Step-1

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Given matrix is $A=\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$

We have to find the inverse of the given matrix.

Step-2

We use Gauss-Jordan elimination process to find the inverse of the given matrix.

Consider

$$\begin{bmatrix} A & e_1 & e_2 & e_3 & e_4 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 & 0 & 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-3

Subtracting $\frac{1}{4}$ times row 1 from row 2, $\frac{1}{3}$ times row 1 from row 3 and

Subtracting $\frac{1}{2}$ times row 1 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & \frac{1}{3} & 1 & 0 & -\frac{1}{3} & 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 1 \end{bmatrix}$$

Step-4

Subtracting $\frac{1}{3}$ times row 2 from row 3 and $\frac{1}{2}$ times row 2 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 1 & -\frac{3}{8} & -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

Step-5

Subtracting $\frac{1}{2}$ times row 3 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{4} & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{3} & -\frac{1}{2} & 1 \end{bmatrix}$$

Hence the inverse of the given matrix is $\boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{4} & 1 & 0 & 0 \\ -\frac{1}{4} & -\frac{1}{3} & 1 & 0 \\ -\frac{1}{4} & -\frac{1}{3} & -\frac{1}{2} & 0 \end{bmatrix}}$