Step-1

Consider that a matrix $A=\begin{bmatrix}1 & i & 0\\ i & 0 & 1\end{bmatrix}$.

The objective is to find A^H and $C=A^H A$.

Step-2

Clearly know that the conjugate transpose of a matrix is called a Hermitian matrix.

Now the conjugate of A is $\overline{A}=\begin{bmatrix}1 & -i & 0\\ -i & 0 & 1\end{bmatrix}$

And the transpose of conjugate of A is $\overline{A}^T=\begin{bmatrix}1 & -i\\ -i & 0\\ 0 & 1\end{bmatrix}$

Therefore, $\boxed{A^H=\begin{bmatrix}1 & -i\\ -i & 0\\ 0 & 1\end{bmatrix}}$

Step-3

Now find the matrix $C=A^H A$ as,

Consider,

$$C=A^H A$$

$$=\begin{bmatrix}1 & -i\\ -i & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & i & 0\\ i & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1(1)-i(i) & 1(i)-i(0) & 1(0)-i(1)\\ -i(1)+0(i) & -i(i)+0(0) & -i(0)+0(1)\\ 0(1)+1(i) & 0(i)+1(0) & 0(0)+1(1)\end{bmatrix}$$

Step-4

Continuation to the above,

$$= \begin{bmatrix} 1-i^2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix} \text{ (Since } i^2 = -1\text{)}$$

Therefore, $\boxed{C = \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}}$.

Step-5

Since $C = A^H A$

And

$$\begin{aligned} C^H &= \left(A^H A\right)^H \\ &= A^H \left(A^H\right)^H \\ &= A^H A \qquad \left(\text{Since } \left(A^H\right)^H = A\right) \\ &= C \end{aligned}$$

Therefore, $\boxed{C^H = C}$ and $\boxed{C^H = \begin{bmatrix} 2 & i & -i \\ -i & 1 & 0 \\ i & 0 & 1 \end{bmatrix}}$.