Step-1

Consider a following matrix:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Let following are the classes of matrices:

Orthogonal, Invertible, Hermitian, Unitary, factorizable into LU, factorizable into QR. Determine P belongs to which classes of matrices.

Step-2

Orthogonal: If $PP^T = I$, then P is said to be orthogonal matrix.

$$\begin{aligned} PP^T &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= I \end{aligned}$$

Therefore, $\boxed{\text{matrix } P \text{ is orthogonal matrix}}$.

Invertible: Orthogonal matrices are always invertible. As $P^{-1} = P^T$. Therefore matrix P is also invertible.

Step-3

Hermitian: If $P^H = P$, then P is said to be Hermitian matrix.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} P^H &= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\ &\neq P \end{aligned}$$

Therefore, matrix $\boxed{P \text{ is not Hermitian}}$.

Step-4

Unitary: If columns of matrix are orthonormal, then matrix is Unitary.

Matrix P has orthogonal columns with unitary length of each column vector. Therefore, matrix P $\boxed{\text{is unitary}}$.

Step-5

Factorization into LU: Factorization of matrix P into lower and upper triangular matrix requires no row exchanges. But to avoid zeros and to make the pivots of upper triangular matrix 1, row transformation is required.

Therefore, matrix P $\boxed{\text{can not be factorised into}}$ LU $\boxed{\text{matrices}}$.

Step-6

Factorization into QR: Matrix P has independent columns which can be orthogonalised into matrix Q by the Gram-Schmidt process.

Therefore, matrix P $\boxed{\text{can be factorised into}}$ QR $\boxed{\text{matrices}}$.