Step-1

If the columns of A are orthogonal to each other, then we have to say about $A^T A$, and also we have to say about $A^T A$ if the columns of A are orthonormal to each other.

Step-2

Suppose $a = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, b = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ are the columns of A

Now

$$a^T b = (1,2)\begin{pmatrix} -2 \\ 1 \end{pmatrix}$$
$$= -2 + 2$$
$$= 0$$

Therefore a and b are orthogonal columns of A.

Step-3

$$A = [a, b]$$
$$= \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$
$$A^T A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Hence $A^T A$ is a diagonal matrix.

Since diagonal elements are equal, $A^T A$ is also a scalar matrix.

Step-4

Suppose $a = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, b = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ are the columns of A

Now

$$a^T b = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$
$$= 0$$

Step-5

$$\|a\| = \|b\| = 1$$

$$A = [a, b]$$
$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Step-6

$$A^T A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$= I$, which is the identity matrix

Therefore $A^T A$ is the identity matrix.