

Step-1

We have to show that the following modified Gram-Schmidt steps produce the same

$$C = c - (q_1^T c)q_1 - (q_2^T c)q_2 \text{ of Gram-Schmidt Process.}$$

$$C^* = c - (q_1^T c)q_1, \text{ and } C = C^* - (q_2^T C^*)q_2$$

Step-2

$$\begin{aligned} C &= C^* - (q_2^T C^*)q_2 \\ &= c - (q_1^T c)q_1 - (q_2^T [c - (q_1^T c)q_1])q_2 \\ &= c - (q_1^T c)q_1 - (q_2^T c)q_2 + q_2^T (q_1^T c)q_1 \cdot q_2 \\ &= c - (q_1^T c)q_1 - (q_2^T c)q_2 + 0 \\ &= c - (q_1^T c)q_1 - (q_2^T c)q_2 \end{aligned}$$

Because q_1, q_2 are orthonormal columns, that is, $q_1^T q_2 = 0$

$$\text{Therefore } C = c - (q_1^T c)q_1 - (q_2^T c)q_2$$

Hence the required result is proved.