Step-1

Singular Value Decomposition (SVD) for any m by n matrix A is as follows

$$\begin{aligned} A &= U\Sigma V^T \\ &= \begin{pmatrix} \text{orthogonal} \\ U \text{ is } m\times m \end{pmatrix} \begin{pmatrix} m\times n \text{ matrix } \Sigma \\ \sigma_1 \cdots \sigma_r \text{ on diagonal} \end{pmatrix} \begin{pmatrix} \text{orthogonal} \\ V \text{ is } n\times n \end{pmatrix} \end{aligned}$$

Here eigenvectors of AA^T are in U, eigenvectors of $A^T A$ are in V, and the $\sigma_i = \sqrt{\lambda_i}$.

Step-2

Let A be an m by n zero matrix.

So, AA^T be m by m zero matrix, whose eigenvalues are $\lambda = 0$.

We know that eigenvectors of AA^T are in U.

Since eigenvalues are zero, U will be m by m identity matrix.

$$\begin{aligned} U &= \left[u_1 \cdots u_m\right] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\ &= I_{m\times m} \end{aligned}$$

Step-3

Now $A^T A$ be n by n zero matrix, whose eigenvalues are $\lambda = 0$.

We know that eigenvectors of $A^T A$ are in V.

Since eigenvalues are zero, V will be n by n identity matrix.

$$\begin{aligned} V &= \left[v_1 \cdots v_n\right] \\ &= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \\ &= I_{n\times n} \end{aligned}$$

Step-4

The diagonal of Σ are the square roots of the nonzero eigenvalues of both AA^T.

So, Σ is *m* by *n* zero matrix.

$$\Sigma = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

The SVD for *m* by *n* zero matrix is as follows:

$$\begin{aligned} A &= U \Sigma V^T \\ &= I_{m\times m} \Sigma_{m\times n} I_{n\times n} \\ &= \Sigma_{m\times n} I_{n\times n} \\ &= \Sigma_{m\times n} \end{aligned}$$

Therefore, SVD for *m* by *n* zero matrix is $\boxed{m \text{ by } n \text{ zero matrix}}$.

Step-5

If SVD of *m* by *n* matrix *A* is $A = U \Sigma V^T$, then the pseudoinverse of *A* is

$$A^+ = V \Sigma^+ U^T$$

For *m* by *n* zero matrix, we have

$$\begin{aligned} U &= I_{m\times m} \\ V &= I_{n\times n} \end{aligned}$$

Since Σ is *m* by *n* zero matrix, so Σ^+ is *n* by *m* zero matrix.

Hence, pseudoinverse of *m* by *n* zero matrix *A* is

$$\begin{aligned} A^+ &= V \Sigma^+ U^T \\ &= I_{n\times n} \Sigma^+{}_{n\times m} I_{m\times m} \\ &= \Sigma^+{}_{n\times m} \\ &= A^T \end{aligned}$$

Therefore, the pseudoinverse of an zero *n* by *m* matrix is $\boxed{\text{equal to its transpose}}$.