

## Step-1

The columns of the matrix  $A$  are:  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .

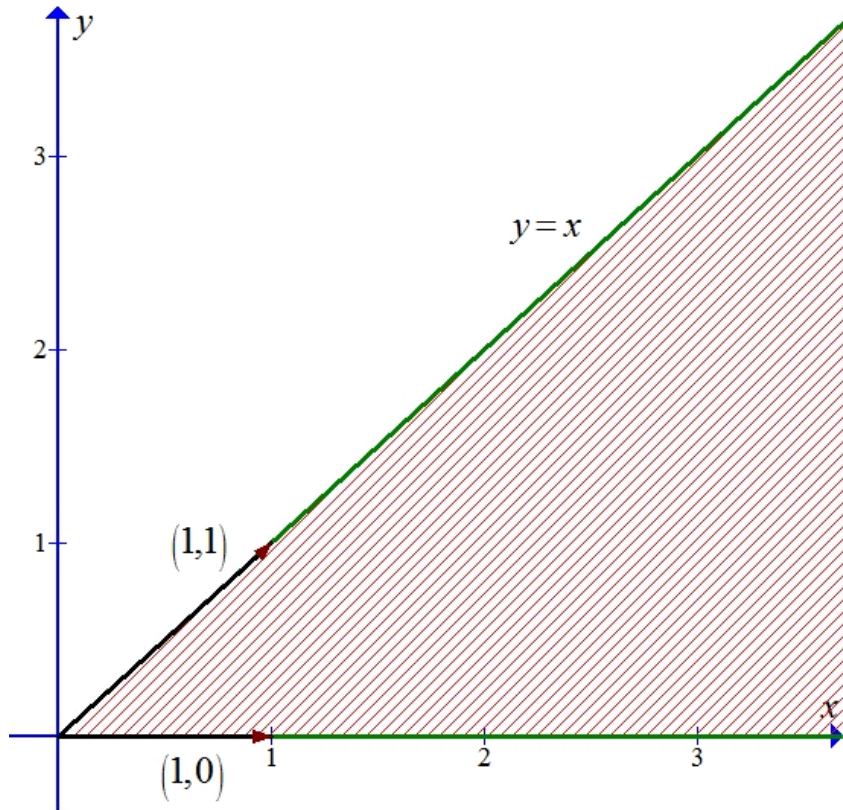
Thus, the nonnegative combinations of the columns of the matrix  $A$  is given by

$$\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Here,  $\alpha \geq 0$  and  $\beta \geq 0$ .

## Step-2

The vector  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  is along the  $x$ -axis and the vector  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  makes the angle of  $45^\circ$  with the positive direction of  $x$ -axis. Thus, the space is the region in the first quadrant between the  $x$ -axis and the line  $y = x$ . The region is as drawn below:



### Step-3

Consider the vector  $b = (3, 2)$ , which lies inside the space.

Let  $Ax = b$ , where  $x = (x_1, x_2)$ . Then we have

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$
$$\begin{bmatrix} x_1 + x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Therefore,  $x_2 = 2$  and this implies that  $x_1 = 1$ .

Therefore,  $\boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$ .

### Step-4

Suppose  $b = (0, 1)$ . Let us write,  $Ax = b$ . This gives,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\begin{bmatrix} y_1 + y_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Therefore,  $y_2 = 1$  and this implies that  $y_1 = -1$ .

Therefore,  $\boxed{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$ .