EECS 455: Solution to Problem Set 5

1. (a) A communication system with data rate of 30Mbps is desired using a bandwidth of 10MHz. What is the minimum received signal-to-noise ratio (E_b/N_0) required to achieve reliable (arbitrarily small error probability) with some modulation technique. Express your answer in dB. Assume an additive white Gaussian noise channel.

   Solution: With $R/W = 3$ the minimum received signal-to-noise ratio is

$$\begin{aligned} E_b/N_0 &> \frac{2^3-1}{3} = 2.333 \\ (E_b/N_0)[dB] &> 10\log_{10}(2.333) = 3.68[dB] \end{aligned}$$

   (b) A communication system using BPSK modulation is allocated a (null-to-null) bandwidth of 20MHz. The null-to-null bandwidth of BPSK is $2/T$ where $R = 1/T$ is the data rate in bits/second and T is the duration of a data bit. The required bit error probability is $Q(\sqrt{20})$. The noise power is $N_0 = 4 \times 10^{-21}$ Watts/Hz. Find the smallest received power P (in Watts) in order to have the required bit error probability.

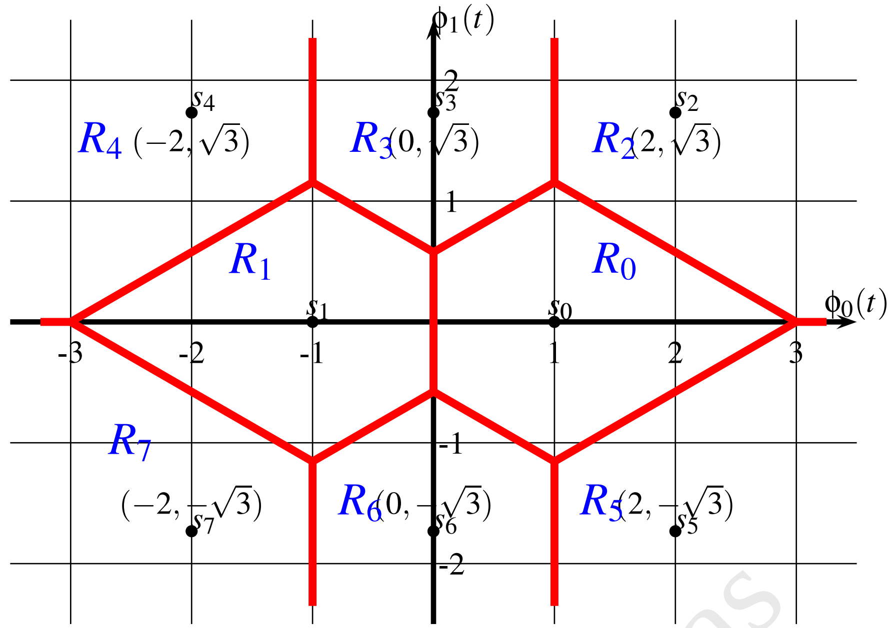
   Solution: With $E_b/N_0 = 10$ then $PT/N_0 = 10$. The received power must satisfy

$$\begin{aligned} 10 &= \frac{PT}{N_0} \\ P &= \frac{10N_0}{T} \\ &= (\frac{2}{T})\frac{10}{2}N_0 \\ &= (20 \times 10^6)(5)(4 \times 10^{-21}) \\ &= 4 \times 10^8 \times 10^{-21} \\ &= 4 \times 10^{-13}\text{Watts}. \end{aligned}$$

2. The eight constellation points for an equal probable signal set received are

$$\begin{array}{cccc} s_0 = (1,0), & s_1 = (-1,0), & s_2 = (2,\sqrt{3}), & s_3 = (0,\sqrt{3}) \\ s_4 = (-2,\sqrt{3}), & s_5 = (2,-\sqrt{3}), & s_6 = (0,-\sqrt{3}), & s_7 = (-2,-\sqrt{3}) \end{array}$$

   shown below.



(a) Determine the average energy of this signal set.

Solution: The average energy is determined as follows.

$$\begin{array}{cccc} E_0 = 1 & E_1 = 1 & E_2 = 7 & E_3 = 3 \\ E_4 = 7 & E_5 = 7 & E_2 = 3 & E_3 = 7 \end{array}$$

$$\begin{aligned} E &= \frac{1}{8}\sum_{i=0}^{7} E_i \\ &= \frac{1}{8}36 \\ &= 4.5 \end{aligned}$$

(b) Determine the average energy per bit.

Solution:

$$E_b = 4.5/3 = 1.5$$

(c) Determine the distance between signal s_0 and every other signal.

Solution:

$$
\begin{aligned}
d_E^2(s_0,s_1) &= 4+0=4\\
d_E^2(s_0,s_2) &= 1+3=4\\
d_E^2(s_0,s_3) &= 1+3=4\\
d_E^2(s_0,s_4) &= 9+3=12\\
d_E^2(s_0,s_5) &= 1+3=4\\
d_E^2(s_0,s_6) &= 1+3=4\\
d_E^2(s_0,s_7) &= 9+3=12
\end{aligned}
$$

(d) Determine the rate of communication in bits/dimension.

Solution:

$$R = 3/2\text{bits/dimension}$$

(e) Determine the conditional probability that given s_0 is transmitted that the received signal is closer to signal s_2.

Solution: The probability that the received signal is closer to signal s_2 than s_0 given that s_0 is transmitted is sometimes called the "pairwise" error probability. It is the probability that the noise in the direction of s_2 from s_0 is larger than half the distance between s_0 and s_2. The distance between s_0 and s_2 is 2. The variance of the noise in the horizontal and vertical directions is $\sigma^2 = N_0/2$ assuming white Gaussian noise with power spectral density $N_0/2$ and orthonormal filtering to generate the horizontal and vertical representations of the signals. In addition, the noise in any direction has the same variance ($N_0/2$). So the pairwise error probability is

$$
\begin{aligned}
P(s_0 \to s_2) &= P\{\text{received signal closer to } s_2 \text{ than } s_0 | s_0 \text{trans}\}\\
&= Q\left(\frac{d(s_0,s_2)}{2\sigma}\right)\\
&= Q\left(\sqrt{\frac{2}{N_0}}\right).
\end{aligned}
$$

3. TRUE or FALSE and short answer.

(a) The capacity of a channel is the smallest rate at which reliable communication is possible. [FALSE: The capacity is the largest rate which reliable communication is possible].

(b) For a linear time-invariant system the output never depends on time. [FALSE: The output will depend on time in general].

(c) If we mix two low pass signals onto a carrier frequency f_c by multiplying by $\cos(2\pi f_c t)$ and $-\sin(2\pi f_c t)$ we can recover the two signals exactly by mixing down to baseband by doing a similar multiplication followed by a low pass filter. [TRUE].

(d) If the input to a linear time-invariant filter is a zero mean Gaussian noise process each sample of the output is a Gaussian random variable. [TRUE]

(e) Sinusoidal pulse shapes are always more bandwidth efficient than rectangular pulse shapes. [FALSE: This depends on the definition of bandwidth].

(f) A large rate in bits/dimension corresponds to a small rate in terms of bits/second/Hz. [FALSE: The rate in bits/second per Hertz is 2W times the rate in bits per dimension]

(g) Increasing the number of signals with a fixed number of dimensions and fixed total energy makes the error probability smaller. [FALSE: More signals with the same energy makes the signals closer and thus would increase the error probability]

(h) Claude Shannon's middle name is <u>Elwood</u>.