Step-1

Consider that the matrix A is a 1×3 matrix whose null space consists of all vectors in $\mathbf{R}^3$ such that $x_1+2x_2+4x_3=0.$

The objective is to find the matrix A.

Step-2

The null space $N(A)$ of the matrix A is the solution space of the system $A\mathbf{x}=\mathbf{0}.$

Let $\mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\in N(A).$

Then $x_1+2x_2+4x_3=0.$

This can be written as,

$$x_1+2x_2+4x_3=0$$
$$\begin{bmatrix} 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

Hence, the required 1×3 matrix is $\boxed{A=\begin{bmatrix} 1 & 2 & 4 \end{bmatrix}}$ whose null space consists of all vectors in $\mathbf{R}^3$ such that $x_1+2x_2+4x_3=0.$

Step-3

Now find a 3×3 matrix whose null space consists of all vectors in $\mathbf{R}^3$ such that $x_1+2x_2+4x_3=0.$

The null space of the matrix A consists of all vectors in $\mathbf{R}^3$ such that $x_1+2x_2+4x_3=0.$

That is,

$$\begin{aligned} N(A) &= \left\{\mathbf{x}\in \square^3 / A\mathbf{x}=\mathbf{0}\right\} \\ &= \left\{\mathbf{x}\in \square^3 / x_1+2x_2+4x_3=0\right\} \end{aligned}$$

Step-4

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a 3×3 matrix whose null space consists of all vectors in $\mathbf{R}^3$ such that $x_1 + 2x_2 + 4x_3 = 0.$

Then

$$A\mathbf{x} = \mathbf{0}$$
$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step-5

From this, the following equations are obtained:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0,\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0$, $a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0$

As these are the equations in null space of A and the null space of the matrix A consists all the vectors of the form $x_1 + 2x_2 + 4x_3 = 0.$

Compare the equations $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = 0,\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = 0$, and

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = 0$ with the equation $x_1 + 2x_2 + 4x_3 = 0.$

If the first row of the matrix A is $(1,2,4)$ then the second and third rows of the matrix A are the multiples of the row $(1,2,4).$

Hence, the required 3×3 matrix is $\boxed{A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \\ 3 & 6 & 12 \end{bmatrix}}.$