Step-1

Let x be such that $Ax = b$.

Consider again

$$\begin{aligned} Q(x) &= \frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}Ax - x^{\mathrm{T}}A^{\mathrm{T}}b + \frac{1}{2}b^{\mathrm{T}}b \\ &= \frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}b - x^{\mathrm{T}}A^{\mathrm{T}}b + \frac{1}{2}b^{\mathrm{T}}b \\ &= -\frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}b + \frac{1}{2}b^{\mathrm{T}}b \end{aligned}$$

That is, $Q(x)$ is minimum whenever $Ax = b$.

Step-2

We have $Q(x) = \frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}Ax - x^{\mathrm{T}}A^{\mathrm{T}}b + \frac{1}{2}b^{\mathrm{T}}b$. If we ignore the last term, that is $\frac{1}{2}b^{\mathrm{T}}b$, we get $Q(x) = \frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}Ax - x^{\mathrm{T}}A^{\mathrm{T}}b$.

Now let x be such that $Ax = b$. Then we get

$$\begin{aligned} Q(x) &= \frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}Ax - x^{\mathrm{T}}A^{\mathrm{T}}b \\ &= \frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}b - x^{\mathrm{T}}A^{\mathrm{T}}b \\ &= -\frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}b \end{aligned}$$

Step-3

Thus, we get $\boxed{Q(x) = -\frac{1}{2}x^{\mathrm{T}}A^{\mathrm{T}}b}$ at the minimum. These equations are called as normal equations in the theory of least squares.