Step-1

We have to construct a matrix whose column space contains $(1,1,1)$ and whose null space is the line of multiples of $(1,1,1,1)$.

Given the null space is the line of multiples of $(1,1,1,1)$

So, let x_4 be free, remaining are pivots. $(x_1, x_2, x_3$ are pivots$)$.

Step-2

Then

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_4 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow x_1 - x_4 = 0$
$x_2 - x_4 = 0$
$x_3 - x_4 = 0$

Step-3

Therefore the matrix for the system is $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

The combination of columns 1,2,3 is

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Therefore the construction is impossible, because the column space contains three coordinates R^3.