Control Systems
Lecture 10: Hand Waving Introduction to PID Control

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Outline

The study materials of this lecture can be found at : https://github.com/Daro12/Control-Systems

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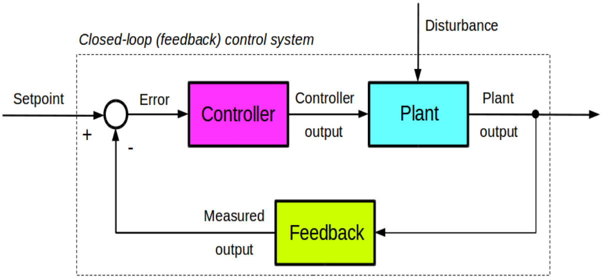

fig source: x-engineer.org

http://www.comtecswiss.com

https://www.goeke-group.com

https://www.mprnews.org

and much more...

- On/off- Controller
- PID controller

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau)d\tau + K_D \frac{d}{dt} e(t)$$

or

$$u(t) = K \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau + T_d \frac{d}{dt} e(t) \right)$$

where $K_p = K, K_I = K/T_i$ and $K_D = K/Td$

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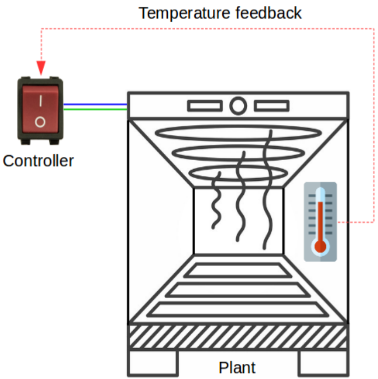
On/Off Control

Example: Oven



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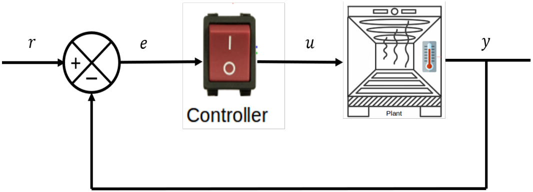
On/Off Control



- y is the output or measurement temperature
- $r = 120c^o$ is the reference or desired temperature
- u, heating effect ($0 \leqq u \leqq 1$), is the control input

On/Off Control

In the theory of control systems, the industrial oven is defined as a first order process with dead time. The transfer function of a first order process with dead time is

$$\frac{K}{Ts+1}e^{-\tau s} \tag{1}$$

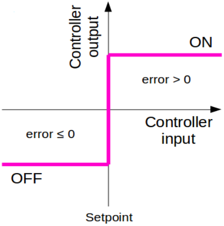
where

- K is gain
- τ is the dead time
- T is the time constant

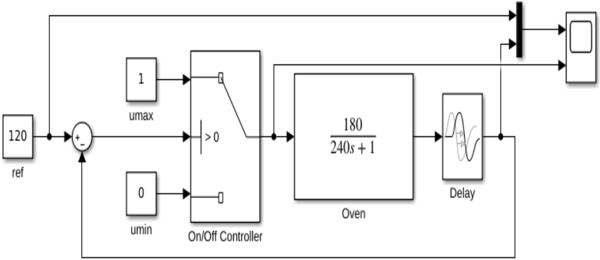
On/Off Control

$$u = \begin{cases} u_{max} & \text{if } e(t) > 0 \\ u_{min} & \text{if } e(t) < 0 \end{cases} \tag{2}$$

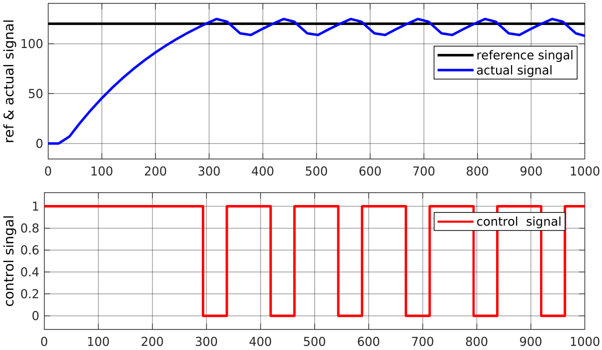
where $e(t) = r(t) - y(t)$



- $r = 120C^o, \tau = 30s$ and $T = 240$



On/Off Control



- Wear on actuators
- Oscillations
- Works only for processes with
  - Simple dynamics
  - low performance requirements

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P-control

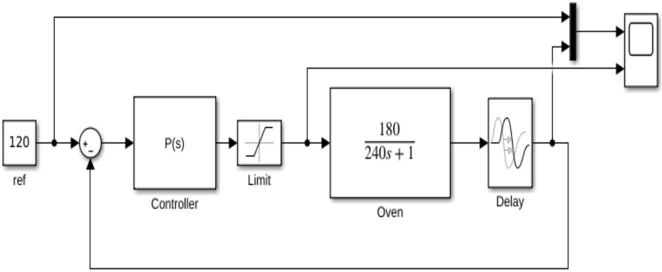
- Use proportional control

$$u(t) = u_0 + Ke(t)$$

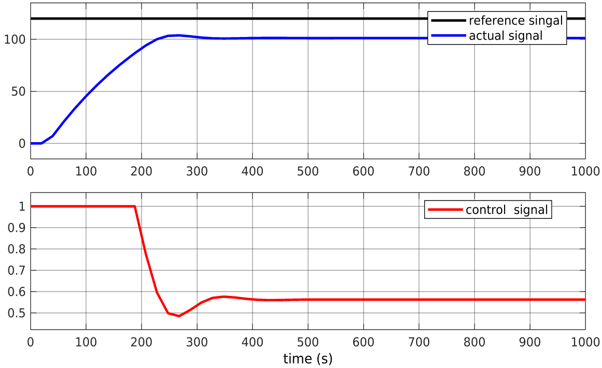
  where

  - u_0 is the control signal corresponding to the zero control error
  - K is proportional gain
  - $e(t) = r(t) - y(t)$

P-Control



Choose $u_0 = 0, K_p = 0.03$, then



There is a steady-state error ($y(t) \neq r(t)$)

P-Control

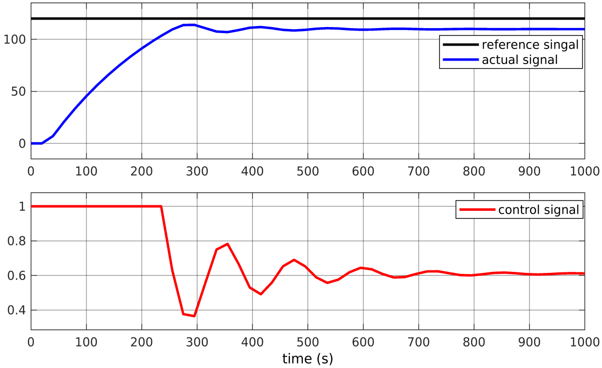
The steady-state error when using a P controller is

$$e = \frac{u - u_0}{K}$$

Two ways to eliminate stationary error:

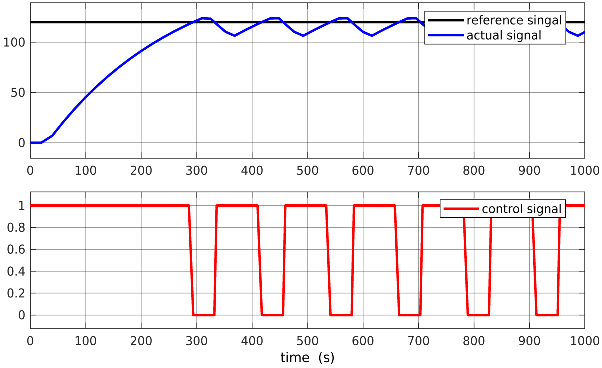
- Let $K \to \infty$
- Select u_0 such that $e = 0$ in stationarity (difficult to find such u_0)

Chose $K_P = 0.06$, then



Can improve the steady-error, but result in the control input with oscillation.

Chose $K = 0.5$, then



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PI Control

- Are there other ways to remove the steady-state error?
- Update u_0 automatically: Replace the constant term u_0 with the integral part:

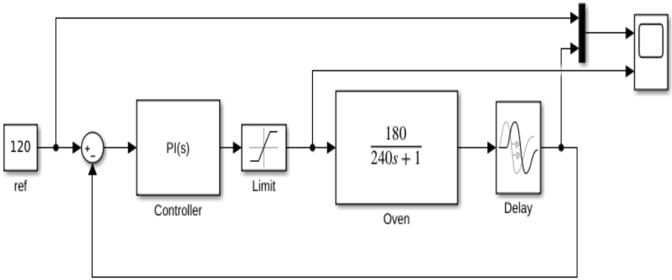
$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right)$$

or

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau)d\tau$$

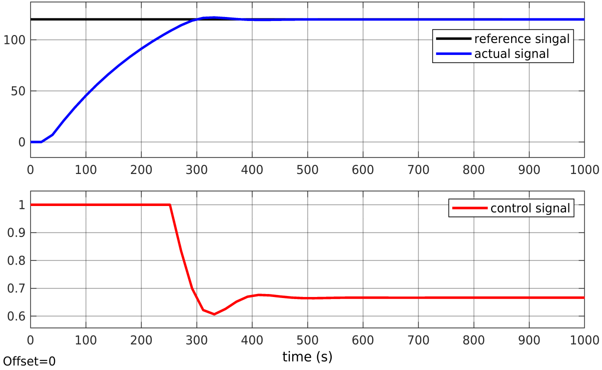
where $K_I = K/T_i$, T_i is the integral time.

PI Control



PI Control

Choose $K_p = 0.03, K_I = 0.00004$

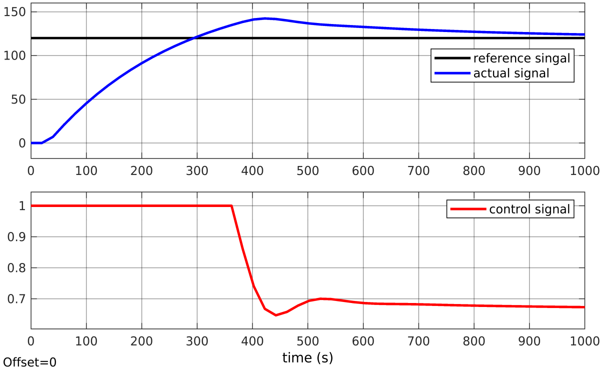


- Can eliminate the steady state error
- If more integral action is added, then there is oscillation

PI Control

Decreasing T_i

Choose $K_p = 0.03, K_I = 0.00009$



Generally, if you increase K_I more, the signal will reach the target value faster, but it creates more oscillation. If you keep increase K_I to a certain value, it can result in an unstable closed loop system.

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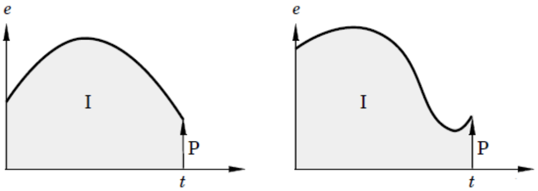
Want something that can react on predicted future errors



PID Control

This can be achieved by adding a derivative term to PI controller

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right) \tag{3}$$

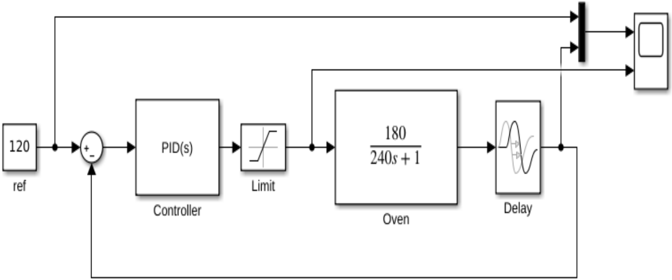
or

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau)d\tau + K_D \frac{d}{dt}e(t)$$

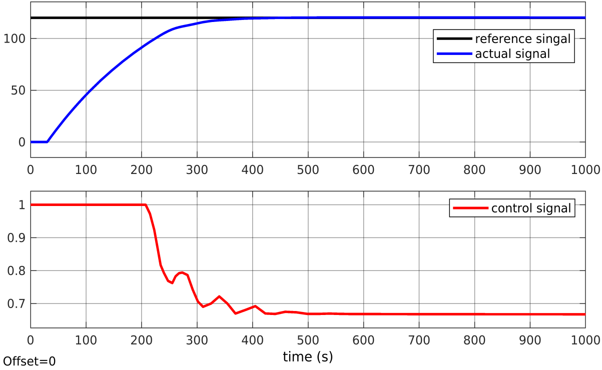
where $K_p = K, K_I = K/T_i$,$K_D = K/Td$, T_d is the derivative time

The derivative part tires to estimate the error change in T_d time units:

$$e(t + T_d) - e(t) \approx T_d\frac{de(t)}{dt} \tag{4}$$

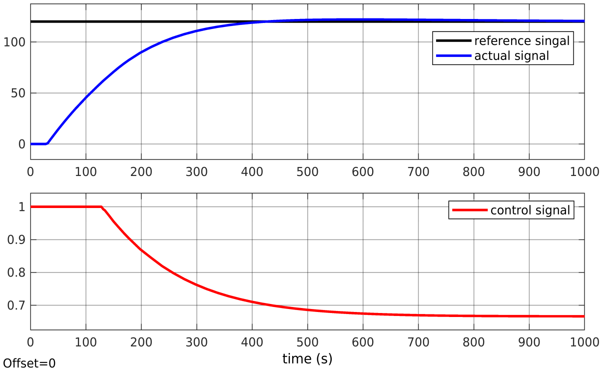

120
ref
PID(s)
Controller
Limit
$\frac{180}{240s+1}$
Oven
Delay

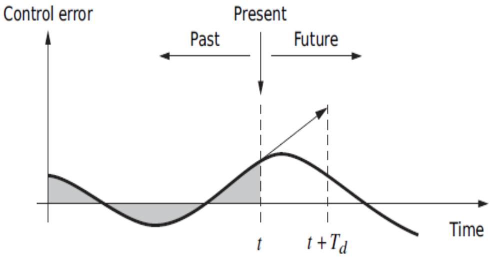
Choose $K_P = 0.03, K_I = 0.00004$ and $K_D = 1$



PID Control

Choose $K_P = 0.03, K_I = 0.00004$ and $K_D = 0.05$





Control error
Present
Past
Future
t
$t+T_d$
Time