

Step-1

Let P_1 = the projection of b onto the line through $a_1 = \frac{a_1^T b}{a_1^T a_1} a_1$

$$\begin{aligned}a_1^T b &= (1 \ 0) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\&= 1 + 0 \\&= 1\end{aligned}$$

$$\begin{aligned}a_1^T a_1 &= (1 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\&= 1 + 0 \\&= 1\end{aligned}$$

$$\begin{aligned}P_1 &= \frac{1}{1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\&= \begin{pmatrix} 1 \\ 0 \end{pmatrix}\end{aligned}$$

Step-2

Let P_2 = the projection of b onto the line through $a_2 = \frac{a_2^T b}{a_2^T a_2} a_2$

$$\begin{aligned}a_2^T b &= (1 \ 2) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\&= 1 + 2 \\&= 3\end{aligned}$$

$$\begin{aligned}a_2^T a_2 &= (1 \ 2) \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\&= 1 + 4 \\&= 5\end{aligned}$$

$$\begin{aligned}P_2 &= \frac{3}{5} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\&= \begin{pmatrix} 3/5 \\ 6/5 \end{pmatrix}\end{aligned}$$

Step-3

$$P_1 + P_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3/5 \\ 6/5 \end{pmatrix} \\ = \begin{pmatrix} 8/5 \\ 6/5 \end{pmatrix}$$

$P_1 + P_2 \neq b$ because a_1, a_2 are not orthogonal

This is established with the help of $a_1^T a_2 = (1 \ 0) \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$= 1 + 0$$

$$= 1 \neq 0$$

