Step-1

We have to prove that $\|x+y\|_\infty \le \|x\|_\infty + \|y\|_\infty$ and $\|x+y\|_1 \le \|x\|_1 + \|y\|_1$.

We k now that the ℓ^1 norm is defined by $\|x\|_1 = |x_1| + |x_2| + \ldots + |x_n|$ and the ℓ^∞ norm is defined by $\|x\|_\infty = \max\{|x_i| : 1 \le i \le n, x = (x_1, x_2, \ldots, x_n)\}$

Also, the Hilbert norm is $\|x\| = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}$

Step-2

We have $\|x+y\|_\infty = \max\{|x_i + y_i| : 1 \le i \le n, x = (x_1, x_2, \ldots, x_n), y = (y_1, y_2, \ldots, y_n)\}$

We know by the properties of scalars that $|x_i + y_i| \le |x_i| + |y_i|$

Using this, we can write

$\max\{|x_i + y_i| : 1 \le i \le n, x = (x_1, x_2, \ldots, x_n), y = (y_1, y_2, \ldots, y_n)\}$

$\le \max\{|x_i| + |y_i| : 1 \le i \le n, x = (x_1, x_2, \ldots, x_n), y = (y_1, y_2, \ldots, y_n)\}$
$\le \max\{|x_i| : 1 \le i \le n, x = (x_1, x_2, \ldots, x_n)\} + \max\{|y_i| : 1 \le i \le n, y = (y_1, y_2, \ldots, y_n)\}$
$\le \|x\|_\infty + \|y\|_\infty$

Therefore, $\boxed{\|x+y\|_\infty \le \|x\|_\infty + \|y\|_\infty}$

Step-3

Now

$\|x+y\|_1 = |x_1 + y_1| + |x_2 + y_2| + \ldots + |x_n + y_n|$

$\le (|x_1| + |x_2| + \ldots + |x_n|) + (|y_1| + |y_2| + \ldots + |y_n|)$

$\le \|x\|_1 + \|y\|_1$

Therefore, $\boxed{\|x+y\|_1 \le \|x\|_1 + \|y\|_1}$