

## Step-1

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Pu$$

Here, projection matrix  $P$  is defined as follows:

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

To show the following from the infinite series:

## Step-2

$$e^P = I + 1.718P$$

## Step-3

Recall the following:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \dots$$

Substitute the following:

$$A = P$$
$$t = 1$$

Thus,  $e^{At}$  becomes:

$$e^P = I + P + \frac{(P)^2}{2!} + \dots$$

## Step-4

A calculation on projection matrix shows the following:

$$P \cdot P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$P^2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= P$$

## Step-5

Substitute the above result  $P^2 = P$  in  $e^P$ :

$$e^P = I + P + \frac{P}{2!} + \frac{P}{3!} \dots$$

$$= I + P \left( 1 + \frac{1}{2!} + \frac{1}{3!} \dots \right)$$

$$= I + P(e^1 - 1)$$

$$= I + P(2.718 - 1)$$

$$e^P = I + 1.718P$$

## Step-6

Therefore,  $\boxed{e^P = I + 1.718P}$ .