Step-1

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The given non homogeneous system of linear equations is

$$kx + y = 1$$
$$x + ky = 1$$

The augmented matrix is $\left[\begin{array}{cc|c} k & 1 & 1 \\ 1 & k & 1 \end{array}\right]$

We apply row operations on this as

$$R_2 \to kR_2 - R_1 \Rightarrow \left[\begin{array}{cc|c} k & 1 & 1 \\ 0 & k^2-1 & k-1 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} k & 1 & 1 \\ 0 & (k-1)(k+1) & k-1 \end{array}\right]$$ â€¦â€¦ (1)

Step-2

We easily see that when $k = 1$, this matrix becomes $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$

This is the reduced matrix and so, we rewrite the non homogeneous equations from this.

That is $x + y = 1$

So, $y = 1 - x$ and thus, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} t \\ -t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ where $x = t$ is the parameter.

$$= t\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

So, for infinite values of t, there will be infinite solutions to the system. â€¦â€¦ (2)

Step-3

When $k = -1$, we see the matrix (1) becomes $\left[\begin{array}{cc|c} k & 1 & 1 \\ 0 & 0 & -2 \end{array}\right]$

This is the reduced matrix and so, writing the non homogeneous equation using 2nd row, we get $0x + 0y = -2$

This is an absurdity.

So, we follow that the given system is inconsistent or the system has no solutionâ€¦â€¦ (3)

Step-4

When k neither takes 1 nor â€“ 1, (1) becomes $\left[\begin{array}{cc|c} k & 1 & 1 \\ 0 & (k+1) & 1 \end{array}\right]$

Rewriting the non homogeneous equations from this, we have

$$\begin{aligned} (k+1)y &= 1 \\ kx + y &= 1 \end{aligned}$$

So, $y = \dfrac{1}{k+1}$, consequently, $x = \dfrac{1}{k} - \dfrac{1}{k(k+1)}$

$$= \frac{1}{k+1}$$

Therefore, $\begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{k+1}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

So, for each value of k, this solution has unique opportunity.

Thus, the system has only one solution when k is neither 1 nor -1. â€¦â€¦ (4)