

Step-1

Given that V and W are the orthogonal subspaces.

That means if v is in V and w is in W , then $v^T w = 0$

Suppose x is any vector in $V \cap W$

Then we follow that $x^T x = 0$

Conveniently, we can write this as $x_1^2 + x_2^2 + \dots + x_n^2 = 0$ where $x = (x_1, x_2, \dots, x_n)$

We know that sum of the squares is zero if and only if each of the components is zero.

i.e, $x_1 = 0, x_2 = 0, \dots, x_n = 0$

We observed that if x is in $V \cap W$, then $x = (x_1, x_2, \dots, x_n) = (0, 0, \dots, 0)$

Therefore, the zero vector is the only vector in the intersection of orthogonal subspaces.