Step-1

Let $b=(b_1,...,b_n)$ and $c=(c_1,...,c_n)$.

We need to find a matrix M such that $b=Mc$.

Step-2

Let

$$\begin{aligned} b_1v_1+...+b_nv_n &= Vb \\ &= c_1w_1+...+c_nw_n \\ &= Wc \end{aligned}$$

Step-3

Therefore, we get

$$\begin{aligned} Vb &= VMc \\ &= Wc \end{aligned}$$

Thus, $VMc=Wc$

Step-4

Thus, we get $VM=W$.

Therefore, $M=V^{-1}W$.