From the Hot Universe to the Dark Matter

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Chapter 1

Introduction

Tout est sur le moodle
Examen: presenter un article comme si on l'avait fait, slides etc
Chaque cours est independant des autres.

Dark universe = tout ce qu'on voit pas (neutrino, matiere noire, primordial black holes (PBH), etc...)

Chapter 2

Historical aspects of the dark matter

Don't hesitate to read historical articles.
It is important to smell the numbers

2.1 Some numbers

The age of the (visible) universe: 13.8 billion years old (it is big and not big at the same time)
Cosmological principle: everyone in the universe sees the same universe.
The hubble law is the only law which preserves the cosmological principle
When was produced the CMB (cosmological microwave background)? 380 000 years (after the big bang)
What is the size of the Universe (radius of the universe): 93 billion light-years
Can galaxies recede faster than light? Yes. The galaxy doesn't feel like receding faster than light, but it can through the added effects of a warping/everchanging spacetime. There is no limit on the warping of spacetime
Cosmological horizon: horizon beyond which we cannot see light, because the metric extends so fast that the light doesn't move forward to us (the spacetime extends and pushes the light farther)
Value of the Hubble constant 70km/s/Mpc
Anything at more than 10 Mpc recedes faster than they come to us to the gravity: they are not gravity bound. Andromeda is at 700kpc. In 40 million years, Everything which is not Andromeda nor us will cross the cosmological horizon. In 4 million years, the sun will explode but at approximately the same time, the Milky way will merge with Andromeda: it will become the Milkomeda
A parsec is 3 lightyears
Andromeda is the nearest galaxy, at around 780kpc
THe Milky way has a size of around 60 000 lyrs, for a mass of $10^1$2 the mass of the sun
Distance of the sun to the galaxy center: 8.5 kpc
Velocity of the earth around the sun: 30km/s
Velocity of the sun in the galaxy: 220km/s

Velocity of the Galaxy in the Local cluster: 650 km/s
Typical kinetic energy of a proton: mv^2 $1keV$ typical energy of a gas in a galaxy
Mean density of photon in the Universe: 411 cm^{-3} (so 10^90 photons in the universe)
Density of protons around the sun 1 cm^{-3}
Density of protons in the Milky way 10^{-3} cm^{-3}
Density of protons between galaxies 1 m^{-3}
Mean density of baryons in the Universe 2,4 $\times$ 10^{-17} cm^{-3}
Density of DM around the sun 0.3 GeV cm^{-3}
Mean density of DM in the Universe 10^{-6} GeV cm^{-3}
Mean density of neutrino in the Universe 112 cm^{-3}
Mass of the Higgs boson 125 GeV
EW cross section 10^{-9} GeV^{-2}
Actual limit on DM direct detection
$\sigma_{xp} \leq 10^{-46}$ cm^2
Number of protons in 1 gram of matter $\mathcal{N}_A \simeq 10^24$
Critical density 10^-5 GeV cm^{-3}
Scale + time of Λ dominance $10Mpc$
Density of proton in earth 1 g cm^{-1}
1 GeV in Kelvin : 10^13 K
Universe is 1000 times larger now than when the CMB was made. The size of the universe is inversely proportional to the temperature, the age of the universe is given by its temperature
Is energy conserved in the universe? No

2.2 History of observations

Different bits in the history of the universe

1. The inflation era, dominated by the inflaton. Inflaton scale is a little bit more than the planck scale. The Hubble rate is constant, the universe is exponentially expanding. The universe is dominated by the dark energy.

2. The reheating era, where the inflaton is oscillating. Energy scale $10^19 \rightarrow 10^10$ GeV

3. The thermal era, dominated by radiation, with energy scales $10^10 \rightarrow 1$ GeV

4. The neutrino and CMB era, from scale of the GeV to the eV, dominated by dark matter

5. Then the quintissence, our current era, dominated by dark energy once again, with constant hubble rate and with exponentially expanding universe.

We usually have P the pressure, ρ the density of energy, and the equation of state is usually written in term of $W = P/\rho$. For matter, $P = 0$, and we get a dilution of the field $\rho \propto \frac{1}{a^3}$. For radiation, $W = 1/3$ and we get an even bigger dilution $\rho \propto \frac{1}{a^4}$ which also gives a redshift. For inflaton/quintissence, $W = -1$, and $\rho \propto 1$. The Hubble rate $H = \frac{\dot{a}}{a}$, we have $H^2 = \frac{\rho}{3M_p^2}$ Inflaton scale is a little bit more than the planck scale. This is why the universe doesn't evolve the same way depending on the field considered.

2.2.1 Poincare point of view

Proxima is the nearest sun, he knew the distance between the sun and proxima $R_s = 10^6 r_E$ where r_E is the distance of earth to the sun. ρ_s is the density of matter around the sun, ρ_p the same around proxima.

$$\rho_s = \frac{M_s}{\frac{4}{3}\pi r_E^3} \tag{2.1}$$

Writing the conservation of energy for the gravitational potential,

$$v_E = \sqrt{\frac{8\pi G}{3}}\sqrt{\rho_S} r_E \tag{2.2}$$

And we have the same for proxima around the galactic center. But noticing that in orders of magnitude, $v_s \sim v_E$, (where v_S is the velocity of the sun and is approximately the same as the velocity of Proxima) we have

$$r_P = 10^9 r_E \tag{2.3}$$

He thus computed the size of the galaxy. But knowing the density of stars (assuming the distance between the sun and proxima is an average distance between stars), he thus was able to compute the number of stars in the galaxy, $\sim 10^9$

Chapter 3

An expanding Universe

The metric changes with time. Universe is almost homogeneous => only dependance on time. Dependance on space at a scale 10^{-5}.

3.1 The Friedmann equation

This is the way of Milne. We suppose a gas made of non-relativistic particles of mass density ρ_m and total energy per mass k. We can write the conservation of energy

$$E = m\phi(r) + \frac{1}{2}mv^2 = mk \tag{3.1}$$

We could write a potential as in gravity $\phi = -\frac{GM}{r}$ or a potential as $\phi_\Lambda = \frac{\Lambda r^2}{6}$. Using the first potential,

$$-\frac{GM}{r} + \frac{v^2}{2} = k \tag{3.2}$$

We can also write $v = \dot{r}$

$$-\frac{GM}{r} + \frac{\dot{r}^2}{2} = k \tag{3.3}$$

We also write the mass density $M = \frac{4\pi}{3}r^3\rho_m$, and the planck mass M_p given by $G = \frac{1}{M_p^2}$. The reduced planck mass is $M_{pr} = \frac{M_p}{\sqrt{8\pi}}$.

$$\left(\frac{\dot{r}}{r}\right)^2 = H^2 = \frac{\rho_m}{3M_{pl}^2} - \frac{k}{r^2} \tag{3.4}$$

If we add the second potential, it also adds a "cosmological constant term", it adds a term in $\frac{\Lambda}{6}$. The first potential is attractive, and creates the Schwarchild radius $r_S = \frac{M}{M_{pl}^2}$. Any light inside the Schwarchild radius is kept inside the radius and cannot escape, due to the attractive force a gavity. On the other side, the second potential is repulsive, and corresponds to the expansion of the universe. It also lead to a radius, $r_? =?$. Any light farther than this radius will never enter it. The light is so far that it cannot keep up with the expansion of the universe, and can never come close to us.

We place ourselves in the comoving coordinates. χ is the factor of expansion, it is the length of a square which is appropriatly expanded with time. We take our position $r = R \times \chi = R_0 \times a(t) \times \chi$ with R_0 the radius of the universe at the beginning of time. We have 3 sources of matter in the universe, ρ_m the matter, ρ_Λ the cosmological constant (dark matter), and ρ_R radiations, hence $\rho = \rho_m + \rho_\Lambda + \rho_R$.

Once again, we write the conservation of the internal energy (with V the volume, P the pressure)

$$\begin{aligned} U = \rho V, \quad \mathrm{d}U &= \mathrm{d}\rho\ V + \rho \mathrm{d}V = -P\mathrm{d}V \\ &= \frac{4\pi}{3}\mathrm{d}\rho\ R^3 + 4\pi R^2 \mathrm{d}R\ \rho \dots \end{aligned} \tag{3.5}$$

And we end up finding

$$\dot{\rho} + 3H(\rho + P) = 0 \tag{3.6}$$

In this equation, we have no curvature but a pressure. It is nice since we don't have to know the curvature, but we need the pressure, in comparison with (3.4). Derivating the hubble rate of (3.4) and inserting the above equation,

$$\dot{H} = -\frac{\rho + P}{2M_p^2} + \frac{k}{R^2} \tag{3.7}$$

But since $H = \left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{\dot{a}}{a}\right)^2$, we get

$$\dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \tag{3.8}$$

But then

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} = -\frac{\rho + 3P}{6M_p^2} \tag{3.9}$$

Note that curvature is an output of the Hubble constant.

Einstein introduced the cosmological constant in order to cancel $\ddot{a}$, as to have a static universe. He did so because he observed a static universe. But Edington noticed that this would make an unstable universe.

Equation of state: $P = w\rho$. For non-relativistic matter, $w = 0$. for the cosmological constant, $w = -1$. For relativistic particles, since the pressure is the quantity of momentum transferred on a unit of surface per second. For $E = pc$, $w = \frac{1}{3}$. For $E = \frac{1}{2}mv^2$, $w = \frac{2}{3}$.

A better, Einstein based approach is given in the slides. We write some general curved metric, the Friedmann metric which only depends on the curvature. Then we try to solve the Einstein equations.

From (3.6), we find

$$\rho \propto a^{-3(1+w)} \tag{3.10}$$

For dust, $\rho \propto a^{-3}$ is obvious since when the universe expand, the matter gets diluted. For radiation, $\rho \propto a^{-4}$ since the radiations gets diluted and redshifted, so they get diluted while loosing energy at the same time. For the cosmological constant, it stays constant. Notice that k (the raw curvature, also possible to interpret as a field), evolves as $\rho \propto a^{-2}$. This means that at the beginning of time, there was no curvature. It is the flatness problem. The Hubble constant evolves as the square root of ρ, so

$$\dot{a} \propto a^{-\frac{1}{2} - \frac{3w}{2}} \Rightarrow a \propto t^{\frac{2}{3(1+w)}} \tag{3.11}$$

Chapter 4

The inflaton field

L'inflaton est un champs qui depend uniquement du temps. On peut soit considerer un champs classique, soit prendre le champs classique pour un background field et faire un champs quantique par dessus, soit quantifier aussi le background field (typiquement la gravite quantique).

The idea of Starobinski was to add a potential to the formula, creating the inflation. Slow-roll means ϕ is going slow, meaning the density is constant, meaning the universe expands a lot.

The inflation tries to solve mainly 3 problems:

1. Entropy problem: We have 10^90 particles, how can we have so much entropy if there was no entropy at the big bang? (411 photons/cm^3, universe is 10^{29} cm long)

2. Horizon (causality) problem : how can independant photons have the exactly same energy?

3. Flatness problem

The action for the inflaton

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{4.1}$$

We have $\phi = \phi(t)$, the inflaton field acts as a Bose Einstein condensate. Everything acts as one, the field is constant, hence it is a classical field. There is no momentum, everything commutes. We want $\delta\mathcal{S} = 0$ so

Chapter 5

Reheating

Chapter 6

A thermal universe: FIMP and WIMP

Chapter 7

BBN, CMB and warm dark matter

Chapter 8

Direct detection

Chapter 9

Indirect detection

Chapter 10

Candidates