Step-1

Let A and B be two matrices defined as follows:

$$A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$
$$B = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

Step-2

Consider the following Eigen values of matrix A and B given as follows:

For matrix A:

$$\lambda_1 = 1$$
$$\lambda_2 = 9$$

For matrix B:

$$\lambda_1 = -1$$
$$\lambda_2 = 9$$

Step-3

Let $R = S\Lambda^{1/2}S^{-1}$, then to find the matrix square root of A from R. Also explain that why there is no real matrix square root of B.

Step-4

Let A be the following matrix:

$$A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Compute $S\Lambda^{1/2}S^{-1}$ to get the desired result.

Step-5

Eigen vectors corresponding to the Eigen values are calculated as follows:

For $\lambda_1 = 1$

$$(A-\lambda_1 I)x_1 = 0$$

$$\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Step-6

For $\lambda_2 = 9$

$$(A-\lambda_2 I)x_2 = 0$$

$$\begin{bmatrix} -4 & 4 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Step-7

Thus, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

Step-8

Now, do the following calculations to get *R*:

$$R = S\Lambda^{1/2}S^{-1}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Next, compute the following:

$$R^2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$
$$= A$$

Step-9

Therefore, $\boxed{A = R^2}$. This shows that R is a matrix square root of A.

Step-10

Next, to find matrix square root of B, first calculate the following:

$$\Lambda^{1/2} = \begin{bmatrix} \sqrt{-1} & 0 \\ 0 & 3 \end{bmatrix}$$

This shows that one of the Eigen values of matrix square root of B is imaginary. In other words trace is not real.

Step-11

Therefore, there can not be any real matrix square root of B.