Step-1

We know that $Ax = b$ has a nonnegative solution or there exists a vector y, such that $yA \geq 0$ and $yb < 0$.

Let $y = (1, -1)$.

Therefore,

$$\begin{aligned} yA &= (1,-1)\begin{bmatrix} 1 & 3 & -5 \\ 1 & -4 & -7 \end{bmatrix} \\ &= (1\times 1 + (-1)\times 1, 1\times 3 + (-1)\times(-4), 1\times(-5) + (-1)\times(-7)) \\ &= (0, 7, 2) \end{aligned}$$

Also,

$$\begin{aligned} yb &= (1,-1)\begin{bmatrix} 2 \\ 3 \end{bmatrix} \\ &= 1\times 2 + (-1)\times 3 \\ &= -1 \\ &< 0 \end{aligned}$$

Step-2

Thus, we have produced a vector $y = (1, -1)$ such that $yA \geq 0$ and $yb < 0$. Therefore, for $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & -4 & -7 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, the equation $Ax = b$ has no nonnegative solution.