

Step-1

Let A be any $n \times n$ matrix. Without loss of generality (W.L.O.S) let B be a matrix formed by throwing away last two rows and columns of A . So, B is $(n-2) \times (n-2)$ matrix. Let μ be the smallest eigenvalue of matrix B and λ be the smallest eigenvalue of matrix A . Note that Rayleigh's quotient $R(x)$ is given by

$$R(x) = \frac{x^T A x}{x^T x}$$

Step-2

The Rayleigh's quotient for B agrees with the Rayleigh's quotient for A , whenever $x = (x_1, x_2, x_3, \dots, x_{n-2}, 0, 0)$. These are the vectors perpendicular to n -tuples, $z_1 = (0, 0, \dots, 0, 0, 1)$ and $z_2 = (0, 0, \dots, 0, 1, 0)$ and for them we have $x^T A x = x^T B x$.

The minimum over these x 's is the smallest eigenvalue μ . It cannot be below the smallest eigenvalue λ of A , because that is the minimum over all x 's.

Therefore, $\lambda \leq \mu$.