Step-1

Given system is

$$u\begin{pmatrix}1\\2\\3\end{pmatrix}+v\begin{pmatrix}1\\0\\1\end{pmatrix}+w\begin{pmatrix}1\\3\\4\end{pmatrix}=b$$

We have to give two examples for b in which the system is solvable, and the system is not solvable.

Step-2

Letting $b=(3,5,8)$ then the column picture for the given equation is

$$u\begin{pmatrix}1\\2\\3\end{pmatrix}+v\begin{pmatrix}1\\0\\1\end{pmatrix}+w\begin{pmatrix}1\\3\\4\end{pmatrix}=\begin{pmatrix}3\\5\\8\end{pmatrix}$$

In the above equation, the second row can be obtained by subtracting the first row from the third row, so the system has infinite solution; hence in this case the system is solvable when $b=(3,5,8)$

Step-3

Letting $b=(1,2,3)$ then the column picture for the given equation is

$$u\begin{pmatrix}1\\2\\3\end{pmatrix}+v\begin{pmatrix}1\\0\\1\end{pmatrix}+w\begin{pmatrix}1\\3\\4\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$$

In the above equation, the second row can be obtained by subtracting the first row from the third row, so the system has infinite solution; hence in this case the system is solvable when $b=(1,2,3)$

Step-4

Letting $b=(3,5,7)$, then the column picture is

$$u\begin{pmatrix}1\\2\\3\end{pmatrix}+v\begin{pmatrix}1\\0\\1\end{pmatrix}+w\begin{pmatrix}1\\3\\4\end{pmatrix}=\begin{pmatrix}3\\5\\7\end{pmatrix}$$

In the above equation, the left-side of the second row can be obtained by subtracting the first row from the third row, but not on the right-side $(\text{since } 7-3\neq 5)$ so the system has no solution; hence in this case the system is not solvable when $b=(3,5,7)$

Step-5

Letting $b=(1,2,2)$, then the column picture is

$$u\begin{pmatrix}1\\2\\3\end{pmatrix}+v\begin{pmatrix}1\\0\\1\end{pmatrix}+w\begin{pmatrix}1\\3\\4\end{pmatrix}=\begin{pmatrix}1\\2\\2\end{pmatrix}$$

In the above equation, the left-side of the second row can be obtained by subtracting the first row from the third row, but not on the right-side $(\text{since } 2-1\neq 2)$ so the system has no solution; hence in this case the system is not solvable when $\boxed{b=(1,2,2)}$