

Step-1

Consider the matrix;

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The column space of A is the linear combination of all columns of A .

Let,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$b_1 = c_1 + 2c_2$$

$$b_2 = 0$$

$$b_3 = 0$$

Therefore $\left\{ \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \right\}$ is linear combinations of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

Column space of A is $\boxed{\mathbf{C}(A) = \{(c_1 + 2c_2, 0, 0) / c_1, c_2 \in \mathbf{R}\}}$

Therefore, column space of A is x -axis.

Step-2

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

Given that

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

Column space of B is linear combinations of vectors

$$c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} c_1 \\ 2c_2 \\ 0 \end{bmatrix}$$

Now,

Therefore, the column space of B is;

$$\boxed{\mathbf{C}(B) = \{(c_1, 2c_2, 0) / c_1, c_2 \in \mathbf{R}\}}$$

Therefore the column space of B is $x-y$ plane;

= all vectors $(x, y, 0)$

Step-3

$$C = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \end{bmatrix}$$

Given that

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The column space of C is linear combinations of columns

Let

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 \\ 2c_1 \\ 0 \end{bmatrix}$$

Therefore the column space of C is;

$$\boxed{\mathbf{C}(C) = \{(c_1, 2c_1, 0) / c_1 \in \mathbf{R}\}}$$

$$= \{c(1, 2, 0) / c \in \mathbf{R}\}$$

Therefore the column space of C is the line of vectors containing $(1, 2, 0)$.