Step-1

Given that $x = 2 + i$ and $y = 1 + 3i$

We have to find $\bar{x}, x\bar{x}, xy, \frac{1}{x}$ and $\frac{x}{y}$.

Step-2

Now

$$\begin{aligned}\bar{x} &= \overline{2+i} \\ &= 2 - i\end{aligned}$$

Therefore, $\boxed{\bar{x} = 2 - i}$

Step-3

Now

$$\begin{aligned}x.\bar{x} &= (2+i)(2-i) \\ &= 4 - (i)^2 \qquad \left(\text{Since } (a+b)(a-b) = a^2 - b^2\right) \\ &= 4 - (-1)^2 \qquad \left(\text{Since } i^2 = -2\right) \\ &= 5\end{aligned}$$

Therefore, $\boxed{x\bar{x} = 5}$

Step-4

Now

$$\begin{aligned}xy &= (2+i)(1+3i) \\ &= 2 + 6i + i + 3i^2 \\ &= 2 + 7i - 3 \qquad \left(\text{Since } i^2 = -1\right) \\ &= -1 + 7i\end{aligned}$$

Therefore, $\boxed{xy = -1 + 7i}$

Step-5

Now

$$\frac{1}{x}=\frac{1}{2+i}$$

$$=\frac{1}{2+i}\times\frac{2-i}{2-i} \quad \text{(Rationalising the denominator)}$$

$$=\frac{2-i}{2^2-i^2} \quad \left(\text{Since } (a+b)(a-b)=a^2-b^2\right)$$

$$=\frac{2-i}{4+1} \quad \left(\text{Since } i^2=-1\right)$$

Step-6

Continuation to the above

$$=\frac{2-i}{5}$$

$$=\frac{1}{5}(2-i)$$

$$=\frac{2}{5}-\frac{1}{5}i$$

Therefore, $\boxed{\frac{1}{x}=\frac{2}{5}-\frac{1}{5}i}$

Step-7

Now

$$\frac{x}{y}=\frac{2+i}{1+3i}$$

$$=\frac{2+i}{1+3i}\times\frac{1-3i}{1-3i}$$

$$=\frac{(2+i)(1-3i)}{(1+3i)(1-3i)}$$

$$=\frac{2-6i+i-3i^2}{(1)^2-(3i)^2}$$

Step-8

Continuation to the above

$$= \frac{(2+3)-5i}{1+9}$$
$$= \frac{5-5i}{10}$$
$$= \frac{1}{2} - \frac{1}{2}i$$

Therefore, $\boxed{\frac{x}{y} = \frac{1}{2} - \frac{1}{2}i}$

Step-9

We have to verify that $|xy| = |x||y|$.

Now

$$\begin{aligned} |xy| &= |(x+i)(1+3i)| \\ &= |-1+7i| \\ &= \sqrt{(-1)^2 + 7^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \end{aligned}$$

Step-10

And

$$\begin{aligned} |x| &= |2+i| \\ &= \sqrt{2^2 + 1^2} \\ &= \sqrt{4+1} \\ &= \sqrt{5} \end{aligned}$$

Step-11

And

$$\begin{aligned} |y| &= |1+3i| \\ &= \sqrt{1^2 + 3^2} \\ &= \sqrt{1+9} \\ &= \sqrt{10} \end{aligned}$$

Now

$$|x||y| = \sqrt{5}\cdot\sqrt{10}$$
$$= \sqrt{50}$$

Hence $\boxed{|xy| = |x|.|y|}$

Step-12

We have to verify that $\left|\frac{1}{x}\right| = \frac{1}{|x|}$

Now

$$\left|\frac{1}{x}\right| = \left|\frac{1}{2+i}\right|$$
$$= \left|\frac{2}{5} - \frac{1}{5}i\right|$$
$$= \sqrt{\left(\frac{2}{5}\right)^2 + \left(-\frac{1}{5}\right)^2}$$
$$= \sqrt{\frac{4}{25} + \frac{1}{25}}$$
$$= \frac{1}{\sqrt{5}}$$

Step-13

And

$$\frac{1}{|x|} = \frac{1}{|2+i|}$$
$$= \frac{1}{\sqrt{4+1}}$$
$$= \frac{1}{\sqrt{5}}$$

Hence $\boxed{\left|\frac{1}{x}\right| = \frac{1}{|x|}}$