EECS 455: Problem Set 7

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Due: Wednesday, November 3, 2021, 11pm.

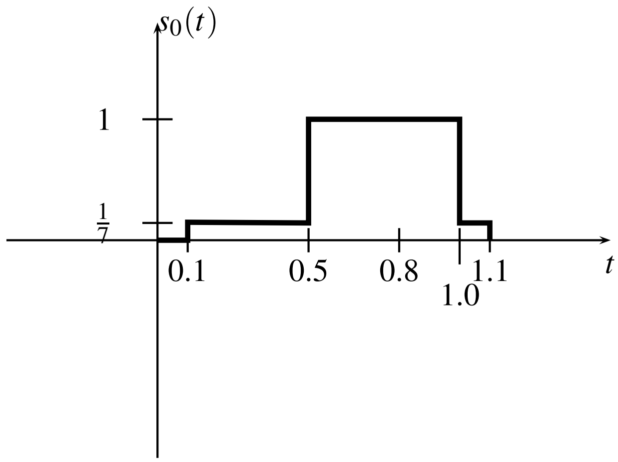
1. (a) The following four signals are used in WWVB to transmit information. Signals used for clocks to tell time are transmitted in the 60kHz frequency band. The data contain information regarding the current the year, day, hour, and minute. The signal itself tells the second within the minute. These signals are transmitted from Boulder CO and can be heard across much of the US. Similar signals are transmitted in some other countries. (See the internet for more information about signals in WWVB). Find two orthonormal signals for which these four signals can be represented as four vectors of dimension 2. Plot the four signals in a plane with the horizontal axis representing the first component and the vertical axis representing the second component.

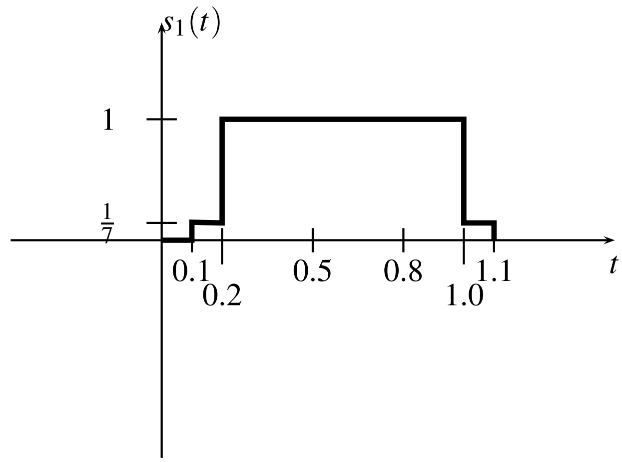
   (b) Assume there is an amplifier that multiplies the transmitted signal by A. Find the energy of each of the signals and the average energy. The answer should be in terms of A.

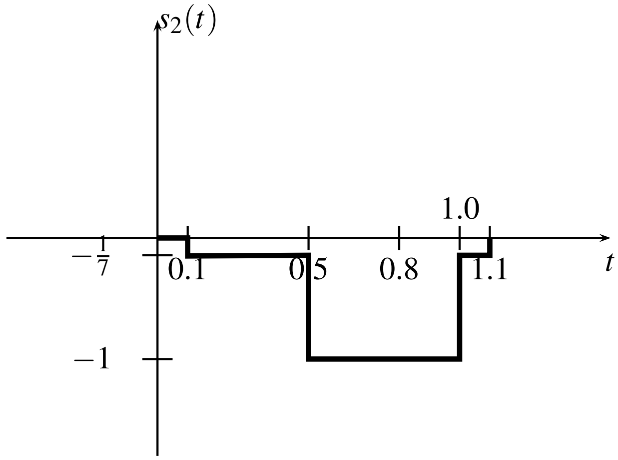
   (c) Find the squared Euclidean distance between all the vectors. The answer should be in terms of A.

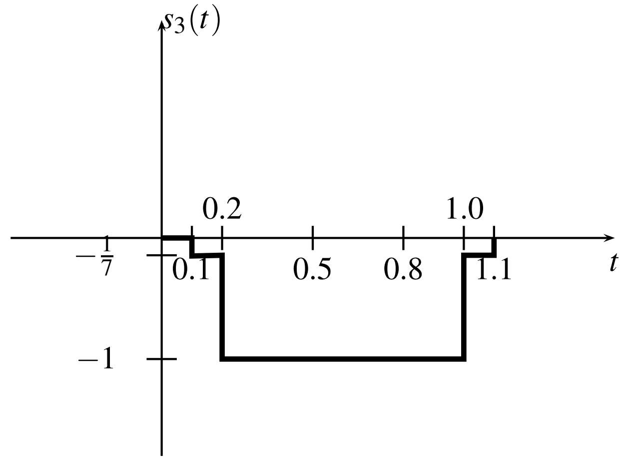
   (d) Describe the optimal receiver for deciding between these four signals.

   (e) Determine a bound on the symbol (2 bits) error probability of these signals in additive white Gaussian noise.



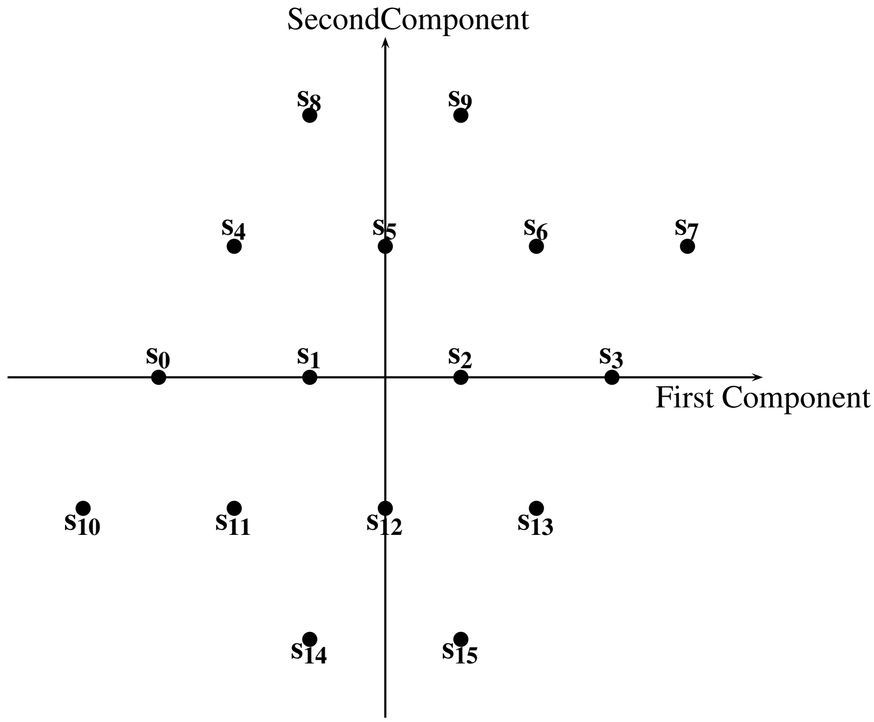






2. Consider the following 16 signal vectors.

$$
\begin{array}{ll}
\mathbf{s_0} = (-3, 0) & \mathbf{s_8} = (-1, 2\sqrt{3}) \\
\mathbf{s_1} = (-1, 0) & \mathbf{s_9} = (+1, 2\sqrt{3}) \\
\mathbf{s_2} = (+1, 0) & \mathbf{s_{10}} = (-4, -\sqrt{3}) \\
\mathbf{s_3} = (+3, 0) & \mathbf{s_{11}} = (-2, -\sqrt{3}) \\
\mathbf{s_4} = (-2, \sqrt{3}) & \mathbf{s_{12}} = (0, -\sqrt{3}) \\
\mathbf{s_5} = (0, \sqrt{3}) & \mathbf{s_{13}} = (+2, -\sqrt{3}) \\
\mathbf{s_6} = (+2, \sqrt{3}) & \mathbf{s_{14}} = (-1, -2\sqrt{3}) \\
\mathbf{s_7} = (+4, \sqrt{3}) & \mathbf{s_{15}} = (+1, -2\sqrt{3})
\end{array}
$$

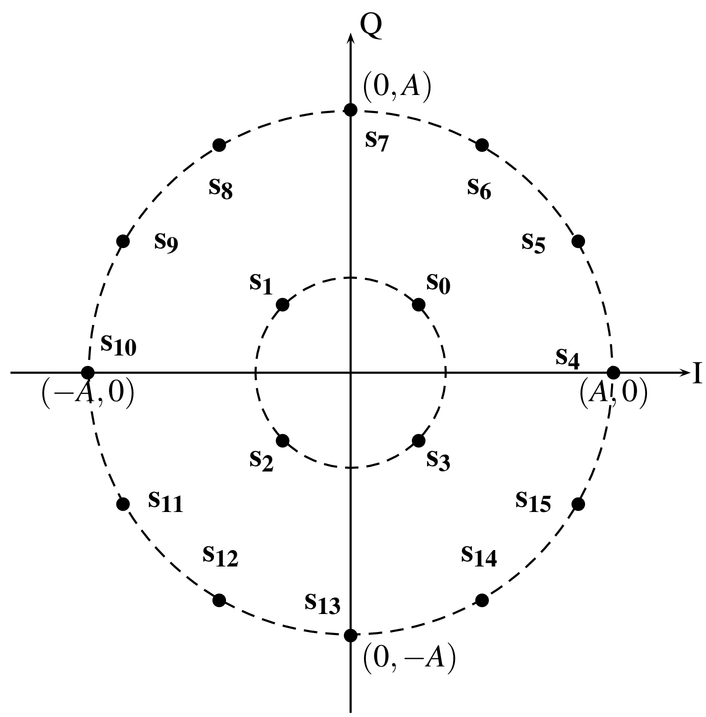


(a) Draw the optimum decision regions for this set of 16 signals.

(b) Determine and plot the union bound on the error probability for this signal set. You should plot the error probability as a function of E_b/N_0 in dB on a log scale for the vertical axis. The horizontal axis should go from 0dB to 15dB and the vertical axis should go from 10^{-6} to 1.

3. A communication system transmits one of 16 equally likely signals. The signals (waveforms) are represented by the vectors shown below. where $A = \sqrt{4(2+\sqrt{3})}$ so that the minimum Euclidean distance between any two signals is at least 2.

$$
\begin{aligned}
\mathbf{s_0} &= (+1,+1)\\
\mathbf{s_1} &= (-1,+1)\\
\mathbf{s_2} &= (-1,-1)\\
\mathbf{s_3} &= (+1,-1)\\
\mathbf{s_4} &= (A,0)\\
\mathbf{s_5} &= (A\cos(2\pi/12), A\sin(2\pi/12))\\
\mathbf{s_6} &= (A\cos(2\pi 2/12), A\sin(2\pi 2/12))\\
\mathbf{s_7} &= (0,A)\\
\mathbf{s_8} &= (A\cos(2\pi 4/12), A\sin(2\pi 4/12))\\
\mathbf{s_9} &= (A\cos(2\pi 5/12), A\sin(2\pi 5/12))\\
\mathbf{s_{10}} &= (-A,0)\\
\mathbf{s_{11}} &= (A\cos(2\pi 7/12), A\sin(2\pi 7/12))\\
\mathbf{s_{12}} &= (A\cos(2\pi 8/12), A\sin(2\pi 8/12))\\
\mathbf{s_{13}} &= (0,-A)\\
\mathbf{s_{14}} &= (A\cos(2\pi 10/12), A\sin(2\pi 10/12))\\
\mathbf{s_{15}} &= (A\cos(2\pi 10/12), A\sin(2\pi 10/12))
\end{aligned}
$$



(a) Determine (draw) the optimum decision regions.

(b) Determine the union bound on the symbol error probability. Plot the union bound versus E_b/N_0 in dB for values of E_b/N_0 between 0 and 18dB.

4. Consider a communication system that transmits an infinite sequence of data bits $\{b_l\}_{l=-\infty}^{\infty}$ using two signals of duration T: $s_0(t) = -s_1(t) = Ap_T(t)$. Thus

$$s(t) = \sum_{l=-\infty}^{\infty} b_l A p_T(t-lT)$$

The signal $s(t)$ is transmitted over an additive white Gaussian noise channel with spectral density $N_0/2$. The receiver consists of a filter $h(t)$ the output of which is sampled at time iT and compared with a threshold of 0. If the output at time $iT > 0$ the receiver decides $b_{i-1} = +1$ otherwise the receiver decides $b_{i-1} = -1$. It is know that the filter is such that $\int_{-\infty}^{\infty} h^2(t)dt = 16$. It is also known that if the input to the filter is $p_T(t)$ ($n(t) = 0$, noise is 0) then the output at time iT is

$$Z(iT) = \begin{cases} 8 & i = 1 \\ 2 & i = 2 \\ 1 & i = 3 \\ 0 & i < 1,\ i > 3 \end{cases}$$

(a) Find the possible values for the output due to the desired signal (no noise) for different data bits.

(b) Find the upper and lower bounds for the $E[Z(iT)|b_{i-1} = +1, b_{i-2}, b_{i-3}, ...]$. That is, find the largest possible value for the output due to signal alone (no noise) at time iT for all possible previous data bits. Find upper and lower bounds on the probability of error for data bit b_{i-1} given that $b_{i-1} = +1$.

(c) Give an expression for the average probability of error for the data bit b_{i-1} if each data bit is equally likely to be +1 or -1 independently of all other data bits.

5. A communication uses the following eight signals where $f_cT = n/2$ for some large n.

$$\begin{aligned}
s_0(t) &= A\cos(2\pi f_c t),\ 0 \le t \le T \\
s_1(t) &= \frac{1}{2}A\cos(2\pi f_c t) + \frac{1}{2}A\sin(2\pi f_c t),\ 0 \le t \le T \\
s_2(t) &= \frac{1}{2}A\sin(2\pi f_c t),\ 0 \le t \le T \\
s_3(t) &= -\frac{1}{2}A\cos(2\pi f_c t) + \frac{1}{2}A\sin(2\pi f_c t),\ 0 \le t \le T \\
s_4(t) &= -A\cos(2\pi f_c t),\ 0 \le t \le T \\
s_5(t) &= -\frac{1}{2}A\cos(2\pi f_c t) - \frac{1}{2}A\sin(2\pi f_c t),\ 0 \le t \le T \\
s_6(t) &= -\frac{1}{2}A\sin(2\pi f_c t),\ 0 \le t \le T \\
s_7(t) &= \frac{1}{2}A\cos(2\pi f_c t) - \frac{1}{2}A\sin(2\pi f_c t),\ 0 \le t \le T
\end{aligned}$$

(a) Determine a suitable set of orthonormal functions to represent these 8 signals.

(b) Express the signals as a linear combination of the orthonormal signals.

(c) Plot the signals (in signal space) and show the optimum decision regions when these signals are used with equal probability in an additive white Gaussian noise channel with two-side power spectral density $N_0/2$.

(d) Find the probability of correct given signal 0 is transmitted in terms of the Φ function or Q function, A, T and N_0.