

Step-1

(a) Given that A is invertible

$$\text{i.e. } AA^{-1} = I$$

Taking transpose on both sides, we get

$$\begin{aligned}(AA^{-1})^T &= I^T \\ (A^{-1})^T A^T &= I\end{aligned}$$

Hence inverse of A^T = Transpose of A^{-1}

$$\text{that is } (A^T)^{-1} = (A^{-1})^T$$

Step-2

(b) A is symmetric, then $A = A^T$

Taking inverses on both sides,

$$\begin{aligned}A^{-1} &= (A^T)^{-1} \\ A^{-1} &= (A^{-1})^T\end{aligned}$$

Step-3

$$\text{(c) Given that } A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\Rightarrow A^T = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\Rightarrow (A^T)^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{-1}$$

$$\Rightarrow (A^T)^{-1} = \frac{1}{2 \cdot 1 - 1 \cdot 1} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow (A^T)^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

Step-4

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{2 \cdot 1 - 1 \cdot 1} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow (A^{-1})^T = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^T$$

$$\Rightarrow (A^{-1})^T = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^T$$

$$= (A^T)^{-1}$$

Step-5

Also A is symmetric and

$$A^{-1} = (A^{-1})^T$$

$$= \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^T$$