

## Step-1

Given that  $A, B, C$  are three invertible square matrices and  $M = ABC$ . To find a formula for  $B^{-1}$  that involves  $M^{-1}$  and  $A$  and  $C$ .

See that as  $A, B, C$  are three invertible square matrices and  $M = ABC$  so  $M$  is also invertible and  $M^{-1} = C^{-1}B^{-1}A^{-1}$  as  $C^{-1}B^{-1}A^{-1}ABC = C^{-1}B^{-1}BC = C^{-1}C = I$ .

## Step-2

So,  $M^{-1} = C^{-1}B^{-1}A^{-1}$  (1)

Now, multiplying the equation (1) from left by  $C$  gives,  $CM^{-1} = CC^{-1}B^{-1}A^{-1} = IB^{-1}A^{-1} = B^{-1}A^{-1}$ .

So,  $CM^{-1} = B^{-1}A^{-1}$  (2)

Now, multiplying the equation (2) from right by  $A$  gives,  $CM^{-1}A = B^{-1}A^{-1}A = B^{-1}I = B^{-1}$ .

Hence,  $B^{-1} = CM^{-1}A$ .