

Step-1

We have to write down the 2 by 2 matrices A and B that have the entries $a_{ij} = i + j$ and

$b_{ij} = (-1)^{i+j}$. Also we have to find AB and BA .

Step-2

Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$.

With the given conditions $a_{ij} = i + j$ and $b_{ij} = (-1)^{i+j}$, the entries become

$$\begin{aligned} a_{11} &= 1 + 1 \\ &= 2, \\ a_{12} &= 1 + 2 \\ &= 3 \end{aligned}$$

$$\begin{aligned} a_{21} &= 2 + 1 \\ &= 3, \\ a_{22} &= 2 + 2 \\ &= 4 \end{aligned}$$

Step-3

$$\begin{aligned} b_{11} &= (-1)^{1+1} \\ &= 1 \\ b_{12} &= (-1)^{1+2} \\ &= -1 \end{aligned}$$

$$\begin{aligned} b_{21} &= (-1)^{2+1} \\ &= -1 \\ b_{22} &= (-1)^{2+2} \\ &= 1 \end{aligned}$$

Step-4

Therefore $A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

$$\begin{aligned}
 AB &= \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 2-3 & -2+3 \\ 3-4 & -3+4 \end{pmatrix} \\
 &= \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} 2-3 & 3-4 \\ -2+3 & -3+4 \end{pmatrix} \\
 &= \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}
 \end{aligned}$$