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Roll No.

B.Tech. (SEM-I) THEORY EXAMINATION 2018-19 MATHEMATICS-I

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions.

Q no.	Question	Marks	CO
a.	Find the rank of the matrix $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$	2	l

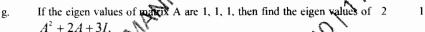
b. Find the stationary point of
$$f(x,y) = x^3 + y^3 + 3axy, a > 0$$

c. If
$$x = r\cos\theta$$
, $y = r\sin\theta$, $z = z$ then find $\frac{\partial(r, \theta, z)}{\partial(x, y, z)}$

d. Define del
$$\nabla$$
 operator and gradient.

If
$$\phi = 3x^2y - y^3z^2$$
, find grad 1 at point (2, 0, -2).

f. Evaluate
$$\int_{0}^{1} \int_{0}^{x^{2}} \frac{y}{e^{x}} dxdy$$



If
$$u = x^3 y^2 \sin^{-1}(y/x)$$
, then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.

SECTION B

2. Attempt any three of the following:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$$

Also express the polynomial $B = A^8-11A^7-4A^6+A^5+A^4-11A^3-3A^2+2A+I$ as a quadratic polynomial in A and hence find B.

b. If
$$y = Sin(m sin^{-1}x)$$
, prove that : $(1 - x^2) y_{n+2} - (2n + 1)x y_{n+1} - (n^2 - 10) 2$
 $m^2)y_n = 0$ and find y_n at $x = 0$.

c. If
$$u$$
, v , w are the roots of the equation $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$, $(x-b)^3 + (x-c)^3 = 0$, $(x-c)^3 = 0$,

d. Evaluate
$$\int_{0}^{\infty} \int_{0}^{\infty} e^{-(x^2 + y^2)} dx dy$$
 by changing to polar coordinates.

Hence show that
$$\int_{0}^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

e. Verify the divergence theorem for
$$\vec{F} = (x^3 - yz)\hat{i} + (y^3 - zx)j + (z^3 - xy)\hat{k}, \text{ taken over the cube bounded by planes } x = 0, y = 0, z = 0, x = 1, y = 1, z = 1.$$

SECTION C

5

3. Attempt any one part of the following:

Q no.	Question					Marks	CO
a.		[3	-3	4]		10	1
	Find inverse employing elementary transformation A	= 2	-3	4			
	Find inverse employing elementary transformation A	0	-1	1			
b.		[1	2	-1	4]	10	l
	Reduce the matrix A to its normal form when A =	2	4	3	4 .		
		1	2	3	4		
	No. of	-1	-2	6	-7		

Hence find the rank of A.

4. Attempt any one part of the following:

Q no.	Question	Marks	CO
a.	If $\sin^{-1} y = 2\log(x+1)$ show that	10	2
b.	$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$ Verify Lagrange's Mean value Theorem for the function $f(x) = x^3$ in $[-2.2]$	10	2

5. Attempt any one part of the following:

Q no.	Question	Marks	CO
a.	Find the maximum or minimum distance of the point (1, 2, -1) from the	10	3
b.	sphere $x^2 + y^2 + z^2 = 24$. If $u = \cos^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}})$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$	10	3

6. Attempt any one part of the following:

Q no. Question Marks CO a. 10 4

Change the order of integration and then evaluate: $\int_{0}^{2} \int_{x^{2}}^{3} x y \, dy \, dx$

Calculate the volume of the solid bounded by the surface x=0, y=0, 10 x+y+z=1 & z=0.

7. Attempt any one part of the following:

Q no. Question Marks CO a. Prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both 10 5 Solenoidal and Irrotational.

b. Find the directional derivative of $\Phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point $\frac{10}{5}$

P(1, 1, 1) in the direction of the line

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}.$$