Step-1

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Let the matrix A be such that column1 + column 2 = column 3.

Let $A=\begin{bmatrix} a & d & a+d \\ b & e & b+e \\ c & f & c+f \end{bmatrix}$

We have to show that A has no inverse.

Step-2

We have $A=\begin{bmatrix} a & d & a+d \\ b & e & b+e \\ c & f & c+f \end{bmatrix}$

Subtracting column 1 from column 3, we get

$$A\approx\begin{bmatrix} a & d & d \\ b & e & e \\ c & f & f \end{bmatrix}$$

Subtracting column 2 from column 3, we get

$$A\approx\begin{bmatrix} a & d & 0 \\ b & e & 0 \\ c & f & 0 \end{bmatrix}$$

Since the last column of A is zero.

So its determinant becomes zero.

Hence the matrix A does not possesses inverse.

Step-3

(a) We have to find a nonzero solution x to $Ax=0$.

The given system $Ax=0$ is

$$\begin{bmatrix} a & d & a+d \\ b & e & b+e \\ c & f & c+f \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The augmented matrix is

$$\begin{bmatrix} a & d & a+d & 0 \\ b & e & b+e & 0 \\ c & f & c+f & 0 \end{bmatrix}$$

Step-4

Subtracting column 1 from column 3, we get

$$A\approx\begin{bmatrix} a & d & d & 0 \\ b & e & e & 0 \\ c & f & f & 0 \end{bmatrix}$$

Subtracting column 2 from column 3, we get

$$A\approx\begin{bmatrix} a & d & 0 & 0 \\ b & e & 0 & 0 \\ c & f & 0 & 0 \end{bmatrix}$$

Therefore, the system $Ax=0$ becomes

$$\begin{bmatrix} a & d & 0 & 0 \\ b & e & 0 & 0 \\ c & f & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

This system has no solution.

Hence this system has no nonzero solution.

Step-5

(b) We have to explain why there is no third pivot.

We have $$A=\begin{bmatrix} a & d & a+d \\ b & e & b+e \\ c & f & c+f \end{bmatrix}$$

Subtracting column 1 from column 3, we get

$$A \approx \begin{bmatrix} a & d & d \\ b & e & e \\ c & f & f \end{bmatrix}$$

Subtracting column 2 from column 3, we get

$$A \approx \begin{bmatrix} a & d & 0 \\ b & e & 0 \\ c & f & 0 \end{bmatrix}$$

From this we observe that column 3 becomes a column of zeros after elimination and that no third pivot.