Step-1

(a)

Consider the following matrix.

$$A=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix}$$

Real Eigen values of the matrix A is 0 only.

Eigen vector corresponding to the Eigen value 0 is $(1,0,0)$.

Therefore, the only Eigen vectors of a matrix A are multiples of the vectors $x=(1,0,0)$.

Note that matrix A is singular matrix.

That is, matrix A is **not invertible**.

Step-2

Now consider the following matrix.

$$A=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix}$$

The only Real Eigen values of the matrix A is 1.

Eigen vector corresponding to the Eigen value 1 is $(1,0,0)$.

Therefore, the only Eigen vectors of a matrix A are multiples of the vectors $x=(1,0,0)$.

Note that matrix A is non-singular matrix.

That is, matrix A is **invertible**.

Therefore, there are **invertible** as well as **not invertible** matrices so that the only Eigen vectors of the matrix are multiple of the vector $x=(1,0,0)$.

So, given statement is False.

Step-3

(b)

Consider the following matrix.

$$A=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix}$$

The Eigen values of the matrix A are $0, \pm i$.

Eigen vector corresponding to the Eigen value 0 is $(1,0,0)$.

Therefore, the only Eigen vectors of a matrix A are multiples of the vectors $x=(1,0,0)$.

Note that matrix A has **no repeated Eigen value**.

Step-4

Now consider the following matrix.

$$A=\begin{pmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}$$

The Eigen values of the matrix A are 1, 1, 1.

Eigen vector corresponding to the Eigen value 1 is $(1,0,0)$.

Therefore, the only Eigen vectors of a matrix A are multiples of the vectors $x=(1,0,0)$.

Note that matrix A has **a repeated Eigen value**.

Therefore, there are matrices which have **no repeated Eigen value** as well as **a repeated Eigen value** with the property that the only Eigen vectors of a matrix are multiples of the vectors $x=(1,0,0)$.

So, given statement is False.

Step-5

(c)

Given that the only Eigen vectors of a matrix A are multiples of the vectors $x=(1,0,0)$, so Eigen space of the matrix A is

$\{c(1,0,0): c\in\mathbb{R}\}$

Therefore, dimension of Eigen space of the matrix A is 1.

That means the Eigen space of the matrix A does not span $\mathbb{R}^3$.

Hence, matrix A is not diagonalizable.

Therefore, the given statement is true.