Step-1

Since A is assumed to be uv^{T} and that the rank of A is 1, it is clear that A is not the zero matrix.

Therefore, neither u not v^{T} can be a zero matrix.

Therefore, the requirement is that the matrix $v^{T}u$ be the zero matrix.

Step-2

$$u = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad v = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}.$$
the tent of the properties of the proper

Therefore,

$$A = uv^{T}$$

$$= \begin{bmatrix} a_{1} \\ a_{2} \\ \vdots \\ a_{n} \end{bmatrix} (b_{1}, b_{2}, \dots, b_{n})$$

$$= \begin{bmatrix} a_{1}b_{1} & a_{1}b_{2} & \dots & a_{1}b_{n} \\ a_{2}b_{1} & a_{2}b_{2} & \dots & a_{2}b_{n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n}b_{1} & a_{n}b_{2} & \dots & a_{n}b_{n} \end{bmatrix}$$

Step-3

Similarly, consider

$$v^{\mathsf{T}} u = (b_1, b_2, ..., b_n) \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$
$$= [b_1 a_1 + b_2 a_2 + ... + b_n a_n]$$

Thus, if $b_1a_1 + b_2a_2 + ... + b_na_n = 0$, the matrix A will have the rank 1 and $A^2 = 0$.

Step-4

For example, let $a_1 = 2$, $a_2 = 3$, and $a_3 = -4$. Let $b_1 = 1$, $b_2 = 6$, $b_3 = 5$.

Then

$$b_1 a_1 + b_2 a_2 + b_3 a_3 = 1 \cdot 2 + 6 \cdot 3 + 5 \cdot (-4)$$
$$= 2 + 18 - 20$$
$$= 0$$

Consider

$$A = \begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix} (2,3,-4)$$
$$= \begin{bmatrix} 2 & 3 & -4 \\ 12 & 18 & -24 \\ 10 & 15 & -20 \end{bmatrix}$$

Step-5

Note that there is only one independent row in the matrix A. Therefore, rank of A is 1. As per the above discussion, A^2 should be zero.

$$A^{2} = \begin{bmatrix} 2 & 3 & -4 \\ 12 & 18 & -24 \\ 10 & 15 & -20 \end{bmatrix} \begin{bmatrix} 2 & 3 & -4 \\ 12 & 18 & -24 \\ 10 & 15 & -20 \end{bmatrix}$$

$$= \begin{bmatrix} 4+36-40 & 6+54-60 & -8-72+80 \\ 24+216-240 & 36+324-360 & -48-432+480 \\ 20+180-200 & 30+270-300 & -40-360+400 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$