Lecture 2 Laplace Transforms

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1 Laplace Transform

- One of the most efficient ways to solve differential equations.
- Laplace transforms convert differential equations into algebraic equations.
- Solution to algebraic equation gives solution to differential equation when transformation is reversed.
- Laplace transform of $f(t)$ given by:

- Find the function $f(t)$ from the Laplace transform $F(s)$ called talking the inverse Laplace.

Example 1
Let $f(t) = e^{at}$ where $a = const$. Find $\mathcal{L}[f(t)]$

Linearity Property

Example 2
Using the Laplace Transforms table, find $\mathcal{L}[3t^5 - t^8 + 4 - 5e^{2t} + 6\cos(3t)]$.

Shifting Property
Use when multiplying $f(t)$ by e^{-at}

Example 3
Find $\mathcal{L}[e^{2t}\cos(3t)]$

Differentiation Theorem

Example 4
Find $\mathcal{L}[sin^2(at)]$

Integration Theorem

Inverse Laplace Transforms

- Finding $f(t)$ from corresponding $F(s)$
  - Use Table of Laplace Transforms
  - Use partial fraction expansion. It simplifies the problem.
- Partial Fraction Review
  - Distinct roots in denominator (example 5)

* Repeated roots in denominator (example 6)
* Complex roots in denominator (example 7)

Example 5
Find the partial fraction expansion of:

$$\frac{1}{s^2 - 5s + 6}$$

Example 6
Find the partial fraction expansion of:

$$\frac{5s^2 + 20s + 6}{s^3 + 2s^2 + s}$$

Example 7
Find the partial fraction expansion of:

$$\frac{2s^3 - 4s - 8}{(s^2 - s)(s^2 + 4)}$$

Solving Linear Differential Equations

- Initial value problems can be solved with the differentiation theorem
  - Take Laplace transform of each term
  - Solve for dependent variable - will be fraction that's function of s
  - If the form is not found in Laplace tables, find the partial fraction
  - Take inverse Laplace

Example 8
Find solution to the initial value problem:

$$\ddot{x} + 4x = 0, x(0) = 1, \dot{x}(0) = 2$$

Example 9 Find the solution to the initial value problem:

$$\ddot{x} - 3\dot{x} + 2x = 4t - 6, x(0) = 1, \dot{x}(0) = 3$$

Example 10 Solve the system of equations

$$\dot{x}_1 + 2x_1 - x_2 = 2e^{-3t}, x_1(0) = 0$$

$$\dot{x}_2 - x_1 + 3x_2 = 0, x_2(0) = 0$$