

## Step-1

The objective is to determine matrix due to the changes the basis.

## Step-2

(a)

Obtain the matrix that transforms  $(1,0)$  into  $(2,5)$  and transforms  $(0,1)$  into  $(1,3)$ .

$$T(1,0) = (2,5)$$

$$T(0,1) = (1,3)$$

The transition matrix is  $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ , use to the provided vectors.

Then,

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$x = 2$$

$$z = 5$$

And

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$y = 1$$

$$w = 3$$

Now, the matrix is:

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

Hence, the matrix of the linear transformation under the basis  $(1,0), (0,1)$  is  $\boxed{\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}}$ .

### Step-3

(b)

Matrix that transforms  $(2,5)$  into  $(1,0)$  and transforms  $(1,3)$  into  $(0,1)$ .

Since, the linear transformation of the matrix is the inverse of the previous part.

So, take the inverse of the matrix.

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

Thus, there is no another matrix is founded by the provided statement. It's only defined by the inverse of the result matrix of previous part.

The matrix of the linear transformation under the basis  $(2,5), (1,3)$  is  $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ .

### Step-4

(c)

Objective is to explain why, no matrix is formed by the  $(2,6)$  into  $(1,0)$  and transforms  $(1,3)$  into  $(0,1)$ .

The matrix transformation is not linear.

$$T(2,6) = (1,0)$$

$$T(1,3) = (0,1)$$

$$T(2,6) = T(2(1,3))$$

$$= (1,0)$$

So,

$$\begin{bmatrix} 2 \\ 6 \end{bmatrix} = 2 \times \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

This is linear transformations of the linear operators. That is:

$$T(\vec{a}) = \vec{b}$$

Then,

$$T(\alpha \vec{a}) = \alpha \vec{b}$$

So,

$$T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 2 \\ 6 \end{bmatrix}\right) = T\left(2 \begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$$

$$= 2T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 2 \\ 6 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Thus, itâ€™s not true. It wants

Hence, there is **no linear transformation of the provided statements.**