

Step-1

Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} = 0$$

$$= (a - \lambda)(d - \lambda) - bc$$

$$= \lambda^2 - (a + d)\lambda + (ad - bc)$$

Step-2

Given that $|A - \lambda I| = \lambda^2 - 9\lambda + 20$ and $a + d = 9, ad - bc = 20$

We get, $a + d = 9, ad - bc = 20$

There can be infinitely many integers which satisfy these equations.

Let us consider the ordered pairs of values of a and d

$(1, 8), (2, 7), (3, 6), \dots$ which are positive and integer choices.

When $a = 1, d = 8$, we have the choices for b and c obtained by $bc = 12$

This gives $b = -6, c = 2$ or $b = -3, c = 4$ or $b = -4, c = 3$ or $b = 6, c = -2$

So, some possible choices of matrices are $\begin{bmatrix} 1 & -4 \\ 3 & 8 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ -3 & 8 \end{bmatrix}, \begin{bmatrix} 1 & -6 \\ 2 & 8 \end{bmatrix}, \begin{bmatrix} 1 & 6 \\ -2 & 8 \end{bmatrix}$