Step-1

Given that $A=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$ yields shearing transformation which leaves the y-axis unchanged.

We have to sketch the effect of A on x-axis by indicating what happens to $(1,0)$, $(2,0)$ and $(-1,0)$ and verify how the whole axis is transformed.

Step-2

Let $x_1=(1,0),\ x_2=(2,0),\ x_3=(-1,0)$

Then

$$\begin{aligned}Ax_1&=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}\\ &=\begin{bmatrix}1(1)+0(0)\\ 3(1)+1(0)\end{bmatrix}\\ &=\begin{bmatrix}1\\ 3\end{bmatrix}\end{aligned}$$

Step-3

And

$$\begin{aligned}Ax_2&=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}\begin{bmatrix}2\\ 0\end{bmatrix}\\ &=\begin{bmatrix}1(2)+0(0)\\ 3(2)+1(0)\end{bmatrix}\\ &=\begin{bmatrix}2\\ 6\end{bmatrix}\end{aligned}$$

Step-4

And

$$Ax_3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1(-1)+0(0) \\ 3(-1)+1(0) \end{bmatrix}$$
$$= \begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

Therefore, the vectors $(1,0),(2,0),(-1,0)$ transformed to $(1,3),(2,6),(-1,-3)$.

The *x*-axis turns vertical lines shift up/down but stay vertical.

The sketch is as shown below.

Step-5