Step-1

When a matrix P has left inverse, it is given by,

$$P_{\text{left}}^{-1} = \left(P^{\mathrm{T}}P\right)^{-1} P^{\mathrm{T}}$$

In this case, A is a 3 by 5 matrix and therefore, A^{T} is a 5 by 3 matrix. This gives $A^{\mathrm{T}}A$ to be a 5 by 5 matrix. Therefore, $\left(A^{\mathrm{T}}A\right)^{-1}$ too will be a 5 by 5 matrix. Finally, $\left(A^{\mathrm{T}}A\right)^{-1} A^{\mathrm{T}}$ will be a 5 by 3 matrix. Thus, $A_{\text{left}}^{-1}A$ will be a 5 by 5 matrix and thus, it cannot be an identity matrix.

Therefore, A cannot have a left inverse.

Step-2

When a matrix P has right inverse, it is given by,

$$P_{\text{right}}^{-1} = P^{\mathrm{T}}\left(PP^{\mathrm{T}}\right)^{-1}$$

In this case, A is a 3 by 5 matrix and therefore, A^{T} is a 5 by 3 matrix. This gives AA^{T} to be a 3 by 3 matrix. Therefore, $\left(AA^{\mathrm{T}}\right)^{-1}$ too will be a 3 by 3 matrix. Finally, $\left(A^{\mathrm{T}}A\right)^{-1} A^{\mathrm{T}}$ will be a 5 by 3 matrix. Thus, AA_{right}^{-1} will be a 3 by 3 matrix. Since e_1, e_2, and e_3 are in the column space of A, it is clear that A can have a right inverse.