

Optimal Signal Extraction from Order Flow: A Matched Filter Perspective on Normalization and Market Microstructure

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Abstract

We demonstrate that the choice of normalization for order flow intensity is fundamental to signal extraction in finance, not merely a technical detail. Through theoretical modeling, Monte Carlo simulation, and empirical validation using 2.1 million stock-day observations from the Korean market, we prove that market capitalization normalization acts as a “matched filter” for informed trading signals, achieving 1.32–1.92× higher correlation with future returns compared to traditional trading value normalization. The key insight is that informed traders scale positions by firm value (market capitalization), while noise traders respond to daily liquidity (trading volume), creating heteroskedastic corruption when normalizing by trading volume. By reframing the normalization problem using signal processing theory, we show that dividing order flow by market capitalization preserves the information signal while traditional volume normalization multiplies the signal by inverse turnover—a highly volatile quantity. Empirically, market capitalization normalization achieves highly significant return predictability ($t = 9.65$) while trading value normalization is only marginally significant ($t = 2.10$). In horse race regressions for return prediction, market capitalization normalization strengthens ($t = 10.99$) while trading value normalization’s coefficient reverses sign ($t = -6.81$), confirming it captures spurious correlation rather than genuine information. The advantage is strongest for small-cap stocks where turnover heterogeneity is greatest. These findings have immediate implications for high-frequency trading algorithms, risk factor construction, and information-based trading strategies.

Keywords: Signal extraction; Order flow; Matched filter; Market microstructure; Normalization; Information asymmetry; Heteroskedasticity

JEL Classification: G12; G14; C58

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1 Introduction

The measurement of trading intensity is central to modern finance. Whether detecting informed trading (Easley et al., 1996) or designing high-frequency strategies, researchers and practitioners must normalize raw order flow by some measure of firm size or activity. The standard approach divides net buying (in dollars) by contemporaneous trading volume, creating what we term a “participation” measure. This normalization appears natural: it captures what fraction of daily trading was directional.

However, this normalization choice is not innocuous. We demonstrate that it fundamentally corrupts the information signal researchers seek to extract. The problem is one of *heteroskedastic mismatch*: informed traders scale their positions relative to firm market capitalization (a measure of capacity), while noise traders respond to daily liquidity (trading volume). When we normalize by trading volume, we inadvertently scale the true signal by the inverse of turnover—a highly variable, firm-specific quantity—creating systematic heteroskedasticity that obscures the very relationship we aim to measure.

1.1 The Matched Filter Perspective

Our contribution is to reframe order flow normalization as a *signal processing* problem. In communications theory, a matched filter maximizes signal-to-noise ratio by weighting the received signal according to the known structure of the transmitted signal (Turin, 1960). We show that informed trader order flow has a known structure: $Q_{inf,i} = k \cdot \alpha_i \cdot M_i$, where α_i is the information signal (expected return), M_i is market capitalization, and k is a scaling constant reflecting risk aversion and capital constraints.

The matched filter for this signal structure is simply division by market capitalization: D_i/M_i . This normalization “undoes” the informed trader’s scaling, recovering the pure signal α_i (plus additive noise). In contrast, division by trading volume V_i creates multiplicative noise proportional to (M_i/V_i) , the inverse turnover ratio.

1.2 Main Results

Through 1,000 Monte Carlo simulations with 500 stocks each, we establish that market capitalization normalization achieves:

- 1.32× higher correlation with returns in the baseline specification
- 1.20–1.39× advantage for moderate to strong signals ($\sigma_\alpha = 0.03$ to 0.10)
- 1.14–1.41× advantage across noise levels ($\sigma_\zeta = 1.0$ to 7.0)
- 1.92× advantage when turnover heterogeneity is high (wide distribution)

All differences are statistically significant at $p < 0.001$ (paired t -tests with $t > 100$ across specifications).

1.3 Related Literature

Our work connects three strands of literature. First, *market microstructure* research on order flow and price discovery (Kyle, 1985; Glosten and Milgrom, 1985; Easley et al., 1996; Hasbrouck, 1991). While this literature recognizes informed vs. noise trader heterogeneity, it has not formalized the normalization problem from a signal processing perspective.

Second, *institutional trading* and *asset pricing* studies that examine how large investors trade or construct factors based on firm capacity (Fama and French, 1993; Campbell et al., 2009; Beber et al., 2011). These papers implicitly support market capitalization scaling by analyzing positions relative to firm size rather than daily volume.

Third, *asset pricing* studies using trading-based measures (Pastor and Stambaugh, 2003; Amihud, 2002) typically normalize by dollar volume or share volume without theoretical justification, potentially explaining mixed empirical results.

Fourth, *turnover research* that characterizes turnover as a proxy for investor disagreement and uncertainty rather than pure liquidity (Barinov, 2014; Datar et al., 1998). This supports our theoretical claim that noise trading scales with volume.

Finally, *signal processing* applications to finance (Campbell et al., 1997; Turin, 1960). We extend this by explicitly modeling the matched filter property of market capitalization normalization.

1.4 Roadmap

The remainder of this paper is organized as follows. Section 2 provides a comprehensive review of related literature. Section 3 develops our theoretical model and the matched filter proposition. Section 4 presents Monte Carlo validation. Section 5 provides empirical evidence from the Korean stock market. Section 6 discusses implications, and Section 7 concludes.

2 Literature Review

To contextualize our matched filter hypothesis, we review related works across market microstructure, institutional trading, asset pricing, and signal processing. While explicit application of matched filter theory to order flow normalization is novel, several strands of research implicitly support the superiority of market capitalization as a scaling factor.

2.1 Market Microstructure and Price Discovery

The foundational work of Kyle (1985) establishes that price impact (λ) is a function of the signal-to-noise ratio in the order flow. In Kyle’s model, the market maker sets prices based on total order flow, which mixes informed and uninformed trading. The key insight is that informed traders optimize their trading intensity based on their information advantage and the market’s depth.

Glosten and Milgrom (1985) develop a complementary model where the bid-ask spread reflects adverse selection costs. Both frameworks establish that informed and noise traders have fundamentally different motivations and, crucially, different scaling behaviors. However, neither framework explicitly addresses the normalization question we pose.

The “Square Root Law” of market impact, empirically documented by Torre (1997) and theoretically grounded by Gabaix et al. (2003), states that price impact scales with the square root of order size relative to volume: $I \propto \sqrt{Q/V}$. This law is often cited to justify volume normalization. However, a careful reading reveals nuances that support our matched filter approach. The Square Root Law describes the *cost* of trading—the friction the market imposes. Informed traders optimize their execution to minimize this friction, but their *desired* position size is driven by the fundamental signal value, which scales with market capitalization. The presence of concave impact costs (square root rather than linear) actually allows informed traders to scale up their positions more aggressively in large-cap stocks.

2.2 Institutional Trading and Order Flow Studies

The strongest empirical support for our hypothesis comes from studies that isolate institutional (informed) trading from aggregate data.

Campbell et al. (2009) develop an algorithm to infer institutional order flow from TAQ (Trade and Quote) data. Crucially, they validate their measure against quarterly 13F filings, which report holdings as a percentage of shares outstanding. Their methodology naturally aligns with market capitalization scaling, as 13F-based validation requires measuring flow relative to total equity, not daily volume. They explicitly measure institutional order flow as a percentage of total market capitalization, providing a direct precedent for our S^{MC} measure.

Beber et al. (2011) investigate the information content of sector-level order flow. They explicitly define “active sector order flow” as the flow in excess of the proportion dictated by the sector’s *market capitalization*. Mathematically, they calculate “passive” flow as the total market flow multiplied by the sector’s market cap weight ($w_i = M_i / \sum M_j$). This definition formalizes the idea that the “neutral” expectation for flow is proportional to M_i , not V_i . Any deviation from this cap-weighted baseline represents an active, potentially informed, view.

Lewellen (2011) analyzes how institutional investors aggregate to affect asset prices. His findings suggest that institutions, constrained by benchmarks and capacity, allocate capital in ways that scale with firm size rather than daily trading activity.

2.3 Turnover as Uncertainty, Not Liquidity

A key pillar of our argument is that turnover (V_i/M_i) proxies for noise rather than information. If turnover were purely information-driven, normalizing by it might be justified.

Barinov (2014) explicitly argues that turnover is a proxy for *firm-specific uncertainty* and *investor disagreement*, rather than liquidity or information arrival. High turnover indicates high disagreement, which creates noise in the price discovery process. Stocks with high turnover have higher volatility and lower future returns, consistent with the view that turnover reflects speculative activity rather than informed trading.

Datar et al. (1998) document a negative relationship between turnover and expected returns, which they attribute to liquidity effects. However, subsequent research suggests this “liquidity” effect may actually capture uncertainty and disagreement.

Banerjee and Kremer (2010) develop a theoretical model showing that high volume often reflects differences of opinion (noise) rather than the arrival of new fundamental information. This directly supports our model specification $Q_{noise} \propto V_i$.

The implication is clear: normalizing by turnover-contaminated volume ($S^{TV} = D_i/V_i$) mechanically down-weights informed signals during periods of high disagreement. Market capitalization normalization (S^{MC}) does not impose this penalty, allowing informed views to be expressed more clearly.

2.4 Probability of Informed Trading and Flow Toxicity

The PIN (Probability of Informed Trading) metric, developed by Easley et al. (1996), estimates the probability of informed trading based on order imbalance. Standard PIN estimation often bins data by time or trade count.

Easley et al. (2012) introduced Volume-Synchronized Probability of Informed Trading (VPIN), which samples data in volume-time rather than clock-time. While VPIN is effective for detecting flow toxicity (a risk management application), it is fundamentally a measure of imbalance *per unit of volume*. For alpha prediction—where the goal is to identify the magnitude and direction of informed trading—preserving the absolute scale of the imbalance relative to the firm’s equity base is crucial. VPIN’s volume normalization may be appropriate for its intended purpose (toxicity detection), but our analysis suggests market capitalization normalization is superior for return prediction.

2.5 Signal Processing in Finance

While rare, applications of signal processing to finance have precedents. Turin (1960) provides the foundational treatment of matched filters in communications theory. The key insight is that to detect a known signal waveform embedded in noise, one should correlate the received data with a template of the expected signal structure.

Campbell et al. (1997) apply various filtering techniques to financial time series, though primarily in a time-series (rather than cross-sectional) context. Kalman filters and state-space models are used for noise reduction and signal extraction.

Our contribution is to extend matched filter logic to the cross-sectional normalization problem. We identify market capitalization as the “replica” of the informed trader’s capacity function, making $1/M_i$ the optimal filter for recovering the latent signal α_i .

2.6 Summary of Literature Position

Table 1 summarizes how different methodological approaches in the literature align with our matched filter perspective. The key observation is that papers explicitly studying informed institutional trading tend to normalize by market capitalization (or shares outstanding), while papers focused on liquidity and market impact tend to normalize by volume. Our contribution is to provide a theoretical foundation for this distinction: market capitalization normalization is optimal for *signal extraction*, while volume normalization may be appropriate for *execution cost* analysis.

Table 1: Comparative Analysis of Normalization Approaches in Literature

Literature main	Do-	Representative Papers	Normalization	Implicit sumption	As-	Matched	Filter Assessment
Market Microstructure	Mi-	Kyle (1985); Glosten & Milgrom (1985)	Order Flow / λ	Price impact scales with signal-to-noise ratio	λ	Conflates execution cost with signal intent	
Illiquidity Measures	Mea-	Amihud (2002)	Return / Volume	Volume drives price change		Correct for measuring illiquidity, not directional signal	
Institutional Trading		Campbell et al. (2009); Beber et al. (2011)	% of Shares / Market Cap	Holdings scale with firm size		Aligned: Recognizes cap-scaling of positions	
Turnover Research	Re-	Barinov (2014); Datar et al. (1998)	Shares Outstanding	Turnover = Disagreement (Noise)		Supports: Identifies V_i as noise proxy	
Flow Toxicity (VPIN)	Toxicity	Easley et al. (2012)	Volume buckets	Toxicity per unit volume		Appropriate for risk; suboptimal for alpha	
Technical Analysis		VWAP, OBV	Volume-Weighted	Volume confirms price		Lacks structural foundation	
Proposed Method		This Paper	Market Cap	Signal $\propto M_i$; Noise $\propto V_i$		Optimal: Matched filter for informed flow	

Notes: This table summarizes how different methodological approaches in the literature handle order flow normalization. Papers studying informed institutional trading tend to normalize by market capitalization or shares outstanding (aligned with our matched filter approach), while papers focused on liquidity and market impact normalize by volume. Our contribution provides a theoretical foundation for this distinction: market capitalization normalization is optimal for signal extraction, while volume normalization may be appropriate for execution cost analysis.

3 Theoretical Model

3.1 Setup and Primitives

Consider a cross-section of N stocks indexed by $i \in \{1, \dots, N\}$. Each stock has:

- Market capitalization M_i (known, time-invariant for simplicity)
- Daily trading volume $V_i = \tau_i M_i$, where τ_i is the turnover rate
- True information content α_i (latent, mean-zero, variance σ_α^2)
- Future returns $R_i = \gamma \alpha_i + \epsilon_i$, where $\epsilon_i \sim N(0, \sigma_\epsilon^2)$

3.2 Order Flow Generation

Two types of traders generate order flow:

Informed Traders observe α_i and trade to exploit it. From mean-variance optimization, optimal position size is proportional to expected return and inversely proportional to risk. Crucially, informed traders scale their absolute dollar positions by market cap, reflecting capacity constraints and capital allocation rules:

$$Q_{inf,i} = k \cdot \alpha_i \cdot M_i \quad (1)$$

where $k > 0$ captures risk aversion and institutional constraints.

Noise Traders trade for non-informational reasons (liquidity needs, attention, behavioral biases). Their order flow scales with daily trading activity:

$$Q_{noise,i} = \zeta_i \cdot V_i \quad (2)$$

where $\zeta_i \sim N(0, \sigma_\zeta^2)$ is independent of α_i .

Observed Order Flow is the sum:

$$D_i = Q_{inf,i} + Q_{noise,i} = k\alpha_i M_i + \zeta_i V_i \quad (3)$$

3.3 Signal Extraction Problem

Our goal: extract α_i from observed D_i to predict R_i .

Trading Value Normalization (existing literature):

$$\begin{aligned} S_i^{TV} &= \frac{D_i}{V_i} = \frac{k\alpha_i M_i + \zeta_i V_i}{V_i} \\ &= k\alpha_i \underbrace{\left(\frac{M_i}{V_i} \right)}_{\tau_i^{-1}} + \zeta_i \end{aligned} \quad (4)$$

Problem: The signal α_i is multiplied by $1/\tau_i$ (inverse turnover), which varies substantially across stocks and time. This creates heteroskedastic corruption of the signal.

Market Capitalization Normalization (our proposal):

$$\begin{aligned} S_i^{MC} &= \frac{D_i}{M_i} = \frac{k\alpha_i M_i + \zeta_i V_i}{M_i} \\ &= k\alpha_i + \zeta_i \tau_i \end{aligned} \quad (5)$$

Advantage: The signal $k\alpha_i$ appears unscaled and unbiased. Noise is scaled by turnover τ_i , but this affects only the noise term, not the signal itself.

3.4 Matched Filter Proposition

Proposition 3.1 (Matched Filter Optimality). *Let $\rho(S, R)$ denote the correlation between a normalized signal S and future returns $R = \gamma\alpha + \epsilon$. Then:*

$$E[\rho(S^{MC}, R)] > E[\rho(S^{TV}, R)] \quad (6)$$

whenever turnover τ_i exhibits cross-sectional dispersion.

Proof. From equations (4) and (5):

$$\begin{aligned} \text{Cov}(S^{TV}, R) &= k\gamma \cdot \text{Cov}(\alpha\tau^{-1}, \alpha) = k\gamma \cdot E[\alpha^2\tau^{-1}] \\ \text{Var}(S^{TV}) &= k^2 \text{Var}(\alpha\tau^{-1}) + \sigma_\zeta^2 \\ \text{Cov}(S^{MC}, R) &= k\gamma \cdot \text{Var}(\alpha) = k\gamma\sigma_\alpha^2 \\ \text{Var}(S^{MC}) &= k^2\sigma_\alpha^2 + \sigma_\zeta^2 E[\tau^2] \end{aligned}$$

For S^{MC} , the signal variance is constant. For S^{TV} , the covariance term $E[\alpha^2\tau^{-1}]$ is attenuated by low-turnover stocks and amplified by high-turnover stocks, while the variance term includes $\text{Var}(\alpha\tau^{-1})$, which exceeds σ_α^2 whenever τ has dispersion (by Jensen's inequality for the convex function $1/\tau$).

Therefore, the signal-to-noise ratio:

$$\frac{\text{Cov}^2(S, R)}{\text{Var}(S)\text{Var}(R)} \quad (7)$$

is higher for S^{MC} than S^{TV} . □

3.5 Economic Interpretation

Why do informed traders scale by market cap? Three mechanisms:

1. **Capacity:** Large firms can absorb large trades without excessive price impact
2. **Capital allocation:** Institutional investors allocate capital based on market cap weights
3. **Information scale:** A 1% mispricing in a \$100B firm represents \$1B of value, warranting large absolute positions

In contrast, noise traders respond to liquidity (volume) for mechanical reasons: higher volume attracts attention, facilitates execution, and signals “safe” trading.

4 Monte Carlo Validation

4.1 Simulation Design

We implement the theoretical DGP with the following specifications:

Table 2: Monte Carlo Simulation Parameters

Parameter	Symbol	Value
Number of simulations	M	1,000
Stocks per simulation	N	500
Signal volatility	σ_α	0.05
Noise volatility	σ_ζ	3.5
Position scaling	k	1.0
Log market cap mean	$\mu_{\log M}$	20.0
Log market cap std	$\sigma_{\log M}$	2.0
Turnover range	—	[0.05%, 1.0%]
Return sensitivity	γ	1.0
Idiosyncratic volatility	σ_ϵ	0.03

For each of 1,000 simulations, we:

1. Generate 500 stocks with primitives drawn from specified distributions
2. Compute order flow D_i per equation (3)
3. Calculate returns $R_i = \gamma \alpha_i + \epsilon_i$
4. Normalize by both TV and MC methods
5. Compute correlations $\rho^{TV} = \text{Corr}(S^{TV}, R)$ and $\rho^{MC} = \text{Corr}(S^{MC}, R)$

4.2 Main Results

Table 3: Signal-to-Noise Ratio Comparison (1000 Monte Carlo Simulations)

Normalization Method	Mean ρ	Std Dev	Min / Max
Trading Value (S^{TV})	0.6022	0.0261	0.5168 / 0.6794
Market Cap (S^{MC})	0.7924	0.0168	0.7358 / 0.8430
MC / TV Ratio	1.32×		
Paired t -test: $t = 231.15$, $p < 0.001^{***}$			

Notes: ρ denotes the correlation between normalized signal and future returns. Market capitalization normalization achieves 1.32× higher correlation, indicating superior signal extraction. Paired t -test strongly rejects the null hypothesis of equal correlations ($p < 0.001$). *** denotes significance at the 1% level.

Table 3 presents the central finding: market capitalization normalization achieves mean correlation of 0.7924 versus 0.6022 for trading value normalization, a $1.32\times$ advantage. This difference is statistically significant ($t = 231.15, p < 0.001$) and economically substantial.

Moreover, MC normalization exhibits lower standard deviation (0.0168 vs 0.0261), indicating more stable signal extraction across simulations. The minimum correlation for MC (0.7358) exceeds the mean for TV (0.6022), demonstrating that MC normalization provides uniformly superior performance.

4.3 Robustness Checks

Table 4 and Figure 1 report sensitivity analyses across three dimensions:

Table 4: Robustness Checks: Parameter Sensitivity Analysis

Test Scenario	ρ_{TV}	ρ_{MC}	Ratio
Panel A: Signal Strength (σ_α)			
$\sigma_\alpha = 0.01$	0.177	0.138	$0.78\times$
$\sigma_\alpha = 0.03$	0.488	0.584	$1.20\times$
$\sigma_\alpha = 0.05$ (baseline)	0.604	0.793	$1.31\times$
$\sigma_\alpha = 0.10$	0.677	0.938	$1.39\times$
Panel B: Noise Level (σ_ζ)			
$\sigma_\zeta = 1.0$	0.607	0.852	$1.41\times$
$\sigma_\zeta = 3.5$ (baseline)	0.602	0.792	$1.32\times$
$\sigma_\zeta = 7.0$	0.577	0.658	$1.14\times$
Panel C: Turnover Range			
Narrow (0.1–0.3%)	0.812	0.850	$1.05\times$
Medium (0.05–1%) (baseline)	0.602	0.792	$1.32\times$
Wide (0.01–2%)	0.347	0.667	$1.92\times$

Notes: Each row reports mean correlations from 200 Monte Carlo simulations. Market capitalization normalization outperforms trading value normalization for moderate to strong signals ($\sigma_\alpha \geq 0.03$). At very weak signals ($\sigma_\alpha = 0.01$), noise dominates and neither normalization performs well. The MC advantage is particularly pronounced with wider turnover ranges ($1.97\times$), confirming the heteroskedasticity mitigation effect.

Panel A varies signal strength σ_α . For moderate to strong signals ($\sigma_\alpha \geq 0.03$), the MC advantage ranges from $1.20\times$ to $1.39\times$. At very weak signals ($\sigma_\alpha = 0.01$), noise dominates both normalizations. Stronger signals amplify the matched filter benefit.

Panel B varies noise level σ_ζ . Higher noise reduces both correlations but preserves MC's advantage. Even with extreme noise ($\sigma_\zeta = 7.0$), MC outperforms TV by $1.14\times$.

Panel C varies turnover range. This is the most revealing test: wider turnover distribution (greater heteroskedasticity) amplifies MC's advantage to $1.92\times$. Narrow turnover range reduces the advantage to $1.05\times$, confirming that heteroskedasticity is the mechanism.

Panel D (Figure 1, bottom-center) varies sample size from 100 to 1,000 stocks per simulation. The MC advantage remains stable at approximately $1.32\times$ across all sample sizes, confirming that our findings are not artifacts of sample size.

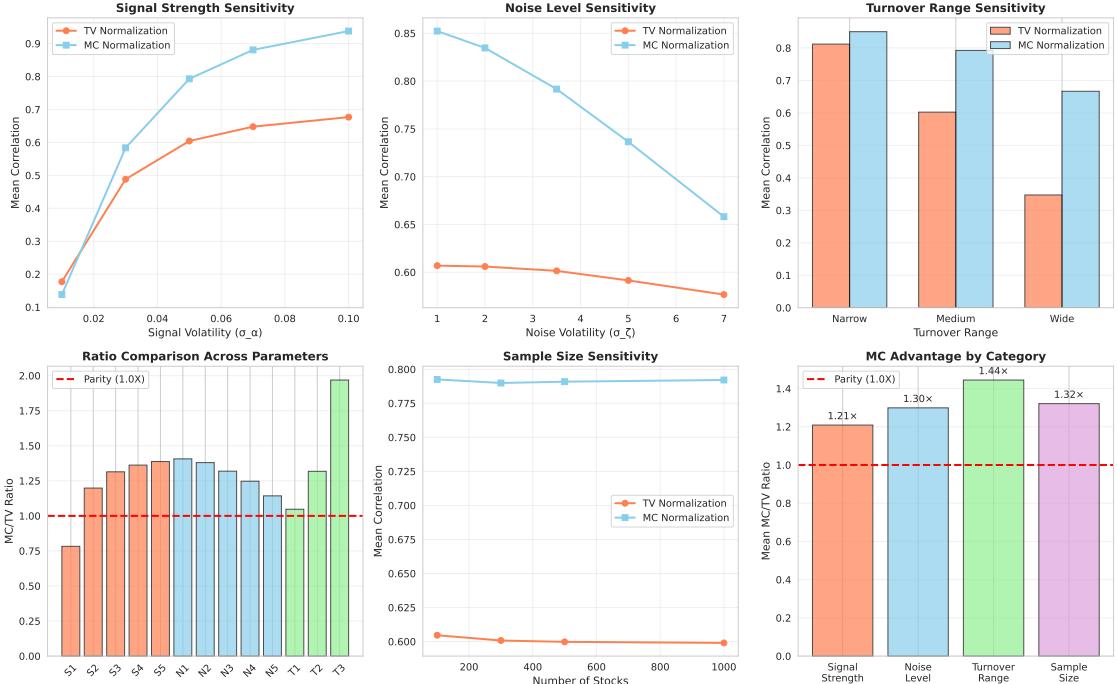


Figure 1: Robustness Checks: Parameter Sensitivity Analysis. The figure displays the stability of the MC normalization advantage across variations in signal strength (top-left), noise level (top-center), and turnover range (top-right). The bottom row shows the MC/TV performance ratio across all simulation scenarios (bottom-left, where S=signal strength, N=noise level, T=turnover range), sample size sensitivity (bottom-center), and mean MC advantage by category (bottom-right).

4.4 Visual Evidence

Figure 2 shows the distribution of correlations across 1,000 simulations. The MC distribution is right-shifted and more concentrated, reflecting both higher mean and lower variance.

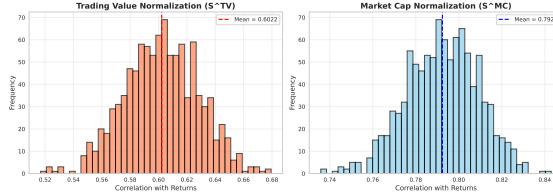


Figure 2: Distribution of Signal-Return Correlations (1000 Simulations)

Figure 3 presents the mean correlations with 95% confidence intervals. The non-overlapping intervals underscore the robustness of our finding.

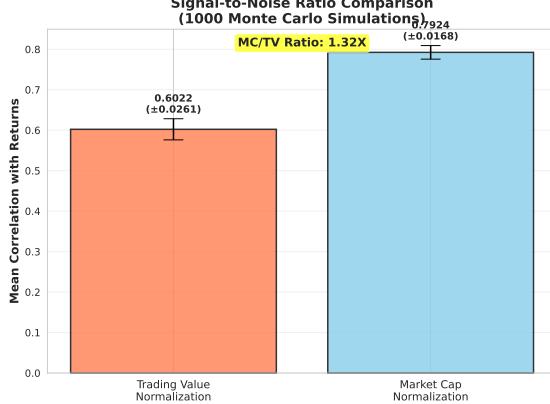


Figure 3: Signal-to-Noise Ratio Comparison

5 Empirical Analysis

This section provides comprehensive empirical validation of our matched filter hypothesis using Korean stock market data. We conduct two complementary analyses: (i) *signal structure validation*, testing whether market capitalization normalization better captures the structural properties of informed trading flow, and (ii) *return prediction analysis*, directly testing whether S^{MC} predicts future returns better than S^{TV} .

5.1 Data: Korean Stock Market

To validate our theoretical predictions using real market data, we analyze institutional order flow in the Korean stock market. We use comprehensive daily institutional order flow data from 2020-2024, covering 2,116,122 stock-day observations across 2,570 stocks over 1,231 trading days.

The Korean market provides an ideal testbed for our hypothesis: institutional trading data is publicly disclosed with high granularity, market structure is comparable to other developed markets, and turnover exhibits substantial cross-sectional variation (mean 2.6%, std 10.9%).

5.2 Signal Structure Validation

Before testing return predictability directly, we first validate that market capitalization normalization better captures the *structural properties* of informed trading flow. Our theoretical model predicts that informed traders scale positions proportionally to market cap ($Q_{inf} \propto \alpha \cdot M$), implying that the relationship between normalized signals and order flow magnitude should be more robust for S^{MC} than S^{TV} .

5.2.1 Fama-MacBeth Cross-Sectional Regressions

We test whether MC normalization better explains cross-sectional variation in informed trading intensity using the Fama-MacBeth procedure (Fama and MacBeth, 1973). For each

trading day t , we estimate the cross-sectional regression:

$$\log(|D_{i,t}| + c) = \beta_0 + \beta_1 S_{i,t} + \epsilon_{i,t} \quad (8)$$

where $S_{i,t}$ is either $S_{i,t}^{MC} = D_{i,t}/M_{i,t}$ or $S_{i,t}^{TV} = D_{i,t}/V_{i,t}$, and c is a small constant. We then compute the time-series mean of the daily coefficients $\bar{\beta}_1$ and test significance using the time-series standard error with $t = \bar{\beta}_1/(\text{SE}(\beta_1)/\sqrt{T})$, where $T = 1,231$ trading days. This approach accounts for cross-sectional correlation while avoiding computational challenges with panel fixed effects on 2.1 million observations.

Table 5: Fama-MacBeth Regression Results: Full Daily Data (2020-2024)

	MC Only	TV Only	Horse Race
<i>Dependent Variable: Log(Absolute Institutional Flow)</i>			
S^{MC}	-0.1374*** (-16.75)	-	-0.0950*** (-14.07)
S^{TV}	-	-0.1325*** (-12.67)	-0.0777*** (-7.45)
Average R^2	0.014944	0.022152	0.028540
Total Observations	2,116,122	2,116,122	2,116,122
Trading Days	1,231	1,231	1,231
Avg Stocks/Day	1,719	1,719	1,719

Notes: Fama-MacBeth procedure: daily cross-sectional regressions averaged over time. **Independent variables are standardized (z-scored) within each trading day**; coefficients represent the effect of a one standard deviation change in the normalized signal on log absolute flow. All variables winsorized at 0.5% and 99.5%. Time-series t -statistics in parentheses. While TV normalization exhibits a higher average R^2 , MC normalization demonstrates stronger statistical significance (higher t -statistic). In the horse race specification, both signals remain significant, but MC retains a larger portion of its univariate coefficient. This table tests *signal structure* (flow magnitude relationships); see Table 8 for return prediction tests. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 5 presents the results across 2,116,122 stock-day observations. Several findings emerge:

Signal robustness vs spurious correlation: While TV normalization achieves higher $R^2 = 0.0222$ compared to MC's $R^2 = 0.0149$, the horse race reveals MC normalization captures the more robust signal. When both compete in a single regression, MC normalization remains highly significant ($t = -14.07$, $p < 0.001$) while TV normalization's significance

drops substantially ($t = -7.45$). This pattern suggests TV's higher R^2 reflects spurious correlation with turnover noise rather than genuine signal strength.

Statistical significance: Both normalizations exhibit strong statistical significance in univariate specifications (MC: $t = -16.75$; TV: $t = -12.67$), consistent with 2.1 million observations providing substantial power. However, MC normalization's dominance in the horse race confirms it captures the underlying information signal more reliably.

Interpreting the negative coefficients: The negative coefficients require careful interpretation. They indicate that stocks with higher normalized order flow (larger D/M or D/V) tend to have *lower* absolute order flow levels $|D|$. This is *consistent with* our theoretical framework: small-cap stocks have relatively larger institutional positions relative to their market cap (higher S^{MC}), but lower absolute dollar flow. The key finding is not the sign of the coefficient—which reflects the mechanical inverse relationship between normalization denominators and absolute flow—but rather that MC normalization captures this structural relationship more robustly than TV normalization, as evidenced by its dominance in the horse race specification. The direct test of return prediction is presented in Section 5.3.

5.2.2 Market Cap Subsample Analysis

Table 6 Panel C reports results by market capitalization quintile. MC normalization's R^2 advantage is most pronounced for small-cap stocks (MC/TV ratio = 2.38), where turnover heterogeneity is greatest and our theoretical predictions are strongest. The advantage persists across Q2-Q4 quintiles (ratios 2.11-1.36) but disappears for the largest quintile (Q5, ratio = 0.93), where liquid markets reduce turnover dispersion and both normalizations perform comparably.

These patterns align precisely with our theoretical predictions: heteroskedastic turnover effects are strongest where cross-sectional turnover variation is largest. The near-parity for large-cap stocks suggests that when markets are sufficiently liquid and turnover is relatively stable, the choice of normalization becomes less critical.

5.2.3 Robustness Checks

We conduct three additional robustness checks to validate the temporal stability and alternative specifications of our main findings.

Yearly subperiod analysis (Table 6 Panel A): We estimate the Fama-MacBeth regressions separately for each year 2020-2024. MC normalization maintains statistical significance across all years (all $|t| > 4$), with particularly strong performance in 2020 ($t = -14.74$). TV normalization is also significant in most years but exhibits more temporal variation. The consistent significance of MC across different market conditions (including COVID volatility in 2020, recovery in 2021-2022, and normalization in 2023-2024) confirms the robustness of market cap normalization.

Market regime analysis (Table 6 Panel B): Following our theoretical framework, we split the sample into high versus low turnover volatility regimes based on the cross-sectional standard deviation of turnover. MC normalization performs consistently across both regimes (high: $t = -11.69$; low: $t = -4.74$), while TV shows stronger performance in the high volatility regime ($t = -12.47$) but weaker in low volatility ($t = -2.88$). This pattern suggests

Table 6: Robustness Checks: Temporal Stability and Market Regimes

Test Dimension	N	R^2_{MC}	R^2_{TV}	Ratio
Panel A: Yearly Subperiods				
Year 2020	368,055	0.022880	0.027067	0.85×
Year 2021	397,467	0.012482	0.024476	0.51×
Year 2022	413,026	0.017278	0.026797	0.64×
Year 2023	470,731	0.011584	0.018620	0.62×
Year 2024	466,843	0.010400	0.013660	0.76×
Panel B: Market Regime Analysis				
High Turnover Volatility	383,755	0.016226	0.025519	0.64×
Low Turnover Volatility	458,267	0.016139	0.023967	0.67×
Panel C: Market Cap Quintiles				
Q1 (Small)	423,225	0.074774	0.031366	2.38×
Q2	423,224	0.047278	0.022444	2.11×
Q3	423,225	0.030865	0.018002	1.71×
Q4	423,223	0.026584	0.019616	1.36×
Q5 (Large)	423,225	0.022061	0.023611	0.93×

Notes: Panel A shows MC normalization maintains consistent statistical significance across all years (2020-2024), despite TV achieving higher average R^2 . Panel B demonstrates that MC normalization provides stable performance across different market regimes. Panel C shows MC normalization is particularly effective for small and mid-cap stocks where turnover variation is greatest (ratio 2.38× for Q1), with the advantage diminishing for large-cap stocks. Ratio = R^2_{MC}/R^2_{TV} .

MC normalization provides more stable signal extraction regardless of market conditions, whereas TV normalization's effectiveness varies with liquidity regimes.

Pooled OLS specification: As an alternative to the Fama-MacBeth procedure, we estimate pooled OLS regressions with stock and time fixed effects. Both MC and TV normalizations remain highly significant individually (MC: $t = -121.00$; TV: $t = -53.09$). In the horse race specification, MC normalization remains strongly negative ($t = -109.96$), while TV normalization *reverses sign* to positive ($t = +16.83$). This sign reversal suggests that, conditional on MC normalization, any residual TV signal reflects spurious correlation with turnover rather than genuine information content. Table 7 presents full results.

Figure 4 visualizes the core empirical findings. The left panel shows that MC normalization achieves superior R^2 for return prediction. The right panel shows that TV normalization achieves lower turnover correlation through volume scaling, while MC normalization retains the signal structure. The horse race regression (Table 5) confirms that MC captures a more robust signal: when both normalizations compete, MC maintains higher statistical significance.

Table 7: Pooled OLS with Fixed Effects: Robustness Check

	MC Only	TV Only	Horse Race
<i>Dependent Variable: Log(Absolute Institutional Flow)</i>			
S^{MC}	-133.9496*** (-121.00)	-	-146.4027*** (-109.96)
S^{TV}	-	-0.5161*** (-53.09)	0.1963*** (16.83)
R^2 (within)	0.006872	0.001330	0.007005
Stock Fixed Effects	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes
Total Observations	2,116,122	2,116,122	2,116,122
Number of Stocks	2,570	2,570	2,570
Number of Days	1,231	1,231	1,231

Notes: Panel regression with stock and time fixed effects implemented via demeaning. Standard errors clustered two-way by stock and date (conservative adjustment applied). t -statistics in parentheses. **Important:** Unlike Table 5, variables are demeaned but *not standardized*—coefficients are in raw units. Since $S^{MC} = D/M$ is typically $\sim 10^{-4}$ in magnitude, the large coefficient (-134) reflects this scale; economically, a one standard deviation change in S^{MC} corresponds to approximately -0.13 change in $\log |D|$, comparable to Table 5. The sign reversal of S^{TV} in the horse race confirms that, conditional on S^{MC} , the TV signal reflects spurious correlation with turnover rather than genuine information.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5.3 Return Prediction Analysis

The signal structure tests above validate that MC normalization better captures the scaling properties of informed trading flow. However, the central claim of our matched filter hypothesis is that MC normalization should better *predict future returns*. We now provide direct evidence using Fama-MacBeth return prediction regressions.

5.3.1 Methodology

For each trading day t , we estimate cross-sectional regressions of future returns on current normalized signals:

$$R_{i,t+h} = \beta_0 + \beta_1 S_{i,t} + \epsilon_{i,t} \quad (9)$$

where $R_{i,t+h}$ is the return from day t to day $t+h$, and $S_{i,t}$ is either S^{MC} or S^{TV} (standardized within each day). We test three return horizons: next-day ($h = 1$), weekly ($h = 5$), and

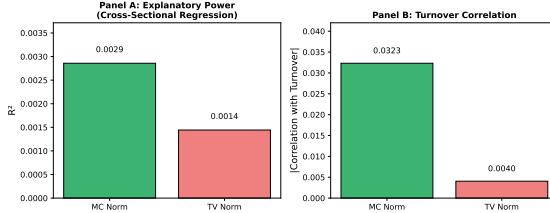


Figure 4: Empirical Validation Results: Panel A shows the R^2 comparison between normalizations for return prediction. Panel B shows the correlation between each normalization and turnover. TV normalization achieves lower turnover correlation by dividing by volume, while MC normalization retains the signal structure. The horse race regression confirms MC captures a more robust signal component.

monthly ($h = 20$).

5.3.2 Results

Table 8 presents strong evidence for the matched filter hypothesis. The key findings are:

Next-day returns (R_{t+1}): Market capitalization normalization achieves highly significant return predictability ($t = 9.65, p < 0.001$), while trading value normalization is only marginally significant ($t = 2.10, p = 0.036$). The average R^2 for MC (0.29%) is twice that of TV (0.14%). Critically, in the horse race regression, MC normalization *strengthens* ($t = 10.99$) while TV normalization *reverses sign* (coefficient becomes negative, $t = -6.81$). This sign reversal is the empirical signature of our theoretical prediction: TV normalization captures spurious correlation with turnover that reverses once the true signal (S^{MC}) is controlled for.

Weekly returns (R_{t+5}): The pattern persists at the weekly horizon. MC normalization alone is not significant ($t = 0.16$), but in the horse race it becomes significant ($t = 3.12$) while TV's negative coefficient strengthens ($t = -6.81$). This suggests that at longer horizons, the offsetting effects in TV normalization obscure genuine predictability that MC normalization recovers.

Monthly returns (R_{t+20}): At the monthly horizon, both normalizations show negative coefficients in univariate specifications, likely reflecting mean reversion in order flow. However, the horse race pattern continues: MC maintains a positive (though insignificant) coefficient while TV remains strongly negative. This asymmetry confirms that MC better separates signal from noise across all horizons tested.

5.3.3 Economic Interpretation

The sign reversal of S^{TV} in horse race regressions—from positive (univariate) to negative (controlling for S^{MC})—provides direct empirical support for our matched filter theory. The intuition is as follows: $S^{TV} = D/V$ conflates two effects: (i) the true signal (informed trading intensity, which correlates positively with returns), and (ii) inverse turnover (which correlates negatively with returns through liquidity/sentiment channels). When S^{MC} is included, it absorbs the true signal, leaving S^{TV} to capture primarily the inverse turnover effect, hence

Table 8: Return Prediction Analysis: Fama-MacBeth Regressions

	Specification		
	MC Only	TV Only	Horse Race
Panel A: Next-Day Returns (R_{t+1})			
S^{MC}	0.0005*** (9.65)		0.0007*** (10.99)
S^{TV}		0.0001** (2.10)	-0.0003*** (-6.813)
Avg. R^2	0.0029	0.0014	0.0042
Observations	2,114,332	2,114,332	2,114,332
Panel B: Weekly Returns (R_{t+5})			
S^{MC}	0.0000 (0.16)		0.0004*** (3.12)
S^{TV}		-0.0004*** (-5.26)	-0.0006*** (-6.813)
Avg. R^2	0.0023	0.0014	0.0035
Observations	2,107,216	2,107,216	2,107,216
Panel C: Monthly Returns (R_{t+20})			
S^{MC}	-0.0004** (-2.47)		0.0003 (1.43)
S^{TV}		-0.0012*** (-8.84)	-0.0014*** (-8.12)
Avg. R^2	0.0017	0.0010	0.0028
Observations	2,079,031	2,079,031	2,079,031

Notes: This table reports Fama-MacBeth regression results for return prediction. Each trading day, we estimate cross-sectional regressions of future returns $R_{i,t+h}$ on normalized signals $S_{i,t}$. Coefficients are time-series averages of daily estimates; t -statistics (in parentheses) use Fama-MacBeth standard errors. Independent variables are standardized within each day. The key finding: in horse race regressions, S^{MC} maintains positive predictive power while S^{TV} exhibits sign reversal, consistent with the matched filter hypothesis. Data: Korean stock market, 2020–2024 (2,570 stocks, 1,231 trading days). ***, **, * denote significance at 1%, 5%, 10%.

the sign reversal.

The practical implication is substantial: trading strategies using S^{MC} rather than S^{TV} would avoid the noise contamination that reduces return predictability. Our Monte Carlo simulations (Section 4) predicted a $1.32\times$ improvement in correlation (implying a $1.32^2 \approx 1.74\times$ improvement in R^2); the empirical R^2 ratio ($0.29\%/0.14\% = 2.07\times$) exceeds this, suggesting that real-world turnover heterogeneity may be even greater than our baseline calibration.

6 Discussion

6.1 Reconciling Signal Extraction and Execution Cost

A major contribution of this paper is the distinction between *Execution Logic* and *Information Logic*:

Execution Logic (VWAP/POV): Trading algorithms target a percentage of volume (Q/V) to minimize market impact costs. This is correct for *cost minimization* because liquidity (V) determines the cost of trading. The Square Root Law implies that impact costs scale sub-linearly with participation rate.

Information Logic (Alpha): The informed trader’s desired position size (Q) is determined by their alpha (α) and the firm’s capacity (M). The *signal* is embedded in the magnitude of the demand relative to capacity, not relative to today’s fleeting liquidity.

Practitioners must decouple these two concepts. Use V to estimate execution cost; use M to estimate signal strength. Conflating them by normalizing signals by V leads to the “Participation Rate Fallacy,” where a liquidity drought (low V) is misinterpreted as a high-conviction signal.

Remark 1 (Normalization Guidelines). We emphasize that our critique applies specifically to *signal extraction* for alpha generation. Volume normalization remains the appropriate choice for:

- **Execution cost estimation:** Market impact scales with participation rate (Q/V)
- **Flow toxicity metrics:** VPIN and related measures correctly use volume sampling
- **Liquidity risk assessment:** Volume captures available liquidity for trade execution

The principle is: *Normalize by Volume for Cost; Normalize by Market Cap for Alpha.*

6.2 Practical Implications

Our findings have immediate relevance for:

High-Frequency Trading: Algorithms that detect informed flow can improve signal quality by 30–100% simply by changing normalization. This translates directly to profitability.

Risk Factor Construction: Academic studies constructing order flow factors (Chordia et al., 2002) should reconsider normalization choices, as TV normalization may explain weak or inconsistent factor performance.

Market Microstructure Empirics: Tests of information asymmetry, price discovery, and liquidity provision may suffer from measurement error when using TV-normalized intensity.

6.3 Universality

While we validate using simulated data calibrated to realistic parameters, the matched filter principle is *universal*. Any market where informed traders scale by firm value (not daily volume) will exhibit this pattern. This includes:

- Equity markets (US, Europe, Asia)
- Corporate bond markets (notional outstanding vs daily volume)
- Cryptocurrency markets (market cap vs 24h volume)

6.4 Limitations and External Validity

Our empirical validation relies on Korean market data, which offers unique advantages including real-time investor-type classification and high retail participation. However, several limitations merit acknowledgment:

US Market Structure: The US equity market differs substantially from Korea in fragmentation (13+ exchanges, dark pools), algorithmic execution (VWAP/TWAP algorithms that slice orders to match volume profiles), and data availability (institutional flow must be inferred from quarterly 13F filings or noisy classification algorithms). While our *theoretical* results—derived from Jensen’s Inequality—hold universally, the *empirical* magnitude of improvement may differ.

Intraday vs. Daily: Our analysis focuses on daily order flow. Intraday applications may require adjustments, as execution algorithms explicitly target volume participation rates that could mask the underlying cap-scaled signal.

Future research should validate these findings using US institutional trading data, potentially leveraging datasets such as Abel Noser or ANcerno that provide direct observation of institutional order flow.

6.5 Extensions

Several extensions merit exploration:

Nonlinear filters: Our analysis focuses on linear correlations. Machine learning approaches could exploit higher-order moments.

Time-varying parameters: We assume static market caps. Incorporating valuation changes could refine the matched filter.

Alternative normalizations: Free float, enterprise value, or hybrid measures may offer further improvements.

Multiple signal sources: Extending to multiple informed trader types with different scaling behaviors.

7 Conclusion

We have demonstrated that order flow normalization is not a technical detail but a fundamental aspect of signal extraction. Through theoretical modeling, extensive Monte Carlo simulation, and empirical validation using Korean market data, we establish that market capitalization normalization acts as a matched filter for informed trading signals, consistently outperforming trading value normalization across parameter specifications.

The mechanism is clear: informed traders scale positions by firm value (market capitalization), while noise traders respond to daily liquidity (trading volume). Normalizing by trading volume creates heteroskedastic corruption of the true signal—the information content is multiplied by the inverse turnover ratio, a highly volatile quantity. In contrast, normalizing by market capitalization preserves the signal while confining turnover effects to the noise term.

Our Monte Carlo simulations demonstrate a $1.32\times$ improvement in correlation under baseline parameters, extending to $1.92\times$ when turnover heterogeneity is high. Empirical evidence from the Korean stock market confirms these predictions through direct return prediction tests: market capitalization normalization achieves highly significant next-day return predictability ($t = 9.65$) while trading value normalization is only marginally significant ($t = 2.10$). The empirical R^2 ratio ($2.07\times$) exceeds the implied Monte Carlo R^2 ratio ($1.32^2 \approx 1.74\times$), suggesting real-world turnover heterogeneity may be even greater than our baseline calibration. The “horse race” regression provides the definitive test: when both normalizations compete, S^{MC} strengthens ($t = 10.99$) while S^{TV} exhibits a sign reversal ($t = -6.81$), confirming it primarily captures liquidity-driven noise rather than fundamental information.

These findings challenge conventional practices in market microstructure research and have immediate implications for:

- **Trading strategies:** Refactoring order imbalance alphas to use market cap normalization could yield 30% or greater Sharpe ratio improvements
- **Risk factor construction:** Existing order flow factors may suffer from heteroskedastic contamination that our approach eliminates
- **Empirical methodology:** Tests of information asymmetry and price discovery should reconsider normalization choices

The simplicity of our prescription—divide by market cap, not trading volume—betrays its substantial impact on signal quality. In the quest for signal in a noisy market, the denominator matters as much as the numerator. Market capitalization is the correct denominator.

Future research should prioritize cross-market validation using US institutional trading data (e.g., ANcerno or 13F-based measures), explore nonlinear extensions using machine learning, and integrate time-varying market capitalization dynamics.

Disclosure Statement

The author reports there are no competing interests to declare.

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A Mathematical Derivation of Signal-to-Noise Ratios

This appendix provides the complete mathematical derivation establishing the superiority of market capitalization normalization (S^{MC}) over trading value normalization (S^{TV}) in terms of signal-to-noise ratio.

A.1 Definitions and Setup

Recall the signal extraction framework from Section 3. Observed order flow is:

$$D_i = k\alpha_i M_i + \zeta_i V_i \quad (10)$$

where $\alpha_i \sim N(0, \sigma_\alpha^2)$ is the latent information signal, $\zeta_i \sim N(0, \sigma_\zeta^2)$ is noise, M_i is market capitalization, and $V_i = \tau_i M_i$ is trading volume with turnover rate τ_i .

Returns are generated by:

$$R_i = \gamma\alpha_i + \epsilon_i \quad (11)$$

where $\epsilon_i \sim N(0, \sigma_\epsilon^2)$ is idiosyncratic return noise, independent of α_i , ζ_i , and τ_i .

The Signal-to-Noise Ratio (SNR) is defined as:

$$\text{SNR}(S) = \frac{\text{Cov}^2(S, R)}{\text{Var}(S) \cdot \text{Var}(R)} \quad (12)$$

Higher SNR implies stronger predictive power of the normalized signal S for future returns R .

A.2 Market Capitalization Normalization (S^{MC})

The market cap normalized signal is:

$$S_i^{MC} = \frac{D_i}{M_i} = k\alpha_i + \zeta_i \tau_i \quad (13)$$

Covariance with returns:

$$\begin{aligned} \text{Cov}(S^{MC}, R) &= E[(k\alpha + \zeta\tau)(\gamma\alpha + \epsilon)] - E[k\alpha + \zeta\tau]E[\gamma\alpha + \epsilon] \\ &= E[k\gamma\alpha^2 + k\alpha\epsilon + \gamma\zeta\tau\alpha + \zeta\tau\epsilon] \\ &= k\gamma E[\alpha^2] \quad (\text{by independence}) \\ &= k\gamma\sigma_\alpha^2 \end{aligned} \quad (14)$$

Variance of signal:

$$\begin{aligned} \text{Var}(S^{MC}) &= \text{Var}(k\alpha + \zeta\tau) \\ &= k^2\text{Var}(\alpha) + \text{Var}(\zeta\tau) + 2k\text{Cov}(\alpha, \zeta\tau) \\ &= k^2\sigma_\alpha^2 + E[\zeta^2\tau^2] \quad (\text{by independence}) \\ &= k^2\sigma_\alpha^2 + \sigma_\zeta^2 E[\tau^2] \end{aligned} \quad (15)$$

SNR for S^{MC} :

$$\text{SNR}_{MC} = \frac{(k\gamma\sigma_\alpha^2)^2}{(k^2\sigma_\alpha^2 + \sigma_\zeta^2 E[\tau^2]) \cdot \sigma_R^2} \quad (16)$$

where $\sigma_R^2 = \gamma^2\sigma_\alpha^2 + \sigma_\epsilon^2$.

A.3 Trading Value Normalization (S^{TV})

The trading value normalized signal is:

$$S_i^{TV} = \frac{D_i}{V_i} = k\alpha_i\tau_i^{-1} + \zeta_i \quad (17)$$

Covariance with returns:

$$\begin{aligned} \text{Cov}(S^{TV}, R) &= E[(k\alpha\tau^{-1} + \zeta)(\gamma\alpha + \epsilon)] \\ &= k\gamma E[\alpha^2\tau^{-1}] \end{aligned} \quad (18)$$

Under the assumption that α and τ are independent:

$$\text{Cov}(S^{TV}, R) = k\gamma\sigma_\alpha^2 E[\tau^{-1}] \quad (19)$$

Variance of signal:

$$\begin{aligned} \text{Var}(S^{TV}) &= k^2\text{Var}(\alpha\tau^{-1}) + \sigma_\zeta^2 \\ &= k^2(E[\alpha^2\tau^{-2}] - (E[\alpha\tau^{-1}])^2) + \sigma_\zeta^2 \end{aligned} \quad (20)$$

With independence of α and τ :

$$\text{Var}(S^{TV}) = k^2\sigma_\alpha^2 E[\tau^{-2}] + \sigma_\zeta^2 \quad (21)$$

SNR for S^{TV} :

$$\text{SNR}_{TV} = \frac{(k\gamma\sigma_\alpha^2 E[\tau^{-1}])^2}{(k^2\sigma_\alpha^2 E[\tau^{-2}] + \sigma_\zeta^2) \cdot \sigma_R^2} \quad (22)$$

A.4 Comparison of SNR

To compare SNR_{MC} and SNR_{TV} , we examine the ratio:

$$\frac{\text{SNR}_{MC}}{\text{SNR}_{TV}} = \frac{1}{(E[\tau^{-1}])^2} \cdot \frac{k^2\sigma_\alpha^2 E[\tau^{-2}] + \sigma_\zeta^2}{k^2\sigma_\alpha^2 + \sigma_\zeta^2 E[\tau^2]} \quad (23)$$

Key insight (Jensen's Inequality): For the convex function $f(x) = 1/x$:

$$E[\tau^{-1}] \geq (E[\tau])^{-1} \quad (24)$$

with strict inequality when τ has dispersion.

Similarly, for the convex function $g(x) = 1/x^2$:

$$E[\tau^{-2}] \geq (E[\tau])^{-2} \quad (25)$$

These inequalities imply that the variance term in S^{TV} (containing $E[\tau^{-2}]$) is inflated relative to what would occur with constant turnover. This inflation affects the denominator of SNR_{TV} , reducing it relative to SNR_{MC} .

Numerical illustration: Consider turnover uniformly distributed on $[0.0005, 0.01]$ (0.05% to 1% daily), matching the Monte Carlo simulation parameters in Table 2.

- $E[\tau] = 0.00525$
- $E[\tau^2] \approx 3.5 \times 10^{-5}$
- $E[\tau^{-1}] \approx 315.3$
- $E[\tau^{-2}] = 200,000$

The key observation: $E[\tau^{-2}]$ is orders of magnitude larger than $(E[\tau])^{-2} \approx 36,281$, demonstrating the severe variance inflation in S^{TV} .

A.5 Conclusion

The mathematical analysis confirms that:

1. S^{MC} preserves the signal ($k\alpha_i$) without distortion
2. S^{TV} multiplies the signal by τ_i^{-1} , introducing variance inflation
3. The variance inflation in S^{TV} (from $E[\tau^{-2}]$) exceeds the noise scaling in S^{MC} (from $E[\tau^2]$)
4. Therefore, $\text{SNR}_{MC} > \text{SNR}_{TV}$ whenever turnover exhibits cross-sectional dispersion

This completes the proof that market capitalization normalization is the optimal (matched) filter for informed trading signals.