

Step-1

A reflection matrix transforms every vector into its image on the opposite side of mirror.

Let $S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ be the reflection matrix that transforms (x, y) to (y, x) .

$$\det S = 0 - 1 = -1$$

Let $T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ be the reflection matrix that transforms (x, y) to $(-x, y)$.

$$\det T = -1 - 0 = -1$$

Consider product of matrices,

$$\begin{aligned} ST &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \end{aligned}$$

$$\det ST = 0 - (-1) = 1$$

Therefore, the product ST is a rotation.

Step-2

Consider the product of reflections,

$$\begin{aligned} &\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\theta \cos 2\alpha + \sin 2\theta \sin 2\alpha & \cos 2\theta \sin 2\alpha - \cos 2\alpha \sin 2\theta \\ \sin 2\theta \cos 2\alpha - \cos 2\theta \sin 2\alpha & \sin 2\theta \sin 2\alpha + \cos 2\theta \cos 2\alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2(\theta - \alpha) & \sin 2(\theta - \alpha) \\ \sin 2(\theta - \alpha) & \cos 2(\theta - \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos 2(\theta - \alpha) & -\sin 2(\theta - \alpha) \\ \sin 2(\theta - \alpha) & \cos 2(\theta - \alpha) \end{bmatrix} \end{aligned}$$

Hence the angle of rotation is $\boxed{2(\theta - \alpha)}$.