期末考題庫答案整理

第一題

1. (15%) Arithmetic Coding and Huffman Coding are two popular lossless compression methods. Suppose the alphabet is [*A*,*B*,*C*], and the known probability distribution is $P_A = 0.5$, $P_B=0.4$, $P_C = 0.1$. For simplicity, let's also assume that both encoder and decoder know that the length of the messages is always 3, so there is no need for a terminator. **(7-5b)**

(1) How many bits are needed to encode the message BBB by Huffman coding?

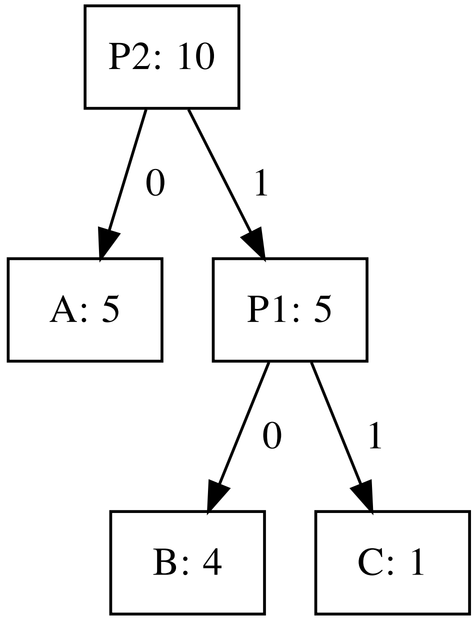
(2) How many bits are needed to encode the message BBB by arithmetic coding?

1. Huffman Coding

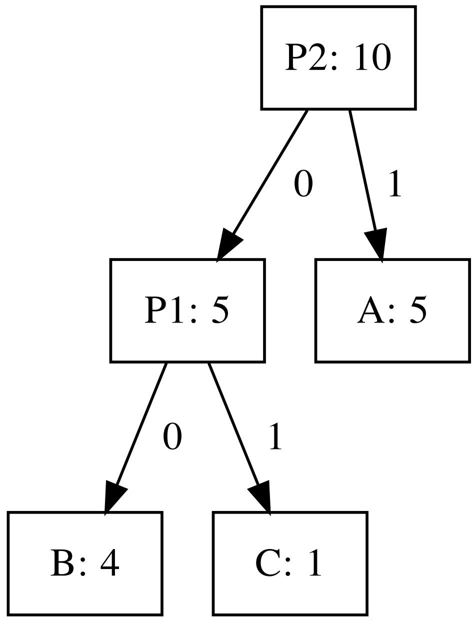
   以機率來當作字元出現次數：

Char.	Cnt.
A	5
B	4
C	1

   Huffman tree 如下：



> 另一種可能的 Huffman tree 如下：
>
> 雖然課本**並未提及同一 level 的節點順序為何**。
> 但**依照解答所表現出來的情況**來說，字典序較小者應該放置在左方。



編碼為： `10 10 10`

共需 6 bits。

2. Arithmetic Coding

   範圍為：

Symbol	Low	High	Range
	0.0	1.0	1.0
B	0.5	0.9	0.4
B	0.7	0.86	0.16
B	0.78	0.844	0.064

   產生編碼：

Binary	Decimal
1	0.5
11	0.75
111	0.875
110	0.75
1101	0.8125

   編碼為： `0.8125`

   共需 4 bits。

第二題

2. **(10%)** Given a very simple dictionary, initially containing only three characters, with the codes as follows:

Code	String
1	A
2	B
3	C

Now if the input string is ABABBABCABABBA, what is the output code based on the LZW compression algorithm. Please show how the LZW works. **(Example 7.2)**

s	c	output	code	string
			1	A
			2	B
			3	C
A	B	1	4	AB
B	A	2	5	BA
A	B			
AB	B	4	6	ABB
B	A			
BA	B	5	7	BAB
B	C	2	8	BC
C	A	3	9	CA
A	B			
AB	A	4	10	ABA
A	B			
AB	B			
ABB	A	6	11	ABBA
A	EOF	1		

編碼為： 1 2 4 5 2 3 4 6 1

> 解碼過程 (單純練習)
>
> | s | k | output | code | string |
> |---|---|---|---|---|
> | | | | 1 | A |
> | | | | 2 | B |
> | | | | 3 | C |
> | NIL | 1 | A | | |
> | A | 2 | B | 4 | AB |
> | B | 4 | AB | 5 | BA |
> | AB | 5 | BA | 6 | ABB |
> | BA | 2 | B | 7 | BAB |
> | B | 3 | C | 8 | BC |
> | C | 4 | AB | 9 | CA |
> | AB | 6 | ABB | 10 | ABA |
> | ABB | 1 | A | 11 | ABBA |
> | A | EOF | | | |
>
> 解碼後： A B A B B A B C A B A B B A

第三題

3. **(15%)** In Arithmetic Coding, suppose the alphabet is [A, B, C, D, E, F, $], in which $ is a special symbol used to terminate the message, and the known probability distribution is shown in the following table.

Symbol	Probability	Range
A	0.2	[0, 0.2)
B	0.1	[0.2, 0.3)
C	0.2	[0.3, 0.5)
D	0.05	[0.5, 0.55)
E	0.3	[0.55, 0.85)
F	0.05	[0.85, 0.9)
$	0.1	[0.9, 1.0)

What is the generated binary code word for the string of symbols BED$ after Arithmetic Coding? **(Page 189)**

範圍為：

Symbol	Low	High	Range
	0.0	1.0	1.0
B	0.2	0.3	0.1
E	0.255	0.285	0.03
D	0.27	0.2715	0.0015
$	0.27135	0.2715	0.00015

產生編碼：

Binary	Decimal
1	0.5
0	0.0
01	0.25
011	0.375
010	0.25
0101	0.3125
0100	0.25
01001	0.28125
01000	0.25
010001	0.265625
0100011	0.2734375
0100010	0.265625
01000101	0.26953125
010001011	0.271484375

編碼為： `0.271484375`

Binary code 為 0.010001011

第四題

4. **(15%)** Consider the dictionary-based LZW compression algorithm. Suppose the alphabet is the set of symbols {0, 1}. Show the dictionary (symbol sets plus associated codes) and output for LZW compression of the input 0 1 1 0 0 1 1. **(7-8)**↵

s	c	output	code	string
			0	0
			1	1
0	1	0	2	01
1	1	1	3	11
1	0	1	4	10
0	0	0	5	00
0	1			
01	1	2	6	011
1	EOF	1		

編碼為： `0 1 1 0 2 1`

> 解碼過程 (單純練習)
>
> | s | k | output | code | string |
> |---|---|---|---|---|
> | | | | 0 | 0 |
> | | | | 1 | 1 |
> | NIL | 0 | 0 | | |
> | 0 | 1 | 1 | 2 | 01 |
> | 1 | 1 | 1 | 3 | 11 |
> | 1 | 0 | 0 | 4 | 10 |
> | 0 | 2 | 01 | 5 | 00 |
> | 01 | 1 | 1 | 6 | 011 |
> | 1 | EOF | | | |
>
> 解碼後： `0 1 1 0 0 1 1`

第五題

5. **(5%)** The logarithmic **MV** search method is suboptimal, in that it relies on continuity in the residual frame.

(a) Explain why that assumption is necessary, and offer a justification for it.

(b) Does the hierarchical search method suffer from suboptimality too? **(10-5)**

中文並未完全依照原文翻譯，是以概述的方式呈現。
如有更好解釋歡迎補充。

1. Explain why that assumption is necessary, and offer a justification for it.

   Continuity assumption 主要是基於「短時間內，目前 macroblock 與其周遭的 macroblock 中的視覺內容變化是連續的」。

   這樣的假設，使得它具有單調性。在某一次的搜尋時，如果有一個 macroblock 具有 minimum error，則在下一次的搜尋中，minimum error 會是發生在附近的 macroblock 中。

   如此一來，logarithmic 的搜尋方法才能夠成立。

   > 原文如下
   >
   > The continuity assumption is based on the observation that visual content of the macroblock and the surrounding macroblocks usually change continuously within a short distance, e.g., a couple dozen pixels away, especially within a short time, e.g., 33 milliseconds for 30 fps.
   >
   > This often turns into monotonicity, i.e., the neighborhood that yielded minimal error in the previous pass will indeed yield another (global) minimum in the current pass and beyond.

2. Does the hierarchical search method suffer from suboptimality too?

   YES，但影響較小。

當影像解析度改變的時候，容易造成其中的紋理也跟著改變或消失。

因此，在較低解析度中有 minimum error 的 macroblock，在高解析度中不一定會是最佳解。

> 原文如下
>
> Yes, but less so. The textured frame will again be a possible example.
>
> When image resolution changes, certain texture patterns change (or disappear).
>
> Hence, the area that yielded minimal error at a lower resolution may not necessarily be a good area to search at the current resolution.

3. 《補充題》 Give an example where this assumption fails.

Continuity assumption 的假設太過強烈，以至於並不是所有的情況都適用。

若輸入的影像具有明顯的紋理，這樣的假設可能會使得 minimum error 發生在錯誤的 macroblock 中。

> 原文如下
>
> The above assumption is too strong to be true all the time.
>
> In a highly textured frame, it can easily be the case that the (global) minimum actually is in the neighborhood that did not yield minimal error in the previous pass.

第六題

6. **(10%)** Assume that the Adaptive Huffman Coding is used to code an information source *S* with a vocabulary of four letters (a, b, c, d). Before any transmission, the initial coding is a = 00, b = 01, c = 10, d = 11. As in the example illustrated in Fig. 7.7, a special symbol NEW will be sent before any letter if it is to be sent the first time. Fig. 7.11 is the Adaptive Huffman Tree after sending letters **aabb**. After that, the additional bitstream received by the decoder for the next few letters is 01010010101.

(1) What are the additional letters received?

(2) Draw the adaptive Huffman trees after each of the additional letters is received. **(7-6)**

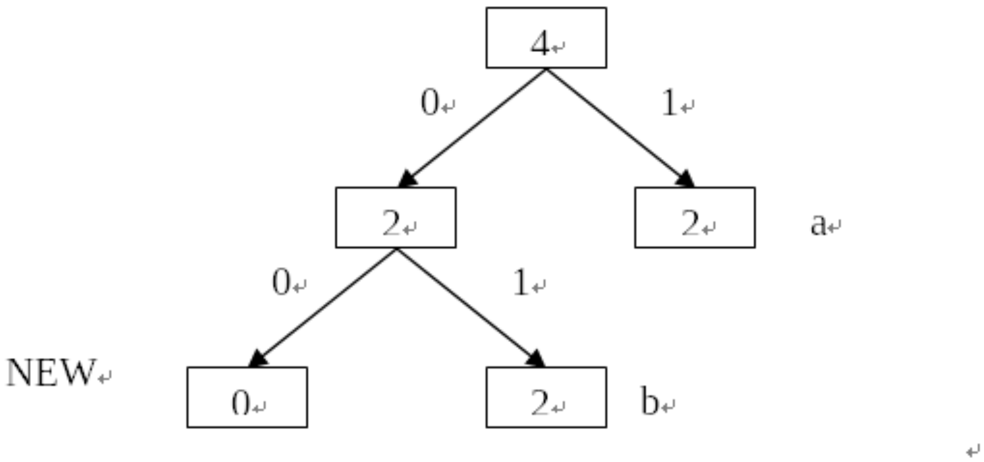
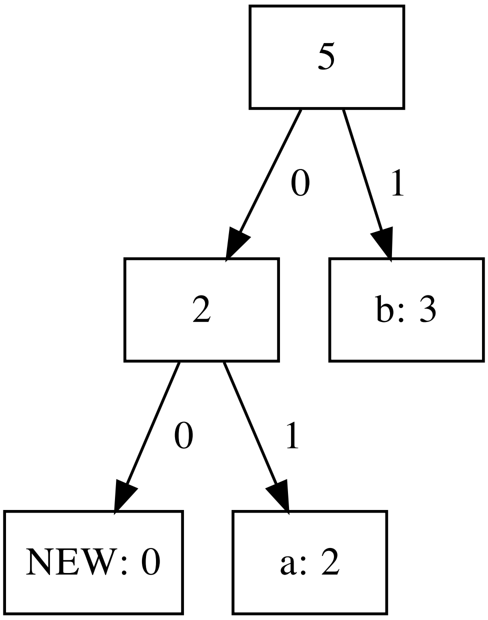
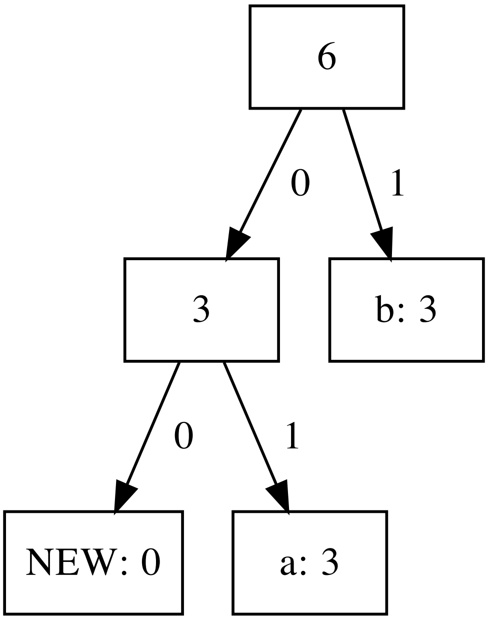


Fig. 7.11: Adaptive Huffman Tree.

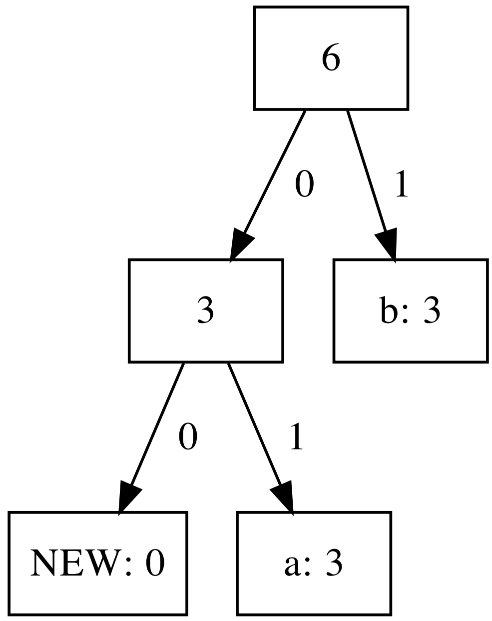
- 收到 01 → b



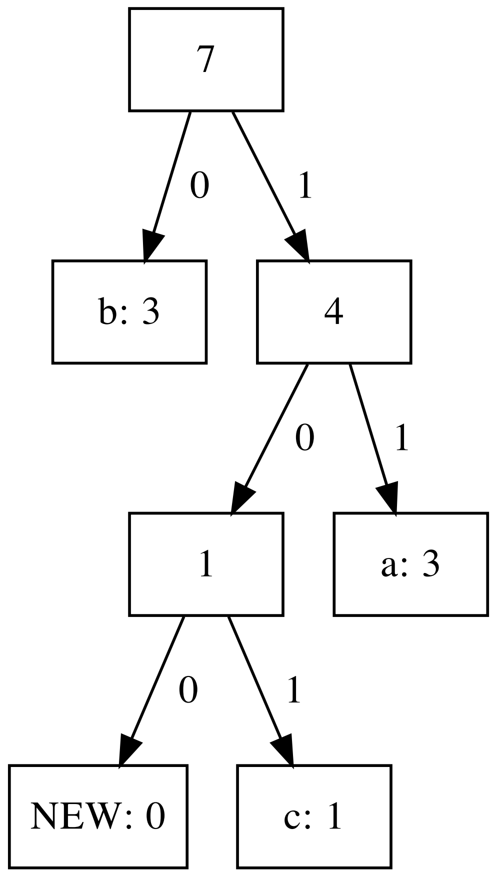
- 收到 01 → a



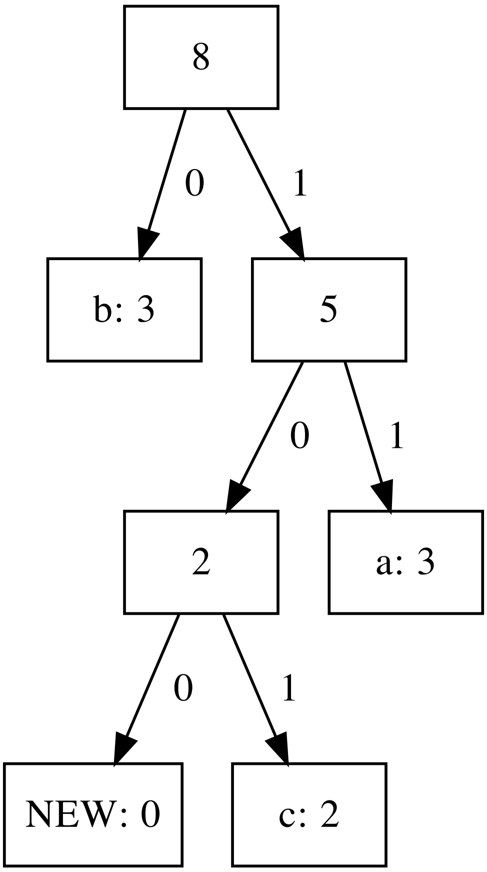
- 收到 00 → NEW



- 收到 10 → c



- 收到 101 → c



1. What are the additional letters received?

   請參考上方過程。

2. Draw the adaptive Huffman trees after each of the additional letters is received.

   請參考上方過程。

第七題

7. **(20%)** The Huffman coding algorithm requires prior statistical knowledge about the information source and such information is often not available. The solution is to use Adaptive Huffman coding algorithm. Let's assume the initial code assignment for both the encoder and decoder as follows.

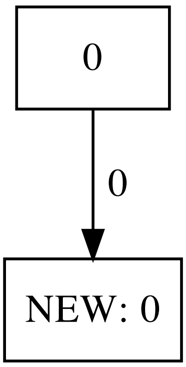
Initial	***Code***
NEW:	0
A:	00001
B:	00010
C:	00011
D:	00100
:	:

For the symbol string AADCC,

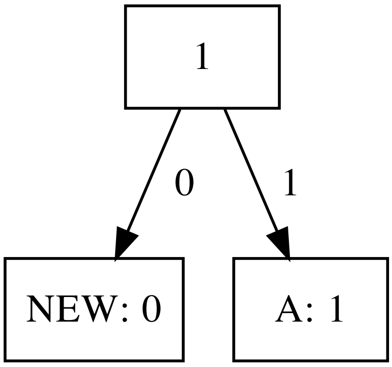
(a) Please provide the Huffman tree after each step using the Adaptive Huffman coding algorithm.

(b) Please give the sequence of symbols and code (zeros and ones) being sent to the decoder. **(Example 7.1)**

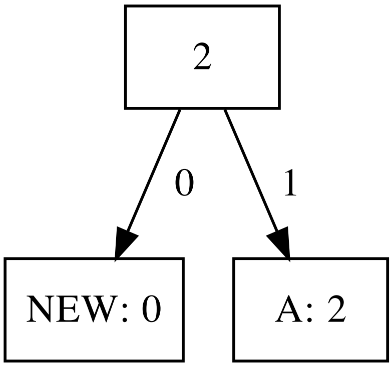
- 處理 NEW → 0



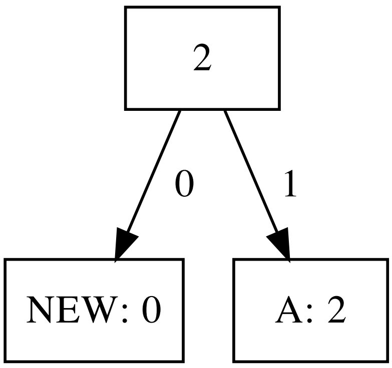
- 處理 A → 00001



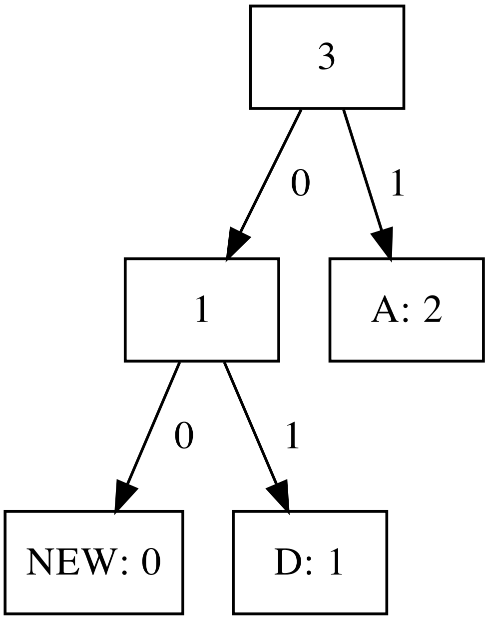
- 處理 A → 1



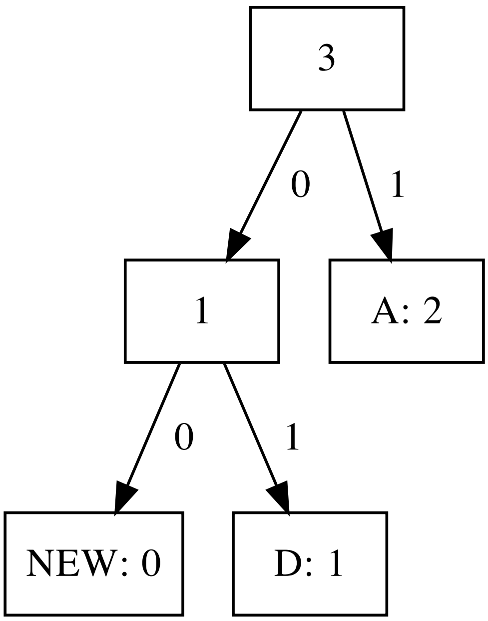
- 處理 NEW → 0



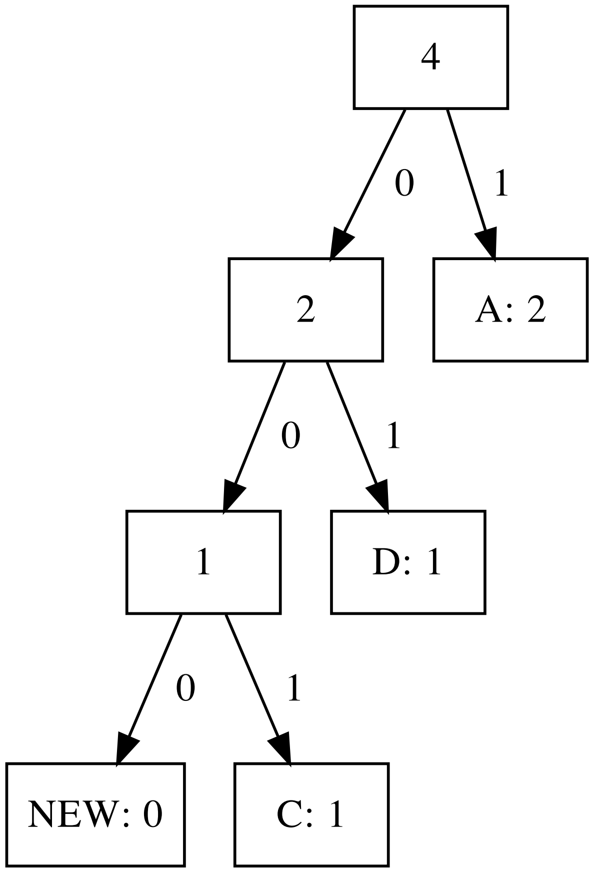
- 處理 D → 00100



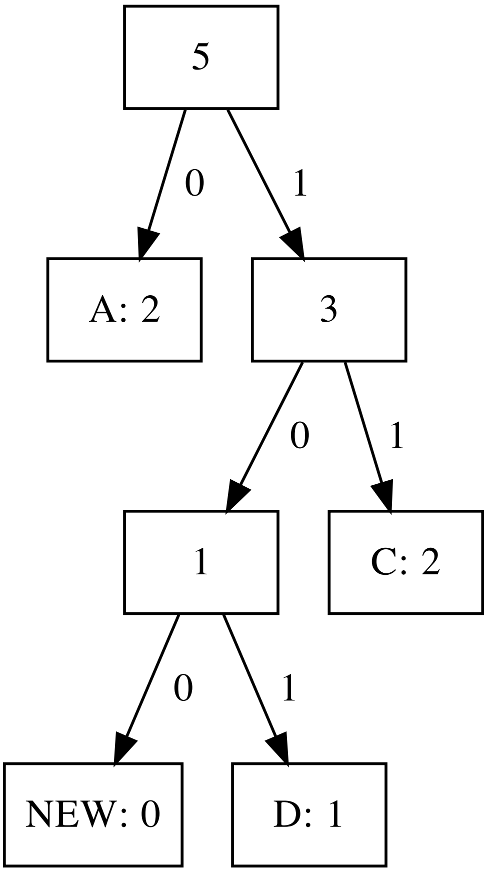
- 處理 NEW → 00



- 處理 C → 00011



- 處理 C → 001



1. Please provide the Huffman tree after each step using the Adaptive Huffman coding algorithm.

   請參考上方過程。

2. Please give the sequence of symbols and code (zeros and ones) being sent to the decoder.

   請參考上方過程。

第八題

8. (10%) Given a very simple dictionary, initially containing only three characters, with the codes as follows:

Code	String
4	A
5	B
6	C

Now if the input string is ABABBABCA, what is the output code based on the LZW compression algorithm. Please show how the LZW works. **(Example 7.2)**

s	c	output	code	string
			4	A
			5	B
			6	C
A	B	4	7	AB
B	A	5	8	BA
A	B			
AB	B	7	9	ABB
B	A			
BA	B	8	10	BAB
B	C	5	11	BC
C	A	6	12	CA
A	EOF	4		

編碼為： 4 5 7 8 5 6 4

解碼過程 (單純練習)

s	k	output	code	string
			4	A
			5	B
			6	C
NIL	4	A		
A	5	B	7	AB
B	7	AB	8	BA
AB	8	BA	9	ABB
BA	5	B	10	BAB
B	6	C	11	BC
C	4	A	12	CA
A	EOF			

解碼後： `A B A B B A B C A`

第九題

9. **(10%)** Consider the Haar Wavelet Transform that replace the original sequence with its pairwise average $x_{n-1,i}$ and difference $d_{n-1,i}$ defined as follows.

$$x_{n-1,i} = \frac{x_{n,2i} + x_{n,2i+1}}{2}, \qquad d_{n-1,i} = \frac{x_{n,2i} - x_{n,2i+1}}{2}$$

For the following input image, what is the output of the first level of the 2D Haar Wavelet Transform? **(Example8.6)**

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	120	60	60	120	0	0
0	0	60	200	200	60	0	0
0	0	60	200	200	60	0	0
0	0	120	60	60	120	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

- Intermediate output

	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	90	90	0	0	30	-30	0
4	0	130	130	0	0	-70	70	0
5	0	130	130	0	0	-70	70	0
6	0	90	90	0	0	30	-30	0
7	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0

- First level output

	1	2	3	4	5	6	7	8
1	0	0	0	0	0	0	0	0
2	0	110	110	0	0	-20	20	0
3	0	110	110	0	0	-20	20	0
4	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
6	0	-20	-20	0	0	50	-50	0
7	0	20	20	0	0	-50	50	0
8	0	0	0	0	0	0	0	0

第十題

10. (15%) For the coefficients of a three-stage wavelet transform shown in the following figure, we attempt to code the coefficients using the EZW algorithm.

57	-37	39	-20	3	7	9	10
-29	30	17	33	8	2	1	6
14	6	15	13	9	-4	2	3
10	19	-7	9	-7	14	12	-9
12	15	33	20	-2	3	1	0
0	7	2	4	4	-1	1	1
4	1	10	3	2	0	1	0
5	6	0	0	3	1	2	1

(a) What is the list of coefficients visited, in the order of the scan?

(b) What is the dominant pass outputs D_0?

(c) What is the subordinate pass output S_0? **(8.8.3 example)**

- 初始化

57	-37	39	-20	3	7	9	10
-29	30	17	33	8	2	1	6
14	6	15	13	9	-4	2	3
10	19	-7	9	-7	14	12	-9
12	15	33	20	-2	3	1	0
0	7	2	4	4	-1	1	1
4	1	10	3	2	0	1	0
5	6	0	0	3	1	2	1

- 初始 Threshold

  最大值為 57，因此 $T_0 = \frac{64}{2} = 32$。

- 與 Threshold 比較

57	**-37**	**39**	*-20*	*3*	*7*	9	10
-29	*30*	*17*	**33**	*8*	*2*	1	6
14	*6*	15	13	9	-4	*2*	*3*
10	*19*	-7	9	-7	14	*12*	*-9*
12	15	**33**	*20*	-2	3	1	0
0	7	*2*	*4*	4	-1	1	1
4	1	10	3	2	0	1	0
5	6	0	0	3	1	2	1

1. What is the list of coefficients visited, in the order of the scan?

   結果如下：

```
{ 57, -37, -29, 30, 39, -20, 17, 33, 14, 6, 10, 19, 3, 7, 8, 2, 2, 3, 1
```

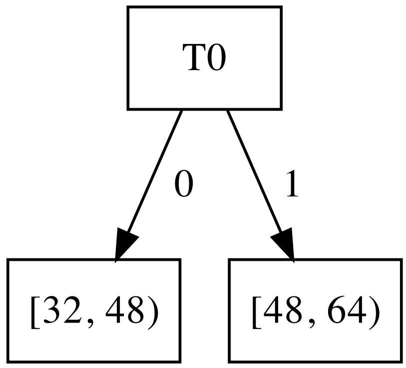
2. What is the dominant pass outputs D_0?

   結果如下：

```
57, -37, -29, 30, 39, -20, 17, 33, 14, 6, 10, 19, 3, 7, 8, 2, 2, 3, 12,
------------------------------------------------------------------------
 p,   n,   z,  t,  p,   t,  t,  p,  t, z,  t,  t, t, t, t, t, t,  t,  t
```

3. What is the subordinate pass output S_0?

   Output tree 如下：



   結果如下：

```
57, -37, 39, 33, 33
-------------------
 1,   0,  0,  0,  0
```

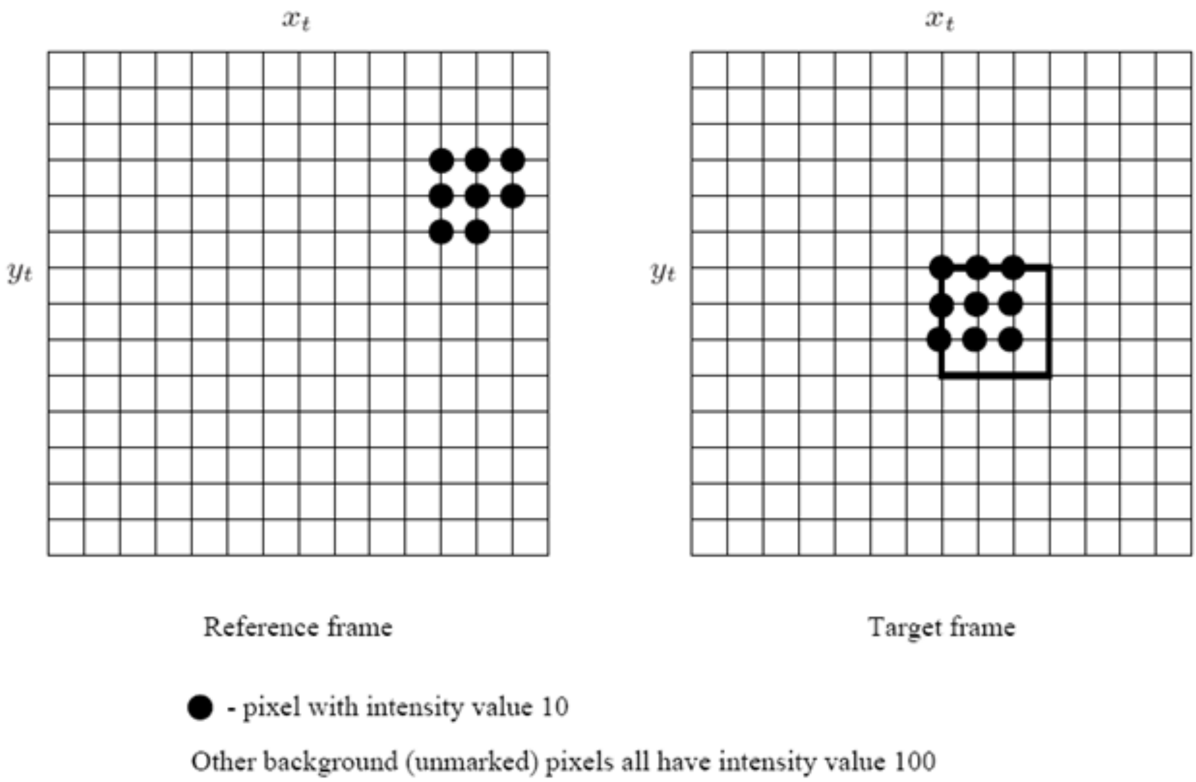
第十一題

11. (20%) Work out the following problem of 2D Logarithmic Search for motion vectors in detail (see the following figure).

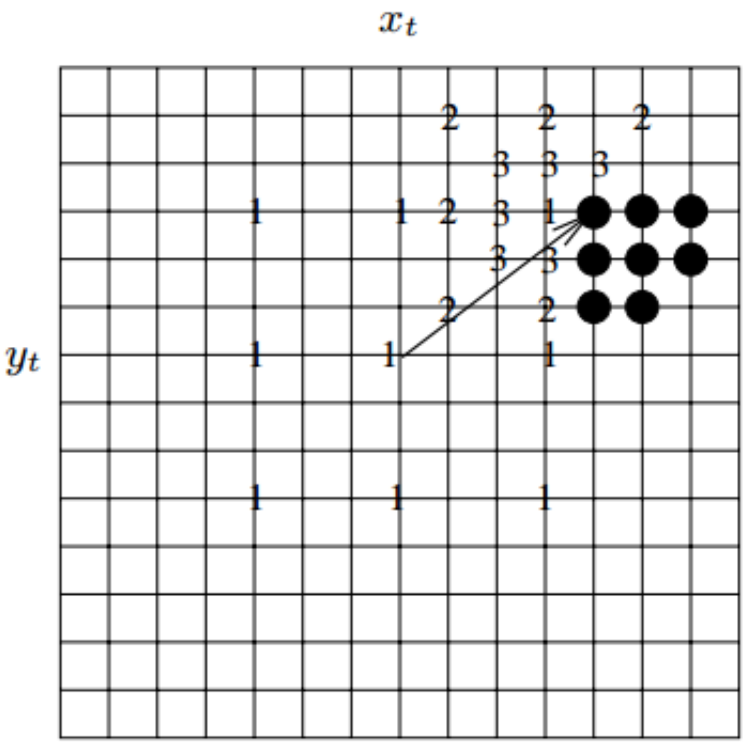
The target (current) frame is a P-frame. The size of macroblocks is 4 x 4. The motion vector is **MV**(Δx,Δy), in which $\Delta x \in [-p, p]$, $\Delta y \in [-p, p]$. In this question, assume $p \equiv 5$. The macroblock in question (darkened) in the frame has its upper left corner at (x_t, y_t). It contains 9 dark pixels, each with intensity value 10; the other 7 pixels are part of the background, which has a uniform intensity value of 100. The reference (previous) frame has 8 dark pixels.

(a) What is the best Δx, Δy, and Mean Absolute Error (MAE) for this macroblock?

(b) Show step by step how the 2D Logarithmic Search is performed, including the locations and passes of the search and all intermediate Δx, Δy, and MAEs. **(10-4)**



- Step by step 示意圖



- Pass 1

  Step size: $\lceil \frac{p}{2} \rceil = 3$
  Displacement: $\Delta x = 3, \Delta y = 3$
  MAE: $\frac{3|10-100|+2|100-10|}{16} = 28.125$

- Pass 2

  Step size: $\lceil \frac{p}{4} \rceil = 2$
  Displacement: $\Delta x = 3, \Delta y = 3$
  MAE: $\frac{3|10-100|+2|100-10|}{16} = 28.125$

- Pass 3

  Step size: $\lceil \frac{p}{8} \rceil = 1$
  Displacement: $\Delta x = 4, \Delta y = 3$
  MAE: $\frac{|10-100|}{16} = 5.625$

1. What is the best Δx, Δy, and Mean Absolute Error (MAE) for this macroblock?

   請參考上方過程。

2. Show step by step how the 2D Logarithmic Search is performed.

請參考上方過程。

第十二題

12. (5%) When the block size is 8, the definition of the DCT is given in the following equation

$$F(u,v)=\frac{2C(u)C(v)}{\sqrt{MN}}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\cos\frac{(2i+1)u\pi}{2M}\cos\frac{(2j+1)v\pi}{2N}f(i,j)$$

$$C(\xi)=\begin{cases}\frac{\sqrt{2}}{2}, & \text{for } \xi=0\\ 1, & \text{otherwise}\end{cases}$$

If an 8 x 8 grayscale image is in the range 0… 255, what is the largest value a DCT coefficient could be? Also, state *all* the DCT coefficient values for that image. (Hint: **When the image is all WHITE, all pixels have** I = 255) **(8-5)**

當圖片為全白時，僅剩下 DC component，意即最大值為 F(0, 0)。

$$\begin{aligned}F(0,0) &= \frac{2C(0)C(0)}{\sqrt{8\times 8}}\sum_{i=0}^{7}\sum_{j=0}^{7}cos(0)cos(0)f(i,j)\\ &= \frac{1}{8}\sum_{i=0}^{7}\sum_{j=0}^{7}cos(0)cos(0)f(i,j)\\ &= \frac{1}{8}\sum_{i=0}^{7}\sum_{j=0}^{7}255\\ &= \frac{1}{8}\times(8\times 8\times 255)\\ &= 2040\end{aligned}$$

而 AC 的係數皆為 0。

> 原文如下
>
> When the image is all WHITE, i.e., all pixels have I = 255.
> The largest coefficient is the DC value which is 8 × 255 = 2040.
> All others (AC values) are zero.

第十三題

13. (20%) Suppose that a 3-bit image (L=8) of size 64x64 pixels has the intensity distributions as follows.

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64 × 64 digital image.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

What is the intensity distribution S_0~S_7 after histogram equalization?

$$S_i = (L-1)\sum_{j=0}^{i} p_r(r_i)$$

$$S_0 = round(7 \times 0.19) = 1$$
$$S_1 = round(7 \times 0.44) = 3$$
$$S_2 = round(7 \times 0.65) = 5$$
$$S_3 = round(7 \times 0.81) = 6$$
$$S_4 = round(7 \times 0.89) = 6$$
$$S_5 = round(7 \times 0.95) = 7$$
$$S_6 = round(7 \times 0.98) = 7$$
$$S_7 = round(7 \times 1.00) = 7$$

第十四題

14. (10%) Considering the effect of Frequency Masking, which descriptions are correct for the general situation in regard to masking?

( ) 1. A lower tone can effectively mask (make us unable to hear) a higher tone

( ) 2. A masking tone with greater power can mask the tone which is widely separated from the masking tone.

Frequency Masking 為第十四章之內容。

第十五題

15. (10%) Please give descriptions on the following speech coders.
(a) ADPCM
(b) Linear predictive coding

1. ADPCM

   假設
   (1) 原訊號有 $f_1, f_2, ..., f_N$。
   (2) $\hat{f}$ 是預測值，$\tilde{f} = \hat{f} + \tilde{e}$ 是重建後的值。
   (3) e_i 是誤差值，$\tilde{e_i}$ 是量化後的誤差值。

   > 預測值公式如下：
   >
   > $$\hat{f_n} = \sum_{i=1}^{M} a_i \tilde{f_{n-i}}$$
   >
   > 其中 M 為往前看多少個訊號，a_i 會 $min\ \sum_{n=1}^{N}(f_n - \hat{f_n})^2$

   ADPCM 最終會傳送出 $f_1, \tilde{e_1}, \tilde{e_2}, ..., \tilde{e_N}$。

2. Linear Predictive Coding (LPC)

   假設
   (1) 原訊號有 $f_1, f_2, ..., f_N$。
   (2) $s(n)$ 為輸出值，$f(n)$ 為輸入數值。

> 輸出值公式如下：
>
> $$s(n) = \sum_{i=1}^{p} a_i s(n-i) + Gf(n)$$
>
> 其中 G 為 gain factor coefficients，p 為往前看多少個訊號，a_i 會 $min\ E\{[s(n) - \sum_{j=1}^{p} a_j s(n-j)]^2\}$

LPC 最終會傳送出 $s(1), s(2), s(3), \ldots, s(N)$。

第十六題

16. (5%) In the text, we study an adaptive quantization scheme for ADPCM. We can also use an adaptive prediction scheme. We consider the case of one tap prediction, $s(n) = a \cdot s(n-1)$. Show how to estimate the parameter a in an open-loop method. (13-5)

Error function 如下：

$$E[e^2] = E[(s(n) - as(n-1))^2]$$

對 a 偏微分後得：

$$\frac{\partial E[e^2]}{\partial a} = E[(s(n) - as(n-1))(-s(n-1))]$$

解方程式：

$$E[(s(n) - as(n-1))(-s(n-1))] = 0$$

$$E[-s(n)s(n-1) + as^2(n-1)] = 0$$

$$a = \frac{E[s(n)s(n-1)]}{E[s^2(n-1)]}$$