

## Step-1

We claim that the subspace spanned by the  $E$  is the subspace of 3 by 3 lower triangular matrices with the same entry along the diagonal.

$$\begin{bmatrix} p & 0 & 0 \\ q & p & 0 \\ r & s & p \end{bmatrix}$$

A 3 by 3 lower triangular matrix with the same entry along the diagonal is equal to  $\begin{bmatrix} p & 0 & 0 \\ q & p & 0 \\ r & s & p \end{bmatrix}$ . Let us show that any such matrix can be expressed as a linear combination of the matrices:  $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix},$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

## Step-2

Consider the following:

$$\begin{bmatrix} p & 0 & 0 \\ q & p & 0 \\ r & s & p \end{bmatrix} = \frac{q}{a} \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{r}{b} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix} + \frac{s}{c} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix} - \left( \frac{q}{a} + \frac{r}{b} + \frac{s}{c} - p \right) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, any lower triangular matrix with same number along the diagonal can be expressed as a linear combination of  $\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix},$  and  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

## Step-3

Therefore, the subspace spanned is the space of lower triangular matrices with same entry along the diagonal elements.