Step-1

Given that n columns of S (eigen vectors of A) are independent.

a) We can see that the eigen vector corresponding to the eigen value $\lambda = 0$ is also non zero and can be linearly independent with the other eigen vectors of A.

So, the matrix S whose columns are eigen vectors of A may be invertible.

But due to the zero eigen value of A, we follow that the determinant of A is the product of eigen values of $A = 0$

So, A is not invertible.

Therefore, the linear independence of eigen vectors cannot confirm the invertibility of A.

Step-2

b) If the eigen vectors of A are the linearly independent columns of the matrix S, then S is non singular and so invertible such that $S^{-1}AS = \Lambda$ the diagonal matrix whose diagonal entries are the eigen values of A.

Thus, A is diagonalizable.

So, the given statement is true.

Step-3

c) A is diagonalizable then the diagonal matrix Λ exists such that

$\Lambda = SAS^{-1}$

Hence S^{-1} is also exists.

Therefore S is invertible.

Step-4

d) If S is a matrix whose columns are linearly independent, then S is non singular.

This confirms that the eigen values of S are non zero.

But it is not necessary that the sum of the algebraic multiplicities of the eigen values is equal to the sum of the geometric multiplicities.

So, S is not necessarily invertible.

So, the statement is false.