Step-1

The objective is to determine a basis for the orthogonal complement of the row space of matrix A.

Step-2

Consider the matrix $A=\begin{bmatrix}1 & 0 & 2\\ 1 & 1 & 4\end{bmatrix}$ which is split $x=(3,3,3)$ into row a row space component x_r and a null space component x_n.

The orthogonal complement of the row space of A is the column null space of A.

By definition of null space $Ax=0$;

$$\begin{bmatrix}1 & 0 & 2\\ 1 & 1 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{aligned} x_1+2x_3&=0\\ x_1+x_2+4x_3&=0 \end{aligned}$$

Substitutes $x_3=a$.

So,

$$x_1=-2a$$

And

$$\begin{aligned} x_2&=-x_1-4x_3\\ &=2a-4a\\ &=-2a \end{aligned}$$

Step-3

Therefore,

$$\begin{aligned} \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}&=\begin{bmatrix}-2a\\ -2a\\ a\end{bmatrix}\\ &=a\begin{bmatrix}-2\\ -2\\ 1\end{bmatrix} \end{aligned}$$

Hence, a basis for the orthogonal complement of row space of matrix is $\boxed{A = \left\{ \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix} \right\}}$.

Step-4

The matrix is split $x = (3,3,3)$ into row a row space component x_r and a null space component x_n.

For, null space $x_n = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$ let $x = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

Compute $x_r = x - x_n$

Then,

$$\begin{aligned} x_r &= x - x_n \\ &= \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} - \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix} \end{aligned}$$

Therefore, $x = \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix} + \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$.

Hence, the row space component is $\boxed{x_r = \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix}}$.