Discrete Mathematics

Deadline: 2021/04/08

共 **15** 題

1. Determine the number of paths in the xy-plane from (-3,-2) to (5,4), and must pass (0,0) on the way where each such path is made up of individual steps going one unit to the right (R) or one unit upward (U).

Ans:$\frac{5!}{3!2} \times \frac{9!}{5!4} = 1260$

2. How many bit string of length 10 contain

a)exactly seven 1s

b)at most nine 1s

c)exactly three 1s and no consecutive 1s

(a)$\frac{10!}{7!3}$=120

(b)$2^{10} - 1 = 1023$

(c)$C_3^8 = 56$

3. How many arrangements of the letters in MISSISSIPPI have no consecutive S's ?

計算$\binom{8}{4} \times \frac{7!}{4!2!}$錯誤扣 3 分，結果要算出來(不扣分)

4. In how many ways can the symbols a, b, c, d, e, e, e, e, e be arranged so that no e is adjacent to another e?

Ans: 4! (6 分)

5. How many triangles are determined by the vertices of a regular polygon of n sides? How many if no side of the polygon is to be a side of any triangle?

Ans: (1) C_3^n (3 分) (2) $C_3^n - n - n(n-4)$ (3 分)

Exercise 2.1

4. Let p, q, r, s denote the following statements:

p: I finish writing my computer program before lunch.
q: I shall play tennis in the afternoon.
r: The sun is shining.
s: The humidity is low.

Write the following in symbolic form.

a) If the sun is shining, I shall play tennis this afternoon.

b) Finishing the writing of my computer program before lunch is necessary for my playing tennis this afternoon.

c) Low humidity and sunshine are sufficient for me to play tennis this afternoon.

(a) $r \to q$ (2 分)

(b) $q \to p$ $(\neg p \to \neg q)$ (2 分)

(c) $(r \wedge s) \to q$ (3 分)

6. Determine the truth value of each of the following implications.

a) If $3 + 4 = 12$, then $3 + 2 = 6$.

b) If $3 + 3 = 6$, then $3 + 4 = 9$.

c) If Thomas Jefferson was the third president of the United States, then $2 + 3 = 5$.

少寫過程扣 1 分

(a) $0 \to 0 = 1$ (True) (2 分)

(b) $1 \to 0 = 0$ (False) (2 分)

(c) $1 \to 1 = 1$ (True) (2 分)

Exercise 2.2

4. For primitive statements p, q, r, and s, simplify the compound statement

$$[\{[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \neg r]\} \vee \neg q] \to s.$$

$$\Big(\Big(\big((p \wedge q) \wedge r\big) \vee \big((p \wedge q) \wedge \neg r\big)\Big) \vee \neg q\Big) \to s$$

$$\Rightarrow \Big(\big((p \wedge q) \wedge (r \vee \neg r)\big) \vee \neg q\Big) \to s$$

$$\Rightarrow \Big(\big((p \wedge q) \wedge \text{True}\big) \vee \neg q\Big) \to s$$

$$\Rightarrow \big((p \wedge q) \vee \neg q\big) \to s$$

$$\Rightarrow \big((p \vee \neg q) \wedge (q \vee \neg q)\big) \to s$$

$$\Rightarrow \big((p \vee \neg q) \wedge \text{True}\big) \to s$$

$$\Rightarrow (p \vee \neg q) \to s \;\text{ or }\; (\neg p \wedge q \vee s) \;\text{ or }\; q \to p \to s \;\; (7\text{ 分})$$

沒過程斟酌扣分

8. Write the dual for (a) $q \to p$, (b) $p \to (q \wedge r)$, (c) $p \leftrightarrow q$, and (d) $p \veebar q$, where p, q, and r are primitive statements.

(a) $q \to p \Leftrightarrow \neg q \vee p$

dual of $q \to p$ is $\neg q \wedge p$

(b) $p \to (q \wedge r) \Leftrightarrow \neg p \vee (q \wedge r)$

dual of $p \to (q \wedge r)$ is $\neg p \wedge (q \vee r)$

(c) $p \leftrightarrow q \Leftrightarrow (p \to q) \wedge (q \to p)$

dual of $p \leftrightarrow q$ is $(\neg p \wedge q) \vee (\neg q \wedge p)$

(d) $p \veebar q \Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$

dual of $p \veebar q$ is $(p \vee \neg q) \wedge (\neg p \vee q)$

沒有 rule 或 law 的話要先證兩者相等

沒過程斟酌扣分，錯一小題扣 2 分

10. Determine whether each of the following is true or false. Here p, q are arbitrary statements.

a) An equivalent way to express the converse of "p is sufficient for q" is "p is necessary for q."

b) An equivalent way to express the inverse of "p is necessary for q" is "$\neg q$ is sufficient for $\neg p$."

c) An equivalent way to express the contrapositive of "p is necessary for q" is "$\neg q$ is necessary for $\neg p$."

(a) True

(b) True

(c) True

講義 p.31

每小題 2 分，整題錯扣 7 分

Exercise 2.3

4. For each of the following pairs of statements, use Modus Ponens or Modus Tollens to fill in the blank line so that a valid argument is presented.

a) If Janice has trouble starting her car, then her daughter Angela will check Janice's spark plugs.
Janice had trouble starting her car.
$\therefore$ ______________________________

b) If Brady solved the first problem correctly, then the answer he obtained is 137.
Brady's answer to the first problem is not 137.
$\therefore$ ______________________________

c) If this is a **repeat-until** loop, then the body of this loop is executed at least once.
____
$\therefore$ The body of the loop is executed at least once.

d) If Tim plays basketball in the afternoon, then he will not watch television in the evening.
____
$\therefore$ Tim didn't play basketball in the afternoon.

(a) Janice's daughter Angela will check Janice's spark plugs.

(b) Brady did not solve the first problem correctly.

(c) This is a repeat-until loop.

(d) Tim watched television in the evening.

每小題 2 分

6. For primitive statements p, q, and r, let P denote the statement

$$[p \wedge (q \wedge r)] \vee \neg[p \vee (q \wedge r)],$$

while P_1 denotes the statement

$$[p \wedge (q \vee r)] \vee \neg[p \vee (q \vee r)].$$

a) Use the rules of inference to show that

$$q \wedge r \Rightarrow q \vee r.$$

b) Is it true that $P \Rightarrow P_1$?

(a)(4 分)

	Steps	Reasons
(1)	$q \wedge r$	Premise
(2)	q	By step(1) and Rule of Conjunctive Simplification
(3)	$q \vee r$	By step(2) and Rule of Disjunctive Amplification

So $q \wedge r \to q \vee r$ is a tautology, then $q \wedge r \Rightarrow q \vee r$ is valid.

(b)(3 分)使用真值表證明，沒寫的話斟酌扣分。沒寫結論扣 1 分。

8. Give the reasons for the steps verifying the following argument.

$$\begin{array}{l} (\neg p \vee q) \rightarrow r \\ r \rightarrow (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \rightarrow \neg t \\ \hline \therefore p \end{array}$$

Steps		**Reasons**
1)	$\neg s \wedge \neg u$	
2)	$\neg u$	
3)	$\neg u \rightarrow \neg t$	
4)	$\neg t$	
5)	$\neg s$	
6)	$\neg s \wedge \neg t$	
7)	$r \rightarrow (s \vee t)$	
8)	$\neg(s \vee t) \rightarrow \neg r$	
9)	$(\neg s \wedge \neg t) \rightarrow \neg r$	
10)	$\neg r$	
11)	$(\neg p \vee q) \rightarrow r$	
12)	$\neg r \rightarrow \neg(\neg p \vee q)$	
13)	$\neg r \rightarrow (p \wedge \neg q)$	
14)	$p \wedge \neg q$	
15)	$\therefore p$	

1)premise

2)step (1) and Rule of conjunctive simplification

3)premise

4)step (2) and step (3), Rule of Detachment (Modus Ponens)

5)step (1) and Rule of conjunctive simplification

6)step (4) and step (5), Rule of conjunction

7)premise

8)step (7) logically equivalent

9)step (8), DE Morgan's law

10)step (6) and step (9), Rule of Detachment (Modus Ponens)

11)premise

12)step (12) and contrapositive

13)step (13) and DE Morgan's law

14)step (10) and step (13), Rule of Detachment (Modus Ponens)

15)step (14) and Rule of conjunctive simplification

Exercise 2.4

18. Negate and simplify each of the following.

a) $\exists x\,[p(x) \vee q(x)]$ **b)** $\forall x\,[p(x) \wedge \neg q(x)]$

c) $\forall x\,[p(x) \rightarrow q(x)]$

d) $\exists x\,[(p(x) \vee q(x)) \rightarrow r(x)]$

a) $\forall x\,[\neg p(x) \wedge \neg q(x)]$

b) $\exists x[\neg p(x) \vee q(x)]$

c) $\exists x[p(x) \vee \neg q(x)]$

d) $\forall x\,[(p(x) \vee q(x)) \wedge \neg r(x)]$

Exercise 2.5

10. Provide the missing reasons for the steps verifying the following argument:

$$\begin{array}{l} \forall x\,[p(x) \vee q(x)] \\ \exists x\,\neg p(x) \\ \forall x\,[\neg q(x) \vee r(x)] \\ \underline{\forall x\,[s(x) \rightarrow \neg r(x)]} \\ \therefore \exists x\,\neg s(x) \end{array}$$

Steps	**Reasons**
1) $\forall x\,[p(x) \vee q(x)]$	Premise
2) $\exists x\,\neg p(x)$	Premise
3) $\neg p(a)$	Step (2) and the definition of the truth for $\exists x\,\neg p(x)$. [Here a is an element (replacement) from the universe for which $\neg p(x)$ is true.] The reason for this step is also referred to as the *Rule of Existential Specification.*
4) $p(a) \vee q(a)$	
5) $q(a)$	
6) $\forall x\,[\neg q(x) \vee r(x)]$	
7) $\neg q(a) \vee r(a)$	
8) $q(a) \rightarrow r(a)$	
9) $r(a)$	
10) $\forall x\,[s(x) \rightarrow \neg r(x)]$	
11) $s(a) \rightarrow \neg r(a)$	
12) $r(a) \rightarrow \neg s(a)$	
13) $\neg s(a)$	
14) $\therefore \exists x\,\neg s(x)$	Step (13) and the definition of the truth for $\exists x\,\neg s(x)$. The reason for this step is also referred to as the *Rule of Existential Generalization.*

4)step (1) and Rule of Universal Specification

5)step (3) and step (4), Rule of Disjunctive Syllogism

6)Premise

7)step (6) and Rule of Universal Specification

8)step (7), logically equivalent

9)step (5) and step (8), Rule of Detachment (Modus Ponens)

10)premise

11) step (10) and Rule of Universal Specification

12)step (11) and logically equivalent

13)step(9) and step(12), Rule of detachment