

Step-1

To fit $y = C + D t$ leads to four equations in two unknowns

These are $C - 2D = -4$

$C - 1D = -3$

$C + 1D = -1$

$C + 2D = 0$

$$\text{Or } \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \\ -1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} C \\ D \end{bmatrix}, \quad b = \begin{bmatrix} -4 \\ -3 \\ -1 \\ 0 \end{bmatrix}$$

This is equivalent to the system $Ax = b$ where

Step-2

$$a_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -2 \\ -1 \\ 1 \\ 2 \end{bmatrix}$$

(a) The columns of A are

$$\begin{aligned} a_1^T a_2 &= \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ -1 \\ 1 \\ 2 \end{bmatrix} \\ &= -2 - 1 + 1 + 2 = 0 \end{aligned}$$

The columns of coefficient matrix are orthogonal.

Step-3

$$(b) \quad \hat{C} = \frac{\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & -3 & -1 & 0 \end{bmatrix}^T}{\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}}$$

$$\begin{aligned}
& \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ -3 \\ -1 \\ 0 \end{bmatrix} \\
&= \frac{1+1+1+1}{4} \\
&= \frac{-4-3-1+0}{4} \\
&= -2
\end{aligned}$$

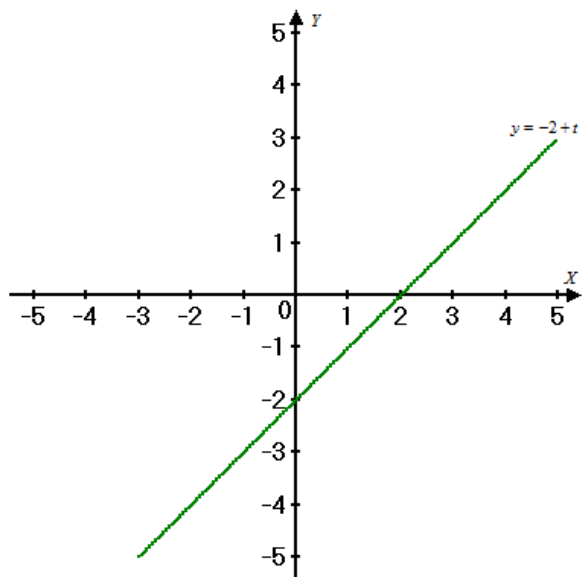
$$\hat{D} = \frac{\begin{bmatrix} -2 & -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} -4 & -3 & -1 & 0 \end{bmatrix}^T}{\begin{bmatrix} -2 & -1 & 1 & -2 \end{bmatrix}^T \begin{bmatrix} -2 & -1 & 1 & -2 \end{bmatrix}}$$

$$\begin{aligned}
& \begin{bmatrix} -2 & -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} -4 \\ -3 \\ -1 \\ 0 \end{bmatrix} \\
&= \frac{4+1+1+4}{10} \\
&= \frac{8+3-1-0}{10} \\
&= 1
\end{aligned}$$

Using $\hat{C} = -2$, $\hat{D} = 1$, the optimal straight line is $\boxed{y = -2 + t}$

Step-4

The graph of the straight line is



$$\begin{aligned}
 E^2 &= (y_1 - C - Dt_1)^2 + (y_2 - C - Dt_2)^2 + (y_3 - C - Dt_3)^2 + (y_4 - C - Dt_4)^2 \\
 &= (-4 + 2 + 2)^2 + (-3 + 2 + 1)^2 + (-1 + 2 - 1)^2 + (0 + 2 - 2)^2 = 0
 \end{aligned}$$

Therefore, $\boxed{E^2 = 0}$

Step-5

Interpret the zero error in terms of the original system of equations in two unknowns:

The right hand side $(-4, -3, -1, 0)$ is in the column space.