Step-1

We have to find three 2 by 2 matrices A, other than $A = I$ and $A = -I$, that are their own inverses $A^2 = I$.

Step-2

Let the 2 by 2 matrices be $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Now $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0(0)+1(1) & 0(1)+1(0) \\ 1(0)+0(1) & 1(1)+0(0) \end{bmatrix}$$
$$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= I$$

Since $A^2 = I$

So $\Rightarrow A = A^{-1}$

Step-3

Let the 2 by 2 matrices be $A = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$

Now

$$A^2 = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right) & \frac{\sqrt{3}}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right) \\ \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) & \frac{1}{2}\left(\frac{1}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) \end{bmatrix}$$

Step-4

Continuation to the above

$$= \begin{bmatrix} \frac{3}{4} + \frac{1}{4} & \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} & \frac{1}{4} + \frac{3}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4} & 0 \\ 0 & \frac{4}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Since $A^2 = I$

So $\Rightarrow A = A^{-1}$

Step-5

Let the 2 by 2 matrices be $A = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

Now

$$A^2 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\left(\frac{1}{2}\right) & \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)+\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) \\ \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) & \frac{1}{2}\left(\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \end{bmatrix}$$

Step-6

Continuation to the above

$$= \begin{bmatrix} \frac{3}{4}+\frac{1}{4} & -\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4} & \frac{1}{4}+\frac{3}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4} & 0 \\ 0 & \frac{4}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Since $A^2 = I$

So $\Rightarrow A = A^{-1}$

Hence the required matrices are $\boxed{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}}$.