

## Step-1

If  $P$  is the projection matrix onto a  $k$ -dimensional subspace  $S$  of the whole space  $\mathbf{R}^n$ , then we have to find the column space of  $P$  and its rank.

If  $\bar{x} \in \mathbf{R}^n$ , then  $P\bar{x} \in S$ , since  $P$  projects  $\bar{x}$  to  $S$ .

Therefore column space  $P$  contained  $S$ , that is,  $\text{col}(P) \subset S$  (1)

## Step-2

On the other hand, if  $\bar{b} \in S$ , then  $P\bar{b} = \bar{b}$

So  $S \subset \text{col}(P)$  (2)

From (1) and (2),  $\text{col}(P) = S$

Therefore the rank of  $P$  is equal to the dimension of  $\text{col}(P)$

That is, since  $S$  is  $k$ -dimensional the rank of  $P$  is  $k$