

# Aufgaben 7

a)  $P_n(x) = \sum_{k=0}^n f_k \cdot L_k(x)$ , mit  $L_k(x) = \prod_{\substack{j=0 \\ j \neq k}}^{h-1} \frac{x - x_j}{x_k - x_j}$

$$(x_0, y_0) = (-1, -2) \quad L_0 = \frac{x - 0}{-1 - 0} = \frac{x}{-1} = -x \quad -\frac{x - 1}{-1 - 1} = \frac{1-x}{-2} = -\frac{1}{2}(1-x)(3-x)x$$

$$(x_1, y_1) = (0, 4) \quad L_1 = \frac{x - (-1)}{0 - (-1)} = \frac{x + 1}{1} = x + 1 \quad -\frac{x - 3}{0 - 3} = \frac{x - 3}{3} = \frac{1}{3}(1-x)(3-x)(x+1)$$

$$(x_2, y_2) = (1, 6) \quad L_2 = \frac{x - (-1)}{1 - (-1)} = \frac{x + 1}{2} = \frac{x}{2} + \frac{1}{2} \quad -\frac{x - 3}{1 - 3} = \frac{x - 3}{2} = \frac{1}{2}(3-x)(x+1)$$

$$(x_3, y_3) = (3, 22) \quad L_3 = \frac{x - (-1)}{3 - (-1)} = \frac{x + 1}{4} = \frac{x}{4} + \frac{1}{4} \quad -\frac{x - 1}{3 - 1} = \frac{x - 1}{2} = \frac{1}{2}(x+1)(x-1)$$

$$P_3(x) = \frac{2}{4} \cdot (1-x)(3-x)x + \frac{4}{3} \cdot (1-x)(3-x)(x+1) + \frac{6}{4} \cdot (3-x)(x+1) \\ + \frac{22}{24} (x+1)x(x-1)$$

$$= x^3 - 2x^2 + 3x + 4 =$$

b)

$$P_h(x) = \sum_{k=0}^h c_k \cdot N_k(x), \quad \text{mit } N_k(x) = \prod_{j=0}^{k-1} (x - x_j)$$

$$(x_0, y_0) = (-1, -2) \rightarrow \frac{4 - (-2)}{0 - (-1)} = \frac{6}{1} = 6 \rightarrow \frac{2 - 6}{1 - 1} = \frac{-4}{0} = -2 \rightarrow \frac{2 - (-2)}{3 - (-1)} = \frac{4}{2} = 2$$

$$(x_1, y_1) = (0, 4) \rightarrow \frac{6 - 4}{1 - 0} = 2 \rightarrow \frac{1 - 2}{1 - 1} = \frac{-1}{0} = -1 \rightarrow \frac{2 - (-1)}{3 - (-1)} = \frac{3}{2} = 1.5$$

$$(x_2, y_2) = (1, 6) \rightarrow \frac{22 - 6}{3 - 1} = 8 \rightarrow \frac{6 - 8}{3 - 1} = \frac{-2}{2} = -1 \rightarrow \frac{8 - (-1)}{3 - (-1)} = \frac{9}{2} = 4.5$$

$$(x_3, y_3) = (3, 22) \rightarrow \frac{22 - 6}{3 - 1} = 8 \rightarrow \frac{6 - 8}{3 - 1} = \frac{-2}{2} = -1 \rightarrow \frac{8 - (-1)}{3 - (-1)} = \frac{9}{2} = 4.5$$

$$P_3(x) = -2 + 6 \cdot (x+1) + (-2) \cdot (x+1)(x-0) + 1 \cdot (x+1)(x-0)(x-1)$$

$$= x^3 - 2x^2 + 3x + 4$$

Aufgabe 3. a)

$$a_0 = 4$$

$$a_0 - b_0 + c_0 - d_0 = 16$$

$$b_0 - b_1 = 0 \Rightarrow b_0 = b_1 = b$$

$$c_0 - c_1 = 0 \Rightarrow c_0 = c_1 = c$$

$$2c_0 - 6d_0 = 0$$

$$a_1 = 4$$

$$a_1 + 2b_1 + 4c_1 + 8d_1 = 16$$

$$2c_1 + 12d_1 = 0$$


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$$8 - b + c - d_0 = 16$$

$$2c - 6d_0 = 0 \Rightarrow d_0 = \frac{c}{3}$$

$$8 + 2b + 4c + 8d_1 = 16$$

$$2c + 12d_1 = 0 \Rightarrow d_1 = -\frac{c}{6}$$


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$$8 - b + c - \frac{c}{3} = 16 \Rightarrow \frac{2}{3}c - b = 8 \rightarrow b = \frac{2}{3}c - 8$$

$$8 + 2b + 4c - \frac{8}{6} = 16 \Rightarrow 2b + \frac{4}{3}c = 8$$


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$$\frac{4}{3}c - 16 + \frac{4}{3}c = 8$$

$$\Rightarrow 4c = 24$$

$$\Rightarrow c = \frac{1}{6}$$

$$\Rightarrow l = -\frac{71}{9}$$

$$\Rightarrow d_0 = \frac{1}{10}$$

$$\Rightarrow d_1 = \frac{-1}{36}$$

$$f(x) = \begin{cases} 4 - \frac{71}{9}x + \frac{1}{6}x^2 + \frac{4}{6}x^3 & \text{für } x \leq 0 \\ 4 - \frac{71}{9}x + \frac{1}{6}x^2 - \frac{1}{36}x^3 & \text{für } x > 0 \end{cases} //$$

3. b)

$$h_0 = 0 - (-1) = 1$$

$$h_1 = 2 - 0 = 2$$

$$A = (6)$$

$$Y_1 = 6 \left( \frac{16-8}{2} - \frac{4-16}{1} \right) = 6 \cdot 12 = 72$$

$$6 \cdot \beta_1 = 72$$

$$\beta_1 = 12 //$$