

Step-1

Let following be the differential equation of matrices:

$$\frac{du}{dt} = Au$$

Here, matrix A is defined as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Step-2

To show that the infinite series produces the following, when $A^2 = A$:

$$e^{At} = I + (e^t - 1)A$$

Use the above relation to find e^{At} .

Step-3

Infinite series for e^{At} is given as follows:

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \dots$$

To get the relation put $A^2 = A$ and do the following calculations:

$$\begin{aligned} e^{At} &= I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \frac{(At)^4}{4!} + \dots \\ &= I + At + \frac{A(t)^2}{2!} + \frac{A(t)^3}{3!} + \frac{A(t)^4}{4!} + \dots \\ &= I + A \left(t + \frac{(t)^2}{2!} + \frac{(t)^3}{3!} + \frac{(t)^4}{4!} + \dots \right) \\ &= I + A(e^t - 1) \end{aligned}$$

Step-4

Therefore, $\boxed{e^{At} = I + (e^t - 1)A}$.

Step-5

Now, find e^{At} with the help of above relation and matrix A .

$$\begin{aligned}e^{At} &= I + (e^t - 1)A \\&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (e^t - 1) \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \\&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} (e^t - 1) & (e^t - 1) \\ 0 & 0 \end{bmatrix} \\&= \begin{bmatrix} e^t & (e^t - 1) \\ 0 & 1 \end{bmatrix}\end{aligned}$$

Step-6

Therefore,

$$e^{At} = \boxed{\begin{bmatrix} e^t & (e^t - 1) \\ 0 & 1 \end{bmatrix}}.$$