Step-1

(a) For any *A, b, x* and *y,* if $Ax = b$ and $y^T A = 0$, then we have to show that $y^T b = 0$.

Now

$$\begin{aligned} y^T b &= y^T (Ax) \\ &= (y^T A)x \\ &= 0.x \\ &= 0 \end{aligned}$$

Therefore, $y^T b = 0$

Hence *y* is perpendicular to *b*

Step-2

(b) For any *A, b, x* and *y,* if $Ax = 0$ and $A^T y = b$, then we have to show that $x^T b = 0$, and we have to find that what theorem does this prove about the fundamental subspaces.

Now

$$\begin{aligned} x^T b &= x^T (A^T y) \\ &= (x^T A^T) y \\ &= (Ax)^T .y \end{aligned}$$

Step-3

$$\begin{aligned} &= 0^T y \\ &= 0.y \\ &= 0 \end{aligned}$$

Therefore, $x^T b = 0$

Hence *x* is perpendicular to *c.*

Step-4

Here we used the theorem (1): $(AB)^T = B^T A^T$, and theorem (2): $(AB)C = A(BC)$

to prove the above results.