

Note: If a question asks you to design an algorithm, full marks will be given if your algorithm runs with optimal time and space complexity

Problem 1 [20 points] Element Shifting

Given an array A with n integers and an integer t ($t < n$). Design an algorithm to shift the sequence in A by t positions. For example, if $A = \{10, 7, 12, 18, 16, 20, 30\}$ and $t=3$, then the correct output of your algorithm is $\{18, 16, 20, 30, 10, 7, 12\}$.

Please describe your algorithm idea first, then write pseudocode and analysis its time and space complexity.

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Problem 2 [20 points] Minimum Distance

We define the distance of tuple (a, b, c) (a, b , and c are positive integers) as $D = |a - b| + |b - c| + |c - a|$. Suppose there are three sorted arrays X_1, X_2 , and X_3 (in ascending order). Please design an algorithm to find the minimum distance among all possible tuple (a, b, c) where $a \in X_1, b \in X_2, c \in X_3$. Please describe your algorithm idea and analysis its time complexity.

Problem 3 [20 points] Sorting Algorithm

ShellSort is mainly a variation of Insertion Sort. In insertion sort, we move elements only one position ahead. When an element has to be moved far ahead, many movements are involved. The idea of shellSort is to allow exchange of far items. In shellSort, we make the array h-sorted for a large value of h. We keep reducing the value of h until it becomes 1. An array is said to be h-sorted if all sublists of every h'th element is sorted.

- (a) [2 points] Records $A[1], A[2], A[3], \dots, A[N]$ are said to be h-sorted, if _____
(A) $A[i] \leq A[i+h]$ for $1 \leq i \leq N$
(B) $A[h] \leq A[i+h]$ for $1 \leq i \leq N$
(C) $A[i] \leq A[h]$ for $1 \leq i \leq h$
(D) $A[i] \leq A[i+h]$ for $1 \leq i \leq N-h$
- (b) [2 points] An array that is first 7-sorted, then 5-sorted becomes _____
(A) 7-ordered
(B) 5-ordered
(C) both 2-ordered and 5-ordered
(D) both 7-ordered and 5-ordered
- (c) [4 points] In the worst case, the quick sort algorithm and shell sort algorithm will degenerate to _____ and _____ sort algorithm, respectively.
- (d) [3 points] Shell sort is more efficient than insertion sort if the length of input arrays is small. True or False? Why?
- (e) [9 points] Fill the following table to show the running steps of Shell-Sort Algorithm.

	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]	A[7]	A[8]
Input	13	26	18	53	7	17	95	86
4-Sorted								
2-Sorted								
1-Sorted								


Problem 4 [30 points] Filling blank questions

(a) [5 points] The time complexity of the following function is $O(n \log n)$

```
int foo(int n){
```

```
    count ← 0;
```

```
    for(k=1 ; k <=n; k*=2)
```

```
        for(j=1; j<=n; j++)
```

```
            count++
```

```
}
```

B

(b) [5 points] Suppose the sequence 11, 12, 13, 7, 8, 23, 4, 5 is the immediate sorting result after the second iteration of a sorting algorithm. Which sorting algorithm is used: _____.

A. Bubble sort B. Insertion sort C. Selection sort D. Quick sort

(c) [5 points] Let $f(n)$ be a function of positive integer n . We know:

$$T(1) = 1$$

$$T(n) = T(\lceil n/5 \rceil) + T(7/10 n + 6) + O(n):$$

then $T(n) = \underline{O(n)}$, recall that $\lceil x \rceil$ is the ceiling operator that returns the smallest integer at least x .

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(d) [5 points] Which of the following function is not $O(n^{2.5})$ (_____)

A. $\frac{n^{100}}{2^n}$

B. $(\log_2 n)^{98}$

C. $938593729n^2$

D. $n^{2.6}/\log^2 n$

(e) [10 points] The time complexity of the following function is:

$$T(n) = \underline{T(n-1)} \text{ (recursion expression)} = \underline{O(n)} \text{ (Big-O notation).}$$

```
int func(int n){
```

```
    if n = 1 return 1;
```

```
    return n*func(n-1)
```

```
}
```

↑

$$T(n-1) + O(1)$$