EPIDEMIC AND DISEASE MODELLING

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EPIDEMIC MODELLING EXPONENTIAL GROWTH

- THE MOST BASIC COMPONENT OF A EPIDEMIC MODEL IS **THE REPRODUCTIVE NUMBER (R0):** WHICH IS THE NUMBER OF SECONDARY INFECTED INDIVIDUALS THAT YOU CAN EXPECT TO HAVE ON AN AVERAGE IN A POPULATION FROM A SINGLE SOURCE OF PRIMARY INFECTION.
- IT BASICALLY FORMS THE SEED FOR THE EPIDEMIC.
- IF R0 <1 THEN THE DISEASE WILL DIE DOWN IE., EACH PERSON CAN INFECT LESS THAN 1 PERSON THEN DISEASE CANNOT SUSTAIN
- IF R0 =1 THEN DISEASE STAYS ALIVE AND STABLE BUT WILL NOT OUTBREAK AS EPIDEMIC
- IF R0>1 THEN DISEASE CAN CAUSE AN EPIDEMIC
- ITS ALSO OBSERVED THAT DIFFERENT VARIANTS OF COVID HAVE DIFFERENT REPRODUCTIVE NUMBER
- R0 FOR DIAMOND PRINCESS CRUISE SHIP WAS AROUND 2.28 IN EARLY FEBRUARY
- R0 FOR THE TRIPLE MUTATING STRAIN FOUND IN INDIA IS ABOVE 3.00

SIR- MODEL

- THE MODEL HAS 3 COMPONENTS:
- ❑ S = SUSCEPTIBLE
- ❑ I = INFECTED
- ❑ R = REMOVED (RECOVERED OR DIED)

- THREE MAJOR ASSUMPTIONS OF THE MODEL:

1. THE TOTAL POPULATION REMAINS CONSTANT
2. THE RATE OF INFECTION WHICH IS PROPORTIONAL TO THE CONTACT BETWEEN THE INFECTED AND SUSCEPTIBLES IS A CONSTANT
3. THE REMOVAL RATE IS CONSTANT IE., THE DEATH AND RECOVERY RATE ARE CONSTANT

MATHEMATICAL REPRESENTATION:

- BASED ON THE ABOVE MAJOR ASSUMPTION WE MAKE THE FOLLOWING MATHEMATICAL REPRESENTATION :

- $\frac{dS}{dt} = -rIS$ RATE OF CHANGE OF SUSCEPTIBLES WRT TIME
- $\frac{dI}{dt} = rIS - aI$ RATE OF CHANGE OF INFECTED WRT TIME
- $\frac{dR}{dt} = aI$ RATE OF CHANGE OF REMOVAL WRT TIME

CONSTANTS USED:

- r – RATE OF CONTACT
- a – RATE OF REMOVAL (RECOVERED/DYING)
- I – NUMBER OF INFECTED INDIVIDUALS
- S – NUMBER OF SUSCEPTIBLES

IMPORTANT QUESTIONS:

1. WILL THE DISEASE SPREAD?

2. WHAT WILL BE THE NUMBER OF INFECTED?

3. HOW MANY PEOPLE WILL END UP CATCHING THE DISEASE?

WILL THE DISEASE SPREAD?

- INITIAL ASSUMPTION:

1. $(S = S_0, I = I_0, R = 0)$
2. RATE OF CHANGES WRT TIME IS CONSTANT $\frac{d}{dt}(S + I + R) = 0$
3. $S + I + R = S_0 + I_0$

- NUMBER OF SUSCEPTIBLES IS ALWAYS LESS THAN THE INITIAL POPULATION $(S \leq S_0)$
- WILL THE DISEASE SPREAD COMES DOWN TO THE FOLLOWING INEQUALITY:

$$\frac{dI}{dt} \leq I(rS_0 - a)$$

WILL THE DISEASE SPREAD?

- FROM THE ABOVE EQUATION WE CAN DEDUCE THAT:
- $S_0 > \frac{a}{r}$ i.e., THE INI.POPULATION IS ALWAYS GREATER THAN RATIO OF RATE OF REMOVAL TO RATE OF CONTACT

WE CAN REARRANGE THIS TERM TO GET CONTACT RATIO AS: $q = \frac{r}{a}$

- SO BASED OF ABOVE EQUATION WE CAN DEDUCE THAT DISEASE SPREADS IF: $S_0 > \frac{1}{q}$
- REARRANGING ABOVE EQUATION WE GET THE REPRODUCTION NUMBER WHICH TELLS US IF EPIDEMIC WILL OCCUR? AS $R_0 = \frac{rS_0}{a}$ $\quad IF\ THIS\ R_0 > 1$ THEN EPIDEMIC OCCURS

WHAT WILL BE NUMBER OF INFECTED?

- BY TAKING THE EQUATIONS OF $\frac{dS}{dt}$, $\frac{dI}{dt}$ AND DIVIDING THEM WE OBSERVE THAT THE RATE OF CHANGE OF INFECTION WITH RESPECT TO RATE OF CHANGE OF SUSCEPTIBLES IS:

$$\frac{d_I}{d_S} = \frac{rIS - aI}{-rIS} = -1 + \frac{a}{rS}$$

- WE CAN NOW REPHRASE THE ABOVE EQUATION WRT q AS: $\frac{d_I}{d_S} = -1 + \frac{1}{qS}$
- NOW INTEGRATING ABOVE EQUATION WE GET:

$$I + S - \frac{1}{q}\log S = I_0 + S_0 - \frac{1}{q}\log S_0$$

- I_{max} OCCURS WHEN $S = \frac{1}{q}$ AND IS GIVEN AS $I_{max} = I_0 + S_0 - \frac{1}{q}(1 + \log(qS_0))$
- BY TAKING ABOVE EQUATION AS A FUNCTION OF q WHEN WE PLOT THE EQUATION WE GET THE GRAPH OF INFECTION RATE(AS A FUNCTION OF CONTACT RATIO) VS q (CONTACT RATIO)

OBSERVATIONS:

FROM THE GIVEN GRAPH ITS OBSERVED THAT AS q INCREASES THE FUNCTION OF q(INFECTION RATE) BECOMES VERY SMALL.. WHICH TELLS US THAT THE

MAXIMUM NUMBER OF PEOPLE THAT CAN HAVE INFECTION AT ANY TIME IS:

TOTAL_POPULATION – f(q)

WHICH IS QUITE BAD FOR A DISEASE WITH LARGE q VALUES AS f(q) IS QUITE SMALL MEANING ALMOST ENTIRE POPULATION IS SUSCEPTIBLE TO INFECTION.

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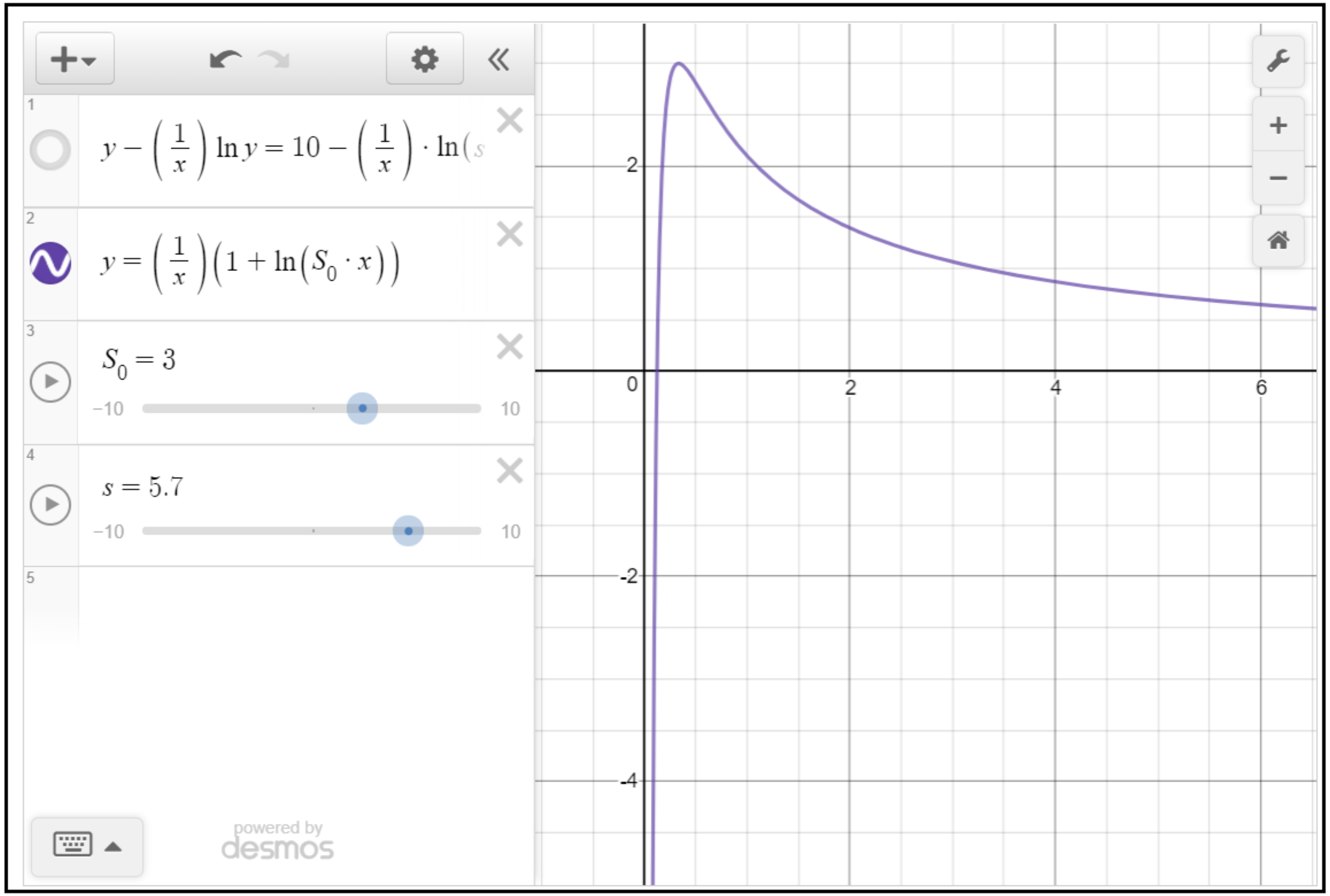

HOW MANY PEOPLE WILL END UP CATCHING THE DISEASE?

- SINCE THE POPULATION IS CONSTANT WE KNOW:

$$R + I + S = I_0 + S_0$$

- AT THE END OF PANDEMIC/EPIDEMIC NO OF INFECTIVES GOES TO 0 THEREFORE $R_{END} = (-S_{END}) + I_0 + S_0$
- HERE S_{END} IS UNKNOWN.
- TO CALCULATE S_{END} FOLLOWING EQUATION IS DERIVED
- $S_{END} - \frac{1}{q} * (\log S(END)) = I_0 + S_0 - \frac{1}{q} \log S_0$
- NOW PLOTTING THIS WE GET THE EFFECT OF q ON S_{END}

OBSERVATIONS:

FROM THE GIVEN GRAPH ITS OBSERVED THAT AS q INCREASES THE FUNCTION OF q(SUSCEPTIBLE RATE AT END) BECOMES VERY NEGLIGIBLE.

$$R_{END} = TOTAL\ POPULATION\ (I_0 + S_0) - S_{END}$$

HERE S_{END} IS NEGLIGIBLE FOR VERY LARGE VALUES OF q WHICH MEANS THAT ALMOST ALL OF THE POPULATION IS SUSCEPTIBLE TO THE INFECTION

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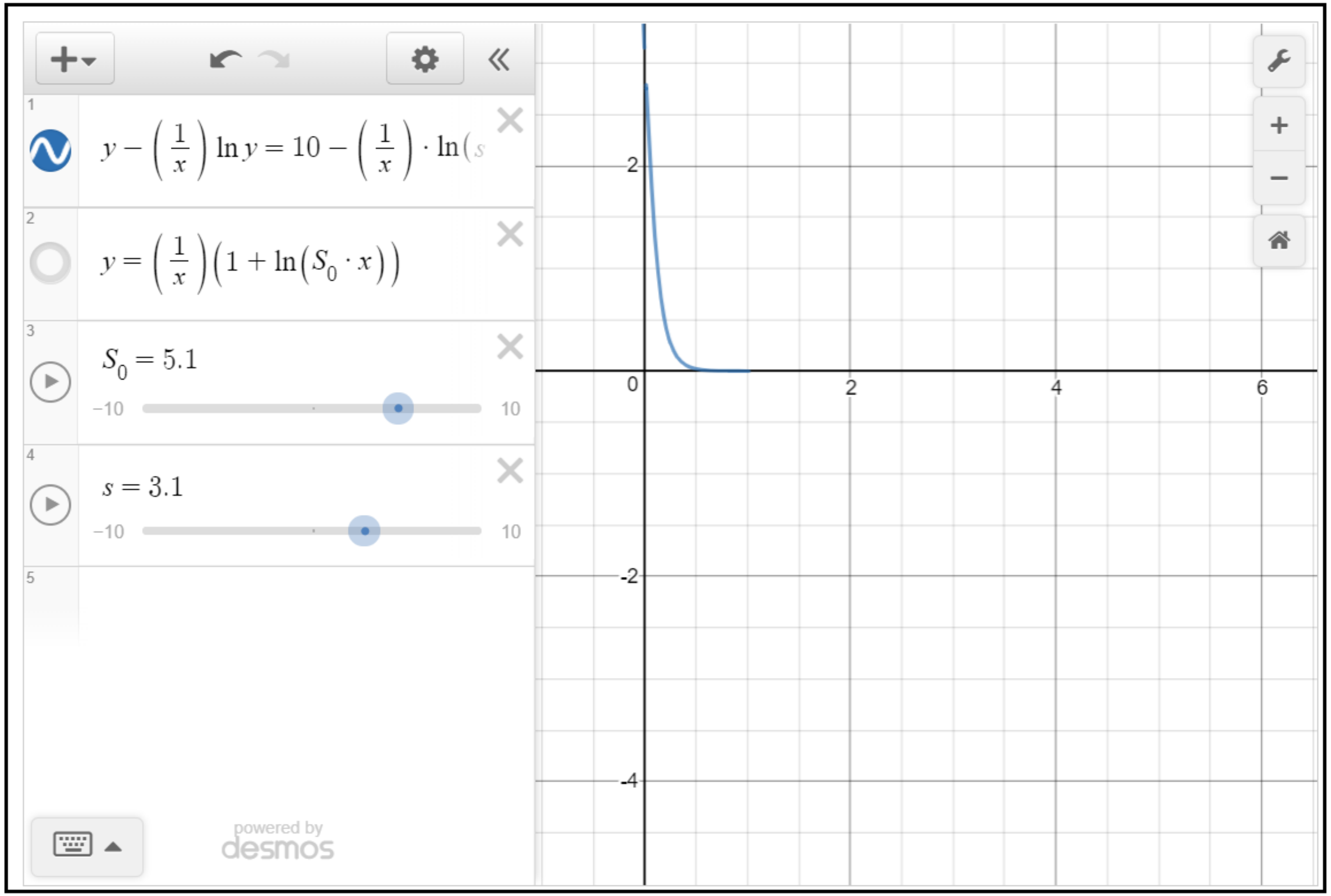

TRAVELLING WAVES

- UNTIL NOW WE HAD TAKEN THE ASSUMPTION OF A CONSTANT MOVEMENT NOW HERE WE MAKE AN ASSUMPTION THAT ONLY THE INFECTED MOVE.
- BASIC ASSUMPTIONS:

1. SUSCEPTIBLES DO NOT MOVE
2. INFECTED PERSONS MIGRATE AT A CONSTANT RATE
3. REMOVED DO NOT MOVE

    - $\frac{\partial S}{\partial t} = -rIS$
    - $\frac{\partial I}{\partial t} = rIS - aI + D\frac{\partial I^2}{\partial x^2}$
    - $\frac{\partial R}{\partial t} = aI$

TRAVELLING WAVES

- TO ANALYZE THE ABOVE EQUATION WE USE DIMENSIONALIZATION WE GET $R_0 = S_0 * q(contact\ ratio) \Rightarrow S_0\ * (\frac{r}{a})$ (fraction of susceptible that come into contact with infected individuals)
- ADDING NEW VARIABLE Y=X-Ct WE CAN CHANGE THIS EQUATION TO A DIFFERENTIAL EQUATION AS
  - $c\frac{dS}{dy} - IS = 0$
  - $\frac{d^2I}{dy^2} + c\frac{dI}{dy} + I\left(1 - \frac{1}{R_0}\right) = 0$
  - $\frac{\partial R}{\partial t} = aI$
- THE MODEL GIVES US ADDITIONAL POWERS AS WE CAN INTEGRATE THE EQUATION TO + AND – INFINITY TO MODEL CHANGES IN PAST AND FUTURE

TRAVELLING WAVES

- BY USING PHASE PLANE ANALYSIS ON THE ABOVE EQUATIONS WE CAN GET THE SOLUTION FOR THE MINIMUM POSSIBLE WAVE SPEED FOR A SOLUTION TO EXIST AND THIS IS GIVEN BY:

$$c \geq 2\sqrt{1 - \frac{1}{R_0}}$$

- THE MINIMUM WAVE SPEED IS GIVEN BY:

$$c = 2\sqrt{1 - \frac{1}{R_0}}$$

- IMPLICATION:

IN ORDER FOR SPEED OF THE INFECTION WAVE TO REDUCE WE NEED TO REDUCE THE R_0 VALUE. THIS IS INTURN DEPENDENT ON THE CONTACT RATIO(q)

INCUBATION PERIOD

- THE UNDERLYING ASSUMPTION IS THAT ONCE THE SUSCEPTIBLE IS IN CONTACT WITH THE INFECTED THERE IS A DELAY IN THE TIME THAT FROM THE TIME WHEN HE IS SUSCEPTIBLE TO THE TIME HE IS INFECTED.
- THE EQUATIONS CAN BE MODELLED AS:
- $\frac{dS}{dt} = -rI(t-\tau)S(t-\tau)$
- $\frac{dI}{dt} = rI(t-\tau)S(t-\tau) - aI(t)$
- $\frac{dR}{dt} = aI(t)$

INCUBATION PERIOD

- BY SOLVING THE ABOVE EQUATIONS USING TAYLOR SERIES EXPANSION WE CAN DEDUCE THAT:

$$\frac{dI}{dS} = \frac{-(rIS - aI)}{rIS} + o(a\tau, \tau^2)$$

IMPLICATION:

- THE RATE OF CHANGE OF THE SPREAD IS SIMILAR TO THE BASIC SIR MODEL AND DOES NOT EFFECT THE SPREAD A WHOLE LOT. THE τ AND τ^2 TERMS CAN BE IGNORED IN CASE THE INCUBATION PERIOD IS SMALL. BUT IF INCUBATION PERIOD IS LARGE THEN THE TERMS ADD THE NECESSARY FACTORS.

VACCINATION

- WE CAN STUDY THE EFFECT OF VACCINATION ON THE DISEASE SPREAD THROUGH TWO MAIN QUESTIONS:

1. CONDITION FOR DISEASE TO STOP AND HOW MANY PEOPLE DO WE NEED TO VACCINATE?

2. WHAT HAPPENS IF THE VACCINE IS NOT 100 PERCENT EFFECTIVE?

CONDITION FOR DISEASE TO STOP AND HOW MANY PEOPLE DO WE NEED TO VACCINATE?

- WE KNOW THAT $\frac{dI}{dt} \leq I(rs - a)$ AND ALSO WE KNOW THAT FOR THE PANDEMIC TO STOP WE NEED $\frac{dI}{dt} < 0$ AND FOR THAT TO HAPPEN WE NEED $R_0 = \frac{rs}{a} < 1$.
- HERE WE HAVE 3 FACTORS TO ANALYZE (r,S,a): FROM WHICH WE CAN CONTROL WITH A HIGHER DEGREE OF CERTANITY THE FACTOR S.
- WE KNOW $\frac{rs}{a} < 1$ SO THIS IMPLIES $S^* < \frac{a}{r}$ WHICH IS $\frac{1}{3}$ FOR COVID.
- WHICH MEANS WE NEED TO VACCINATE $\frac{2}{3}$ OF THE POPULATION.

HOW MANY VACCINES ARE NEEDED IN GENERAL?

- NUMBER OF VACCINATIONS NEEDED IN GENERAL CAN BE GIVEN BY::

$$S^* < \frac{1}{R_0}$$

- WE ALSO KNOW $S^* + V = 1$ SO WE CAN DEDUCE $V = 1 - S^*$
- FROM THE ABOVE EQUATION WE GET $V > 1 - \frac{1}{R_0}$
- SO WE NEED TO VACCINATE ATLEAST $1 - \frac{1}{R_0}$ OF POPULATION TO STOP THE PANDEMIC.
- LARGER THE VALUE OF R_0 MORE YOU NEED TO VACCINATE

WHAT HAPPENS IF VACCINES ARE NOT 100% EFFECTIVE?

- WE CAN ASSUME $V_{eff} > \left(1 - \frac{1}{R_0}\right)$ AS THE EFFECTIVENESS OF VACCINE
- ALSO $V_{eff} = eV$ (SOME FACTOR e * NUMBER OF VACCINATIONS)
- THEREFORE WE CAN CONCLUDE $V > \frac{1}{e}\left(1 - \frac{1}{R_0}\right)$
- LETS TAKE EFFECTIVENESS OF VACCINE AS 100 PERCENT -> WE NEED TO VACCINATE 67% OF POPULATION
- LETS TAKE EFFECTIVENESS OF VACCINE AS 95 PERCENT -> WE NEED TO VACCINATE 70% OF POPULATION
- LETS TAKE EFFECTIVENESS OF VACCINE AS 79 PERCENT -> WE NEED TO VACCINATE 84% OF POPULATION

THE END

THANK YOU SO MUCH.....

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