Step-1

Given

$$S_1 = |3|,\ S_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix},\ S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

Now changing 3 to 2 in the upper left corner of these matrices, we get

$$A_1 = |2|$$

$$\begin{aligned} A_2 &= \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \\ &= 6 - 1 \\ &= \boxed{5} \end{aligned}$$

Step-2

Now

$$\begin{aligned} A_3 &= \begin{vmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix} \\ &= 2\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \end{aligned}$$

$$= 2\left[\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix}\right] - \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix}$$

$$\begin{aligned} &= 2(5) + 2(3) - 3 \\ &= 16 - 3 \\ &= \boxed{13} \end{aligned}$$

Step-3

In general

$$S_n = \begin{vmatrix} 2 & 1 & 0 & \ldots\ldots 0 \\ 1 & \mathrm{S}_{n-1} & & \\ \vdots & & & \end{vmatrix} + \begin{vmatrix} 1 & 0 & \ldots\ldots 0 \\ 1 & \mathrm{S}_{n-1} & \\ \vdots & & \end{vmatrix}$$

$(\text{by linearity on row } 1)$

$|S_1| = |A_n| + |S_{n-1}|$

$|A_n| = |S_n| - |S_{n-1}|$

$$\begin{aligned}|A_n| &= F_{2n+2} - F_{2n} \\ &= F_{2n+1}\end{aligned}$$

Step-4

We have in Fibonacci series

$$\begin{aligned}F_k &= F_{k-1} + F_{2n-1} \\ &= F_{2n} + F_{2n-1}\end{aligned}$$

So

$$\begin{aligned}F_{2n-1} &= F_{2n} + F_{2n-1} \\ &= F_{2n} + F_{2n} - F_{2n-2} \\ &= 2F_{2n} - F_{2n-2}\end{aligned}$$

Step-5

As in earlier problem we can note that

$A_n = F_{2n+1}$ For each n

So when problem 31 gives all even terms

(Starting from 4^{th} term) of Fibonacci sequence $(1,1,2,3,5,8,13,21.......)$ gives all odd terms (starting from 3^{rd} term) of Fibonacci sequence $(1,1,2,3,5,8,13,21,......)$