Step-1

Let $x=(x_1,x_2,x_3,...,x_n)$

$$x^T=\begin{pmatrix}x_1\\x_2\\-\\-\\-\\x_n\end{pmatrix}$$

Suppose $x^T y=0$ for every y

Step-2

Let $y_1=(1,0,0,...,0)$

$x^T y_1=0\Rightarrow$

$$\begin{pmatrix}x_1\\x_2\\-\\-\\-\\x_n\end{pmatrix}(1,0,0,...,0)=0$$

$\Rightarrow \quad x_1=0$

Step-3

Similarly, $y_2=(0,1,...,0)$ is such that $x^T y_2=0$ gives $x_2=0$

In the same way, $x_3=x_4=x_5=...=x_n=0$

Putting these things together, we can say $x^T y=0$ for every y, then $x=0$