Step-1

Given $A=\begin{pmatrix}2 & 1 & 4\\ 0 & -1 & 1\end{pmatrix}, B=\begin{pmatrix}1 & 1\\ 0 & 1\\ 1 & 0\end{pmatrix}$ and the first row of AB is a linear combination of all the rows of B.We have to find the coefficients in this combination and the first row of AB.

Step-2

$$AB=\begin{pmatrix}2 & 1 & 4\\ 0 & -1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 0 & 1\\ 1 & 0\end{pmatrix}$$

In A the first row is $\begin{pmatrix}2 & 1 & 4\end{pmatrix}$ and the first column of B is $\begin{pmatrix}1 & 0 & 1\end{pmatrix}^T$ and the second column is $\begin{pmatrix}1 & 0 & 1\end{pmatrix}^T$ so the first row of AB is

$$=\left[\begin{pmatrix}2 & 1 & 4\end{pmatrix}\begin{pmatrix}1\\ 0\\ 1\end{pmatrix} \quad \begin{pmatrix}2 & 1 & 4\end{pmatrix}\begin{pmatrix}1\\ 1\\ 0\end{pmatrix}\right]$$

$$=\begin{pmatrix}2.1+1.0+4.1 & 2.1+1.1+4.0\end{pmatrix}$$
$$=\begin{pmatrix}2+0+4 & 2+1+0\end{pmatrix}$$
$$=\begin{pmatrix}6 & 3\end{pmatrix}$$

The first row of AB is $\boxed{\begin{pmatrix}6 & 3\end{pmatrix}}$.

Step-3

Let the linear combination of all the rows of B is $a(1,1)+b(0,1)+c(1,0)$ where a,b and c are the coefficients which we want to find.

Since the first row of AB is a linear combination of all the rows of B,we have

$$a(1,1)+b(0,1)+c(1,0)=\begin{pmatrix}6 & 3\end{pmatrix}$$

$$(a+c,a+b)=\begin{pmatrix}6 & 3\end{pmatrix}$$

Therefore $\boxed{a=2,b=1 \text{ and } c=4}$.