Step-1

Given matrix is $\begin{bmatrix} 100 & 0 \\ 0 & 2 \end{bmatrix}$.

We have to find the norm $\lambda_{\max}$ and the condition number $\frac{\lambda_{\max}}{\lambda_{\min}}$ of the given matrix.

Step-2

Let $A = \begin{bmatrix} 100 & 0 \\ 0 & 2 \end{bmatrix}$

The characteristic equation of A is

$$|A - \lambda I| = 0$$
$$\Rightarrow \begin{vmatrix} 100-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (100-\lambda)(2-\lambda) = 0$$
$$\Rightarrow \lambda = 2,100$$

So, the eigenvalues of A are 100, 2.

Step-3

Now the norm of the given matrix is $\lambda_{\max} = 100$

And the condition number is

$$\frac{\lambda_{\max}}{\lambda_{\min}} = \frac{100}{2}$$

$$= 50$$

Hence the norm of the matrix $\begin{bmatrix} 100 & 0 \\ 0 & 2 \end{bmatrix}$ is 100 and the condition number is 50.

Step-4

Given matrix is $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

We have to find the norm $\lambda_{\max}$ and the condition number $\frac{\lambda_{\max}}{\lambda_{\min}}$ of the given matrix.

Step-5

Let $A=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$

The characteristic equation of A is

$$|A-\lambda I|=0$$
$$\Rightarrow\begin{vmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{vmatrix}=0$$
$$\Rightarrow(2-\lambda)(2-\lambda)-1=0$$

$$\Rightarrow\lambda^2-4\lambda+3=0$$
$$\Rightarrow(\lambda-3)(\lambda-1)=0$$
$$\Rightarrow\lambda=1,3$$

So, the eigenvalues of A are 3, 1.

Step-6

Now the norm of the given matrix is $\lambda_{\max}=3$

And the condition number is

$$\frac{\lambda_{\max}}{\lambda_{\min}}=\frac{3}{1}$$

$$=3$$

Hence the norm of the matrix $\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$ is 3 and the condition number is 3.

Step-7

Given matrix is $\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}$.

We have to find the norm $\lambda_{\max}$ and the condition number $\frac{\lambda_{\max}}{\lambda_{\min}}$ of the given matrix.

Step-8

Let $A=\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}$

The characteristic equation of A is

$$|A-\lambda I|=0$$
$$\Rightarrow\begin{vmatrix}3-\lambda & 1\\ 1 & 1-\lambda\end{vmatrix}=0$$
$$\Rightarrow(3-\lambda)(1-\lambda)-1=0$$
$$\Rightarrow\lambda^2-4\lambda+2$$
$$\Rightarrow\lambda=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}\qquad\left(\begin{array}{l}\text{Since by the quadratic}\\ \text{equation fromula}\end{array}\right)$$
$$=\frac{4\pm\sqrt{16-8}}{2}$$
$$=\frac{4\pm\sqrt{8}}{2}$$
$$=\frac{4\pm2\sqrt{2}}{2}$$
$$=2\pm\sqrt{2}$$

So, the eigenvalues of A are $2+\sqrt{2}$ and $2-\sqrt{2}$.

Step-9

Now the norm of the given matrix is $\lambda_{\max}=2+\sqrt{2}$

And the condition number is

$$\frac{\lambda_{\max}}{\lambda_{\min}}=\frac{2+\sqrt{2}}{2-\sqrt{2}}$$
$$=\frac{2+\sqrt{2}}{2-\sqrt{2}}\times\frac{2+\sqrt{2}}{2+\sqrt{2}}\qquad\left(\begin{array}{l}\text{Rationalising the}\\ \text{denominator}\end{array}\right)$$
$$=\frac{\left(2+\sqrt{2}\right)^2}{4-2}$$

Step-10

Continuation to the above

$$= \frac{4+2+2\sqrt{2}}{2}$$
$$= \frac{6+2\sqrt{2}}{2}$$
$$= 3+\sqrt{2}$$

Hence the norm of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ is $\boxed{2+\sqrt{2}}$ and the condition number is $\boxed{3+\sqrt{2}}$.