

Step-1

We have to explain that why the following statements are false.

(a) $(1,1,1)$ is perpendicular to $(1,1,-2)$, so the planes $x+y+z=0$ and $x+y-2z=0$ are orthogonal subspaces.

Step-2

Given planes are $x+y+z=0$ and $x+y-2z=0$

If $z=0$, $x+y=0$

Put $x=k$

$\Rightarrow y=-k$

If $k=1$ then $x=1, y=-1$

Therefore intersecting point of the above planes is $P=(1,-1,0)$.

Step-3

Let $a=(1,1,1)$, $b=(1,1,-2)$

Direction ratios of normal of plane $x+y+z=0$ are 1, 1, 1 and

Direction ratios of normal of plane $x+y-2z=0$ are 1, 1, -2

Here $1(1)+1(1)+1(-2)=0$

Therefore normal vectors are perpendicular and given planes intersect at $P=(1,-1,0)$

That is, planes still intersect.

So the given statement is false.

Step-4

(b) The subspace spanned by $(1,1,0,0,0)$ and $(0,0,0,1,1)$ is the orthogonal complement of the subspace spanned by $(1,-1,0,0,0)$ and $(2,-2,3,4,-4)$

Step-5

Need three orthogonal vectors to span the whole orthogonal complement in \mathbf{R}^5 .

So the given statement is false.

Step-6

(c) Two subspaces that meet only in the zero vector are orthogonal.

We know that two subspaces (Lines) can meet without being orthogonal.

So the given statement is false.