Step-1

Given that $A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$

$$A - 3I = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

The determinant of this 2×2 matrix is zero.

So, rank $A - 3I < 2$

But we know that rank of the matrix with at least one non zero entry is greater than or equal to 1.

So, putting the above statements together, we get rank $A - 3I$ = 1.

Step-2

Changing any entry 0 in the given matrix, it can be diagonalizable.

For instance, $A = \begin{bmatrix} 3 & 1 \\ 16 & 3 \end{bmatrix}$ has the eigen values â€“ 1 and 7 which are distinct and so, the corresponding eigen vectors will be linearly independent and so, it is diagonalizable.