Step-1

(a)

Consider the following matrices:

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

The objective is to describe the smallest subspace of the matrix space **M** that contains those matrices.

According to the definition of a subspace, the vector addition is closure on *S* and scalar multiplication is also closure on *S*. So, the smallest subspace of the matrix space **M** that contains those matrices will be the set of all linear combinations of those matrices.

So, the smallest subspace of the matrix space **M** that contains $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is,

$$\begin{aligned} S &= \left\{ a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} / a, b \in \mathbf{F} \right\}, \quad \text{where, } \mathbf{F} \text{ is a field.} \\ &= \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} / a, b \in \mathbf{F} \right\} \\ &= \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} / a, b \in \mathbf{F} \right\} \end{aligned}$$

Therefore, the smallest subspace of the matrix space **M** that contains the indicated matrices is,

$$\boxed{S = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} / a, b \in \mathbf{F} \right\}}$$

Step-2

(b)

Consider the following matrices:

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The objective is to describe the smallest subspace of the matrix space **M** that contains those matrices.

According to the definition of a subspace, the vector addition is closure on *S* and scalar multiplication is also closure on *S*. So, the smallest subspace of the matrix space **M** that contains those matrices will be the set of all linear combinations of those matrices.

So, the smallest subspace of the matrix space **M** that contains $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is,

$$
\begin{aligned}
T &= \left\{ a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} / \, a,b \in \mathbf{F} \right\}, \quad \text{where, } \mathbf{F} \text{ is a field.} \\
&= \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} / \, a,b \in \mathbf{F} \right\} \\
&= \left\{ \begin{bmatrix} a+b & 0 \\ 0 & b \end{bmatrix} / \, a,b \in \mathbf{F} \right\}
\end{aligned}
$$

Therefore, the smallest subspace of the matrix space **M** that contains the indicated matrices is,

$$\boxed{T = \left\{ \begin{bmatrix} a+b & 0 \\ 0 & b \end{bmatrix} / \, a,b \in \mathbf{F} \right\}}$$

Step-3

(c)

Consider the following matrix:

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

The objective is to describe the smallest subspace of the matrix space **M** that contains those matrices.

According to the definition of a subspace, the vector addition is closure on S and scalar multiplication is also closure on S. So, the smallest subspace of the matrix space **M** that contains those matrices will be the set of all linear combinations of those matrices.

So, the smallest subspace of the matrix space **M** that contains $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is,

$$
\begin{aligned}
R &= \left\{ a\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} / \in \mathbf{F} \right\}, \quad \text{where, } \mathbf{F} \text{ is a field.} \\
&= \left\{ \begin{bmatrix} a & a \\ 0 & 0 \end{bmatrix} / \, a \in \mathbf{F} \right\}
\end{aligned}
$$

Therefore, the smallest subspace of the matrix space **M** that contains the indicated matrices is,

$$\boxed{R = \left\{ \begin{bmatrix} a & a \\ 0 & 0 \end{bmatrix} / \, a \in \mathbf{F} \right\}}$$

Step-4

(d)

Consider the following matrices:

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

The objective is to describe the smallest subspace of the matrix space **M** that contains those matrices.

According to the definition of a subspace, the vector addition is closure on *S* and scalar multiplication is also closure on *S*. So, the smallest subspace of the matrix space **M** that contains those matrices will be the set of all linear combinations of those matrices.

So, the smallest subspace of the matrix space **M** that contains $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ is,

$$\begin{aligned} Q &= \left\{ a\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} / a,b,c \in \mathbf{F} \right\}, \quad \text{where, } \mathbf{F} \text{ is a field.} \\ &= \left\{ \begin{bmatrix} a & a \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} + \begin{bmatrix} 0 & c \\ 0 & c \end{bmatrix} / a,b,c \in \mathbf{F} \right\} \\ &= \left\{ \begin{bmatrix} a+b & a+c \\ 0 & b+c \end{bmatrix} / a,b,c \in \mathbf{F} \right\} \end{aligned}$$

Therefore, the smallest subspace of the matrix space **M** that contains the indicated matrices is,

$$\boxed{Q = \begin{bmatrix} a+b & a+c \\ 0 & b+c \end{bmatrix} / a,b,c \in \mathbf{F}}$$