

Step-1

We need to prove that A is not positive definite.

$$A = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 5 \end{pmatrix}.$$

Given matrix is

Now,

$$\begin{aligned} x^T Ax &= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 4 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= 4x_1^2 + 5x_3^2 + 2x_1x_2 + 2x_1x_3 + 4x_2x_3 \end{aligned}$$

Now for $(x_1, x_2, x_3) = (0, 1, 0)$

$$x^T Ax = 0 + 0 + 0 + 0 + 0 = 0$$

Therefore, A is not positive definite when $(x_1, x_2, x_3) = (0, 1, 0)$, because positive definite matrix cannot have a zero on its diagonal.