

## Step-1

Let  $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$$\Rightarrow c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} c_1 + c_2 + c_3 \\ c_2 + c_3 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

## Step-2

i.e.  $c_3 = 0$

Plug this in the following equation.

$$c_2 + c_3 = 0$$
$$\Rightarrow c_2 = 0$$

Plug these values in the following equation.

$$c_1 + c_2 + c_3 = 0$$
$$\Rightarrow c_1 = 0$$

Therefore,  $\boxed{c_1 = c_2 = c_3 = 0}$

Therefore  $v_1, v_2, v_3$  are linearly independent.

## Step-3

Now,

let  $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$

$$\Rightarrow c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_4 \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} c_1 + c_2 + c_3 + 2c_4 \\ c_2 + c_3 + 3c_4 \\ c_3 + 4c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

## Step-4

i.e.  $c_3 + 4c_4 = 0$

$$\Rightarrow c_3 = -4c_4$$

Plug this value in the following equation.

$$\begin{aligned}c_2 + c_3 + 3c_4 &= 0 \\c_2 &= -c_3 - 3c_4 \\&= +4c_4 - 3c_4 \\&= c_4\end{aligned}$$

Plug this value in the following equation.

$$\begin{aligned}c_1 + c_2 + c_3 + 2c_4 &= 0 \\c_1 &= -c_2 - c_3 - 2c_4 \\&= -c_4 + 4c_4 - 2c_4 \\c_1 &= c_4\end{aligned}$$

If  $c_4 = 1$ , then  $c_1 = 1, c_2 = 1, c_3 = -4$

$$v_1 + v_2 - 4v_3 + v_4 = 0$$

Therefore,  $\boxed{v_1 + v_2 - 4v_3 + v_4 = 0}$

$\Rightarrow v_1, v_2, v_3, v_4$  are linearly dependent