Step-1

Permutation matrix: Matrix P has single 1 in every row and every column. It has the rows of identity matrix I in any order.

Step-2

Let matrix P is a rotation matrix which multiplies to a vector (x, y, z) to give (z, y, x). Determine P and P^3.

Matrix P if multiplied to the vector (x, y, z) puts last column first by shifting other columns. This gives the permutation matrix P defined as follows. Therefore,

$$\boxed{P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}$$

Step-3

Now do the following calculations:

$$P \cdot P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P^2 \cdot P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^3 = I$$

Therefore, $\boxed{P^3 = I}$.

Step-4

Next consider the rotation axis $a = (1,1,1)$ that doesnâ€™t move but equals to Pa. determine the angle of rotation from $v = (2,3,-5)$ to $Pv = (-5,2,3)$.

Matrix P if multiplied to the vector $v = (2,3,-5)$ puts last column first by shifting other columns. This gives the permutation matrix P as defined above.

Also, above calculation, $\boldsymbol{P}^3 = \boldsymbol{I}$, shows that after taking three rotations matrix equals to identity matrix. This implies that three rotations equals to $\mathbf{360^\circ}$, so one rotation will make an angle of $\mathbf{360^\circ/3 = 120^\circ}$.

Therefore, angle of rotation from *v* to $\boldsymbol{P}\cdot\boldsymbol{v}$ around $\boldsymbol{a} = (1,1,1)$ is $\boxed{\mathbf{120^\circ}}$.