

Step-1

Consider the 5 by 5 Jordan block defined as below:

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, Eigen values are zero, $\lambda = 0$. Find value of J^2 and its Eigen vectors. Also find its Jordan form in two blocks.

Step-2

Jordan block powers can be given as follows:

$$\begin{aligned} (J_i)^k &= \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}^k \\ &= \begin{bmatrix} \lambda^k & k\lambda^{k-1} & \frac{1}{2}k(k-1)\lambda^{k-2} \\ 0 & \lambda^k & k\lambda^{k-1} \\ 0 & 0 & \lambda^k \end{bmatrix} \end{aligned}$$

Step-3

To find the value of J^2 do the following calculations:

$$\begin{aligned}
J^2 &= \begin{bmatrix} \mathbf{0} & 1 & 0 & 0 & 0 \\ 0 & \mathbf{0} & 1 & 0 & 0 \\ 0 & 0 & \mathbf{0} & 1 & 0 \\ 0 & 0 & 0 & \mathbf{0} & 1 \\ 0 & 0 & 0 & 0 & \mathbf{0} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{0} & 2 \cdot \mathbf{0}^{2-1} & \frac{1}{2} 2(2-1) \mathbf{0}^{2-2} & 0 & 0 \\ 0 & \mathbf{0} & 2 \cdot \mathbf{0}^{2-1} & \frac{1}{2} 2(2-1) \mathbf{0}^{2-2} & 0 \\ 0 & 0 & \mathbf{0} & 2 \cdot \mathbf{0}^{2-1} & \frac{1}{2} 2(2-1) \mathbf{0}^{2-2} \\ 0 & 0 & 0 & \mathbf{0} & 2 \cdot \mathbf{0}^{2-1} \\ 0 & 0 & 0 & 0 & \mathbf{0} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{0} & 1 & 0 \\ 0 & 0 & \mathbf{0} & \mathbf{0} & 1 \\ 0 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 & 0 & \mathbf{0} \end{bmatrix}
\end{aligned}$$

Step-4

Therefore,

$$J^2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & 1 & 0 & 0 \\ 0 & \mathbf{0} & \mathbf{0} & 1 & 0 \\ 0 & 0 & \mathbf{0} & \mathbf{0} & 1 \\ 0 & 0 & 0 & \mathbf{0} & \mathbf{0} \\ 0 & 0 & 0 & 0 & \mathbf{0} \end{bmatrix}$$

Step-5

For every missing Eigen vector Jordan form will have 1 in the matrix. Here in J^2 matrix three 1 can be seen. This implies that three Eigen vectors are missing. So, from 5 Eigen vectors corresponding to 5 repeated Eigen values 3 are missing.

Therefore, J^2 have 2 independent Eigen vectors corresponding to Eigen value $\lambda = 0$.

Step-6

Jordan form of J^2 can be written as follows in two blocks:

$$\begin{bmatrix} J_3 & \\ & J_2 \end{bmatrix}$$

Here, both blocks are defined as:

$$J_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$