Step-1

The first column of A is the vector $\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$. Its length is 1.

Therefore, the first column of the matrix Q is $\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$only. The second column of the matrix Q must be of the length 1 and it should be orthogonal to the first column. Thus, the second column of Q must be $\begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix}$.

Thus, $Q=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$.

Step-2

If we assume that the column of the matrix A are a and b, then the matrix R is given by $R=\begin{bmatrix}q_1^{\mathrm{T}}a & q_1^{\mathrm{T}}b\\ 0 & q_2^{\mathrm{T}}b\end{bmatrix}$.

Thus, we get

$$R=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta\\ 0 & -\sin^2\theta\end{bmatrix}$$
$$=\begin{bmatrix}1 & \cos\theta\sin\theta\\ 0 & -\sin^2\theta\end{bmatrix}$$

Step-3

Thus, we have

$$A=Q_0R_0$$
$$=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}1 & \cos\theta\sin\theta\\ 0 & -\sin^2\theta\end{bmatrix}$$

Step-4

Let $A_1=R_0Q_0$

Thus, we get

$$
\begin{aligned}
A_1 &= R_0 Q_0 \\
&= \begin{bmatrix} 1 & \cos\theta\sin\theta \\ 0 & -\sin^2\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta + \cos\theta\sin^2\theta & -\sin\theta + \sin\theta\cos^2\theta \\ -\sin^2\theta & -\sin^2\theta\cos\theta \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta\left(1+\sin^2\theta\right) & -\sin\theta\left(1-\cos^2\theta\right) \\ -\sin^2\theta & -\sin^2\theta\cos\theta \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= \begin{bmatrix} \cos\theta\left(1+\sin^2\theta\right) & -\sin\theta\left(\sin^2\theta\right) \\ -\sin^2\theta & -\sin^2\theta\cos\theta \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta\left(1+\sin^2\theta\right) & -\sin^3\theta \\ -\sin^2\theta & -\sin^2\theta\cos\theta \end{bmatrix}
\end{aligned}
$$

Step-5

Thus, we see that the off-diagonal entries of the matrix A change from $\sin\theta$ to $-\sin^3\theta$.