Step-1

Given four points are

$y = 3$ at $t = 1,\ z = 1$
$y = 6$ at $t = 0,\ z = 3$
$y = 5$ at $t = 2,\ z = 1$
$y = 0$ at $t = 0,\ z = 0$

We have to fit a plane $y = C + Dt + Ez$ to the given four points.

Step-2

First we write the equations that would hold if a line could go through the given four points.

Then every $y = C + Dt + Ez$ would agree exactly with *b*,

We get the equations as

$$C + D(1) + E(1) = 3$$
$$C + D(0) + E(3) = 6$$
$$C + D(2) + E(1) = 5$$
$$C + D(0) + E(0) = 0$$

Step-3

These are 4 equations in 3 unknowns to pass a plane through the points

The matrix form of above system is

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 5 \\ 0 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix},\ x = \begin{bmatrix} C \\ D \\ E \end{bmatrix},\ b = \begin{bmatrix} 3 \\ 6 \\ 5 \\ 0 \end{bmatrix}$

Step-4

We know that the least-squares fitting is

$A^T A\hat{x} = A^T b$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 5 \\ 0 \end{bmatrix}$$

Step-5

Continuation to the above

$$\Rightarrow \begin{bmatrix} \begin{pmatrix} 1(1)+1(1) \\ +1(1)+1(1) \end{pmatrix} & \begin{pmatrix} 1(1)+1(0) \\ +1(2)+1(0) \end{pmatrix} & \begin{pmatrix} 1(1)+1(3) \\ +1(1)+1(0) \end{pmatrix} \\ \begin{pmatrix} 1(1)+0(1) \\ +2(1)+0(1) \end{pmatrix} & \begin{pmatrix} 1(1)+0(0) \\ +2(2)+0(0) \end{pmatrix} & \begin{pmatrix} 1(1)+0(3) \\ +2(1)+0(0) \end{pmatrix} \\ \begin{pmatrix} 1(1)+3(1) \\ +1(1)+0(1) \end{pmatrix} & \begin{pmatrix} 1(1)+3(0) \\ +1(2)+0(0) \end{pmatrix} & \begin{pmatrix} 1(1)+3(3) \\ +1(1)+0(0) \end{pmatrix} \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 1(3)+1(6) \\ +1(5)+1(0) \end{pmatrix} \\ \begin{pmatrix} 1(3)+0(6) \\ +2(5)+0(0) \end{pmatrix} \\ \begin{pmatrix} 1(3)+3(6) \\ +1(5)+0(0) \end{pmatrix} \end{bmatrix}$$

Step-6

Continuation to the above

$$\Rightarrow \begin{bmatrix} 1+1+1+1 & 1+0+2+0 & 1+3+1+0 \\ 1+0+2+0 & 1+0+4+0 & 1+0+2+0 \\ 1+3+1+0 & 1+0+3+0 & 1+9+1 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} 3+6+5 \\ 3+0+10+0 \\ 3+18+5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 3 & 5 \\ 3 & 5 & 3 \\ 5 & 3 & 11 \end{bmatrix} \begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 26 \end{bmatrix}$$

Step-7

Continuation to the above

$$\Rightarrow 4\hat{C} + 3\hat{D} + 5\hat{E} = 14$$
$$3\hat{C} + 5\hat{D} + 3\hat{E} = 13$$
$$5\hat{C} + 3\hat{D} + 11\hat{E} = 26$$

Hence the three equations in 3 unknowns for the best least â€“ square solutions are

$$\boxed{\begin{array}{l} 4\hat{C} + 3\hat{D} + 5\hat{E} = 14 \\ 3\hat{C} + 5\hat{D} + 3\hat{E} = 13 \\ 5\hat{C} + 3\hat{D} + 11\hat{E} = 26 \end{array}}$$