Step-1

(a)

A square matrix has no free variables.

For example, $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$\xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

Step-2

Second column is free column to find special solution for $Ax = 0$.

$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

x_1 Is pivot, x_2 is free variable.

Hence the given statement is false.

Step-3

(b)

An invertible matrix has no free variables.

The reduced row echelon form of an invertible matrix is identity matrix.

Therefore there are no free variables.

So the given statement is true.

Step-4

(c)

An *m* by *n* matrix has no more than *n* pivot variables.

There are only *n* columns in the *m* by *n* matrix. Hence it does not have more than *n* pivot variables.

So the given statement is true

Step-5

(d)

An m by n matrix has no more than m pivot variables.

There are only m columns in the m by n matrix. Hence it does not have more than m pivot variables.

So the given statement is true