Step-1

a) The given system of equations is

$$\begin{aligned} 2x_1 + 5x_2 &= 1 \\ x_1 + 4x_2 &= 2 \end{aligned}$$

We need to solve the given system by cramerâ€™s rule

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Step-2

Replacing the first and second columns of A with b, we get the matrices B_1 and B_2

$$B_1 = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}, B_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Now

$$\det(A) = |A| = \begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix}$$

$$= (2)(4) - (5)(1)$$

$$= 3$$

Step-3

And

$$\det(B_1) = |A_1| = \begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix}$$

$$\begin{aligned} &= (1)(4) - (2)(5) \\ &= -6 \end{aligned}$$

$$\det(B_2) = |A_2| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$\begin{aligned} &= (2)(2) - (1)(1) \\ &= 3 \end{aligned}$$

Step-4

Thus, by cramers rule we have

$$x_1 = \frac{\det(B_1)}{\det(A)}$$

$$= \frac{-6}{3}$$

$$= -2$$

$$x_2 = \frac{\det(B_2)}{\det(A)}$$

$$= \frac{3}{3}$$

$$= 1$$

Thus, the solution for the given system is $\boxed{x_1 = -2 \text{ and } x_2 = 1}$

Step-5

b) The given system of equations is

$$\begin{aligned} 2x_1 + x_2 &= 1 \\ x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 0 \end{aligned}$$

We need to solve the given system by cramerâ€™s rule

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Step-6

Replacing the first, second and third columns of A with b we get the matrices B_1, B_2 and B_3

$$B_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} B_2 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \text{And} \quad B_3 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Now

$$\det(A)=|A|=\begin{vmatrix}2 & 1 & 0\\1 & 2 & 1\\0 & 1 & 2\end{vmatrix}$$

$$=2(3)-(2)$$
$$=4$$

Step-7

And

$$\det(B_1)=|B_1|=\begin{vmatrix}1 & 1 & 0\\0 & 2 & 1\\0 & 1 & 2\end{vmatrix}$$

$$=(3)-(0)$$
$$=3$$

$$\det(B_2)=|B_2|$$

Step-8

$$=\begin{vmatrix}2 & 1 & 0\\1 & 0 & 1\\0 & 0 & 2\end{vmatrix}$$

$$=2(0)-(2)$$
$$=-2$$

$$\det(B_3)=|B_3|$$

$$=\begin{vmatrix}2 & 1 & 1\\1 & 2 & 0\\0 & 1 & 0\end{vmatrix}$$

$$=2(0)-(0)+(1)$$

$$=1$$

Step-9

Thus, by cramers rule we have

$$x_1 = \frac{\det(B_1)}{\det(A)}$$
$$= \frac{3}{4}$$

$$x_2 = \frac{\det(B_2)}{\det(A)}$$

$$= \frac{-2}{4}$$
$$= \frac{-1}{2}$$

$$x_3 = \frac{\det(B_3)}{\det(A)}$$

$$= \frac{1}{4}$$

Step-10

Thus, the solution for the given system is

$$\boxed{x_1 = \frac{3}{4}, x_2 = -\frac{1}{2}, x_3 = \frac{1}{4}}$$