

## Step-1

i) Given matrix

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 6 \end{bmatrix}$$

Given

$$C_{ij} = (-1)^{i+j} \det M_{ij}$$

Then  $C_A = \begin{bmatrix} 6 & -3 \\ -1 & 2 \end{bmatrix}$  and  $C_A^T = \begin{bmatrix} 6 & -1 \\ -3 & 2 \end{bmatrix}$

## Step-2

Now

$$\begin{aligned} A.C_A^T &= \begin{bmatrix} 2 & 1 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 6 & -1 \\ -3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 2(6)+1(-3) & 2(-1)+1(2) \\ 3(6)+6(-3) & 3(-1)+6(2) \end{bmatrix} \\ &= \begin{bmatrix} 12-3 & -2+2 \\ 18-18 & -3+12 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} \end{aligned}$$

Note that

$$\begin{aligned} \det A &= 12 - 3 \\ &= 9 \end{aligned}$$

## Step-3

So, we get

$$A.C_A^T = \det A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \det A \cdot I$$

## Step-4

$$\text{ii) } B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 0 \end{bmatrix}$$

Then

$$C_B = \begin{bmatrix} 0 & 42 & -35 \\ 0 & -21 & 14 \\ -3 & 6 & -3 \end{bmatrix}$$

$$C_B^T = \begin{bmatrix} 0 & 0 & -3 \\ 42 & -21 & 6 \\ -35 & 14 & -3 \end{bmatrix}$$

## Step-5

Now

$$\begin{aligned} B.C_B^T &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -3 \\ 42 & -21 & 6 \\ -35 & 14 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 84 - 105 & -42 + 42 & -3 + 12 - 9 \\ 210 - 210 & -105 + 84 & -12 + 30 - 18 \\ 0 & 0 & -21 \end{bmatrix} \end{aligned}$$

## Step-6

And

$$\begin{aligned} &= \begin{bmatrix} -21 & 0 & 0 \\ 0 & -21 & 0 \\ 0 & 0 & -21 \end{bmatrix} \\ &= -21 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= -21I \end{aligned}$$

And note that  $\det B = \boxed{-21}$