Step-1

Consider the matrix,

$$A = \begin{bmatrix} 0 & 3 \\ 4 & 6 \end{bmatrix}$$

The objective is to find the inverse of this matrix directly or from the $2 \text{ by } 2$ formula.

Step-2

Use the inverse of matrix formula.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Compare the matrix $A = \begin{bmatrix} 0 & 3 \\ 4 & 6 \end{bmatrix}$ with $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$a = 0$, $b = 3$, $c = 4$, and $d = 6$.

Step-3

Inverse of the matrix can be calculated as,

$$A^{-1} = \frac{1}{(0)\cdot(6) - (3)\cdot(4)}\begin{pmatrix} 6 & -3 \\ -4 & 0 \end{pmatrix}$$

$$\begin{aligned} A^{-1} &= \frac{1}{0 - 12}\begin{pmatrix} 6 & -3 \\ -4 & 0 \end{pmatrix} \\ &= \frac{1}{-12}\begin{pmatrix} 6 & -3 \\ -4 & 0 \end{pmatrix} \\ &= \begin{pmatrix} \frac{6}{-12} & \frac{-3}{-12} \\ \frac{-4}{-12} & \frac{0}{-12} \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 \end{pmatrix} \end{aligned}$$

Hence, the inverse of matrix $A=\begin{bmatrix}0 & 3\\ 4 & 6\end{bmatrix}$ is $A^{-1}=\boxed{\begin{pmatrix}-\frac{1}{2} & \frac{1}{4}\\ \frac{1}{3} & 0\end{pmatrix}}$.

Step-4

Consider the matrix,

$$B=\begin{bmatrix}a & b\\ b & 0\end{bmatrix}$$

The objective is to find the inverse of this matrix directly or from the 2 by 2 formula.

Compare the matrix $B=\begin{bmatrix}a & b\\ b & 0\end{bmatrix}$ with $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$.

$a=a$, $b=b$, $c=b$, and $d=0$.

Step-5

Inverse of the matrix can be calculated as,

$$B^{-1}=\frac{1}{a(0)-(b)(b)}\begin{bmatrix}0 & -b\\ -b & a\end{bmatrix}$$

$$\begin{aligned}B^{-1}&=\frac{1}{0-b^2}\begin{pmatrix}0 & -b\\ -b & a\end{pmatrix}\\ &=\frac{1}{-b^2}\begin{pmatrix}0 & -b\\ -b & a\end{pmatrix}\\ &=\begin{pmatrix}\frac{0}{-b^2} & \frac{-b}{-b^2}\\ \frac{-b}{-b^2} & \frac{a}{-b^2}\end{pmatrix}\end{aligned}$$

$$B^{-1}=\begin{pmatrix}0 & \frac{1}{b}\\ \frac{1}{b} & -\frac{a}{b^2}\end{pmatrix}$$

Hence, the inverse of the matrix $B=\begin{bmatrix} a & b \\ b & 0 \end{bmatrix}$ is $\boxed{B^{-1}=\begin{pmatrix} 0 & \frac{1}{b} \\ \frac{1}{b} & -\frac{a}{b^2} \end{pmatrix}}$.

Step-6

Consider the matrix,

$$C=\begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$$

The objective is to find the inverse of this matrix directly or from the 2 by 2 formula.

Compare the matrix $C=\begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ with $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$a=3$, $b=4$, $c=5$, and $d=7$.

Step-7

Inverse of the matrix can be calculated as,

$$C^{-1}=\frac{1}{(3)\cdot(7)-(4)\cdot(5)}\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$$

$$\begin{aligned} C^{-1} &= \frac{1}{21-20}\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix} \\ &= \frac{1}{1}\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix} \end{aligned}$$

Hence, the inverse of matrix $C=\begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ is $\boxed{C^{-1}=\begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}}$.