**Discrete Mathematics**

Homework 3

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4-1 18.

Assume general formula:

Proof:

4-1 24.

a)

b)

For n=1, is true.

For n=2,

Assume for all , k>=2, is true.

For n=k+1,

Is still true

By mathematical induction,

4-2 12.

For n=0,

Set for n=k () , is also true.

Then, when n=k+1,

Is still true.

By Principle of Mathematical induction, is true.

4-2 16.

a)

even integer: 2, 4, 6, 8, 10, …

in recursive:

=>

1.

2.

b)

nonnegative even integer: 0, 2, 4, 6, 8, 10, …

in recursive:

=>

1.

2.

4-3 10.

Let

Since one of k, k+1 must be even,

4-3 12.

a)

b)

In positive dividend,

=>

, r > b-r > 0

so,

c)

d)

4-3 18.

251+445 = 696 in base 10

However, it has 6 in the base, but no 9 in here.

And 9 change into 2, assume the base is 9-2 =7

Pf:

So, they are in base 7.

4-4 12.

Let and

by theorem 4.3

by theorem 4.3

Hence since , it follows that . Consequently, we find that

4-4 16.

(=>)

Suppose that there exist, with cd = a and , since and, it could expressed as and . Consequently, , so .

(<=)

, let then , and

4-5 8.

a)

# of positive divisor:

(14+1)(9+1)(8+1)(10+1)(3+1)(5+1)(10+1)=3920400

b)

i)

is divisible by ,

then , , , , ,

,

# of positive divisor:

ii)

is divisible by , then , , , , ,

,

# of positive divisor:

iii)

prefect square:

2: take 0, 2, 4, …, 14 => 8 7: take 0, 2, …, 10 =>6

3: take 0, 2, 4, …, 8 => 5 11: take 0, 2 =>2

5: take 0, 2, 4, 6, 8 => 5 13: take 0, 2, 4 =>3

37: take 0, 2, 4, …, 10 =>6

iv)

prefect square divisible by

2: take 2, 4, …, 14 =>7 7: take 0, 2, …, 10 =>6

3: take 4, 6, 8 =>3 11: take 2 =>1

5: take 2, 4, 6, 8 => 4 13: take 0, 2, 4 =>3

37: take 0, 2, 4, …, 10 =>6

v)

prefect cubes:

2: take 0, 3, 6, 9, 12 => 5 7: take 0, 3, 6, 9 => 4

3: take 0, 3, 6, 9 => 4 11: take 0, 3 => 2

5: take 0, 3, 6 => 3 13: take 0, 3 => 2

37: take 0, 3, 6, 9 => 4

vi)

prefect cubes divisible by :

2: take 12 => 1 7: take 6, 9 => 2

3: take 9 => 1 11: take 3 => 1

5: take 3, 6 => 2 13: take 3 => 1

37: take 3, 6, 9 => 3

vii)

prefect squares and cubes:

2: take 0, 6, 12 => 3 7: take 0, 6 => 2

3: take 0, 6 => 2 11: take 0 => 1

5: take 0, 6 => 2 13: take 0 => 1

37: take 0, 6 => 2