

Step-1

If A is a square matrix with integer entries and determinant A is not zero, then $A^{-1} = \frac{C^T}{\det A}$

If $\det A$ is ± 1 or 1 , then $A^{-1} = \frac{C^T}{(-1)}$ or $A^{-1} = \frac{C^T}{1}$

In either case, we have $A^{-1} = C^T$ or $A^{-1} = -C^T$

We know that the cofactor of each entry in A with integer entries is also an integer and so, $A^{-1} = C^T$ or $A^{-1} = -C^T$ are matrices with integer entries.

Step-2

$$A = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix}$$

$$\det A = 6 - 5$$

$$= 1$$

the cofactors of A are $C_{11} = 2, C_{12} = -1, C_{21} = -5, C_{22} = 3$ which are all integers

$$A^{-1} = \frac{1}{\det A} \cdot C^T \\ = C^T$$

$$= \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$