Step-1

Given that $A=\begin{bmatrix}2 & -2\\ 2 & -2\end{bmatrix}$ is a vector in the space **M** of all 2 by 2 matrices.

We have to write the zero vector in this space.

Step-2

Given that,

$$M=\left\{\begin{bmatrix}a & b\\ c & d\end{bmatrix}\middle| a,b,c,d\in\mathbf{R}\right\}$$

Let,

$$A=\begin{bmatrix}2 & -2\\ 2 & -2\end{bmatrix}$$

If we take matrix addition is the vector addition of the space **M**.

Then, $D=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ is the zero vectors in **M.**

Step-3

Since,

$$\begin{aligned}D+\begin{bmatrix}a & b\\ c & d\end{bmatrix}&=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}+\begin{bmatrix}a & b\\ c & d\end{bmatrix}\\ &=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\end{aligned}$$

Similarly,

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Thus the zero vector in the space **M** is $D=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

Step-4

We have to find $\frac{1}{2}A$ and $-A$.

Now

$$\begin{aligned}\frac{1}{2}A &= \frac{1}{2}\begin{bmatrix}2 & -2\\ 2 & -2\end{bmatrix}\\ &= \begin{bmatrix}\frac{1}{2}\cdot(2) & \frac{1}{2}\cdot(-2)\\ \frac{1}{2}\cdot(2) & \frac{1}{2}\cdot(-2)\end{bmatrix}\\ &= \begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}\end{aligned}$$

Therefore, $\boxed{\frac{1}{2}A = \begin{bmatrix}1 & -1\\ 1 & -1\end{bmatrix}}$

Step-5

And

$$\begin{aligned}-A &= (-1)A\\ &= \begin{bmatrix}(-1)2 & (-1)(-2)\\ (-1)2 & (-1)(-2)\end{bmatrix}\\ &= \begin{bmatrix}-2 & 2\\ -2 & 2\end{bmatrix}\end{aligned}$$

Therefore, $\boxed{-A = \begin{bmatrix}-2 & 2\\ -2 & 2\end{bmatrix}}$

Step-6

The smallest subspace containing

$$\begin{aligned}A &= \{rA \mid r \in B\}\\ &= \left\{\begin{bmatrix}2r & -2r\\ 2r & -2r\end{bmatrix} \mid r \in \mathbf{R}\right\}\end{aligned}$$

If $r = 1$, the matrix $\begin{bmatrix}2r & -2r\\ 2r & -2r\end{bmatrix} = \begin{bmatrix}2 & -2\\ 2 & -2\end{bmatrix}$

The smallest subspace containing $A = \{rA \mid r \in R\}$