EECS 455: Problem Set 10
Submit via Gradescope via link on Canvas

Due: Wednesday, December 8, 2021, 11pm.

1. A (7,5) Reed Solomon code is used on a channel. The code operates on the finite field $GF(2^3)$ shown below using the polynominal $\alpha^3 = \alpha + 1$. The code has minimum distance 3 and can correct 1 error.

$-$	0	$(0,0,0)$
1	1	$(0,0,1)$
α	α	$(0,1,0)$
α^2	α^2	$(1,0,0)$
α^3	$\alpha+1$	$(0,1,1)$
α^4	$\alpha^2+\alpha$	$(1,1,0)$
α^5	$\alpha^2+\alpha+1$	$(1,1,1)$
α^6	α^2+1	$(1,0,1)$

The code has generator polynomial

$$\begin{aligned} g(x) &= (x-\alpha)(x-\alpha^2) \\ &= x^2 - (\alpha+\alpha^2)x + \alpha^3 \\ &= x^2 + \alpha^4 x + \alpha^3 \end{aligned}$$

A codeword is generated from an information polynomial $i(x)$ by

$$c(x) = g(x)i(x)$$

If $r(x) = x^6 + \alpha^4 x^5$ determine the codeword transmitted.

2. A (7,5) Reed Solomon code is used on a channel. The code operates on the finite field $GF(2^3)$ shown below using the polynominal $\alpha^3 = \alpha + 1$. The code has minimum distance 3 and can correct 1 error.

$-$	0	$(0,0,0)$
1	1	$(0,0,1)$
α	α	$(0,1,0)$
α^2	α^2	$(1,0,0)$
α^3	$\alpha+1$	$(0,1,1)$
α^4	$\alpha^2+\alpha$	$(1,1,0)$
α^5	$\alpha^2+\alpha+1$	$(1,1,1)$
α^6	α^2+1	$(1,0,1)$

The code has generator polynomial

$$\begin{aligned} g(x) &= (x-\alpha)(x-\alpha^2) \\ &= x^2 - (\alpha+\alpha^2)x + \alpha^3 \\ &= x^2 + \alpha^4 x + \alpha^3 \end{aligned}$$

A codeword is generated from an information polynomial $i(x)$ by

$$c(x) = g(x)i(x)$$

The channel drops two symbols (erases them) but the rest of the symbols are received correctly with no errors. If the received vector is $r(x) = ?x^4 + ?x^2 + \alpha^5 x + \alpha^3$ determine the codeword transmitted.

3. Consider a Reed Solomon code with 24 information symbols and 28 coded symbols. So there are 4 redundant symbols. The distance of the code is $d = n - k + 1 = 28 - 24 + 1 = 5$. So the code can correct up to 2 errors. Each symbol is an eight bit byte. Each 8 bit symbol is transmitted over a binary symmetric channel with crossover probability p. Errors occur independently from one bit to the next. Assume that if more than 2 symbol errors occur the decder fails. Determine the probability that the decoder fails. Plot this as a function of p for p. The vertical scale (for probability of decoder failure) should be between 10^{-10} and 1. The horizontal scale between 10^{-6} and 1.

4. Consider the rate 1/3 convolutional code with memory 2 and generator polynomials

$$\begin{aligned} g_0 &= [101] \\ g_1 &= [111] \\ g_2 &= [111] \end{aligned}$$

The input to the encoder is a sequence of 5 information bits followed by two zeros in order to clear out the encoder. For each input bit to the encoder the encoder output is first the output from generator g_0, then from g_1, then from g_2. This repeats for every input bit.

(a) Suppose the information bits are $u = [01101]$. Find the output of the encoder. The output should be a vector of length $3(5+2) = 21$.

(b) Draw the state transition diagram for the encoder and label the output the branches by the input and output.

(c) Draw the trellis diagram from one state to the next labeling the transition with the input bit and the output bits.

(d) The transmitted signal is found by encoding the information, mapping the bits to levels ± 1 ($0 \rightarrow +1$, $1 \rightarrow -1$). Suppose the received signal is

$$\begin{array}{l||l||l||l} rcvd[0] = -2.56 & rcvd[3] = +0.37 & rcvd[6] = +1.13 & rcvd[9] = +4.51 \\ rcvd[1] = +1.69 & rcvd[4] = -0.14 & rcvd[7] = +1.05 & rcvd[10] = +2.17 \\ rcvd[2] = -1.56 & rcvd[5] = +2.19 & rcvd[8] = +1.78 & rcvd[11] = -0.41 \end{array}$$

$$\begin{array}{l||l||l} rcvd[12] = -0.46 & rcvd[15] = +0.93 & rcvd[18] = +0.13 \\ rcvd[13] = -0.92 & rcvd[16] = +1.54 & rcvd[19] = -1.66 \\ rcvd[14] = +0.68 & rcvd[17] = +2.34 & rcvd[20] = -0.82 \end{array}$$

Find the most likely information sequence at the input to the encoder.

(e) If the demodulator made a hard decision on each bit find the most likely encoder input (for the same sequence received out of the demodulator except that a hard decision is made on each of the signals received whether it corresponds to a +1 or -1).