TEST SIMULATION 2

Propositional logic

$$T \begin{cases} \varphi_1 := p \to q \\ \varphi_2 := q \to (s \wedge t) \\ \varphi_3 := r \to (s \wedge t) \\ \varphi_4 := p \vee r \end{cases}$$

- Is $s \wedge t$ a logical consequence of T?

L.C.

- This is the case iff $T \cup \{\neg(s \wedge t)\}$ is inconsistent
- This is the case iff

SAT

$$\psi := \bigwedge_{i=1}^{4} \varphi_i \wedge \neg(s \wedge t) \text{ is unsatisfiable}$$

To obtain a CNF for ψ we convert to CNF each φ_i and $\neg(s \wedge t)$

$\varphi_1 := p \to q \qquad \boxed{CNF(\varphi_1)} := \boxed{\neg p \vee q}$

$\varphi_2 := q \to (s \wedge t) \qquad \boxed{CNF(\varphi_2)} := \neg q \vee (s \wedge t) \Longrightarrow \boxed{(\neg q \vee s) \wedge (\neg q \vee t)}$

$\varphi_3 := r \to (s \wedge t) \qquad \boxed{CNF(\varphi_3)} := \neg r \vee (s \wedge t) \Longrightarrow \boxed{(\neg r \vee s) \wedge (\neg r \vee t)}$

$\varphi_4 := p \vee r$ Already a CNF (a single clause)

$\neg(s \wedge t) \Longrightarrow \neg s \vee \neg t$ (a single clause)

Putting it all together

$$\underbrace{\bigwedge_{i=1}^{4} \varphi_i \wedge \neg(s \wedge t)}_{\psi} \Longrightarrow \left.\begin{array}{l} \cdot\ \neg p \vee q \\ \cdot\ \neg q \vee s \\ \cdot\ \neg q \vee t \\ \cdot\ \neg r \vee s \\ \cdot\ \neg r \vee t \\ \cdot\ p \vee r \\ \cdot\ \neg s \vee \neg t \end{array}\right\} CNF(\psi)$$

① $\neg p \vee q$

② $\neg q \vee s$

③ $\neg q \vee t$

④ $\neg r \vee s$

⑤ $\neg r \vee t$

⑥ $p \vee r$

⑦ $\neg s \vee \neg t$

no unit clauses
no pure literals
need to SPLIT!

$p := T$

① simplified
⑥ satisfied

① q
② $\neg q \vee s$
③ $\neg q \vee t$
④ $\neg r \vee s$
⑤ $\neg r \vee t$
⑦ $\neg s \vee \neg t$

$q := T$

② s
③ t
④ $\neg r \vee s$
⑤ $\neg r \vee t$
⑦ $\neg s \vee \neg t$

$s := T$
$t := T$

②, ③, ④, ⑤ satisfied

⑦ contradiction

$p := \perp$

① satisfied
⑥ simplified

② $\neg q \vee s$
③ $\neg q \vee t$
④ $\neg r \vee s$
⑤ $\neg r \vee t$
⑥ r
⑦ $\neg s \vee \neg t$

$r := T$

② $\neg q \vee s$
③ $\neg q \vee t$
④ s
⑤ t
⑦ $\neg s \vee \neg t$

$t := T$
$s := T$

②, ③, ④, ⑤ satisfied

⑦ contradiction

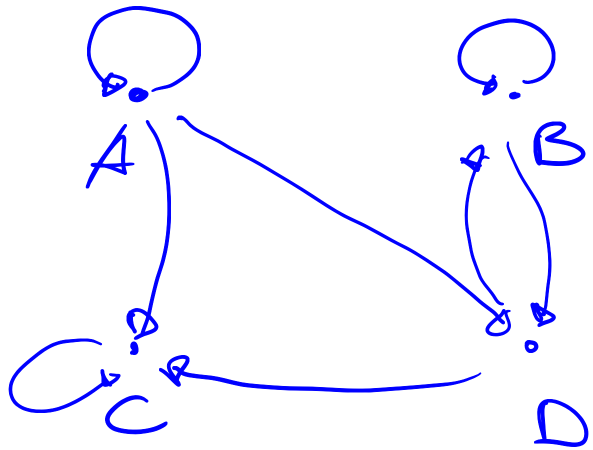
By DPLL we can conclude that CNF($\neg\psi$) is UNSAT

Therefore also $\neg\psi$ is UNSAT and thus $\boxed{T \models s \wedge t}$

First Order Logic

I $\quad D = \{\text{Abby}, \text{Bess}, \text{Cody}, \text{Dana}\}$

$g(\text{likes})$ ⇝ see Table
or directed
graph



nodes := individuals

edges := interpretation
of "likes"

Q $\models^{?}_{I} \forall x.\ \text{likes}(x,x)$ A NO, because $\neg\,\text{likes}(\text{dana}, \text{dana})$

Q $\models^{?}_{I} \forall x.\exists y.\ \text{likes}(x,y)$ A YES, $\text{likes}(\text{abby}, \text{abby})$ $\text{likes}(\text{dana}, \text{cody})$

$\text{likes}(\text{bess}, \text{bess})$

$\text{likes}(\text{cody}, \text{cody})$

Q $\models_I^{?} \exists y . \forall x . likes(x,y)$ "someone that everybody else likes"

A NO, there is no constant c such that $likes(x,c)$ for all objects x in the domain.

Q $\models_I^{?} \forall x . \forall y\ likes(x,y) \to likes(y,x)$ "likes is symmetric"

A NO, e.g. $likes(dana, cody)$ but $\neg likes(cody, dana)$

Q $\models_I^{?} \forall x . \forall y (\exists z\ likes(x,z) \wedge likes(z,y) \to likes(x,y))$ "likes is transitive"

A NO, e.g. $likes(bess, dana)$ $\neg likes(bess, cody)$

$likes(dana, cody)$