

Step-1

The give inequality is,

$$\max_x \min_y yAx = \min_y yAx^* \leq y^* Ax^* \leq \max_x y^* Ax = \min_y \max_x yAx$$

Step-2

In a payoff matrix A, the total expected payoff of X will be $\sum_j \sum_i x_j a_{ij} y_i$ where a_{ij} is the element of the matrix where option j is chosen by X and option i is chosen by Y, with the probabilities of x_j, y_i .

Step-3

As it is the payoff for X, it wants to maximize this payoff yAx but the player Y wants to minimize it. Hence, player Y will minimize the value of $\max_x yAx$.

Similarly, X will maximize the value of $\min_y yAx$.

The maximum of the minimized value of $\min_y yAx$ will be smaller than or equal to minimum of maximized value of $\max_x yAx$.

In between these two values there will a point which is optimal for both X and Y, that is, $y^* Ax^*$.

This can be written as $\min_y yAx^* \leq y^* Ax^* \leq \max_x y^* Ax$.

Using minimax theorem we get,

$$\max_x \min_y yAx = \min_y \max_x yAx$$

Using these two equations we get,

$$\max_x \min_y yAx = \min_y yAx^* \leq y^* Ax^* \leq \max_x y^* Ax = \min_y \max_x yAx$$

Step-4

There will be optimal values of x and y known as x^* and y^* .

The maximum value player X wants to attain is yAx^* .

The minimum value payer Y wants to attain is $y^* Ax$.

Hence, the equation $\min_y yAx^* \leq y^* Ax^* \leq \max_x y^* Ax$ can be written as

$$y^* Ax \leq y^* Ax^* \leq yAx^*$$

