Aufgabe 1.

Berechnen Sie für die Punkte $(x_i; y_i)$, $i = 0, \dots, 3$,

| $i$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $x_i$ | -2 | 0 | 1 | 2 |
| $y_i$ | 4 | 0 | 1 | -4 |

das Ausgleichspolynom der Form $p(x) = a + bx^2$.

$$A^T A = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 4 & 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 0 & 0 \\ 1 & 1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 9 & 17 \\ 17 & 33 \end{pmatrix}$$

$$A^T b = \begin{pmatrix} 2 & 0 & 1 & 2 \\ 4 & 0 & 1 & 4 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 17 \\ 17 & 33 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

I $\quad 9a + 17b = 1$

II $\quad 17a + 33b = 1$

I $\quad 9a = 1 - 17b \quad | :9$

$a = \frac{1-17b}{9}$

$a = \frac{18}{9}$

$a = 2$

II $\quad 17 \cdot \left(\frac{1-17b}{9}\right) + 33b = 1$

$\frac{17}{9}(1-17b) + 33b = 1$

$b = -1$

$$f(x) = 2 - 1x^2$$

## Aufgabe 2

$$AA^T = \begin{array}{cc} & \begin{pmatrix}1&1\\2&0\\-1&-1\end{pmatrix} \\ \begin{pmatrix}1&2&1\\1&0&-1\end{pmatrix} & \begin{pmatrix}6&0\\0&2\end{pmatrix}\end{array}$$

$$A^TA = \begin{array}{cc} & \begin{pmatrix}1&2&1\\1&0&-1\end{pmatrix} \\ \begin{pmatrix}1&1\\2&0\\1&-1\end{pmatrix} & \begin{pmatrix}2&2&0\\2&4&2\\0&2&2\end{pmatrix}\end{array}$$

$$\lambda E_2 - AA^T = \begin{pmatrix}\lambda-6 & 0\\ 0 & \lambda-2\end{pmatrix}$$

$$\det(\lambda E_2 - AA^T) = (\lambda-6)(\lambda-2) = \lambda^2 - 8\lambda + 12$$

$\lambda = 4 \pm \sqrt{4^2-12}$ $\quad \lambda_1 = 2 \quad \lambda_2 = 6$

$$\lambda E_3 - A^TA = \begin{pmatrix}\lambda-2 & -2 & 0\\ -2 & \lambda-4 & -2\\ 0 & -2 & \lambda-2\end{pmatrix}$$

$$\det(\lambda E_3 - A^TA) = (\lambda-2)^2\cdot(\lambda-4) - 4\cdot(\lambda-2)\cdot 2$$
$$= (\lambda^2-4\lambda+4)(\lambda-4) - 8(\lambda-2)$$
$$= \lambda^3 - 8\lambda^2 + 16\lambda + 4\lambda \cancel{-16} - 8\lambda \cancel{+16}$$
$$= \lambda^3 - 8\lambda^2 + 12\lambda \qquad \lambda_1 = 0$$
$$\lambda^2 - 8\lambda + 12 \qquad \lambda_2 = 2 \quad \lambda_3 = 6$$

$\sigma_1 = \sqrt{6} \quad \sigma_2 = \sqrt{2} \quad \sigma_3 = 0$