

Step-1

Consider the step function; $y(x) = \begin{cases} 1, 0 \leq x \leq \pi \\ 0, \pi < x < 2\pi \end{cases}$

And Fourier coefficients;

$$a_0 = \frac{(y, 1)}{(1, 1)}$$
$$a_1 = \frac{(y, \cos x)}{(\cos x, \cos x)}$$
$$b_1 = \frac{(y, \sin x)}{(\sin x, \sin x)}$$

To find the Fourier coefficients a_0, a_1, b_1 of the above step function.

Step-2

Now, calculate inner product as follows;

$$(y, 1) = \int_0^{2\pi} y(x) \cdot 1 dx$$
$$= \int_0^{\pi} 1 \cdot 1 dx + \int_{\pi}^{2\pi} 0 \cdot 1 dx$$
$$= [x]_0^{\pi} + 0$$
$$= \pi - 0$$
$$= \pi$$

Also,

$$(1, 1) = \int_0^{2\pi} 1 \cdot 1 dx$$
$$= [x]_0^{2\pi}$$
$$= 2\pi - 0$$
$$= 2\pi$$

Step-3

Thus, obtain Fourier coefficient by substitute values;

$$\begin{aligned}
 a_0 &= \frac{(y, 1)}{(1, 1)} \\
 &= \frac{\pi}{2\pi} \\
 &= \frac{1}{2}
 \end{aligned}$$

Therefore, $\boxed{a_0 = \frac{1}{2}}$

Step-4

Now, again calculate inner product;

$$\begin{aligned}
 (y, \cos x) &= \int_0^{2\pi} y(x) \cos x dx \\
 &= \int_0^{\pi} 1 \cdot \cos x dx + \int_{\pi}^{2\pi} 0 \cdot \cos x dx \\
 &= [\sin x]_0^{\pi} + 0 \\
 &= \sin \pi - \sin 0 \\
 &= 0
 \end{aligned}$$

Step-5

And,

$$\begin{aligned}
 (\cos x, \cos x) &= \int_0^{2\pi} \cos x \cos x dx \\
 &= \int_0^{2\pi} \frac{1 + \cos 2x}{2} dx \\
 &= \frac{1}{2} \left[x + \frac{\sin 2x}{2} \right]_0^{2\pi} \\
 &= \frac{1}{2} \left[2\pi + \frac{\sin 4\pi}{2} \right] - \frac{1}{2} [0 + 0] \\
 &= \pi
 \end{aligned}$$

Step-6

Therefore, substitute values obtained above to get;

$$\begin{aligned}
 a_1 &= \frac{(y, \cos x)}{(\cos x, \cos x)} \\
 &= \frac{0}{\pi} \\
 &= 0
 \end{aligned}$$

Hence, $\boxed{a_1 = 0}$

Step-7

Now,

$$\begin{aligned}
 (y, \sin x) &= \int_0^{2\pi} y(x) \sin x dx \\
 &= \int_0^{\pi} 1 \cdot \sin x dx + \int_{\pi}^{2\pi} 0 \cdot \sin x dx \\
 &= [-\cos x]_0^{\pi} + 0 \\
 &= -\cos \pi + \cos 0 \\
 &= 2
 \end{aligned}$$

Step-8

And,

$$\begin{aligned}
 (\sin x, \sin x) &= \int_0^{2\pi} \sin x \sin x dx \\
 &= \int_0^{2\pi} \frac{1 - \cos 2x}{2} dx \\
 &= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{2\pi} \\
 &= \frac{1}{2} \left[2\pi - \frac{\sin 4\pi}{2} \right] - \frac{1}{2} [0 + 0] \\
 &= \pi
 \end{aligned}$$

Step-9

Therefore,

$$b_1 = \frac{(y, \sin x)}{(\sin x, \sin x)}$$

$$= \frac{2}{\pi}$$

Hence, $\boxed{b_1 = \frac{2}{\pi}}$