Lecture 9

Goals

- Be able to determine received power from transmitted power.
- Be able to determine possible data rate.
- Global Positioning System (GPS)

Propagation Equation: Free Space

Suppose a transmitter radiates with an antenna a power of P watts. The receiver is located a distance ~~r~~ d meters from the transmitter. Then the received power is

$f_c = \frac{c}{\lambda}$ → speed of light

λ → wavelength

$$P_r = \frac{P_t G_T G_R}{(4\pi d/\lambda)^2}$$

— free space propagation loss (FSPL)

where λ is the wavelength, G_t is the gain of the transmitting antenna, G_r is the gain of the receiving antenna.

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$$(P_r)_{dB} = (P_t)_{dB} + (G_T)_{dB} + (G_r)_{dB} - \underbrace{20\log_{10}(4\pi d/\lambda)}_{\text{FSPL}}$$

Example: Problem

$10 \log(1/1W) = 0\,dBW$

$10 \log(1\times10^3/1mW) = 30\,dBm$

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I wanted to get internet access at my house which is 4 miles away from the EECS building (as the crow flies). I install a 24dB gain antenna on the roof of the EECS building. I install a 24dB gain antenna on the roof of my house (or in the attic). I use the ISM band at 2.4GHz for which the allowed transmitted power is 1W (=0dBW=30dBm). What data rate could I expect if the propagation was that of free space? Assume noise temperature of 290^oK and BPSK modulation. Also assume a bit error probability of 0.00001 is acceptable.

Example: Solution

Either convert to non dB units to dB or from dB units to non dB units.

$$\lambda = c/f = \frac{3 \times 10^8}{2.4 \times 10^9} = 0.125m$$

$$\begin{aligned} P_r &= \frac{(1)(251)(251)}{(4\pi(6.4 \times 10^3)/0.125)^2} \\ &= 152n\text{Watts} \end{aligned}$$

$$\begin{aligned} (P_r)_{dB} &= 0 + 24 + 24 - 20\log_{10}(6.43 \times 10^5) \\ &= 0 + 24 + 24 - 116.17 \\ &= -68.2\text{dBW} = -38.2\text{dBm} \end{aligned}$$

Example: Solution

In order to get error probability 0.00001 it is required that I have $E_b/N_0(dB)$=9.6dB. This means $E_b/N_0 = 9.12$. Now,

$$N_0 = kT_{temp} = 1.38 \times 10^{-23}(290) = 4 \times 10^{-21}$$

$Q\left(\sqrt{\frac{2E}{N_0}}\right) = 10^{5}$

Thus

$$E_b = 9.12 \times (4 \times 10^{-21}) = 3.65 \times 10^{-20}$$

$\frac{E}{N_0} = 9.6$ dB

So in order to get that much energy from a signal with power 152nWatts the duration of the information must be

$$\begin{aligned} PT_b &= 3.65 \times 10^{-20}\text{Joules} \\ T_b &= 3.65 \times 10^{-20}/P \\ &= 2.40 \times 10^{-13}(s) \\ R_b &= 1/T_b = 4.16 \times 10^{12} = 4.16\,\textit{Terrabits/second} \end{aligned}$$

Example: Solution

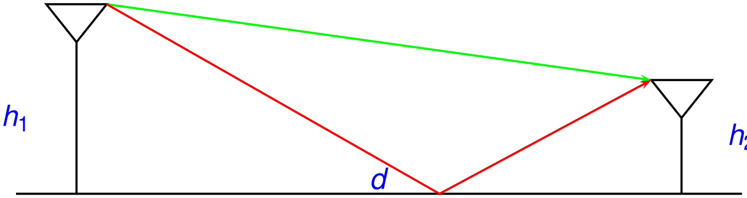
Clearly this is not possible since the 2.4GHz spectrum for ISM (e.g. wireless LANS etc.) only has about 83MHz of bandwidth. So it appears that I am really bandwidth limited and not power limited.

Propagation Equation: Single Reflection

Now consider the case where there is an additional path due to a signal reflection from the ground. The multipath has a different phase from the direct path. If we assume the reflection from the ground causes a 180 degree phase change then for large distances the relation between the transmitted power and the received power changes to

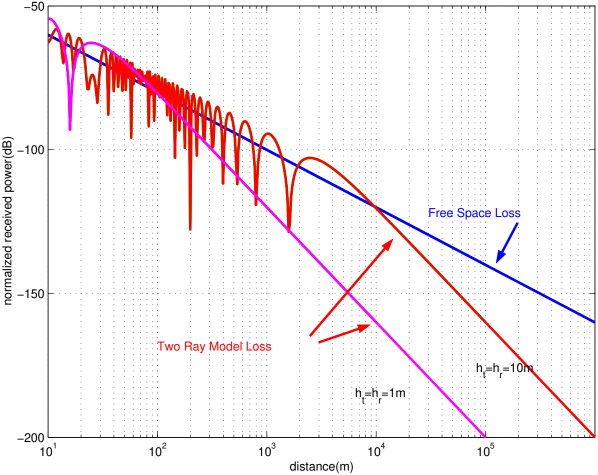
$$P_r \approx P_t G_r G_t \frac{h_1^2 h_2^2}{d^4} \quad (d \gg \max(h_1, h_2))$$

Propagation Equation: Single Reflection



Thus the relation of received power to distance becomes an inverse fourth power law or equivalently the power decreases 40dB per decade of distance. Generally, for a wireless channel the decrease in power is 20dB per decade near the base station but as the receiver moves away the rate of decrease increases. Note that the relation does not depend on frequency or wavelength.

Example: $f_c = 2.4$GHz



Example

Consider the previous example. We need to recompute the received power. Assume the antennas are at a height of 1m and 1m. Then the received power is

$$\begin{aligned} P_r &\approx P_t G_r G_t \frac{h_1^2 h_2^2}{d^4} \\ &= (1)(251)(251)\frac{(1)^2(1)^2}{(6.4 \times 10^3)^4} \\ &= 37.5pWatts = -104.3dBW = -74.3dBm \end{aligned}$$

In this case the data rate possible would be approximately 10Gbps. Again, we are still bandwidth limited rather than energy limited.

Global Positioning System

Goals

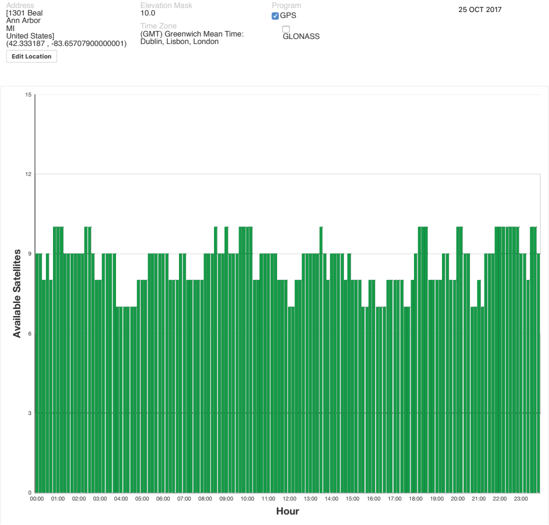
- Understand the GPS System

Global Positioning System

(Location, Location, Location)

- Uses 24 satellites
- 20,200km (12,000 miles) with half day orbit
- Uses one-way ranging from GPS
- Uses two frequencies: L1 (1575.42 MHz), L2 (1227.6 MHz)
- Using the range measurements from 4 satellites it is possible to estimate 3 positions (latitude, longitude, altitude) and a correction to the user's clock.
- Transmits data regarding satellite location to enable users to do ranging.

GPS Satellites



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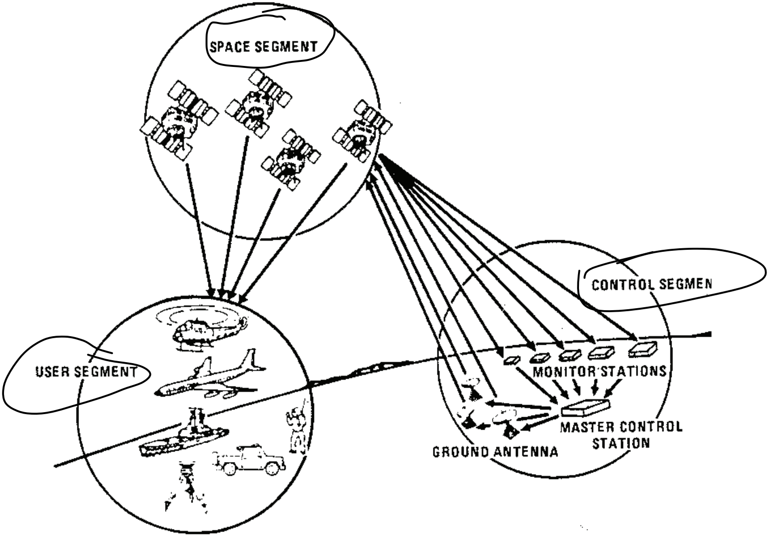
GPS Components



GPS Components

- Control Segment
  - Ground system
  - Uploads corrections to satellite position and time
- Space Segment
  - 24 satellites
  - Low earth orbit; period 12 hours
  - Each satellite contains accurate clocks (2 cesium clocks, 2 rubidium clocks)
  - Each transmits a CDMA signal
  - Data transmitted is location of satellite, time stamp
- User Segment
  - Receiver tracks the range to various satellites
  - Contains a clock
  - Determines position, time

GPS

- The GPS user equipment determines the range to at least four satellites
- Using these four values the GPS unit determines the users clock bias (error) and the location (x,y,z)
- Differential GPS uses ground stations to transmit correction errors due to atmospheric effects at known locations.

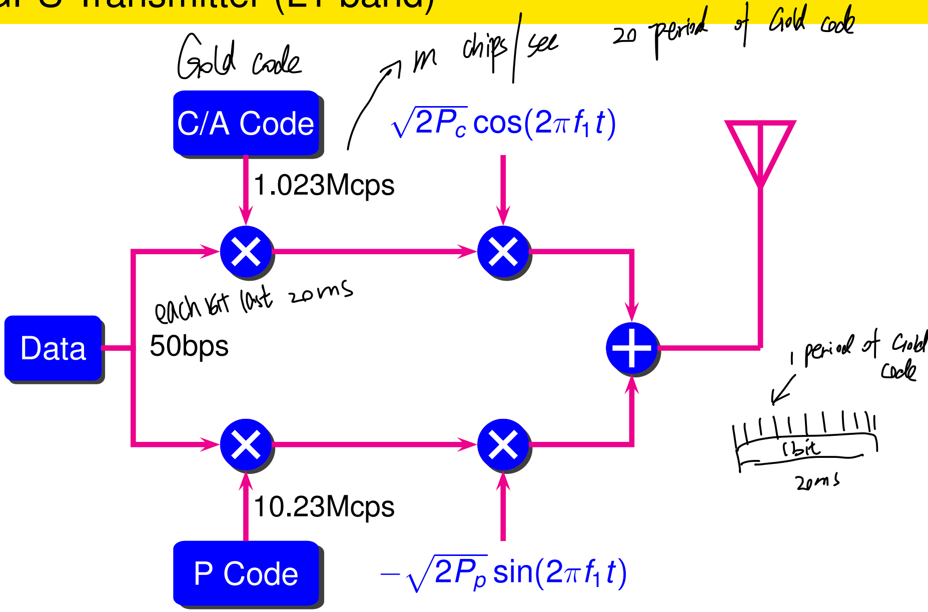
GPS

1540 cycles/chip

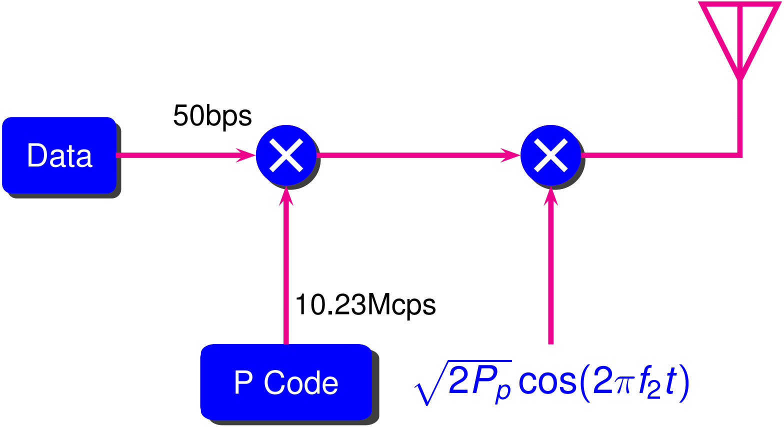
f_1

- Two frequencies:
  - L1=1575.42 MHz=154 $\times$ 10.23 MHz
  - L2=1227.60 MHz=120 $\times$ 10.23 MHz
- Two different spreading codes: Coarse aquistion code C/A, Precision code P.
- C/A (coarse acquisition) code (Gold code) operates at 1.023 MHz and modulates the in-phase carrier.
- P code operates at 10.23 MHz and modulates the quadrature-phase carrier.
- Two bands can be used to measure the ionospheric delay.
- P code encrypted for military.

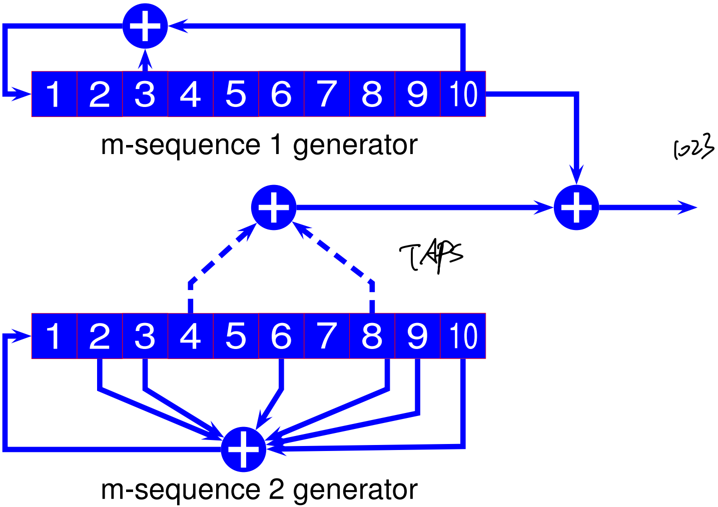
GPS Transmitter (L1 band)



GPS Transmitter (L2 band)



C/A Gold Code Generator



C/A code (Gold Code) Generator Notes

$$\frac{1023 \text{ chips}}{1.023 \text{ cps}} = 1 \text{ms}$$

- Initial state of shift registers is the all one state.
- Addition is done mod 2 (exclusive or).
- The taps of the lower shift register (shown dotted) are different for different satellites.
- The sequence has a period of 1023 chips.
- Period of sequence is 1 ms.
- Sequence of 0's and 1's mapped to +1 and -1 ($0 \rightarrow 1, \ 1 \rightarrow -1$) before modulation.
- Data bit duration is 20ms (50bps) which means the Gold code repeats 20 times per data bit duration.

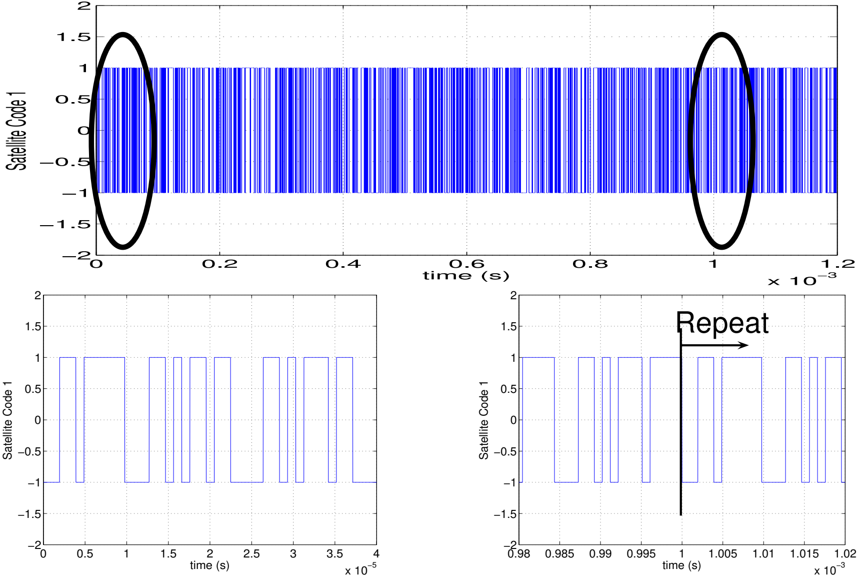
Gold Code Generator Notes

Satellite	Connections	First 10 chips (octal)	First 10 chips (binary)
1	$2 \oplus 6$	1440	1 100 100 000
2	$3 \oplus 7$	1620	1 110 010 000
3	$4 \oplus 8$	1710	1 111 001 000
4	$5 \oplus 9$	1744	1 111 100 100
.	.	.	.
.	.	.	.
.	.	.	.

GPS C/A Code Assignments

SV PRN ID	G2 Phase Taps	First 10 Chips
1	2 & 6	1100100000
2	3 & 7	1110010000
3	4 & 8	1111001000
4	5 & 9	1111100100
5	1 & 9	1001011011
6	2 & 10	1100101101
7	1 & 8	1001011001
8	2 & 9	1100101100
9	3 & 10	1110010110
10	2 & 3	1101000100
11	3 & 4	1110100010
12	5 & 6	1111101000
13	6 & 7	1111110100
14	7 & 8	1111111010
15	8 & 9	1111111101
16	9 & 10	1111111110
17	1 & 4	1001101110
18	2 & 5	1100110111
19	3 & 6	1110011011
20	4 & 7	1111001101
21	5 & 8	1111100110
22	6 & 9	1111110011
23	1 & 3	1000110011
24	4 & 6	1111000110
25	5 & 7	1111100011
26	6 & 8	1111110001
27	7 & 9	1111111000
28	8 & 10	1111111100
29	1 & 6	1001010111
30	2 & 7	1100101011
31	3 & 8	1110010101
32	4 & 9	1111001010

Gold Code Waveform

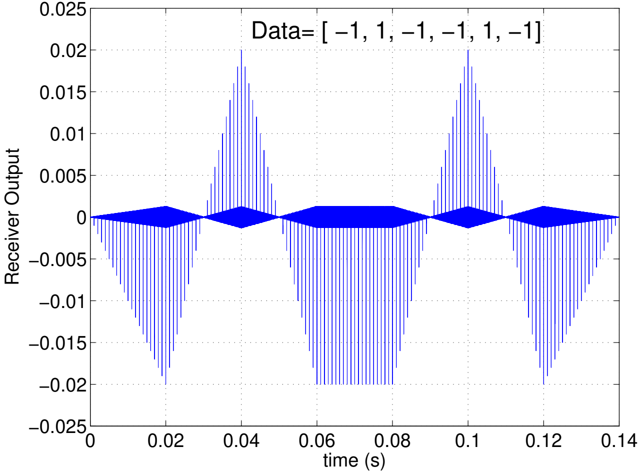


GPS C/A Link Analysis

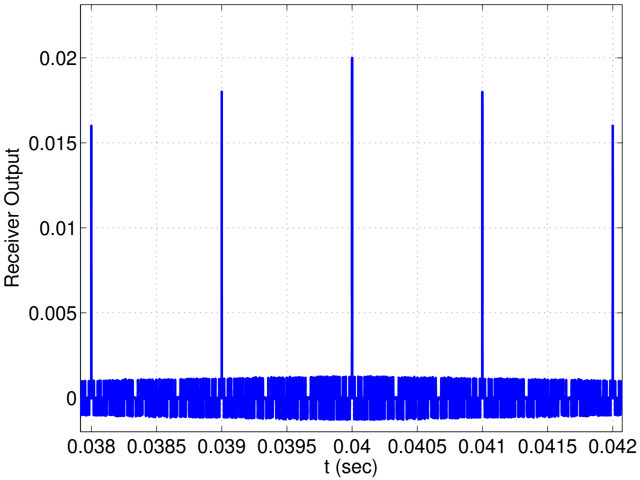
BPSK

- Transmitted power (C/A code) is P_t=479W (26.8dBW).
- Frequency 1575.42 MHz, wavelength $\lambda = .1904$m. $\lambda = \frac{c}{f_c}$
- Range $d = 2 \times 10^7$m.
- Path loss $(\lambda/(4\pi d))^2$= 182.4dB. FSPL
- Atmospheric attenuation: 2 dB.
- Received signal power (P): -157.6 dBW.
- Received signal power density (P/W): -220.6 dBW/Hz.
- Noise power spectral density N_0= -201.5dBW/Hz (including equipment noise).
- P/N_0=43.9dB.
- Data rate $R =$50bps.
- $E_b/N_0 = (P/R)/N_0$=26.9 dB.

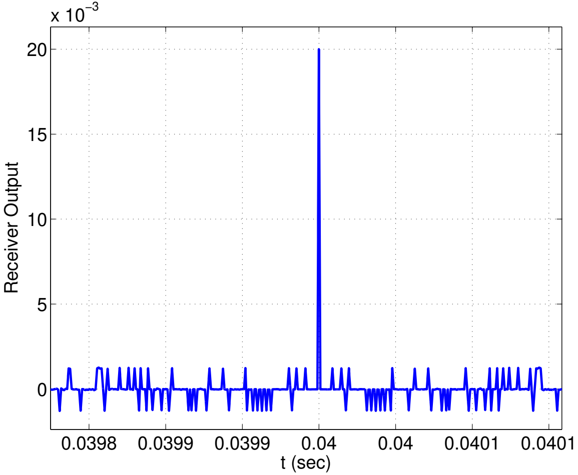
GPS Receiver Output (Single Satellite) C/A code



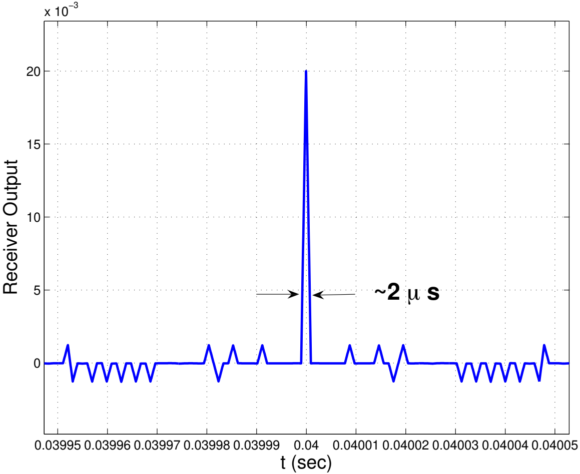
GPS Receiver Output (Single Satellite) C/A code

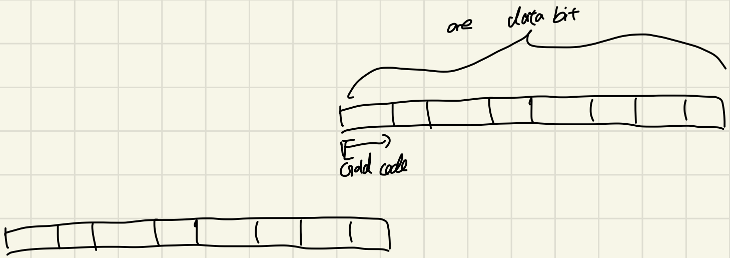


GPS Receiver Output (Single Satellite) C/A code

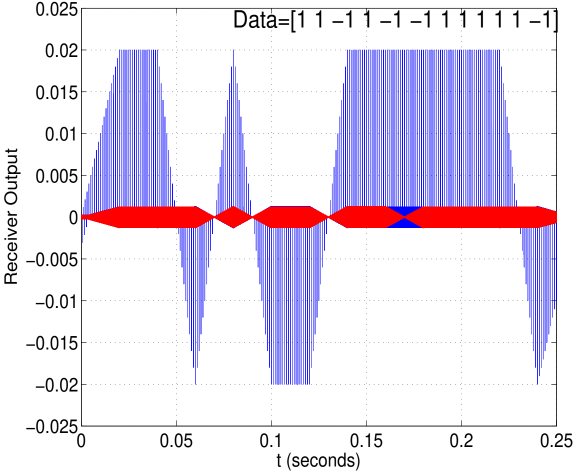


GPS Receiver Output (Single Satellite) C/A code

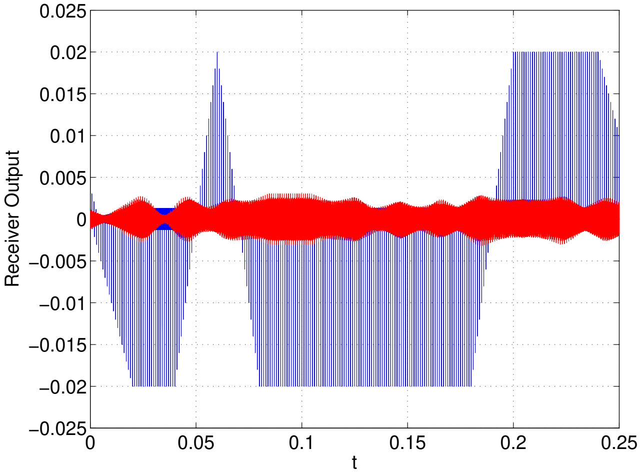



are data bit
Gold code

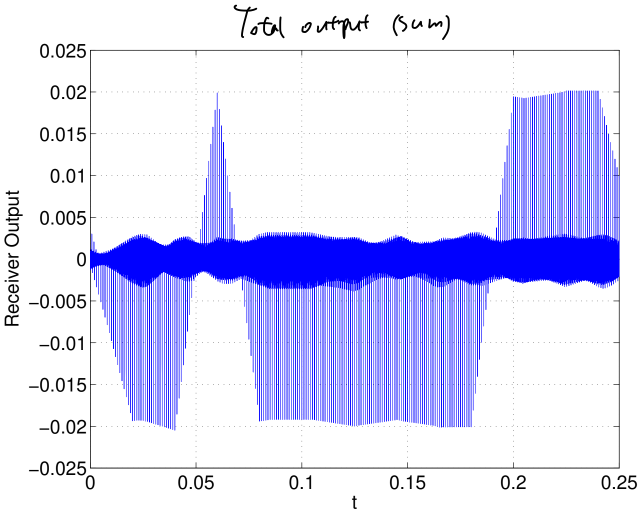
GPS Receiver Output (Two Satellites) C/A code



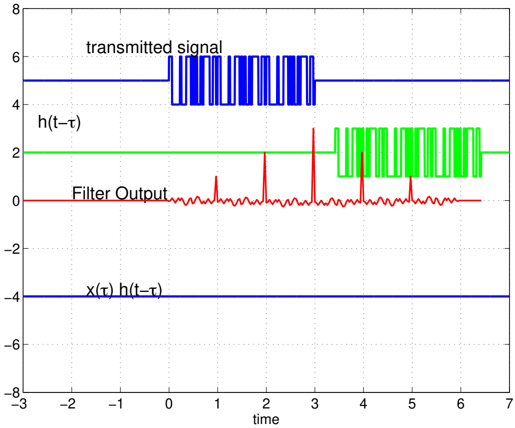
GPS Receiver Output (Four Satellites) C/A code



GPS Receiver Output (Four Satellites) C/A code



GPS Example


transmitted signal
h(t–τ)
Filter Output
x(τ) h(t–τ)
8
6
4
2
0
–2
–4
–6
–8
–3
–2
–1
0
1
2
3
4
5
6
7
time

GPS C/A code notes

- 20 repeats of Gold code per data bit
- Width of receiver output "blip" is twice the chip duration ($2T_c = .977517\mu s$).
- Data transmitted contains information about satellite location and correction of the satellite clock.

GPS P code

- Operates at 10.23 M chips/second (Mcps)
- P code is exclusive or ($\oplus$) of two sequences (length 15,345,000 chips and 15,345,037 chips)
- Codes are reset every week (period =1 week)
- Each satellite has a separate code obtained by delaying one of the two sequences by different amounts

GPS Timing

- Time divided into 1024 weeks (repeats every 19 years, 36 weeks).
- Zero time is midnight Saturday January 5, 1980 or 12:00am Sunday Jan 6, 1980, August 22, 1999 12:00am,...
- Weeks are divided into 1.5 second "X1" epochs.
- X1 epoch occurs every 1.5 seconds (403,200 X1 epochs every week).
- Z count is the number of X1 epochs since zero GPS time (max=412,876,800=$2^{28.62}$)
  - 10 bits for the week number (0 to 1023).
  - 19 bits for the number of X1 epochs since start of time of week (TOW) count.
  - 17 MSB are included in the HOW message indicating number of 6 second subframes since the beginning of the week.

GPS Data

- GPS Data is a 50bps stream
- Same information on the C/A code and P(Y) code.
- Synchronous with the C/A code epochs
- Data are formated into 30 bit words (0.6 seconds)
- 10 words is a subframe (6 seconds, 300 bits)
- 5 subframes is a frame (30 seconds, 1500 bits)
- 25 frames or pages (12.5 minutes) for complete set of data

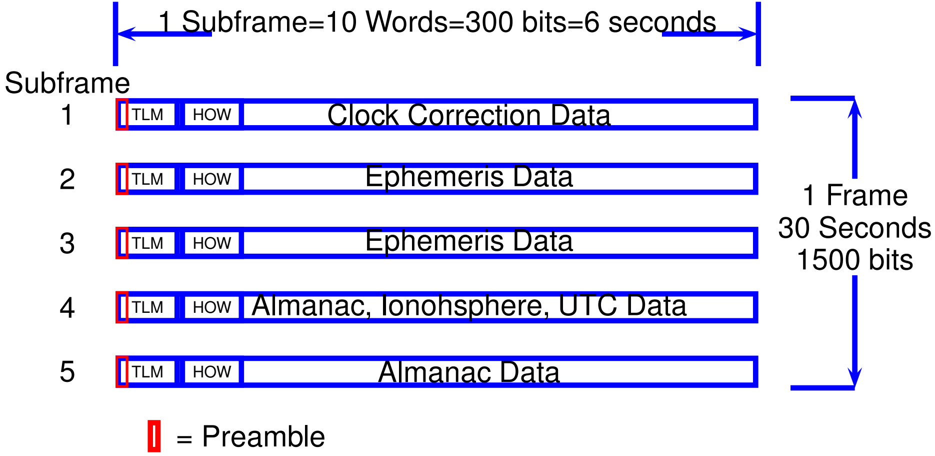
GPS Data

- Word 1 and 2 of all subframes contain a preamble for synchronization and a handover word (HOW).
- First subframe describes the satellite clock information relative to GPS time
- Ephemeris data giving the satellites position (updated every 2 hours)
- Almanac data is the coarse orbit and status information for each satellite in the constellation.
  - Repeats every 12.5 minutes

GPS Data

- Cold Start. A GPS receiver without information in memory that requires it to find the satellites. Must search for satellite signal. More than 30 seconds.
- Warm Start. A GPS receiver with almanac data, old ephemeris data and approximate position in memory that allows it rapidly find the necessary satellites. About 35-40 seconds.
- Hot Start. A GPS receiver without ephemeris data and approximate position. Takes 5-10 seconds to collect valid time satellites.

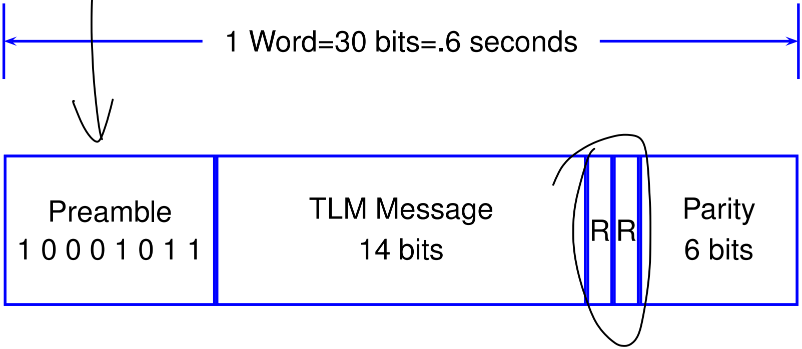
GPS Data


1 Subframe=10 Words=300 bits=6 seconds
Subframe
1
TLM
HOW
Clock Correction Data
2
TLM
HOW
Ephemeris Data
3
TLM
HOW
Ephemeris Data
4
TLM
HOW
Almanac, Ionohsphere, UTC Data
5
TLM
HOW
Almanac Data
1 Frame
30 Seconds
1500 bits
= Preamble

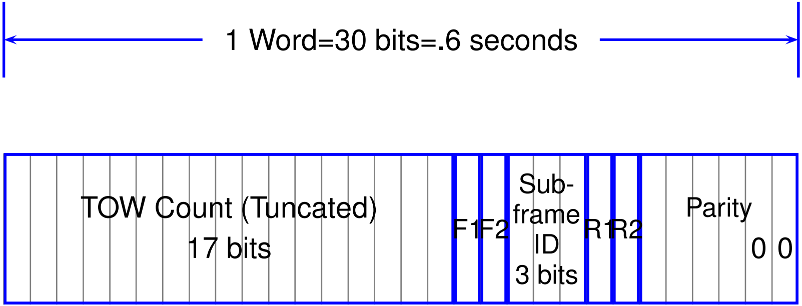
GPS Telemetry Word

homework

used for syn



GPS Handover Word



GPS Handover Word

- Flags F1 and F2 are for synchronization, anti-spoofing and momentum flags.
- The last two parity bits are forced to zero by choosing R1 and R2 appropriately.
- Parity is determined by using an extended Hamming code with 24 information bits and 6 parity check bits.
- Time of week (TOW) should increment by 1 each subframe (this can be used as a check that synchronization is achieved).

GPS Data

- Subframe 1: Clock Correction and Space Vehicle Accuracy
  - Clock correction for the satellite as uploaded from the GPS control segment
  - Delay correction term for L1 and L2 bands
  - Space vehicle-user range accuracy index
- Subframe 2 and 3: Ephemeris Parameters
  - Satellite position vs. time
  - Fit interval for ephemeris data
- Subframe 4 and 5: Almanac Data, Space Vehicle Health Data, Ionoshpere Models
  - Position information (truncated) for up to 32 satellites
  - Health of each satellite
  - Data relating GPS time to UTC time
  - Ionoshpere delay model parameters
  - Corrects for relativistic effects

GPS Source of Errors

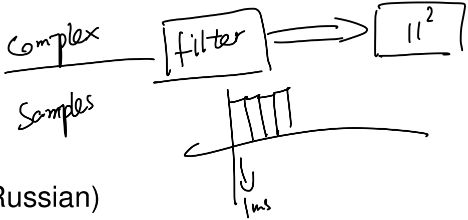
Effect	Errors
Troposheric effects	± 1m
Ionospheric effects	± 0.5m
Satellite clock errors	± 2m
Multipath distortion	± 1m
Ephemeris errors	± 2.5m

Total accuracy on the order of 15m (50 feet).

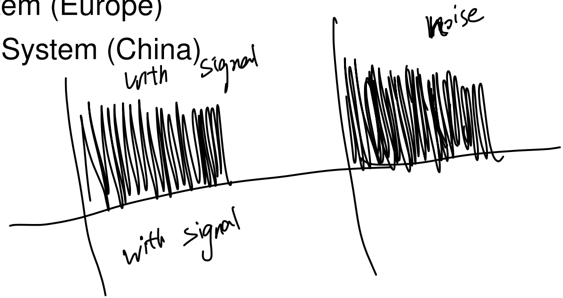
GPS Time to Synch vs. Complexity

- To determine range, a correlation with each satellite must be accomplished
- To find the starting point for each signal many correlations must be completed
- We can speed up the process by doing many correlations in parallel.
- This requires more hardware. Tradeoff between complexity and performance.

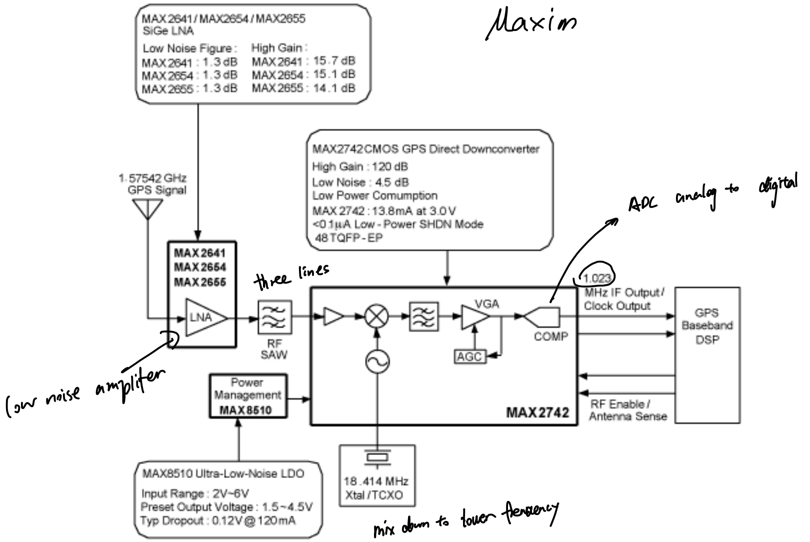
Other Global Positioning Systems



- GPS III
- GLONASS (Russian)
- Galileo Positioning System (Europe)
- COMPASS Navagation System (China) Beidou
- IRNSS (India)
- DORIS (France)



GPS Receiver Front End



GPS Receiver Front End