Step-1

Given that $B = \begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$

We have to show that the eigenvalues of B are $\pm\sigma_i$, the singular values of A.

Step-2

We have $B = \begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$

So, $B^2 = \begin{bmatrix} AA^T & 0 \\ 0 & A^T A \end{bmatrix}$

We know that the ***singular values*** of A in the singular value decomposition are the *square roots* of the eigenvalues of $A^T A$

Step-3

Observe that the upper left portion of B^2 is a $n \times n$ matrix AA^T and the right side is the zero matrix while the lower left is the square matrix of zeroes and the lower right is the square matrix $A^T A$

So, the eigenvalues of B^2 are the eigenvalues of AA^T and those of $A^T A$.

So, the eigenvalues of B are the + or â€“ the square roots of the eigenvalues of $A^T A$.

By the above result, these are nothing but the + or â€“ singular values of A.

Therefore, the eigenvalues of B are nothing but the + or â€“ singular values of A denoted by $\pm\sigma_i$.

Hence the eigenvalues of B are $\boxed{\text{the singular values of}}$ A.