Step-1

Consider P is a projection matrix.

The projection matrix has two properties:

1. P is a symmetric matrix.

2. Its square is itself: $P^2 = P.$

Step-2

Use the second condition to show that P is not invertible.

Consider,

$P^2 = P$

Take determinant on both sides

$\det P^2 = \det P$

$\det^2 P - \det P = 0$

$\det P(\det P - 1) = 0$

$\det P = 0$ or $\det P = 1.$

Step-3

Case1: If $\det P = 0.$ In this case P is called singular matrix.

Thus, P is not invertible.

Step-4

Case2: If $\det P = 1.$

In this case P is called non-singular matrix.

Thus, P is invertible.

So, clearly projection matrices are usually singular.

Therefore, projection matrix P is not invertible.