

Step-1

(a) Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Step-2

Here the columns of matrix A are dependent but matrix A has only one row, so row is independent.

Step-3

Therefore, if the columns of a matrix are dependent, so are the rows, is ☐ false.

Step-4

(b) Consider the following matrix

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

The row space of B contains multiples of $\begin{bmatrix} 1 & 0 \end{bmatrix}$ while the column space of B contains multiples of $\begin{bmatrix} 1 & 1 \end{bmatrix}$.

Therefore, the column space of a 2 by 2 matrix is the same as its row space is ☐ false.

Step-5

(c) We know that, the dimension of column space equals to the dimension of row space and both equal the rank of the matrix.

Therefore, the column space of a 2 by 2 matrix has the same dimension as its row space is ☐ true.

Step-6

(d) We know that, the columns that contain pivots are a basis for the column space.

Since the columns might be linearly dependent, so the columns of a matrix are a basis for the column space is ☐ false.