University of Michigan FALL 2021

EECS501: Homework 11

Assigned: November 29, 2021
Due: December 7, 2021 at 11:59 on gradescope
Text: “Probability and random processes” by J. A. Gubner

Reading assignment:

1. Poisson Process: Direct Calculation [5 points each]

Consider a Poisson Process with intensity λ. Let Y_K denote the time to see the Kth arrival. Let us use the following direct approach to calculate the joint PDF of $Y_1, Y_2, \ldots, Y_K$.

(a) Consider any two positive numbers $y_1 < y_2$. Observe that $(X(y_2)-X(y_1))$ is the number of arrivals in the interval $(y_1, y_2]$, and hence $X(y_2)-X(y_1)$ is independent of $X(y_1)$. Using this show

$$P(X(y_1) < 1, X(y_2) < 2) = e^{-\lambda y_2}(1 + \lambda(y_2 - y_1))$$

(b) Use part (a) to find $P(Y_1 > y_1, Y_2 > y_2)$, and then find the joint PDF of Y_1 and Y_2 using the inclusion-exclusion principle.

(c) To extend this idea to the general case, first prove the following result (again using the inclusion-exclusion principle) for any K-dimensional random vector $\mathbf{Z}$:

$$f_{\mathbf{Z}}(\mathbf{z}) = (-1)^K \frac{\partial^K}{\partial \mathbf{z}} P(\mathbf{Z} > \mathbf{z}).$$

(d) Using ideas from (a), (b) and (c), find the joint PDF of $Y_1, Y_2, \ldots, Y_K$.
Hint: Use the inclusion-exclusion principle one more time and evaluate only that term in $P(\mathbf{Y} \geq \mathbf{y})$ that contains all variables $y_1, y_2, \ldots, y_K$. The rest of the terms will be canceled by the partial derivatives.

2. Wide-Sense Stationary [15 points]

Consider a random process $\{X_t\}$ such that

$$X_t = A\sin(t + \Theta),$$

where A is a Bernoulli random variable with mean $1/4$, Θ is uniformly distributed over $[0, 2\pi]$, and A and Θ are independent.

- Is $\{X_t\}$ WSS?

3. Wide-Sense Stationary 2 [15 points]

Consider a random process $\{X_t\}$ such that

$$X_t = A\sin(t + \Theta) + B,$$

where A is a Bernoulli random variable with mean $1/4$, B is a Bernoulli random variable with mean $1/2$, Θ is uniformly distributed over $[0, 2\pi]$, and A, B, and Θ are independent from each other.

- Is $\{X_t\}$ WSS?