Step-1

Given matrix is in $PA = LU$ form

$$A = \begin{bmatrix} 1 & & & \\ 2 & 1 & & \\ 2 & 1 & 1 & \\ 3 & 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} \boxed{1} & 2 & 0 & 1 & 2 & 1 \\ 0 & 0 & \boxed{2} & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step-2

(a)

Since U matrix is just matrix obtained after row elimination, it gives no of pivots as 3. Thus rank $\boxed{\text{rank of } A \text{ is } 3}$

Step-3

(b)

Dimension for row space is same as rank. Hence $\text{dimension of rowspace is } 3$

Rows with 3 pivots are independent and just combination of other rows, thus they form basis of rows pace.

These are $\boxed{\begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}}$

Step-4

(c)

Given rows 1, 2 and 3 are $\boxed{\text{linearly independent}}$ as they contains 3 pivots

Step-5

(d)

Dimension for column space is same as rank. Hence $\text{dimension of column space is } 3$

3 linearly independent columns are basis of column space of A matrix, which can be obtained by product LU.

These are $\boxed{\begin{bmatrix}2\\0\\2\\4\end{bmatrix},\begin{bmatrix}4\\1\\5\\9\end{bmatrix},\begin{bmatrix}1\\2\\5\\4\end{bmatrix}}$

Step-6

(e)

Dimension of left null-space is n â€“ r . Thus $\boxed{\text{dimension of null space is 2}}$.

Step-7

(f)

It can be solved in 2 steps, by first finding y such that

$Ly = b$

Then finding x which is original solution such that

$Ux = y$

Since L has 4 pivots only possible solution for y is $\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

Now, we find x.

$$\begin{bmatrix}2 & -1 & 4 & 2 & 1\\0 & 0 & 1 & -3 & 2\\0 & 0 & 0 & 0 & 2\\0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

Free variables are x_2, x_4

Finding general and particular solutions

We get particular solution, for x to be $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Now finding 2 homogenous solutions, which are $\begin{bmatrix} 1/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$

Step-8

Hence ,general solution for $Ax = 0$ is $\boxed{x_2 \begin{bmatrix} 1/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}}$