Step-1

Consider the plane $x-2y+3z=0$ in R^3.

The plane is the null space of the following matrix:

$$A=\begin{bmatrix}1 & -2 & 3\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

To find the null space of matrix *A*, solve the following system of equations:

$$\begin{aligned}x-2y+3z&=0\\0&=0\\0&=0\end{aligned}$$

This system has infinitely many solutions.

$$\begin{aligned}x&=2y-3z\\y&=s\\z&=t\end{aligned}$$

Here *s* and *t* are any real numbers.

Step-2

The solutions can be written in vector form as follows:

$c_1=\begin{bmatrix}2\\1\\0\end{bmatrix}$ and $c_2=\begin{bmatrix}-3\\0\\1\end{bmatrix}$

Step-3

Therefore, a basis for the null space of matrix *A* or the plane $x-2y+3z=0$ is

$\boxed{c_1=\begin{bmatrix}2\\1\\0\end{bmatrix}}$ and $\boxed{c_2=\begin{bmatrix}-3\\0\\1\end{bmatrix}}$

Step-4

The intersection of the plane $x-2y+3z=0$ with the *xy*-plane contains c_1 but it does not contain c_2, so we get the intersection as a line.

We know that, c_1 lies in the *xy*-plane. Hence, c_1 lies on the line.

Therefore, the basis for the intersection of the plane with the *xy*-plane is

Step-5

$$\boxed{c_1=\begin{bmatrix}2\\1\\0\end{bmatrix}}$$

Step-6

By using cross product we will find the vector c_3, which is perpendicular to the vectors c_1 and c_2.

$$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 1 & 0\\ -3 & 0 & 1\end{vmatrix}=\mathbf{i}(1-0)-\mathbf{j}(2-0)+\mathbf{k}(0-(-3))$$
$$=\mathbf{i}-2\mathbf{j}+3\mathbf{k}$$

Or

$$c_3=\begin{bmatrix}1\\-2\\3\end{bmatrix}$$

So, vector c_3 is perpendicular to the plane.

Therefore, a basis for all vectors perpendicular to the plane is

$$\boxed{c_3=\begin{bmatrix}1\\-2\\3\end{bmatrix}}$$