Step-1

A be an n by n matrix with every entry equal to 1.

$$A=\begin{bmatrix}1 & 1 & - & - & 1\\ 1 & 1 & - & - & 1\\ 1 & 1 & - & - & 1\\ - & - & - & - & -\\ 1 & 1 & - & - & 1\end{bmatrix}$$

By using the *n* â€“ 1 row operations $R_n-R_1, R_{n-1}-R_1, R_{n-2}-R_1,\ldots,R_2-R_1$ upon this matrix, we get $A\approx\begin{bmatrix}1 & 1 & - & - & 1\\ 0 & 0 & - & - & 0\\ 0 & 0 & - & - & 0\\ - & - & - & - & -\\ 0 & 0 & - & - & 0\end{bmatrix}$

Rank of the matrix is the number of non zero rows in the reduced matrix = 1

Step-2

B be the ehexer board matrix with

$$a_{ij}=\begin{cases}1 \text{ if } i+j \text{ is even}\\ 0 \text{ if } i+j \text{ is odd}\end{cases}$$

If n is even

$$B=\begin{bmatrix}1 & 0 & 1 & - & 0\\ 0 & 1 & 0 & - & 1\\ 1 & 0 & 1 & - & 0\\ - & - & - & - & -\\ 0 & 1 & 0 & - & 1\end{bmatrix}$$

Step-3

By using the row operations $R_3-R_1, R_5-R_1, R_7-R_1,\ldots,R_4-R_2, R_6-R_2,\cdots$ we get

$$B\approx\begin{bmatrix}1 & 0 & 1 & - & 0\\ 0 & 1 & 0 & - & 1\\ 0 & 0 & 0 & - & 0\\ - & - & - & - & -\\ 0 & 0 & 0 & - & 0\end{bmatrix}$$

Further, any operation on R_2 using R_1 does not make R_2 completely zero.

In other words, B has two linearly independent rows.

Therefore rank of $B = 2$.