Step-1

Given that a_1, a_2 and b are orthogonal vectors. We have to find $A^T A$ and $A^T b$, and the projection of b onto the plane of a_1, a_2

Step-2

Write $A = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$ and $A^T = \begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix}$

$$A^T A = \begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix} \begin{bmatrix} a_1 & a_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1^T a_1 & a_1^T a_2 \\ a_2^T a_1 & a_2^T a_2 \end{bmatrix}$$

Step-3

As a_1, a_2 are orthogonal

$$\Rightarrow a_1^T a_2 = 0 = a_2^T a_1$$

Therefore $A^T A = \begin{bmatrix} a_1^T a_1 & 0 \\ 0 & a_2^T a_2 \end{bmatrix}$

Step-4

And

$$A^T b = \begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix} b$$

$$= \begin{bmatrix} a_1^T b \\ a_2^T b \end{bmatrix}$$

Since a_1, a_2 and b are orthogonal

$$\Rightarrow a_1^T b = 0, a_2^T b = 0$$

Therefore $A^T b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Step-5

The projection of b onto the plane of a_1 and a_2

$= Pb$, where $P = A\left(A^T A\right)^{-1} A^T$