Step-1

We need to tell whether the given statement is true or false with a reason or a counter example.

(a)

The given statement is â€œ A and A^T has the same number of pivotsâ€.

Letâ€™s consider the 3 by 3 non-singular matrix. So, the rank of the matrix A is 3(because, the number of non-zeros are 3) and the rank of the matrix A^T is 3(because, the number of non-zero are 3).

Therefore, rank of A = rank of A^T

Therefore, the given statement is $\boxed{\text{true}}$.

Step-2

(b)

The given statement is â€œ A and A^T has the same left null spaceâ€.

Letâ€™s consider $A = [1\ \ 0]$ then null space basis of A is $\{(0,1)\}$ But, the left null space basis is $\{(0)\}$.

$$\text{i.e.} A^T X = 0$$
$$\Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} [x_1] = 0$$
$$\Rightarrow x_1 = 0$$

Therefore, the given statement is $\boxed{\text{false}}$.

Step-3

(c)

The given statement is â€œIf the row space equals the column space then $A^T = A$ â€.

If the matrix A is invertible and un-symmetric then $A^T \neq A$

For example $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

The above matrix is invertible and un-symmetric. But, row space equals to the column space. i.e. $\{(1,0),(0,1)\}$. But, the transpose the matrix is nor equals to the matrix. i.e. $A^T \neq A$

Therefore, the given statement is $\boxed{\text{false}}$.

Step-4

(d)

The given statement is â€œIf $A^T = -A$ then row space of A equals the column spaceâ€.

$$\begin{aligned} A^T &= -A \\ \text{col}\left(A^T\right) &= \text{col}(-A) \\ &= -\text{col}(A) \\ &= \text{col}(A) \end{aligned}$$

Therefore, row space of A = column space of A

Therefore, the given statement is $\boxed{\text{true}}$.