University of Michigan Fall 2021

EECS501: Homework 9

Assigned: Nov 7, 2021
Text: "Probability and random processes" by J. A. Gubner

1. Sense of Convergence

Assume the sample space is $\Omega = [0,1]$, and we have $P([a,b]) = b - a$ for any $0 \le a \le b \le 1$. Consider the following sequence of random variables. In what sense and to what random variable do the following sequences converge.

(a) $X_n(w) = \frac{(-1)^n}{n\sqrt{w}}$

(b) $X_n(w) = nw^n$.

2. Convergence in Distribution and in Probability

(a) Show that if a sequence of random variables X_n converges in distribution to X, and if X is a constant random variable, say $X \equiv c$, then X_n converges in probability to c.

(b) Let $X_n \sim N(0, 1/n^2)$. Show that $\sqrt{n}X_n$ converges in probability to zero.

3. Almost Sure and Mean Square Convergence

Let $Y_n \sim \mathit{Bernoulli}(p_n)$, and put $X_n := X + n(-1)^n Y_n$, where $X \sim N(0,1)$.

(a) Determine whether or not there is a sequence p_n such that X_n converges almost surely to X but not in mean square.

(b) Determine whether or not there is a sequence p_n such that X_n converges almost surely and in mean square to X.

Hint: use the fact that if $\sum_{n=1}^{\infty} P(A_n(\epsilon)) < \infty$ holds for every ϵ, then $X_n \to X$ a.s., where we define $A_n(\epsilon) = \{|X_n - X| \ge \epsilon\}$ (the event that $|X_n - X| \ge \epsilon$).