

Step-1

Consider the vectors: $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $v_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Clearly observe that

$$\begin{aligned} v_1 - v_2 &= \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= V_1 \end{aligned}$$

And also, $V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix} = v_2$

Step-2

Now,

$$\begin{aligned} V_1 &= v_1 - v_2 \\ V_2 &= v_2 \end{aligned}$$

These vectors can be written as,

$$\begin{Bmatrix} V_1 = (1)v_1 + (-1)v_2 \\ V_2 = (0)v_1 + (1)v_2 \end{Bmatrix}$$

From the data, the columns of M comes from expressing V_1 and V_2 as a combinations of v_1, v_2 .

So, the matrix M can be written as $M = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$.

Hence, the required matrix to change the basis is $M = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$.