

Step-1

Given that $Ax = \lambda x$

If $\lambda = 0$ then x is in the null space.

If $\lambda \neq 0$ then x is in the column space.

Also given that the dimension of the null space is $n - r$ and the dimension of the column space is r .

So the dimension of the given space is n .

We have to explain why every square matrix has no n linearly independent eigenvectors.

Step-2

Because there are two problems.

The null space and column space can overlap.

So x could be in both.

Second problem is if there may not be r independent eigenvectors in the column space.

If suppose λ have repeated eigenvalues then there may not be r independent eigenvectors.