Step-1

Consider the following matrices:

$$A=\begin{bmatrix}0 & 1-i\\ i+1 & 1\end{bmatrix}$$
$$K=\begin{bmatrix}0 & -1+i\\ 1+i & i\end{bmatrix}$$

Diagonalize it to reach $A=U\Lambda U^H$ and $K=U\Lambda U^H$.

Step-2

First step is to find the Eigen values and Eigen vectors of matrix A. To calculate the Eigen values do the following calculations;

$$A-\lambda I=\begin{bmatrix}0-\lambda & 1-i\\ i+1 & 1-\lambda\end{bmatrix}$$
$$\det(A-\lambda I)=0$$
$$(1-\lambda)(-\lambda)-(1-i^2)=0$$
$$(\lambda^2-\lambda-2)=0$$

After solving following values are obtained:

$$\lambda_1=2$$
$$\lambda_2=-1$$

Step-3

To calculate Eigen vectors do the following calculations:

$$(A-\lambda I)x=0$$
$$\begin{bmatrix}0-2 & 1-i\\ i+1 & 1-2\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$
$$\begin{bmatrix}-2 & 1-i\\ i+1 & -1\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda=2$ are as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1+i \end{bmatrix}$$

Step-4

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -1$ is as follows:

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} 0+1 & 1-i \\ i+1 & 1+1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 1-i \\ i+1 & 2 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} -1+i \\ 1 \end{bmatrix}$$

Step-5

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned} \|x\|^2 &= \left|(1)^2\right| + \left|(1+i)^2\right| \\ &= |1| + |(2i)| \\ &= 3 \end{aligned}$$

Let the length be L. So $L = \sqrt{3}$.

Step-6

Now, the diagonalization of the matrix can be written as follows:

$$\begin{aligned} A &= U \Lambda U^H \\ &= \frac{1}{L} \begin{bmatrix} 1 & -1+i \\ 1+i & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \frac{1}{L} \begin{bmatrix} 1 & 1-i \\ -1-i & 1 \end{bmatrix} \end{aligned}$$

Here, $L = \sqrt{3}$

Step-7

Therefore, matrix A is diagonalize to reach $\boxed{A = U\Lambda U^H}$.

Step-8

To diagonalize matrix K, do the following calculations. First step is to find the Eigen values and Eigen vectors of matrix K. To calculate the Eigen values do the following calculations;

$$\begin{aligned} K - \lambda I &= \begin{bmatrix} 0-\lambda & -1+i \\ 1+i & i-\lambda \end{bmatrix} \\ \det(K-\lambda I) &= 0 \\ (i-\lambda)(-\lambda)-(-1+i^2) &= 0 \\ (\lambda^2 - i\lambda + 2) &= 0 \end{aligned}$$

After solving following values are obtained:

$$\lambda_1 = 2i$$
$$\lambda_2 = -i$$

Step-9

To calculate Eigen vectors do the following calculations:

$$\begin{aligned} (K-\lambda I)x &= 0 \\ \begin{bmatrix} 0-2i & -1+i \\ 1+i & i-2i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} -2i & -1+i \\ 1+i & -i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned}$$

On solving, values of y and z corresponding to $\lambda = 2i$ are as follows:

$$\begin{aligned} x_1 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1-i \end{bmatrix} \end{aligned}$$

Step-10

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -i$ is as follows:

$$(K-\lambda I)x=0$$

$$\begin{bmatrix} 0+i & -1+i \\ 1+i & i+i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i & -1+i \\ 1+i & 2i \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} -1-i \\ 1 \end{bmatrix} \end{aligned}$$

Step-11

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\begin{aligned} \|x\|^2 &= \left|(1)^2\right|+\left|(1-i)^2\right| \\ &= |1|+|(-2i)| \\ &= 3 \end{aligned}$$

Let the length be L. So $L=\sqrt{3}$.

Step-12

Now, the diagonalization of the matrix can be written as follows:

$$\begin{aligned} K &= U\Lambda U^H \\ &= \frac{1}{L}\begin{bmatrix} 1 & -1-i \\ 1-i & 1 \end{bmatrix}\begin{bmatrix} 2i & 0 \\ 0 & -i \end{bmatrix}\frac{1}{L}\begin{bmatrix} 1 & 1+i \\ -1+i & 1 \end{bmatrix} \end{aligned}$$

Here, $L=\sqrt{3}$

Step-13

Therefore, matrix K is diagonalize to reach $\boxed{K=U\Lambda U^H}$.