Step-1

The second order differential equation $y'' = 5y' + 4y$ can be written into a vector equation for $u(t) = (y(t), y'(t))$ and as a first order system by introducing the velocity y'.

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} y' \\ y'' \end{bmatrix}$$

Step-2

Above differential equation can also be written as follows:

$$\frac{d}{dt}\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} y \\ y' \end{bmatrix}$$

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$\boxed{A = \begin{bmatrix} 0 & 1 \\ 4 & 5 \end{bmatrix}}$$

Step-3

Find the Eigen values of matrix A. Substitute $y = e^{\lambda t}$ into the scalar equation $y'' = 5y' + 4y$ to get the Eigen values again.

Step-4

First step is to find the Eigen values of matrix A. Do the following calculations:

$$A - \lambda I = \begin{bmatrix} -\lambda & 1 \\ 4 & 5-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(-\lambda)(5-\lambda) - 4 = 0$$

$$\lambda^2 - 5\lambda - 4 = 0$$

After solving following values are obtained:

$$\lambda_1 = \frac{5+\sqrt{41}}{2}$$
$$\lambda_2 = \frac{5-\sqrt{41}}{2}$$

Step-5

Therefore, Eigen values are $\boxed{\frac{5\pm\sqrt{41}}{2}}$

Step-6

Eigen values can also be obtained by substituting $y = e^{\lambda t}$ into the scalar equation $y'' = 5y' + 4y$. For the same do the following calculations:

$$y = e^{\lambda t}$$
$$y' = \lambda e^{\lambda t}$$
$$y'' = \lambda^2 e^{\lambda t}$$

Step-7

Substitute the values in the following equation:

$$y'' = 5y' + 4y$$
$$\lambda^2 e^{\lambda t} = 5\lambda e^{\lambda t} + 4e^{\lambda t}$$
$$\lambda^2 - 5\lambda - 4 = 0$$

After solving value of λ is:

$$\lambda = \frac{5\pm\sqrt{41}}{2}$$

Step-8

Therefore, Eigen values are as follows:

$$\boxed{\lambda = \frac{5\pm\sqrt{41}}{2}}$$