Step-1

Given that every m by n matrix of rank r reduces to (m by r) times (r by n):

$$A = (\text{pivot columns of } A)(\text{first } r \text{ rows of } R)$$

We have to write the 3 by 4 matrix A at the start of this section as the product of the 3 by 2 matrix from the pivot columns and the 2 by 4 matrix from R:

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$$

Step-2

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$$

$$\xrightarrow{R_3 + R_1} \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ 0 & 0 & 6 & 6 \end{bmatrix}$$

$$\xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 6 & 6 \end{bmatrix}$$

Step-3

$$\xrightarrow{R_3 - 2R_1} \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 - 3R_2} \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore 1,3 are pivot columns.

Rank of A is 2.

Step-4

Now $\left(\text{pivot columns of A}\right)\left(\text{first r rows of R}\right)$

$$=\begin{bmatrix} 1 & 3 \\ 2 & 9 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}=A$$

Therefore $A=\left(\text{pivot columns of } A\right)\left(\text{first } r \text{ rows of } R\right)$