Step-1

If the special solutions to $Rx = 0$ are in the columns of the following *N*, then we have to go backward to find the nonzero rows of the reduced matrices *R*:

$N = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and $N = \begin{bmatrix} \; \end{bmatrix}$ (empty 3 by 1)

Step-2

Now

$$N = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow x_1 = 2x_2 + 3x_3$$

$$\Rightarrow x_1 - 2x_2 + 3x_3$$

Here x_2, x_3 are free variables.

Step-3

Therefore the coefficient matrix is

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Any zero rows comes after the row $R = \begin{bmatrix} 1 & -2 & -3 \end{bmatrix}$.

Step-4

Now

$$N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Let $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\Rightarrow x_1 = 0$$
$$x_2 = 0$$

Step-5

Therefore the matrix of form $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Any zero rows comes after the row $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

Step-6

Now

$$N = \begin{bmatrix} \\ \\ \\ \end{bmatrix} (\text{empty 3 by 1})$$

Let $N = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore any nonzero row comes after $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$