Homework 2

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Q2.5.4

a) $P(gas\ leak \mid failures) = \frac{87}{107}$

```
87/107
```

```
## [1] 0.8130841
```

b) $P(electrical\ fail \mid gas\ leak) = \frac{32}{87}$

```
32/87
```

```
## [1] 0.3678161
```

c) $P(gas\ leak \mid electrical\ fail) = \frac{55}{72}$

```
55/72
```

```
## [1] 0.7638889
```

Q2.5.6

a) $P(second\ defective \mid first\ is\ defective) = \frac{4}{499} \approx 0.008016032$

```
(4/499)
```

```
## [1] 0.008016032
```

b) $P(defective) \cdot P(second\ defective \mid first\ is\ defective) = \frac{5}{500} \cdot \frac{4}{499} \approx 0.00008016032$

```
(5/500) * (4/499)
```

```
## [1] 8.016032e-05
```

c) $P(acceptable) \cdot P(second\ acceptable \mid first\ is\ acceptable) = \frac{495}{500} \cdot \frac{494}{499} \approx 0.9800802$

```
(495/500) * (494/499)
```

```
## [1] 0.9800802
```

Extra)
P(A) = probability first card is defective = 5/500
P(A') = probability first card isn't defective = 495/500
P(B|A) = probability second card is defective given first card is defective = 4/499
P(B|A') = probability second card is defective given first card isn't defective = 5/499
P(B) = probability second card is defective

$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|A')P(A') \\ &= \frac{4}{499} \cdot \frac{5}{500} + \frac{5}{499} \cdot \frac{495}{500} \\ &= \frac{20}{499 \cdot 500} + \frac{2475}{499 * 500} \\ &= \frac{2495}{499 * 500} \\ &= 0.01 \end{aligned}$$

```
(5*495)/(499*500) + 20/(499*500)
```

```
## [1] 0.01
```

Q2.6.3

$0.9 \cdot P(failure \mid dry) + 0.1(failure \mid wet) = 0.9 \cdot 0.01 + 0.1 \cdot 0.05 = 0.014$

```
0.9*0.01 + 0.1*0.05
```

```
## [1] 0.014
```

Q2-141

a) First we calculate P(B'):

$$\begin{aligned} P(B') &= P(\Omega) - P(B) \\ &= 1 - \frac{10^8}{62^8} \end{aligned}$$

Then we can calculate P(A|B'):

$$\begin{aligned} P(A|B') &= \frac{P(A \cap B')}{P(B')} \\ &= \frac{P(A)}{P(B')} \\ &= \frac{\frac{52^8}{62^8}}{1 - \frac{10^8}{62^8}} \\ &\approx 0.2448462 \end{aligned}$$

```
(52**8/62**8) / (1-(10**8/62**8))
```

```
## [1] 0.2448462
```

b) We know that B is a subset of A′ since A′ is simply all combinations that contains at least 1 letter. Therefore $B = A' \cap B$
Then we simply calculate P(B):

$$\begin{aligned} P(B) &= \frac{|B|}{|\Omega|} \\ &= \frac{10^8}{62^8} \\ &\approx 4.580011e-07 \end{aligned}$$

```
(10**8) / (62**8)
```

```
## [1] 4.580011e-07
```

c)

$$\begin{aligned} P(contains\ exactly\ 2\ int \mid contains\ at\ least\ 1\ int) &= \frac{P(contains\ exactly\ 2\ int\ \cap\ contains\ at\ least\ 1\ int)}{P(contains\ at\ least\ 1\ int)} \\ &= \frac{P(contains\ exactly\ 2\ int)}{P(contains\ at\ least\ 1\ int)} \\ &= \frac{\frac{10^2 \cdot C_2^8 \cdot 52^6}{62^8}}{1 - P(A)} \\ &= \frac{\frac{10^2 \cdot C_2^8 \cdot 52^6}{62^8}}{1 - \frac{52^8}{62^8}} \\ &= \frac{10^2 \cdot \frac{8!}{2!6!} \cdot 52^6}{62^8 - 52^8} \\ &\approx 0.3357447 \end{aligned}$$

```
((10**2)*(factorial(8)/(factorial(2)*factorial(6)))*(52**6))/(62**8 - 52**8)
```

```
## [1] 0.3357447
```

Problem 2.7.4

If P(A) ≠ P(A|B) then we can say that the two aren't independent:

$$P(A) = \frac{86}{100}$$

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{\frac{70}{100}}{\frac{79}{100}} \\ &= \frac{70}{79} \end{aligned}$$

Since $\frac{86}{100} \neq \frac{70}{79}$, we know that the two events A and B are not independent.

Problem 2.8.9

a) $0.2 \cdot P(free\ in\ spam) + 0.8 \cdot P(free\ in\ valid) = 0.2 \cdot 0.6 + 0.8 \cdot 0.04 = 0.152$

```
0.2*0.6+0.8*0.04
```

```
## [1] 0.152
```

b)

$$
\begin{aligned}
P(is\ spam \mid contains\ free) &= \frac{P(is\ spam\ \cap\ contains\ free)}{P(contains\ free)} \\
&= \frac{P(\ contains\ free \mid is\ spam) \cdot P(is\ spam)}{P(contains\ free)} \\
&= \frac{0.6 \cdot 0.2}{0.152} \\
&\approx 0.7894737
\end{aligned}
$$

```
0.6 * 0.2/0.152
```

```
## [1] 0.7894737
```

c)

$$
\begin{aligned}
P(is\ valid \mid not\ contains\ free) &= \frac{P(is\ valid\ \cap\ not\ contains\ free)}{P(not\ contains\ free)} \\
&= \frac{P(not\ contains\ free \mid is\ valid) \cdot P(is\ valid)}{P(not\ contains\ free)} \\
&= \frac{0.96 \cdot 0.8}{1 - 0.152} \\
&\approx 0.1886792
\end{aligned}
$$

```
(0.96-0.8) / (1-0.152)
```

```
## [1] 0.1886792
```