

# Homework 1 - ISYE 3770

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## PART 1 - Textbook Questions

### Q2.1.13

- a)  $3 + 5 + 6 + 8$
- b)  $1 + 2$
- c)  $1 + 2 + 3 + 4 + 5$
- d)  $7 + 8$
- e)  $3 + 4 + 5$

### Q2.1.21

- a)  $A' = \{x \mid x \geq 72.5\}$
- b)  $B' = \{x \mid 0 < x \leq 52.5\}$
- c)  $A \cap B = \{x \mid 52.5 < x < 72.5\}$
- d)  $A \cup B = \{x \mid x = S\}$

*Note:* Sample space is positive real numbers.

### Q2.2.13

$$\begin{aligned} C_4^5 \times C_6^{45} + C_5^5 \times C_5^{45} &= \frac{5!}{4!1!} \times \frac{45!}{6!39!} + \frac{5!}{5!0!} \times \frac{45!}{5!40!} \\ &= 41,947,059 \end{aligned}$$

```
choose(5, 4) * choose(45, 6) + choose(5, 5) * choose(45, 5)
```

```
## [1] 41947059
```

### Q2.3.1

- a)  $P(A) = 0.1 + 0.1 + 0.2 = 0.4$
- b)  $P(B) = 0.2 + 0.4 + 0.2 = 0.8$
- c)  $P(A') = 0.4 + 0.2 = 0.6$
- d)  $P(A \cup B) = 0.1 + 0.1 + 0.2 + 0.4 + 0.2 = 1$
- e)  $P(A \cap B) = 0.2$

### Q2.3.17

a)  $A = \frac{52^8}{62^8}$

```
52**8 / 62**8
```

```
## [1] 0.2448461
```

b)  $B = \frac{10^8}{62^8}$

```
10**8 / 62**8
```

```
## [1] 4.580011e-07
```

c)  $\frac{62^8 - 52^8}{62^8}$

```
(62**8 - 52**8) / 62**8
```

```
## [1] 0.7551539
```

d)  $\frac{10^2 \cdot 52^6}{52^8}$

```
(10**2 * 52**6) / 52**8
```

```
## [1] 0.03698225
```

### Q2.4.7

b)  $\frac{36^5 \times C_1^5}{36^6} = \frac{5 \times 36^5}{36^6}$

```
choose(5, 1) * (36**5) / 36**6
```

```
## [1] 0.1388889
```

b)  $\frac{36^5 \times C_1^5}{36^6} = \frac{36^5 \times 5}{36^6}$

```
(36**5) * choose(5, 1) / 36**6
```

```
## [1] 0.1388889
```

c)  $\frac{C_1^5 \times 36^4 \times C_1^5}{36^6} = \frac{5 \times 36^4 \times 5}{36^6}$

```
choose(5, 1) * (36**4) * choose(5, 1) / 36**6
```

```
## [1] 0.01929012
```

d)  $\frac{2 \times C_1^5 \times 36^5 - C_1^5 \times 36^4 \times C_1^5}{36^6} = \frac{2 \times 5 \times 36^5 - 5 \times 36^4 \times 5}{36^6}$

```
(2 * choose(5, 1) * 36**5 - choose(5, 1) * 36**4 * choose(5, 1)) / 36**6
```

```
## [1] 0.2584877
```

## PART 2

Find the numbers of combinations for the following 5-card Poker hand.

a) Three of a Kind It contains three cards of the same number(rank), plus two cards which are not of this number nor the same as each other. For example: AAA83, KKK72, . . .

solution:

$$\begin{aligned} C_1^{13} \times C_3^4 \times C_2^{12} \times C_1^4 \times C_1^4 &= \frac{13!}{1!12!} \times \frac{4!}{3!1!} \times \frac{12!}{2!10!} \times \frac{4!}{1!3!} \times \frac{4!}{1!3!} \\ &= 54,912 \end{aligned}$$

```
choose(13, 1) * choose(4, 3) * choose(12, 2) * choose(4, 1) * choose(4, 1)
```

```
## [1] 54912
```

## PART 3

Four Candidates: A, B, C, and D run for the president of GT Elementary School. Suppose we have 100 voters (students) and everyone needs to vote for exactly one candidate from A, B, C, and D and the votes are anonymous. How many different voting results (combinations) can we have here? [5 pts]

solution:

$$\begin{aligned} C_{4-1}^{100+4-1} &= \frac{(100+4-1)!}{3!(103-3)!} \\ &= \frac{103!}{3!100!} \\ &= 176,851 \end{aligned}$$

```
choose(100+4-1, 4-1)
```

```
## [1] 176851
```