Step-1

Now consider the following:

$$F_{1024} = \begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix} \begin{bmatrix} F_{512} & \\ & F_{512} \end{bmatrix} \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix}$$

By using the above rule, we get

$$\begin{aligned} F_{1024}{}^{\mathrm{T}} &= \left(\begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix} \begin{bmatrix} F_{512} & \\ & F_{512} \end{bmatrix} \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix} \right)^{\mathrm{T}} \\ &= \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} F_{512} & \\ & F_{512} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix}^{\mathrm{T}} \end{aligned}$$

Step-2

Note that the transpose of an even permutation is another even permutation. Similarly, transpose of an odd permutation is another odd permutation.

Thus, we get

$$\boxed{F_{1024}{}^{\mathrm{T}} = \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix} \begin{bmatrix} & F_{512} \\ F_{512} & \end{bmatrix} \begin{bmatrix} I_{512} & I_{512} \\ D_{512} & -D_{512} \end{bmatrix}}$$