Step-1

Given system is

$$2x+y=0$$
$$x+2y+z=0$$
$$y+2z+t=0$$
$$z+2t=5$$

We have to find the fifth pivot and the nth pivot by finding the first four pivots of the system.

Step-2

Given system can be written as

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{pmatrix}$$

Subtract â€˜ $\frac{1}{2}$ â€™ times the row 1 from the row 2

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{pmatrix}$$

Step-3

Subtract â€˜ $\frac{2}{3}$ â€™ times the row 2 from the row 3.

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{pmatrix}$$

Step-4

Subtract â€˜ $\frac{3}{4}$ â€™ times the row 2 from the row 3.

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & 1 & 0 \\ 0 & 0 & 0 & \frac{5}{4} & 5 \end{pmatrix}$$

This is upper triangular form.

Step-5

The pivots are boxed in the following matrix:

$$\begin{pmatrix} \boxed{2} & 1 & 0 & 0 & 0 \\ 0 & \boxed{\frac{3}{2}} & 1 & 0 & 0 \\ 0 & 0 & \boxed{\frac{4}{3}} & 1 & 0 \\ 0 & 0 & 0 & \boxed{\frac{5}{4}} & 5 \end{pmatrix}$$

And the difference between the numerator and the denominator in the fractions (pivots) is 1, so the fifth pivot is $\frac{6}{5}$ and the nth pivot is $\boxed{\frac{n+1}{n}}$