

作業 2.

Midterm Copy 2
Suggested solutions

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Question 1: (1) A (2) D (3) A (4) A (5) C.

Question 2: (1) $\begin{bmatrix} 1 & -\frac{x}{2} & \frac{x^2-2y}{8} \\ 0 & \frac{1}{2} & -\frac{x}{8} \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ (2) 2. (3) $\begin{bmatrix} \frac{2}{15} \\ \frac{7}{15} \end{bmatrix}$

(4) -1 (5) $\begin{bmatrix} 0 & -A^2 \\ I_n & 0 \end{bmatrix}$

Question 3: $A = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}}_{L''} \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & -24 \end{bmatrix}}_{U''}$

Question 4: (a) A basis for the column space is: $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

(b) No. $C(A) \perp N(A^T)$.

(c) $C(A) = C(B)$ $B = \begin{bmatrix} -1 & -2 & 0 & 1 \end{bmatrix}$.

Question 5: (a) $f(A+B) = f(A) + f(B)$, for all $A, B \in \mathbb{R}^{2 \times 2}$.
 $f(\lambda A) = \lambda f(A)$, for all $A \in \mathbb{R}^{2 \times 2}$, $\lambda \in \mathbb{R}$.

(b) Verify that $\ker(f)$ is closed under addition and scalar multiplication.

$A \in \ker(f) \Leftrightarrow A$ is symmetric.

A basis of $\ker(f)$ is: $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

(c) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 \\ -1 & 0 & -1 & 1 \end{bmatrix}$ (d) $a=2$, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

Question 6: Case 1: Xiaomeng can get to SUSTech by hot air balloon.
Case 2: Xiaomeng can't get to SUSTech.

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Question 7.

$$(a) R = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, T = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$S = RT$$

(b) Yes, for example

$$T' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, R' = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

Question 8:

$$\begin{aligned} (a) \underbrace{\begin{bmatrix} I_n - AB & \\ 0 & I_n \end{bmatrix}}_C M &= \begin{bmatrix} I_n - AB & \\ 0 & I_n \end{bmatrix} \begin{bmatrix} A & B \\ B^{-1} & A^{-1} \end{bmatrix} \\ &= \begin{bmatrix} A - A & B - ABA^{-1} \\ B^{-1} & A^{-1} \end{bmatrix} \\ &\stackrel{AB=BA}{=} \begin{bmatrix} 0 & B - BAA^{-1} \\ B^{-1} & A^{-1} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ B^{-1} & A^{-1} \end{bmatrix}}_D \end{aligned}$$

$$CM = D, \quad C \text{ is invertible}$$

D is NOT invertible

$\Rightarrow M$ is NOT invertible.

$$(b) \text{rank}(CM) = \text{rank}(D) \leq \text{rank } M$$

$$\text{rank}(M) = \text{rank}(C^{-1}CM) \leq \text{rank}(CM) = \text{rank } D$$

$$\Rightarrow \text{rank } M = \text{rank } D.$$