

Step-1

Explain two reasons that why the exponential matrix e^{At} can never be singular.

(a) *Inverse*: A square matrix that is not invertible is called as singular matrix. However, determinant of e^{At} exists, so inverse will also exist. Inverse of e^{At} is given as follows:

Inverse: $\boxed{e^{-At}}$.

Determinant of the exponential is given as follows:

$$\begin{aligned}\det(e^{At}) &= e^{\lambda_1 t} \cdot e^{\lambda_2 t} \cdot \dots \cdot e^{\lambda_n t} \\ &= e^{(\lambda_1 + \lambda_2 + \dots + \lambda_n)t} \\ &= e^{\text{trace}(At)}\end{aligned}$$

Step-2

(b) Eigen values: If $Ax = \lambda x$ shows that λ is an Eigen values of A, then $e^{\lambda t}$ is an Eigen value matrix of e^{At} . That means $e^{\lambda t}$ can never be zero.

$$e^{At}x = \boxed{e^{\lambda t}x}.$$

Step-3

Therefore, exponential matrix e^{At} can never be singular.