

## Step-1

### Properties of the determinant

1. The determinant of the identity matrix is 1
2. The determinant changes sign when two rows are exchanged
3. The determinant depends linearly on the first row.
4. If two rows of  $A$  are equal, then  $\det A = 0$
5. Subtracting a multiple of one row from another row leaves the same determinant.
6. If  $A$  has a row of zeros, then  $\det A = 0$
7. If  $A$  is triangular, then  $\det A$  is the product  $a_{11}a_{22}a_{33}\dots a_{nn}$  of the diagonal entries.
8. If  $A$  is singular, then  $\det A = 0$ . If  $A$  is invertible, then  $\det A \neq 0$ .
9. The determinant of  $AB$  is product of  $\det A$  times  $\det B$
10. The transpose of  $A$  has the same determinant as  $A$  itself;  $\det A^T = \det A$

## Step-2

We have

$$A = \begin{bmatrix} 3 & 3 & 4 \\ 6 & 8 & 7 \\ -3 & 5 & -9 \end{bmatrix} = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Here

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix} \quad \text{And} \quad U = \begin{bmatrix} 3 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

## Step-3

Both  $L$  and  $U$  are triangular matrices

$$\begin{aligned}\det L &= (1)(1)(1) \\ &= 1 \\ \det U &= (3)(2)(-1) \\ &= -6\end{aligned}$$

## Step-4

Therefore

$$\begin{aligned}\det A &= \det L \times \det U \\ &= (1)(-6) \\ &= -6\end{aligned}$$

## Step-5

And

$$\begin{aligned}A^{-1} &= (LU)^{-1} \\ &= U^{-1}L^{-1} \\ \det U^{-1}L^{-1} &= \det A^{-1} \\ &= \frac{1}{\det A} \\ &= \frac{-1}{6}\end{aligned}$$

## Step-6

Further

$$\begin{aligned}U^{-1}L^{-1}A &= A^{-1}A = I \\ \det(U^{-1}L^{-1}A) &= \det I = 1\end{aligned}$$

Thus

$\begin{aligned}\det L &= 1 \\ \det U &= -6 \\ \det A &= -6 \\ \det U^{-1}L^{-1} &= \frac{-1}{6} \\ \det(U^{-1}L^{-1}A) &= 1\end{aligned}$
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