

Step-1

If P is a matrix, λ is the eigen value and x is the respective eigen vector, then we have $Px = \lambda x$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

In view of this, we consider (1)

We consider the characteristic equation of this matrix $|P - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda(\lambda^2 - 0) - 1(-1) + 0$$

$$\Rightarrow -\lambda^3 + 1 = 0$$

$$\Rightarrow (\lambda - 1)(\lambda^2 + \lambda + 1) = 0$$

$$\Rightarrow \lambda = 1 \text{ and } \lambda^2 + \lambda + 1 = 0$$

$$\lambda = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$$

$$\lambda = 1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$$

The other eigen values are $\lambda = \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$

Step-2

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Similarly, we consider $(P - \lambda I) = 0$ for the matrix

$$\Rightarrow \begin{vmatrix} -\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda(-\lambda(1-\lambda))-0+1(-1-\lambda)=0$$

$$\Rightarrow -\lambda(-\lambda+\lambda^2)-1+\lambda=0$$

$$\Rightarrow \lambda^2-\lambda^3-1+\lambda=0$$

$$\Rightarrow \lambda^3-\lambda^2+1=0$$

$$\Rightarrow \lambda^2(\lambda-1)-1(\lambda-1)=0$$

$$\Rightarrow (\lambda^2-1)(\lambda-1)=0$$

$$\Rightarrow \lambda=1, \lambda^2-1=0$$

$$\Rightarrow \lambda=\pm 1$$

$$\Rightarrow \lambda=1, 1, -1$$

The remaining λ values are $-1, 1$.