Step-1

Given system is $u+w=4$

$$u+v=3$$
$$u+v+w=3$$

We have to solve this system by elimination and back substitution.

Step-2

Given system in matrix form is

$$\begin{bmatrix} 1 & 0 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 1 & 1 & 1 & 6 \end{bmatrix}$$

apply $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

apply $R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

(which is upper triangular matrix)

that is $u+w=4$

$$v-w=-1$$
$$w=3$$

Step-3

By back-ward substitution, we have

$\boxed{w=3}$

$v-w=-1$

$\Rightarrow v-3=-1$

$\Rightarrow \boxed{v=2}$

$u+w=4$

$\Rightarrow u+3=4$

$\Rightarrow \boxed{u=1}$

Solution are $\boxed{u=1, v=2, w=3}$

Step-4

(b)

Given system is $v+w=0$

$$u+w=3$$
$$u+v=6$$

We have to solve this system by elimination and back substitution.

Step-5

Given system in matrix form is

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 6 \end{bmatrix}$$

apply $R_2 \leftrightarrow R_1$

$$\square \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 6 \end{bmatrix}$$

apply $R_2 \leftrightarrow R_3$

$$\square \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 6 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

apply $R_2 \rightarrow R_2 - R_1$

$$\square \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 6 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

apply $R_2 \to R_2 - R_1$

$$\square \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 2 & -6 \end{bmatrix}$$

(which is upper triangular matrix)

that is $u + w = 0$

$v - w = 6$

$2w = -6$

Step-6

By back-ward substitution, we have

$\boxed{w = -3}$

$v - w = 6$

$\Rightarrow v + 3 = 6$

$\Rightarrow \boxed{v = 3}$

$u + w = 0$

$\Rightarrow u - 3 = 0$

$\Rightarrow \boxed{u = 3}$

Solution are $\boxed{u = 3, v = 3, w = -3}$