Step-1

Consider a matrix A with Eigen values $|\lambda_i| > 1$ and $|\lambda_i| < 1$. The powers A^k approach zero if all $|\lambda_i| < 1$, and they blow up if any of the $|\lambda_i| > 1$.

Let B and C be two matrices defined as follows:

$$B = \begin{bmatrix} 3 & 2 \\ -5 & -3 \end{bmatrix}$$
$$C = \begin{bmatrix} 5 & 7 \\ -3 & -4 \end{bmatrix}$$

Determine the Eigen values of B and C and show the following:

$$B^4 = I$$
$$C^3 = -I$$

Step-2

To find the Eigen values of B do the following calculations:

$$\begin{aligned} \det(B - \lambda I) &= 0 \\ (3-\lambda)(3-\lambda)+10 &= 0 \\ \lambda^2 + 1 &= 0 \\ \lambda &= \pm i \end{aligned}$$

Therefore, Eigen values of matrix B is $\boxed{\lambda = \pm i}$.

Step-3

Eigen values of matrix B^4 will be λ^4.

$$\lambda^4 = 1$$

This shows that $\boxed{B^4 = I}$.

Step-4

To find the Eigen values of C do the following calculations:

$$\det(C - \lambda I) = 0$$
$$(5-\lambda)(-4-\lambda) + 21 = 0$$
$$\lambda^2 - \lambda + 1 = 0$$
$$\lambda = e^{\pm i\pi/3}$$

Therefore, Eigen values of matrix C is $\boxed{\lambda = e^{\pm i\pi/3}}$.

Step-5

Eigen values of matrix C^3 will be λ^3.

$$\lambda^3 = \left(e^{\pm i\pi/3}\right)^3$$
$$= e^{\pm i\pi}$$
$$\lambda^3 = -1$$

This shows that $\boxed{C^3 = -I}$.