Step-1

If A is 5 by 5 matrix with all $|a_{ij}| \le 1$, then we have to find $\det A \le$ ___ .

Suppose $\vec{v_i}$ be the ith column of A. Then we have

$$|\vec{v_i}| = \sqrt{a_{1i}^{\;2} + a_{2i}^{\;2} + a_{3i}^{\;2} + a_{4i}^{\;2} + a_{5i}^{\;2}}$$

We know that $|a_{ij}| \le 1$ for all i and j, so $a_{ij}^{\;2} \le 1$ is true.

Step-2

Since $a_{ij}^{\;2} \le 1$, so

$$\begin{aligned}\sqrt{a_{1i}^{\;2} + a_{2i}^{\;2} + a_{3i}^{\;2} + a_{4i}^{\;2} + a_{5i}^{\;2}} &\le \sqrt{1+1+1+1+1} \\ &\le \sqrt{5}\end{aligned}$$

Now consider $|\vec{v_i}| = \sqrt{a_{1i}^{\;2} + a_{2i}^{\;2} + a_{3i}^{\;2} + a_{4i}^{\;2} + a_{5i}^{\;2}}$, then we can write

$$|\vec{v_i}| \le \sqrt{5}$$

Step-3

Now we have $|\vec{v_i}| \le \sqrt{5}$, hence the edges of a 5-dimensional box spanned by the columns of A is less than $\sqrt{5}$.

The volume of such box, V, can be calculate as follows

$$\begin{aligned}V &\le \left(\sqrt{5}\right)^5 \\ &\le 25\sqrt{5} \\ &\le 55.091\end{aligned}$$

Step-4

We know that $|\det A|$ is equivalent to the volume of 5-dimensional box spanned by the columns of A .

So, by using this fact we get

$$|\det A| \le \boxed{55.091}$$

Step-5

Now, to find an upper bound on the determinant, we use the Big formula for the determinant

$$\det A = \sum_{all\,p's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$$

The sum determinant of the whole matrix is the sum of $n!$.

Step-6

We have considered A is 5 by 5 matrix with all $|a_{ij}| \le 1$, so the number of terms in the sum is

$$5! = 120$$

Since $|a_{ij}| \le 1$, so that all of the 120 terms in the sum is smaller than 1 .

Therefore, $|\det A| \le \boxed{120}$.