Step-1

To find rank and null space of the matrix as follows,

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Interchange the first and third row, that is $R_3 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Apply the row operation: $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, first and third column has the number of pivot position, so these columns are linearly independent.

Therefore, rank of matrix A is $\boxed{2}$.

Step-2

Now it is known that rank nullity theorem;

$$\begin{aligned} \text{rank } A + \dim N(A) &= \text{number of columns of } A \\ 2 + \dim N(A) &= 3 \\ \dim N(A) &= 1 \end{aligned}$$

To find null space of matrix A;

$$Ax = 0$$

This implies;

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This implies;

$$x_3 = 0$$
$$x_1 + x_2 + x_3 = 0$$

Thus,

$$x_1 = -x_2$$
$$x_3 = 0$$

Step-3

Hence, vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ can be written as,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_2 \\ 0 \end{bmatrix}$$
$$= x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

Thus, null space of $\boxed{A = \left\{ x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}; x_2 \in \mathrm{R} \right\}}$.

Step-4

Consider matrix B as follows,

$$B = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Interchange the first and the second row, that is $R_3 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Apply the row operation: $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix has 2 pivot columns which is 1st and 3rd column and the r columns are free because the matrix has 2 points.

Hence, rank of matrix B is $\boxed{2}$.

Step-5

Now it is known that rank nullity theorem;

$$\begin{aligned} \text{rank } A + \dim N(A) &= \text{number of columns of } A \\ 2 + \dim N(A) &= 4 \\ \dim N(A) &= 4 - 2 \\ \dim N(A) &= 2 \end{aligned}$$

To find null space of matrix B;

$$BX = 0$$

This implies;

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This implies that,

$$\begin{aligned} x_3 + 2x_4 &= 0 \\ x_1 + x_2 + x_3 &= 0 \end{aligned}$$

This implies that,

$$\begin{aligned} x_3 &= -2x_4 \\ x_1 &= -x_2 - x_3 \end{aligned}$$

This implies that,

$x_3 = -2x_4$
$x_1 = -x_2 + 2x_4$

Step-6

Thus, vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ can be written as,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -x_2 + 2x_4 \\ x_2 \\ -2x_4 \\ x_4 \end{bmatrix}$$
$$= x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

Hence, the null space of matrix B is spanned by $\boxed{\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}}$.