Discrete Mathematics

HW5

E94086107 張娟鳴

11-1 2.

(a)

Trail: can have repeated vertices but no repeat edges, open.

b-d not a trail: {b,e} {e,f} {e,g} {g,e} {e,b} {b,c} {c,d}

(b)

Path: no repeated vertices and edges, open.

b-d trail and not a path: {b,e} {e,f} {f,g} {g,e} {e,d}

(c )

b-d path: {b,e} {e,d}

(d)

Closed walk: can have repeated vertices and edges, closed.

Circuit: can have repeated vertices but no repeat edges, closed.

b-b closed but not a circuit: {b,e} {e,f} {f,g} {g,e} {e,b}

(e)

Cycle: no repeated edges and circuits, closed.

b-b circuit but not cycle: {b,c} {c,d} {d,e} {e,g} {g,f} {f,e} {e,b}

(f )

b-b cycle: {b,a} {a,c} {c,b}

11-1 5.

(1) path from a to h needs to go through {b,g},

so the answer = # of path a-b \* # of path g-h.

a-b path:　　　　　　g-h path:

{a,b}　　　　　　　　{g,h}

{a,c} {c,b}　　　　　　{g,f} {f,h}

{a,c} {c,d} {d,b} 　　　{g,e} {e,f} {f,h}

a-h path:

(2) length 5: length a-b + length g-h + {b,g} = 5

Length a-b + length g-h = 4 = 1 + 3 = 2 + 2 = 3 + 1

上面三種情況都各只有一種搭配可以符合，所以共三種

a-h and length 5 path: 3 kinds.

11-2 4.

Spanning graph: , no restrict with E

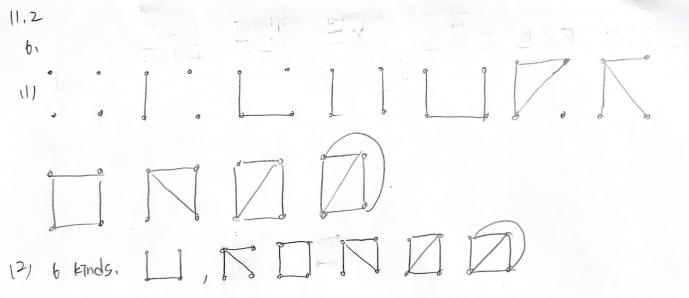
Induced graph: if , induced subgraph’s edges contain all edges (from G)

因為Spanning graph的點包含了原先G的所有點，如果他要是induced subgraph的話，就需要包含所有點在G裡面構成的邊，也就只有G本身自己可以滿足這個條件。

A: 1種，G本身

11-2 6.

(1) 11種，如下圖



11-3 2.

希望 |V| 越大，則deg(v)應該越小越好，所以可以的話盡量讓 deg(v)=3

And by theorem, we know

Then, , maximum |V|=11 ( 3\*11+1，有10個vertex deg(v)=3，一個deg(v)=4 )

11-3 9.

(a)

# of edges of a cube of dimension n is .

So,

(b)

# of vertices of n dimension hypercube is

# of vertices

11-4 8.

(a)

: edge in G {a,b}, 且 每個點都有對應到

所以在這個case裡面最多可以有4個edge，從 到 的四個點

而最長path可為2，從 出發到 再回到

(b)

In this case, 因為比較少點的那一邊有3個點，最好情況是 每個點(假設有 a, b, c)都與 每個點(1,2,3…,7)相連

最長path就可以從的 出發，到 的a，接著連回 的2， 的 b， 的3， 的c， 的4，共7個點6條邊

其中a,b,c和1,2,3,4......並非指特定點，只是代表不同集合的不同點

(c )

同理，最長的path應該是從有12個vertices的 出發，然後拜訪 全部共7個，來回總共15個點14條邊（=2\*7=14）

(d)

Since m<n, the longest path in

11-5 7.

(a)

我們可以視為有n個點要排成列，共有n!種排法，但因為要排成圓型，1->2->3->1和2->3->1->2會變成同一種，需除n，且因為沒有方向概念，1->2->3->1和1->3->2->1視為同一種，要再除2，所以共 種。

(b)

For a , we have edges. And each Hamilton cycles has n edges, so we will have kinds of edge-disjoint Hamilton cycles.

To have a better answer, n should be odd.

When n=21,

(c)

Since they need to hold hands form a cycle, we can think it as edge-disjoint Hamilton cycle, so just like we need to find total number of edge-disjoint Hamilton cycle on a . when n=19, , we have 9 edge-disjoint Hamilton cycle in .

所以9天內所有學生旁邊的人都不是重複的。

11-6 2.

Each committees is a vertex. If someone attends two committees, for example Then draw the edge joining the vertices for , and we will get graph G. Then the least number of meeting times is the chromatic number of this graph (in hours) .