

Step-1

Given

B_n is still the same as A_n except for $b_{11} = 1$

Using linearity in 1st row of determinants we get

$$|B_n| = \begin{vmatrix} 1 & -1 & 0 \\ -1 & & \\ 0 & & A_{n-1} \end{vmatrix} = \begin{vmatrix} 2 & -1 & 0 \\ -1 & & \\ 0 & & A_{n-1} \end{vmatrix} - \begin{vmatrix} 1 & 0 & 0 \\ -1 & & \\ 0 & & A_{n-1} \end{vmatrix}$$

Step-2

On solving

$$= |A_n| - |A_{n-1}| \text{ expanding 2nd determinant by 1st row}$$

$$= (n+1) - n$$

$$= 1 \text{ For all } n \in \mathbb{N}$$

Thus

$$|B_n| = \boxed{1}$$