

Step-1

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Given system is $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

We have to show that $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ has no inverse by trying to solve for the column (x, y) in the system $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x & t \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step-2

We have the system $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Subtracting 3 times row 1 from row 2 gives

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

By row column multiplication of matrices, we have $\begin{bmatrix} x+2y \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

The second column shows that $0 = -3$ case which is impossible

Therefore, $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ has no solution.

Step-3

Given system is $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Subtracting 3 times row 1 from row 2 gives

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

By row column multiplication of matrices, we have $\begin{bmatrix} t+2z \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

The second column shows that $0 = 1$ case which is impossible

Therefore, $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ has no solution.

Hence the given matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ has no inverse.