Step-1

Consider a 2 by 2 symmetric matrix A with unit eigenvectors u_1 and u_2. If its eigenvalues are $\lambda_1 = 3, \lambda_2 = -2$

The objective is to find U, Σ and V^T.

Step-2

Assume that $A = A^T$ since A is symmetric.

Therefore, the eigenvectors of $A^T A = A^2$ are the same as for A.

The eigenvalues $A^T A$ are 9 and 4.

Therefore,

$$\Sigma = \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

Step-3

The matrix A is symmetric so the matrix U becomes the matrix of eigenvectors of the matrix A and V^T is the transpose of U.

So the U is;

$$U = \begin{bmatrix} u_1 & u_2 \end{bmatrix}$$

And,

$$V^T = U^T$$
$$= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Hence the required matrices are $\boxed{\Sigma = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}, U = \begin{bmatrix} u_1 & u_2 \end{bmatrix}, V^T = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}$