

## Step-1

We have to factor the following matrices into  $A = LU$  and  $A = LDV$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \text{ and } A = \begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix}$$

## Step-2

Given

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Subtracting 1 times row 1 from row 2 gives

$$= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Subtracting row 2 from row 3, we get

$$U = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

## Step-3

To get  $L$ , we have to do reverse operations on the identity matrix which are held on  $A$ ;

Adding 1 times row 1 to row 2 and adding row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\text{Therefore } A = LU \text{ factorization is } \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

## Step-4

The diagonal elements of  $D$  are the pivots of  $U$  and  $V$  is obtained by the each row of  $U$ , by the pivots of  $U$ .

The factorization  $A = LDV$  is 
$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

## Step-5

Given 
$$A = \begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix}$$

Subtracting row 1 from row 2 gives

$$= \begin{pmatrix} a & a & 0 \\ 0 & b & b \\ 0 & b & b+c \end{pmatrix}$$

## Step-6

Subtracting row 2 from row 3 gives

$$U = \begin{pmatrix} a & a & 0 \\ 0 & b & b \\ 0 & 0 & c \end{pmatrix}$$

To get  $L$ , we have to do reverse operations on the identity matrix which are held on  $A$ ;

Adding 1 times row 1 to row 2 and adding row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

## Step-7

Therefore  $A = LU$  factorization is

$$\begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} a & a & 0 \\ 0 & b & b \\ 0 & 0 & c \end{pmatrix}$$

## Step-8

The diagonal elements of  $D$  are the pivots of  $U$  and  $V$  is obtained by the each row of  $U$ , by the pivots of  $U$ .

The factorization  $A = LDV$  is

$$\begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$