Step-1

If A is an m by n matrix, using row operations, we can reduce A to r non zero rows and m â€“ r zero rows. Then

1. $C(A) =$ Column space of A; dimension r

2. $N(A)$ = null space of A; dimension n â€“ r

3. $C(A^T)$ = row space of A; dimension r

4. $N(A^T)$ = left null space of A; dimension m â€“ r

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -1 & -2 \end{bmatrix}$$

Using the elementary row operations on A, we can reduce it to the equivalent matrices.

$$A \xrightarrow[R_3 - R_1]{R_2 + R_1} \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -3 & -9 & -3 \end{bmatrix}$$

$$\xrightarrow{R_3 + 3R_2} \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step-2

We see that one row is made zero by row operations and two rows are left with at least one non zero entry.

So, the dimension of row space of A is $\dim C(A^T) = 2$ and $\dim N(A) = 5 - 2 = 3$

Consequently, dim $C(A) = 2$

But the number of rows of A = 3

From this, $\dim N(A^T) = 1$