Step-1

a) We have to describe a subspace of **M** that contains $A=\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ but not $B=\begin{bmatrix}0 & 0\\0 & -1\end{bmatrix}$

Let **M** be the subspace that contains $A=\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ but not $B=\begin{bmatrix}0 & 0\\0 & -1\end{bmatrix}$

Let $M=\left\{\begin{bmatrix}a & 0\\0 & 0\end{bmatrix} \mid a\in\mathbf{R}\right\}$

Clearly **M** is a vector space where vector addition is matrix addition, whose scalar multiplication is a constant multiple of a 2 by 2 matrix.

$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\in\mathbf{M}$ And also $\begin{bmatrix}0 & 0\\0 & -1\end{bmatrix}\notin\mathbf{M}$

i.e., the **M** is the smallest subspace containing $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$

Step-2

b) Yes, it **M** contains *I*.

Since the linear combination

$$1\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}+(-1)\begin{bmatrix}0 & 0\\0 & -1\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$
$$=I$$

Therefore if a subspace **M** contains *A* and *B*, must b contain *I*.

Step-3

c) Consider the subspace of all matrices where main diagonal is all zero

Let $M=\left\{\begin{bmatrix}0 & a\\b & 0\end{bmatrix} \middle| a,b\in\mathbf{R}\right\}$

Then *M* is a subspace of the vector space of 2×2 matrices.