Step-1

Consider the given matrix $A=\begin{pmatrix}1 & 2\\ 2 & 9\end{pmatrix}$.

Let $X^T=\begin{pmatrix}x & y\end{pmatrix}$ so that $X=\begin{pmatrix}x\\ y\end{pmatrix}$.

Now X^TAX

$=\begin{pmatrix}x & y\end{pmatrix}\begin{pmatrix}1 & 2\\ 2 & 9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$

$=\begin{pmatrix}x+2y & 2x+9y\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$

$=(x+2y)x+(2x+9y)y$

$=x^2+4xy+9y^2$

Step-2

Thus the quadratic for the given metro A is $x^2+4xy+9y^2$.

Now,

$f=x^2+4xy+9y^2$

$=x^2+2(x)(2y)+4y^2-4y^2+9y^2$

$=(x+2y)^2+5y^2$

$=d_1(x+2y)^2+d_2y^2$

So that $d_1=1$ and $d_2=5$.

Therefore, the quadratic $f\boxed{=x^2+4xy+9y^2=(x+2y)^2+5y^2}$.

Step-3

Now consider the second matrix, $A=\begin{pmatrix}1 & 3\\ 3 & 9\end{pmatrix}$.

$$X^T AX = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} x+3y & 3x+9y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= (x+3y)\, x\, (3x+9y)\, y$$

$$= x^2 + 6xy + 9y^2$$

Step-4

Thus the quadratic for the given matrix A is, $x^2 + 6xy + 9y^2$.

Now,

$$f = x^2 + 6xy + 9y^2$$

$$= x^2 + 2(x)(2y) + (3y)^2$$

$$= (x+3y)^2$$

Therefore, the quadratic $f \boxed{= x^2 + 6xy + 9y^2 = (x+3y)^2}$.