Step-1

Given that the matrix A has eigen values 2, 2, and 5

a) The statement is true.

The det A = product of eigen values

$$=(2)(2)(5)$$
$$=20\neq 0$$

Therefore A is invertiable.

Step-2

b) We observe that the algebraic multiplicity of the eigen value 2 is 2 (number of times repeated)

But we donâ€™t know whether the corresponding eigen vectors are also two in number (geometric multiplicity).

So, A is diagonalizable if and only if the sum of the algebraic multiplicities is equal to the sum of the geometric multiplicities.

In our case, we have no confirmation about the geometric multiplicity of the eigen value 2. So, we cannot confirm that A is diagonalizable.

Therefore, the statement is false.

Step-3

c) If the homogeneous equation from $(A-\lambda I)x=0$ when the eigen value $\lambda=2$ is substituted is $x+y+z=0,$ then the solution set is $x=-k-m$ where $y=k, z=m$ are the parameters.

So, it can be written as $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = k\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + m\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Putting k = 1,m = 1, the corresponding eigen vectors for $\lambda=2$ are $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

So, the geometric multiplicity of the eigen value λ=2 is 2.

While the algebraic multiplicity = geometric multiplicity, we can see the given matrix A is diagonalizable.

So, the statement is false.