

## Step-1

(b) Let  $Nx = \lambda x$ . Then we get the following:

$$\begin{aligned} Nx &= \lambda x \\ N(Nx) &= N(\lambda x) \\ &= \lambda Nx \\ &= \lambda^2 x \end{aligned}$$

Thus,  $N^2 x = \lambda^2 x$ . Further,

$$\begin{aligned} N^2 x &= \lambda^2 x \\ N(N^2 x) &= N(\lambda^2 x) \\ &= \lambda^2 (Nx) \\ &= \lambda^3 x \end{aligned}$$

## Step-2

Thus, we have  $N^3 x = \lambda^3 x$ .

But  $N^3 = 0$ . This gives us  $N^3 x = 0$ , for any  $x$ .

Therefore,  $\lambda^3 x = 0$ , for any  $x$ .

This indicates that  $\lambda^3 = 0$ . This is possible if and only if  $\lambda = 0$ .

Therefore,  $\boxed{\lambda = 0}$ .

## Step-3

(c) Suppose, if possible,  $N$  is a similar matrix of order  $k$  by  $k$ . Then  $n_{ij} = n_{ji}$ , for each  $i, j$ . Consider the product of  $i^{\text{th}}$  row and  $i^{\text{th}}$  column in  $N^2$ .

$$\begin{aligned} [n_{i1}, n_{i2}, \dots, n_{ik}] \begin{bmatrix} n_{1i} \\ n_{2i} \\ \vdots \\ n_{ki} \end{bmatrix} &= n_{i1}n_{1i} + n_{i2}n_{2i} + \dots + n_{ik}n_{ki} \\ &= n_{i1}^2 + n_{i2}^2 + \dots + n_{ik}^2 \end{aligned}$$

This indicates that  $n_{ij} = 0$ , for each  $i$  and  $j$ . But  $N$  is assumed to be non zero matrix. Thus, our assumption that  $N$  could be a similar matrix is false. So  $N$  is not a similar matrix.