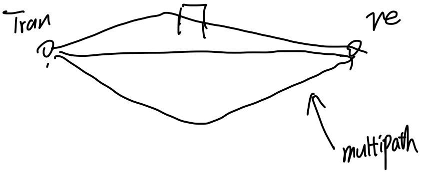
Lecture Notes 14

10-27 Lec 最后



Goals

- Multipath (Fast) Fading
- Understand Orthogonal Frequency Division Multiplexing (OFDM)
- Understand various applications of OFDM

Solve intersymbol interference

half time

Two problem: ① It creates constructive and destructive interference

② (Fading cause Error Prob much larger)

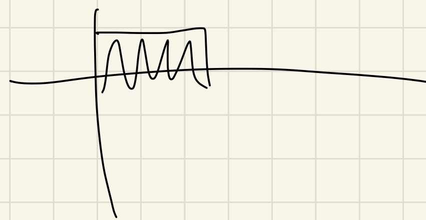
$\bar{P}_e = \frac{1}{2}(0) + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$ ← very bad → can led to loss of 30–40 dB in SNR

half constructive

half time destructive

① If these paths add destructively, we've lost our signal (Fading)

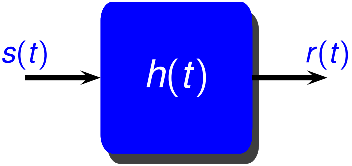
② The 2nd path will cause interference with the receive signal



Intersymbol interference

OFDM with Multipath

linear time invariant



Channel

longer path, a ↓

multipath

L paths betweens trans and recv

delay

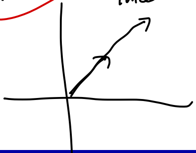
$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l)$$

amplitial.

$$H(f) = \sum_{l=0}^{L-1} \alpha_l \exp(-j2\pi f \tau_l)$$

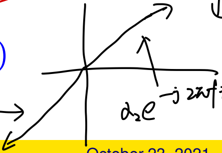
Constuctive

twice



destructive

L=2



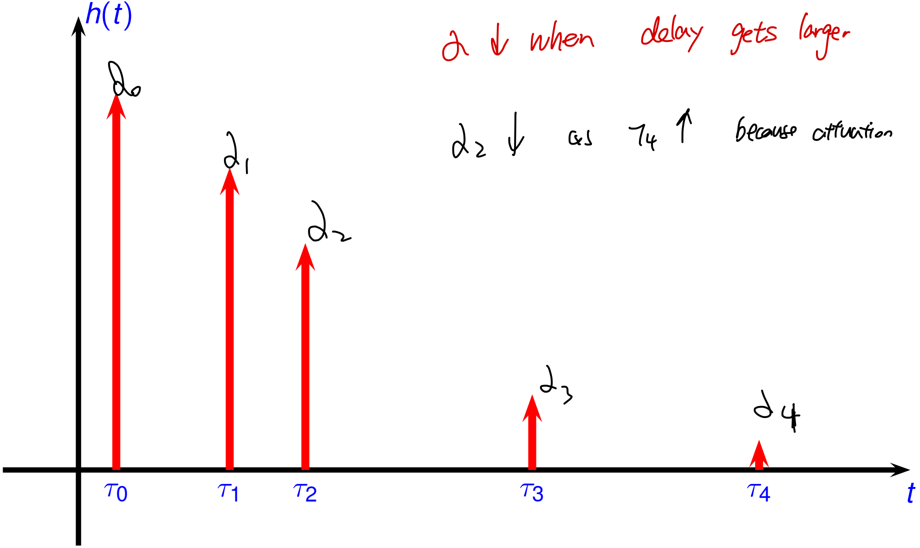
$\alpha_1 e^{-j2\pi f\tau_1}$

$\alpha_2 e^{-j2\pi f\tau_0}$

OFDM with Multipath

$\alpha \downarrow$ when delay gets larger

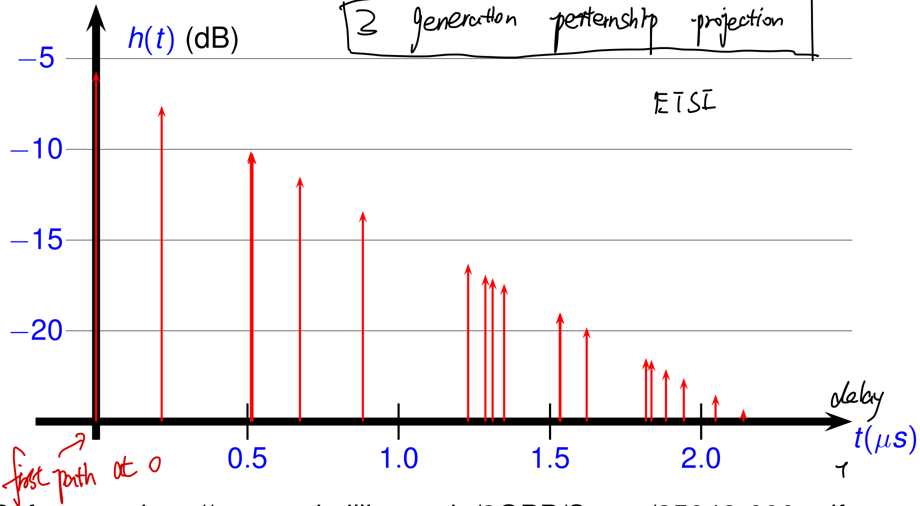
$\alpha_2 \downarrow$ as $\tau_4 \uparrow$ because attuation



3GPP Standard: Typical Urban Multipath

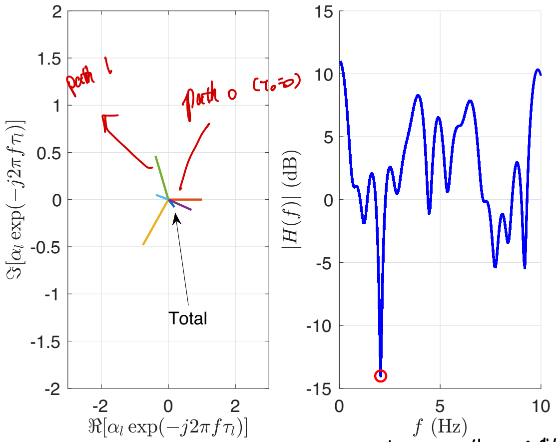
$\boxed{\text{3 generation partnership projection}}$

ETSI



Reference: http://www.quintillion.co.jp/3GPP/Specs/25943-600.pdf

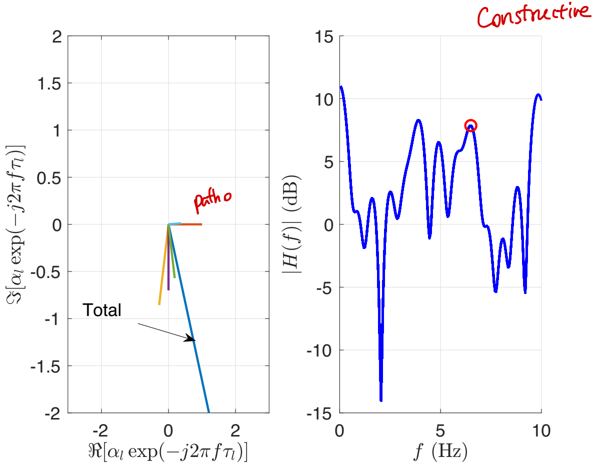
Multipath



different path

Change f $\rightarrow$ change the amplitude of the received signal
relative phase of phase change

Multipath

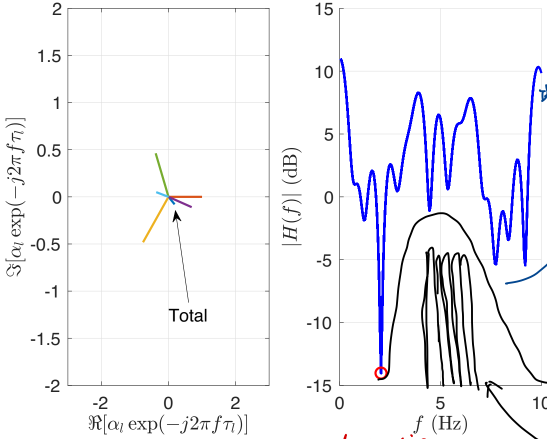


Constructive
path 0
Total
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
0
2
$\Im[\alpha_l \exp(-j2\pi f \tau_l)]$
$\Re[\alpha_l \exp(-j2\pi f \tau_l)]$
15
10
5
0
-5
-10
-15
0
5
10
$|H(f)|$ (dB)
f (Hz)

Multipath

depends on f

frequency seletive Fading



If signal is wide,
Distortion in f Domain,
means ISI in Time Domain

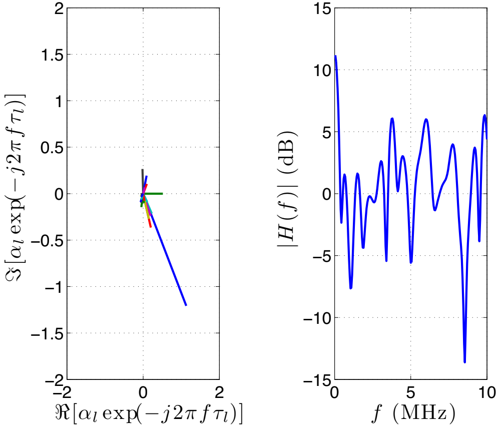
bandwidth larger
(lots of distortions)
cause ISI

destructive

bad!

OFDM
(Narrow bandwidth
Signal will not be
distorted)

Multipath



2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2
0
2
$\Im[\alpha_l \exp(-j2\pi f \tau_l)]$
$\Re[\alpha_l \exp(-j2\pi f \tau_l)]$
15
10
5
0
−5
−10
−15
0
5
10
$|H(f)|$ (dB)
f (MHz)

- The coherence bandwidth B_c of a channel is the bandwidth such that the frequency response of the channel does not change significantly. That is $H_c(f_1) \approx H_c(f_2)$ for $|f_2 - f_1| < B_c$.
- In a frequency nonselective model the bandwidth of the signal is less than the coherence bandwidth of the channel.
- Consider a complex lowpass representation of a bandpass signal. That is

$$s(t) = \Re[s_0(t)\sqrt{2}\underbrace{e^{j2\pi f_c t}}]$$

mix up to carrier frequency

where $s_0(t)$ is the lowpass complex representation.
- The fact that the bandwidth of $s(t)$ is less than the coherence bandwidth, B_c means that $s_0(t)$, does not vary much in a time duration of $\max \tau_j < 1/B_c$. That is,

$$s_0(t - \tau_j) \approx s_0(t) \text{ for } \max \tau_j < 1/B_c$$

The W of signal s(t) is narrow, so does $s_0(t)$, so it doesn't change quickly

The received signal is

$r(t) = h_c(t) * s(t)$

$$
\begin{aligned}
r(t) &= \sum_{l=0}^{L-1} \alpha_l s(t-\tau_l) \\
&= \sum_{l=0}^{L-1} \alpha_l \Re[s_0(t-\tau_l)\sqrt{2}e^{j2\pi f_c(t-\tau_l)}] \\
&= \Re[\sum_{l=0}^{L-1} \alpha_l s_0(t-\tau_l) e^{-j2\pi f_c \tau_j}\sqrt{2}e^{j2\pi f_c t}] \\
&\approx \Re[\sum_{l=0}^{L-1} \alpha_l s_0(t) e^{-j2\pi f_c \tau_j}\sqrt{2}e^{j2\pi f_c t}] \\
&= \Re[s_0(t)[\sum_{l=0}^{L-1} \alpha_l e^{-j2\pi f_c \tau_j}]\sqrt{2}e^{j2\pi f_c t}]
\end{aligned}
$$

If have a narrow signal, if change f, the a and phase changed, but I receive the same signal, there is no distortion

- Letting $r_0(t)$ be the lowpass complex representation of the received signal we see that bce narrow bandwidth signal

$$r_0(t) = s_0(t)[\sum_{l=0}^{L-1} \alpha_l e^{-j2\pi f_c \tau_j}]$$

- This shows that the effect of multipath fading in a frequency nonselective environment is to change the amplitude and phase of the signal but not distort the signal.

- For a reasonably large (e.g. more than 5) paths the factor

$$X = \sum_{l=0}^{L-1} \alpha_l e^{-j2\pi f_c \tau_j}$$

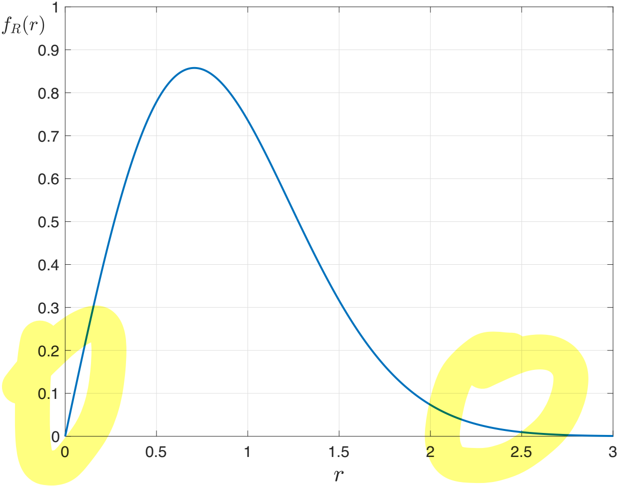
  can be approximated by a complex Gaussian random variable.
- That is, the real part and the imaginary part are Gaussian. This is a result of the central limit theorem.

- The magnitude of a complex Gaussian random variable X is a Rayleigh distributed random variable R with density

$$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/(2\sigma^2)}, \quad r \geq 0.$$

- Here σ^2 is the variance of the real part and imaginary part of X.
- The distribution of R is due to the fact that sometimes the paths add constructively and sometimes the paths add destructively.
- Typically, the fading level is normalized to have $E[R^2] = 1$. This normalization means that $2\sigma^2 = 1$ or $\sigma^2 = 1/2$.

Rayleigh density function, $\sigma^2 = 1/2$.



Performance Analysis

- First consider a modulator transmitting a BPSK signal with rectangular pulses and received with a faded amplitude. The transmitted signal is

$$s(t) = \sqrt{2P}b(t)\cos(2\pi f_c t)$$

where $b(t)$ is the usual data bit signal consisting of a sequence of rectangular pulses of amplitude +1 or -1.

- That is,

$$b(t) = \sum_l b_l p_T(t - lT).$$

- The received signal is

$$r(t) = \underbrace{R}_{\text{fade amplitude}}\sqrt{2P}b(t)\cos(2\pi f_c t + \overbrace{\theta}^{\text{phase}}) + n(t)$$

Performance Analysis

recieved Energy

- Assuming the receiver can accurately estimate the phase θ the coherent demodulator (matched filter) output at time kT is

$$z_k = \underbrace{R}_{\text{fading}}\sqrt{E}\underbrace{b_{k-1}}_{\text{data}} + \eta_k \quad \text{noise}$$

  where $E = PT$.
- The random variable R represents the fading and has density

$$p_R(r) = \begin{cases} 0, & r < 0 \\ \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} & r \geq 0 \end{cases}$$

$E[R^2]=1$

$\sigma^2 = \frac{1}{2}$

- The conditional error probability (conditioned on the value of R) is

$$P_e(R) = Q\left(\sqrt{\frac{2ER^2}{N_0}}\right).$$

Performance Analysis

Let $\alpha = 2E/N_0$, $\beta = \sigma^2\alpha = \bar{E}/N_0$ and

$$\begin{aligned}\gamma &= \sqrt{\frac{\beta}{1+\beta}} \\ &= \sqrt{\frac{\bar{E}/N_0}{1+\bar{E}/N_0}}\end{aligned}$$

Performance Analysis

The unconditional error probability is

$$
\begin{aligned}
P_e &= \int_{r=0}^{\infty} \underbrace{p_R(r)}_{\text{density for } R} \underbrace{Q(\sqrt{\frac{2Er^2}{N_0}})dr}_{\text{conditional error prob given } R=r} \\
&= \int_{r=0}^{\infty} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} Q(\sqrt{\frac{2Er^2}{N_0}})dr \\
&= \int_{r=0}^{\infty} \frac{r}{\sigma^2} \int_{u=\sqrt{\alpha r^2}}^{\infty} e^{-r^2/2\sigma^2} \frac{\exp\{-u^2/2\}}{\sqrt{2\pi}} du dr \\
&= \int_{u=0}^{\infty} \frac{\exp\{-u^2/2\}}{\sqrt{2\pi}} \int_{r=0}^{u/\sqrt{\alpha}} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} du dr \\
&= \int_{u=0}^{\infty} \frac{\exp\{-u^2/2\}}{\sqrt{2\pi}} \int_{r=0}^{u/\sqrt{\alpha}} \frac{r}{\sigma^2} \exp\{-r^2/2\sigma^2\} dr du \\
&= \int_{u=0}^{\infty} \frac{\exp\{-u^2/2\}}{\sqrt{2\pi}} (1 - \exp\{-u^2/(2\alpha\sigma^2)\}) du \\
&= \frac{1}{2} - \int_{u=0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\{-\frac{u^2}{2}(1 + \frac{1}{\alpha\sigma^2})\} du \\
&= \frac{1}{2} - \gamma \int_{u=0}^{\infty} \frac{1}{\gamma\sqrt{2\pi}} \exp\{-\frac{u^2}{2\gamma^2}\} du \\
&= \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\bar{E}/N_0}{1 + \bar{E}/N_0}}
\end{aligned}
$$

Performance Analysis

- The last integral is evaluated by recognizing the integrand to be a Gaussian density function with zero mean which when integrated from 0 to ∞ is 1/2.
- For large E/N_0 the error probability is approximately

$$P_e \simeq \frac{1}{4E/N_0}.$$

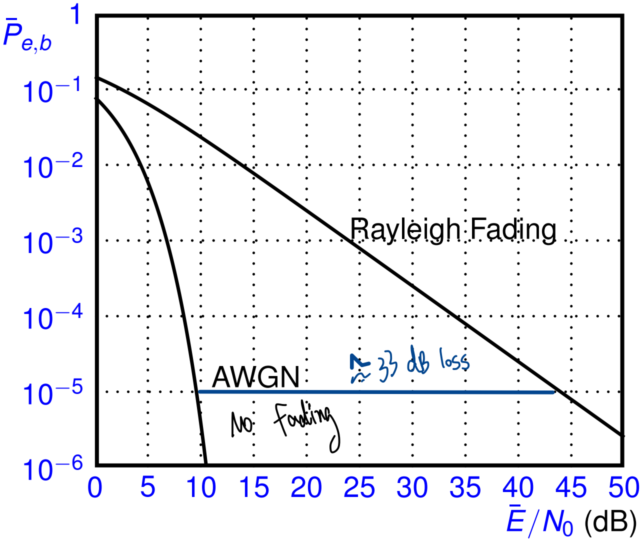


Figure: Bit error probability for BPSK with Rayleigh Fading

Performance Analysis

- Thus for high E/N_0 the error probability decreases inverse linearly with signal-to-noise ratio.
- To achieve error probability of 10^{-5} requires a signal-to-noise ratio of 44.0dB whereas in additive white Gaussian noise the required signal-to-noise ratio for the same error probability is 9.6dB.
- Thus fading causes a loss in signal-to-noise ratio of 34.4dB.
- This loss in performance is at the same average received power.

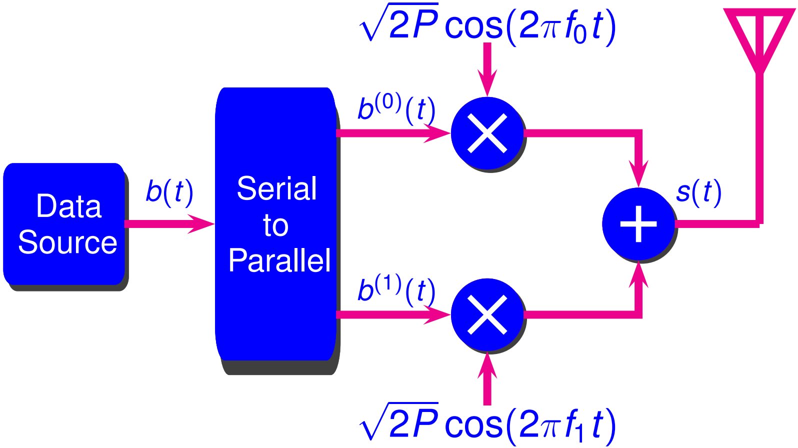
Performance Analysis

- The cause of this loss is the fact that the signal amplitude sometimes is very small and causes the error probability to be close to 1/2.
- Of course, sometimes the signal amplitude is large and results in very small error probability (say 0).
- However when we average the error probability the result is going to be much larger than the error probability at the average signal-to-noise ratio because of the nonlinear nature of the error probability in just Gaussian noise without fading as a function of signal amplitude.
- The way to mitigate the effect of fading is through diversity. Examples of diversity include 1) time diversity, 2) frequency diversity, 3) space diversity or suitable combinations. MIMO. multiple input, multiple output.
- Diversity means transmitting the same information multiple times at different times, frequencies, or with different antennas where each transmission experiences independent fading. T.F diversity reduce rate of transmission (bandwidth efficiency)

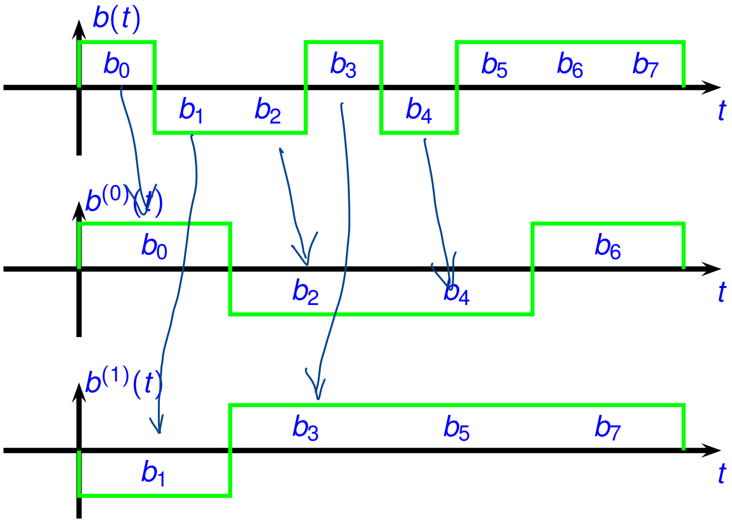
Orthogonal Frequency Division Multiplexing (OFDM)

- OFDM is a method of multiplexing several different modulated carriers into one signal
- The different carriers are separated in frequency by an amount to make the different carriers orthogonal
- OFDM has several benefits
  - Narrower spectrum than a single carrier system with the same data rate
  - Intersymbol interference from frequency selective fading is mitigated using a sufficient guard interval without an equalizer
- Also called multi-carrier modulation, discrete multi-tone modulation (DTM)
- Used in DSL, 802.11a, LTE,...

Transmitter for two carriers



Serial-to-Parallel Converter



Orthogonal Frequency Division Multiplexing (OFDM)

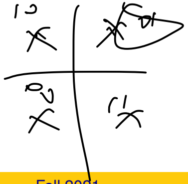
Consider the time interval $[0, T]$ where the input to the mixers are b_0 and b_1. The transmitted signal is then

$$\begin{aligned} s(t) &= \sqrt{2P}\,(\Re\,[b_0 \exp(j2\pi f_0 t) + b_1 \exp(j2\pi f_1 t)]),\ 0 \le t \le T \\ &= \sqrt{2P}\,\left(\Re\left[\sum_{i=0}^{1} b_i \exp(j2\pi f_i t)\right]\right),\ 0 \le t \le T \end{aligned}$$

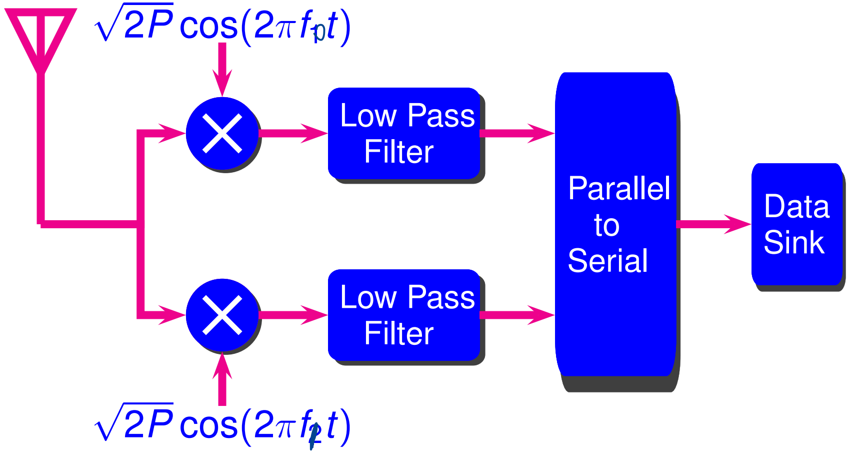
where $\Re(x)$ is the real part of the complex number x.

Orthogonal Frequency Division Multiplexing (OFDM)

- Serial-to-parallel converter reduces the bit rate on each output by factor equal to the number of carriers.
- Addition of two modulated carriers with different frequencies creates a nonconstant envelope signal.
- Frequency separation between two carriers sufficient to make the signals, over a symbol duration orthogonal.



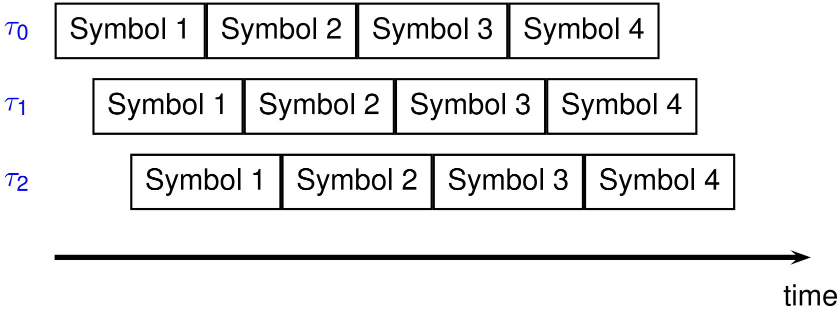
Receiver for two carriers



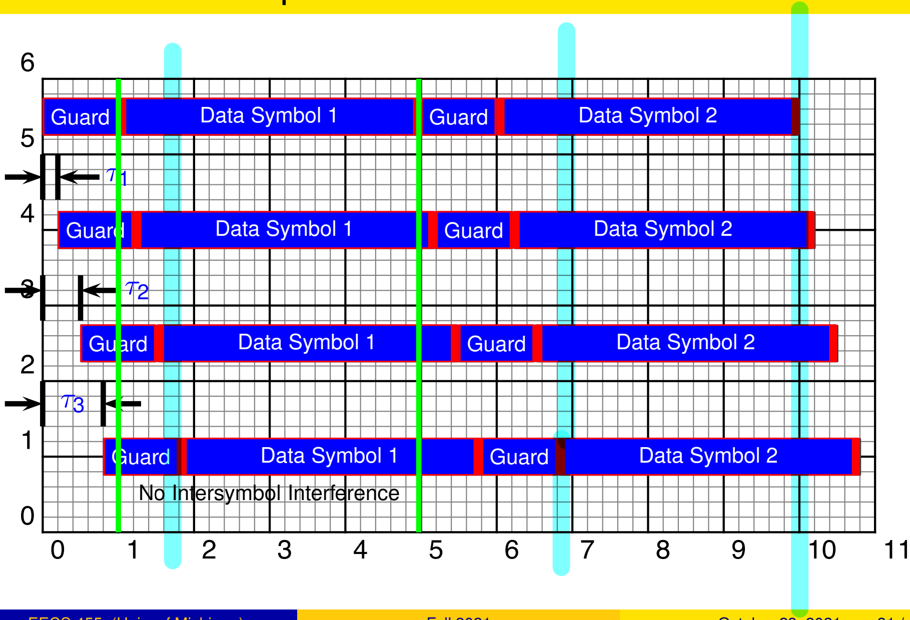
OFDM

- Two data streams transmitted on two orthogonal carriers.
- The output (ideally) of each would only depend on the corresponding transmitted signal at that frequency.
- The bandwidth of each carrier is relatively narrow so there is not significant distortion due to frequency selective fading.
- Guard intervals (cyclic prefix) are added to mitigate the multipath interference from one data symbol to the next data symbol.

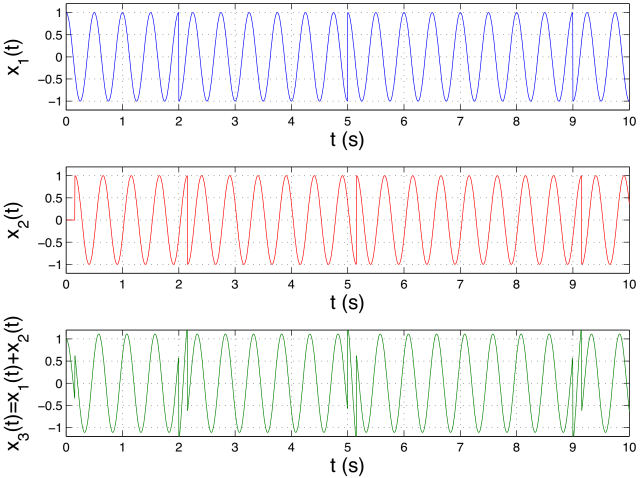
OFDM with Multipath



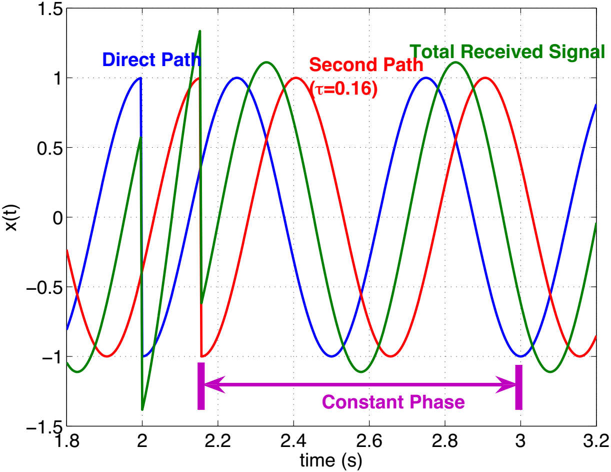
OFDM with Multipath



OFDM with Multipath



OFDM with Multipath



Orthogonal Frequency Division Multiplexing (OFDM)

- Multipath propagation causes intersymbol interference because the delay spread is significant relative to the bit duration.
- By making the bit duration longer the effect of intersymbol interference can be decreased.
- Longer bit duration reduces the bandwidth and the data rate.
- Adding modulated carriers at other frequencies allows the data rate to be increased.

Orthogonal Frequency Division Multiplexing (OFDM)

- Different carriers are chosen so that one carrier does not interfere with other carriers.
- Another way of looking at OFDM is that the bandwidth of each carrier is narrow enough so that the channel frequency response does not change significantly.
- IFFT produces signals that are orthogonal. Need extra time to let multipath die out.
- OFDM has worse peak-to-average power ratio compared to single carrier (unfiltered system).

OFDM Signals

- Consider using I-Q modulation (two dimensional) on each carrier.
- Let $(I_k + jQ_k)$ represent the complex data symbol transmitted on each carrier for the data interval $[0, T]$.
- If on each carrier we use both cosine and sine, then for each carrier the generated signal for the interval $[0, T]$ is

$$\begin{aligned} s_k(t) &= [I_k \cos(2\pi k f_0 t) - Q_k \sin(2\pi k f_0 t)] \\ &= \Re\left[(I_k + jQ_k)\exp(j2\pi k f_0 t)\right]. \end{aligned}$$

$f_0 T$ = integer for orthogonal

- The total transmitted signal is then

$$\begin{aligned} s(t) &= \sum_{k=0}^{N-1} s_k(t) \\ &= \Re\left[\sum_{k=0}^{N-1}(I_k + jQ_k)\exp(j2\pi k f_0 t)\right] \end{aligned}$$

OFDM Signals

Consider now generating samples of this signal. Assume we generate N samples during the time interval $[0, T]$. Suppose also that $f_0 T = 1$. Then

$$\begin{aligned} s[n] &= s((n/N)T)) = \Re\left[\sum_{k=0}^{N-1}(I_k + jQ_k)\exp(j2\pi k f_0 (n/N)T)\right] \\ &= \Re\left[\sum_{k=0}^{N-1}(I_k + jQ_k)\exp(j2\pi kn/N)\right] \\ &= N\left(\text{IFFT}\left[I_k + jQ_k\right]\right). \end{aligned}$$

inverse fast Fourier transform

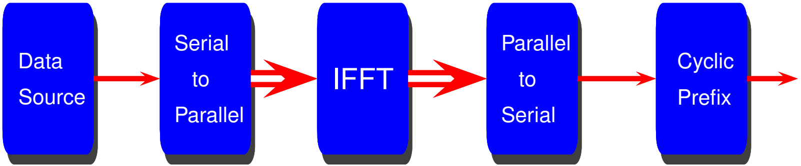
regular $NN = N^2$

Fast $N \log N$

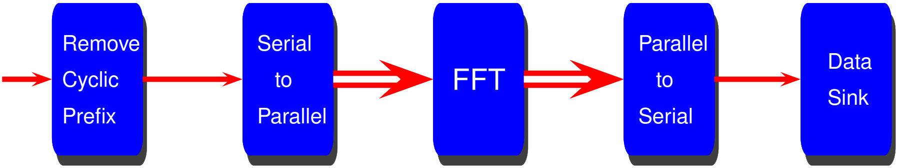
OFDM Parameters

- N carriers
- Total data rate is R_b.
- Data rate on each carrier is R_b/N.
- Duration of each data bit is N/R_b.
- Carrier Separation is R_b/N Hz.
- Cyclic Prefix of Length P added

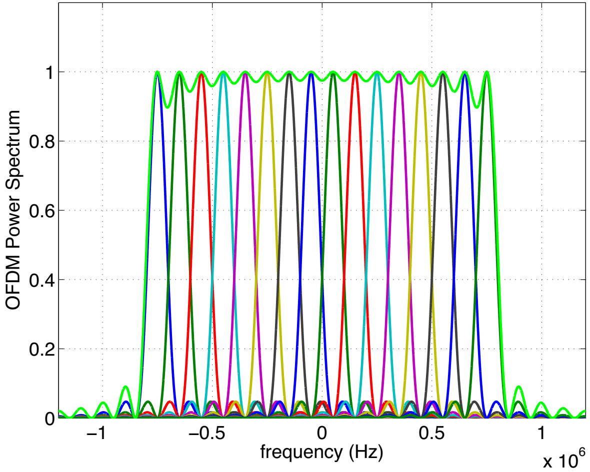
OFDM Transmitter Implementation



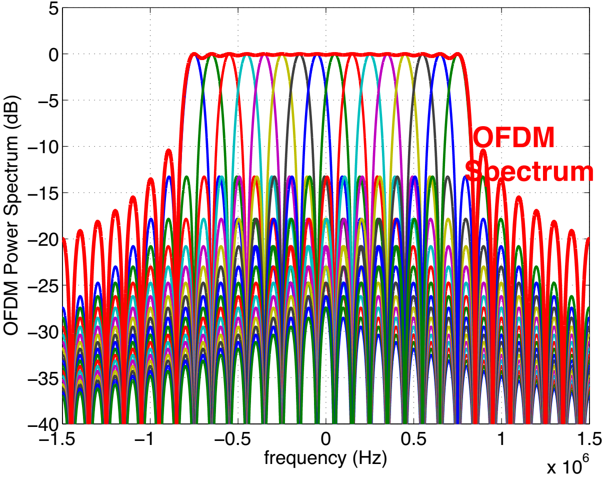
OFDM Receiver



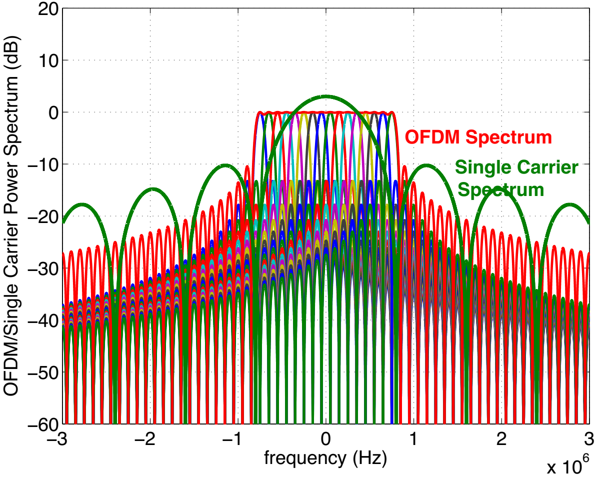
OFDM Spectrum



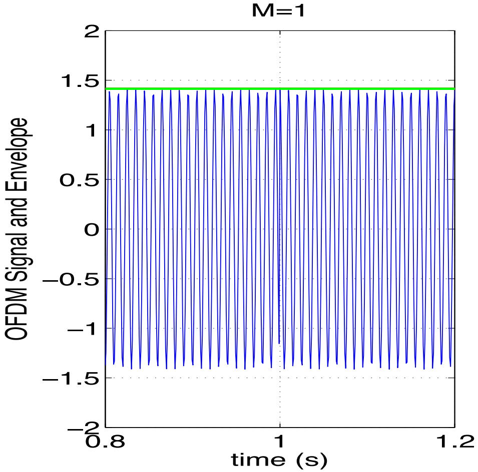
OFDM Spectrum

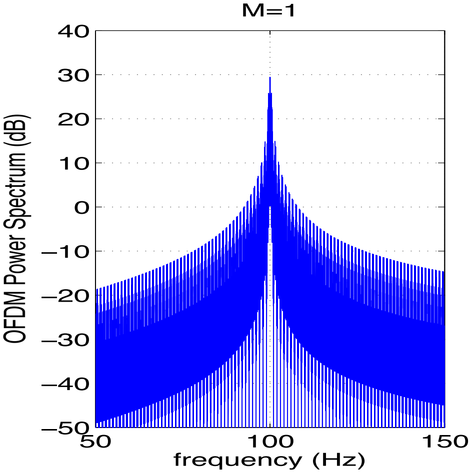


OFDM Spectrum

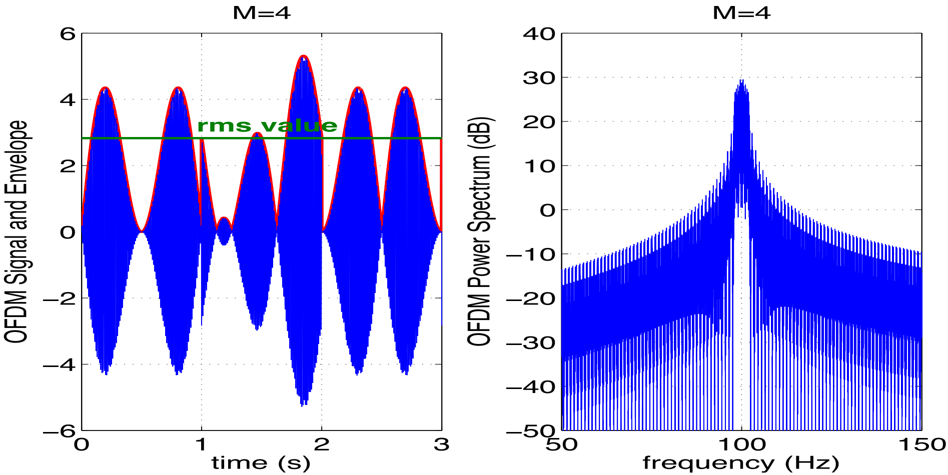


OFDM Signal and Spectrum (M=1)

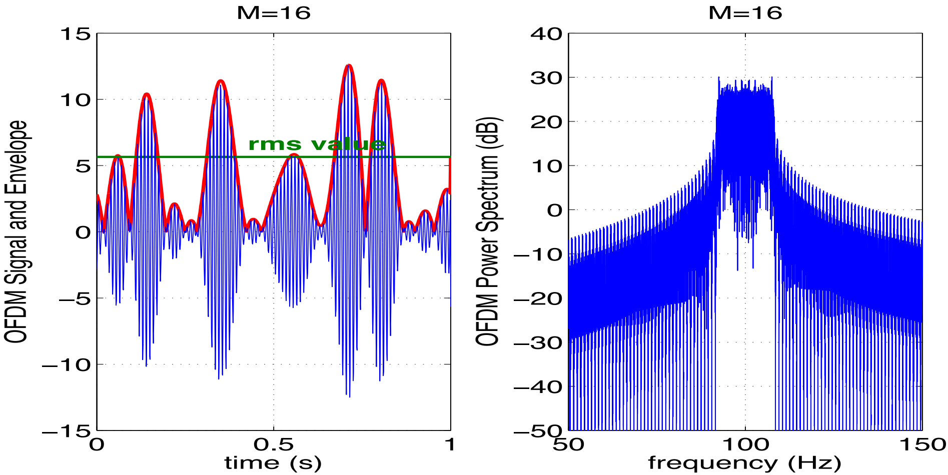




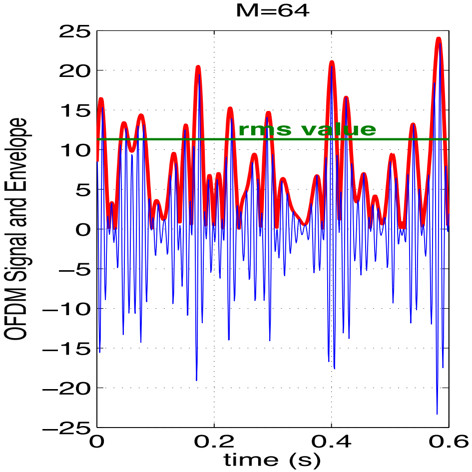
OFDM Signal and Spectrum (M=4)

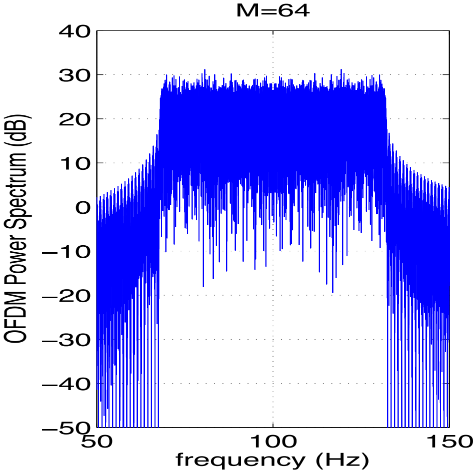


OFDM Signal and Spectrum (M=16)



OFDM Signal and Spectrum (M=64)





OFDM Summary

- OFDM has narrow spectrum (relative to single carrier with same data rate)
- OFDM has larger peak-to-average power ratio
    - Requires larger backoff of amplifier to avoid nonlinear distortion compared to single carrier
    - Larger backoff usually implies a smaller power efficiency for the amplifier
- Frequency selective fading can be mitigated in OFDM by appropriate channel estimation, coding and interleaving
- OFDM has lower sidelobes (smaller high frequency components) and thus can not make fast transitions of the amplitude or phase compared to unfiltered BPSK/QPSK.

OFDM in Wireless Systems

Parameter	802.11a (WiFi)	UWB (Multi-band)	LTE (10 MHz)	WiMax (10MHz)
Bandwidth	20MHz	528 MHz	10MHz	10MHz
Sample Time	50ns	1.8939ns	65.104 ns	1024
Number of Active Carriers	52	112	601	up to 914
Number of Guard Carriers	12	10	423	–
Data Carriers	48	100		1024
Pilots Carriers	4	12		–
FFT Size	64	128	1024	1024
Subcarrier frequency spacing	0.3125MHz (20MHz/64)	4.125MHz (528MHz/128)	15kHz	10.9375kHz
IFFT period	3.2 μs	242.42ns	66.67 μ s	91.429 μ s
Guard Interval	0.8 μs (16 samples)	70.08ns (37 samples)	4.69-5.21 μ s	11.428 μ s
OFDM Symbol Duration	4.0 μs	312.5ns	71.35-71.88 μs	102.857 μs