MA215 Probability Theory

Assignment 15

1. Suppose that a discrete random variable X has finite kth moment, i.e., $E(|X|^k) < \infty$ ($k > 0$, but k may not be a positive integer). Show that for any $\varepsilon > 0$,

$$P\{|X| \geqslant \varepsilon\} \leqslant \frac{E(|X|^k)}{\varepsilon^k}.$$

2. Suppose that $\{X_1, X_2, \cdots, X_n, \cdots\}$ is a sequence of independent r.v.s (not necessarily with the same distribution), each with finite (but not necessarily with the same) mean and uniformly bounded variance by $M < \infty$ (i.e., $\mathrm{Var}\,(X_i) \leqslant M \ \forall i \geqslant 1$). Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$ be the sample mean. Show that for any $\varepsilon > 0$, we have

$$\lim_{n\to\infty} P\left\{|\bar{X}_n - E\bar{X}_n| > \varepsilon\right\} = 0.$$

3. Suppose that $\{X_1, X_2, \cdots\}$ is a sequence of i.i.d. r.v.s with common mean 1 and variance 16. Let n be sufficiently large and $Y = X_1 + X_2 + \cdots + X_n$. Estimate the value of $P\{2.608 < Y \leqslant 4.4124\}$. (Hint: using the central limit theorem and normal approximation method.)