Step-1

The objective is to solve the equation as two triangular systems, without multiplying LU to find A:

$$LUx = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

Here,

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, U = \begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } b = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}.$$

Step-2

First solve the equation, $Lc = b$ by forward to determine the column matrix c.

Assume $c = [c_1 \quad c_2 \quad c_3]^T$ and $b = [2 \quad 0 \quad 2]^T$

Consider,

$$Lc = b$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

Solve the equations,

$$\begin{aligned} c_1 &= 2 &&......(1) \\ c_1 + c_2 &= 0 &&......(2) \\ c_1 + c_3 &= 2 &&......(3) \end{aligned}$$

From (2),

$$\begin{aligned} c_2 &= -c_1 \\ c_2 &= -2 \qquad \text{since } c_1 = 2 \end{aligned}$$

From (3),

$$\begin{aligned} c_3 &= 2 - c_1 \\ &= 2 - 2 \end{aligned}$$

$= 0$

Therefore, the solution of $Lc = b$ gives $\boxed{c = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}}$.

Step-3

Solve the equation, $Ux = c$ by backward to determine the column matrix x.

Assume $x = [u \quad v \quad w]^T$

$$\begin{pmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$$

The systems of equations are,

$$\begin{aligned} 2u + 4v + 4w &= 2 \qquad \text{......}(4) \\ v + 2w &= -2 \qquad \text{......}(5) \\ w &= 0 \qquad \text{.......}(6) \end{aligned}$$

From (5),

$$\begin{aligned} v &= -2 - 2 \times 0 \\ v &= -2 \end{aligned}$$

From (4),

$$\begin{aligned} 2u &= 2 - 4v - 4w \\ u &= 1 - 2v - 2w \\ u &= 1 - 2(-2) - 2 \times 0 \\ u &= 5 \end{aligned}$$

Therefore, the solution of $Ux = c$ gives $\boxed{x = \begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix}}$.