

Step-1

Fibonacci sequence: Fibonacci sequence is given as follows:

0, 1, 1, 2, 3, 5, 8, 13, â€¦

Let the Fibonacci matrix be as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

To compute A^2 , A^3 and A^4 . Also, calculate F_{20} using the text and calculator.

Step-2

To compute A^2 , A^3 and A^4 do the following calculations:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A \cdot A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$A^3 \cdot A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

Step-3

Therefore, values of A^2 , A^3 and A^4 are as follows:

$$A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

Step-4

To calculate Fibonacci 20th number put $n = 20$ in the following:

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$

And solve it using calculator:

$$\begin{aligned} F_{20} &= \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^{20} - \left(\frac{1-\sqrt{5}}{2} \right)^{20} \right] \\ &= 6764.99 \\ &\approx 6765 \end{aligned}$$

Step-5

Therefore, $\boxed{F_{20} = 6765}$.