Step-1

Consider the triangular system,

$$\left. \begin{array}{r} u - v - w = b_1 \\ v + w = b_2 \\ w = b_3 \end{array} \right\} \quad \ldots\ldots (1)$$

The objective is to solve for the elements of b using back substation method.

Step-2

From the system, the last equation is $w = b_3$

Substitute $w = b_3$ in second equation $v + w = b_2$

$$v + b_3 = b_2$$
$$v = b_2 - b_3$$

Substitute $w = b_3, v = b_2 - b_3$ in first equation to find u.

$$u - (b_2 - b_3) - b_3 = b_1$$
$$u - b_2 + b_3 - b_3 = b_1$$
$$u - b_2 = b_1$$
$$u = b_1 + b_2$$

Step-3

Column form of system (1) can be written as.

$$u \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + v \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + w \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Substitute u, v and w in the column form that gives the linear combination of the vectors of b.

Hence, the column combination as in b is

$$\boxed{(b_1 + b_2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + (b_2 - b_3) \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + (b_3) \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}$$

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