EECS 455: Problem Set 4

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Due: Wednesday, September 29, 2021, 11pm.

1. A binary communication system transmits one of two equally likely signals $s_0(t)$ and $s_1(t)$ of duration T given by

$$s_i(t) = \sqrt{2P}(-1)^i \cos(2\pi f_c t) p_T(t), \quad i = 0, 1.$$

The noise in the system is white Gaussian noise with power spectral density $N_0/2$. The receiver shown in Figure 1 is used to demodulate the signal. The filter impulse response is a rectangular pulse, $h(t) = p_T(t)$. However, as shown, the phase of the received signal is not know completely accurately. In fact, there is a discrepancy of θ radians between the received signal (in the absence of noise) and the local reference signal. Determine the error probability at the output of the demodulator as a function of θ. (Assume $2\pi f_c T = 2\pi n$ for some integer n or $f_c T \gg 1$. That is, ignore double frequency terms).

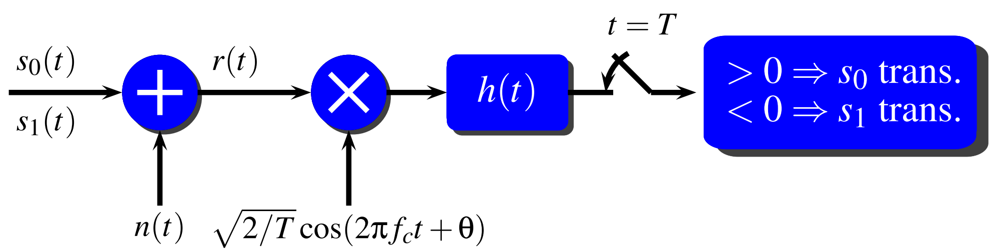


Figure 1: Receiver with Phase Offset.

2. A transmitter uses one of four equally likely signals to convey two bits of information. The signals are $s_0(t), s_1(t), s_2(t)$, and $s_3(t)$. The following table indicates the mapping between information bits and signals.

Information bits	Signals
00	$s_0(t)$
01	$s_1(t)$
11	$s_2(t)$
10	$s_3(t)$

The signals are received in the presence of white Gaussian noise with power spectral density $N_0/2$. The receiver consist of a filter, a sampler and a threshold device. The sampled output is denoted by $Z(T)$. The threshold device uses the following table to make a decision.

$Z(T) > 2$	decide $s_0(t)$ transmitted
$0 < Z(T) < 2$	decide $s_1(t)$ transmitted
$-2 < Z(T) < 0$	decide $s_2(t)$ transmitted
$Z(T) < -2$	decide $s_3(t)$ transmitted

It is known that the output of the filter due to the signals alone at the sampling time is

$$\begin{aligned}\hat{s}_0(T) &= +3\\ \hat{s}_1(T) &= +1\\ \hat{s}_2(T) &= -1\\ \hat{s}_3(T) &= -3\end{aligned}$$

It is also known that the variance of the output due to noise alone is $\sigma^2 = 4$.

(a) Determine the probability of error given signal $s_i(t)$ is transmitted for $i = 0, 1, 2, 3$. (Express your answers in terms of the Q function).

(b) Determine the probability that the receiver makes an error in the first bit given the first bit is 0. (The first bit being 0 means either signal $s_0(t)$ was transmitted or $s_1(t)$ was transmitted).

Hint: Let b_0 represent the first bit. Let $\hat{b}_0$ represent the decision on the first bit. The probability of error for the first bit is then $P\{\hat{b}_0 = 1 | b_0 = 0\}$.

3. Show that the raised cosine pulse shape satisfies the Nyquist criteria for zero intersymbol interference. That is, show if

$$H(f) = \begin{cases} T, & |f| < \frac{1-\alpha}{2T} \\ \frac{T}{2}\left\{1 + \cos\left(\frac{\pi T}{\alpha}\left[|f| - \frac{(1-\alpha)}{2T}\right]\right)\right\}, & \frac{1-\alpha}{2T} < |f| < \frac{1+\alpha}{2T} \\ 0, & |f| > \frac{1+\alpha}{2T} \end{cases}$$

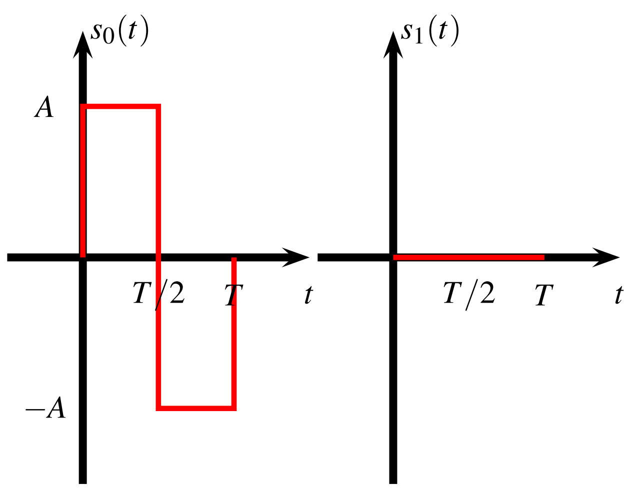
then

$$H(f) + H(f - 1/T) + H(f + 1/T) = T, \text{ for } -1/(2T) < f < 1/(2T).$$

4. A binary communications system operates over an AWGN channel with power spectral density $N_0/2$. The transmitted signals are given by

$$s_0(t) = Ap_{T/2}(t) - Ap_{T/2}(t - T/2)$$

$$s_1(t) = 0$$



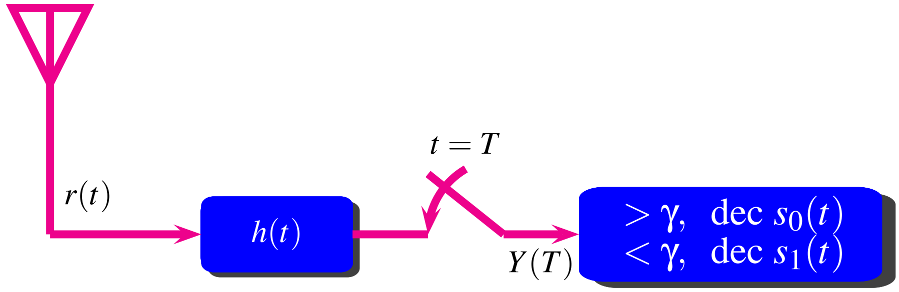


Figure 2: Receiver Structure

The receiver has the following structure.

(a) Give an expression (in terms of A, T, N_0, and $Q(x)$) for the minimum average error probability when the two signals are transmitted with equal probabilities (i.e. $\pi_0 = \pi_1$).

(b) What is the optimum filter for minimizing the error probability and the optimum threshold?

(c) Assume that the optimum filter $h(t)$ and the optimum threshold is used for the signals above (i.e. the answer to part (b)) but the signal $s_0(t)$ is actually given by

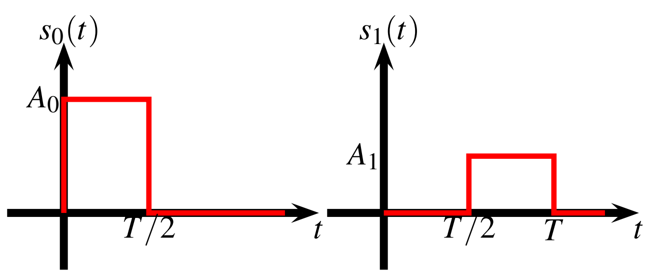
$$s_0(t) = cAp_{T/2}(t) - cAp_{T/2}(t - T/2)$$

instead of the one given above while $s_1(t)$ is the same where $c > 0$. Give an expression (in terms of c, A, T, N_0 and Φ or Q) for the average error probability if $\pi_0 = \pi_1$.

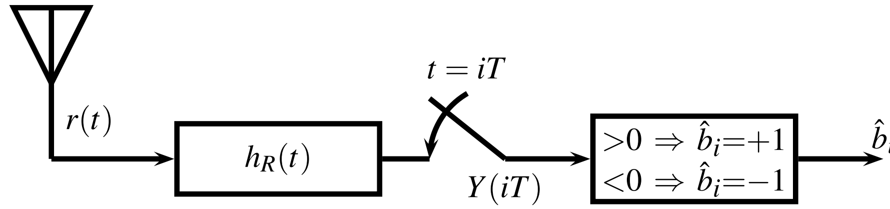
5. Consider a binary communication system that transmits one of two signal $s_0(t)$ and $s_1(t)$ over an additive white Gaussian noise channel (power spectral density $N_0/2$) where

$$s_0(t) = A_0 p_{T/2}(t), \quad s_1(t) = A_1 p_{T/2}(t - T/2);$$

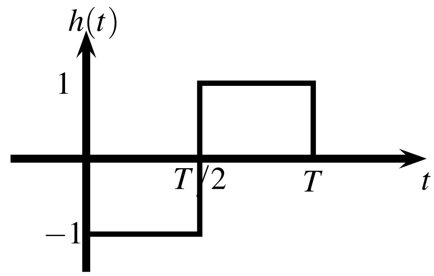
that is $s_0(t)$ is a pulse of amplitude A_0 from 0 to $T/2$ and $s_1(t)$ is a pulse of amplitude A_1 from $T/2$ to T as shown below.



The received signal, $r(t)$, is the transmitted signal with additive white Gaussian noise. The receiver shown below consist of a filter $h(t)$ which is sampled at time T and a threshold device.



(a) If $h_R(t) = -p_{T/2}(t) + p_{T/2}(t - T/2)$ shown below, find the output of the filter $\hat{s}_0(T)$ due to signal $s_0(t)$ at time T and the output of the filter $\hat{s}_1(T)$ due to signal $s_1(t)$ at time T



(b) Find the threshold γ that will minimize the average of the error probabilities $P_{e,0}$ and $P_{e,1}$ for the given signals and filter. Assume $\pi_0 = \pi_1$.

(c) Find the error average error probability $\bar{P}_e = \pi_0 P_{e,0} + \pi_1 P_{e,1}$ for the threshold found in the previous part. Assume $\pi_0 = \pi_1$.

(d) Find the matched filter for the same signals and find the corresponding threshold that minimizes $\bar{P}_e$. Assume $\pi_0 = \pi_1$.

(e) Find $\bar{P}_e$ for the matched filter with the optimum threshold. Assume $\pi_0 = \pi_1$.