Step-1

We have to construct a 3 by 3 matrix A with no zero entries whose columns are mutually perpendicular, and we have to compute A^TA, and then we have to explain that why is it a diagonal matrix.

Step-2

Let

$$A=\begin{bmatrix} a & b & c \end{bmatrix}$$
$$=\begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix}$$

So

$$a=\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix},\ b=\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix},\ c=\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

Step-3

And

$$a^Tb=\begin{bmatrix} 2 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$
$$=4-2-2$$
$$=0$$

Step-4

$$b^Tc=\begin{bmatrix} 2 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$
$$=-2+4-2$$
$$=0$$

Step-5

$$c^T a = \begin{bmatrix} -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

$$= -2 - 2 + 4$$

$$= 0$$

Therefore a, b, c are mutually perpendicular

Step-6

And

$$A^T A = \begin{bmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \boxed{9I}$$

Hence $A^T A$ is a diagonal matrix, because its all non-diagonal elements are 0

an

That is, column 1 of A^{-1} is orthogonal to the space spanned by 2nd, 3rd,â€¦â€¦.., n th rows of A.