EECS 455 Homework 3 YUZHAN JIANG

P1:

① Baseband signal:

$$\tilde{x}(t) = x_I(t) + j x_Q(t) \quad = S_b(t)$$

The freqency content of $x_I(t)$ and $x_Q(t)$ is

$$X_I(f) \text{ and } X_Q(f)$$

Energy of Baseband:

$$E_L = \int |\tilde{x}(t)|^2 dx$$

② Passband signals:

$$x(t) = x_I(t)\sqrt{2}\cos(2\pi f_c t) - x_Q(t)\sqrt{2}\sin(2\pi f_c t)$$

Energy: $E_p = \int |x(t)|^2 dt$

proof: $E_L = E_p$

We can write $x(t)$ in this way:

$$x(t) = \Re\{\tilde{x}(t)\sqrt{2}e^{j2\pi f_c t}\}$$

$$= \Re\{(x_I(t) + j x_Q(t))\sqrt{2}e^{j2\pi f_c t}\}$$

We can write $x(t)$ in this way:

$$x(t) = x_e(t)\cos(2\pi f_c t + \theta(t))$$

where $x_e(t) = \sqrt{2(x_I^2(t) + x_Q^2(t))}$

$\theta(t) = \tan^{-1}\left(\frac{x_Q(t)}{x_I(t)}\right)$

Firstly, we have $E_L = \int |\tilde{x}(t)|^2 dt$

$$= \int |x_I(t) + j x_Q(t)|^2 dt$$

$$= \int [x_I^2(t) + x_Q^2(t)]\, dt$$

$$= \int x_I^2(t)\, dt + \int x_Q^2(t)\, dt$$

For $x(t)$:

$$\|x(t)\|^2 = x_I^2(t)\, 2\cos^2(2\pi f_c t) + x_Q^2(t)\, 2\sin^2(2\pi f_c t) - 4x_I(2)x_I(t)\sin(4\pi f_c t)$$

$$\Rightarrow \quad = x_I^2(t) + x_Q^2(t) + x_I^2(t)\cos(4\pi f_c t) - x_Q^2(t)\cos(4\pi f_c t) - 2x_Q(t)x_I(t)\cdot\sin(4\pi f_c t)$$

$$\therefore E_p = \int |x(t)|^2 dt$$

$$= \int \left(x_I^2(t) + x_Q^2(t) + x_I^2(t)\cos(4\pi f_c t) - x_Q^2(t)\cos(4\pi f_c t) - 2x_Q(t)x_I(t)\sin(4\pi f_c t) \right)$$

$$= \int x_I^2(t)\, dt + \int x_Q^2(t) + \int x_I^2(t)\cos(4\pi f_c t) - \int x_Q^2(t)\cos(4\pi f_c t)\, dt -$$

$$\int 2x_Q(t)\, x_I(t)\sin(4\pi f_c t)\, dt$$

By Parseval's Theorem,

$$\int u(t)\, v^*(t)\, dt = \int U(f)\, V^*(f)\, df$$

let $u(t) = x_I(t)$ and $v^*(t) = x_I(t)\cos(2\pi(2f_c)t)$

$$\Rightarrow \int x_I(t)\cdot x_I(t)\cos(2\pi(2f_c)t)\, dt$$

$$= \int |x_I(t)|^2 \cos(2\pi(2f_c)t)\, dt$$

$$= \int U(f)\, V^*(f)\, df$$

And Since $X(t)\cos(2\pi f_0 t) = \frac{1}{2}[X(f-f_0) + X(f+f_0)]$

$$\therefore \int U(f)\, V^*(f)\, df = \frac{1}{2}\int X_I(f)\, X_I(f-2f_c)\, df + \frac{1}{2}\int X_I(f)\, X_I(f+2f_c)\, df$$

$$= \frac{1}{2}\int X_I(f)\cdot 0 + \frac{1}{2}\int X_I(f)\cdot 0\, df$$

$$= 0$$

$$\therefore \int |x_I^2(t)|\cdot\cos(4\pi f_c t)\, dt = 0$$

Similarly, $\int x_I^2(t)\cos(4\pi f_c(t))\, dt = \int x_Q^2(t)\cos(4\pi f_c t)\, dt = \int 2x_Q(t)\, x_I(t)\sin(4\pi f_c t)\, dt$

$$= 0$$

$$\therefore E_p = \int |x(t)|^2 dt = \int x_I^2(t) + x_Q^2(t)\, dt$$

$$= E_c$$