

Step-1

$$U = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Consider the matrix,

$$\begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 9 \\ 0 \end{bmatrix}.$$

The three independent columns are

$$= \left\{ \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} / a, b, c \in R \right\}$$

The columns are base for the columns space

Step-2

Now,

$$A = \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 4 & 6 & 8 & 2 \end{bmatrix}$$

Apply $R_4 \rightarrow R_4 - 2R_1$

$$= \begin{bmatrix} 2 & 3 & 4 & 1 \\ 0 & 6 & 7 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ = U$$

Therefore U is echelon form of A therefore the linearly dependent columns of U are same as the linear independent columns of A .