## Step-1

Given

$$S_1 = |3|, \quad S_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}, \quad S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

We need to compute the determinants of $S_1, S_2, S_3$ of these 1, 3, 1 tridiagonal matrices.

## Step-2

Now

$$\begin{aligned} S_1 &= |3| \\ &= 3 \end{aligned}$$

$$\begin{aligned} S_2 &= \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} \\ &= 9 - 1 \\ &= 8 \end{aligned}$$

## Step-3

Then

$$\begin{aligned} S_3 &= \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix} \\ &= 3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \text{ expanding I row} \end{aligned}$$

$$\begin{aligned} &= 3S_2 - 3 \\ &= 3S_2 - S_1 \\ &= 24 - 3 \\ &= 21 \end{aligned}$$

## Step-4

So, we can guess that

$$\begin{aligned}S_4 &= 3S_3 - S_2\\ &= 3(21) - 8\\ &= 63 - 8\\ &= \boxed{55}\end{aligned}$$

## Step-5

Calculating $S_4$ directly we get

$$S_4 = \begin{vmatrix}3 & 1 & 0 & 0\\ 1 & 3 & 1 & 0\\ 0 & 1 & 3 & 1\\ 0 & 0 & 1 & 3\end{vmatrix}$$

$$= 3\begin{vmatrix}3 & 1 & 0\\ 1 & 3 & 1\\ 0 & 1 & 3\end{vmatrix} - \begin{vmatrix}1 & 1 & 0\\ 0 & 3 & 1\\ 0 & 1 & 3\end{vmatrix}$$

## Step-6

On solving

$$= 3.S_3 - \left[\begin{vmatrix}3 & 1\\ 1 & 3\end{vmatrix} - \begin{vmatrix}0 & 1\\ 0 & 3\end{vmatrix}\right]$$

$$\begin{aligned}&= 3(21) - [9 - 1]\\ &= 63 - 8\\ &= \boxed{55}\end{aligned}$$