Step-1

Given $P_C = A(A^TA)^{-1}A^T$ is the projection onto the column space of A, we have to find the projection P_R onto the row space.

Suppose V is the column space of A

Then V is the row space of A^T

So replace A by A^T in P_C we get P_R

Step-2

The projection P_R onto the row space

$$= A^{T} \left[\left(A^{T} \right)^{T} A^{T} \right]^{-1} \left(A^{T} \right)^{T}$$
$$= \left[A^{T} \left(A A^{T} \right)^{-1} A \right]$$

$$= A^T (AA^T)^{-1} A$$