# Step-1

Solve the following equations for $u$, $v$, $w$ and $z$.

$$\begin{aligned} u+v+w+z&=6 \\ u+w+z&=4 \\ u+w&=2 \end{aligned}$$

$$\left.\begin{aligned} u+w+z&=4 \\ u+w&=2 \end{aligned}\right\} \Rightarrow z=2$$

$$\left.\begin{aligned} u+v+w+z&=6 \\ u+w&=2 \\ z&=2 \end{aligned}\right\} \Rightarrow v=2$$

Hence the obtained equations are

$$\begin{aligned} u+w&=2 \\ z&=2 \\ v&=2 \end{aligned}$$

These equations contain one dependent variable, one independent variable and two constants, so these three equations represent a $\boxed{\text{line}}$ in four-dimensional space.

# Step-2

Include the equation $u=-1$ in the given set of equations and solve them for $u$, $v$, $w$ and $z$.

$$\begin{aligned} u+v+w+z&=6 \\ u+w+z&=4 \\ u+w&=2 \\ u&=-1 \end{aligned}$$

$$\left.\begin{aligned} u+w&=2 \\ u&=-1 \end{aligned}\right\} \Rightarrow w=3$$

$$\left.\begin{aligned} u+w+z&=4 \\ w&=3 \\ u&=-1 \end{aligned}\right\} \Rightarrow z=2$$

$$\left.\begin{aligned} u+v+w+z&=6 \\ z&=2 \\ w&=3 \\ u&=-1 \end{aligned}\right\} \Rightarrow v=2$$

Hence the solution to the set of equations is $(-1,2,3,2)$ which represents a point in a four-dimensional space.

Therefore the intersection of the given four planes is the point $(-1,2,3,2)$.

## Step-3

Many equations can be found as fourth equation to make the given system of equations without a solution.

For example include the equation $z=1$ in the given set of equations.

$$\begin{aligned} u+v+w+z&=6 \\ u+w+z&=4 \\ u+w&=2 \\ z&=1 \end{aligned}$$

Now put $z=1$ into the equation $u+w+z=4$ then the equation obtained will be $u+w=3$ and the new system of equations will be

$$\begin{aligned} u+v+w+z&=6 \\ u+w&=3 \\ u+w&=2 \\ z&=1 \end{aligned}$$

Obviously the second and third equations are cannot be solved so this system has no solution if the fourth equation $\boxed{z=1}$ included.