Step-1

Consider the matrix $A_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Step 1: we transpose it.

$A_1^T = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Step 2: we assume the entries of the rows as the coefficients of the linear combinations of the given basis of the co domain vector space $\mathbb{R}^2$

$1(1,0)+0(0,1)=(1,0)$

$0(1,0)-1(0,1)=(0,-1)$

Step 3: we assume that these are the images of the basis of the domain under the linear transformation T_1

i.e., $T(1,0)=(1,0); T(0,1)=(0,-1)$

Step 4: we write the general vector in the domain as a linear combination of basis vectors.

i.e., $T_1(x,y)=T_1(x(1,0)+y(0,1))$

$=xT_1(1,0)+yT_1(0,1)$ Since T is linear

$=x(1,0)+y(0,-1)$

$=(x,0)+(0,-y)$
$=(x,-y)$

Thus, the suitable linear transformation for A_1 is $T_1(x,y)=(x,-y)$

Step-2

$A_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

Step 1: $A_2^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Step 2: $1(1,0)+2(0,1)=(1,2)$

$0(1,0)+1(0,1)=(0,1)$

Step 3: $T_2(1,0)=(1,2);T_2(0,1)=(0,1)$

Step 4: $T_2(x,y)=T_2(x(1,0)+y(0,1))$

$=xT_2(1,0)+yT_2(0,1)$

$=x(1,2)+y(0,1)$

$=(x,2x+y)$

Step-3

$A_3=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$

Step 1: $A_3^T=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

Step 2: $0(1,0)-1(0,1)=(0,-1)$

$1(1,0)+0(0,1)=(1,0)$

Step 3: $T_3(1,0)=(0,-1);T_3(0,1)=(1,0)$

Step 4: $T_3(x,y)=T_3(x(1,0)+y(0,1))$

$=xT_3(1,0)+yT_3(0,1)$

$=x(0,-1)+y(1,0)$

$=(y,-x)$