Step-1

Given $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $A+B=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$

Now $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

$$\begin{aligned}A-\lambda I &= \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &= \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &= \begin{bmatrix}1-\lambda & 0\\ 1 & 1-\lambda\end{bmatrix}\end{aligned}$$

Step-2

Then

$$\begin{aligned}|A-\lambda I| &= \begin{vmatrix}1-\lambda & 0\\ 1 & 1-\lambda\end{vmatrix}\\ &= (1-\lambda)^2\\ &= 1+\lambda^2-2\lambda\end{aligned}$$

Step-3

Becomes

$$\begin{aligned}\lambda^2-2\lambda+1 &= 0\\ \lambda^2-\lambda-\lambda+1 &= 0\\ \lambda(\lambda-1)-1(\lambda-1) &= 0\\ (\lambda-1)(\lambda-1) &= 0\\ \lambda &= 1,1\end{aligned}$$

That is $\lambda_1=1, \lambda_2=1$

Therefore the Eigen values of A is 1, 1

Step-4

Now $B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

$$\begin{aligned}A-\lambda I&=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ &=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\\ &=\begin{bmatrix}1-\lambda & 1\\ 0 & 1-\lambda\end{bmatrix}\end{aligned}$$

Step-5

Then

$$\begin{aligned}|A-\lambda I|&=\begin{vmatrix}1-\lambda & 1\\ 0 & 1-\lambda\end{vmatrix}\\ &=(1-\lambda)^2\\ &=1+\lambda^2-2\lambda\end{aligned}$$

Step-6

Becomes

$$\begin{aligned}\lambda^2-2\lambda+1&=0\\ \lambda^2-\lambda-\lambda+1&=0\\ \lambda(\lambda-1)-1(\lambda-1)&=0\\ (\lambda-1)(\lambda-1)&=0\\ \lambda&=1,1\end{aligned}$$

That is $\lambda_1=1,\lambda_2=1$

Therefore the Eigen values of B is 1, 1

Step-7

Now $A+B=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$

$$A-\lambda I=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}-\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$$
$$=\begin{bmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{bmatrix}$$

Step-8

Then

$$|A-\lambda I|=\begin{vmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{vmatrix}$$
$$=(2-\lambda)(2-\lambda)-1$$
$$=4-2\lambda-2\lambda+\lambda^2-1$$
$$=\lambda^2-4\lambda+3$$

$$\lambda^2-4\lambda+3=0$$
$$\lambda^2-3\lambda-\lambda+3=0$$
$$\lambda(\lambda-3)-1(\lambda-3)=0$$
$$(\lambda-3)(\lambda-1)=0$$

That is $\lambda_1=3,\lambda_2=1$

Therefore the Eigen values of $A+B$ is 3, 1

Eigen values of $A+B$ are not equal to Eigen values of A plus Eigen values of B.

Because here the Eigen values of $A+B$ is 3, 1

Step-9

Which are not equal to the Eigen values of A is 1, 1 plus the Eigen values of B is 1, 1

Here Eigen values of A is equal to the Eigen values of B

But Eigen values of $A+B$ are not equal to Eigen values of A plus Eigen values of B.