Step-1

Consider the given matrix $R = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$.

The objective is to write R^2 and check that it is positive definite unless R is singular.

Step-2

Here,

$$R = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$$
$$R^2 = \begin{bmatrix} p & q \\ q & r \end{bmatrix}\begin{bmatrix} p & q \\ q & r \end{bmatrix}$$
$$R^2 = \begin{bmatrix} p^2+q^2 & pq+qr \\ pq+qr & q^2+r^2 \end{bmatrix}$$

Therefore, $\boxed{R^2 = \begin{bmatrix} p^2+q^2 & pq+qr \\ pq+qr & q^2+r^2 \end{bmatrix}}$.

Step-3

Note that;

If R is singular, then $\det R = 0$, that is

$$R = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$$
$$\det(R) = pr - q^2$$

That is, $\det(R) = pr - q^2 = 0.$

Step-4

Observe the following;

$p^2 + q^2 > 0$,

Consider $(p^2+q^2)(q^2+r^2) - (pq+qr)^2$

Simply the above expression and obtain the following;

$$\Rightarrow p^2q^2 + p^2r^2 + q^4 + q^2r^2 - q^2(p+r)^2$$

$$\Rightarrow p^2q^2 + p^2r^2 + q^4 + q^2r^2 - q^2p^2 - q^2r^2 - 2prq^2$$

$$\Rightarrow p^2r^2 - 2prq^2 + q^4$$
$$\Rightarrow p^2r^2 - prq^2 - prq^2 + q^4$$

$$\Rightarrow pr(pr - q^2) - q^2(pr - q^2)$$

$$\Rightarrow (pr - q^2)(pr - q^2)$$

$$\Rightarrow (pr - q^2)^2$$

If R is singular, then $pr - q^2 = 0$ which implies $(pr - q^2)^2 = 0$.

This implies R^2 is singular whenever R is.

Therefore, R^2 is positive **definite** unless R is singular.