Control Systems
L7: Transient and Steady State Response Analyses

Daro VAN

Phnom Penh, May 17th, 2020

Outline

Reading: Modern Control Engineering by Katsuhiko Ogata, 5th edition (chapter 5)

Outline

Reading: Modern Control Engineering by Katsuhiko Ogata, 5th edition (chapter 5)

Mathematical Models

Mathematical Models

$\Downarrow$

Analysis of system performance
(various control systems)

The commonly used test input signals are:

- Step functions
- Ramp functions
- Impulse functions
- Acceleration functions
- Sinusoidal functions
- White noise

The commonly used test input signals are:

- Step functions
- Ramp functions
- Impulse functions
- Acceleration functions
- Sinusoidal functions
- White noise

With these test signals, mathematical and experimental analyses of control systems can be carried out easily, since the signals are very simple functions of time.

Note: We will use the signals marked blue.

The time response of a control system consists of two parts:

- Transient response (from initial state to final state)
- Steady-state response (t approach infinity)

The time response of a control system consists of two parts:

- Transient response (from initial state to final state)
- Steady-state response (t approach infinity)

Thus, the system response $c(t)$ may be written as

$$c(t) = c_{tr}(t) + c_{ss}(t) \tag{1}$$

where the first term on the right-hand side of the equation is the transient response and the second term is the steady-state response.

knowledge of the components

knowledge of the components

$$\Downarrow$$

Dynamic behavoir of the system

knowledge of the components

$\Downarrow$

Dynamic behavoir of the system

$\Downarrow$

absolute stability

(whether the system is stable or unstable?)

knowledge of the components

$\Downarrow$

Dynamic behavoir of the system

$\Downarrow$

absolute stability

(whether the system is stable or unstable?)

- A control system is in equilibrium if, in the absence of any disturbance or input, the output stays in the same state.

knowledge of the components

$\Downarrow$

Dynamic behavoir of the system

$\Downarrow$

absolute stability

(whether the system is stable or unstable?)

- A control system is in equilibrium if, in the absence of any disturbance or input, the output stays in the same state.
- A LTI system is stable if the output eventually comes back to its equilibrium state when the system is subjected to an initial condition.

knowledge of the components

$\Downarrow$

Dynamic behavoir of the system

$\Downarrow$

absolute stability

(whether the system is stable or unstable?)

- A control system is in equilibrium if, in the absence of any disturbance or input, the output stays in the same state.
- A LTI system is stable if the output eventually comes back to its equilibrium state when the system is subjected to an initial condition.
- A LIT is critically stable if oscillations of the output continue forever.

knowledge of the components

$\Downarrow$

Dynamic behavoir of the system

$\Downarrow$

absolute stability

(whether the system is stable or unstable?)

- A control system is in equilibrium if, in the absence of any disturbance or input, the output stays in the same state.
- A LTI system is stable if the output eventually comes back to its equilibrium state when the system is subjected to an initial condition.
- A LIT is critically stable if oscillations of the output continue forever.
- It is unstable if the output diverges without bound from its equilibrium state when the system is subjected to an initial condition.

Important system behavior (other than absolute stability) to which we must give careful consideration includes relative stability and steady-state error.

Important system behavior (other than absolute stability) to which we must give careful consideration includes relative stability and steady-state error.

Since a physical control system involves energy storage, the output of the system, when subjected to an input, cannot follow the input immediately but exhibits a transient response before a steady state can be reached.

Important system behavior (other than absolute stability) to which we must give careful consideration includes relative stability and steady-state error.

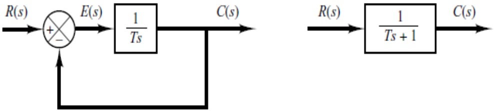
Since a physical control system involves energy storage, the output of the system, when subjected to an input, cannot follow the input immediately but exhibits a transient response before a steady state can be reached. If

the output of a system at steady state does not exactly agree with the input, the system is said to have steady-state error. This error is indicative of the accuracy of the system.

Outline

Reading: Modern Control Engineering by Katsuhiko Ogata, 5th edition (chapter 5)

Consider the first-order system shown below

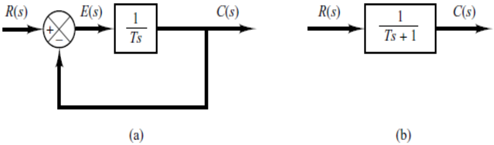


(a)

(b)

First Order systems

Consider the first-order system shown below



The input-output relationship is given by

$$\frac{C(s)}{R(s)} = \frac{1}{Ts + 1} \tag{2}$$

In the following, we shall analyze the system responses to such inputs as the unit-step, unit-ramp, and unit-impulse functions.The initial conditions are assumed to be zero.

Since the Laplace transform of the unit-step function is $1/s$, substituting $R(s) = 1/s$ into Equation (2) , we obtain

$$C(s) = \frac{1}{Ts+1}\frac{1}{s}$$

Since the Laplace transform of the unit-step function is $1/s$, substituting $R(s) = 1/s$ into Equation (2) , we obtain

$$C(s) = \tfrac{1}{Ts+1}\tfrac{1}{s}$$

Expanding $C(s)$ into partial fractions gives

$$C(s) = \frac{1}{s} - \frac{T}{Ts+1} = \frac{1}{s} - \frac{1}{s+(1/T)} \tag{3}$$

Since the Laplace transform of the unit-step function is $1/s$, substituting $R(s) = 1/s$ into Equation (2) , we obtain

$$C(s) = \tfrac{1}{Ts+1}\tfrac{1}{s}$$

Expanding $C(s)$ into partial fractions gives

$$C(s) = \frac{1}{s} - \frac{T}{Ts+1} = \frac{1}{s} - \frac{1}{s+(1/T)} \tag{3}$$

Taking the inverse Laplace transform, we obtain

$$c(t) = 1 - e^{-t/T}, t \geqq 0 \tag{4}$$

when	$t = 0$	$c(t) = 0$
when	$t = T$	$c(t) = 0.632$

Since the Laplace transform of the unit-step function is $1/s$, substituting $R(s) = 1/s$ into Equation (2) , we obtain

$$C(s) = \tfrac{1}{Ts+1}\tfrac{1}{s}$$

Expanding $C(s)$ into partial fractions gives

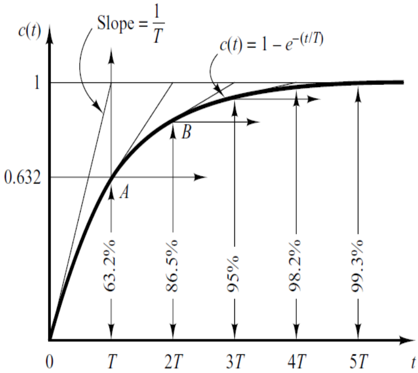
$$C(s) = \frac{1}{s} - \frac{T}{Ts+1} = \frac{1}{s} - \frac{1}{s+(1/T)} \tag{3}$$

Taking the inverse Laplace transform, we obtain

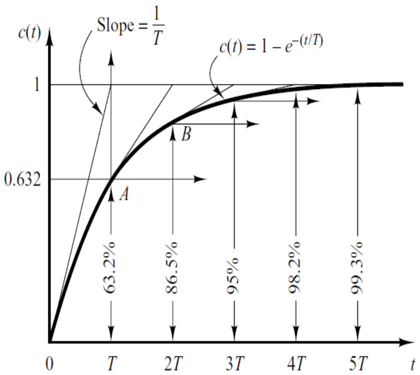
$$c(t) = 1 - e^{-t/T}, t \geqq 0 \tag{4}$$

when	$t = 0$	$c(t) = 0$
when	$t = T$	$c(t) = 0.632$

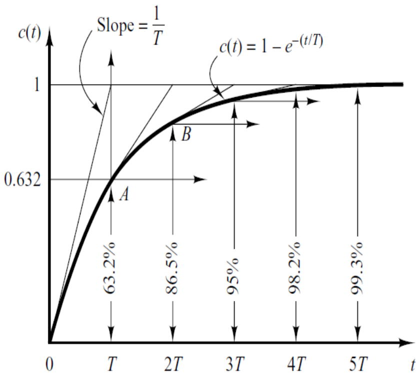
The response $c(t)$ reaches 63.2% of its total change.



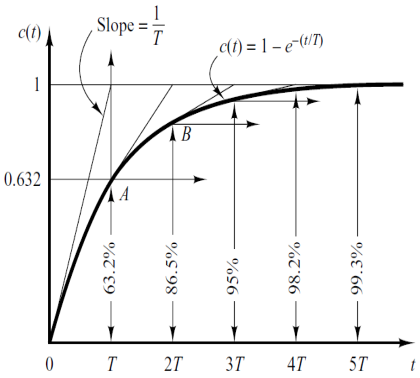
$c(t)$
Slope = $\frac{1}{T}$
$c(t) = 1 - e^{-(t/T)}$
1
0.632
B
A
63.2%
86.5%
95%
98.2%
99.3%
0
T
$2T$
$3T$
$4T$
$5T$
t



- The smaller the time constant T, the faster the system response



- The smaller the time constant T, the faster the system response
- The slope of the response curve $c(t)$ decrease monotonically from $1/T$ at $t = 0$ to 0 at $t = \infty$.



- The smaller the time constant T, the faster the system response
- The slope of the response curve $c(t)$ decrease monotonically from $1/T$ at $t = 0$ to 0 at $t = \infty$.
- In practice, a reasonable estimate of the response time is the length of time the response curve needs to reach and stay within the 2%line of the final value.

Since the Laplace Transform of the unit-ramp function is $1/s^2$, we obtain the output of the system as

$$C(s) = \frac{1}{Ts+1}\frac{1}{s^2} \tag{5}$$

Since the Laplace Transform of the unit-ramp function is $1/s^2$, we obtain the output of the system as

$$C(s) = \frac{1}{Ts+1}\frac{1}{s^2} \tag{5}$$

Expanding $C(s)$ into partial fractions gives

$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{Ts+1} \tag{6}$$

Since the Laplace Transform of the unit-ramp function is $1/s^2$, we obtain the output of the system as

$$C(s) = \frac{1}{Ts+1}\frac{1}{s^2} \tag{5}$$

Expanding $C(s)$ into partial fractions gives

$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{Ts+1} \tag{6}$$

Taking the inverse Laplace transform, we obtain

$$c(t) = t - T + Te^{-t/T}, t \geqq 0 \tag{7}$$

Since the Laplace Transform of the unit-ramp function is $1/s^2$, we obtain the output of the system as

$$C(s) = \frac{1}{Ts+1}\frac{1}{s^2} \tag{5}$$

Expanding $C(s)$ into partial fractions gives

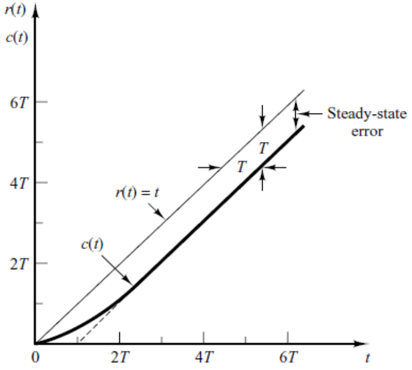
$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{Ts+1} \tag{6}$$

Taking the inverse Laplace transform, we obtain

$$c(t) = t - T + Te^{-t/T}, t \geqq 0 \tag{7}$$

The error signal $e(t)$ is then

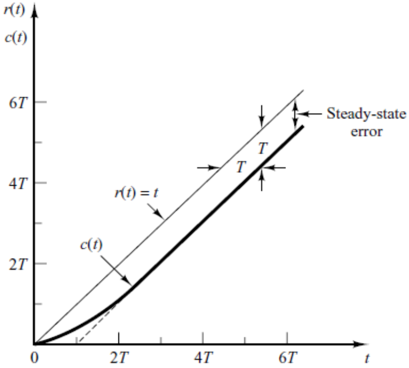
$$\begin{aligned} e(t) &= r(t) - c(t) \\ &= T(1 - e^{-t/T}) \end{aligned}$$



r(t)
c(t)
6T
4T
2T
0
2T
4T
6T
t
r(t) = t
c(t)
T
T
Steady-state
error

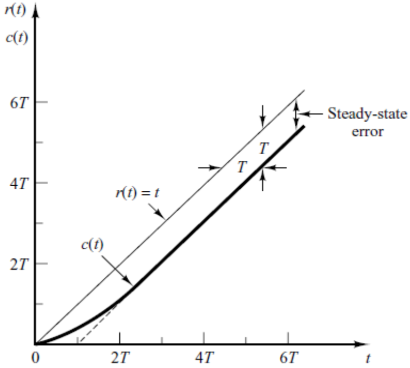
First Order Systems

Unit-Ramp Response of First-Order Systems



- As t approaches infinity, $e^{-t/T}$ approaches zero, and thus the error signal $e(t)$ approaches T or

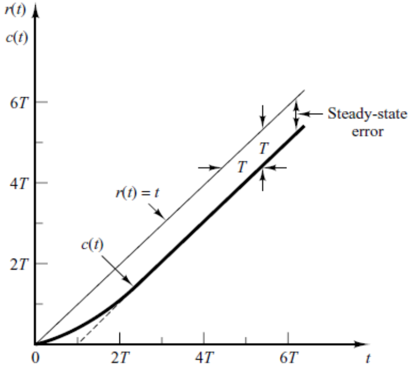
$$e(\infty) = T$$



- As t approaches infinity, $e^{-t/T}$ approaches zero, and thus the error signal $e(t)$ approaches T or

$$e(\infty) = T$$

- The error in following the unit-ramp input is equal to T for sufficiently large t.



- As t approaches infinity, $e^{-t/T}$ approaches zero, and thus the error signal $e(t)$ approaches T or

$$e(\infty) = T$$

- The error in following the unit-ramp input is equal to T for sufficiently large t.

  The smaller the time constant T, the smaller the steady-state error in following the ramp input.

For the unit-impulse input, $R(s) = 1$ and the output of the system can be obtained as

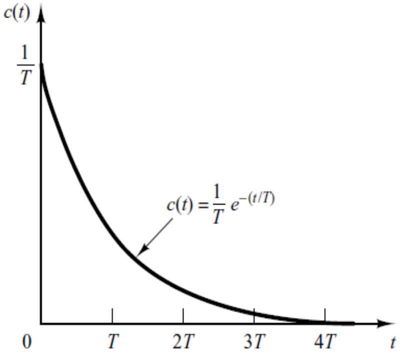
$$C(s) = \frac{1}{Ts + 1} \tag{8}$$

For the unit-impulse input, $R(s) = 1$ and the output of the system can be obtained as

$$C(s) = \frac{1}{Ts + 1} \tag{8}$$

The inverse Laplace Transform of Equation (8) gives

$$c(t) = \frac{1}{T}e^{-t/T}, t \geqq 0 \tag{9}$$



It has been shown that the response of

- unit-ramp input is $c(t) = t - T + Te^{-t/T}, t \geqq 0$
- unit-step input is $c(t) = 1 - e^{-t/T}, \geqq 0$
- unit-impulse input is $c(t) = \frac{1}{T}e^{-t/T}, t \geqq 0$

Note:This is a property of linear time-invariant systems. Linear time-varying systems and nonlinear systems do not possess these properties.

It has been shown that the response of

- unit-ramp input is $c(t) = t - T + Te^{-t/T}, t \geqq 0$
- unit-step input is $c(t) = 1 - e^{-t/T}, \geqq 0$
- unit-impulse input is $c(t) = \frac{1}{T}e^{-t/T}, t \geqq 0$

It can be seen that

- The response to the derivative of an input signal can be obtained by differentiating the response of the system to the original signal.

Note:This is a property of linear time-invariant systems. Linear time-varying systems and nonlinear systems do not possess these properties.

It has been shown that the response of

- unit-ramp input is $c(t) = t - T + Te^{-t/T}, t \geqq 0$
- unit-step input is $c(t) = 1 - e^{-t/T}, \geqq 0$
- unit-impulse input is $c(t) = \frac{1}{T}e^{-t/T}, t \geqq 0$

Note:This is a property of linear time-invariant systems. Linear time-varying systems and nonlinear systems do not possess these properties.

It can be seen that

- The response to the derivative of an input signal can be obtained by differentiating the response of the system to the original signal.
- The response to the integral of the original signal can be obtained by integrating the response of the system to the original signal and by determining the integration constant from the zero-output initial condition.

Outline

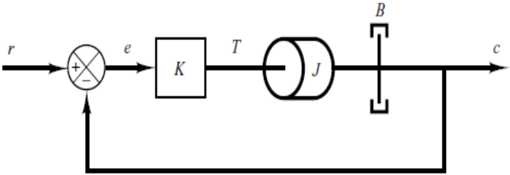
Reading: Modern Control Engineering by Katsuhiko Ogata, 5th edition (chapter 5)

Second Order System

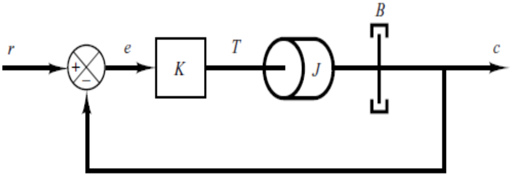
In this section, we shall obtain the response of a typical second-order control system to

- Step input
- Ramp input
- Impulse input

The servo system consists of a proportional controllers and load elements (inertia and viscous-friction elements)



The servo system consists of a proportional controllers and load elements (inertia and viscous-friction elements)



Suppose that we wish to control the output position c in accordance with the input position r.The equation for the load elements is

$$J\ddot{c} + B\dot{c} = T$$

where T is the torque produced by the proportional controller whose gain is K.

By taking Laplace transforms of both sides of this last equation, assuming the zero initial conditions, we obtain

$$Js^2C(s) + BsC(s) = T(s)$$

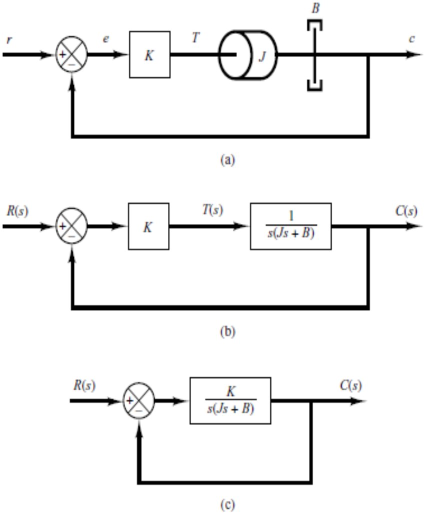
By taking Laplace transforms of both sides of this last equation, assuming the zero initial conditions, we obtain

$$Js^2C(s) + BsC(s) = T(s)$$

then,

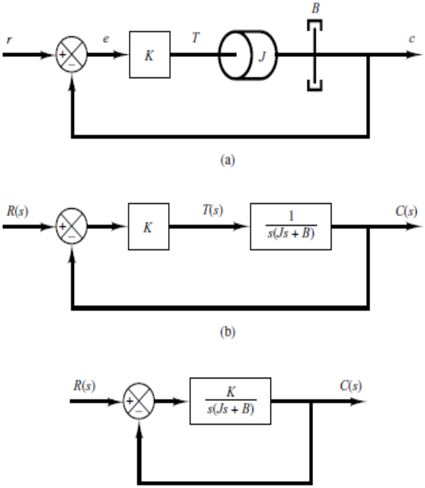
$$\frac{C(s)}{T(s)} = \frac{1}{s(Js+B)}$$

By using this transfer function, can you find $\frac{C(s)}{R(s)}$?



The closed-loop transfer function is then obtained as

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Bs + K} = \frac{K/J}{s^2 + (B/J)s + (K/J)}$$



(a) (b) (c)

The closed-loop transfer function is then obtained as

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Bs + K} = \frac{K/J}{s^2 + (B/J)s + (K/J)}$$

Such a system where the closed-loop transfer function possesses two poles is called a second-order system.

The closed-loop transfer function of the system shown in Figure (c) is

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Bs + K} \tag{10}$$

The closed-loop transfer function of the system shown in Figure (c) is

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Bs + K} \tag{10}$$

which can be rewritten as

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{J}}{(s + \frac{B}{2J} + \sqrt{(\frac{B}{2J})^2 - \frac{K}{J}})(s + \frac{B}{2J} - \sqrt{(\frac{B}{2J})^2 - \frac{K}{J}})} \tag{11}$$

The closed-loop poles are complex conjugate if $B^2 - 4JK < 0$ and they are real if $B^2 - 4JK \geqq 0$

In the transient-response analysis, it is convenient to write

$$\frac{K}{J} = \omega_n^2, \frac{B}{J} = 2\zeta\omega_n = 2\sigma$$

where σ is called the attenuation, ω_n is called the undamped natural frequency and ζ is called damping ratio of the system.

In the transient-response analysis, it is convenient to write

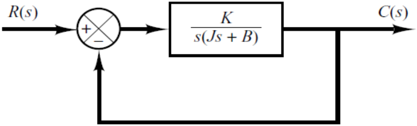
$$\frac{K}{J} = \omega_n^2, \frac{B}{J} = 2\zeta\omega_n = 2\sigma$$

where σ is called the attenuation, ω_n is called the undamped natural frequency and ζ is called damping ratio of the system.

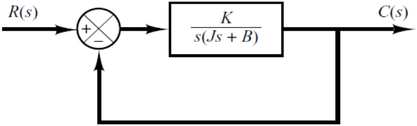
The damping ratio ζ is the ratio of the actual damping B to the critical damping $B_c = 2\sqrt{JK}$ or

$$\zeta = \frac{B}{B_c} = \frac{B}{2\sqrt{JK}}$$

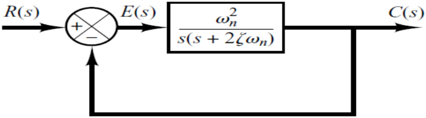
In term of ζ and ω_n, the system (c) can be modified to a generic form



In term of ζ and ω_n, the system (c) can be modified to a generic form



$$\Downarrow$$



and the closed loop transfer function given in equation (10) can be written as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{12}$$

This form is called standard form of the second-order system.

and the closed loop transfer function given in equation (10) can be written as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{12}$$

This form is called standard form of the second-order system.

The dynamic behavior of the second-order system can then be described in terms of two parameters ζ and ω_n.

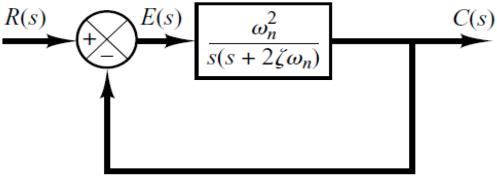
- If $0 < \zeta < 1$, the closed-loop poles are complex conjugates and lie in the left-half s plane. The system is called underdamped, and the transient response is oscillatory.

- If $0 < \zeta < 1$, the closed-loop poles are complex conjugates and lie in the left-half s plane. The system is called underdamped, and the transient response is oscillatory.
- If $\zeta = 0$, the transient response does not die out.

- If $0 < \zeta < 1$, the closed-loop poles are complex conjugates and lie in the left-half s plane. The system is called underdamped, and the transient response is oscillatory.
- If $\zeta = 0$, the transient response does not die out.
- if $\zeta = 1$, the system is called critically damped.

- If $0 < \zeta < 1$, the closed-loop poles are complex conjugates and lie in the left-half s plane. The system is called underdamped, and the transient response is oscillatory.
- If $\zeta = 0$, the transient response does not die out.
- if $\zeta = 1$, the system is called critically damped.
- If $\zeta > 1$, the system is overdamped.

We shall now solve for the response of the system shown below



- Underdamped case $(o < \zeta < 1)$: In the case, $C(s)/R(s)$ can be written
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\zeta\omega_n+j\omega_d)(s+\zeta\omega_n-j\omega_d)}$$
where $\omega_d = \omega_n\sqrt{1-\zeta^2}$. ω_d is called damped natural frequency.

- Underdamped case ($o < \zeta < 1$) : In the case, $C(s)/R(s)$ can be written
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\zeta\omega_n+j\omega_d)(s+\zeta\omega_n-j\omega_d)}$$
where $\omega_d = \omega_n\sqrt{1-\zeta^2}$. ω_d is called damped natural frequency.For unit-step input, $C(s)$ can be written as

$$C(s) = \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)s} \tag{13}$$

- Underdamped case ($o < \zeta < 1$) : In the case, $C(s)/R(s)$ can be written
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\zeta\omega_n+j\omega_d)(s+\zeta\omega_n-j\omega_d)}$$
where $\omega_d = \omega_n\sqrt{1-\zeta^2}$. ω_d is called damped natural frequency.For unit-step input, $C(s)$ can be written as
$$C(s) = \frac{\omega_n^2}{(s^2+2\zeta\omega_n s+\omega_n^2)s} \tag{13}$$
The inverse Laplace transform can be obtained easily if $C(s)$ is written in the following form:
$$\begin{aligned} C(s) &= \frac{1}{s} - \frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2} \\ &= \frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} \end{aligned}$$

Referring to the Laplace transform Table, it can be shown that

$$\mathcal{L}^{-1}\left[\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}\right] = e^{-\zeta\omega_n t}\cos\omega_d t$$
$$\mathcal{L}^{-1}\left[\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right] = e^{-\zeta\omega_n t}\sin\omega_d t$$

Referring to the Laplace transform Table, it can be shown that

$$\mathcal{L}^{-1}[\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}] = e^{-\zeta\omega_n t}\cos\omega_d t$$
$$\mathcal{L}^{-1}[\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}] = e^{-\zeta\omega_n t}\sin\omega_d t$$

Hence, the inverse Laplace transform of $C(s)$ is obtained

$$\begin{aligned}\mathcal{L}^{-1}[C(s)] &= c(t) \\ &= 1 - e^{-\zeta\omega_n t}(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t) \\ &= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}), t \geqq 0\end{aligned} \tag{14}$$

It can be seen that the frequency of transient oscillation is the damped natural frequency ω_d and thus varies with the damping ratio ζ. The error signal for this system is the difference between the input and output and is

$$
\begin{aligned}
e(t) &= r(t) - c(t) \\
&= e^{-\zeta\omega_n t}(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t), t \geqq 0
\end{aligned}
$$

It can be seen that the frequency of transient oscillation is the damped natural frequency ω_d and thus varies with the damping ratio ζ. The error signal for this system is the difference between the input and output and is

$$\begin{aligned} e(t) &= r(t) - c(t) \\ &= e^{-\zeta\omega_n t}(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t), t \geqq 0 \end{aligned}$$

This error signal exhibits a damped sinusoidal oscillation.

It can be seen that the frequency of transient oscillation is the damped natural frequency ω_d and thus varies with the damping ratio ζ. The error signal for this system is the difference between the input and output and is

$$\begin{aligned} e(t) &= r(t) - c(t) \\ &= e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right), t \geqq 0 \end{aligned}$$

This error signal exhibits a damped sinusoidal oscillation.

- At steady state, or at $t = \infty$, no error exists between the input and output.

It can be seen that the frequency of transient oscillation is the damped natural frequency ω_d and thus varies with the damping ratio ζ. The error signal for this system is the difference between the input and output and is

$$\begin{aligned} e(t) &= r(t) - c(t) \\ &= e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right), t \geqq 0 \end{aligned}$$

This error signal exhibits a damped sinusoidal oscillation.

- At steady state, or at $t = \infty$, no error exists between the input and output.
- If the damping ratio ζ is equal to zero, the response becomes undamped oscillations continue indefinitely. The response $c(t)$ for zero damping can be found as

$$c(t) = 1 - \cos\omega_n t, t \geqq 0 \tag{15}$$

- Critically damped case ($\zeta = 1$) : If the two poles of $C(s)/R(s)$ are equal, the system is said to be critically damped one.

- Critically damped case ($\zeta = 1$) : If the two poles of $C(s)/R(s)$ are equal, the system is said to be critically damped one.

  For a unit-step input, $R(s) = 1/s$ and $C(s)$ can be written as

$$C(s) = \frac{\omega_n^2}{(s + \omega_n)^2 s} \tag{16}$$

- Critically damped case ($\zeta = 1$) : If the two poles of $C(s)/R(s)$ are equal, the system is said to be critically damped one.

  For a unit-step input, $R(s) = 1/s$ and $C(s)$ can be written as

$$C(s) = \frac{\omega_n^2}{(s + \omega_n)^2 s} \tag{16}$$

  The inverse Laplace transform can be found as

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t), t \geqq 0 \tag{17}$$

- Overdamped case ($\zeta > 1$): In this case, the two poles of $C(s)/R(s)$ are negative real and unequal. For a unit-step input, $R(s) = 1/s$ and $C(s)$ can be written

$$C(s) = \frac{\omega_n^2}{(s + \zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1})(s + \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1})s} \tag{18}$$

- Overdamped case ($\zeta > 1$): In this case, the two poles of $C(s)/R(s)$ are negative real and unequal. For a unit-step input, $R(s) = 1/s$ and $C(s)$ can be written

$$C(s) = \frac{\omega_n^2}{(s+\zeta\omega_n+\omega_n\sqrt{\zeta^2-1})(s+\zeta\omega_n-\omega_n\sqrt{\zeta^2-1})s} \qquad (18)$$

The inverse Laplace transform of equation (18) is

$$c(t) = 1 + \frac{1}{2\sqrt{\zeta^2-1}(\zeta+\sqrt{\zeta^2-1})} e^{-(\zeta+\sqrt{\zeta^2-1})\omega_n t} - \frac{1}{2\sqrt{\zeta^2-1}(\zeta-\sqrt{\zeta^2-1})} e^{-(\zeta-\sqrt{\zeta^2-1})\omega_n t}$$

$$c(t) = 1 + \frac{\omega_n}{2\sqrt{\zeta^2-1}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right), t \geqq 0$$

where $s_1 = (\zeta + \sqrt{\zeta^2-1})\omega_n$ and $s_2 = (\zeta - \sqrt{\zeta^2-1})\omega_n$

- When ζ is appreciably greater than unity, one of the two decaying exponentials decreases much faster than the other, so the faster-decaying exponential term may be neglected.That is, if s_2 is located very much closer to the $j\omega$ axis than s_1 (which means $|s_2| \gg |s_1|$), then for an approximate solution we may neglect s_1.

- When ζ is appreciably greater than unity, one of the two decaying exponentials decreases much faster than the other, so the faster-decaying exponential term may be neglected.That is, if s_2 is located very much closer to the $j\omega$ axis than s_1 (which means $|s_2| \gg |s_1|$), then for an approximate solution we may neglect s_1.
- This is permissible because the effect of s_1 on the response is much smaller than that of s_2 , since the term involving s_1 decays much faster than the term involving s_2 .

- When ζ is appreciably greater than unity, one of the two decaying exponentials decreases much faster than the other, so the faster-decaying exponential term may be neglected.That is, if s_2 is located very much closer to the $j\omega$ axis than s_1 (which means $|s_2| \gg |s_1|$), then for an approximate solution we may neglect s_1.
- This is permissible because the effect of s_1 on the response is much smaller than that of s_2 , since the term involving s_1 decays much faster than the term involving s_2 .
- Once the faster-decaying exponential term has disappeared, the response is similar to that of a first-order system, and $C(s)/R(s)$ may be approximated by

$$\frac{C(s)}{R(s)} = \frac{\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}}{s+\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}} = \frac{s_2}{s+s_1}$$

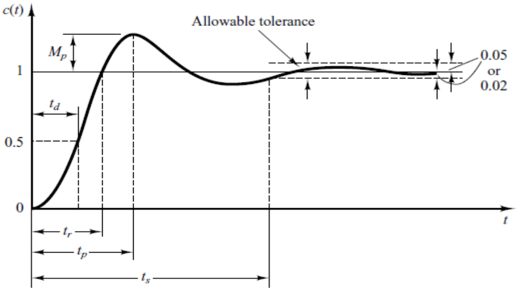
With the appropriate transfer function $\frac{C(s)}{R(s)}$, the unit-step response can be obtained as

$$C(s) = \frac{\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}}{(s + \zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1})s}$$

The time response $c(t)$ is then

$$c(t) = 1 - e^{-(\zeta - \sqrt{\zeta^2 - 1})\omega_n t}, \text{ for } t \geqq 0$$

This gives an approximate unit-step response when one of the poles of $C(s)/R(s)$ can be neglected.



Unit-step response curve showing t_d, t_r,t_p,M_p and t_s

- Delay time,t_d: the time required for the response to reach half the final value the very first time.

- Delay time,t_d: the time required for the response to reach half the final value the very first time.
- Rise time, t_r: the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value. For underdamped secondorder systems, the 0% to 100% rise time is normally used.For overdamped systems, the 10% to 90% rise time is commonly used.

- Delay time,t_d: the time required for the response to reach half the final value the very first time.
- Rise time, t_r: the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value. For underdamped secondorder systems, the 0% to 100% rise time is normally used.For overdamped systems, the 10% to 90% rise time is commonly used.
- Peak time, t_p:the time required for the response to reach the first peak of the overshoot.

Definition of transient-Response Specification

- Maximum overshoot, M_p: the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

Definition of transient-Response Specification

- Maximum overshoot, M_p: the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

- Settling time, t_s:the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%).

Definition of transient-Response Specification

- Maximum overshoot, M_p: the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

- Settling time, t_s:the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%).

Note: Not all these specifications necessarily apply to any given case. For example, for an overdamped system, the terms peak time and maximum overshoot do not apply.

Except for certain applications where oscillations cannot be tolerated, it is desirable that the transient response be sufficiently fast and be sufficiently damped. Thus, for a desirable transient response of a second-order system, the damping ratio must be between 0.4 and 0.8.

Except for certain applications where oscillations cannot be tolerated, it is desirable that the transient response be sufficiently fast and be sufficiently damped. Thus, for a desirable transient response of a second-order system, the damping ratio must be between 0.4 and 0.8.

- Small value of $\zeta(\zeta < 0.4)$ yield excessive overshoot in the transient response.

Except for certain applications where oscillations cannot be tolerated, it is desirable that the transient response be sufficiently fast and be sufficiently damped. Thus, for a desirable transient response of a second-order system, the damping ratio must be between 0.4 and 0.8.

- Small value of $\zeta(\zeta < 0.4)$ yield excessive overshoot in the transient response.
- Large value of ζ $(\zeta > 0.8)$ responds sluggishly.

Except for certain applications where oscillations cannot be tolerated, it is desirable that the transient response be sufficiently fast and be sufficiently damped. Thus, for a desirable transient response of a second-order system, the damping ratio must be between 0.4 and 0.8.

- Small value of $\zeta(\zeta < 0.4)$ yield excessive overshoot in the transient response.
- Large value of ζ $(\zeta > 0.8)$ responds sluggishly.

Note: The maximum overshoot and the rise time conflict with each other. Both the maximum overshoot and the rise time cannot be made smaller simultaneously. If one of them is made smaller, the other necessarily becomes larger (you will see later)

- Rise time t_r: Referring to equation (14), we obtain the rise time by letting $c(t_r) = 1$.

$$c(t_r) = 1 = 1 - e^{\zeta\omega_n t_r}(\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r)$$

since $e^{\zeta\omega_n t_r} \neq 0$, we obtain the following equation:

$$\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r = 0$$

Since $\omega_n\sqrt{1-\zeta^2} = \omega_d$ and $\zeta\omega_n = \sigma$, we have

$$\tan\omega_d t_r = -\frac{\sqrt{1-\zeta^2}}{\zeta} = -\frac{\omega_d}{\sigma}$$

- Rise time t_r: Referring to equation (14), we obtain the rise time by letting $c(t_r) = 1$.

$$c(t_r) = 1 = 1 - e^{\zeta\omega_n t_r}(\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r)$$

since $e^{\zeta\omega_n t_r} \neq 0$, we obtain the following equation:

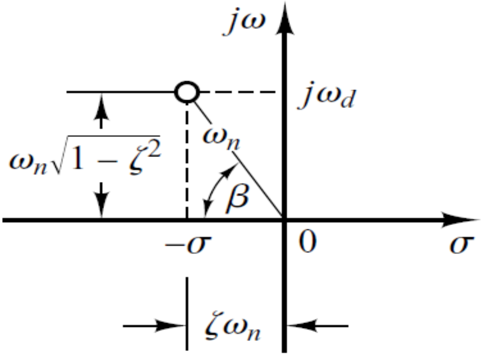
$$\cos\omega_d t_r + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t_r = 0$$

Since $\omega_n\sqrt{1-\zeta^2} = \omega_d$ and $\zeta\omega_n = \sigma$, we have

$$\tan\omega_d t_r = -\frac{\sqrt{1-\zeta^2}}{\zeta} = -\frac{\omega_d}{\sigma}$$

Thus, the rise time t_r is

$$t_r = \frac{1}{\omega_d}\tan^{-1}(\frac{\omega_d}{-\sigma}) = \frac{\pi-\beta}{\omega_d}$$



Clearly, for small value of t_r,ω_d, angle β must be large.

- Peak time t_p: Referring to equation (14), we may obtain peak time by differentiating $c(t)$ with respect to time and letting this derivative equal to zero

$$\frac{dc}{dt} = \zeta\omega_n e^{-\zeta\omega_n t}(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t)$$
$$+e^{-\zeta\omega_n t}(\omega_d \sin\omega_d t + \frac{\zeta\omega_d}{\sqrt{1-\zeta^2}}\cos\omega_d t)$$

and the cosine terms in this last equation cancel each other, dc/dt, evaluated at $t = t_p$, can be simplified to

$$(\sin\omega_d t_p)\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t_p} = 0$$

- Peak time t_p: Referring to equation (14), we may obtain peak time by differentiating $c(t)$ with respect to time and letting this derivative equal to zero

$$\frac{dc}{dt} = \zeta\omega_n e^{-\zeta\omega_n t}(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t) + e^{-\zeta\omega_n t}(\omega_d \sin\omega_d t + \frac{\zeta\omega_d}{\sqrt{1-\zeta^2}}\cos\omega_d t)$$

and the cosine terms in this last equation cancel each other, dc/dt, evaluated at $t = t_p$, can be simplified to

$$(\sin\omega_d t_p)\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t_p} = 0$$

This last equation yields the following equation:

$$\sin\omega_d t_p = 0 \text{ or } \omega_d t_p = 0, \pi, 2\pi, 3\pi, \ldots$$

- Peak time t_p: Referring to equation (14), we may obtain peak time by differentiating $c(t)$ with respect to time and letting this derivative equal to zero

$$\frac{dc}{dt} = \zeta\omega_n e^{-\zeta\omega_n t}(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t) + e^{-\zeta\omega_n t}(\omega_d \sin\omega_d t + \frac{\zeta\omega_d}{\sqrt{1-\zeta^2}}\cos\omega_d t)$$

and the cosine terms in this last equation cancel each other, dc/dt, evaluated at $t = t_p$, can be simplified to

$$(\sin\omega_d t_p)\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t_p} = 0$$

This last equation yields the following equation:

$$\sin\omega_d t_p = 0 \text{ or } \omega_d t_p = 0, \pi, 2\pi, 3\pi, \ldots$$

Since the peak time corresponds to the first peak overshoot, Hence

$$t_p = \frac{\pi}{\omega_d}$$

- Maximum overshoot M_p:occurs at the peak time or at $t = t_p = \pi/\omega_d$. Assuming that the final value of the output is unity,M_p is obtained from Equation (14) as

$$
\begin{aligned}
M_p &= c(t_p) - 1 \\
&= -e^{-\zeta\omega_n(\pi/\omega_d)}(\cos\pi + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\pi) \\
&= e^{-(\sigma/\omega_d)\pi} = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}
\end{aligned}
\tag{19}
$$

- Maximum overshoot M_p:occurs at the peak time or at $t = t_p = \pi/\omega_d$. Assuming that the final value of the output is unity,M_p is obtained from Equation (14) as

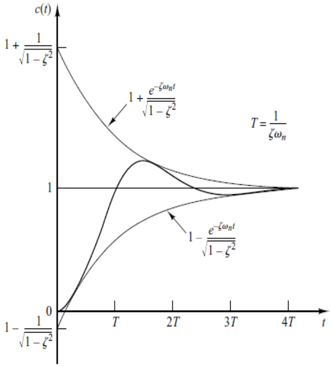
$$\begin{aligned} M_p &= c(t_p) - 1 \\ &= -e^{-\zeta\omega_n(\pi/\omega_d)}(\cos\pi + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\pi) \\ &= e^{-(\sigma/\omega_d)\pi} = e^{-(\zeta/\sqrt{1-\zeta^2})\pi} \end{aligned} \tag{19}$$

The maximum percent overshoot is $e^{-(\sigma/\omega_d)\pi} \times 100\%$.
If the final value $c(\infty)$ of the output is not unity, then we need to use the following equation:

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)}$$

- Setting time t_s: For an underdamped second-order system, the transient response is obtained from Equation (14) as

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}), t \geqq 0$$



For convenience in comparing with the responses of systems, we commonly define the settling time t_s to be

$$t_s = 4T = \frac{4}{\sigma} = \frac{4}{\zeta\omega_n} \text{ (2\% criterion)}$$

or

$$t_s = 3T = \frac{3}{\sigma} = \frac{3}{\zeta\omega_n}$$

Outline

Reading: Modern Control Engineering by Katsuhiko Ogata, 5th edition (chapter 5)

Servo System with Velocity Feedback

- The derivative of the output signal can be used to improve system performance. In obtaining the derivative of the output position signal, it is desirable to use a tachometer instead of physically differentiating the output signal.

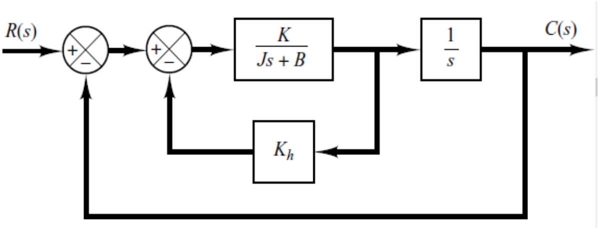
Servo System with Velocity Feedback

- The derivative of the output signal can be used to improve system performance. In obtaining the derivative of the output position signal, it is desirable to use a tachometer instead of physically differentiating the output signal.

Note: The differentiation amplifies noise effects.In fact, if discontinuous noises are present, differentiation amplifies the discontinuous noises more than the useful signal.

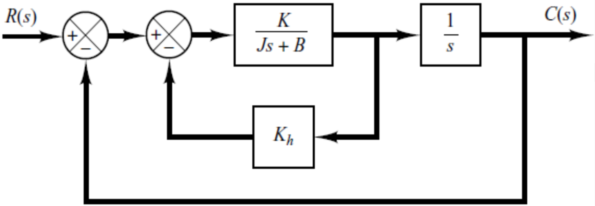
Servo Systems with Velocity Feedback

In this system, the velocity signal, together with the position signal, is fed back to the input to produce the actuating error signal.

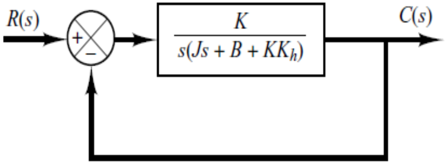


Servo Systems with Velocity Feedback

In this system, the velocity signal, together with the position signal, is fed back to the input to produce the actuating error signal.



The block diagram can then be simplified as



Servo Systems with Velocity Feedback

giving

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2+(B+KK_h)s+K}$$

Comparing with equation (10), the damping ratio ζ becomes

$$\zeta = \frac{B + KK_h}{2\sqrt{KJ}} \tag{20}$$

The undamped natural frequency $\omega_n = \sqrt{K/J}$ is not affected by velocity feedback.

giving

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2+(B+KK_h)s+K}$$

Comparing with equation (10), the damping ratio ζ becomes

$$\zeta = \frac{B + KK_h}{2\sqrt{KJ}} \tag{20}$$

The undamped natural frequency $\omega_n = \sqrt{K/J}$ is not affected by velocity feedback. Note: the maximum overshoot for a unit-step input can be controlled by controlling the value of the damping ratio ζ, we can reduce the maximum overshoot by adjusting the velocity-feedback constant K_h so that ζ is between 0.4 and 0.7.

Servo Systems with Velocity Feedback

giving

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2+(B+KK_h)s+K}$$

Comparing with equation (10), the damping ratio ζ becomes

$$\zeta = \frac{B + KK_h}{2\sqrt{KJ}} \tag{20}$$

The undamped natural frequency $\omega_n = \sqrt{K/J}$ is not affected by velocity feedback. Note: the maximum overshoot for a unit-step input can be controlled by controlling the value of the damping ratio ζ, we can reduce the maximum overshoot by adjusting the velocity-feedback constant K_h so that ζ is between 0.4 and 0.7.
It is important to remember that velocity feedback has the effect of increasing the damping ratio without affecting the undamped natural frequency of the system.

For a unit-impulse input $r(t)$, the corresponding Laplace transform is unity, or $R(s) = 1$. The unit-impulse response $C(s)$ of the second-order system is

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{21}$$

For a unit-impulse input $r(t)$, the corresponding Laplace transform is unity, or $R(s) = 1$. The unit-impulse response $C(s)$ of the second-order system is

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{21}$$

The inverse Laplace transform of this equation yields the time solution for the response $c(t)$ as follows:
For $0 \leqq \zeta \leqq 1$

$$c(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t, t \geqq 0 \tag{22}$$

For a unit-impulse input $r(t)$, the corresponding Laplace transform is unity, or $R(s) = 1$. The unit-impulse response $C(s)$ of the second-order system is

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{21}$$

The inverse Laplace transform of this equation yields the time solution for the response $c(t)$ as follows:

For $0 \leqq \zeta \leqq 1$

$$c(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t, t \geqq 0 \tag{22}$$

For $\zeta = 1$

$$c(t) = \omega_n^2 t e^{-\omega_n t}, t \geqq 0 \tag{23}$$

For a unit-impulse input $r(t)$, the corresponding Laplace transform is unity, or $R(s) = 1$. The unit-impulse response $C(s)$ of the second-order system is

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{21}$$

The inverse Laplace transform of this equation yields the time solution for the response $c(t)$ as follows:
For $0 \leqq \zeta \leqq 1$

$$c(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t, t \geqq 0 \tag{22}$$

For $\zeta = 1$

$$c(t) = \omega_n^2 t e^{-\omega_n t}, t \geqq 0 \tag{23}$$

For $\zeta > 1$,

$$c(t) = \frac{\omega_n}{2\sqrt{\zeta^2-1}} e^{-(\zeta-\sqrt{\zeta^2-1})\omega_n t} - \frac{\omega_n}{2\sqrt{\zeta^2-1}} e^{-(\zeta+\sqrt{\zeta^2-1})\omega_n t}, t \geqq 0 \tag{24}$$

Outline

Reading: Modern Control Engineering by Katsuhiko Ogata, 5th edition (chapter 5)

The transfer function of a system is represented by two arrays of numbers. Consider the system

$$\frac{C(s)}{R(s)} = \frac{2s + 25}{s^2 + 4s + 25} \tag{25}$$

The transfer function of a system is represented by two arrays of numbers. Consider the system

$$\frac{C(s)}{R(s)} = \frac{2s+25}{s^2+4s+25} \tag{25}$$

This system can be represented as two arrays, each containing the coefficients of the polynomials in decreasing powers of s as follows:

```
num = [2,25]
den = [1,4,25]
```

The transfer function of a system is represented by two arrays of numbers. Consider the system

$$\frac{C(s)}{R(s)} = \frac{2s + 25}{s^2 + 4s + 25} \tag{25}$$

This system can be represented as two arrays, each containing the coefficients of the polynomials in decreasing powers of s as follows:

```
num = [2,25]
den = [1,4,25]
```

If `num` and `den` of the closed-loop transfer function are known, commands such as

```
step(num,den),step(num,den,t)
```

will generate plots of unit-step responses.

For a control system defined in a state-space form, where state matrix A, control matrix B, output matrix C, and direct transmission matrix D of state-space equations are known, the commands

```
step(A,B,C,D), step(A,B,C,D,t)
```

will generate plots of unit-step responses.

For a control system defined in a state-space form, where state matrix A, control matrix B, output matrix C, and direct transmission matrix D of state-space equations are known, the commands

`step(A,B,C,D)`, `step(A,B,C,D,t)`

will generate plots of unit-step responses.

Note: the command `step(sys)` may be used to obtain the unit-step response of a system. First, define the system by

`sys = tf(num,den)` or `sys=ss(A,B,C,D)`

Then, to obtain, for example, the unit-step response, enter

```
step(sys)
```

Consider the following systems

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Obtain the unit-step response curves.

Consider the following systems

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Obtain the unit-step response curves.
After doing the Maths, we obtain, the transfer matrix $G(s)$ given as

$$G(s) = \frac{1}{s^2+s+6.5} \begin{bmatrix} s-1 & s \\ s+7.5 & 6.5 \end{bmatrix}$$

and

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{s-1}{s^2+s+6.5} & \frac{s}{s^2+s+6.5} \\ \frac{s+7.5}{s^2+s+6.5} & \frac{6.5}{s^2+s+6.5} \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

Since this system involves two inputs and two outputs, four transfer functions may be defined, depending on which signals are considered as input and output.

Since this system involves two inputs and two outputs, four transfer functions may be defined, depending on which signals are considered as input and output.

Note: when considering the signal u_1 as the input, we assume that signal u_2 is zero, and vice versa.The four transfer functions are

$$\frac{Y_1(s)}{U_1(s)} = \frac{s-1}{s^2+s+6.5} \qquad \frac{Y_1(s)}{U_2(s)} = \frac{s}{s^2+s+6.5}$$
$$\frac{Y_2(s)}{U_1(s)} = \frac{s+7.5}{s^2+s+6.5} \qquad \frac{Y_2(s)}{U_1(s)} = \frac{6.5}{s^2+s+6.5}$$

Since this system involves two inputs and two outputs, four transfer functions may be defined, depending on which signals are considered as input and output.

Note: when considering the signal u_1 as the input, we assume that signal u_2 is zero, and vice versa.The four transfer functions are

$$\frac{Y_1(s)}{U_1(s)} = \frac{s-1}{s^2+s+6.5} \qquad \frac{Y_1(s)}{U_2(s)} = \frac{s}{s^2+s+6.5}$$
$$\frac{Y_2(s)}{U_1(s)} = \frac{s+7.5}{s^2+s+6.5} \qquad \frac{Y_2(s)}{U_1(s)} = \frac{6.5}{s^2+s+6.5}$$

In MATLAB, the plot can be obtained by using the following code

```
A = [1,-1;6.5,0];
B = [1,1;1,0];
C = [1,0;0,1];
D = [0,0;0,0]
step(A,B,C,D)
```

The second-order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{26}$$

is called the standard second-order system. Given ω_n and ζ, the command

`printsys(num,den)` or `printsys(num,den,s)`

The second-order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{26}$$

is called the standard second-order system. Given ω_n and ζ, the command

`printsys(num,den)` or `printsys(num,den,s)`

Consider the case where $\omega_n = 5rad/sec$ and $\zeta = 0.4$, MATLAB program generate the standard second-order system.

The second-order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{26}$$

is called the standard second-order system. Given ω_n and ζ, the command

`printsys(num,den)` or `printsys(num,den,s)`

Consider the case where $\omega_n = 5rad/sec$ and $\zeta = 0.4$, MATLAB program generate the standard second-order system.
MATLAB Program:

```
wn = 5;
dampingratio = 0.4;
[num0,den] = ord2(wn,dampingratio);
num = 25 * num0;
printsys(num,den's')
```

The unit-impulse response of a control system may be obtained by using any of the impulse commands such as `impulse(num,den)`

```
impulse(A,B,C,D)
[y,x,t] = impulse(num,den)
[y,x,t]= impulse(num,den,t)
[y,x,t]= impulse(A,B,C,D)
[y,x,t] = impulse(A,B,C,D,iu)
[y,x,t] = impulse(A,B,C,D,iu,t)
```

Reading: Modern Control Engineering by Katsuhiko Ogata, 5th edition (page 183-211)

To obtain the response to an arbitrary input, the command lsim may be used. The commands like

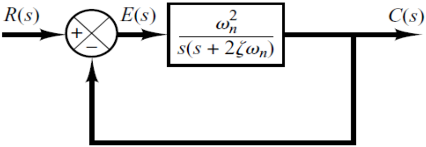
```
lsim(num,den,r,t)
lsim(A,B,C,D,u,t)
y = lsim(num,den,r,t)
y = lsim(A,B,C,D,u,t)
```

will generate the response to input time function r or u.

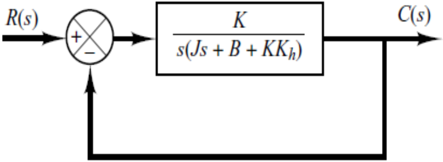
Reading: Modern Control Engineering by Katsuhiko Ogata, 5th edition (page 183-211)

Homework

- Consider the system shown below, where $\zeta = 0.6$ and $\omega_n = 5rad/sec$, obtain the rise time t_r, peak time t_p, maximum overshoot M_p and setting time t_s. The system is subjected to the step input.



- Consider the system shown below, determine the value of gain K and velocity feedback constant K_h so that the maximum overshoot is the unit-step response is 0.2 and the peak time is $1sec$. With these values of K and K_h, obtain the rise time and settling time. Assume that $J = 1kg - m^2$ and $B = 1N - m/rad/sec$.



Reference

- Katsuhiko Ogata, Modern Control Engineering, Fifth Edition, Pearson,