P1:

① Baseband Signal:

$$\tilde{x}(t) = x_I(t) + j x_Q(t) \qquad = S_b(t)$$

The frequency content of $x_I(t)$ and $x_Q(t)$ is

$$X_I(f) \text{ and } X_Q(f)$$

Energy of Baseband:

$$E_L = \int |\tilde{x}(t)|^2 dx$$

② Passband Signals:

$$x(t) = x_I(t) \sqrt{2} \cos(2\pi f_c t) - x_Q(t) \sqrt{2} \sin(2\pi f_c t)$$

Energy: $E_p = \int |x(t)|^2 dt$

proof: $E_L = E_p$

We can write $x(t)$ in this way:

$$x(t) = R\{\tilde{x}(t) \sqrt{2} e^{j 2\pi f_c t}\}$$
$$= R\{(x_I(t) + j x_Q(t)) \sqrt{2} e^{j 2\pi f_c t}\}$$

We can write $x(t)$ in this way:

$$x(t) = x_e(t) \cos(2\pi f_c t + \theta(t))$$

where $\quad x_e(t) = \sqrt{2(x_I^2(t) + x_Q^2(t))}$

$$\theta(t) = \tan^{-1}\left(\frac{x_Q(t)}{x_I(t)}\right)$$

Firstly, we have $\quad E_L = \int |\tilde{x}(t)|^2 dt$

$$= \int |x_I(t) + j x_Q(t)|^2 dt$$
$$= \int [x_I^2(t) + x_Q^2(t)] dt$$
$$= \int x_I^2(t) dt + \int x_Q^2(t) dt$$

For $x(t)$:

$$\| x(t)\|^2 = x_I^2(t) \, 2\cos^2(2\pi f_c t) + x_Q^2(t) \, 2\sin^2(2\pi f_c t) - 4x_I(a)x_I(t)\sin(4\pi f_c t)$$

$$\Rightarrow = x_I^2(t) + x_Q^2(t) + x_I^2(t)\cos(4\pi f_c t) - x_Q^2(t)\cos(4\pi f_c t) - 2x_Q(t)x_I(t)\cdot\sin(4\pi f_c t)$$

$\therefore E_p = \int |x(t)|^2 \, dt$

$$= \int \left(x_I^2(t) + x_Q^2(t) + x_I^2(t)\cos(4\pi f_c t) - x_Q^2(t)\cos(4\pi f_c t) - 2x_Q(t)x_I(t)\sin(4\pi f_c t) \right)$$

$$= \int x_I^2(t)\,dt + \int x_Q^2(t) + \int x_I^2(t)\cos(4\pi f_c t) - \int x_Q^2(t)\cos(4\pi f_c t)\,dt -$$

$$\int 2x_Q(t)\,x_I(t)\sin(4\pi f_c t)\,dt$$

By Parseval's Theorem,

$$\int u(t)\,v^*(t)\,dt = \int U(f)\,v^*(f)\,df$$

let $u(t) = x_I(t)$ and $v(t) = x_I(t)\cos(2\pi(2f_c)t)$

$$\Rightarrow \int x_I(t)\cdot x_I(t)\cos(2\pi(2f_c)t)\,dt$$

$$= \int |x_I(t)|^2 \cos(2\pi(2f_c)t)\,dt$$

$$= \int U(f)\,v^*(f)\,df$$

And Since $x(t)\cos(2\pi f_0 t) = \tfrac{1}{2}\left[x(f-f_0) + x(f+f_0)\right]$

$$\therefore \int U(f)\,v^*(f)\,df = \tfrac{1}{2}\int x_I(f)\,x_I(f-2f_c)df + \tfrac{1}{2}\int x_I(f)\,x_I(f+2f_c)\,df$$

$$= \tfrac{1}{2}\int x_I(f)\cdot 0 + \tfrac{1}{2}\int x_I(f)\cdot 0 \, df$$

$$= 0$$

$\therefore \int |x_I^2(t)|\cdot \cos(4\pi f_c t)\,dt = 0$

Similarly, $\int x_I^2(t)\cos(4\pi f_c(t))\,dt = \int x_Q^2(t)\cos(4\pi f_c t)\,dt = \int 2x_Q(t)\,x_I(t)\sin(4\pi f_c t)\,dt$
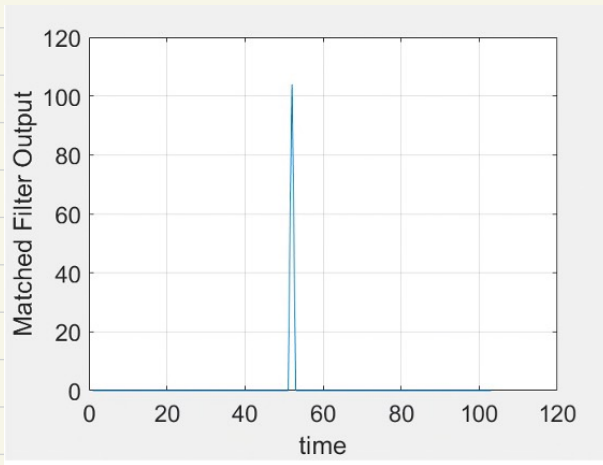
$$= 0$$

$\therefore E_p = \int |x(t)|^2\,dt = \int x_I^2(t) + x_Q^2(t)\,dt$

$$= E_b$$

P3.

(a)

$$y(n) = R\left[\sum_{L=0}^{N-1} h(n-L) \times (L)\right]$$

(b)