Step-1

Given vectors $(1,3,1),(2,7,2)$.We have to find all the vectors that are perpendicular to these vectors by making those the rows of A and solving $Ax=0$.

Step-2

Let $a=(1,3,1),b=(2,7,2)$

Let $\alpha=(x,y,z)$ is perpendicular to a and b.

Therefore $a^T\alpha=0$ and $b^T\alpha=0$

$$\Rightarrow (1,3,1)\begin{pmatrix}x\\y\\z\end{pmatrix}=0 \text{ and } (2,7,2)\begin{pmatrix}x\\y\\z\end{pmatrix}=0$$
$$\Rightarrow x+3y+z=0 \text{ and } 2x+7y+2z=0$$

Step-3

Matrix form of above equations is

$$\begin{bmatrix}1&3&1\\2&7&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$
apply $R_2\to R_2-2R_1$
$$\begin{bmatrix}1&3&1\\0&1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{aligned}\Rightarrow x+3y+z&=0\\ y&=0\end{aligned}$$

$x+z=0$

Put $z=k$
$\Rightarrow x=-k$

Then $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-k\\0\\k\end{pmatrix}$

where k is a real number.

Hence $\alpha = (-k, 0, k)$ for all k, is a vector perpendicular to both a and b.

Step-4

Verification:

If $k = 1, \alpha = (-1, 0, 1)$ is perpendicular to both a and b.

Since $a^T \alpha = -1 + 3(0) + 1$

$= 0$ and

$$\begin{aligned} b^T \alpha &= 2(-1) + 7(0) + 2(1) \\ &= 0 \end{aligned}$$