

Step-1

Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} 3 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

Suppose that,

$$B = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 2 & 7 \end{bmatrix}$$

And

$$C = \begin{bmatrix} 3 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

Step-2

We need to find the basis of $\text{row}(C)$,

$$C = \begin{bmatrix} 3 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

By using $R_1 \rightarrow \frac{R_1}{3}$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

By using $R_2 \rightarrow R_2 - R_1$, we get

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Therefore,

$$\text{Row space basis} = ([1 \ 0 \ 1], [0 \ 1 \ 1])$$

Step-3

Now, we shall find the basis of $\text{col}(B)$,

$$B = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 2 & 7 \end{bmatrix}$$

By using $R_2 \rightarrow R_2 - 4R_1$ and $R_3 \rightarrow R_3 - 2R_1$ we get.

$$\begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 0 & 3 \end{bmatrix}$$

By using $R_3 \rightarrow R_3 + R_2$, we get.

$$\begin{bmatrix} 1 & 2 \\ 0 & -3 \\ 0 & 0 \end{bmatrix}$$

Therefore,

$$\text{Column space basis} = \left(\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix} \right)$$

Step-4

We know that, an inverse exists only when the rank is as large as possible.

Here the rank of $\text{row}(C)$ is 2 and the rank of $\text{col}(B)$ is 2.

Now, matrix A is 3×3 and $\text{rank}(A) = \text{rank}(B) < 3$, therefore matrix A is not invertible.