Step-1

The objective is to show that the matrices A and B are similar by finding a matrix M so that $B = M^{-1}AM$.

Above equation can be written as follows:

$$MB = MM^{-1}AM$$
$$MB = AM$$

Step-2

(a)

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Now assume $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Consider

$$MB = AM$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$\begin{bmatrix} 0 & a+b \\ 0 & c+d \end{bmatrix} = \begin{bmatrix} a & b \\ a & b \end{bmatrix}$$

$$a = 0$$
$$a + b = b$$
$$c + d = b$$

Step-3

Therefore, for $MB = AM$, one requires that

$$a = 0, b = c + d$$

Take the values for b, c and d so that the matrix M should be invertible.

Since $a = 0$, so one of possible values for b, c and d is

$$b = 1, c = 1, d = 0$$

Hence the matrices A and B are similar and one of possible for the matrix M is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ so that $MB = AM$.

Step-4

(b)

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Now assume $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Consider

$$MB = AM$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$\begin{bmatrix} a-b & -a+b \\ c-d & -c+d \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ a+c & b+d \end{bmatrix}$$

$$\begin{aligned} a-b &= a+c \\ -a+b &= b+d \\ c-d &= a+c \\ -c+d &= b+d \end{aligned}$$

Step-5

Solve these equations for the values a, b, c and d.

$$\begin{aligned} -b &= c \\ -a &= d \\ -d &= a \\ -c &= b \end{aligned}$$

Therefore, for $MB = AM$, one requires that

$$a = -d, b = -c$$

Take the values for a, b, c and d so that the matrix M should be invertible.

One of possible values for a, b, c and d is

$a=1, b=2, c=-2, d=-1$

Hence the matrices A and B are similar and one possible for the matrix M is $\boxed{\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}}$, so that $MB = AM$.

Step-6

(c)

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

Now assume $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Consider

$$\begin{aligned} MB &= AM \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} &= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \\ \begin{bmatrix} 4a+2b & 3a+b \\ 4c+2d & 3c+d \end{bmatrix} &= \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 4a+2b &= a+2c \\ 4c+2d &= 3a+4c \\ 3a+b &= b+2d \\ 3c+d &= 3b+4d \end{aligned}$$

Step-7

Solve these equations for the values a,b,c and d.

$$\begin{aligned} 3a+2b &= 2c \\ 2d &= 3a \\ 3a &= 2d \\ c &= b+d \end{aligned}$$

Therefore, for $MB = AM$, one requires that

$3a = 2d,\ c = b + d$

Take the values for a, b, c and d so that the matrix M should be invertible.

One of possible values for a, b, c and d is

$a = 2,\ b = 1,\ c = 4,\ d = 3$

Hence the matrices A and B are similar and one of possible for the matrix M is $\boxed{\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}}$ so that $MB = AM$.