EECS 455: Problem Set 8
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Due: Wednesday, November 17, 2021, 11pm.

1. In this problem we compare the performance of several communication systems to understand the benefit of diversity in a communication system subject to fading. Consider a communication system using BPSK modulation. The channel is a Rayleigh faded channel. However, each bit is transmitted 3 times (on three different frequencies or three different antennas). After a decision is made on each of the three bits individually a majority vote is taken. So if the three individual decisions are (0,0,1) then the majority vote is 0. If the three individual decisions are (1,0,1) then the majority vote is 1. The fading and the event of error are independent on each transmission. The energy per information bit must be split into three parts for each transmission.

   (a) Suppose that each bit is in error with probability p. Determine the probability that the majority vote is incorrect.

   (b) Given that each transmission is faded, determine the probability of error for each transmission as a function of the energy used for each bit. (Look it up in the notes).

   (c) Determine the relation between the energy per information bit and the energy per transmitted signal.

   (d) Plot the error probability versus E_b/N_0 (dB). The axis should be from -10 to 40 dB on the horizontal axis and 10^{-5} to 1 on the vertical scale.

   (e) On the same graph also plot the error probability for the case of just one transmission.

   (f) On the same graph also plot the error probability for the case of one transmission without fading (additive white Gaussian noise).

2. In this problem you will generate samples of an OFDM waveform in Matlab and then determine the output of a multipath channel. Finally you will process the received signal.

   The OFDM system consists of 64 subcarriers. Each subcarrier will be modulated with a 16QAM signal.

   (a) First we will do the case of no channel (noise or fading).

   - Generate a vector (length 64) of symbols from the set $\{0, 1, \ldots, 15\}$ using the Matlab command `randi` or some other (deterministic) method. You will want to make sure that you eventually generate all 16 different symbols.
   - Using the Matlab command `qammod` to generate a vector (length 64) of 16QAM modulated symbols.
   - Using Matlab command `IFFT` generate the time samples of an OFDM signal.
   - Add a cyclic prefix of size 16 to your OFDM signal. You signal should be 80 samples now.
   - To start we will not have any channel so the received is exactly the transmitted.
   - Remove the first 16 samples (the cyclic prefix) from the transmitted signal.

- Using Matlab command `FFT` regenerate the 16QAM modulated symbols.

To show your results make a plot of the constellation after doing the demodulation. For this part (and the next) you should make sure that all 16 points are plotted.

(b) Now do the fading case.

- Copy your code to a second file. Add the following discrete channel to your code.

$$h = (1, \alpha_1 e^{j\phi_1}, \alpha_2 e^{j\phi_2}, \alpha_3 e^{j\phi_3})$$

  whre

$$\begin{aligned} \alpha_1 &= 0.9; \\ \alpha_2 &= 0.35; \\ \alpha_3 &= 0.15; \\ \phi_1 &= 2\pi(0.9051); \\ \phi_2 &= 2\pi(0.5338); \\ \phi_3 &= 2\pi(0.1092); \end{aligned}$$

- Using the command `conv` to process the transmitted signal (after the inserting the cyclic prefix) to generate the received signal (no noise).
- Remove the cyclic prefix and process with Matlab `FFT` command;
- Determine which of the 64 16QAM symbols has the smallest amplitude (deepest faded) and which has the largest amplitude.
- Plot the constellation for each of these two symbols. For this part you should make sure that all 16 points are plotted. First plot all constellation points using the same marker symbol. Then for symbol 0 (which corresponds to 16QAM symbol $-3+3j$) make an additional marker on the same plot for that symbol and similarly for symbol 10 (which corresponds to constellation point $+3-3j$. This will help identify the phase rotation that the channel causes on each symbol.