Step-1

Consider the matrix,

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{pmatrix}$$

and $E_{32}E_{31}E_{21}A = U$.

The objective is to determine the matrices, $E_{21}, E_{31}, E_{32},$ and multiply the matrices of E, to obtain the matrix M such that $MA = U$.

Step-2

Consider the matrix,

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{pmatrix}$$

Apply the row operation, $R_2 \rightarrow R_2 - 4R_1$.

$$= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -2 & 2 & 0 \end{pmatrix} \text{â€¦â€¦ (1)}$$

Apply the same row operation $R_2 \rightarrow R_2 - 4R_1$ on identity matrix I, obtained as,

$$E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Apply the row operation, $R_3 \rightarrow R_3 + 2R_1$ on matrix (1), obtained as,

$$= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{pmatrix} \text{â€¦â€¦ (2)}$$

Apply the same row operation $R_3 \rightarrow R_3 + 2R_1$ on identity matrix I, obtained as,

$$E_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

Apply the row operation, $R_3 \rightarrow R_3 - 2R_2$ on matrix (2), obtained as,

$$U = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{pmatrix}$$

Apply the same row operation $R_3 \rightarrow R_3 - 2R_2$ on identity matrix I, obtained as,

$$E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Step-3

Multiply the matrices, E_{32}, E_{31}, and E_{21}, obtained as,

$$E_{32}E_{31}E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{pmatrix}$$

$$= M$$

Now, verify that, $MA = U$.

$$E_{32}E_{31}E_{21}A = MA$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{pmatrix}$$

$$= U$$

Therefore,

$$E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

$$E_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

$$E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$