Step-1

Given that the first of these equations plus the second equals to the third:

$$x+y+z=2$$
$$x+2y+z=3$$
$$2x+3y+2z=5$$

And given that the fist two planes meet along a line. If x,y,z satisfy the first two equations then we have to find that the plane in which they lie. And we have to find the three solutions of the infinite number of solutions of the given system.

Step-2

Form the given planes; the third plane occurs by the sum of the first two planes, so if x,y,z satisfy the first two equations then they will lies on the third plane.

Step-3

The given three planes can be written as

$$\begin{bmatrix}1&1&1\\1&2&1\\2&3&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\3\\5\end{bmatrix}$$

Apply $R_3 \to R_3-(R_1+R_2)$

$$\begin{bmatrix}1&1&1\\1&2&1\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\3\\0\end{bmatrix}$$

Step-4

Apply $R_2 \to R_2-R_1$

$$\begin{bmatrix}1&1&1\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\0\end{bmatrix}$$

Apply $R_1 \to R_1-R_2$

$$\begin{bmatrix}1&0&1\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}$$

Step-5

$\Rightarrow x+z=1$
$y=1$

Put $z=k$

$\Rightarrow x=1-k$

Step-6

Therefore the solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-k \\ 1 \\ k \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

For $k=1$, $(x,y,z)=(0,1,1)$

Therefore the three solution of the given equations are $\boxed{(1,1,0),(-1,0,1),(0,1,1)}$