

## Step-1

We need to apply row operations to produce an upper triangular U to compute the determinants of the following matrices:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

## Step-2

(a) So, first consider

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \leftarrow \text{subtract 2 times the first row from the second row}$$

## Step-3

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 2 & 3 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} \leftarrow \text{Adding first row to the third}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 2 & 0 & 7 \end{bmatrix} \leftarrow \text{Subtracting the second row from the third row}$$

## Step-4

$$= \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix} \leftarrow \text{Subtracting the second row from the fourth row}$$

$$= (1)(2)(3)(6)$$

$$= 36$$

Note: if A is triangular, then  $\det A$  is the product  $a_{11}a_{22}\dots a_{nn}$  of the diagonal entries.

## Step-5

(b) Now consider

$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 2 \end{vmatrix} \leftarrow \text{subtracting the fourth row from the second and third rows}$$

## Step-6

$$= \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{2} \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the first row to the fourth.}$$

$$= \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & \frac{1}{2} & 2 \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the second row to the fourth.}$$

## Step-7

$$= \begin{vmatrix} 2 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & \frac{5}{2} \end{vmatrix} \leftarrow \text{adding } -\frac{1}{2} \text{ times the third row to the fourth.}$$

$$= (2)(1)(1)\left(\frac{5}{2}\right)$$

$$= 5$$

## Step-8

Note: if A is triangular, then  $\det A$  is the product  $a_{11}a_{22}\dots a_{nn}$  of the diagonal entries.

## Step-9

$$\det \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix} = 36$$

Thus,

And

$$\det \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = 5$$