HW9

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2024-11-23

9.1.10

a) We want to find:

$$\begin{aligned}\alpha &= P(\text{reject}|H_0 \text{ is true}) \\ &= P(\bar{X} \leq 4.85) + P(\bar{X} \geq 5.15)\end{aligned}$$

Given that $H_0 : \mu = 5$, we get $\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{4.85-5}{\frac{0.25}{\sqrt{8}}} \approx -1.69706$.

So we're thus solving for $P(Z \leq -1.69706) + P(Z \geq 1.69706)$, which is simply $2P(Z \leq -1.69706) \approx 0.08968532$.

Thus, $\alpha = 0.08968532$

b) The power is the probability of rejecting H_0 given that the alternative hypothesis: $H_1 : \mu = 5.1$ is true. Thus, we're solving for $P(\bar{X} \leq 4.85) + P(\bar{X} \geq 5.15)$ with $\mu = 5.1$.

Here, we'll normalized $\bar{X} = 4.85$, the lower bound to find $P(\bar{X} \leq 4.85) + P(\bar{X} \geq 5.15) \ = \ 2P(\bar{X} \leq 4.85)$

$\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{4.85-5.1}{\frac{0.25}{\sqrt{8}}} \approx -2.82843$

We then get $P(Z \leq -2.82843) = 0.0023388$, thus $2(Z \leq -2.82843) = 0.004677693$.

Thus, the power of the test is 0.004677693.

```r
2 * pnorm(-1.69706)
```

```
## [1] 0.08968532
```

```r
2 * pnorm(-2.82843)
```

```
## [1] 0.004677693
```

9-23

Given are given $H_0 : \mu = 5$.

a) $5.2 > 5$, so we'll take it as the upper bound. Thus, we're looking for $P(\frac{\bar{X}-5}{\frac{0.25}{\sqrt{8}}} \geq \frac{5.2-5}{\frac{0.25}{\sqrt{8}}})$.

We get $\frac{5.2-5}{\frac{0.25}{\sqrt{8}}} = 2.26274$, and $P(Z > 2.26274) \approx 0.0118258$.

Since the CI is two-sided, our p-value is thus $2P(Z > 1.13137) \approx 0.02365172$

b) $4.7 < 5$, so we take the observed statistic as the lower bound. Thus, we're looking for $P(\frac{\bar{X}-5}{\frac{0.25}{\sqrt{8}}} \leq \frac{4.7-5}{\frac{0.25}{\sqrt{8}}})$.

We get $\frac{4.7-5}{\frac{0.25}{\sqrt{8}}} = -3.394112$, and $P(Z < -3.394112) \approx 0.0003442576$.

Since the CI is two-sided, our p-value is thus $2P(Z < -3.394112) \approx 0.0003442576$

c) $5.1 > 5$, so we'll take it as the upper bound. Thus, we're looking for $P(\frac{\bar{X}-5}{\frac{0.25}{\sqrt{8}}} \geq \frac{5.1-5}{\frac{0.25}{\sqrt{8}}})$.

We get $\frac{5.1-5}{\frac{0.25}{\sqrt{8}}} = 1.131371$, and $P(Z > 1.131371) \approx 0.257899$.

Since the CI is two-sided, our p-value is thus $2P(Z > 1.13137) = 1.742101$

```
2 * (1 - pnorm(2.26274))
```

```
## [1] 0.02365172
```

```
2 * pnorm(-3.394112)
```

```
## [1] 0.0006885153
```

```
2 * (1 - pnorm(1.131371))
```

```
## [1] 0.257899
```

9.2.6

a) Our $t_0 = \frac{2.78-3}{\frac{0.9}{\sqrt{15}}} = -0.9467293$

Since the interval is two-sided, our p-value for this test statistic is $2 \times P(t_{14} > 0.9467293)$, which is approximately 0.3598322.

Since the p-value is greater than our α ($0.3598322 > 0.05$), we fail to reject the null hypothesis H_0.

b) $power = P(Reject\ H_0 \mid H_1 \text{ is true})$ is what we're looking for. We will retrieve the upper and lower critical regions using the fact that we know $\alpha = 0.05$ and $H_0 = 3$

$1.96 = \frac{\bar{X}-3}{\frac{0.9}{\sqrt{15}}}$, thus $\bar{X} \approx 3.45546$ for the upper critical region.

$-1.96 = \frac{\bar{X}-3}{\frac{0.9}{\sqrt{15}}}$, thus $\bar{X} \approx 2.54454$ for the lower critical region.

```
-0.22/(0.9/sqrt(15))
```

```
## [1] -0.9467293
```

```
2 * pt(-0.946729, 15-1)
```

```
## [1] 0.3598322
```

9.3.8

a) $H_0 : \mu_0 = 25$, and our calculated mean is 26.035. We can then calculate our test statistic:

$t_0 = \frac{26.035-25}{\frac{4.784765}{\sqrt{20}}} \approx 0.96737$

Using our p-test function, we get that our p-value is ≈ 0.1727562. Since this is larger than our α, we fail to reject our null hypothesis.

b) Since our rainfall roughly matches the qqline, we can say that its normally distributed.

c) Constructing Our CI:

$z = \bar{x} + z_{0.01}\frac{\sigma}{\sqrt{n}} = 26.035 + 2.33\frac{4.784765}{\sqrt{20}} \approx 28.52788$

Since this upper bound is larger than 25, we fail to reject our null hypothesis.

```r
rainfall <- c(18.0, 30.7, 19.8, 27.1, 22.3, 18.8, 31.8,
              23.4, 21.2, 27.9, 31.9, 27.1, 25.0, 24.7,
              26.9, 21.8, 29.2, 34.8, 26.7, 31.6)

mean(rainfall)
```

```
## [1] 26.035
```

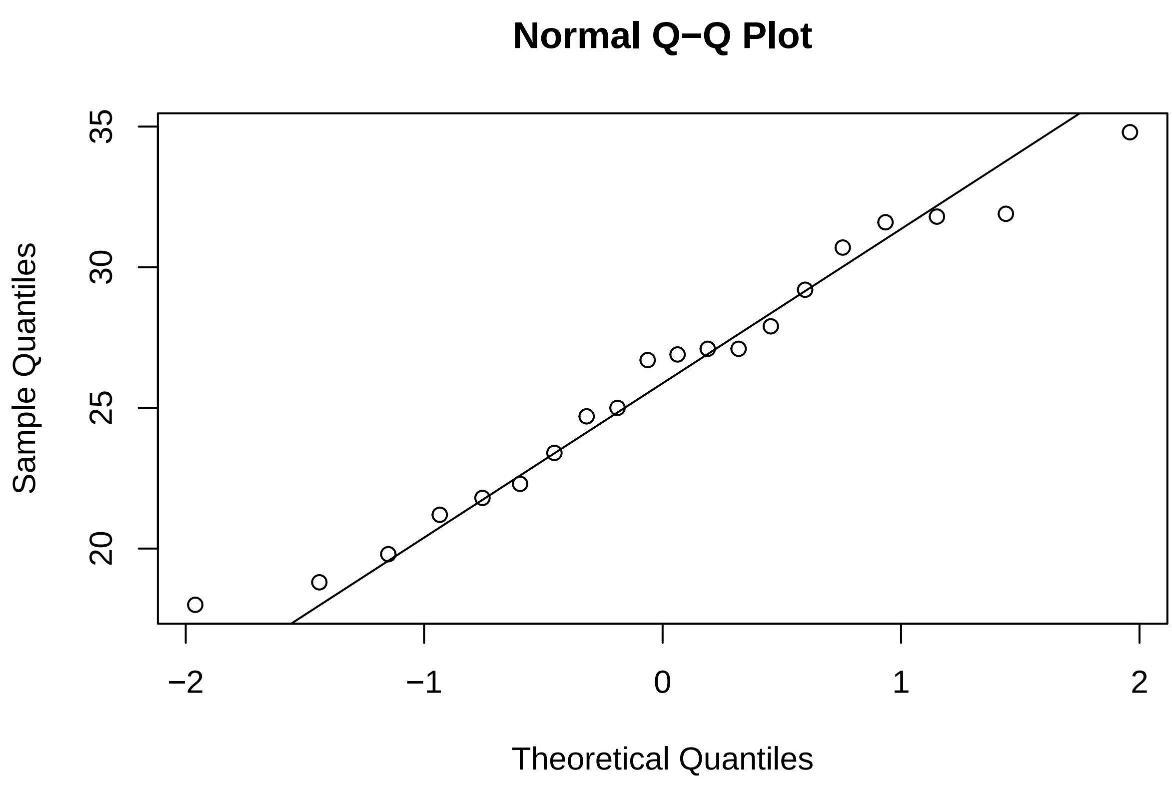
```r
sd(rainfall)
```

```
## [1] 4.784765
```

```r
pt(0.96737, 20-1, lower.tail = FALSE)
```

```
## [1] 0.1727562
```

```r
qqnorm(rainfall)
qqline(rainfall)
```



9.4.3

s = 0.09, $\sigma = 0.75$, $\alpha = 0.05$

a) $\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(17-1)0.09^2}{0.75^2} = 0.2304$

Our p-value is ≈ 1, which is greater than our $alpha$, thus, we fail to reject the null hypothesis.

b) We want to create an CI where:

$\sigma = \sqrt{\frac{(17-1)(0.09^2)}{\chi^2_{1-0.05,17-1}}} = \sqrt{\frac{(17-1)(0.09^2)}{26.296}} \approx 0.070203$

```
pchisq(0.2304, 16, lower.tail = FALSE)
```

```
## [1] 1
```

9.5.4

a) This is a binomial statistic with a large sample, therefore, our test statistic will be:

$z_0 = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\frac{117}{484}-(0.5)}{\sqrt{\frac{0.5(1-0.5)}{484}}} \approx -11.363636$

$p - val = 2 \cdot P(Z < -11.363636) \approx 0.$

Since our p-value is so much smaller than our $alpha = 0.05$, thus we reject our null hypothesis made by Fortune.

b) To construct a two-sided CI on p, we want:

$\hat{p} \pm z_{0.025} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{117}{484} \pm 1.96 \cdot \sqrt{\frac{\frac{117}{484}(1-\frac{117}{484})}{484}}$

Our upper bound is 0.279878 and our lower bound is 0.203593. Since the CI does not contain 0.5, we thus reject H_0.