

Step-1

(a)

The objective is to find the relation between the determinant of A^H and the determinant of A .

It is known that $\det A^T = \det A$

(Since $A^H = \text{Transpose of conjugate matrix}$)

But, $\det A^H = \overline{(\det A)}$

Hence, $\boxed{\det(A^H) = \overline{(\det A)}}$

Step-2

(b)

The objective is to prove that the determinant of a Hermitian matrix is real.

Let A be any Hermitian matrix

Then $A^H = A$

Take determinant on both sides, to get;

$$\det(A^H) = \det A$$

$$\left(\text{Since } \det A^H = \overline{(\det A)} \right)$$

$$= \overline{(\det A)}$$

$$\left(\begin{array}{l} \text{Since} \\ a + ib = a - ib \\ \text{this implies;} \\ ib = -ib \\ \text{this implies;} \\ b = 0 \end{array} \right)$$

= real number

Hence, the determinant of a Hermitian matrix is **real**.