

## Step-1

Given the matrix  $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ , with Eigen values  $\lambda_1 = 1$  and  $\lambda_2 = 3$  and the inverse of this matrix  $A^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ . The initial guess is  $u_0 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

We need to compare three inverse power steps and need to find the limiting vector.

The inverse power method is  $u_{k+1} = A^{-1}u_k$ ,

When  $k = 0$  then,

$$\begin{aligned} u_1 &= A^{-1}u_0 \\ &= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} 10 \\ 11 \end{bmatrix} \\ &= \begin{bmatrix} 3.333 \\ 3.667 \end{bmatrix} \end{aligned}$$

Therefore,  $u_1 = \begin{bmatrix} 3.333 \\ 3.667 \end{bmatrix}$ .

## Step-2

When  $k = 1$  then,

$$\begin{aligned} u_2 &= A^{-1}u_1 \\ &= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3.333 \\ 3.667 \end{bmatrix} \\ &= \begin{bmatrix} 3.444 \\ 3.556 \end{bmatrix} \end{aligned}$$

Therefore,  $u_2 = \begin{bmatrix} 3.444 \\ 3.556 \end{bmatrix}$ .

## Step-3

When  $k = 2$  then,

$$\begin{aligned}
 u_3 &= A^{-1}u_2 \\
 &= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3.444 \\ 3.556 \end{bmatrix} \\
 &= \begin{bmatrix} 3.481 \\ 3.518 \end{bmatrix}
 \end{aligned}$$

Therefore, 
$$u_3 = \begin{bmatrix} 3.481 \\ 3.518 \end{bmatrix}.$$

## Step-4

We follow that (1, 1) and (1, 2) entries of  $u_k$  approaches equally to 3.5 as  $k$  becoming alarmingly high and tends to infinity.

So,

$$\begin{aligned}
 \lim_{k \rightarrow \infty} u_k &= \begin{bmatrix} 3.5 \\ 3.5 \end{bmatrix} \\
 &= 3.5 \begin{bmatrix} 1 \\ 1 \end{bmatrix}
 \end{aligned}$$

Therefore, the multiple of the Eigen vector  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  is  $\boxed{3.5}$ .