

## Step-1

(a)

Given statement is every basis for  $\mathbf{S}$  can be extended to a basis for  $\mathbf{R}^6$  by adding one more vector.

The basis for  $\mathbf{S}$  is a linearly independent set and we know that any linearly independent set in  $\mathbf{R}^6$  can be extended to a basis, by adding more vectors if necessary.

As,  $\mathbf{S}$  is a five dimensional subspace of six dimensional vector space  $\mathbf{R}^6$ , by adding one more vector, every basis for  $\mathbf{S}$  can be extended to a basis for  $\mathbf{R}^6$ .

Hence the given statement is true.

## Step-2

(b)

Given statement that every basis for  $\mathbf{R}^6$  can be reduced to a basis for  $\mathbf{S}$  by removing one more vector is false.

For example,

$$\beta = \left\{ \begin{array}{l} (1, 0, 0, 0, 0, 0), (0, 1, 0, 0, 0, 0), (0, 0, 1, 0, 0, 0), (0, 0, 0, 1, 0, 0) \\ (0, 0, 0, 0, 1, 0), (0, 0, 0, 0, 0, 1) \end{array} \right\} \text{ is a basis for } \mathbf{R}^6$$

Let  $\mathbf{S}$  be a subspace spanned by  $(1, 2, 3, 4, 5, 6)$ .

## Step-3

Therefore  $\mathbf{S}$  is a subspace for  $\mathbf{R}^6$ . But it is not by removing one of the bases

Vector, remaining those cannot span  $\mathbf{S}$ . (Infact no vector in the basis is longs to  $\mathbf{S}$ )

Therefore the given statement is false.