Step-1

Given matrix is $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$.

We have to find the norm and condition number from the square roots of $\lambda_{\max}\left(A^T A\right)$ and $\lambda_{\min}\left(A^T A\right)$ of the given matrix.

Step-2

We have $\lambda_{\max}\left(A^T A\right) = \|A\|^2$ â€¦â€¦ (1)

So, the root of $\lambda_{\max}\left(A^T A\right)$ is nothing but the norm of the matrix A.

Also, $\lambda_{\min}\left(A^T A\right) = \dfrac{1}{\left\|A^{-1}\right\|^2}$

In other words,

The reciprocal of the square root of $\lambda_{\min}\left(A^T A\right)$ is nothing but the norm of A^{-1}.

Further, the ***conditional number*** of A is

$$c = \|A\|\left\|A^{-1}\right\|$$

$$= \sqrt{\lambda_{\max}\left(A^T A\right) \times \frac{1}{\lambda_{\min}\left(A^T A\right)}}$$ â€¦â€¦ (2)

Step-3

Let $A = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$

So, $A^T = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$ and $A^T A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ are the positive definite matrices.

So, we can proceed with the details of the positive definite matrix.

The characteristic equation of $A^T A$ is

$$\left|A^T A - \lambda I\right| = 0$$
$$\Rightarrow \begin{vmatrix} 4-\lambda & 0 \\ 0 & 4-\lambda \end{vmatrix} = 0$$
$$\Rightarrow (4-\lambda)(4-\lambda) = 0$$
$$\Rightarrow \lambda = 4, 4$$

So, the eigenvalues are 4, 4.

Step-4

Now $\lambda_{\max}\left(A^T A\right) = 4$, and so, $\|A\| = 2$ by (1)

And $\lambda_{\min}\left(A^T A\right) = 4$

In view of (2) above, we get $c = \sqrt{4 \times \frac{1}{4}}$

Since the norm of a vector is non negative, we consider the positive square root of this and thus, $c = 1$.

Hence the norm of the given matrix $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$ is 4 and the condition is $\boxed{c = 1}$.

Step-5

Given that $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

So, $A^T = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ and $A^T A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

The characteristic equation of $A^T A$ is

$$\left|A^T A - \lambda I\right| = 0$$
$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$
$$\Rightarrow (1-\lambda)(1-\lambda) - 1 = 0$$
$$\Rightarrow \lambda^2 - 2\lambda = 0$$
$$\Rightarrow \lambda = 0, 2$$

So, the eigenvalues are 0, 2

Step-6

Now we have $\lambda_{\max}\left(A^T A\right)=2,$

So by (1), we get $\|A\|=\sqrt{2}$

And $\lambda_{\min}\left(A^T A\right)=0$

In view of (2) above, we get $c=\sqrt{4\times\frac{1}{0}}$

This is an infinite number and so, a singular value.

Hence the norm of the given matrix $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ is $\boxed{\sqrt{2}}$ and the condition number is an infinite number.

Step-7

Given that $A=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$

So, $A^T=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$ and $A^T A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$ a positive definite matrix and so, we can proceed with the details of the positive definite matrix.

The characteristic equation of $A^T A$ is

$$\left|A^T A-\lambda I\right|=0$$
$$\Rightarrow\begin{vmatrix}2-\lambda & 0\\ 0 & 2-\lambda\end{vmatrix}=0$$
$$\Rightarrow(2-\lambda)(2-\lambda)=0$$
$$\Rightarrow\lambda=2,2$$

So, the eigenvalues are 2, 2.

Step-8

Now $\lambda_{\max}\left(A^T A\right)=2$ and so, $\|A\|=\sqrt{2}$ by (1)

And $\lambda_{\min}\left(A^T A\right)=2$

In view of (2) above, we get $c=\sqrt{2\times\frac{1}{2}}$

Since the norm of a vector is non negative, we consider the positive square root of this and thus, $c = 1$.

Hence the norm of the given matrix $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ is $\boxed{\sqrt{2}}$ and the condition number is $\boxed{c=1}$.