EECS 455 Homework 5

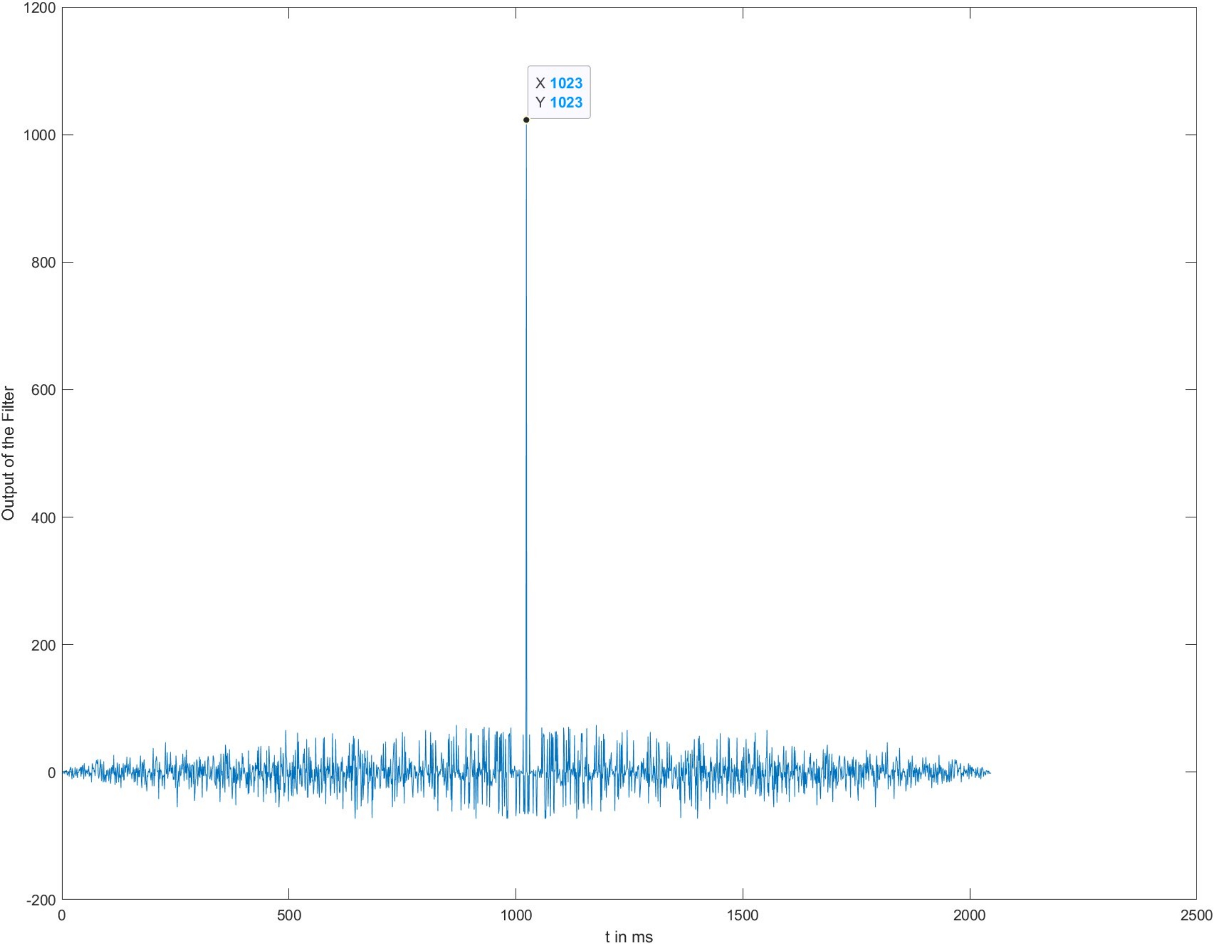
1. (a)

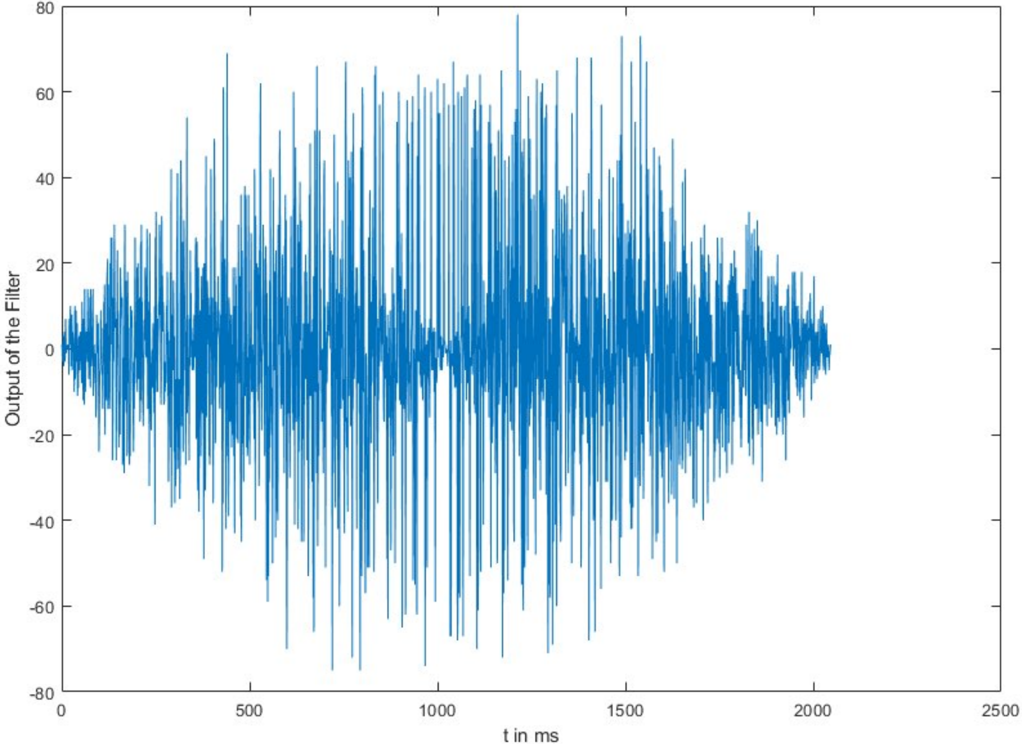
I use matlab to compute the first 20 bits of the output

1 1 1 1 0 0 1 0 0 0 1 1 1 0 0 0 1 0 0 0 0

$\#$ of ones $= 2^{m-1} = 2^{10-1} = 2^9 = 512$ ones

$\#$ of zeros $= 2^{m-1} - 1 = 2^9 - 1 = 511$ zeros


X 1023
Y 1023
Output of the Filter
t in ms
1200
1000
800
600
400
200
0
-200
0
500
1000
1500
2000
2500


80
60
40
20
0
-20
-40
-60
-80
Output of the Filter
0
500
1000
1500
2000
2500
t in ms

1. (c)

The output of the same filter is shown above.

The maximum output is 79

The minimum output is -76

2. We are given that bandwidth is 7.5 GHz $= 7.5 \times 10^9$ Hz

$f_c = 6.85$ GHz $= 6.85 \times 10^9$ Hz

Baseband $= \pm 3.75$ GHz

$$X(f) = \begin{cases} \sqrt{T}, & 0 \le |f| \le \frac{1-\alpha}{2T} \\ \sqrt{\frac{T}{2}[1-\sin(\pi T(f-\frac{1}{2T})/\alpha)}, & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0 & \text{otherwise} \end{cases}$$

Since $S(f) = \frac{1}{T}|X(f)|^2$

$$\therefore \quad S_Y(f) = \begin{cases} 1, & 0 \le |f| \le \frac{1-\alpha}{2T} \\ \frac{1}{2}[1-\sin(\pi T(|f|-\frac{1}{2T})/\alpha], & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0, & \text{otherwise} \end{cases}$$

When $S(f)$ is 3.75×10^9 Hz and $\alpha = 0.35$

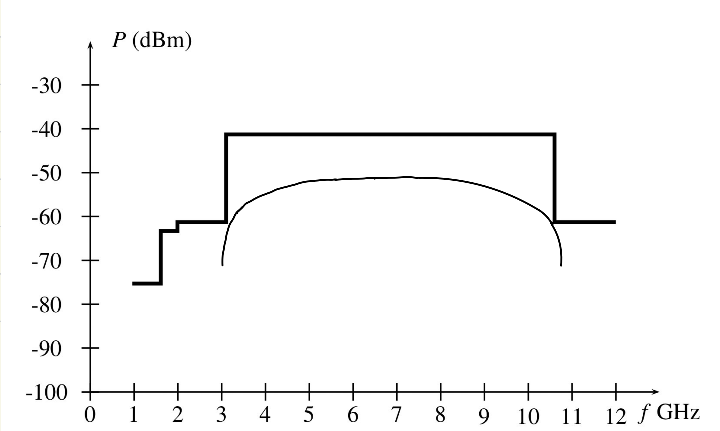
$$\frac{1}{2}[1-\sin(\pi T(|f|-\frac{1}{2T})/0.35)] = 0.01$$

$$1-\sin(\pi T(|f|-\frac{1}{2T})/0.35)] = 0.02$$

$$\sin(\pi T(|f|-\frac{1}{2T})/0.35) = 0.98$$

$$\sin(\pi T(3.75\times10^9-\frac{1}{2T})/0.35) = 0.98$$

$$\Rightarrow \quad T = 0.174 \text{ ns}$$



P3:

```
%=====================================================================%
%                            My filter                                %
%=====================================================================%
x2t_as_3_tuple = [x2t, x2t, x2t];
my_filter = conj(fliplr(x1t(1 : 512)));

conv_output = conv(x2t_as_3_tuple, my_filter);
tt = 0 : dt : ((length(conv_output) - 1) * dt);

figure;
plot(tt, conv_output);
xlabel('time (s)');
ylabel('Output');

[A, B] = max(abs(conv_output));
delay_started = ((B - 1) * dt - 1) * 1000;
delay_ended = (B * dt - 1) * 1000;
```

```
>> delay_started

delay_started =

  109.3750

>> delay_ended

delay_ended =

  111.3281
```

P4:

$$x_n = e^{jM\pi n(n+1)/N}, \quad n = 0, 1, \dots, N-1$$

(a) for N odd

$$x_{n-k-N} = e^{jM\pi(n-k-N)(n-k-N+1)/N}$$

$$= e^{jM\pi[(n-k)^2 + (n-k)(-N) + (n-k) - N(n-k) + N^2 - N)]/N}$$

$$= e^{jM\pi(n-k)^2/N} e^{jM\pi(n-k)/N} \cdot e^{jM\pi(-2N(n-k))/N} e^{jM\pi N^2/N} e^{jM\pi(-N)/N}$$

$$= e^{jM\pi(n-k)^2/N} e^{jM\pi(n-k)/N} \cdot \underline{e^{jM\pi(-2N(n-k))/N}} \cdot \underline{e^{jM\pi(N^2-N)/N}}$$

$$= e^{jM\pi(n-k+1)(n-k)/N}$$

$$= x_{n-k}$$

(Since N is odd, $-2N$ and $N-1$ are even, thus $e^{jM\pi(-2N(n-k))/N} = e^{jM\pi(N^2-N)/N} = 1$

(b) $$x_n x_{n-k}^* = e^{jM\pi n(n+1)/N} e^{-jM\pi(n-k)(n-k+1)/N}$$

$$= e^{jM\pi(n(n+1) - (n-k)(n-k+1))/N}$$

$$= e^{jM\pi(n^2+n-(n-k)^2-(n-k))/N}$$

$$= e^{jM\pi(n^2+n-n^2+2nk-k^2-n+k)/N}$$

$$= e^{jM\pi(2nk-k^2+k)/N}$$

$$= e^{-jM\pi(k^2-k)/N} e^{jM\pi(2nk)/N}$$

$$= e^{-jM\pi(k^2-k)/N} e^{jM2\pi kn/N}$$

(c) $$\theta_x(k) = \sum_{n=0}^{N-k-1} x_n x_{n-k}^* + \sum_{n=N-k}^{N-1} x_n x_{n-k-N}^*$$

$$= e^{-jM\pi(k^2-k)/N} \sum_{n=0}^{N-1} \left(e^{jM2\pi kn}\right)^n$$ (Based on part(a) and part(b))

When $k=0$, $$\theta_x(k) = 1 \cdot \sum_{n=0}^{N-1} 1 = N-1$$

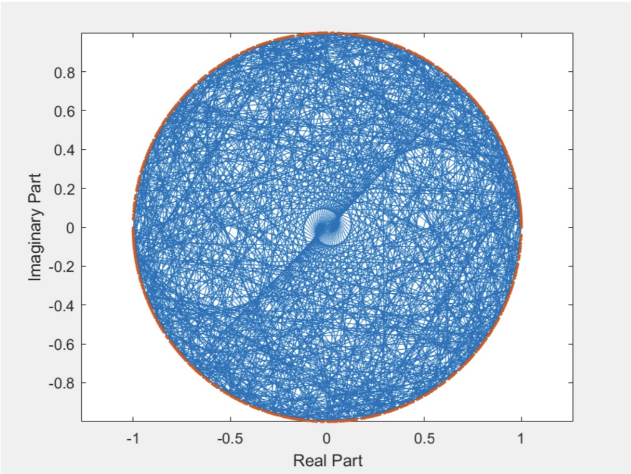
when $k \neq 0$, $$\theta_x(k) = e^{-jM\pi(k^2-k)/N} \frac{e^{jM2\pi k} - 1}{e^{jM2\pi k/N} - 1}$$

$$= e^{-jM\pi(k^2-k)/N} \cdot 0$$

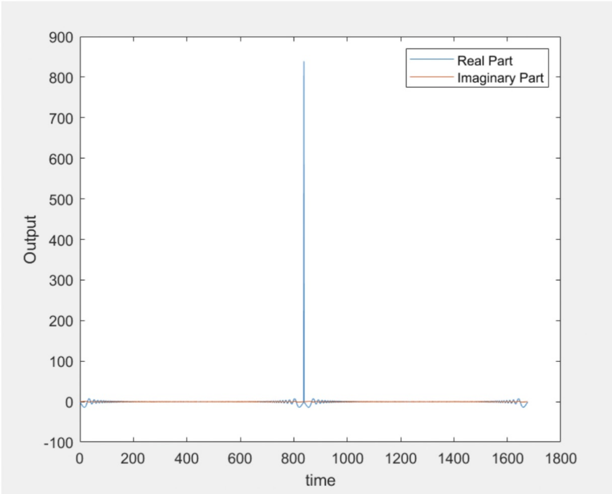
$$= 0$$

P5.

(a)

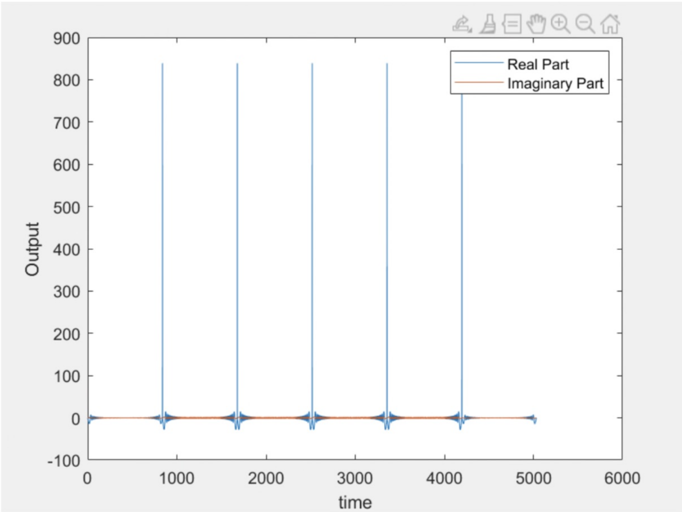


(b)



Real Part
Imaginary Part
Output
time
900
800
700
600
500
400
300
200
100
0
-100
0
200
400
600
800
1000
1200
1400
1600
1800

(c)



Real Part
Imaginary Part
Output
time
900
800
700
600
500
400
300
200
100
0
-100
0
1000
2000
3000
4000
5000
6000