Step-1

Let unitary matrix U is defined as follows:

$$U = \begin{bmatrix} 1/\sqrt{3} & i/\sqrt{2} & a \\ 1/\sqrt{3} & 0 & b \\ i/\sqrt{3} & 1/\sqrt{2} & c \end{bmatrix}$$

Step-2

Find the values of a, b and c. Also find the freedom in column 3.

Step-3

Recall that Unitary matrix has orthonormal columns. So, each column of unitary matrix must be orthogonal to each other columns. Let C_1, C_2 and C_3 be the column of unitary matrix. Dot product of each column will be zero.

$$\begin{aligned} C_1^H \cdot C_2 &= \left(1/\sqrt{3}\right)\cdot\left(i/\sqrt{2}\right) + \left(1/\sqrt{3}\right)\cdot(0) + \left(-i/\sqrt{3}\right)\cdot\left(1/\sqrt{2}\right) \\ &= i/\sqrt{6} - i/\sqrt{6} \\ &= 0 \end{aligned}$$

$$\begin{aligned} C_2^H \cdot C_3 &= \left(-i/\sqrt{2}\right)\cdot a + (0)\cdot b + \left(1/\sqrt{2}\right)\cdot c \\ &= \frac{-ia + c}{\sqrt{2}} \end{aligned}$$

However, $C_2^H \cdot C_3 = 0$. So,

$$\begin{aligned} C_2^H \cdot C_3 &= 0 \\ \frac{-ia + c}{\sqrt{2}} &= 0 \\ -ia + c &= 0 \\ c &= ia \end{aligned}$$

This gives one equation in unknown values.

Step-4

Similarly, dot product of other two columns will also give an equation in unknown values.

$$C_1^H \cdot C_3 = \left(1/\sqrt{3}\right)\cdot(a) + \left(1/\sqrt{3}\right)\cdot(b) + \left(-i/\sqrt{3}\right)\cdot(c)$$
$$= \frac{a+b-ic}{\sqrt{3}}$$

However, $C_1^H \cdot C_3 = 0$. So,

$$C_1^H \cdot C_3 = 0$$
$$\frac{a+b-ic}{\sqrt{3}} = 0$$
$$a+b-ic = 0$$
$$a+b = ic$$

Or

$$a+b = i(ia)$$
$$b = -a-a$$
$$= -2a$$

Step-5

Every vector in unitary matrix is orthonormal. Therefore,

$$C_3 \cdot C_3 = 1$$
$$|a|^2 + |b|^2 + |c|^2 = 1$$

Substitute the values of b and c from the above two equations and solve them to get the value of one unknown:

$$|a|^2 + |-2a|^2 + |ia|^2 = 1$$
$$6|a|^2 = 1$$
$$a = \frac{1}{\sqrt{6}}$$

Step-6

Therefore, values of other unknown are:

$$a = \boxed{\frac{1}{\sqrt{6}}}$$
$$b = -2a$$
$$= \boxed{\frac{-2}{\sqrt{6}}}$$

$c = ia$

$= \boxed{\frac{i}{\sqrt{6}}}$

Step-7

To find the freedom of a column 3 find the independent constraints.

First constraint: Dot product with column C_1 will be zero, $C_1 \cdot C_3 = 0$.

Second constraint: Dot product with column C_2 will be zero, $C_2 \cdot C_3 = 0$.

Third constraint: Unitary length $C_3 \cdot C_3 = 1$.

Step-8

Total, 3 constraints are applicable. Therefore, degree of freedom of a column 3 is $\boxed{3.}$