EECS 455: Problem Set 3

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Due: Wednesday, September 22, 2021, 11pm.

1. Consider a baseband signal

$$\tilde{x}(t) = x_I(t) + jx_Q(t)$$

where $x_I(t)$ and $x_Q(t)$ are baseband signals with frequency content limited to $[-W, +W]$. Let $X_I(f)$ and $X_Q(f)$ be the frequency content of the signals. So

$$X_I(f) = X_Q(f) = 0 \text{ for } f \notin [-W, W]$$

The energy of the lowpass complex signal is

$$E_l = \int |\tilde{x}(t)|^2 dt$$

The passband signal is

$$x(t) = x_I(t)\sqrt{2}\cos(2\pi f_c t) - x_Q(t)\sqrt{2}\sin(2\pi f_c t)$$

where $f_c > W$. The energy of the passband signals is

$$E_p = \int |x(t)|^2 dt$$

Show that $E_l = E_p$.

Hint: Derive expressions for the energy in the frequency domain for $x(t)$. Use Parseval's Theorem

$$\int u(t)v^*(t)dt = \int U(f)V^*(f)df.$$

where $u(t) = x_I(t)$ and $v(t) = x_I(t)\cos(2\pi(2f_c)t)$.

2. Suppose you want to implement a square-root raised cosine pulse shape. The pulse shape is continuous and the pulse shape theoretically last forever. You generate samples of the pulse in the time domain and truncate the pulse to some number of samples. Suppose you generated 8 samples per T seconds where T is the time between pulses. You can assume $T = 1$. You can use Matlab. Do the following for $\alpha = 0.05, 0.15, 0.25$.

(a) Determine how many samples you need for the maximum sample you ignore is 40dB down relative to the peak sample. That is the amplitude of the any sample you ignore is 0.01 times as small as the peak sample.

(b) Determine the frequency content of the signal (plot the frequency content in dB) versus f.

(c) Suppose you add samples to the truncated pulse. You want to add samples so that the last sample is as close as possible to 0. What is your new pulse length in samples. Compare the frequency response to that in part (b).

For this problem you should produce 3 graphs (one for each value of α) that shows the frequency content for two different number of samples.

3. (a) Consider a complex sequence of length $N = 26$;

$$
\begin{aligned}
x = (x_0, ..., x_{25}) = \quad & (-1+1j, -1+1j, -1+1j, 1-1j, 1-1j, -1+1j, \\
& -1+1j, -1+1j, 1-1j, -1+1j, 1-1j, 1-1j, \\
& 1+1j, 1-1j, 1+1j, -1-1j, 1+1j, -1-1j, \\
& -1-1j, 1+1j, 1+1j, -1-1j, 1+1j, 1+1j, 1+1j, 1+1j)
\end{aligned}
$$

You can load this vector into Matlab from a file on Canvas HW3P1.mat. Now consider a matched fiiter $h(n) = x^*((N-1)-n) = x^*(25-n)$. Use Matlab functions `fliplr` and `conj`. Determine the output of the matched filter when the input is x. That is determine y where

$$
y(n) = \Re\left[\sum_{l=0}^{N-1} h(n-l)x(l)\right]
$$

and $\Re(x)$ is the real part of x. Use Matlab's `conv` function. Plot the result.

(b) Consider the sequence of length 52

$$
\begin{aligned}
\Re(u) &= [\Re(x), \quad +\Im(x)] \\
\Im(u) &= [\Re(x), \quad -\Im(x)] \\
u &= \Re(u) + j\Im(u)
\end{aligned}
$$

That is, the first half of the real part of u is the real part of x from part (a). The second half of the real part of u is the imaginary part of x. Similarly for the imaginary part of u. Determine the real part of the output of a filter matched to the sequence u. Plot the result.

4. A communication system uses BPSK modulation to transmit data bits $b_l, l = 0, 1, 2, ...$ In the transmitter a sequence of rectangular pulses is mixed to a carrier frequency by multiplying the rectangular pulses by $\sqrt{2P}\cos(2\pi f_1 t)$.

$$
s(t) = \sqrt{2P} \sum_{l=0,1,...} b_l p_T(t-lT)\cos(2\pi f_1 t)
$$

At the receiver the received signal is first mixed down to baseband by multiplying by $\sqrt{2/T}\cos(2\pi f_2 t)$ where $f_2 - f_1 = \Delta f$ is the offset of the two oscillators. After the signal is mixed down it is filtered with a matched filter (that is $h(t) = p_T(t)$). The filter is sampled at time $t = iT$ for $i = 1, 2, ...$ In addition the signal is mixed down by multiplying by $-\sqrt{2/T}\sin(2\pi f_2 t)$. Let $y_c(iT)$ denote the first output and $y_s(iT)$ denote the second output. Then

$$
\begin{aligned}
y_c(iT) &= \int_{-\infty}^{\infty} s(\tau)\sqrt{\frac{2}{T}}\cos(2\pi f_2 \tau)h(iT-\tau)d\tau \\
y_s(iT) &= -\int_{-\infty}^{\infty} s(\tau)\sqrt{\frac{2}{T}}\sin(2\pi f_2 \tau)h(iT-\tau)d\tau
\end{aligned}
$$

(a) In the absence of noise evaluate the outputs $y_c(iT)$ and $y_s(iT)$ in terms of b_{i-1}, $E = PT$, ΔfT and i. Ignore double frequency terms in evaluating the output. That is, derive an expression for $y_c(iT)$ and $y_s(iT)$. (Useful trig identity $\sin(u) - \sin(v) = 2\cos(\frac{u+v}{2})\sin(\frac{u-v}{2})$).

(b) Assume you buy two crystal oscillators at a 10MHz nominal frequency that have $\pm$ 10 PPM accuracy. That is, $f_{\text{actual}} = f_{\text{nominal}}(1 \pm 10/10^6)$. Assume that the data rate is 100kbps ($T = 10^{-5}$), that the data bits are all positive ($b_i = 1, i = 0, 1, 2, ..., 500$) and that $E = 1$.

   i. Are the double frequency terms negligible?

   ii. Plot the output of the filters $y_c(iT)$ and $y_s(iT)$ as a function of i for $1 \leq i \leq 500$. Assume all the data bits are +1 and $f_2 - f_1 = \Delta f = 200$ Hz and $T = 10^{-5}$.

   iii. Plot $y_s(iT)$ versus $y_c(iT)$ for the case where $\Delta fT = 0.01$, the data bits are all 1 for $i = 0, ..., 39$. Plot each point with a "marker". Use the Matlab command `plot(yc,ys,'+')` where yc is a vector of samples representing $y_c(iT)$ and `ys` is a vector of samples representing $y_s(iT)$ for $i = 0, 1, ..., 39$.