

EECS 455 Homework 4 YUZHAN JIANG

$$1. \quad r(t) = s_c(t) + n(t)$$

$$\hat{S}_i(t) = \int_{-\infty}^{\infty} \left(\sqrt{2P} (-1)^i \cos(2\pi f_c t) p_T(t) \right) \cdot \sqrt{\frac{2}{T}} \cos(2\pi f_c t + \theta) h(T-t) \cdot dt$$

$$\begin{aligned} \therefore \hat{S}_i(t) &= (-1)^i 2 \sqrt{\frac{P}{T}} \int_{-\infty}^{\infty} \cos(2\pi f_c t) p_T(t) \cdot \cos(2\pi f_c t + \theta) p_T(T-t) \cdot dt \\ &= (-1)^i 2 \sqrt{\frac{P}{T}} \int_{-\infty}^{\infty} \frac{1}{2} \left[\cos(2\pi f_c t - 2\pi f_c t - \theta) + \cos(2\pi f_c t + \theta) \right] p_T(t) dt \end{aligned}$$

$$= (-1)^i 2 \sqrt{\frac{P}{T}} \int_{-\infty}^{\infty} \frac{1}{2} [\cos(-\theta)] \cdot dt$$

$$= (-1)^i \sqrt{\frac{P}{T}} \int_0^T \cos \theta \, dt$$

$$= (-1)^i \sqrt{\frac{P}{T}} \cos \theta \, T$$

$$= (-1)^i \sqrt{P T} \cdot \cos \theta$$

$$\therefore \hat{S}_0(t) = \sqrt{P T} \cdot \cos \theta \quad \hat{S}_1(t) = -\sqrt{P T} \cdot \cos \theta$$

$$\therefore \bar{P}_e = \pi_{00} \cdot Q\left(\frac{\hat{S}_0(t) - \gamma}{\sigma_N}\right) + \pi_{11} \cdot Q\left(\frac{\gamma - \hat{S}_1(t)}{\sigma_N}\right) \quad \text{where } \gamma = 0$$

$$= \pi_{00} \cdot Q\left(\frac{\sqrt{P T} \cdot \cos \theta}{\sigma_N}\right) + \pi_{11} \cdot Q\left(\frac{\sqrt{P T} \cos \theta}{\sigma_N}\right)$$

$$\begin{aligned} \text{where } \sigma_N^2 &= \frac{N_0}{2} \int_{-\infty}^{\infty} h^2(t) \, dt = \frac{N_0}{2} \int_0^T p_T^2(t) \cdot dt \\ &= \frac{N_0}{2} T \end{aligned}$$

P_2 ,

(a) By def of probability of error,

$$\bar{P}_e = \pi_0 P_{e,0} + \pi_1 P_{e,1} + \pi_2 P_{e,2} + \pi_3 P_{e,3}$$

The probability of error given signal 0 transmitted is then

$$P_{e,0} = P\{\hat{S}_0(T) + n < 2\}$$

$$= P\{n < 2-3\}$$

$$\sigma_n^2 = 4$$

$$= P\{n < -1\}$$

$$\sigma_n = 2$$

$$= \Phi\left(\frac{-1}{2}\right)$$

$$= Q\left(\frac{1}{2}\right)$$

Similarly, $P_{e,1} = 1 - P\{0 < \hat{S}_1(T) + n < 2\}$

$$= 1 - P\{0 < 1 + n < 2\}$$

$$\hat{S}_1(T) = +1$$

$$= 1 - P\{-1 < n < 1\}$$

$$= 1 - \left(\Phi\left(\frac{1}{2}\right) - \Phi\left(-\frac{1}{2}\right)\right)$$

$$= 2Q\left(\frac{1}{2}\right)$$

$$\hat{S}_2(T) = -1 \quad P_{e,2} = 1 - P\{-2 < \hat{S}_2(T) + n < 0\}$$

$$= 1 - P\{-2 < -1 + n < 0\}$$

$$= 1 - P\{-1 < n < 1\}$$

$$= 1 - \left(\Phi\left(\frac{1}{2}\right) - \Phi\left(-\frac{1}{2}\right)\right)$$

$$= 2Q\left(\frac{1}{2}\right)$$

$$\hat{S}_3(T) = -3 \quad P_{e,3} = P(\hat{S}_3(T) + n > -2)$$

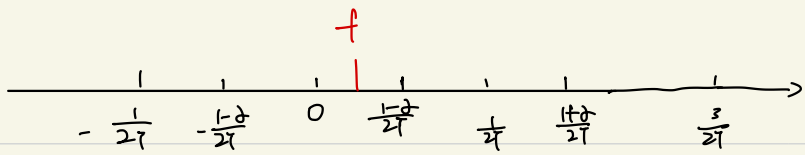
$$= P(-3 + n > -2)$$

$$= P(n > 1)$$

$$= Q\left(\frac{1}{2}\right)$$

P2 (b) Let b_0 represent the first bit. Let \hat{b}_0 represent the decision on the first bit.
Then we need to compute:

$$\begin{aligned} & P(\text{decision on } S_2, S_3 \mid S_0, S_1 \text{ transmitted}) \\ &= P\{\hat{b}_0 = 1 \mid b_0 = 0\} \\ &= P\{Z(T) < 0 \mid b_0 = 0\} \\ &= P\{S_0(T) + \eta < 0\} + P\{S_1(T) + \eta < 0\} \\ &= P\{\eta < -3\} + P\{\eta < -1\} \\ &= \Phi(-\frac{3}{\sigma}) + \Phi(-\frac{1}{\sigma}) \\ &= Q(\frac{1}{\sigma}) + Q(\frac{3}{\sigma}) \end{aligned}$$



P3:

① For $-\frac{1}{2T} < f < \frac{1}{2T}$, $\alpha \in [0, 1]$

$$0 < |f| < \frac{1}{2T}$$

$$H(f) = \begin{cases} 0 & , 0 < |f| < \frac{1-\delta}{2T} \\ \frac{T}{2} \left\{ 1 + \cos\left(\frac{\pi T}{2} \left[f - \frac{1-\delta}{2T} \right] \right) \right\} & , \frac{1-\delta}{2T} < |f| < \frac{1}{2T} \end{cases}$$

② Let $f_1 = f - \frac{1}{T}$.

For f_1 ,

$$-\frac{1}{2T} - \frac{1}{T} < f_1 < \frac{1}{2T} - \frac{1}{T}$$

$$-\frac{3}{2T} < f_1 < -\frac{1}{2T}$$

$$\frac{1}{2T} < |f_1| < \frac{3}{2T}$$

$$H(f_1) = H\left(f - \frac{1}{T}\right) = \begin{cases} \frac{T}{2} \left\{ 1 + \cos\left(\frac{\pi T}{2} \left[f - \frac{1}{T} - \frac{1-\delta}{2T} \right] \right) \right\} & , \frac{1}{2T} < |f_1| < \frac{1+\delta}{2T} \\ 0 & , \frac{1+\delta}{2T} < |f_1| < \frac{3}{2T} \end{cases}$$

③ Let $f_2 = f + \frac{1}{T}$.

$$-\frac{1}{2T} + \frac{1}{T} < f_2 < \frac{1}{2T} + \frac{1}{T}$$

$$\frac{1}{2T} < f_2 < \frac{3}{2T}$$

$$\therefore \frac{1}{2T} < |f_2| < \frac{3}{2T}$$

$$\therefore H(f_2) = H\left(f + \frac{1}{T}\right) = \begin{cases} \frac{T}{2} \left\{ 1 + \cos\left(\frac{\pi T}{2} \left[f + \frac{1}{T} - \frac{1-\delta}{2T} \right] \right) \right\} & , \frac{1}{2T} < |f_2| < \frac{1+\delta}{2T} \\ 0 & , \frac{1+\delta}{2T} < |f_2| < \frac{3}{2T} \end{cases}$$

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

Case ①: If $0 < |f| < \frac{1-\delta}{2T}$, $\Rightarrow |f - \frac{1}{T}| < \frac{1+\delta}{2T}$

$$H(f) = 0$$

$$H(f_1) = \frac{T}{2} \left\{ 1 + \cos\left(\frac{\pi T}{2} \left[f - \frac{3+\delta}{2T}\right]\right) \right\}$$

$$H(f_2) = \frac{T}{2} \left\{ 1 + \cos\left(\frac{\pi T}{2} \left[f + \frac{1+\delta}{2T}\right]\right) \right\}$$

$$\therefore H(f) + H(f_1) + H(f_2) = T + \frac{T}{2} \left\{ \cos\left(\frac{\pi T}{2} \left[f - \frac{3+\delta}{2T}\right]\right) + \cos\left(\frac{\pi T}{2} \left[f + \frac{1+\delta}{2T}\right]\right) \right\}$$

$$= T + \frac{T}{2} \left\{ \cos\left(\frac{\pi T}{2} \left[2f - \frac{2}{2T}\right]\right) \cos\left(\frac{\pi T}{2\delta} \left(-\frac{4}{2T}\right)\right) \right\}$$

$$= T$$

Case ②: If $-\frac{1-\delta}{2T} < f < \frac{1-\delta}{2T}$

then $\frac{1+\delta}{2T} < |f| < \frac{3}{2T}$ and $\frac{1+\delta}{2T} < |f_2| < \frac{3-\delta}{2T}$

$$\therefore H(f) + H(f_1) + H(f_2) = T + \frac{T}{2} \left\{ \cos\left(\frac{\pi T}{2} \left[f - \frac{3-\delta}{2T}\right]\right) + \cos\left(\frac{\pi T}{2} \left[f - \frac{1+\delta}{2T}\right]\right) \right\}$$

$$= T + \frac{T}{2} \left\{ \cos\left(\frac{\pi T}{2\delta} \left(2f - \frac{4}{2T}\right)\right) \cdot \cos\left(\frac{\pi T}{2\delta} \left(-\frac{2+2\delta}{2T}\right)\right) \right\}$$

$$= T + \frac{T}{2} \left\{ \cos\left(\frac{\pi T f}{2} - \frac{\pi}{\delta}\right) \cos\left(-\frac{\pi}{2\delta} + \frac{\pi}{2}\right) \right\}$$

$$= T$$

\therefore Therefore, $H(f) + H(f - 1/T) + H(f + 1/T) = T$, for $-\frac{1}{2T} < f < \frac{1}{2T}$

P4:

$$S_0 = A p_{T/2}(t) - A p_{T/2}(t - T/2), \quad S_1(t) = 0$$

$$\text{We } \bar{P}_e = \tau_{00} Q\left(\frac{\hat{S}_0(T) - \gamma}{\sigma_N}\right) + \tau_{01} Q\left(\frac{\gamma - \hat{S}_1(T)}{\sigma_N}\right) \quad \text{where } \tau_{00} = \tau_{01} = \frac{1}{2}$$

$$\hat{S}_0(T) = \int_{-\infty}^{\infty} h(T-\tau) S_0(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) S_0(T-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} S_0^2(T-\tau) d\tau$$

$$\begin{aligned} \hat{S}_1(T) &= \int_{-\infty}^{\infty} h(T-\tau) S_1(\tau) d\tau \\ &= 0 \end{aligned}$$

$$\sigma_N^2 = \frac{N_0}{2} \int k^2(t) dt = \frac{N_0}{2} \int |H(f)|^2 df$$

When $\tau_{00} = \tau_{01} = \frac{1}{2}$, optimise with respect to $h(t)$

$$\bar{P}_e = Q(\lambda) \quad \text{where } \lambda = \sqrt{\frac{E(1-r)}{N_0}}$$

$$E_1 = 0 \quad \text{since } S_1'(t) = 0$$

$$E_0 = \int_{-\infty}^{\infty} S_0^2(t) \cdot dt$$

$$= \int_{-\infty}^{\infty} \left(A p_{T/2}(t) - A p_{T/2}(t - \frac{T}{2}) \right)^2 dt$$

$$= A^2 \int_{-\infty}^{\infty} \left(p_{T/2}^2(t) - 2 p_{T/2}(t) p_{T/2}(t - \frac{T}{2}) + p_{T/2}^2(t - \frac{T}{2}) \right) dt$$

$$= A^2 \left[\int_0^{\frac{T}{2}} p_{T/2}^2(t) dt - 0 + \int_{\frac{T}{2}}^T p_{T/2}^2(t - \frac{T}{2}) dt \right]$$

$$= A^2 \left(\frac{T}{2} + \frac{T}{2} \right)$$

$$= A^2 T$$

$$\therefore \bar{E} = \frac{E_0 + E_1}{2}$$

$$= \frac{A^2 T}{2}$$

$$\therefore \gamma = \frac{(S_0, S_1)}{\bar{E}}$$

$$= 0 \quad \text{since } (S_0, S_1) = \int_{-\infty}^{\infty} S_0(t) S_1(t) dt = 0$$

$$\therefore Q = (2) \quad \text{where } 2 = \sqrt{\frac{A^2 T}{2N_0}}$$

(4b)

Optimum filter:

$$h_{opt} = S_0(T-t) - S_1(T-t)$$

$$= A p_{T/2}(T-t) - A p_{T/2}(T/2-t)$$

Optimum Threshold.

$$\gamma_{opt} = \frac{1}{2} (E_0 - E_1)$$

$$= \frac{A^2}{2} T$$

$$4c) \quad S_0(t) = CA P_{\frac{T}{2}}(t) - CA P_{\frac{T}{2}}(t - \frac{T}{2})$$

$$S_1(t) = 0$$

$$\bar{P}_e(r, h(t), S_0(t), S_1(t)) = \pi_0 Q\left(\frac{S_0(t) - \gamma}{\sigma_N}\right) + \pi_1 Q\left(\frac{r - S_1(t)}{\sigma_N}\right)$$

$$\hat{S}_0(t) = \int_{-\infty}^{\infty} h(\tau) S_0(T - \tau) \cdot d\tau$$

$$= \int_{-\infty}^{\infty} \left[A P_{\frac{T}{2}}(T - \tau) - A P_{\frac{T}{2}}(\frac{T}{2} - \tau) \right] \left[C A P_{\frac{T}{2}}(T - \tau) - C A P_{\frac{T}{2}}(\frac{T}{2} - \tau) \right] d\tau$$

$$= CA^2 \int_{-\infty}^{\infty} \left[P_{\frac{T}{2}}^2(T - \tau) + P_{\frac{T}{2}}^2(\frac{T}{2} - \tau) - 2 P_{\frac{T}{2}}(\frac{T}{2} - \tau) P_{\frac{T}{2}}(T - \tau) \right] d\tau$$

$$= CA^2 \left(\frac{T}{2} + \frac{T}{2} + 0 \right)$$

$$= CA^2 T$$

$$\hat{S}_1(t) = 0, \quad \gamma_{opt} = \frac{A^2}{2} T \quad (\text{from part (b)})$$

$$\sigma_N^2 = \frac{N_0}{2} \int_{-\infty}^{\infty} h^2(t) dt$$

$$= \frac{N_0}{2} \int_{-\infty}^{\infty} \left[A P_{\frac{T}{2}}(T - t) - A P_{\frac{T}{2}}(\frac{T}{2} - t) \right]^2 dt$$

$$= \frac{N_0}{2} A^2 \int_{-\infty}^{\infty} \left[P_{\frac{T}{2}}^2(T - t) - 2 P_{\frac{T}{2}}(T - t) P_{\frac{T}{2}}(\frac{T}{2} - t) + P_{\frac{T}{2}}^2(\frac{T}{2} - t) \right] dt$$

$$= \frac{N_0}{2} A^2 \left(\frac{T}{2} + \frac{T}{2} \right)$$

$$= \frac{A^2}{2} N_0 T$$

$$\therefore \bar{P}_e = \frac{1}{2} Q\left(\frac{CA^2 T - \frac{A^2 T}{2}}{\sigma_N}\right) + \frac{1}{2} Q\left(\frac{\frac{A^2 T}{2}}{\sigma_N}\right)$$

$$\text{where } \sigma^2 = \frac{A^2}{2} N_0 T$$

4(b) Assume $\pi_0 = \pi_1$

$$\gamma_{opt} = \frac{\sigma_n^2 \ln \frac{\pi_0}{\pi_1}}{\hat{S}_0(T) - \hat{S}_1(T)} + \frac{\hat{S}_0(T) + \hat{S}_1(T)}{2}$$

$$= \frac{\hat{S}_0(T) + \hat{S}_1(T)}{2}$$

$$= 0 \quad \text{the optimum threshold}$$

$$\lambda = \frac{C(h, S_T)}{\|h\| \|S_T\|}$$

From Schwartz's inequality:

$$- \|h\| \|S_T\| = C(h, S_T) \leq \|h\| \|S_T\|$$

Thus $-1 \leq \lambda \leq 1$ with equality if $h = S_T$. choose $\lambda = 1$

$$h = S_T = S_0(T-t) - S_1(T-t)$$

$$\therefore h = A_{PT/2}(T-t) - A_{PT/2}(T/2-t) \quad \text{the optimum filter}$$

4(c) $P_e = \pi_0 Q\left(\frac{\hat{S}_0(T) - \gamma}{\sigma_n}\right) + \pi_1 Q\left(\frac{\gamma - \hat{S}_1(T)}{\sigma_n}\right)$ where $\pi_0 = \pi_1 = \frac{1}{2}$

$$\hat{S}_0(T) = \int_{-\infty}^{\infty} h(T-\tau) S_0(\tau) d\tau$$

$$= \int_0^{\frac{T}{2}} h(T-\tau) C A_{PT/2}(\tau) d\tau - \int_{\frac{T}{2}}^T h(T-\tau) C A_{PT/2}(\tau - T/2) d\tau$$

$$= \int_0^{\frac{T}{2}} h(T-\tau) C A d\tau - \int_{\frac{T}{2}}^T h(T-\tau) C A d\tau$$

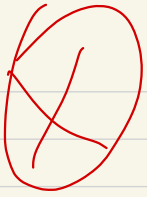
$$= 0$$

$$\hat{S}_1(T) = \int_{-\infty}^{\infty} h(T-\tau) S_1(\tau) d\tau$$

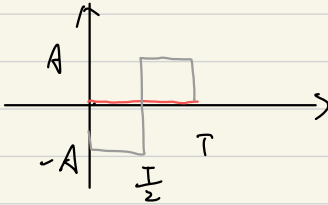
$$= 0$$

$$\therefore \bar{P}_e = \frac{1}{2} Q\left(\frac{-\gamma}{\sigma_n}\right) + \frac{1}{2} Q\left(\frac{\gamma}{\sigma_n}\right)$$

$$\bar{P}_e = Q(0)$$



$$S_T = S_0(T-\tau) - S_1(T-\tau):$$



$$\begin{aligned} C(h, S_T) &= \int_{-\infty}^{\infty} h(\tau) (S_0(T-\tau) - S_1(T-\tau)) d\tau \\ &= \int_{\frac{T}{2}}^T h(\tau) (-A) d\tau + \int_{\frac{T}{2}}^T h(\tau) A d\tau \\ &= 0 \end{aligned}$$

$$\bar{P}_e = \pi_0 Q\left(\lambda_2 - \frac{\beta}{\lambda}\right) + \pi_1 Q\left(\lambda_2 + \frac{\beta}{\lambda}\right)$$

Assume that $\pi_1 = \pi_0$, then $\beta = \frac{\sqrt{N_0}}{\sqrt{E(C+H)}} \ln \frac{\pi_1}{\pi_0} = 0$

$$\bar{P}_e = Q(\lambda_2) \text{ where } \lambda = \frac{C(h, S_T)}{\|h\| \|S_T\|}, \quad \bar{E} = \frac{E_0 + E_1}{2}, \quad r = \frac{(S_0, S_1)}{\bar{E}}$$

The noise at the input to the receiver is assumed to be white Gaussian noise with power spectral density $N_0/2$. The output of the filter due to noise alone is $y = \int h(T-\tau) n(\tau) d\tau$

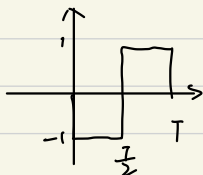
P5

$$S_0(t) = A_0 P_{T/2}(t)$$

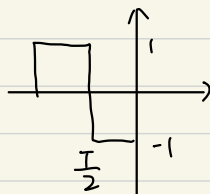
$$S_1(t) = A_1 P_{T/2}(t - T/2)$$

a) $h_p(t) = -P_{T/2}(t) + P_{T/2}(t - T/2)$

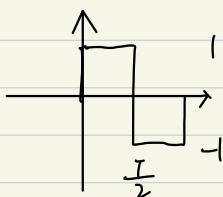
$$h(t) =$$



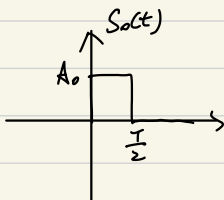
$$\Rightarrow h(-t)$$



$$h(T-t) =$$



$$S_0(t) =$$



$$S_0(t) = \int_{-\infty}^{\infty} h(T-\tau) S_0(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) S_0(T-\tau) d\tau$$

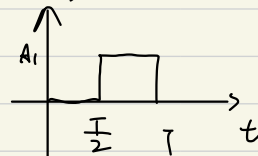
$$= \int_0^{T/2} A_0 d\tau + \int_{T/2}^T 0(-1) d\tau$$

$$= \frac{A_0 T}{2}$$

$$\therefore \hat{S}_0(T) = \frac{A_0}{2} T \quad \text{and} \quad \hat{S}_1(T) = -\frac{A_1}{2} T$$

$$S_1(t)$$

$$S_1(t) =$$



$$\hat{S}_1(t) = \int_{-\infty}^{\infty} h(T-\tau) S_1(\tau) d\tau$$

$$= \int_0^{T/2} (-1) \cdot 0 d\tau + \int_{T/2}^T (-1) \cdot A_1 d\tau$$

$$= -A_1 T - \left(-\frac{A_1 T}{2}\right)$$

$$= -\frac{A_1 T}{2}$$

5(b) Assume $\pi_0 = \pi_1$

$$\begin{aligned}\gamma_{opt} &= \frac{\sigma_n^2 \ln \frac{\pi_0}{\pi_1}}{\hat{S}_0(T) - \hat{S}_1(T)} + \frac{\hat{S}_0(T) + \hat{S}_1(T)}{2} \\ &= \frac{\hat{S}_0(T) + \hat{S}_1(T)}{2} \\ &= \frac{\frac{A_0 T}{2} - \frac{A_1 T}{2}}{2} \\ &= \frac{(A_0 - A_1)}{4} T\end{aligned}$$

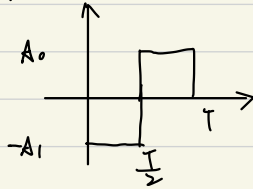
5(c) Assume that $\pi_0 = \pi_1$

$$\bar{P}_e = \pi_0 P_{e,0} + \pi_1 P_{e,1}$$

$$= Q(\alpha \lambda) \quad \text{where} \quad \lambda = \frac{(h, S_T)}{\|h\| \|S_T\|}, \quad \alpha = \sqrt{\frac{\bar{E}(1-r)}{N_0}}, \quad \bar{E} = \frac{E_0 + E_1}{2}, \quad r = \frac{(S_0, S_1)}{E}$$

$$S_T = S_0(T-\tau) - S_1(T-\tau)$$

$$S_0(T-\tau) - S_1(T-\tau):$$



$$\begin{aligned}(h, S_T) &= \int_{-\infty}^{\infty} h(\tau) [S_0(T-\tau) - S_1(T-\tau)] d\tau \\ &= \int_0^{\frac{T}{2}} (-1)(-A_1) d\tau + \int_{\frac{T}{2}}^T A_0 d\tau \\ &= A_1 \frac{T}{2} + \frac{1}{2} A_0 T \\ &= \left(\frac{A_0 + A_1}{2} \right) T\end{aligned}$$

(continued)

5(c)

$$\|h\| = \sqrt{\int_{-\infty}^{\infty} h^2(t) dt}$$

$$= \sqrt{\int_0^T 1 dt}$$

$$= \sqrt{T}$$

$$\|S_T\| = \sqrt{\int_{-\infty}^{\infty} S_T^2(t) dt}$$

$$= \sqrt{\int_{-\frac{T}{2}}^{\frac{T}{2}} A_1^2 dt + \int_{\frac{T}{2}}^T A_0^2 dt}$$

$$= \sqrt{\frac{(A_0^2 + A_1^2) T}{2}}$$

Therefore, we get $\lambda = \frac{(A_0 + A_1) T}{\sqrt{T} \cdot \sqrt{\frac{(A_0^2 + A_1^2) T}{2}}}$

$$\alpha = \sqrt{\frac{E(t-r)}{N_0}}$$

$$E_0 = \int_{-\infty}^{\infty} S_0(t)^2 dt = \int_0^{\frac{T}{2}} A_0^2 dt = \frac{A_0^2}{2} T$$

$$E_1 = \int_{-\infty}^{\infty} S_1(t)^2 dt = \int_{\frac{T}{2}}^T A_1^2 dt = \frac{A_1^2}{2} T$$

$$\bar{E} = \frac{E_0 + E_1}{2}$$

$$= \left(\frac{A_0^2 + A_1^2}{4} \right) T$$

$$(S_0, S_1) = \int_{-\infty}^{\infty} S_0(t) S_1(t) dt$$

$$= 0$$

$$\therefore \alpha = \sqrt{\frac{(A_0^2 + A_1^2) T}{4 N_0}}$$

Overall, $\bar{P}_e = Q(\alpha \lambda)$, $\alpha = \sqrt{\frac{(A_0^2 + A_1^2) T}{4 N_0}}$, $\lambda = \frac{(A_0 + A_1) T}{\sqrt{T} \cdot \sqrt{\frac{(A_0^2 + A_1^2) T}{2}}}$

5(d)

$h_{opt} = S_0(T-t) - S_1(T-t)$ is the matched filter

$$= A_0 P_{T/2}(T-t) - A_1 P_{T/2}(t)$$

$$Y_{opt} = C(h_{opt}, s_0, s_1)$$

$$= \frac{1}{2}(E_0 - E_1) + \underbrace{\frac{1}{2} N_0 \ln \frac{\pi_1}{\pi_0}}_0$$

$$= \frac{1}{2}(E_0 - E_1)$$

$$= \left(\frac{A_0^2 - A_1^2}{4} \right) T$$

5(E)

Assume that $\pi_0 = \pi_1$,

$$\bar{P}_e = Q(\alpha) \quad \text{for the matched filter} \quad \text{where } \alpha = \sqrt{\frac{(A_0^2 + A_1^2)}{4N_0}} T \quad \text{(using } \alpha \text{ in part (c))}$$