

## Step-1

Let a projection matrix have  $n$  rows and  $n$  columns. Consider a matrix  $A$ , which too has  $n$  rows and  $n$  columns.

Let the projection matrix be denoted by  $P_{ij}$ . This means, in the  $i^{\text{th}}$  row of the same, we have  $\cos \theta$  and  $-\sin \theta$  in the  $i^{\text{th}}$  column and  $j^{\text{th}}$  column respectively. Also, the matrix has  $\sin \theta$  and  $\cos \theta$  in the  $j^{\text{th}}$  row in the  $i^{\text{th}}$  column and  $j^{\text{th}}$  column respectively.

## Step-2

Consider the matrix  $P_{ij}A$ .

It is clear that all the entries in this matrix will be same as that of  $A$ , except in the  $i^{\text{th}}$  and  $j^{\text{th}}$  rows.

Thus, if  $b_{kl}$  denotes the entry in the  $k^{\text{th}}$  row and  $l^{\text{th}}$  column of the matrix  $P_{ij}A$ , and if  $a_{kl}$  denotes the entry in the  $k^{\text{th}}$  row and  $l^{\text{th}}$  column of the matrix  $A$ , then we get the following:

$$b_{kl} = a_{kl} \text{ when } k \neq i \text{ and } k \neq j$$

## Step-3

Let the inverse projection matrix be denoted by  $P_{ij}^{-1}$ . This means, in the  $i^{\text{th}}$  row of the same, we have  $\cos \theta$  and  $\sin \theta$  in the  $i^{\text{th}}$  column and  $j^{\text{th}}$  column respectively. Also, the matrix has  $-\sin \theta$  and  $\cos \theta$  in the  $j^{\text{th}}$  row in the  $i^{\text{th}}$  column and  $j^{\text{th}}$  column respectively.

Consider the matrix  $P_{ij}AP_{ij}^{-1}$ .

It is clear that all the entries in this matrix will be same as that of  $P_{ij}A$ , except in the  $i^{\text{th}}$  and  $j^{\text{th}}$  columns.

Thus, if  $b_{kl}$  denotes the entry in the  $k^{\text{th}}$  row and  $l^{\text{th}}$  column of the matrix  $P_{ij}AP_{ij}^{-1}$ , and if  $a_{kl}$  denotes the entry in the  $k^{\text{th}}$  row and  $l^{\text{th}}$  column of the matrix  $P_{ij}A$ , then we get the following:

$$b_{kl} = a_{kl} \text{ when } l \neq i \text{ and } l \neq j$$