Step-1

Let $A=\begin{bmatrix}0&0\\1&0\end{bmatrix}, B=\begin{bmatrix}0&1\\0&0\end{bmatrix}, C=\begin{bmatrix}-1&1\\-1&1\end{bmatrix}$

$$A^2=\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}0&0\\1&0\end{bmatrix}$$
$$=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$B^2=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}0&1\\0&0\end{bmatrix}$$
$$=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$C^2=\begin{bmatrix}-1&1\\-1&1\end{bmatrix}\begin{bmatrix}-1&1\\-1&1\end{bmatrix}$$
$$=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

Therefore $A^2=B^2=C^2=0$

Step-2

For the eigen values of these matrices, we consider $|A-\lambda I|=0$

$$\begin{vmatrix}-\lambda&0\\1&-\lambda\end{vmatrix}=0$$
$$\Rightarrow \lambda^2=0$$
$$\Rightarrow \boxed{\lambda_A=0,0}$$

Similarly, we get the eigen values of *B* are 0, 0.

Step-3

We consider $|C-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} -1-\lambda & 1 \\ -1 & 1-\lambda \end{vmatrix} = 0$$
$$\Rightarrow -(1+\lambda)(1-\lambda)+1=0$$
$$\Rightarrow -1-\lambda^2+1=0$$
$$\Rightarrow \lambda^2=0$$
$$\Rightarrow \boxed{\lambda_C = 0,0}$$

Step-4

Trace of A = sum of the diagonal entries = 0+ 0 =0

Trace of B = 0+0=0

Trace of C = $-1+1=0$

Also,

Determinant of A = Product of eigen value

$$=(0)(0)=0$$

Determinant of B = Product of eigen value

$$=(0)(0)=0$$

Determinant of C = Product of eigen value

$$=(0,0)$$
$$=0$$

Step-5

Thus, $A=\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, B=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C=\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$ satisfy all the required conditions.