Step-1

Consider a matrix $A=\begin{bmatrix}0 & -1\\ i & 0\end{bmatrix}$ and the complex number $c=a+ib$.

To show the matrix $A+cI$ is invertible, we have to show $\det(A+cI)\neq 0$.

$$\begin{aligned}\det(A+cI)&=\det\left(\begin{bmatrix}0 & -1\\ i & 0\end{bmatrix}+c\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\\ &=\det\begin{bmatrix}c & -1\\ i & c\end{bmatrix}\\ &=c^2+i\end{aligned}$$

Step-2

Substitute $c=a+bi$ in the equation $\det(A+cI)=c^2+i$.

$$\begin{aligned}\det(A+cI)&=(a+ib)^2+i\\ &=\left(a^2+2abi-b^2\right)+i\\ &=\left(a^2-b^2\right)+(2ab+1)i\end{aligned}$$

Since $\left(a^2-b^2\right)+(2ab+1)i$ cannot be zero for any real numbers a and b.

So,

$$\begin{aligned}\det(A+cI)&=\left(a^2-b^2\right)+(2ab+1)i\\ &\neq 0\end{aligned}$$

Thus, the matrix $A+cI$ is $\boxed{\text{invertible}}$ for all complex numbers $c=a+ib$.

Step-3

Consider a matrix $A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ and $r\in R$.

To show the matrix $A+rI$ is invertible, we have to show $\det(A+rI)\neq 0$.

$$\begin{aligned}\det\left(A+rI\right) &= \det\left(\begin{bmatrix}0 & -1\\ i & 0\end{bmatrix}+r\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\\ &= \det\begin{bmatrix}r & -1\\ 1 & r\end{bmatrix}\\ &= r^2+1\end{aligned}$$

Step-4

Since r^2+1 cannot be zero for any real numbers r.

So,

$$\begin{aligned}\det\left(A+cI\right) &= r^2+1\\ &\neq 0\end{aligned}$$

Thus, the matrix $A+rI$ is $\boxed{\text{invertible}}$ for all real numbers r.