

## Step-1

Suppose  $\mathbf{P}$  is a plane through  $(0,0,0)$  and  $\mathbf{L}$  is a line through  $(0,0,0)$ .

The smallest vector space containing both  $\mathbf{P}$  and  $\mathbf{L}$  is either  $\mathbf{P}$  or  $\mathbf{R}^3$ .

Because,

1. If the line  $\mathbf{L}$  is in the plane  $\mathbf{P}$ , then  $\mathbf{P}$  is itself a smallest vector space containing both  $\mathbf{L}$  and  $\mathbf{P}$
2. If the line  $\mathbf{L}$  is not in the plane  $\mathbf{P}$  then dimension of the vector space is greater than the maximum of dimension of  $\mathbf{L}$  dimension of  $\mathbf{P}$ .

Therefore it must have dimension 3

(Since  $\dim \mathbf{L} = 1, \dim \mathbf{P} = 2$ )

Therefore, it must be  $\mathbf{R}^3$ .