Step-1

Now let y be any vector from $\mathbf{S}^{\perp}$.

By definition of $\mathbf{S}^{\perp}$, y is perpendicular to every vector from **S**. Therefore, the projection of y into **S** must be a zero vector.

That is, $Py = 0$.

This can also be written as $Py = 0y$.

This shows that each vector from $\mathbf{S}^{\perp}$ is also an eigenvector of the projection matrix P.