Step-1

Consider the following matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The new matrix $\begin{pmatrix} a-mc & b-md \\ c-la & d-lb \end{pmatrix}$ is obtained by applying the two row operations

$$R_1 - mR_2 \rightarrow R_{1}$$
 and $R_2 - lR_1 \rightarrow R_{2}$ on $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Step-2

Now, to find the determinant of the new matrix directly:

$$\begin{vmatrix} a-mc & b-md \\ c-la & d-lb \end{vmatrix} = (a-mc)\cdot(d-lb)-(c-la)\cdot(b-md)$$

$$= ad-alb-mcd+mclb-cb+cmd+lab-lamd$$

$$= ad-cb+mclb-lamd$$

$$= ad(1-lm)-cb(1-lm)$$

$$= (ad-cb)\cdot(1-lm)$$

Therefore, the determinant of the new matrix comes out to be $(ad-cb)\cdot (1-lm)$