Step-1

By the Figure 3.4, $Ax_n = 0$, $x = \begin{bmatrix} 2 \\ 0 \end{bmatrix} = x_r + x_n$ such that $Ax = Ax_r$ â€¦â€¦ (1)

Using $Ax_n = 0$ where $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, we get $x_1 - x_2 = 0$

$$\Rightarrow x_1 = x_2 = k(say)$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Therefore, $x_n = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Step-2

Also, by the Figure 3.4, we have $Ax_r = b$ â€¦â€¦ (2)

Using (1), we get

$$b = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

We now use (2) to give $\begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

$$\Rightarrow y_1 - y_2 = 2$$

In view of (1), we get $y_1 = 1, y_2 = -1$

Therefore, $x_r = (1, -1)$