EECS 455: Problem Set 1
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Due: Wednesday, September 8, 2021, 11pm.

1. A communication system transmits one of 8 equally likely signals. The signals (waveforms) are represented by the vectors shown below.

$$
\begin{aligned}
s_0 &= (+1,+1,+1,+1)\\
s_1 &= (+1,-1,+1,-1)\\
s_2 &= (+1,+1,-1,-1)\\
s_3 &= (+1,-1,-1,+1)\\
s_4 &= (-1,-1,-1,-1)\\
s_5 &= (-1,+1,-1,+1)\\
s_6 &= (-1,-1,+1,+1)\\
s_7 &= (-1,+1,+1,-1)
\end{aligned}
$$

   (a) Determine how many information bits can be sent using these signals.

   (b) Determine the energy of each of the signals and the average energy per information bit.

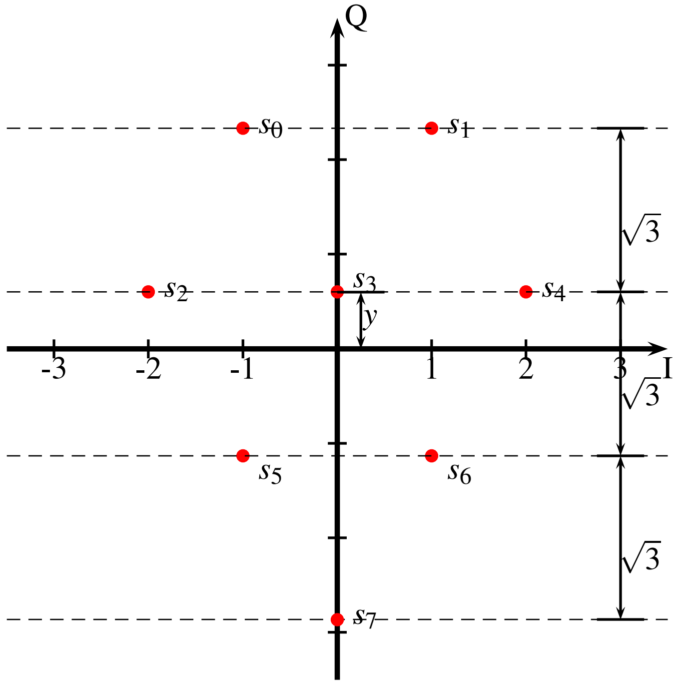
   (c) Determine the squared Euclidean distance between signal $s_0$ and all the other signals.

   (d) Determine the rate of communication in bits/dimension for these signals.

2. A modulator transmits 3 bits of information using 8 equally likely signals in two dimensions. The signal vectors are given as

$$
\begin{aligned}
s_0 &= A(-1, y+\sqrt{3})\\
s_1 &= A(1, y+\sqrt{3})\\
s_2 &= A(-2, y)\\
s_3 &= A(0, y)\\
s_4 &= A(2, y)\\
s_5 &= A(-1, y-\sqrt{3})\\
s_6 &= A(1, y-\sqrt{3})\\
s_7 &= A(0, y-2\sqrt{3})
\end{aligned}
$$

   (a) Determine the optimum value of the parameter $y$ to minimize the average signal energy transmitted.

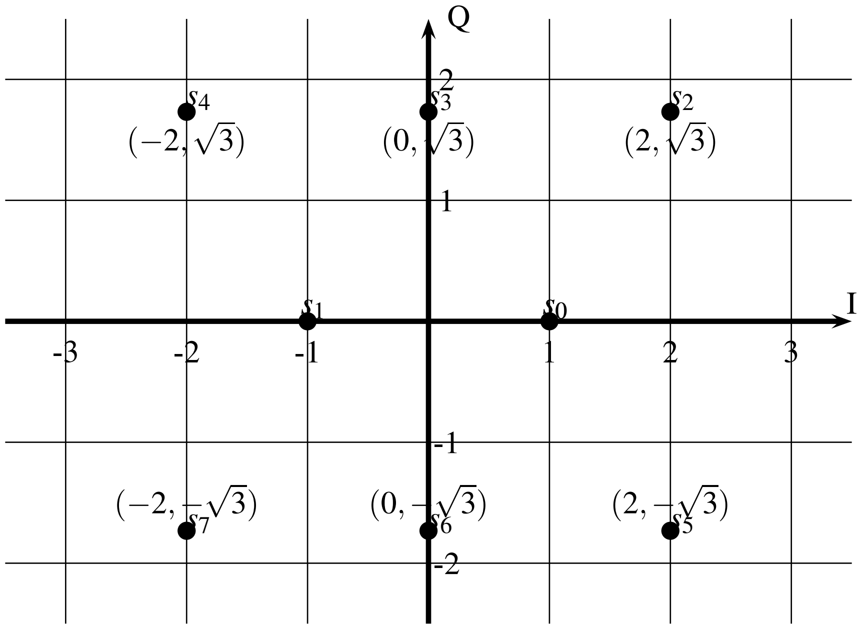



(b) Determine the minimum squared Euclidean distance between any two signals.

(c) Determine the rate of communication in bits/dimension.

3. The eight constellation points for an equal probable signal set are shown below.

$$s_0 = (1,0), \quad s_1 = (-1,0), \quad s_2 = (2,\sqrt{3}), \quad s_3 = (0,\sqrt{3})$$
$$s_4 = (-2,\sqrt{3}), \quad s_5 = (2,-\sqrt{3}), \quad s_6 = (0,-\sqrt{3}), \quad s_7 = (-2,-\sqrt{3})$$




(a) Determine the average energy of this signal set.

(b) Determine the average energy per bit.

(c) Determine the distance between signal $s_0$ and every other signal.

(d) Determine the rate of communication in bits/dimension.

4. Consider the UWB channel which goes from 3.1GHz to 10.6 GHz. Suppose the noise power spectral density is $N_0 = kT = (1.38 \times 10^{-23})(290) = 4 \times 10^{-21}$ Watts/Hz. Here $k$ is Boltzmann's constant and $T$ is the temperature in Kelvin. A temperature of 290 K corresponds to 62 degrees Fahrenheit. The allowed transmitted power *density* is -41.3dBm/MHz =-71.3dB/MHz. (Note 0dBm=1mW, 30dBm=1W, -30dBm=1 $\mu$W).

   (a) For the given frequency band determine the total power that can be transmitted.

   Suppose the received power is related to the transmitted power by

$$P_r = P_t h_t^2 h_r^2 / d^4$$

   where the $d$ is the distance in meters (independent of frequency), $h_t$ is the height of the transmitting antenna (in meters) and $h_r$ is the height of the receiving antenna (in meters).

   (b) Compute the largest possible data rate that can be communicated reliably with both antennas at a height of 1m at a distance of 100 m and 1000 m.

5. (a) A communication system is to be designed. The allocated (absolute) bandwidth is 100kHz. It is desired to communicate 300kbits/sec very reliably (error probability close to zero). What is the smallest value of $E_b/N_0$ (in dB) for which this is possible?

   (b) A channel with absolute bandwidth $W = 100$kHz, power $P = 5$ watts= 5 Joules/sec and two sided noise power spectral density $N_0/2 = 1.778 \times 10^{-3}$ Watts/ Hz is used. The source is an i.i.d. Gaussian source with mean 0 and variance 1. What is the minimum possible distortion (mean square error) if the source is sampled at rate 4000 samples/sec.