Step-1

Given data is $b=0$ at $t=0$

$b=1$ at $t=1$

$b=2$ at $t=3$

$b=5$ at $t=4$

We have to best fit this data with a straight line $C+Dt$.

Step-2

Let $C+Dt=b$ be the required straight line that best fits the given data, then

$C+0.D=0$

$C+1.D=1$

$C+3.D=2$

$C+4.D=5$

In matrix form,

$$\begin{pmatrix}1&0\\1&1\\1&3\\1&4\end{pmatrix}\begin{pmatrix}C\\D\end{pmatrix}=\begin{pmatrix}0\\1\\2\\5\end{pmatrix}$$

$\Rightarrow Ax=b$

where $A=\begin{bmatrix}1&0\\1&1\\1&3\\1&4\end{bmatrix}, x=\begin{bmatrix}C\\D\end{bmatrix}$ and $b=\begin{bmatrix}0\\1\\2\\5\end{bmatrix}$

Step-3

Let $\hat{x}=\begin{bmatrix}\hat{C}\\\hat{D}\end{bmatrix}$

For least squares fit, we have

$$A^T A\hat{x} = A^T b$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 8 \\ 8 & 26 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 8 \\ 27 \end{bmatrix}$$

Step-4

apply $R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 4 & 8 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

$\Rightarrow 4\hat{C} + 8\hat{D} = 8$ and $10\hat{D} = 11$

$\Rightarrow \hat{D} = \frac{11}{10}$ and

$$\hat{C} = \frac{8 - 8\left(\frac{11}{10}\right)}{4}$$

$$= \frac{-8}{40}$$

$$= \frac{-1}{5}$$

Therefore $\hat{x} = \begin{bmatrix} \frac{-1}{5} \\ \frac{11}{10} \end{bmatrix}$

Hence required straight line $\boxed{b = -\frac{1}{5} + \left(\frac{11}{10}\right)t}$