Field of sec 16

$x^4+x+1=0$

$\begin{pmatrix} 0000 \\ \vdots \\ 1111 \end{pmatrix}$

1010

0101

let $x^4+x+1=0$ $\quad 0+0+1\neq 0$ $\quad 1+1+1\neq 0$

let α be the solution

$\alpha^4=\alpha+1$

ax^3+bx^2+cx+d

(a,b,c,d)

0000	0	0
0001	1	1
0010	x	α
0100	x^2	α^2
1000	x^3	α^3
0011	x^4	$x^4=x+1$ $\quad \alpha^4=\alpha+1$
0110	$x^5=x^4x=(x+1)\cdot x=x^2+x$	$\alpha^5=\alpha^4\cdot\alpha=(\alpha+1)\cdot\alpha=\alpha^2+\alpha=$
1100	$x^6=x^5\cdot x=(x^2+x)x=x^3+x^2$	$\alpha^5\cdot\alpha=(\alpha^2+\alpha)\cdot\alpha=\alpha^3+\alpha^2=$
1011	$x^7=x^6x=(x^3+x^2)x=x^4+x^3=x^3+x+1$	cancel $\alpha^6\cdot\alpha=\alpha^4+\alpha^3=\alpha^3+\alpha+1$
0101	$x^8=x^7\cdot x=(x^3+x+1)\cdot x=x^4+x^2+x=x+1+x^2+x=x^2+1$	
1010	$x^9=x^8\cdot x=(x^2+1)\cdot x=x^3+x$	
0111	$x^{10}=x^9\cdot x=(x^3+x)x=x^4+x^2=x+1+x^2$	
1110	$x^{11}=x^{10}\cdot x=(x+1+x^2)x=x^3+x^2+x$	
1111	$x^{12}=x^{11}\cdot x=(x^3+x^2+x)\cdot x=x^4+x^3+x^2=x^3+x^2+x+1$	
1101	$x^{13}=x^{12}\cdot x=(x^3+x^2+x+1)x=x^4+x^3+x^2+x=x^3+x^2+x+x+1=$	
1001	$x^{14}=x^{13}\cdot x=(x^3+x^2+1)x=x^4+x^3+x=x^3+x+x+1=x^3+1$	
0001	$x^{15}=x^{14}\cdot x=(x^3+1)x=(x^4+x)=x+x+1=1$	

$\dfrac{1010}{0101}=\dfrac{x^9}{x^8}=x=0010$

$(1101)\times(0110)=x^{18}$

$x^{13}\cdot x^5$

$=x^{15}\cdot x^3$

$=x^3$

$=(1000)$