

EECS 487: Introduction to Natural Language Processing

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Outline

- 
- Logistic Regression
 - Feedforward Neural Networks

[Some slides are borrowed from Dan Jurafsky and Hugo Larochelle]

Logistic Regression (LogReg)

- • Generative and Discriminative Classifiers
 - Classification in LogReg (test)
 - An example with sentiment analysis
 - Learning in LogReg (training)

Generative and Discriminative Classifiers

- Naïve Bayes is a **generative** classifier

by contrast:

- Logistic regression is a **discriminative** classifier

Generative and Discriminative Classifiers

Suppose we're distinguishing cat from dog images



imagenet



imagenet

Generative Classifier:

- Build a model of what's in a cat image
 - Knows about whiskers, ears, eyes
 - Assigns a probability to any image:
 - how cat-y is this image?



Also build a model for dog images

Now given a new image:

Run both models and see which one fits better

Discriminative Classifier

Just try to distinguish dogs from cats



Oh look, dogs have collars!
Let's ignore everything else

Finding the correct class c from a document d in Generative vs Discriminative Classifiers

- Naive Bayes

$$\hat{c} = \operatorname{argmax}_{c \in C} \frac{\text{likelihood}}{\text{prior}} P(d|c) P(c)$$

- Logistic Regression

$$\hat{c} = \operatorname{argmax}_{c \in C} \frac{\text{posterior}}{P(c|d)}$$

Components of a probabilistic machine learning classifier

Given m input/output pairs $(x^{(i)}, y^{(i)})$:

1. A **feature representation** of the input. For each input observation x , a vector of features $[x_1, x_2, \dots, x_n]$. Feature i for input x is x_i , or sometimes $f_i(x)$.
2. A **classification function** that computes y , the estimated class, via $p(y|x)$, like the **sigmoid** or **softmax** functions.
3. An **objective function** for learning, like **cross-entropy loss**.
4. An algorithm for optimizing the objective function: **stochastic gradient descent**.

The two phases of Logistic Regression

- **Training:** we learn weights w and bias b using **stochastic gradient descent** and **cross-entropy loss**.
- **Test:** Given a test example x we compute $p(y|x)$ using learned weights (or parameters), and return whichever label ($y = 1$ or $y = 0$) is higher probability

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Binary Classification in Logistic Regression

- Given a series of input/output pairs:
 - $(x^{(i)}, y^{(i)})$
- For each observation $x^{(i)}$
 - We represent $x^{(i)}$ by a **feature vector** $[x_1, x_2, \dots, x_n]$
 - We compute an output: a predicted class $\hat{y}^{(i)} \in \{0,1\}$

Features in logistic regression

- For feature x_i , weight w_i tells “**how important is x_i** ”
 - $x_i = \text{"review contains 'awesome'"}: w_i = +10$
 - $x_j = \text{"review contains 'abysmal'"}: w_j = -10$
 - $x_k = \text{"review contains 'mediocre'"}: w_k = -2$

Logistic Regression for one observation x

- Input observation: vector $x = [x_1, x_2, \dots, x_n]$
- Weights: one per feature: $W = [w_1, w_2, \dots, w_n]$
 - Sometimes we call the weights θ
- Output: a predicted class $\hat{y} \in \{0,1\}$

How to do classification

- For each feature x_i , weight w_i tells us the importance of x_i
 - Plus we'll have a bias b
- We'll sum up all the weighted features and the bias

$$z = \left(\sum_{i=1}^n w_i x_i \right) + b$$

$$\longrightarrow z = w \cdot x + b$$

- If this sum is high, we say $y=1$; if low, then $y=0$

But we want a probabilistic classifier

- We need to formalize “sum is high”.
- We’d like a **principled** classifier that gives us a **probability**, just like Naive Bayes did
- Concretely, we want a model that can tell us:
 $p(y=1|x)$
 $p(y=0|x)$

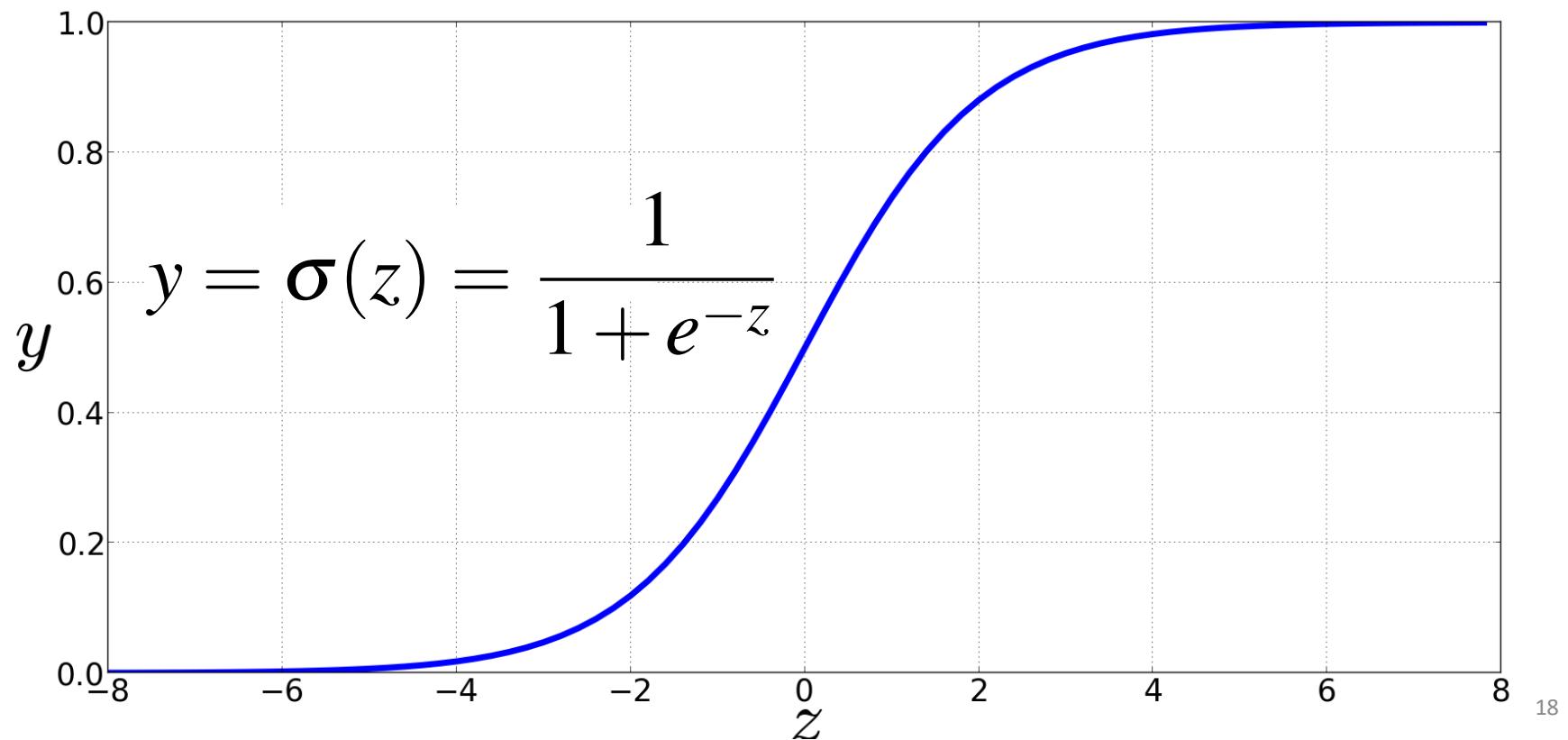
The problem: z isn't a probability, it's just a number!

$$z = w \cdot x + b$$

- Solution: use a function of z that goes from 0 to 1

$$y = \sigma(z) = \frac{1}{1 + e^{-z}} = \frac{1}{1 + \exp(-z)}$$

The very useful sigmoid (or logistic) function



Idea of logistic regression

- We'll compute $w \cdot x + b$
- And then we'll pass it through the sigmoid function:
 $\sigma(w \cdot x + b)$
- And we'll just treat it as a probability

Making probabilities with sigmoids

$$\begin{aligned} P(y = 1) &= \sigma(w \cdot x + b) \\ &= \frac{1}{1 + \exp(-(w \cdot x + b))} \end{aligned}$$

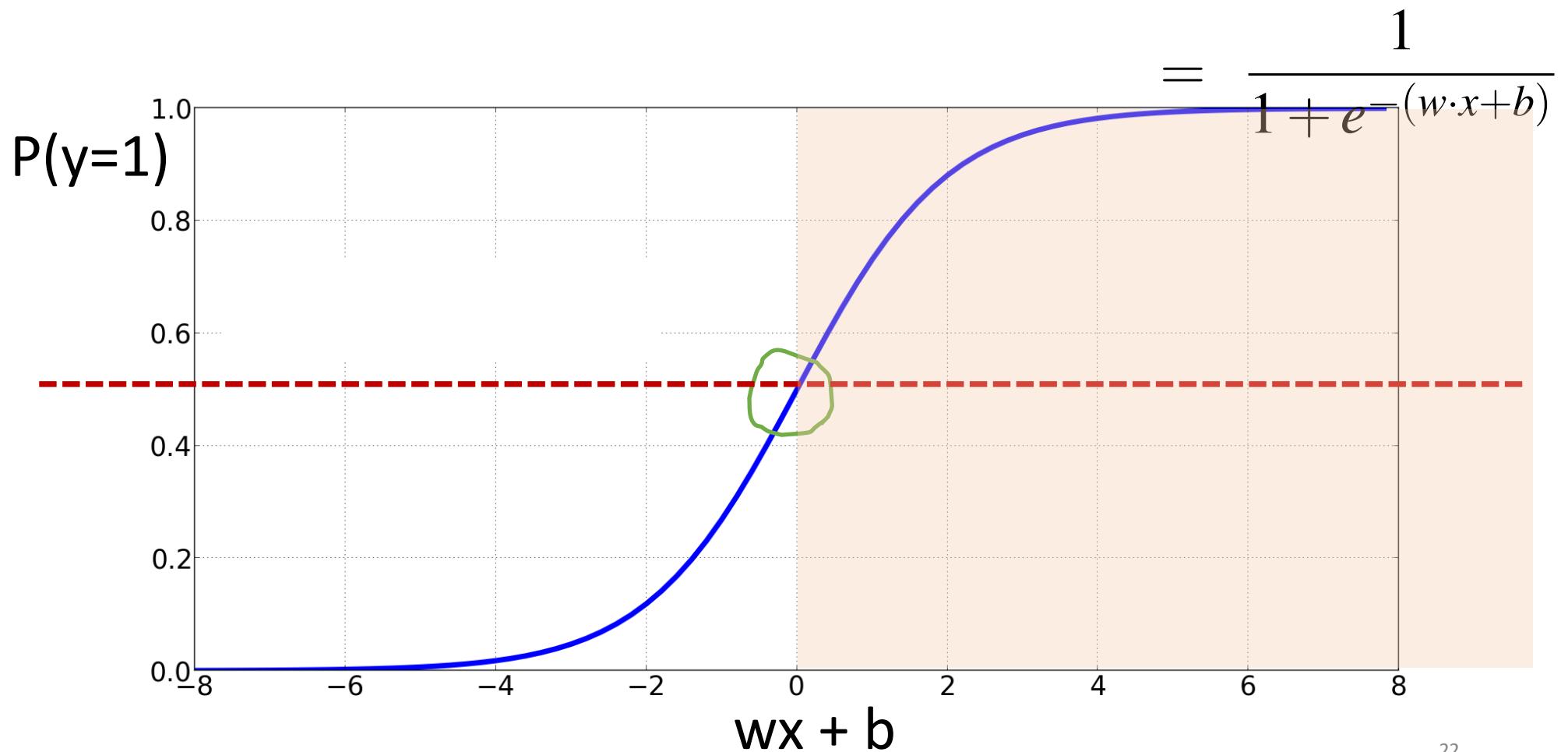
$$\begin{aligned} P(y = 0) &= 1 - \sigma(w \cdot x + b) \\ &= 1 - \frac{1}{1 + \exp(-(w \cdot x + b))} \\ &= \frac{\exp(-(w \cdot x + b))}{1 + \exp(-(w \cdot x + b))} \end{aligned}$$

Turning a probability into a classifier

$$\hat{y} = \begin{cases} 1 & \text{if } P(y = 1|x) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

0.5 here is called the **decision boundary**

The probabilistic classifier $P(y=1) = \sigma(w \cdot x + b)$



Turning a probability into a classifier

$$\hat{y} = \begin{cases} 1 & \text{if } P(y=1|x) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

if $w \cdot x + b > 0$
if $w \cdot x + b \leq 0$

Logistic Regression (LogReg)

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- ➡ • An example with sentiment analysis
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Sentiment example: does $y=1$ or $y=0$?

- It's hokey . There are virtually no surprises , and the writing is second-rate . So why was it so enjoyable ? For one thing , the cast is great . Another nice touch is the music . I was overcome with the urge to get off the couch and start dancing . It sucked me in , and it'll do the same to you .

$x_1=3$ $x_2=2$ $x_3=1$ $x_4=3$ $x_5=0$ $x_6=4.19$

It's **hokey**. There are virtually **no** surprises , and the writing is **second-rate**.
 So why was it so **enjoyable**? For one thing , the cast is
great. Another **nice** touch is the music **I** was overcome with the urge to get off
 the couch and start dancing . It sucked **me** in, and it'll do the same to **you** .

Feature	Definition	Value of the Feature
x_1	count(positive lexicon) \in doc)	3
x_2	count(negative lexicon) \in doc)	2
x_3	$\begin{cases} 1 & \text{if “no”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns \in doc)	3
x_5	$\begin{cases} 1 & \text{if “!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0
x_6	$\log(\text{word count of doc})$	$\ln(66) = 4.19$

Classifying sentiment for input x

Feature	Definition	Value of the Feature
x_1	$\text{count}(\text{positive lexicon}) \in \text{doc}$)	3
x_2	$\text{count}(\text{negative lexicon}) \in \text{doc})$	2
x_3	$\begin{cases} 1 & \text{if “no”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	$\text{count}(1\text{st and 2nd pronouns} \in \text{doc})$	3
x_5	$\begin{cases} 1 & \text{if “!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0
x_6	$\log(\text{word count of doc})$	$\ln(66) = 4.19$

Suppose $w = [2.5, -5.0, -1.2, 0.5, 2.0, 0.7]$ $b = 0.1$

Classifying sentiment for input x

Suppose $w = [2.5, -5.0, -1.2, 0.5, 2.0, 0.7]$ $b = 0.1$

$$\begin{aligned} p(+|x) = P(Y = 1|x) &= \sigma(w \cdot x + b) \\ &= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\ &= \sigma(.833) \\ &= 0.70 \end{aligned}$$

$$\begin{aligned} p(-|x) = P(Y = 0|x) &= 1 - \sigma(w \cdot x + b) \\ &= 0.30 \end{aligned}$$

Classification in (binary) logistic regression: summary

- Given:

- a set of classes: (+ sentiment, - sentiment)
- a vector \mathbf{x} of features $[x_1, x_2, \dots, x_n]$
 - $x_1 = \text{count}(\text{"awesome"})$
 - $x_2 = \log(\text{number of words in review})$
- A vector \mathbf{w} of weights $[w_1, w_2, \dots, w_n]$
 - w_i for each feature f_i

$$\begin{aligned} P(y=1) &= \sigma(w \cdot x + b) \\ &= \frac{1}{1 + e^{-(w \cdot x + b)}} \end{aligned}$$

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Wait, where did the W and b come from?

- Supervised classification: we know the correct label y (either 0 or 1) for each x .
- What the system produces is an estimate, \hat{y}
- We want to set w and b to minimize the **distance** between our estimate $\hat{y}^{(i)}$ and the true $y^{(i)}$.
 - We need a distance estimator: a **loss function** or a **cost function**
 - We need an **optimization algorithm** to update w and b to minimize the loss.

Learning components

- A loss function:
 - **cross-entropy loss**
- An optimization algorithm:
 - **stochastic gradient descent** (not covered in the lecture)

The distance between \hat{y} and y

We want to know how far is the classifier output:

$$\hat{y} = \sigma(w \cdot x + b)$$

from the true output:

$$y \text{ [= either 0 or 1]}$$

We'll call this difference:

$$L(\hat{y}, y) = \text{how much } \hat{y} \text{ differs from the true } y$$

Cross-entropy loss

- We choose the parameters w, b that maximize the probability of the true y labels in the training data given the observations x

Deriving cross-entropy loss for a single observation x

- **Goal:** maximize probability of the correct label $p(y|x)$
- Since there are only 2 discrete outcomes (0 or 1) we can express the probability $p(y|x)$ from our classifier (the thing we want to maximize) as

$$p(y|x) = \hat{y}^y (1 - \hat{y})^{1-y}$$

- if the gold-standard label $y=1$, this simplifies to \hat{y} (*the predicted probability for x having a label of 1*)
- if the gold-standard label $y=0$, this simplifies to $1 - \hat{y}$

Deriving cross-entropy loss for a single observation x

Goal: maximize probability of the correct label $p(y|x)$

Maximize:

$$p(y|x) = \hat{y}^y (1 - \hat{y})^{1-y}$$

- Now take the **log** of both sides (mathematically handy)

Maximize:

$$\begin{aligned} \log p(y|x) &= \log [\hat{y}^y (1 - \hat{y})^{1-y}] \\ &= y \log \hat{y} + (1 - y) \log (1 - \hat{y}) \end{aligned}$$

- Whatever values maximize $\log p(y|x)$ will also maximize $p(y|x)$

Deriving cross-entropy loss for a single observation x

Goal: maximize probability of the correct label $p(y|x)$

$$\begin{aligned}\text{Maximize: } \log p(y|x) &= \log [\hat{y}^y (1 - \hat{y})^{1-y}] \\ &= y \log \hat{y} + (1 - y) \log(1 - \hat{y})\end{aligned}$$

- Now flip sign to turn this into a **loss**: something to **minimize**

Cross-entropy loss

$$L_{\text{CE}}(\hat{y}, y) = -\log p(y|x) = -[y \log \hat{y} + (1 - y) \log(1 - \hat{y})]$$

- Or, plugging in definition of \hat{y} :

$$L_{\text{CE}}(\hat{y}, y) = -[y \log \sigma(w \cdot x + b) + (1 - y) \log(1 - \sigma(w \cdot x + b))]$$

Let's see if this works for our sentiment example

- We want loss to be:
 - smaller** if the model's prediction is close to the correct label
 - bigger** if model is confused
- Let's first suppose the true label of this is $y=1$ (positive)

It's hokey . There are virtually no surprises , and the writing is second-rate . So why was it so enjoyable ? For one thing , the cast is great . Another nice touch is the music . I was overcome with the urge to get off the couch and start dancing . It sucked me in , and it'll do the same to you .

Let's see if this works for our sentiment example

- True value is $y=1$. How well is our model doing?

$$\begin{aligned} p(+|x) = P(Y = 1|x) &= \sigma(w \cdot x + b) \\ &= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\ &= \sigma(.833) \\ &= 0.70 \end{aligned}$$

- Pretty well! What's the loss?

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log(1 - \sigma(w \cdot x + b))] \\ &= -[\log \sigma(w \cdot x + b)] \\ &= -\log(.70) \\ &= .36 \end{aligned}$$

Let's see if this works for our sentiment example

- Suppose true value instead was $y=0$.

$$\begin{aligned} p(-|x) = P(Y = 0|x) &= 1 - \sigma(w \cdot x + b) \\ &= 0.30 \end{aligned}$$

- What's the loss?

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log(1 - \sigma(w \cdot x + b))] \\ &= -[\log(1 - \sigma(w \cdot x + b))] \\ &= -\log(.30) \\ &= 1.2 \end{aligned}$$

Let's see if this works for our sentiment example

- The loss when model was right (if true $y=1$)

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log(1 - \sigma(w \cdot x + b))] \\ &= -[\log \sigma(w \cdot x + b)] \\ &= -\log(.70) \\ &= .36 \end{aligned}$$

- Is lower than the loss when model was wrong (if true $y=0$):

$$\begin{aligned} L_{\text{CE}}(\hat{y}, y) &= -[y \log \sigma(w \cdot x + b) + (1 - y) \log(1 - \sigma(w \cdot x + b))] \\ &= -[\log(1 - \sigma(w \cdot x + b))] \\ &= -\log(.30) \\ &= 1.2 \end{aligned}$$

- Sure enough, loss was bigger when model was wrong!

Our goal: minimize the loss

- Let's make explicit that the loss function is parameterized by weights $\theta = (w, b)$
- We want the weights that minimize the loss, averaged over **all examples**:

$$\hat{\theta} = \operatorname{argmin}_{\theta} \frac{1}{m} \sum_{i=1}^m L_{\text{CE}}(f(x^{(i)}; \theta), y^{(i)})$$

Problem of Overfitting

- A model that perfectly match the training data has a problem.
- It will also **overfit** to the data, e.g.,
 - A random word that perfectly predicts y (it happens to only occur in one class) will get a very high weight.
 - Failing to generalize to a test set without this word.
- A good model should be able to **generalize**.

Problem of Overfitting

+

This movie drew me in, and
it'll do the same to you.

—

I can't tell you how much I
hated this movie. It sucked.

Useful or harmless features

X1 = "this"

X2 = "movie"

X3 = "hated"

X4 = "drew me in"

4gram features that just "memorize"
training set and might cause
problems

X5 = "the same to you"

X6 = "tell you how much"

Problem of Overfitting

- 4-gram model on tiny data will just memorize the data
 - 100% accuracy on the training set
- But it will be surprised by the novel 4-grams in the test data
 - Low accuracy on test set
- Models that are too powerful can **overfit** the data
 - Fitting the details of the training data so exactly that the model doesn't generalize well to the test set
 - How to avoid overfitting?
 - **Regularization** in logistic regression
 - **Dropout** in neural networks

Regularization

- A solution for overfitting
- Add a regularization term $R(\theta)$ to the loss function (for now written as maximizing log probability rather than minimizing loss)

$$\hat{\theta} = \operatorname{argmax}_{\theta} \sum_{i=1}^m \log P(y^{(i)} | x^{(i)}) - \alpha R(\theta)$$

- Idea: choose an $R(\theta)$ that **penalizes large weights**
 - fitting the data well with lots of big weights not as good as fitting the data a little less well, with small weights

L2 Regularization

- The sum of the squares of the weights: **L2 norm** $\|\theta\|_2$
 - i.e., the square of the **Euclidean distance** of θ to the origin.
- L2 regularized objective function:

$$R(\theta) = \|\theta\|_2^2 = \sum_{j=1}^n \theta_j^2$$

$$\hat{\theta} = \operatorname{argmax}_{\theta} \left[\sum_{i=1}^m \log P(y^{(i)} | x^{(i)}) \right] - \alpha \sum_{j=1}^n \theta_j^2$$

L1 Regularization

- The sum of the absolute value of the weights: **L1 norm** $\|\theta\|_1$
- L1 regularized objective function:

$$R(\theta) = \|\theta\|_1 = \sum_{i=1}^n |\theta_i|$$
$$\hat{\theta} = \operatorname{argmax}_{\theta} \left[\sum_{1=i}^m \log P(y^{(i)} | x^{(i)}) \right] - \alpha \sum_{j=1}^n |\theta_j|$$

Outline

- Logistic Regression
- • Feedforward Neural Networks

Neural Network Learning

- Learning approach based on modeling adaptation in biological neural systems.
- **Perceptron**: Initial algorithm for learning simple neural networks (single layer) developed in the 1950's.
- **Backpropagation**: More complex algorithm for learning multi-layer neural networks developed in the 1980's. (not required for this class)

ARTIFICIAL NEURON

Topics: connection weights, bias, activation function

- Neuron pre-activation (or input activation):

$$a(\mathbf{x}) = b + \sum_i w_i x_i = b + \mathbf{w}^\top \mathbf{x}$$

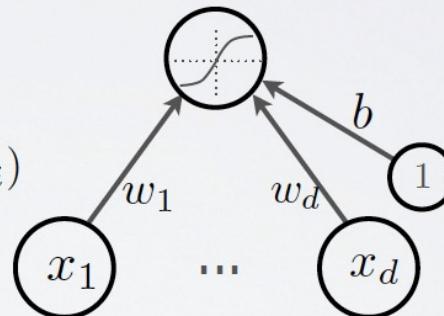
- Neuron (output) activation

$$h(\mathbf{x}) = g(a(\mathbf{x})) = g(b + \sum_i w_i x_i)$$

- \mathbf{w} are the connection weights

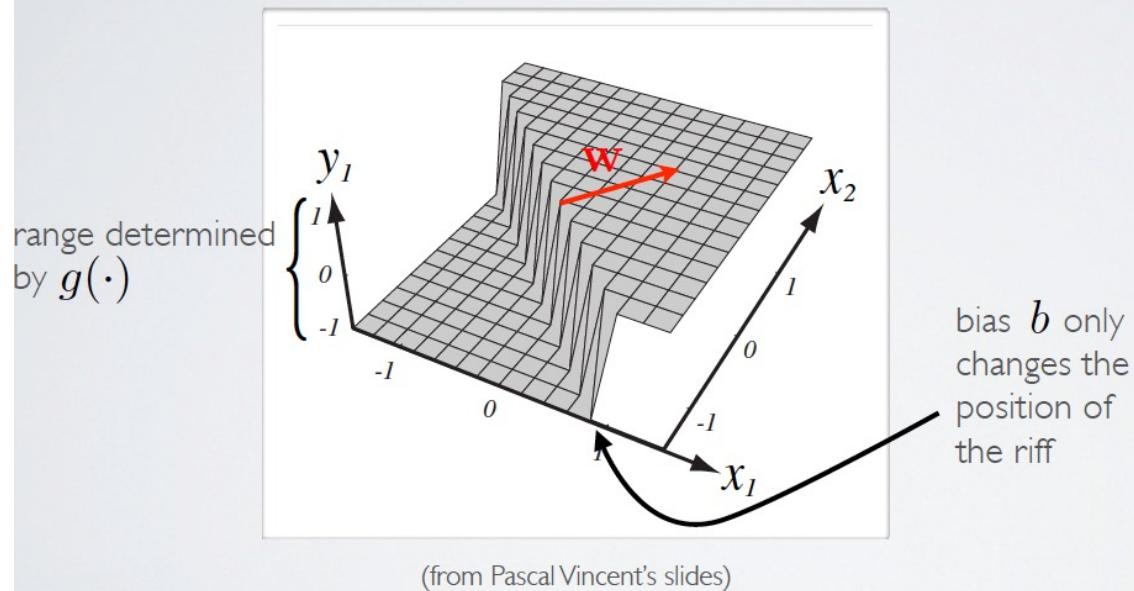
- b is the neuron bias

- $g(\cdot)$ is called the activation function



ARTIFICIAL NEURON

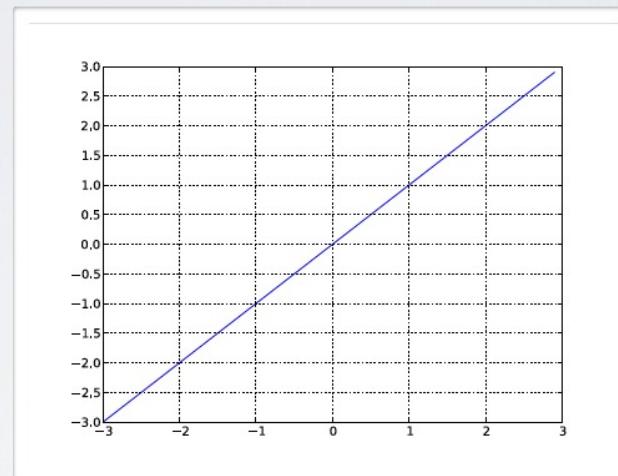
Topics: connection weights, bias, activation function



ACTIVATION FUNCTION

Topics: linear activation function

- Performs no input squashing
- Not very interesting...

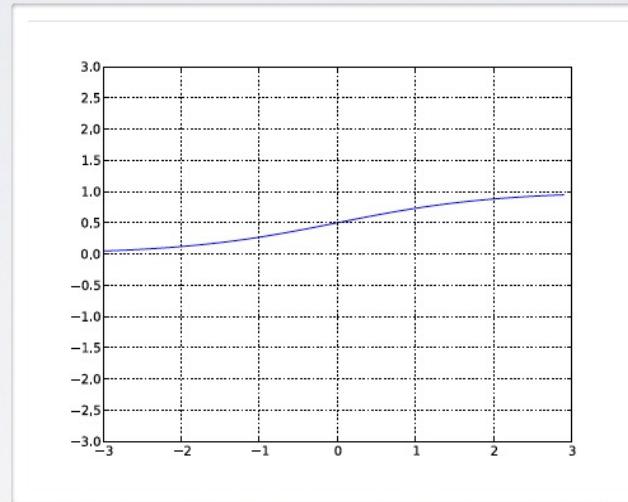


$$g(a) = a$$

ACTIVATION FUNCTION

Topics: sigmoid activation function

- Squashes the neuron's pre-activation between 0 and 1
- Always positive
- Bounded
- Strictly increasing

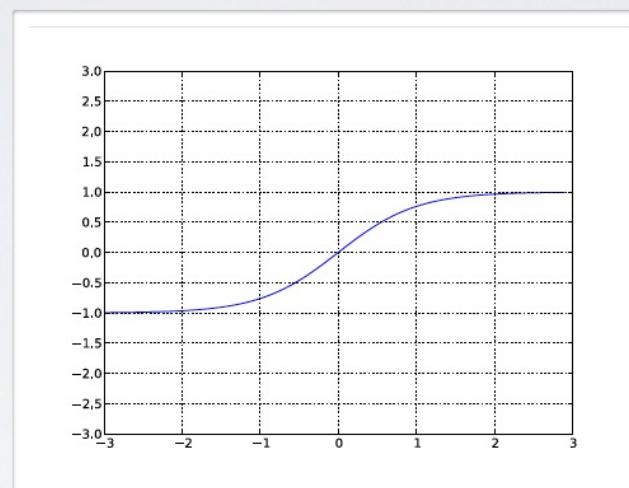


$$g(a) = \text{sigm}(a) = \frac{1}{1+\exp(-a)}$$

ACTIVATION FUNCTION

Topics: hyperbolic tangent ("tanh") activation function

- Squashes the neuron's pre-activation between -1 and 1
- Can be positive or negative
- Bounded
- Strictly increasing

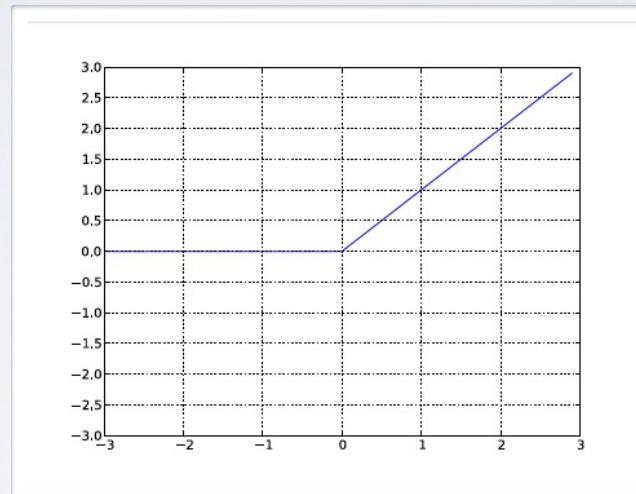


$$g(a) = \tanh(a) = \frac{\exp(a) - \exp(-a)}{\exp(a) + \exp(-a)} = \frac{\exp(2a) - 1}{\exp(2a) + 1}$$

ACTIVATION FUNCTION

Topics: rectified linear activation function

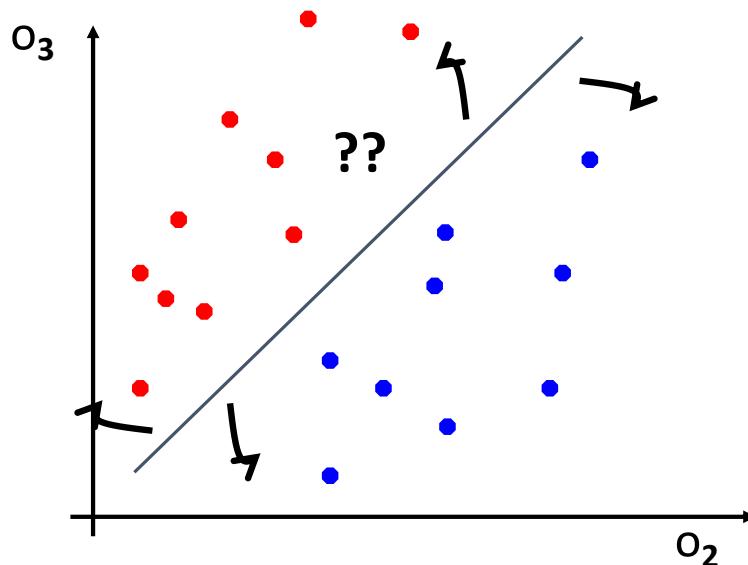
- Bounded below by 0
(always non-negative)
- Not upper bounded
- Strictly increasing
- Tends to give neurons
with sparse activities



$$g(a) = \text{reclin}(a) = \max(0, a)$$

Linear Separator

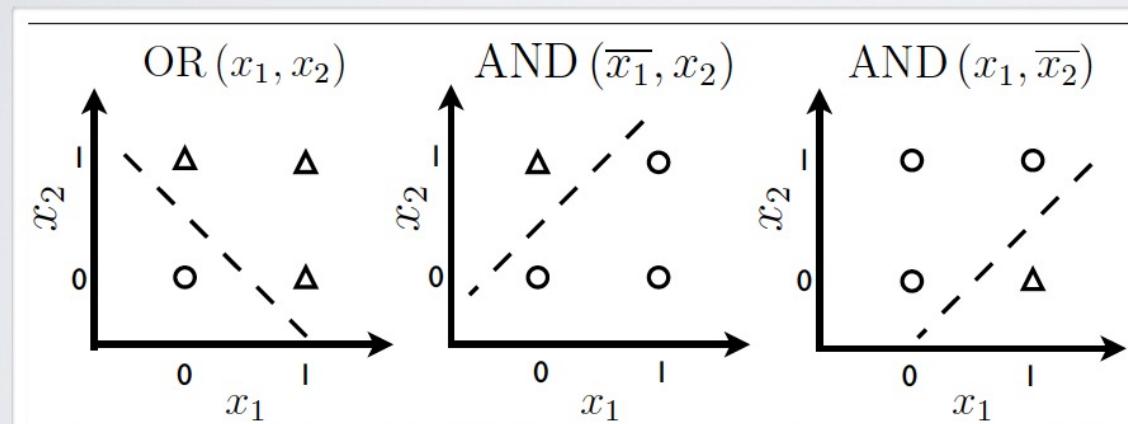
- Since one-layer neuron (aka perceptron) uses linear threshold function, it is searching for a linear separator that discriminates the classes.



ARTIFICIAL NEURON

Topics: capacity of single neuron

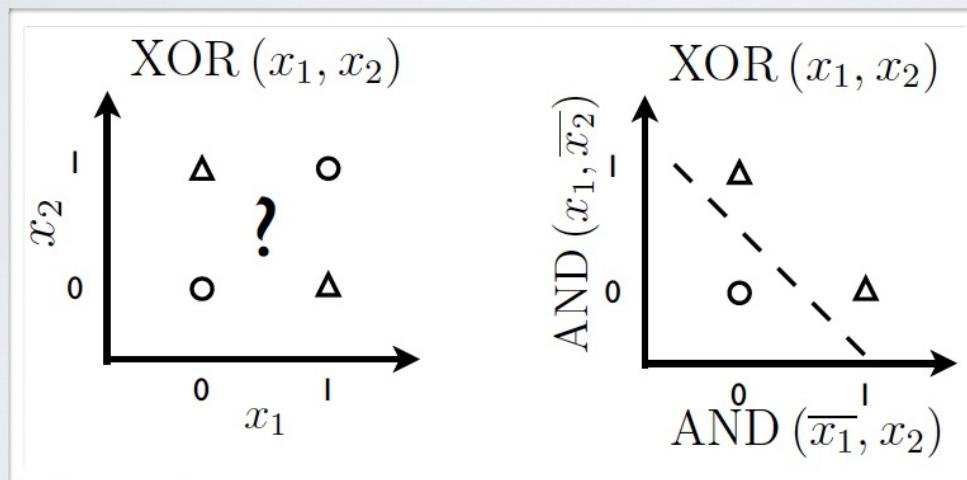
- Can solve linearly separable problems



ARTIFICIAL NEURON

Topics: capacity of single neuron

- Can't solve non linearly separable problems...



- ... unless the input is transformed in a better representation

NEURAL NETWORK

Topics: single hidden layer neural network

- Hidden layer pre-activation:

$$\mathbf{a}(\mathbf{x}) = \mathbf{b}^{(1)} + \mathbf{W}^{(1)}\mathbf{x}$$

$$(a(\mathbf{x})_i = b_i^{(1)} + \sum_j W_{i,j}^{(1)}x_j)$$

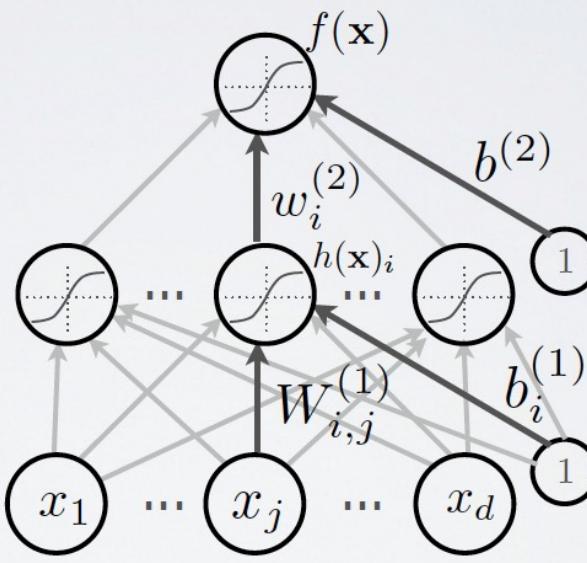
- Hidden layer activation:

$$\mathbf{h}(\mathbf{x}) = \mathbf{g}(\mathbf{a}(\mathbf{x}))$$

- Output layer activation:

$$f(\mathbf{x}) = o(b^{(2)} + \mathbf{w}^{(2) \top} \mathbf{h}^{(1)} \mathbf{x})$$

output activation function



NEURAL NETWORK

Topics: softmax activation function

- For multi-class classification:
 - we need multiple outputs (1 output per class)
 - we would like to estimate the conditional probability $p(y = c|\mathbf{x})$
- We use the softmax activation function at the output:
$$\mathbf{o}(\mathbf{a}) = \text{softmax}(\mathbf{a}) = \left[\frac{\exp(a_1)}{\sum_c \exp(a_c)} \cdots \frac{\exp(a_C)}{\sum_c \exp(a_c)} \right]^\top$$
 - strictly positive
 - sums to one
- Predicted class is the one with highest estimated probability

NEURAL NETWORK

Topics: multilayer neural network

- Could have L hidden layers:

- layer pre-activation for $k > 0$ ($\mathbf{h}^{(0)}(\mathbf{x}) = \mathbf{x}$)

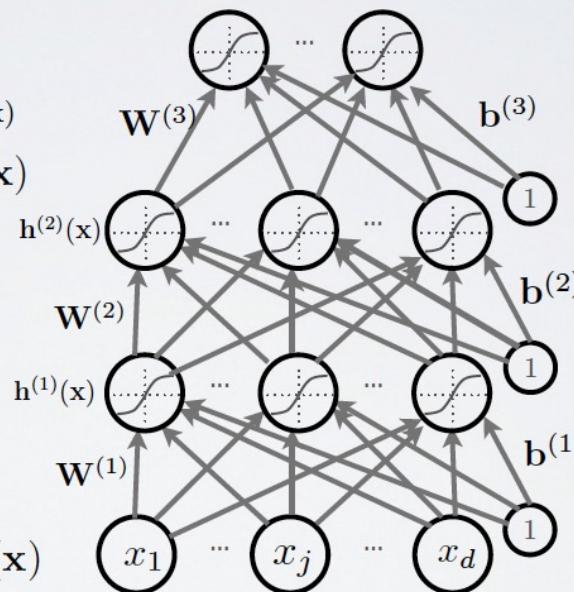
$$\mathbf{a}^{(k)}(\mathbf{x}) = \mathbf{b}^{(k)} + \mathbf{W}^{(k)}\mathbf{h}^{(k-1)}(\mathbf{x})$$

- hidden layer activation (k from 1 to L):

$$\mathbf{h}^{(k)}(\mathbf{x}) = \mathbf{g}(\mathbf{a}^{(k)}(\mathbf{x}))$$

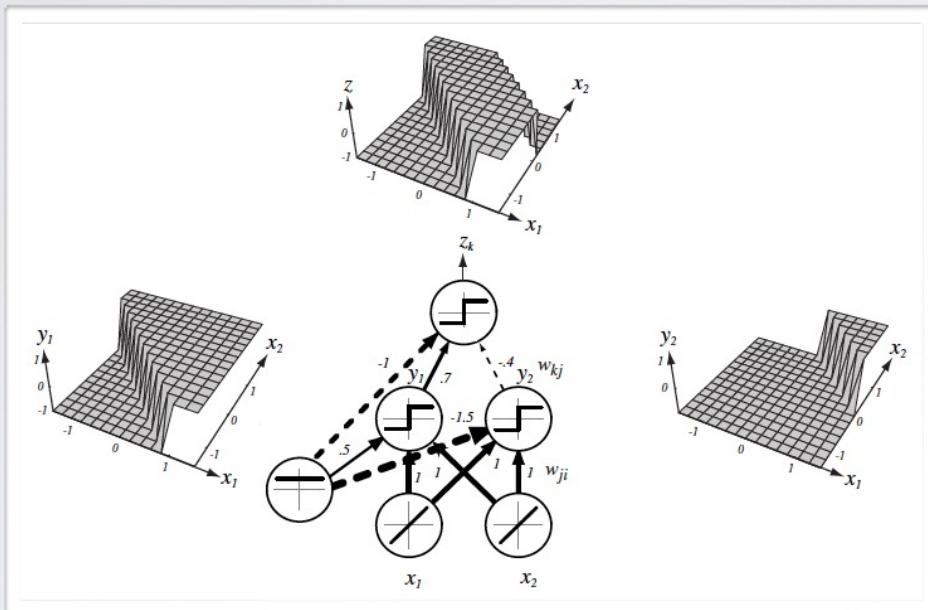
- output layer activation ($k=L+1$):

$$\mathbf{h}^{(L+1)}(\mathbf{x}) = \mathbf{o}(\mathbf{a}^{(L+1)}(\mathbf{x})) = \mathbf{f}(\mathbf{x})$$



CAPACITY OF NEURAL NETWORK

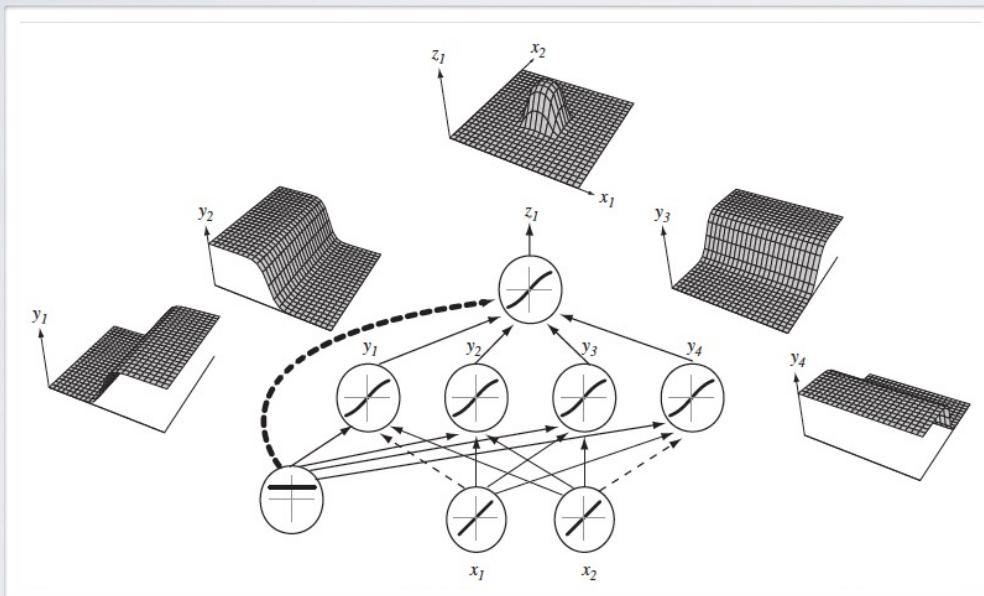
Topics: single hidden layer neural network



(from Pascal Vincent's slides)

CAPACITY OF NEURAL NETWORK

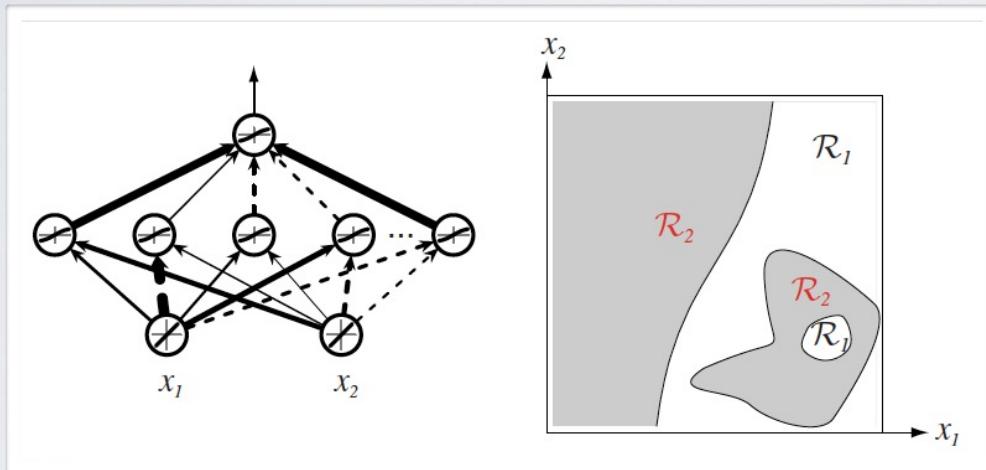
Topics: single hidden layer neural network



(from Pascal Vincent's slides)

CAPACITY OF NEURAL NETWORK

Topics: single hidden layer neural network



(from Pascal Vincent's slides)

How to train a neural network?

Topics: multilayer neural network

- Could have L hidden layers:

- layer input activation for $k > 0$ ($\mathbf{h}^{(0)}(\mathbf{x}) = \mathbf{x}$)

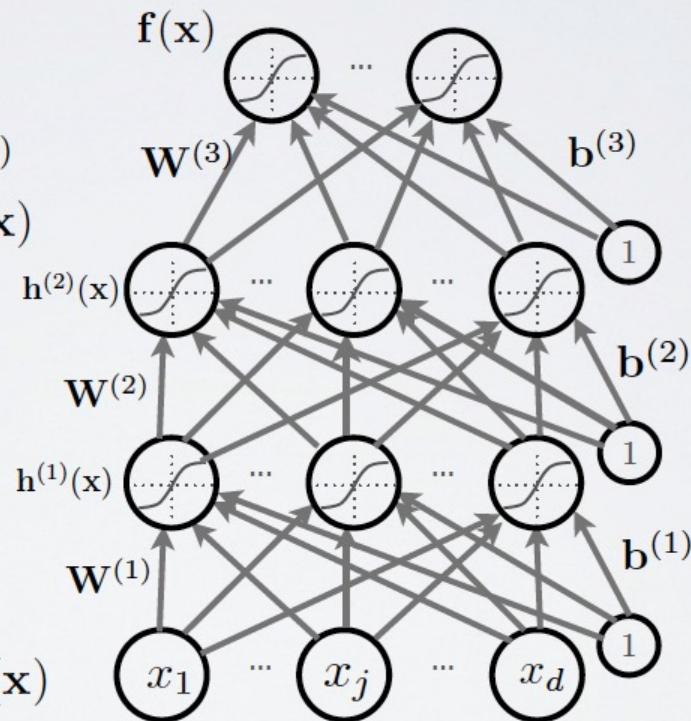
$$\mathbf{a}^{(k)}(\mathbf{x}) = \mathbf{b}^{(k)} + \mathbf{W}^{(k)}\mathbf{h}^{(k-1)}(\mathbf{x})$$

- hidden layer activation (k from 1 to L):

$$\mathbf{h}^{(k)}(\mathbf{x}) = \mathbf{g}(\mathbf{a}^{(k)}(\mathbf{x}))$$

- output layer activation ($k=L+1$):

$$\mathbf{h}^{(L+1)}(\mathbf{x}) = \mathbf{o}(\mathbf{a}^{(L+1)}(\mathbf{x})) = \mathbf{f}(\mathbf{x})$$



Empirical Risk Minimization and Regularization

Topics: empirical risk minimization, regularization

- Empirical risk minimization
 - framework to design learning algorithms

$$\arg \min_{\boldsymbol{\theta}} \frac{1}{T} \sum_t l(f(\mathbf{x}^{(t)}; \boldsymbol{\theta}), y^{(t)}) + \lambda \Omega(\boldsymbol{\theta})$$

- $l(f(\mathbf{x}^{(t)}; \boldsymbol{\theta}), y^{(t)})$ is a loss function
- $\Omega(\boldsymbol{\theta})$ is a regularizer (penalizes certain values of $\boldsymbol{\theta}$)
- Learning is cast as optimization
 - ideally, we'd optimize classification error, but it's not smooth
 - loss function is a surrogate for what we truly should optimize (e.g. upper bound)

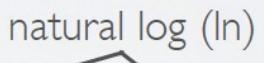
Loss Function

Topics: loss function for classification

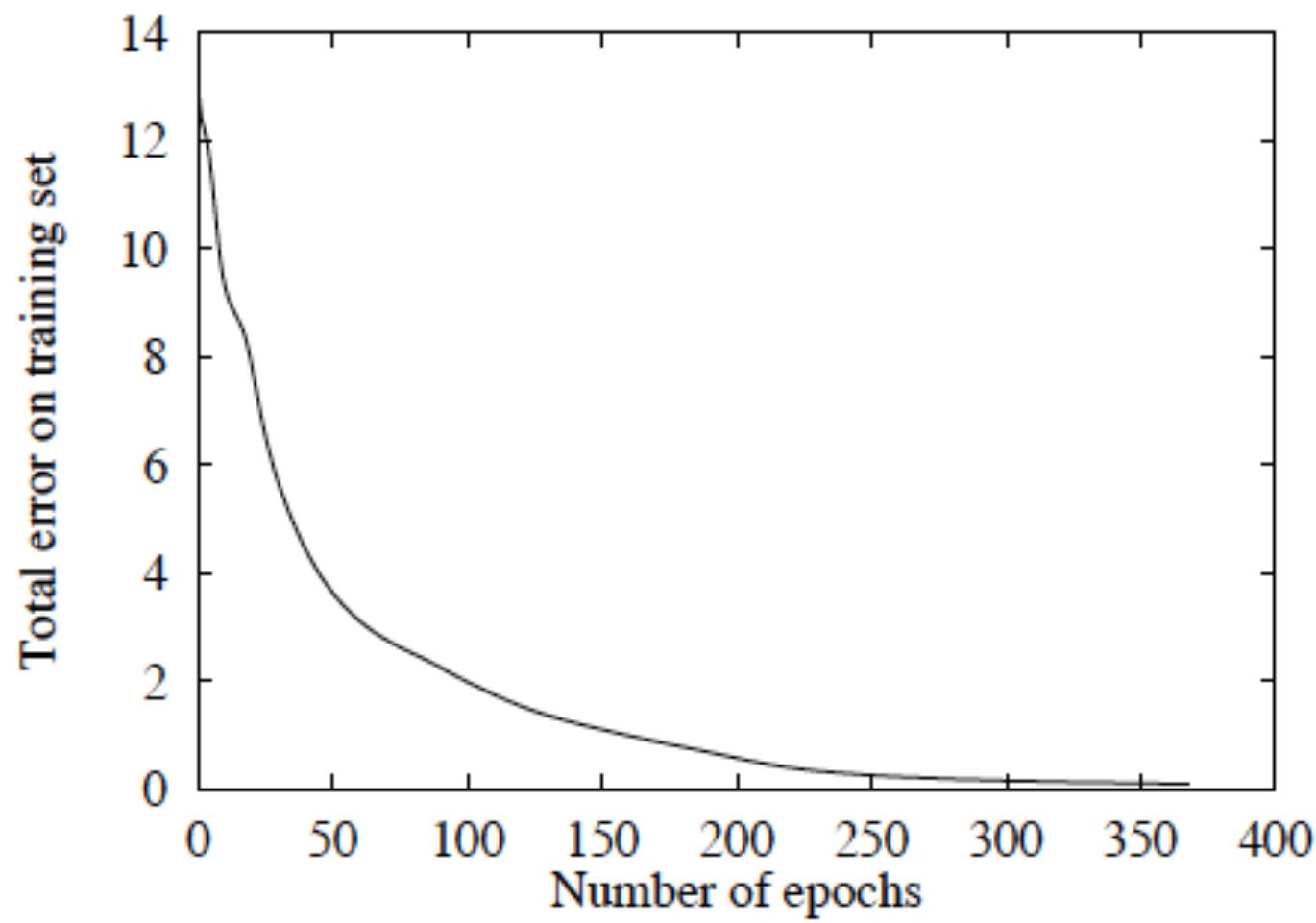
- Neural network estimates $f(\mathbf{x})_c = p(y = c|\mathbf{x})$
 - we could maximize the probabilities of $y^{(t)}$ given $\mathbf{x}^{(t)}$ in the training set
- To frame as minimization, we minimize the negative log-likelihood

$$l(\mathbf{f}(\mathbf{x}), y) = - \sum_c 1_{(y=c)} \log f(\mathbf{x})_c = - \log f(\mathbf{x})_y$$

natural log (ln)



```
graph TD; A["natural log (ln)"] --> B["- log f(x)_y"]
```



[figure from Greg Mori's slides]

Regularization

Topics: L2 regularization

$$\Omega(\boldsymbol{\theta}) = \sum_k \sum_i \sum_j \left(W_{i,j}^{(k)} \right)^2 = \sum_k \|\mathbf{W}^{(k)}\|_F^2$$

It's call squared Frobenius norm when \mathbf{W} is a matrix

Empirical Risk Minimization

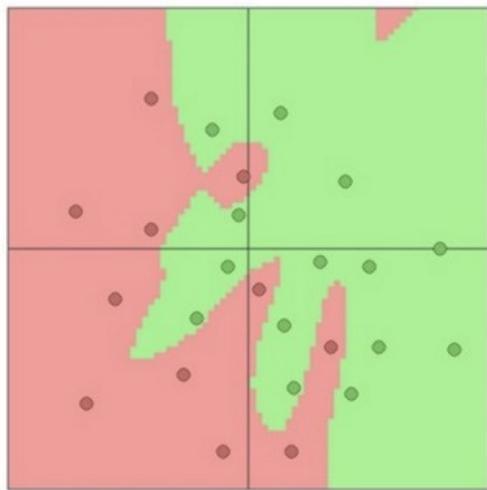
Topics: empirical risk minimization, regularization

- Empirical risk minimization
 - framework to design learning algorithms

$$\arg \min_{\boldsymbol{\theta}} \frac{1}{T} \sum_t l(f(\mathbf{x}^{(t)}; \boldsymbol{\theta}), y^{(t)}) + \lambda \Omega(\boldsymbol{\theta})$$

- $l(f(\mathbf{x}^{(t)}; \boldsymbol{\theta}), y^{(t)})$ is a loss function
- $\Omega(\boldsymbol{\theta})$ is a regularizer (penalizes certain values of $\boldsymbol{\theta}$)

$\lambda = 0.001$



$\lambda = 0.01$



$\lambda = 0.1$



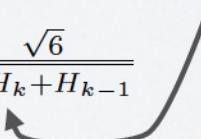
[<http://cs231n.github.io/neural-networks-1/>]

INITIALIZATION

Topics: initialization

- For biases
 - ▶ initialize all to 0
- For weights
 - ▶ Can't initialize weights to 0 with tanh activation
 - we can show that all gradients would then be 0 (saddle point)
 - ▶ Can't initialize all weights to the same value
 - we can show that all hidden units in a layer will always behave the same
 - need to break symmetry
 - ▶ Recipe: sample $\mathbf{W}_{i,j}^{(k)}$ from $U[-b, b]$ where $b = \frac{\sqrt{6}}{\sqrt{H_k + H_{k-1}}}$
 - the idea is to sample around 0 but break symmetry
 - other values of b could work well (not an exact science) (see Glorot & Bengio, 2010)

size of $\mathbf{h}^{(k)}(\mathbf{x})$



Learning the Parameters (weights and bias)

- Backpropagation (BP) algorithm (not required for this course)
- Further reading on BP:
 - <https://towardsdatascience.com/understanding-backpropagation-algorithm-7bb3aa2f95fd>