Step-1

We have to project the vector $b=(1,2)$ onto vectors that are not orthogonal, $a_1=(1,0)$ and $a_2=(1,1)$. And we have to show that, unlike the orthogonal case, the sum of the two one dimensional projections does not equal *b*.

Step-2

Let P_1= the projection of *b* onto the line through $a_1 = \frac{a_1^T b}{a_1^T a_1} a_1$

Now

$$a_1^T b = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$
$$= 1+0$$
$$= 1$$

Step-3

$$a_1^T a_1 = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$= 1+0$$
$$= 1$$

Step-4

Therefore

$$P_1 = \frac{1}{1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$= \boxed{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

Step-5

Let P_2= the projection of b onto the line through $a_2 = \frac{a_2^T b}{a_2^T a_2} a_2$

Now

$$a_2^T b = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$
$$= 1 + 2$$
$$= 3$$

Step-6

$$a_2^T a_2 = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$= 1 + 1$$
$$= 2$$

Step-7

Therefore

$$P_2 = \frac{3}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$= \boxed{\begin{pmatrix} 3/2 \\ 3/2 \end{pmatrix}}$$

Step-8

Hence $P_1 + P_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3/2 \\ 3/2 \end{pmatrix} = \begin{pmatrix} 5/2 \\ 3/2 \end{pmatrix}$

So $P_1 + P_2 \neq b$