Step-1

Given $b=(0,8,8,20)$, we have to check that $e=b-p=(-1,3-5,3)$ is perpendicular to both the columns of *A* and we have to find the shortest distance $\|e\|$ from *b* to the column space of *A*.

Step-2

We have

$$e=b-p$$
$$=\begin{bmatrix}0\\8\\8\\20\end{bmatrix}-\begin{bmatrix}1\\5\\13\\17\end{bmatrix}$$
$$=\begin{bmatrix}-1\\3\\-5\\3\end{bmatrix}$$

Step-3

Let a_1,a_2 are the columns of *A,* where

$$a_1=\begin{bmatrix}1\\1\\1\\1\end{bmatrix},\ a_2=\begin{bmatrix}0\\1\\3\\4\end{bmatrix}$$

$$a_1^Te=\begin{bmatrix}1&1&1&1\end{bmatrix}\begin{bmatrix}-1\\3\\-5\\3\end{bmatrix}$$
$$=-1+3-5+3$$
$$=0$$

Therefore *e* is perpendicular to a_1

Step-4

And

$$a_2^T e = \begin{bmatrix} 0 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -5 \\ 3 \end{bmatrix}$$
$$= 0 + 3 - 15 + 12$$
$$= 0$$

Therefore e is perpendicular to a_2

Hence e is perpendicular to both columns of A

Step-5

$\|e\|$ is the shortest distance of vector b to the column space of A.

$$\|e\|^2 = e^T e$$
$$= \begin{bmatrix} -1 & 3 & -5 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -5 \\ 3 \end{bmatrix}$$
$$= 1 + 9 + 25 + 9$$
$$= 44$$

Hence the required shortest distance $= \sqrt{44} = 2\sqrt{11}$