

Step-1

Consider the following linear programming problem

Minimize: $3x + y$

Subject to

$$x + 2y \geq 6$$

$$2x + y \geq 6$$

Step-2

Now, consider the problem with four unknown (x, y , and two slack variables)

Therefore, we get the following vectors.

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

$$c = [3 \quad 1 \quad 0 \quad 0]$$

Step-3

Thus, the original cost and the constraints gives

$$\begin{bmatrix} A & b \\ c & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 & 0 & 6 \\ 2 & 1 & 0 & -1 & 6 \\ 3 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Now, we exchange column 1 and 3 to put basic variable before free variables.

Tableau at point P is shown below

$$T = \begin{bmatrix} -1 & 2 & 1 & 0 & 6 \\ 0 & 1 & 2 & -1 & 6 \\ 0 & 1 & 3 & 0 & 0 \end{bmatrix}$$

Perform the row transformation to get a fully reduced form

Perform multiplication of first row with 1, to give a unit pivot and use the second row to produce zeros in the second column as shown below

$$R = \begin{bmatrix} 1 & 0 & 3 & -2 & 6 \\ 0 & 1 & 2 & -1 & 6 \\ 0 & 0 & 1 & 1 & -6 \end{bmatrix}$$

Step-4

Now, consider $r = [1 \ 1]$ at the bottom row.

It is observed that it has all positive entries. Therefore, the stopping test is passed.

Thus, point P is optimum. And the optimum cost is 6