MA215 Probability Theory

Assignment 06

1. Suppose a probability density function $f(x)$ takes the form of

$$f(x) = cx^3, \quad (0 < x < 1).$$

   (a) Find the value of the constant c.

   (b) Sketch $f(x)$.

   (c) Obtain the cumulative distribution function $F(x)$.

   (d) Find $P(0.25 < X < 0.75)$.

2. A continuous random variable X is said to have a *memoryless* property if

$$P(X > s + t \mid X > t) = P(X > s)$$

   is true for all $s > 0$ and $t > 0$. Show that any exponential random variable has the memoryless property.

3. For a certain type of electrical component, the lifetime X (in thousands of hours) has an exponential distribution with rate parameter $\lambda = 0.5$. What is the probability that a new component will last longer than 1000 hours? If a component has already lasted 1000 hours, what is the probability that it will last at least 1000 hours more?

4. The number of phone calls received at a certain residence in any period of t hours is a Poisson random variable with parameter $\lambda = \mu t$ for some $\mu > 0$. What is the probability that no calls are received during a period of t hours? Denoting by T the time (in hours) at which the first call after time zero is received, write down an expression for $P(T \leqslant t)$. What is the name of the distribution of the random variable T?

5. The Weibull distribution with parameters $\alpha > 0$ and $\beta > 0$ has cumulative distribution function

$$F(x) = 1 - \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, \quad x > 0.$$

   (a) Find the median of the distribution in terms of the parameters α and β (The median of a random variable X is the value m such that $P(X \leqslant m) = 0.5$).

   (b) From the Weibull distribution function given above, derive an expression for the corresponding probability density function.