Intro to AdS/CFT correspondance

Buisine Léo
Ecole Normale Superieure of Paris

February 20, 2025

Contents

1 Introduction **5**

2 Conformal invariance **7**
2.1 Non abelian gauge theories 8
2.2 Large N limit 9
2.3 Correlation functions in (large N) CFTs 10

3 Anti de Sitter spacetime **13**
3.1 Geometry 13
3.2 The Scalar field 15

Chapter 1

Introduction

A duality is a relation between two theories with physically different degrees of freedom. AdS/CFT correspondance is a strong-weak coupling correspondance.

There are multiple kinds of dualities

- A weak duality is a correspondance between physically and mathematically two different theories, where the correspondance exist only in a certain limit.
- A strong duality is a true equivalence between two theories, it is an isomorphism of theories.

The AdS/CFT is a strong duality, between a theory in 10D with gravity and a theory in 4D without gravity. What would be a hint for a duality? If one theory has a global (not a gauge symmetry, since a gauge symmetry is unphysical) symmetry, then the other theory should also have it.

We call it the AdS/CFT correspondance because even though we have many exemples now, the first exemple was the correspondance between a CFT in 4D and a theory on the spacetime $\mathrm{AdS}_5 \times \mathbb{S}^5$, the product of a 5 dimensional Anti de Sitter space and a 5 dimensional sphere.

We don't need to take notes since the notes are on the moodle.

Chapter 2

Conformal invariance

The conformal Killing equation is

$$\partial_\rho \epsilon_\sigma + \partial_\sigma \epsilon_\rho = \frac{2}{d}(\partial^\mu \epsilon_\mu)\eta_{\rho\sigma} \tag{2.1}$$

It defines a general Killing vector.

We can locally modify the stress-energy momentum tensor to make it symmetric, thanks to Loentz invariance. Moreover,

$$T_\mu{}^\mu = 0 \Leftrightarrow \text{ conformal invariance} \tag{2.2}$$

Classically, this means also that all couplings are dimensionless, and reciprocally the dimensionless of all couplings imply conformal invariance.

How can we compute $T_\mu{}^\mu$ easily? Suppose we have a generic Lagrangian

$$\mathcal{S} = \int \mathrm{d}^4 x \sum_i g_i \mathcal{O}_i(x) \tag{2.3}$$

where the $O_i(x)$ are any monomial in the fields, which we will call operators. Let's assume that the operators have a given conformal weight Δ_i.

$$x \to \lambda x \qquad \mathcal{O}_i \to \lambda^{-\Delta_i}\mathcal{O}_i \tag{2.4}$$

The dimension of g_i is $d - \Delta_i = \Delta_{g_i}$. Scale invariance is preserved iff $\Delta_{g_i} = 0$.

The spurion trick: we can take the g_i to be dynamical fields (which are physically constants), which we pretend to have conformal weight Δ_{g_i}. In this fake world, there is conformal invariance. It is equivalent to scale the coupling when we rescale the theory, which is trivially invariant under scale transformation. In this fake world,

$$0 = \delta S = \int \sum_{\text{fields}} \frac{\delta S}{\delta \phi_i(x)} \delta\phi_i(x) + \int \frac{\partial \mathcal{L}}{\partial g_i} \delta g_i \ \mathrm{d}^d x \tag{2.5}$$

which gives

$$T_\mu{}^\mu(x) = -\sum_i \Delta_{g_i} g_i \mathcal{O}_i(x) \tag{2.6}$$

showing by how much scale invariance is broken locally. To get to this fake world, we can introduce a field μ transforming as

$$\begin{aligned} x^\mu &\to \lambda x^\mu \\ \mu &\to \lambda^{-1}\mu \end{aligned} \tag{2.7}$$

And then give a dependance of the coupling in μ.

$$g_i(\mu) = g_i \left(\frac{\mu}{\mu_0}\right)^{-\Delta_{g_i}} \tag{2.8}$$

But since $g_i(\mu)$ is fixed in the fake world (that is why we introduced the fake world in the first place),

$$\frac{\partial g_i}{\partial \log(\mu)} = \Delta_{g_i} g_i \equiv \beta_i(g_i) \tag{2.9}$$

We can obtain a dependance in μ of the coupling parameters in the real world too, with the right renormalization scheme (introduce a cutoff and cancel it by adding an interaction term, which normalizes the cutoff using μ). Similarly, looking at δZ, we find

$$< T^\mu_\mu >= -\sum \beta_i(g_i) < \mathcal{O}_i > \tag{2.10}$$

We name $\gamma = \frac{1}{g}\beta(g)$ the anomalous dimension. Theories with scalars or fermions can only have positive beta functions. What about theories with vectors, such as non abelian gauge theories?

2.1 Non abelian gauge theories

Just like QED but we replace the $U(1)$ symmetry by a non abelian group.

$$\Phi^i(x) \to U^i_j(x)\Phi^j(x) \tag{2.11}$$

THe Yang Mills action

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \tag{2.12}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \tag{2.13}$$

THis describes a theory already interacting by itself. The theory is classically conformal invariant, and renormalizable (although it is hard to show that it is renormalizable). However, it is not conformal at the quantum level. In pure Yang-Mills, without any matter,

$$\beta(g) = -\frac{11}{3}g^3\frac{N}{16\pi^2} \tag{2.14}$$

where N is the Casimir invariant.

In SYM, g is a true physical parameter of the theory. In QCD, we replace the coupling constant g by a running coupling $g(\mu)$, so 1 QFT is actually a curve of QFTs in the parameter space, and we can follow the flow from high energy to low energy. In other words, the coupling constant depends on the scale of the experiment/measurement. In SYM, g is a constant coupling, since it is invariant under the RG flow. In other words, CFT are fixed points of the RG flow. In other other words, CFTs are the end points of trajectories in RG space. A continuous family of CFTs correspond to an (absorbing?) line of fixed point in RG space, a horizon. From the point of view of operators, a CFT is a theory where all coupling which are positive in mass dimension are set to 0. RG gives the universality of physics at low energy. An operator which moves from a CFT to a CFT is called an (exactly) marginal operator.

We can move by infinitesimal transformations from SYM with weak coupling (which is solvable from perturbation theory) to SYM with strong coupling. The ground-breaking discovery is that SYM at strong coupling is dual to string theory (quantum gravity). 't Hoft discovered that we can reorganize the feynman diagram expansion of the correlation functions as feynman diagrams of string theory, in the large N limit.

2.2 Large N limit

We are taking the SYM for $SU(N)$ with $N \to +\infty$, so A_μ is a matrix with $N^2 - 1$ real parameters.

$$\Lambda = \mu e^{-\frac{1}{b_0 g^2(\mu)}} \qquad b_0 = \frac{11N}{3} \qquad \beta(g) = -b_0 g^3 \tag{2.15}$$

Is there a way to bring g to 0, while keeping Λ constant? In other words, is there a way to approach a free field theory but still with strongly coupled behaviour? We want to define a new running coupling, which we want to keep finite:

$$Ng^2(\mu) = \lambda(\mu) \tag{2.16}$$

If we compute the beta function of λ, we see its linear expansion is independant of N.

$$\beta(\lambda) = -\frac{22}{3}\lambda^2 \tag{2.17}$$

Large-N theories: we take our elementary field to be a matrix, which we take to be in the adjoint of $SU(N)$ say.

$$M(x), \quad M^\dagger = M, \quad \{M_i^j(x)\}_{i,j=1\ldots N} \text{ being scalar fields} \tag{2.18}$$

$$S = \int \frac{1}{g^2}\left[\mathrm{Tr}(\partial_\mu M^\dagger \partial^\mu M) + V(\{\mathrm{Tr}(M^n)\})\right] \tag{2.19}$$

the sum of the kinetic term and some potential, it is the most basic action invariant under $M \to UMU^{-1}$. When we have cubic potential term, we can always pull the coupling constant in fron as in above by a redifinition of the field. The field is not canonical, but idc we can always renormalize it. In YM, we can also use the same trick since the coupling in front of the quartic term is in g^2.

Consider a cubic lagrangian

$$\begin{aligned} S &= \int \frac{1}{g^2}\mathrm{Tr}\left\{[\partial_\mu M^\dagger \partial^\mu M] + M^3\right\} \\ &= \int \frac{N}{\lambda}[(\partial M)^2 + M^3] \end{aligned} \tag{2.20}$$

So in the large N limit, S diverges, but we don't care. Expliciting the trace, we have

$$\frac{N}{\lambda}\left[\partial_\mu M_i{}^j \partial^\lambda M_k{}^l \delta_j^k \delta_l^i + M_j^i M_l{}^k M_m{}^l \delta_i^l \delta_k^m \delta_l^j\right] \tag{2.21}$$

We can directly write Feynman rules for this thing. For the propagator between M_j^i and M_k^l, we would have $\frac{\delta_{jk}\delta^{il}}{p^2}\frac{\lambda}{N}$. But j only connects with k, and i with l. The idea of 't Hooft was to write the propagator as two lines, oriented in different directions. Each line is associated with a delta δ^{il} and δ_{jk}, and the total double line is associated with $\frac{\lambda}{N}\frac{1}{p^2}$. We can see the propagator of the whole field as two propagators of field in the fundamental representations, of fields with indices between 1 and N. Interestingly enough, the exact same thing happens for the 3 point interaction. We can draw it as 3 lines which never cross, with consistency of the orientation of the propagators. In fact, the same thing would work for a quartic interaction: whenever we have a trace, the lines never cross.

Now, suppose we have a Feyman diagram, with V vertices, P propagators and L loops. Vertices bring a factor $\left(\frac{N}{\lambda}\right)^V$, propatagors a factor $\left(\frac{N}{\lambda}\right)^{-P}$, and loops bring a factor N^L. So we would have an overall factor $\left(\frac{N}{\lambda}\right)^{V-P} N^L$. The number $V-P+L$ is the Euler index of the manifold associated to the Feyman diagram !!! wtf?

We have a double expansion (possible), in λ and in N. We can keep λ 1, but we can remove all diagrams which are of order lower than N^2 (leading order). At each order in N, we can then expand in λ.

The powerful result: terms in N^2 can be put on the 2-sphere, terms in N^0 can be put on the torus, etc etc. Actually, the power in N is equal to $2-2g$, with g the genus. Since we are looking at the vacuum diagrams, we are looking at a vacuum string propagation. The worldsheet of a string starting from the vacuum (a point) and ending similarly is a closed 2-manifold. The genus of this manifold is the number of splits of the string. So our expansion in the genus is exactly the expansion of the string propagation in the number of splittings. In a string theory, the probability for a string to split is g_S. The idea from 't Hooft is to see the duality between SYM in the large N limit and weakly coupled string theory, with $g_s = \frac{1}{N}$. In general, the idea is hard to put into practice because the correlation functions are hard to compute. But in $N=4$ SYM, we can control the correlation functions pretty well, to the point that we can recognize the string theory.

2.3 Correlation functions in (large N) CFTs

The basic objects of a CFT are the operators $\hat{O}(x)$. Then, the "observables" are correlation functions of these operators. We call them green functions, depending on the string of operators.

$$G_n(x) = \langle \hat{O}_i(x_i) \dots \hat{O}_n(x_n) \rangle \tag{2.22}$$

The conformal symmetry will give a huge quantity of constraints on these green functions. A primary operator is a primary weight of the representation of the

Virasoro algebra. Iff we have

$$[\hat{D},\hat{O}(x)] = -i(x^\mu\partial_\mu + \Delta)O(x) \tag{2.23}$$

aka $\hat{O}'(x') = \lambda^{-\Delta}(x)O(x)$, then $\hat{O}(x)$ is a quasi primary operator of weight Δ.

Since

$$\begin{aligned} [D,P_\mu] &= -iP_\mu \\ [D,K_\mu] &= iK_\mu \end{aligned} \tag{2.24}$$

we see that we have a ladder algebra, where P_μ transforms a quasi-primary field into a quasi-primary field with conformal dimension $\Delta - 1$ (it lowers the weight) and similarly for K_μ but raising the weight by 1. Since we want a physical representation, we must have a lowest weight operator. This is a primary operator.

Definition 2.3.1. A primary operator O_Δ is such that $[K_\mu, O_\Delta] = 0$

The primary operator, when acted upon by the P_μ and K_μ, generates a Verma module. In particular, the stress energy tensor is a quasi-primary field. But composite fields without derivatives (P_μ acts with derivatives) are primary. Note that we want a lowest weight and not a highest weight because if we were to choose a highest weight, it would become a nightmare in momentum space (taking the derivative annihilates the field).

"Theorem": if we do a local conformal transformation on the spacetime coordinates $x'_\mu = f_\mu(x_\nu)$, $\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = \Lambda^2(x)\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu$, the green function transforms as

$$G_n(x_1,\ldots,x_n) = \Lambda(x_1)^{\Delta_1}\ldots\Lambda(x_n)^{\Delta_n}G_n(x'_1,\ldots,x'_n) \tag{2.25}$$

where the conformal weights are those of the associated operators. The operators must be quasi-primary for it to work. This is a highly non trivial statement: if we know the green function at some point, we know it everywhere.

Let's see the consequences for a 2-point function.

$$\langle O_I(x_1)O_J(x_2)\rangle = G_{IJ}((x_1-x_2)^2) = \frac{1}{(x_1-x_2)^{2\Delta}} \tag{2.26}$$

where Δ is the common conformal weight of both fields (if the conformal weights are not equal, the propagator must vanish). We obtain the factor 1 on top by a proper redefinition of the fields.

For the 3-point function, we have

$$< O_i(x_1)O_j(x_2)O_k(x_3) >= \frac{C_{ijk}}{(x_1-x_2)^{\#}(x_3-x_2)^{\#}(x_1-x_3)^{\#}} \tag{2.27}$$

where the C_{ijk} are entirely determined by the conformal field theory. Now let's look at the stress energy tensor.

$$< T_{\mu\nu}(x)T_{\rho\sigma}(y) >= \Pi_{\mu\nu\rho\sigma}\frac{c}{(x-y)^{2d}} \tag{2.28}$$

The stress-energy tensor is symmetric, traceless, and obeys conservation equations. This puts a whole lot of conditions on $\Pi_{\mu\nu\rho\sigma}$. c is a constant, and is a property of the theory (fixed by the action, since the stress energy tensor is directly fixed by the action). It is often called the central charge (it is the central charge in $d = 2$, the story is more difficult in general).

Now we want to define correlation functions in the large N limit. We want to compute the N point function. We can take operators as

$$O_1(x) = \mathrm{Tr}(M(x)\dots M(x)) \tag{2.29}$$

or we could take

$$O_2(x) = \mathrm{Tr}(M(x)\dots M(x))\mathrm{Tr}(M(x)\dots M(x)) \tag{2.30}$$

Taking the trace is almost mandatory for $SU(N)$ invariance. But there are many ways to do it, we can write in general $O_n(x)$ a n-trace operator.

Let's consider first 1-trace operators.

$$< O_1(x)O_1(y) >= \frac{\delta}{\delta J(x)}\frac{\delta}{\delta J(y)}\log\left[\int \mathcal{D}[M]e^{i\frac{N}{\lambda}\int L[M]+i\int J(x)O_1(x)}\right] \tag{2.31}$$

But if J is fixed, it gets suppressed by N. We thus want to keep $\tilde{J} = \frac{J}{N}$ fixed, aka we want to take the large J limit together with the large N limit.

$$< O_1(x)O_1(y) >= \left(\frac{\delta\tilde{J}}{\delta J}\right)^2 \frac{\delta}{\delta\tilde{J}(x)}\frac{\delta}{\delta\tilde{J}(y)}\log\left[N^2 f_0(\lambda,\tilde{J}) + f_1(\lambda,\tilde{J}) + o(1/N)\right] \tag{2.32}$$

with $\left(\frac{\delta\tilde{J}}{\delta J}\right)^2 = \frac{1}{N^2}$, which shows that the 2-point correlation function doesn't scale with N, in the large N limit. f_0 is the contribution from planar diagrams, f_1 the contribution from genus 1 diagrams, etc...

This generalizes to n-point functions.

$$< O_1(x_1)\dots O_n(x_n) > \Big|_{\tilde{J}=0} = \frac{1}{N^{n-2}}\sum h_0(\lambda) + \frac{1}{N^2}h_1(\lambda) + \dots \tag{2.33}$$

with $\sum h_0(\lambda)$ corresponding to planar diagrams. In particular, we see from this formula that in $N \to +\infty$, all connected correlation functions vanish except for the propagator, just like in free field theory. The theory is not free, since the propagator takes the form $\sim \frac{1}{|x-y|^{2\Delta}}$. Hence, we say it is a generalization of free field theories. We should see the states created by operators acting on the vacuum as "bound states" (though not exactly).

We can also notice that the theory, although not free, has interactions with coupling of the order $\frac{1}{N}$ for the 3-point interaction, $\frac{1}{N^2}$ for the 4-point correlation, etc... So we can see this theory as a weakly coupled theory, with coupling $\frac{1}{N}$ (hints of the duality with strings).

What we just did was for single-trace operators. For multi-trace operators, let's take

$$O(x) = O_1(x)\dots O_n(x) \tag{2.34}$$

where $O_i(x)$ are single trace operators. Suppose we take an n-point function of O.

$$\langle O(x_1)O(x_p)\rangle \simeq (np)\text{-point function} \sim \frac{1}{N^{np}} << \frac{1}{N^p} \tag{2.35}$$

Chapter 3

Anti de Sitter spacetime

3.1 Geometry

$$\mathrm{d}s^2 = \Omega^2(\tau, \theta)[-\mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\Omega_{d-2}^2] \tag{3.1}$$

with $\theta \in [0, \pi]$, $|\tau \pm \theta| < \pi$. This is the flat Minkowski spacetime in d-dimension, but it is convenient because all infinities are at a finite distance. This is nice for physics because we can see physics in this space as physics in a box, except the boundaries can be reached only by massless particles, in infinite time.

Now, $d+1$ anti de Sitter space is a space naturally embedded in $\mathbb{R}^{2,d} \equiv < x_{-1}, x_0, x_1, \dots x_d >$ with metric $\mathrm{d}s^2 = -\mathrm{d}x_{-1}^2 - \mathrm{d}_0^2 + \sum_i \mathrm{d}x_i^2$. Inside this space, the AdS space is defined by the equation

$$-x_{-1}^2 - x_0^2 + \sum_i x_i^2 = -R \tag{3.2}$$

. Note that it is some kind of hyperboloid space. If the radius was positive, this would be some kind of hypersphere. We can hope to write global coordinates in the form

$$\begin{aligned} x_{-1} &= F(\rho)\sin(\tau) \\ x_0 &= F(\rho)\cos(\tau) \\ x_i &= G(\rho)\Omega_i \end{aligned} \tag{3.3}$$

Where the Ω_i are coordinates on $\mathbb{S}_{d-1}$, verifying $\sum_i \Omega_i^2 = 1$. Solving the equation of AdS for these equations, we get

$$-F^2(\rho) + G(\rho)^2 = -R^2 \tag{3.4}$$

Such that we can write

$$\begin{aligned} F(\rho) &= R\cosh(\rho) \\ G(\rho) &= R\sinh(\rho) \end{aligned} \tag{3.5}$$

If we take all coordinates from $-\infty$ to $+\infty$, we would cover the space multiple times. Actually, a minimal range of coordinates is $\rho \in [0, +\infty[$ and $\tau \in [0, 2\pi]$. For these coordinates, the metric becomes

$$\mathrm{d}s^2 = R^2\left(\cosh^2(\rho)\mathrm{d}\tau^2 + \mathrm{d}\rho^2 + \sinh^2(\rho)\mathrm{d}\Omega_{d-1}^2\right) \tag{3.6}$$

We see it is a Lorentzian spacetime. Moreover, we have a time direction, τ. However, we have closed time-like curves because $\tau \in [0, 2\pi]$. We don't want to do physics in such a space, so we take the universal cover, meaning we take $\tau \in]-\infty, +\infty[$. This removes any closed time-like curve.

Now, we would like to compactify the space through conformal transformations. We write

$$\mathrm{d}\rho = \sinh(\rho)\mathrm{d}\theta \tag{3.7}$$

so

$$\tan(\theta) = \sinh(\rho) \tag{3.8}$$

with $\theta \in [0, \pi/2]$. The metric becomes

$$\mathrm{d}s^2 = \frac{R^2}{\cos(\theta)}\left[-\mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \sin(\theta)^2\mathrm{d}\Omega_{d-1}^2\right] \tag{3.9}$$

This is just like a cylinder, except it is cut in 2 since $\theta \in [0, \pi/2]$. It is thus half a cylinder. Each time slice is half a sphere, a sphere cut at the equatorial line, a curved disk. This spacetime has a boundary, which is time-like. This means that massless particles really live in a box. They are confined by a boundary, and we need to define how they act upon reaching this boundary: are they reflected? absorbed? However, we will see that massless particle do not see the boundary, they never reach it (see TD).

So AdS is conformally half a cylinder, but it is not really a cylinder. Seeing the factor in front of the metric, we see that $\theta = \frac{\pi}{2}$ is out of the spacetime, since there is a coordinate singularity there. The equation giving AdS hints at the fact that AdS is a maximally symmetric spacetime. AdS is completely invariant under Lorentz transformations, but not under Poincare transformations. Since it is maximally symmetric, we can also guess it has constant curvature, in this case negative constant curvature. Btw this spacetime has a boundary, but is geodesic complete. Particles can reach the boundary only in infinite time.

In these coordinates, we see that AdS is conformally a half cylinder. But let's look at another set of coordinates, highlighting another property of the space. Let's define the Poincare coordinates $(z, \chi^\mu)_{\mu=0,\ldots,d-1}$ with $z > 0$. We define in the embedding space $\mathbb{R}^{2,d} =< X_{-1}, \ldots, X_d >$

$$\begin{aligned}
X_{-1} &= \frac{R^2}{2z}(1 + \frac{z^2 + \chi_\mu\chi^\mu}{R^2}) \\
X_0 &= \frac{R}{z}\chi^0 \\
X_i &= \frac{R}{z}\chi^i \qquad i = 1, \ldots, d-1 \\
X_d &= \frac{R^2}{2z}(1 + \frac{\chi_\mu\chi^\mu - z^2}{R^2})
\end{aligned} \tag{3.10}$$

Using these, we find the metric

$$\mathrm{d}s^2 = \frac{R^2}{z^2}\left[\mathrm{d}z^2 + \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu\right] \tag{3.11}$$

We see that the AdS space here is conformally half a Minkowski space. We can clearly see Poincare invariance in d dimensions (along every coordinates except

z). In other words, we have a whole minkowski space times a half line. But these definitions do not cover all of the AdS space. We only see a part of the space, this is a local AdS. Note that before, the boundaries of the space where $\theta = \frac{\pi}{2}$ or in the other coordinates $\rho = +\infty$. Now, the boundary is $z = 0$.

After a few computations, we see that the algebra of symmetries of the AdS_{d+1} space is the conformal algebra of a d dimensional flat space. In term of vocabulary, the bulk is everything that is not the boundary. The d dimensional conformal algebra acts in the bulk with some action, but act on the boundary (which is d dimensional) exactly as it should, and leaves it invariant. In other words, the symmetry algebra of the space acts on the boundary as general conformal transformations.

A small variation of AdS is EAdS, Euclidian anti de Sitter space. Instead of taking the space embedded in $\mathbb{R}^{2,d}$, we take it inside $\mathbb{R}^{1,d+1}$. The space gets a euclidian metric, and the space of symmetries is the euclidian conformal algebra, acting on the boundary which is an Euclidian space by conformal transformations. To find the de Sitter space, we take the same embedding space as for EAdS but we put a R^2 instead of $-R^2$ on the right side of the equation. Looking at the Poincare metric, we can analytically go from AdS to EAdS by switching the Minkowski metric with the euclidian metric, and go from AdS to dS by switching the minkowski metric with a Euclidian one but flipping the sign in fron of the $\mathrm{d}z^2$.

Now, the real question is does this space physically matter? The answer is yes, it is a solution of the Einstein equations in the absence of matter and with only a cosmological constant, which is negative. For a negative cosmological constant Λ, we have

$$R^2 = -\frac{d(d-1)}{2\Lambda} \tag{3.12}$$

In other words, AdS_{d+1} is the maximally symmetric spacetime with constant negative curvature. Now, let's try to see the dynamics of physics inside anti de Sitter.

3.2 The Scalar field

To put a scalar field in anti de Sitter, we can try to write an action

$$\mathcal{S} = \int \mathrm{d}^{d+1}x\sqrt{g}\{g^{ab}\partial_a\phi\partial_b\phi - m^2\phi^2\} \tag{3.13}$$

The solution for the boundary is adding the Gibbon-Hawking term, which leads to the Israel Junction condition. For now, we will consider the boundary field fixed, $\delta\varphi_-(x^\mu) = 0$. If we want to look at the quantum theory, we have the partition function

$$Z[\varphi_-] = \int \mathcal{D}\phi e^{iS[\phi]} \tag{3.14}$$

where $\phi(x^\mu, z) \to z^{\Delta_-}\varphi_-(x^\mu)$.

We define

$$G_n(x_1, \ldots, x_n) = \frac{1}{Z}\frac{\delta}{\delta\varphi_-(x_1)}\cdots\frac{\delta}{\delta\varphi(x_n)}Z[\varphi_-] \tag{3.15}$$