Step-1

Let A and B be any two 5×5 permutation matrices.

Consider $\alpha A + \beta B = 0$, where 0 is the zero matrix.

Thus, the matrix $\alpha A + \beta B$ has each entry 0. Now αA can be thought of as the matrix, obtained from A by replacing each 1 by *and keeping 0's as they are. Similarly,* βB *can be thought of as the matrix, obtained from B by replacing each 1 by and keeping 0's as they are.*

Step-2

It is clear that $a_{ij} \neq b_{ij}$ *, for some i and j.*

Therefore, if not both and are zero, then $\alpha A + \beta B \neq 0$.

Therefore, A and B are linearly independent matrices. But A and B are any two matrices. Therefore, all the matrices are linearly independent.

Step-3

These 120 matrices do not span the space of all 5 by 5 matrices.

To understand this, let us show that the following matrix A cannot be expressed as any linear combination of these 120 matrices.

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

If we want to write A as a linear combination of some of the 120 matrices, it is obvious that one of the matrices must have the entry in the first row and first column as 1. But the inclusion of such a matrix will have entry 1 in each of the 5 rows. When we try to cancel these 1's by having another matrix, multiplied by â€“1, this adds â€“1 somewhere. This process goes on and on.

Thus, by no way it is possible to express A as a linear combination of any of the 120 matrices.