

Step-1

If A is 2 by 3 and C is 3 by 2, then we have to show that from its rank, $CA \neq I$ and we have to give an example in which $AC = I$. For $m < n$, a right inverse is not a left inverse.

Step-2

$$\text{Let } A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 0 & 0 \end{bmatrix}$$

Now A is 2 by 3 matrix, and C is 3 by 2 matrix,

$$\begin{aligned} CA &= \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

Step-3

Therefore the rank of $CA = 2$

The rank of $I = 3$ where I is 3 by 3 matrix. Therefore $CA \neq I$

Step-4

Now

$$\begin{aligned} AC &= \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Therefore $AC = I$, where I is 2 by 2 identity matrix C is right inverse of A , but C is not left inverse of A .