

Step-1

We have to fill the following blanks:

If the columns of A are linearly independent (A has m by n), then the rank is ———, the null space is ———, the row space is ———, and there exists a ———-inverse.

Step-2

If the columns of A are linearly independent (A has m by n), then the rank is number of linearly independent columns. So we have the rank of A is n , and we know that and if full column rank $r = n$ then A has a left inverse B such that $BA = I_n$ and this is possible only if $m \geq n$

Step-3

Therefore we have the following:

If the columns of A are linearly independent (A has m by n), then the rank is n , the null space is $\{0\}$, the row space is \mathbf{R}^n , and there exists a left-inverse.