Step-1

(a)

Let A be a triangular matrix.

To find the inverse of triangular matrix is triangular or not.

Let $A=\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$ be a triangular matrix.

Then

$$\begin{aligned} A^{-1} &= \frac{1}{2(1)-0(3)}\begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Hence, $\boxed{A^{-1}\text{is also a triangular matrix}}$

Step-2

(b)

Let A be a symmetric matrix.

To find the inverse of symmetric matrix is symmetric or not.

The inverse of symmetric matrices is also a symmetric.

Let $A=\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, which a symmetric matrix

Since,

$$\begin{aligned} A^{T} &= A \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \end{aligned}$$

Now

$$A^{-1} = \frac{1}{2(3)-0(0)}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$
$$= \frac{1}{6}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

Now it is clear that $\left(A^{-1}\right)^T = A^{-1}$

Hence, $\boxed{A^{-1} \text{is also a symmetric matrix}}$

Step-3

(c)

Let A be a tridiagonal matrix.

To find the inverse of tridiagonal matrix is tridiagonal or not.

The inverse of tridiagonal matrices is also tridiagonal.

Let $A = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$ be a tridiagonal matrix.

Then,

$$A^{-1} = \frac{1}{0(0)-2(3)}\begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix}$$
$$= \frac{1}{-6}\begin{bmatrix} 0 & -2 \\ -3 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & \frac{1}{3} \\ \frac{1}{2} & 0 \end{bmatrix}$$

Hence, $\boxed{A^{-1} \text{is also a tridiagonal matrix}}$ since its anti diagonal consists of nonzero elements.

Step-4

(d)

Let A be a matrix in which all the entries are whole numbers.

Find all the entries of inverse of A.

The inverse of the matrix A in which all entries is whole numbers need not consists all whole numbers.

For instance let $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

Step-5

Then,

$$\begin{aligned} A^{-1} &= \frac{1}{4(5)-2(3)}\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} \\ &= \frac{1}{20-6}\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} \\ &= \frac{1}{14}\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{5}{14} & -\frac{3}{14} \\ -\frac{1}{7} & \frac{2}{7} \end{bmatrix} \end{aligned}$$

Here the elements of A^{-1} are rational numbers.

Hence, $\boxed{\text{all entries of } A^{-1} \text{need not to be whole numbers}}$

Step-6

(e)

Let A be a matrix in which all entries are fractions.

The inverse of the matrices A in which all entries of A are rational numbers consists of all rational numbers.

Since every whole number is also a rational number.

So it follows from the above example that

Hence, $\boxed{A^{-1}\text{is a matrix in which all entries are rational numbers}}$