## Step-1

Suppose $A$ is a linear transformation from the $x$-$y$ plane to itself.

We have to verify why $A^{-1}(x+y)=A^{-1}x+A^{-1}y$.

## Step-2

Let $u,v\in \mathrm{R}^2$ and $Au=x,\ Av=y$

$\Rightarrow\ u=A^{-1}x,\ v=A^{-1}y$

And $A^{-1}x,\ A^{-1}y$ are also in $x$-$y$ plane.

Since $A$ is linear transformation.

So

$A\left(A^{-1}x+A^{-1}y\right)=A\left(A^{-1}x\right)+A\left(A^{-1}\right)y$

$=\left(AA^{-1}\right)x+\left(AA^{-1}\right)y$

$=x+y\ \left(\text{Since } AA^{-1}=A^{-1}A=I\right)$

Thus $\boxed{A^{-1}(x+y)=A^{-1}x+A^{-1}y}$

Suppose $A$ is represented by matrix $M$.

If $A^{-1}$ exists, then $A^{-1}$ is represented by $M^{-1}$

Hence $A^{-1}$ is represented by $M^{-1}$.