

Step-1

$w = 6^{\text{th}}$ root of one in complex plane.

$$w = \sqrt[6]{1} \Rightarrow w = 1^{1/6}$$

$$\Rightarrow w = (\cos 2k\pi + i \sin 2k\pi)^{1/6}, k = 0, 1, 2, 3, 4, 5.$$

$$\Rightarrow w = \left(e^{i2k\pi} \right)^{1/6}$$

$$= \left(e^{\frac{2i\pi}{6}} \right)^k$$

$$= e^{\frac{2ik\pi}{6}}, k = 0, 1, 2, 3, 4, 5.$$

$w = e^{\frac{2i\pi}{6}}, w^2 = e^{\frac{4i\pi}{6}}, w^3 = e^{\frac{6i\pi}{6}}, w^4 = e^{\frac{8i\pi}{6}}, w^5 = e^{\frac{10i\pi}{6}}, w^6 = e^{\frac{12i\pi}{6}} = 1$ are the six roots of 1.

Step-2

$$D = \begin{bmatrix} 1 & & \\ & e^{\frac{2i\pi}{6}} & \\ & & e^{\frac{4i\pi}{6}} \end{bmatrix}$$

Using $w = e^{\frac{2i\pi}{6}}$, we get

$$= \begin{bmatrix} 1 & & \\ & w & \\ & & w^2 \end{bmatrix}$$

Step-3

$$\begin{aligned}
F_3 &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w^4 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w^2 & w \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2i\pi}{6}} & e^{\frac{4i\pi}{6}} \\ 1 & e^{\frac{4i\pi}{6}} & e^{\frac{2i\pi}{6}} \end{bmatrix}
\end{aligned}$$