Step-1

Given if the orthogonal vectors $q_1 = \begin{bmatrix} 2/3 \\ 2/3 \\ -1/3 \end{bmatrix},\ q_2 = \begin{bmatrix} -1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$ are the columns of Q, then we have to find Q^TQ and QQ^T, and also we have to show that QQ^T is a projection matrix(onto the plane of q_1 and q_2)

Step-2

Given

$$Q = [q_1, q_2] = \begin{bmatrix} 2/3 & -1/3 \\ 2/3 & 2/3 \\ -1/3 & 2/3 \end{bmatrix}$$

$$Q^TQ = \begin{bmatrix} 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{bmatrix} \begin{bmatrix} 2/3 & -1/3 \\ 2/3 & 2/3 \\ -1/3 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4+4+1}{9} & \frac{-2+4-2}{9} \\ \frac{-2+4-2}{9} & \frac{1+4+4}{9} \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

Step-3

And

$$QQ^T = \begin{bmatrix} 2/3 & -1/3 \\ 2/3 & 2/3 \\ -1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 5/9 & 2/9 & -4/9 \\ 2/9 & 8/9 & 2/9 \\ -4/9 & 2/9 & 5/9 \end{bmatrix}}$$

Step-4

Let $P = QQ^T$

We know that P is a projection matrix if and only if $P^2 = P$

$$P^2 = \frac{1}{9}\begin{bmatrix} 5 & 2 & -4 \\ 2 & 8 & 2 \\ -4 & 2 & 5 \end{bmatrix} \frac{1}{9}\begin{bmatrix} 5 & 2 & -4 \\ 2 & 8 & 2 \\ -4 & 2 & 5 \end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix} 45 & 18 & -36 \\ 18 & 72 & 18 \\ -36 & 18 & 45 \end{bmatrix}$$

$$= \begin{bmatrix} 5/9 & 2/9 & -4/9 \\ 2/9 & 8/9 & 2/9 \\ -4/9 & 2/9 & 5/9 \end{bmatrix}$$

$$= P$$

Hence $P = QQ^T$ is a projection matrix.
