

Step-1

Given functions are $f(x) = x^2$, $g(x) = 5x$ are vectors in the vector space \mathbf{F} of all real functions.

The combination $3f(x) - 4g(x)$ is $h(x)$.

Therefore,

$$\begin{aligned}h(x) &= 3f(x) - 4g(x) \\&= 3 \cdot x^2 - 4 \cdot (5x) \\&= 3x^2 - 20x\end{aligned}$$

Hence $h(x) = 3x^2 - 20x$

Step-2

The scalar multiplication of the vector space \mathbf{F} of all real function is $(cf)(x) = cf(x)$

If we define $(cf)(x) = f(cx)$ which is not equal to $cf(x)$

For example $f(x) = x^2$

$$\begin{aligned}(3f)(x) &= 3 \cdot f(x) \\&= 3 \cdot x^2\end{aligned}$$

If we define $(cf)(x) = f(cx)$

$$\begin{aligned}f(3x) &= (3x)^2 \\&= 9x^2\end{aligned}$$

Therefore, $3 \cdot f(x) \neq f(3x)$

Therefore scalar multiplication rule is broken.