Step-1

Linear dependence:

A set of vectors is said to be linearly dependent if one vector in the set can be written as linear combination of the others.

Step-2

Given;

$$A=\begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \end{bmatrix}$$

Assume;

$$A=\begin{bmatrix} a & b & c & d & e \\ p & q & r & s & t \\ 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & z & f \\ 0 & 0 & 0 & g & h \end{bmatrix}$$

$a,b,c,d,e,p,q,r,s,t,x,y,z,f,g,h$ Can be any real numbers

Step-3

(a)

The objective is to give the reason for linear dependence of the above matrix.

The last three rows of A are **linearly dependent**.

Since if any of x,z,g is non zero, remaining two places can be made zero by suitable subtraction of a constant multiple of the vector from remaining vectors.

For example if $x \neq 0$, apply row operations as row 4 goes to $row4+\left(-x^{-1}z\right)row3$ and row 5 goes to $row4+\left(-x^{-1}z\right)\text{row 3}$ and row 5 goes to $row5+\left(-x^{-1}g\right)\text{row 3}$,given matrix is reduced in the form;

$$B=\begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \\ 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & 0 & \otimes \\ 0 & 0 & 0 & 0 & \otimes \end{bmatrix}$$

Once again if at least one of places marked as $\otimes$ is non zero, by another row operation, a matrix of zero rows is obtained.

Hence $\det A$ is always zero for any values inplaces marked as letters.

Step-4

(b)

Big Formula is;

$$\det A = \sum_{all\, P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$$

In the big formula each term is forward taking exactly one number from each column and each row, now take non zero entries from 4,5th rows and choose zero from 3rd row compulsorily and hence the product of three terms (any term in big formula) is invariably zero.