

Step-1

When the edge vectors a, b, c are perpendicular, the volume of the box is $\|a\|$ time $\|b\|$ times $\|c\|$.and

When $A = \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$ is 3 by 3 matrix.

Step-2

With mutually perpendicular vectors $\vec{a}, \vec{b}, \vec{c}$ we have

$$\begin{aligned} A^T A &= \begin{bmatrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{bmatrix} \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix} \\ &= \begin{bmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{bmatrix} \\ &= \begin{bmatrix} \|a\|^2 & 0 & 0 \\ 0 & \|b\|^2 & 0 \\ 0 & 0 & \|c\|^2 \end{bmatrix} \end{aligned}$$

Step-3

And hence

$$\begin{aligned} \det(A^T A) &= \det A^T \cdot \det A \\ &= \det A \cdot \det A \\ \det(A^T A) &= (\det A)^2 \end{aligned}$$

Step-4

$$\begin{aligned} \det(A^T A) &= \|a\|^2 \|b\|^2 \|c\|^2 \\ \Rightarrow \det A &= \|a\| \|b\| \|c\| \end{aligned}$$

Thus,

$$\det A^T A = \|a\|^2 \|b\|^2 \|c\|^2$$

$$\det A = \|a\| \|b\| \|c\|$$