

## Step-1

Let  $p_1$  = the projection of  $b$  onto the line through  $a_1 = \frac{a_1^T b}{a_1^T a_1} a_1$

$$\begin{aligned} a_1^T b &= (-1, 2, 2) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ &= -1 + 0 + 0 \\ &= -1 \end{aligned}$$

$$\begin{aligned} a_1^T a_1 &= \begin{bmatrix} -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \\ &= 1 + 4 + 4 \\ &= 9 \end{aligned}$$

$$\begin{aligned} p_1 &= \frac{-1}{9} \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 1/9 \\ -2/9 \\ -2/9 \end{pmatrix} \end{aligned}$$

## Step-2

$p_2$  = The projection of  $b$  onto the line through  $a_2 = \frac{a_2^T b}{a_2^T a_2} a_2$

$$\begin{aligned} a_2^T b &= (2, 2, -1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ &= 2 + 0 + 0 \\ &= 2 \end{aligned}$$

$$\begin{aligned} a_2^T a_2 &= \begin{bmatrix} 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \\ &= 4 + 4 + 1 \\ &= 9 \end{aligned}$$

$$\begin{aligned}
 p_2 &= \frac{2}{9} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \\
 &= \begin{bmatrix} 4/9 \\ 4/9 \\ -2/9 \end{bmatrix}
 \end{aligned}$$

### Step-3

$P_3$  = The projection of  $b$  onto the line through  $a_3 = \frac{a_3^T b}{a_3^T a_3} a_3$

$$\begin{aligned}
 a_3^T b &= (2, -1, 2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\
 &= 2 + 0 + 0 \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 a_3^T a_3 &= \begin{bmatrix} 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \\
 &= 4 + 1 + 4 \\
 &= 9
 \end{aligned}$$

$$\begin{aligned}
 p_3 &= \frac{2}{9} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \\
 &= \begin{bmatrix} 4/9 \\ -2/9 \\ 4/9 \end{bmatrix}
 \end{aligned}$$

### Step-4

$$\begin{aligned}
 P_1 + P_2 + P_3 &= \begin{bmatrix} 1/9 \\ -2/9 \\ -2/9 \end{bmatrix} + \begin{bmatrix} 4/9 \\ 4/9 \\ -2/9 \end{bmatrix} + \begin{bmatrix} 4/9 \\ -2/9 \\ 4/9 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
 \end{aligned}$$

Therefore  $P_1 + P_2 + P_3 = b$