Step-1

Given that the matrix $A=\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ is positive definite

We need to test $A^{-1}=\begin{bmatrix} p & q \\ q & r \end{bmatrix}$ for positive definiteness.

So that $a>0$ and $ac-b^2>0$.

Also it implies that $c>0$.

Step-2

Now,

$$A^{-1}=\frac{1}{ac-b^2}\begin{pmatrix} c & -b \\ -b & a \end{pmatrix}$$

$$=\begin{pmatrix} \frac{c}{ac-b^2} & \frac{-b}{ac-b^2} \\ \frac{-b}{ac-b^2} & \frac{a}{ac-b^2} \end{pmatrix}$$

$$=\begin{bmatrix} p & q \\ q & r \end{bmatrix}$$

Step-3

As $c>0$ and $ac-b^2>0,$

So, $A=\frac{c}{ac-b^2}>0$,

And $AC-B^2=\left(\frac{c}{ac-b^2}\right)\left(\frac{a}{ac-b^2}\right)-\left(\frac{-b}{ac-b^2}\right)^2$

$$=\frac{ac}{\left(ac-b^2\right)^2}-\frac{b^2}{\left(ac-b^2\right)^2}$$

$$=\frac{1}{ac-b^2}$$

$>0 \quad \left(\text{ since } ac-b^2>0\right)$

Therefore, $\boxed{\text{the matrix } A^{-1} = \begin{bmatrix} p & q \\ q & r \end{bmatrix} \text{ is also positive definite.}}$