Step-1

Consider the matrix:

$$A=\begin{bmatrix}3 & 1 & 1\\ 0 & 4 & 1\\ 2 & 2 & 5\end{bmatrix}$$

Thus, matrix D is given by,

$$D=\begin{bmatrix}3 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 5\end{bmatrix}$$

We know that,

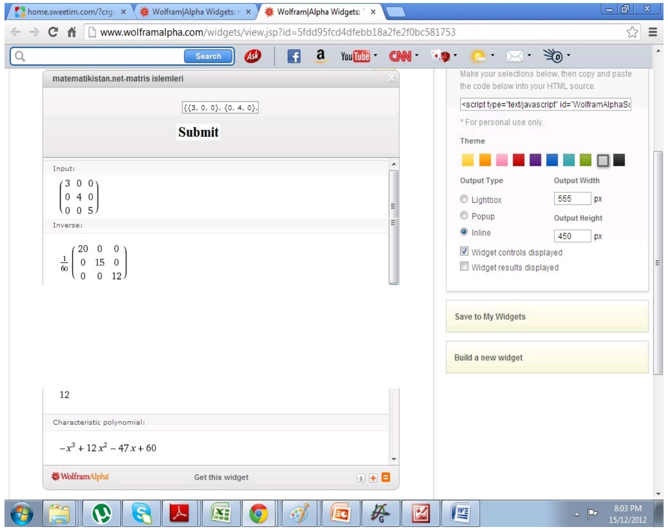
$$A=D+L+U$$

Therefore, we get,

$$\begin{aligned}L+U&=A-D\\ &=\begin{bmatrix}3 & 1 & 1\\ 0 & 4 & 1\\ 2 & 2 & 5\end{bmatrix}-\begin{bmatrix}3 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 5\end{bmatrix}\\ &=\begin{bmatrix}0 & 1 & 1\\ 0 & 0 & 1\\ 2 & 2 & 0\end{bmatrix}\end{aligned}$$

Step-2

By using matrix calculator (the screenshot is given below), the inverse of D is given by,



Therefore,

$$D^{-1} = \frac{1}{60}\begin{bmatrix} 20 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

Step-3

By multiplying D^{-1} and $L + U$, the Jacobi matrix J for the diagonally dominant A is given by,

$$\begin{aligned} J &= D^{-1}(L+U) \\ &= \frac{1}{60}\begin{bmatrix} 20 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 12 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 2 & 2 & 0 \end{bmatrix} \\ &= \frac{1}{60}\begin{bmatrix} 0 & 20 & 20 \\ 0 & 0 & 15 \\ 24 & 24 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{4} \\ \frac{2}{5} & \frac{2}{5} & 0 \end{bmatrix} \end{aligned}$$

Step-4

Let C_1 , C_2, and C_3 are the three Gershgorin circles for J.

The center of C_1 is at the point (0, 0).

The radius of C_1 is given by,

$$\begin{aligned} r_1 &= \left|\frac{1}{3}\right| + \left|\frac{1}{3}\right| \\ &= \frac{2}{3} \end{aligned}$$

The center of C_2 is at the point (0, 0).

The radius of C_2 is given by,

$$\begin{aligned} r_2 &= |0| + \left|\frac{1}{4}\right| \\ &= \frac{1}{4} \end{aligned}$$

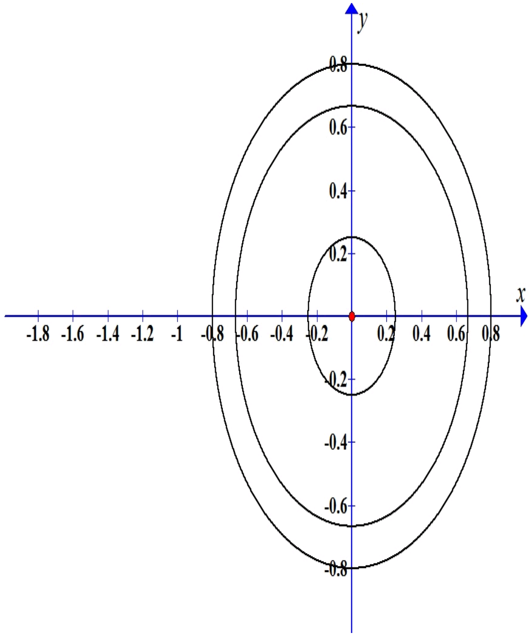
The center of C_3 is at the point (0, 0).

The radius of C_3 is given by,

$$\begin{aligned} r_3 &= \left|\frac{2}{5}\right| + \left|\frac{2}{5}\right| \\ &= \frac{4}{5} \end{aligned}$$

Step-5

The graph of three Gershgorin circles C_1, C_2, and C_3 is given below.



Step-6

Thus, $\boxed{\omega_{opt} = 4 - 2\sqrt{2}}$ and $\boxed{\lambda_{\max} = 3 - 2\sqrt{2} \approx 0.2}$.