## Step-1

Consider the following four fundamental subspaces:

The row space is  $C(A^T)$ , a subspace of  $\mathbb{R}^n$ .

The column space C(A), a subspace of  $\mathbb{R}^m$ .

The nullspace is N(A), a subspace of  $\mathbb{R}^n$ .

The left nullspace is  $N(A^T)$ , a subspace of  $\mathbb{R}^m$ .

Find the system Ax = b has a solution if and only if b is orthogonal to which of the given four fundamental subspaces.

## Step-2

Suppose the system Ax = b has a solution.

Then the vector **b** is lie in the column space C(A).

Thus, **b** is orthogonal to the left nullspace  $N(A^T)$ .

Hence, if the system  $A\mathbf{x} = \mathbf{b}$  has a solution, then the vector  $\mathbf{b}$  is orthogonal to  $N(A^T)$ .

## Step-3

Conversely,

Suppose the vector  $\mathbf{b}$  is orthogonal to the left nullspace  $N(A^T)$ .

Then the vector **b** is lie in the column space C(A).

Thus, the system  $A\mathbf{x} = \mathbf{b}$  has a solution.

Hence, if the vector **b** is orthogonal to  $N(A^T)$ , then the system  $A\mathbf{x} = \mathbf{b}$  has a solution.