Step-1

If B is positive definite, then, $x^T Bx > 0$ for all nonzero vectors x.

The Rayleigh quotient is given by,

$$R(x) = \frac{x^T Ax}{x^T x}$$

The Rayleigh quotient for $A+B$ is given by,

$$R(x) = \frac{x^T (A+B)x}{x^T x}$$

We know that,

$$x^T (A+B)x > x^T Ax$$

Thus, the Rayleigh quotient for $A+B$ is greater the Rayleigh quotient for A.

Step-2

Therefore, $\boxed{\frac{x^T (A+B)x}{x^T x} > \frac{x^T Ax}{x^T x}}$.