Step-1

We have

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

We find $\det(A)$ by cofactor expansion along the first column.

$$\begin{aligned} \det(A) &= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{vmatrix} \\ &= \begin{vmatrix} 4 & 0 \\ 0 & 5 \end{vmatrix} \\ &= (4)(5) - 0 \\ &= 20 \end{aligned}$$

Step-2

Second method: since A is a triangular matrix, we find $\det(A)$ by taking the product of the diagonal entries.

$$\det(A) = |A| = (1)(4)(5)$$

$$= 20$$

Step-3

The cofactor of A are

$$C_{11} = \begin{vmatrix} 4 & 0 \\ 0 & 5 \end{vmatrix}$$

$$= 20$$

$$\begin{aligned} C_{12} &= -\begin{vmatrix} 0 & 0 \\ 0 & 5 \end{vmatrix} \\ &= 0 \end{aligned}$$

$$\begin{aligned} C_{13} &= \begin{vmatrix} 0 & 4 \\ 0 & 0 \end{vmatrix} \\ &= 0 \end{aligned}$$

Step-4

And

$$C_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$$
$$= -10$$

$$C_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix}$$
$$= 5$$

$$C_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix}$$
$$= 0$$

Step-5

Then

$$C_{31} = \begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix}$$
$$= -12$$

$$C_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix}$$
$$= 0$$

$$C_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix}$$
$$= 4$$

Step-6

So, all the nine cofactors c_{ij} of A are

$$C_{11} = 20, \quad C_{12} = 0, C_{13} = 0,$$
$$C_{21} = -10, C_{22} = 5, C_{23} = 0,$$
$$C_{31} = -12, C_{32} = 0, C_{33} = 4$$

The matrix of cofactor is

$$C = \begin{bmatrix} 20 & 0 & 0 \\ -10 & 5 & 0 \\ -12 & 0 & 4 \end{bmatrix}$$

Step-7

Now

$$C^T = \begin{bmatrix} 20 & -10 & -12 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

We need to verify that $AC^T = (\det A) I$

So, consider

$$A.C^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 20 & -10 & -12 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Step-8

$$= \begin{bmatrix} 20+0+0 & -10+10+0 & -12+0+12 \\ 0+0+0 & 0+20+0 & -0+0+0 \\ 0+0+0 & -0+0+0 & -0+0+20 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$= 20 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (\det A) I$$

Step-9

Thus, $AC^T = \dfrac{C^T}{\det(A)}$

$$=\frac{1}{20}\begin{bmatrix} 20 & -10 & -12 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -\frac{1}{2} & \frac{-3}{5} \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$$

Step-10

Thus

$$A^{-1} = \boxed{\begin{bmatrix} 1 & -\frac{1}{2} & \frac{-3}{5} \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}}$$

Note: The inverse of an upper triangular matrix is also an upper triangular matrix.