Step-1

Thus, we get $2\lambda x^{H}Ax = -x^{H}x$.

Note that $x^{H}x$ is a positive real number, since *x* is a non zero vector. Thus, in the above equation, the right hand side is negative. If *x* is a real vector, then $x^{H}Ax$ too will be a positive real number. Therefore, the real part of the eigenvalue *?* will be negative.

Step-2

Let us show that the real part of the eigenvalue *?* is less than zero. Consider the following:

$$
\begin{aligned}
AM + M^{H}A &= -I \\
x^{H}\left(AM + M^{H}A\right) &= -x^{H}I \\
x^{H}\left(AM + M^{H}A\right)x &= -x^{H}Ix \\
x^{H}AMx + x^{H}M^{H}Ax &= -x^{H}x
\end{aligned}
$$

Step-3

We have $Mx = \lambda x$. Also, we can write $x^{H}M^{H} = (Mx)^{H}$. Therefore, we get

$$
\begin{aligned}
x^{H}AMx + x^{H}M^{H}Ax &= -x^{H}x \\
x^{H}A(\lambda x) + (Mx)^{H}Ax &= -x^{H}x \\
\lambda x^{H}Ax + (\lambda x)^{H}Ax &= -x^{H}x \\
\lambda x^{H}Ax + \lambda x^{H}Ax &= -x^{H}x
\end{aligned}
$$