Step-1

We have to factor $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & 0 \end{bmatrix}$ into QR

Let $A = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$, where $a_1 = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$, $a_2 = \begin{bmatrix} \sin\theta \\ 0 \end{bmatrix}$

Â We know that

$$A = \begin{bmatrix} a_1 & a_2 \end{bmatrix} = \begin{bmatrix} q_1 & q_2 \end{bmatrix} \begin{bmatrix} q_1^T a_1 & q_1^T a_2 \\ 0 & q_2^T a_2 \end{bmatrix}$$

Step-2

Now

$$\|a_1\| = \sqrt{\cos^2\theta + \sin^2\theta}$$
$$= \sqrt{1}$$
$$= 1$$

Step-3

$$q_1 = \frac{a_1}{\|a_1\|}$$
$$= \frac{1}{1}\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

Step-4

Now

$$q_2 = \frac{\beta}{\|\beta\|} \text{ where } \beta = a_2 - \left(q_1^T a_2\right) q_1$$

$$q_1^T a_2 = \begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \sin\theta \\ 0 \end{bmatrix}$$
$$= \cos\theta \sin\theta$$

Step-5

$$\left(q_1^T a_2\right) q_1 = \cos\theta \sin\theta \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta \sin\theta \\ \cos\theta \sin^2\theta \end{bmatrix}$$

$$\beta = \begin{bmatrix} \sin\theta \\ 0 \end{bmatrix} - \begin{bmatrix} \cos^2\theta \sin\theta \\ \cos\theta \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^3\theta \\ -\cos\theta \sin^2\theta \end{bmatrix}$$

Step-6

$$\|\beta\| = \sqrt{\sin^6\theta + \cos^2\theta \sin^4\theta}$$

$$= \sqrt{\left(\sin^4\theta\right)(1)}$$

$$= \sin^2\theta$$

Step-7

Therefore

$$q_2 = \frac{1}{\sin^2\theta} \begin{bmatrix} \sin^3\theta \\ -\cos\theta \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin\theta \\ -\cos\theta \end{bmatrix}$$

Step-8

And

$$q_1^T a_1 = \begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$= \cos^2\theta + \sin^2\theta$$

Â $=1$ Â

Step-9

$$q_1^T a_2 = \begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \sin\theta \\ 0 \end{bmatrix}$$

$$= c\,os\theta \sin\theta$$

Step-10

$$q_2^T a_2 = \begin{bmatrix}\sin\theta & -\cos\theta\end{bmatrix}\begin{bmatrix}\sin\theta \\ 0\end{bmatrix}$$
$$= \sin^2\theta + 0$$
$$= \sin^2\theta$$

$$A = \begin{bmatrix}a_1 & a_2\end{bmatrix}$$
$$= \begin{bmatrix}q_1 & q_2\end{bmatrix}\begin{bmatrix}q_1^T a_1 & q_1^T a_2 \\ 0 & q_2^T a_2\end{bmatrix}$$
$$= \begin{bmatrix}\cos\theta & \sin\theta \\ \sin\theta & -\cos\theta\end{bmatrix}\begin{bmatrix}1 & \cos\theta\sin\theta \\ 0 & \sin^2\theta\end{bmatrix}$$
$$= QR$$

Therefore $\boxed{\begin{bmatrix}\cos\theta & \sin\theta \\ \sin\theta & 0\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta \\ \sin\theta & -\cos\theta\end{bmatrix}\begin{bmatrix}1 & \cos\theta\sin\theta \\ 0 & \sin^2\theta\end{bmatrix} = QR}$