Step-1

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1) Given statement is â€˜If $L_1U_1 = L_2U_2$ (upper triangular *U*â€™s with nonzero diagonal, lower triangular *L*â€™s with unit diagonal), then $L_1 = L_2$ and $U_1 = U_2$. The *LU* factorization is uniqueâ€™.

We have to determine whether the given statement is true or false.

Step-2

The given statement is **true**.

Since the factorization is unique.

Consider $L_1U_1 = L_2U_2$

Multiplying both sides by L_1^{-1}, we get

$$L_1^{-1}(L_1U_1) = L_1^{-1}(L_2U_2)$$
$$(L_1^{-1}L_1)U_1 = (L_1^{-1}L_2)U_2$$
$$U_1 = (L_1^{-1}L_2)U_2$$

Step-3

Similarly multiplying $L_1U_1 = L_2U_2$ by $L^{-1}{}_2$, we get

$$L_2^{-1}(L_1U_1) = L_2^{-1}(L_2U_2)$$
$$(L_2^{-1}L_1)U_1 = (L_2^{-1}L_2)U_2$$
$$(L_2^{-1}L_1)U_1 = U_2$$

But L_1, L_2 are elementary matrices and its inverses exists and $L_1^{-1}L_2 = L_2L_1^{-1}$ becomes identity.

Hence $U_1 = U_2$

Similarly we can prove that $L_1 = L_2$

Hence the given statement is **true**.

Step-4

2) Given statement is â€˜IF $A^2+A=I$ then $A^{-1}=A+I$ â€™.

We have to determine whether the given statement is true or false.

Step-5

The given statement is **true**.

Consider

$$\begin{aligned}
&A^2+A=I \\
&\Rightarrow A^{-1}\left(A^2+A\right)=A^{-1}(I) \qquad \left(\text{Multiplying both sides with } A^{-1}\right) \\
&\Rightarrow A^{-1}A^2+A^{-1}A=A^{-1} \\
&\Rightarrow \left(A^{-1}A\right)A+A^{-1}A=A^{-1} \\
&\Rightarrow IA+I=A^{-1} \qquad \left(\text{Since } A^{-1}A=AA^{-1}=I\right) \\
&\Rightarrow A+I=A^{-1}
\end{aligned}$$

Hence $A^{-1}=A+I$

Step-6

(c) Given statement is â€˜If all diagonal entries of A are zero, then A is singular.

We have to determine whether the given statement is true or false.

Step-7

The given statement is **false.**

Since let $A=\begin{bmatrix}0 & 1\\ 2 & 0\end{bmatrix}$

Then

$$\begin{aligned}
\det A &= 0(0)-1(2) \\
&= 0-2 \\
&= -2
\end{aligned}$$

Since $\det A \neq 0$

So A is nonsingular.

Hence the given statement is **false**.