Aufgabe 3

für den Fehler gilt:

$$E = -\frac{1}{12} h^3 f''(\xi)$$

Damit gilt dann für E_l und E_r:

$$E_l = -\frac{1}{12}\left(\frac{h}{2}\right)^3 f''(\xi_l)$$

$$E_r = -\frac{1}{12}\left(\frac{h}{2}\right)^3 f''(\xi_r)$$

$$\tilde{E} = -\left(\frac{1}{2}^2\right)\frac{1}{12} h^3 f''(\xi) = \frac{1}{4} E$$

Damit gilt dann:

$$\tilde{T} - T = E - \tilde{E} \approx \frac{3}{4} E \approx 3\tilde{E}$$

und damit

$$\tilde{F} = \frac{1}{3} |\tilde{T} - T|$$

$T = \frac{b_i - a_i}{2}(f(a_i) + f(b_i))$ folgt direkt aus der Trapez-Regel.

Durch die nochmalige Teilung in zwei Intervalle für $\tilde{T}$ mit $m_i = \frac{b_i + a_i}{2}$ gilt dann:

$$\tilde{T} = \frac{b_i - m_i}{2}(f(m_i) + f(b_i)) + \frac{m_i - a_i}{2}(f(m_i) + f(a_i)) = \frac{b_i - a_i}{4}(f(b_i) + 2f(m_i) + f(a_i))$$

$$= \frac{b_i - a_i}{4}\left(f(b_i) + 2f\left(\frac{b_i + a_i}{2}\right) + f(a_i)\right)$$

b)

$$T_1 = \frac{5-1}{2}\left(\frac{1}{5} + 1\right) \quad \tilde{T}_1 = \frac{4}{4}\left(1 + \frac{2}{3} + \frac{1}{5}\right)$$

$$\tilde{F}_1 = \frac{\left|\frac{12}{5} - \frac{28}{15}\right|}{3} = \frac{\frac{6}{15}}{3} = \frac{2}{15} > 0{,}02$$

$$T_{11} = \frac{3-1}{2}\left(1 + \frac{1}{3}\right) \quad \tilde{T}_{11} = \frac{2}{4}\left(1 + \frac{2}{2} + \frac{1}{3}\right)$$

$$\tilde{F}_{11} = \frac{\left|\frac{4}{3} - \frac{7}{6}\right|}{3} = \frac{\frac{1}{6}}{3} = \frac{1}{18} > 0{,}02$$

$$T_{12} = \frac{5-3}{2}\left(\frac{1}{3} + \frac{1}{5}\right) \quad \tilde{T}_{1,2} = \frac{2}{4}\left(\frac{1}{3} + \frac{2}{4} + \frac{1}{5}\right)$$

$$\tilde{F}_{1,2} = \frac{\left|\frac{8}{15} - \frac{31}{60}\right|}{3} = \frac{\frac{1}{60}}{3} = \frac{1}{180} < 0{,}02$$

$\Rightarrow$ Abbruch

$$T_{111} = \frac{2-1}{2}\left(1 + \frac{1}{2}\right) \qquad \tilde{T}_{111} = \frac{2-1}{4}\left(1 + \frac{4}{3} + \frac{1}{2}\right)$$

$$\tilde{F} = \frac{\left|\frac{3}{4} - \frac{15}{24}\right|}{3} = \frac{\frac{3}{24}}{3} = \frac{1}{24} > 0{,}02$$

$$T_{112} = \frac{3-2}{2}\left(\frac{1}{2} + \frac{1}{3}\right) \qquad \tilde{T}_{112} = \frac{1}{4}\left(\frac{1}{2} + \frac{4}{5} + \frac{1}{3}\right)$$

$$\tilde{F}_{112} = \frac{\left|\frac{5}{12} - \frac{49}{120}\right|}{3} = \frac{1}{360} < 0{,}02 \Rightarrow \text{Abbruch}$$

$$T_{1111} = \frac{1{,}5-1}{2}\left(1 + \frac{2}{3}\right) \qquad \tilde{T}_{1111} = \frac{1}{8}\left(1 + \frac{8}{5} + \frac{2}{3}\right)$$

$$\tilde{F}_{1111} = \frac{\left|\frac{5}{12} - \frac{44}{120}\right|}{3} = \frac{1}{360} < 0{,}02 \Rightarrow \text{Abbruch}$$

$$T_{1112} = \frac{2-1{,}5}{2}\left(\frac{2}{3} + \frac{1}{2}\right) \qquad \tilde{T}_{1112} = \frac{1}{8}\left(\frac{2}{3} + \frac{8}{7} + \frac{1}{2}\right)$$

$$\tilde{F}_{1112} = \frac{\left|\frac{7}{24} - \frac{47}{336}\right|}{3} = \frac{1}{1008} < 0{,}02 \Rightarrow \text{Abbruch}$$

$$T = T_{12} + T_{112} + T_{1112} + T_{111} = \frac{8}{15} + \frac{5}{12} + \frac{5}{12} + \frac{7}{24} = \frac{199}{120} \approx 1{,}658$$

$$\int_1^5 \frac{1}{x}\,dx = [\ln(x)]_1^5 = \ln(5) - \ln(1) = 1{,}609 - 0$$

$$|1{,}609 - 1{,}658| = \underline{\underline{0{,}049}}$$