## Step-1

We have to find the ranks of *AB* and *AM*:

$A=\begin{bmatrix}1&2\\2&4\end{bmatrix}$ and $B=\begin{bmatrix}2&1&4\\3&1.5&6\end{bmatrix}$, and $M=\begin{bmatrix}1&b\\c&bc\end{bmatrix}$

## Step-2

Given

$$AB=\begin{bmatrix}8&4&16\\16&8&32\end{bmatrix}$$
$$\xrightarrow{R_2-2R_1}\begin{bmatrix}8&4&16\\0&0&0\end{bmatrix}$$
$$=R$$

*R* is row reduced echelon form of *AB*. Therefore rank of *AB* = number of non- zero rows in *R*.

Therefore rank of *AB* = $\boxed{1}$

## Step-3

Now

$$AM=\begin{bmatrix}1+2c&b+2bc\\2+4c&2b+4bc\end{bmatrix}$$
$$\xrightarrow{R_2-2R_1}\begin{bmatrix}1+2c&b+2bc\\0&0\end{bmatrix}$$

If $1+2c=0$ then

$$b+2bc=b(1+2c)$$
$$=b.0$$
$$=0$$

## Step-4

Therefore $AM=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

So if $1+2c=0$ then the rank of $AM=0$

That is if $c = -\frac{1}{2}$ then the rank of $AM = 0$

## Step-5

Let $C \neq \frac{-1}{2}$

Since the row reduced echelon form is $\left(\frac{1}{1+2c}R_1\right)$

$AM \square \begin{bmatrix} 1 & b \\ 0 & 0 \end{bmatrix}$, the rank of $AM = 1$