

Step-1

Consider the equation:

$$\begin{aligned} Ax &= \lambda Mx \\ \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} x &= \lambda \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} x \\ \left\{ \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda m_1 & 0 \\ 0 & \lambda m_2 \end{bmatrix} \right\} x &= 0 \\ \begin{bmatrix} 2 - \lambda m_1 & -1 \\ -1 & 2 - \lambda m_2 \end{bmatrix} x &= 0 \end{aligned}$$

By substituting $m_1 = 1$ and $m_2 = 2$, we get,

$$\begin{aligned} \begin{bmatrix} 2 - \lambda & -1 \\ -1 & 2 - 2\lambda \end{bmatrix} x &= 0 \\ \begin{bmatrix} 2 - \lambda & -1 \\ -1 & 2 - 2\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= 0 \\ (2 - \lambda)x_1 - x_2 &= 0 \\ -x_1 + x_2(2 - 2\lambda) &= 0 \end{aligned}$$

Therefore, the normal modes are M-orthogonal.