Step-1

Let matrices u_{k+1}, A, and u_k be defined as follows:

$$A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$$
$$u_{k+1} = \begin{bmatrix} y_{k+1} \\ z_{k+1} \end{bmatrix}$$
$$u_k = \begin{bmatrix} y_k \\ z_k \end{bmatrix}$$

Let following be the difference equation of matrices:

$$u_{k+1} = Au_k$$
$$\begin{bmatrix} y_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} y_k \\ z_k \end{bmatrix}$$

Step-2

Find the Eigen values and Eigen vectors of matrix A:

To calculate the Eigen values do the following calculations;

$$\det(A - \lambda I) = 0$$
$$(0.8-\lambda)(0.7-\lambda) - (0.3 \times 0.2) = 0$$
$$\lambda^2 - 1.5\lambda + 0.5 = 0$$

After solving following values are obtained:

$$\lambda_1 = 0.5$$
$$\lambda_2 = 1$$

Therefore, Eigen values are 0.5 and 1.

Step-3

To calculate Eigen vectors do the following calculations:

$$(A - \lambda I)x = 0$$
$$\begin{bmatrix} 0.8-\lambda & 0.3 \\ 0.2 & 0.7-\lambda \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z corresponding to $\lambda_1 = 0.5$ is as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Step-4

Similarly, Eigen vectors corresponding to Eigen value $\lambda_2 = 1$ is as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Step-5

Find the limit of the following matrices when $n \to \infty$.

$$A^n = S\Lambda^n S^{-1}$$

Matrix A can be written as follows:

$$A = S\Lambda S^{-1}$$
$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix}$$

Power matrix:

$$A^k = S\Lambda^k S^{-1}$$
$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix}$$

Step-6

By using the initial condition $\begin{bmatrix} y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$, we have

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = A^k \begin{bmatrix} y_0 \\ z_0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} -2/5 & 3/5 \\ 1/5 & 1/5 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0.5^k & 0 \\ 0 & 1^k \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \times 0.5^k \\ 1^k \end{bmatrix}$$

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} -3 \times 0.5^k + 3 \times 1^k \\ 3 \times 0.5^k + 2 \times 1^k \end{bmatrix}$$

Therefore, the formulas are $y_k = \boxed{-3 \times 0.5^k + 3 \times 1^k}$ and $z_k = \boxed{3 \times 0.5^k + 2 \times 1^k}$.

Step-7

Take the limit $k \to \infty$. Value of $(0.5)^k$ becomes very small, so neglect it, so we get

$$\begin{bmatrix} y_k \\ z_k \end{bmatrix} = \begin{bmatrix} -3 \times 0.5^k + 3 \times 1^k \\ 3 \times 0.5^k + 2 \times 1^k \end{bmatrix}$$

$$\begin{bmatrix} y_\infty \\ z_\infty \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Therefore, when $k \to \infty$, the limiting values of $y_k = \boxed{3}$ and $z_k = \boxed{2}$.