

① PROPOSITIONAL LOGIC

$$f_1 := A \wedge C$$

$$f_2 := B \vee D$$

$$f_3 := (A \rightarrow \bar{C})$$

$$f_4 := (\bar{C} \rightarrow \neg D)$$

$$f_5 := (B \rightarrow \neg(C \leftrightarrow A))$$

CONVERT EACH f_i TO CNF

$$f_1 := A \wedge C$$

f_1 ALREADY IN CNF.

$$f_2 := B \vee D$$

f_2 ALREADY IN CNF.

$$f_3 := (A \rightarrow \bar{C}) \Rightarrow \neg A \vee \bar{C} \text{ CNF.}$$

$$f_4 := (\bar{C} \rightarrow \neg D) \Rightarrow \neg \bar{C} \vee \neg D \text{ CNF.}$$

$$f_5 := (B \rightarrow (\neg C \vee \neg A)) \Rightarrow (\neg B \vee \neg C \vee \neg A) \text{ CNF}$$

DPL

①, ② SAT.

④, ⑦ SAT

- ① A
- ② C
- ③ B ∨ D
- ④ $\neg A \vee E$
- ⑤ $\neg E \vee \neg D$
- ⑥ $\neg B$
- ⑦ $\neg A \vee \neg B \vee \neg C \vee \neg D \vee \neg E$

$$\begin{array}{c} A := \top \\ C := \top \end{array}$$

- ③ B ∨ D
- ④ E
- ⑤ $\neg E \vee \neg D$
- ⑥ $\neg B$
- ⑦ $\neg B \vee \neg D \vee \neg E$

$$\begin{array}{l} E := \top \\ \neg B := \top \end{array}$$

- ③ D
- ⑤ $\neg D$
- ⑥ {}

CONTRADICTION

$$\begin{array}{l} A := \perp \\ C := \perp \end{array}$$

FOR THE
EMPTY CLAUSE
SOMEBODY IS
GOING TO THE
PARTY.

② FIRST ORDER LOGIC

THEORY

1. $\forall x. (\text{LOVES}(x, \text{NEGRONI}) \rightarrow \text{LOVES}(x, \text{AMERICANO}))$
2. $\forall x. (\text{LOVES}(x, \text{STINGER}) \vee \text{LOVES}(x, \text{NEGRONI}))$.
3. $\exists x. \neg \text{LOVES}(x, \text{AMERICANO})$
4. $\exists x. (\text{LOVES}(x, \text{STINGER}) \vee \text{LOVES}(x, \text{NEGRONI}))$

SENTENCES

① $\exists x. (\neg \text{LOVES}(x, \text{NEGRONI}) \wedge \neg \text{LOVES}(x, \text{STINGER}))$

FALSE, BECAUSE ALL X LOVES NEGRONI OR STINGER.

2: $\forall x. (\text{LOVES}(x, \text{STINGER}) \vee \text{LOVES}(x, \text{NEGRONI}))$

② $\forall x. (\neg \text{LOVES}(x, \text{STINGER}) \rightarrow \neg \text{LOVES}(x, \text{AMERICANO}))$

FALSE, BECAUSE IF EXIST X THAT NOT LOVES STINGER THEN LOVES NEGRONI (4.) AND IF X LOVES NEGRONI THEN LOVES AMERICANO (1.).

③ $\forall x. (\text{LOVES}(x, \text{NEGRONI}) \vee \text{LOVES}(x, \text{AMERICANO}))$.

FALSE, BECAUSE IF EXIST X THAT NOT LOVES AMERICANO THEN LOVES NEGRONI, BUT

FOR 1. THEN MUST LOVES AMERICANO.

④ $\forall x. (\text{LOVES}(x, \text{AMERICANO})$)

FALSE, BECAUSE FOR 3. EXIST AT LEAST ONE X THAT NOT LOVES AMERICANO.

⑤ $\forall x. ((\text{LOVES}(x, \text{NEGRONI}) \wedge \text{LOVES}(x, \text{AMERICANO})) \rightarrow \text{LOVES}(x, \text{STINGER}))$

FALSE

FALSE, BECAUSE FOR 1. FOR ALL X LOVES NEGRONI,
THEN LOVES AMERICANO.

TO PROVE THAT LET'S FIND A MODEL THAT
SATISFY THE THEORY AND THE NEGATION
OF SENTENCE.

$\neg ① \Rightarrow \forall x (\text{LOVES}(x, \text{NEGRONI}) \vee \text{LOVES}(x, \text{STINGER}))$

TRUE, BECAUSE 2. EVERY X LOVES NEGRONI OR
STINGER, SO:

① IS NOT LOGICAL CONSEQUENCE.

$\neg ② \Rightarrow \exists x. \neg (\neg \text{LOVES}(x, \text{STINGER}) \rightarrow \neg \text{LOVES}(x, \text{AMERICANO}))$

$\Rightarrow \exists x. \neg (\neg \text{LOVES}(x, \text{STINGER}) \vee \neg \text{LOVES}(x, \text{AMERICANO}))$

$\Rightarrow \exists x. (\neg \text{LOVES}(x, \text{STINGER}) \wedge \text{LOVES}(x, \text{AMERICANO}))$

TRUE, BECAUSE IF EXIST X THAT NOT LOVES

STINGER, FOR 2. LOVES NEGRONI THAN
FOR 1. LOVES AMERICANO

② IS NOT LOGICAL CONSEQUENCE.

$$\neg \textcircled{3} \Rightarrow \exists x. (\neg \text{Loves}(x, \text{NEGRONI}) \wedge \neg \text{Loves}(x, \text{AMERICANO}))$$

TRUE, BECAUSE IF EXIST X THAT NOT LOVES NEGRONI, THEN LOVES STINGER FOR 4.
ANYTHING ABOUT AMERICANO IN THE THEORY, SO:

③ IS NOT LOGICAL CONSEQUENCE

$$\neg \textcircled{4} \Rightarrow \exists x. (\neg \text{Loves}(x, \text{AMERICANO}))$$

TRUE, BY 3. SO:

④ IS NOT LOGICAL CONSEQUENCE

$$\neg \textcircled{5} \Rightarrow \exists x. \neg ((\text{Loves}(x, \text{NEGRONI}) \wedge \neg \text{Loves}(x, \text{AMERICANO}) \rightarrow \\ \rightarrow \neg \text{Loves}(x, \text{STINGER})) \Rightarrow$$

$$\exists x. \neg (\neg (\text{Loves}(x, \text{NEGRONI}) \wedge \neg \text{Loves}(x, \text{AMERICANO})) \vee$$

$$\vee \neg \text{Loves}(x, \text{STINGER})). \Rightarrow$$

$$\exists x. \neg ((\neg \text{Loves}(x, \text{NEGRONI}) \vee \text{Loves}(x, \text{AMERICANO})) \vee$$

$\vee \neg \text{LOVES}(x, \text{STINGER}) \Rightarrow$

$\exists x. (\neg(\neg \text{LOVES}(x, \text{NEGRONI}) \vee \text{LOVES}(x, \text{AMERICANO})) \wedge$

$\wedge \text{LOVES}(x, \text{STINGER})) \Rightarrow$

$\exists x. \text{LOVES}(x, \text{NEGRONI}) \wedge \neg \text{LOVES}(x, \text{AMERICANO}) \wedge$

$\wedge \text{LOVES}(x, \text{STINGER})$

FALSE, BECAUSE IF X LOVES NEGRONI
THEN LOVES AMERICANO.

⑤ IS LOGICAL CONSEQUENCE.