Step-1

Given equation is $x_1 + x_2 + x_3 + x_4 = 0$

Suppose *S* is the subspace of $\mathbf{R}^4$ spanned by this equation.

This can be easily followed that $\mathbf{P} = \{u = (1,1,1,1) : 1 \in \mathbf{R}\}$

By the definition of the orthogonal complement, we have $x \cdot 1 + y \cdot 1 + z \cdot 1 + w \cdot 1 = 0$

$\Rightarrow w = -x - y - z$

$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} + p \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} + q \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ where *x* = *t, y* = *p, z* = *q* are the parameters.

Therefore, $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ span **P** and $\mathbf{P}^{\perp}$ is spanned by {(1, 1, 1, 1)}.

Observe that dim *S* = 1 and dim $S^{\perp}$ = 3

Dim *S* + dim $S^{\perp}$ = 4 = dim $\mathbf{R}^4$

Step-2

The matrix in which $\mathbf{P}$ is the null space is $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$