

Step-1

We need to choose the value of θ , so that $(2,1)$ entry in the matrix $R = P \frac{1}{3} A$ will be 0. We will consider only PA .

Consider

$$\begin{aligned} PA &= \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -\cos \theta - 2 \sin \theta & 2 \cos \theta + \sin \theta & 2 \cos \theta - 2 \sin \theta \\ -\sin \theta + 2 \cos \theta & 2 \sin \theta - \cos \theta & 2 \sin \theta + 2 \cos \theta \\ 2 & 2 & -1 \end{bmatrix} \end{aligned}$$

Step-2

The $(2,1)$ entry corresponds to the element in the 2nd row and 1st column of the product. Since, this has to be zero, we want $-\sin \theta + 2 \cos \theta = 0$.

Consider

$$\begin{aligned} -\sin \theta + 2 \cos \theta &= 0 \\ 2 \cos \theta &= \sin \theta \\ 2 &= \tan \theta \\ \theta &= \tan^{-1}(2) \end{aligned}$$

Therefore, $\theta = 63.43^\circ$.