Step-1

Let

$$A=\begin{bmatrix}1 & 3 & 4\\ 3 & 1 & 0\\ 4 & 0 & 0\end{bmatrix}$$

$$x=\begin{bmatrix}3\\ 4\end{bmatrix}$$

$$z=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\sigma=5$$

$$v=\begin{bmatrix}8\\ 4\end{bmatrix}$$

$$H=\begin{bmatrix}-\frac{3}{5} & -\frac{4}{5}\\ -\frac{4}{5} & \frac{3}{5}\end{bmatrix}$$

Therefore, the matrix U_1 is given by,

$$U_1=\begin{bmatrix}1 & 0 & 0\\ 0 & \mathrm{H} & \\ 0 & & \end{bmatrix}$$

$$U_1=\begin{bmatrix}1 & 0 & 0\\ 0 & -\frac{3}{5} & -\frac{4}{5}\\ 0 & -\frac{4}{5} & \frac{3}{5}\end{bmatrix}$$

$$U_1^{-1}=\begin{bmatrix}1 & 0 & 0\\ 0 & -\frac{3}{5} & -\frac{4}{5}\\ 0 & -\frac{4}{5} & \frac{3}{5}\end{bmatrix}$$

Therefore, $U_1^{-1}AU_1$ is given by,

$$U_1^{-1}AU_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & 0 \\ 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 4 \\ 0-\frac{9}{5}-\frac{16}{5} & -\frac{3}{5} & 0 \\ 0-\frac{12}{5}+\frac{12}{5} & -\frac{4}{5} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 4 \\ -5 & -\frac{3}{5} & 0 \\ 0 & -\frac{4}{5} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\frac{9}{5}-\frac{16}{5} & -\frac{12}{5}+\frac{12}{5} \\ -5 & \frac{9}{25} & \frac{12}{25} \\ 0 & \frac{12}{25} & \frac{16}{25} \end{bmatrix}$$

$$U_1^{-1}AU_1 = \begin{bmatrix} 1 & -5 & 0 \\ -5 & \frac{9}{25} & \frac{12}{25} \\ 0 & \frac{12}{25} & \frac{16}{25} \end{bmatrix}$$

Step-2

Thus, the tridiagonal $U_1^{-1}AU_1$ is $\boxed{U_1^{-1}AU_1 = \begin{bmatrix} 1 & -5 & 0 \\ -5 & \frac{9}{25} & \frac{12}{25} \\ 0 & \frac{12}{25} & \frac{16}{25} \end{bmatrix}}$.