Step-1

(a)

Let λ be an eigenvalue of *A* and the respective eigenvector is *x*.

Then, $Ax = \lambda x$

Multiply with *A* on both sides to get,

$A^2 x = A(\lambda x)$

$= \lambda(Ax)$ while λ is the real number, it commutes with the matrix *A*.

$= \lambda(\lambda x)$ Since $Ax = \lambda x$

$= \lambda^2 x$

Given that, $A^2 = I$.

So, $Ix = \lambda^2 x$ then $x = \lambda^2 x$ and thus, $\lambda^2 = 1$ implies $\lambda = \pm 1$.

Thus, the possible eigenvalues of *A* are $\boxed{\text{â€“ 1 and/or 1}}$.

Step-2

(b)

Suppose *A* is 2×2 matrix and not equal to I or $-I$.

Since *A* is 2×2 matrix with $A^2 = I$, so the possible eigenvalues are â€“ 1 and/or 1.

But *A* is not equal to I or $-I$, so the eigenvalues are not equal to 1, 1 or â€“1, â€“1.

Hence the eigenvalues of *A* are â€“1, 1.

Recollect that, the trace of *A* is the sum of the eigenvalues and the determinant of *A* is nothing but the product of the eigenvalues.

Thus, the trace of the matrix *A* is $-1+1=\boxed{0}$

And the determinant of the matrix *A* is $-1\cdot 1=\boxed{-1}$.

Step-3

(c)

The first row of matrix A is $(3,-1)$.

The objective is to find the second row of matrix A.

Let $A=\begin{bmatrix}3 & -1\\ x & y\end{bmatrix}$

Since $A^2=I$ so,

$$\begin{bmatrix}3 & -1\\ x & y\end{bmatrix}\begin{bmatrix}3 & -1\\ x & y\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$\begin{bmatrix}9-x & -3-y\\ 3x+xy & -x+y^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Then,

$$\begin{aligned}9-x&=1 \qquad & -3-y&=0\\ x&=8 & y&=-3\end{aligned}$$

Hence, the required matrix is $A=\begin{bmatrix}3 & -1\\ 8 & -3\end{bmatrix}$.

Thus, the second row of matrix A is $\boxed{(8,-3)}$.

Check:

The characteristic equation of A is,

$$\begin{aligned}\begin{vmatrix}3-\lambda & -1\\ 8 & -3-\lambda\end{vmatrix}&=0\\ (3-\lambda)(-3-\lambda)-(8)(-1)&=0\\ \lambda^2-9+8&=0\\ \lambda^2&=1\\ \lambda&=\pm 1\end{aligned}$$

So, the eigenvalues of A are â€“1 and 1.

Hence all the conditions of part A and B are satisfied.