Step-1

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We have to compute $\begin{bmatrix} 1 & 0 & 0 \\ l & 1 & 0 \\ m & 0 & 1 \end{bmatrix}^n$ by experiment or by Gauss-Jordan method.

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ l & 1 & 0 \\ m & 0 & 1 \end{bmatrix}^n$

If $n = 1$

Then

$$\begin{aligned} A &= \begin{bmatrix} 1 & 0 & 0 \\ l & 1 & 0 \\ m & 0 & 1 \end{bmatrix}^1 \\ &= \begin{bmatrix} 1 & 0 & 0 \\ l & 1 & 0 \\ m & 0 & 1 \end{bmatrix} \end{aligned}$$

Step-2

If $n = 2$

Then

$$A^2=\begin{bmatrix}1&0&0\\l&1&0\\m&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\l&1&0\\m&0&1\end{bmatrix}$$
$$=\begin{bmatrix}1(1)+0(l)+0(m)&1(0)+0(1)+0(0)&1(0)+0(0)+0(1)\\l(1)+1(l)+0(m)&l(0)+1(1)+0(0)&l(0)+1(0)+0(1)\\m(1)+0(l)+1(m)&m(0)+0(1)+1(0)&m(0)+0(0)+1(1)\end{bmatrix}$$
$$=\begin{bmatrix}1+0+0&0+0+0&0+0+0\\l+l+0&0+1+0&0+0+0\\m+0+m&0+0+0&0+0+1\end{bmatrix}$$
$$=\begin{bmatrix}1&0&0\\2l&1&0\\2m&0&1\end{bmatrix}$$

Step-3

By the way of induction, we can get

$$A=\begin{bmatrix}1&0&0\\l&1&0\\m&0&1\end{bmatrix}^n$$
$$=\begin{bmatrix}1&0&0\\nl&1&0\\nm&0&1\end{bmatrix}$$

Hence $\boxed{\begin{bmatrix}1&0&0\\l&1&0\\m&0&1\end{bmatrix}^n=\begin{bmatrix}1&0&0\\nl&1&0\\nm&0&1\end{bmatrix}}$

Step-4

Let $A=\begin{bmatrix}1&0&0\\l&1&0\\m&0&1\end{bmatrix}$.

We can find A^{-1} by using Gauss-Jordan elimination method.

Consider

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ l & 1 & 0 & 0 & 1 & 0 \\ m & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Subtracting l times row 1 from row 2 and m times row 1 from row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -l & 1 & 0 \\ 0 & 0 & 1 & -m & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Hence $\boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -l & 1 & 0 \\ -m & 0 & 1 \end{bmatrix}}$

Step-5

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ l & 1 & 0 \\ 0 & m & 1 \end{bmatrix}$

We can find A^{-1} by using Gauss-Jordan elimination method.

Consider

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ l & 1 & 0 & 0 & 1 & 0 \\ 0 & m & 1 & 0 & 0 & 1 \end{bmatrix}$$

Subtracting l times row 1 from row 2 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -l & 1 & 0 \\ 0 & m & 1 & 0 & 0 & 1 \end{bmatrix}$$

Subtracting m times row 2 from row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -l & 1 & 0 \\ 0 & 0 & 1 & lm & -m & 1 \end{bmatrix} \approx \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Hence $\boxed{A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -l & 1 & 0 \\ lm & -m & 1 \end{bmatrix}}$