Homework 3

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3.3.6

a)

$P(X \leq 0) = f(0) = 0.17$
$P(X \leq 2) = f(0) + f(2) = 0.17 + 0.35 = 0.52$
$P(X \leq 3) = f(0) + f(2) + f(3) = 0.17 + 0.35 + 0.33 = 0.85$
$P(X \leq 4) = f(0) + f(2) + f(3) + f(4) = 0.17 + 0.35 + 0.33 + 0.17 = 1$

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.17 & \text{if } 0 \leq x < 2 \\ 0.52 & \text{if } 2 \leq x < 3 \\ 0.85 & \text{if } 3 \leq x < 3 \\ 1 & \text{if } x \geq 2 \end{cases}$$

b)

$$\begin{aligned} Mean &= \sum_{x=0}^{4} x \cdot f(x) \\ &= 0(0.17) + 2(0.35) + 3(0.33) + 4(0.15) \\ &= 2.29 \end{aligned}$$

```r
0*.17+2*.35+3*.33+4*.15
```

```
## [1] 2.29
```

$$\begin{aligned} Variance &= [\sum_{x}^{4} x^2 f(x)] - E[X]^2 \\ &= 0^2(0.17) + 2^2(0.35) + 3^2(0.33) + 4^2(0.15) - 2.29^2 \\ &= 1.5259 \end{aligned}$$

```r
(0.17 * 0**2) + (0.35 * 2**2) + (0.33 * 3**2) + (0.15 * 4 ** 2) - (2.29 ** 2)
```

```
## [1] 1.5259
```

3.4.3

We first need to find the mean, variance, and standard deviation for X.

$$\begin{aligned} mean\ of\ X = E[X] &= \sum_{x=0}^{9} x \cdot f(x) \\ &= \frac{a+b}{2}, where\ a = upper\ bound\ and\ b = lower\ bound \\ &= \frac{9+0}{2} \\ &= 4.5 \end{aligned}$$

$$\begin{aligned} variance\ of\ X = V[X] &= E[X^2] - (E[X])^2 \\ &= \frac{(b-a+1)^2 - 1}{12} \\ &= \frac{(9-0+1)^2 - 1}{12} \\ &= \frac{100}{12} \\ &= 8.25 \end{aligned}$$

$$\begin{aligned} Standard\ Deviation\ of\ X &= \sigma \\ &= \sqrt{\sigma^2} \\ &= \sqrt{8.25} \\ &\approx 2.872281 \end{aligned}$$

```
(9 + 0)/2
```

```
## [1] 4.5
```

```
(((9 - 0 + 1)**2)-1)/12
```

```
## [1] 8.25
```

```
sqrt(8.25)
```

```
## [1] 2.872281
```

Then using X, we can calculate the Mean, Variance, and Standard Deviation of $Y = 5X$:

Mean of Y = $E[5X] = 5E[X] = 5 \cdot 4.5 = 22.5$
Variance of Y = $V[5X] = 25 \cdot V[X] = 25 \cdot 8.25 = 206.25$
Standard Dev. of Y = $\sqrt{\sigma^2} = \sqrt{206.25} \approx 14.36141$

```
5 * 4.5
```

```
## [1] 22.5
```

```
25 * 8.25
```

```
## [1] 206.25
```

```
sqrt(206.25)
```

```
## [1] 14.36141
```

3.5.13

a)

$$\begin{aligned}P(X \leq 120) &= 1 - P(X \geq 121) \\ &= 1 - \sum_{x=121}^{125} \binom{125}{x} (0.9)^x (0.1)^{125-x} \\ &\approx 1 - 0.003858595 \\ &\approx 0.9961414\end{aligned}$$

```r
pbinom(120, 125, 0.9)
```

```
## [1] 0.9961414
```

b) $P(X = 0) = \binom{125}{0} 0.9^0 (0.1)^{125} = 1 \times 10^{125}$

```r
dbinom(0, 125, 0.9)
```

```
## [1] 1e-125
```

3.6.6

a) PMF $= f(x_i) = (0.8)^{x_i}(0.2)$
b) f(2) = 1 - [P(X=0) + P(X=1)] = 1 - $[0.8 \cdot 0.2 + 0.2] = 0.64$
c) E[X] = $\frac{1}{p} = \frac{1}{0.8} = 1.25$
d)

$$\begin{aligned}P(X \geq 3) &= 1 - [P(X = 0) + P(X = 1) + P(X = 2)] \\ &= 1 - [(0.8^0 0.2 + 0.8^1 0.2 + 0.8^2 0.2)] \\ &= 1 - 0.488 \\ &= 0.512\end{aligned}$$

```r
1 - 0.8*0.2 - 0.2
```

```
## [1] 0.64
```

```r
1/0.8
```

```
## [1] 1.25
```

```r
1 - ((0.2*0.8**0)+(0.2*0.8**1)+(0.2*0.8**2))
```

```
## [1] 0.512
```

3.6.13

a) $P(X = 2) = \binom{10-1}{2-1} \cdot (0.8)^8 (0.2)^2 \approx 0.06039798$

b) $P(X \leq 2) = \binom{4-1}{2-1} \cdot (0.8)^2 (0.2)^2 + \binom{3-1}{2-1} \cdot (0.8)^1 (0.2)^2 + \binom{2-1}{2-1} (0.8)^0 (0.2)^2 = 0.1808$

C) $E[X = 3] = \frac{r}{p} = \frac{3}{0.2} = 15$

```r
dnbinom(8, 2, 0.2)
```

```
## [1] 0.06039798
```

```r
pnbinom(2, 2, 0.2)
```

```
## [1] 0.1808
```

```r
3/0.2
```

```
## [1] 15
```

3.7.6

a) P(X=6) $= \frac{\binom{6}{6}\binom{34}{0}}{\binom{40}{6}} \approx 2.605266 \times 10^{-7}$

b) P(X=5) $= \frac{\binom{6}{5}\binom{34}{1}}{\binom{40}{6}} \approx 5.314742 \times 10^{-5}$

c) P(X=4) $= \frac{\binom{6}{4}\binom{34}{2}}{\binom{40}{6}} \approx 0.002192331$

d) $\frac{1}{P(X=6)} = \frac{1}{\frac{\binom{6}{6}\binom{34}{0}}{\binom{40}{6}}} = 3838380$

```r
choose(6, 6) * choose(34, 0) / choose(40, 6)
```

```
## [1] 2.605266e-07
```

```r
choose(6, 5) * choose(34, 1) / choose(40, 6)
```

```
## [1] 5.314742e-05
```

```r
choose(6, 4) * choose(34, 2) / choose(40, 6)
```

```
## [1] 0.002192331
```

```r
1 / (choose(6, 6) * choose(34, 0) / choose(40, 6))
```

```
## [1] 3838380
```