Step-1

Suppose A is a 3×3 upper triangular matrix with the diagonal entries 1, 2, and7.

Then the characteristic equation is $|A-\lambda I| = \begin{vmatrix} 1-\lambda & b & c \\ 0 & 2-\lambda & d \\ 0 & 0 & 7-\lambda \end{vmatrix} = 0$ where *b, c, d* are any real numbers.

We use the first column to find the determinant of this matrix.

i.e., $(1-\lambda)\begin{vmatrix} 2-\lambda & d \\ 0 & 7-\lambda \end{vmatrix} - 0\begin{vmatrix} b & c \\ 0 & 7-\lambda \end{vmatrix} + 0\begin{vmatrix} b & c \\ 2-\lambda & d \end{vmatrix} = 0$

$= (1-\lambda)(2-\lambda)(7-\lambda) - 0 + 0$

So, the eigen values are $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 7$

Step-2

We know that the eigen vectors of the distinct eigen values are linearly independent.

While 1, 2, and 7 are the distinct eigen values, the respective eigen vectors x_1, x_2, x_3 are linearly independent.

So, the matrix S whose columns are x_1, x_2, x_3 is a non singular matrix.

So, S^{-1} exists such that $A = S\Lambda S^{-1}$ where Λ is the diagonal matrix whose diagonal entries are respectively the eigen values of A.

So, $\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$