Step-1

Given that $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$$A.B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 2+0 \\ 2+0 & 4+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

$$B.A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} . \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+4 & 0+2 \\ 0+2 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$

Step-2

We know that $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\Rightarrow A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{1(1) - 2(0)} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

Step-3

$$B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow B^{-1} = \frac{1}{1(1)-2(0)} \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow B^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

Step-4

$$A.B = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$$

$$\Rightarrow (A.B)^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}^{-1}$$

$$\Rightarrow (A.B)^{-1} = \frac{1}{1.5-2.2} \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

$$\Rightarrow (A.B)^{-1} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

Step-5

$$B.A = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$$

$$\Rightarrow (B.A)^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}^{-1}$$

$$\Rightarrow (B.A)^{-1} = \frac{1}{5.1-2.2} \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$$

$$\Rightarrow (B.A)^{-1} = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$$