Step-1

Consider the following matrix:

$A = QR$

Then, $A^T A$ can be written as,

$$\begin{aligned} A^T A &= (QR)^T \cdot (QR) \\ &= R^T Q^T QR \end{aligned}$$

Since Q is an orthogonal matrix, then $Q^T Q = I$.

$$\begin{aligned} A^T A &= R^T Q^T QR \\ &= R^T R \end{aligned}$$

Step-2

Thus, the projection P onto the column space of A is given by,

$P = A\left(A^T A\right)^{-1} A^T$.â€¦.. (1)

Substitute $A = QR$ and $A^T A = R^T R$ in (1), to get

$$\begin{aligned} P &= (QR)\left(R^T R\right)^{-1} (QR)^T \\ &= QRR^{-1}\left(R^T\right)^{-1} R^T Q^T \\ &= QRR^{-1} I Q^T \\ &= QRR^{-1} Q^T \end{aligned}$$

This implies that,

$$\begin{aligned} P &= QIQ^T \\ &= QQ^T \end{aligned}$$

Hence, the formula for the projection matrix P is $\boxed{P = QQ^T}$ provided that R is invertible matrix.