Step-1

(a)

Consider the matrix A,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} d_1 & & \\ & d_2 & \\ & & d_3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

The objective is to find the conditions if the matrix A is non-singular.

Step-2

The matrix A is nonsingular if $\det(A) \neq 0.$

Here, the matrix $\begin{bmatrix} d_1 & & \\ & d_2 & \\ & & d_3 \end{bmatrix}$ is a diagonal matrix.

If any one of d_1, d_2, d_3 is 0 then $\det(A) = 0$

Therefore, the matrix A is nonsingular when, $d_1 d_2 d_3 \neq 0.$

Step-3

(b)

Consider the system $Lc = b$:

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = b$$

The objective is to solve the system $Ax = b,$ starts with $Lc = b.$

Step-4

Consider the matrix,

$$A=\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}\begin{bmatrix}d_1&0&0\\0&d_2&0\\0&0&d_3\end{bmatrix}\begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}$$
$$=\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}\begin{bmatrix}d_1&-d_1&0\\0&d_2&-d_2\\0&0&d_3\end{bmatrix}$$
$$=\begin{bmatrix}d_1&-d_1&0\\-d_1&d_1+d_2&-d_2\\0&-d_2&d_2+d_3\end{bmatrix}$$

Step-5

Consider the system,

$$Ax=b$$
$$\begin{bmatrix}d_1&-d_1&0\\-d_1&d_1+d_2&-d_2\\0&-d_2&d_2+d_3\end{bmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}$$
$$\xrightarrow[R_2\to R_2+R_1]{R_3\to R_1+R_2+R_3,}\begin{bmatrix}d_1&-d_1&0\\0&d_2&-d_2\\0&0&d_3\end{bmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

The equations form is,

$$\begin{aligned}d_1u-d_1v&=0\\d_2v-d_2w&=0\\d_3w&=1\end{aligned}$$

Step-6

Solve by back substitution,

$d_3w=1$

From part (a), $d_3\neq 0$.

$w=\frac{1}{d_3}$.

From second equation,

$$
\begin{aligned}
d_2(v-w) &= 0 \\
v-w &= 0, \\
v &= w \\
&= \frac{1}{d_3}
\end{aligned}
$$

Since, $d_3 \neq 0$, and $d_2 \neq 0$.

From first equation,

$$
\begin{aligned}
d_1(u-v) &= 0 \\
u-v &= 0 \\
u &= v \\
&= \frac{1}{d_3}
\end{aligned}
$$

Since, $d_3 \neq 0$, and $d_1 \neq 0$.

Step-7

Therefore,

$$u = v = w = \frac{1}{d_3}, d_3 \neq 0.$$

Hence, the solution of the system is,

$$\boxed{x = \begin{bmatrix} 1/d_3 \\ 1/d_3 \\ 1/d_3 \end{bmatrix}}, d_3 \neq 0.$$