Step-1

We have to compute L and U for the symmetric matrix:

$$A = \begin{pmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{pmatrix}$$

Also we have to find four conditions on a,b,c,d to get $A = LU$ with four pivots.

Step-2

Subtracting row 1 from row 2 and row 3 and row 4 gives

$$A \to \begin{pmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{pmatrix}$$

Subtracting row 2 from row 3 and row 4 gives

$$A \to \begin{pmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{pmatrix}$$

Step-3

Subtracting row 3 from row 4 gives

$$A \to \begin{pmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{pmatrix} = U$$

Step-4

The lower triangular matrix is

$$L=\begin{bmatrix}1&0&0&0\\l_{21}&1&0&0\\l_{31}&l_{32}&1&0\\l_{41}&l_{42}&l_{43}&1\end{bmatrix}$$

Fix $l_{21}=1$ for the row operation, subtracting row 1 from row 2, so from the above row operations we get $l_{21}=l_{31}=l_{41}=1,\ l_{32}=l_{42}=1, l_{43}=1$

Step-5

Therefore

$$L=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}$$

We can observe that

$$A=\begin{pmatrix}a&a&a&a\\a&b&b&b\\a&b&c&c\\a&b&c&d\end{pmatrix}=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}\begin{pmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&0&c-b&c-b\\0&0&0&d-c\end{pmatrix}$$

The four pivots are $a,b-a,c-b,d-c$ (the diagonal elements of U) and the factorization is possible if $a\neq 0,b\neq a,c\neq b,d\neq c$