Step-1

Given system is $ax+3y=-3$

$4x+6y=6$

Given system can be written as

$$\begin{pmatrix} a & 3 & -3 \\ 4 & 6 & 6 \end{pmatrix}$$

Step-2

(a)

Subtract â€˜4â€™ times the first row fromâ€˜aâ€™times the second row

$$\begin{pmatrix} a & 3 & -3 \\ 0 & 6a-12 & 6a-12 \end{pmatrix}$$

If $6a-12=0$ i.e. $a=2$,then we cannot proceed for the elimination.

Hence, the system has no solution.

Therefore the elimination breaks down permanently when $a=2$.

Step-3

(b)

If $a=0$,then

$$\begin{pmatrix} a & 3 & -3 \\ 4 & 6 & 6 \end{pmatrix} = \begin{pmatrix} 0 & 3 & -3 \\ 4 & 6 & 6 \end{pmatrix}$$

If we exchange the rows then only we can proceed for the elimination.

So we have $\begin{pmatrix} 4 & 6 & 6 \\ 0 & 3 & -3 \end{pmatrix}$

By back ward substitution, we have

$3y=-3$

$\Rightarrow \boxed{y=-1}$

And $4x+6y=6$

$\Rightarrow 4x-6=6$

$\Rightarrow 4x=12$

$\Rightarrow \boxed{x=3}$

So if $a=0$,elimination stops for a row exchange and the solution is $(3,-1)$.