Step-1

Consider the values of $b$at different t values.

$b=4$ at $t=-2$
$b=3$ at $t=-1$
$b=1$ at $t=0$
$b=0$ at $t=2$.

The objective is to find the best-straight line fit for the given measurements.

Step-2

From these data points, write the matrices A and x as,

$$A=\begin{bmatrix}1 & -2\\ 1 & -1\\ 1 & 0\\ 1 & 2\end{bmatrix} \text{ and } \boldsymbol{b}=\begin{bmatrix}4\\ 3\\ 1\\ 0\end{bmatrix}.$$

First find $A^T A$.

$$\begin{aligned}
A^T A &= \begin{bmatrix}1 & 1 & 1 & 1\\ -2 & -1 & 0 & 2\end{bmatrix}\begin{bmatrix}1 & -2\\ 1 & -1\\ 1 & 0\\ 1 & 2\end{bmatrix}\\
&= \begin{bmatrix}1(1)+1(1)+1(1)+1(1) & 1(-2)+1(-1)+1(0)+1(2)\\ (-2)(1)+(-1)(1)+0(1)+2(1) & (-2)(-2)+(-1)(-1)+0(0)+2(2)\end{bmatrix}\\
&= \begin{bmatrix}1+1+1+1 & -2-1+0+2\\ -2-1+0+2 & 4+1+0+4\end{bmatrix}\\
&= \begin{bmatrix}4 & -1\\ -1 & 9\end{bmatrix}
\end{aligned}$$

Step-3

Now find $\left(A^T A\right)^{-1}$.

$$\left(A^T A\right)^{-1} = \frac{1}{4(9)-(-1)(-1)}\begin{bmatrix} 9 & 1 \\ 1 & 4 \end{bmatrix}$$
$$= \frac{1}{36-1}\begin{bmatrix} 9 & 1 \\ 1 & 4 \end{bmatrix}$$
$$= \frac{1}{35}\begin{bmatrix} 9 & 1 \\ 1 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{9}{35} & \frac{1}{35} \\ \frac{1}{35} & \frac{4}{35} \end{bmatrix}$$

Therefore, $$\left(A^T A\right)^{-1} = \begin{bmatrix} \frac{9}{35} & \frac{1}{35} \\ \frac{1}{35} & \frac{4}{35} \end{bmatrix}.$$

Step-4

The solution to the best straight line fit can be calculated as,

$$x = \left(A^T A\right)^{-1} A^T b$$
$$= \begin{bmatrix} \frac{9}{35} & \frac{1}{35} \\ \frac{1}{35} & \frac{4}{35} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{9}{35} & \frac{1}{35} \\ \frac{1}{35} & \frac{4}{35} \end{bmatrix}\begin{bmatrix} 1(4)+1(3)+1(1)+1(0) \\ -2(4)+(-1)(3)+0(1)+(-2)(0) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{9}{35} & \frac{1}{35} \\ \frac{1}{35} & \frac{4}{35} \end{bmatrix}\begin{bmatrix} 4+3+1+0 \\ -8-3+0-0 \end{bmatrix}$$

Step-5

Further simplification is as follows:

$$=\begin{bmatrix} \frac{9}{35} & \frac{1}{35} \\ \frac{1}{35} & \frac{4}{35} \end{bmatrix}\begin{bmatrix} 8 \\ -11 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{9}{35}(8)+\frac{1}{35}(-11) \\ \frac{1}{35}(8)+\frac{4}{35}(-11) \end{bmatrix}$$

Step-6

Further simplification is as follows:

$$=\begin{bmatrix} \frac{72}{35}-\frac{11}{35} \\ \frac{8}{35}-\frac{44}{35} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{72-11}{35} \\ \frac{8-44}{35} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{61}{35} \\ \frac{-36}{35} \end{bmatrix}$$

Hence, the required solution is $\boxed{x=\begin{bmatrix} \frac{61}{35} \\ \frac{-36}{35} \end{bmatrix}}$.

From the solution vector $x=\begin{bmatrix} \frac{61}{35} \\ \frac{-36}{35} \end{bmatrix}$, the equation that represents the best line is $\boxed{y=\frac{61}{35}-\frac{36}{35}t}$.

Step-7

Now find the projection of $b=(4,3,1,0)$ onto the column space of the matrix A.

The projection p of the vector b onto the column space of the matrix A is computed by using the formula $p=Ax=A\left(A^TA\right)^{-1}A^Tb$.

Thus, the projection is,

$$\begin{aligned}
p &= Ax \\
&= \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \frac{61}{35} \\ \frac{-36}{35} \end{bmatrix} \\
&= \begin{bmatrix} 1\left(\frac{61}{35}\right) + (-2)\left(\frac{-36}{35}\right) \\ 1\left(\frac{61}{35}\right) + (-1)\left(\frac{-36}{35}\right) \\ 1\left(\frac{61}{35}\right) + (0)\left(\frac{-36}{35}\right) \\ 1\left(\frac{61}{35}\right) + (2)\left(\frac{-36}{35}\right) \end{bmatrix}
\end{aligned}$$

Step-8

Further simplification is as follows:

$$=\begin{bmatrix} \frac{61}{35}+\frac{72}{35} \\ \frac{61}{35}+\frac{36}{35} \\ \frac{61}{35}+0 \\ \frac{61}{35}-\frac{72}{35} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{61+72}{35} \\ \frac{61+36}{35} \\ \frac{61}{35} \\ \frac{61-72}{35} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{113}{35} \\ \frac{97}{35} \\ \frac{61}{35} \\ \frac{-11}{35} \end{bmatrix}$$

Hence, the projection p of b onto the column space of A is $\boxed{p=\begin{bmatrix} \frac{113}{35} \\ \frac{97}{35} \\ \frac{61}{35} \\ \frac{-11}{35} \end{bmatrix}}$.