Step-1

Consider the matrices $A=\begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix}, B=\begin{pmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{pmatrix}$

The objective is to find the range of a, b such that these matrices become positive definite matrices.

Step-2

The following test is necessary and sufficient condition for the real symmetric matrix A to be positive definite:

The entire upper left sub matrices A_k have positive determinants.

Step-3

Take matrix $A=\begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix}$

Use the test: The upper left submatrices has positive determinants, to find the range of a for the matrix A.

For positive definiteness of the matrix A, the following conditions should be satisfied.

$|a|>0$, $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix}>0$ and $\begin{vmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{vmatrix}>0$

Now, find the determinant $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix}>0$ as follows.

$$\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix}>0$$
$$a^2-4>0$$
$$(a-2)(a+2)>0$$
$$a>2, a<-2$$

So, here obtained that: $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix}>0$, when $a>2$ or $a<-2$.

But here take the range, $a>2$ because $|a|>0$

Now, find the determinant $\begin{vmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{vmatrix} > 0$ as follows.

On solving:

$$\begin{aligned} a\left(a^2-4\right)-2(2a-4)+2(4-2a) &> 0 \\ a^3-12a+16 &> 0 \\ (a-2)(a+4) &> 0 \end{aligned}$$

Which will give: $a > 2$ or $a < -4$.

But here take the range, $a > 2$ because $|a| > 0$

Hence, the matrix A is positive definite if $\boxed{a > 2}$.

Step-4

Similarly, for the matrix $B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{pmatrix}$

For positive definiteness of the matrix A, the following conditions should be satisfied.

$|1| > 0$, $\begin{vmatrix} 1 & 2 \\ 2 & b \end{vmatrix} > 0$ and $\begin{vmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{vmatrix} > 0$

Now, find the determinant $\begin{vmatrix} 1 & 2 \\ 2 & b \end{vmatrix} > 0$ as follows.

$$\begin{aligned} \begin{vmatrix} 1 & 2 \\ 2 & b \end{vmatrix} &> 0 \\ b-4 &> 0 \\ b &> 4 \end{aligned}$$

Now, find the determinant $\begin{vmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{vmatrix} > 0$ as follows.

On solving:

$$\begin{aligned}1(7b-64)-2(14-32)+4(16-4b)&>0\\ -9b+36&>0\\ 9b&<36\\ b&<4\end{aligned}$$

But here we get two range, $b>4$ and $b<4$, therefore, there is no common range:

Hence, the matrix B can $\boxed{\text{never be positive definite}}$.