Step-1

Given that $b=\begin{bmatrix}0.217\\0.254\end{bmatrix}, A=\begin{bmatrix}0.780 & 0.563\\0.913 & 0.659\end{bmatrix}, y=\begin{bmatrix}0.341\\-0.087\end{bmatrix}, z=\begin{bmatrix}0.999\\-1.0\end{bmatrix}$

We have to compute $b-Ay$ and $b-Az$

Step-2

Now

$$Ay=\begin{bmatrix}0.780 & 0.563\\0.913 & 0.659\end{bmatrix}\begin{bmatrix}0.341\\-0.087\end{bmatrix}$$
$$=\begin{bmatrix}0.780(0.341)+(0.563)(-0.087)\\0.913(0.341)+(0.659)(-0.087)\end{bmatrix}$$
$$=\begin{bmatrix}0.21699\\0.254\end{bmatrix}$$

Therefore,

$$b-Ay=\begin{bmatrix}0.217\\0.254\end{bmatrix}-\begin{bmatrix}0.216999\\0.254\end{bmatrix}$$

$$=\begin{bmatrix}0.000001\\0\end{bmatrix}$$ â€¦â€¦ (1)

Hence $\boxed{b-Ay=\begin{bmatrix}0.000001\\0\end{bmatrix}}$

Step-3

Now

$$Az=\begin{bmatrix}0.780 & 0.563\\0.913 & 0.659\end{bmatrix}\begin{bmatrix}0.999\\-1.0\end{bmatrix}$$
$$=\begin{bmatrix}0.780(0.999)+(0.563)(-1.0)\\0.913(0.999)+(0.659)(-1.0)\end{bmatrix}$$
$$=\begin{bmatrix}0.216220\\0.253087\end{bmatrix}$$

Step-4

Therefore,

$$b - Az = \begin{bmatrix} 0.217 \\ 0.254 \end{bmatrix} - \begin{bmatrix} 0.21622 \\ 0.253087 \end{bmatrix}$$

$$= \begin{bmatrix} 0.00078 \\ 0.000913 \end{bmatrix}$$ â€¦â€¦ (2)

Hence $\boxed{b - Az = \begin{bmatrix} 0.00078 \\ 0.000913 \end{bmatrix}}$

Step-5

If $Ax = b$ is a non homogeneous system of equations in two variables whose solutions are *y* and *z*, then we follow that $Ay = b$, $Az = b$.

Consequently, $b - Ay$ and $b - Az$

In view of (1) and (2), we see that *y* is the better solution than *z*.

Step-6

On the other hand, comparing the solutions *y* and *z* to the true solution $x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, *z* is the better solution than *y*.

The corresponding residuals $x - y = \delta y = \begin{bmatrix} 0.659 \\ -0.913 \end{bmatrix}$ and $x - z = \delta z = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix}$ confirms this.