EECS 455 H10 YUZHAN JIANG

Problem 1:

$n=7, \quad k=5 \quad \alpha^3=\alpha+1 \quad \alpha^7=\alpha^6\cdot\alpha=(\alpha^2+1)\alpha=\alpha^3+\alpha$

$g(x)=(x-\alpha)(x-\alpha^2)$

$=\alpha+1+\alpha$

$=1$

$S_1 = r(\alpha) = c(\alpha)+e(\alpha) = e(\alpha) = e_j\alpha^j$ (since $g(\alpha)=0$)

$S_2 = r(\alpha^2) = c(\alpha^2)+e(\alpha^2) = e(\alpha^2) = e_j\alpha^{2j}$

$\frac{S_2}{S_1} = \frac{e_j\cdot\alpha^{2j}}{e_j\alpha^j} = \alpha^j$ where $j$ is the error position

$S_1 = r(\alpha) = \alpha^6+\alpha^4\cdot\alpha^5 = \alpha^6+\alpha^9 = \alpha^2+1+\alpha^7\cdot\alpha^2$

$= \alpha^2+1+\alpha^2$

$=1$

$S_2 = r(\alpha^2) = \alpha^{12}+\alpha^4\cdot\alpha^{10} = \alpha^7\cdot\alpha^5+\alpha^7\cdot\alpha^7$

$=\alpha^2+\alpha+1+1$

$=\alpha^2+\alpha$

$\therefore \frac{S_2}{S_1} = \frac{\alpha^2+\alpha}{1} = \frac{\alpha^4}{1} = \alpha^4$

$\therefore$ error position is 4

Then we need to find error magnitude,

$e_j = \frac{S_1}{\alpha^4} = \frac{1}{\alpha^4} = \frac{\alpha^7}{\alpha^4} = \alpha^3$

$\therefore$ therefore, $\hat{e}(x) = \alpha^3\cdot x^4$

$\therefore$ therefore, $c(x) = r(x)-\hat{e}(x)$

$= x^6+\alpha^4x^5-\alpha^3x^4$

$= x^6+\alpha^4x^5-\alpha^3x^4$

$= x^6+\alpha^4x^5+\alpha^3x^4$

Problem 2:

let received vector be $ax^4 + bx^2 + \alpha^5 x + \alpha^3$

$$\left.\begin{aligned} S_1 &= r(\alpha) = c(\alpha) + e(\alpha) = m(\alpha)g(\alpha) + e(\alpha) = 0 \\ S_2 &= r(\alpha^2) = c(\alpha^2) + e(\alpha^2) = m(\alpha^2)\cdot g(\alpha^2) + e(\alpha^2) = 0 \end{aligned}\right\}$$ since the symbols are received correctly

$$S_1 = r(\alpha) = a\cdot\alpha^4 + b\alpha^2 + \alpha^5\alpha + \alpha^3 = a\cdot\alpha^4 + b\alpha^2 + \alpha^6 + \alpha^3 = 0 \quad ①$$
$$S_2 = r(\alpha^2) = a\cdot\alpha^8 + b\alpha^4 + \alpha^5\cdot\alpha^2 + \alpha^3 = a\cdot\alpha + b\alpha^4 + \alpha^7 + \alpha^3 = 0 \quad ②$$

We simplify these two equation ① and ②

①: $a\cdot(\alpha^2+\alpha) + b(\alpha^2) + (\alpha^2+1)\ \alpha+1 = 0$

$\Rightarrow a\alpha^2 + a\alpha + b\alpha^2 + \alpha^2 + 1 + \alpha + 1 = 0$

$\Rightarrow (a+b+1)\alpha^2 + (a+1)\alpha = 0$

② $a\alpha + b\alpha^4 + \alpha^7 + \alpha^3 = 0$

$\Rightarrow a\cdot\alpha + b(\alpha^2+\alpha) + 1 + \alpha + 1 = 0$

$\Rightarrow a\cdot\alpha + b\alpha^2 + b\alpha + \alpha = 0$

$\Rightarrow b\alpha^2 + (a+b+1)\alpha = 0$

$$\begin{cases} a+b+1 = b \\ a+1 = \alpha + b + 1 \end{cases} \Rightarrow \begin{cases} a = -1 \\ b = 0 \end{cases}$$

$\therefore\ r(x) = -x^4 + \alpha^5 x + \alpha^3$

$= x^4 + \alpha^5 x + \alpha^3$

P3:

28 code symbols — 24 information symbols.

Each symbols is eight bits byte

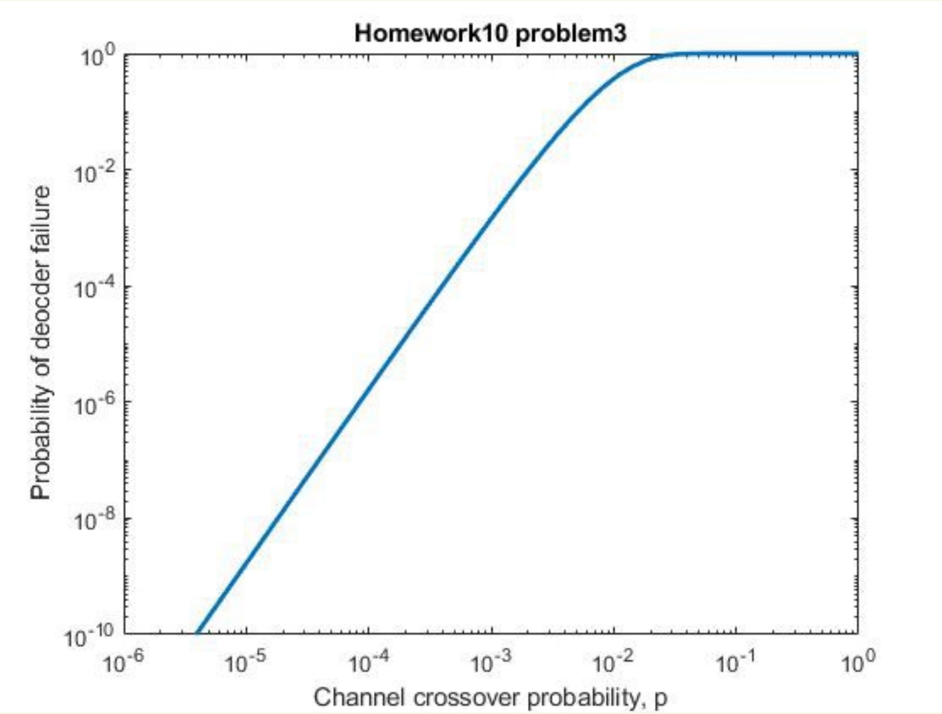
$$P_b = \underset{\sim\sim\sim}{1-(1-P)^8}$$

$$P_e = \sum_{i=3}^{28} \binom{28}{i} \cdot P_b^{i} (1-P_b)^{28-i}$$

$P$ represent the crossover probability

$P_b$ represent the probability of symbol error (Since at least one bit error causes the symbol error)

$P_e$ is the probability of decoding failure.

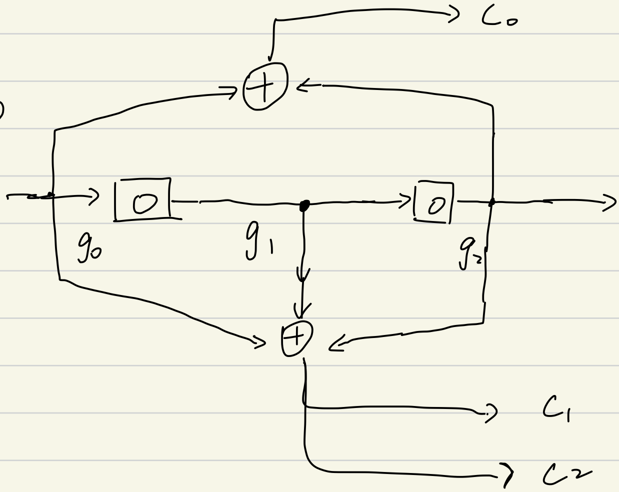



```
p = logspace(-6, 0);
p_fail = 0;
for i = 3:28
    pe = 1 - (1-p).^8
    pe_symbol = nchoosek(28,i) * pe.^i .* (1-pe).^(28-i)
    p_fail = p_fail + pe_symbol
end


loglog(p, p_fail, Linewidth = 2)
ylim([10.^(-10), 1])
ylabel('Probability of deocder failure')
xlabel('Channel crossover probability, p')
title('Homework10 problem3')
```

P4:

(a)




$g_0 = [1\ 0\ 1]$

$g_1 = [1\ 1\ 1]$

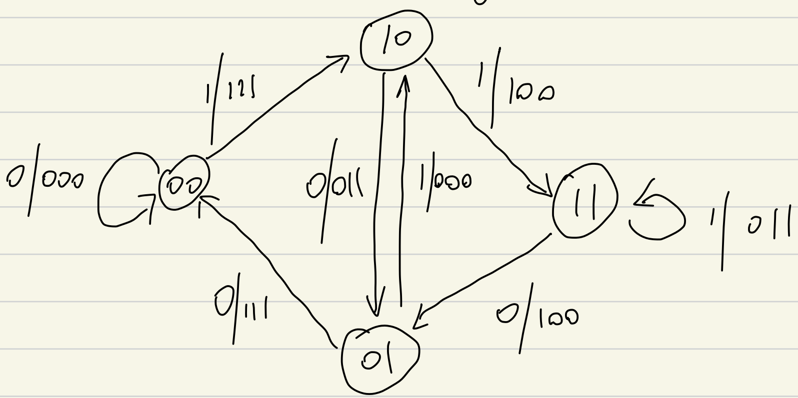
$g_2 = [1\ 1\ 1]$

Our info + tail bits is $[\ \underline{0\ 1\ 1\ \ 0\ 1}\ \ \underline{0\ 0}\ ]$

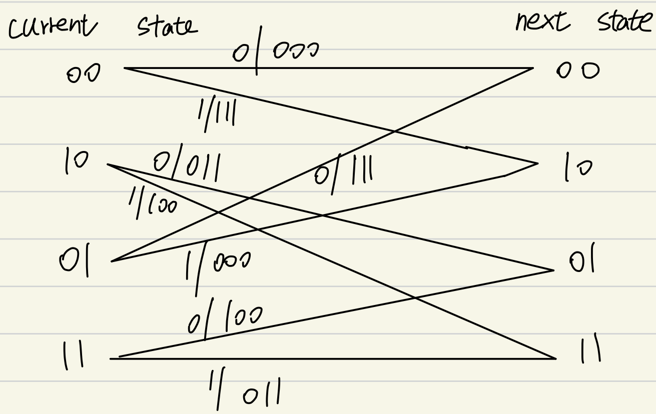
| input | State | Output |
|---|---|---|
| 0 | 00 → 00 | 000 |
| 1 | 00 → 10 | 111 |
| 1 | 10 → 11 | 100 |
| 0 | 11 → 01 | 100 |
| 1 | 01 → 10 | 000 |
| 0 | 10 → 01 | 011 |
| 0 | 01 → 00 | 111 |

Therefore, the output is [000 111 100 100 000 011 111]

(b) State transition diagram

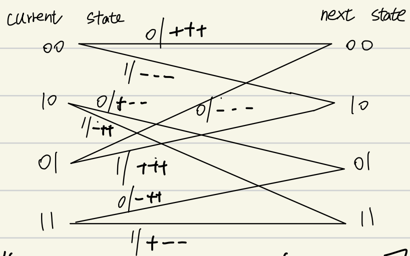



(c) Trellis diagram



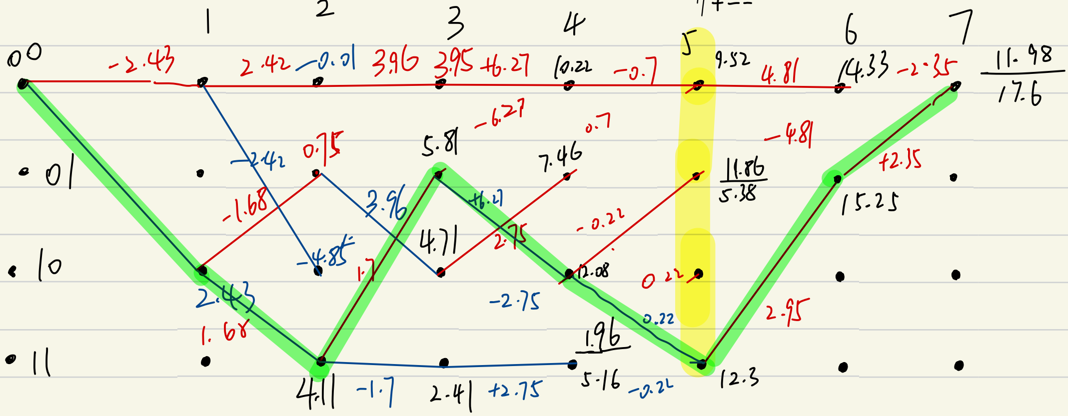
$(-1)^0 = 1$

$(-1)^1 = -1$




(d) $0 \rightarrow +1$ $1 \rightarrow -1$

Relabel the trellis diagram




① 0/+++: $-2.56 + 1.69 - 1.56 = -2.43$

1/---: $2.56 - 1.69 + 1.56 = 2.43$

Input: [1 1 0 1 1 0 0]

0.23

② 10→01/+--: $+0.37 + +0.14 - 2.19 = -1.68$

11/---: $-0.37 + 0.14 - 2.19 = -2.42$

10→11: 1/-++ $= -0.37 - 0.14 + 2.19 = 1.68$

③ 0/+++: $1.13 + 1.05 + 1.78 = 3.96$

1/---: $-1.13 - 1.05 - 1.78 = -3.96$

01→00: 0/---: $-3.96$

01→10: 1/+++: $3.96$

10→11: 1/-++: $-1.13 + 1.05 + 1.78 = 1.7$

11→11, 1/+--: $1.13 - 1.05 - 1.78 = -1.7$

11→01, 0/-++: $1.7$

⑭ 00 → 00 : 0/ +++ : 4.51 + 2.17 - 0.41 = 6.27
01 → 00 : 0/ --- : -6.27
01 → 10 : 1/ +++ : 6.27
10 → 01 : 0/ +-- : 4.51 - 2.17 + 0.41 = 2.75
10 → 11 : 1/ -++ : -2.75

11 → 11 : 1/ +-- : 2.75

⑮ 00 → 00 : 0/ +++ : - 0.46 - 0.92 + 0.68 = -0.7
01 → 00 : 0/ --- : 0.7
10 → 01 : 0/ +-- : - 0.22
11 → 11 : 1/ +-- : - 0.46 + 0.92 - 0.68 = -0.22
11 → 01 : 0/ -++ : 0.22
10 → 11 : 1/ -++ : 0.22

⑯ 00 → 00 : 0/ +++ : 0.93 + 1.54 + 2.34 = 4.81
01 → 00 : 0/ --- : -4.81

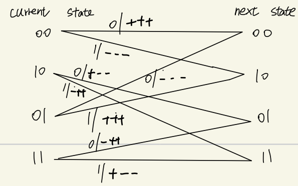
11 → 01 : 0/ -++ : - 0.93 + 1.54 + 2.34 = 2.95

⑰ 00 → 00 : 0/ +++ : 0.15 - 1.66 - 0.82 = -2.35
01 → 00 : 0/ --- : 2.35

Therefore, input bits are [ 1 1 0 1 1 $\underbrace{0 \quad 0}_{\text{Tail bits}}$ ]

$(-1)^0 = 1 \qquad 0 \to 1$

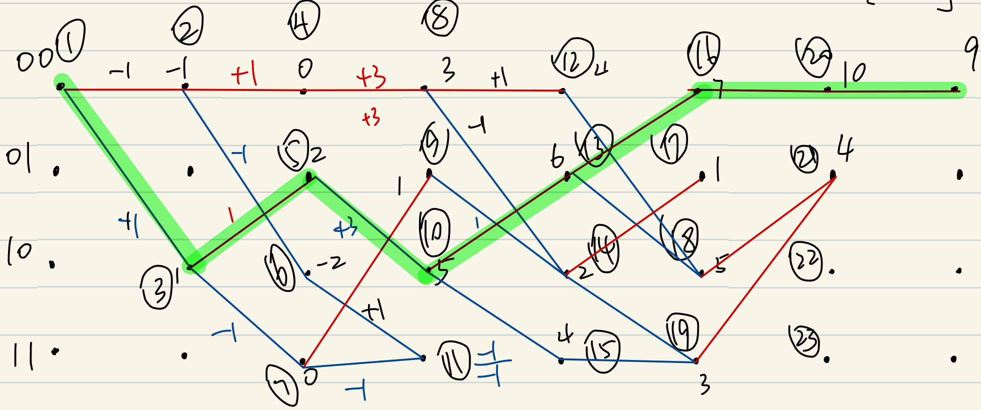
$(-1)^1 = -1 \qquad 1 \to -1$




e) For hard decision,

recd

① $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ ② $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ ③ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ④ $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ ⑤ $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ ⑥ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ⑦ $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$




+++
② 0: 1
1: -1

+++
④ 0: 3
1: -3

⑧: 0: 1
1: -1

⑫ 0: -1
1: 1

⑯ 0: 3
1: -3

⑳: 0: -1
1: 1

---
⑤ 0: -3
1: 3

⑨ 0: -1
1: 1

⑬ 0: 1
1: -1

⑰ 0: -3
1: 3

㉑ 0: 1
1: -1

+--
③ 0: 1
1: -1

⑥ 0: -1
1: 1

⑩ 0: 1
1: -1

⑭ 0: -1
1: 1

⑱ 0: -1
1: 1

㉒ 0: 1
1: -1

-++
⑦ 0: 1
1: -1

⑪ 0: -1
1: 1

⑮ 0: 1
1: -1

⑲: 0: 1
1: -1

㉓ 0: -1
1: 1

∴ Therefore, the input is $[\,1 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; \underbrace{0 \;\; 0}_{\text{Tail bits}}\,]$