

## Step-1

We consider a  $5 \times 5$  matrix by name  $A$ .

A cofactor of each entry  $a_{ij}$  of  $A$  is nothing but the determinant  $B_{ij}$  of  $4 \times 4$  sub matrix obtained by leaving the  $i^{\text{th}}$  row and  $j^{\text{th}}$  column in which  $a_{ij}$  exists.

Also,  $B_{ij}$  is to be multiplied with  $(-1)^{i+j}$  and the entire matrix is to be transposed to get the adjoint of the given matrix  $A$ .

While there are 25 entries in  $A$ , we see that there are 25 cofactors in the adjoint matrix.

## Step-2

Again, each  $B_{ij}$  is a  $4 \times 4$  matrix and so, it has 16 entries.

Leaving each entry  $B_{ij}$  and the row and column in which it is standing, we get a sub matrix  $C_{ij}$  of order  $3 \times 3$  whose determinant is the cofactor of  $B_{ij}$ .

Also, we have to multiply each cofactor with  $(-1)^{i+j}$  where  $i$  is the number of row and  $j$  is the number of column in which the entry standing.

## Step-3

The above discussion repeats and each  $3 \times 3$  sub matrix has nine  $2 \times 2$  matrices of the form  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  whose determinant is  $ad - bc$  is the cofactor of each entry in the  $3 \times 3$  matrix.

Each entry has to be multiplied with  $(-1)^{i+j}$  as discussed above.

We observe that  $(-1)^{i+j}(ad - bc)$  requires one additive operation and three multiplications.

One column or row of any  $3 \times 3$  sub matrix is enough to calculate the determinant of the sub matrix of size  $3 \times 3$

## Step-4

In a column, there will be 3 entries and so, 3 additions and 9 multiplications are required.

In a  $4 \times 4$  matrix, there will be 4 entries in a column or a row and so, 4 times the above process is required. So, the number of operations is  $4 \times 3$  additions and  $4 \times 9$  multiplication.

In a  $5 \times 5$  matrix, there will be 5 entries in a column or a row and so,  $5 \times 4 \times 3$  additions and  $5 \times 4 \times 9$  multiplications are required

So, the total number of calculations required to find the determinant of the  $5 \times 5$  matrix is 240.

## Step-5

On the other hand, each  $4 \times 4$  sub matrix of the given matrix requires  $4 \times 3 = 12$  additions and  $4 \times 9 = 36$  multiplications.

So, all the 25 entries require  $25(12+36) = 1200$  calculations.

Transposing requires 10 operations.

Putting all the number of calculations and operations together we require 1440 additive and multiplicative operations and 10 entry replacing operations to find the inverse of the  $5 \times 5$  matrix.

So, when compared to the Gauss-Jordan method to find the inverse, this method requires considerably more operations.