Step-1

We have to verify which of the given transformations satisfy $T(v+w)=T(v)+T(w)$ and which satisfy $T(cv)=cT(v)$

(a) Given transformation is $T(v)=\frac{v}{\|v\|}$.

Now

$$T(v+w)=\frac{v+w}{\|v+w\|}$$

$$T(v)+T(w)=\frac{v}{\|v\|}+\frac{w}{\|w\|}$$

Therefore, $T(v+w)\neq T(v)+T(w)$

Step-2

And also $T(cv)\neq cT(v)$

Since let $v=(1,4)$

Then $2v=(2,8)$

$$T(2v)=\frac{(2,8)}{\sqrt{4+64}}$$

$$=\frac{(2,8)}{2\sqrt{17}}$$

$$=\frac{(1,4)}{\sqrt{17}}$$

Step-3

$$2T(v)=2.\frac{(1,4)}{\sqrt{1+16}}$$

$$=\frac{2}{\sqrt{17}}(1,4)$$

Therefore, $T(2v) \neq 2T(v)$

Hence the given transformation does not satisfy $T(v+w)=T(v)+T(w)$ and $T(cv)=cT(v)$.

Step-4

(b) Given transformation is $T(v)=v_1+v_2+v_3$.

Let $v=(v_1,v_2,v_3)$ and $w=(w_1,w_2,w_3)$

Then $v+w=(v_1+w_1,v_2+w_2,v_3+w_3)$

Now

$$\begin{aligned}
T(v+w) &= (v_1+w_1,v_2+w_2,v_3+w_3) \\
&= (v_1+w_1)+(v_2+w_2)+(v_3+w_3) \\
&= (v_1+v_2+v_3)+(w_1+w_2+w_3) \\
&= T(v)+T(w)
\end{aligned}$$

Therefore, $T(v+w)=T(v)+T(w)$

Step-5

And

$$T(cv)=T(cv_1,cv_2,cv_3)$$

$$=cv_1+cv_2+cv_3$$

$$=cT(v)$$

Hence the given transformation T satisfies both $T(v+w)=T(v)+T(w)$ and $T(cv)=cT(v)$.

Step-6

(c) Given transformation is $T(v)=(v_1,2v_2,3v_3)$.

Let $v=(v_1,v_2,v_3)$ and $w=(w_1,w_2,w_3)$

Now

$T(v+w) = T(v_1 + w_1, (v_1 + w_2), (v_3 + w_3))$

$= (v_1 + w_1, 2(v_2 + w_2), 3(v_3 + w_3))$

$= T(v) + T(w)$

$T(cv) = (cv_1, cv_2, cv_3)$

$= c(v_1, v_2, v_3)$

$= cT(v)$

Hence the given transformation T satisfies both $T(v+w) = T(v) + T(w)$ and $T(cv) = cT(v)$.

Step-7

(d) Given transformation is $T(v) =$ largest component of v

Let $T(1,2,3) = 3$ and $T(1,2,-3) = 2$

Then

Step-8

$$T(1,2,3) + T(1,2,-3) = 3 + 2$$
$$= 5$$

And

$T((1,2,3) + (1,2,-3)) = T(2,4,0)$

$= 4$

Therefore, $T((1,2,3) + (1,2,-3)) \neq T(1,2,3) + T(1,2,-3)$

Hence the given transformation does not satisfy $T(v+w) = T(v) + T(w)$ and $T(cv) = cT(v)$.