Step-1

We know that,

$$\det A = \left(\det C\right)^{\frac{1}{n-1}}$$

In this case $n = 4$.

Therefore, we get,

$$A^{-1} = \frac{C^T}{\det A}$$

The matrix A can be obtained by inverting A^{-1}.

Consider the matrix A

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

Therefore, we get,

$$C = \begin{bmatrix} \begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & 1 \\ -6 & 2 & 1 \end{bmatrix}$$

By using the formula $A^{-1} = \frac{C^T}{\det A}$, we get,

$$A^{-1} = \frac{C^T}{\det A}$$

$$= \frac{\begin{bmatrix} 6 & -3 & 0 \\ 6 & 1 & 0 \\ -6 & -2 & 1 \end{bmatrix}^T}{\det \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} 6 & 6 & -6 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}{\begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}}$$

$$= \frac{\begin{bmatrix} 6 & 6 & -6 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}}{6-3+0}$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 6 & 6 & -6 \\ -3 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Step-2

Thus, we can A^{-1} by using the formula $A^{-1} = \frac{C^T}{\det A}$ and by inverting A^{-1}