Step-1

The area of triangle with corners $(0,0),(6,0)$ and $(1,4)$ is

$$=\frac{1}{2}\begin{vmatrix}6 & 0\\1 & 4\end{vmatrix}$$

$$=\frac{1}{2}\left[6(4)-0\right]$$

$$=\frac{1}{2}(24)$$

= 12 sq.units

Step-2

When the axes are rotated by $\theta = 60°$

The rotation matrix has determinant=1

Since $$\begin{vmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{vmatrix}=\begin{vmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{vmatrix}$$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$=\frac{1}{4}+\frac{3}{4}$$

=1

Step-3

The new coordinates of the corners of the triangle are

$$\left(6\cos 60°,-6\sin 60°\right),\ \left(\cos 60°+4\sin 60°,-\sin 60°+4\cos 60°\right)$$

$$=\left(6.\frac{1}{2},-6.\frac{\sqrt{3}}{2}\right),\left(\frac{1}{2}+4\left(\frac{\sqrt{3}}{2}\right),-\frac{\sqrt{3}}{2}+4\left(\frac{1}{2}\right)\right)$$

$$=\left(3,-3\sqrt{3}\right),\left(\frac{1+4\sqrt{3}}{2},\frac{4-\sqrt{3}}{2}\right)$$

Step-4

Area of triangle with new coordinates is

$$A=\frac{1}{2}\begin{vmatrix} 3 & -3\sqrt{3} \\ \frac{1+4\sqrt{3}}{2} & \frac{4-\sqrt{3}}{2} \end{vmatrix}$$

$$=\frac{1}{2}\left[\frac{12-3\sqrt{3}-\left(-3\sqrt{3}-36\right)}{2}\right]$$

Step-5

$$A=\frac{1}{2}\left[\frac{12+36}{2}\right]$$

$$=\frac{1}{2}(24)$$

$$\boxed{=12\text{ sq.units}}$$

So, by rotating the axes by 60° the area of the triangle is unaltered.