Step-1

The objective is to determine the basis of each four subspaces of the provided matrix.

Step-2

A basis for the subspaces: column space $\mathbf{C}(\mathbf{A})$, null space $\mathbf{N}(\mathbf{A})$, row space $\mathbf{C}(\mathbf{A}^T)$, and left null space $\mathbf{N}(\mathbf{A}^T)$ of A:

$$A=\begin{bmatrix}0&1&2&3&4\\0&1&2&4&6\\0&0&0&1&2\end{bmatrix}$$

Step-3

Consider the matrix:

$$\begin{aligned}A&=\begin{bmatrix}0&1&2&3&4\\0&1&2&4&6\\0&0&0&1&2\end{bmatrix}\\&=\begin{bmatrix}1&0&0\\1&1&0\\0&1&1\end{bmatrix}\begin{bmatrix}0&1&2&3&4\\0&0&0&1&2\\0&0&0&0&0\end{bmatrix}\end{aligned}$$

Here the $RREF(A)$ of the matrix A is:

$$RREF(A)=\begin{bmatrix}0&1&2&3&4\\0&0&0&1&2\\0&0&0&0&0\end{bmatrix}$$

There are only two nonzero rows. The vector in the null space $\begin{bmatrix}x_1&x_2&x_3&x_4\end{bmatrix}$. Let $x_2=r$ and $x_4=t$.

Then,

$$\begin{aligned}x_3&=-2t\\x_1&=-2r+2t\end{aligned}$$

Thus, the null space is:

$$\left\{\begin{bmatrix}-2\\1\\0\\0\end{bmatrix},\begin{bmatrix}2\\0\\-2\\1\end{bmatrix}\right\}$$

From $RREF(A)$,

The basis for column space is:

$$\left\{\begin{bmatrix}1\\1\\0\end{bmatrix},\begin{bmatrix}3\\4\\1\end{bmatrix}\right\}$$

Step-4

The transpose of the matrix A is:

$$A^T=\begin{bmatrix}0&0&0\\1&1&0\\2&2&0\\3&4&1\\4&6&2\end{bmatrix}$$

Use row method.

Replace,

$$R_3 \to R_3 - 2R_2$$
$$R_4 \to R_4 - 3R_2$$
$$R_5 \to R_5 - 4R_2$$

So,

$$A^T=\begin{bmatrix}0&0&0\\1&1&0\\0&0&0\\0&1&1\\0&2&2\end{bmatrix}$$

Replace $R_5 \to R_5 - 2R_4$.

$$A^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$RREF\left(A^T\right) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Step-5

Thus, in the transpose matrix only two rows are non-zero rows. Since only one variable for the vector in the null space $\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$.

Let $y_3 = s$

Then,

$$y_2 = -s$$
$$y_2 = s$$

Step-6

For matrix $RREF\left(A^T\right)$, the column space is expressed by the one of the second and third column and the first one.

Thus, the basis for the column space is:

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$$

Hence, the basis for the column space is $\boxed{\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}}$.