

## Step-1

Given quadratic is  $z = 4x^2 + 12xy + cy^2$ .

Comparing with  $ax^2 + 2bxy + cy^2$ ,

So,  $a = 4$ ,  $2b = 12$ ,  $c = c$ .

For saddle point the condition is,

$$ac - b^2 < 0$$

$$\Rightarrow (4)(c) - (6)^2 < 0$$

$$\Rightarrow 4c - 36 < 0$$

$$\Rightarrow c < 9$$

Thus if  $\boxed{c > 9}$ ,  $4x^2 + 12xy + cy^2$  is positive definite and hence graph of  $z$  is a bowl.

## Step-2

If  $c < 9$ ,

$$z = 4x^2 + 12xy + cy^2$$

$$= (2x + 3y)^2$$

Thus if  $\boxed{c = 9}$ , then the graph of  $z$  is a trough staying at zero on this line  $2x + 3y = 0$ .