Step-1

Suppose, we know the values of $c_0 + c_2$, $c_0 - c_2$, $c_1 + c_3$, and $c_1 - c_3$. In order to obtain Fc, we need not calculate c_0, c_1, c_2, and c_3.

We know the following properties of the imaginary number $i = \sqrt{-1}$.

$$
\begin{aligned}
i^1 &= i \\
i^2 &= -1 \\
i^3 &= -i \\
i^4 &= 1
\end{aligned}
$$

We also know that if $0 \le k \le 4$, then $i^{4n+k} = i^k$

Step-2

Therefore, we have the following:

$$
\begin{aligned}
i^6 &= i^2 \\
&= -1 \\
i^9 &= i^1 \\
&= i
\end{aligned}
$$

Step-3

Consider the following:

$$
\begin{aligned}
Fc &= \begin{bmatrix} c_0 + c_1 + c_2 + c_3 \\ c_0 + ic_1 + i^2c_2 + i^3c_3 \\ c_0 + i^2c_1 + i^4c_2 + i^6c_3 \\ c_0 + i^3c_1 + i^6c_2 + i^9c_3 \end{bmatrix} \\
&= \begin{bmatrix} c_0 + c_1 + c_2 + c_3 \\ c_0 + ic_1 - 1c_2 - ic_3 \\ c_0 - 1c_1 + c_2 - 1c_3 \\ c_0 - ic_1 - c_2 + ic_3 \end{bmatrix} \\
&= \boxed{\begin{bmatrix} (c_0 + c_2) + (c_1 + c_3) \\ (c_0 - c_2) + i(c_1 - c_3) \\ (c_0 + c_2) - (c_1 + c_3) \\ (c_0 - c_2) - i(c_1 - c_3) \end{bmatrix}}
\end{aligned}
$$

Step-4

Thus, suppose we have the following relations:

$$c_0 + c_2 = p$$
$$c_0 - c_2 = q$$
$$c_1 + c_3 = r$$
$$c_1 - c_3 = s$$

Then, $\boxed{Fc = \begin{bmatrix} p+r \\ q+is \\ p-r \\ q-is \end{bmatrix}}$.