

## Step-1

Note the following;

$$\begin{aligned}P(PA) &= P^2 A \\P(P(PA)) &= P^3 A \\&\vdots \\P(P(\dots(PA))) &= P^n A\end{aligned}$$

When a Permutation Matrix  $P$  is multiplied by itself, the resultant matrix is also a Permutation Matrix.

Thus,  $P, P^2, P^3, \dots, P^n$  are all Permutation Matrices.

## Step-2

When we consider  $n$  by  $n$  Permutation Matrices, we know that there are only  $n!$  distinct Permutation Matrices.

Thus, if we go on multiplying by Permutation Matrices, there are bound to be some  $P^k$  and  $P^r$  such that  $P^k = P^r$ . Without loss of generality, let  $k > r$ . Thus, we have

$$\begin{aligned}P^r &= P^k \\&= P^{r+k-r} \\&= P^r P^{k-r}\end{aligned}$$

This clearly indicates that  $P^{k-r} = I$ .

## Step-3

Thus, when we multiply  $A$  by  $P$ , as many as  $k-r$  times, we get

$$\begin{aligned}P^{k-r} A &= IA \\&= A\end{aligned}$$

Thus, the first row of the matrix  $P^{k-r} A$  must be same as that of the matrix  $A$ .