University of Michigan Fall 2021

EECS501: Homework 2

Assigned: September 13, 2021
Due: September 21 , 2021 at 11:59PM on gradescope
Text: "Probability and random processes" by J. A. Gubner

Reading assignment: Please read Chapter 1. In lecture we are covering the material in this order: 1.1 - 1.6.

1. Dice Game [10 points]

Consider the following experiment. Keep rolling a fair die sequentially. What is the probability of observing "1" any time before the pattern "$5, 6$" (i.e., 5 followed immediately by 6)?

2. Badminton (10 points)

In the game of badminton, one player starts a rally by serving the shuttlecock or "birdie." The two players rally back and forth until the birdie hits the ground.

Let's assume the traditional scorekeeping system for badminton, where you score a point when you win a rally as the server. If you win a rally as the receiver, the score remains unchanged, but you get to serve and thus the opportunity to score.

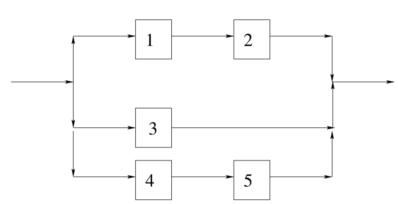
Suppose Ann wins a rally against Bob with probability p, regardless of who serves. Assume rallies are won independently of each other. If Ann is currently the server, what is the probability that she wins the next point? Express your answer as a function of p.

3. Dinner party problem [10 points]

4 couples (husband and wife) are to be seated at a long rectangular table for a dinner party. The table has 4 chairs on each side. The seating arrangement is done randomly. Find the probability that at least one couple is facing directly across from each other.

4. Reliability [5 points each]

There is a system consisting of components. Each component functions with probability $p = 0.9$, independent of the other components. The system as a whole functions provided that there is at least one path (from left to right) through functioning elements. Give your answers in terms of the variable p, and the numeric value when $p = 0.9$.



(a) Given that the system shown in the figure works, what is the probability that either component 2 or component 3 functions?

(b) Let A_i denote the event that component i works, and let E denote the event that the system works. Find $P(E|A_2 \cup A_3)$.

5. Battery Testing (15 points)

Itsibitsi Motor Company manufactures two kinds of batteries: B-1 and B-2. They are produced in 2 separate but synchronously operating assembly lines which are located nearby. Each assembly line manufactures one battery in one second. Each B-1 battery is faulty with probability p_1 and each B-2 battery is faulty with probability p_2. The faults in B-1 and B-2 occur independently of each other. The company has a battery testing equipment, which can test one battery per second. The setup is such that it chooses an assembly line at random and tests the corresponding battery. This procedure is repeated until a faulty B-1 is found. Let N denote the number of tests conducted. Find the PMF of N.

6. MAP Rule[15 points]

We have two boxes, each containing three balls: one black and two white in box 1; two black and one white in box 2. We choose one of the boxes at random, where the probability of choosing box 1 is equal to some given p, and then draw a ball.

(a) Describe the MAP rule for deciding the identity of the box based on whether the drawn ball is black or white.

(b) Assuming that $p = 1/2$, find the probability of an incorrect decision and compare it with the probability of error if no ball had been drawn.