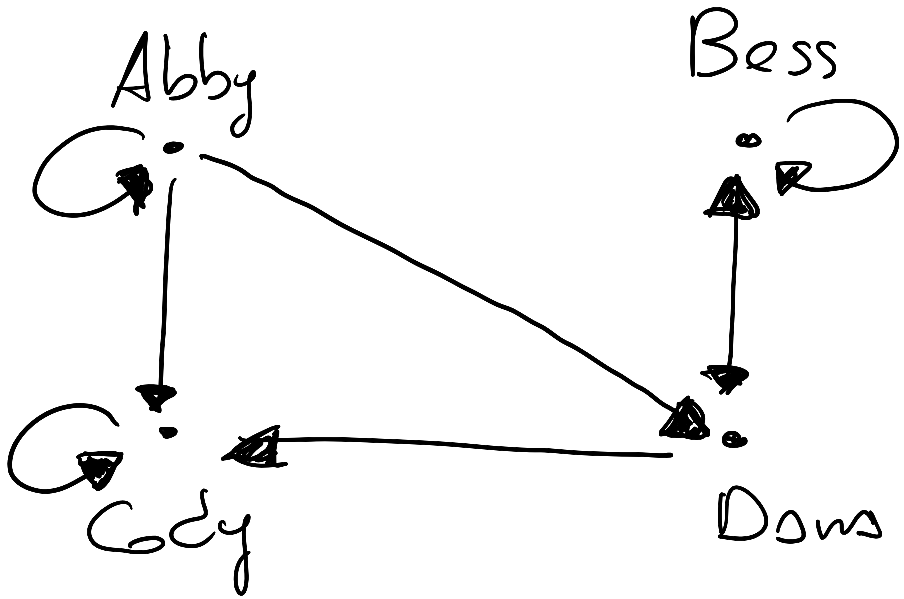
① Propositional Logic: same approach as Test of Sept. 17th, 2022

② First order Logic

We are given an interpretation $I = (D, \varrho)$

$D := \{\text{Abby}, \text{Bess}, \text{Cody}, \text{Dana}\}$

$\varrho(\text{likes}) :=$



$\longrightarrow$ "likes"

① $\forall x. \mathrm{likes}(x,x)$ not satisfied by I

$\neg \mathrm{likes}(\mathrm{Dana}, \mathrm{Dana})$ is satisfied by I

② $\forall x. \exists y. \mathrm{likes}(x,y)$

Intuitively this is satisfied "There is always an outgoing Arrow"

Formally

$$\left\{\begin{array}{l} \mathrm{likes}(\mathrm{Abby}, \mathrm{Abby}) \\ \mathrm{likes}(\mathrm{Bess}, \mathrm{Bess}) \\ \mathrm{likes}(\mathrm{Cody}, \mathrm{Cody}) \\ \mathrm{likes}(\mathrm{Dana}, \mathrm{Bess}) \end{array}\right.$$

for every possible instantiation of x I can find an instantiation of y s.t. $\models_I \mathrm{likes}(x,y)$

(3) $\exists y . \forall x . likes(x,y)$ A "celebrity", someone liked by everyone else, including herself.

This is not satisfiable in I because for all possible instantiations of y we can find someone that does not like y.

$$\neg \exists y \, \forall x \, likes(x,y) \equiv \forall y \, \exists x \, \neg likes(x,y)$$

$\neg likes(Bess, Abby)$

$\neg likes(Abby, Bess)$

$\neg likes(Bess, Cody)$

$\neg likes(Dana, Dana)$

every possible instantiation of y

exists an instantiation of x

④ $\forall x \forall y\,(likes(x,y) \to likes(y,x))$

Intuitively this is not satisfied because "likes" is not symmetric

Given our intuition we try to show that

$$\models_I \neg \forall x \forall y\,(likes(x,y) \to likes(y,x))$$

$$\models_I \exists x \exists y\,(likes(x,y) \wedge \neg likes(y,x))$$

Need to find a "counterexample"

$$likes(Abby, Cody) \wedge \neg likes(Cody, Abby)$$

(5) $\forall x. \forall y. (\exists z (likes(x,z) \wedge likes(z,y)) \rightarrow likes(x,y))$

By looking at the interpretation, we think there is a case that does not satisfy $\forall x. \forall y$ ....

Let's prove that $\models_I \neg \forall x \forall y (\exists z (likes(x,z) \wedge likes(z,y)) \rightarrow likes(x,y))$

$$\boxed{\neg(\square \rightarrow \square) \equiv \square \wedge \neg \square}$$

$\models_I \exists x \exists y (\exists z (likes(x,z) \wedge likes(z,y)) \wedge \neg likes(x,y))$

$x := Abby$ $y := Bess$ $z := Dow$

likes(Abby, Dow)

likes (Dow, Bess)

$\neg$ likes (Abby, Bess)