

## Step-1

**Solvable equation;** an equation is said to be solvable if value of constant of coefficient can be easily computed and then solving the equation by manipulation.

Let  $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$

$$c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} + c_4 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

That is;

$$c_1 + c_2 = 0 \quad \text{--- (1)}$$

$$c_1 + c_4 = 0 \quad \text{--- (2)} \quad c_2 + c_3 = 0 \quad \text{--- (3)} \quad c_3 + c_4 = 0 \quad \text{--- (4)}$$

## Step-2

Now solve, by subtract (1) - (2)

$$\begin{aligned} (c_1 + c_2) - (c_1 + c_4) &= 0 \\ c_2 - c_4 &= 0 \\ c_2 &= c_4 \end{aligned}$$

From (4),

$$\begin{aligned} -c_4 - c_3 &= 0 \\ c_3 &= -c_4 \end{aligned}$$

So,

$$\begin{aligned} c_1 &= -c_4 \\ c_2 &= -c_1 \\ &= c_4 \end{aligned}$$

$$\boxed{\begin{matrix} c_1 = -c_4 \\ c_2 = c_4 \\ c_3 = -c_4 \end{matrix}}.$$

Therefore,

## Step-3

Therefore  $-c_4v_1 + c_4v_2 - c_4v_3 + c_4v_4 = 0$

If  $c_4 = 1$  then  $-v_1 + v_2 - v_3 + v_4 = 0$

Therefore  $v_1, v_2, v_3, v_4$  are dependent.

They do not span  $R^4$  because dimension of  $R^4$  is 4.

## Step-4

Now,  $(0, 0, 0, 1)$

$$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = (0, 0, 0, 1)$$

Now,  $(0, 0, 0, 1)$

$$c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = (0, 0, 0, 1)$$

That is;

$$c_1 + c_2 = 0 \quad \text{--- (5)}$$

$$c_1 + c_4 = 0 \quad \text{--- (6)}$$

$$c_2 + c_3 = 0 \quad \text{--- (7)}$$

$$c_3 + c_4 = 1 \quad \text{--- (8)}$$

Equation (5) --- Equation (6)

$$c_2 - c_4 = 0$$

Equation (3) --- Equation (4)

$$c_2 - c_4 = -1$$

Therefore this is not possible to find  $c_2, c_4$ .

Hence the equation is not solved.