EECS 501

Solutions to Homework 2

1 Conditional Probability, independence and Bayes' rule

1.1 *Gubner 1.53*

Solution

From the definition of conditional probability we have

$$P(A|B\cap C)P(B|C) = \frac{P(A\cap(B\cap C))}{P(B\cap C)}\frac{P(B\cap C)}{P(C)} \tag{1}$$
$$= \frac{P(A\cap(B\cap C))}{P(C)} \tag{2}$$
$$= \frac{P((A\cap B)\cap C)}{P(C)} \tag{3}$$
$$= P(A\cap B|C) \tag{4}$$

Also from the above equations we have

$$P(A|B\cap C)P(B|C) = \frac{P((A\cap B)\cap C)}{P(C)} \tag{5}$$

and solving for $P(P(A\cap B\cap C))$ we get the final result.

End Solution

1.2 *Gubner 1.68*

Solution

We know that $A,\ B$ are independent, i.e.,

$$P(A \cap B) = P(A)P(B), \tag{1}$$

and that $A,\ C$ are independent, i.e.,

$$P(A \cap C) = P(A)P(C). \tag{2}$$

From the fact that $C \subset B$ we have that

$$P(B) = P(C) + P(B \setminus C), \tag{3}$$

since C and $B \setminus C$ are disjoint.

For the independence of A and $B \setminus C = B \cap C^c$ we need to establish that

$$P(A \cap (B \setminus C)) = P(A)P(B \setminus C). \tag{4}$$

We have

$$\begin{aligned} P(A)P(B \cap C^c) &= P(A)[P(B) - P(C)] && (5)\\ &= P(A)P(B) - P(A)P(C) && (6)\\ &= P(A \cap B) - P(A \cap C), && (7) \end{aligned}$$

where the first equality is from (3), and the third equality is due to (1), and (2).

Consider now the set $A \cap B$. We can write

$$\begin{aligned} A \cap B &= (A \cap B \cap C) \cup (A \cap B \cap C^c) && (8)\\ &= (A \cap C) \cup (A \cap B \cap C^c), && (9) \end{aligned}$$

and since the last two sets are disjoint we have

$$P(A \cap B) = P(A \cap C) + P(A \cap B \cap C^c). \tag{10}$$

Substituting this equation into (11) we get

$$P(A)P(B \cap C^c) = P(A \cap B \cap C^c), \tag{11}$$

which establishes the independence of A and $B \cap C^c$.

End Solution

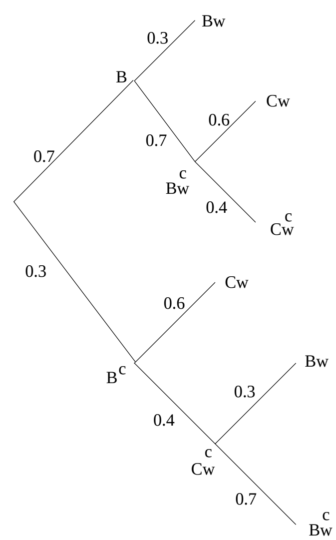
1.3 *Bo and Ci are the only two people who will enter the Rover Dog Fod jingle contest. Only one entry is allowed per contestant, and the judge (Rover) will declare the one winner as soon as he receivers a suitably inane entry, which may be never.*

Bo writes inane jingles rapidly but poorly. He has probabilitly 0.7 of submitting his entry first. If Ci has not already won the contest, Bo's entry will be declared the winner with probability 0.3. Ci writes slowly, but he has a gift for this sort of thing. If Bo has not already won the contest by the time of Ci's entry, Ci will be declared the winner with probability 0.6.

1. *What is the probability that the prize will never be awarded?*
2. *What is the probability that Bo will win?*
3. *Given that Bo wins, what is the probability that Ci's entry arrived first?*
4. *What is the probability that the first entry wins the contest?*
5. *Suppose that the probability that Bo's entry arrived first were P instead of 0.7. Can you find a value of P for which "First entry wins" and "Second entry does not win" are independent events?*

Solution

Let B be the event that Bo enters firsts. Then B^c is the event that Ci enters first. Let B_w be the event that Bo wins the contest and C_w be the event that Ci wins the contest. We have the following probability tree



1. The event that the prize is never awarded is $(B \cap B_w^c \cap C_w^c) \cup (B^c \cap C_w^c \cap B_w^c)$. Thus

$$\begin{aligned}
P(\text{no prize awarded}) &= P(B \cap B_w^c \cap C_w^c) + P(B^c \cap C_w^c \cap B_w^c) &(1)\\
&= P(B)P(B_w^c \mid B)P(C_w^c \mid B \cap B_w^c) + P(B^c)P(C_w^c \mid B^c)P(B_w^c \mid B^c \cap C_w^c) &(2)\\
&= (0.7)\cdot(0.7)\cdot(0.4) + (0.3)\cdot(0.4)\cdot(0.7) = \quad 0.28 &(3)
\end{aligned}$$

2. The event that Bo wins is given by $(B \cap B_w) \cup (B^c \cap C_w^c \cap B_w)$. Thus

$$
\begin{aligned}
P(\text{Bo wins}) &= P(B \cap B_w) + P(B^c \cap C_w^c \cap B_w) && (4)\\
&= P(B)P(B_w \,|\, B) + P(B^c)P(C_w^c \,|\, B^c)P(B_w \,|\, B \cap C_w^c) && (5)\\
&= (0.7) \cdot (0.3) + (0.3) \cdot (0.4) \cdot (0.3) = \quad 0.246 && (6)
\end{aligned}
$$

3.

$$
\begin{aligned}
P(\text{Ci entered first} \,|\, \text{Bo wins})) &= \frac{P(\text{ Ci entered first and Bo wins})}{P(\text{Bo wins})} && (7)\\
&= \frac{P(B^c \cap C_w^c \cap B_w)}{P(\text{Bo wins})} = \frac{(0.3) \cdot (0.4) \cdot (0.3)}{0.246} = 0.146 && (8)
\end{aligned}
$$

4.

$$
\begin{aligned}
P(\text{first entry wins}) &= P(B \cap B_w) + P(B^c \cap C_w) = P(B)P(B_w \,|\, B) + P(B^c)P(C_w \,|\, B^c) && (9)\\
&= (0.7) \cdot (0.3) + (0.3) \cdot (0.6) = \quad 0.39 && (10)
\end{aligned}
$$

5. Let E_1 be the event that "First entry wins" and E_2 be the event that "Second entry does not win". Note that $E_1 \subset E_2$. By definition, events E_1 and E_2 are independent if and only if $P(E_1 \cap E_2) = P(E_1)P(E_2)$. However, note that $E_1 \subset E_2$ implies that $P(E_1 \cap E_2) = P(E_1)$. Thus, events E_1 and E_2 are independent if and only if $P(E_1) = P(E_1)P(E_2)$. This is only possible if either $P(E_1) = 0$ or $P(E_2) = 1$. We check both of these cases as follows:

   (a) $P(E_1) = 0 \Leftrightarrow P \cdot (0.3) + (1 - P) \cdot (0.6) = 0 \Leftrightarrow P = 2$ which is not possible.

   (b) $P(E_2^c) = 0 \Leftrightarrow P \cdot (0.7) \cdot (0.6) + (1 - P) \cdot (0.4) \cdot (0.3)) = 0 \Leftrightarrow P = -0.4$ which is not possible.

   Thus we have shown that $P(E_1) \neq P(E_1)P(E_2)$ for all P, which means there is no value of P such that the two events are independent.

End Solution

1.4 *The disc containing the only copy of your term project just got corrupted, and the disc got mixed up with three other corrupted discs that were lying around. It is equally likely that any of the four discs holds the corrupted remains of your term project. Your computer expert friend offers to have a look, and you know from past experience that his probability of finding your term project from any disc is 0.4 (assuming the term project is there). Given that he searches on disc 1 but cannot find your thesis, what is the probability that your thesis is on disc i for $i = 1, 2, 3, 4$?*

Solution

Let D_i be the event that the thesis in disc i and
F be the event that your friend finds the thesis in *disc 1*. Now, $P(D_i) = 0.25,\ i = 1, 2, 3, 4$ and

$$P(F \mid D_1) = 0.4 \qquad P(F^c \mid D_1) = 0.6 \tag{1}$$
$$P(F \mid D_2) = 0 \qquad P(F^c \mid D_2) = 1 \tag{2}$$
$$P(F \mid D_3) = 0 \qquad P(F^c \mid D_3) = 1 \tag{3}$$
$$P(F \mid D_4) = 0 \qquad P(F^c \mid D_4) = 1 \tag{4}$$

We want to find $P(D_i \mid F^c)$ for $i = 1, 2, 3, 4$. Using Bayes' Rule

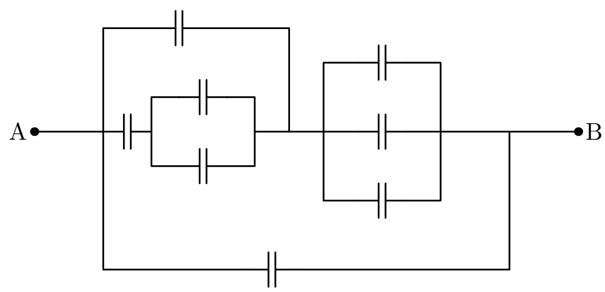
$$P(D_i \mid F^c) = \frac{P(D_i)P(F^c \mid D_i)}{\sum_{k=1}^{4} P(D_k)P(F^c \mid D_k)} \tag{5}$$

Substituting the values, we get

$$P(D_1 \mid F^c) = 0.1666 \tag{6}$$
$$P(D_2 \mid F^c) = P(D_3 \mid F^c) = P(D_4 \mid F^c) = 0.2777 \tag{7}$$

End Solution

1.5 *Each $-||-$ represents one communication link. Link failures are independent, and each link has a probabilty of 0.5 of being out of service. Towns A and B can communicate as long as they are connected in the communication network by a least one path which contains only in-service links. Determine, in an eficient manner, the probability that A and B can communicate.*



Solution

Consider a similar problem of two communication links in series, with failure probabilities p_1 and p_2. We can replace it by an equivalent communication link with failure probability p given by

$$p = P(G_1^c \cup G_2^c) = P(G_1^c) + P(G_2^c) - P(G_1^c \cap G_2^c) = p_1 + p_2 - p_1 p_2 \tag{1}$$

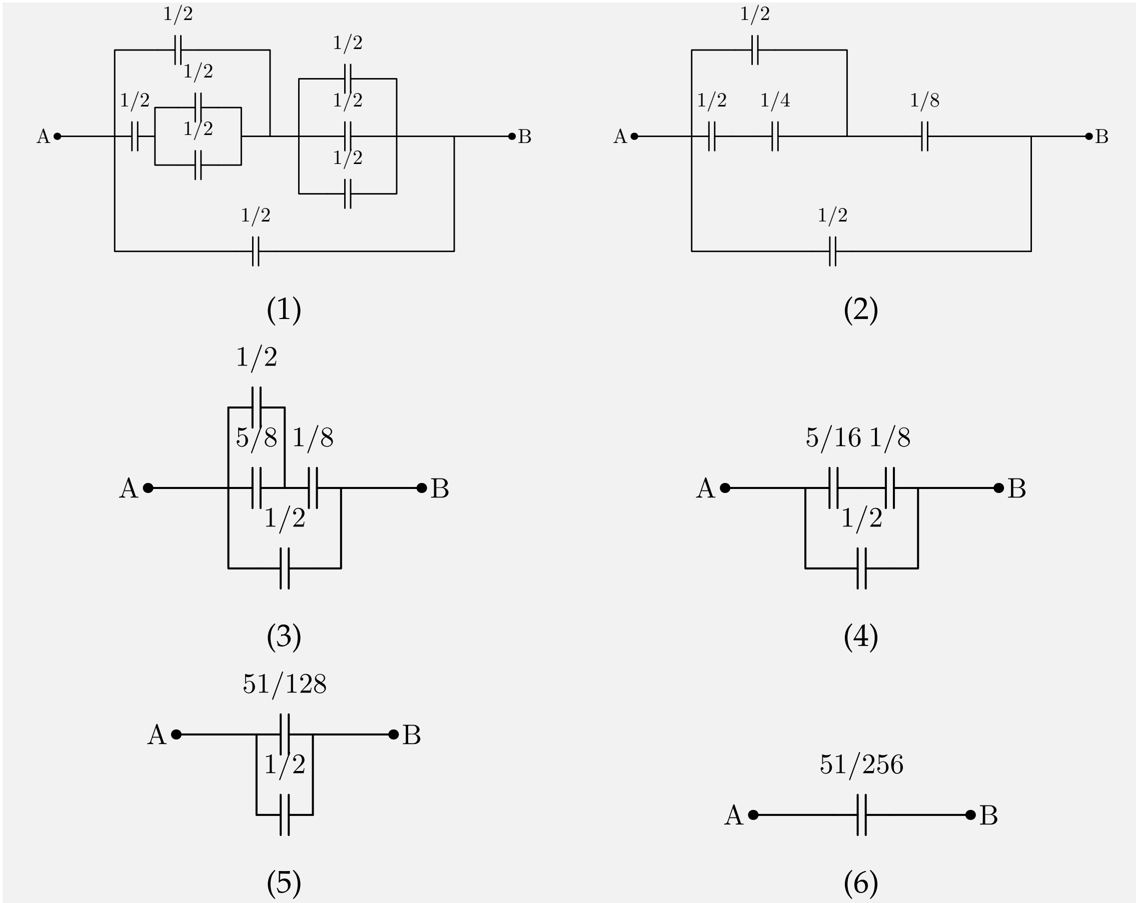
For two communication links in parallel, we can replace it by an equivalent communication link with failure probability p given by

$$p = P(G_1^c \cup G_2^c) = P(G_1^c)P(G_2^c) = p_1 p_2 \tag{2}$$

Similarly three communication links in parallel can be replaced by an equivalent communication link with failure probability p given by

$$p = p_1 p_2 p_3 \tag{3}$$

Thus we can simplify the network as follows



Thus the probability that A and B can communicate is $1 - \frac{51}{256} = \frac{205}{256} = 0.8008$

End Solution

1.6 *You enter a special kind of chess tournament, whereby you play one game with each of three opponents, but you get to choose the order in which you play your opponents. You win the tournament if you win two games in a row. You know your probability of a win against each of the three opponents. What is your probability of winning the tournament, assuming that you choose the optimal order of playing the opponents? Note that the underlying probability model is not explicitly given so you will have to define it. Give reasons why your model is a reasonable one.*

Solution

We assume the plays with different players are independent. Define W_i and L_i $(i = 1, 2, 3)$ as winning and losing against the ith players respectively, and let p_i denote the probability of wining against the ith player, then

$$\Omega = \{W_1W_2W_3, W_1W_2L_3, W_1L_2W_3, W_1L_2L_3, L_1W_2W_3, L_1W_2L_3, L_1L_2W_3, L_1L_2L_3\} \tag{1}$$

The probability law is

$$P(\{W_1W_2W_3\}) = p_1p_2p_3, \quad P(\{W_1W_2L_3\}) = p_1p_2(1-p_3), \ldots \tag{2}$$

We claim that the optimal order is to play the weakest player second (the order in which the other two opponents are played makes no difference). To see this, let q_i be the probability of winning against the opponent played in the ith turn. Then you will win the tournament if you win against the 2nd player (prob. q_2) and also you win against at least one of the two other players [prob. $q_1 + (1-q_1)q_3 = q_1 + q_3 - q_1q_3$]. Thus the probability of winning the tournament is

$$q_2(q_1 + q_3 - q_1q_3), \tag{3}$$

The order is optimal if and only if the above probability is no less than the probabilities corresponding to the two alternative orders:

$$q_2(q_1 + q_3 - q_1q_3) \geq q_1(q_2 + q_3 - q_2q_3), \tag{4}$$
$$q_2(q_1 + q_3 - q_1q_3) \geq q_3(q_2 + q_1 - q_2q_1) \tag{5}$$

It can be seen that the first inequality above is equivalent to $q_2 \geq q_1$, while the second inequality above is equivalent to $q_2 \geq q_3$.

End Solution

1.7 *A coin is tossed twice. Nick claims that the event of two heads is at least as likely if we know that the first toss is a head than if we know that at least one of the tosses is a head. Is he right? Does it make a difference if the coin is fair or unfair? How can we generalize Nick's reasoning? Note that the underlying probability model is not explicitly given so you will have to define it. Give reasons why your model is a reasonable one.*

Solution

The sample space is $\Omega = \{HH, HT, TH, TT\}$. A reasonable model to assume is that the two tosses are independent of each other and the probability of head in the first toss is the same as the probability of head in the second toss. Let us assume that $P(Head) = p$. Thus the probability of each event of the sample space is given by $P(\{HH\}) = p^2$, $P(\{HT\}) = P(\{TH\}) = p(1-p)$ and $P(\{TT\}) = (1-p)^2$. Let A be the event that the outcome is two heads, i.e. $A = \{HH\}$. Let B be the event that the first toss is a head, i.e. $B = \{HH, HT\}$. Let C be the event at least one of the tosses is head, i.e. $C = \{HH, HT, TH\}$. As stated in the problem, Nick claims that $P(A \mid B) \geq P(A \mid C)$. Now we can calculate both conditional probabilities to see if the claim is true or not.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{p^2}{p^2 + p(1-p)} \tag{1}$$

and

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{p^2}{p^2 + 2p(1-p)} \tag{2}$$

Now comparing the R.H.S of (1) with the R.H.S of (2), we observe that the numerators are the same, while the denominator of (1) is less than the denominator of (2). Thus, (1) greater than (2), that is,

$$P(A \mid B) \geq P(A \mid C) \tag{3}$$

Generalization: The above holds for any sets A, B, C such that $A \subseteq B \subseteq C$.

End Solution

1.8 ***Sequential madness!*** *Oscar has lost his dog in either forest A (with a priori probability* 0.4*) or in forest B (with a priori probability* 0.6*). If the dog is alive and not found by the* n^{th} *day of the search, it will die that evening with probability* $n/(n+2)$*.*

If the dog is in A (either dead or alive) and Oscar spends a day searching for it in A, the conditional probability that he will find the dog that day is 0.25*. Similarly, if the dog is in B and Oscar spends a day looking for it there, he will find the dog that day with probability 0.15.*

The dog cannot go from one forest to the other. Oscar can search only in the daytime, and he can travel from one forest to the other only at night.

All parts of this problem are to be worked separately.

1. *In which forest should Oscar look to maximize the probability he finds his dog on the first day of search?*
2. *Given that Oscar looked in A on the first day but didn't find his dog, what is the probability that the dog is in A?*
3. *If Oscar flips a fair coin to determine where to look on the first day and finds the dog on the first day, what is the probability that he looked in A?*
4. *Oscar has decided to look in A for the first two days. What is the a priori probability that he will find a live dog for the first time on the second day?*
5. *Oscar has decided to look in A for the first two days. Given the fact that he was unsuccessful on the first day, determine the probability that he does not find a dead dog on the second day.*
6. *Oscar finally found his dog on the fourth day of the search. He looked in A for the first 3 days and in B on the fourth day. What is the probability he found his dog alive?*
7. *Oscar finally found his dog on the fourth day of the search. If the only other thing we know is that he looked in A for 2 days and in B for 2 days. What is the probability that he found his dog alive?*

Hint: *Don't draw a tree! (it will be a big one). Instead consider the sequential nature of the experiment having in mind that each day* n *consists of two periods: morning and evening. In the morning the sequence of events are: first you make a decision where to search, then you search, then you observe the results of your search. Then in the evening the only thing that may happen is the dog dying.*

Solution

Define the following events.

$$A = \text{“the dog was lost in forest A”} \tag{1a}$$

$$A^c = \text{“the dog was lost in forest B”} \tag{1b}$$

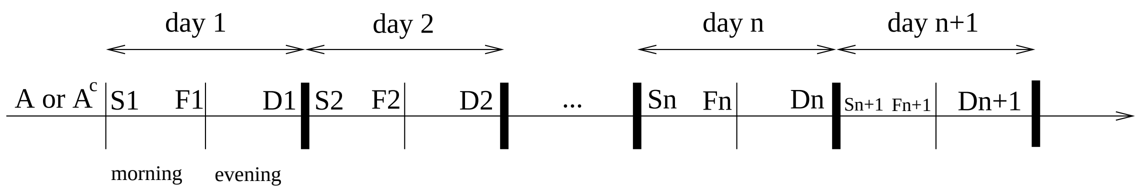
$$D_n = \text{“the dog dies at the evening of the } n\text{th day”} \tag{1c}$$

$$F_n = \text{“the dog is found on the } n\text{th day”} \tag{1d}$$

$$S_n = \text{“Oscar searches forest A on } n\text{th day”} \tag{1e}$$

$$S_n^c = \text{“Oscar searches forest B on } n\text{th day”}. \tag{1f}$$

In the following figure we sketch the sequence of events of interest. Please note that events are sets and so depicting them on a time scale is not appropriate. So, please interpret this sketch more as a way to understand the cause/effect relationship between events so that you can evaluate the correct conditional probabilities.



From the problem we can deduce the following

$$P(A) = 0.4 \tag{2a}$$

$$P(D_n | D_{n-1}^c, \cap_{i=1}^n F_i^c) = \frac{n}{n+2} \tag{2b}$$

$$P(F_n \,|\, A, S_n) = 0.25 \tag{2c}$$

$$P(F_n \,|\, A^c, S_n^c) = 0.15. \tag{2d}$$

Note: We very commonly use the notation $P(A|B,C) = P(A|B \cap C)$.

1. We want to find

$$P(F_1 \,|\, S_1) \gtreqless P(F_1 \,|\, S_1^c)$$

We'll use a version of the total probability law for conditional probabilities (recall conditional probabilities behave exactly as standard probabilities). So in the following total probability formula the event S_1 is given.

$$P(F_1 \,|\, S_1) = P(F_1 \,|\, A, S_1)P(A|S_1) + \underbrace{P(F_1 \,|\, A^c, S_1)}_{=0} P(A^c|S_1) \tag{3a}$$

$$= P(F_1 \,|\, A, S_1)P(A) \qquad \text{(independence of } A \text{ and } S_1) \tag{3b}$$

$$= 0.25 \times 0.4 \tag{3c}$$

$$= 0.1 \tag{3d}$$

Similarly (conditioned on S_1^c)

$$P(F_1 \,|\, S_1^c) = \underbrace{P(F_1 \,|\, A, S_1^c)}_{=0} P(A|S_1^c) + P(F_1 \,|\, A^c, S_1^c)P(A^c|S_1^c) \tag{4a}$$

$$= P(F_1 \,|\, A^c, S_1^c)P(A^c) \qquad \text{(independence of } A^c \text{ and } S_1^c) \tag{4b}$$

$$= 0.15 \times 0.6 \tag{4c}$$

$$= 0.09 \tag{4d}$$

Thus $P(F_1 \,|\, S_1) > P(F_1 \,|\, S_1^c)$, so Oscar should search in forest A.

Note: If you didn't want to use total probability you could also write (eg for the first one) $F_1 = (F_1 \cap A) \cup (F_1 \cap A^c)$ which is the union of two disjoint events, and thus

$$\begin{aligned}
P(F_1 \mid S_1) &= P(F_1 \cap A|S_1) + \underbrace{P(F_1 \cap A^c|S_1)}_{=0} && \text{(5a)}\\
&= \frac{P(F_1 \cap A \cap S_1)}{P(S_1)} \qquad \text{(def. of cond. prob.)} && \text{(5b)}\\
&= \frac{P(F_1|A \cap S_1)P(A|S_1)P(S_1)}{P(S_1)} \qquad \text{(chain rule)} && \text{(5c)}\\
&= P(F_1 \mid A, S_1)P(A|S_1), && \text{(5d)}
\end{aligned}$$

and then proceed as above.

2. We'll use a version of the Bayes' law for conditional probabilities (recall conditional probabilities behave exactly as standard probabilities). So in the following Bayes formula the event S_1 is given.

$$\begin{aligned}
P(A \mid F_1^c, S_1) &= \frac{P(F_1^c \mid A, S_1)P(A|S_1)}{P(F_1^c \mid S_1)} && \text{(6a)}\\
&= \frac{P(F_1^c \mid A, S_1)P(A)}{1 - P(F_1 \mid S_1)} \qquad \text{(independence of } A \text{ and } S_1) && \text{(6b)}\\
&= \frac{(1 - 0.25) \times 0.4}{1 - 0.1} && \text{(6c)}\\
&= 0.333\ldots && \text{(6d)}
\end{aligned}$$

3. Here we are given that $P(S_1) = P(S_1^c) = 0.5$. We want to find $P(S_1 \mid F_1)$. We can work as follows using Bayes' law

$$\begin{aligned}
P(S_1 \mid F_1) &= \frac{P(F_1|S_1)P(S_1)}{P(F_1|S_1)P(S_1) + P(F_1|S_1^c)P(S_1^c)} && \text{(7a)}\\
&= \frac{0.1 \times 0.5}{0.1 \times 0.5 + 0.09 \times 0.5} && \text{(7b)}\\
&= 0.5263. && \text{(7c)}
\end{aligned}$$

4. Probability that Oscar will find a live dog for the first time on the second day is

$$
\begin{aligned}
P(D_1^c \cap F_2 | S_1, S_2) &= P(D_1^c \cap F_2 \cap F_1^c | S_1, S_2) && \text{(8a)} \\
&= P(D_1^c \cap F_2 \cap F_1^c | S_1, S_2, A) P(A | S_1, S_2) \\
&\quad + \underbrace{P(D_1^c \cap F_2 \cap F_1^c | S_1, S_2, A^c)}_{=0} P(A^c | S_1, S_2) && \text{(8b)} \\
&= P(D_1^c \cap F_2 | S_1, S_2, A, F_1^c) P(F_1^c | S_1, S_2, A) P(A) && \text{(8c)} \\
&= P(F_2 | S_1, S_2, A, F_1^c, D_1^c) P(D_1^c | S_1, S_2, A, F_1^c) P(F_1^c | S_1, A) P(A) && \text{(8d)} \\
&= P(F_2 | S_2, A) P(D_1^c | F_1^c) P(F_1^c | S_1, A) P(A) && \text{(8e)} \\
&= 0.25 \times (1 - \frac{1}{1+2}) \times 0.75 \times 0.4 && \text{(8f)} \\
&= 0.05, && \text{(8g)}
\end{aligned}
$$

where $P(D_1^c \cap F_2 \cap F_1^c | S_1, S_2, A^c) = 0$ since the dog cannot be found if he is in B and you search in A; the third equality is due to decomposition of an intersection of events and the fact that A and $S_1 \cap S_2$ are independent; and the remaining equalities are due to independence of events and standard "causal" decomposition of intersection of events.

5. Probability that Oscar does not find a dead dog at the end of the second day is $P\Big((F_2 \cap D_1)^c | F_1^c, S_1, S_2\Big) = 1 - P(F_2 \cap D_1 | F_1^c, S_1, S_2)$. Following the logic of earlier solutions, we'll introduce the event A through a total probability law and then interchange the position of events F_1^c and A through Bayes law as follows

$$
\begin{aligned}
P(F_2 \cap D_1 | F_1^c, S_1, S_2) &= P(F_2 \cap D_1 \cap A | F_1^c, S_1, S_2) + \underbrace{P(F_2 \cap D_1 \cap A^c | F_1^c, S_1, S_2)}_{=0} && \text{(9a)} \\
&= \frac{P(F_2 \cap D_1 \cap A \cap F_1^c | S_1, S_2)}{P(F_1^c | S_1, S_2)}. && \text{(9b)}
\end{aligned}
$$

Regarding the numerator we have

$$
\begin{aligned}
P(F_2 \cap D_1 \cap A \cap F_1^c | S_1, S_2) &= P(F_2 | D_1, F_1^c, A, S_1, S_2) \\
&\quad P(D_1 | F_1^c, A, S_1, S_2) P(F_1^c | A, S_1, S_2) P(A) && \text{(10a)} \\
&= P(F_2 | S_2, A) P(D_1 | F_1^c) P(F_1^c | A, S_1) P(A) && \text{(10b)} \\
&= 0.25 \times \frac{1}{3} \times 0.75 \times 0.4. && \text{(10c)}
\end{aligned}
$$

For the denominator we have

$$
P(F_1^c | S_1, S_2) = P(F_1^c | S_1) = 1 - 0.1 = 0.9, \quad \text{(11a)}
$$

which is a result of the first question. So finally we have $P\Big((F_2 \cap D_1)^c | F_1^c, S_1, S_2\Big) = 1 - \frac{0.25 \times \frac{1}{3} \times 0.75 \times 0.4}{0.9} = 0.9722$.

6. Probability that the dog is alive conditioned on the given information is

$$
\begin{aligned}
&P(D_3^c \,|\, F_4, F_3^c, F_2^c, F_1^c, S_1, S_2, S_3, S_4^c) \\
&\quad = P(D_3^c \cap D_2^c \cap D_1^c | F_4, F_3^c, F_2^c, F_1^c, S_1, S_2, S_3, S_4^c) && \text{(12a)}\\
&\quad = P(D_3^c | D_2^c, D_1^c, F_4, F_3^c, F_2^c, F_1^c, S_1, S_2, S_3, S_4^c) && \text{(12b)}\\
&\qquad \times P(D_2^c | D_1^c, F_4, F_3^c, F_2^c, F_1^c, S_1, S_2, S_3, S_4^c) && \text{(12c)}\\
&\qquad \times P(D_1^c | F_4, F_3^c, F_2^c, F_1^c, S_1, S_2, S_3, S_4^c) && \text{(12d)}\\
&\quad = P(D_3^c | D_2^c, D_1^c, F_3^c) P(D_2^c | D_1^c, F_2^c) P(D_1^c | F_1^c) && \text{(12e)}\\
&\quad = (1 - \frac{3}{5})(1 - \frac{2}{4})(1 - \frac{1}{3}) = \frac{2}{15} = 0.13333. && \text{(12f)}
\end{aligned}
$$

7. Let L be the event that he searches twice in forest A and twice in forest B. Then

$$
\begin{aligned}
&P(D_3^c | F_4, F_3^c, F_2^c, F_1^c, L) \\
&\quad = P(D_3^c \cap D_2^c \cap D_1^c | F_4, F_3^c, F_2^c, F_1^c, L) && \text{(13a)}\\
&\quad = P(D_3^c | D_2^c, D_1^c, F_4, F_3^c, F_2^c, F_1^c, L) \\
&\qquad \times P(D_2^c | D_1^c, F_4, F_3^c, F_2^c, F_1^c, L) \\
&\qquad \times P(D_1^c | F_4, F_3^c, F_2^c, F_1^c, L) && \text{(13b)}\\
&\quad = P(D_3^c | D_2^c, D_1^c, F_3^c) P(D_2^c | D_2^c, F_2^c) P(D_1^c | F_1^c) && \text{(13c)}\\
&\quad = (1 - \frac{3}{5})(1 - \frac{2}{4})(1 - \frac{1}{3}) = \frac{2}{15} = 0.13333, && \text{(13d)}
\end{aligned}
$$

which is the same as in the previous question. The reason is that conditioned on the sequence of events describing the finding or not of the dog, the events of the dog dying and where we search are independent.

End Solution