MA215 Probability Theory

Assignment 14

1. Suppose that the m.g.f. of X is $M_X(t) = \frac{2}{\sqrt{4-t}}$, $(t < 4)$.

   (i) Find $E(X)$, $E(X^2)$ and $\mathrm{Var}\,(X)$.

   (ii) Suppose that X and Y are independent and both with this m.g.f. (i.e., $M_X(t) = M_Y(t) = \frac{2}{\sqrt{4-t}}$). Find the m.g.f. of of $X+Y$ and identify the distribution of $X+Y$.

2. (a) If the m.g.f. of X is $M_X(t) = \frac{a^2}{a^2-t^2}$, then find the kth moment $E(X^k)$, $k \in \mathbb{N}_+$.

   (b) Suppose the m.g.f. of X can be expressed as a power series
   $$M_X(t) = \sum_{k=0}^{\infty} a_k t^k = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots$$
   and assume further that $a_0 = 1$, $a_1 = 3$ and $a_2 = 7$. Find $E(X)$ and $\mathrm{Var}\,(X)$.

3. Suppose the m.g.f. of X has the Maclaurin series
   $$M_X(t) = 1 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots .$$
   Find the variance and the third central moment $E[(X - E(X))^3]$ of X in terms of a_1, a_2, and a_3.

4. Let $X_1, X_2, \cdots, X_n$ be i.i.d., each having the normal distribution with parameters μ and σ^2.

   (i) Find the m.g.f.s of the sample sum $S_n = \sum_{i=1}^n X_i$ and sample average $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$.

   (ii) What are the distributions of these two random variables?

5. Suppose X is a discrete random variable taking values of non-negative integers (or subset of non-negative integers) with p.m.f $\{p_k; k \geqslant 0\}$. Define the probability generating function (p.g.f.) of X, denoted by $\prod_X(t)$, as
   $$\prod\nolimits_X(t) = E\left(t^X\right).$$

   (i) Write down the form $\prod_X(t)$ in terms of the p.m.f. $\{p_k; k \geqslant 0\}$.

   (ii) Investigate the problem as how to get $E(X)$ and $\mathrm{Var}\,(X)$ by using $\prod_X(t)$.

   (iii) Find the p.g.f. of the Binomial Random Variable X with parameter n and p.

   (iv) Find the p.g.f. of the Poisson Random Variable X with parameter λ.