q14

Due Oct 14 at 9am **Points** 5 **Questions** 5
Available Oct 13 at 9am - Oct 14 at 9am 1 day **Time Limit** 20 Minutes

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	4 minutes	4.8 out of 5

Correct answers will be available on Oct 14 at 9:01am.

Score for this quiz: **4.8** out of 5
Submitted Oct 13 at 11:04pm
This attempt took 4 minutes.

Question 1

1 / 1 pts

The nonzero singular values of a matrix A are $0.1, 0.1, 0.2, 0.5, 1$. Let $\hat{x}$ denote the minimum-norm LS minimizer of $||Ax - y||_2$. Let $\hat{x}_3$ denote the corresponding truncated SVD solution for $K = 3$. Let u_4 denote the 4th left singular vector of A. Which of the following statements are true?

- [x] The difference $\hat{x} - \hat{x}_3$ is in a 2-dimensional subspace.
- [x] The solution $\hat{x}_3$ is in a 3-dimensional subspace.
- [x] $u_4{}' A\hat{x}_3 = 0$
- [x] The solution $\hat{x}$ is in a 5-dimensional subspace.
- [ ] $A\hat{x} = A\hat{x}_3$
- [ ] $u_4{}' A\hat{x} = 0$

- [ ] $A\hat{x}_3 = y$
- [ ] $A\hat{x} = y$
- [x] An upper bound on $\|\hat{x} - \hat{x}_3\|_2$ is $10\|y\|_2$.
- [ ] $\hat{x} = \hat{x}_3$

The MNLS solution is $\hat{x} = A^+ y = \sum_{k=1}^{5} 1/\sigma_k v_k u_k' y$. And $\hat{x}_3 = A_3^+ y = \sum_{k=1}^{3} 1/\sigma_k v_k u_k' y3$ = A_3^+ y = \sum{k=1}^3 1/\sigma_k v_k u_k'y, so $A\hat{x}_3 \perp u_4$ and $\hat{x}_3 \in \mathrm{span}(\{v_1, v_2, v_3\})$. The difference is $\hat{x} - \hat{x}_3 = (A^+ - A_3^+)y$ so $\|\hat{x} - \hat{x}_3\|_2 \leq \|A^+ - A_3^+\|_2 \|y\|_2 = 10\|y\|_2$

Question 2

1 / 1 pts

Every projection matrix has a unitary eigendecomposition.

- ( ) True
- (•) False

No, consider $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$. It is idempotent but not normal.

Partial

Question 3

0.8 / 1 pts

Which of the following statements about an orthogonal projection matrix are always true?

- [x] It has a unitary eigendecomposition.
- [ ] Its range is a subspace.
- [x] Its range is the orthogonal complement of some subspace.
- [x] It is diagonalizable.
- [ ] None of these.
- [x] Its range and its null space are both subspaces of the same vector space.

Because it is Hermitian it is square so its range and nullspace are both subspaces of $\mathbb{F}^N$.

- [ ] It is invertible.

It is Hermitian so normal and thus diagonalizable. The range of any matrix is a subspace. The range of any matrix A is the orthogonal complement of the subspace $\mathscr{R}^{\perp}(A)$.

Question 4

1 / 1 pts

Let

$$A = \begin{bmatrix} U_r & U_0 \end{bmatrix} \Sigma \begin{bmatrix} V_r & V_0 \end{bmatrix}'$$

where U_r and V_r have r columns where r denotes the rank of A. The projection matrix for the orthogonal complement of $\mathscr{R}(A')$ is:

- [ ] $I - U_0 U_0{}'$
- [ ] $U_0 U_0{}'$
- [x] $I - V_r V_r{}'$
- [ ] None of these
- [ ] $I - U_r U_r{}'$
- [ ] $U_r U_r{}'$
- [ ] $I - V_0 V_0{}'$
- [x] $V_0 V_0{}'$
- [ ] $V_r V_r{}'$

$$P^{\perp}_{\mathscr{R}(A')} = P_{\mathscr{R}(V_0)} = V_0 V_0{}' = I - V_r V_r{}'$$

Question 5

1 / 1 pts

Let

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $b = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$. An SVD is $A = U \Sigma V'$. The projection matrix for $\mathscr{R}(A)$ is:

- [ ] None of these
- [x] The product $U_r U_r{}'$, where $U_r = \begin{bmatrix} u_1 & \dots & u_r \end{bmatrix}$ and A has rank r.
- [x] I_3
- [x] AA^+

This A has full rank so $U_r U_r{}' = UU' = AA^+ = I$.

Quiz Score: **4.8** out of 5