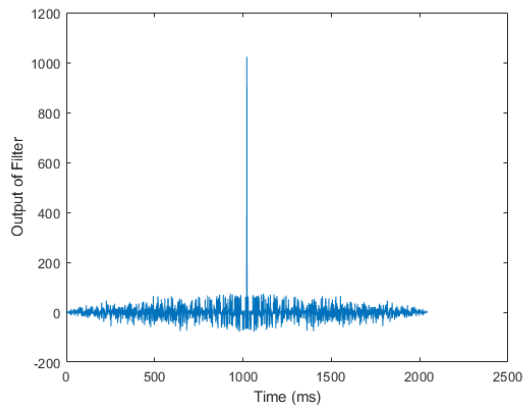
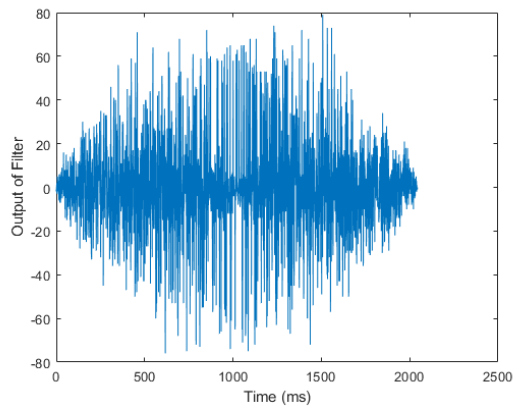


1. a) first 20 bits are 1111001000110001000. In the first 1023 bits,
#0s is $2^9 - 1 = 511$ and #1s is $2^9 = 512$.

b)



c)



max : 79

min : -76

```
reg_1 = ones(1,10);
reg_2 = ones(1,10);
output = [1];

%% a)
for i=1:19
    reg_1 = shift_1(reg_1);
    reg_2 = shift_2(reg_2);
    output = [output, mod(reg_1(end)+reg_2(4)+reg_2(8), 2)];
end

output

%% b)
output = [1];

for i=1:1022
    reg_1 = shift_1(reg_1);
    reg_2 = shift_2(reg_2);
    output = [output, mod(reg_1(end)+reg_2(4)+reg_2(8), 2)];
end

output(output == 1) = -1;
output(output == 0) = 1;

filter = fliplr(output);
y = conv(output, filter);
y = y ./ max(y) .* 1023;
t = 0 : length(y) - 1;

plot(t,y);
xlabel("Time (ms)");
ylabel("Output of Filter");

%% c)
reg_1 = ones(1,10);
reg_2 = ones(1,10);
output = [1];

for i=1:1022
    reg_1 = shift_1(reg_1);
    reg_2 = shift_2(reg_2);
    output = [output, mod(reg_1(end)+reg_2(2)+reg_2(6), 2)];
end

output(output == 1) = -1;
output(output == 0) = 1;

y_c = conv(output, filter);
y_c = y_c ./ max(y) .* 1023;
```

```
figure;
plot(t,y_c);
xlabel("Time (ms)");
ylabel("Output of Filter");

max(y_c)
min(y_c)

%% helper functions
function output_1 = shift_1(x)
    b = mod(x(3)+x(end), 2);
    x = x(1:9);
    x = [b,x];
    output_1 = x;
end
function output_2 = shift_2(x)
    b = mod(x(2)+x(3)+x(6)+x(8)+x(9)+x(10), 2);
    x = x(1:9);
    x = [b,x];
    output_2 = x;
end
```

2. Bandwidth = 7.5×10^9 Hz $f_c = 6.85 \times 10^9$ Hz

down conversion to $f_c = 0$, baseband = $\pm 3.75 \times 10^9$ Hz

$$S(f) = \frac{1}{T} |X(f)|^2$$

$$X(f) = \begin{cases} \sqrt{T}, & 0 \leq |f| \leq \frac{1-\alpha}{2T} \\ \sqrt{\frac{T}{2} [1 - \sin(\pi T(|f| - \frac{1}{2T})/\alpha)]}, & \frac{1-\alpha}{2T} \leq |f| \leq \frac{1+\alpha}{2T} \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore S(f) = \begin{cases} 1, & 0 \leq |f| \leq \frac{1-\alpha}{2T} \\ \frac{1}{2} [1 - \sin(\pi T(|f| - \frac{1}{2T})/\alpha)], & \frac{1-\alpha}{2T} \leq |f| \leq \frac{1+\alpha}{2T} \\ 0, & \text{otherwise} \end{cases}$$

$$S(f) \text{ at } 3.75 \times 10^9 \text{ Hz} = \frac{1}{2} [1 - \sin(\pi T(|f| - \frac{1}{2T})/\alpha)] = 0.01 \times (S(f) \text{ at } 0 = 1)$$

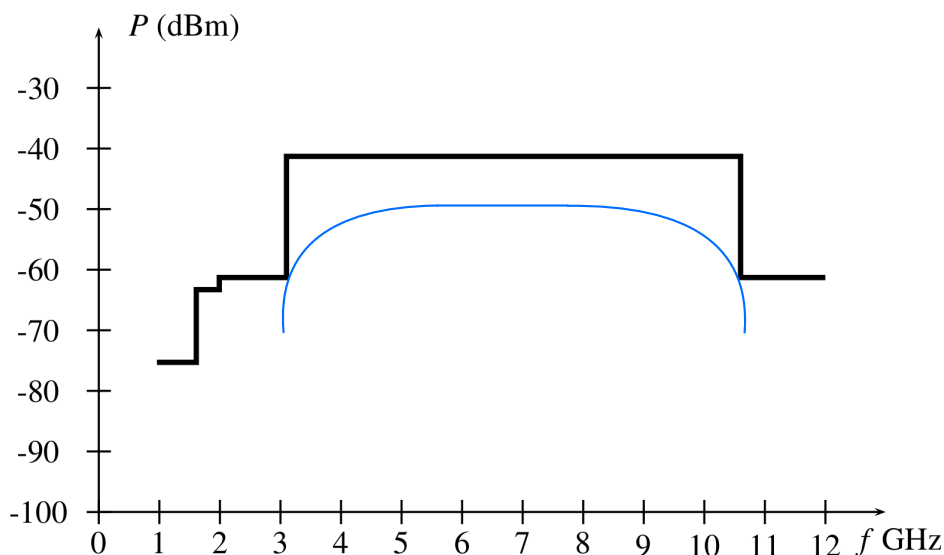
$$\therefore 1 - \sin(\pi T(|f| - \frac{1}{2T})/\alpha) = 0.02$$

$$\sin(\pi T(|f| - \frac{1}{2T})/\alpha) = 0.98 \text{ where } \alpha = 0.35$$

$$\sin(\pi T(3.75 \times 10^9 - \frac{1}{2T})/0.35) = 0.98$$

$$3.75 \times 10^9 T = \sin^{-1}(0.98) \cdot 0.35 / \pi + 0.5$$

$$T = 0.174 \text{ ns}$$



```
3. x2t_triple = [x2t, x2t, x2t];  
   filter = conj(fliplr(x1t(1 : 512)));  
   output = conv(x2t_triple, filter);  
   t_plot = 0 : dt : ((length(output) - 1) * dt);  
   figure;  
   plot(t_plot, output);  
   xlabel('Time(s)');  
   ylabel('Output Amplitude');  
  
   [a, b] = max(abs(output));  
   disp('Delay from 0 = ' + string((b - 1) * dt - 1) * 1000) + ' ms');  
   disp('Delay from dt = ' + string((b * dt - 1) * 1000) + ' ms');
```

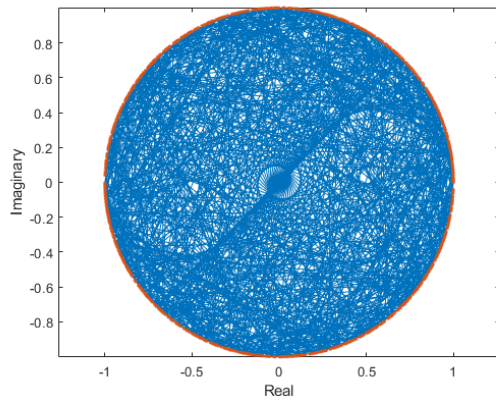
} delay estimate

$$\begin{aligned}
 4. \quad a) \quad X_{n-k-N} &= e^{j\pi(n-k-N)(n-k-N+1)/N} \\
 &= e^{j\pi(n-k)^2/N} \cancel{e^{j\pi(n-k)(-N)/N}} e^{j\pi(n-k)/N} \cancel{e^{j\pi(-N)(n-k)/N}} \frac{e^{j\pi N^2/N} e^{j\pi N/N}}{e^{j\pi N^2/N} e^{j\pi N/N}} \\
 &= e^{j\pi(n-k)^2/N} e^{j\pi(n-k)/N} \quad \text{if } N \text{ is odd} \\
 &= e^{j\pi(n-k+1)(n-k)/N} = e^{j\pi(N-1)} = e^{\text{even}} = 1 \\
 &= X_{n-k}
 \end{aligned}$$

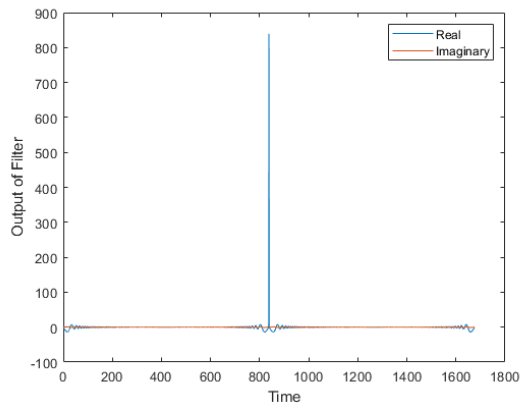
$$\begin{aligned}
 b) \quad X_n X_{n-k}^* &= e^{j\pi n(n+1)/N} e^{-j\pi(n-k)(n-k+1)/N} \\
 &= e^{j\pi(n(n+1) - (n-k)(n-k+1))/N} \\
 &= e^{j\pi(n^2 + n - (n^2 - kn + n - kn + k^2 - k))/N} \\
 &= e^{j\pi(k^2 - k)/N} \\
 &= e^{-j\pi(k^2 - k)/N} e^{j\pi 2kn/N}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad \theta_x(k) &= \sum_{n=0}^{N-1} X_n X_{n-k}^* = e^{-j\pi(k^2 - k)/N} \sum_{n=0}^{N-1} (e^{j\pi 2kn/N})^n \\
 \text{if } k=0, \quad \theta_x(k) &= \sum_{n=0}^{N-1} 1 = N \\
 \text{if } k \neq 0, \quad \theta_x(k) &= e^{-j\pi(k^2 - k)/N} \frac{e^{j\pi 2kN/N} - 1}{e^{j\pi 2k/N} - 1} = 0
 \end{aligned}$$

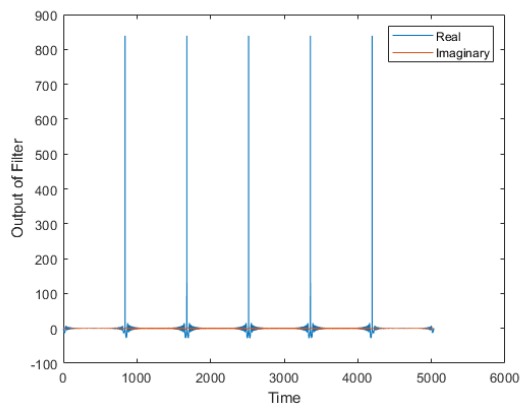
5. a)



b)



c)



```
M = 1;
N = 839;
n = 0 : 1 : (N - 1);
x = exp(1j .* M .* pi .* n.^2 ./ N);

%% a)
figure;
plot(real(x), imag(x), real(x), imag(x), '.');
xlabel('Real');
ylabel('Imaginary');
axis equal;

%% b)
h = conj(fliplr(x));
output = conv(x, h);
t = 0 : 1 : (length(output) - 1);

figure;
plot(t, real(output), t, imag(output));
xlabel('Time');
ylabel('Output of Filter');
legend('Real', 'Imaginary');

%% c)
u = [0,x,x,x,x,x,0];
output_c = conv(u, h);

figure;
t_c = 0 : 1 : (length(output_c) - 1);
plot(t_c, real(output_c), t_c, imag(output_c));
xlabel('Time');
ylabel('Output of Filter');
legend('Real', 'Imaginary');
```