

Step-1

$$(a) \quad \frac{dy}{dx} - 2y = 0$$

$$\frac{dy}{dx} = 2y$$

$$\Rightarrow \frac{dy}{2y} = dx$$

Integrating both sides

$$\int \frac{1}{2y} dy = \int dx$$

$$\Rightarrow \log y = 2x + C$$

$$\Rightarrow y = e^{2x+C} = e^C \cdot e^{2x}$$

Therefore $\Rightarrow y = ke^{2x}$ where $e^C = K$

The basis for functions $\{e^{2x}\}$

Dimension of the space is infinite while $e^{2x} = \sum_0^{\infty} \frac{(2x)^n}{n!}$ and each x in the summand is spanned by one dimension.

Step-2

$$(b) \quad \frac{dy}{dx} - \frac{y}{x} = 0$$

$$\Rightarrow xdy - ydx = 0$$

$$\Rightarrow \frac{dy}{y} - \frac{dx}{x} = 0$$

Integrating both sides

$\log y - \log x = \log c$ where $\log c$ is constant

$$\Rightarrow \log \frac{y}{x} = \log c$$

$$\Rightarrow \frac{y}{x} = c$$

$$\Rightarrow y = cx$$

The basis for the functions $\{x\}$

Dimension of the space = 1.