Step-1

a) Given that $x = re^{i\theta}$

We have to find x^2, x^{-1} and $\bar{x}$ in polar coordinates.

Step-2

We have $x = re^{i\theta}$

Now

$$x^2 = \left(re^{i\theta}\right)^2$$
$$= r^2 e^{2i\theta}$$

Therefore, $\boxed{x^2 = r^2 e^{2i\theta}}$

Step-3

Now

$$x^{-1} = \frac{1}{x}$$
$$= \frac{1}{re^{i\theta}}$$
$$= \frac{1}{r}.e^{-i\theta}$$

Therefore, $\boxed{x^{-1} = \frac{1}{r}.e^{-i\theta}}$

Step-4

We can write $x = re^{i\theta} = r\left(\cos\theta + i\sin\theta\right)$

Now

$$\bar{x} = \overline{\left(r\left(\cos\theta + i\sin\theta\right)\right)}$$
$$= r\left(\cos\theta - i\sin\theta\right)$$
$$= re^{-i\theta}$$

Therefore, $\boxed{\bar{x} = re^{-i\theta}}$

Step-5

We have to find the points at which $x^{-1} = \bar{x}$

Let us take $x^{-1} = \bar{x}$

$$\Rightarrow \frac{1}{r}e^{-i\theta} = r.e^{-i\theta}$$
$$\Rightarrow \frac{1}{r} = r$$
$$\Rightarrow r^2 = 1$$
$$\Rightarrow |x|^2 = 1$$

Therefore, on the unit circle, we have $\boxed{x^{-1} = \bar{x}}$.

Step-6

b) We have to sketch the path of the complex number $e^{-(1+i)t}$ at $t = 0$.

At $t = 0$, the complex number

$$e^{(-1+i)t} = e^0$$
$$= 1$$

Step-7

If $t = \frac{\pi}{2}$

$$\Rightarrow e^{(-1+i)\frac{\pi}{2}} = e^{\frac{-\pi}{2}+i\frac{\pi}{2}}$$
$$= e^{\frac{-\pi}{2}}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$
$$= e^{\frac{-\pi}{2}}(i)$$

Step-8

If $t = \pi$

$$\Rightarrow e^{-\pi+i\pi} = e^{-\pi}(\cos\pi + i\sin\pi)$$
$$= e^{-\pi}(-1+0)$$
$$= -e^{-\pi}$$

Step-9

If $t = \dfrac{3\pi}{2}$

Then

$$e^{(-1+i)\frac{3\pi}{2}} = e^{\frac{-3\pi}{2}}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$$
$$= e^{\frac{-3\pi}{2}} i$$

Step-10

If $t = 2\pi$

Then

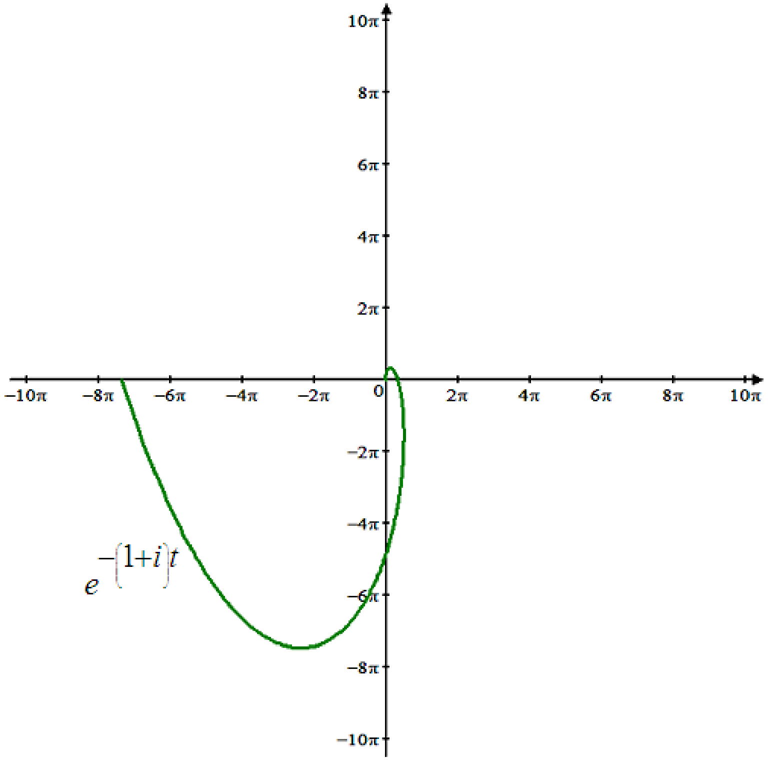
$$e^{(1+i)2\pi} = e^{-2\pi}(\cos 2\pi + i\sin 2\pi)$$
$$= e^{-2\pi}$$

Step-11

The path is a curve passing through the points $(1,0),\left(e^{\frac{-\pi}{2}},0\right),\left(-e^{-\pi},0\right),\left(e^{\frac{-3\pi}{2}},0\right),\left(e^{-2\pi},0\right)$.

The sketch of the path of the given number in the complex plane as t increases from 0 to 2π.

Step-12


10π
8π
6π
4π
2π
0
−2π
−4π
−6π
−8π
−10π
−10π
−8π
−6π
−4π
−2π
2π
4π
6π
8π
10π
$e^{-(1+i)t}$