Control Systems
Lecture 5 Block Diagram

Daro VAN

Laboratory of Dynamics and Control
Department of Industrial and Mechanical Engineering
Institute of Technology of Cambodia

Phnom Penh, April 30th, 2020

Outline

Outline

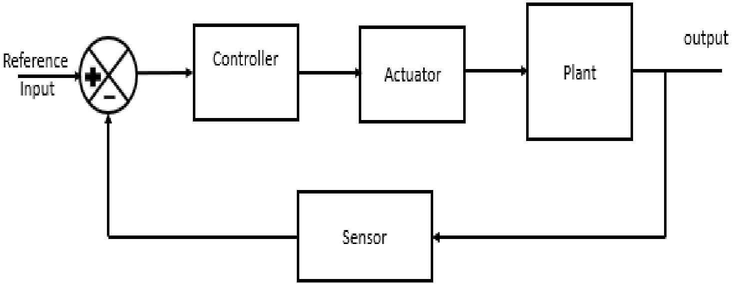
Introduction

definition

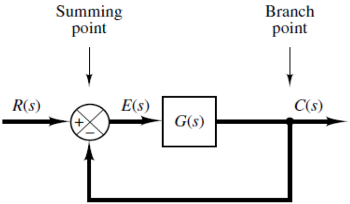
Block Diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals.

definition

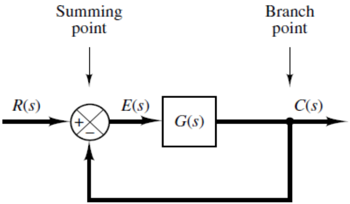
Block Diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals.



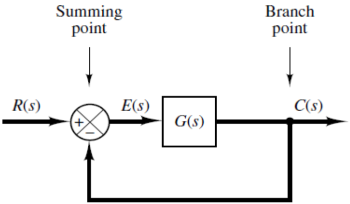
Block Diagram of a Simplified Industrial Control System



Summing point
Branch point
R(s)
+
−
E(s)
G(s)
C(s)



- Summing Point: it is important that the quantities being added or subtracted have the same dimensions and the same units.



- Summing Point: it is important that the quantities being added or subtracted have the same dimensions and the same units.
- Branch Point: point from which the signal from a block goes concurrently to other blocks or summing points.

- The concepts of open-loop and closed-loop system in term of transfer function
- Block diagram operations to represent control systems

- The concepts of open-loop and closed-loop system in term of transfer function
- Block diagram operations to represent control systems
- Block diagram transformations

- The concepts of open-loop and closed-loop system in term of transfer function
- Block diagram operations to represent control systems
- Block diagram transformations
- Block diagram reduction

Outline

definition

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input under the assumption that all initial conditions are zero.

definition

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input under the assumption that all initial conditions are zero.

Consider the linear time-invariant system defined by the following differential equation:

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1}\dot{y} + a_n y = b_0 x^{(m)} + b_1 x^{(m-1)} + \ldots + b_{m-1}\dot{x} + b_m x$$

where $n \geq m$, y is the output of the system and x is the input.

definition

The transfer function of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input under the assumption that all initial conditions are zero.

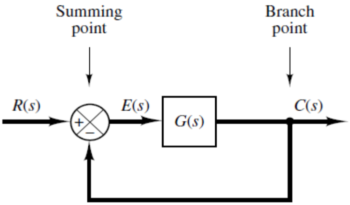
Consider the linear time-invariant system defined by the following differential equation:

$$a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1}\dot{y} + a_n y = b_0 x^{(m)} + b_1 x^{(m-1)} + \ldots + b_{m-1}\dot{x} + b_m x$$

where $n \geq m$, y is the output of the system and x is the input.Then,

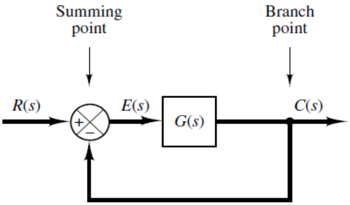
Transfer Function $= G(s) = \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1}s + b_m}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n}$

Block Diagram of a Closed-Loop System



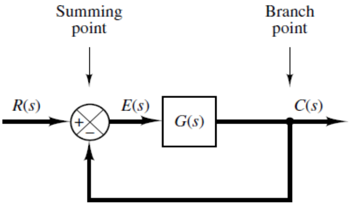
- The output $C(s)$ is fed back to the summing point, where it is compared with the reference input $R(s)$.

Block Diagram of a Closed-Loop System



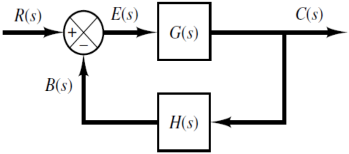
- The output $C(s)$ is fed back to the summing point, where it is compared with the reference input $R(s)$.
- The output of the block, $C(s)$, is obtained by multiplying $G(s)$ by $E(s)$.

Block Diagram of a Closed-Loop System

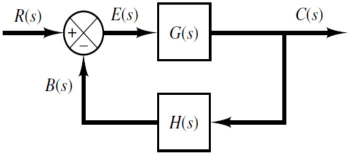


- The output $C(s)$ is fed back to the summing point, where it is compared with the reference input $R(s)$.
- The output of the block, $C(s)$, is obtained by multiplying $G(s)$ by $E(s)$.

Block Diagram of a Closed-Loop Systems

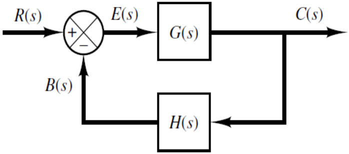


Block Diagram of a Closed-Loop Systems



- When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of the output signal to that of the input signal.

Block Diagram of a Closed-Loop Systems



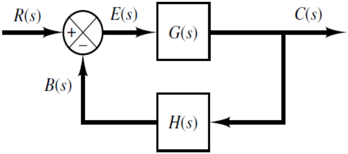
- When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of the output signal to that of the input signal.
- The role of the feedback element is to modify the output before it is compared with the input.

Outline

Open-Loop Transfer Function and Feedforward Transfer Function

- The open-looped transfer function, defined as

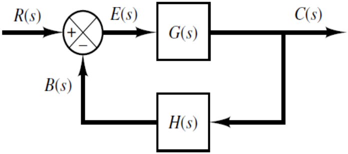
$$\frac{B(s)}{E(s)} = G(s)H(s) \tag{1}$$



Open-Loop Transfer Function and Feedforward Transfer Function

- The open-looped transfer function, defined as

$$\frac{B(s)}{E(s)} = G(s)H(s) \tag{1}$$



- The feedforward transfer function, defined as

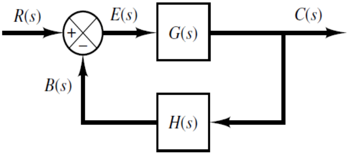
$$\frac{C(s)}{E(s)} = G(s) \tag{2}$$

Outline

Closed-Loop Transfer Function

The output $C(s)$ and input $R(s)$ are related as follows:

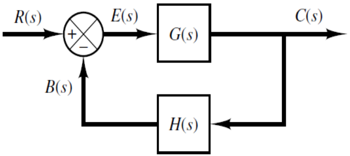
$$
\begin{aligned}
C(s) &= G(s)E(s) \\
E(s) &= R(s) - B(s) \\
&= R(s) - H(s)C(s)
\end{aligned}
$$



Closed-Loop Transfer Function

The output $C(s)$ and input $R(s)$ are related as follows:

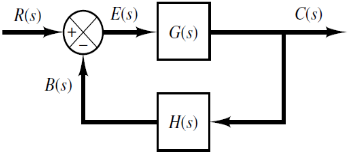
$$
\begin{aligned}
C(s) &= G(s)E(s) \\
E(s) &= R(s) - B(s) \\
&= R(s) - H(s)C(s)
\end{aligned}
$$



eliminating $E(s)$ from these equations gives

The output $C(s)$ and input $R(s)$ are related as follows:

$$\begin{aligned} C(s) &= G(s)E(s) \\ E(s) &= R(s) - B(s) \\ &= R(s) - H(s)C(s) \end{aligned}$$



eliminating $E(s)$ from these equations gives

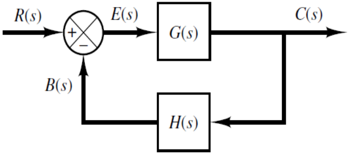
$$C(s) = G(s)[R(s) - H(s)C(s)]$$

or,

Closed-Loop Transfer Function

The output $C(s)$ and input $R(s)$ are related as follows:

$$
\begin{aligned}
C(s) &= G(s)E(s) \\
E(s) &= R(s) - B(s) \\
&= R(s) - H(s)C(s)
\end{aligned}
$$



eliminating $E(s)$ from these equations gives

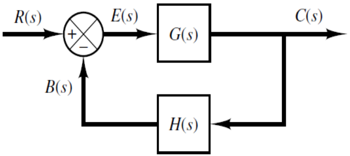
$$C(s) = G(s)[R(s) - H(s)C(s)]$$

or,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{3}$$

The output $C(s)$ and input $R(s)$ are related as follows:

$$\begin{aligned} C(s) &= G(s)E(s) \\ E(s) &= R(s) - B(s) \\ &= R(s) - H(s)C(s) \end{aligned}$$



eliminating $E(s)$ from these equations gives

$$C(s) = G(s)[R(s) - H(s)C(s)]$$

or,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \tag{3}$$

The transfer function relating $C(s)$ and $R(s)$ is called closed-loop transfer function. It relates the closed loop system dynamics to the dynamics of the feedforward elements and feedback elements.

Outline

Obtaining Transfer Functions Using MATLAB

MATLAB has convenient commands to obtain the cascaded, parallel, and feedback (closed-loop) transfer functions.

Obtaining Transfer Functions Using MATLAB

MATLAB has convenient commands to obtain the cascaded, parallel, and feedback (closed-loop) transfer functions.

Suppose that there are two components $G_1(s)$ and $G_2(s)$ connected differently where

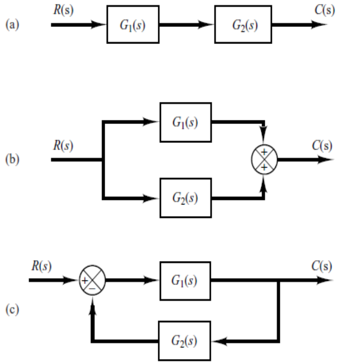
$$G_1(s) = \frac{num1}{den1} \text{ and } G_1(s) = \frac{num2}{den2}$$

then,

Obtaining Transfer Functions Using MATLAB

To obtain the transfer functions of the cascaded system, parallel system, or feedback (closed-loop) system, the following commands may be used:

```
[num, den] = series(num1,den1,num2,den2)
[num, den] = parallel(num1,den1,num2,den2)
[num, den] = feedback(num1,den1,num2,den2)
```



Obtaining Transfer Functions Using MATLAB

As an example, consider the case where,

$$G_1(s) = \frac{10}{s^2+2s+10} = \frac{num1}{den1}, G_2(s) = \frac{5}{s+5} = \frac{num2}{den2}$$

In MATLAB, $\frac{C(s)}{R(s)}$ for each arrangement of $G_1(s)$ and $G_2(s)$ can be achieved with the following script

Obtaining Transfer Functions Using MATLAB

As an example, consider the case where,

$$G_1(s) = \frac{10}{s^2+2s+10} = \frac{num1}{den1}, G_2(s) = \frac{5}{s+5} = \frac{num2}{den2}$$

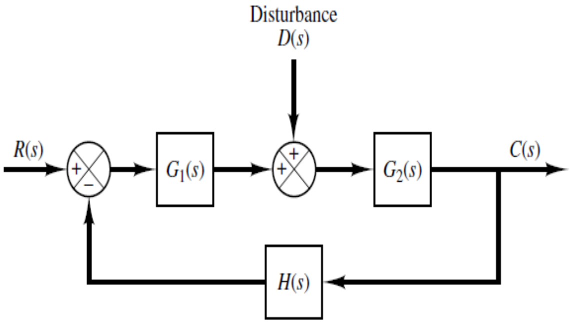
In MATLAB, $\frac{C(s)}{R(s)}$ for each arrangement of $G_1(s)$ and $G_2(s)$ can be achieved with the following script ----------------

```
num1 = [10];
den1 = [1 2 10];
num2 = [5];
den2 = [1 5];
[num, den]= series(num1,den1,num2,den2);
printsys(num,den)
[num, den]= parallel(num1,den1,num2,den2);
printsys(num,den)
[num3, den3]= feedback(num1,den1,num2,den2);
printsys(num3,den3)
```

Outline

Closed-Loop System Subjected to a Disturbance

Consider the system shown below



Closed-Loop System Subjected to a Disturbance

In examining the effect of the disturbance $D(s)$, we may assume that the reference input is zero; we may then calculate the response $C_D(s)$to the disturbance only. This response can be found from

Closed-Loop System Subjected to a Disturbance

In examining the effect of the disturbance $D(s)$, we may assume that the reference input is zero; we may then calculate the response $C_D(s)$to the disturbance only. This response can be found from

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{4}$$

Closed-Loop System Subjected to a Disturbance

In examining the effect of the disturbance $D(s)$, we may assume that the reference input is zero; we may then calculate the response $C_D(s)$to the disturbance only. This response can be found from

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{4}$$

On the other hand, in considering the response to the reference input $R(s)$, we may assume that the disturbance is zero.Then the response $C_R(s)$ to the reference input $R(s)$ can be obtained from

Closed-Loop System Subjected to a Disturbance

In examining the effect of the disturbance $D(s)$, we may assume that the reference input is zero; we may then calculate the response $C_D(s)$to the disturbance only. This response can be found from

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{4}$$

On the other hand, in considering the response to the reference input $R(s)$, we may assume that the disturbance is zero.Then the response $C_R(s)$ to the reference input $R(s)$ can be obtained from

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{5}$$

Closed-Loop System Subjected to a Disturbance

In examining the effect of the disturbance $D(s)$, we may assume that the reference input is zero; we may then calculate the response $C_D(s)$to the disturbance only. This response can be found from

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{4}$$

On the other hand, in considering the response to the reference input $R(s)$, we may assume that the disturbance is zero.Then the response $C_R(s)$ to the reference input $R(s)$ can be obtained from

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{5}$$

The response to the simultaneous application of the reference input and disturbance can be obtained by adding the two individual responses.

Closed-Loop System Subjected to a Disturbance

In examining the effect of the disturbance $D(s)$, we may assume that the reference input is zero; we may then calculate the response $C_D(s)$to the disturbance only. This response can be found from

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{4}$$

On the other hand, in considering the response to the reference input $R(s)$, we may assume that the disturbance is zero.Then the response $C_R(s)$ to the reference input $R(s)$ can be obtained from

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} \tag{5}$$

The response to the simultaneous application of the reference input and disturbance can be obtained by adding the two individual responses.

$$\begin{aligned} C(s) &= C_R(s) + C_D(s) \\ &= \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}[G_1(s)R(s) + D(s)] \end{aligned} \tag{6}$$

Closed-Loop System Subjected to a Disturbance

We have

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}, \frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

Consider now the case where $|G_1(s)H(s)| \gg 1$ and $|G_1(s)G_2(s)H(s)| \gg 1$:

Closed-Loop System Subjected to a Disturbance

We have

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}, \frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

Consider now the case where $|G_1(s)H(s)| \gg 1$ and $|G_1(s)G_2(s)H(s)| \gg 1$:

- The closed-loop transfer function $C_D(s)/D(s)$ becomes almost zero, and the effect of the disturbance is suppressed.

Closed-Loop System Subjected to a Disturbance

We have

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}, \frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

Consider now the case where $|G_1(s)H(s)| \gg 1$ and $|G_1(s)G_2(s)H(s)| \gg 1$:

- The closed-loop transfer function $C_D(s)/D(s)$ becomes almost zero, and the effect of the disturbance is suppressed.
  This is an advantage of the closed-loop system.

Closed-Loop System Subjected to a Disturbance

We have

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}, \frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

Consider now the case where $|G_1(s)H(s)| \gg 1$ and $|G_1(s)G_2(s)H(s)| \gg 1$:

- The closed-loop transfer function $C_D(s)/D(s)$ becomes almost zero, and the effect of the disturbance is suppressed.
  This is an advantage of the closed-loop system.
- The closed-loop transfer function $C_R(s)/R(s)$ approaches $1/H(s)$ as the gain of $G_1(s)G_2(s)H(s)$ increases.

Closed-Loop System Subjected to a Disturbance

We have

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}, \quad \frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

Consider now the case where $|G_1(s)H(s)| \gg 1$ and $|G_1(s)G_2(s)H(s)| \gg 1$:

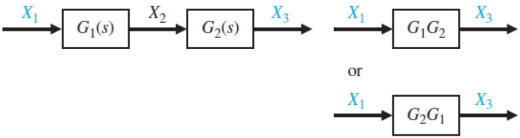
- The closed-loop transfer function $C_D(s)/D(s)$ becomes almost zero, and the effect of the disturbance is suppressed.
  This is an advantage of the closed-loop system.
- The closed-loop transfer function $C_R(s)/R(s)$ approaches $1/H(s)$ as the gain of $G_1(s)G_2(s)H(s)$ increases.

  This means that if $|G1(s)G2(s)H(s)| \gg 1$, then the closed-loop transfer function $C_R(s)/R(s)$ becomes independent of $G_1(s)$ and $G_2(s)$ and inversely proportional to $H(s)$, so that the variations of $G_1(s)$ and $G_2(s)$ do not affect the closed-loop transfer function $C_R(s)/R(s)$.

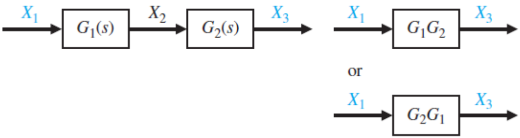
Note: Both cases are good for closed-loop systems.

Outline

1. Combining blocks in cascade

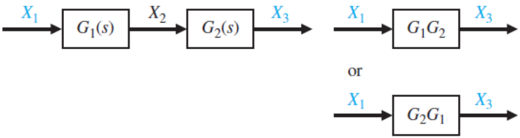


1. Combining blocks in cascade

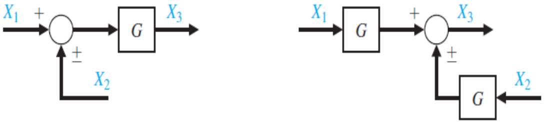


2. Moving a summing point behind a block

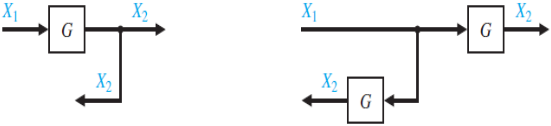
1. Combining blocks in cascade



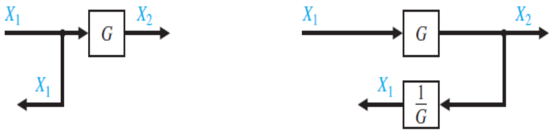
2. Moving a summing point behind a block



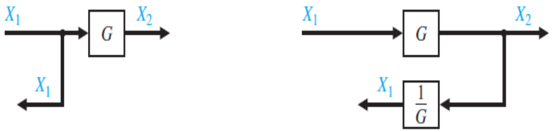
3. Moving a pickoff point ahead of a block



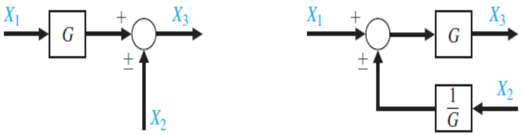
4. Moving a pickoff point behind a block



4. Moving a pickoff point behind a block



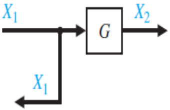
5. Moving a summing point ahead of a block

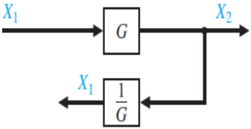


Block Diagram Reduction

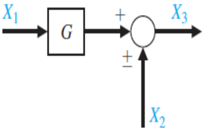
Block Diagram Transformation

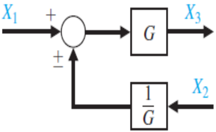
4. Moving a pickoff point behind a block



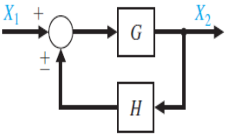


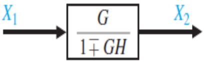
5. Moving a summing point ahead of a block





6. Eliminating a feedback loop





To draw a block diagram for a system, we need to:

To draw a block diagram for a system, we need to:

- write the equations that describe the dynamic behavior of each component.

To draw a block diagram for a system, we need to:

- write the equations that describe the dynamic behavior of each component.
- take the Laplace transforms of these equations, assuming zero initial conditions.

To draw a block diagram for a system, we need to:

- write the equations that describe the dynamic behavior of each component.
- take the Laplace transforms of these equations, assuming zero initial conditions.

Note:

- Blocks can be connected in series only if the output of one block is not affected by the next following block.

Recap:

Recap:

- Resistor
  $v = iR \Longleftrightarrow V(s) = RI(s)$

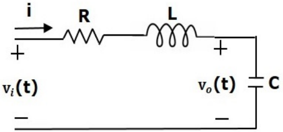
Recap:

- Resistor
  $v = iR \Longleftrightarrow V(s) = RI(s)$
- Inductor
  $v = L\frac{di}{dt} \Longleftrightarrow V(s) = LsI(s)$ (assume zero initial condition)

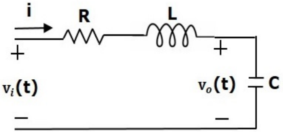
Recap:

- Resistor
  $v = iR \Longleftrightarrow V(s) = RI(s)$
- Inductor
  $v = L\frac{di}{dt} \Longleftrightarrow V(s) = LsI(s)$ (assume zero initial condition)
- Capacitor
  $i = C\frac{dv}{dt} \Longleftrightarrow I(s) = C(sV(s))$ (assume zero initial condition)

Consider a series of *RLC* circuit as shown below.



Consider a series of *RLC* circuit as shown below.

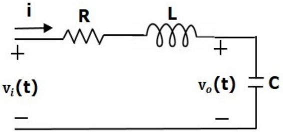


$\Big\|$

Take Laplace Transform

$\Downarrow$

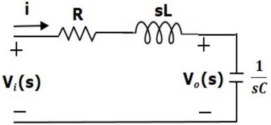
Consider a series of *RLC* circuit as shown below.



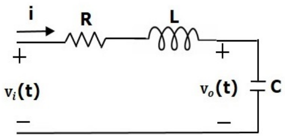
$\Downarrow$

Take Laplace Transform

$\Downarrow$



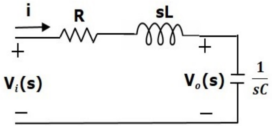
Consider a series of *RLC* circuit as shown below.



||

Take Laplace Transform

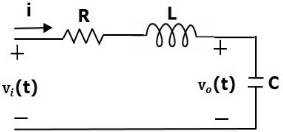
$\Downarrow$



From the circuit, we obtain

$$I(s) = \frac{V_i(s) - V_0(s)}{R + sL}$$

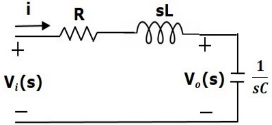
Consider a series of *RLC* circuit as shown below.



$||$

Take Laplace Transform

$\Downarrow$



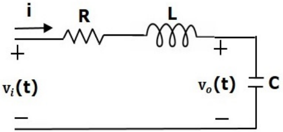
From the circuit, we obtain

$$I(s) = \frac{V_i(s) - V_0(s)}{R + sL}$$

and

$$V_0(s) = \frac{1}{sC} I(s)$$

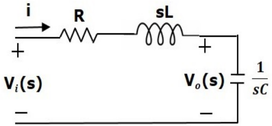
Consider a series of *RLC* circuit as shown below.



$\Downarrow$

Take Laplace Transform

$\Downarrow$



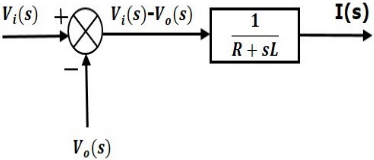
From the circuit, we obtain

$$I(s) = \frac{V_i(s) - V_0(s)}{R + sL}$$

and

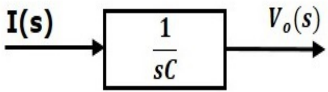
$$V_0(s) = \frac{1}{sC} I(s)$$

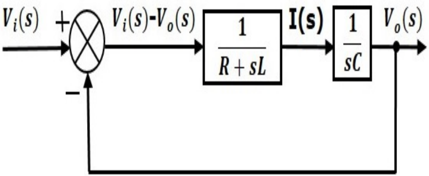
The block diagram can be drawn as



Procedure for Drawing Block Diagram

Example: RLC circuit





Recap from Block Diagram Transformation

Recap from Block Diagram Transformation

1. Check for the blocks connected in series and simplify.

Block Diagram Reduction

Recap from Block Diagram Transformation

1. Check for the blocks connected in series and simplify.
2. Check for the blocks connected in parallel and simplify.

Block Diagram Reduction

Recap from Block Diagram Transformation

1. Check for the blocks connected in series and simplify.
2. Check for the blocks connected in parallel and simplify.
3. Check for the blocks connected in feedback loop and simplify.

Block Diagram Reduction

Recap from Block Diagram Transformation

1. Check for the blocks connected in series and simplify.
2. Check for the blocks connected in parallel and simplify.
3. Check for the blocks connected in feedback loop and simplify.
4. If there is difficulty with take-off point while simplifying, shift it towards right.

Block Diagram Reduction

Recap from Block Diagram Transformation

1. Check for the blocks connected in series and simplify.
2. Check for the blocks connected in parallel and simplify.
3. Check for the blocks connected in feedback loop and simplify.
4. If there is difficulty with take-off point while simplifying, shift it towards right.
5. If there is difficulty with summing point while simplifying, shift it towards left.

Block Diagram Reduction

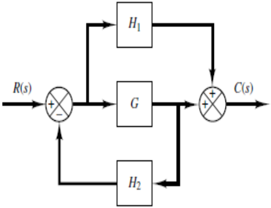
Recap from Block Diagram Transformation

1. Check for the blocks connected in series and simplify.
2. Check for the blocks connected in parallel and simplify.
3. Check for the blocks connected in feedback loop and simplify.
4. If there is difficulty with take-off point while simplifying, shift it towards right.
5. If there is difficulty with summing point while simplifying, shift it towards left.
6. Repeat the above steps till you get the simplified form, i.e., single block.

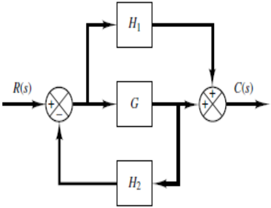
Block Diagram Reduction

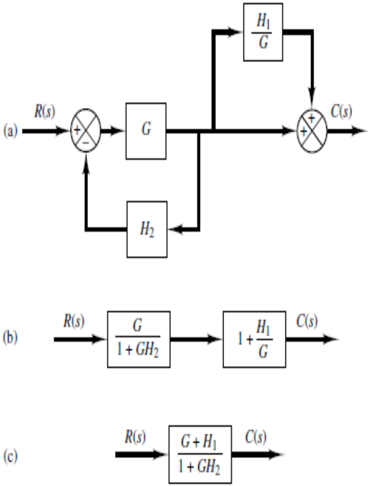
Example 1

Simplify the block diagram

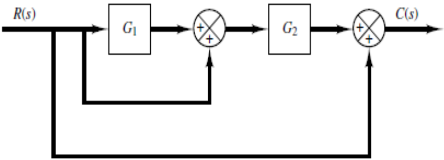


Simplify the block diagram



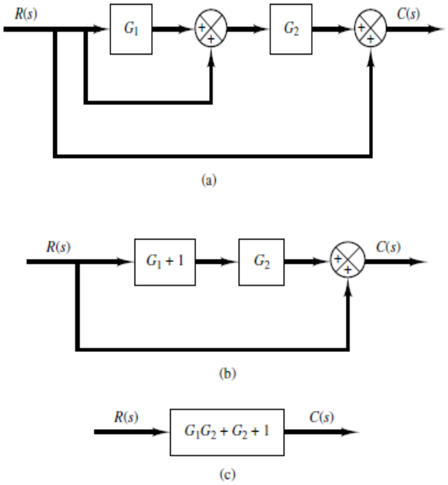


Simplify the block diagram

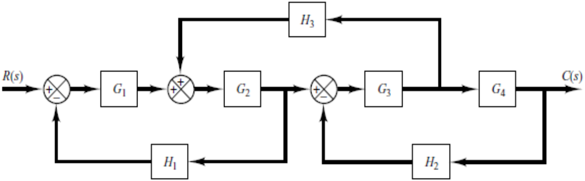


(a)

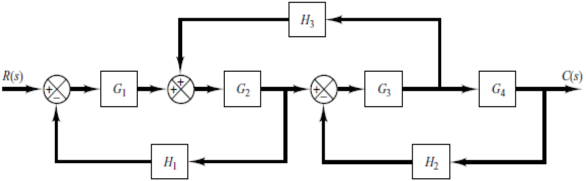
Simplify the block diagram

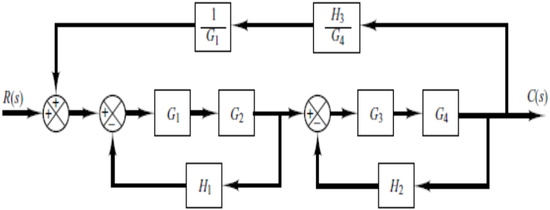


Simplify the block diagram



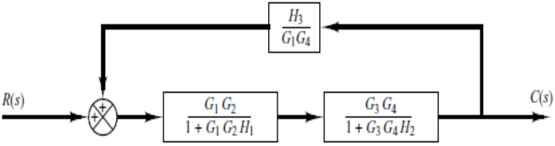
Simplify the block diagram





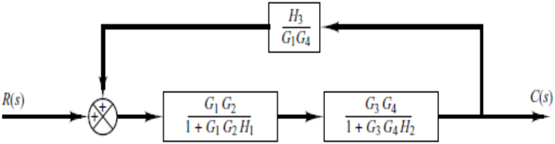
(a)

Simplify the block diagram

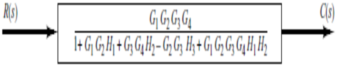


(b)

Simplify the block diagram

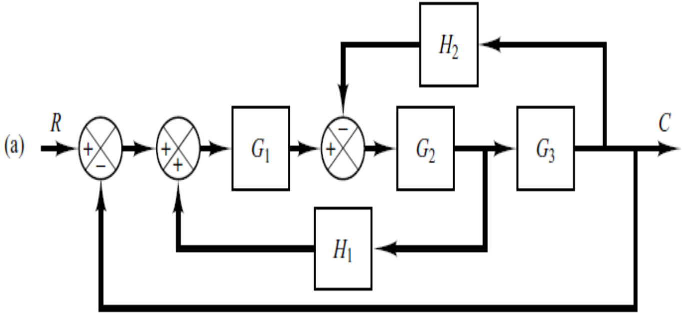


(b)

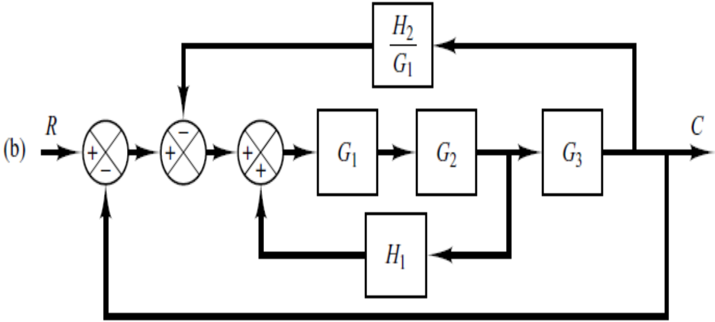


(c)

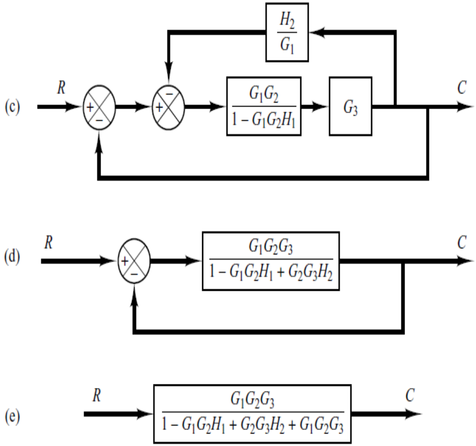
Consider the system shown below, simplify the diagram.



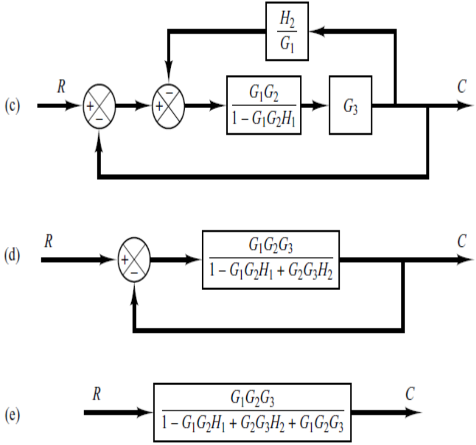
By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1, we obtain



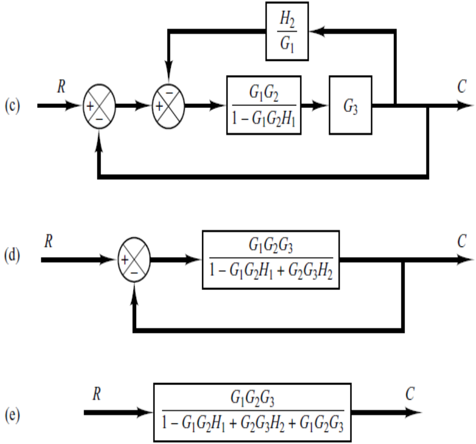
Eliminating the positive feedback loop, we have (c)



The elimination of the loop containing H_2/G_1 give (d)



Finally eliminating the feedback loops result in (e)



Notice that the numerator of the closed-loop transfer function $C(s)/R(s)$ is the product of the transfer functions of the feedforward path. The denominator of $C(s)/R(s)$ is equal to

$$1 + \Sigma(\text{product of the transfer functions around each loop})$$

$$= 1 + (-G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3)$$

$$= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

The positive feedback loop yields a negative term in the denominator.

Reference

- Katsuhiko Ogata, Modern Control Engineering, Fifth Edition, Pearson,