Step-1

Consider the matrices: $A=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 2\\ 1 & 2 & 3\end{bmatrix}$.

The cofactors of A are as follows.

$$\begin{aligned}C_{11}&=\begin{vmatrix}2 & -1\\ -1 & 2\end{vmatrix}\\ &=4-1\\ &=3\end{aligned}$$

$$\begin{aligned}C_{12}&=-\begin{vmatrix}-1 & -1\\ 0 & 2\end{vmatrix}\\ &=-(-2-0)\\ &=2\end{aligned}$$

$$\begin{aligned}C_{13}&=\begin{vmatrix}-1 & 2\\ 0 & -1\end{vmatrix}\\ &=1-0\\ &=1\end{aligned}$$

Step-2

And also,

$$\begin{aligned}C_{21}&=-\begin{vmatrix}-1 & 0\\ -1 & 2\end{vmatrix}\\ &=-(-2-0)\\ &=2\end{aligned}$$

$$\begin{aligned}C_{22}&=\begin{vmatrix}2 & 0\\ 0 & 2\end{vmatrix}\\ &=4-0\\ &=4\end{aligned}$$

$$\begin{aligned}C_{23}&=-\begin{vmatrix}2 & -1\\ 0 & -1\end{vmatrix}\\ &=-(-2-0)\\ &=2\end{aligned}$$

Step-3

Now,

$$C_{31} = \begin{vmatrix} -1 & 0 \\ 2 & -1 \end{vmatrix}$$
$$= 1 - 0$$
$$= 1$$

$$C_{32} = -\begin{vmatrix} 2 & 0 \\ -1 & -1 \end{vmatrix}$$
$$= -(-2 - 0)$$
$$= 2$$

$$C_{33} = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$
$$= 4 - 1$$
$$= 3$$

So, the matrix of cofactors of A is $C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.

And also, the transpose of the matrix C is $C^T = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.

Step-4

Find determinant of the matrix is,

$$\det(A) = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix}$$
$$= 2(3) - 2$$
$$= 4$$

Now, the inverse of the matrix A, using cofactor matrix is,

$$A^{-1} = \frac{C^T}{\det(A)}$$
$$= \frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Step-5

Now, take the matrix $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.

Calculate the cofactors of the matrix B are as follows.

$$C_{11} = \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix}$$
$$= 6 - 4$$
$$= 2$$

$$C_{12} = -\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$
$$= -(3 - 2)$$
$$= -1$$

$$C_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$$
$$= 2 - 2$$
$$= 0$$

Step-6

And also,

$$C_{21} = -\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$
$$= -(3 - 2)$$
$$= -1$$

$$C_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix}$$
$$= 3 - 1$$
$$= 2$$

$$C_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= -(2 - 1)$$
$$= -1$$

Step-7

Now

$$C_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}$$
$$= 2 - 2$$
$$= 0$$

$$C_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= -(2 - 1)$$
$$= -1$$

$$C_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= 2 - 1$$
$$= 1$$

Step-8

The matrix of cofactors of B is $C = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$.

And also, $C^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$.

Now,

$$\det(B) = \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$$
$$= (2) - (1)$$
$$= 1$$

Find the inverse, using cofactor matrix as follows.

The inverse of the matrix B is,

$$B^{-1} = \frac{C^T}{\det(A)}$$
$$= \frac{1}{1}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Step-9

Hence, the inverses of the given matrices A and B are

$$A^{-1} = \boxed{\frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}},\quad B^{-1} = \boxed{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}}.$$