Step-1

Consider the following matrices,

$$\begin{bmatrix}0&1&0&1\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\begin{bmatrix}1&1&1&1\\1&i&i^2&i^3\\1&i^2&i^4&i^6\\1&i^3&i^6&i^9\end{bmatrix}=\begin{bmatrix}0&1&0&1\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\begin{bmatrix}\lambda_0&&&\\&\lambda_1&&\\&&\lambda_3&\\&&&\lambda_4\end{bmatrix}$$

$$\begin{bmatrix}1+1&i+i^3&i^2+i^6&i^3+i^9\\1&i^2&i^4&i^6\\1&i^3&i^6&i^9\\1&1&1&1\end{bmatrix}=\begin{bmatrix}0&1&0&1\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\begin{bmatrix}\lambda_0&&&\\&\lambda_1&&\\&&\lambda_3&\\&&&\lambda_4\end{bmatrix}$$

$$\begin{bmatrix}2&0&-2&0\\1&-1&1&-1\\1&-i&-1&i\\1&1&1&1\end{bmatrix}=\begin{bmatrix}0&1&0&1\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\begin{bmatrix}\lambda_0&&&\\&\lambda_1&&\\&&\lambda_3&\\&&&\lambda_4\end{bmatrix}$$

Therefore, we get,

$\lambda_0 = 2$
$\lambda_1 = -1$
$\lambda_2 = -1$
$\lambda_4 = 1$

Step-2

Thus, the eigenvalues are $\boxed{\lambda_0 = 2,\ \lambda_1 = -1,\ \lambda_2 = -1,\ \text{and}\ \lambda_4 = 1}$.