Step-1

Kernel of a linear transformation $T:\mathbf{R}^2 \to \mathbf{R}^2$ is $\ker T = \{(v_1,v_2)\in \mathbf{R}^2 / T(v_1,v_2)=0\}$

And the **range of** T is $\text{Range } T = \{T(v_1,v_2)/(v_1,v_2)\in \mathbf{R}^2\}$

Step-2

(a)

Given that $T:\mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(v_1,v_2)=(v_2,v_1)$.

Now the kernel is;

$$\ker T = \{(v_1,v_2)\in \mathbf{R}^2 / T(v_1,v_2)=0\}$$

This implies;

$$\begin{aligned}
&= \{(v_1,v_2)\in \mathbf{R}^2 / (v_2,v_1)=0\} \\
&= \left\{\begin{matrix}(v_1,v_2)\in \mathbf{R}^2 / v_2 = 0 \\ v_1 = 0\end{matrix}\right\} \\
&= \{(v_1,v_2)\in \mathbf{R}^2 / (v_1,v_2)=(0,0)\} \\
&= \{(0,0)\}
\end{aligned}$$

Step-3

And the range of T is

$$\begin{aligned}
\text{Range } T &= \{T(v_1,v_2)/(v_1,v_2)\in \mathbf{R}^2\} \\
&= \{(v_2,v_1)/v_1,v_2 \in \mathbf{R}\} \\
&= \mathbf{R}^2
\end{aligned}$$

Hence the kernel of the given transformation is $\boxed{\ker T = \{(0,0)\}}$ and the range is $\boxed{\text{Range} T = \mathbf{R}^2}$.

Step-4

(b)

Given that $T:\mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(v_1,v_2,v_3)=(v_1,v_2)$.

Now the kernel is;

$$\ker T = \left\{(v_1, v_2, v_3) \in \mathbf{R}^3 / T(v_1, v_2, v_3) = 0\right\}$$

This implies;

$$\begin{aligned}
&= \left\{(v_1, v_2, v_3) \in \mathbf{R}^3 / (v_1, v_2) = 0\right\} \\
&= \left\{(v_1, v_2, v_3) \in \mathbf{R}^3 / v_1 = 0, v_2 = 0\right\} \\
&= \left\{(v_1, v_2, v_3) \in \mathbf{R}^3 / (v_1, v_2, v_3) = (0, 0, v_3)\right\} \\
&= \left\{(0, 0, v_3) / v_3 \in \mathbf{R}\right\}
\end{aligned}$$

And the range of T is;

$$\begin{aligned}
\text{Range } T &= \left\{T(v_1, v_2, v_3) / (v_1, v_2, v_3) \in \mathbf{R}^3\right\} \\
&= \left\{(v_1, v_2) / v_1, v_2 \in \mathbf{R}\right\} \\
&= \mathbf{R}^2
\end{aligned}$$

Hence the kernel of the given transformation is $\boxed{\ker T = \left\{(0, 0, v_3) / v_3 \in \mathbf{R}\right\}}$ and the range is $\boxed{\text{Range} T = \mathbf{R}^2}$.

Step-5

(c)

Given that $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(v_1, v_2) = (0, 0)$.

Now the kernel is;

$$\begin{aligned}
\ker T &= \left\{(v_1, v_2) \in \mathbf{R}^2 / T(v_1, v_2) = 0\right\} \\
&= \left\{(v_1, v_2) \in \mathbf{R}^2 / (0, 0) = 0\right\} \\
&= \left\{(v_1, v_2) \in \mathbf{R}^2 / v_1, v_2 \in \mathbf{R}\right\} \\
&= \mathbf{R}^2
\end{aligned}$$

Step-6

And the range of T is;

$$\begin{aligned}
\text{Range } T &= \left\{T(v_1, v_2) / (v_1, v_2) \in \mathbf{R}^3\right\} \\
&= \left\{(0, 0) / v_1, v_2 \in \mathbf{R}\right\} \\
&= \left\{(0, 0)\right\}
\end{aligned}$$

Hence the kernel of the given transformation is $\boxed{\ker T = \mathbf{R}^2}$ and the range is $\boxed{\text{Range}T = \{(0,0)\}}$.

Step-7

(d)

Given that $T:\mathbf{R}^2 \to \mathbf{R}^2$ defined by $T(v_1,v_2)=(v_1,v_1)$.

Now the kernel is;

$$\begin{aligned}\ker T &= \{(v_1,v_2)\in\mathbf{R}^2 / T(v_1,v_2)=0\}\\ &= \{(v_1,v_2)\in\mathbf{R}^2 / (v_1,v_1)=0\}\\ &= \{(v_1,v_2)\in\mathbf{R}^2 / v_1=0\}\\ &= \{(0,v_2) / v_2\in\mathbf{R}\}\end{aligned}$$

Step-8

And the range of T is;

$$\begin{aligned}\text{Range } T &= \{T(v_1,v_2)/(v_1,v_2)\in\mathbf{R}^3\}\\ &= \{(v_1,v_1)/v_1,v_2\in\mathbf{R}\}\\ &= \{(v_1,v_2)/v_1=v_2 \text{ and } v_1,v_2\in\mathbf{R}\}\end{aligned}$$

Hence the kernel of the given transformation is $\boxed{\ker T = \{(0,v_2)/v_2\in\mathbf{R}\}}$ and the range is $\boxed{\text{Range}T = \{(v_1,v_2)/v_1=v_2 \text{ and } v_1,v_2\in\mathbf{R}\}}$.