Step-1

$Q^TQ = \left(I - 2uu^T\right)^T \left(I - 2uu^T\right)$

$= \left[I^T - 2\left(uu^T\right)^T\right]\left(I - 2uu^T\right)$
$= \left[I - 2\left(u^T\right)^T u^T\right]\left(I - 2uu^T\right)$
$= \left(I - 2uu^T\right)\left(I - 2uu^T\right)$

Since $(kA)^T = kA^T, (AB)^T = B^T A^T$ and $\left(A^T\right)^T = A$, we get the above product as

$Q^TQ = I - 2uu^T - 2uu^T + 4uu^Tuu^T$

$= I - 4uu^T + 4u\left(u^Tu\right)u^T$

Since u is the unit vector, we follow that $u^Tu = 1$

So, the above equation becomes $= I - 4uu^T + 4uu^T$

$\Rightarrow Q^TQ = I$

Therefore, Q is an orthogonal matrix.

Step-2

Also, $Q^T = \left(I - 2uu^T\right)^T$

$= I^T - 2\left(uu^T\right)^T$

$= I - 2\left(u^T\right)^T u^T$ By the properties of transposing matrices

$= I - 2uu^T$
$= Q$

Therefore, Q is a symmetric matrix.

Step-3

Given that $u = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} \end{pmatrix}$

$$u^T u = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{pmatrix}$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$
$$= 1$$

Step-4

Also, $$uu^T = \begin{pmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{-1}{2} \end{pmatrix}$$

$$= \begin{bmatrix} 1/4 & 1/4 & -1/4 & -1/4 \\ 1/4 & 1/4 & -1/4 & -1/4 \\ -1/4 & -1/4 & 1/4 & 1/4 \\ -1/4 & -1/4 & 1/4 & 1/4 \end{bmatrix}$$

$$2uu^T = \begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix}$$

Step-5

$$Q = I - 2uu^T$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix}$$

Therefore, $$Q = \begin{bmatrix} 1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & 1/2 \end{bmatrix}$$