Step-1

All possible 2×2 matrices with entries $1,-1$ are

$$\begin{bmatrix}1&1\\1&1\end{bmatrix},\begin{bmatrix}-1&1\\1&1\end{bmatrix},\begin{bmatrix}1&-1\\1&1\end{bmatrix},\begin{bmatrix}1&1\\-1&1\end{bmatrix},\begin{bmatrix}1&1\\1&-1\end{bmatrix},\begin{bmatrix}-1&-1\\1&1\end{bmatrix},$$
$$\begin{bmatrix}-1&1\\1&-1\end{bmatrix},\begin{bmatrix}-1&1\\-1&1\end{bmatrix},\begin{bmatrix}1&-1\\-1&1\end{bmatrix},\begin{bmatrix}1&-1\\1&-1\end{bmatrix},\begin{bmatrix}1&1\\-1&-1\end{bmatrix},$$
$$\begin{bmatrix}-1&-1\\-1&-1\end{bmatrix},\begin{bmatrix}-1&-1\\-1&1\end{bmatrix},\begin{bmatrix}-1&1\\-1&-1\end{bmatrix},\begin{bmatrix}1&-1\\-1&-1\end{bmatrix},\begin{bmatrix}-1&-1\\1&-1\end{bmatrix},$$

We can observe that the values of determinants of all these matrices are zero $-2,2$ only.

Step-2

So that maximum values of determinant in 2×2 case is 2.

Now getting to 3×3 case, we should compute $a_{22}c_{11}+a_{12}c_{12}+a_{13}c_{13}$ to get $\det A$ where c_{ij} cofactors are a_{ij}

Exploring possibilities suppose $a_{11}=1, a_{12}=1\ a_{13}=1$

The other two rows should not be proportional to this row and proportional to each other so that determinant is non zero so they can be

$$\begin{bmatrix}1&-1&1\\-1&1&1\end{bmatrix}or\begin{bmatrix}-1&1&1\\1&-1&1\end{bmatrix}or\begin{bmatrix}1&1&-1\\-1&1&1\end{bmatrix}or$$
$$\begin{bmatrix}-1&1&1\\1&1&-1\end{bmatrix}or\begin{bmatrix}1&-1&1\\1&1&-1\end{bmatrix}or\begin{bmatrix}1&1&-1\\1&-1&1\end{bmatrix}$$

Step-3

The remaining cases are negatives of rows of these versions.

Clearly at least one factor vanishes.

So, the determinant value can not exceed 4

Similarly we can dispose all cases where exactly one entry of 1st row is -1 and exactly two entries of 1st row are -1â€™s etc.

In any case we note that atleast one cofactor vanishes and hence determinant of 3×3 matrix of entries 1 or -1 cannot exceed 4.