Step-1

We have to prove that the trace of $P = \frac{aa^T}{a^T a}$ is always 1.

Let $a = (a_1, a_2, ..., a_n)$

$$aa^T = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}$$

$$= \begin{bmatrix} a_1^2 & a_1 a_2 & \cdots & a_1 a_n^2 \\ a_2 a_1 & a_2^2 & \cdots & a_2 a_n \\ \cdots & \cdots & \cdots & \cdots \\ a_n a_1 & a_n a_1 & \cdots & a_n^2 \end{bmatrix}$$

Step-2

$$a^T a = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$= a_1^2 + a_2^2 + \cdots + a_n^2$$

$$= \sum a_1^2$$

Step-3

The projection matrix

$$P = \frac{aa^T}{a^T a}$$

$$= \frac{1}{\sum a_1^2} \begin{bmatrix} a_1^2 & a_1 a_2 & \cdots & a_1 a_n^2 \\ a_2 a_1 & a_2^2 & \cdots & a_2 a_n \\ \cdots & \cdots & \cdots & \cdots \\ a_n a_1 & a_n a_1 & \cdots & a_n^2 \end{bmatrix}$$

Step-4

Trace P = Sum of its diagonal elements

$$= \frac{a_1 a_1}{\sum a_1^2} + \frac{a_2 a_2}{\sum a_1^2} + \cdots + \frac{a_n a_n}{\sum a_1^2}$$

$$= \frac{a_1^2 + a_2^2 + \cdots + a_n^2}{\sum a_1^2}$$

$$= \frac{\sum a_1^2}{\sum a_1^2}$$

$$= 1$$

Hence trace of P is $\boxed{1}$