Step-1

By the least squares method, we have the equation $A^T A\hat{x} = A^T b$

is $\begin{bmatrix} (1,1) & (1,x) \\ (x,1) & (x,x) \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} (1,x^4) \\ (x,x^4) \end{bmatrix}$ â€¦â€¦ (1)

$$\begin{aligned}(1,1) &= \int_0^1 1.1dx \\ &= x|_0^1 \\ &= 1-0 \\ &= 1\end{aligned}$$

$$\begin{aligned}(x,1) &= \int_0^1 x.1dx \\ &= \left[\frac{x^2}{2}\right]_0^1 \\ &= \frac{1}{2}-0 \\ &= \frac{1}{2}\end{aligned}$$

Step-2

Similarly $(1,x) = \frac{1}{2}$

$$\begin{aligned}(x,x) &= \int_0^1 x^2dx \\ &= \left[\frac{x^3}{3}\right]_0^1 \\ &= \frac{1}{3}-0 \\ &= \frac{1}{3}\end{aligned}$$

$$(1, x^4) = \int_0^1 1x^4 dx$$

$$= \left[\frac{x^5}{5}\right]_0^1$$

$$= \frac{1}{5} - 0$$

$$= \frac{1}{5}$$

$$(x, x^4) = \int_0^1 x^5 dx$$

$$= \left[\frac{x^6}{6}\right]_0^1$$

$$= \frac{1}{6} - 0$$

$$= \frac{1}{6}$$

Step-3

Using these results in (1), we get

$$\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1/3 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1/5 \\ 1/6 \end{bmatrix}$$

$$C + \frac{1}{2}D = \frac{1}{5}, \frac{1}{2}C + \frac{1}{3}D = \frac{1}{6}$$

$$\Rightarrow C = \frac{-1}{5}, D = \frac{4}{5}$$

Therefore, the closest straight line to the parabola $y = x^4$ is

$$y = C + Dx$$

$$= \frac{-1}{5} + \frac{4}{5}x$$

$$= \boxed{\frac{4x-1}{5}}$$