

## Math 209 Homework/Project #7

**Due Date: Dec. 29, 2020**

1. (2 points) Problem 5, page 428 of the textbook
2. (2 points) Problem 6, page 428 of the textbook
3. (2 points) Problem 4, page 431 of the textbook
4. (2 points) Problem 1, page 432 of the textbook
5. (2 points) Problem 5, page 441 of the textbook
6. (2 points) Problem 7, page 441 of the textbook
7. (8 points) Do this problem by using Sage. Lagrange's theorem says that every positive integer is a sum of 4 squares. One of my research collaborators conjectured the following refinement of Lagrange's theorem: Every positive integer  $n$  can be written as  $x^2 + y^2 + z^2 + w^2$ , where  $x, y, z, w$  are nonnegative integers, such that  $x + 3y + 5z$  is a perfect square. Write a program (by using Sage) to check the conjecture for all positive integers up to  $10^6$ .