Step-1

Consider that ***P*** is the plane in 3-space with equation $x+2y+z=6$.

The objective is to find the equation of the plane $\mathbf{P}_0$ through the origin parallel to **P.**

Further objective is to verify whether **P** and $\mathbf{P}_0$ are subspaces of $\mathbb{R}^3$.

Step-2

Recall the following;

Let $ax+by+cz=d$ be the plane. Then, the equation of the plane $\mathbf{P}_0$ through the origin parallel to P is given as follows;

$\vec{r}\cdot\vec{n}=0$.

Here, $\vec{n}$ is quite normal to the plane ***P.***

And $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$.

Step-3

Here, given plane equation is $x+2y+z=6$.

So, $\vec{n}=\vec{i}+2\vec{j}+\vec{k}$, and $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$.

Step-4

Now use the result discussed above and write the equation of plane $\mathbf{P}_0$ through the origin parallel to P as follows;

$$\vec{r}\cdot\vec{n}=0$$
$$\Rightarrow x+2y+z=0$$

Therefore, equation of plane $\mathbf{P}_0$ through the origin parallel to P is $\boxed{x+2y+z=0}$.

Step-5

Note that, the points $(1,2,1),(2,1,2)\in\mathbf{P}$,as these points satisfy the equation $x+2y+z=6$

$(1,2,1)+(2,1,2)=(3,3,3)\notin\mathbf{P}$.

Since, $(3,3,3)$ does not satisfy the equation of plane $x+2y+z=6$.

So, the vector addition is not closed in **P.**

Therefore, **P** is not a subspace of $\mathbb{R}^3$.

Step-6

Note the following;

$(0,0,0)\in \mathbf{P}_0$ as it satisfies the equation of plane $x+2y+z=0$.

So, $\mathbf{P}_0$ is non-empty and contains the additive identity.

Next, suppose $(x_1,y_1,z_1),(x_2,y_2,z_2)\in \mathbf{P}_0$ and verify the following;

$$\Rightarrow x_1+2y_1+z_1=0, \quad x_2+2y_2+z_2=0$$
$$\Rightarrow (x_1+x_2)+2(y_1+y_2)+(z_1+z_2)=0$$
$$\Rightarrow (x_1+x_2,\ y_1+y_2,\ z_1+z_2)\in \mathbf{P}_0$$

Step-7

Next, verify that, $c(x_1,y_1,z_1)\in \mathbf{P}_0$ for any scaler c as follows;

$$cx_1+2cy_1+cz_1=0$$
$$c(x_1+cy_1+z_1)=0 \qquad \{As,(x_1,y_1,z_1)\in P_0\}$$

This Implies, $(cx_1,cy_1,cz_1)\in \mathbf{P}_0 \Rightarrow c(x_1,y_1,z_1)\in \mathbf{P}_0$.

Therefore $\mathbf{P}_0$ is a subspace of $\mathbb{R}^3$.