University of Michigan Fall 2021

EECS501: Homework 3

Assigned: September 18, 2021
Due: September 28, 2021 at 11:59PM on gradescope
Text: "Probability and random processes" by J. A. Gubner

Reading assignment: Please read Chapter 1 and Chapter 2. In lectures we are covering the material in this order: 2.1 - 2.4 till Indicator functions.

1. Dice Game 1 [10 points]

A fair die is rolled sequentially until you observe 5 followed immediately by 6 when you win or 6 followed immediately by 1 when you lose. Let X denote the number of rolls. Find $E(X)$.

2. Data Packets over a Network [10 points]

In a packet data transmission system, a source sends packets containing computer data to a receiver. Because transmission error occasionally occurs, an acknowledgment (ACK) or a nonacknowledgement (NACK) is transmitted back on a feedback channel to the source to indicate the status of each received packet. Let X denote the number of times the packet is transmitted until successfully received. If the network is CONGESTED, then each time a packet is sent, it is received successfully with probability $\frac{5}{10}$. If the network is NOT CONGESTED, then each time a packet is sent, it is received successfully with probability $\frac{9}{10}$. The probability that the network is CONGESTED is $\frac{1}{5}$. Given that a packet was sent 3 times to be received successfully, what is the probability that the network in CONGESTED?

3. Dice Game 2 [5 points, 10 points]

Here is a dice game. You roll a dice once. If you get 6 you win. If you get 1 you lose. Otherwise you keep rolling until you either roll a 2 or a 3 or a 4 or a 5. If you roll a 2 or a 3 or a 4 you win, and if you roll a 5 you lose. Let X denote the number of rolls.

(a) Find $E(X)$ using the law of total expectation.

(b) Find a set of difference equations that characterize the PMF of X along with initial conditions.

4. Airline Booking (10 points, 5 points, 5 points)

An airline company wants to book reservations for its popular flight at 7.00 AM from Detroit to New York. There are 100 openings for passengers. A person with a confirmed ticket will not show up for the flight with probability 0.08. For each passenger (person with a booking and who shows up), the airline makes a profit of 75 Dollars. The airline overbooks the flight. Let N denote the number of bookings for the flight. Note that $N \geq 100$. Treat N as a parameter that the airline would like to choose. Let K denote the number of persons (among N) with confirmed ticket who show up for the flight. If K is greater than 100, $(K - 100)$ people are randomly selected, and will be forced to travel on the next flight, and each of them is given a coupon of 100 Dollars. In other words, the airline makes a loss of 100 Dollars for each such person. Let D denote the profit that airline makes for each flight.

(a) For a fixed parameter N, find the PMFs of random variables K and D.

(b) For a fixed parameter N, find the expected value of D.

(c) Find N that will maximize the expected profit. Please use a computer to solve this part of the problem.

5. PMF [5 points each]

For the following problems, use your intuition rather than direct computation. You must obtain a closed form expression for the solutions to get credit. Do not leave your answers in the form of summations.

(a) Let X and Y be two independent geometric random variables with parameters p_1 and p_2, respectively. Let $Z = \min\{X, Y\}$. Find the PMF of Z.

(b) Let X and Y be two independent binomial random variables with parameters (n_1, p) and (n_2, p). Let $Z = X + Y$. Find the PMF of Z.

(c) Let X_1, X_2 and X_3 be three independent geometric random variables all with parameters p. Let $Z = X_1 + X_2 + X_3$. Find the PMF of Z.