Control Systems
Lecture 4 Modeling of Electrical Systems

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Outline

Basic Definitions

- **Current** – I (Amperes - A): The rate at which charge is flowing. A measure of the transfer of electric charge (coulombs per second).
- **Voltage** ($\Delta V = J/C$): The difference in charge between two specified points. Amount of potential energy between two specified points in an electric circuit.
- **Resistance** (Ohms - Ω): The material's tendency to resist the flow of charge (current)
- **Power** (Watts - W or Joule per Sec): A rate of doing work. It is a function of both current and voltage: $P = VI = I^2R = V^2/R$, where $V = IR$.

Basic Definitions

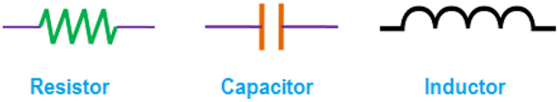
- Water = Charge
- Pressure = Voltage
- Flow = Current
- Hose Width = Resistance

More water in the tank means more pressure at the end of the hose. The battery is similar to the tank, It stores a certain amount of charge, and then releases it

Basic Definitions

Electrical systems consist of three basic elements

- Resistor (R)
- Capacitor (C)
- Inductor (L)



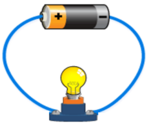
Voltage across resistors is proportional to the current passing through it. The constant of proportionality is known as Resistance.

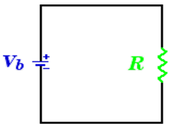
For a wire of length L, cross-sectional area A and resistivity ρ:

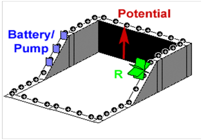
$$R = \rho L A$$

Basic electrical circuits are analogous to hydraulic systems(Ex. water flow analogy).
Ohm's law governs the relationship between current, resistance and voltage

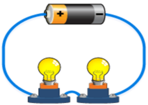
$$V = IR$$

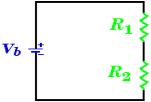


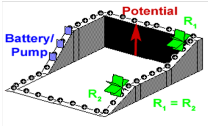




Resistors connected in series have the same current.



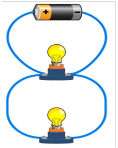


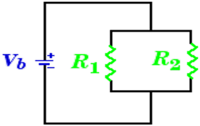


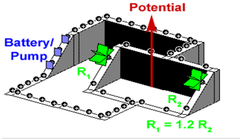
Resistors connected in parallel have the same potential but different current values.

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- Adding more resistors to a parallel circuit results of having less overal resistance.
- Since there are multiple pathways by which charge can flow, adding another resistor in a separate branch adds additional pathway to the charges.







- Capacitors are electrical elements used to store electrostatic energy.
- Voltage across a capacitor of capacitance(C) changes according to the following equation:

$$V(t) = \frac{1}{C}\int_0^t i dt$$

- Energy stored in the capacitor is expressed as:

$$E = \frac{1}{2}CV^2$$

- the difference between a capacitor and battery is that a capacitor can dump its entire charge extremely fast whereas a battery would take minutes to completely discharge.

Basic Electrical Elements

Inductor

- An inductor is capable of storing electrical energy in a form of magnetic energy.
- Inductors refer to coiled conductors where a variable current generates voltage.
- The voltage (V) across an inductor is given by:

$$V = L\frac{di}{dt}$$

  Where, L refers to the inductance of the coil.
- The energy stored by the conductor is given by:

$$E = 1/2(Li^2(t))$$

- The induced inductor voltage will oppose the cause, ie; the current flow through it, and hence the voltage applied.
- So, if we provide A.C to an inductor, the change in current of AC induces a voltage in the inductor that opposes the applied voltage, and this blocks AC by inductors

- Resistors consume both AC and DC, while the inductor will block AC and allow only DC to pass through, and the capacitor will block DC and only allow AC to pass through.
- A reactance is simply the resistance that an inductor would offer to AC, or the resistance that a capacitor would offer to DC.
- The resistance of a resistor does not depend on the frequency of the AC, while the reactance of an inductor or capacitor depends on the frequency of the AC.

Kirchhoff's Circuit

- Current Law (KCL)- Sum of currents at a node = 0
  - Current is conserved, no loss of current.
  - At any node, the summation of the current flowing into the node is equal to the summation of currents flowing out of the node

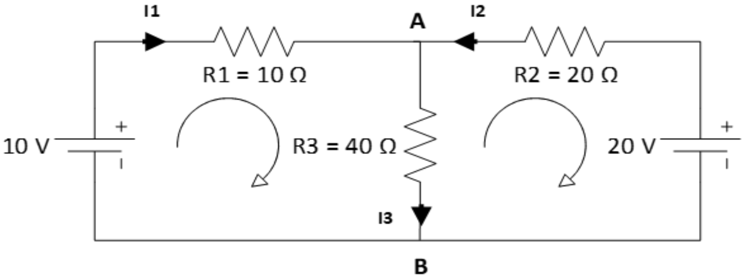
$$\Sigma I_{in} = \Sigma I_{out}$$

- Voltage Law (KVL) - Total voltage change in loop = 0
  - Summation of voltages around any closed circuit is zero

$$\Sigma \Delta V_{closedloop} = 0$$

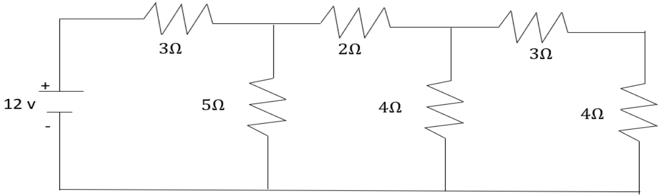
STEP:

- Assume a loop direction
- Assume current direction in every closed loop
- Apply junction rule at each node
- Apply voltage sign convention
    - Assume (+) voltage source if the loop direction flows from (–) to (+).
    - Assume (–) voltage source if the loop direction flows from (+) to (–).
    - Assume (+) drop across an element if the loop direction opposes the current direction.
    - Assume (–) drop across an element if the loop direction travels with the current direction.



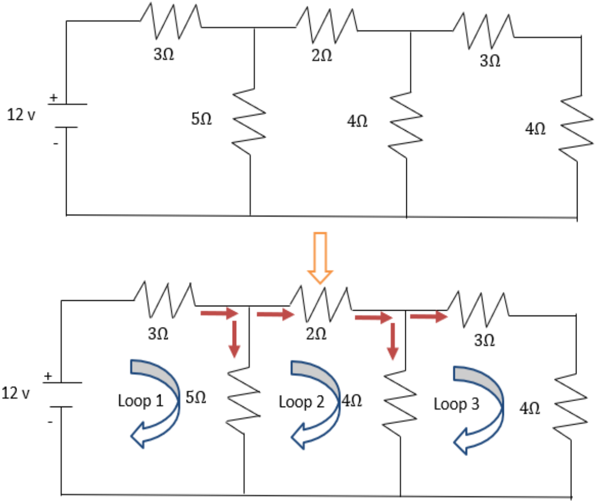
Node A : $I_1 + I_2 = I_3$
Loop 1 = $10 - I_1R_1 - I_3R_3 = 0$
Loop 2 = $-20 + I_2R_2 + I_3R_3 = 0$

By substitution:
$I_1 = -0.143A$
$I_2 = 0.429A$
$I_3 = I_1 + I_2 = 0.286A$



For the circuit shown above, find:

- Current through each resistor
- Voltage drop across each resistor

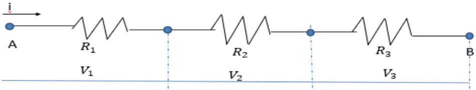

3Ω
2Ω
3Ω
12 v
+
-
5Ω
4Ω
4Ω
3Ω
2Ω
3Ω
12 v
+
-
Loop 1
5Ω
Loop 2
4Ω
Loop 3
4Ω

- Ohm's law - Current in a circuit is proportional to the voltage acting in the circuit and inversely proportional to the resistance of the ciruit

$$i = \frac{V}{R}$$

where , i is current, V is voltage, R resistance

- Series circuits



where, $V_1 = iR_1, V_2 = iR_2, V_3 = iR_3$ then,

$$\frac{V}{i} = R_1 + R_2 + R_3$$

- Series equivalents:
  - Resistors:

$$R_{eq} = \Sigma_{k=1}^{n} R_k$$

  - Capacitor

$$C_{eq} = \frac{1}{\Sigma_{k=1}^{n} C_k}$$

  - Inductor

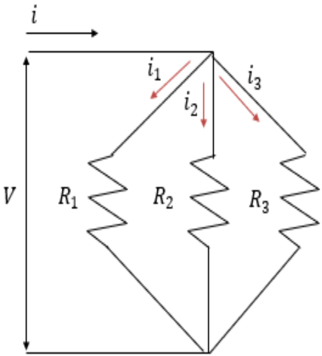
$$L_{eq} = \Sigma_{k=1}^{n} L_k$$

we have
$i_1 = V/R_1, i_2 = V/R_2, i_3 = V/R_3$
$i = i_1 + i_2 + i_3 = V/R_1 + V/R_2 + V/R_3$
then,

$$1/R = 1/R_1 + 1/R_2 + 1/R_3$$

Solve for R, then we obtain,

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$



Circuit Reduction

parallel Circuits

- Resistors

$$R_{eq} = \frac{1}{\Sigma_{k=1}^{n}(1/R_k)}$$

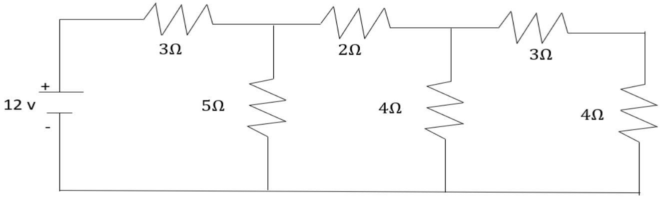
- Capacitors

$$C_{eq} = \Sigma_{k=1}^{n} C_k$$

- Inductors

$$L_{eq} = \frac{1}{\Sigma_{k=1}^{n}(1/L_k)}$$

Replace the resistors in the circuit with an equivalent resistor.



Result: $R_{eq} = 5.38\Omega$

Transfer Functions

$$V(s) = Z(s)I(s)$$

where,
$V(s)$ is the Laplace transform of the voltage
$I(s)$ is the Laplace transform of the current
$Z(s)$ is the Laplace transform of the impedance

Capacitor :	$v_c = \frac{1}{C}\int idt$	$V_c(s) = \frac{I(s)}{Cs}$
Inductor :	$v_L = L\frac{di}{dt}$	$LsI(s)$
Resistor :	$v_R = Ri$	$RI(s)$

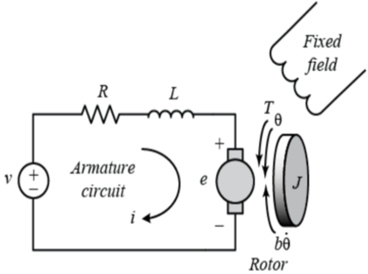
Obtain the transfer function when L, R and C are connected in series.

$v = v_L + v_R + v_c$
$V(s) = LsI(s) + RI(s) + \frac{1}{Cs}I(s)\; V(s) = (Ls + R + \frac{1}{Cs})I(s)$
then,
$Z(s) = \frac{V(s)}{I(s)} = (Ls + R + \frac{1}{Cs})$

- A motor is an electrical machine which converts electrical energy into mechanical energy.
- Take a wire and put it between the poles of a powerful, permanent horseshoe magnet.
- Connect the two ends of the wire to a battery, the wire will move briefly.
- When current pass through, it creates a magnetic field around it.
- If we place the wire near a magnet, this magnetic field interacts with the permanent magnet's field.
- It's like putting two magnets near one another, they will repel or attract.
- In the same way, the temporary magnetism around the wire attracts or repels the permanent magnetism from the magnet, and that's what causes the wire to move.

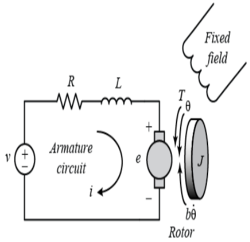
- The principle of working of a DC motor is that "whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force".
- Magnetic field may be provided by field winding (electromagnetism) or by using permanent magnets.

The electric circuit of the stator and the free-body diagram of the rotor are as shown



- Input: voltage applied to the motor's armature (rotor) (V).
- Model Output: angular position of the shaft (θ)

- The physical parameters for our example are:
  - J: moment of inertia of the rotor $3.2284E-6kg.m^2$
  - b motor viscous friction constant $3.5077E-6Nms$
  - K_e: electromotive force constant $0.0274V/rad/sec$
  - K_t : motor torque constant $0.0274Nm/Amp$
  - R: electric resistance $4ohm$
  - L: electric inductance $2.75E-6H$



- The motor torque is proportional to the armature current i by a constant K_T as shown in the relation below

$$T = K_T i$$

- the back emf is proportional to the angular velocity of the shaft by a constant factor as follows

$$e = K_b \omega$$

- Applying Newton's second law:

$$J\frac{d^2\theta}{dt^2} = T - b\frac{d\theta}{dt} \Rightarrow \frac{d^2\theta}{dt^2} = \frac{1}{J}(K_t i - b\frac{d\theta}{dt})$$

- Applying Kirchhoff's law:

$$L\frac{di}{dt} = -Ri + V - e \Rightarrow \frac{di}{dt} = \frac{1}{L}(-Ri + V - K_b\frac{d\theta}{dt})$$

Can you simulate this in Simulink?