Step-1

Let us consider the positive definite and symmetric matrix of order 2 and extend the result to order *n*.

Suppose $A=\begin{bmatrix} a & c \\ c & b \end{bmatrix}$ is the positive definite symmetric matrix with $\lambda_{\max}=\lambda_{\min}=1$

Bt definition, $\lambda_{\max}\left(A^T A\right)=\|A\|^2$

So, we follow $\|A\|=1$

Similarly, $\lambda_{\min}\left(A^T A\right)=\dfrac{1}{\left\|A^{-1}\right\|^2}$

Step-2

Consequently, $\left\|A^{-1}\right\|=1$

With the help of Cholesky decomposition or by singular value decomposition, we get

$$\begin{aligned} c(A) &= \frac{\lambda_{\max}}{\lambda_{\min}} \\ &= 1 \\ &= c\left(R^T\right)c(R) \end{aligned}$$

Here $R=\sqrt{D}L^T$, *D* is the diagonal matrix and *L* is the lower triangular matrix.

We know that the minimum value of the conditional number is 1 and thus, $c\left(R^T\right)=c(R)=1$

Thus, the only possibility is $L=L^T=I, D=I$

Therefore, $A = I$.

Hence I is the only symmetric positive definite matrix that has $\lambda_{\max}=\lambda_{\min}=1$.

Step-3

From the entire discussion, it follows that $A^T=A^{-1}$ and thus, A is an orthogonal matrix.