Step-1

If **S** is the subspace of $\mathbf{R}^3$ containing only the zero vector, then we have to find $\mathbf{S}^{\perp}$. If **S** is spanned by $(1,1,1)$, we have to find $\mathbf{S}^{\perp}$, and if **S** is spanned by $(2,0,0)$ and $(0,0,3)$, then we have to find $\mathbf{S}^{\perp}$

Step-2

Let $\mathbf{0}=(0,0,0)$ belongs to **S** is a zero vector because **S** is a subspace of $\mathbf{R}^3$

By definition of orthogonal complement,

$\mathbf{S}^{\perp}=\left\{\alpha \in \mathbf{R}^3 / \alpha^T \mathbf{0}=0 \text{ for } \mathbf{0} \in \mathbf{S}\right\}$ â€¦â€¦ (1)

Let $\alpha=(x,y,z)$

Step-3

Now

$\alpha^T \mathbf{0}=0$

$\Rightarrow \begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}=0$

$\Rightarrow 0x+0y+0z=0$

x,y,z are any real values satisfies above equation.

Hence $\mathbf{S}^{\perp}=\left\{(x,y,z)\in \mathbf{R}^3 / x,y,z \in R\right\}$

Step-4

S is spanned by the vector $x=(1,1,1)$

By (1), $\alpha^T x=0$

$\Rightarrow \begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=0$

$\Rightarrow x+y+z=0$

Step-5

Put $y = a, z = b$, for $a, b \in R$.

$\Rightarrow x = -a - b$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -a-b \\ a \\ b \end{bmatrix}$$

$$= a\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Hence $\boxed{\mathbf{S}^{\perp} = \{(-1,1,0),(-1,0,1)\}}$

Step-6

S is spanned by the vectors $x = (2,0,0)$ *and* $y = (0,0,3)$

By (1), $\alpha^T x = 0$ and $\alpha^T y = 0$

$$\Rightarrow \begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = 0 \text{ and} \begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = 0$$

$\Rightarrow 2x = 0$ and $3z = 0$

$\Rightarrow x = 0$ and $z = 0$

Step-7

Above equations does not depends on y

There y is any arbitrary, put $y = k$

Hence $\boxed{\mathbf{S}^{\perp} = \{(0,k,0) / k \in R\}}$