Step-1

We have given that the proof of (AB)C = A(BC).

We use the column vectors $b_1, b_2, ..., b_n$ of B.

Let
$$B = [b_1, b_2, ..., b_n]$$
 and $C = c = [c_1, c_2, ..., c_n]^T$, then

$$AB = [Ab_1, Ab_2, ..., Ab_n]^T$$
 and $Bc = [c_1b_1 + ... + c_nb_n]$

And then
$$(AB)c = c_1Ab_1 + ... + c_nAb_n$$

$$= A(c_1b_1 + \dots + c_nb_n)$$
$$= A(Bc)$$

Step-2

The same is true for all other Elements (entries) of C.

Therefore
$$(AB)C = A(BC)$$
.