Step-1

Given system is $2x+y=0$

$x+2y+z=0$

$y+2z+t=0$

$z+2t=5$

We have to find the pivots and the solution for these equations.

Step-2

Given system can be written as

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{pmatrix}$$

Subtract â€˜ $\frac{1}{2}$ â€™ times the row 1 from the row 2

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{pmatrix}$$

Step-3

Subtract â€˜ $\frac{2}{3}$ â€™ times the row 2 from the row 3.

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & 1 & 0 \\ 0 & 0 & 1 & 2 & 5 \end{pmatrix}$$

Subtract â€˜ $\frac{3}{4}$ â€™ times the row 2 from the row 3.

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & \frac{3}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{4}{3} & 1 & 0 \\ 0 & 0 & 0 & \frac{5}{4} & 5 \end{pmatrix}$$

Step-4

which is upper triangular form.

The pivots are circled in $\begin{pmatrix} \boxed{2} & 1 & 0 & 0 & 0 \\ 0 & \boxed{\frac{3}{2}} & 1 & 0 & 0 \\ 0 & 0 & \boxed{\frac{4}{3}} & 1 & 0 \\ 0 & 0 & 0 & \boxed{\frac{5}{4}} & 5 \end{pmatrix}$

That is $\boxed{2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}}$

Step-5

Back ward substitution:-

From upper triangular form, We have

$$\begin{aligned} 2x + y &= 0 \\ \frac{3}{2}y + z &= 0 \\ \frac{4}{3}z + t &= 0 \\ \frac{5}{4}t &= 5 \end{aligned}$$

Step-6

$$\frac{5}{4}t = 5$$

$$\Rightarrow \boxed{t = 4}$$

$$\frac{4}{3}z + t = 0$$

$$\Rightarrow \frac{4}{3}z + 4 = 0$$

$$\Rightarrow \boxed{z = -3}$$

$$\frac{3}{2}y + z = 0$$

$$\Rightarrow \frac{3}{2}y - 3 = 0$$

$$\Rightarrow \boxed{y = 2}$$

$$2x + y = 0$$

$$\Rightarrow 2x + 2 = 0$$

$$\Rightarrow \boxed{x = -1}$$

Step-7

Solutions are $\boxed{x = -1, y = 2, z = -3, t = 4}$

Step-8

Operations are

(i) Subtract â€˜ $\frac{1}{2}$ â€™ times the row 1 from the row 2

(ii) Subtract â€˜ $\frac{2}{3}$ â€™ times the row 2 from the row 3

(iii) Subtract â€˜ $\frac{3}{4}$ â€™ times the row 2 from the row 3