Step-1

$$\begin{bmatrix} a_1^T a_1 & -a_1^T a_2 \\ -a_1^T a_2 & a_2^T a_2 \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} a_1^T b \\ -a_2^T b \end{bmatrix}$$ â€¦â€¦ (1)

$$a_1^T a_1 = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1+1+0 = 2$$

$$a_1^T a_2 = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0+1+0 = 1$$

$$a_2^T a_2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0+1+0 = 1$$

$$a_1^T b = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = 2+1+0 = 3$$

$$a_2^T b = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = 0+1+0 = 1$$

Step-2

By (1), we get $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Applying $R_2 \to 2R_2 + R_1$

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\Rightarrow \hat{x}_2 = 1$ and $2\hat{x}_1 - \hat{x}_2 = 3$

$$\Rightarrow \hat{x}_1 = \frac{3+1}{2}$$

$$= 2$$

Therefore, $\hat{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$