

Step-1

To find the square and the square root of w_{128} , the primitive 128th root of 1.

It is already known that;

$$\begin{aligned}w_n &= e^{2\pi i/n} \\&= \cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}\end{aligned}$$

This implies;

$$\begin{aligned}w_{128} &= \cos \frac{2\pi}{128} + i \sin \frac{2\pi}{128} \\&= \cos \frac{\pi}{64} + i \sin \frac{\pi}{64} \\&= e^{i\pi/64}\end{aligned}$$

Hence $\boxed{(w_{128})^2 = e^{i\pi/32}}$

Step-2

And,

$$\begin{aligned}\sqrt{w_{128}} &= \sqrt{\cos \frac{\pi}{64} + i \sin \frac{\pi}{64}} \\&= \sqrt{e^{i\pi/64}} \\&= \boxed{e^{i\pi/128}}\end{aligned}$$

Step-3

The primitive 128th root of 1 is $\cos \frac{2\pi}{128} + i \sin \frac{2\pi}{128}$

$$= \cos \frac{\pi}{64} + i \sin \frac{\pi}{64}$$

Hence, the primitive 128th root of 1 = $\boxed{e^{i\pi/64}}$