

Step-1

Consider the following;

$$\begin{aligned}(y, 1-y)A &= (y, 1-y) \begin{bmatrix} 2 & 8 \\ 6 & -6 \end{bmatrix} \\ &= (2y + 6(1-y), 8y - 6(1-y)) \\ &= (6-4y, 14y-6)\end{aligned}$$

Step-2

Equate $6-4y$ and $14y-6$. This gives,

$$\begin{aligned}6-4y &= 14y-6 \\ 6+6 &= 14y+4y \\ 12 &= 18y \\ y &= \frac{2}{3}\end{aligned}$$

Step-3

Thus, the optimal strategy involves $y = \frac{2}{3}$.

When $y = \frac{2}{3}$, we have $6-4y = \frac{10}{3}$.

Thus, the average gain on each play is $\frac{10}{3}$.