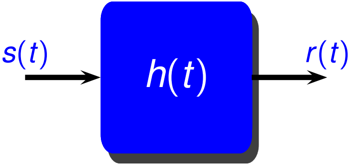
Lecture Notes 14

Goals

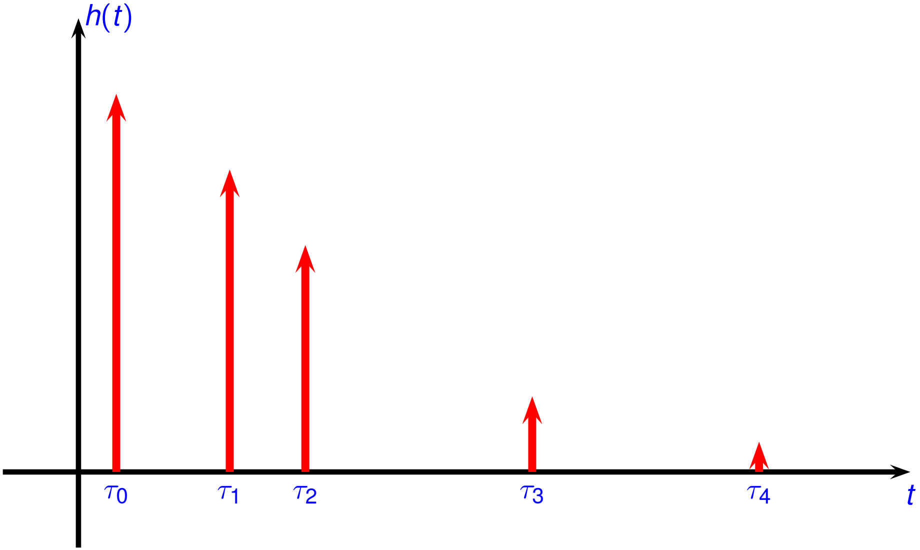
- Multipath (Fast) Fading
- Understand Orthogonal Frequency Division Multiplexing (OFDM)
- Understand various applications of OFDM

OFDM with Multipath

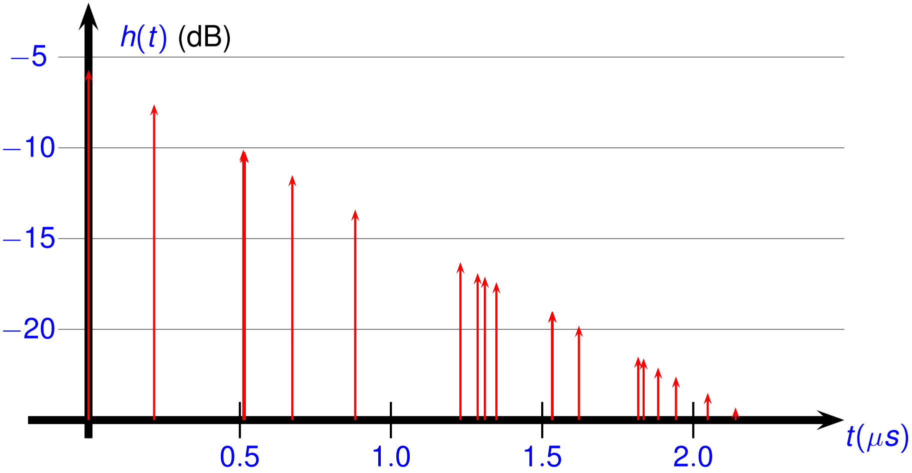


$$
\begin{aligned}
h(t) &= \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l) \\
H(f) &= \sum_{l=0}^{L-1} \alpha_l \exp(-j2\pi f \tau_l)
\end{aligned}
$$

OFDM with Multipath

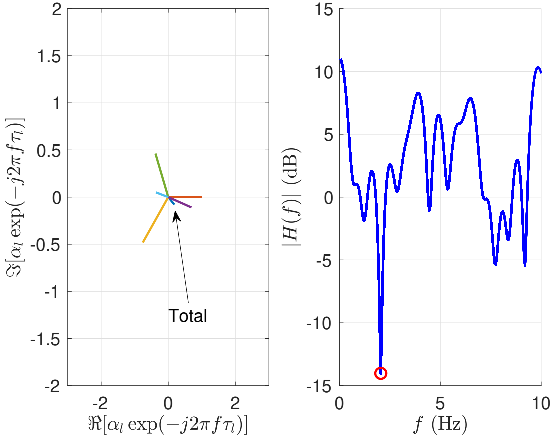


3GPP Standard: Typical Urban Multipath

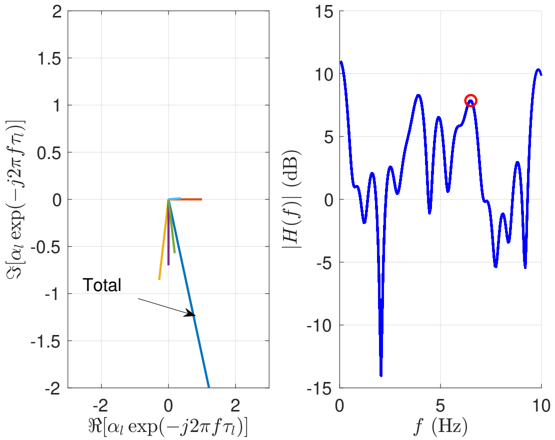


Reference: http://www.quintillion.co.jp/3GPP/Specs/25943-600.pdf

Multipath

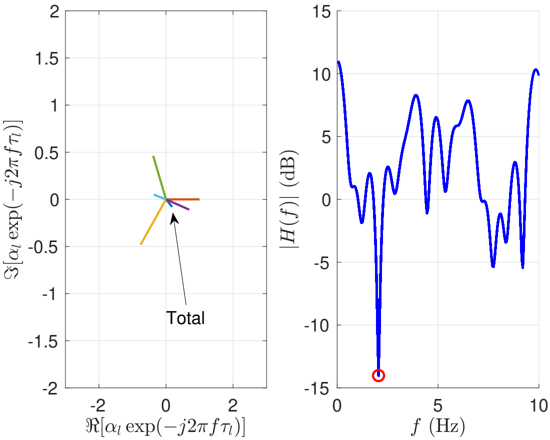

Total
$\Re[\alpha_l \exp(-j2\pi f \tau_l)]$
$\Im[\alpha_l \exp(-j2\pi f \tau_l)]$
$|H(f)|$ (dB)
f (Hz)

Multipath



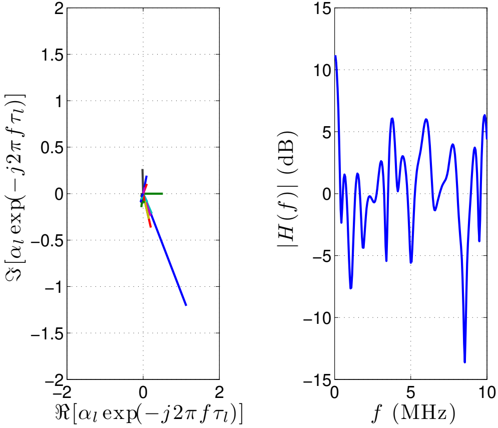
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Total
-2
0
2
$\Re[\alpha_l \exp(-j2\pi f \tau_l)]$
$\Im[\alpha_l \exp(-j2\pi f \tau_l)]$
15
10
5
0
-5
-10
-15
0
5
10
f (Hz)
$|H(f)|$ (dB)

Multipath



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
0
2
$\Im[\alpha_l \exp(-j2\pi f \tau_l)]$
$\Re[\alpha_l \exp(-j2\pi f \tau_l)]$
Total
15
10
5
0
-5
-10
-15
0
5
10
$|H(f)|$ (dB)
f (Hz)

Multipath



2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
0
2
$\Im[\alpha_l \exp(-j2\pi f \tau_l)]$
$\Re[\alpha_l \exp(-j2\pi f \tau_l)]$
15
10
5
0
−5
−10
−15
0
5
10
$|H(f)|$ (dB)
f (MHz)

- The coherence bandwidth B_c of a channel is the bandwidth such that the frequency response of the channel does not change significantly. That is $H_c(f_1) \approx H_c(f_2)$ for $|f_2 - f_1| < B_c$.
- In a frequency nonselective model the bandwidth of the signal is less than the coherence bandwidth of the channel.
- Consider a complex lowpass representation of a bandpass signal. That is

$$s(t) = \Re[s_0(t)\sqrt{2}e^{j2\pi f_c t}]$$

  where $s_0(t)$ is the lowpass complex representation.
- The fact that the bandwidth of $s(t)$ is less than the coherence bandwidth, B_c means that $s_0(t)$, does not vary much in a time duration of $\max \tau_j < 1/B_c$. That is,

$$s_0(t - \tau_j) \approx s_0(t) \text{ for } \max \tau_j < 1/B_c$$

The received signal is

$$
\begin{aligned}
r(t) &= \sum_{l=0}^{L-1} \alpha_l s(t-\tau_l) \\
&= \sum_{l=0}^{L-1} \alpha_l \Re[s_0(t-\tau_l)\sqrt{2}e^{j2\pi f_c(t-\tau_l)}] \\
&= \Re[\sum_{l=0}^{L-1} \alpha_l s_0(t-\tau_l)e^{-j2\pi f_c \tau_j}\sqrt{2}e^{j2\pi f_c t}] \\
&\approx \Re[\sum_{l=0}^{L-1} \alpha_l s_0(t)e^{-j2\pi f_c \tau_j}\sqrt{2}e^{j2\pi f_c t}] \\
&= \Re[s_0(t)[\sum_{l=0}^{L-1} \alpha_l e^{-j2\pi f_c \tau_j}]\sqrt{2}e^{j2\pi f_c t}]
\end{aligned}
$$

- Letting $r_0(t)$ be the lowpass complex representation of the received signal we see that

$$r_0(t) = s_0(t)[\sum_{l=0}^{L-1} \alpha_l e^{-j2\pi f_c \tau_j}]$$

- This shows that the effect of multipath fading in a frequency nonselective environment is to change the amplitude and phase of the signal but not distort the signal.

- For a reasonably large (e.g. more than 5) paths the factor

$$X = \sum_{l=0}^{L-1} \alpha_l e^{-j2\pi f_c \tau_j}$$

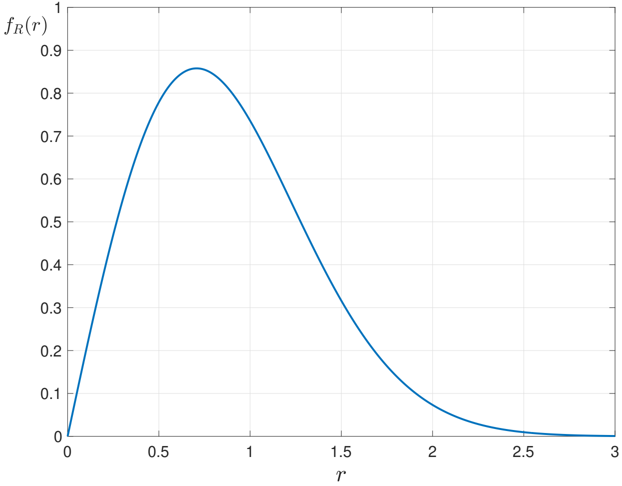
can be approximated by a complex Gaussian random variable.
- That is, the real part and the imaginary part are Gaussian. This is a result of the central limit theorem.

- The magnitude of a complex Gaussian random variable X is a Rayleigh distributed random variable R with density

$$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/(2\sigma^2)}, \quad r \geq 0.$$

- Here σ^2 is the variance of the real part and imaginary part of X.
- The distribution of R is due to the fact that sometimes the paths add constructively and sometimes the paths add destructively.
- Typically, the fading level is normalized to have $E[R^2] = 1$. This normalization means that $2\sigma^2 = 1$ or $\sigma^2 = 1/2$.

Rayleigh density function, $\sigma^2 = 1/2$.



Performance Analysis

- First consider a modulator transmitting a BPSK signal with rectangular pulses and received with a faded amplitude. The transmitted signal is

$$s(t) = \sqrt{2P}b(t)\cos(2\pi f_c t)$$

  where $b(t)$ is the usual data bit signal consisting of a sequence of rectangular pulses of amplitude +1 or -1.
- That is,

$$b(t) = \sum_l b_l p_T(t - lT).$$

- The received signal is

$$r(t) = R\sqrt{2P}b(t)\cos(2\pi f_c t + \theta) + n(t)$$

Performance Analysis

- Assuming the receiver can accurately estimate the phase θ the coherent demodulator (matched filter) output at time kT is

$$z_k = R\sqrt{E}b_{k-1} + \eta_k$$

where $E = PT$.

- The random variable R represents the fading and has density

$$p_R(r) = \begin{cases} 0, & r < 0 \\ \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} & r \geq 0 \end{cases}$$

- The conditional error probability (conditioned on the value of R) is

$$P_e(R) = Q\left(\sqrt{\frac{2ER^2}{N_0}}\right).$$

Performance Analysis

Let $\alpha = 2E/N_0$, $\beta = \sigma^2\alpha = \bar{E}/N_0$ and

$$\begin{aligned} \gamma &= \sqrt{\frac{\beta}{1+\beta}} \\ &= \sqrt{\frac{\bar{E}/N_0}{1+\bar{E}/N_0}} \end{aligned}$$

Performance Analysis

The unconditional error probability is

$$
\begin{aligned}
P_e &= \int_{r=0}^{\infty} p_R(r) Q(\sqrt{\frac{2Er^2}{N_0}})dr \\
&= \int_{r=0}^{\infty} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} Q(\sqrt{\frac{2Er^2}{N_0}})dr \\
&= \int_{r=0}^{\infty} \frac{r}{\sigma^2} \int_{u=\sqrt{\alpha r^2}}^{\infty} e^{-r^2/2\sigma^2} \frac{\exp\{-u^2/2\}}{\sqrt{2\pi}} dudr \\
&= \int_{u=0}^{\infty} \frac{\exp\{-u^2/2\}}{\sqrt{2\pi}} \int_{r=0}^{u/\sqrt{\alpha}} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} dudr \\
&= \int_{u=0}^{\infty} \frac{\exp\{-u^2/2\}}{\sqrt{2\pi}} \int_{r=0}^{u/\sqrt{\alpha}} \frac{r}{\sigma^2} \exp\{-r^2/2\sigma^2\} drdu \\
&= \int_{u=0}^{\infty} \frac{\exp\{-u^2/2\}}{\sqrt{2\pi}} (1 - \exp\{-u^2/(2\alpha\sigma^2)\}) du \\
&= \frac{1}{2} - \int_{u=0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\{-\frac{u^2}{2}(1+\frac{1}{\alpha\sigma^2})\} du \\
&= \frac{1}{2} - \gamma \int_{u=0}^{\infty} \frac{1}{\gamma\sqrt{2\pi}} \exp\{-\frac{u^2}{2\gamma^2}\} du \\
&= \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\bar{E}/N_0}{1+\bar{E}/N_0}}
\end{aligned}
$$

Performance Analysis

- The last integral is evaluated by recognizing the integrand to be a Gaussian density function with zero mean which when integrated from 0 to ∞ is 1/2.
- For large E/N_0 the error probability is approximately

$$P_e \simeq \frac{1}{4E/N_0}.$$

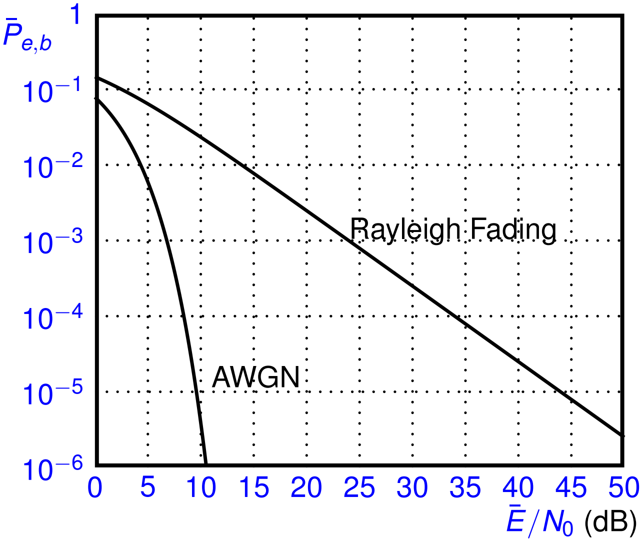


Figure: Bit error probability for BPSK with Rayleigh Fading

Performance Analysis

- Thus for high E/N_0 the error probability decreases inverse linearly with signal-to-noise ratio.
- To achieve error probability of 10^{-5} requires a signal-to-noise ratio of 44.0dB whereas in additive white Gaussian noise the required signal-to-noise ratio for the same error probability is 9.6dB.
- Thus fading causes a loss in signal-to-noise ratio of 34.4dB.
- This loss in performance is at the same average received power.

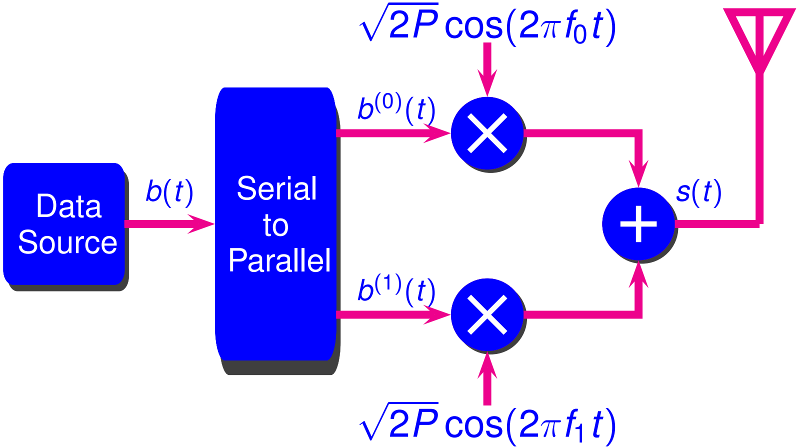
Performance Analysis

- The cause of this loss is the fact that the signal amplitude sometimes is very small and causes the error probability to be close to 1/2.
- Of course, sometimes the signal amplitude is large and results in very small error probability (say 0).
- However when we average the error probability the result is going to be much larger than the error probability at the average signal-to-noise ratio because of the nonlinear nature of the error probability in just Gaussian noise without fading as a function of signal amplitude.
- The way to mitigate the effect of fading is through diversity. Examples of diversity include 1) time diversity, 2) frequency diversity, 3) space diversity or suitable combinations.
- Diversity means transmitting the same information multiple times at different times, frequencies, or with different antennas where each transmission experiences independent fading.

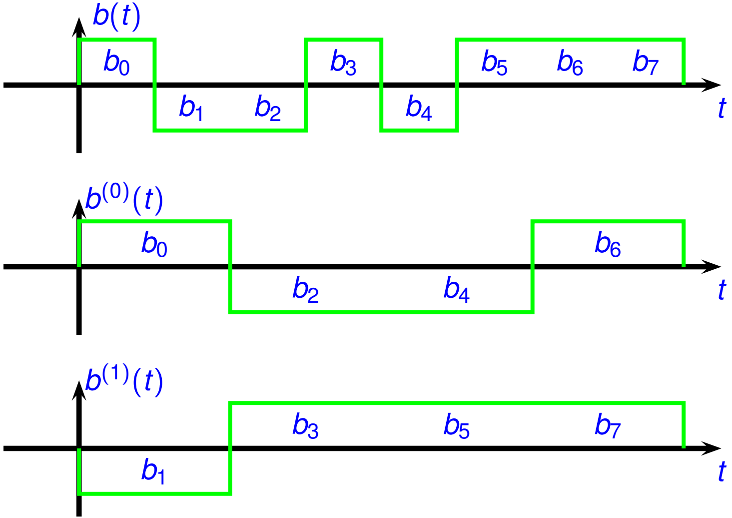
Orthogonal Frequency Division Multiplexing (OFDM)

- OFDM is a method of multiplexing several different modulated carriers into one signal
- The different carriers are separated in frequency by an amount to make the different carriers orthogonal
- OFDM has several benefits
  - Narrower spectrum than a single carrier system with the same data rate
  - Intersymbol interference from frequency selective fading is mitigated using a sufficient guard interval without an equalizer
- Also called multi-carrier modulation, discrete multi-tone modulation (DTM)
- Used in DSL, 802.11a, LTE,...

Transmitter for two carriers



Serial-to-Parallel Converter



Orthogonal Frequency Division Multiplexing (OFDM)

Consider the time interval $[0, T]$ where the input to the mixers are b_0 and b_1. The transmitted signal is then

$$\begin{aligned} s(t) &= \sqrt{2P}\left(\Re\left[b_0 \exp(j2\pi f_0 t) + b_1 \exp(j2\pi f_1 t)\right]\right),\ 0 \le t \le T \\ &= \sqrt{2P}\left(\Re\left[\sum_{i=0}^{1} b_i \exp(j2\pi f_i t)\right]\right),\ 0 \le t \le T \end{aligned}$$

Duration of each data

where $\Re(x)$ is the real part of the complex number x.

Data bit duration at input of S/P conv is T_b

Data duration at output of of S/P conv is NT_b => Bandwidth is reduced by N

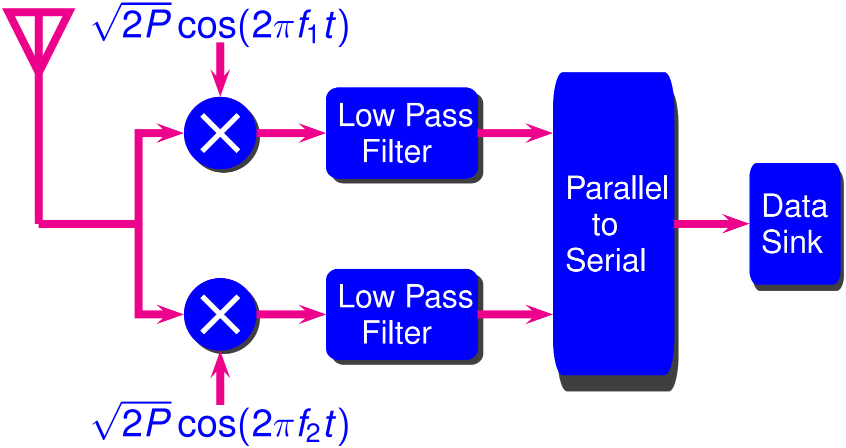
$R = \frac{2}{T}$ (2 bits last in two second)

if ($R = \frac{N}{T}$) ← bits

Orthogonal Frequency Division Multiplexing (OFDM)

- Serial-to-parallel converter reduces the bit rate on each output by factor equal to the number of carriers.
- Addition of two modulated carriers with different frequencies creates a nonconstant envelope signal.
- Frequency separation between two carriers sufficient to make the signals, over a symbol duration orthogonal.

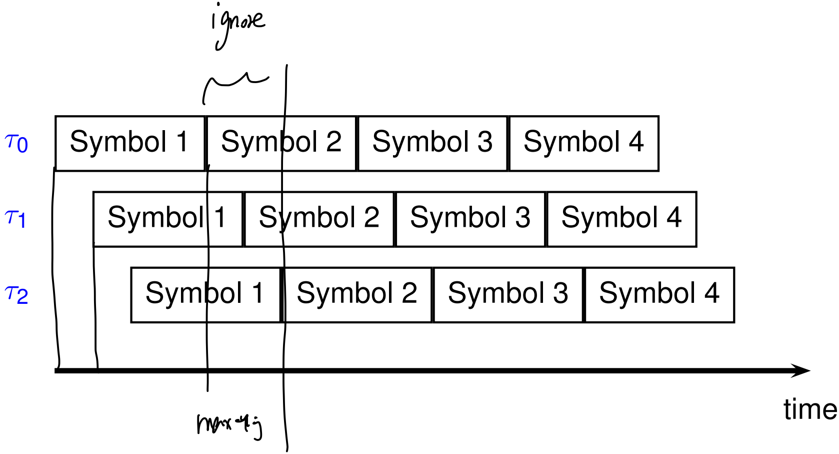
Receiver for two carriers



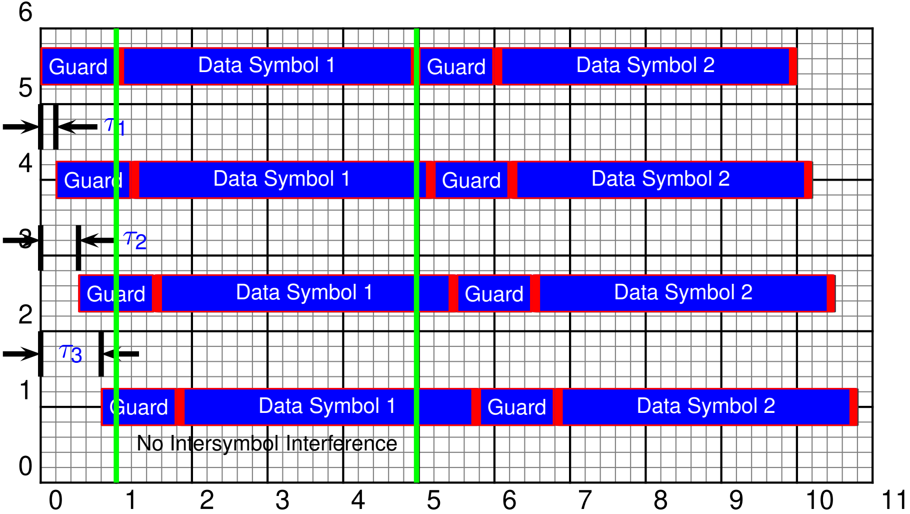
OFDM

- Two data streams transmitted on two orthogonal carriers.
- The output (ideally) of each would only depend on the corresponding transmitted signal at that frequency.
- The bandwidth of each carrier is relatively narrow so there is not significant distortion due to frequency selective fading.
- Guard intervals (cyclic prefix) are added to mitigate the multipath interference from one data symbol to the next data symbol.

OFDM with Multipath



OFDM with Multipath

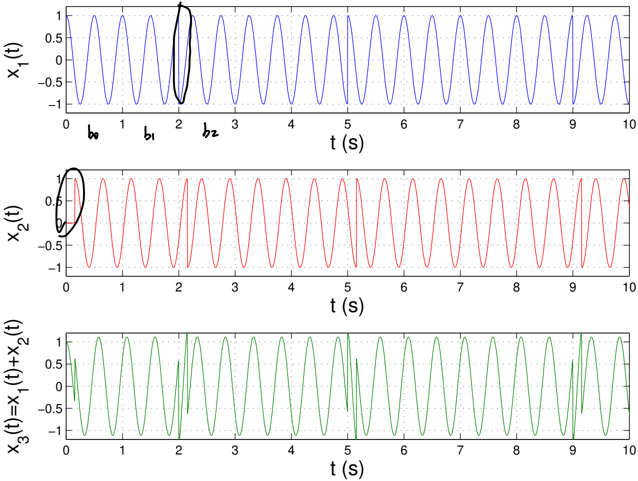


OFDM with Multipath

change data bit

$\tau_0 = 0$

$\tau_1 \neq 0$

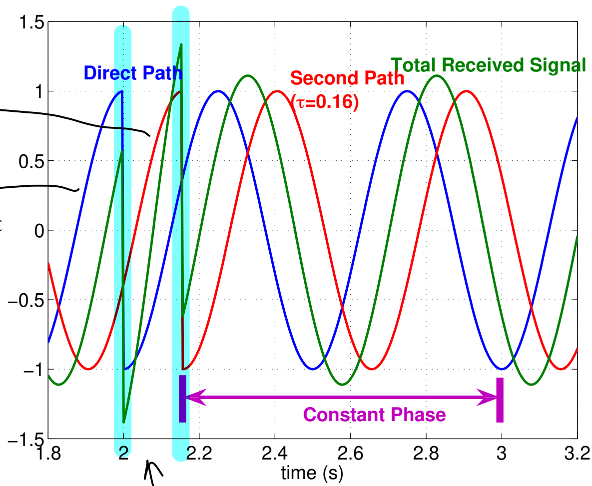


OFDM with Multipath

ISI

second path
from first symbol

direct path
from second symbol



Orthogonal Frequency Division Multiplexing (OFDM)

- Multipath propagation causes intersymbol interference because the delay spread is significant relative to the bit duration.
- By making the bit duration longer the effect of intersymbol interference can be decreased. Ignore small window
- Longer bit duration reduces the bandwidth and the data rate.
- Adding modulated carriers at other frequencies allows the data rate to be increased.

Orthogonal Frequency Division Multiplexing (OFDM)

- Different carriers are chosen so that one carrier does not interfere with other carriers.
- Another way of looking at OFDM is that the bandwidth of each carrier is narrow enough so that the channel frequency response does not change significantly.
- IFFT produces signals that are orthogonal. Need extra time to let multipath die out.
- OFDM has worse peak-to-average power ratio compared to single carrier (unfiltered system).

OFDM Signals

- Consider using I-Q modulation (two dimensional) on each carrier.
- Let $(I_k + jQ_k)$ represent the complex data symbol transmitted on each carrier for the data interval $[0, T]$.
- If on each carrier we use both cosine and sine, then for each carrier the generated signal for the interval $[0, T]$ is

$$\begin{aligned} s_k(t) &= [I_k \cos(2\pi k f_0 t) - Q_k \sin(2\pi k f_0 t)] \\ &= \Re\left[(I_k + jQ_k)\exp(j2\pi k f_0 t)\right]. \end{aligned}$$

$k f_0$ $(k+1) f_0$

- The total transmitted signal is then

$$\begin{aligned} s(t) &= \sum_{k=0}^{N-1} s_k(t) \\ &= \Re\left[\sum_{k=0}^{N-1} (I_k + jQ_k)\exp(j2\pi k f_0 t)\right] \end{aligned}$$

OFDM Signals

Consider now generating samples of this signal. Assume we generate N samples during the time interval $[0, T]$. Suppose also that $f_0 T = 1$. Then

$$
\begin{aligned}
s[n] &= s((n/N)T)) = \Re\underbrace{\left[\sum_{k=0}^{N-1}(I_k + jQ_k)\exp(j2\pi k f_0 (n/N)T)\right]} \\
&= \Re\left[\sum_{k=0}^{N-1}(I_k + jQ_k)\exp(j2\pi kn/N)\right] \\
&= N\left(\text{IFFT}\left[I_k + jQ_k\right]\right).
\end{aligned}
$$

Scaling constant

- Consider a channel with impulse response $h_c(t)$.
- This impulse response could include any filtering at the front ends of either the transmitter or receiver.
- Consider transmitting a sum of infinite duration signals that are a sum of complex exponentials with different coefficients.

The received signal in the absence of noise is

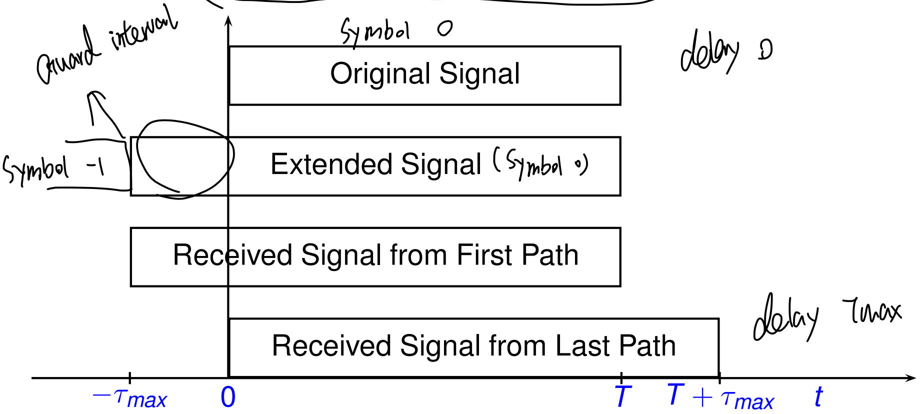
Assume infinite duration

$$
\begin{aligned}
r(t) &= \int_{-\infty}^{\infty} h_c(t-\tau)s(\tau)d\tau \\
&= \int_{-\infty}^{\infty} h_c(t-\tau)\underbrace{\left[\sum_k s_k e^{j2\pi k f_0 \tau}\right]} d\tau \\
&= \sum_k s_k \int_{-\infty}^{\infty} h_c(t-\tau)e^{j2\pi k f_0 \tau} d\tau \\
&= \sum_k s_k \int_{-\infty}^{\infty} h_c(v)e^{j2\pi k f_0 (t-v)} dv \\
&= \sum_k s_k e^{j2\pi k f_0 t} \int_{-\infty}^{\infty} h_c(v)e^{-j2\pi k f_0 v} dv \\
&= \sum_k s_k e^{j2\pi k f_0 t} H_c(k f_0) \\
&= \sum_k \underbrace{H_c(k f_0)}_{\text{channel}} s_k \underbrace{e^{j2\pi k f_0 t}}
\end{aligned}
$$

data.

- This shows that if we put in a sum of complex exponentials (infinite in duration) what comes out is a sum of complex exponentials at the same frequencies but with amplitudes and phase shifts that are possibly different at different frequencies.
- This is the basic principle of linear time-invariant systems:
  - if the input is a sine wave the output is a sine wave
  - if the input is the sum of infinite duration sinusoidal signals with a set of frequencies, the output is a sum of infinite duration sinusoidal signals with the same set of frequencies but different amplitudes and phases.

Now consider finite duration signals but extended.



orthogonal

The output of the receiver for the n-th frequency y_n is given by

$h_c(\tau) = 0, \ \tau > \tau_{max}$

$$
\begin{aligned}
y_n &= \int_0^T r(t) e^{-j2\pi n f_0 t} dt \\
&= \int_0^T \left[\int_{\tau=0}^{\tau_{max}} s(t-\tau) h_c(\tau) d\tau \right] e^{-j2\pi n f_0 t} dt \\
&= \int_{\tau=0}^{\tau_{max}} h_c(\tau) \left[\int_0^T s(t-\tau) e^{-j2\pi n f_0 t} dt \right] d\tau \\
&= \int_{\tau=0}^{\tau_{max}} h_c(\tau) \left[\int_{-\tau}^{T-\tau} s(u) e^{-j2\pi n f_0 (u+\tau)} du \right] d\tau, \quad u = t - \tau
\end{aligned}
$$

$$
\begin{aligned}
y_n &= \int_{\tau=0}^{\tau_{max}} h_c(\tau) e^{-j2\pi n f_0 \tau} \left[\int_{-\tau}^{T-\tau} s(u) e^{-j2\pi n f_0 u} du \right] d\tau \\
&= \int_{\tau=0}^{\tau_{max}} h_c(\tau) e^{-j2\pi n f_0 \tau} \left[\int_{-\tau}^{T-\tau} \sum_{l=0}^{N-1} s_l e^{j2\pi l f_0 u} e^{-j2\pi n f_0 u} du \right] d\tau \\
&= \int_{\tau=0}^{\tau_{max}} h_c(\tau) e^{-j2\pi n f_0 \tau} \left[\sum_{l=0}^{N-1} s_l \int_{-\tau}^{T-\tau} e^{j2\pi f_0 (l-n) u} du \right] d\tau
\end{aligned}
$$

Note that $f_0 T$ is an integer so

$$\int_{-\tau}^{T-\tau} e^{j2\pi f_0(l-n)t} du = \begin{cases} T, & l = n \\ 0, & l \neq n. \end{cases}$$

Using this we obtain,

$$y_n = T \int_{\tau=0}^{\tau_{max}} h_c(\tau) e^{-j2\pi n f_0 \tau} s_n d\tau = s_n T H_c(n f_0)$$

- Here

$$H_c(f) = \int_{\tau=0}^{\tau_{max}} h_c(\tau) e^{-j2\pi f \tau} d\tau.$$

  is the frequency response of the channel.
- As a result of this we can see that the output y_n depends only on s_n so there is no ISI.
- Furthermore, any signal sent before time $-\tau_{max}$ will not arrive after time $t = 0$ and thus have no effect on the received signal in the time interval $[0, T]$.

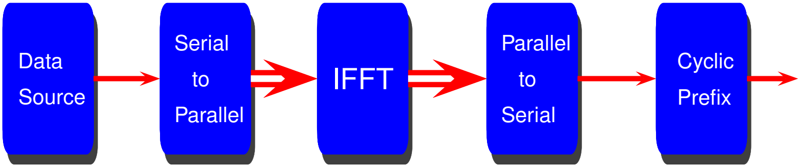
OFDM Parameters

- N carriers
- Total data rate is R_b.
- Data rate on each carrier is R_b/N.
- Duration of each data bit is N/R_b.
- Carrier Separation is R_b/N Hz.
- Cyclic Prefix of Length P added

OFDM Transmitter Implementation

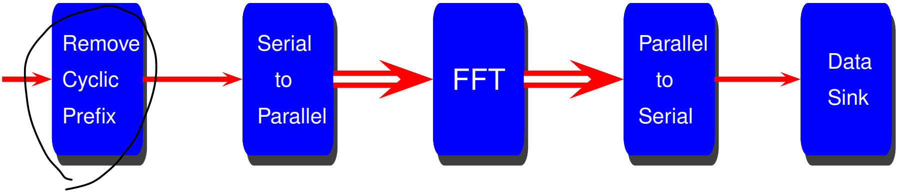
vector of frequency sample

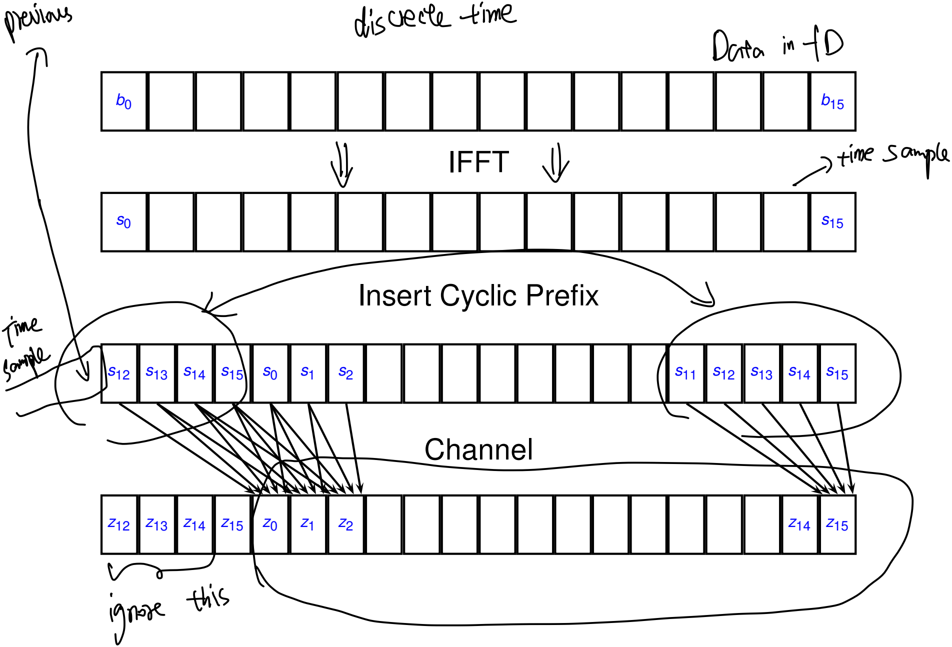
vector of time sample



OFDM Receiver

part of rece signals we ignore




previous
discrete time
Data in FD
b_0
b_{15}
IFFT
time sample
s_0
s_{15}
Insert Cyclic Prefix
time sample
s_{12} s_{13} s_{14} s_{15} s_0 s_1 s_2
s_{11} s_{12} s_{13} s_{14} s_{15}
Channel
z_{12} z_{13} z_{14} z_{15} z_0 z_1 z_2
z_{14} z_{15}
ignore this

Discrete analysis

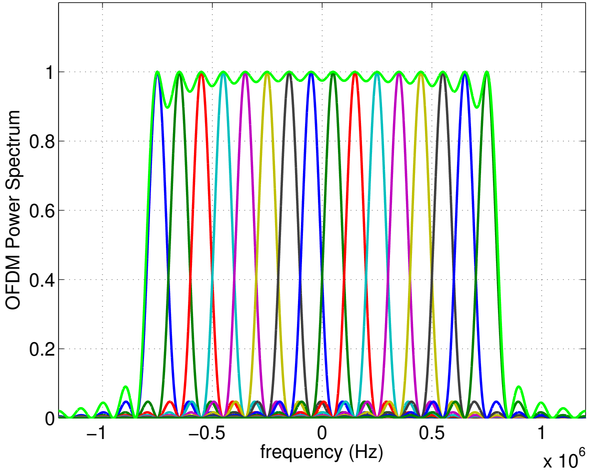
$$
\begin{aligned}
y_n &= \sum_{k=0}^{M-1} z_k e^{-j2\pi nk/M} \quad \text{Fourier Transform} \\
&= \sum_{k=0}^{M-1} \sum_{m=0}^{\nu} c_{k-m} h_m e^{-j2\pi nk/M} \\
&= \sum_{m=0}^{\nu} h_m \sum_{k=0}^{M-1} c_{k-m} e^{-j2\pi nk/M} \\
&= \sum_{m=0}^{\nu} h_m \sum_{u=-m}^{M-1-m} c_u e^{-j2\pi n(u+m)/M} \qquad (u = k-m, k = u+m) \\
&= \sum_{m=0}^{\nu} h_m e^{-j2\pi nm/M} \sum_{u=-m}^{M-1-m} \left[\sum_{l=0}^{M-1} b_l e^{j2\pi lu/M} \right] e^{-j2\pi nu/M}
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{m=0}^{\nu} h_m e^{-j2\pi nm/M} \sum_{u=-m}^{M-1-m} \left[\sum_{l=0}^{M-1} b_l e^{j2\pi lu/M} \right] e^{-j2\pi nu/M} \\
&= \sum_{m=0}^{\nu} h_m e^{-j2\pi nm/M} \sum_{l=0}^{M-1} b_l \sum_{u=-m}^{M-1-m} e^{j2\pi (l-n)u/M} \\
&= \sum_{m=0}^{\nu} h_m e^{-j2\pi nm/M} b_n M \\
&= H_n b_n M
\end{aligned}
$$

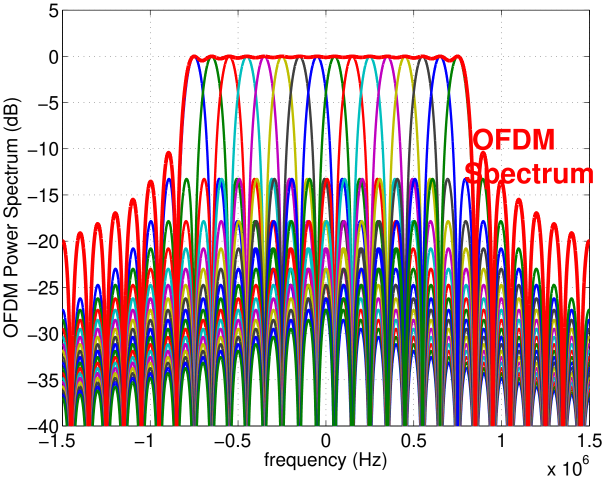
where

$$
H_n = \sum_{m=0}^{\nu} h_m e^{-j2\pi nm/M}
$$

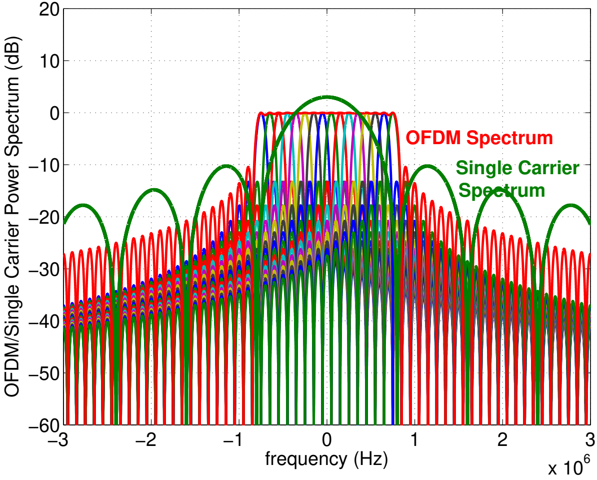
OFDM Spectrum



OFDM Spectrum

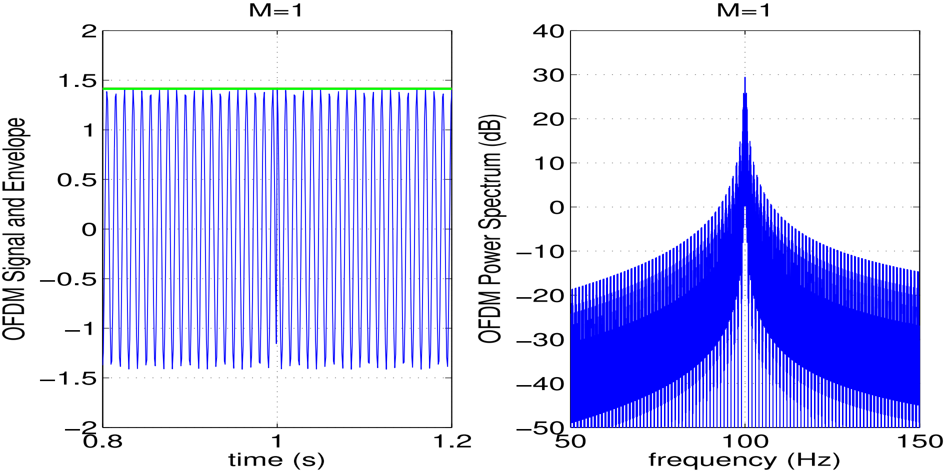


OFDM Spectrum

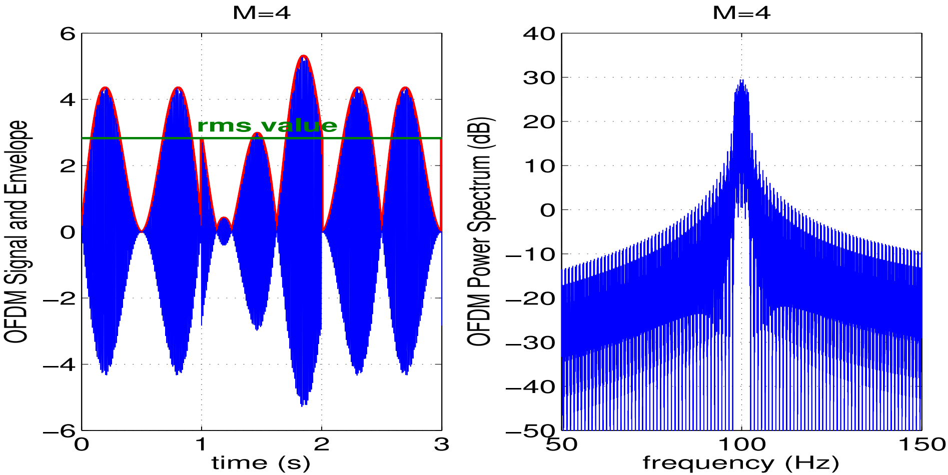


OFDM Signal and Spectrum (M=1)

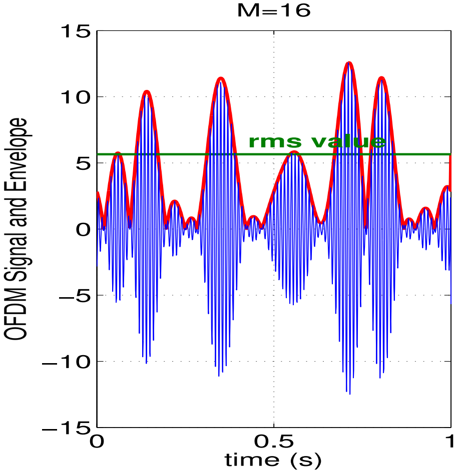
缺点: non-constant envelope

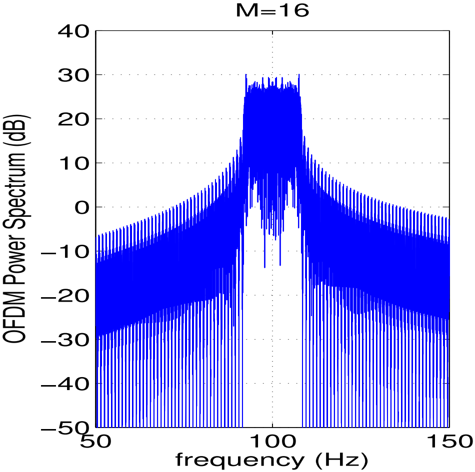


OFDM Signal and Spectrum (M=4)

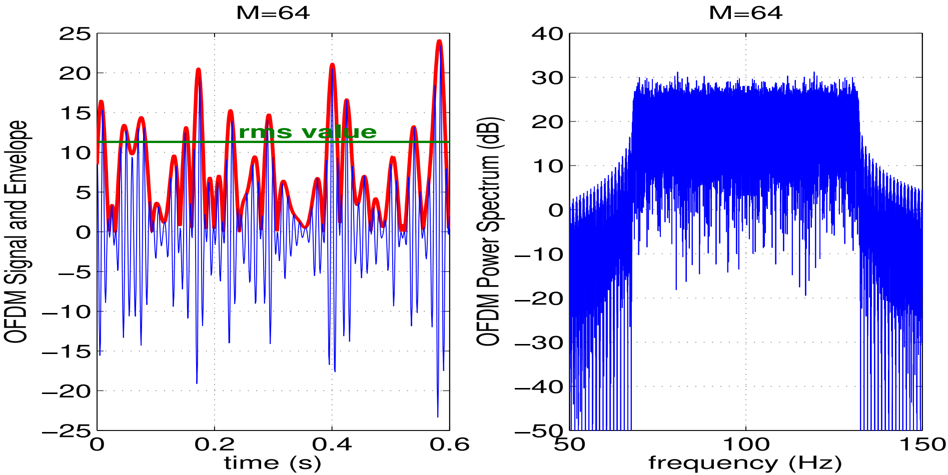


OFDM Signal and Spectrum (M=16)





OFDM Signal and Spectrum (M=64)



OFDM Summary

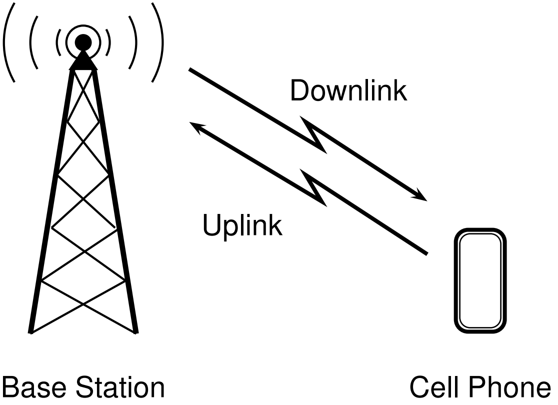
- OFDM has narrow spectrum (relative to single carrier with same data rate)
- OFDM has larger peak-to-average power ratio
  - Requires larger backoff of amplifier to avoid nonlinear distortion compared to single carrier
  - Larger backoff usually implies a smaller power efficiency for the amplifier
- Frequency selective fading can be mitigated in OFDM by appropriate channel estimation, coding and interleaving
- OFDM has lower sidelobes (smaller high frequency components) and thus can not make fast transitions of the amplitude or phase compared to unfiltered BPSK/QPSK.

OFDM in Wireless Systems

Parameter	802.11a (WiFi)	UWB (Multi-band)	LTE (10 MHz)	WiMax (10MHz)
Bandwidth	20MHz	528 MHz	10MHz	10MHz
Sample Time	50ns	1.8939ns	65.104 ns	1024
Number of Active Carriers	52	112	601	up to 914
Number of Guard Carriers	12	10	423	–
Data Carriers	48	100		1024
Pilots Carriers	4	12		–
FFT Size	64	128	1024	1024
Subcarrier frequency spacing	0.3125MHz (20MHz/64)	4.125MHz (528MHz/128)	15kHz	10.9375kHz
IFFT period	3.2 μs	242.42ns	66.67 μ s	91.429 μ s
Guard Interval	0.8 μs (16 samples)	70.08ns (37 samples)	4.69-5.21 μ s	11.428 μ s
OFDM Symbol Duration	4.0 μs	312.5ns	71.35-71.88 μs	102.857 μs

Ceullar

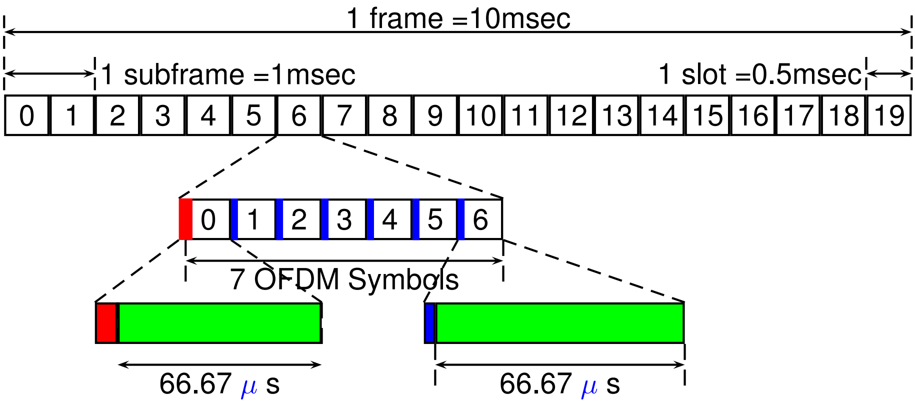
- In cellular there is an uplink (mobile/user equipment (UE)/cell phone to basestation) and a downlink (base station to phone).



FDD vs TDD

- It is hard (costly) for each radio (base station/mobile) to simultaneously transmit on the same frequency and time.
- There will be a signal received at the base station due to the transmitted signal at the base station that will be 100 dB or more stronger than the signal from the mobile to the base station.
- So generally either the base and mobile transmit at different times or on different frequencies.
- These are known as frequency division duplexing (FDD) or time division duplexing (TDD).

LTE Frame Structure 4G



Cyclic prefix for OFDM symbol 0 = 5.20833 μ sec= $160/(30.72 \times 10^6)$ sec.

Cyclic prefix for OFDM symbol 1,2,..,6.= 4.6875 μ sec $144/(30.72 \times 10^6)$ sec.

66.67 μ sec=$2048/(30.72 \times 10^6)$ sec.

LTE Physical Layer Uplink Channels

- **Physical Uplink Shared Channel (PUSCH):** Used to carry user data
- **Physical Uplink Control Channel (PUCCH):** Used to carry control information such as acknowledgement (ACK) messages or negative acknowledgement (NACK) messages, channel quality information (CQI), and scheduling requests (SR).
- **Physical Random Access Channel (PRACH):** Used to carry random access preamble when a user is requesting access to PUCCH or PUSCH

LTE Physical Layer Downlink Channels

- **Physical Downlink Shared Channel (PDSCH):** Used to carry user data and the channel is shared between users. Resource blocks in one frame can be used for one user and in a different frame for another user.
- **Physical Downlink Control Channel (PDCCH):** Used to carry control information such as resource allocation (which users can use which resource blocks for uplink data and which resource blocks on the downlink are used for which user.
- **Physical Broadcast Channel (PBCH):** This channel transmits system information. A user turning on their phone can listen to this channel to find out information about the system such as the current frame number, the number of antennas used by the base station.
- **Physical Control Format Indicator Channel (PCFICH):** This is a downlink channel containing the number of control symbols in a subframe.
- **Physical HARQ Indicator Channel (PHICH):** This is a channel for sending acknowledgement messages to a user regarding a successful demodulation/decoding of user data.

OFDM

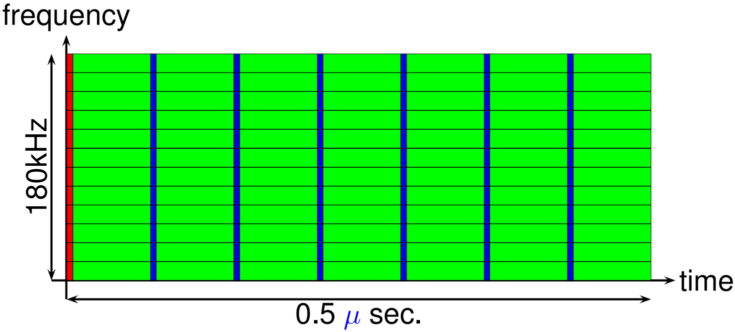
- The duration of each OFDM symbol is 66.67 μ seconds.
- In order for different frequencies to be orthogonal over that time duration (1/15000 seconds) the frequency separation should be 15kHz.

$$\begin{aligned}
\rho &= \int_0^T \cos(2\pi f_1 t)\sin(2\pi f_0 t)dt \\
&= \frac{1}{2}\int_0^T \sin(2\pi(f_1+f_0)t) - \sin(2\pi(f_1-f_0)t)dt \\
&= \frac{T}{2}\left[\frac{\cos(2\pi(f_1-f_0)T)-1}{2\pi(f_1-f_0)T} - \frac{\cos(2\pi(f_0+f_1)T)-1}{2\pi(f_0+f_1)T}\right] \\
&= \frac{T}{2}\left[\frac{\cos(2\pi(f_1-f_0)T)-1}{2\pi(f_1-f_0)T}\right]
\end{aligned}$$

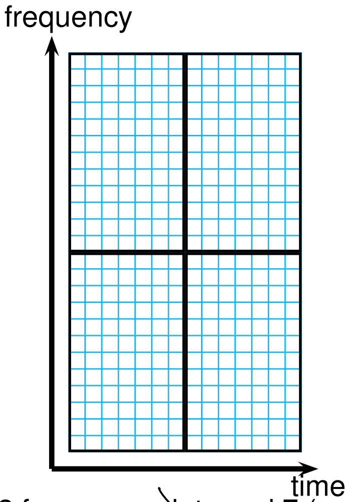
This integral will be zero if $2\pi(f_1-f_0)T = 2\pi, 4\pi, 6\pi, ...$. The minimum frequency spacing is $f_1 - f_0 = 1/(T)$.

LTE: One Resource Block

- A resource block consists of 12 different frequencies (180kHz) and 7 (for normal cyclic prefix) OFDM symbols (0.5 μ s).



LTE: Resource Block

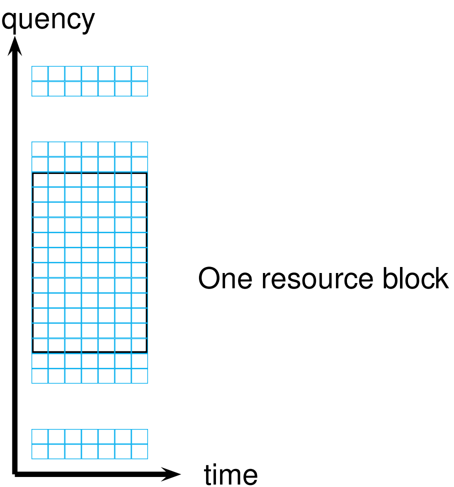


Symbols

One resource block = 12 frequency ~~slots~~ and 7 (or 6) time ~~slots~~.

Subcarriers

LTE: Resource Block



One resource block = 12 frequency slots and 7 (or 6) time slots.

Resource Blocks

- Depending on the bandwidth there are different numbers of resource blocks.
- The Table below shows the number of resource blocks as a function of bandwidth

Bandwidth	Number of Resource Blocks
1.4 MHz	6
3 MHz	15
5 MHz	25
10 MHz	50
15 MHz	75
20 MHz	100

PRACH Signal

za seff cuv

- There are several different PRACH signals (formats 1,2,3,and 4).
- Format 1,2 and 3 uses a ZC sequence of length 839.
- Format 4 uses a ZC sequence of 139.
- The PRACH signal is based on a Zadoff-Chu sequence of length 139 or 839.

$$x_u(n) = e^{-j\frac{\pi un(n+1)}{N_{ZC}}} \quad n = 0, 1, ..., N_{ZC} - 1$$

- In this equation N_{ZC} is either 139 or 839, u is a parameter that can vary.

PRACH Signal

- Different preamble sequences are defined by cyclic shifts of $x_u(n)$, i.e. $x_u(n-l)$.
- The 839 values of the sequence are applied to the frequency domain signals with frequency separation of 1.25kHz.
- The 139 values of the sequence are applied to the frequency domain signals with frequency separation of 7.5kHz.
- The duration of the signal in the time domain (without the cyclic prefix) can be 800 or 1600 μ sec for formats 1-3 and have different length cyclic prefixes.
- For format 4, the duration is 133 μ seconds.

- Once a user turns on, it listens for system information that is broadcast. This allows the user to know the frame timing information .
- The eNodeB (base station) broadcast the Network ID, Cell ID and other information.
- Once the UE is synchronized on the downlink it also needs to synchronize on the uplink. The signals from different users need to roughly arrive at the same time (frame synchronized). But different users have different delays between the UE and the eNodeB.
- The UE transmits a PRACH signal. The eNodeB responds to this transmission. The response indicates resources allocated to the UE for transmitting a connection request message.
- It is possible that two users simultaneously transmit a PRACH signal in which case most likely there is a collision and neither receive a response. In this case the users wait a random amount of time to retry.

- LTE allocates resources to users based on the channel quality.
- For downlink transmission the UE must send channel quality information (CQI) to the base station.
- The base station can then allocate resource blocks to users based on the channel quality.
- Base stations and users can employ multiple antennas to improve the possible data rate (MIMO).

5G New Radio (NR)

- Introduces new frequencies (millimeter wavelengths), for example 28GHz, and 39 GHz.
- Same basic structure as LTE (OFDM, Resource Blocks)
- Numerology for 5G
  - Frequency spacing between OFDM subcarriers: 15, 30, 60, 120, 240 KHz.
  - Slots/subframe: 1,2,4,8,16.

Example

- 30kHz subcarrier spacing.
- 2 slots/subframe, 20 slots/frame
- Symbol duration 33.33 μ s.
- Symbols/slot 14
- Cyclic prefix 2.3 μ s
- 4096 point FFT
- 3300 subcarriers for maximum bandwidth of 400MHz.

https://www.rfwireless-world.com/5G/5G-NR-m-sequence.h
https://www.rfwireless-world.com/5G/5G-NR-Zadoff-chu-s