

Step-1

Consider the system,

$$2x + 5y + z = 0$$

$$4x + dy + z = 2$$

$$y - z = 3$$

The objective is to find the number d forces a row exchange, and what is the triangular system (not singular) for that d . Which d makes this system singular (no third pivot).

Step-2

Write the system in matrix notation as shown:

$$\underbrace{\begin{bmatrix} 2 & 5 & 1 \\ 4 & d & 1 \\ 0 & 1 & -1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}}_{\mathbf{b}}$$

Augmented matrix associated with the above system is,

$$[\mathbf{A} \mid \mathbf{b}] = \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 4 & d & 1 & 2 \\ 0 & 1 & -1 & 3 \end{array} \right]$$

Step-3

To eliminate the element in the position $(2,1)$, multiply the first row with 2 and subtract it from the second row. Then the obtained matrix is,

$$\approx \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & d-10 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array} \right] \quad \text{â€œâ€œ (i)}$$

For the case $d-10=0$ gives $d=10$. The above matrix becomes,

$$\approx \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array} \right] \quad [\text{For the case, } d=10]$$

Interchange the rows 2 and 3 to obtain the triangular form as shown:

$$\approx \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & -1 & 2 \end{array} \right]$$

System associated with the above notation is,

$$2x + 5y + z = 0$$

$$y - z = 3$$

$$-z = 2$$

This is in triangular (upper triangular) form.

Hence, rows 2 and 3 are interchanged to obtained **triangular** system if $\boxed{d=10}$

Step-4

For the case $d-10=1$ gives $d=11$.

Augmented matrix in (i) becomes,

$$\approx \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 11-10 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array} \right]$$

$$\approx \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array} \right]$$

Observe that, rows 2 and 3 are identical (for the matrix **A**).

Hence determinant of matrix **A** is zero for $d=11$

Therefore, the system is **singular** for $\boxed{d=11}$.