Step-1

Suppose T is a linear transformation that transforms $(1,1)$ to $(2,2)$ and $(2,0)$ to $(0,0)$.

We have to find $T(v)$ for the given vectors.

We know that a transformation T is said to be a linear transformation if $T(x+y)=T(x)+T(y)$ and $T(cx)=cT(x)$, where x, y are vectors and a, b are scalars.

We have $T(1,1)=(2,2)$ â€¦â€¦ (1)

$T(2,0)=(0,0)$ â€¦â€¦ (2)

Step-2

a) Given vector is $v=(2,2)$

Now

$$
\begin{aligned}
T(2,2) &= T(2(1,1)) \\
&= 2T(1,1) && (\text{Since } T(cx)=cT(x)) \\
&= 2(2,2) && (\text{By } (1)) \\
&= (4,4)
\end{aligned}
$$

Therefore, $\boxed{T(2,2)=(4,4)}$

Step-3

b) Given vector is $v=(3,1)$

Now

$$
\begin{aligned}
T(3,1) &= T((1,1)+(2,0)) \\
&= T(1,1)+T(2,0) && (\text{Since } T(x+y)=T(x)+T(y))
\end{aligned}
$$

$$
\begin{aligned}
&= (2,2)+(0,0) && (\text{By } (1) \text{ and } (2)) \\
&= (2,2)
\end{aligned}
$$

Therefore, $\boxed{T(3,1)=(2,2)}$

Step-4

c) Given vector is $v=(-1,1)$

Now

$$\begin{aligned}T(-1,1)&=T((1,1)-(2,0))\\&=T(1,1)-T(2,0) \qquad (\text{Since } T(x+y)=T(x)+T(y))\\&=(2,2)-(0,0) \qquad (\text{By } (1) \text{ and } (2))\\&=(2,2)\end{aligned}$$

Therefore, $\boxed{T(-1,1)=(2,2)}$

Step-5

d) Given vector is $v=(a,b)$.

We have to find $T(v)$.

Step-6

Let $(a,b)=x(1,1)+y(2,0)$

$$\begin{aligned}&\Rightarrow (a,b)=(x+2y,x)\\&\Rightarrow x=b, x+2y=a\\&\Rightarrow x=b \quad y=\frac{a-b}{2}\end{aligned}$$

Therefore, $(a,b)=b(1,1)+\frac{(a-b)}{2}(2,0)$

Step-7

Now

$$T(a,b) = T\left(b(1,1) + \frac{(a-b)}{2}(2,0)\right)$$

$$= b\ T(1,1) + \frac{(a-b)}{2} T(2,0)$$

$$= b\ (2,2) + \frac{(a-b)}{2}(0,0)$$

$$= (2b, 2b)$$

Hence $\boxed{T(a,b) = (2b, 2b)}$