

Step-1

Let λ be the Eigen value of matrix A and $\lambda \neq c$. Here, c is any constant and not an Eigen value of A . Let

$$u = e^{ct} v$$

Find v to solve the following:

$$du/dt = Au - e^{ct} b$$

Also explain how it breaks down when c is an Eigen value.

Step-2

Substitute the $u = e^{ct} v$ in the following and solve:

$$\begin{aligned}\frac{du}{dt} &= Au - e^{ct} b \\ \frac{d(e^{ct} v)}{dt} &= A(e^{ct} v) - e^{ct} b \\ c(e^{ct} v) &= A(e^{ct} v) - e^{ct} b \\ cv &= Av - b \\ (A - cI)v &= b \\ v &= (A - cI)^{-1} b\end{aligned}$$

Therefore, $\boxed{v = (A - cI)^{-1} b}$. This gives the particular solution.

Step-3

If c is Eigen value then $c = \lambda$ and following must be true:

$$\det(A - cI) = 0$$

This shows that $(A - cI)$ is not invertible. Therefore, solution for v will not be possible.