Step-1

Consider F_n be the determinant of the 1, 1,-1 tridiagonal matrix (n by n):

$$F_n = \det\begin{vmatrix} 1 & -1 & & & \\ 1 & 1 & -1 & & \\ & 1 & 1 & -1 & \\ & & . & . & . \\ & & & 1 & 1 \end{vmatrix}$$

If $n = 1$,

$$\begin{aligned} F_1 &= \det[1] \\ &= 1 \end{aligned}$$

Step-2

If $n = 2$,

$$\begin{aligned} F_2 &= \det\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

Step-3

If $n = 3$,

$$F_3 = \det\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \det\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} + \det\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$= F_2 + \det[1]$ Expanding 2^{nd} determinant by 1^{st} column

$$= F_2 + F_1$$

$$\begin{aligned} &= 2 + 1 \\ &= 3 \end{aligned}$$

Step-4

If $n = 4$,

$$F_4 = \det\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \det\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} + \det\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$= F_3 + \det\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ Expanding 2nd determinant by 1st column

$= F_3 + F_2$

$= 3 + 2$
$= 5$

$\neq 4$

Step-5

The 1, 1 cofactor of the n by n matrix is F_{n-1}.

The 1, 2 cofactor has a 1 in column 1, with cofactor F_{n-2}.

Multiply by $(-1)^{1+2}$ and also (-1) from the 1, 2 entry to find $F_n = F_{n-1} + F_{n-2}$.

So, these determinants are Fibonacci numbers.