Step-1

Given that $A=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}$

We have

$$A=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}$$

$$\Rightarrow A^T=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$$

$$B=\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}$$

$$\Rightarrow B^T=\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}$$

Step-2

$$A.B=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}.\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}$$

$$A.B=\begin{pmatrix}1 & 3\\ 2 & 7\end{pmatrix}$$

$$\Rightarrow (A.B)^T=\begin{pmatrix}1 & 2\\ 3 & 7\end{pmatrix}$$

Step-3

$$B^T.A^T=\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}.\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1 & 2\\ 3 & 7\end{pmatrix}$$

$$=(A.B)^T$$

$$A^T.B^T = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$$
$$\neq (A.B)^T$$

Step-4

$$B^T.A^T = (A.B)^T$$
$$= (B.A)^T$$
$$= A^T.B^T \quad (\because AB = BA)$$