Homework 7

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Problem 7.2.6

$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1.5}{7}$

a) $z = \frac{X-\mu}{\sigma_{\bar{X}}} = \frac{10-8.2}{\frac{1.5}{7}} = 8.4$

$P(Z < 8.4) \approx 1$ since the z-score is so high.

Thus, the probability that the average time waiting in line is less than 10 minutes is approximately 1.

b) $z = \frac{X-\mu}{\sigma_{\bar{X}}} = \frac{5-8.2}{\frac{1.5}{7}} \approx -14.9333$

$P(-14.933 < X < 8.4) = P(X < 8.4) - P(X > -14.933) \approx 1 - 0 = 1$

Thus, the probability that the average time waiting in line between 5 and 10 minutes is approximately 1.

c) $z = \frac{X-\mu}{\sigma_{\bar{X}}} = \frac{6-8.2}{\frac{1.5}{7}} \approx -10.267$

$P(Z < -10.267) \approx 4.979314 \times 10^{-25} \approx 0$

Thus, the probability that the average waiting time in line is less than 6 minutes is approximately 0.

```r
pnorm(10, mean=8.2, sd=(1.5/7))
```

```
## [1] 1
```

```r
pnorm(10, mean=8.2, sd=(1.5/7)) - pnorm(5, mean=8.2, sd=(1.5/7))
```

```
## [1] 1
```

```r
pnorm(6, mean=8.2, sd=(1.5/7))
```

```
## [1] 4.979314e-25
```

Problem 7.3.7

a) $\hat{\theta} = \frac{425+431+416+419+421+436+418+410+431+433+423+426+410+435+436+428+411+426+409+437+422+428+413+416}{24} \approx 423.33$

b) $S = \sqrt{S^2} = \sqrt{\frac{\sum_{i=0}^{24}(X_i-\bar{X})^2}{n-1}} = \sqrt{\frac{(425-423.33)^2+(431-423.33)^2+...+(413-423.33)^2+(416-423.33)^2}{23}} \approx \sqrt{82.49275} \approx 9.082552$

c) $\sigma X = \frac{S}{\sqrt{n}} = \sqrt{\frac{S^2}{n}} \approx \sqrt{\frac{82.49275}{24}} \approx 1.853968$

d) $\frac{423+425}{2} = 424$

e) $\frac{7}{24}$ Notably, the following thicknesses are larger than 430: 431 431 433 435 436 436 437

```r
data <- c(425, 431, 416, 419, 421, 436, 418, 410, 431, 433, 423, 426, 410, 435, 436, 428, 411, 426, 409
mean(data)
```

```
## [1] 423.3333
```

```r
sqrt(var(data))
```

```
## [1] 9.082552
```

```r
sqrt(var(data)/length(data))
```

```
## [1] 1.853968
```

```r
sort(data)
```

```
##  [1] 409 410 410 411 413 416 416 418 419 421 422 423 425 426 426 428 428 431 431
## [20] 433 435 436 436 437
```

```r
median(data)
```

```
## [1] 424
```

Problem 7.3.9

a)

$$\begin{aligned}
E[\bar{X}_1 - \bar{X}_2] &= E[\bar{X}_1] - E[\bar{X}_2] \\
&= E[\frac{X_{1,1} + X_{1,2} + ... + X_{1,n_1}}{n_1}] - E[\frac{X_{2,1} + X_{2,2} + ... + X_{2,n_2}}{n_2}] \\
&= \frac{1}{n_1}E[X_{1,1} + X_{1,2} + ... + X_{1,n_1}] - \frac{1}{n_2}E[X_{2,1} + X_{2,2} + ... + X_{2,n_1}] \\
&= \frac{1}{n_1}(\mu_1 + \mu_1 + ... + \mu_1) - \frac{1}{n_2}(\mu_2 + \mu_2 + ... + \mu_2) \\
&= \frac{1}{n_1}n_1\mu_1 - \frac{1}{n_2}n_2\mu_2 \\
&= \mu_1 - \mu_2
\end{aligned}$$