Step-1

Consider that the row space of a matrix contains $(1,2,1)$ and the null space contains $(1,-2,1)$.

The objective is to verify whether such a matrix exists or not.

Step-2

Use the result that the null space of a matrix is the orthogonal complement of the row space in $\mathbf{R}^n$.

This means that the vectors in the nullspace $N(A)$ of the matrix A are orthogonal to the vectors in the row space $C(A)$ of the matrix.

Thus, the dot product of the vectors in nullspace $N(A)$ and row space $C(A)$ is zero.

That is, if $\mathbf{u} \in N(A)$ and $\mathbf{v} \in C(A)$ then $\mathbf{u} \cdot \mathbf{v} = 0$.

Step-3

Here, the vector $(1,-2,1) \in N(A)$ and $(1,2,1) \in C(A)$.

Now find the dot product of the vectors $(1,-2,1) \in N(A)$ and $(1,-2,1) \in C(A)$.

$$\begin{aligned}
\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} &= 1(1) + (-2)2 + 1(1) \\
&= 1 - 4 + 1 \\
&= -2
\end{aligned}$$

As the dot product of these vectors is $-2 \neq 0$, so the vectors are not orthogonal.

Thus, the matrix whose nullspace contains $(1,-2,1)$ and the row space contains $(1,2,1)$ does not exist.

Hence, such a matrix **does not exist**.