

## Step-1

The quantity  $V'(x)$  gives the slope of the graph.

From  $x = 0$  to  $x = \frac{1}{2}$ ,  $V'(x) = 2$ .

From  $x = \frac{1}{2}$  to  $x = 1$ ,  $V'(x) = -2$ .

Thus,

$$\begin{aligned}\int (V')^2 dx &= \int_0^1 (V')^2 dx \\ &= \int_0^{\frac{1}{2}} (2)^2 dx + \int_{\frac{1}{2}}^1 (-2)^2 dx \\ &= 4 \left( \frac{1}{2} \right) + 4 \left( \frac{1}{2} \right) \\ &= 4\end{aligned}$$

Thus,  $A = 4$ .

## Step-2

Let  $M = \int V^2 dx$ .

From  $x = 0$  to  $x = \frac{1}{2}$ ,  $V(x) = 2x$ .

From  $x = \frac{1}{2}$  to  $x = 1$ ,  $V(x) = -2x$ .

Thus,

$$\begin{aligned}\int (V)^2 dx &= \int_0^1 (V)^2 dx \\ &= \int_0^{\frac{1}{2}} (2x)^2 dx + \int_{\frac{1}{2}}^1 (-2x)^2 dx \\ &= 4 \left[ \frac{x^3}{3} \right]_0^{\frac{1}{2}} + 4 \left[ \frac{x^3}{3} \right]_{\frac{1}{2}}^1\end{aligned}$$

$$\begin{aligned}\int (V)^2 \, dx &= \frac{4}{24} + \frac{4}{24} \\ &= \frac{8}{24} \\ &= \frac{1}{3}\end{aligned}$$

Therefore,  $M = \frac{1}{3}$ .

### Step-3

Consider the following:

$$\begin{aligned}\frac{A}{M} &= \frac{4}{\frac{1}{3}} \\ &= 12\end{aligned}$$

We know that  $\pi^2 = 9.8696$

Thus,  $\lambda = \frac{A}{M}$  is larger than the true eigenvalue  $\lambda = \pi^2$ .