

## Step-1

(i)

Given system is

$$u + v + w = 6$$

$$u + 2v + 2w = 11$$

$$2u + 3v - 4w = 3$$

We have to find the solution to this system by applying elimination.

## Step-2

Given system can be written as

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 2 & 11 \\ 2 & 3 & -4 & 3 \end{bmatrix}$$

apply  $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - 2R_1$

$$\square \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 1 & -6 & -9 \end{bmatrix}$$

apply  $R_3 \rightarrow R_3 - R_2$

$$\square \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & -7 & -14 \end{bmatrix}$$

which is upper triangular form.

that is  $u + v + w = 6$

$$v + w = 5$$

$$-7w = -14$$

## Step-3

By back-ward substitution, we have

$$-7w = -14$$

$$\Rightarrow \boxed{w = 2}$$

$$v + w = 5$$

$$\Rightarrow v + 2 = 5$$

$$\Rightarrow \boxed{v = 3}$$

$$u + v + w = 6$$

$$\Rightarrow u + 3 + 2 = 6$$

$$\Rightarrow \boxed{u = 1}$$

Solutions are  $\boxed{u = 1, v = 3, w = 2}$

## Step-4

(ii)

Given system is

$$u + v + w = 7$$

$$u + 2v + 2w = 10$$

$$2u + 3v - 4w = 3$$

We have to find the solution to this system by applying elimination.

## Step-5

Given system can be written as

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 1 & 2 & 2 & 10 \\ 2 & 3 & -4 & 3 \end{bmatrix}$$

apply  $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & -6 & -11 \end{bmatrix}$$

apply  $R_3 \rightarrow R_3 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -7 & -11 \end{bmatrix}$$

which is upper triangular form.

that is  $u + v + w = 7$

$$v + w = 3$$

$$-7w = -14$$

## Step-6

By back-ward substitution, we have

$$-7w = -14$$

$$\Rightarrow \boxed{w = 2}$$

$$v + w = 3$$

$$\Rightarrow v + 2 = 3$$

$$\Rightarrow \boxed{v = 1}$$

$$u + v + w = 7$$

$$\Rightarrow u + 1 + 2 = 7$$

$$\Rightarrow \boxed{u = 4}$$

solution are  $\boxed{u = 4, v = 1, w = 2}$