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5. **Proof for 3-qubit XOR**

<u>Proof by Contradiction:</u>

Assume that there is a solution for XOR for 3 qubits.

x	y	z	Obj
0	0	0	0
0	0	1	$a_3 > 0$
0	1	0	$a_2 > 0$
0	1	1	$a_2 + a_3 + b_{23} = 0$
1	0	0	$a_1 > 0$
1	0	1	$a_1 + a_3 + b_{13} = 0$
1	1	0	$a_1 + a_2 + b_{12} = 0$
1	1	1	$a_1 + a_2 + a_3 + b_{12} + b_{13} + b_{23} > 0$

From row 1, we can say that the ground state = 0.

Equations from each row(the penalty function):
Row 1: 0, Ground State
Row 2: $a_3 > 0$
Row 3: $a_2 > 0$
Row 4: $a_2 + a_3 + b_{23} = 0$
Row 5: $a_1 > 0$
Row 6: $a_1 + a_3 + b_{13} = 0$
Row 7: $a_1 + a_2 + b_{12} = 0$
Row 8: $a_1 + a_2 + a_3 + b_{12} + b_{13} + b_{23} > 0$

Consider the equations from row 2, row 3, and row 5 where
$a_1 > 0$
$a_2 > 0$
$a_3 > 0$

Therefore, we can say that
$a_1 + a_2 + a_3 > 0$ (Eq. 1)

Consider the equations from row 4, row 6, and row 7 where
$a_2 + a_3 + b_{23} = 0 \rightarrow b_{23} = -(a_2 + a_3)$
$a_1 + a_3 + b_{13} = 0 \rightarrow b_{13} = -(a_1 + a_3)$
$a_1 + a_2 + b_{12} = 0 \rightarrow b_{12} = -(a_1 + a_2)$

Consider the equation from row 8:
$a_1 + a_2 + a_3 + b_{12} + b_{13} + b_{23} > 0$
$a_1 + a_2 + a_3 - (a_1 + a_2) - (a_1 + a_3) - (a_2 + a_3) > 0$
$- a_1 - a_2 - a_3 > 0$
$a_1 + a_2 + a_3 < 0$ (Eq. 2)

The Equation 2 contradicts the equation 1 hence, we can say that the solution does not exist by proof of contradiction,