Step-1

The objective is to find the factors of L and U using elimination for the following matrices.

Let $A=\begin{bmatrix}2 & 1\\ 8 & 7\end{bmatrix}$.

The first step of the elimination process is to subtract 4 times row one from row two.

$$\underbrace{\begin{bmatrix}1 & 0\\ -4 & 1\end{bmatrix}}_{E_{21}}\underbrace{\begin{bmatrix}2 & 1\\ 8 & 7\end{bmatrix}}_{A}=\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}$$

The upper triangle is obtained $E_{21}A=U$.

That is $U=\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}$.

Use the result $E_{21}A=U \Rightarrow A=E_{21}^{-1}U$, from this the factor for L is $L=E_{21}^{-1}$.

Step-2

Now determine the factor L using $L=E_{21}^{-1}$.

$$\begin{aligned}L&=\begin{bmatrix}1 & 0\\ -4 & 1\end{bmatrix}^{-1}\\ &=\frac{1}{1-0}\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}\\ &=\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}\end{aligned}$$

Therefore, the factors of L and U for A is $\boxed{\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}\text{ and }\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}}$.

Step-3

Let $A=\begin{bmatrix}3 & 1 & 1\\ 1 & 3 & 1\\ 1 & 1 & 3\end{bmatrix}$.

The first step of the elimination process is to subtract $\frac{1}{3}$ times row one from row two.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_{21}} \underbrace{\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 3 & 1 & 1 \\ 0 & \frac{8}{3} & \frac{2}{3} \\ 1 & 1 & 3 \end{bmatrix}}_{E_{21}A}.$$

The second step is to subtract $\frac{1}{3}$ times row one from row three.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{3} & 0 & 1 \end{bmatrix}}_{E_{31}} \underbrace{\begin{bmatrix} 3 & 1 & 1 \\ 0 & \frac{8}{3} & \frac{2}{3} \\ 1 & 1 & 3 \end{bmatrix}}_{E_{21}A} = \underbrace{\begin{bmatrix} 3 & 1 & 1 \\ 0 & \frac{8}{3} & \frac{2}{3} \\ 0 & \frac{2}{3} & \frac{8}{3} \end{bmatrix}}_{E_{31}E_{21}A}$$

Step-4

Now subtract $\frac{1}{4}$ times row 2 from row 3.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{4} & 1 \end{bmatrix}}_{E_{32}} \underbrace{\begin{bmatrix} 3 & 1 & 1 \\ 0 & \frac{8}{3} & \frac{2}{3} \\ 0 & \frac{2}{3} & \frac{8}{3} \end{bmatrix}}_{E_{31}E_{21}A} = \underbrace{\begin{bmatrix} 3 & 1 & 1 \\ 0 & \frac{8}{3} & \frac{2}{3} \\ 0 & 0 & \frac{5}{2} \end{bmatrix}}_{E_{32}E_{31}E_{21}A}.$$

The upper triangle is obtained $E_{32}E_{31}E_{21}A = U$.

That is

$$U = \begin{bmatrix} 3 & 1 & 1 \\ 0 & \frac{8}{3} & \frac{2}{3} \\ 0 & 0 & \frac{5}{2} \end{bmatrix}.$$

Use the result $E_{32}E_{31}E_{21}A = U \Rightarrow A = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U$, from this the factor for L is $L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$.

Step-5

Now determine the factor L using $L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{3} & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{4} & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{4} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{1}{4} & 1 \end{bmatrix}$$

Therefore, the factors of L and U for A is $\boxed{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{1}{4} & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 3 & 1 & 1 \\ 0 & \frac{8}{3} & \frac{2}{3} \\ 0 & 0 & \frac{5}{2} \end{bmatrix}}$.

Step-6

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{bmatrix}$.

The first step of the elimination process is to subtract row 1 from row 2.

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_{21}} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 1 & 4 & 8 \end{bmatrix}}_{E_{21}A}.$$

The second step is to subtract row 1 from row 3.

$$\underbrace{\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\-1 & 0 & 1\end{bmatrix}}_{E_{31}}\underbrace{\begin{bmatrix}1 & 1 & 1\\0 & 3 & 3\\1 & 4 & 8\end{bmatrix}}_{E_{21}A}=\underbrace{\begin{bmatrix}1 & 1 & 1\\0 & 3 & 3\\0 & 3 & 7\end{bmatrix}}_{E_{31}E_{21}A}$$

Step-7

Now subtract times row 2 from row 3.

$$\underbrace{\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & -1 & 1\end{bmatrix}}_{E_{32}}\underbrace{\begin{bmatrix}1 & 1 & 1\\0 & 3 & 3\\0 & 3 & 7\end{bmatrix}}_{E_{31}E_{21}A}=\underbrace{\begin{bmatrix}1 & 1 & 1\\0 & 3 & 3\\0 & 0 & 4\end{bmatrix}}_{E_{32}E_{31}E_{21}A}.$$

The upper triangle is obtained $E_{32}E_{31}E_{21}A=U$.

That is $U=\begin{bmatrix}1 & 1 & 1\\0 & 3 & 3\\0 & 0 & 4\end{bmatrix}.$

Use the result $E_{32}E_{31}E_{21}A=U \Rightarrow A=E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}U$, from this the factor for L is $L=E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$.

Now determine the factor L using $L=E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$.

$$\begin{aligned}L&=\begin{bmatrix}1 & 0 & 0\\-1 & 1 & 0\\0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\-1 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & -1 & 1\end{bmatrix}^{-1}\\&=\begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 1 & 1\end{bmatrix}\\&=\begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\1 & 1 & 1\end{bmatrix}\end{aligned}$$

Therefore, the factors of L and U for A is $\boxed{\begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\1 & 1 & 1\end{bmatrix}\text{ and }\begin{bmatrix}1 & 1 & 1\\0 & 3 & 3\\0 & 0 & 4\end{bmatrix}}.$