Step-1

Stability: Real parts of the Eigen values govern the stability. The differential equation $du/dt = Au$ is:

Stable: If $\operatorname{Re}(\lambda_i) < 0$

Neutrally stable: If all $\operatorname{Re}(\lambda_i) \le 0$ and $\lambda_1 = 0$.

Unstable: If any Eigen value has $\operatorname{Re}(\lambda_i) > 0$.

Step-2

A matrix is stable if it passes two tests:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Tests:

The trace: $T = a + d$ must be negative.

The determinant: $D = ad - bc$ must be positive.

If Eigen values are complex numbers then determinant will be always positive and trace will be negative if real part of complex number is negative. So, negative trace means that the real part is negative and the matrix is stable.

Step-3

Evaluate the value of *t* for which the following matrix is stable and instable. Stability can be with real and complex Eigen values. Use the concept of trace and determinant of the matrix.

Let matrix A_1 is defined as follows:

$$A_1 = \begin{bmatrix} 1 & -1 \\ t & -1 \end{bmatrix}$$

Step-4

For stability, trace and determinant of matrix is as follows:

Trace: $1 - 1 = 0$

Determinant: $-1 + t > 0$ or $t > 1$.

As trace is zero so matrix will be neutrally stable.

Therefore, A_1 will be neutrally stable if $\boxed{t \geq 1}$ and unstable if $\boxed{t < 1}$.

Step-5

Let another matrix A_2 is defined as follows:

$$A_2 = \begin{bmatrix} 0 & 4-t \\ 1 & -2 \end{bmatrix}$$

Step-6

For stability, trace and determinant of matrix is as follows:

Trace:

$$\begin{aligned} 0-2 &= -2 \\ &< 0 \end{aligned}$$

Determinant: $-4+t>0$ or $t>4$.

As trace is negative so matrix is stable.

Step-7

Therefore, A_2 will be neutrally stable if $\boxed{t=4}$ and unstable if $\boxed{t<4}$. Also, it will be stable if $\boxed{t>4}$.

Step-8

Let another matrix A_3 is defined as follows:

$$A_3 = \begin{bmatrix} t & -1 \\ 1 & t \end{bmatrix}$$

Step-9

For stability, trace and determinant of matrix is as follows:

Trace:

$$\begin{aligned} t+t &= 2t \\ &< 0 \end{aligned}$$

Determinant: $t^2+1>0$.

As determinant is positive so matrix can be stable.

Step-10

Therefore, A_3 will be unstable if $\boxed{t > 0}$ and it will be stable if $\boxed{t < 0}$.