Step-1

We have to find that why canâ€™t a 1 by 3 system have $x_p = (2,4,0)$ and

$x_n =$ any multiple of $(1,1,1)$.

Let $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the solution of the systems and x_2, x_3 are free variables.

Then $x_1 =$ linear combinaton of x_2, x_3

Let $x_1 = ax_2 + bx_3 + c$

Step-2

Therefore

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ax_2 + bx_3 + c \\ x_2 \\ x_3 \end{bmatrix}$$
$$= \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} a \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} b \\ 0 \\ 1 \end{bmatrix}$$

Step-3

Therefore the particular solution is in the form $\begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix}$

Therefore a 1 by 3 system can not have

$x_p = (2,4,0)$ and $x_n =$ any multiple of $(1,1,1)$