MA215 Probability Theory

Assignment 08

1. Suppose that the continuous random variable X has p.d.f

$$f_X(x) = \begin{cases} kx(1-x), & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

   Evaluate the constant k, and then find the non-zero range of Y and the p.d.f $f_Y(y)$ of Y when

   (a) $Y = -3X + 3$;

   (b) $Y = \frac{1}{X}$.

2. Suppose that the random variable X has c.d.f.

$$F_X(x) = \begin{cases} 0, & x < 0, \\ \frac{1-\cos(x)}{2}, & 0 \leqslant x \leqslant \pi, \\ 1, & x > \pi. \end{cases}$$

   and that $Y = \sqrt{X}$. What is the non-zero range of Y? Find the c.d.f. $F_Y(y)$ of Y, and hence find the p.d.f of Y.

3. Suppose that the two random variables X and Y have joint probability c.d.f. $F(x, y)$. Show that $F(x, y)$ possesses the following properties:

   (a) For any fixed x, $F(x, y)$ is a non-decreasing function of y and, similarly, for any fixed y, $F(x, y)$ is a non-decreasing function of x.

   (b) $F(x, y) \to 1$ when both $x \to +\infty$ and $y \to +\infty$.

   (c) $F(x, y) \to 0$ when either $x \to -\infty$ or $y \to -\infty$.

   (d) If $x_1 < x_2$ and $y_1 < y_2$, then

$$P(x_1 < X \leqslant x_2, y_1 < Y \leqslant y_2) = F(x_2, y_2) - F(x_2, y_1) - F(x_1, y_2) + F(x_1, y_1).$$

4. Suppose that the two discrete random variables X and Y have joint p.m.f. given by

Y	$Y = 1$	$Y = 2$	$Y = 3$	$Y = 4$
$X = 1$	2/32	3/32	4/32	5/32
$X = 2$	3/32	4/32	5/32	6/32

   Obtain the marginal p.m.f. of X.