Last (family) name:______________________

First (given) name:______________________

I have neither given nor received aid on this examination, nor concealed any violation of the Honor Code. The only online resources I have used during this exam are those listed below as well as the following sites (list URL and exam problem #):

Signature: ______________________

ID Number: ______________________

EECS 551 Midterm 1, 2020-09-29 5PM EDT (24 hour online)

- There are 8 problems for a total of 100 points.
- This part of the exam has 12 pages. Make sure your copy is complete.
- This is a 24-hour "online" exam. Many of the exam questions will be on Canvas; this pdf has additional questions that you should answer by submitting to gradescope unless instructed otherwise in each problem.
- This is an "open book" exam. During the exam, you may use all of the course materials on the Canvas EECS 551 site including the course notes on google drive, as well as wikipedia, the built-in JULIA help, and the online JULIA manual. If you use resources from any other web site then you must cite the source along with your honor code statement above.
- You may use without rederiving any of the results presented in the course notes.
- You must complete the exam entirely on your own.
- If you need an exam question to be clarified, post a private question to the instructors on piazza.
- Clearly $\boxed{\text{box}}$ your final answers. For full credit, show your complete work clearly and legibly.
  Answers must be submitted properly to gradescope to earn credit.
- For multiple-choice questions, select *all* correct answers.
- To "disprove" any statement, provide a concrete counter-example. For maximum credit, make that counter-example as small and simple as possible, *e.g.*, having the smallest possible matrix dimensions and using the simplest numbers like "0" and "1" as much as possible. For example, to disprove the statement "any square matrix $\boldsymbol{A}$ is invertible," the smallest and simplest counter-example is the 1×1 matrix $\boldsymbol{A} = [0]$.

(Solutions)

Regrade policy

- For any student who submits a regrade request, we may regrade the entire exam, and your final score may increase **or decrease**.
- Submit any regrade request on gradescope, with a clear description of why you think the problem should be regraded. Saying just "please look at it again" is insufficient and will not be considered.
- Regrade requests must be within 3 days (72 hours) of when the exam scores were released on gradescope.
- After scores are returned, discussing your solutions with a professor or GSI nullifies the opportunity to submit a regrade request. Your request needs to be based on your answer at the time of the exam. (Wait to discuss until *after* requesting a regrade, if you plan to make such a request.) (Submitting reports of errors in the solution to Canvas is fine.)
- Some questions were graded by student graders who may not have been very discriminating about the precision of your justifications for your answers. If you submit a regrade request, that means you think you were unfairly given too few points on some part of the exam, so it is logical for us to see if you were unfairly given too many points on some other part of the exam! Of course we want fairness overall. Minor mistakes are inevitable when grading numerous exams, and those mistakes go in both directions. One point on any midterm is only 0.2% of your overall final score, and unlikely to affect your final grade. You should look over the solutions and your answers to all problems carefully *before* submitting a regrade request to make sure that you really want to have your whole exam reevaluated.
- For elaboration on these solutions, please come to office hours.

---

1. (4 points)

```
0. Let $\A$ and $\C$ denote $M \times N$ matrices
   and $\B$ and $\D$ denote $N \times M$ matrices
   with $M < N$.
   Which Julia expression is guaranteed to compute $\A\B\C\D$ most efficiently?
%... `(A*B)*(C*D)` makes the two smaller $M \times M$ matrices first
a) `A*(B*C*D)`
b) `A*(B*(C*D))`
%... 3M^2N > 2M^N+M^3
c) `A*((B*C)*D)`
d) `(A*B*C)*D`
e) `((A*B)*C)*D`
f) `(A*(B*C))*D`
g) `A*(B*C)*D`
h) `A*B*(C*D)`
i) `(A*B)*C*D`
*j) `(A*B)*(C*D)`
%... 2M^2N + M^3
k) `(A*B*C*D)`
i) None of these
```

....
(HW 1.11) [93% correct]

---

2. (4 points)

```
0. Which of the following properties of the matrix determinant is/are TRUE?
   Assume that $\A$ and $\B$ are square with the same size.
[*] Only square matrices have a determinant
[*] For the identity matrix, $\det(\I) = 1$
[*] $\det(\A\B) = \det(\A) \det(\B)$
[*] $\det(\A^T) = \det(\A)$
[ ] $\det(\A') = \det(\A)$
[ ] $\det(\A+\B) = \det(\A) + \det(\B)$
[ ] For the identity matrix, $\det(3 \I) = 3$
```

....
(HW 1.2 1.3) [89% correct]

---

3. (4 points)

```
0. If $N \times N$ matrix $\A$ is square and has linearly independent columns,
   then the cardinality of the set of solutions
   to the linear system of equations
   $\A \x = \b$ for $\b \in \FN$
%... There is a unique solution for any $\b$.
[ ] is uncountably infinite for all $\b$
[ ] is uncountably infinite for some $\b$ but not all $\b$
```

```
[*] is never uncountably infinite
[ ] is 0 for all $\b$
[ ] is 0 for some $\b$ but not all $\b$
[*] is never 0
[*] is 1 for any $\b$
[ ] is 1 for some $\b$ but not all $\b$
[ ] is never 1
[ ] depends on additional unspecified properties of $\A$
```

....

(HW 1.2 1.3) [62% correct]

---

4. (4 points)

```
0. The set of $N \times N$ diagonal matrices with constant diagonals is a field,
   with the usual definition of matrix addition and multiplication.
%... Yes, it is closed under "vector" addition and scalar multiplication,
%    and the "inverse element" for $a I$ is $(1/a) I$ when $a \neq 0.
*a) true
b) false
```

....

(HW 2.7) [82% correct]

---

5. (4 points)

```
0. If $\A$ is a $N \times N$ normal matrix with eigendecomposition
   $\A = \V \Lam \V'$,
   and $\S$ is a $N \times N$ diagonal matrix
   whose diagonal elements each have unit magnitude,
   then $\W \Lam \W'$ is a unitary eigendecomposition of $\A$
   where (i) $\W = \V \S$ or (ii) $\W = \S \V$.
%... $\V \Lam \V' = (\V \S) (\S' \Lam \S) (\V \S)'$
a) both (i) and (ii) are true
b) neither (i) nor (ii) are true
*c) (i) is true but (ii) is false
d) (ii) is true but (i) is false
```

....

(HW 3.5 3.11) [45% correct] (48% chose both)

---

6. (4 points)

```
0. If $\P$ denotes any $N \times N$ permutation matrix
   and $\I$ denotes the $N \times N$ identity matrix,
   then the matrix $\I + \P$
% ... Not invertible or PSD or upper triangular:
% ... consider $\P = \bmat 0 & 1 \\ 1 & 0 \emat$
[ ] is invertible
```

```
[ ] is unitary
[ ] is positive semidefinite
[ ] is symmetric
[ ] is upper triangular
[*] None of these
```

....

() [56% correct] 27% chose psd, 21% chose invertible

---

7. (4 points)

```
0. Counting possible repeated values,
   the number of nonzero singular values of a square matrix $\A$
   is the same as the number of nonzero eigenvalues of $\A$
   if $\A$ (select sufficient condition(s)):
[ ] is square
%... No, consider $\bmat 0 1 \\ 0 0 \emat$
[*] is square and diagonal
[*] is normal
[*] is Hermitian
[ ] is singular
%... No, consider $\bmat 0 1 \\ 0 0 \emat$
[*] is invertible
[*] is a permutation matrix
[ ] has orthogonal columns
%... No, because there could be columns of zeros, like [0 1; 0 0]
%... (This one was incorrect in the Canvas grading.)
[ ] None of these
```

....

(HW 3.5 3.11) ["17% correct"] only 53% chose correct "is invertible"
38% chose "orthogonal columns" but it includes [0 1; 0 0]

---

8. (4 points)

```
0. If $\A$ denotes any $M \times N$ matrix
   and $\P_1$ denotes any $M \times M$ permutation matrix
   and $\P_2$ denotes any $N \times N$ permutation matrix,
   then
%... Equality for all A, because permutation matrices are unitary
%    and these norms are unitarily invariant.
[*] $\| \P_1 \A \P_2 \|_2 = \| \A \|_2$ for all $\A$
[ ] $\| \P_1 \A \P_2 \|_2 = \| \A \|_2$ for some $\A$, but not all $\A \in \FMN$
[*] $\| \P_1 \A \P_2 \|_F = \| \A \|_F$ for all $\A$
[ ] $\| \P_1 \A \P_2 \|_F = \| \A \|_F$ for some $\A$, but not all $\A \in \FMN$
[*] $\| \P_1 \A \P_2^{-1} \|_2 = \| \A \|_2$ for all $\A$
[ ] $\| \P_1 \A \P_2' \|_2 = \| \A \|_2$ for some $\A$, but not all $\A \in \FMN$
```

....

(HW 2.3 2.4) [79% correct]

---

9. (4 points)

```
0. Let $\Q$ denote a $N \times N$ unitary matrix
   and $\A$ denote a $N \times N$ invertible matrix
   having set of singular values
   $\cup_{n=1}^N \sigma_n = \{ 0.2, 0.5, 1, 8, 10 \}$.
   Determine $\max_{\v \in \CN \, : \, \|\v\|_2 = 1} \| \Q \A^{-1} \v \|_2$.
%... The spectral norm of $\A^{-1}$ is $1/\sigma_N$
=  5
```

....

(HW 2.14 2.15) [90% correct]

---

10. (4 points)

```
0. The second singular value of the matrix
   $\A = \bmat -a & 0 \\ 0 & -b \emat$, where $0 < a < b$
   are real numbers, is
%... The eigenvalues of $\A'\A$ are $a^2$ and $b^2$.
*a) $a$
b) $b$
c) $a^2$
d) $b^2$
e) $\sqrt{a}$
f) $\sqrt{b}$
g) $-a$
h) $-b$
i) None of these
```

....

(HW 3.5) [93% correct]

---

11. (4 points)

```
0. Which of the following sets of $N \times N$ real matrices
   are subspaces of $\RNN$?
[*] The set whose diagonal entries are all zero.
[*] The set whose trace is zero.
[ ] The set whose trace is one.
[ ] The set whose determinant is zero.
[ ] The set whose determinant is one.
[ ] The set of invertible matrices.
[ ] The set of positive semidefinite matrices.
[*] The set of pentadiagonal matrices.
[*] The set of matrices whose spectral norm is zero.
[ ] The set of matrices whose spectral norm is one.
[ ] None of these.
```

. . . .

(Ch3 Q5,6,7,9) [43% correct] (PSD was chosen by 33%)

misc/trace2

1. (6 points)
Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be vectors in $\mathbb{F}^9$ with Euclidean norms 5 and 2, respectively, and define the matrices $\boldsymbol{A} = \boldsymbol{x}\boldsymbol{y}'$ and $\boldsymbol{B} = \boldsymbol{A}\boldsymbol{A}'$. Either determine trace$\{\boldsymbol{B}\}$, or explain why more information is needed to determine it.

. . . .

$\operatorname{trace}\{\boldsymbol{B}\} = \operatorname{trace}\{\boldsymbol{A}\boldsymbol{A}'\} = \operatorname{trace}\{\boldsymbol{x}\boldsymbol{y}'\boldsymbol{y}\boldsymbol{x}'\} = \operatorname{trace}\{\boldsymbol{x}'\boldsymbol{x}\boldsymbol{y}'\boldsymbol{y}\} = \|\boldsymbol{x}\|_2^2 \, \|\boldsymbol{y}\|_2^2 = 5^2 \cdot 2^2 = 100$
(HW 1.4) [87% correct]

misc/det-block

2. (6 points)
Write a JULIA expression that evaluates $\det\left\{\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{0} & \boldsymbol{D} \end{bmatrix}' \begin{bmatrix} \boldsymbol{E} & \boldsymbol{F} \\ \boldsymbol{0} & \boldsymbol{H} \end{bmatrix}\right\}$ as efficiently as possible.
(Assume that $\boldsymbol{A}$,$\boldsymbol{D}$,$\boldsymbol{E}$,$\boldsymbol{H}$ are square, and that $\boldsymbol{B}$ and $\boldsymbol{F}$ and the two $\boldsymbol{0}$ matrices are **conformable**.)

. . . .

$$\det\left\{\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{0} & \boldsymbol{D} \end{bmatrix}' \begin{bmatrix} \boldsymbol{E} & \boldsymbol{F} \\ \boldsymbol{0} & \boldsymbol{H} \end{bmatrix}\right\} = \det\{\boldsymbol{A}'\}\det\{\boldsymbol{D}'\}\det\{\boldsymbol{E}\}\det\{\boldsymbol{H}\} = \det{}^*\{\boldsymbol{A}\}\det{}^*\{\boldsymbol{D}\}\det\{\boldsymbol{E}\}\det\{\boldsymbol{H}\}$$

```
conj(det(A)*det(D))*det(E)*det(H)
```

(HW 1.13) [69% correct (16% missing conjugates)]

tf/eig/diag-r1

3. (6 points)
Prove or disprove. Every square rank-1 matrix is diagonalizable.

. . . .

False. Counter example (some students used stackexchange): $\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

(HW 3.6) [68% correct]

misc/frob-ainv-ux

4. (6 points)
Let $\boldsymbol{A} \in \mathbb{F}^{N \times N}$ be invertible with SVD $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}'$ and let $\boldsymbol{x} \in \mathbb{F}^K$ denote a vector with $\|\boldsymbol{x}\|_2 = 7$. Express $\left\|\boldsymbol{A}^{-1}\boldsymbol{u}_1\boldsymbol{x}'\right\|_{\mathrm{F}}$ concisely in terms of the given ingredients, or explain why more information would be needed for a concise expression.

. . . .

$\left\|\boldsymbol{A}^{-1}\boldsymbol{u}_1\boldsymbol{x}'\right\|_{\mathrm{F}} = \left\|\boldsymbol{V}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}'\boldsymbol{u}_1\boldsymbol{x}'\right\|_{\mathrm{F}} = \|\boldsymbol{v}_1/\sigma_1\boldsymbol{x}'\|_{\mathrm{F}} = \|\boldsymbol{x}\|_2/\sigma_1 = 7/\sigma_1$.
(HW 2.14, 2.15) [55% correct.]

`misc/s1-from-eig`

5. (6 points)
Let $\boldsymbol{A}$ be a diagonalizable matrix with eigenvalues -2, $3+4\imath$. Determine σ_1 or explain why more information is needed to determine it.

. . . .

Being diagonalizable is insufficient for having a relationship between the eigenvalues and the singular values. Consider $\boldsymbol{A} = \begin{bmatrix} -2 & a \\ 0 & 3+4\imath \end{bmatrix}$. This matrix is diagonalizable (because its eigenvalues are distinct), and it has the two specified eigenvalues. But the singular values depend on a and can be arbitrarily large. $\boldsymbol{G} = \boldsymbol{A}\boldsymbol{A}' = \begin{bmatrix} -2 & a \\ 0 & 3+4\imath \end{bmatrix} \begin{bmatrix} -2 & 0 \\ a & 3-4\imath \end{bmatrix} = \begin{bmatrix} 4+a^2 & a(3-4\imath) \\ a(3+4\imath) & 25 \end{bmatrix} \Longrightarrow \det\{z\boldsymbol{I} - \boldsymbol{G}\} = (z-4-a^2)(z-25) - 25a^2 = z^2 - (29+a^2)z + 100 \Longrightarrow z = (29+a^2 \pm \sqrt{(29+a^2)^2 - 400})/2 \Longrightarrow \sigma_1 = \sqrt{(29+a^2+\sqrt{(29+a^2)^2-400})/2}$, which is ever increasing in a.
(HW 3.5, 3.6) [61% correct with good explanation, another 20% correct with insufficient explanation.]

`sub/span-ball`

6. (6 points)
The unit ball $\mathcal{B}_1$ in a vector space $\mathcal{V}$ is the set of vectors having norm less than unity: $\mathcal{B}_1 = \{\boldsymbol{x} \in \mathcal{V} \,:\, \|\boldsymbol{x}\|_2 \leq 1\}$. Determine the span of the unit ball in $\mathbb{F}^N$, *i.e.*, $\mathsf{span}(\mathcal{B}_1)$, or explain why more information is needed to determine it.

. . . .

$\mathsf{span}(\mathcal{B}_1) = \mathbb{F}^N$ because any nonzero $\boldsymbol{x} \in \mathbb{F}^N$ can be written as $\boldsymbol{x} = \alpha \boldsymbol{b}$ where $\alpha = \|\boldsymbol{x}\|_2$ and $\boldsymbol{b} = \boldsymbol{x}/\|\boldsymbol{x}\|_2 \in \mathcal{B}_1$.
Alternative explanation: all the standard unit vectors $\{\boldsymbol{e}_n \,:\, n = 1, \ldots, N\}$ are in $\mathcal{B}_1$, and their span is all of $\mathbb{F}^N$.
(HW 3.9) [77% correct with clear explanation, and further 12% correct with incomplete explanation.]

`sub/orthcomp-xy`

7. (6 points)
Let $\boldsymbol{A} = \boldsymbol{x}\boldsymbol{y}'$, where $\boldsymbol{x}$ and $\boldsymbol{y}$ are two nonzero vectors. Express the orthogonal complement of the nullspace of $\boldsymbol{A}$ in terms of the vectors $\boldsymbol{x}$ and/or $\boldsymbol{y}$.

. . . .

$\mathcal{N}^{\perp}(\boldsymbol{A}) = \mathsf{span}(\boldsymbol{y})$
(HW 3.9) [69% correct and fully simplified. 10% insufficiently simplified, e.g., $\mathcal{R}(\boldsymbol{A}')$ (-1).]

`code/second_diff_2d`

8. (14 points)
Some image processing operations require discrete approximations to *second* derivatives. For unit sample spacing, in 1D the most popular approach is

$$\ddot{f}(x) \approx 2f(x) - f(x-1) - f(x+1).$$

For a 1D signal sampled at $x_1, \ldots, x_N$, with $x_n - x_{n-1} = 1$, we can express this relation for all x_i samples via a matrix-vector product with a $N \times N$ second finite-difference matrix $\boldsymbol{S}_N$ as follows:

$$\begin{bmatrix} \ddot{f}(x_1) \\ \ddot{f}(x_2) \\ \vdots \\ \ddot{f}(x_N) \end{bmatrix} \approx \boldsymbol{S}_N \begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{bmatrix}, \qquad \boldsymbol{S}_N \triangleq \begin{bmatrix} -1 & 2 & -1 & & & & \\ -1 & 2 & -1 & & & & \\ & -1 & 2 & -1 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & -1 & 2 & -1 & \\ & & & & -1 & 2 & -1 \\ & & & & -1 & 2 & -1 \end{bmatrix}.$$

Note that the first and last rows use a slightly different approximation due to the lack of a $f(0)$ or $f(N+1)$ sample.

In this problem you will derive and implement the analog of $\boldsymbol{S}_N$ for 2D differentiation. Let $f(x, y)$ be a function of two variables. We can approximate its 2nd partial derivatives using finite differences as follows:

$$\frac{\partial^2}{\partial x^2} f(x, y) \approx 2f(x, y) - f(x+1, y) - f(x-1, y) \tag{1}$$

$$\frac{\partial^2}{\partial y^2} f(x, y) \approx 2f(x, y) - f(x, y+1) - f(x, y-1). \tag{2}$$

To simplify notation, define the $m \times n$ matrices `FXY`, `SFDX`, and `SFDY` having elements as follows:

$$\texttt{FXY}[m,n] = f(m,n), \qquad \texttt{SFDX}[m,n] = \frac{\partial^2}{\partial x^2} f(m,n), \qquad \texttt{SFDY}[m,n] = \frac{\partial^2}{\partial y^2} f(m,n), \qquad \begin{array}{l} m = 1, \ldots, M \\ n = 1, \ldots, N. \end{array}$$

The x coordinate is along the column of `FXY` and the y coordinate is along the row of `FXY`, so we can think `FXY[x,y]`. Define corresponding vectors `fxy`, `sfdx`, and `sfdy` in $\mathbb{R}^{MN}$ to be vectorized versionsof `FXY`, `SFDX`, and `SFDY`. With this notation, we succinctly express (1) and (2) in terms of an appropriately sized matrix $\boldsymbol{A}$ as:

$$\begin{bmatrix} \texttt{sfdx} \\ \texttt{sfdy} \end{bmatrix} = \boldsymbol{A}\, \texttt{fxy}.$$

- Write a function `second_diff_2d` that takes as input the dimensions M and N of $M \times N$ array `FXY` and returns the appropriate $\boldsymbol{A}$ matrix. Your code should work even when M and N are quite large. To save memory, the elements of $\boldsymbol{A}$ should be of type `Int8`. Here is a docstring and template for your code.

```julia
"""
   A = second_diff_2d(M, N)

In:
- `M` and `N` are positive integers

Out:
- `A` is a appropriate matrix such that `A * X[:]` computes the
second finite differences down the columns (along x direction)
and across the (along y direction) of the `M x N` matrix `X`,
where `eltype(A) == Int8`
"""
function second_diff_2d(M, N)
```

- [9 points] Upload your code for the function `second_diff_2d` as a single, complete `uniquename.jl` file to Canvas under the "Midterm1code" assignment. (Do not upload any test or plotting code to Canvas.)
- [5 points] You should test your code yourself by writing and running your own test routine *before* uploading your solution. Submit to gradescope a screenshot of your test code, along with some plot(s) or image(s) that provide evidence that your code was working before you uploaded it to Canvas.

. . . .

Function code:

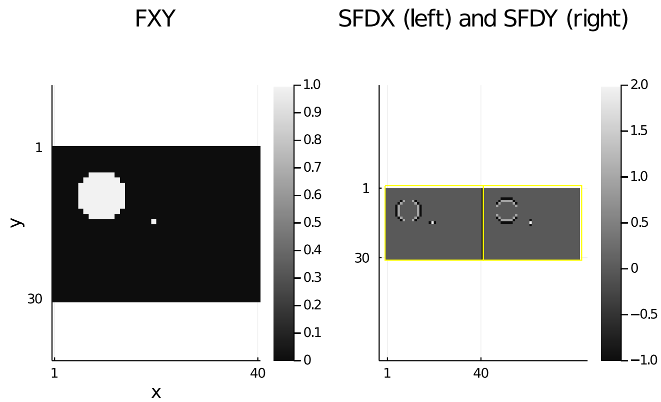
```julia
using LinearAlgebra: I
using SparseArrays: sparse, spdiagm
```

```
"""
    A = second_diff_2d(M, N)

In:
- m and n are positive integers

Out:
- `A` is a `2MN x MN` sparse matrix such that `A * X[:]` computes the
second differences down the columns (along x direction)
and across the (along y direction) of the `M x N` matrix `X`
"""
function second_diff_2d(M, N)
    D = [kron(I(N), S(M)); # 2nd differences down columns (x)
        kron(S(N), I(M))] # 2nd differences across rows (y)
    return D
end


"""
- In: `N` is a positive integer
- Out: `S` is an `N x N` sparse 2nd differences matrix
"""
function S(N)
    out = spdiagm(0 => 2*ones(N), 1 => -ones(N-1), -1 => -ones(N-1))
    out[1,:] .= out[2,:]
    out[N,:] .= out[N-1,:]
    return Int8.(out)
end
```

Test code:

```
using MIRT: jim
using Plots
include("second_diff_2d-ans.jl")
M = 40; N = 30
X = Float64.([(x-M/4)^2+(y-N/3)^2 < 5^2 for x=1:M, y=1:N])
X[Int(M/2),Int(N/2)] = 1 # Kronecker impulse
A = second_diff_2d(M, N)
s = A * vec(X)
pic = reshape(s, M, N, 2)
plot(jim(X, "FXY", xlabel="x", ylabel="y"),
    jim(pic, "SFDX (left) and SFDY (right)"))
#savefig("second_diff_2d.pdf")
```

Test results:



(HW 2.10)

- *[code: 38/107 completely correct. mean: 6.3/9*

  This problem was graded with a custom autograder. Points deducted as follows:
  -1 wrong file name (9 students)
  -2 submitting a .doc file renamed as a .jl file (1 student)
  -1 including (uncommented) test code within file, contrary to instructions (2 students)
  -3 file would not even `include` *(1 student)*
  -1 wrong function name (5 students)
  -1 missing `include SparseArrays` *(3 students)*
  -1 output `eltype` *not* `Int8` *(many students)*
  -2 output type not `SparseMatrixCSC` *(27 students)*
  -6 code does not produce correct output (28 students)
  -9 no submission (3 students)

  There were 50 (!) students who wrote code with `for` *loops, apparently disregarding the HW solutions.*

  Only 80 students downloaded HW03 solutions from server.

  In a class this size, having to manually handle incorrect filenames and function names is unsustainable. For future exams, only submissions that follow the specifications will earn credit.]

- *[self test: mean 3.82/5; 63% submitted test code with results that looked like plausible.*
  Many used nice calculus examples like $f(x, y) = x^2 + y^3$.
  Many had x,y clearly reversed in the figures.
  Testing with $M = N$ *is insufficient because some buggy code can pass when* $M = N$ *but fails when* $M \neq N$.
  -0.1 point to make sure you remember this!]

Exam scores with *approximate* grades.

Gradescope part (excluding code) 107 students: min 15, max 47/47, std dev 7.4, mean 39.6, median 41
Canvas part (excluding one 0): min 18, max 44/44, std dev 6.5, mean 34.3, median 35,

Overall, 107 students: median=81.1, mean=82.3, std=14.4, range 39-100