Step-1

Then we get $M^{-1}AM$ as follows:

$$M^{-1}AM = \begin{bmatrix} \frac{1}{d} & 0 & 0 \\ 0 & \frac{1}{d^2} & 0 \\ 0 & 0 & \frac{1}{d^3} \end{bmatrix} \begin{bmatrix} a & b & c \\ e & f & g \\ h & i & j \end{bmatrix} \begin{bmatrix} d & 0 & 0 \\ 0 & d^2 & 0 \\ 0 & 0 & d^3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{d} & 0 & 0 \\ 0 & \frac{1}{d^2} & 0 \\ 0 & 0 & \frac{1}{d^3} \end{bmatrix} \begin{bmatrix} ad & bd^2 & cd^3 \\ ed & fd^2 & gd^3 \\ hd & id^2 & jd^3 \end{bmatrix}$$

$$= \begin{bmatrix} a & bd & cd^2 \\ \frac{e}{d} & f & gd \\ \frac{h}{d^2} & \frac{i}{d} & j \end{bmatrix}$$

Step-2

The matrix $M^{-1}AM$ is similar to the matrix A. Moreover, the determinant of A is equal to the determinant of $M^{-1}AM$.

We have to obtain the eigenvalues of $M^{-1}AM$, in case $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

We know that the similar matrices have the same eigenvalues. Thus, the eigenvalues of $M^{-1}AM$ are same as that of A.

Step-3

Let us obtain the eigenvalues of A. For this, consider $\det(A - \lambda I) = 0$.

$$
\begin{aligned}
0 &= \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} \\
&= (1-\lambda)^3 + 1 + 1 - 3(1-\lambda) \\
&= 3\lambda^2 - \lambda^3 \\
&= \lambda^2 (3-\lambda)
\end{aligned}
$$

Therefore, the eigenvalues of A are 0 and 3. This gives us that the eigenvalues of $M^{-1}AM$ are 0 and 3.