EECS 501. PROBABILITY AND RANDOM PROCESSES

FALL 2016

MIDTERM EXAM

THURSDAY, OCTOBER 27, 2016.

6:30 p.m – 8:30 p.m

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Problem 1 (14 points)

Consider the function

$$X : (\Omega, \mathcal{F}, P) \longrightarrow (\mathcal{D}, 2^{\mathcal{D}}, P_X)$$

where

$$\Omega = \{a, b, c, d, e, f\}$$

$$\mathcal{F} = \{\Omega, \{a, b\}, \{c\}, \{d, e, f\}, \{a, d, e, f, b\}, \{a, b, c\}, \{c, d, e, f\}, \phi\},$$

$$\mathcal{D} = \{l_1, l_2, l_3, l_4\}$$

$$P(\{a, b\}) = 1/3, \quad P(\{c\}) = 1/4, \quad P(\{d, e, f\}) = 5/12.$$

(i) Assume that X is a random variable. What restrictions does this assumption impose on $X(a), X(b), X(c), X(d), X(e), X(f)$?

(7 pts)

(ii) Suppose $X(a) = X(b) = X(c) = l_1$, $X(d) = X(e) = X(f) = l_4$.

Determine $P_X(\{l_1, l_2, l_3\})$.

(7 pts)

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Problem 2 (12 points)

The following experiment is performed. A die is rolled twice. You are told whether or not the sum of the two outcomes (of the roll of the die) is equal to 7. Determine the sample space Ω and the σ-field $\mathcal{F}$ on Ω for this experiment.

Problem 3 (12 points)

Two individuals decide to meet for dinner at a certain location. If each individual independently arrives at a time that is uniformly distributed between 7 p.m and 8 p.m, determine the probability that the first to arrive has to wait longer than 10 minutes.

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Problem. 4 (12 points)

A bus going from Central Campus to North Campus carries 35 passengers. There are 7 stops between the Central Campus station and the North Campus station (these 7 stops do not include the Central Campus and North Campus stations). In how many different ways can the bus discharge the 35 passengers among the 7 stops and the North Campus station?

Problem 5 (12 points)

In the card game of bridge, 52 cards are dealt equally to 4 players called A, B, C, D. If A and B have a total of 7 hearts among them, what is the probability that player C has 4 of the remaining 6 hearts?

Note: The deck of 52 cards consists of 13 cards that are hearts, 13 cards that are diamonds, 13 cards that are clubs, and 13 cards that are spades.

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Problem 6 (13 points)

Die A has 4 red and 2 white faces, die B has 2 red and 4 white faces. A fair coin is flipped once. If it lands on "heads" the game continues with rolling die A all the time; if it lands on "tails" the game continues with rolling die B all the time. Each face appears with equal probability on any roll of die A or die B.

(i) Compute the probability that a red face will appear at any roll. (6 pts)

(ii) If the first two rolls result in a red face, what is the probability that a red face will result in at the third roll? (7 pts)

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Problem 7 (12 points)

Suppose that it is Known that the number of items produced in a factory during a week is a random variable X with $E(X) = 50$ and $Var(X) = 25$. Determine a lower bound on the probability that this week's production is between 40 and 60.

Problem 8 (13 points)

Suppose there are N different types of coupons and each time one obtains a coupon it is equally likely to be any one of the N types. Find the expected number of different types of coupons that are contained in a set of n coupons

Hint: Associate an appropriate indicator function with each type of coupon.

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