## Step-1

Given that $\mathbf{V} = \mathbf{R}^2$ and $\mathbf{W} = \mathbf{R}^2$

a) Given $T : \mathbf{V} \to \mathbf{W}$ such that $T(v) = -v$

We have to find $T(T(v))$

## Step-2

Now $T(v) = -v$

So

$$\begin{aligned} T(T(v)) &= T(-v) \\ &= -(-v) \\ &= v \end{aligned}$$

Hence $\boxed{T(T(v)) = v}$

## Step-3

b) Given $T : \mathbf{V} \to \mathbf{W}$ such that $T(v) = v + (1,1)$

We have to find $T(T(v))$

## Step-4

Let $v = (v_1, v_2)$

Then

$$\begin{aligned} T(T(v)) &= T(v_1 + 1, v_2 + 1) \\ &= (v_1 + 1, v_2 + 1) + (1,1) \\ &= (v_1 + 2, v_2 + 2) \\ &= (v_1, v_2) + (2,2) \\ &= v + (2,2) \end{aligned}$$

Hence $\boxed{T(T(v)) = v + (2,2)}$

## Step-5

c) Given $T:\mathbf{V}\to\mathbf{W}$ such that $T(v)=90°$ rotation$=(-v_2,v_1)$

We have to find $T(T(v))$

## Step-6

Let $v=(v_1,v_2)$

Then

$$T(v_1,v_2)=(-v_2,v_1)$$

$$\begin{aligned}T(T(v))&=T(T(v_1,v_2))\\&=T(-v_2,v_1)\\&=(-v_1,-v_2)\\&=-(v_1,v_2)\\&=-v\end{aligned}$$

Hence $\boxed{T(T(v))=-v}$

## Step-7

d) Given $T:\mathbf{V}\to\mathbf{W}$ such that $T(v)=\text{projection}=\left(\frac{v_1+v_2}{2},\frac{v_1+v_2}{2}\right)$.

We have to find $T(T(v))$

## Step-8

Let $v=(v_1,v_2)$

Then

$$T(T(v_1,v_2))=T\left(\frac{v_1+v_2}{2},\frac{v_1+v_2}{2}\right)$$

$$=\left(\frac{v_1+v_2+v_1+v_2}{4},\frac{v_1+v_2+v_1+v_2}{4}\right)$$

$$=\left(\frac{v_1+v_2}{2},\frac{v_1+v_2}{2}\right)$$

Hence $\boxed{T\left(T\left(v\right)\right)=T\left(v\right)}$