Step-1

false

Step-2

Given that x and y are orthogonal

So, $x^T y = 0$

P is a projection matrix

Therefore $P^2 = P$ and $P^T = P$

Now $(Px)^T Py = \left(x^T P^T\right) Py$

$= \left(\left(x^T P^T\right) P\right) y$

$= x^T \left((PP) y\right)$ By the associativity of matrix multiplication and $P^T = P$

$= x^T Py$ Using $P^2 = P$

Step-3

We know that the inner product of x with $P y$ equal to the inner product of $P x$ with y.

In other words, $x^T Py = y^T Px$

So, $(Px)^T Py$ is not necessarily equal to 0

Therefore, Px and Py are not orthogonal