Step-1

Consider the system of three variables and three equations;

$$\begin{aligned} u+v+w&=2 \\ u+2v+3w&=1 \\ v+2w&=0 \end{aligned}$$

The objective is to explain why this system is singular. Also, to determine the suitable substitution for 0 on the right-hand side of the last equation, such that the system have solutions and then find solutions.

Step-2

Add first and third equations;

$$u+2v+3w=2$$

Right hand side of this equation is same of the second equation but left side is not equal.

So, equation 1 plus equation 3 minus equation 2 gives;

$$\begin{aligned} (u+2v+3w)-(u+2v+3w)&=2-1 \\ 0&=1 \end{aligned}$$

Step-3

Suppose the system is;

$$\begin{aligned} u+v+w&=2 \\ u+2v+3w&=1 \\ v+2w&=a \end{aligned}$$

Add first and third equations;

$$u+2v+3w=2+a$$

Now, subtract second equation from above equation;

$$\begin{aligned} (u+2v+3w)-(u+2v+3w)&=2+a-1 \\ 0&=1+a \\ a&=-1 \end{aligned}$$

Hence, $\boxed{-1}$ should replace the last zero.

Step-4

Now, consider system in which -1 in place of zero;

$$
\begin{aligned}
u+v+w&=2\\
u+2v+3w&=1\\
v+2w&=-1
\end{aligned}
$$

So, this is now consistent and so have infinite solutions.

The 2nd equation indicates that $v=-1-2w$, consequently the 1st row turn out to be

$$
\begin{aligned}
u+(-1-2w)+w&=2\\
u-w&=3
\end{aligned}
$$

The two equations $u-w=3$ and $v+2w=-1$ require a line of solutions; to determine one solution, then suppose $w=0$ and solve for u and v. This results the solution

$\boxed{(u,v,w)=(3,-1,0)}$.