

## Step-1

Consider the matrix equation,

$$u \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + v \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + w \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 7 \end{pmatrix} \quad \dots\dots (1)$$

The objective is to find the solution  $(u, v, w)$  for the above system other than  $(1, 0, 1)$ .

## Step-2

The matrix equation (1) has a solution, when the linear combination of columns in left hand side of equation (1) satisfies the right hand side part of the equation (1).

The system of equations as,

$$u + v + w = 2,$$

$$2u + 3w = 5,$$

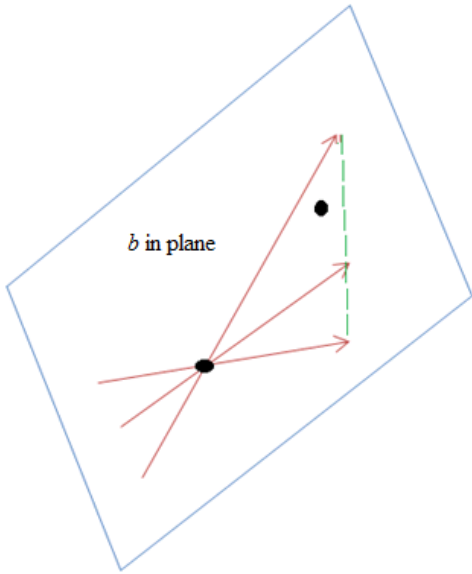
$$3u + v + 4w = 7.$$

Add first two equations and subtract from the third equation, obtained as  $0 = 0$ .

Thus, the equations are consistent.

## Step-3

The system has infinitely many solutions shown in the following diagram



## Step-4

Now find the solution by Gaussian elimination process.

Consider the system,

$$u + v + w = 2,$$

$$2u + 3w = 5,$$

$$3u + v + 4w = 7.$$

Subtract two times the first equation from second, and subtract 3 times the first equation from third, obtained as follows:

$$u + v + w = 2,$$

$$-2v + w = 1,$$

$$-2v + w = 1.$$

Subtract  $-1$  times the second equation from third equation, obtained as,

$$u + v + w = 2,$$

$$-2v + w = 1,$$

$$0 = 0.$$

Put,

$$w = k.$$

$$v = \frac{k-1}{2}$$

$$u = 2 - \frac{k-1}{2} - k$$

$$= \frac{5-3k}{2}$$

So, the solution is,

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{5-3k}{2} \\ \frac{k-1}{2} \\ k \end{bmatrix}$$

These are infinitely many solutions.

Take  $k = -1$ , obtained the solution as follows:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{5-3(-1)}{2} \\ \frac{-1-1}{2} \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}$$

## Step-5

That is,  $4(\text{column1}) + (-1)(\text{column2}) + (-1)(\text{column3}) = b$

Hence, the solution is,  $\boxed{(u, v, w) = (4, -1, -1)}$ .