

## Step-1

Suppose  $A_n$  is the  $n$  by  $n$  tridiagonal matrix with 1s on the three diagonals:

$$A_1 = [1], \quad A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \text{ etc.}$$

Let  $D_n$  be the determinant of  $A_n$ .

## Step-2

$$A_4 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \text{ the five terms are non zero in } \det A_4 \text{ are given by}$$

$$a_{11}a_{22}a_{33}a_{44} - a_{12}a_{21}a_{33}a_{44} - a_{11}a_{22}a_{34}a_{43} + a_{12}a_{21}a_{34}a_{43} - a_{11}a_{23}a_{32}a_{44}$$

## Step-3

Since  $a_{11} = 1 = a_{12}, a_{21} = a_{22} = a_{23} = 1$

$$a_{32} = a_{33} = a_{34} = 1, a_{43} = a_{44} = 1$$

And remaining entries are zero.

Hence

$$\det A_4 = 1 - 1 - 1 + 1 - 1$$

$$\det A_4 = \boxed{-1}$$