Step-1

Matrix form of given system is $Ax = b$

Where $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 5 \\ 9 \end{bmatrix}$

Considering A and applying $R_1 + R_2 - R_3$, we get $(1+2-3, 2+2-4, 2+3-5) = (0,0,0)$

In other words, multiplying the 1st row with 1, 2nd row with 1, 3rd with -1 and then adding, we get 0.

Step-2

Letting $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, we write

$$y^T Ax = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x+2y+2z \\ 2x+2y+3z \\ 3x+4y+5z \end{bmatrix}$$

$$= 1(x+2y+2z) + 1(2x+2y+3z) - 1(3x+4y+5z)$$

$= 0$ â€¦â€¦ (1)

Step-3

On the other hand, $y^T b = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 9 \end{bmatrix}$

$= 5 + 5 - 9$

$= 1$ â€¦â€¦ (2)

Therefore, $y^T Ax = y^T b$ reduces to 0 = 1

This is an absurdity.

Therefore, the given system $Ax = b$ has no solution

Step-4

We have seen that $y^T Ax = 0$

That means the inner product $\langle y, Ax \rangle = 0$

So, y is perpendicular to Ax

Therefore, the vector y is in the null space of A.