

Step-1

We have to fill the following blanks:

Multiplying the rank 1 matrices, $A = uv^T$ and $B = wz^T$ gives uz^T times the number ----.

AB has rank 1 unless ---- = 0.

Step-2

$A = uv^T$ and $B = wz^T$, and $\text{rank } A = 1$, $\text{rank } B = 1$.

$$\begin{aligned}\text{Rank}(AB) &= \text{rank of } (uv^T.wz^T) \\ &= \text{rank of } (u(v^T.w)z^T)\end{aligned}$$

Multiplying the rank 1 matrices A, B gives uz^T times $v^T.w$.

And hence rank of $AB = 1$ unless $v^T.w = 0$.