

Step-1

Matrix A can be factorized into product of lower and upper triangular matrices.

$$A = LU$$

Here, matrix L is a lower triangular matrix with 1 at the diagonal position and matrix U is the upper triangular matrix with pivots at the diagonal position.

Step-2

(a) The third row of matrix U is the third pivot that comes from the third row of matrix A by subtracting multiples of row 1 and 2 of matrix U .

Row 3 of matrix $U = \text{row 3 of } A - l_{31}(\text{row 1 of matrix } U) - l_{32}(\text{row 2 of matrix } U)$.

Here, rows of matrix U is subtracted and not rows of matrix A . Because by the time a pivot row is used, original rows of matrix A are changed by elimination.

Step-3

(b) Above equation can be written as follows:

$$\text{Row 3 of matrix } A = (\text{row 1 of matrix } U) + l_{32}(\text{row 2 of matrix } U) + (\text{row 3 of } U)$$

Here, multiplication seems to be done row wise, rows of matrix L is multiplied by rows of matrix U , however row 3 of matrix A is obtained by row times column multiplication rule. Actual multiplication after solving will give the same result as the following equation:

$$\text{Row 3 of matrix } A = (\text{row 3 of matrix } L) \cdot (\text{column 1 of } U),$$

$$(\text{row 3 of matrix } L) \cdot (\text{column 2 of matrix } U),$$

$$(\text{row 3 of matrix } L) \cdot (\text{column 3 of } U).$$

Therefore, matrix multiplication rule followed is row times column.