Feedback Control Systems
Lecture 1 Introduction

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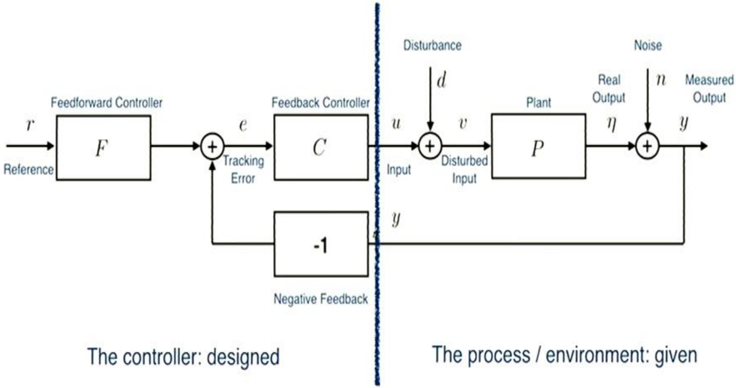
Outline

Feedback Systems

"I think it's very important to have a feedback loop, where you're constantly thinking about what you've done and how you could be doing it better."
Elon Musk

Have you ever seen this schematic and what it is all about?



- Uncertainty enters the systems through noise in sensing and actuation subsystems.
- Disturbances from the environment enters the systems.
- Uncertain dynamic or model uncertainty caused by parameters errors, unmodeled effects and so on.
- The algorithm that computes the control actions as the function of the sensor values is often called a control law.
- The system can be influenced externally by an operator who introduced command signals to the systems.

Control appears almost 99% in industrial applications, in nature and also in life.

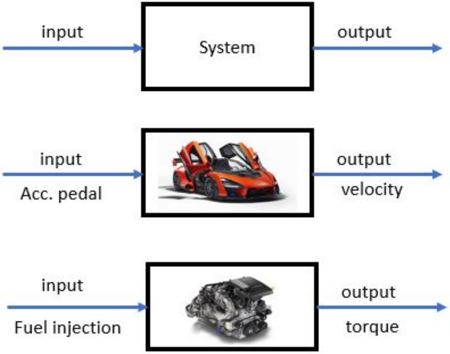
Control is everywhere $\Rightarrow$ No Control, No Life!

...and feedback is the key to success !

What is (automatic) control?

Control of dynamical system via feedback

- Control: For control, we talk about cause which is something that we can manipulate. Cause $\rightarrow$ Effect.



What is (automatic) control?

Control of dynamical system via feedback

- Dynamics system = a system whose behavior changes over time. The output depends not only the current input but also the previous input.

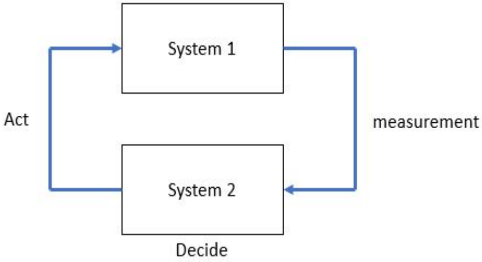
For simplicity, the dynamics is described by differential equation as follow

$$\frac{dv}{dt} = -\frac{a}{m}v + \frac{b}{m}u$$
$$y = v$$

What is (automatic) control?

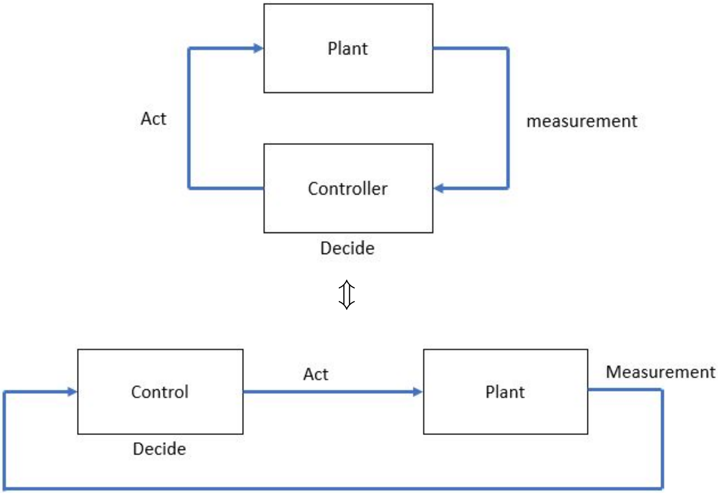
Control of dynamical system via feedback

- Feedback: Refers to a situation in which two (or more) dynamical systems are connected together such that each system influences the other and their dynamics are thus strongly coupled.



Feedback Systems

Feedback Principle



Control

Control is a core part of robotics.

Control system design concerns with the question: Given the desired trajectory, how could we actually activate its various inputs so that the vehicle moves to the desired trajectory in the continuous space.

For flying robot, a quadrotor for example, we are mainly interested in controlling its position and attitude. What do you think we should control for the quadrotor?

Goal: Design a controller so that the system has some desired characteristic.

- Stabilize the system (Stabilization)
- Regulate the system about some design point (regulation)
- Follow a given class of command signals (Tracking)
- Reduce response to disturbances (Disturbance Rejection)

To achieve them, we require tools for

- Analysis: given a controller, how can we check if the above is satisfied?
- Synthesis: given a plant, how to design a controller that satisfies the above?

- Lyapunov stability: a system is called Lyapunov stable if, for any bounded initial condition, an zero, input, the state remained bounded, i.e.,

$$||x_0|| < \epsilon \text{ and } u = 0 \Rightarrow ||x(t)|| < \delta, \forall t \geq 0$$

- A system is called asymptomatically stable if, for any bounded initial condition, and zero input, the state converges to zero, i.e.,

$$||x_0|| < \epsilon \text{ and } u = 0 \Rightarrow \lim_{t \to +\infty} ||x(t)|| = 0$$

- Bounded-Input, Bounded-Output stability: A system is called BIBO-stable if , for any bounded input, the output remains bounded, i.e.,

$$||u(t)|| < \epsilon, \forall t \geq 0, \text{ and } x_0 \Rightarrow ||y(t)|| < \delta, \forall t \geq 0.$$

- A system is called unstable if it is not stable.

- Regulation problem: determine an input such that the system maintains a reference value despite disturbances.
- Tracking / servo problem: find the input that allows the system output to closely follow a time varying reference signal.

Main challenges are disturbance and noise attenuation

A control system is robust when it is insensitive to model uncertainties (bounded variation in the model parameters)

There is a saying:

"All models are wrong, but some are useful"

Typical model mismatch cause:

- Aging: Plant model parameters change with time / use.
- System identification: poorly modeled systems, open loop unstable plants.

Robust stability and robust performance

- Robust stability(RS): The system is stable for all perturbed plants about the nominal model up to the worst-case model uncertainty.
- Robust performance(RP): The system satisfies the performance for all perturbed plants about the nominal model up to the worst-case model uncertainty.

Control

A typical control system includes the sensors, actuators and control law.

- The sensors and actuators need not always be physical devices. (e.g, economic systems)
- A good selection of the sensor and actuator can greatly simplify the control design process.
- We will study the design of the control law given the rest of the system.

Basic questions that we will address about state-space approach:

- What are the state-space model?
- Why should we use them?
- How are they related to transfer functions used in classical control design?
- How do we develop a state-space model?
- How do we design a controller using a state-space model?

- What are the state-space model?
  The representation of the dynamic of an n^{th} order system using n first order differential equation
  Example

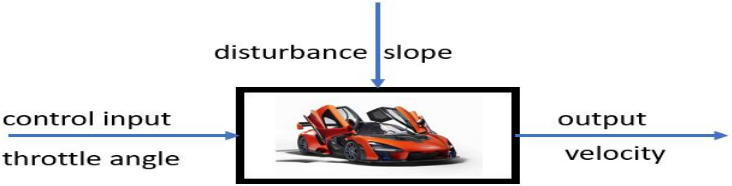
$$m\ddot{q} + c\dot{q} + kq = F$$
$$\Rightarrow \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

- Why should we use them?
  - State variable form a convenient way to work with complex dynamics. Matrix format is easy to use on computers.
  - Transfer function only deal with input/output behavior, but state-space form provides easy access to the "internal" feature.
  - Great for MIMO systems which are very hard to work with using transfer function.
- How do we develop a state-space model?
  - There are variety of ways to develop state-space models.
    - Linearization
    - Derivation from simple linear dynamics
- How do we design a controllers using state-space model?
  - Full-state feedback
  - Observer/estimator design

Feedback Control Approach

Example: Simplified Cruise Control

- Specification: keep the vehicle's velocity constant
- Control signal: throttle angle
- Disturbance: road slope
- Output: velocity

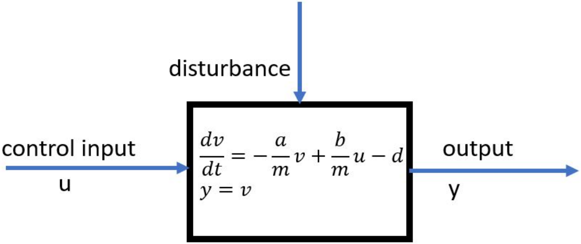


The plant is given by

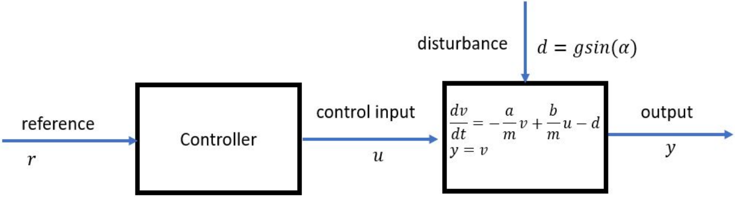
$$\frac{dv}{dt} = -\frac{a}{m}v + \frac{b}{m}u - g\sin\alpha$$
$$y = v$$

Feedback Control Approach

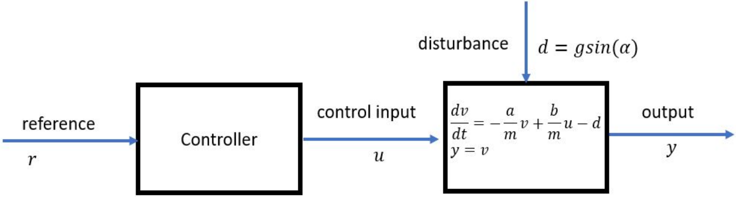
Example: Simplified Cruise Control



The open loop control looks like this



Open loop control



Specification: $v = r$

- Reference velocity $r = 30m/s$

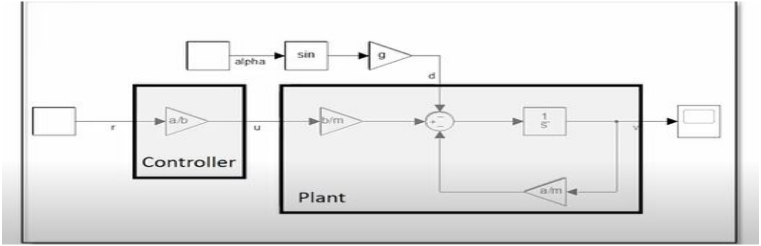
Assumptions for control design:

- Flat road: $\alpha = 0$
- At steady state: $0 = -\frac{a}{m}v + \frac{b}{m}u$

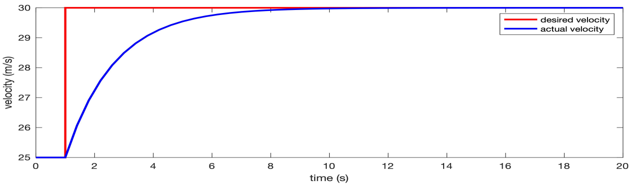
Control design
$u = \frac{a}{b}r \rightarrow v = r$

Try to use Simulink for verification

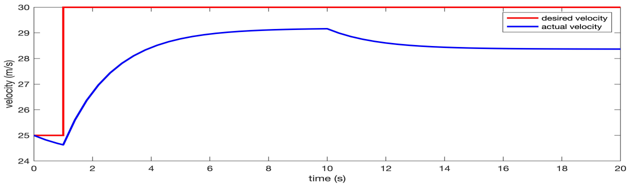
- Open loop



- Step response: 25 m/s to 30m/s,
  $m = 1000kg$, $a = 600Ns/m$, $b = 10kN/rad$ and $g = 9.82m/s^2$



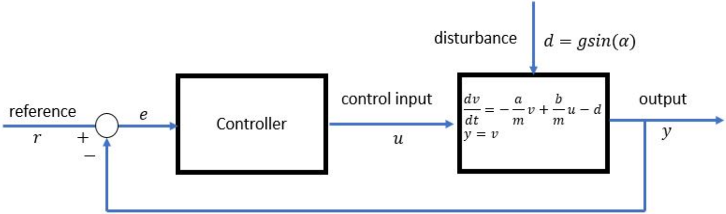
what will happen if there is slop?



Feedback Control Approach

Example: Simplified Cruise Control

- Specification: Keep the vehicle's velocity constant
- Closed loop control
- Measurement: velocity



Specification: $v = r$

- Reference velocity $r = 30m/s$

control design: P-Controller

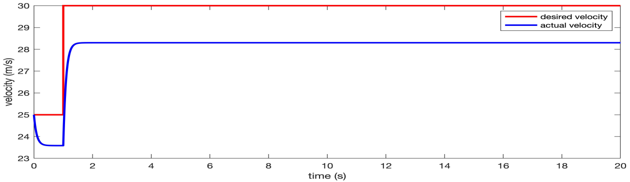
$u = K_p e = K_p(r - y)$

Control design: PI-controller

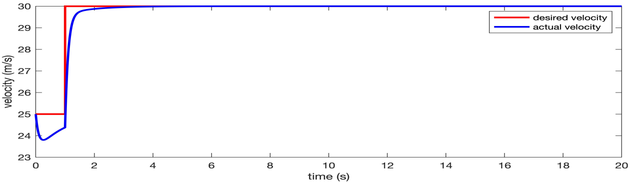
$u = K_p(r - y) + K_i \int_0^t e d\tau$

Closed loop Control

- Proportional control (P Control). For some value of k, we obtain the following result.



- Proportional and Integral Control (PI control)



Remark: This example illustrated the possibilities with closed loop control.

- Design of dynamics (stabilization, speed-up the response time).
- Robust to uncertainty (disturbance, parameter variations)

...but there are some challenges

- Destabilization
- Measurement noise

...Furthermore, open loop control can be considered when we have a stable system and we have good knowledge about the disturbances and model parameters.

Feedback Control Approach

- Advantages
  - Many unnecessary disturbances and noise signals from outsides can be rejected.
  - The change in the performance of the system due to parameter variations in reduced.
  - The steady-state error of the system can be relatively small.
  - The transient behavior of the process can be easily manipulated.
- Disadvantages
  - The system is complicated by the increased number of components.
  - The system may not stable (it may oscillate or depart greatly from the desired output), even though the comparable open-loop system is stable.
  - If there is a change in an output, it will affect the input.

Feedback control is a hidden technology.

Control objectives (specification)

- Qualitative - minimize energy (achieving as good result as possible, do not use much fuel,)
- Quantitative - response time (time should be less than a certain value,

Description of the system/plant

- Level abstraction (system level, component level, or even more details)
- Modeling physical modeling or from the measurement data

Feedback Control Approach

Design controller

- Select technique - open loop or closed loop
- Classical methods or state-space methods
- Choose parameters (trail-and-error, design method, optimization)

Analyze the performance

- Analysis
- Simulation
- Experiments

(meet objective ? Yes->done, No ->iteration)

Classical control methods (Ex. PID)

- works well for simple systems
- can be tuned based on trail-and-error or engineering intuition.
- do not require model of the systems

...but

- are typical iterative
- are difficult to use for larger-scale systems (complex systems) with multi inputs and outputs(MIMO)

State-space method

- Can easily handle larger-scale systems (complex systems) with multi inputs and outputs (MIMO)
- tuning can be formed as an optimization problem
- are easy to implement
- require a mathematical model of the system

and some others...

Simple Form of Feedback

Recall...

"I think it's very important to have a feedback loop, where you're constantly thinking about what you've done and how you could be doing it better."
Elon Musk

The idea of feedback to make corrective actions based on the difference between the desired and the actual values of a quantity can be implemented in many different ways.

A simple feedback mechanism can be described as follows:

$$u = \begin{cases} u_{max} & \text{if } e > 0 \\ u_{min} & \text{if } e < 0, \end{cases} \tag{1}$$

where the control error $e = r - y$ is the different between the reference signal and the output of the system, and u is the actuation command.

- Chief advantages
  - Simple and no parameter to choose
  - keep the process variable close to the reference
    Ex. Simple thermostat to maintain the temperature of the room.
  - Acceptable if the oscillation is sufficiently small

The reason why on-off control often give rise to oscillations is that the system overreacts since a small change in the error makes the actuated variable change over the full range. This effect is avoided in proportional control, where the characteristic of the controller is proportional to the control error for small errors. This can be achieved with the control law

$$u = \begin{cases} u_{max} & \text{if } e \geqq e_{max} \\ k_p e & \text{if } e_{min} < e < e_{max} \\ u_{min} & \text{if} e \leqq e_{min}, \end{cases} \tag{2}$$

where k_p is the controller gain, $e_{min} = u_{min}/k_p$ and $e_{max} = u_{max}/k_p$. The interval (e_{min}, e_{max}) is called proportional band because the behavior of the controller is linear when the error is in this interval:

$$u = k_p(r - y) = k_p e \text{ if } e_{min} \leqq e \leqq e_{max} \tag{3}$$

Simple Form of Feedback

Integral Control (I Control)

P Control has the drawback that the process variable often deviates from its reference value. In particular, if some level of control signal is required for the system to maintain a desired value, then we must have $e \neq 0$ in order to generate the requisite input.

This can be avoided by making the control action proportional to the integral of the error:

$$u(t) = k_i \int_0^t e(\tau) d\tau \tag{4}$$

where k_i is the integral gain.

An additional refinement is to provide the controller with an anticipation ability by using the predictive error. A simple prediction is given by the linear extrapolation

$$e(t + T_d) \approx e(t) + T_d \frac{de(t)}{dt}, \tag{5}$$

which predicts the error T_d time units ahead.

Combining proportional, integral and derivative control, we obtain a controller that can be expressed mathematically as

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt} \tag{6}$$

The control action is thus a sum of three terms: the past as represented by the integral of the error, the present as represented by the proportional term and the future as represented by the linear extrapolation of the error (the derivative term).

Note

More than 95 % of all industrial control problems are solved by PID control.

Assumption and Prerequisite

- Assumption
  - Will not focus on the classical design in this course. However you should review your previous course on control systems.
  - Our focus in this class is on state space methods.
    - More systematic design tools exist - can be easily codified and implemented numerically.
    - Easily handle large scale systems
- Prerequisite
  - Differential equations and linear algebra are crucial for the studies of control theory in general. (please review some).
  - MATLAB/Simulink (please review some).

References

Materials used in this lecture draws heavily from the book entitled **Feedback Systems: An Introduction for Scientists and Engineers** by Karl Johan Astrom and Richard M. Murray.