Step-1

Stability: Real parts of the Eigen values govern the stability. The differential equation $du/dt = Au$ is:

Stable: If $\mathrm{Re}(\lambda_i) < 0$

Neutrally stable: If all $\mathrm{Re}(\lambda_i) \le 0$ and $\lambda_1 = 0$.

Unstable: If any Eigen value has $\mathrm{Re}(\lambda_i) > 0$.

Step-2

Define the matrix *A* to illustrate the following unstable region.

(a) When Eigen values are:

$\lambda_1 < 0$ and $\lambda_2 > 0$

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

Step-3

To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1-\lambda & 2 \\ 3 & -1-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (1-\lambda)(-1-\lambda) - 6 &= 0 \\ \lambda^2 - 7 &= 0 \end{aligned}$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= -\sqrt{7} \\ &< 0 \\ \lambda_2 &= \sqrt{7} \\ &> 0 \end{aligned}$$

Step-4

Therefore, following matrix satisfies $\lambda_1 < 0$ and $\lambda_2 > 0$

$$\boxed{A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}}$$

Step-5

(b) When Eigen values are:

$\lambda_1 > 0$ and $\lambda_2 > 0$

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$$

Step-6

To calculate the Eigen values do the following calculations;

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$
$$(1-\lambda)(3-\lambda) = 0$$
$$\lambda^2 - 4\lambda + 3 = 0$$

After solving following values are obtained:

$$\begin{aligned} \lambda_1 &= 1 \\ &> 0 \\ \lambda_2 &= 3 \\ &> 0 \end{aligned}$$

Step-7

Therefore, following matrix satisfies $\lambda_1 > 0$ and $\lambda_2 > 0$

$$\boxed{A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}}$$

Step-8

(c) When Eigen values complex λ's real part is greater than zero.

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Step-9

To calculate the Eigen values do the following calculations;

$$\begin{aligned} A - \lambda I &= \begin{bmatrix} 1-\lambda & 1 \\ -1 & 1-\lambda \end{bmatrix} \\ \det(A - \lambda I) &= 0 \\ (1-\lambda)(1-\lambda)+1 &= 0 \\ \lambda^2 - 2\lambda + 2 &= 0 \end{aligned}$$

After solving following values are obtained:

$$\lambda_1 = 1+i$$
$$\lambda_2 = 1-i$$

Step-10

Therefore, following matrix satisfies the condition that real part is greater than zero.

$$\boxed{A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}}$$