

Step-1

Matrix A can be factorized into product of lower and upper triangular matrices.

$$\mathbf{A} = \mathbf{LU}$$

Here, matrix L is a lower triangular matrix with 1 at the diagonal position and matrix U is the upper triangular matrix with pivots at the diagonal position.

Step-2

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} x & x & x & x \\ x & x & x & 0 \\ 0 & x & x & x \\ 0 & 0 & x & x \end{bmatrix}$$
$$\mathbf{B} = \begin{bmatrix} x & x & x & 0 \\ x & x & 0 & x \\ x & 0 & x & x \\ 0 & x & x & x \end{bmatrix}$$

Here, nonzero positions are marked by x . Determine which zeros will be still zero in their factors L and U .

Step-3

First take matrix A .

$$\mathbf{A} = \begin{bmatrix} x & x & x & x \\ x & x & x & 0 \\ 0 & x & x & x \\ 0 & 0 & x & x \end{bmatrix}$$

It can be seen that a part (3×3) of the (4×4) matrix is already in lower triangular form. Now recall that if a row of matrix A starts with zeros, so does that row of matrix L and if a column starts with zero so does that column of matrix U .

Therefore, matrix L will contain three zeros at the position $\boxed{a_{31}, a_{41}, a_{42}}$ however matrix U may not contain zero at the position $\boxed{a_{24}}$.

Step-4

Similarly, in the case of matrix B , bottom left side zero will be in matrix L and top right side zero will be in matrix U . Rest zeros may be filled in by the non zeros.

Therefore, matrix L will contain one zero at the position $\boxed{a_{41}}$ and matrix U will contain zero at the position $\boxed{a_{44}}$.