Step-1

a) Given that λ is an Eigen value of the matrix A and also its Eigen vector is x.

That is $Ax = \lambda x$.

Now,

$$Ax = \lambda x$$

 $Ax - 7x = \lambda x - 7x$ Add $-7x$ on both sides
 $Ax - 7Ix = \lambda x - 7x$
 $(A - 7I)x = (\lambda - 7)x$ Take x as common on both sides

 $(A-7I)x = (\lambda - 7)x$ Take x as common on both sides $Bx = (\lambda - 7)x$ Since A-7I = B

Therefore x is an Eigen vector of B = A - 7I, and also $\lambda - 7$ is also an Eigen value.

Here, observe that Eigen values are reduced by 7 to λ with unchanged Eigen vectors.

Step-2

b) Assume that $\lambda \neq 0$, and also $Ax = \lambda x$.

Multiply both sides with A^{-1} .

$$A^{-1}(Ax) = A^{-1}(\lambda x)$$

$$(A^{-1}A)x = \lambda (A^{-1}x)$$

$$Ix = \lambda (A^{-1}x) \qquad \text{Since } A^{-1}A = I$$

$$x = \lambda A^{-1}x$$

$$A^{-1}x = \frac{1}{\lambda}x$$

$$A^{-1}x = \left(\frac{1}{\lambda}\right)x$$

Observe that the vector x satisfies $A^{-1}x = \left(\frac{1}{\lambda}x\right)$.

Hence, the Eigen value of A^{-1} is $\frac{1}{\lambda}$ and the Eigen vector does not change.

Thus, conclude that the Eigen value of A^{-1} is the reciprocal of the Eigen value of A and the respective Eigen vectors are one and the same.