Step-1

We have to find the projection matrix onto the space spanned by $a_1 = (1,0,1), a_2 = (1,1,-1)$.

Write $A = \begin{bmatrix} a_1 & a_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$

So the required projection matrix is $P = A\left(A^T A\right)^{-1} A^T$

Step-2

Now

$$A^T A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)+0(0)+1(1) & 1(1)+0(1)+1(-1) \\ 1(1)+1(0)-1(1) & 1(1)+1(1)-1(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

Step-3

Now the inverse of $\left(A^T A\right)^{-1}$ is

$$\left(A^T A\right)^{-1} = \frac{1}{6-0} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A\left(A^T A\right)^{-1} = \frac{1}{6} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \frac{1}{6} \begin{bmatrix} 3 & 2 \\ 0 & 2 \\ 3 & -2 \end{bmatrix}$$

Step-4

Therefore, the projection matrix is

$$P = A\left(A^T A\right)^{-1} A^T$$
$$= \frac{1}{6}\begin{bmatrix} 3 & 2 \\ 0 & 2 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Hence $P = \frac{1}{6}\begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{bmatrix}$

Step-5

Verification:

$$P^2 = \frac{1}{36}\begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{bmatrix}\begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{bmatrix} = \frac{1}{36}\begin{bmatrix} 30 & 12 & 6 \\ 12 & 12 & -12 \\ 6 & -12 & 30 \end{bmatrix} = P$$
$$P^T = \frac{1}{6}\begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{bmatrix} = P$$

Hence $\boxed{P = \frac{1}{6}\begin{bmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{bmatrix}}$ is the required projection matrix.