Step-1

Consider a matrix A of order 6 by 4. Another matrix B is of order 4 by 6. Matrix AB (6×6) and BA (4×4) are of different sizes. Then consider the matrix G formed as below:

$$
\begin{aligned}
G &= \begin{bmatrix} I & -A \\ 0 & I \end{bmatrix} \begin{bmatrix} AB & 0 \\ B & 0 \end{bmatrix} \begin{bmatrix} I & A \\ 0 & I \end{bmatrix} \\
&= M^{-1}FM \\
&= \begin{bmatrix} 0 & 0 \\ B & BA \end{bmatrix}
\end{aligned}
$$

Step-2

(a) Sizes of blocks are same in each matrix. Determine the sizes of blocks in G.

Following equation shows that matrices F and G are similar matrices.

$$G = M^{-1}FM$$

They have the same 10 Eigen values. Matrix G has one block of BA (4×4) on main diagonal corresponding to 4 Eigen values. Matrix F has one block of AB (6×6) starting from main diagonal corresponding to 6 Eigen values. Total Eigen values are 10 in number and F and G are similar matrices, this implies that another block in matrix G is of (6×6).

Therefore, sizes of blocks in matrix G are: $\boxed{(6\times 6)}$ and $\boxed{(4\times 4)}$.

Step-3

(b) Matrix G and F have the same 10 Eigen values Matrix F has the Eigen values of AB plus 4 zeros and matrix G has the Eigen values of matrix BA plus 6 zeros.

Matrix F has 4 zeros so remaining $(10-4)$ Eigen values must be Eigen values of matrix AB. Therefore, matrix AB has 6 Eigen values.

Similarly, matrix G has 6 zeros so remaining $(10-6)$ Eigen values must be Eigen values of matrix BA. Therefore, matrix BA has 4 Eigen values.

Therefore, matrix AB has the same Eigen values as matrix BA plus $\boxed{6-4=2}$ zeros.