

Step-1

Therefore,

$$\begin{aligned} v &= y^* A x^* \\ &= (y_1, y_2, y_3) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &= (y_1, y_2, y_3) \begin{bmatrix} x_1 \\ 2x_2 \\ 3x_3 \end{bmatrix} \\ &= x_1 y_1 + 2x_2 y_2 + 3x_3 y_3 \end{aligned}$$

Step-2

In order to make very small payments, if Y decides to go for $(1, 0, 0)$, X will understand his policy and will have his policy $(1, 0, 0)$. Thus, Y will have to pay 1 to X . On the other hand if X decides to go for $(0, 0, 1)$ to have maximum amount from Y , Y will have $y_3 = 0$.

Thus, the optimal strategy will be to have $x^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ and $y^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

Therefore,

$$\begin{aligned} v &= x_1 y_1 + 2x_2 y_2 + 3x_3 y_3 \\ &= \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) \\ &= \frac{6}{9} \\ &= \frac{2}{3} \end{aligned}$$

Step-3

Thus, $x^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, $y^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, $v = \frac{2}{3}$.