

## Step-1

We have to decide whether the following systems are singular or nonsingular, and whether they have no solution, one solution or infinitely many solutions:

$$\begin{array}{lll} v-w=2 & v-w=0 & v+w=1 \\ u-v=2 & u-v=0 & u+v=1 \\ u-w=2, u-w=0, \text{ and } & & u+w=1 \end{array}$$

## Step-2

Consider the system

$$\begin{array}{l} v-w=2 \\ u-v=2 \\ u-w=2 \end{array}$$

Converting the given equations into  $Ax=b$  form gives

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

## Step-3

First coefficient is zero, so it needs row exchange hence exchange row 1 and row 2, then

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

Subtracting row 1 from row 3, we have

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

## Step-4

Subtracting row 2 from row 3 gives

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$$

From the above the last equation is  $0 = -2$ ,

So the system has no solution, and hence it is singular.

## Step-5

Consider the second system

$$v - w = 0$$

$$u - v = 0$$

$$u - w = 0$$

Converting the given equations into  $Ax = b$  form gives

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

## Step-6

First coefficient is zero, so it needs row exchange hence exchange row 1 and row 2, then

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Subtracting row 1 from row 3 gives

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

## Step-7

Subtracting row 2 from row 3 gives

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Now from the system, we obtained the last equation as  $0 = 0$  and no further elimination is possible, so we have two equations in three variables.

Hence the system has infinitely many solutions, and hence it is nonsingular.

## Step-8

Consider the third system

$$v + w = 1$$

$$u + v = 1$$

$$u + w = 1$$

Converting the given equations into  $Ax = b$  form gives

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

## Step-9

First coefficient is zero, so it needs row exchange hence exchange row 1 and row 2

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Subtracting row 1 from row 3, we have

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Adding row 2 to row 3 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

## Step-10

Now the pivot is 2, so subtracting  $\frac{1}{2}$  times row 3 from row 2 gives

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1/2 \\ 1 \end{pmatrix}$$

Subtracting row 2 from row 1 and divide row 3 by 2 gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

Hence the system has unique solution which is  $u = v = w = \frac{1}{2}$

Hence it is a nonsingular system.