

# Probability and Statistics Fall 2020

## Final exam review note

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**Coverage:** Chapters 5~8 (excluding 8.7)

**Please note:** This note is made available to help you get an overview of most important concepts in covered chapters. There is no guarantee that final exam problems will be from this note. Please see textbook and lecture slides for complete course contents.

### Chapter 5 Some Discrete Probability Distributions

1. What is a discrete probability distribution? How to find probability values using a discrete probability distribution?
2. What are the properties of the Bernoulli process? What is a binomial distribution?
3. In general, given a reasonable large  $n$  (e.g.,  $n = 30$ ), how does the shape of a binomial distribution change when  $p$  varies from 0 to 1?
4. What are the differences between a hypergeometric distribution and a binomial distribution? When will these two distributions become very close to each other? Why?
5. What are the differences between a negative binomial distribution and a binomial distribution?
6. What are the properties of a Poisson process? Can you give an example of a Poisson distribution?
7. In general, how does the shape of a Poisson distribution change when  $\mu$  varies from 0 to larger values?
8. When can we approximate a binomial distribution with a Poisson distribution? Why? Do you know how to do it?
9. In general, how can we derive mean and variance of a random variable given its probability distribution?
10. How can we find probability values from tables for Binomial and for Poisson distributions?

### Chapter 6 Some Continuous Probability Distributions

1. What is a continuous probability distribution? How to find probability values using a continuous probability distribution?
2. What are the properties of a normal curve?

3. How can we find probability values of a normal distribution with a normal distribution table? What is the meaning of converting a normal distribution to a standard normal distribution?
4. When can we approximate a binomial distribution with a normal distribution? Why? Do you know how to do it? (Compare this with point 8 of Chapter 5.)
5. What is the relationship between a Gamma/Exponential distribution to a Poisson distribution? When to apply which?
6. What is a Chi-Squared distribution? How can we find probability values from a Chi-squared distribution table?
7. What is a lognormal distribution? How can we find probability values from a lognormal distribution?

## **Chapter 7 Functions of Random Variables**

1. Can you explain the process of transforming random variables in a simple case? For example, if we know a discrete probability distribution  $f(x)$ , how can we find  $g(y)$  when  $Y=2X$ ?
2. In general, what are the steps to transform random variables? In case of discrete random variables? In case of continuous random variables? Between 1D spaces ( $X \rightarrow Y$ )? Between 2D spaces ( $[X_1, X_2] \rightarrow [Y_1, Y_2]$ )?
3. What is the  $r$ th moment about the origin of a random variable? What is the moment-generating function of a random variable?
4. What is the uniqueness theorem for moment-generating function?
5. To make use of the uniqueness theorem, please see Theorem 7.7~7.12. Please try to understand their meanings instead of memorizing them.

## **Chapter 8 Fundamental Sampling Distributions and Data Distributions**

1. In general, can you explain sampling process? What are distributions for population, an observation (i.e., a sample point), and a sample of size  $n$  (i.e., with  $n$  observations)?
2. What is the (approximated) probability distribution of sample mean  $\bar{X}$ ? What is the Central Limit Theorem?
3. Why can we use  $\bar{X}$  to estimate  $\mu$ ? Do you get the same amount of error for each estimation? How can we control errors?
4. What is the sampling distribution of  $S^2$  from a normal population? In this case, why can we use  $S^2$  to estimate  $\sigma^2$ ?

5. What is the difference between a normal distribution and a t-distribution? When do we usually apply a t-distribution?
6. When does a t-distribution approximate to a z-distribution? Why?
7. How can we find probability values from a t-distribution table?
8. What is a quantile plot?
9. What is a normal quantile-quantile plot? In general, how can we make use of a normal quantile-quantile plot?