Step-1

Given that $x=(x_1,x_2,x_3,x_4)$ is transformed to $Ax=(x_2,x_3,x_4,x_1)$.

We have to find the 4 by 4 cyclic permutation matrix that satisfies the above transformation.

Step-2

We have $A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ x_1 \end{bmatrix}$

From this, we get the matrix A as $A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Hence the required permutation matrix that satisfies the given transformation is $\boxed{A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}}$.

Step-3

We have to find the effect of A^2.

Now $A^2x=A(Ax)$

$=A(x_2,x_3,x_4,x_1)$
$=(x_3,x_4,x_1,x_2)$

Therefore, $\boxed{A^2x=(x_3,x_4,x_1,x_2)}$

Step-4

Now

$A^4x=A^2A^2x$

$= A^2(x_3, x_4, x_1, x_2)$
$= (x_1, x_2, x_3, x_4)$

Thus $A^4 x = x = Ix$

Now

$A^4 = I$
$\Rightarrow A.A^3 = I$

Step-5

Now we have to show that $A^3 = A^{-1}$.

Since A is a permutation matrix.

So A^{-1} exists

Now multiplying both sides with A^{-1}, we get

$$A^{-1}(AA^3) = IA^{-1}$$
$$\Rightarrow (A^{-1}A)A^3 = A^{-1} \qquad (\text{Since } IA = A)$$
$$\Rightarrow (I)A^3 = A^{-1} \qquad (\text{Since } A^{-1}A = AA^{-1} = I)$$

$$\Rightarrow A^3 = A^{-1}$$

Hence $\boxed{A^3 = A^{-1}}$