Step-1

Given that the eigen values of A are 1 and 0.

Trace of A = sum of eigen values

$=1+0=1$

$\det A$ = Product of eigen values

$=1(0)$

A is singular matrix.

Therefore A is not invertiable.

Step-2

A has distinct eigen values. So, the eigen vectors corresponding to the distinct eigen values are linearly independent.

So, the matrix S whose columns are the eigen vectors of A is non singular and so, S^{-1} exists such that $S^{-1}AS=\Lambda=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$ the diagonal matrix.

Therefore, A is diagonalizable.

Further, $A^n=S\Lambda^n S^{-1}$ and Λ^n is the diagonal matrix whose diagonal entries are the n^{th} powers of the eigen values 1 and 0.

So, $A^n=S\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}S^{-1}$ for every positive integer n.

Step-3

While the determinant of A is zero, it is not invertible.

So, $A^n=S\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}S^{-1}$ does not hold for other than positive integers.