EECS 455: Problem Set 5
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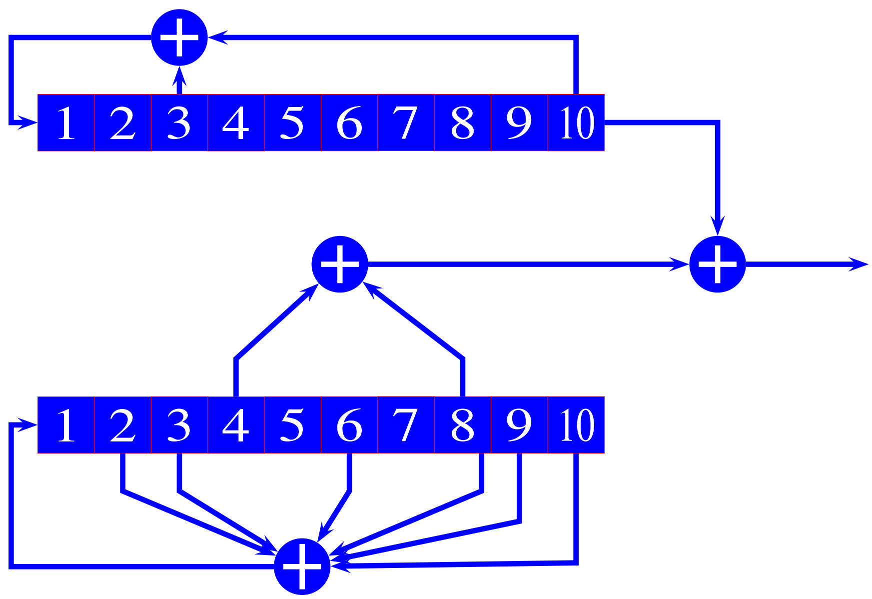
Due: Wednesday, October 13, 2021, 11pm.

1. A signal $s(t)$ of duration $T = 1ms$ from one of the GPS satellites consists of 1023 consectuive pulses (of duration $T/1023$) of amplitude ± 1. The sequence of amplitudes are determined by a pair of 10 stage shift registers shown below. Every $T/1023$ the binary (0 or 1) contents are shifted right and a new bit is produced at the output.



(a) The contents of the shift register at the start (time 0) are both all ones. The addition is done modulo 2 (0+0=0,0+1=1,1+0=1,1+1=0). Generate the sequence of bits from this shift register. The first 5 bits are 11110. What are the first 20 bits of the output? How many zeros and how many ones are produced in the sequence of 1023 bits.

(b) Assume that a signal consisting of pulses with amplitude determined by this shift register generates a signal $s(t)$. A zero bit at the output is mapped to $+1$ while a one bit at the output is mapped to -1. This signal is input to a linear time-invariant system (filter) with impulse response $h(t) = s(T-t)$. Find (plot) the output of the filter as a function of time (time going from 0 to 2 ms). (Use Matlab). Normalize so that the maximum output value is 1023.

(c) Now consider a signal from a different satellite whereby the sequence is produced as before except that contents of the second and sixth stage of the bottom shift register are added (exclusive or) with the output of the top shift register to produce the sequence. Determine (plot) the output of the same filter as in part (b) to the new signal. Determine the maximum (positive) output and the minimum (negative) output with the same normalization used in part (b).

2. The FCC allows for handheld "ultrawideband" systems a radiated power of -41.3 dBm per 1MHz bandwidth in the frequency band 3.1GHz to 10.6GHz that follows the mask shown in Figure 1 below. (see also FCC Rules, Title 47, Part 15.519). The radiated power outside

this band must be less than -61.3dBm per 1MHz bandwidth. Using this restriction determine the minimum symbol duration using square-root raised cosine pulses with roll-off of 0.35 that will fit in this spectral window. Plot the spectrum of the square-root raised cosine pulse along with the FCC Mask.

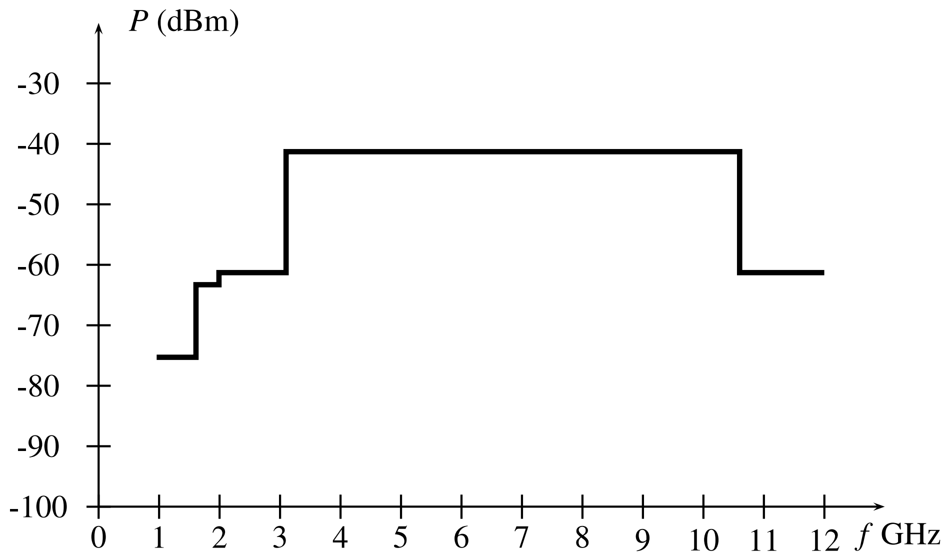


Figure 1: FCC Mask for Hand Held UWB Systems.

3. A communication system (GSM) sends a signal that is used for synchronization. The signal consists of 64 pulses of amplitude either 1 or -1. The file on the Canvas web page contains samples of the signal except that noise has been added as well as a random delay (to model the delay between the transmitter and receiver). Use an appropriate filter to determine when the signals started and ended. Matlab code (p107.m) is provided on the Canvas web site to show how the signal was generated. In your solution provide your Matlab code and your estimate of when the signal started and ended. The file p107.m is the Matlab code I used to generate the data that is contained in p107.mat. You need to load p107.mat (Matlab command "load p107") into Matlab and then filter the signal (x2t) to remove as much of the noise as possible and determine the delay that was used in the Matlab execution of the program.

4. Consider the odd length Zadoff Chu sequence

$$x_n = e^{jM\pi n(n+1)/N},\ n = 0, 1, \ldots, N-1.$$

(a) Show that for N odd

$$\begin{aligned} x_{n-k-N} &= x_{n-k} \\ &= e^{jM\pi(n-k)(n-k+1)/N}. \end{aligned}$$

(b) Show that $x_n x^*_{n-k} = e^{-jM\pi(k^2-k)/N} e^{jM2\pi kn/N}$.

(c) Using part (a) and part (b) show that the periodic autocorrelation

$$\theta_x(k) = \sum_{n=0}^{N-k-1} x_n x^*_{n-k} + \sum_{n=N-k}^{N-1} x_n x^*_{n-k-N}$$

is

$$\theta_x(k) = \begin{cases} N, & k = 0 \\ 0, & k \neq 0 \end{cases}$$

5. Generate a Zadof-Chu sequence of length 839. Let x denote the sequence.

   (a) Generate the sequence and plot the imaginary part vs the real part. (e.g. `plot(real(x),imag(x))`

   (b) Plot the output of a filter matched to x.

   (c) A signal u is the concatenation of 5 copies of x. $u = [0, x, x, x, x, x, 0]$ padded by a zero at each end. Suppose u is filtered with the same filter as in part a (i.e. matched to x). Plot the output of the filter when u is the input.