EECS 487: Introduction to Natural Language Processing

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Today's Outline

- Probabilistic language model and n-grams
- Estimating n-gram probabilities
- Language model evaluation and perplexity
- Generalization and zeros
- Smoothing: add-one
- Interpolation, backoff, and web-scale LMs
- Smoothing: Kneser-Ney Smoothing

[Modified from slides by Dan Jurafsky and Joyce Chai]

Probabilistic Language Models

- Assign a probability to a sentence
  - Machine Translation:
    - P(**high** winds tonight) > P(**large** winds tonight)
  - Spell Correction
    - The office is about fifteen **minuets** from my house
      - P(about fifteen **minutes** from) > P(about fifteen **minuets** from)
  - Speech Recognition
    - P(I saw a van) >> P(eyes awe of an)
  - Text Generation in general:
    - Summarization, question-answering, dialogue systems …

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words:

  $P(W) = P(w_1,w_2,w_3,w_4,w_5 \ldots w_n)$

- Related task: probability of an upcoming word:

  $P(w_5|w_1,w_2,w_3,w_4)$

- A model that computes either of these:

  $P(W)$ or $P(w_n|w_1,w_2 \ldots w_{n-1})$ is called a **language model**.

- Better: **the grammar**
- But **language model (**or **LM)** is standard

How to compute P(W)

- How to compute this joint probability:
    - P(its, water, is, so, transparent, that)

How to compute P(W)

- How to compute this joint probability:
  - P(its, water, is, so, transparent, that)
- Intuition: let’s rely on the Chain Rule of Probability

Quick Review: Probability

- Recall the definition of conditional probabilities

  p(B|A) = P(A,B)/P(A) Rewriting: **P(A,B) = P(A)P(B|A)**

- More variables:

  P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)

- The Chain Rule in General

$$P(x_1,x_2,x_3,\ldots,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)\ldots P(x_n|x_1,\ldots,x_{n-1})$$

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P("its water is so transparent") =

P(its) × P(water|its) × P(is|its water)

× P(so|its water is) × P(transparent|its water is so)

How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

How to estimate these probabilities

- Could we just count and divide?

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

- No! Too many possible sentences!
- We'll never see enough data for estimating these

Markov Assumption

- Simplifying assumption:

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

- Or maybe

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

Markov Assumption

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

- In other words, we approximate each component in the product

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

```
fifth, an, of, futures, the, an, incorporated, a, a,
the, inflation, most, dollars, quarter, in, is, mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
```

Bigram model

Condition on the previous word:

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-1})$$

texaco, rose, one, in, this, issue, is, pursuing, growth, in, a, boiler, house, said, mr., gurria, mexico, 's, motion, control, proposal, without, permission, from, five, hundred, fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november

N-gram models

- We can extend to trigrams, 4-grams, 5-grams

N-gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
  - because language has **long-distance dependencies**:

    "The computer(s) which I had just put into the machine room on the fifth floor is (are) crashing."

- But we can often get away with N-gram models

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Estimating bigram probabilities

- The Maximum Likelihood Estimate for bigram probability

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

$P(\texttt{I} \mid \texttt{<s>}) = \frac{2}{3} = .67$ $P(\texttt{Sam} \mid \texttt{<s>}) = \frac{1}{3} = .33$ $P(\texttt{am} \mid \texttt{I}) = \frac{2}{3} = .67$

$P(\texttt{</s>} \mid \texttt{Sam}) = \frac{1}{2} = 0.5$ $P(\texttt{Sam} \mid \texttt{am}) = \frac{1}{2} = .5$ $P(\texttt{do} \mid \texttt{I}) = \frac{1}{3} = .33$

More examples: Berkeley Restaurant Project sentences

- can you tell me about any good cantonese restaurants close by
- mid priced thai food is what i’m looking for
- tell me about chez panisse
- can you give me a listing of the kinds of food that are available
- i’m looking for a good place to eat breakfast
- when is caffe venezia open during the day

Raw bigram counts

- Out of 9222 sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw bigram probabilities

- Normalize by unigrams:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

- Result:

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

P(<s> I want english food </s>) =

P(I|<s>)

× P(want|I)

× P(english|want)

× P(food|english)

× P(</s>|food)

= .000031

Knowledge

- P(english|want) = .0011
- P(chinese|want) = .0065
- P(to|want) = .66
- P(eat | to) = .28
- P(food | to) = 0
- P(want | spend) = 0
- P (i | <s>) = .25

Practical Issues

- We do everything in log space
  - Avoid underflow
  - (also adding is faster than multiplying)

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Language Modeling Toolkits

- SRILM
  - http://www.speech.sri.com/projects/srilm/

- Neural language models (will be discussed later)
  - Word2vec
  - RNN language model
  - BERT
  - GPT

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Evaluation: How good is our model?

- Does our language model prefer good sentences to bad ones?
  - Assign higher probability to “real” or “frequently observed” sentences
    - Than “ungrammatical” or “rarely observed” sentences?

Evaluation: How good is our model?

- Does our language model prefer good sentences to bad ones?
  - Assign higher probability to “real” or “frequently observed” sentences
    - Than “ungrammatical” or “rarely observed” sentences?
- We train parameters of our model on a **training set**.
- We test the model’s performance on data we haven’t seen.
  - A **test set** is an unseen dataset that is different from our training set, totally unused.
  - An **evaluation metric** tells us how well our model does on the test set.

Training on the test set

- We can't allow test sentences into the training set
- We will assign it an artificially high probability when we set it in the test set
- "Training on the test set"
- Bad science!

Extrinsic evaluation of N-gram models

- Best evaluation for comparing language models A and B
  - Put each model in a task
    - spelling corrector, speech recognizer, MT system
  - Run the task, get an accuracy for A and for B
    - How many misspelled words corrected properly
    - How many words translated correctly
  - Compare accuracy for A and B

Difficulty of extrinsic evaluation of N-gram models

- Extrinsic evaluation
  - Time-consuming; can take days or weeks
- So
  - Sometimes use **intrinsic** evaluation: **perplexity**

Difficulty of extrinsic evaluation of N-gram models

- Extrinsic evaluation
  - Time-consuming; can take days or weeks
- So
  - Sometimes use **intrinsic** evaluation: **perplexity**
  - Bad approximation
    - unless the test data looks **just** like the training data
    - So **generally only useful in pilot experiments**
  - But is helpful to think about.

Intuition of Perplexity

- The Shannon Game:
  - How well can we predict the next word?

    I always order pizza with cheese and ____

    The 33^{rd} President of the US was ____

    I saw a ____

  - Unigrams are terrible at this game. (Why?)
- A better model of a text
  - is one which assigns a higher probability to the word that actually occurs

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mushrooms 0.1

pepperoni 0.1

anchovies 0.01

….

fried rice 0.0001

….

and 1e-100

Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest P(sentence)

Perplexity is the inverse probability of the test set, normalized by the number of words:

$$
\begin{aligned}
PP(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\
&= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}}
\end{aligned}
$$

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$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}}$$

Chain rule:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_1 \ldots w_{i-1})}}$$

For bigrams:

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For bigrams:

$$\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability

Perplexity as branching factor

- Let's suppose a sentence consisting of random digits
- What is the perplexity of this sentence according to a model that assign P=1/10 to each digit?

Perplexity as branching factor

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- What is the perplexity of this sentence according to a model that assign P=1/10 to each digit?

$$
\begin{aligned}
\mathrm{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\
&= (\frac{1}{10}^N)^{-\frac{1}{N}} \\
&= \frac{1}{10}^{-1} \\
&= 10
\end{aligned}
$$

Lower perplexity = better model

- Training 38 million words, test 1.5 million words, WSJ

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

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The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
  - In real life, it often doesn't
  - We need to train robust models that generalize!

The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
  - In real life, it often doesn't
  - We need to train robust models that generalize!
  - One kind of generalization: Zeros!
    - Things that don't ever occur in the training set
      - But occur in the test set

Zeros

In training set, we see
… denied the allegations
… denied the reports
… denied the claims
… denied the request

P("offer" | denied the) = 0

But in test set,
… denied the offer
… denied the loan

Zero probability bigrams

- Bigrams with zero probability
  - mean that we will assign 0 probability to the test set!
- And hence we cannot compute perplexity (can't divide by 0)!

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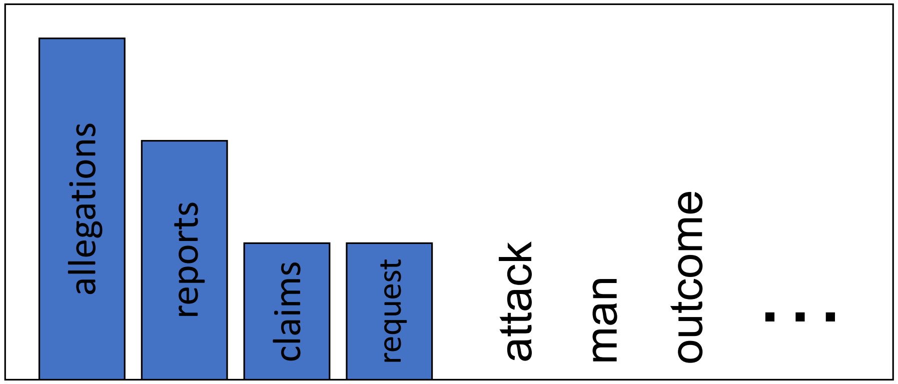
The intuition of smoothing (from Dan Klein)

- When we have sparse statistics:

  P(w | denied the)
  3 allegations
  2 reports
  1 claims
  1 request

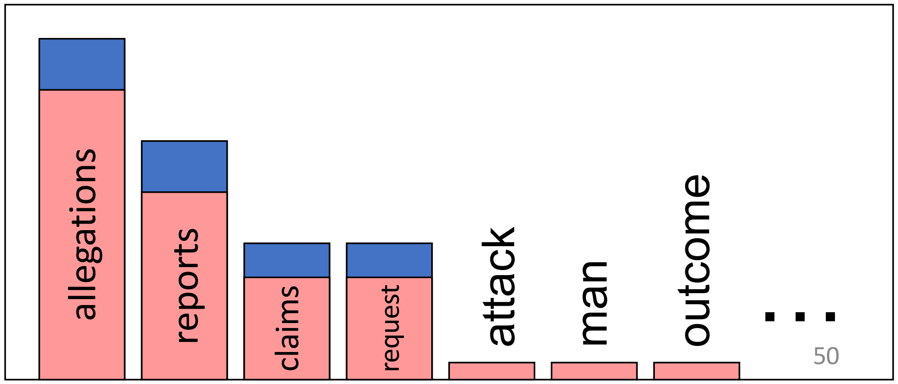
  7 total



- Steal probability mass to generalize better

  P(w | denied the)
  2.5 allegations
  1.5 reports
  0.5 claims
  0.5 request
  2 other

  7 total



Add-one estimation

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts! (*Instead of taking away counts*)

- MLE estimate: $$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate: $$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

V is the size of vocabulary

Add-one estimation

- Also called Laplace smoothing
- Pretend we saw each word one more time than we did
- Just add one to all the counts! (*Instead of taking away counts*)
- MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

Why add V?

Berkeley Restaurant Corpus: Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)+1}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Add-1 estimation is a blunt instrument

- So add-1 isn't used for N-grams:
  - We'll see better methods
  - (nowadays, neural LM becomes popular, will discuss later)
- But add-1 is used to smooth other NLP models
  - For text classification (coming soon!)
  - In domains where the number of zeros isn't so huge.
- Add-1 can be extended to add-k (k can be any positive real number, sometimes also called add-alpha)

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Backoff and Interpolation

- Sometimes it helps to use less context
    - Condition on less context for contexts you haven't learned much about
- Backoff:
    - use trigram if you have good evidence (e.g. the trigram is observed in training)
    - otherwise bigram
    - otherwise unigram
- Interpolation:
    - mix unigram, bigram, trigram

- In general, interpolation works better

Backoff

$$P_{backoff}(w_i \mid w_{i-2}w_{i-1}) = \begin{cases} P_{ML}(w_i \mid w_{i-2}w_{i-1}), & if \;\; C(w_{i-2}w_{i-1}w_i) > 0 \\ P_{ML}(w_i \mid w_{i-1}), & if \;\; C(w_{i-2}w_{i-1}w_i) = 0 \\ & and \;\; C(w_{i-1}w_i) > 0 \\ P_{ML}(w_i), & otherwise \end{cases}$$

Backoff

$$P_{backoff}(w_i \mid w_{i-2}w_{i-1}) = \begin{cases} P_{ML}(w_i \mid w_{i-2}w_{i-1}), & if \;\; C(w_{i-2}w_{i-1}w_i) > 0 \\ P_{ML}(w_i \mid w_{i-1}), & if \;\; C(w_{i-2}w_{i-1}w_i) = 0 \\ & and \;\; C(w_{i-1}w_i) > 0 \\ P_{ML}(w_i), & otherwise \end{cases}$$

- However, this doesn't make a true probability distribution

Katz Backoff for Trigram

$$P_{Katz}(w_i \mid w_{i-2}w_{i-1}) = \begin{cases} P^*(w_i \mid w_{i-2}w_{i-1}), \text{ if } C(w_{i-2}w_{i-1}w_i) > 0 \\ \alpha(w_{i-2}w_{i-1})P_{Katz}(w_i \mid w_{i-1}), \text{ otherwise} \end{cases}$$

$$P_{Katz}(w_i \mid w_{i-1}) = \begin{cases} P^*(w_i \mid w_{i-1}), \text{ if } C(w_{i-1}w_i) > 0 \\ \alpha(w_{i-1})P(w_i), \text{ otherwise} \end{cases}$$

Katz Backoff for Trigram

- (After class practice)

$$\alpha(w_{i-2}w_{i-1}) = \frac{1 - \sum_{w_i: C(w_{i-2}, w_{i-1}, w_i) > 0} P^*(w_i \mid w_{i-2}, w_{i-1})}{\sum_{w_i: C(w_{i-2}, w_{i-1}, w_i) = 0} P_{katz}(w_i \mid w_{i-1})}$$

$$\alpha(w_{i-1}) = \frac{1 - \sum_{w_i: C(w_{i-1}, w_i) > 0} P^*(w_i \mid w_{i-1})}{\sum_{w_i: C(w_{i-1}, w_i) = 0} P(w_i)}$$

Linear Interpolation

- Simple interpolation

$$
\begin{aligned}
P_{LI}(w_i \mid w_{i-2}, w_{i-1}) = {} & \lambda_1 \times P_{ML}(w_i \mid w_{i-2}, w_{i-1}) \\
& + \lambda_2 \times P_{ML}(w_i \mid w_{i-1}) \\
& + \lambda_3 \times P_{ML}(w_i)
\end{aligned}
$$

$\lambda_1 + \lambda_2 + \lambda_3 = 1$, and $\lambda_i >= 0$ for all i

Linear Interpolation

- Why does this estimation correctly define a distribution?

$$
\begin{aligned}
&\sum\nolimits_{w_i \in V} P_{LI}(w_i \mid w_{i-2}, w_{i-1}) \\
&= \sum\nolimits_{w_i \in V} [\lambda_1 \times P_{ML}(w_i \mid w_{i-2}, w_{i-1}) + \lambda_2 \times P_{ML}(w_i \mid w_{i-1}) + \lambda_3 \times P_{ML}(w_i)] \\
&= \lambda_1 \sum\nolimits_{w_i \in V} P_{ML}(w_i \mid w_{i-2}, w_{i-1}) + \lambda_2 \sum\nolimits_{w_i \in V} P_{ML}(w_i \mid w_{i-1}) + \lambda_3 \sum\nolimits_{w_i \in V} P_{ML}(w_i) \\
&= \lambda_1 + \lambda_2 + \lambda_3 \\
&= 1
\end{aligned}
$$

How to set the lambdas?

- Use a **held-out** corpus

Training Data | Held-Out Data | Test Data

- Choose λs to maximize the probability of held-out data:
  - Fix the N-gram probabilities (on the training data)
  - Then search for λs that give largest probability to held-out set:

$$\log P(w_1 \dots w_n \mid M(\lambda_1 \dots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \dots \lambda_k)}(w_i \mid w_{i-1})$$

An assignment of λs

A Common Method – Grid Search

- Take a list of possible values, e.g. [0.1, 0.2, … ,0.9]
- Try all combinations

Unknown words: Open versus closed vocabulary tasks

- If we know all the words in advance
  - Vocabulary V is fixed
  - Closed vocabulary task
- Often we don't know this
  - **Out Of Vocabulary** = OOV words
  - Open vocabulary task

Unknown words: Open versus closed vocabulary tasks

- If we know all the words in advanced
  - Vocabulary V is fixed
  - Closed vocabulary task
- Often we don't know this
  - **Out Of Vocabulary** = OOV words
  - Open vocabulary task
- Instead: create an unknown word token <UNK>
  - Training of <UNK> probabilities
    - Create a fixed lexicon L of size V (e.g. selecting high frequency words)
    - At text normalization phase, any training word not in L changed to <UNK>
    - Now we train its probabilities like a normal word
  - At test time
    - Use UNK probabilities for any word not in training

Smoothing for Web-scale N-grams

- "Stupid backoff" (Brants *et al*. 2007)
- No discounting, just use relative frequencies

$$S(w_i \mid w_{i-k+1}^{i-1}) = \begin{cases} \dfrac{\text{count}(w_{i-k+1}^{i})}{\text{count}(w_{i-k+1}^{i-1})} & \text{if } \text{count}(w_{i-k+1}^{i}) > 0 \\ 0.4S(w_i \mid w_{i-k+2}^{i-1}) & \text{otherwise} \end{cases}$$

$$S(w_i) = \frac{\text{count}(w_i)}{N}$$

Until unigram probability

Today's Outline

- Probabilistic language model and n-grams
- Estimating n-gram probabilities
- Language model evaluation and perplexity
- Generalization and zeros
- Smoothing: add-one
- Interpolation, backoff, and web-scale LMs
- Smoothing: Kneser-Ney Smoothing

Absolute discounting: just subtract a little from each count

- Suppose we wanted to subtract a little from a count of 4 to save probability mass for the zeros
- How much to subtract ?

- Church and Gale (1991)'s clever idea
- Divide up 22 million words of AP Newswire
  - Training and held-out set
  - for each bigram in the training set
  - see the actual count in the held-out set!

Bigram count in training	Bigram count in heldout set
1	0.448
2	1.25
3	2.24
4	3.23
5	4.21
6	5.23
7	6.21
8	7.21
9	8.26

- It sure looks like $c^* = (c - .75)$

Absolute Discounting Interpolation

- Save ourselves some time and just subtract 0.75 (or some d)!

discounted bigram

Interpolation weight

$$P_{\text{AbsoluteDiscounting}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})} + \lambda(w_{i-1})P(w_i)$$

unigram

- But should we really just use the regular unigram P(w)?

Kneser-Ney Smoothing I

- Better estimate for probabilities of lower-order unigrams!
  - Shannon game: *I can't see without my reading___________*? *glasses*
  - "Francisco" is more common than "glasses" *Francisco*
  - … but "Francisco" always follows "San"
- The unigram is useful exactly when we haven't seen this bigram!
- Instead of P(w): "How likely is w"
- $P_{continuation}(w)$: "How likely is w to appear as a **novel** continuation?
  - For each word, count the number of **unique** bigrams it completes
  - Every unique bigram was a novel continuation the first time it was seen

$$P_{CONTINUATION}(w) \propto \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

Kneser-Ney Smoothing II

- How many times does w appear as a novel continuation (unique bigrams):

$$P_{CONTINUATION}(w) \propto \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

- Normalized by the total number of word bigram types

$$\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|$$

All unique bigrams in the corpus

$$P_{CONTINUATION}(w) = \frac{\left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|}{\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|}$$

Kneser-Ney Smoothing III

- Alternative metaphor: The number of unique words seen to precede w

$$|\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- normalized by the number of (unique) words preceding all words:

$$P_{CONTINUATION}(w) = \frac{\left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|}{\sum_{w'} \left|\{w'_{i-1} : c(w'_{i-1}, w') > 0\}\right|}$$

- A frequent word (Francisco) occurring in only one context (San) will have a low continuation probability

Kneser-Ney Smoothing IV (after class practice)

$$P_{KN}(w_i \mid w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1}) P_{CONTINUATION}(w_i)$$

λ is a normalizing constant; the probability mass we've discounted

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} \left| \{ w : c(w_{i-1}, w) > 0 \} \right|$$

the normalized discount

The number of word types that can follow w_{i-1}
= # of word types we discounted
= # of times we applied normalized discount