Step-1

Consider the triangular system,

$$\left.\begin{aligned} u-v-w&=b_1\\ v+w&=b_2\\ w&=b_3 \end{aligned}\right\} \quad \text{â€¦â€¦ (1)}$$

The objective is to solve for the elements of b using back substation method.

Step-2

From the system, the last equation is $w=b_3$

Substitute $w=b_3$ in second equation $v+w=b_2$

$$\begin{aligned} v+b_3&=b_2\\ v&=b_2-b_3 \end{aligned}$$

Substitute $w=b_3, v=b_2-b_3$ in first equation to find u.

$$\begin{aligned} u-(b_2-b_3)-b_3&=b_1\\ u-b_2+b_3-b_3&=b_1\\ u-b_2&=b_1\\ u&=b_1+b_2 \end{aligned}$$

Step-3

Column form of system (1) can be written as.

$$u\begin{pmatrix}1\\0\\0\end{pmatrix}+v\begin{pmatrix}-1\\1\\0\end{pmatrix}+w\begin{pmatrix}-1\\1\\1\end{pmatrix}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

Substitute u, v and w in the column form that gives the linear combination of the vectors of b.

Hence, the column combination as in b is

$$\boxed{(b_1+b_2)\begin{pmatrix}1\\0\\0\end{pmatrix}+(b_2-b_3)\begin{pmatrix}-1\\1\\0\end{pmatrix}+(b_3)\begin{pmatrix}-1\\1\\1\end{pmatrix}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}}.$$