Step-1

Consider the symmetric matrices $A=\begin{bmatrix}1 & 2 & 0\\2 & 6 & 4\\0 & 4 & 11\end{bmatrix}$ and $A=\begin{bmatrix}a & b\\b & c\end{bmatrix}$.

The objective is to decompose these matrices as LDL^T.

Step-2

Consider first matrix $A=\begin{bmatrix}1 & 2 & 0\\2 & 6 & 4\\0 & 4 & 11\end{bmatrix}$

Change this matrix by row operations into upper triangular matrix.

Add -2 times row 1 to row 2 then matrix will be;

$$\begin{bmatrix}1 & 2 & 0\\0 & 2 & 4\\0 & 4 & 11\end{bmatrix}$$

Add -2 times row 2 to row 3 then matrix will be;

$$\begin{bmatrix}1 & 2 & 0\\0 & 2 & 4\\0 & 0 & 3\end{bmatrix}$$

So this is an upper triangular matrix U.

Step-3

Now factor matrix U to get DU as;

$$\begin{aligned}U &= \begin{bmatrix}1 & 2 & 0\\0 & 2 & 4\\0 & 0 & 3\end{bmatrix}\\ &= \begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\0 & 1 & 2\\0 & 0 & 1\end{bmatrix}\\ &= DU\end{aligned}$$

Step-4

Since matrix A is symmetric so lower matrix will be transpose of upper matrix.

So,

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

So this lower triangular matrix L.

Thus, the symmetric factorization of matrix $A = LDL^T$ is;

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}$$

Step-5

Now take second symmetric matrix;

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

Now apply row operations to get upper triangular matrix.

Add $\left(-\frac{b}{a}\right)$ times row 1 to row 2;

$$\begin{bmatrix} a & b \\ 0 & c - \dfrac{b^2}{a} \end{bmatrix}$$

This is an upper triangular matrix.

Step-6

Now factor this matrix as;

$$\begin{bmatrix} a & b \\ 0 & c - \dfrac{b^2}{a} \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & c - \dfrac{b^2}{a} \end{bmatrix}\begin{bmatrix} 1 & \dfrac{b}{a} \\ 0 & 1 \end{bmatrix}$$

This is of form DU.

Since matrix A is symmetric so transpose of this upper triangular matrix U will give lower triangular matrix.

$$\begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{bmatrix}$$

Therefore, the factor is;

$$\begin{aligned} A &= \begin{bmatrix} a & b \\ b & c \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & c-\frac{b^2}{a} \end{bmatrix} \begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix} \\ &= LDU \\ &= LDL^T \end{aligned}$$

Hence, the symmetric factorization $A = LDL^T$ is $\boxed{\begin{bmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & c-\frac{b^2}{a} \end{bmatrix} \begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix}}$