Step-1

Consider the following matrices:

$$A = S\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}S^{-1}$$

$$B = S\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}S^{-1}$$

Matrix S contains vectors x_1 and x_2. Find the Eigen values and Eigen vectors of matrices A and B.

Step-2

Letâ€™s consider matrix A first. Rewrite matrix A as follows:

$$A = S\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}S^{-1}$$

$$A = SCS^{-1}$$

Here,

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix C is a diagonal matrix. This implies that matrix A is diagonalizable. So, diagonal of matrix C is Eigen values of A and columns of matrix S is Eigen vectors of matrix A.

Step-3

Therefore, matrix A has Eigen value: $\boxed{\lambda = (2,1)}$ and Eigen vectors x_1 and x_2.

Step-4

Next consider matrix B. Rewrite matrix B as follows:

$$B = S\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}S^{-1}$$

$$B = SDS^{-1}$$

Here,

$$D = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

Matrix D is upper triangular matrix so Eigen values of matrix D are $\lambda = (2,1)$.Matrix $B = SDS^{-1}$ implies that B and D are similar matrices. Recall that similar matrices share the same Eigen values and every Eigen vector x of D corresponds to an Eigen vector $S^{-1}x$ of B.

Step-5

Therefore, Eigen values of matrix B are $\boxed{\lambda = (2,1)}$ and Eigen vectors $\boxed{S^{-1}x_1}$ and $\boxed{S^{-1}x_2}$.