University of Michigan WINTER 2020

EECS501: Homework 2

Assigned: January 17, 2020
Due: January 24, 2020 at 8.00PM on gradescope
Text: "Probability and random processes" by J. A. Gubner

Reading assignment: Please read Chapter 1. In lecture we are covering the material in this order: 1.1 - 1.6.

1. Union Bound Refined (10 points)

Show that for any finite number of events $A_1, A_2, \ldots, A_n$,

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \min_{0\leq\rho\leq 1}\left[\sum_{i=1}^{n} P(A_i)\right]^{\rho}.$$

Hint: Consider two cases: (i) $\sum_{i=1}^{n} P(A_i) \leq 1$ and (ii) $\sum_{i=1}^{n} P(A_i) \geq 1$. Note that $P\left(\bigcup_{i=1}^{n} A_i\right) \leq 1$.

2. Shine or Rain (10 points)

It rains on fridays in Ann arbor with probability 0.2. The sun shines on fridays in Ann Arbor with probability 0.8. It is cloudy and dry on fridays in Ann Arbor with probability 0.1. What is the conditional probability that it is dry given that it is cloudy on fridays in Ann Arbor?

3. Tennis anyone? [10 points]

Suppose Alice and Bob are playing tennis and the score is deuce, meaning that the next player to go ahead by two points wins the game. Suppose that Alice is serving, that Alice wins points when serving with probability p, and that the outcomes of points are independent. What is the probability that Alice wins the game? (Use the law of total probability)

4. Coin Tossing Game (15 points)

A fair coin is tossed until you get THT when you win or HHH when you lose. Find the probability that you win using the law of total probability.

5. World Series [10 points]

Suppose the Rangers and Cardinals play each other in the World Series. The winner of the series is the first team to win 4 games. If the Rangers win each game with probability p (substitute your own concrete value if you wish), independent of other games, what is the probability that they win the series?

6. Dice Game [10 points]

Consider the following experiment. Keep rolling a fair die sequentially. What is the probability of observing "1" any time before the pattern "$5, 6$" (i.e., 5 followed immediately by 6)?

7. Independence (15 points)

Three events A, B and C are said to be mutually independent if every pair of events are independent and $P(A \cap B \cap C) = P(A)P(B)P(C)$, i.e., the events have to satisfy four conditions. Show that if A, B and C are mutually independent, then so are A^c, B^c and C^c.

8. Independence (10 points)

Four fair coins are tossed. Find three events A, B and C that are NOT pairwise independent but satisfy

$$P(A \cap B \cap C) = P(A)P(B)P(C).$$

9. Badminton (10 points)

In the game of badminton, one player starts a rally by serving the shuttlecock or "birdie." The two players rally back and forth until the birdie hits the ground.

Let's assume the traditional scorekeeping system for badminton, where you score a point when you win a rally as the server. If you win a rally as the receiver, the score remains unchanged, but you get to serve and thus the opportunity to score.

Suppose Ann wins a rally against Bob with probability p, regardless of who serves. Assume rallies are won independently of each other. If Ann is currently the server, what is the probability that she wins the next point? Express your answer as a function of p.