Step-1

The objective is to reduce the following matrices to their ordinary echelon forms *U:*

The objective is to find a special solution for each free variable and describe every solution to $Ax=0$ and $Bx=0$.

To reduce echelon forms *U* to R and draw a box around the identity matrix in the pivot rows and pivot columns.

Step-2

(a)

Given

$$A=\begin{bmatrix}1&2&2&4&6\\1&2&3&6&9\\0&0&1&2&3\end{bmatrix}$$

$$\underline{R_2-R_1}\begin{bmatrix}1&2&2&4&6\\0&0&1&2&3\\0&0&1&2&3\end{bmatrix}$$

$$\underline{R_3-R_2}\begin{bmatrix}1&2&2&4&6\\0&0&1&2&3\\0&0&0&0&0\end{bmatrix}$$

$$\underline{R_1-2R_2}\begin{bmatrix}1&2&0&0&0\\0&0&1&2&3\\0&0&0&0&0\end{bmatrix}$$

This is an ordinary echelon form of *A*

Step-3

Now to find special solution for $Ax=0$

$$\begin{bmatrix}1&2&0&0&0\\0&0&1&2&3\\0&0&0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_4 + 2x_2 = 0$$
$$x_3 + 2x_4 + 3x_5 = 0$$

x_1, x_3 are pivot variables remaining are free variables.

$$x_1 = -2x_2$$
$$x_3 = -2x_4 - 3x_5$$

Step-4

Therefore

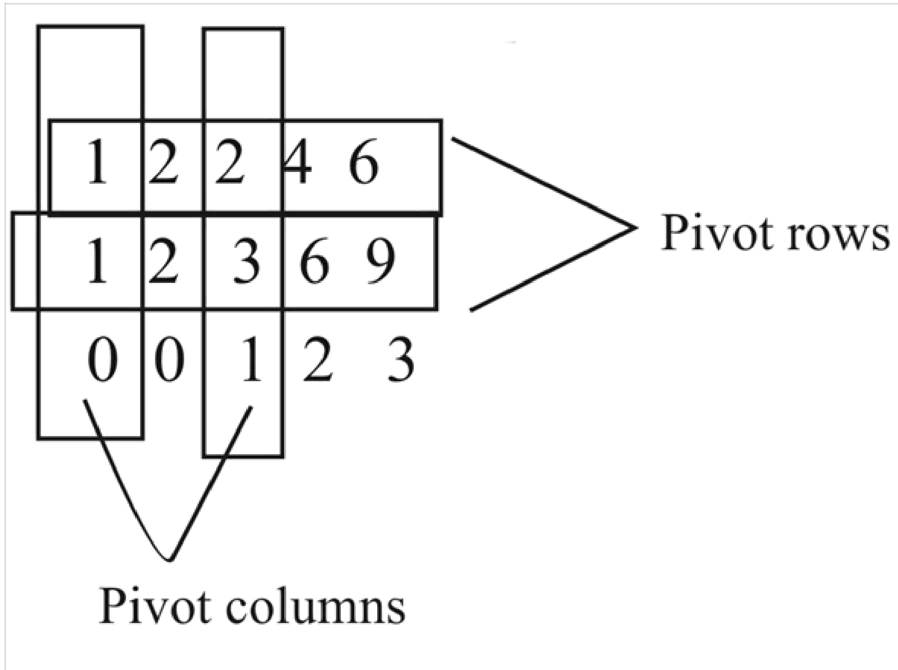
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \\ -2x_4 - 3x_5 \\ x_4 \\ x_5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

Are the special solutions of $Ax = 0$ and the identity matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step-5

The pivot rows, pivot columns:



Step-6

b)

Given

$$B=\begin{bmatrix}2&4&2\\0&4&4\\0&8&8\end{bmatrix}$$

$$\xrightarrow{R_3-2R_1}\begin{bmatrix}2&4&2\\0&4&4\\0&0&0\end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_1}\begin{bmatrix}1&2&1\\0&4&4\\0&0&0\end{bmatrix}$$

$$\xrightarrow{\frac{1}{4}R_2}\begin{bmatrix}1&2&1\\0&1&1\\0&0&0\end{bmatrix}$$

$$\xrightarrow{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

This is row reduced echelon form to find special solution of $Bx = 0$

Step-7

Therefore

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

x_1, x_2 are pivots , x_3 is free variables

$$\begin{aligned} x_1 - x_3 &= 0, \\ x_2 - x_3 &= 0 \\ x_1 &= x_3, \\ x_2 &= x_3 \end{aligned}$$

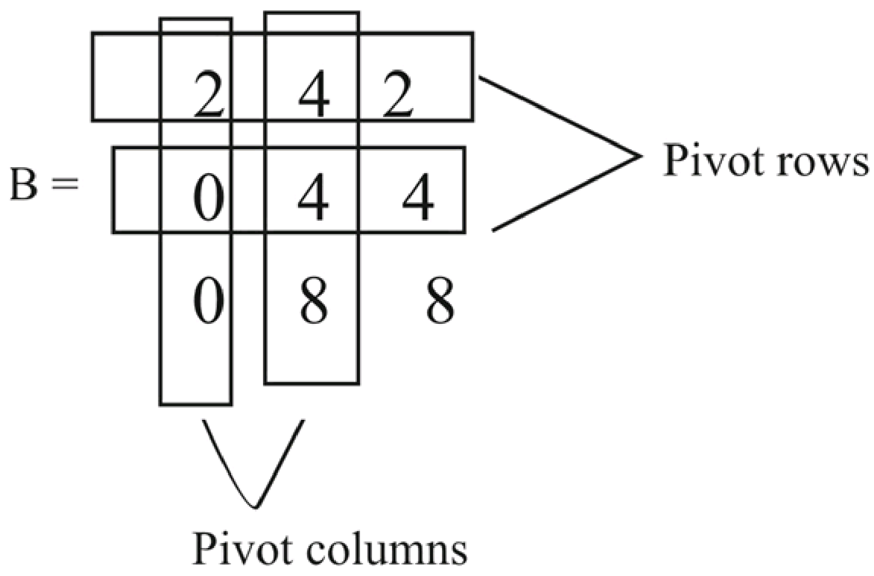
Step-8

Special solutions are $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Therefore, the required identity matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step-9

The pivot rows, pivot columns:


B =
2 4 2
0 4 4
0 8 8
Pivot rows
Pivot columns