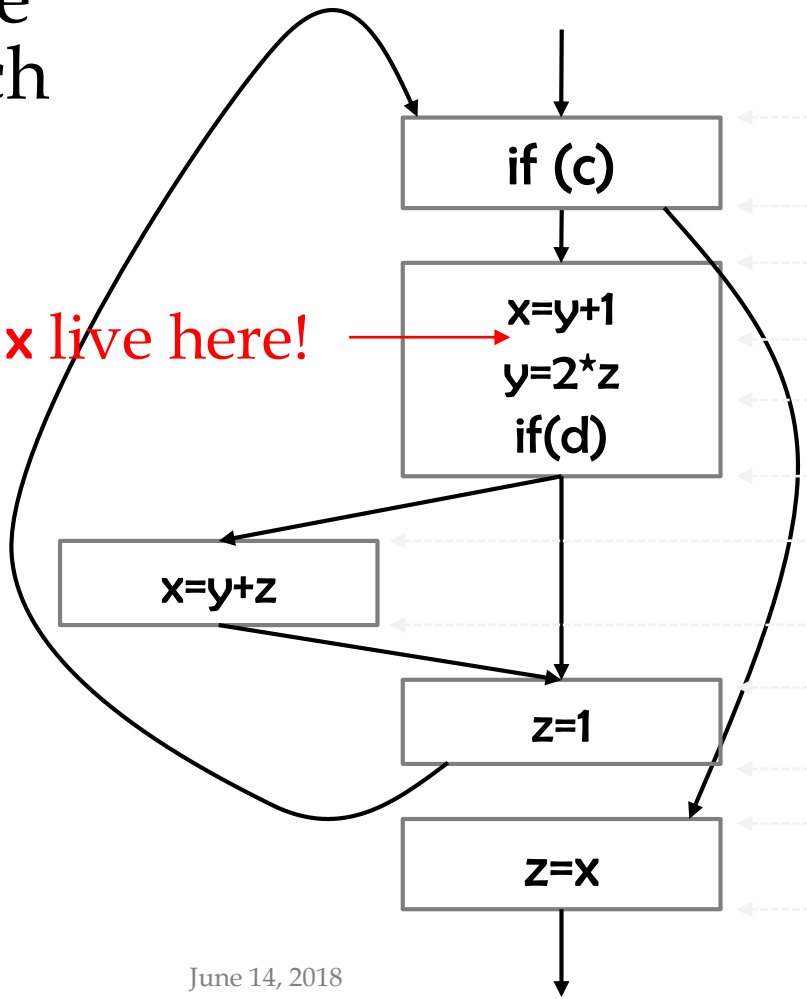
L3 = {y, z, c, d}
L4 = {x, z, c, d}
L5 = {x, y, z, c, d}
L6 = {x, y, z, c, d}
L7 = {y, z, c, d}
L8 = {x, y, c, d}
L9 = {x, y, c, d}
L10 = {x, y, z, c, d}
L11 = {x}
L12 = {}

# Final Result

- Indicate the live variables at each program point




L1 = {x, y, z, c, d}

L2 = {x, y, z, c, d}

L3 = {y, z, c, d}

L4 = {x, z, c, d}

L5 = {x, y, z, c, d}

L6 = {x, y, z, c, d}

L7 = {y, z, c, d}

L8 = {x, y, c, d}

L9 = {x, y, c, d}

L10 = {x, y, z, c, d}

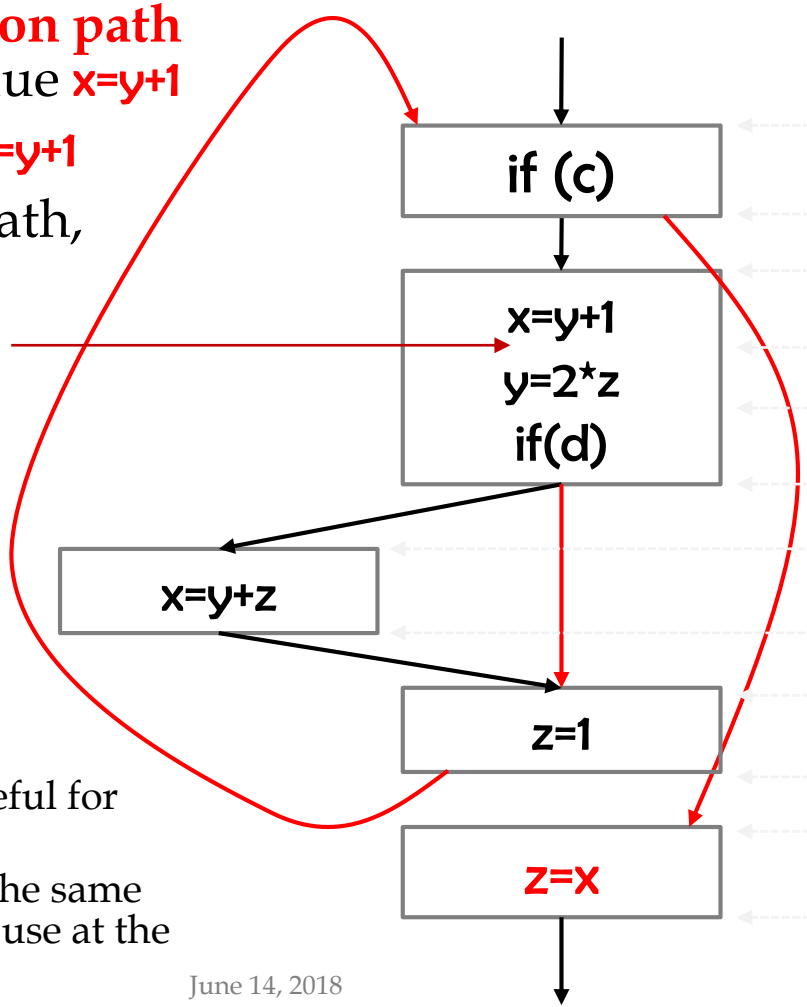
L11 = {x}

L12 = {}

# Characterize All Executions

- The analysis detects that
  - there is **an execution path** which uses the value **x=y+1**
  - in such a path, **z=x=y+1**
  - There is another path, where **x=y+z**

**x live here! (x=y+1)**

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L6 = {x, y, z, c, d}
L7 = {y, z, c, d}
L8 = {x, y, c, d}
L9 = {x, y, c, d}
L10 = {x, y, z, c, d}
L11 = {x}
L12 = {}

- NOTE:
  - Live variable analysis is useful for Register Allocation
  - I.e., two **variables** can use the same register if they are never in use at the same time



# Summary

- What we have learnt here
  - Define some information that they need to compute
  - Build constraints for the information
  - Solve constraints iteratively on the CFG
    - The information always ***increases*** during iteration
    - Eventually, it reaches a fixed-point
- By extending the procedure, we will have a general framework for Data Flow Analysis
  - Computes desired information at each program point
  - The computed information characterizes all possible execution paths of the program
  - Note: it is a **conservative** method to compute the information
- **You may like to search online for examples* of the two analyses to get familiar with the concepts**

*http://www.cs.colostate.edu/~mstrout/CS553/slides/lecture03.pdf



# Reference


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2. Compilers: Principles, Techniques, and Tools, 2nd Edition, by Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman, **ISBN-10:** 0321486811, **ISBN-13:** 978-0321486813, 2006, Addison Wesley



# QUESTIONS?