Step-1

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Given Hilbert matrix is $H=\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{3}\\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4}\\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5}\end{bmatrix}$

One solution for the non homogeneous system $Hx = b$ is $x = (1, 1, 1)$

So, we write $\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{3}\\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4}\\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5}\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

Step-2

$$\Rightarrow\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}=\begin{bmatrix}1+\frac{1}{2}+\frac{1}{3}\\ \frac{1}{2}+\frac{1}{3}+\frac{1}{4}\\ \frac{1}{3}+\frac{1}{4}+\frac{1}{5}\end{bmatrix}=\begin{bmatrix}1.8333\\1.0833\\0.7833\end{bmatrix}$$ â€¦â€¦ (1)

Step-3

Similarly, when $x = (0, 6, -3.6)$, we get $\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{3}\\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4}\\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5}\end{bmatrix}\begin{bmatrix}0\\6\\-3.6\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

Consequently, $\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}=\begin{bmatrix}0+3-1.2\\0+2-0.9\\0+1.5-0.72\end{bmatrix}=\begin{bmatrix}1.8\\1.1\\0.78\end{bmatrix}$ â€¦â€¦ (2)

Step-4

We easily see that the systems (1) and (2) are almost closely written and rounded off to

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} x = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

where x has two clearly distinguishable solutions (1, 1, 1) and (0, 6, -3.6).

This shows that the system has two distinguishable solutions.