Step-1

Let **V** be the subspace of tridiagonal matrices.

Therefore, we get,

$$\mathbf{V} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix}$$

Let **W** be the subspace of upper triangular matrices.

Therefore, we get,

$$\mathbf{W} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

Step-2

Therefore,

$$\mathbf{V} + \mathbf{W} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{bmatrix}$$

Similarly,

$$\mathbf{V}\cap\mathbf{W}=\begin{bmatrix}a_{11}&a_{12}&0&0\\a_{21}&a_{22}&a_{23}&0\\0&a_{32}&a_{33}&a_{34}\\0&0&a_{43}&a_{44}\end{bmatrix}+\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\0&a_{22}&a_{23}&a_{24}\\0&0&a_{33}&a_{34}\\0&0&0&a_{44}\end{bmatrix}$$

$$=\begin{bmatrix}a_{11}&a_{12}&0&0\\0&a_{22}&a_{23}&0\\0&0&a_{33}&a_{34}\\0&0&0&a_{44}\end{bmatrix}$$

Step-3

Therefore, the dimension of $(\mathbf{V}+\mathbf{W})$ is given by,

$$\dim(\mathbf{V}+\mathbf{W})=13$$

Similarly, the dimension of $(\mathbf{V}\cap\mathbf{W})$ is given by,

$$\dim(\mathbf{V}\cap\mathbf{W})=7$$

The dimension of $\mathbf{V}$ is given by,

$$\dim\mathbf{V}=10$$

The dimension of $\mathbf{W}$ is given by,

$$\dim\mathbf{W}=10$$

Step-4

Thus,

$$\dim(\mathbf{V}\cap\mathbf{W})+\dim(\mathbf{V}+\mathbf{W})=\dim\mathbf{V}+\dim\mathbf{W}$$

Step-5

Thus,

$\boxed{\mathbf{V}\cap\mathbf{W}=\begin{bmatrix}a_{11}&a_{12}&0&0\\0&a_{22}&a_{23}&0\\0&0&a_{33}&a_{34}\\0&0&0&a_{44}\end{bmatrix}}$ and $\boxed{\mathbf{V}+\mathbf{W}=\begin{bmatrix}a_{11}&a_{12}&a_{13}&a_{14}\\a_{21}&a_{22}&a_{23}&a_{24}\\0&a_{32}&a_{33}&a_{34}\\0&0&a_{43}&a_{44}\end{bmatrix}}$.