Step-1

Consider the following system of differential equations:

$$\frac{dv}{dt} = w,$$
$$\frac{dw}{dt} = v.$$

Objective is to determine whether the system is stable or not. Also check for the solution that decays.

For the sake of simplicity, express the given system in matrix form as:

$$u = \begin{bmatrix} v \\ w \end{bmatrix}, u' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} u.$$

Step-2

For the stability, calculate the eigenvalues of the obtained matrix $u' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. And the eigenvalues are:

$$\begin{vmatrix} 0-\lambda & 1 \\ 1 & 0-\lambda \end{vmatrix} = 0$$
$$(-\lambda)(-\lambda) - 1 = 0$$
$$\lambda^2 - 1 = 0$$
$$\lambda = 1, -1.$$

Since one eigenvalue is positive for the given system, therefore rule implies that system is unstable.

Step-3

Since $\lambda = 1, -1$, so the corresponding solution for the linear differential equation will be of the form:

$$u = c_1 e^t + c_2 e^{-t},$$

for some constant c_1, c_2. Note that for large value of *t*, the value of e^{-t} will tend to zero. That is, as $t \to \infty$, $e^{-t} \to 0$.

Step-4

Hence, for the given system, there is a solution e^{-t} that decays.