EECS 455: Problem Set 2

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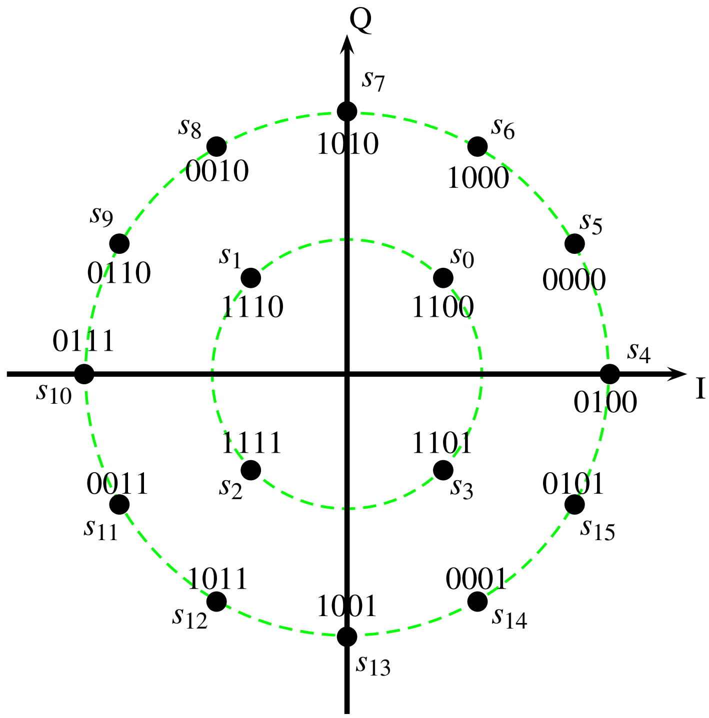
Due: Wednesday, September 15, 2021, 11pm.

1. A communication system transmits one of 16 equally likely signals using 16APSK modulation. The signal vectors of length 2 (or 1 complex dimension) lie on two circles with 4 points on the inner circle spaced evenly in phase and 12 points on the outer circle also evenly spaced as shown in the diagram below.



(a) Suppose the desired minimum squared Euclidean distance is 4 . Determine the radius of the outer circle so that the minimum squared Euclidean distance is 4 between signals on the outer circle.

(b) Determine the radius of the inner circle so that the minimum squared Euclidean distance between any point on the inner circle and points on the outer circle is also 4.

(c) With these radii, is the minimum squared Euclidean distance of two points on the inner circle at least 4?

(d) Determine the average energy, E and the energy per bit E_b.

(e) Modify the simulation for 8PSK to simulate the error probability for 16APSK. Generate the symbol error probability and bit error probability as a function of the signal-to-noise ratio (E_b/N_0).

(f) Compare the fundamental limit on the error probability for the best signal of the same rate (bits/dimension) to the bit error probability of 16APSK. (This will be something similar to Figures 1.19 and 1.20 of the book except for a different rate).

2. Consider the following 16 signals in 2 dimensions.

$$\begin{aligned}
\mathbf{s_0} &= (1.2028, -3.4096) \\
\mathbf{s_1} &= (2.8312, -2.2484) \\
\mathbf{s_2} &= (3.5934, -0.3994) \\
\mathbf{s_3} &= (3.2558, +1.5720) \\
\mathbf{s_4} &= (1.9220, +3.0622) \\
\mathbf{s_5} &= (0.0000, +3.6154) \\
\mathbf{s_6} &= (-1.9220, +3.0622) \\
\mathbf{s_7} &= (-3.2558, +1.5720) \\
\mathbf{s_8} &= (-3.5934, -0.3994) \\
\mathbf{s_9} &= (-2.8312, -2.2484) \\
\mathbf{s_{10}} &= (-1.2028, -3.4096) \\
\mathbf{s_{11}} &= (0.0000, -1.8116) \\
\mathbf{s_{12}} &= (1.0000, +1.2874) \\
\mathbf{s_{13}} &= (1.6054, -0.6188) \\
\mathbf{s_{14}} &= (-1.0000, +1.2874) \\
\mathbf{s_{15}} &= (-1.6054, -0.6188)
\end{aligned}$$

(a) Calculate the Euclidean distance $d_E(i,k)$ between every pair (s_i, s_k) of distinct signals and the minimum Eucidean distance $d_{E,min}$ between distinct signals.

(b) Plot the signals in the plane. On the sample plot draw circles around each signal point with radius being half the minimum distance $d_{E,min}$ calculated in part (a).

(c) Calculate the average energy per information bit, E_b.

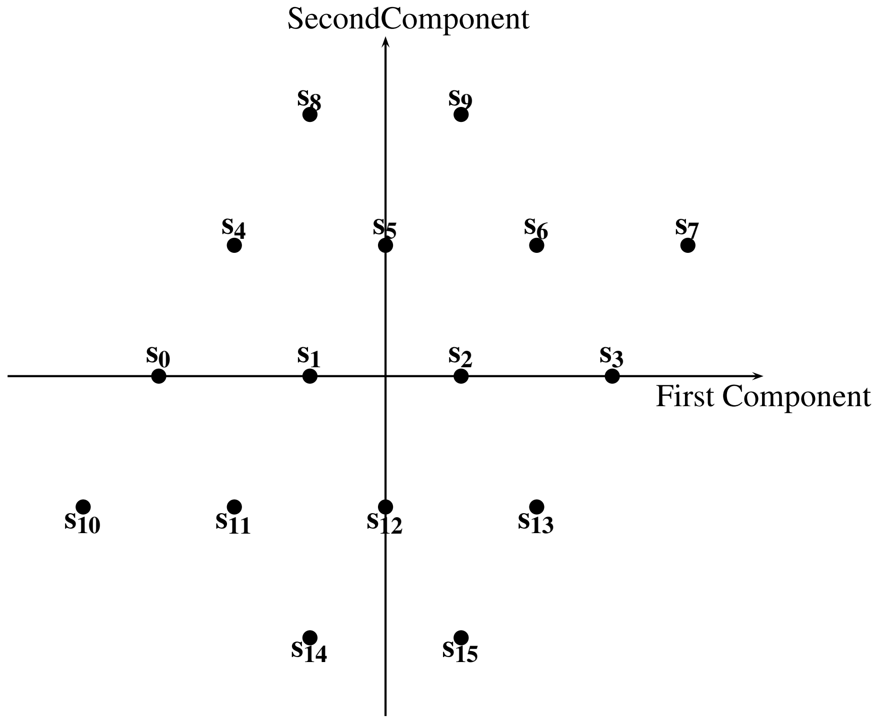
(d) Calculate the normalized squared Euclidean distance $(d^2_{E,min}/E_b)$.

(e) Calculate the peak-to-average power ratio for this constellation.

(f) If $\varphi_0(t) = \sqrt{2/T}\cos(2\pi f_c t)p_T(t)$ and $\varphi_1(t) = -\sqrt{2/T}\sin(2\pi f_c t)p_T(t)$ calculated the peak-to-average power ratio for the set of 16 signal waveforms.

3. Consider the following 16 signal vectors.

$$\begin{array}{ll}
\mathbf{s_0} = (-3, 0) & \mathbf{s_8} = (-1, 2\sqrt{3}) \\
\mathbf{s_1} = (-1, 0) & \mathbf{s_9} = (+1, 2\sqrt{3}) \\
\mathbf{s_2} = (+1, 0) & \mathbf{s_{10}} = (-4, -\sqrt{3}) \\
\mathbf{s_3} = (+3, 0) & \mathbf{s_{11}} = (-2, -\sqrt{3}) \\
\mathbf{s_4} = (-2, \sqrt{3}) & \mathbf{s_{12}} = (0, -\sqrt{3}) \\
\mathbf{s_5} = (0, \sqrt{3}) & \mathbf{s_{13}} = (+2, -\sqrt{3}) \\
\mathbf{s_6} = (+2, \sqrt{3}) & \mathbf{s_{14}} = (-1, -2\sqrt{3}) \\
\mathbf{s_7} = (+4, \sqrt{3}) & \mathbf{s_{15}} = (+1, -2\sqrt{3})
\end{array}$$



(a) Calculate the Euclidean distance $d_E(i,k)$ between every pair $(\mathbf{s_i},\mathbf{s_k})$ of distinct signals and the minimum Euclidean distance $d_{E,min}$ between distinct signals.

(b) Calculate the average energy per information bit, E_b.

(c) Calculate the normalized squared Euclidean distance $(d^2_{E,min}/E_b)$.

(d) Calculate the peak-to-average power ratio Γ_v for this constellation.

(e) If $\varphi_0(t) = \sqrt{2/T}\cos(2\pi f_c t)p_T(t)$ and $\varphi_1(t) = -\sqrt{2/T}\sin(2\pi f_c t)p_T(t)$ calculated the peak-to-average power ratio Γ_w for the set of 16 signal waveforms.