Step-1

Given that

$$f(a,b,c,d) = \ln(ad-bc)$$

$$\Rightarrow \frac{\partial f}{\partial a} = \frac{d}{ad-bc}$$

$$\frac{\partial f}{\partial b} = \frac{1}{ad-bc}(-c)$$

Step-2

$$\frac{\partial f}{\partial c} = \frac{-b}{ad-bc}$$

$$\frac{\partial f}{\partial d} = \frac{a}{ad-bc}$$

Step-3

Therefore $\begin{bmatrix} \frac{\partial f}{\partial a} & \frac{\partial f}{\partial c} \\ \frac{\partial f}{\partial b} & \frac{\partial f}{\partial d} \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \boxed{A^{-1}}$