

## Step-1

Consider the following:

$$Ax = \lambda_1 x$$
$$A^T y = \lambda_2 y$$

To prove that  $x^T y = 0$  or Eigen vectors are perpendicular.

Here, all Eigen values, eigenvectors and matrix  $A$  are all real.

## Step-2

Recall that matrix  $A$  has real Eigen values and real orthogonal Eigenvectors if and only if  $A = A^T$ .

Now do the following calculations:

$$Ax = \lambda_1 x$$

Dot product with vector  $y$  will give:

$$\begin{aligned} (\lambda_1 x)^T y &= (Ax)^T y \\ &= x^T A^T y \\ x^T \lambda_1 y &= x^T \lambda_2 y \\ x^T y (\lambda_1 - \lambda_2) &= 0 \end{aligned}$$

Since,  $\lambda_1 - \lambda_2 \neq 0$ , So,  $x^T y = 0$ .

## Step-3

Therefore, Eigen vectors corresponding to different Eigen values are perpendicular  $\boxed{x^T y = 0}$ .