Step-1

Now consider the following:

$$F_{1024} = \begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix} \begin{bmatrix} F_{512} & \\ & F_{512} \end{bmatrix} \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix}$$

By using the above rule, we get

$$F_{1024}{}^{T} = \left(\begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix} \begin{bmatrix} F_{512} & \\ & F_{512} \end{bmatrix} \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix} \right)^{T}$$

$$= \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix}^{T} \begin{bmatrix} F_{512} & \\ & F_{512} \end{bmatrix}^{T} \begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix}^{T}$$

Step-2

Note that the transpose of an even permutation is another even permutation. Similarly, transpose of an odd permutation is another odd permutation.

Thus, we get

$$F_{1024}{}^{T} = \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix} \begin{bmatrix} F_{512} & \\ & F_{512} \end{bmatrix} \begin{bmatrix} I_{512} & I_{512} \\ D_{512} & -D_{512} \end{bmatrix}$$