HW4

Homework 4

Please answer the following questions about model fitting.

Question 1:

In 1601 the German astronomer Johannes Kepler became director of the Prague Observatory. Kepler had been helping Tycho Brahe in collecting 13 years of observations on the relative motion of the planet Mars. By 1609 Kepler had formulated his first two laws:

i. Each planet moves on an ellipse with the sun at one focus.

ii. For each planet, the line from the sun to the planet sweeps out equal areas in equal times.

Kepler spent many years verifying these laws and formulating a third law, which relates the planets' orbital periods and mean distances from the sun.

a. Plot the period time T versus the mean distance r using the following updated observational data.

Planet	Period (days)	Mean distance from the sun (millions of kilometers)
Mercury	88	57.9
Venus	225	108.2
Earth	365	149.6
Mars	687	227.9
Jupiter	4,329	778.1
Saturn	10,753	1428.2
Uranus	30,660	2837.9
Neptune	60,150	4488.9

b. Assuming a relationship of the form

$$T = Cr^a$$

determine the parameters C and a by plotting $\ln T$ versus $\ln r$. Does the model seem reasonable? Try to formulate Kepler's third law.

Question 2:

In the following data, V represents a mean walking velocity and P represents the population size. We wish to know if we can predict the population size P by observing how fast people walk. Plot the data. What kind of a relationship is suggested? Test the following models by plotting the appropriate transformed data.

a. $P = aV^b$

b. $P = a \ln V$

V	2.27	2.76	3.27	3.31	3.70	3.85	4.31	4.39	4.42
P	2500	365	23700	5491	14000	78200	70700	138000	304500

V	4.81	4.90	5.05	5.21	5.62	5.88
P	341948	49375	260200	867023	1340000	1092759

Question 3:

Solve the two equations given by (3.4) to obtain the values of the parameters given by Equations (3.5) and (3.6), respectively.

$$\left.\begin{aligned} a\sum_{i=1}^{m} x_i^2 + b\sum_{i=1}^{m} x_i &= \sum_{i=1}^{m} x_i y_i \\ a\sum_{i=1}^{m} x_i + mb &= \sum_{i=1}^{m} y_i \end{aligned}\right\} \tag{3.4}$$

The preceding equations can be solved for a and b once all the values for x_i and y_i are substituted into them. The solutions (see Problem 1 at the end of this section) for the parameters a and b are easily obtained by elimination and are found to be

$$a = \frac{m\sum x_i y_i - \sum x_i \sum y_i}{m\sum x_i^2 - \left(\sum x_i\right)^2}, \quad \textbf{the slope} \tag{3.5}$$

and

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{m\sum x_i^2 - \left(\sum x_i\right)^2}, \quad \textbf{the intercept} \tag{3.6}$$

Question 4:

fit the data with the models given, using least squares.

Data for the ponderosa pine

x	17	19	20	22	23	25	28	31	32	33	36	37	39	42
y	19	25	32	51	57	71	113	140	153	187	192	205	250	260

a. $y = ax + b$

b. $y = ax^2$

c. $y = ax^3$

d. $y = ax^3 + bx^2 + c$

Due: 10:00am **Please email your homework to TA.**