Step-1

Given $a=\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$, $b=\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}$, $c=\begin{bmatrix}0\\0\\1\\-1\end{bmatrix}$

We have to find the orthogonal vectors A,B,C by Gram-Schmidt from a,b,c

Step-2

We know that

$$A=a=\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$$

Step-3

And $B=b-P_A$, where

$$P_A=A\frac{A^Tb}{A^TA}$$

Now

$$A^Tb=\begin{bmatrix}1 & -1 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}$$

$=0-1+0+0$

$=-1$

Step-4

$$A^T A = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= 1+1+0+0$$

$$= 2$$

Step-5

$$P_A = \frac{-1}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \\ 0 \end{bmatrix}$$

Step-6

Therefore

$$B = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

Step-7

$C = c - P_A - P_B$, where

$P_A = A\frac{A^T c}{A^T A}, P_B = B\frac{B^T c}{B^T B}$

Step-8

Now

$$A^T c = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

$$= 0 - 0 + 0 - 0$$

$$= 0$$

Step-9

$$A^T A = \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= 1 + 1 + 0 + 0$$

$$= 2$$

$$B^T c = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

$$= 0 + 0 - 1 - 0$$

$$= -1$$

Step-10

$$B^T B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -1 & 0 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4} + \frac{1}{4} + 1 + 0$$

$$= \frac{3}{2}$$

Step-11

$$P_A = \frac{0}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Step-12

$$P_B = \frac{-1}{3/2}\begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

$$= -\frac{2}{3}\begin{bmatrix} 1/2 \\ 1/2 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/3 \\ -1/3 \\ 2/3 \\ 0 \end{bmatrix}$$

Step-13

Therefore

$$C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -1/3 \\ -1/3 \\ 2/3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \\ -1 \end{bmatrix}$$

Hence $A = (1,-1,0,0),\ B = \left(\frac{1}{2},\frac{1}{2},-1,0\right),\ C = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3},-1\right)$ Â