Step-1

Given that the column 5 of U has no pivot. Then we have to fill the following blanks.

Then, x_5 is a _____ variable, the zero vector (is) (is not) the only solution to $Ax = 0$. If $Ax = b$ has a solution, then it has _____ solutions.

Step-2

Given that the column 5 of U has no pivot. So that column contains zeros, and hence x_5 is a free variable. And the system $Ax = b$ has a solution then it has unique solution or infinitely many solutions.

Step-3

For example, consider two systems with augmented forms:

$$[A\ b] = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$[A\ b] = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

For the first system, the rank of A and the rank of $[A\ b]$ are equal and the system has unique solution while in the second system, the rank of A and the rank of $[A\ b]$ are equal and the system has infinite number of solutions.