Step-1

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We have the system $Ax = b$ is $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 11 \end{pmatrix}$

Applying the forward elimination method, we get $Ux = c$ is

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$$

To find the matrix L, we apply the reverse process to identity matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Adding row 1 to row 2 and adding row 1 to row 3

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Adding 2 times row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

Step-2

Now $Lc = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 21 \end{pmatrix}$

Since by matrix multiplication

Lc = b solves for (5, 7, 21)

Step-3

Further, $Ux = c = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$ can be written as the system of equations from below as

$$\begin{aligned} z &= 2 \\ y + 2z &= 2 \\ x + y + z &= 5 \end{aligned}$$

Consequently, $z = 2$, $y = -2$ and $x = 5$

So, $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 2 \end{pmatrix}$ satisfies the equation $Ux = c$.