

Step-1

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Given that

We have to compute P^2, P^3 and P^{100} . And also we have to find the eigenvalues of P .

Step-2

Now

$$\begin{aligned} P^2 &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0(0)+1(0)+0(1) & 0(1)+1(0)+0(0) & 0(0)+1(1)+0(0) \\ 0(0)+0(0)+1(1) & 0(1)+0(0)+1(0) & 0(0)+0(1)+1(0) \\ 1(0)+0(0)+0(1) & 1(1)+0(0)+0(0) & 1(0)+0(1)+0(0) \end{bmatrix} \\ &= \begin{bmatrix} 0+0+1 & 0+0+0 & 0+1+0 \\ 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 1+0+0 & 0+0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

$$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Therefore,

Step-3

Now

$$\begin{aligned}
P^3 &= P^2 P \\
&= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} 0(0)+0(0)+1(1) & 0(1)+0(0)+1(0) & 0(0)+0(1)+1(0) \\ 1(0)+0(0)+0(1) & 1(1)+0(0)+0(0) & 1(0)+0(1)+0(0) \\ 0(0)+1(0)+0(1) & 0(1)+1(0)+0(0) & 0(0)+1(1)+0(0) \end{bmatrix}
\end{aligned}$$

Step-4

Continuation to the above

$$\begin{aligned}
&= \begin{bmatrix} 0+0+1 & 0+0+0 & 0+0+0 \\ 0+0+0 & 1+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= I
\end{aligned}$$

Therefore, $\boxed{P^3 = I}$

Step-5

Now we have to compute P^{100} .

Now

$$\begin{aligned}
P^{100} &= P^{99} P \\
&= (P^3)^{33} P \\
&= (I)^{33} P \quad (\text{Since } P^3 = I) \\
&= IP \\
&= P \quad (\text{Since } IP = PI = P)
\end{aligned}$$

Hence $\boxed{P^{100} = P}$

Step-6

Now we have to find the eigenvalues of P .

The characteristic equation of P is

$$|P - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda(\lambda^2) - 1(-1) + 0 = 0$$

$$\Rightarrow -\lambda^3 + 1 = 0$$

$$\Rightarrow \lambda^3 = 1$$

Therefore the eigenvalues of P are cube roots of unity $\lambda = 1, \omega, \omega^2 = 1, e^{\frac{2\pi i}{3}}, e^{\frac{4\pi i}{3}}$.