Step-1

Prove that A^T is always similar to A in three steps:

(a) For matrix A matrix M_i of permutations is calculated so that $M_i^{-1}J_iM_i = J_i^T$.

(b) Matrix M_0 is constructed from blocks so that $M_0^{-1}JM_0 = J^T$

(c) For any matrix A following is true:

$$\begin{aligned} A &= MJM^{-1} \\ A^T &= \left(MJM^{-1}\right)^T \\ &= \left(M^{-1}\right)^T J^T M^T \end{aligned}$$

Step-2

From step 2 substitutes $M_0^{-1}JM_0 = J^T$.

$$A^T = \left(M^{-1}\right)^T M_0^{-1}JM_0M^T$$

Substitute $M^{-1}AM = J$. Thus,

$$\begin{aligned} A^T &= \left(M^{-1}\right)^T M_0^{-1}M^{-1}AMM_0M^T \\ &= \left(MM_0M^T\right)^{-1} A\left(MM_0M^T\right) \end{aligned}$$

Step-3

Therefore, A^T is always similar to A.