

Step-1

We have to verify directly that reflection matrices satisfy $H^2 = I$ from $c^2 + s^2 = 1$.

Step-2

We know that the Regulation matrix H is $H = \begin{bmatrix} 2c^2 - 1 & 2cs \\ 2cs & 2s^2 - 1 \end{bmatrix}$.

Now

$$\begin{aligned} H^2 &= \begin{bmatrix} 2c^2 - 1 & 2cs \\ 2cs & 2s^2 - 1 \end{bmatrix} \begin{bmatrix} 2c^2 - 1 & 2cs \\ 2cs & 2s^2 - 1 \end{bmatrix} \\ &= \begin{bmatrix} 4c^4 + 1 - 4c^2 + 4c^2s^2 & 4c^3s - 2cs + 4cs^3 - 2cs \\ 4c^3s - 2cs + 4cs^3 - 2cs & 4c^2s^2 + 4s^4 + 1 - 4s^2 \end{bmatrix} \\ &= \begin{bmatrix} 4c^2[c^2 + s^2] + 1 - 4c^2 & 4cs[c^2 + s^2] - 4cs \\ 4cs[c^2 + s^2] - 4cs & 4s^2[c^2 + s^2] + 1 - 4s^2 \end{bmatrix} \end{aligned}$$

Step-3

Continuation to the above

$$\begin{aligned} &= \begin{bmatrix} 4c^2[1] + 1 - 4c^2 & 4cs[1] - 4cs \\ 4cs[1] - 4cs & 4s^2[1] + 1 - 4s^2 \end{bmatrix} \left[\text{Since } c^2 + s^2 = 1 \right] \\ &= \begin{bmatrix} 4c^2 + 1 - 4c^2 & 4cs - 4cs \\ 4cs - 4cs & 4s^2 + 1 - 4s^2 \end{bmatrix} \left[\text{Since } c^2 + s^2 = 1 \right] \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Hence $\boxed{H^2 = I}$