Step-1

Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

We have to show that the identity matrix I is not in the range of T.

Step-2

Suppose I is in the range of T

Then there exists a matrix M such that $T(M) = I$

$\Rightarrow AM = I$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a + 2c = 1, \qquad b + 2d = 0$

$\Rightarrow 3a + 6c = 0, \qquad 3b + 6d = 1$

Now $a + 2c = 1 \Rightarrow 3a + 6c = 3$

And $3a + 6c = 0$

We have $3a + 6c = 1$ and $3a + 6c = 0$ which is impossible.

Hence the identity matrix T is not in the range of T.

Step-3

Now we have to find a nonzero matrix M such that $T(M) = AM$ is zero.

Now $T(M) = 0$

$\Rightarrow AM = 0$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow x_1 + 2x_3 = 0, \ 3x_1 + 6x_3 = 0$

$x_2 + 2x_4 = 0$, $3x_2 + 6x_4 = 0$

$\Rightarrow x_1 = -2x_3,\ x_2 = -2x_4$

$x_3 = 1,\ x_4 = 1,\ x_1 = -2,\ x_2 = -2$

Therefore, $M = \begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix}$ is the nonzero matrix such that $AM = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence the required matrix is $\boxed{M = \begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix}}$