Step-1

Consider the following matrices:

$$A_1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$A_2 = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$
$$A_3 = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$
$$A_4 = \begin{bmatrix} d & c \\ b & a \end{bmatrix}$$

Step-2

Determine the pairs which are similar.

Step-3

Recall that similar matrices have same Eigen values. In other words trace of the similar matrices will be same.

So, calculate the Eigen values of all the matrices. Eigen values of matrix A_1 will be:

$$A_1 - \lambda I = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$
$$\det\left(A_1 - \lambda I\right) = 0$$
$$(a-\lambda)(d-\lambda) - bc = 0$$
$$\lambda^2 - \lambda(a+d) + (ad - bc) = 0$$

On solving above equation following Eigen values are obtained:

$$\lambda_1 = \frac{1}{2}\left((a+d) + \sqrt{a^2 + d^2 - 2ad + 4bc}\right)$$
$$\lambda_2 = \frac{1}{2}\left((a+d) - \sqrt{a^2 + d^2 - 2ad + 4bc}\right)$$

Step-4

Therefore, trace will be as follows:

$$\begin{aligned}\lambda_1+\lambda_2 &= \frac{1}{2}\left((a+d)+\sqrt{a^2+d^2-2ad+4bc}\right)+\frac{1}{2}\left((a+d)-\sqrt{a^2+d^2-2ad+4bc}\right)\\ &= \frac{1}{2}(a+d)+\frac{1}{2}(a+d)\\ &= a+d\end{aligned}$$

Trace of matrix A_1 is: $a+d$.

Step-5

Eigen values of matrix A_2 will be:

$$\begin{aligned}A_2-\lambda I &= \begin{bmatrix} b-\lambda & a \\ d & c-\lambda \end{bmatrix}\\ \det(A_2-\lambda I) &= 0\\ (b-\lambda)(c-\lambda)-ad &= 0\\ \lambda^2-\lambda(c+b)+(bc-ad) &= 0\end{aligned}$$

On solving above equation following Eigen values are obtained:

$$\lambda_1=\frac{1}{2}\left((b+c)+\sqrt{c^2+b^2-2bc+4ad}\right)$$
$$\lambda_2=\frac{1}{2}\left((b+c)-\sqrt{c^2+b^2-2bc+4ad}\right)$$

Step-6

Therefore, trace will be as follows:

$$\begin{aligned}\lambda_1+\lambda_2 &= \frac{1}{2}\left((b+c)+\sqrt{c^2+b^2-2bc+4ad}\right)+\frac{1}{2}\left((b+c)-\sqrt{c^2+b^2-2bc+4ad}\right)\\ &= \frac{1}{2}(b+c)+\frac{1}{2}(b+c)\\ &= (b+c)\end{aligned}$$

Trace of matrix A_2 is: $b+c$.

Step-7

Similarly, trace of matrix A_3 is $b+c$ and of matrix A_4 is $a+d$. Here, trace of matrix A_1 and A_4 are equal. Hence, they form a pair. Also trace of matrix A_2 and A_3 are equal, so they form another pair.

Therefore, following pairs are similar:

$\boxed{\begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} d & c \\ b & a \end{bmatrix}}$ and $\boxed{\begin{bmatrix} b & a \\ d & c \end{bmatrix}, \begin{bmatrix} c & d \\ a & b \end{bmatrix}}$