Step-1

.

The values forÂ c_1 Â andÂ c_2 Â can be determined such that

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}+c_2\begin{bmatrix}1\\2\\1\end{bmatrix}=\begin{bmatrix}1\\3\\2\end{bmatrix}$$

Observing that the last item in the 1st column is 0, so as to

$$c_1\cdot 0+c_2\cdot 1=2\Rightarrow c_2=2$$

From the 1st item in every column it outcomes

$$c_1\cdot 1+c_2\cdot 1=1\Rightarrow c_1+2=1\Rightarrow c_1=-1$$

Lastly, from the 2nd entry in every column it outcomes

$$c_1\cdot 1+c_2\cdot 2=-1\cdot 1+2\cdot 2=-1+4=3$$

Therefore the 3rd column can be represented as a linear combination of the 1st two columns:

$$-1\begin{bmatrix}1\\1\\0\end{bmatrix}+2\begin{bmatrix}1\\2\\1\end{bmatrix}=\begin{bmatrix}1\\3\\2\end{bmatrix}$$

Step-2

At the present suppose $b=(0,0,0)$ in the original system:

$$u\begin{bmatrix}1\\1\\0\end{bmatrix}+v\begin{bmatrix}1\\2\\1\end{bmatrix}+w\begin{bmatrix}1\\3\\2\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

From the above equation the result obtained is

$$-1\begin{bmatrix}1\\1\\0\end{bmatrix}+2\begin{bmatrix}1\\2\\1\end{bmatrix}=\begin{bmatrix}1\\3\\2\end{bmatrix}\Rightarrow -1\begin{bmatrix}1\\1\\0\end{bmatrix}+2\begin{bmatrix}1\\2\\1\end{bmatrix}-1\begin{bmatrix}1\\3\\2\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Therefore one solution for (u,v,w) is $(-1,2,-1)$.

Alternative solution is $(0,0,0)$. the equation above can be multiplied by an arbitrary constant c and the right-hand side will keep on zero, therefore the whole set of solutions is altogether vectors of the form $c(-1,2,-1)$, that is, a line through the points $(-1,2,-1)$ and the origin.