ODE-A Midterm Exam

Sunday, April 9, 2023, **16:00–18:00**

No lecture notes, textbooks, calculators, cell phones, or anything else is allowed.

1. (10 points) Find the value of y_0 for which the solution of the initial value problem
$$y' - y = 1 + 3\sin t, \quad y(0) = y_0,$$
remains finite as $t \to \infty$.

2. (10 points) Solve the initial value problem
$$y' = \frac{1+3x^2}{3y^2 - 6y}, \quad y(0) = 1$$
and determine the interval in which the solution is valid.
Hint: To find the interval of definition, look for points where the integral curve has a vertical tangent.

3. (10 points) Solve the differential equation
$$y' = \frac{4y - 3x}{2x - y}.$$

4. (10 points) Consider the initial value problem
$$y' = y^{1/3}, \quad y(0) = 0$$
for $t \geq 0$. Apply the existence and uniqueness theorem to this initial value problem and then solve the problem.

5. (10 points) Solve the differential equation
$$(3xy + y^2) + (x^2 + xy)y' = 0.$$

$c_1 e^{r_1 x} + c_2 e^{r_2 x}$

6. (10 points) Find the solution of the initial value problem
$$y'' - y = 0, \quad y(0) = \frac{5}{4}, \quad y'(0) = -\frac{3}{4}.$$

$c_1 + c_2 = \frac{5}{4}$

$c_2 = \frac{3}{4}$

7. (10 points) Consider the differential equation
$$ay'' + by' + cy = g(t),$$
where a, b, and c are positive constants.
If $g(t) = d$, a constant, show that every solution of this equation approaches d/c as $t \to \infty$. What happens if $c = 0$? What if $b = 0$ also?

8. (10 points) Verify that $y_1(t) = t$ is a solution of

$$t^2y'' - 2ty' + 2y = 0,$$

and then solve the differential equation

$$t^2y'' - 2ty' + 2y = 4t^2, \quad t > 0.$$

9. (10 points) For the differential equation

$$y''' - 2y'' + y' = t^3 + 2e^t$$

determine a suitable form for its particular solution $Y(t)$ if the method of undetermined coefficients is to be used.

10. (10 points) Find a formula involving integrals for a particular solution of the differential equation

$$y''' - 3y'' + 3y' - y = g(t).$$

If $g(t) = t - 2e^t$, determine $Y(t)$.