University of Michigan Fall 2021

EECS501: Homework 1

Assigned: Sep 5, 2021
Due: Sep 14, 2021 at 11:59PM on gradescope
Text: "Probability and random processes" by J. A. Gubner

Reading assignment: Please read Chapter 1. In lecture we are covering the material in this order: 1.1 - 1.4.

For all questions asked in this course, unless the question is simply asking about definitions, please justify all answers. Stating the correct answer without justification will earn no credit. For example, if you write down a string of equations, each successive equation should be accompanied by some justification such as "by the additivity axiom" or "because the outcomes are equally likely" or "by the distributive property of set operations."

1. Consistency of Subjective Probabilities (2.5 points each)

People are asked to assign probabilities to the following events: "rain on Saturday," "rain on Sunday," "rain on both days," and "rain on neither day." Which of the following answers are consistent with the axioms of probability? Explain. **(a)** 90%, 80%, 70%, 30%, **(b)** 60%, 80%, 70%, 30%, **(c)** 80%, 70%, 60%, 10%, **(d)** 70%, 60%, 50%, 90%.

2. High Probability Events (5 points)

Using the axioms of probability show that if $P(A) > 1-\delta$ and $P(B) > 1-\delta$, then $P(A \cap B) \geq 1 - 2\delta$. In other words, if two events have probability nearly 1, then their intersection has probability nearly one.

3. The Inclusion-Exclusion Principle for Three Events

(a) (10 points) The inclusion-exclusion principle for three events A, B, and C is

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Prove this result using the axioms of probability. You are allowed to use the inclusion-exclusion principle for two events.

(b) (5 points) Suppose three fair, six-sided dice are rolled. Use part (a) to calculate the probability that at least one of the dice shows either 4 or 5.

4. Coin Tossing [2.5 points each]

A fair coin is flipped 4 times. What is the probability of observing **(a)** no more than two heads, **(b)** a run of two or more consecutive tails?

5. Daughters [5 points each]

For each of the probability questions below, define the sample space such that outcomes are equally likely. Then determine the answer by counting.

(a) A family with three children is chosen at random. What is the probability that they have exactly one daughter?

(b) A girl having two siblings is chosen at random. What is the probabilty that she has no sisters?

6. Shine or Rain (10 points)

It rains on fridays in Ann arbor with probability 0.2. The sun shines on fridays in Ann Arbor with probability 0.8. It is cloudy and dry on fridays in Ann Arbor with probability 0.1. What is the conditional probability that it is dry given that it is cloudy on fridays in Ann Arbor?

7. Pairwise Independence (5 points) Consider the experiment of throwing two dices. Let A be the event "the sum of the dots is 7", B the event "dice #1 came up 3", and C the event "dice #2 came up 4".

Question. Prove that the events A, B and C are pairwise independent but not independent as a triplet.