Lecture 11

Goals

- Be able to analyze MPSK modulation Same energy
- Be able to analyze QAM modulation
- Be able to quantify the tradeoff between data rate and energy.

Multiphase Shift Keying (MPSK)

$$
\begin{aligned}
s_i(t) &= \sqrt{2P}\cos\left[2\pi f_c t + \frac{2\pi}{M} i\right] p_T(t) \qquad 0 \le t \le T \\
&= A_{c,i}\sqrt{\frac{2}{T}}\cos(2\pi f_c t) p_T(t) - A_{s,i}\sqrt{\frac{2}{T}}\sin(2\pi f_c t) p_T(t) \\
&= A_{c,i}\varphi_0(t) + A_{s,i}\varphi_1(t)
\end{aligned}
$$

for $i = 0, 1, \ldots, M-1$, where $\varphi_0(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t) p_T(t)$ and $\varphi_1(t) = -\sqrt{\frac{2}{T}}\sin(2\pi f_0 t) p_T(t)$.

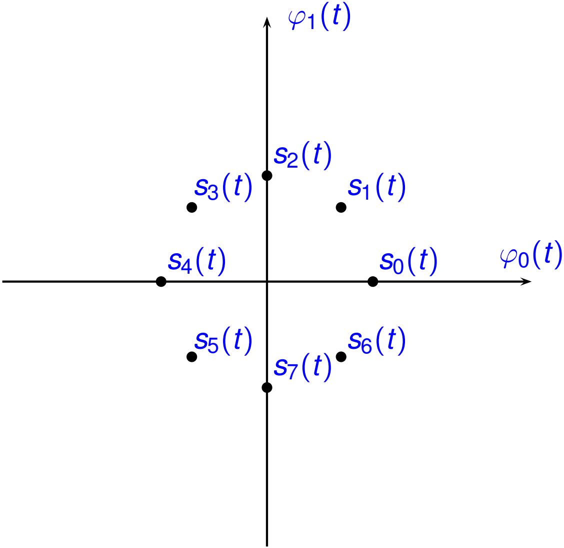
$$
\begin{aligned}
A_{c,i} &= \sqrt{E}\cos(\frac{2\pi i}{M}) \\
A_{s,i} &= \sqrt{E}\sin(\frac{2\pi i}{M})
\end{aligned}
$$

Constellation

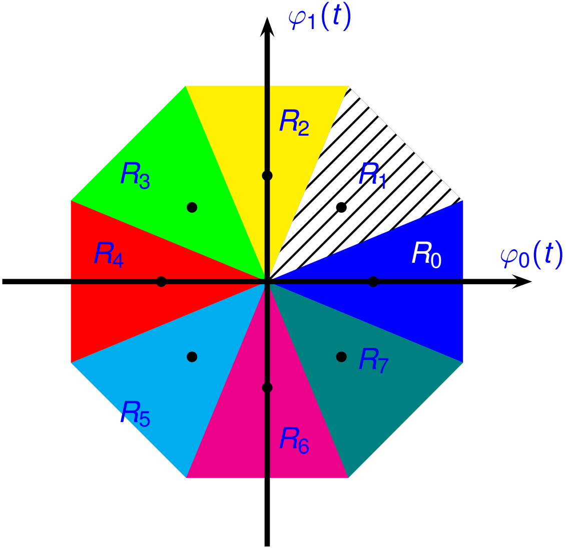
1) $d_E^2(s_i, s_j)$

2) $\frac{3 \text{ bits}}{2 \text{ dims}}$

3 bits/Hz



Decision Regions



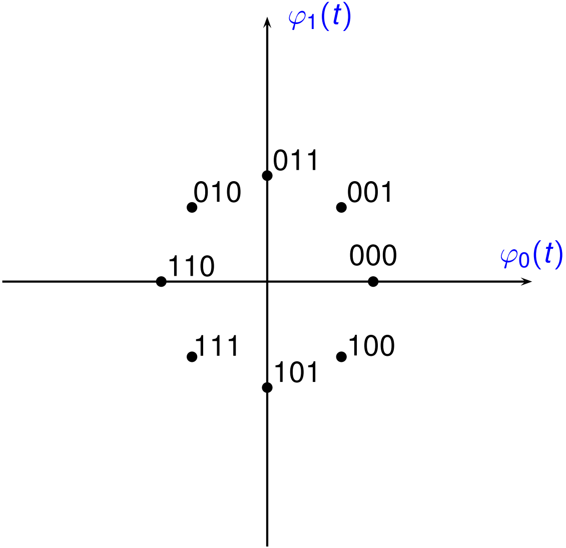
Notes on MPSK

- For this modulation scheme we should use Gray coding to map bits into signals.

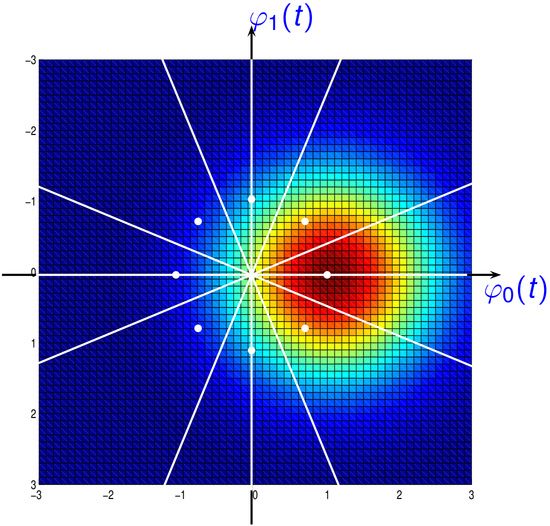
$$M = 2 \Rightarrow \text{BPSK} \qquad M = 4 \Rightarrow \text{QPSK}$$

- This type of modulation has the properties that all signals have the same power thus the use of nonlinear amplifiers (class C amplifiers) affects each signal in the same manner.
- Furthermore if we are restricted to two dimensions and every signal must have the same power then this signal set minimizes the error probability of all such signal sets.
- QPSK and BPSK are special cases of this modulation.

Gray Coding



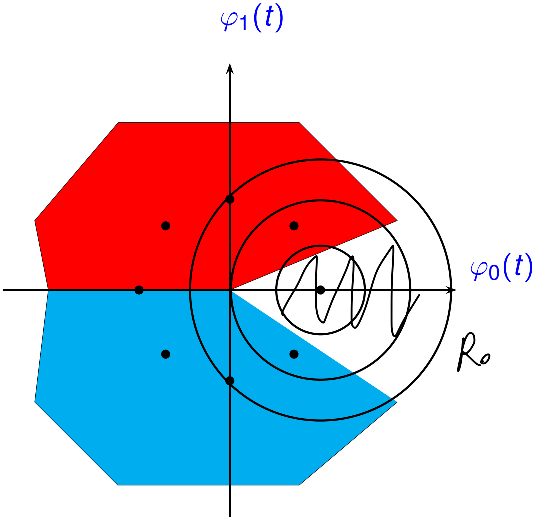
Symbol Error Probability



Symbol Error Probability for MPSK

- The error probability for MPSK can be determined as follows.
- Consider a signal 0 transmitted where with the constellation shown above.
- The probability of error given signal 0 transmitted is the probability that the noise brings the received signal outside the region R_0 where the decision is that signal 0 was transmitted.

Symbol Error Probability for MPSK



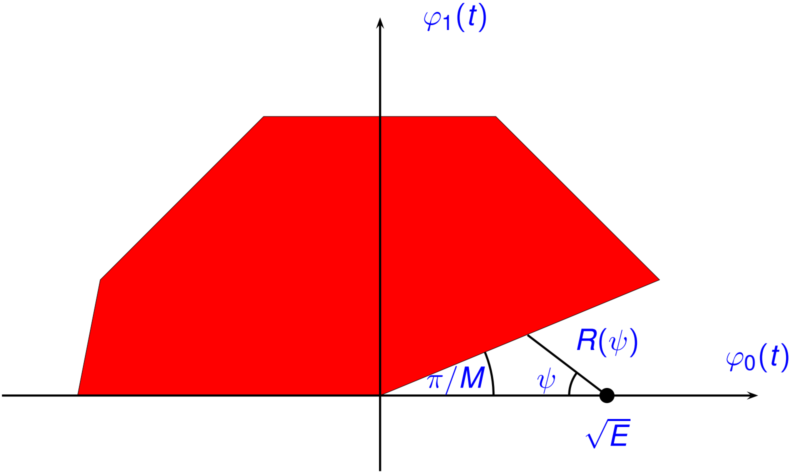
Symbol Error Probability for MPSK

H_0 = signal s_0 trans

$$
\begin{aligned}
P_{e,0} &= \int\int_{R_0^c} f_{r_0|H_0}(r_0|H_0) f_{r_1|H_0}(r_1|H_0) dr_0 dr_1 \\
&= \int\int_{R_0^c} \frac{1}{2\pi\sigma^2} \exp\{-\frac{1}{2\sigma^2}[(r_0-\sqrt{E})^2 + r_1^2]\} dr_0 dr_1 \\
&= \int\int_{R_0^c+(\sqrt{E},0)} \frac{1}{2\pi\sigma^2} \exp\{-\frac{1}{2\sigma^2}[z_0^2 + z_1^2]\} dz_0 dz_1 \\
&= 2\int_{\psi=0}^{\pi(1-\frac{1}{M})} \int_{r=R(\psi)}^{\infty} \frac{1}{2\pi} \frac{r}{2\sigma^2} \exp\{-\frac{r^2}{2\sigma^2}\} dr d\psi
\end{aligned}
$$

Change of variable from line 2 to line 3: $z_0 = r_0 - \sqrt{E}$, $z_1 = r_1$.
Rectangular-to-polar conversion line 3 to line 4.

Symbol Error Probability for MPSK



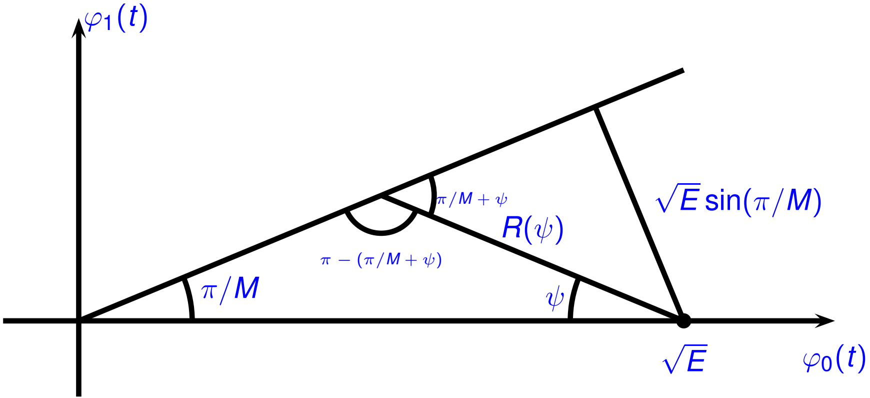
Symbol Error Probability for MPSK

$$
\begin{aligned}
P_{e,0} &= 2\int_{\psi=0}^{\pi(1-\frac{1}{M})} \frac{1}{2\pi} \int_{r=R(\psi)}^{\infty} \frac{r}{2\sigma^2} \exp\{-\frac{r^2}{2\sigma^2}\} dr d\psi \\
&= 2\int_{\psi=0}^{\pi(1-\frac{1}{M})} \frac{1}{2\pi} \int_{u=R^2(\psi)/2\sigma^2}^{\infty} \exp\{-u\} du d\psi \\
&= 2\int_{\psi=0}^{\pi(1-\frac{1}{M})} \frac{1}{2\pi} \exp\{-R(\psi)^2/2\sigma^2\} d\psi
\end{aligned}
$$

Line 1 to line 2: change of variables $u = r^2/(2\sigma^2)$.

$du = \frac{2r \cdot dr}{2\sigma^2}$

Symbol Error Probability for MPSK



$$R(\psi)\sin(\pi/M+\psi)=\sqrt{E}\sin(\pi/M)$$

Symbol Error Probability for MPSK

R_0^c is the complement of R_0 and R is the distance from the signal point s_0 to the line with slope π/M.

$$\begin{aligned} R(\psi) &= \frac{\sqrt{E}\sin(\pi/M)}{\sin(\pi-(\psi+\pi/M))} \\ &= \frac{\sqrt{E}\sin(\pi/M)}{\sin(\psi+\pi/M)}. \end{aligned}$$

Thus

$$\frac{R(\psi)^2}{2\sigma^2} = \frac{E\sin^2(\pi/M)}{N_0\sin^2(\psi+\pi/M)}.$$

Symbol Error Probability for MPSK

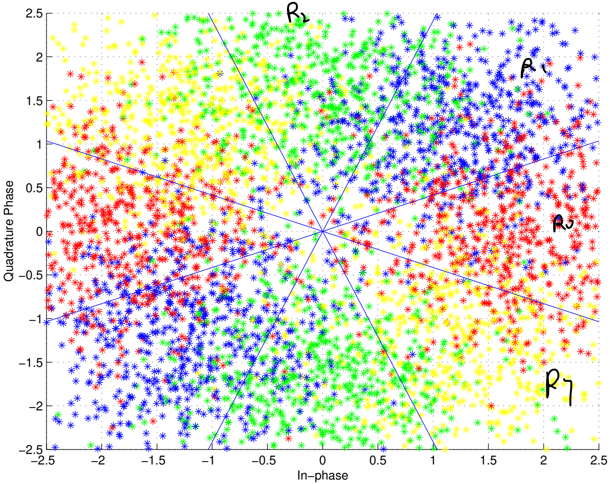
In the derivation of the error probability the last line follows from a change of variables where the point z_0, z_1 is mapped to an angle φ from the horizontal axis with reference point s_0 and a magnitude r from the point s_0. The symbol error probability is thus

$$P_{e,s} = 2\int_{\psi=0}^{\pi(1-\frac{1}{M})} \frac{1}{2\pi} \exp\{-\frac{E\sin^2(\pi/M)}{N_0\sin^2(\psi+\pi/M)}\}d\psi$$

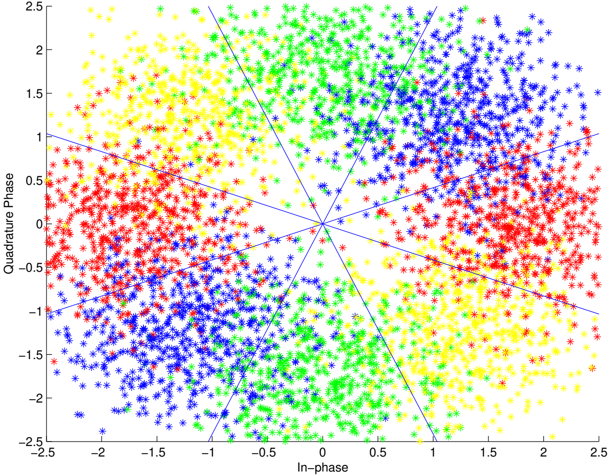
$$E_b = E/\underbrace{\log_2(M)}_{\text{\# of bit per signal}}$$

Constellation ($E_b/N_0 = 0$dB)

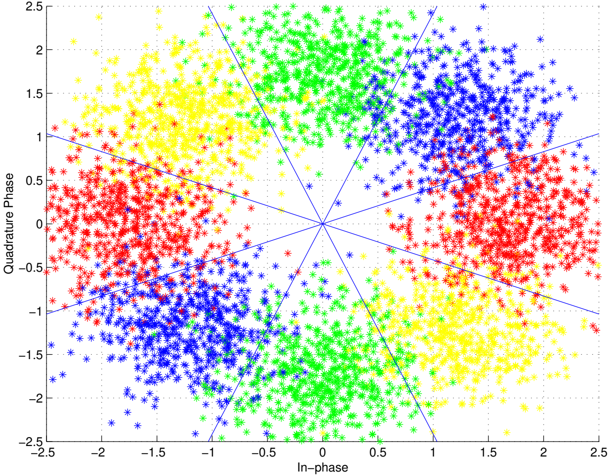
$M=8$



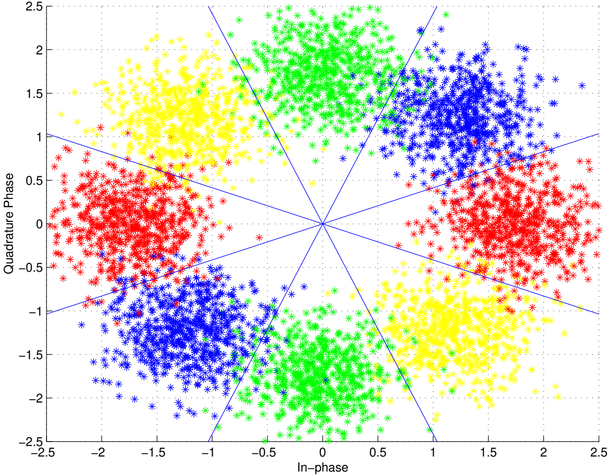
Constellation ($E_b/N_0 = 2$dB)



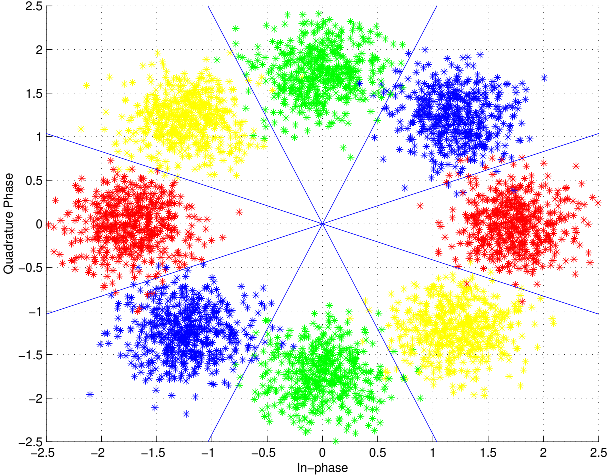
Constellation ($E_b/N_0 = 4$dB)



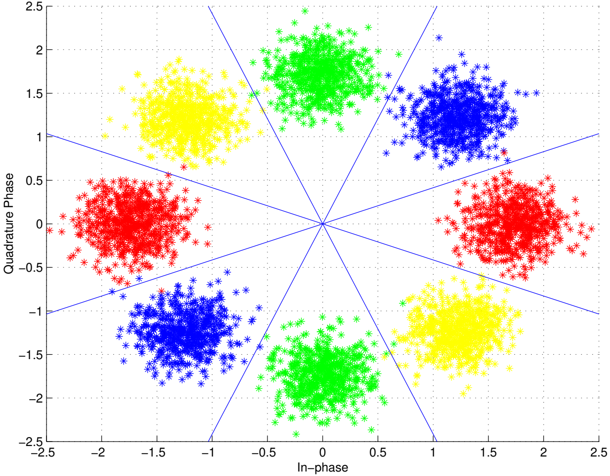
Constellation ($E_b/N_0 = 6$dB)



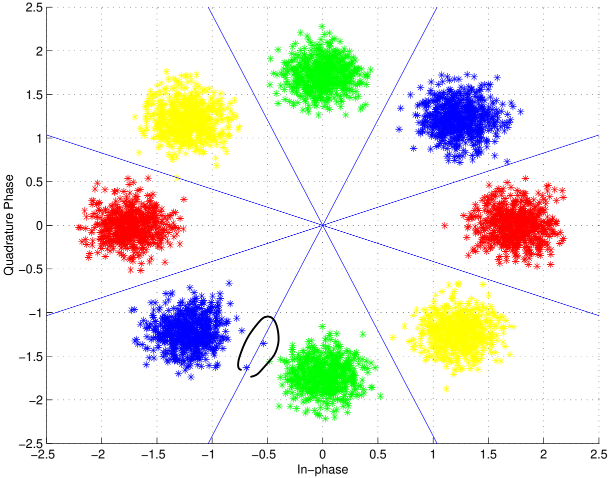
Constellation ($E_b/N_0 = 8$dB)



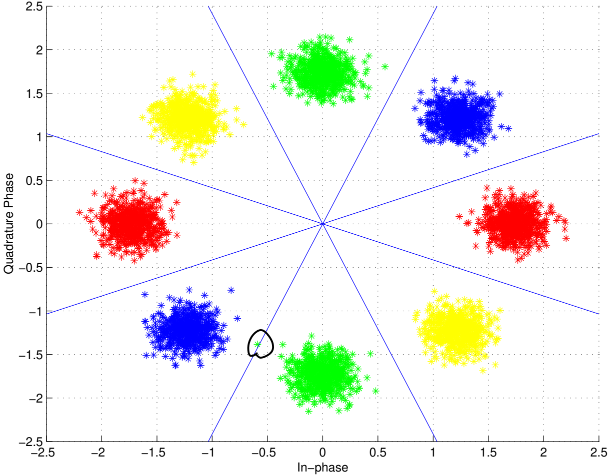
Constellation ($E_b/N_0 = 10$dB)



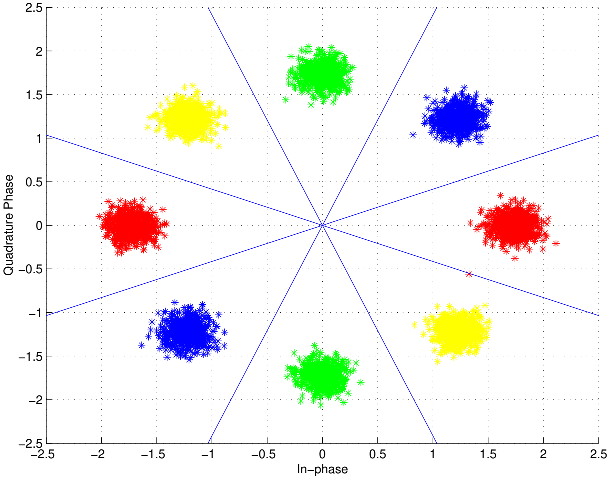
Constellation ($E_b/N_0 = 12$dB)



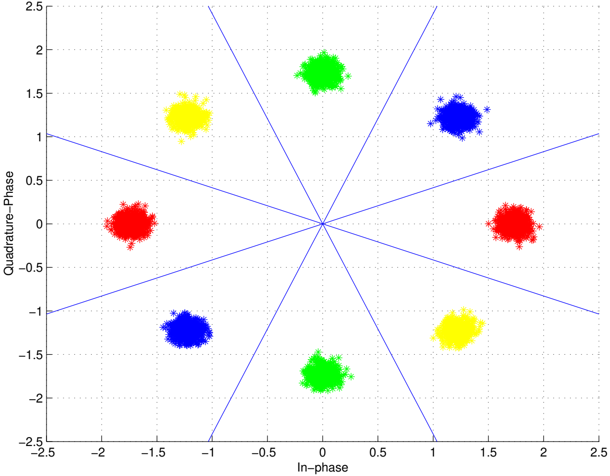
Constellation ($E_b/N_0 = 14$dB)



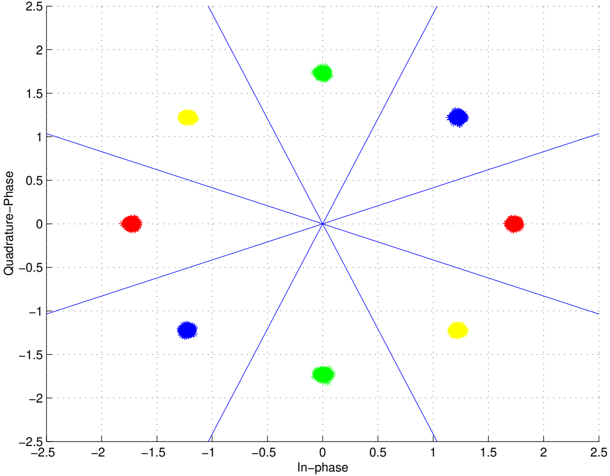
Constellation ($E_b/N_0 = 16$dB)



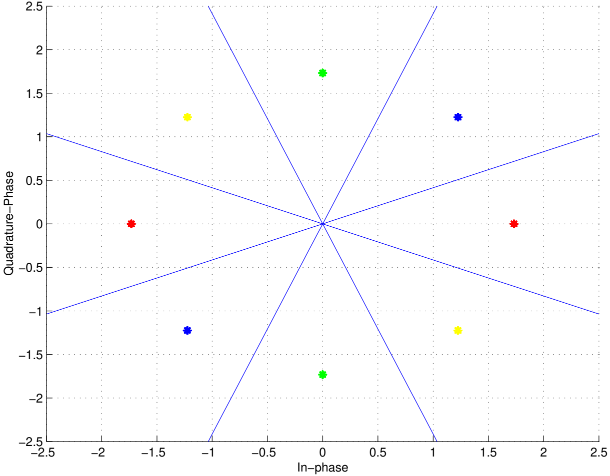
Constellation ($E_b/N_0 = 20$dB)



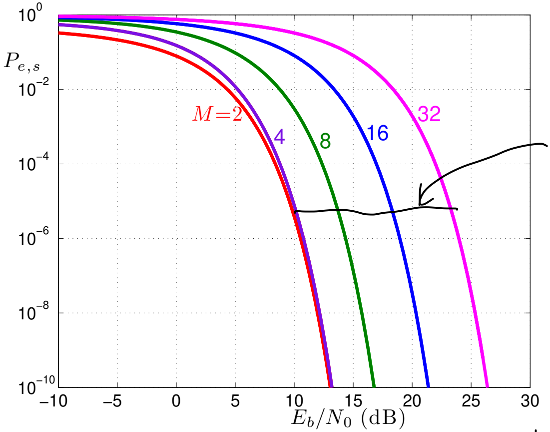
Constellation ($E_b/N_0 = 30$dB)



Constellation ($E_b/N_0 = 50$dB)



Symbol Error Probability for MPSK Signaling



extra Energy you need

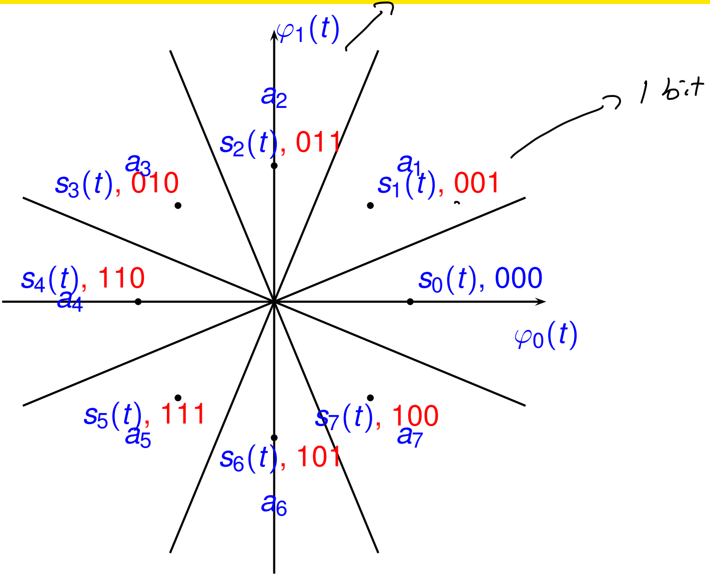
Signal to noise ratio

Bit Error Probability for 8-PSK

- For 8-PSK we can compute the bit error probability without too much difficulty.
- Assume signal $s_0(t)$ is the transmitted signal and corresponds to the information bits being (0 0 0).
- Furthermore assume that the constellation is gray coded.
- Label the decision regions $R_0, \ldots, R_7$ in a circular fashion starting with the signal s_0 which has coordinates $(\sqrt{E}, 0)$.
- The Gray mapping is shown below

Signal	Bits	Signal	Bits
$s_0(t)$	000	$s_4(t)$	110
$s_1(t)$	001	$s_5(t)$	111
$s_2(t)$	011	$s_6(t)$	101
$s_3(t)$	010	$s_7(t)$	100

Bit Error Probability for 8-PSK



Bit Error Probability for 8-PSK

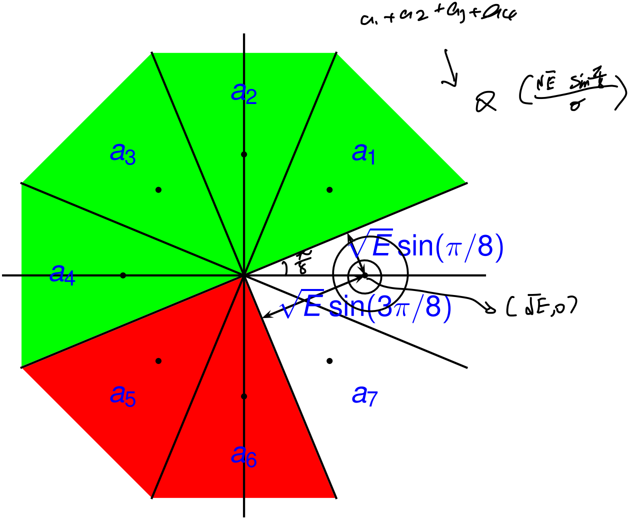
Because of symmetry we assume that $s_0(t)$ is the transmitted signal. Clearly

$$\begin{aligned} a_1 = P\{R_1|s_0\} &= P\{R_7|s_0\} = a_7 \\ a_2 = P\{R_2|s_0\} &= P\{R_6|s_0\} = a_6 \\ a_3 = P\{R_3|s_0\} &= P\{R_5|s_0\} = a_5 \end{aligned}$$

Let $a_i = P\{R_i|s_0\}$. The bit error probability is

$$\begin{aligned} P_{e,b} &= \frac{1}{3}[a_1 + 2a_2 + a_3 + 2a_4 + 3a_5 + 2a_6 + a_7] \\ P_{e,b} &= \frac{1}{3}[2a_1 + 2a_2 + 2a_3 + 2a_4 + 2a_5 + 2a_6] \end{aligned}$$

Bit Error Probability for 8-PSK



Bit Error Probability for 8-PSK

- Now it is clear that $a_1 + a_2 + a_3 + a_4$ is the probability of a half plane and $a_5 + a_6$ is the probability of a quarter plane.
- Also, we have that $E = 3E_b$.
- Thus for 8-ary PSK with Gray mapping the bit error probability is

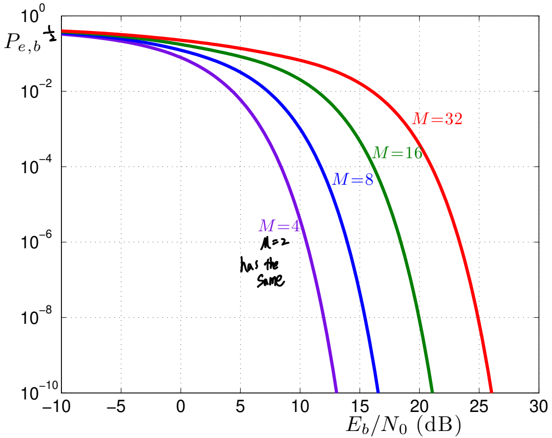
green one

$$P_{e,b} = \frac{2}{3}\left[Q(d_3 \sin(\theta)) + Q(d_3 \sin(3\theta))(1 - Q(d_3 \sin(\theta)))\right]$$

red region

where $d_3 = \sqrt{6E_b/N_0}$ and $\theta = \pi/8$.

Bit Error Probability for MPSK Signaling (with Gray coding)



M-ary Pulse Amplitude Modulation (PAM)

$\mathcal{N}=1$

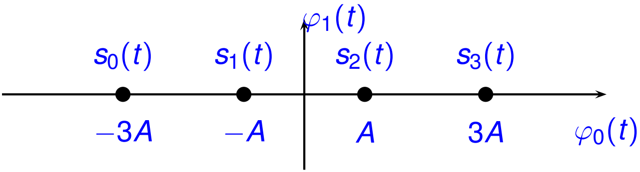
$$s_i(t) = A_i\, \varphi_0(t), \qquad 0 \le t \le T$$

where

$$A_i = (2i + 1 - M)A \qquad i = 0, 1, \ldots, M-1$$

$$E_i = A_i^2$$

$\mathcal{M}=4$



$E_0 = 9A^2$ $\quad E_1 = A^2$ $\quad A^2$ $\quad 9A^2$

M-ary Pulse Amplitude Modulation (PAM)

$$\overline{E} = \frac{1}{M}\sum_{i=0}^{M-1} E_i = \frac{A^2}{M}\sum_{i=0}^{M-1}(2i+1-M)^2 = \frac{M^2-1}{3}A^2$$

where we used the fact

$$\sum_{m=1}^{J} m^2 = J^3/3 + J^2/2 + J/6.$$

The average energy per information bit is

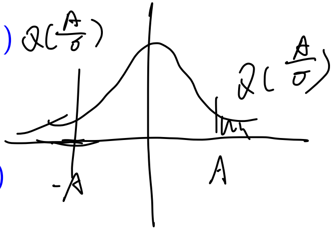
$$\begin{aligned}\overline{E_b} &= \frac{\overline{E}}{\log_2(M)} \\ &= \frac{M^2-1}{3\log_2(M)}A^2.\end{aligned}$$

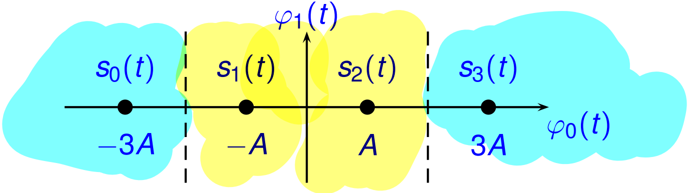
Symbol Error Probability

The error probability (for 4-ary) is

$$P_{e,1} = P_{e,2} = 2Q\left(\frac{A}{\sqrt{N_0/2}}\right)$$

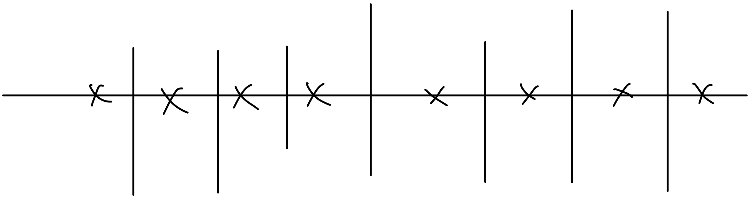
$$P_{e,0} = P_{e,3} = Q\left(\frac{A}{\sqrt{N_0/2}}\right)$$





The average error probability (for 4-ary PAM) is

$$\begin{aligned}
\bar{P}_e &= \frac{1}{4}P_{e,0} + \frac{1}{4}P_{e,1} + \frac{1}{4}P_{e,2} + \frac{1}{4}P_{e,3} \\
&= \frac{3}{2}Q\left(\frac{A}{\sqrt{N_0/2}}\right) = \frac{3}{2}Q\left(\sqrt{\frac{2\bar{E}}{5N_0}}\right)
\end{aligned}$$

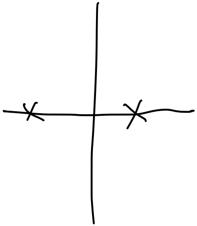


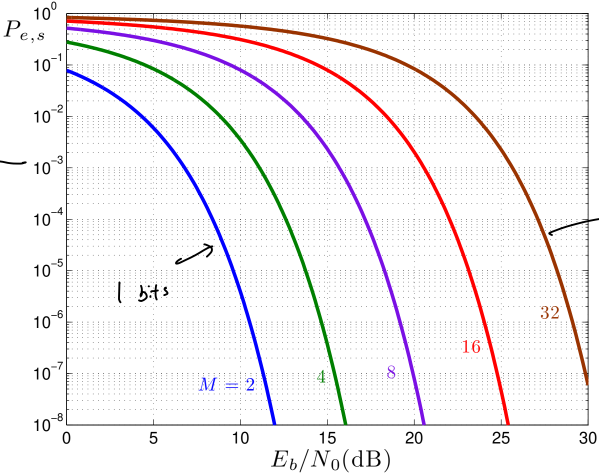
Symbol Error Probability

In general the error probability is

$$\begin{aligned} P_{e,s} &= \left(\frac{2(M-1)}{M}\right) Q\left(\sqrt{\frac{6\overline{E}}{(M^2-1)N_0}}\right) \\ &= \left(\frac{2(M-1)}{M}\right) Q\left(\sqrt{\frac{6\overline{E_b}\log_2(M)}{(M^2-1)N_0}}\right) \end{aligned}$$

Symbol Error Probability for *M*-PAM





1 bits

5 bits

Bit Error Probability for 4-PAM with Gray Coding

rev

trans	00	01	11	10
00	0	1	2	1
01	1	0	1	2
11	2	1	0	1
10	1	2	1	0

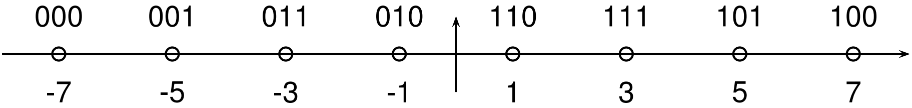
# of bit errors

1 bit error   2 bits error   between 3A and 5A   greater 5A

$$
\begin{aligned}
P_{e,b}(00) &= \frac{1}{2}\left[Q(\frac{A}{\sigma}) - Q(\frac{3A}{\sigma})\right] + \frac{2}{2}\left[Q(\frac{3A}{\sigma}) - Q(\frac{5A}{\sigma})\right] + \frac{1}{2}\left[Q(\frac{5A}{\sigma})\right] = \frac{1}{2}Q(\frac{1A}{\sigma}) + \frac{1}{2}Q(\frac{3A}{\sigma}) - \frac{1}{2}Q(\frac{5A}{\sigma})\\
P_{e,b}(01) &= \frac{1}{2}\left[Q(\frac{A}{\sigma})\right] + \frac{1}{2}\left[Q(\frac{A}{\sigma}) - Q(\frac{3A}{\sigma})\right] + \frac{2}{2}\left[Q(\frac{3A}{\sigma})\right] = \frac{2}{2}Q(\frac{A}{\sigma}) + \frac{1}{2}Q(\frac{3A}{\sigma})\\
P_{e,b}(11) &= \frac{1}{2}\left[Q(\frac{A}{\sigma}) - Q(\frac{3A}{\sigma})\right] + \frac{2}{2}\left[Q(\frac{3A}{\sigma})\right] + \frac{1}{2}\left[Q(\frac{A}{\sigma})\right] = \frac{2}{2}Q(\frac{A}{\sigma}) + \frac{1}{2}Q(\frac{3A}{\sigma})\\
P_{e,b}(10) &= \frac{1}{2}\left[Q(\frac{A}{\sigma}) - Q(\frac{3A}{\sigma})\right] + \frac{2}{2}\left[Q(\frac{3A}{\sigma}) - Q(\frac{5A}{\sigma})\right] + \frac{1}{2}\left[Q(\frac{5A}{\sigma})\right] = \frac{1}{2}Q(\frac{A}{\sigma}) + \frac{1}{2}Q(\frac{3A}{\sigma}) - \frac{1}{2}Q(\frac{5A}{\sigma})
\end{aligned}
$$

$$
\begin{aligned}
P_{e,b} &= \frac{1}{4}\left[P_{e,b}(00) + P_{e,b}(01) + P_{e,b}(11) + P_{e,b}(10)\right]\\
&= \frac{3}{4}Q(\frac{A}{\sigma}) + \frac{2}{4}Q(\frac{3A}{\sigma}) - \frac{1}{4}Q(\frac{5A}{\sigma})
\end{aligned}
$$

Bit Error Probability for 8-PAM with Gray Coding



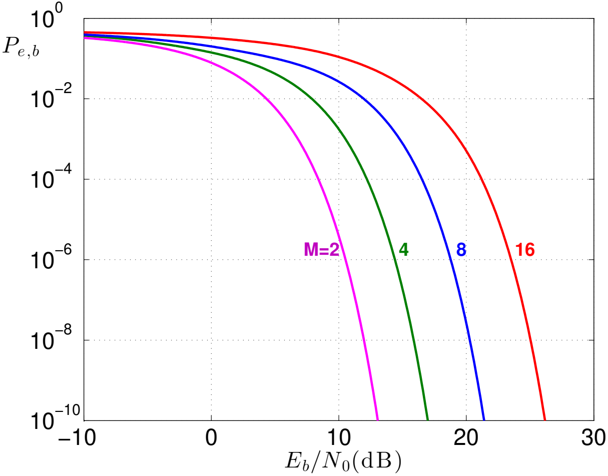
	000	001	011	010	110	111	101	100
000	0	1	2	1	2	3	2	1
001	1	0	1	2	3	2	1	2
011	2	1	0	1	2	1	2	3
010	1	2	1	0	1	2	3	2
110	2	3	2	1	0	1	2	1
111	3	2	1	2	1	0	1	2
101	2	1	2	3	2	1	0	1
100	1	2	3	2	1	2	1	0

Symbol and Bit Error Probabilities for 8-PAM

- The bit error probability depends on the mapping from information bits to signals.
- Gray coded mapping are a natural choice since neighboring signals differ by only a single bit.
- For Gray coded mapping and 8-ary PAM the bit error probability is given by

$$\begin{aligned} P_{e,b} &= \frac{14}{24}Q\left(\sqrt{\frac{2\bar{E}_b}{7N_0}}\right) + \frac{12}{24}Q\left(3\sqrt{\frac{2\bar{E}_b}{7N_0}}\right) - \frac{2}{24}Q\left(5\sqrt{\frac{2\bar{E}_b}{7N_0}}\right) \\ &\quad + \frac{2}{24}Q\left(9\sqrt{\frac{2\bar{E}_b}{7N_0}}\right) - \frac{2}{24}Q\left(13\sqrt{\frac{2\bar{E}_b}{7N_0}}\right). \end{aligned}$$

Bit Error Probability for M-PAM



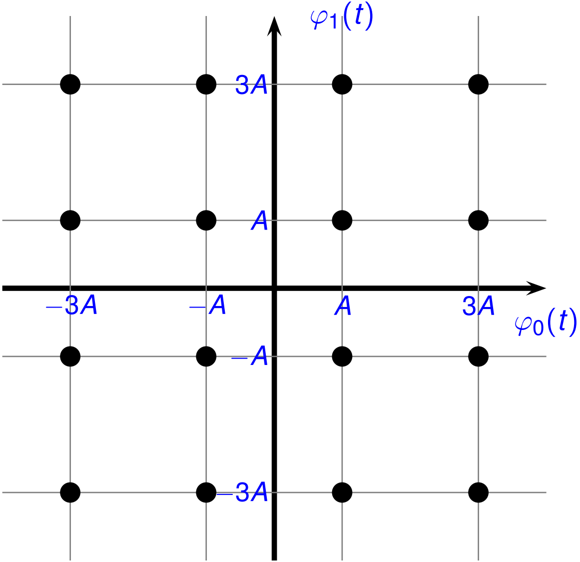
Quadrature Amplitude Modulation

For $i = 0, \ldots, M-1$

$$
\begin{aligned}
s_i(t) &= A_i \cos(2\pi f_c t) + B_i \sin(2\pi f_c t) \quad 0 \leq t \leq T \\
s_i(t) &= A_i \sqrt{\frac{T}{2}} \varphi_0(t) + B_i \sqrt{\frac{T}{2}} \varphi_1(t) \quad 0 \leq t \leq T
\end{aligned}
$$

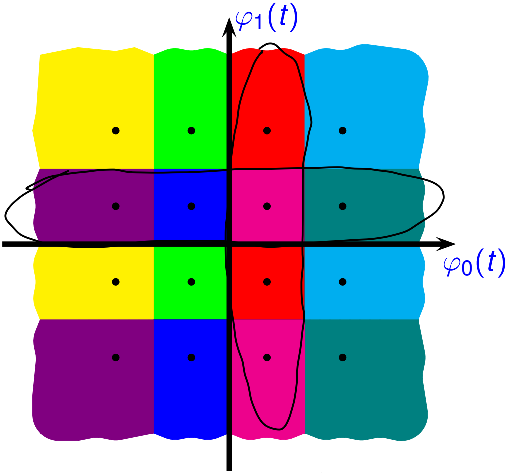
$$A_i, B_i \in \{-3A, -A, A, 3A\}$$

16QAM Constellation



Decision Regions

16 QAM

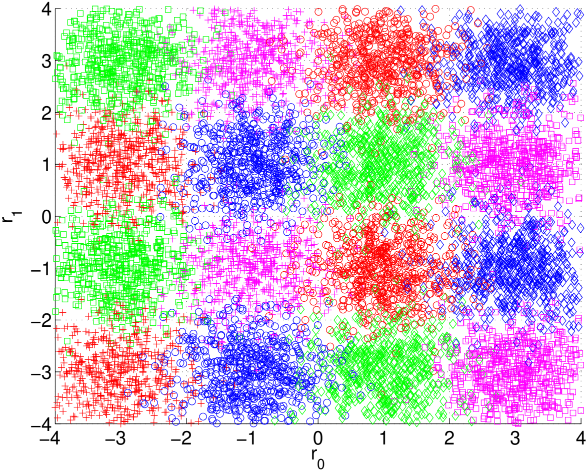


P(Correct) = P(correct in φ_0) P(correct in φ_1)

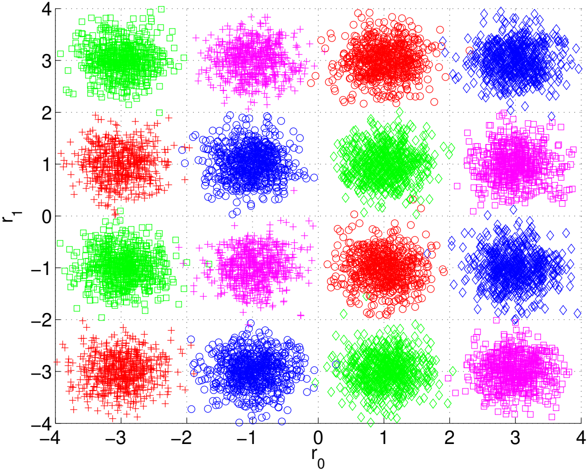
P(error) = 1 − P(correct) = 1 − ((1 − p_0)(1 − p_1))

QAM Constellation ($E_b/N_0 = 15$dB)

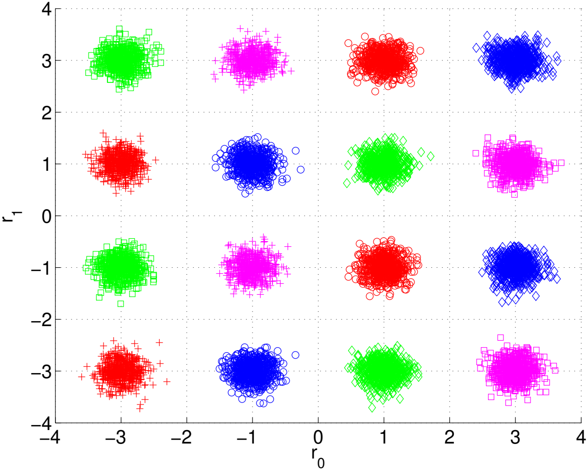
correct in φ_0 correct in φ_1



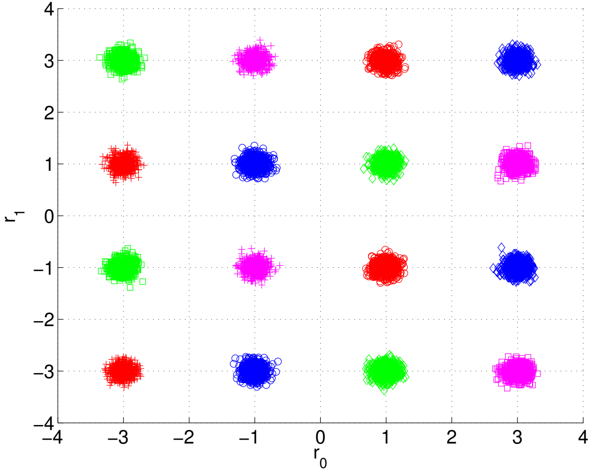
QAM Constellation ($E_b/N_0 = 20$dB)



QAM Constellation ($E_b/N_0 = 25$dB)



QAM Constellation ($E_b/N_0 = 30$dB)



Symbol and Bit Error Probabilities for QAM

independent

Since QAM is two PAM systems in quadrature. $P_{e,2} = 1 - (1 - P_{e,1})^2$ for PAM with $\sqrt{M}$ signals

- The bit error probability depends on the mapping from information bits to signals. $P_e(16\,QAM) = 1 - [1 - Pe(4PAM)]^2$
- Gray coded mapping are a natural choice since neighboring signals differ by only a single bit.
- Gray code is used for each dimension separately.
- For Gray coded mapping and $M = 16$ the bit error probability is given by

$$P_{e,b} = \frac{3}{4} Q\left(\sqrt{\frac{4\bar{E}_b}{5N_0}}\right) + \frac{2}{4} Q\left(3\sqrt{\frac{4\bar{E}_b}{5N_0}}\right) - \frac{1}{4} Q\left(5\sqrt{\frac{4\bar{E}_b}{5N_0}}\right)$$

Symbol and Bit Error Probabilities for QAM

- The bit error probability depends on the mapping from information bits to signals.
- Gray coded mapping are a natural choice since neighboring signals differ by only a single bit.
- For Gray coded mapping and $M = 64$ the bit error probability is the same as 8-ary PAM and is given by

$$
\begin{aligned}
P_{e,b} \;=\; & \frac{14}{24} Q\left(\sqrt{\frac{2\bar{E}_b}{7N_0}}\right) + \frac{12}{24} Q\left(3\sqrt{\frac{2\bar{E}_b}{7N_0}}\right) - \frac{2}{24} Q\left(5\sqrt{\frac{2\bar{E}_b}{7N_0}}\right) \\
& + \frac{2}{24} Q\left(9\sqrt{\frac{2\bar{E}_b}{7N_0}}\right) - \frac{2}{24} Q\left(13\sqrt{\frac{2\bar{E}_b}{7N_0}}\right)
\end{aligned}
$$

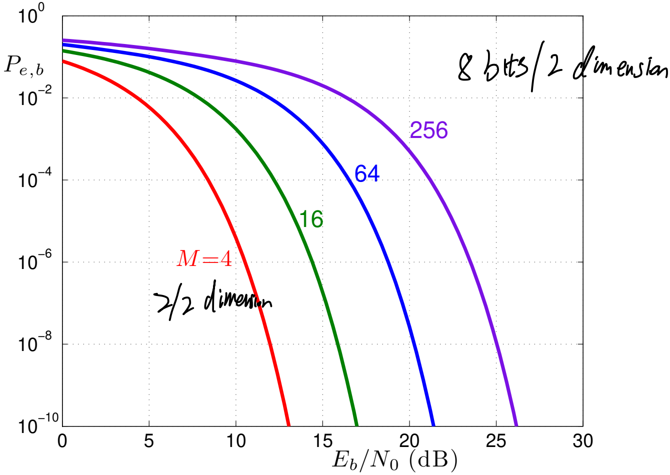
Bit Error Probabilities for QAM, $M = 256$ (Same as 16-PAM)

$$P_{e,b} = \frac{1}{(4 \times 16)} \sum_{i=1}^{15} A_i Q\left((2i-1)\sqrt{\frac{8\bar{E}_b}{85N_0}}\right)$$

where A_i is given in the table below.

i	A_i	i	A_i	i	A_i
1	30	6	8	11	-6
2	28	7	-10	12	-4
3	-2	8	-8	13	2
4	0	9	10	14	0
5	10	10	8	15	-2

BER of QAM



10^{0}
$P_{e,b}$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
5
10
15
20
25
30
E_b/N_0 (dB)
256
64
16
$M=4$
8 bits/2 dimension
2/2 dimension

Applications of QAM

- One application of QAM is to bandwidth constrained channels.
- We can consider as a baseline a two dimensional modulation system transmitting a 2400 symbols per second.
- If each symbol represents 4 bits of information then the data rate is 9600 bits per second.
- So we would like to have more signals per dimension in order to increase the data rate.
- However, we must try to keep the signals as far apart from each other as possible (in oder to keep the error rate low).
- So an increase of the size of the signal constellation for fixed minimum distance would likely increase the total signal energy transmitted.

Applications of QAM

- Transmission of signals for digital TV over cables uses 64 QAM and 256 QAM (Standard ITU-T J.83).
- For the 64 QAM format the symbol rate is 5.056941 Msymbols per second. A square-root raised cosine filter with rolloff of 0.18 is used.
- For the 256 QAM format the symbol rate is 5.360537 Msymbols per second. A square-root raised cosine filter with rolloff of 0.12 is used.

Applications of QAM: HDTV

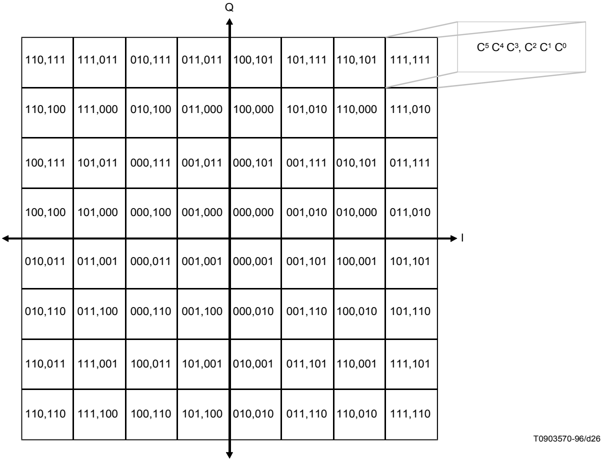
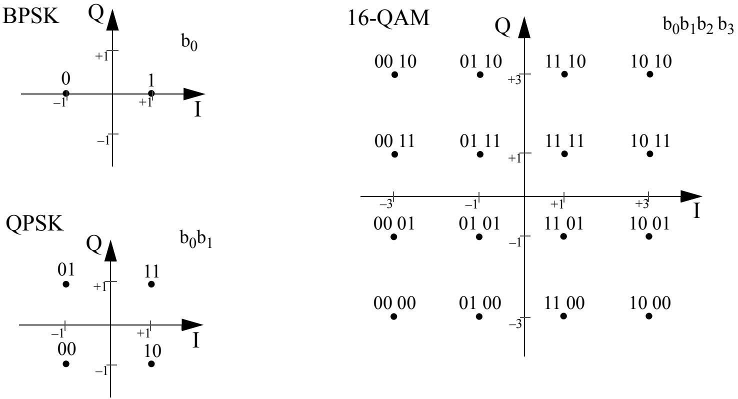


Figure B.18 – 64-QAM constellation

Applications of QAM: 802.11 wifi



Source: IEEE 802.11-2007 Standard, page 608

Applications of QAM: 802.11

64 QAM

wifi

ver dim

hor

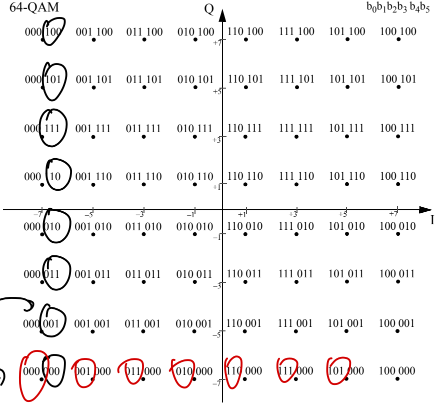


Figure 17-10—BPSK, QPSK, 16-QAM, and 64-QAM constellation bit encoding

Source: IEEE 802.11-2007 Standard, page 608

Applications of 3GPP (Cell Phones)

Release 5 20 3G TS 25.213 V5.0.0 (2002-03)

Table 4: 16 QAM index bits

i	0	1	2	3
b(i)	i_1	q_1	i_2	q_2

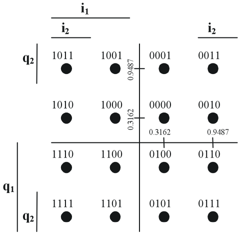


Figure 9: 16 QAM constellation

Source: 3GPP TS25.213 v5.0.0

Applications of 3GPP (Cell Phones)

In the uplink, the complex-valued chip sequence generated by the spreading process is QPSK modulated as shown in Figure 7 below:

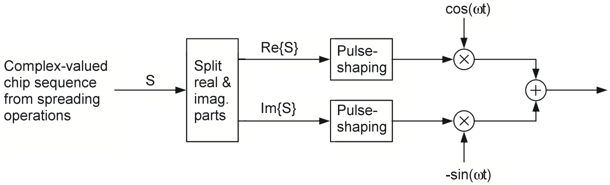


Figure 7: Uplink modulation

Source: 3GPP TS25.104 v5.0.0

Applications of 3GPP (Cell Phones)

6.8.1 Transmit pulse shape filter

The transmit pulse-shaping filter is a root-raised cosine (RRC) with roll-off $\alpha=0.22$ in the frequency domain. The impulse response of the chip impulse filter $RC_0(t)$ is

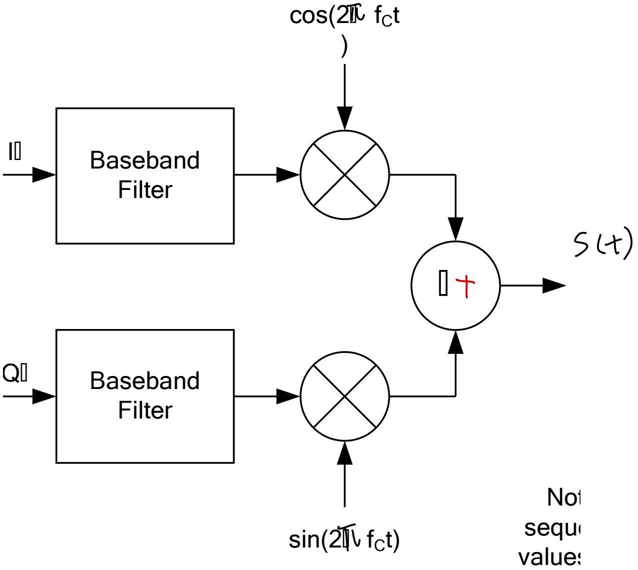
$$RC_0(t)=\frac{\sin\left(\pi\frac{t}{T_C}(1-\alpha)\right)+4\alpha\frac{t}{T_C}\cos\left(\pi\frac{t}{T_C}(1+\alpha)\right)}{\pi\frac{t}{T_C}\left(1-\left(4\alpha\frac{t}{T_C}\right)^2\right)}$$

Where the roll-off factor $\alpha = 0.22$ and the chip duration:

$$T_c = \frac{1}{chiprate} \approx 0.26042\mu\text{s}$$

Source: 3GPP TS25.213 v10.0.0

Applications of 3GPP2 (Cell Phones)



Applications of 3GPP2 (Cell Phones)

The impulse response of the baseband filter, s(t), should satisfy the following equation:

$$\text{Mean Squared Error} = \sum_{k=0}^{\infty} [\alpha s(kT_s - \tau) - h(k)]^2 \leq 0.03,$$

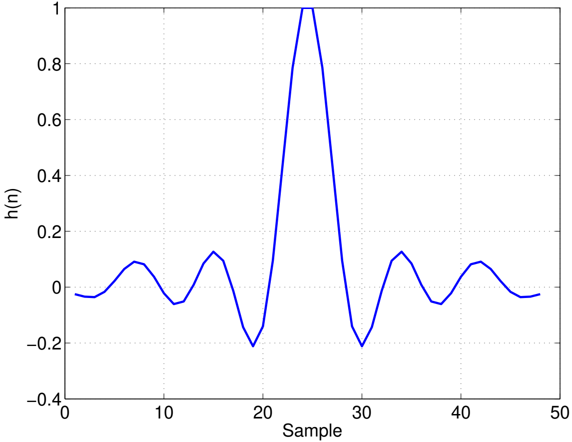
Source: 3GPP2 C.S0024-A v3.0 $T_s = T_c/4 = 1/(4 * 1.2288 \times 10^6)s$

Applications of 3GPP2 (Cell Phones)

Table 12.3.1.3.5.1-1. Baseband Filter Coefficients

k	h(k)
0, 47	–0.025288315
1, 46	–0.034167931
2, 45	–0.035752323
3, 44	–0.016733702
4, 43	0.021602514
5, 42	0.064938487
6, 41	0.091002137
7, 40	0.081894974
8, 39	0.037071157
9, 38	–0.021998074
10, 37	–0.060716277
11, 36	–0.051178658
12, 35	0.007874526
13, 34	0.084368728
14, 33	0.126869306
15, 32	0.094528345
16, 31	–0.012839661
17, 30	–0.143477028
18, 29	–0.211829088
19, 28	–0.140513128
20, 27	0.094601918
21, 26	0.441387140
22, 25	0.785875640
23, 24	1.0

Applications of 3GPP2 (Cell Phones)



Source: 3GPP2 C.S0024-A v3.0

Applications of 3GPP2 (Cell Phones)

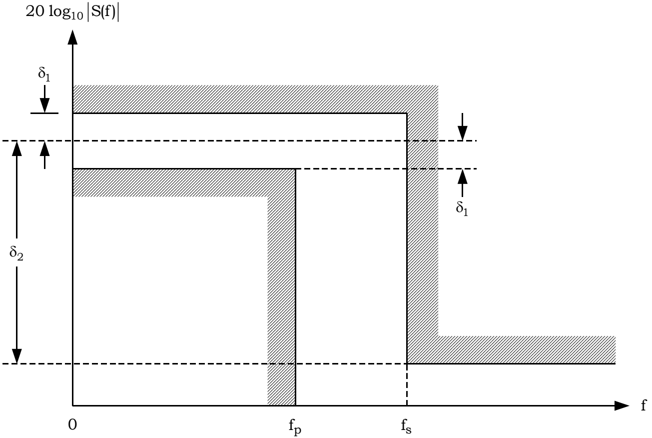
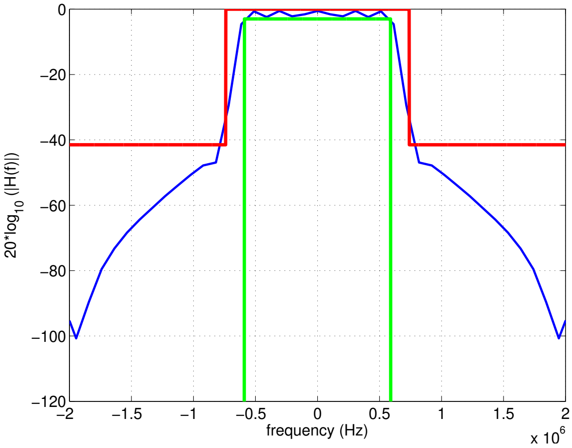


Figure 12.3.1.3.5.1-1. Baseband Filter Frequency Response Limits

Source: 3GPP2 C.S0024-A v3.0

$\delta_1 = 1.5dB, \delta_2 = 40dB, f_p = 590kHz, f_s = 740kHz$

Applications of 3GPP2 (Cell Phones)



Source: 3GPP2 C.S0024-A v3.0

Applications of 3GPP2 (Cell Phones)

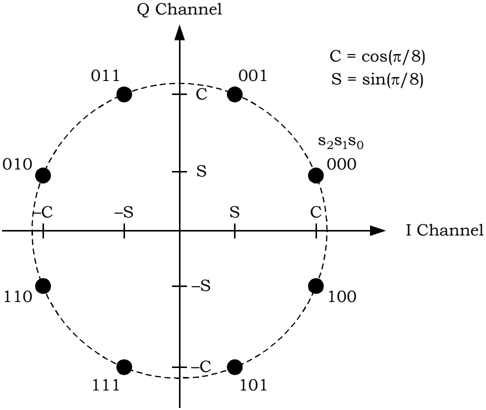
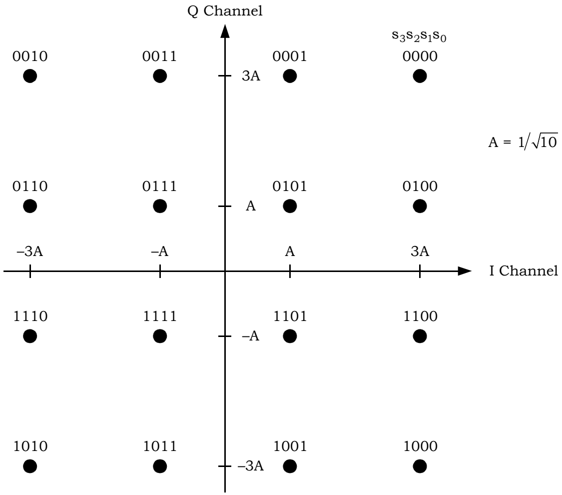


Figure 12.3.1.3.2.3.5.2-1. Signal Constellation for 8-PSK Modulation

Source: 3GPP2 C.S0024-A v3.0

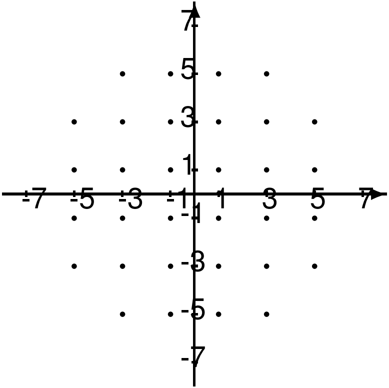
Applications of 3GPP2 (Cell Phones)

3GPP2 C.S0024-A v3.0 Default (Subtype 0) and Subtype 1 Physical Layer Protocol



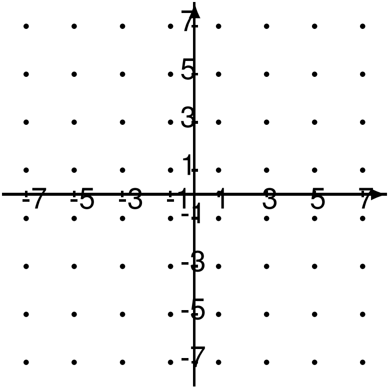
32-ary signal sets

Consider a 32-ary QAM signal set shown below. The average energy is 20. The minimum distance is 2 and the rate is 5 bits/dimension.



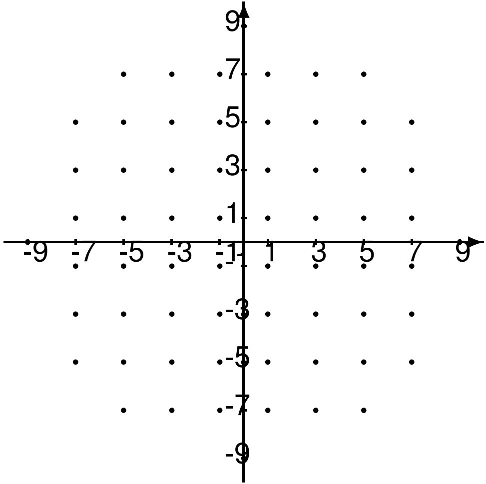
64-ary signal sets

Consider a 64-ary QAM signal set shown below. The average energy is 42.



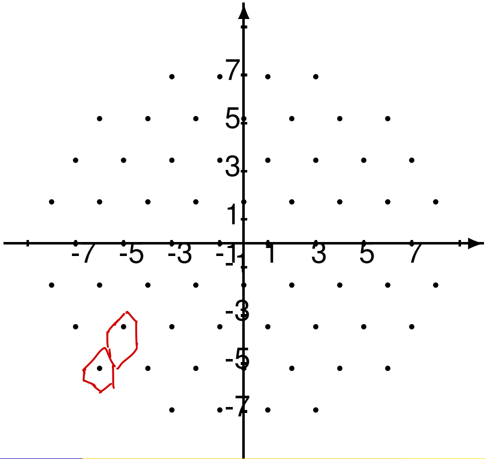
QAM

Modified QAM (used in Paradyne 14.4kbit modem). This has average energy of 40.9375.

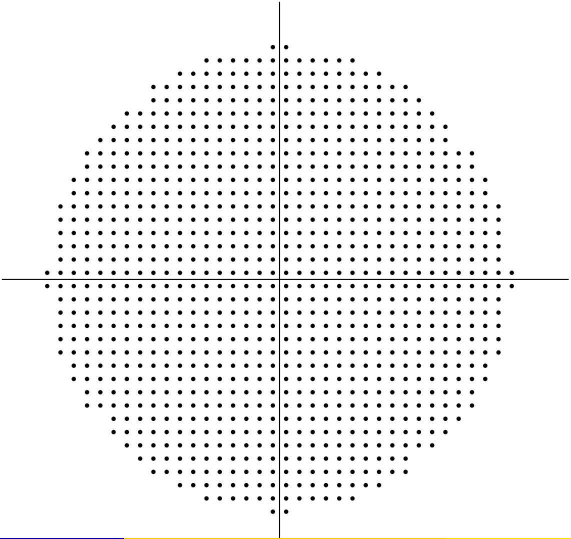


QAM

The following 64 point hexagonal constellation has energy 35.25 but each interior point now has 6 neighbors compared to the four neighbors for the rectangular structures.



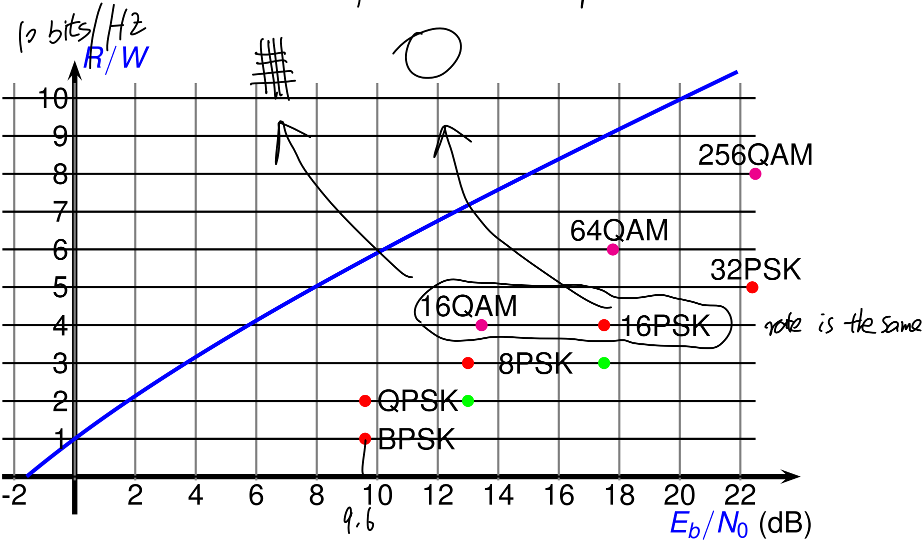
QAM constellation used in v34 28.8kbps modems (960 points)



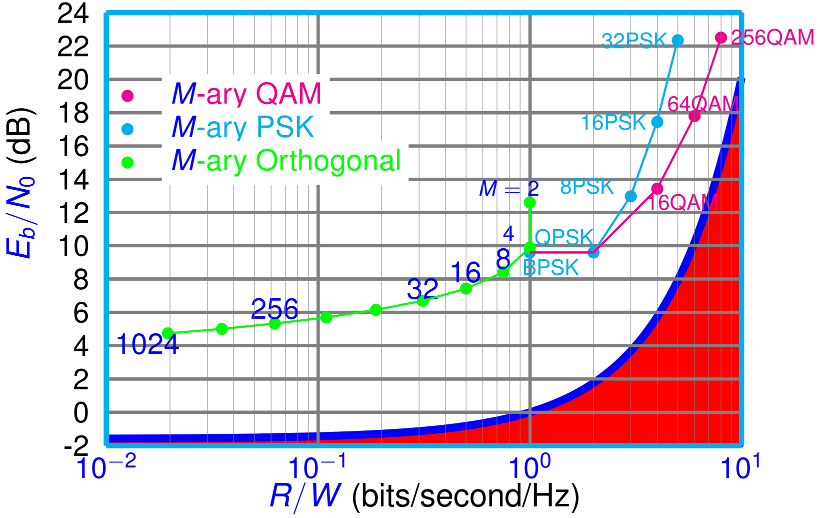
Capacity

Shannon's formula

$Pe = 10^{-5}$



Capacity (vs. Modulation at $P_{e,b} = 10^{-5}$)



Capacity Notes

- The value of E_b/N_0 in dB for achieiving a bit error probability of 10^{-5} is plotted for different modulation schemes.
- There is a gap between the required value of E_b/N_0 in dB for a each modulation technique and the theoretical possible required E_b/N_0 in dB at the same bandwidth efficiency.
- The gap can be reduced by employing various forms of error correcting codes.

E_b/N_0 (dB) for various BER

Modulation	R/W	BER = 10^{-3}	BER= 10^{-5}	BER= 10^{-8}
BPSK	1	6.79	9.58	11.97
QPSK	2	6.79	9.58	11.97
8-PSK	3	10.01	12.97	14.43
16-PSK	4	14.35	17.44	19.96
32-PSK	5	19.14	22.33	24.90
16 QAM	4	10.52	13.43	15.87
64 QAM	6	14.77	17.79	20.28
256 QAM	8	19.38	22.50	25.04
2-ary orthogonal	1	9.80	12.60	14.98
4-ary orthogonal	1	7.30	9.89	12.15
8-ary orthogonal	0.7500	6.00	8.41	10.57
16-ary orthogonal	0.5000	5.17	7.43	9.49
32-ary orthogonal	0.3125	4.58	6.70	8.68
64-ary orthogonal	0.1875	4.13	6.15	8.04
128-ary orthogonal	0.1094	3.77	5.70	7.52
256-ary orthogonal	0.0625	3.49	5.33	7.09
512-ary orthogonal	0.0352	3.24	5.10	6.72
1024-ary orthogonal	0.0195	3.04	4.74	6.40