

Step-1

Let E be the 2 by 2 elementary matrix and it adds the first equation to the second.

We have to find E^2, E^8 and $8E$.

Step-2

Given that the matrix E is 2 by 2 that adds the first row to the second row.

Hence
$$E = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Now

$$\begin{aligned} E^2 &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1(1)+0(1) & 1(0)+0(1) \\ 1(1)+1(1) & 1(0)+1(1) \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & 0+0 \\ 1+1 & 0+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \end{aligned}$$

Therefore,
$$E^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Step-3

Now

$$\begin{aligned} E^4 &= E^2 E^2 \\ &= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1(1)+0(2) & 1(0)+0(1) \\ 2(1)+1(2) & 2(0)+1(1) \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & 0+0 \\ 2+2 & 0+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \end{aligned}$$

Therefore, $E^4 = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

Step-4

Now

$$\begin{aligned} E^8 &= E^4 E^4 \\ &= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1(1)+0(4) & 1(0)+0(1) \\ 4(1)+1(4) & 4(0)+1(1) \end{bmatrix} \\ &= \begin{bmatrix} 1+0 & 0+0 \\ 4+4 & 0+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix} \end{aligned}$$

Therefore, $E^8 = \begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix}$

Step-5

Now

$$\begin{aligned} 8E &= 8 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 0 \\ 8 & 8 \end{bmatrix} \end{aligned}$$

Therefore, $8E = \begin{bmatrix} 8 & 0 \\ 8 & 8 \end{bmatrix}$