University of Michigan Fall 2021

EECS501: Homework 5

Text: "Probability and random processes" by J. A. Gubner

Reading assignment: Please read Chapter 2 and 3 (except Section 3.1-3.3).

1. **Transformations of random variables** [5 points each]

   Determine the pdf of

   (a) $Y = 1/X$, where $X \sim \text{unif}(0, 1)$.

   (b) $Y = 1/\sqrt{X}$, where $X \sim \text{unif}(0, 1)$.

   (c) $Y = \log X$, where $X \sim \text{unif}(0, 1)$.

   (d) $Y = -\log_2 F_X(X)$, where X is a random variable with CDF F_X.

2. **Waiting times** [15 points]

   Suppose you call your cell phone company to address a problem with your service. When you call, it takes 1 minute to navigate through the menus before you can request a live operator. Your wait time for a live operator is exp(0.5) distributed. Once you get a live operator, one of two things happens. With probability 0.2, you get a competent operator who can resolve your problem in time that is exp(0.2). With probability 0.8, you get an incompetent operator who gives you the run-around and eventually returns you to the queue for a live operator, after wasting your time with distribution exp(0.1). What is the expected time before your problem is resolved?

3. **Transformation of Uniform** [5 points]

   Let X be uniformly distributed over the unit interval. It is transformed using a function $Y = g(X)$. Suppose we want Y to have the following PDF:

$$f_Y(y) = \begin{cases} y & 0 \le y \le 1 \\ 3 - y & 2 \le y \le 3 \\ 0 & \text{otherwise} \end{cases}$$

   Find a monotone non-decreasing function $g(\cdot)$ that satisfies this constraint.

4. **Joint Probability Density** [5 points each]

   The joint probability density function of X and Y is given by

$$f(x, y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), & 0 \le x \le 1, \quad 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find $P(X > Y)$

   (b) Find $E(X|Y)$