Step-1

Now consider the following:

$$\begin{aligned} A^{+}b &= \underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\left(\underline{L}^{\mathrm{T}}\underline{L}\right)^{-1}\underline{L}^{\mathrm{T}}b \\ &= \underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}b \\ &= \underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}\right)b \end{aligned}$$

$$\begin{aligned} A^{+}b &= \underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}b \\ &= \underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\underline{U}^{-1}\underline{L}^{-1}b \\ &= \left(\underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\right)\underline{U}^{-1}\underline{L}^{-1}b \end{aligned}$$

$$\begin{aligned} A^{+}b &= \underline{U}^{-1}\underline{L}^{-1}b \\ &= \left(\underline{L}\underline{U}\right)^{-1}b \\ &= A^{-1}b \end{aligned}$$

Therefore, it is clear that $A^{+}b$ is in the row space.

Step-2

Consider next:

$$\begin{aligned} A^{T}AA^{+}b &= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\left(\underline{L}^{\mathrm{T}}\underline{L}\right)^{-1}\underline{L}^{\mathrm{T}}b \\ &= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}b \\ &= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}\left(\left(\underline{L}^{\mathrm{T}}\right)^{-1}\underline{L}^{\mathrm{T}}\right)b \end{aligned}$$

$$\begin{aligned} A^{T}AA^{+}b &= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}\underline{U}^{\mathrm{T}}\right)^{-1}\underline{L}^{-1}b \\ &= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\underline{U}^{-1}\underline{L}^{-1}b \\ &= A^{T}\left(\underline{L}\underline{U}\right)\left(\underline{U}^{\mathrm{T}}\left(\underline{U}^{\mathrm{T}}\right)^{-1}\right)\underline{U}^{-1}\underline{L}^{-1}b \end{aligned}$$

$$\begin{aligned} A^{T}AA^{+}b &= A^{T}\left(\underline{L}\underline{U}\right)\underline{U}^{-1}\underline{L}^{-1}b \\ &= A^{T}\underline{L}\left(\underline{U}\underline{U}^{-1}\right)\underline{L}^{-1}b \\ &= A^{T}\underline{L}\underline{L}^{-1}b \\ &= A^{T}b \end{aligned}$$

Step-3

Therefore, we have shown that $\boxed{A^T AA^+ b = A^T b}$.