Step-1

Consider the following:

$$\begin{aligned}\|ABx\| &\le \|A\|\|Bx\| \\ &\le \|A\|\|B\|\|x\|\end{aligned}$$

$$\frac{\|ABx\|}{\|x\|} \le \|A\|\|B\|$$

Maximizing, we get $\|AB\| \le \|A\|\|B\|$.

Step-2

We have $\|AB\| \le \|A\|\|B\|$. Let $A = B$.

This gives,

$$\|BB\| \le \|B\|\|B\|$$
$$\|B^2\| \le \|B\|^2$$

Suppose, for some positive integer m, we have

$$\|B^m\| \le \|B\|^m$$

This gives,

$$\|BB^m\| \le \|B\|\|B\|^m$$
$$\|B^{m+1}\| \le \|B\|^{m+1}$$

Step-3

Therefore, from induction, it follows that for any integer k, $\boxed{\|B^k\| \le \|B\|^k}$.