

Step-1

Consider system as;

$$ax + by = f$$

$$cx + dy = g$$

The objective is to find formula for the second pivot, find y .

Step-2

This system can be written as;

$$\begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix}$$

Subtract $\frac{c}{a}$ times the first row from the second row to get;

$$\begin{bmatrix} a & b & f \\ 0 & d - \frac{bc}{a} & g - \frac{fc}{a} \end{bmatrix}$$

This is upper triangular form.

Therefore, the multiple is $\boxed{l = \frac{c}{a}}$ and, the second pivot is $\boxed{d - \frac{bc}{a}}$ (if $ad \neq bc$).

Step-3

Now, the above triangular system is;

$$ax + by = f$$

$$\left(d - \frac{bc}{a}\right)y = \left(g - \frac{fc}{a}\right)$$

Apply back-substitution, and get;

$$\left(\frac{ad - bc}{a}\right)y = \left(\frac{ag - fc}{a}\right)$$

$$y = \frac{ag - fc}{a} \cdot \frac{a}{ad - bc}$$

$$y = \frac{ag - fc}{ad - bc}$$

Hence, the value of y is $\frac{fc - ag}{ad - bc}$.

Step-4

Provided,

$$\begin{aligned} a &\neq 0 \\ ad - bc &\neq 0 \end{aligned}$$

If $ad - bc = 0$, then the second pivot becomes;

$$\begin{aligned} \frac{ad - bc}{a} &= \frac{0}{a} \\ &= 0 \end{aligned}$$

That is, the second pivot is missing when $ad - bc = 0$.