Step-1

We have to find the dimensions for the given vector spaces.

(a) Let S_1 be the space of all vectors in $\mathbf{R}^4$ whose components add to zero.

$$S_1 = \left\{ (x_1, x_2, x_3, x_4) \Big/ \begin{matrix} x_1 + x_2 + x_3 + x_4 = 0 \\ x_1, x_2, x_3, x_4 \in R \end{matrix} \right\}$$

$x_1 = -x_2 - x_3 - x_4$

Therefore $S_1 = \left\{ (-x_2 - x_3 - x_4, x_2, x_3, x_4) \Big/ x_2, x_3, x_4 \in R \right\}$

$(-x_2 - x_3 - x_4, x_2, x_3, x_4) = x_2(-1,1,0,0) + x_3(-1,0,1,0) + x_4(-1,0,0,1)$

$\{(-1,1,0,0),(-1,0,1,0)(-1,0,0,1)\}$ is a basis for S_1

Therefore dimension of $S_1 = \boxed{3}$

Step-2

(b) Let $I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

To find the null space $IX = 0$

$\Rightarrow x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0$

Null space of $I = \{(0,0,0,0)\}$

Dimension of Null space is $\boxed{0}$

Step-3

(c) Let S_2 be the space of all 4×4 matrices.

$$e_{11} == \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$e_{12} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

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$$e_{44} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ then

Hence $\{e_{ij}\}_{i=1,j=1}^{4}$ is a basis for S_2

Therefore dim $S_2 = \boxed{16}$.