Step-1

Consider the function,

$$P_1 = \frac{1}{2}x^2 + xy + y^2 - 3y$$

Find the partial derivatives with respect to the variables *x* and *y*.

$$\frac{\partial P_1}{\partial x} = x + y,$$

$$\frac{\partial P_1}{\partial y} = x + 2y - 3$$

Step-2

For the extremum values,

$$\frac{\partial P_1}{\partial x} = 0$$

$x + y = 0$ â€¦â€¦ (1)

And

$$\frac{\partial P_1}{\partial y} = 0$$

$$x + 2y - 3 = 0$$

$x + 2y = 3$ â€¦â€¦ (2)

Solve equations (1) and (2) to obtain,

$x = -3$ and $y = 3$.

Now find the value of the expression,

$$\begin{aligned} P_{1xx}P_{1yy} - P_{1xy}{}^2 &= (1)(2) - 1 \\ &= 1 \\ &> 0 \end{aligned}$$

Thus, P_1 $\boxed{\text{has minimum at } x = -3, y = 3.}$

Step-3

Consider the function,

$$P_2 = \frac{1}{2}x^2 - 3y$$

Find the partial derivatives with respect to the variables *x* and *y*.

$$\frac{\partial P_2}{\partial x} = x,$$

$$\frac{\partial P_2}{\partial y} = -3 \neq 0$$

So, P_2 $\boxed{\text{has no minimum.}}$

Step-4

Now the matrix associated with P_2 is given,

$$A = \begin{pmatrix} \frac{\partial^2 P_2}{\partial x^2} & \frac{\partial^2 P_2}{\partial x \partial y} \\ \frac{\partial^2 P_2}{\partial y \partial x} & \frac{\partial^2 P_2}{\partial y^2} \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

Thus, the semi definite matrix associated with P_2 is,

$$\boxed{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}.$$