# Step-1

Given that $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $v_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

The vector $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = C_1 V_1 + C_2 V_2$

$$= C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 + C_2 \\ C_1 + 4C_2 \end{bmatrix}$$

$C_1 + C_2 = 3$
$C_1 + 4C_2 = 9$

Solving these $C_1 = 1, C_2 = 2$

# Step-2

Also, $\begin{bmatrix} 3 \\ 9 \end{bmatrix} = d_1 v_1 + d_2 v_2$

$$= d_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + d_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2d_1 + d_2 \\ 3d_1 + 4d_2 \end{bmatrix}$$

$\Rightarrow 2d_1 + d_2 = 3$
$\quad 5d_1 + 4d_2 = 9$

# Step-3

Solving these, we get $d_1 = 1, d_2 = 1$

The column of M comes from $V_1, V_2$ as colwntration

$$\sum m_{ij} v_i$$

$V_1 = m_{11} v_1 + v_{21} v_2$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = m_{11} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + m_{21} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2m_{11} + m_{21} \\ 5m_{11} + 4m_{21} \end{bmatrix}$$

$$\Rightarrow\ 2m_{11} + m_{21} = 1$$
$$5m_{11} + 4m_{21} = 1$$

Solving these, we get $m_{11} = 1, m_{21} = -1$

## Step-4

$$V_2 = m_{12}v_1 + m_{22}v_2$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = m_{12} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + m_{22} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2m_{12} + m_{22} \\ 5m_{12} + 4m_{22} \end{bmatrix}$$

$$\Rightarrow 2m_{12} + m_{22} = 1$$
$$5m_{12} + 4m_{22} = 4$$

Solving these, we get $m_{12} = 0$, $m_{22} = 1$

$$M = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$MC = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= d$$

## Step-5

$$m_{21}c_1 + m_{22}c_{22} = (-1)(1) + (1)(2)$$

$$= 1$$
$$= d_2$$

2

$$C_1V_1 + C_2V_2 = 1\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

$$d_1v_1 + d_2v_2 = 1\begin{bmatrix} 2 \\ 5 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

$$C_1V_1 + C_2V_2 = d_1v_1 + d_2v_2$$