EECS 455 Exam II: Fall 2020

Instructions:
Print your name and sign the honor code.

Name ____________________

Honor code: _____________________

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1. A communication system transmits four (equally likely) bits using one of 16 signals in 7 dimensions. The first 8 signals are

$$\begin{aligned}
s_0 &= A(+1,+1,+1,+1,+1,+1,+1) \\
s_1 &= A(-1,+1,+1,+1,-1,+1,-1) \\
s_2 &= A(-1,-1,+1,+1,+1,-1,+1) \\
s_3 &= A(+1,-1,-1,+1,+1,+1,-1) \\
s_4 &= A(-1,+1,-1,-1,+1,+1,+1) \\
s_5 &= A(+1,-1,+1,-1,-1,+1,+1) \\
s_6 &= A(+1,+1,-1,+1,-1,-1,+1) \\
s_7 &= A(+1,+1,+1,-1,+1,-1,-1)
\end{aligned}$$

The last 8 signals ($s_8,\cdots,s_{15}$) are the negatives of the first 8 signals. That is, $s_8 = -s_0$,... $s_{15} = -s_7$. The signal vectors are used with 7 orthonomal waveforms $\varphi_0(t),...,\varphi_6(t)$ to generate a transmitted signal (in the usual way). The received signal is the transmitted signal with additive white Gaussian noise with power spectral density $N_0/2$.

(a) Determine the average energy of the 16 signals in terms of A.

(b) Determine the relation between the energy per bit E_b and the amplitude A.

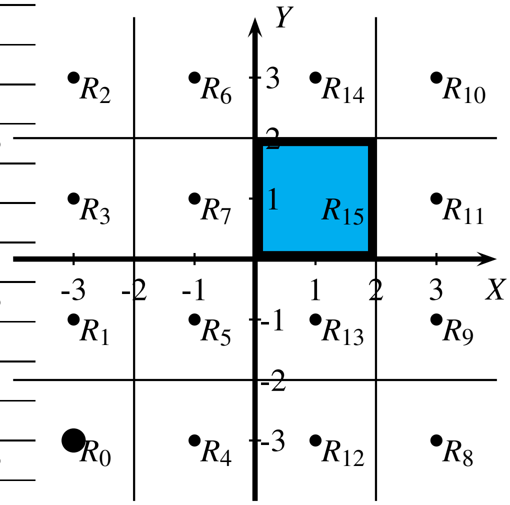
(c) Determine the squared Euclidean distance between s_0 and every other signal.

(d) Describe the optimal decision rule to minimize the probability of choosing the wrong signal.

(e) Determine the union bound on the probability of making an error given signal s_0 is transmitted. Express your answer in terms of E_b/N_0 (not A).

2. Four bits of information are communicated via a signal with one of 16 amplitudes and phases. The receiver output is a pair of random variables (X,Y). Let $H_i, \ \ i = 0,1,...,15$ be the event that a certain sequence of bits is sent as follows

Hypothesis	b_0	b_1	b_2	b_3	$\mu_c(i)$	$\mu_s(i)$
H_0	0	0	0	0	-3	-3
H_1	0	0	0	1	-3	-1
H_2	0	0	1	0	-3	3
H_3	0	0	1	1	-3	1
H_4	0	1	0	0	-1	-3
H_5	0	1	0	1	-1	-1
H_6	0	1	1	0	-1	3
H_7	0	1	1	1	-1	1
H_8	1	0	0	0	3	-3
H_9	1	0	0	1	3	-1
H_{10}	1	0	1	0	3	3
H_{11}	1	0	1	1	3	1
H_{12}	1	1	0	0	1	-3
H_{13}	1	1	0	1	1	-1
H_{14}	1	1	1	0	1	3
H_{15}	1	1	1	1	1	1



Assuming hypothesis i is true the output of the receiver is

$$X = \mu_c(i) + N_c, \qquad Y = \mu_s(i) + N_s$$

where N_c and N_s are independent, zero mean Gaussian random variables with variance σ^2. The receiver decides hypothesis H_i is true if $(X,Y) \in R_i$ where R_i is shown above.

Determine an expression (involving the Q function) for the conditional probability that the receiver decides that hypothesis H_{15} occurred given H_0 is actually occurred. That is, find $P\{(X,Y) \in R_{15} | H_0\}$. Your answer should be in terms of the Q function and σ.

3. A communication system transmits a signal using a power of 1mWatt. The rate of the communication is 100Mbps. The gain of the transmit and receive antennas is 0dB. The signal propagates via free space a distance of 100 meters. The frequency used is 2.4GHz. The noise level at the receiver is $N_0 = 4 \times 10^{-21}$ Watts/Hz.

   (a) What is the signal-to-noise ratio, E_b/N_0, is at the receiver

   (b) If 16QAM is the modulation with rectangular pulses what null-to-null bandwidth is required.

   (c) For the same data rate and bandwidth found in part (b) what is the smallest possible signal-to-noise ratio, E_b/N_0 in dB for any communication system that is reliable.

4. A communication system transmits a bit of information $b \in \{+1, -1\}$ by transmitting one of two signals and by using two different frequencies.

$$\begin{aligned} b = +1, \Rightarrow s_0(t) &= +[\varphi_0(t) + \varphi_1(t)] \\ b = -1, \Rightarrow s_1(t) &= -[\varphi_0(t) + \varphi_1(t)] \end{aligned}$$

where

$$\varphi_i(t) = \sqrt{2/T} \cos(2\pi f_i t) p_T(t), \quad i = 0, 1$$

and f_0, f_1, T are such that $\varphi_0(t)$ and $\varphi_1(t)$ are orthonormal. The transmitted signal can also be expressed as

$$s_i(t) = b[\varphi_0(t) + \varphi_1(t)], \; i = 0, 1$$

where $b = +1$ for $i = 0$ and $b = -1$ for $i = 1$. The channel attenuates different frequencies by different amounts. The received signal is

$$r(t) = \alpha_0 b \varphi_0(t) + \alpha_1 b \varphi_1(t) + n(t)$$

where $n(t)$ is white Gaussian noise with power spectral density $N_0/2$. The receiver knows the values of α_0 and α_1.

(a) Determine the optimum receiver for deciding if the data bit is $b = +1$ or $b = -1$.

(b) Determine the error probability of the optimum receiver. This should depend on α_0 and α_1, and N_0.