

①

$$A x = b$$

$$A = \begin{pmatrix} 0 & 1 & 3 & 1 \\ 1 & 1 & 2 & 0 \\ 4 & 4 & 8 & 2 \\ 2 & 6 & 4 & 8 \end{pmatrix} \quad \begin{matrix} \curvearrowright \\ \curvearrowleft \end{matrix}$$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 4 & 4 & 8 & 2 \\ 2 & 6 & 4 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 4 & 0 & 0 & 2 \\ 2 & 4 & 0 & 8 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 4 & 0 & 0 & 2 \\ 2 & 4 & -12 & 4 \end{pmatrix} \quad \begin{matrix} \curvearrowright \\ \curvearrowleft \end{matrix}$$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 2 & 4 & -12 & 4 \\ 4 & 0 & 0 & 2 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ 2 & 4 & 1 & \\ 4 & 0 & 0 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 1 & 2 & 0 \\ & 1 & 3 & 1 \\ & & -12 & 4 \\ & & & 2 \end{pmatrix} //$$

$$P = \begin{pmatrix} 2 \\ 2 \\ 4 \\ 4 \end{pmatrix}$$

$$a) \quad b = (5, 1, 8, 18)^T$$

$$Pb = (1, 5, 18, 8)^T$$

$$Lx = Pb$$

$$\begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ 2 & 4 & 1 & \\ 4 & 0 & 0 & 1 \end{pmatrix} y = \begin{pmatrix} 1 \\ 5 \\ 18 \\ 8 \end{pmatrix} \Rightarrow \begin{aligned} x_1 &= 1 \\ x_2 &= 5 \\ x_3 &= 18 - 2 \cdot 1 - 4 \cdot 5 = -4 \\ x_4 &= 8 - 4 \cdot 1 = 4 \end{aligned}$$

$$y = \begin{pmatrix} 1 \\ 5 \\ -4 \\ 4 \end{pmatrix}$$

$$Ux = y$$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ & 1 & 3 & 1 \\ & & -12 & 4 \\ & & & 2 \end{pmatrix} x = \begin{pmatrix} 1 \\ 5 \\ -4 \\ 4 \end{pmatrix} \Rightarrow \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$$

$$b) \quad b = (5, 7, 28, 22)^T$$

$$Pb = (7, 5, 22, 28)^T$$

$$L_y = Pb$$

$$\begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ 2 & 4 & 1 & \\ 4 & 0 & 0 & 1 \end{pmatrix} y = \begin{pmatrix} 7 \\ 5 \\ 22 \\ 28 \end{pmatrix} \Rightarrow \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} = \begin{pmatrix} 7 \\ 5 \\ -12 \\ 0 \end{pmatrix}$$

$$Ux = y$$

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ & 1 & 3 & 1 \\ & & -12 & 4 \\ & & & 2 \end{pmatrix} x = \begin{pmatrix} 7 \\ 5 \\ -12 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

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$$a) A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$\begin{array}{cccc} 2 & & & \\ -1 & 2 & & \\ 0 & -1 & 2 & \\ 0 & 0 & -1 & 2 \end{array}$$

$$\Rightarrow \begin{array}{c|ccc} \sqrt{2} & & & \\ -\frac{1}{\sqrt{2}} & & & \\ 0 & & & \\ 0 & & & \end{array} \begin{array}{l} 2-\frac{1}{2} \\ -1 \quad 2 \\ 0 \quad -1 \quad 2 \end{array}$$

$$\Rightarrow \begin{array}{c|ccc} \sqrt{2} & & & \\ \frac{1}{\sqrt{2}} & \sqrt{\frac{2}{3}} & & \\ 0 & -\frac{\sqrt{3}}{2} & 2-\frac{2}{3} & \\ 0 & 0 & -1 & 2 \end{array}$$

$$\Rightarrow \begin{array}{c|ccc} \sqrt{2} & & & \\ \frac{1}{\sqrt{2}} & \sqrt{\frac{2}{3}} & & \\ 0 & -\frac{\sqrt{3}}{2} & \frac{4}{3} & \\ 0 & 0 & -\frac{\sqrt{2}}{2} & 2-\frac{2}{3} \end{array}$$

$$\Rightarrow \begin{array}{c|ccc} \sqrt{2} & & & \\ \frac{1}{\sqrt{2}} & \sqrt{\frac{2}{3}} & & \\ 0 & -\frac{\sqrt{3}}{2} & \frac{4}{3} & \\ 0 & 0 & -\frac{\sqrt{2}}{2} & \frac{4}{3} \end{array}$$