## Step-1

Stability status: Stability depends on the Eigen values. The difference equation $u_{k+1} = Au_k$ will have following stability status corresponding to the Eigen values:

*Stable*: If all Eigen values are $|\lambda_i| < 1$.

*Neutrally stable*: If some $|\lambda_i| < 1$ and some are $|\lambda_i| = 1$

*Unstable*: If atleast one Eigen value has $|\lambda_i| > 1$.

## Step-2

To find the value of $\alpha$ which produces instability in the following:

$$\begin{aligned} v_{n+1} &= \alpha v_n + \alpha w_n \\ w_{n+1} &= \alpha v_n + \alpha w_n \end{aligned}$$

## Step-3

Above defined equations can be converted into the following equation of matrices:

$$u_{k+1} = Au_k$$

Here, matrices $u_{k+1}, A, u_k$ are defined as follows:

$$A = \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \end{bmatrix}$$

$$u_{k+1} = \begin{bmatrix} v_{n+1} \\ w_{n+1} \end{bmatrix}$$

$$u_k = \begin{bmatrix} v_n \\ w_n \end{bmatrix}$$

## Step-4

Calculate the Eigen values of matrix $A$ by doing following calculations:

$$\begin{aligned} \det\left(A - \lambda I\right) &= 0 \\ \alpha^2 + \lambda^2 - 2\alpha\lambda - \alpha^2 &= 0 \\ \lambda\left(\lambda - 2\alpha\right) &= 0 \\ \lambda &= 0 \end{aligned}$$

Or $\lambda = 2\alpha$

## Step-5

Difference equation $u_{k+1} = Au_k$ will have following stability status:

Stable: If $|\lambda_i| < 1$ or

$$|2\alpha| < 1$$
$$|\alpha| < \frac{1}{2}$$

Unstable: If $|\lambda_i| > 1$ or

$$|2\alpha| > 1$$
$$|\alpha| > \frac{1}{2}$$

## Step-6

Therefore, value of $\boxed{|\alpha| > \frac{1}{2}}$ gives the instability in $u_{k+1} = Au_k$.