Step-1

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(a) We have to find a vector x that will make $Ax = \text{column 1 of } A + 2(\text{column 3})$, for a 3 by 3 matrix.

Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

Then

$$\begin{aligned} Ax &= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 2(1)+0(0)+0(2) \\ 0(1)+1(0)+0(2) \\ 0(1)+0(0)+3(2) \end{bmatrix} \\ &= \begin{bmatrix} 2+0+0 \\ 0+0+0 \\ 0+0+6 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix} \end{aligned}$$

This shows that $\boxed{Ax = \text{column 1 of } A + 2(\text{column 3})}$

Step-2

(b) We have to construct a matrix that has $\text{column } 1 + 2(\text{column } 3) = 0$.

Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 1 & -2 \\ 6 & 1 & -3 \end{bmatrix}$

From the matrix it is clear that

$$\text{column } 1+2(\text{column } 3)=\begin{bmatrix}2\\4\\6\end{bmatrix}+2\begin{bmatrix}-1\\-2\\-3\end{bmatrix}$$

$$=\begin{bmatrix}2-2\\4-4\\6-6\end{bmatrix}$$

$$=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Hence the required matrix that has $\text{column } 1+2(\text{column } 3)=0$ is $\boxed{A=\begin{bmatrix}2&1&-1\\4&1&-2\\6&1&-3\end{bmatrix}}$.

Step-3

Now we have to check that A is singular.

Now

$$\begin{aligned}\det A &= 2(-3+2)-1(-12+12)-1(4-6)\\ &= 2(1)-1(0)-1(-2)\\ &= 2-2\\ &= 0\end{aligned}$$

Since $\det A=0$

So A $\boxed{\text{is singular}}$.

Step-4

We have $A=\begin{bmatrix}2&1&-1\\4&1&-2\\6&1&-3\end{bmatrix}$

Subtracting 2 times row 1 from row 2 and 3 times row 1 from row 3 gives

$$\to\begin{bmatrix}2&1&-1\\0&-1&0\\0&-2&0\end{bmatrix}$$

Subtracting 2 times row 2 from row 3

$$\rightarrow \begin{bmatrix} 2 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The last row of the echelon form of A is zero.

So the matrix A is singular.

Hence A has no inverse.