Step-1

The objective is to find the nullspace matrix N containing the special solutions of $R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$ here I is $r \times r$ and F is $r \times (n-r)$ matrices.

As I is $r \times r$ and F is a $r \times (n-r)$ matrix, it follows that R is $n \times n$ matrix.

Thus, there are r pivot columns in R such that $RX = 0$.

$$\begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix} X = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Step-2

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ such that $x_1, x_2, \ldots, x_r$ is a linear combination of $x_{r+1}, x_{r+2}, \ldots, x_n$ and $x_{r+1}, x_{r+2}, \ldots, x_n$ are free variables.

Therefore, $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} L.C \text{ of } x_{r+1}, x_{r+2}, \ldots, x_n \\ L.C \text{ of } x_{r+1}, x_{r+2}, \ldots, x_n \\ \vdots \\ L.C \text{ of } x_{r+1}, x_{r+2}, \ldots, x_n \\ x_{r+1} \\ x_{r+2} \\ \vdots \\ x_n \end{bmatrix}.$

Step-3

If $F = \begin{bmatrix} F_{1(r+1)} & F_{1(r+2)} & \cdots & F_{1n} \\ F_{2(r+1)} & F_{2(r+1)} & \cdots & F_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ F_{r(r+1)} & F_{r(r+2)} & \cdots & F_{rn} \end{bmatrix},$ then

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \\ x_{r+1} \\ x_{r+2} \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} -F_{1(r+1)} \\ F_{2(r+1)} \\ \vdots \\ F_{r(r+1)} \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + x_{r+2} \begin{bmatrix} -F_{1(r+2)} \\ F_{2(r+2)} \\ \vdots \\ F_{r(r+2)} \\ 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix} + \ldots + x_n \begin{bmatrix} -F_{1n} \\ F_{2n} \\ \vdots \\ F_{rn} \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

Therefore, the nullspace is $N = \begin{bmatrix} -F \\ I \end{bmatrix}$, here N is $n \times (n-r)$ matrix and I is an identity matric of order $(n-r) \times (n-r)$.