Step-1

Consider the matrix equation $Ax = b$.

From the values of *b* and *t*, we can write

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 16 \end{bmatrix}$$

Here,

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} C \\ D \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 16 \end{bmatrix}$$

Step-2

Therefore, $A^{\mathrm{T}} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$.

Thus,

$$A^{\mathrm{T}}A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$$

Step-3

The rank of $A^{\mathrm{T}}A = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$ is one and therefore, $\left(A^{\mathrm{T}}A\right)^{-1}$ does not exist.

Thus, the normal equations break down in this case.

Step-4

We have,

$$
\begin{aligned}
E^2 &= (b_1 - C - Dt_1)^2 + (b_2 - C - Dt_2)^2 \\
&= (0 - C - 2D)^2 + (6 - C - 2D)^2 \\
&= C^2 + 4CD + 4D^2 + 36 + C^2 + 4D^2 - 12C - 24D + 4CD \\
&= 2C^2 + 8CD + 8D^2 + 36 - 12C - 24D
\end{aligned}
$$

Step-5

In order to minimize E^2, differentiate it with respect to C and D.

$$
\frac{\partial E^2}{\partial C} = 4C + 8D - 12
$$
$$
\frac{\partial E^2}{\partial D} = 8C + 16D - 24
$$

Equating these two equations to zero, we get

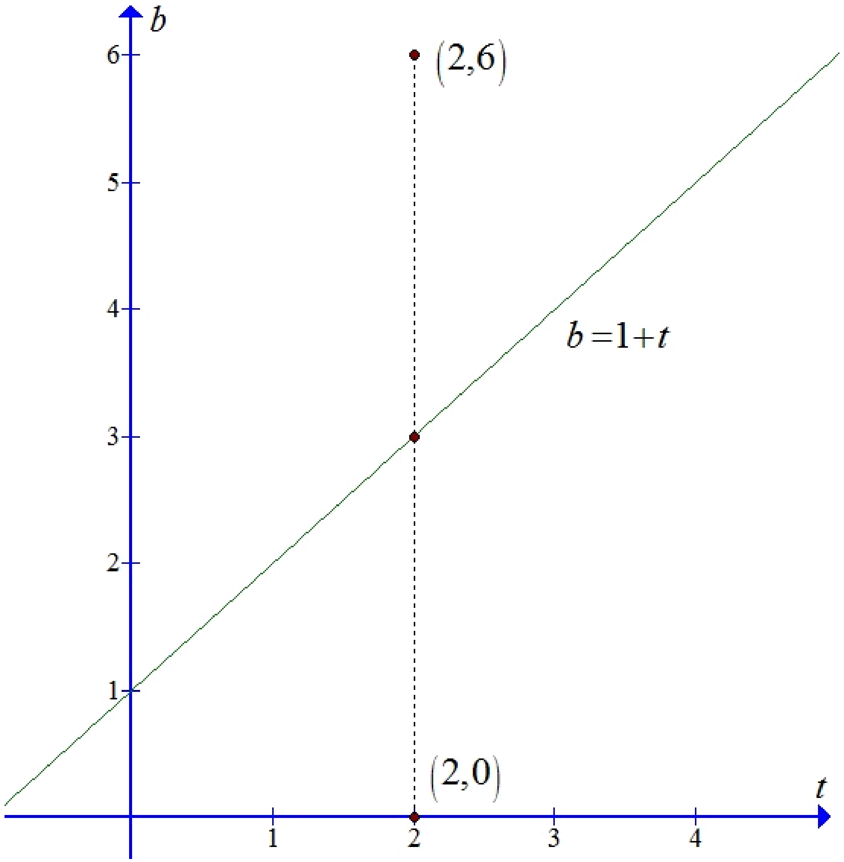
$C + 2D = 3$.

The minimum value of $E^2 = 2C^2 + 8CD + 8D^2 + 36 - 12C - 24D$ occurs when $C = 1$ and $D = 1$.

Step-6

The optimal line has the equation: $\boxed{b = 1 + t}$.

The graph of the line is as drawn below:



b
6
5
4
3
2
1
(2,6)
b = 1 + t
(2,0)
t
1
2
3
4