

Step-1

Let $\hat{\lambda}_1$ be the smallest eigenvalue of the matrix A and the corresponding eigenvector be x_1 . Let $\hat{\lambda}_2$ be the second smallest eigenvalue of the matrix A and let x_2 be the corresponding eigenvector.

Consider the Subspace S_2 , which is spanned by the eigenvectors x_1 and x_2 .

Consider $\max_{x \in S_2} R(x)$.

Step-2

Then the minimum value of $R(x)$ is $\hat{\lambda}_1$ and its maximum value will be $\hat{\lambda}_2$.

Thus, $\max_{x \in S_2} R(x) = \lambda_2$

This gives the following

$$\begin{aligned} \min_{S_j} \left[\max_{x \in S_j} R(x) \right] &= \min_{S_2} \left[\max_{x \in S_2} R(x) \right] \\ &= \min_{S_2} [\lambda_2] \\ &= \lambda_2 \end{aligned}$$

Step-3

Therefore, when the subspace S_2 is spanned by the eigenvectors x_1 and x_2 , we get $\min_{S_j} \left[\max_{x \in S_j} R(x) \right] = \lambda_2$.