

## Step-1

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Consider the matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

Let the matrix be

$$\det A = \sum_{\text{all } P's} (a_{1\alpha} a_{2\beta} \dots a_{m\gamma}) \det P$$

This can be expanded as follows.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} \\ + a_{13} \begin{vmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} - a_{14} \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}$$

## Step-2

Now, apply the formula on the determinant of the matrix  $A$ .

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{vmatrix} = (1) \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} - (0) \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} + (0) \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} - (1) \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} \\ = (1)(-1) - 0 + 0 - (1)(-1) \\ = -1 + 1 \\ = 0$$

$$-a_{11}a_{23}a_{32}a_{44} + a_{44}a_{23}a_{32}a_{41} = (1) \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} - (1) \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

So, the non-zero term is

Also note that last column of  $A$  is sum of 1<sup>st</sup> and 3<sup>rd</sup> columns of  $A$  and 1<sup>st</sup> row, last row of  $A$  are identical.

### Step-3

$$\text{Let } B = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 4 & 5 \\ 5 & 4 & 0 & 3 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$

Observe that the matrix  $B$  also have zero in exactly same places as  $A$  and hence exactly two none zero terms occur in the big formula.

They are  $-b_{11}b_{23}b_{32}b_{44} + b_{44}b_{23}b_{32}b_{41}$

Hence, the required determinant of the matrix is

$$\begin{aligned} \det B &= -1.4.4.1 + 2.4.4.2 \\ &= -16 + 64 \\ &= 48 \end{aligned}$$

From the above results, the determinant of  $A$  is  $-1+1$  and also  $\boxed{\det B = 48}$ .