

Step-1

Let y be the vector $(6,3,6)$.

Consider the following:

$$\begin{bmatrix} 1 & 5 \\ 3 & 0 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+5 \\ 3+0 \\ 2+4 \end{bmatrix} \\ = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix} \\ = y$$

Step-2

Similarly, note the following:

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6+0 \\ 0+3 \\ 0+6 \end{bmatrix} \\ = \begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix} \\ = y$$

Step-3

This gives the following:

$$[A \quad B]x = \begin{bmatrix} 1 & 5 & 3 & 0 \\ 3 & 0 & 0 & 1 \\ 2 & 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix} \\ = \begin{bmatrix} 1+5-6 \\ 3-3 \\ 2+4-6 \end{bmatrix} \\ = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step-4

$$A = \begin{bmatrix} 1 & 5 \\ 3 & 0 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}$$

The column space of each $\begin{bmatrix} 1 & 5 \\ 3 & 0 \\ 2 & 4 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 \\ 0 & 1 \\ 0 & 2 \end{bmatrix}$ has dimension equal to 2. Thus, it is clear that the dimension of $C(A) \cap C(B)$ is equal to 1.