Step-1

Given data is $y = 6$ at $t = 0$

$y = 4$ at $t = 1$

$y = 0$ at $t = 2$

We have to find the best least squares fit for this data with a curve $y = C + D2^t$.

Step-2

If $y = C + D2^t$ passes through the given points, then

$6 = C + D2^0 = C + D$

$4 = C + D2^1 = C + 2D$

$0 = C + D2^2 = C + 4D$

In matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$\Rightarrow Ax = b$

where $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{bmatrix}$, $x = \begin{bmatrix} C \\ D \end{bmatrix}$, $b = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$.

Step-3

Let $\hat{x} = \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix}$

For least squares fit, we have

$A^T A \hat{x} = A^T b$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 7 \\ 7 & 21 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

apply $R_2 \to 3R_2 - 7R_1$

$$\Rightarrow \begin{bmatrix} 3 & 7 \\ 0 & 14 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 18 \\ -28 \end{bmatrix}$$

Step-4

Continuation to the above,

$\Rightarrow 3\hat{C} + 7\hat{D} = 18$ and

$14\hat{D} = -28$

$\Rightarrow \hat{D} = -2$ and

$$\hat{C} = \frac{18+14}{3}$$

$$= \frac{32}{3}$$

Hence $C = \frac{32}{3}$ and $D = -2$

Given curve is $\boxed{y = \frac{32}{3} - 2.2^t}$