

Step-1

Given $A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

Now without multiplying we have to determine the information of A .

(a).

The determinant of the matrix is,

$$\begin{aligned} \det A &= (2)(5) \\ &= 10. \end{aligned}$$

Therefore, the determinant of the matrix is 10.

Step-2

(b).

Clearly, the Eigen values of A are the diagonal elements

i.e. 2 and 5.

Therefore, the Eigen values of A are 2 and 5.

Step-3

(c).

The Eigen vectors of A are $\begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}^T$ and $\begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}^T$.

Step-4

(d).

As the Eigen values of A are both positive, A is positive definite.

Therefore, A is positive definite.