Step-1

The objective is to determine the basis for the space S^1 when subspace consists all vectors orthogonal to S.

Step-2

Consider the provided equation is $x_1 + x_2 + x_3 + x_4 = 0$. Suppose S is the subspace of $\mathbf{R}^4$ spanned by this equation.

So,

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_4 &= -x_1 - x_2 - x_3 \end{aligned}$$

This is a single equation with three unknown variables.

The solution set is $\left[x_1, x_2, x_3, -x_1, -x_2, -x_3\right]$.

Consider the vector $V = \left[P, Q, R, S\right]$ in subspace S^1 which contains all vectors that are orthogonal to subspace.

So,

$$\begin{aligned} Px_1 + Qx_2 + Rx_3 - Sx_1 - Sx_2 - Sx_3 &= 0 \\ x_1\left(P - S\right) + x_2\left(Q - S\right) + x_3\left(R - S\right) &= 0 \end{aligned}$$

For any values,

$$\begin{aligned} P - S &= 0 \\ Q - S &= \\ R - S &= 0 \end{aligned}$$

This implies,

$$\begin{aligned} P &= S \\ Q &= S \\ R &= S \end{aligned}$$

Thus,

$$P = Q = R = S$$

Hence, the basis of the subspace S^1 which contains all vectors are orthogonal to subspace S is $\boxed{[1,1,1,1]}$.