Step-1

Then we consider

$$\begin{aligned}\alpha_1 &= \frac{r_0^{\mathrm{T}} r_0}{p_0^{\mathrm{T}} A p_0} \\ &= \frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0}\end{aligned}$$

Step-2

Then we have

$$\begin{aligned}r_1 &= r_0 - \alpha_1 A p_0 \\ &= r_0 - \frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0} A r_0\end{aligned}$$

Step-3

Consider

$$\begin{aligned}r_0^{\mathrm{T}} r_1 &= r_0^{\mathrm{T}}\left(r_0 - \frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0} A r_0\right) \\ &= r_0^{\mathrm{T}} r_0 - r_0^{\mathrm{T}}\left(\frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0}\right) A r_0 \\ &= r_0^{\mathrm{T}} r_0 - \left(\frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0}\right) r_0^{\mathrm{T}} A r_0\end{aligned}$$

$$\begin{aligned}r_0^{\mathrm{T}} r_1 &= r_0^{\mathrm{T}} r_0 - r_0^{\mathrm{T}} r_0\left(\frac{r_0^{\mathrm{T}} A r_0}{r_0^{\mathrm{T}} A r_0}\right) \\ &= r_0^{\mathrm{T}} r_0 - r_0^{\mathrm{T}} r_0 \\ &= 0\end{aligned}$$

Therefore, r_0 is orthogonal to r_1.

Step-4

Now we show that $p_1^{\mathrm{T}} A p_0 = 0$.

We have $p_0 = r_0$ and $p_1 = r_1 + \beta_1 p_0$

Here, $\beta_1 = \frac{r_1^{\mathrm{T}} r_1}{r_0^{\mathrm{T}} r_0}$.

Step-5

Consider $p_1^{\mathrm{T}} A p_0$

$$p_1^{\mathrm{T}} A p_0 = \left(r_1 + \left(\frac{r_1^{\mathrm{T}} r_1}{r_0^{\mathrm{T}} r_0} \right) r_0 \right)^{\mathrm{T}} A r_0$$

$$= \left(r_0 - \frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0} A r_0 + \left(\frac{r_1^{\mathrm{T}} r_1}{r_0^{\mathrm{T}} r_0} \right) r_0 \right)^{\mathrm{T}} A r_0$$

$$= r_0^{\mathrm{T}} A r_0 - \left(\frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0} \right) \left(A r_0 \right)^{\mathrm{T}} A r_0 + \left(\frac{r_1^{\mathrm{T}} r_1}{r_0^{\mathrm{T}} r_0} \right) r_0^{\mathrm{T}} A r_0$$

$$p_1^{\mathrm{T}} A p_0 = r_0^{\mathrm{T}} A r_0 - \left(\frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0} \right) r_0^{\mathrm{T}} A^{\mathrm{T}} A r_0 + \left(\frac{r_1^{\mathrm{T}} r_1}{r_0^{\mathrm{T}} r_0} \right) r_0^{\mathrm{T}} A r_0$$

$$= r_0^{\mathrm{T}} A r_0 - \left(\frac{r_0^{\mathrm{T}} r_0}{r_0^{\mathrm{T}} A r_0} \right) r_0^{\mathrm{T}} A^2 r_0 + \left(\frac{r_1^{\mathrm{T}} r_1}{r_0^{\mathrm{T}} r_0} \right) r_0^{\mathrm{T}} A r_0$$

$$= 0$$

Step-6

Thus, we have shown that $p_1^{\mathrm{T}} A p_0 = 0$.