

Step-1

a) We have to find A so that the only solution to $Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Let $x = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\Rightarrow x_1 = 0, x_2 = 1$$

Step-2

For these values, we can take the system as

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\frac{1}{2}R_2, \frac{1}{3}R_3 \rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Step-3

$$\frac{R_3 - R_2}{\rightarrow} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 + x_2 = 1$$

$$x_2 = 1$$

$$\Rightarrow x_1 = 0$$

Step-4

Therefore the matrix A satisfying the given condition is $\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 3 \end{bmatrix}$

Step-5

b) We have to find B so that the only solution to $Bx = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Given system is $Bx = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

From this we can observe that there is a system with two equations and three unknowns, so it has infinite number of solutions, but given that system has only one solution.

Therefore the matrix B does not exist for the given system.