Step-1

Therefore, if A is an n by n real matrix, such that $A^2 = -I$, then n must be even.

Step-2

Simplifying we get

$$A^2 x = \lambda^2 x$$
$$-Ix = \lambda^2 x$$
$$-x = \lambda^2 x$$

This shows that $\lambda^2 = -1$.

Therefore, $\lambda = i$ or $\lambda = -i$

Thus, the eigenvalues of A are *i* and â€“*i*.

Step-3

Let A be an n by n real matrix.

If $n = 1$, then $A = [a]$, where a is a real number.

In such case, $A^2 = [a^2]$ and it is clear that $a^2 \neq -1$.

Step-4

Thus, the order of A cannot be 1.

Let A be a 2 by 2 real matrix.

Consider an example, $A = \begin{bmatrix} \sqrt{5} & 2 \\ -3 & -\sqrt{5} \end{bmatrix}$.

Then,

$$
\begin{aligned}
A^2 &= AA \\
&= \begin{bmatrix} \sqrt{5} & 2 \\ -3 & -\sqrt{5} \end{bmatrix}\begin{bmatrix} \sqrt{5} & 2 \\ -3 & -\sqrt{5} \end{bmatrix} \\
&= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \\
&= -I
\end{aligned}
$$

Step-5

Now suppose it were possible to have *A* to be a 3 by 3 matrix.

Consider the following separation of *A*.

$$A = \left[\begin{array}{c|cc} \_ & \_ & \_ \\ \hline & & \\ & & \end{array}\right]$$

This indicates that for the 1 by 1 matrix at the top left corner too should have the property that is square is equal to â€“*I*. But we have shown that it is not possible.

Therefore, for a 3 by 3 matrix *A*, $A^2 \neq I$. Similarly, for any odd ordered matrix *A*, we cannot get $A = -I$.