Step-1

a) D is invertible $\Rightarrow DD^{-1} = D^{-1}D = I$ and $CD = -DC$. â€¦â€¦ (1)

Let us consider $C = IC$

$= \left(D^{-1}D\right)C$

$= D^{-1}\left(DC\right)$ By associativity of multiplication of matrices

$= D^{-1}\left(-CD\right)$ By (1)

$= D^{-1}\left(-C\right)D$

Therefore C is similar to $-C$.

Step-2

b) If two matrices are similar, then their eigen values are equal.

We have C is similar to $-C$.

But we follow that the eigen values are respectively the roots of $|C - \lambda I| = 0, |C + \lambda I| = 0$

So, it follows that if $\lambda_1, \lambda_2, \ldots, \lambda_n$ are eigen values of C then

$-\lambda_1, -\lambda_2, \ldots, -\lambda_n$ are eigen values of â€“C

Therefore, we confirm that the eigen values of the matrices C and *â€“* C are the plus â€“ minus pairs of $\lambda_1, \lambda_2, \ldots \lambda_n$.

Step-3

c) Given that $Cx = \lambda x$

Then $C\left(Dx\right) = \left(CD\right)x$

$$
\begin{aligned}
&= \left(-DC\right)x \\
&= -D\left(Cx\right) \\
&= -D\left(\lambda x\right) \\
&= -\lambda\left(Dx\right)
\end{aligned}
$$

Therefore $C(Dx) = -\lambda(Dx)$