Step-1

The singular value decomposition , $U\Sigma V^T$ expresses A as a sum of r rank-1 matrices because of the block form of multiplication.

If m by p matrix A has r row partitions and s column partitions and p by n matrix B has s row partitions and t column partitions then the block partitioned matrix product can given by m by n matrix C, which has r row partitions and t column partitions.

We know that the block form of multiplication is fact, now the key is in singular values in the Σ.

We have to show how Σ effect it.

Step-2

If the number of columns is more than rows, then by multiplication of Σ reset the rows of the matrix V and remove the bottom ones.

Similarly, if the number of rows is more than columns, then by multiplication of Σ reset the columns of U and remove the the last ones.

Step-3

Using the columns of U times rows of ΣV^T, we get

$$\begin{aligned} U\Sigma V^T &= \begin{bmatrix} u_1 & \cdots & u_r \end{bmatrix} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \begin{bmatrix} v_1 & \cdots & v_r \end{bmatrix}^T \\ &= u_1\sigma_1 v_1^T + \cdots + u_r\sigma_r v_r^T \end{aligned}$$

So, we have found $U\Sigma V^T$ as a sum of r rank-1 matrices $u_1\sigma_1 v_1^T + \cdots + u_r\sigma_r v_r^T$.