

Step-1

We need to find all the odd permutations of $\{1, 2, 3, 4\}$

The odd permutations on four symbols are $\frac{1}{2}(4!) = 12$ in number

They are

One inter change $(1, 2, 4, 3), (1, 3, 4, 2), (1, 4, 3, 2), (2, 1, 3, 4), (3, 2, 1, 4), (4, 2, 3, 1)$

Three interchanges $(2, 3, 4, 1), (2, 4, 1, 3), (3, 1, 4, 2), (3, 4, 2, 1), (4, 1, 2, 3), (4, 3, 1, 2)$

Step-2

The permutations that lead to $\det P = -1$ are odd permutations.

The row interchanges are shown by $\hat{1}\hat{a}\epsilon^{\text{TM}}$ s in the respective places of the 4×4 matrix.

If only one interchange has taken place, then $\det P = (-1)^1 = -1$ and so, P is an odd permutation.

1st and 2nd rows interchange $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3rd and 4th rows interchange $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Step-3

2nd and 4th rows interchange $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

1st and 4th rows interchange

Step-4

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2nd and 3rd rows interchange

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1st and 3rd rows interchange

Step-5

Three row interchange:

$$1234 \rightarrow 1243 \rightarrow 1423 \rightarrow 4123 \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$1234 \rightarrow 2134 \rightarrow 2143 \rightarrow 2413 \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Step-6

$$1234 \rightarrow 1324 \rightarrow 3142 \rightarrow 3412 \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$1234 \rightarrow 2134 \rightarrow 2314 \rightarrow 2134 \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-7

$$1234 \rightarrow 1243 \rightarrow 1423 \rightarrow 1432 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$1234 \rightarrow 1324 \rightarrow 1342 \rightarrow 3142 \Rightarrow \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

We easily see that $\det P = (-1)^{\text{number of row interchanges}}$

Step-8

$$= (-1)^3$$

$$= -1$$