

Step-1

Consider the following system

$$2x + by = 16$$

$$4x + 8y = g$$

The objective is to determine the coefficient b that makes the above system singular.

Also, calculate the variable in right hand side g for which the system is solvable and provide two solutions when the system is singular.

Step-2

Assume the matrix corresponding to the given system is,

$$A = \begin{bmatrix} 2 & b \\ 4 & 8 \end{bmatrix}$$

Recall that a matrix is singular, if $\det(A) = 0$.

That is,

$$\det(A) = 2 \times 8 - 4 \times b$$

$$\det(A) = 16 - 4b$$

This implies, $\det(A) = 0$ if,

$$16 - 4b = 0$$

$$b = 4$$

Thus, the value of b that makes the above system singular is $\boxed{b = 4}$.

Step-3

The system of equation can be rewritten as,

$$2x + 4y = 16$$

$$4x + 8y = g$$

Apply the Gaussian elimination method in the below matrix A' corresponding to the above simplified system of equation.

$$A' = \begin{bmatrix} 2 & 4 & 16 \\ 4 & 8 & g \end{bmatrix}$$

Apply row operation $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 2 & 4 & 16 \\ 4 & 8 & g \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 16 \\ 0 & 0 & g-32 \end{bmatrix}$$

To find the solution for which the above system is solvable (otherwise not solvable as no solution will exist), it is observed that the second row should be 0.

It means that $g - 32 = 0$ or $g = 32$

Thus, for $\boxed{g = 32}$ the system is solvable and has singular solutions.

Step-4

Determine the solutions of the given singular system of equations.

Substitute the value of $b = 4$ and $g = 32$ in the system of equation as,

$$2x + 4y = 16$$

$$4x + 8y = 32 \quad \text{--- (1)}$$

By inspection calculate two solutions of the singular system.

Assume $x = 0$ in equation (1) then the value of y is,

$$4y = 16$$

$$y = \frac{16}{4}$$

$$y = 4$$

Assume $y = 0$ in equation (1) then the value of x is,

$$4x = 32$$

$$x = \frac{32}{4}$$

$$x = 8$$

Hence, the two possible solutions are $\boxed{(0, 4), (8, 0)}$

So, b as $\boxed{4}$ and g as $\boxed{32}$, then the system is solvable, and in that case it is singular.

$$2x + 4y = 16$$

$$4x + 8y = 16$$

The column picture for this system as follows:

The row form of the system as follows:

$$\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 16 \end{bmatrix}$$

Perform the row operation $R_2 \rightarrow R_2 - 2R_1$, and get;

$$\begin{bmatrix} 2 & 4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \end{bmatrix}$$
$$2x + 4y = 16$$

Step-5

By inspection, obtain two solutions as $\boxed{(4, 2) \text{ and } (0, 4)}$