Step-1

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Suppose the boundary conditions are changed to in the same system, then we are instructed that when the boundary conditions are not zero, we have to shift them to the right hand side of the above matrix equation (1).

Suppose the given differential equation is $-\frac{d^2u}{dx^2} = f(x), 0 \le x \le 1$

We have $\frac{d^2u}{dx^2} \approx \frac{\Delta^2 u}{\Delta \mathbf{x}^2} = \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}$

So, considering the mesh points of a matrix as the equi spaced values of $x = jh$, we can write this equation as $-h^2 f(jh) = -u_{j+1} + 2u_j - u_{j-1}$ where j = 1, 2, â€¦, n

Observe that this formula includes u_0, u_{n+1} which are the boundary conditions.

Step-2

By using $u(0) = 1, u(1) = 0$ in this, we get the equations $-h^2 f(h) = -u_{1+1} + 2u_1 - u_{1-1} = -u_2 + 2u_1 - 1$

$-h^2 f(2h) = -u_{2+1} + 2u_2 - u_{2-1} = -u_3 + 2u_2 - u_1$

$-h^2 f(3h) = -u_{3+1} + 2u_3 - u_{3-1} = -u_4 + 2u_3 - u_2$

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Writing the coefficients in the 5 by 5 matrix, the constant in the 1st equation in a separate matrix, the system becomes

$$\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = h^2 \begin{bmatrix} f(h) - \boxed{1} \\ f(2h) \\ f(3h) \\ f(4h) \\ f(5h) \end{bmatrix}$$