Control Systems
Lecture 9: Root Locus Techniques

Daro VAN

Laboratory of Dynamics and Control
Department of Industrial and Mechanical Engineering
Institute of Technology of Cambodia

Phnom Penh, June 4th, 2020

Outline

- Slide can be found at : https://github.com/Daro12/Control-Systems
- Reading:
  - Norman S. Nise, Control Systems Engineering, 7th edition, WILEY (chapter 8: page 382-448)
  - Katsuhiko Ogata, Modern Control Engineering, 5th edition, Pearson (chapter 6: page 290-301)

Outline

Reading:

- Norman S. Nise, Control Systems Engineering, 7th edition, WILEY (chapter 8: page 382-448)
- Katsuhiko Ogata, Modern Control Engineering, 5th edition, Pearson (chapter 6: page 290-301)

Introduction

definition

Root locus is a graphical representation of the closed-loop poles as a system parameter is varied.

Ex. Consider the closed-loop systems defined as

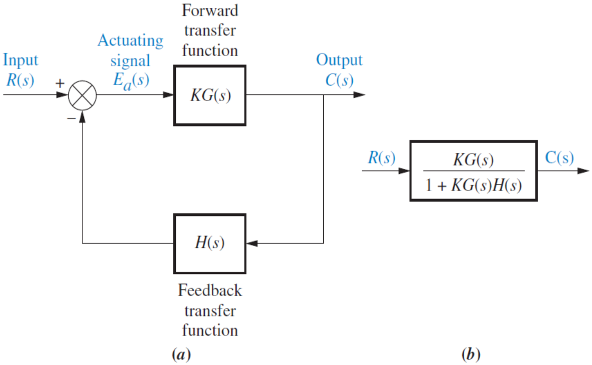
$$\frac{s^2+s+1}{s^3+4s^2+Ks+1}$$

Recall: The stability of the system depends on the location the poles.

- What value of K should we choose to achieve the requirement?
- What is the effect of varying K on the system?

- State the properties of root locus
- Sketch the root locus
- Use the computer program to plot the root locus
- Sketch the root locus for positive feedback systems

Consider the typical closed-loop feedback control system below



From Norman S. Nise (figure 8.1)

The poles change with K

Let define

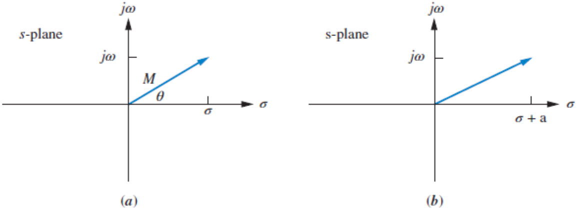
$$G(s) = \frac{N_G(s)}{D_G(s)} \text{ and } H(s) = \frac{N_H(s)}{D_H(s)}$$

then,

$$T(s) = \frac{KN_G(s)D_H(s)}{D_G(s)D_H(s)+KN_G(s)N_H(s)}$$

The system's transient response and stability are dependent upon the poles which is the function of K. We have no knowledge of the systems' performance unless we now the value of K.

We have the complex number $s = \sigma + j\omega$



From Norman S. Nise (Figure 8.2)

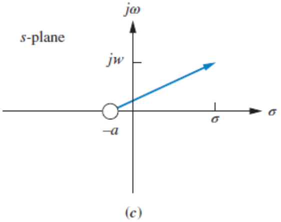
and consider the complex function

$$F(s) = (s + a),$$

then,

$$F(s) = \sigma + j\omega + a = (\sigma + a) + j\omega$$

$F(s)$ has a zero at $-a$. If we translate the vector a units to the left, we have an alternative representation of a complex number originated at zeros of $F(s)$, and terminates on the point $s = \sigma + j\omega$



Conclusion: $(s + a)$ is a complex number and can be represented by a vector drawn from the zero of the function to the point s.

Ex. $(s+7)|_{s\to 5+j2}$ is a complex number drawn from the zero ,-7, to $s = 5 + j2$.

Let apply the concepts to a complicated function. Assume a function

$$F(s) = \frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)} \tag{1}$$

The function defines the complex arithmetic to be performed in order to evaluate $F(s)$ at any point s. The magnitude, M, of $F(s)$ at any point , s, is

$$M = \frac{\prod \texttt{(zeros lengths)}}{\prod \texttt{(pole lengths)}} = \frac{\prod_{i=1}^{m}|(s+z_i)|}{\prod_{j=1}^{n}|(s+p_j)|} \tag{2}$$

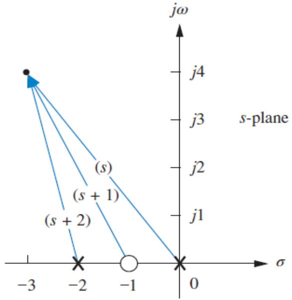
The angle, θ, of $F(s)$ at any point , s, is

$$\begin{aligned} \theta &= \Sigma\texttt{(zero angles)} - \Sigma\texttt{(pole angles)} \\ &= \Sigma_{i=1}^{m}\angle(s+z_i) - \Sigma_{j=1}^{n}\angle(s+p_j) \end{aligned} \tag{3}$$

Ex. Given

$$F(s) = \frac{(s+1)}{s(s+2)}$$

find $F(s)$ at the point $s = (-3 + j4)$



From Norman S. Nise (Figure 8.3)

- At zero -1: $\sqrt{20}\angle 116.6^o$
- At pole 0: $5\angle 126.9^o$
- At pole -2: $\sqrt{17}\angle 104^o$

Thus,

$$M\angle\theta = \frac{\sqrt{20}}{5\sqrt{15}}\angle(116^o - 126.9^o - 104.0^o)$$
$$= 0.217\angle(-114.3^o)$$

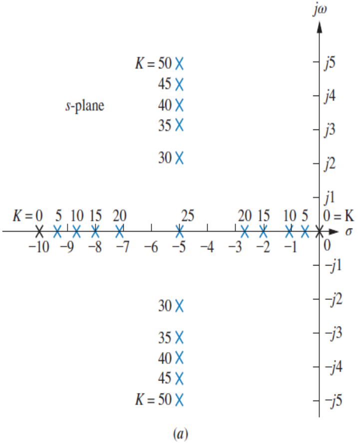
Outline

Reading:

- Norman S. Nise, Control Systems Engineering, 7th edition, WILEY (chapter 8: page 382-448)
- Katsuhiko Ogata, Modern Control Engineering, 5th edition, Pearson (chapter 6: page 290-301)

consider the closed loop transfer function defined as

$$\frac{K}{s^2+10s+K}$$



From Norman S. Nise (Figure 8.5)

Outline

Reading:

- Norman S. Nise, Control Systems Engineering, 7th edition, WILEY (chapter 8: page 382-448)
- Katsuhiko Ogata, Modern Control Engineering, 5th edition, Pearson (chapter 6: page 290-301)

Properties of the Root Locus

Consider the closed-loop transfer function for the system

$$T(s) = \frac{KG(s)}{1+KG(s)H(s)}$$

Poles exist when the characteristic polynomial in the denominator become zero, or

$$KG(s)H(s) = -1 = 1\angle(2k+1)180^o, k = 0, \pm 1, \pm 2, \ldots$$

Since $KG(s)H(s)$ is the complex quantity, It can be written in the form of angle and magnitude.
Angle condition:

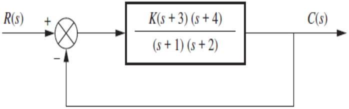
$$\angle KG(s)H(s) = (2k+1)180^o, k = 0, \pm 1, \pm 2, \ldots \tag{4}$$

Magnitude condition:

$$|KG(s)H(s)| = 1 \tag{5}$$

Properties of the Root Locus

Consider the system below



From Norman S. Nise (Figure 8.6)

The closed-loop transfer function $T(s)$ is

$$T(s) = \frac{K(s+3)(s+4)}{(1+K)s^2+(3+7K)s+(2+12K)}$$

If point s is a closed-loop system pole for some value of gain , K, then s must satisfy Eqs. (4),(5)

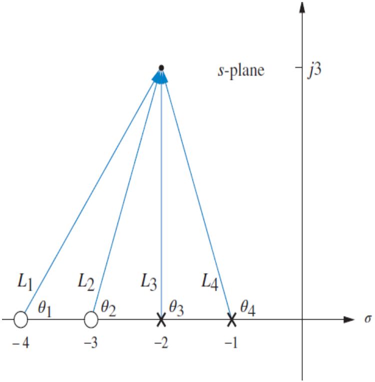
Properties of the Root Locus

Consider the point $-2 + j3$. If this point is a closed-loop pole for some value of gain, then the angles of the zeros minus the angles of the poles must equal an odd multiple of 180^o.

$$\theta_1 + \theta_2 - \theta_3 - \theta_4 = 56.31^o + 71.57^o - 90^o - 108.43^o = -70.55^o$$

Thus, $-2 + j3$ is not a point on the root locus, or alternatively, $-2 + j3$ is not a closed-loop pole for any gain.

If these calculations are repeated for the point $-2 + j(\sqrt{2}/2)$, the angles do add up to 180^o. That is, $-2 + j(\sqrt{2}/2)$ is a point on the root locus for some value of gain. We now proceed to evaluate that value of gain.



From Norman S. Nise (Figure 8.7)

$$K = \frac{1}{|G(s)H(s)|} = \frac{1}{M} = \frac{\prod \texttt{pole length}}{\prod \texttt{zero lengths}} \tag{6}$$

Replace $-2 + j3$ by $-2 + j(\sqrt{2}/2)$, the gain K, is calculated as

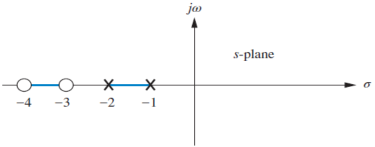
$$K = \frac{L_3 L_4}{L_1 L_2} = \frac{(\sqrt{2}/2)(1.22)}{(2.12)(1.22)} = 0.33$$

Thus, the point $-2 + j(\sqrt{2}/2)$ is a point on the root locus for a gain of 0.33.

Reading:

- Norman S. Nise, Control Systems Engineering, 7th edition, WILEY (chapter 8: page 382-448)
- Katsuhiko Ogata, Modern Control Engineering, 5th edition, Pearson (chapter 6: page 290-301)

- Rule 1: Number of branches
  $\#$ of branches of root locus $=$ $\#$ of branches of closed-loop poles
- Rule 2: Symmetry
  The root locus is symmetrical about the real axis.
- Rule 3: Real-axis segments
  On the real axis, for $K > 0$, the root locus exists to the left of an odd number of real-axis, finite open-loop poles and/or finite open-loop zeros.



From Norman S. Nise (Figure 8.9)

- Rule 4: Starting and ending points
  The root locus begins at the finite and infinite poles of $G(s)H(s)$ and ends at the finite and infinite zeros of $G(s)H(s)$.
  consider again the closed-loop transfer function

$$T(s) = \frac{KN_G(s)D_H(s)}{D_G(s)D_H(s)+KN_G(s)N_H(s)}$$

  - As $K \to 0$ (small gain),

$$T(s) \approx \frac{KN_G(s)D_H(s)}{D_G(s)D_H(s)+\epsilon} \tag{7}$$

    We conclude that the root locus begin at the poles of G(s)H(s).
  - As $K \to \infty$

$$T(s) \approx \frac{KN_G(s)D_H(s)}{\epsilon + KN_G(s)N_H(s)} \tag{8}$$

    We conclude that the root locus ends at the zeros of $G(s)H(s)$

- Rule 5: Behavior at Infinity
  Consider applying Rule 4 to the following open-loop transfer function:
  $$KG(s)H(s) = \frac{K}{s(s+1)(s+2)}$$
  There are three finite poles, at $s = 0, s = -1$ and $s = -2$, and no finite zeros.

  A function can also have infinite poles and zeros.
  - If the function approaches infinity as s approaches infinity, then the function has a pole at infinity.
  - If the function approaches zeros as s approaches infinity, then the function has a zero at infinity
    Ex.
    - $G(s) = s$ has the poles at ∞ since $G(s) \to \infty$ as $s \to \infty$.
    - $G(s) = \frac{1}{s}$ has a zero at ∞ since $G(s) \to 0$ as $s \to \infty$

- Rule 5: (cont.)

  Every function of s has an equal number of poles and zeros if we include the infinite poles and zeros as well as the finite poles and zeros.

  For illustration, consider

  $$KG(s)H(s) \approx \frac{K}{s^3} = \frac{K}{s \cdot s \cdot s}$$

  Each s in the denominator causes the open-loop function, $KG(s)H(s)$, to become 0 as that $s \to \infty$. It has three zeros at infinity

  So, the root locus begins at the finite poles of $KG(s)H(s)$ and ends at the infinite zeros.

  - Where are the infinite zeros?

- Rule 5: (Cont.)

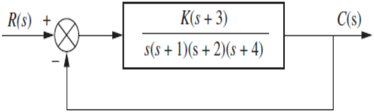
The root locus approaches straight lines as asymptotes as the locus approaches infinity. Further, the equation of the asymptotes is given by the real-axis intercept, σ_a and angle, θ_a as follows:

$$\sigma_a = \frac{\Sigma \texttt{finite poles} - \Sigma \texttt{finite zeros}}{\#\texttt{finite poles} - \#\texttt{finite zeros}} \tag{9}$$

$$\theta_a = \frac{(2k+1)\pi}{\#\texttt{finite poles} - \#\texttt{finite zeros}} \tag{10}$$

where $k = 0, \pm 1, \pm 2, \ldots$ and the angle is given in radians with respect to the positive extension of the real axis.

Consider the system below



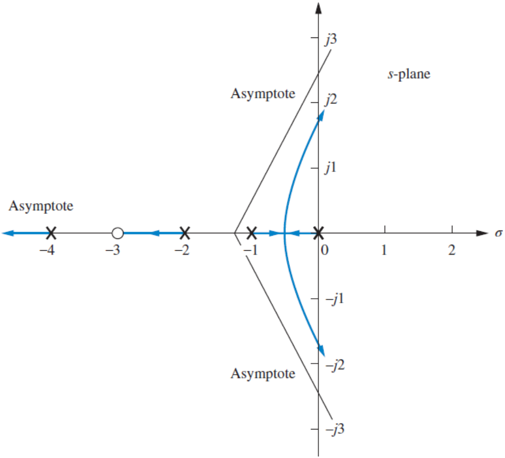
From Nise Figure 8.11

The real-axis intercept is evaluated as

$$\sigma_a = \frac{(-1-2-4)-(-3)}{4-1} = -\frac{4}{3}$$

The angles of the lines that intersect at $-4/3$ is given as

$$\begin{aligned}\theta_a &= \frac{(2k+1)\pi}{\#\texttt{finite poles} - \#\texttt{finite zeros}} \\ &= \pi/3, \qquad k = 0 \\ &= \pi, \qquad k = 1 \\ &= 5\pi/3, \qquad k = 2\end{aligned}$$



From Nise Figure 8.12

- Rule 6: Real axis break-in and breakaway points

  For each $s = \sigma$ on the real-axis segment of the root locus,

$$KG(\sigma)H(\sigma) = -1$$

or

$$K = -\frac{1}{G(\sigma)H(\sigma)} \tag{11}$$

Real-axis break-in and breakaway points are the real values of σ for which

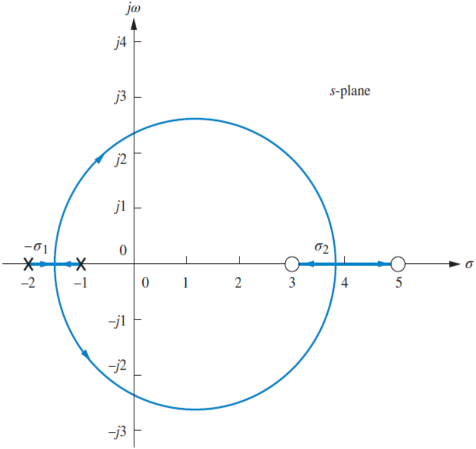
$$\frac{dK(\sigma)}{d\sigma} = 0$$

Ex. consider

$$KG(s)H(s) = \frac{K(s-3)(s-5)}{(s+1)(s+2)}$$

- Rule 6: Real axis break-in and breakaway points (cont.)
  we have
  $$KG(s)H(s) = -1$$
  then,
  $$K(\sigma) = -\frac{\sigma^2+3\sigma+2}{\sigma^2-8\sigma+15}$$
  Take the derivative with respect to σ
  $$\frac{dK}{d\sigma} = -\frac{11\sigma^2-26\sigma-61}{(\sigma^2-8\sigma+15)^2}$$
  Setting $dK/d\sigma = 0$, we find
  $$\sigma_1 = -1.45 \text{ and } \sigma_2 = 3.82$$
  which are the breakaway and break-in points.

- Rule 6: Real axis break-in and breakaway points (cont.)



From Nise Figure 8.13

- Rule 7: The $j\omega$-Axis Crossings
  if $s = j\omega$ is a closed loop pole on the imaginary axis, then

$$KG(j\omega)H(j\omega) = -1 \tag{12}$$

Ex. Consider

$$KG(s)H(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}$$

substitute $s = j\omega$ then setting $KG(j\omega)H(j\omega) = -1$
we obtain

$$-\omega^4 + j7\omega^3 + 14\omega^2 - j(8+K)\omega - 3K = 0$$

Separating real and imaginary parts,

$$\begin{aligned} -\omega^4 + 14\omega^2 - 3K &= 0 \\ 7\omega^3 - (8+K)\omega &= 0 \end{aligned}$$

- Rule 7: The $j\omega$-Axis Crossings: Form the second equation, discard the trivial solution, we obtain

$$\omega^2 = \frac{8+K}{7}$$

Substituting into the first equation. Then

$$K^2 + 65K - 720 = 0$$

Then,

$$K = -74.65 \text{ or } K = 9.65$$

Discard the negative one (negative feedback $K > 0$).
Thus,

$$K = 9.65 \text{ and } \omega = 1.59$$

Outline

Reading:

- Norman S. Nise, Control Systems Engineering, 7th edition, WILEY (chapter 8: page 382-448)
- Katsuhiko Ogata, Modern Control Engineering, 5th edition, Pearson (chapter 6: page 290-301)

Plotting Root Loci with MATLAB

In plotting root loci with MATLAB, we deal with the system equation given in the form

$$1 + K\frac{\mathtt{num}}{\mathtt{den}} = 0$$

A MATLAB command commonly used for plotting root loci is

```
rlocus(num,den)
```

Using this command, the root-locus plot is drawn on the screen. The gain vector K is automatically determined.

For the systems defined in state space, `rlocus(A,B,C,D)` plots the root locus of the system with the gain vector automatically determined.
Note that commands

`rlocus(num,den,K)` or `rlocus(A,B,C,D,K)`

use the user-supplied gain vector K.

Plotting Root Loci with MATLAB

If it is desired to plot the root loci with marks `'o'` or `'x'`, it is necessary to use the following command:

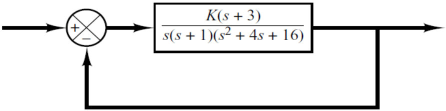
```
r = rlocus(num,den)
plot(r,'o') or plot(r,'x')
```

Plotting root loci using marks o or x is instructive, since each calculated closed-loop pole is graphically shown; in some portion of the root loci those marks are densely placed and in another portion of the root loci they are sparsely placed

Plotting Root Loci with MATLAB

Example 1

Consider the system shown below



From Ogata (Figure 6-15)

Plot root loci with a square aspect ratio so that a line with slope 1 is a true 45^o line. Choose the region of root-locus plot to be

$$-6 \leqq x \leqq 6, -6 \leqq y \leqq 6$$

where x and y are the real-axis coordinate and imaginary-axis coordinate, respectively.

To set the given plot region on the screen to be square, enter the command

```
v = [-6 6 -6 6]; axis (v); axis('square')
```

With this command, the region of the plot is as specified and a line with slope 1 is at a true 45^o, not skewed by the irregular shape of the screen.

Define

$a = s(s+1)$: `a = [1 1 0]`

$b = s^2 + 4s + 16$: `b = [1 4 16]`

Then we use the following command:

```
c = conv(a,b)
```

The denominator polynomial is thus found to be

```
den = [1 5 20 16 0]
```

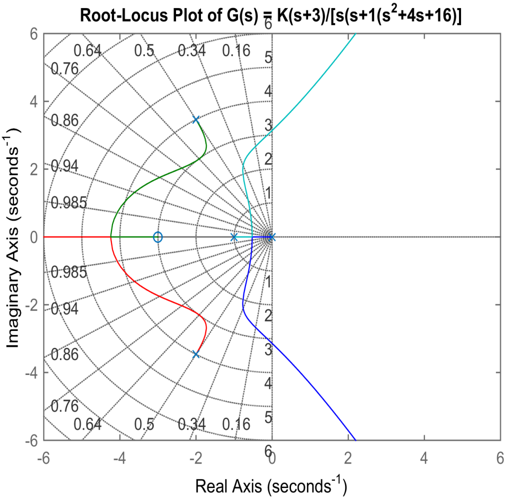
To find the complex-conjugate open-loop poles (the roots of $s^2 + 4s + 16 = 0$), we may enter the roots command as follows:

```
r = roots(b)
```

Thus, the system has the following open-loop zero and open-loop poles:

Open-loop zero:	$s = -3$
Open-loop poles:	$s = 0, s = -1, s = -2 \pm j3.4641$

```
%--------Root-locus plot-----------
num = [1 3];
den = [1 5 20 16 0];
rlocus(num,den)
v= [-6 6 -6 6];
axis(v);axis('square')
grid;
title('Root-Locus Plot of G(s) = K(s+3)/[s(s+1)(s^2+4s+16)]')
```


Root-Locus Plot of G(s) = K(s+3)/[s(s+1(s^2+4s+16)]
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
0.64
0.5
0.34
0.16
0.76
0.86
0.94
0.985
6
5
4
3
2
1
0
-2
-4
-6
2
4
6

Consider the negative feedback system whose open-loop transfer function $G(s)H(s)$ is

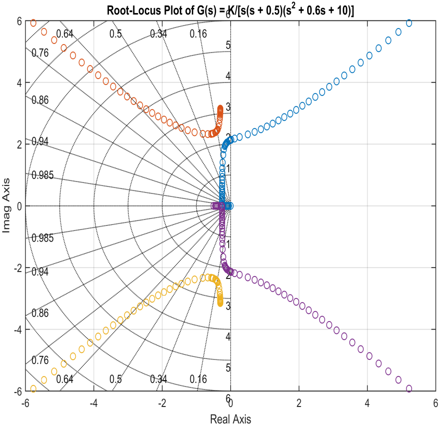
$$G(s)H(s) = \frac{K}{s(s+0.5)(s^2+0.6s+10)}$$

There are no open-loop zeros. Open-loop poles are located at $s = -0.3 + j3.1480, s = -0.3 - j3.1480, s = -0.5$, and $s = 0$

```
%--------Root-locus plot-----------
num = [1];
den = [1 1.1 10.3 5 0];
rlocus(num,den)
plot(r,'o'); v= [-6 6 -6 6];
axis(v);axis('v'); grid
sgrid
title('Root-Locus Plot of G(s) = K/[s(s + 0.5)(s^2 + 0.6s + 10)]')
xlabel('Real Axis')
ylabel('Imag Axis')
```

Plotting Root Loci with MATLAB

Example 2

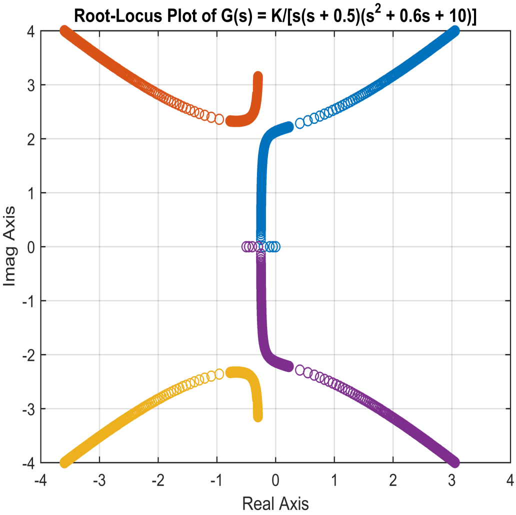


we can also plot the root loci using smaller increments of K in the critical region.

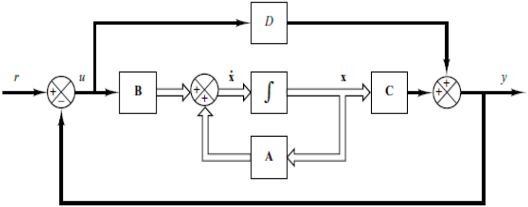
```
-----Root locus plot -----

num = [1]; den = [1 1.1 10.3 5 0];
K1 = 0:0.2:20; K2 = 20:0.1:30;K3 = 30:5:1000;
K = [K1 K2 K3];
r = rlocus(num,den,K);
plot(r, 'o')
v = [-4 4 -4 4]; axis(v)
grid
title('Root-Locus Plot of G(s) = K/[s(s + 0.5)(s^2 + 0.6s + 10)]')
xlabel('Real Axis')
ylabel('Imag Axis')
```

Plotting Root Loci with MATLAB

Exampe 2



Consider the system shown below



From Ogata (figure 6-19)

The system equations are

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$
$$u = r - y$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -160 & -56 & -14 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ -14 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \end{bmatrix}$$

In this example problem, we shall obtain the root-locus diagram of the system defined in state space.

The root-locus plot for this system can be obtained with MATLAB by use of the following command:

```
rlocus(A,B,C,D)
```

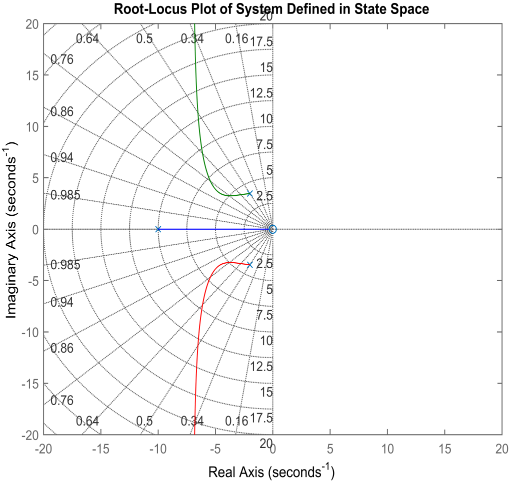
This command will produce the same root-locus plot as can be obtained by use of the rlocus `(num,den)` command, where `num` and `den` are obtained from

```
[num,den]=ss2tf(A,B,C,D)
```

```
% --------Program------------
A = [0 1 0;0 0 1;-160 -56 -14]; B = [0;1;-14];
C = [1 0 0]; D = [0];K = 0:0.1:400;
rlocus(A,B,C,D,K);
v = [-20 20 -20 20]; axis(v)
grid
title('Root-Locus Plot of System Defined in State Space')
```

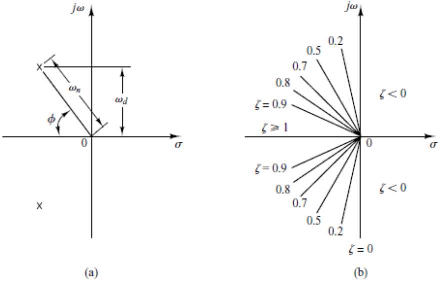
Plotting Root Loci with MATLAB

Example 3



The damping ratio ζ of a pair of complex-conjugate poles can be expressed in terms of the angle ϕ, which is measured from negative real axis with

$$\zeta = \cos\phi$$



From Ogata (figure 6-21)

- The damping ratio determines the angular location of the poles.
- The distance of the pole from the origin is determined by the undamped natural frequency ω_n.

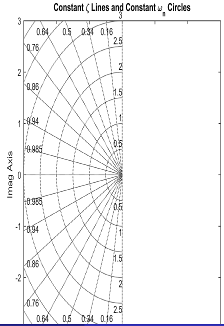
The command

$$\texttt{sgrid}$$

overlays lines of constant damping ratio ($\zeta = 0$ to 1 with 0.1 increment) and circles of constant ω_n on the root-locus plot.

-------Example 4 ------

```
sgrid
v = [-3 3 -3 3];
axis(v); axis('square')
title('Constant ζ Lines
and Constant ω_n
Circles')
xlabel('Real Axis')
ylabel('Imag Axis')
```



If only particular constant ζ lines (such as the $\zeta = 0.5$ line and $\zeta = 0.707$ line) and particular constant ω_n circles (such as the $\omega_n = 0.5$ circle, $\omega_n = 1$ circle, and $\omega_n = 2$ circle) are desired, use the following command:

```
sgrid([0.5, 0.707], [0.5, 1, 2])
```

If we wish to overlay lines of constant ζ and circles of constant ω_n as given above to a root-locus plot of a negative feedback system with

```
num = [0 0 0 1]
den =[1 4 5 0]
```

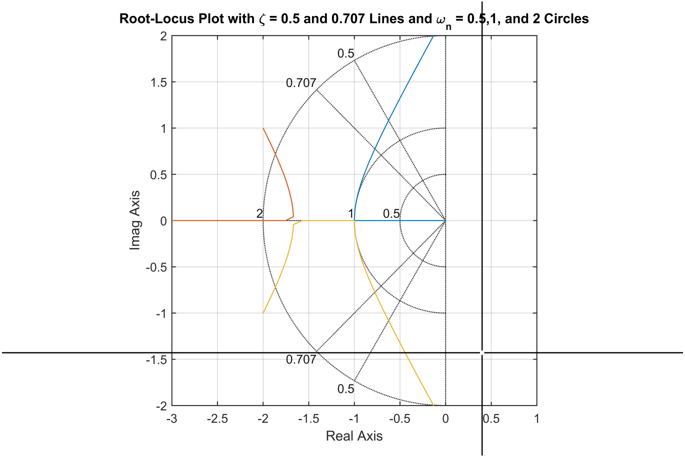
Plotting Root Loci with MATLAB

Plotting polar grid in the root-locus diagram

```
num = [1];
den = [1 4 5 0];
K = 0:0.01:1000;
r = rlocus(num,den,K);
plot(r,'-'); v = [-3 1 -2 2]; axis(v); axis('square')
sgrid([0.5,0.707], [0.5,1,2])
grid
title('Root-Locus Plot with \zeta = 0.5 and 0.707 Lines and
\omega_n = 0.5,1, and 2 Circles')
xlabel('Real Axis'); ylabel('Imag Axis')
gtext('\omega_n = 2')
gtext('\omega_n = 1') gtext('\omega_n = 0.5')
gtext('x')
gtext('x')
gtext('x')
```

Plotting Root Loci with MATLAB

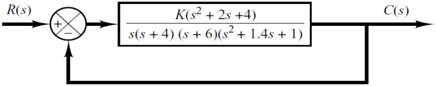
Plotting polar grid in the root-locus diagram



If we want to omit either the entire constant ζ lines or entire constant ω_n circles, we may use empty bracket [] in the arguments of the sgrid command.

For example, if we want to overlay only the constant damping ratio line corresponding to $\zeta = 0.5$ and no constant ω_n circle on the root plot, the we may use this command

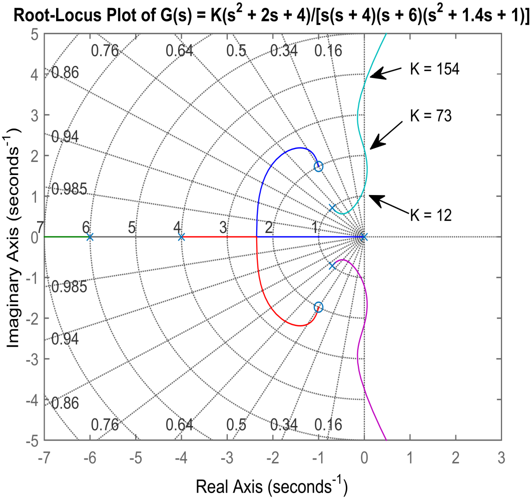
```
sgrid(0.5,[])
```



From Ogata (figure 6-24)

```
-------- Example 6-------
num = [1 2 4];
den = conv(conv([1 4 0],[1 6]), [1 1.4 1]);
rlocus(num, den)
v = [-7 3 -5 5]; axis(v); axis('square')
grid
title('Root-Locus Plot of G(s) = K(s^2 + 2s + 4)/[s(s + 4)(s
+ 6)(s^2 + 1.4s + 1)]')
text(1.0, 0.55,'K = 12')
text(1.0,3.0,'K = 73')
text(1.0,4.15,'K = 154')
```

Conditionally stable systems



- This system is stable only for limited ranges of the value of K –that is, $0 \leq K \leq 12$ and $73 \leq K \leq 154$.
- The system becomes unstable for $12 \leq K \leq 73$ and $154 \leq K$.
- If K assumes a value corresponding to unstable operation, the system may break down or may become nonlinear due to a saturation nonlinearity that may exist. Such a system is called conditionally stable.

In practice, conditionally stable systems are not desirable.

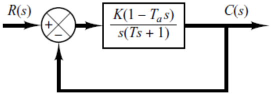
Conditional stability is dangerous but does occur in certain systems—in particular, a system that has an unstable feedforward path. Such an unstable feedforward path may occur if the system has a minor loop.

It is advisable to avoid such conditional stability since, if the gain drops beyond the critical value for any reason, the system becomes unstable.

- If all the poles and zeros of a system lie in the left half s plane, then the system is called minimum phase.
- If a system has at least one pole or zero in the right-half s plane, then the system is called nonminimum phase.

The term nonminimum phase comes from the phase-shift characteristics of such a system when subjected to sinusoidal inputs.

Consider the system shown



From Ogata (figure 6-26)

For this system,

$$G(s) = \frac{K(1-T_a s)}{s(Ts+1)}, (T_a \leq 0), H(s) = 1$$

This is a nonminimum-phase system, since there is one zero in the right-half s plane.

Plotting Root Loci with MATLAB

Nonminimum-Phase Systems

The angle condition becomes

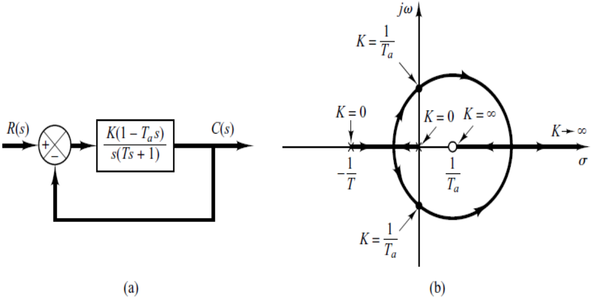
$$\begin{aligned}\angle G(s) &= \angle - \frac{K(T_a s - 1)}{s(Ts + 1)} \\ &= \angle \frac{K(T_a s - 1)}{s(Ts + 1)} + 180^o \\ &= \pm 180^o(2k + 1), (k = 0, 1, 2, \ldots)\end{aligned}$$

or

$$\angle \frac{K(T_a s - 1)}{s(Ts + 1)} = 0^o \tag{13}$$

The root loci can be obtained from Equation (13).

Figure below shows a root-locus plot for this system. From the diagram, we see that the system is stable if the gain K is less than $1/T_a$.



From Ogata (figure 6-26)

Plotting Root Loci with MATLAB

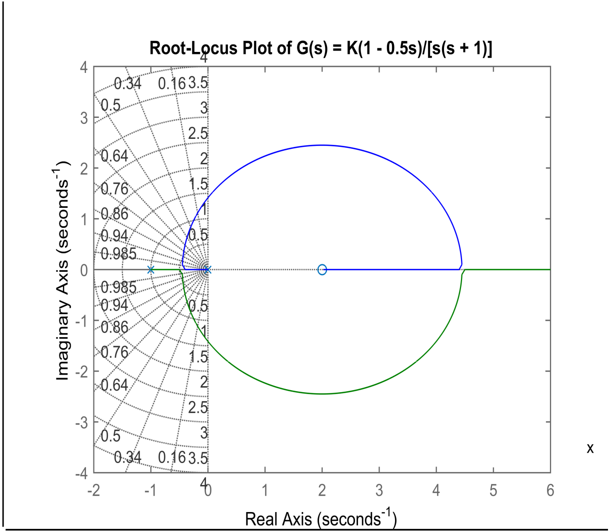
Nonminimum-Phase Systems

For example, if $T = 1$ sec and $T_a = 0.5$ sec, The program is

```
--------example 7 -------
num = [-0.5 1]; den = [1 1 0];
k1 = 0:0.01:30; k2 = 30:1:100;
K3 = 100:5:500; K = [k1 k2
k3];
rlocus(num,den,K)
v = [-2 6 -4 4]; axis(v);
axis('square')
grid
title('Root-Locus Plot of G(s)
= K(1 - 0.5s)/[s(s + 1)]')
gtext('x')
gtext('x')
gtext('o')
```

Plotting Root Loci with MATLAB

Nonminimum-Phase Systems



It is frequently desired to find the gain value K at an arbitrary point on the root locus. This can be accomplished by using the following `rlocfind` command:

```
[K, r] = rlocfind(num, den)
```

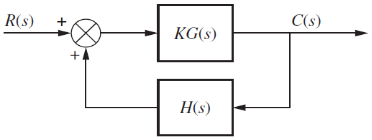
- The `rlocfind` command, which must follow an `rlocus` command, overlays movable $x - y$ coordinates on the screen.
- Using the mouse, we position the origin of the $x - y$ coordinates over the desired point on the root locus and press the mouse button.
- Then MATLAB displays on the screen the coordinates of that point, the gain value at that point, and the closed-loop poles corresponding to this gain value.

Outline

Reading:

- Norman S. Nise, Control Systems Engineering, 7th edition, WILEY (chapter 8: page 382-448)
- Katsuhiko Ogata, Modern Control Engineering, 5th edition, Pearson (chapter 6: page 290-301)

Root Locus for Positive-Feedback Systems



We find that the transfer function for the positive-feedback is given as

$$T(s) = \frac{KG(s)}{1 - KG(s)H(s)} \tag{14}$$

From Eq. (14), obviously, a pole , s, exists when

$$KG(s)H(s) = 1 = 1\angle k360^o, k = 0, \pm 1, \pm 2, \pm 3, \ldots \tag{15}$$

- Rule 1: Number of branches

  The same arguments as for negative feedback apply to this rule. There is no change.

- Rule 2: Symmetry

  The same arguments as for negative feedback apply to this rule. There is no change.

- Rule 3: Real-axis segments

  Real-axis segments: On the real axis, the root locus for positive-feedback systems exists to the left of an even number of real-axis, finite open-loop poles and/or finite open-loop zeros.

- Rule 4: Starting and ending points

  Starting and ending points: The root locus for positive-feedback systems begins at the finite and infinite poles of $G(s)H(s)$ and ends at the finite and infinite zeros of $G(s)H(s)$.

- Rule 5: Behavior at infinity

  The root locus approaches straight lines as asymptotes as the locus approaches infinity. Further, the equations of the asymptotes for positive-feedback systems are given by the real-axis intercept, σ_a, and angle, θ_a , as follows:

$$\sigma_a = \frac{\Sigma \texttt{finite poles} - \Sigma \texttt{finite zeros}}{\#\texttt{finite poles} - \#\texttt{finite zeros}} \tag{16}$$

$$\theta_a = \frac{2k\pi}{\#\texttt{finite poles} - \#\texttt{finite zeros}} \tag{17}$$

- The imaginary axis crossing can be found using the root locus program.
- In a search of the $j\omega$-axis, you are looking for the point were the angles add up to a multiple of 360^o.
- The breakaway points are found by looking for the maximum value of K.
- The break-in points are found by looking for the minimum value of K.

Reference

- Norman S. Nise, Control Systems Engineering, 7th edition, WILEY
- Katsuhiko Ogata, Modern Control Engineering, 5th edition, Pearson