## Step-1

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Given that the voltages of the cities are $x_B, x_C, x_S$.

With unit resistances between the cities, the three currents are in $y$ given as

$y = Ax$ as $\begin{bmatrix} y_{BC} \\ y_{CS} \\ y_{BS} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_B \\ x_C \\ x_S \end{bmatrix}$

Here the matrix $A$ is $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

The transpose of $A$ is $A^T = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$

## Step-2

(a) We have to find the total currents $A^T y$ out of the three cities.

The total currents out of the three cities is $A^T y$.

Therefore,

$$A^T y = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_{BC} \\ y_{CS} \\ y_{BS} \end{bmatrix}$$
$$= \begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}$$

Hence the total currents in three cities is $\boxed{\begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}}$.

## Step-3

(b) We have to verify that $(Ax)^T y$ agrees with $x^T(A^T y)$.

Now

$$\begin{aligned}(Ax)^T y &= (x^T A^T) y && \left(\text{Since } (AB)^T = B^T A^T\right)\\ &= x^T (A^T y) && (\text{By Associative property})\end{aligned}$$

Therefore, $(Ax)^T y$ agrees with $x^T(A^T y)$.

Now

$$\begin{aligned}x^T(A^T y) &= \begin{bmatrix} x_B & x_C & x_S \end{bmatrix}\begin{bmatrix} y_{BC} + y_{BS} \\ -y_{BC} + y_{CS} \\ -y_{CS} - y_{BS} \end{bmatrix}\\ &= \left[x_B y_{BC} + x_B y_{BS} - x_C y_{BC} + x_C y_{CS} - x_A y_{CS} - x_A y_{BS}\right]\end{aligned}$$

Therefore, $x^T(A^T y) = \left[x_B y_{BC} + x_B y_{BS} - x_C y_{BC} + x_C y_{CS} - x_A y_{CS} - x_A y_{BS}\right]$