Step-1

Suppose T transposes every matrix M.

We have to try to find a matrix A that gives $AM = M^T$

Step-2

Let $M = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Then $M^T = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Step-3

Now

$$AM = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} b & 0 \\ d & 0 \end{bmatrix}$$

Therefore, $AM = M^T$ is not possible. $\left(\text{since} \begin{bmatrix} b & 0 \\ d & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ for any } b, d \right)$

So, no matrix A will do it.

Step-4

To professors the matrix space has dimension 4

(The dim of 2 by 2 matrixes is 4)

Linear transformations on the space must come from 4 by 4 matrix. This linear transformation does not come from a matrix.