

Step-1

Given that $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$ and the eigen values are -3, 0, and 3

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ a & b & c - \lambda \end{vmatrix} = 0$$

The characteristic equation of A is

$$\begin{aligned} \Rightarrow -\lambda \begin{vmatrix} -\lambda & 1 \\ b & c - \lambda \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ a & c - \lambda \end{vmatrix} &= 0 \\ \Rightarrow -\lambda (\lambda^2 - c\lambda - b) + a &= 0 \\ \Rightarrow -\lambda^3 + c\lambda^2 + b\lambda + a &= 0 \end{aligned}$$

Step-2

On the other hand, the given characteristic equation is $9\lambda - \lambda^3$

Comparing both the above equations, we follow that $a = 0$, $b = 9$ and $c = 0$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 9 & 0 \end{bmatrix}$$

Therefore