

Step-1

(a)

Consider the following matrices:

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Here the matrix B is obtained from the matrix A by exchanging first and second row.

Eigen values of matrix A are $\pm i$.

Eigen values of B are -1 and 1 .

Since similar matrices have same set of Eigen values and the matrices A and B have different Eigen values so, the matrix B is not similar to matrix A .

Therefore, the given statement is false.

Step-2

(b)

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Characteristic polynomial of the matrix A is

$$(x-1)(x-2) = 0$$

Eigen values of the matrix A are 1 and 2 .

As Characteristic polynomial of the matrix A is product of linear factors and degree of the Characteristic polynomial is 2 , is equal to order of the matrix A , so the matrix A is diagonalizable.

Hence, the matrix A is similar to a diagonal matrix; in fact the diagonal matrix is the matrix B .

Therefore, one can conclude that a triangular matrix A may be similar to a diagonal matrix B , but the triangular matrix may not itself a diagonal matrix.

Therefore, the given statement is false.

Step-3

(c)

Consider the following three statements about a matrix A .

A is Hermitian, A is unitary and $A^2 = I$.

Now one needs to check that any two of three conditions implies third one.

If A is Hermitian then

$$A^H = A \quad (1)$$

If A is unitary, then

$$A^H A = I \quad (2)$$

And

$$A^2 = I \quad (3)$$

First show that (1) and (2) implies (3).

To show this, substitute the value for A^H from (1) into (2).

$$A^H A = I$$

$$A \cdot A = I$$

$$A^2 = I$$

Therefore, the statements (1) and (2) imply the third statement.

Step-4

Next show that (1) and (3) implies (2).

To show this, consider (3).

$$A^2 = I$$

$$A \cdot A = I$$

$$A^H \cdot A = I \quad \text{from (1) } A = A^H$$

Therefore, the statements (1) and (3) imply the second statement.

Step-5

Next show that (2) and (3) implies (1).

To show this, consider (2).

$$A^H \cdot A = I$$

Post multiply with A on both sides.

$$A^H \cdot A \cdot A = I \cdot A$$

$$A^H \cdot A^2 = A$$

$$A^H \cdot I = A \quad \text{from (3) } A^2 = I$$

$$A^H = A$$

Therefore, the statements (2) and (3) imply the first statement.

Therefore, any two of three conditions implies third one.

Therefore, given statement is true.

Step-6

(d)

If matrix A and matrix B are diagonalizable, then matrix AB is also diagonalizable.

Therefore, the given statement is true.