Step-1

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a) We have to find a 3 by 3 matrix B such that $BA = 2A$ for every A.

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Then

$$2A = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Step-2

Now

$$BA = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$
$$= \begin{bmatrix} 2(1)+0(0)+0(0) & 2(0)+0(1)+0(0) & 2(0)+0(0)+0(1) \\ 0(1)+2(0)+0(0) & 0(0)+2(1)+0(0) & 0(0)+2(0)+0(1) \\ 0(1)+0(0)+2(0) & 0(0)+0(1)+2(0) & 0(0)+0(0)+2(1) \end{bmatrix}$$

Step-3

Continuation to the above

$$= \begin{bmatrix} 2+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+2+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+2 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$
$$= 2A$$

Hence the required matrix that satisfies $BA = 2A$ for every A is $\boxed{B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}}$.

Step-4

(b) We have to find a 3 by 3 matrix B such that $BA = 2B$ for every A.

By reversing the matrices A, B in the above result, we get the required result $BA = 2B$

Hence the required matrix that satisfies $BA = 2B$ for every A is $\boxed{B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$.

Step-5

(c) We have to find a 3 by 3 matrix B such that BA has the first and last rows of A reversed.

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Then

$$BA = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0(1)+0(0)+1(0) & 0(0)+0(1)+1(0) & 0(0)+0(0)+1(1) \\ 0(1)+1(0)+0(0) & 0(0)+1(1)+0(0) & 0(0)+1(0)+0(1) \\ 1(1)+0(0)+1(0) & 1(0)+0(1)+1(0) & 1(0)+0(0)+1(1) \end{bmatrix}$$

Step-6

Continuation to the above

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+1 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 0+0+0 & 0+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Therefore, the rows of BA has the first and last rows of A reversed.

Hence the required matrix is $\boxed{B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$

Step-7

d) We have to find a 3 by 3matrix B such that BA has the first and last columns of A reversed.

Step-8

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Then

$$BA = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0(1)+0(0)+1(0) & 0(0)+0(1)+1(0) & 0(0)+0(0)+1(1) \\ 0(1)+1(0)+0(0) & 0(0)+1(1)+0(0) & 0(0)+1(0)+0(1) \\ 1(1)+0(0)+1(0) & 1(0)+0(1)+1(0) & 1(0)+0(0)+1(1) \end{bmatrix}$$

Step-9

Continuation to the above

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+1 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 0+0+0 & 0+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Therefore, the rows of BA has the first and last columns of A reversed.

Hence the required matrix is $\boxed{B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$.