Step-1

We have to describe all 2 by 3 matrices A_1, A_2 with row echelon forms R_1 and R_2 such that $R_1 + R_2$ is the row echelon form of $A_1 + A_2$. We have to find that is it true that $R_1 = A_1$ and $R_2 = A_2$ in this case.

Step-2

Let A_1, A_2 be 2 by 3 matrices A_1 and A_2 with row echelon forms are R_1 and R_2 such that $R_1 + R_2$ is the row echelon form of $A_1 + A_2$.

If possible $R_1 \neq A_1$

Then R_1 is 2×3 matrix with rank either 1 or 2.

R_2 is 2×3 matrix with rank either 1 or 2.

Step-3

Then $R_1 + R_2$ is not in the form of row echelon form of $A_1 + A_2$.

This is a contradiction.

Therefore $R_1 = A_1$, similarly $R_2 = A_2$.

Therefore the given statement is true.