Step-1

Given that the *m* errors e_i are independent with variance σ^2.

Also given that the average of $(b-Ax)(b-Ax)^T=\sigma^2 I$ â€¦â€¦ (1)

We have to show that the average of $\left(\hat{x}-x\right)\left(\hat{x}-x\right)=\sigma^2\left(A^T A\right)^{-1}$.

Step-2

Now

$$\begin{aligned}
\left(\hat{x}-x\right)\left(\hat{x}-x\right) &= \left(A^T A\right)^{-1} A^T\left[(b-Ax)(b-Ax)^T\right]A\left(A^T A\right)^{-1} \\
&= \left(A^T A\right)^{-1} A^T\sigma^2 I\, A\left(A^T A\right)^{-1} && \left(\text{Since by } (1)\right) \\
&= \left(A^T A\right)^{-1} A^T\sigma^2\, A\left(A^T A\right)^{-1} && \left(\text{Since } IA=AI=A\right) \\
&= \sigma^2\left(A^T A\right)^{-1} A^T A A^{-1}\left(A^T\right)^{-1}
\end{aligned}$$

Step-3

Continuation to the above

$$\begin{aligned}
&= \sigma^2\left(A^T A\right)^{-1} A^T I\left(A^T\right)^{-1} && \left(\text{Since } AA^{-1}=A^{-1}A=I\right) \\
&= \sigma^2\left(A^T A\right)^{-1} A^T\left(A^T\right)^{-1} \\
&= \sigma^2\left(A^T A\right)^{-1} I \\
&= \sigma^2\left(A^T A\right)^{-1}
\end{aligned}$$

Hence the average of $\boxed{\left(\hat{x}-x\right)\left(\hat{x}-x\right)=\sigma^2\left(A^T A\right)^{-1}}$