

## Step-1

*Feasible set:* A feasible set is composed of the solutions to a family of linear inequalities, and a feasible point maximizes or minimizes a certain cost function.

## Step-2

To sketch the feasible set with following constraints:

$$x + 2y \geq 6$$

$$2x + y \geq 6$$

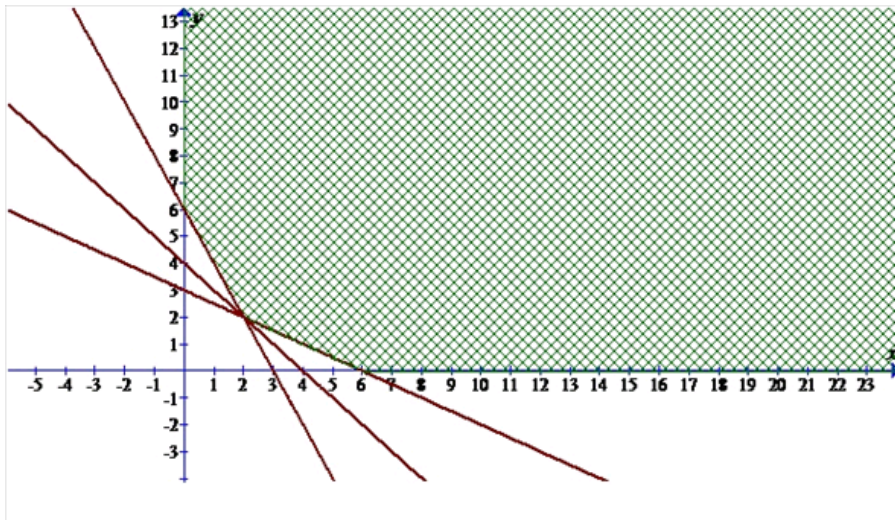
$$x \geq 0$$

$$y \geq 0$$

To find the minimum value of the cost function  $x + y$ , also to sketch the line  $x + y = \text{constant}$  in feasible region that touches the first feasible set. Finally, to check which point minimizes the cost functions  $3x + y$  and  $x - y$ .

## Step-3

Following sketch gives the feasible set:



Here shaded region denotes the feasible region. The line  $x + y = 4$  touches the feasible region at  $(2, 2)$ .

## Step-4

Three corner sets are as follows:

$$(2,2), (0,6), (6,0)$$

## Step-5

Substitute the following points in the function  $x + y$  to get the minimum value:

When point is  $(2,2)$

$$\begin{aligned} x + y &= 2 + 2 \\ &= 4 \end{aligned}$$

When point is  $(0,6)$

$$\begin{aligned} x + y &= 0 + 6 \\ &= 6 \end{aligned}$$

When point is  $(6,0)$

$$\begin{aligned} x + y &= 6 + 0 \\ &= 6 \end{aligned}$$

Above calculations show that at point  $(2,2)$  function  $x + y$  gets minimum value 4.

## Step-6

Substitute the following points in the function  $3x + y$  to get the minimum value:

When point is  $(2,2)$

$$\begin{aligned} 3x + y &= 3 \cdot 2 + 2 \\ &= 8 \end{aligned}$$

When point is  $(0,6)$

$$\begin{aligned} 3x + y &= 3 \cdot 0 + 6 \\ &= 6 \end{aligned}$$

When point is  $(6,0)$

$$\begin{aligned} 3x + y &= 3 \cdot 6 + 0 \\ &= 18 \end{aligned}$$

Above calculations show that at point  $(0,6)$  function  $3x + y$  gets minimum value 6.

Substitute the following points in the function  $x - y$  to get the minimum value:

### Step-7

When point is  $(2, 2)$

$$\begin{aligned}x - y &= 2 - 2 \\ &= 0\end{aligned}$$

When point is  $(0, 6)$

$$\begin{aligned}x - y &= 0 - (-6) \\ &= 6\end{aligned}$$

When point is  $(6, 0)$

$$\begin{aligned}x - y &= 6 - 0 \\ &= 6\end{aligned}$$

Above calculations show that at point  $(2, 2)$  function  $x - y$  gets minimum value 0.

### Step-8

Therefore, minimum values of the functions are:

$x + y = 4$
$3x + y = 6$
$x - y = 0$