Control Systems
Lecture 3 Modeling of Mechanical Systems

Daro VAN

Laboratory of Dynamics and Control
Department of Industrial and Mechanical Engineering
Institute of Technology of Cambodia

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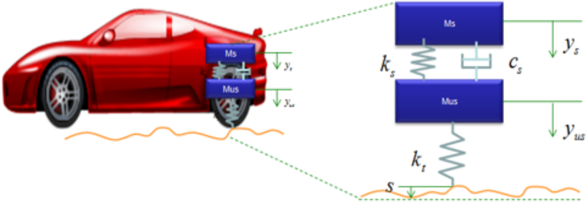
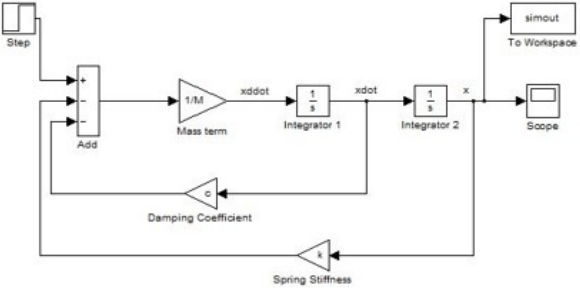
Outline

- Systems modeling is used to map the reality (real physical systems such as drone, car, aircraft, engine,...) into a set of simplified equations (model).
- A system is a set of governing mathematical equations which describe the model's dynamical behavior.



- Once you have a governing mathematical model of equations you can then simulate the system and assess its performances.
- MATLAB/Simulink is a powerful tool to simulate these systems.

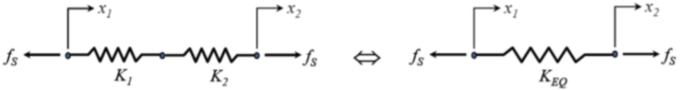


- Attach springs to your feet and start jumping, your body acts as a mass and springs will compress and stores the energy, and then it releases that energy back onto mass.
- You keep jumping and bouncing back until energy is dissipated by friction.
- A damper is a device that resists the springs motion and dissipate the energy
- With damper, you jump and don't bounce back because energy stored and released by the spring are absorbed by the damper.
  Example: car's suspension. The mass is the "sprung mass" (chassis), the spring is the coil, and the damper is the shock absorber.

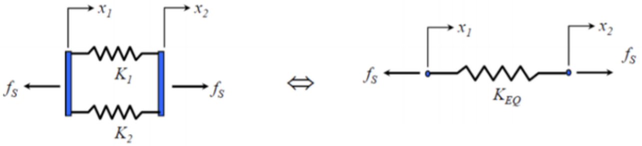
Modeling Simple Systems
Spring

- Springs are stiffness elements used to store and release potential energy
- Assume the spring is ideal; i.e. ignoring its mass and assume zero damping and linear behavior. In real life, springs are not linear.

  - Spring in series: $K_{EQ}=K_1K_2/(K_1+K_2)$



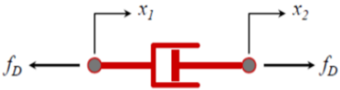
  - Spring in parallel: $K_{EQ}=K_1+K_2$



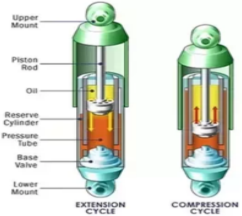
Modeling Simple Systems

Dampers

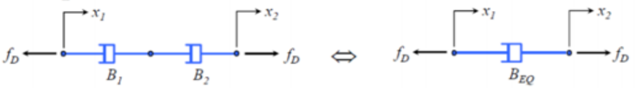
- A damper helps to dissipate energy.
- It absorbs the vibrational energy of the spring of the car.
- If the car does not have a shock absorber, the vehicle would spring up and down till it lost all its energy.
- The shock absorber dissipates the energy of the spring as heat energy.



$$f_D = B(\dot{x}_2 - \dot{x}_1) = B(v_2 - v_1)$$



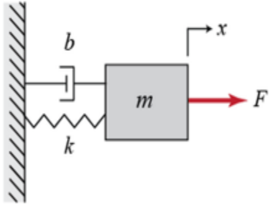
- Dampers in series: $B_{EQ}=B_1B_2/(B_1+B_2)$



- Dampers in parallel: $B_{EQ}=B_1+B_2$

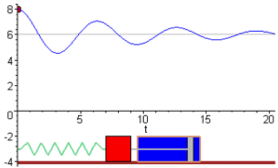
Mass-Spring-Damper Modeling

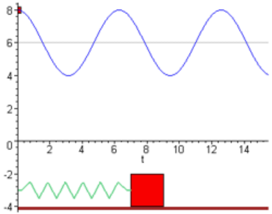
Can you derive the equation of motion of this system?



Spring and damper

No damper





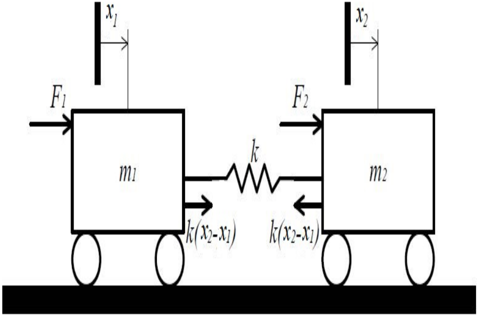
Mass-Spring-Damper in Simulink

- In this section, we model mass-spring-damper system in MATLAB/Simulink.

Mass-Spring-Damper

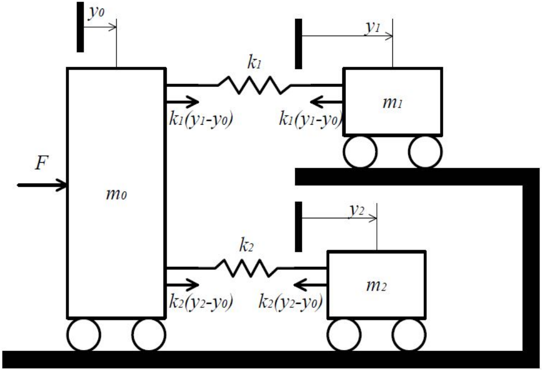
Two masses with Spring Systems

Please derive the equation of motion of this system.



We will get back to this when we study state-space representation.

Please derive the equation of motion of this system.



We will get back to this when we study state-space representation.

The Transfer Function

In a Laplace Transform, if the input is represented by R(s) and the output is represented by C(s), then the transfer function will be:

$$G(s) = \frac{C(s)}{R(s)}$$

So what is the transfer function?

Definition

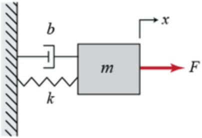
The transfer function of a control system is defined as the ratio of the Laplace transform of the output variable to Laplace transform of the input variable assuming all initial conditions to be zero.

The Transfer Function

Procedure for determining the transfer function of a control system are as follows:

- We form the equations for the system.
- Now we take Laplace transform of the system equations, assuming initial conditions as zero.
- Specify system output and input.
- Lastly we take the ratio of the Laplace transform of the output and the Laplace transform of the input which is the required transfer function.

Note: It is not necessary that output and input of a control system are of same category. For example, in electric motors the input is electrical signal whereas the output is mechanical signal. Similarly in an electric generator, the input is mechanical signal and the output is electrical signal.



Assume
$m = 1kg, b = 10Ns/m, k = 20N/m$