

Step-1

The three steps of the Fast Fourier Transform are

$$c = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} \rightarrow \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \\ c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} \rightarrow \begin{bmatrix} F_4 c^1 \\ F_4 c^{11} \end{bmatrix} \rightarrow y \quad \hat{\in} (1)$$

Step-2

Given $c = (1, 0, 1, 0, 1, 0, 1, 0)$

That is, $\hat{c}_0 = 1, c_1 = 0, c_2 = 1, c_3 = 0, c_4 = 1, c_5 = 0, c_6 = 1, c_7 = 0$.

$$c = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \hat{\in} (2)$$

Step-3

$$\begin{bmatrix} c_{even} \\ c_{odd} \end{bmatrix} = \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \\ c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \hat{\in} (3)$$

Step-4

$$\begin{aligned} y^I &= F_4 c^I = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

Step-5

$$\begin{aligned} y^{II} &= F_4 c^{II} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

Step-6

$$\begin{bmatrix} F_4 c^1 \\ F_4 c^{11} \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in (4)$$

Therefore

Step-7

$$y = F_8 c = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ 1 & w^2 & w^4 & w^6 & w^8 & w^{10} & w^{12} & w^{14} \\ 1 & w^3 & w^6 & w^9 & w^{12} & w^{15} & w^{18} & w^{21} \\ 1 & w^4 & w^8 & w^{12} & w^{16} & w^{20} & w^{24} & w^{28} \\ 1 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} & w^{30} & w^{35} \\ 1 & w^6 & w^{12} & w^{18} & w^{24} & w^{30} & w^{36} & w^{42} \\ 1 & w^7 & w^{14} & w^{21} & w^{28} & w^{35} & w^{42} & w^{49} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Step-8

$$\hat{A} \hat{A} \hat{A} \hat{A} = \begin{bmatrix} 1+0+1+0+1+0+1+0 \\ 1+0+w^2+0+w^4+0+w^6+0 \\ 1+0+w^4+0+w^8+0+w^{12}+0 \\ 1+0+w^6+0+w^{12}+0+w^{18}+0 \\ 1+0+w^8+0+w^{16}+0+w^{24}+0 \\ 1+0+w^{10}+0+w^{20}+0+w^{30}+0 \\ 1+0+w^{12}+0+w^{24}+0+w^{36}+0 \\ 1+0+w^{14}+0+w^{28}+0+w^{42}+0 \end{bmatrix} \in (5)$$

Step-9

$$= \begin{bmatrix} 4 \\ 1+w^2+w^4+w^6 \\ 2+2w^4 \\ 1+w^2+w^4+w^6 \\ 4 \\ 1+w^2+w^4+w^6 \\ 2+2w^4 \\ 1+w^2+w^4+w^6 \end{bmatrix}$$

Step-10

Substituting (2),(3),(4) and (5) in (1), we get

$$c = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} c_{even} = \\ c_{odd} = \end{matrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{matrix} y^I = \\ y^{II} = \end{matrix} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow y = \begin{bmatrix} 4 \\ 1+w^2+w^4+w^6 \\ 2+2w^4 \\ 1+w^2+w^4+w^6 \\ 4 \\ 1+w^2+w^4+w^6 \\ 2+2w^4 \\ 1+w^2+w^4+w^6 \end{bmatrix} \text{ while } w^8 = 1.$$

The three steps of the Fast Fourier Transform are

$$c = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} \rightarrow \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \\ c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} \rightarrow \begin{bmatrix} F_4 c^1 \\ F_4 c^{11} \end{bmatrix} \rightarrow y$$

(1)

Given $c = (0, 1, 0, 1, 0, 1, 0, 1)$

That is, $c_0 = 0, c_1 = 1, c_2 = 0, c_3 = 1, c_4 = 0, c_5 = 1, c_6 = 0, c_7 = 1$.

$$c = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

(2)

$$\begin{bmatrix} c_{even} \\ c_{odd} \end{bmatrix} = \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \\ c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(3)

$$y^I = F_4 c^I = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} c_1 \\ c_3 \\ c_5 \\ c_7 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y^{II} = F_4 c^{II} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} \begin{bmatrix} c_0 \\ c_2 \\ c_4 \\ c_6 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} F_4 c^I \\ F_4 c^{II} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

Therefore

$$y = F_8 c = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 & w^4 & w^5 & w^6 & w^7 \\ 1 & w^2 & w^4 & w^6 & w^8 & w^{10} & w^{12} & w^{14} \\ 1 & w^3 & w^6 & w^9 & w^{12} & w^{15} & w^{18} & w^{21} \\ 1 & w^4 & w^8 & w^{12} & w^{16} & w^{20} & w^{24} & w^{28} \\ 1 & w^5 & w^{10} & w^{15} & w^{20} & w^{25} & w^{30} & w^{35} \\ 1 & w^6 & w^{12} & w^{18} & w^{24} & w^{30} & w^{36} & w^{42} \\ 1 & w^7 & w^{14} & w^{21} & w^{28} & w^{35} & w^{42} & w^{49} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0+1+0+1+0+1 \\ 0+w+0+w^3+0+w^5+0+w^7 \\ 0+w^2+0+w^6+0+w^{10}+0+w^{14} \\ 0+w^3+0+w^9+0+w^{15}+0+w^{21} \\ 0+w^4+0+w^{12}+0+w^{20}+0+w^{28} \\ 0+w^5+0+w^{15}+0+w^{25}+0+w^{35} \\ 0+w^6+0+w^{18}+0+w^{30}+0+w^{42} \\ 0+w^7+0+w^{21}+0+w^{35}+0+w^{49} \end{bmatrix} = \begin{bmatrix} 4 \\ w+w^3+w^5+w^7 \\ 2w^2+2w^6 \\ w+w^3+w^5+w^7 \\ 4w^4 \\ w+w^3+w^5+w^7 \\ 2w^2+2w^6 \\ w+w^3+w^5+w^7 \end{bmatrix} \quad (5)$$

(2),(3),(4) and (5) values substitute in(1)

$$\begin{aligned}
 c = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} &\rightarrow \begin{bmatrix} c_{\text{even}} \\ c_{\text{odd}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \rightarrow \begin{matrix} y^1 = 0 \\ y^{11} = 4 \end{matrix} \rightarrow y = \begin{bmatrix} 4 \\ w + w^3 + w^5 + w^7 \\ 2w^2 + 2w^6 \\ w + w^3 + w^5 + w^7 \\ 4w^4 \\ w + w^3 + w^5 + w^7 \\ 2w^2 + 2w^6 \\ w + w^3 + w^5 + w^7 \end{bmatrix} \text{ since } w^8 = 1
 \end{aligned}$$