

Step-1

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Given that producing x_1 trucks and x_2 planes requires $x_1 + 50x_2$ tons of steel, $40x_1 + 1000x_2$ pounds of rubber, and $2x_1 + 50x_2$ months of labor.

Suppose the unit costs y_1, y_2, y_3 are \$700 per ton, \$3 per pound and \$3000 per month.

We have to find the value of one truck and one plane.

Step-2

Given that the truck and the plane are the components of $A^T y$.

From the given data, we get

$$\begin{aligned} Ax &= \begin{bmatrix} x_1 + 50x_2 \\ 40x_1 + 1000x_2 \\ 2x_1 + 50x_2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned}$$

Step-3

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{pmatrix} 700 \\ 3 \\ 3000 \end{pmatrix}$$

And

$$A = \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix}$$

We have the matrix A is

$$\text{The transpose of } A \text{ is } A^T = \begin{bmatrix} 1 & 40 & 2 \\ 50 & 1000 & 50 \end{bmatrix}.$$

Step-4

The values of one truck and one plane is $A^T y$.

Therefore,

$$\begin{aligned} A^T y &= \begin{bmatrix} 1 & 40 & 2 \\ 50 & 1000 & 50 \end{bmatrix} \begin{bmatrix} 700 \\ 3 \\ 3000 \end{bmatrix} \\ &= \begin{bmatrix} 1(700) + 40(3) + 2(3000) \\ 50(700) + 1000(3) + 50(3000) \end{bmatrix} \\ &= \begin{bmatrix} 700 + 120 + 6000 \\ 35000 + 3000 + 150000 \end{bmatrix} \\ &= \begin{bmatrix} 6820 \\ 188000 \end{bmatrix} \end{aligned}$$

Hence the value of one truck is $\boxed{\$6,820}$ and the value of one plane is $\boxed{\$188,000}$.