

Step-1

Let $Ax=b$ has to be solved for three right hand side b . Let the three solutions be as follows:

$$\begin{aligned}x_1 &= (1,1,1) \\x_2 &= (0,1,1) \\x_3 &= (0,0,1)\end{aligned}$$

These solution form a column of matrix X . if matrix $b = (3,5,8)$ solve $Ax=b$. Challenge problem and find matrix A .

Step-2

Let three right hand sides be:

$$\begin{aligned}b_1 &= [3] \\b_2 &= [5] \\b_3 &= [8]\end{aligned}$$

It can be seen that $x_1, x_2, \text{ and } x_3$ contains only elements 0 and 1. So to get these right hand sides matrix A must be row matrix defined as follows:

$$A = [3 \quad 5 \quad 8]$$

Step-3

Now solve $Ax=b$ as follows:

$$\begin{aligned}Ax &= 3x_1 + 5x_2 + 8x_3 \\&= 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + 8 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\&= \begin{bmatrix} 3 \\ 8 \\ 16 \end{bmatrix} \\&\neq b\end{aligned}$$

This calculation shows that right hand side is not equal to $b = (3,5,8)$. This gives the challenge to the solution in the problem.

Step-4

To calculate matrix A let's consider the result found above. Consider $x_1, x_2, \text{ and } x_3$ be the three solutions, then:

$$\begin{aligned}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} 3 \\ 8 \\ 16 \end{bmatrix} \\
&= \begin{bmatrix} 3 \\ 3+5 \\ 3+5+8 \end{bmatrix} \\
&= \begin{bmatrix} b_1 \\ b_1+b_2 \\ b_1+b_2+b_3 \end{bmatrix}
\end{aligned}$$

Step-5

Write the solution in terms of x as follows:

$$\begin{aligned}
x_1 &= 3 \\
x_2 &= 8 \\
&= x_1 + b_2 \\
-x_1 + x_2 &= b_2 \\
x_3 &= 16 \\
&= x_2 + b_3 \\
-x_2 + x_3 &= b_3
\end{aligned}$$

Step-6

Therefore, solution in the form of matrix will be:

$$\begin{aligned}
A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} x_1 \\ -x_1 + x_2 \\ -x_2 + x_3 \end{bmatrix} \\
\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} x_1 \\ -x_1 + x_2 \\ -x_2 + x_3 \end{bmatrix} \\
&= \begin{bmatrix} 3 \\ 8 \\ 16 \end{bmatrix}
\end{aligned}$$

Therefore, following matrix A gives the solution x_1, x_2 , and x_3 .

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$