

Step-1

We need to choose the value of θ , so that the matrix $R = PA$ will be triangular.

Consider

$$\begin{aligned} R &= PA \\ &= \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix} \\ &= \begin{bmatrix} \cos \theta - 3 \sin \theta & -\cos \theta - 5 \sin \theta \\ \sin \theta + 3 \cos \theta & -\sin \theta + 5 \cos \theta \end{bmatrix} \end{aligned}$$

Step-2

Since, R has to be triangular, we want $\sin \theta + 3 \cos \theta = 0$.

Consider

$$\begin{aligned} \sin \theta + 3 \cos \theta &= 0 \\ \sin \theta &= -3 \cos \theta \\ \tan \theta &= -3 \\ \theta &= \tan^{-1}(-3) \end{aligned}$$

This gives $\theta = -71.565^\circ$.

Step-3

We have

$$\begin{aligned} \sin(-71.565^\circ) &= -0.9487 \\ \cos(-71.565^\circ) &= 0.3162 \end{aligned}$$

Step-4

Therefore, when $\sin(-71.565^\circ) = -0.9487$ and $\cos(-71.565^\circ) = 0.3162$, the matrix $R = PA$ is triangular.