Step-1

Each elementary row matrix is a lower triangular matrix.

An elementary row matrix is obtained by applying the same elementary row operation that was applied on A is applied on the identity matrix.

The inverse of the elementary matrix is also elementary and a lower triangular matrix.

Product of lower triangular matrices is also a lower triangular matrix.

Thus, the given matrix has become the product of lower and upper triangular matrices.

The above procedure is exhibited as;

$$E_k E_{k-1} \ldots E_2 E_1 A = U_1$$

Where E_i â€™s are elementary row matrices and U_1 is the echelon form.

Thus,

$$\begin{aligned} A &= E_1^{-1} E_2^{-1} \ldots E_k^{-1} U_1 \\ &= L_1 U_1 \end{aligned}$$

Step-2

Now write the given matrix A as the product of upper and lower triangular matrices.

So, start to make zero the entries above the principal diagonal.

For this procedure, use the elementary column operations result in elementary column matrices.

Apply an elementary column operation on A is nothing but post multiplying A with an elementary column matrix.

Each elementary column matrix is an upper triangular matrix.

Inverse of an elementary column matrix is also an elementary column matrix.

Product of upper triangular matrices is also an upper triangular matrix.

Step-3

Thus, write the given matrix as the product of upper and lower triangular matrices.

The above procedure is performed as;

$$A F_1 F_2 \ldots F_s = L_2$$

Where each F_i is the post multiplied elementary column matrix with A and ultimately, it is reduced to the lower triangular matrix.

$$
\begin{aligned}
A &= F_s^{-1} F_{s-1}^{-1} \ldots F_1^{-1} L_2 \\
&= U_2 L_2
\end{aligned}
$$

Step-4

Observe that the entire procedure used in this case and the previous case is different. So, the triangularisations are also different.

That is,

$\boxed{U_1 \neq U_2}$ And $\boxed{L_1 \neq L_2}$ but $\boxed{L_1 U_1 = L_2 U_2}$