Last (family) name:____________________

First (given) name:____________________

I have neither given nor received aid on this examination, nor concealed any violation of the Honor Code. The only online resources I have used during this exam are those listed below as well as the following sites (list URL and exam problem #):

Signature: ________________________

ID Number: ________________________

EECS 551 Midterm 2, 2020-10-27 EDT (24 hour online)

- There are 6 problems for a total of 70 points.
- This part of the exam has 8 pages. Make sure your copy is complete.
- This is a 24-hour "online" exam. Many of the exam questions will be on Canvas; this pdf has additional questions that you should answer by submitting to gradescope unless instructed otherwise in each problem.
- This is an "open book" exam. During the exam, you may use all of the course materials on the Canvas EECS 551 site including the course notes on google drive, as well as wikipedia, the built-in JULIA help, and the online JULIA manual. If you use any other resources for solving the CanvasQuiz questions, then you must cite the source along with your honor code statement above. *If you use any other resources for solving the gradescope questions, then cite the source as part of your solution submitted to gradescope.* Be sure to *sign* the honor code above and scan (e.g., photograph) the top part of this page and submit to gradescope.
- You may use without rederiving any of the results presented in the course notes.
- You must complete the exam entirely on your own.
- If you need an exam question to be clarified, post a private question to the instructors on piazza.
- Clearly $\boxed{\text{box}}$ your final answers. For full credit, show your complete work clearly and legibly. Answers must be submitted properly to gradescope to earn credit.
- For multiple-choice questions, select *all* correct answers.
- To "disprove" any statement, provide a concrete counter-example. For maximum credit, make that counter-example as small and simple as possible, *e.g.*, having the smallest possible matrix dimensions and using the simplest numbers like "0" and "1" as much as possible. For example, to disprove the statement "any square matrix $\boldsymbol{A}$ is invertible," the smallest and simplest counter-example is the 1×1 matrix $\boldsymbol{A} = [0]$.

autograder instructions

For any problem involving writing a JULIA function, carefully follow these instructions.

- Submit a mathematical explanation of your solution to gradescope.
- Submit a readable screenshot of your code to gradescope for grading *efficiency*.
- Test your code thoroughly *before* submitting, to maximize credit earned.
  (Passing on the 1st or 2nd submission will earn full credit; partial credit will decrease rapidly after that.)
- Submit your tested code by email attachment to `eecs551@autograder.eecs.umich.edu` as usual.
  Incorporate any necessary `using` statements.
  Name your function correctly.
  Name the file attachment like `thefunction.jl` where `thefunction` is the function name.

[9] **1**. Given a data vector $\boldsymbol{y} \in \mathbb{R}^M$, we can perform polynomial regression, *i.e.*, $y_m \approx f(t_m)$, $m = 1, \ldots, M$, where f is a polynomial, by solving optimization problems of the form $\hat{\boldsymbol{x}} = \underset{\boldsymbol{x} \in \mathbb{R}^N}{\arg\min} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2$ for a suitably chosen $M \times N$ matrix $\boldsymbol{A}$.

Find a simple expression for the residual norm squared $\|\boldsymbol{A}\hat{\boldsymbol{x}} - \boldsymbol{y}\|_2^2$ for the case of fitting a 0-degree polynomial.
A simple expression is one that involves only vectors like $\boldsymbol{y}$ (no matrices).

[14] **2.** Write a JULIA function that determines whether a given vector $\boldsymbol{z} \in \mathbb{F}^M$ is in the range of a given matrix $\boldsymbol{B} \in \mathbb{F}^{M \times N}$. The function should return `true` or `false`, and return value `true` indicates that $\boldsymbol{z}$ is in the range of $\boldsymbol{B}$ to within numerical precision.
You may not use `pinv` or `eigen`. Hint: use `isapprox(x,y)` to see if vectors `x` and `y` are equal to within numerical precision. (Do not try to write your own numerical precision test!) If you use any other JULIA function that has optional tolerance parameter(s), just use JULIA's default setting.
A full credit solution must be as efficient as possible.
Template:

```
"""
    tf = inrange(B, z)
Return `true` or `false` depending on whether `z` is in the range of `B`
to within numerical precision.  Must be as compute efficient as possible.

In:
* `B` a `M × N` matrix
* `z` vector of length `M`
"""
function inrange(B::AbstractMatrix, z::AbstractVector)
```

See submission instructions near front of exam.

[10] **3**. For a given nonzero matrix $\boldsymbol{A} \in \mathbb{F}^{M \times N}$ and measurement $\boldsymbol{y} \in \mathbb{F}^{M}$, the solution to the least-squares problem $\arg\min_{\boldsymbol{x} \in \mathbb{F}^N} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|_2$ is not unique in general, but the *minimum norm* least-squares solution *is* unique and is $\hat{\boldsymbol{x}} = \boldsymbol{A}^+ \boldsymbol{y}$. This problem asks you to think about what other vectors $\boldsymbol{b} \in \mathbb{F}^M$ would lead to the same $\hat{\boldsymbol{x}}$?

- Specifically, describe concisely the set $\mathcal{S} = \left\{ \boldsymbol{b} \in \mathbb{F}^M \ : \ \boldsymbol{A}^+ \boldsymbol{b} = \boldsymbol{A}^+ \boldsymbol{y} \right\}$ in terms of one or more of the four fundamental spaces associated with $\boldsymbol{A}$ and possibly in terms of other problem quantities.
- Discuss whether $\mathcal{S}$ is convex. (Always? Never? Under certain necessary and sufficient conditions?)
- Discuss whether $\mathcal{S}$ is a subspace. (Always? Never? Under certain necessary and sufficient conditions?)

[14] **4**. Let $\mathcal{D}$ denote the set of $N \times N$ diagonal matrices. Complete the following JULIA function so that it returns $\hat{\boldsymbol{D}} = \arg\min_{\boldsymbol{D} \in \mathcal{D}} \|\boldsymbol{X}\boldsymbol{D} - \boldsymbol{Y}\|_{\mathrm{F}}$, when given matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$ in $\mathbb{F}^{M \times N}$, where each column of $\boldsymbol{X}$ is nonzero. Your code must return a variable of type `Diagonal` in the `LinearAlgebra` package.

The `::Diagonal` declaration for the function is a reminder of that.
To earn full credit, the code must be as efficient as possible, and should work for both real and complex inputs.
Template:

```
"""
    D = bestdiag(X::Matrix, Y::Matrix)
Solve `\\arg\\min_{D diagonal} \\| X D - Y \\|_F`
In:
* `X`, `Y` `M × N` matrices
Out:
* `D` `N × N Diagonal`
"""
function bestdiag(X::Matrix, Y::Matrix)::Diagonal
```

See submission instructions near front of exam.

[14] **5**. Write a JULIA function that, given a nonzero matrix $\boldsymbol{A} \in \mathbb{C}^{M \times N}$, returns a tuple of the three matrices in a compact SVD of $\boldsymbol{A}^+$ *without* calling the `eigen` or `pinv` functions. See code template below for specifics.
To earn full credit, your solution must be as efficient as possible.
Template:

```
"""
    (Ur,Sr,Vr) = pseudosvd(A)
Return 3 matrices in a compact SVD of the Moore-Penrose pseudo-inverse of `A`
without calling (or duplicating) Julia's built-in pseudo-inverse function.
The returned triplet of matrices should satisfy `Ur*Sr*Vr' = A^+`
to within appropriate numerical precision.
"""
function pseudosvd(A::AbstractMatrix)
```

See submission instructions near front of exam.

[9] **6**. Express $\|\boldsymbol{x}\boldsymbol{y}'\|_\infty$ in terms of appropriate norms of vectors $\boldsymbol{x}$ and of $\boldsymbol{y}$. Explain.

end