Homework 1

YUZHAN JIANG

U-M ID: 41898102

1. $N=4 \quad M=8$

(a) $\log_2(M) = \log_2(8) = 3$

$\therefore$ 3 information bits can be sent using signals

(b) $E_i = \sum_{i=0}^{N-1} S_i^2$

$E_0 = |1|^2 + |1|^2 + |1|^2 + |1|^2$

$= 4$

$\therefore$ The energy of each of the signals is 4

$\bar{E}_b = \bar{E} / \log_2(M)$ while $\bar{E} = 4$

$= \frac{4}{\log_2(8)}$

$= \frac{4}{3}$

$\therefore$ the average energy per information bit is $\frac{4}{3}$

(c) $d_E^2(S_0, S_i)$ for $i \in 1, M-1 = \sum_{l=0}^{N-1} (S_{0,l} - S_{l,j})^2$

$d_E^2(S_0, S_1) = 2 \times (1-(-1))^2 = 8$

$d_E^2(S_0, S_2) = 2 \cdot (4) = 8$

$d_E^2(S_0, S_3) = 2 \cdot (4) = 8$

$d_E^2(S_0, S_4) = 4(4) = 16$

$d_E^2(S_0, S_5) = 2(2)^2 = 8$

$d_E^2(S_0, S_6) = 2(2)^2 = 8$

$d_E^2(S_0, S_7) = 2(2)^2 = 8$

(d) $r = (\log_2 M) / N$

$= \frac{3}{4}$

$\therefore$ the rate of communication in bits/dimension is $\frac{3}{4}$

2. $N=2$ $M=8$

(a) $E_0 = |-1|^2 + |y+\sqrt{3}|^2 = 1 + y^2 + 3 + 2\sqrt{3}y = y^2 + 2\sqrt{3}y + 4$

$E_1 = y^2 + 2\sqrt{3}y + 4$

$E_2 = y^2 + 4$ $E_5 = y^2 - 2\sqrt{3}y + 4$

$E_3 = y^2$ $E_6 = y^2 - 2\sqrt{3}y + 4$

$E_4 = y^2 + 4$ $E_7 = y^2 - 4\sqrt{3}y + 12$

$$\therefore \bar{E} = \frac{1}{8}\sum_{i=0}^{N-1} E_i$$

$$= \frac{1}{8}(8y^2 + 26 - 4\sqrt{3}y)$$

$$= y^2 - \frac{\sqrt{3}}{2}y + \frac{26}{8}$$

$$= \left(y - \frac{\sqrt{3}}{4}\right)^2 + \frac{49}{16}$$

When $y = \frac{\sqrt{3}}{4}$, we minimize the average signal energy transmitted

(b) $d_E^2(S_0, S_1) = (1-(-1))^2 = 4$ $d_E^2(S_1, S_3) = 1^2 + (\sqrt{3})^2 = 4$

$d_E^2(S_0, S_2) = 1^2 + (\sqrt{3})^2 = 4$ $d_E^2(S_1, S_4) = (1)^2 + (\sqrt{3})^2 = 4$

$d_E^2(S_0, S_3) = 1^2 + (\sqrt{3})^2 = 4$

$d_E^2(S_2, S_3) = 2^2 + 0 = 4$ $d_E^2(S_4, S_3) = 4$

$d_E^2(S_2, S_5) = 4$ $d_E^2(S_4, S_6) = 4$

$d_E^2(S_5, S_3) = 4$ $d_E^2(S_6, S_7) = 4$

$d_E^2(S_5, S_6) = 4$ $d_E^2(S_6, S_3) = 4$

$d_E^2(S_5, S_7) = 4$

$\therefore$ the minimum squared Euclidean distance is 4

(c)

$$r = \frac{\log_2(M)}{N} = \frac{3}{2}$$

3. (a)

$$\bar{E} = \frac{1}{M} \sum_{i=0}^{M-1} E_i$$

$E_0 = 1^2 = 1$

$E_1 = 1^2 = 1$

$E_2 = 2^2 + (\sqrt{3})^2 = 7$

$E_3 = 0^2 + (\sqrt{3})^2 = 3$

$E_4 = (-2)^2 + (\sqrt{3})^2 = 7$

$E_5 = (2^2) + (-\sqrt{3})^2 = 7$

$E_6 = 0^2 + (-\sqrt{3})^2 = 3$

$E_7 = (-2) + (-\sqrt{3})^2 = 7$

$\therefore \quad \bar{E} = \frac{1}{8}(1+1+7+3+7+7+3+7)$

$= \frac{1}{8}(12+24)$

$= \frac{9}{2}$

(b) $\bar{E}_b = \frac{\bar{E}}{\log_2 M} = \frac{\frac{9}{2}}{\log_2 8} = \frac{\frac{9}{2}}{3} = \frac{3}{2}$

(c) $d_E^2(S_0, S_i)$ for $i \in 1 \ldots M-1$

$d_E^2(S_0, S_1) = (1-(-1))^2 = 4$

$d_E^2(S_0, S_2) = (1-2)^2 + (\sqrt{3})^2 = 4$

$d_E^2(S_0, S_3) = 1^2 + (\sqrt{3})^2 = 4$

$d_E^2(S_0, S_4) = (1-(-2))^2 + (-\sqrt{3})^2 = 12$

$d_E^2(S_0, S_5) = (1-2)^2 + (\sqrt{3})^2 = 4$

$d_E^2(S_0, S_6) = 1^2 + (\sqrt{3})^2 = 4$

$d_E^2(S_0, S_7) = (1-(-2))^2 + (\sqrt{3})^2 = 12$

(d) $r = \frac{\log_2(M)}{N} = \frac{\log_2(8)}{2} = \frac{3}{2}$

4. (a) The UWB channel goes from 3.1 GHz to 10.6 GHz

$$10.6 - 3.1 = 7.5 \text{ GHz}$$

We determine the power in a 1MHz bandwidth

$$P_{t|1MHz} = 10^{(-71.3/10)}$$

$$= 7.41 \times 10^{-8} \text{ W/MHz}$$

$$\therefore P_t = 7.41 \times 10^{-8} \cdot 7500$$

$$= 0.556 \times 10^{-3} \text{ W}$$

(b) By the formula:

$$P_r = P_t h_t^2 h_r^2 / d^4 \quad \text{where} \quad h_t = h_r = 1 \text{ m}$$

$$d = 100 \text{ m or } 1000 \text{ m}$$

When $d = 100$ m

$$P_r = 0.556 \times 10^{-3} \text{ W} / (100)^4$$

$$= 5.56 \times 10^{-12} \text{ W}$$

And the largest possible data rate is

$$C = W \log_2 (1 + \frac{P_r}{N_0 W})$$

$$= 1.8 \times 10^9 \text{ bps}$$

When $d = 1000$ m

$$P_r = 0.556 \times 10^{-3} \text{ W} / (1000)^4$$

$$= 5.56 \times 10^{-16}$$

And the largest possible data rate is

$$C = W \log_2 (1 + \frac{P_r}{N_0 W})$$

$$= 200 \text{ kbps}$$

5. (a) $\frac{E_b}{N_0} > \frac{2^{\frac{R}{W}} - 1}{\frac{R}{W}}$ where R is 300 k bits/second, W is 100 k HZ

$$\Rightarrow \frac{E_b}{N_0} > \frac{2^3 - 1}{3} = \frac{7}{3}$$

$$\therefore \frac{E_b}{N_0}(dB) = 10\log_{10}\frac{7}{3} = 10 \times (\log_{10} 7 - \log_{10} 3)$$

$$= 3.68 \text{ dB}$$

(b) Known Quantities:

$W = 100\text{ kHZ}$

$P = 5$ Watts

$\frac{N_0}{2} = 1.778 \times 10^{-3}$ Watts/HZ

$R_s = 4000$ sample/second

By the formula,

$$R = \frac{R_s}{2}\log_2\frac{(\sigma_s^2)}{D} \quad ①$$

where $R < W\log_2\left(1 + \frac{P}{N_0 W}\right)$ ②

Combinations of equations of ① and ② gives us:

$$\frac{R_s}{2}\log_2\frac{(\sigma_s^2)}{D} < W\log_2\left(1 + \frac{P}{N_0 W}\right)$$

$$D > \frac{\sigma_s^2}{\left(1 + \frac{P}{N_0 W}\right)^{2W/R_s}}$$

$$D > \frac{1}{\left(1 + \frac{5}{3.556 \times 10^{-3} \times 100 \times 10^3}\right)^{\frac{2 \cdot 100 \times 10^3}{4000}}}$$

$$D > 0.4975$$