

## Step-1

Note that when the black starts playing, either one of the white pawns has come forward (by one or two places) or one of the white knights has come forward.

Initially  $A$  and  $B$  start the game. Initially,  $A$  places forward one of its pawns.

Before  $B$  could reply to this move,  $C$  places forward one of its knights.  $D$  places the same pawn (as that of  $A$ ) forward. That is,  $D$  copies  $A$ 's move. Then  $C$  takes his knight back. So far  $B$  hasn't made any move.

Therefore, the situation on either boards is same. Only difference is that on one board, a white pawn is moved and on the other board, a similar black pawn is moved.

Now both games start. Whatever move is made by  $B$ , it is copied by  $C$  and whatever move is made by  $A$ , it is copied by  $D$ .

## Step-2

We have assumed that no strategy always wins for black. Therefore, if  $B$  loses the match  $A$ , then  $C$  cannot win against  $D$ . Thus, on one board black loses and on the other board black wins.

Suppose  $A$  moves his knight first, then also  $C$  will place his knight forward. Then  $D$  will move his knight, copying  $A$ 's move. Again  $C$  will take his knight back. Just like the previous case, if  $B$  loses the match  $A$ , then  $C$  cannot win against  $D$ . Thus, on one board black loses and on the other board black wins.

Therefore, our assumption that no strategy always wins for black is wrong. Therefore, it is not proved that no strategy always wins for black.