Step-1

Properties of determinants: determinant is defined for a square matrix A.

1) If two rows of a matrix are equal, then $\det A = 0$

2) Subtracting a multiple of one row from another row leaves the same determinant.

3) If one row or column is having completely zeroes, then $\det A = 0$

4) the sign is reversed by a row exchange

In view of these properties, we find the determinant of the given matrices.

Step-2

i) $\det\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 1 & 1 & 2\\ 1 & 1 & 3 & 1\\ 1 & 4 & 1 & 1\end{bmatrix} = \begin{vmatrix}1 & 1 & 1 & 1\\ 1 & 1 & 1 & 2\\ 1 & 1 & 3 & 1\\ 1 & 4 & 1 & 1\end{vmatrix}$

In view of property 2, we use $\left.\begin{matrix}R_2 \to R_2 - R_1,\\ R_3 \to R_3 - R_1,\\ R_4 \to R_4 - R_1\end{matrix}\right\} \Rightarrow \begin{vmatrix}1 & 1 & 1 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 2 & 0\\ 0 & 3 & 0 & 0\end{vmatrix}$

Step-3

$R_2 \leftrightarrow R_4 \Rightarrow -\begin{vmatrix}1 & 1 & 1 & 1\\ 0 & 3 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 1\end{vmatrix}$ in view of the property 4.

This is an upper triangular matrix and so, the determinant is the product of diagonal entries.

Therefore, $\det\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 1 & 1 & 2\\ 1 & 1 & 3 & 1\\ 1 & 4 & 1 & 1\end{bmatrix} = -6$

Step-4

ii) $\det\begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$

$$\left.\begin{array}{l} R_2 \to 2R_2 + R_1, \\ R_4 \to 2R_4 + R_1 \end{array}\right\} \Rightarrow \begin{vmatrix} 2 & -1 & 0 & -1 \\ 0 & 3 & -2 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -2 & 3 \end{vmatrix}$$

Step-5

$$\left.\begin{array}{l} R_3 \to 3R_3 + R_2, \\ R_4 \to 3R_4 + R_2 \end{array}\right\} \Rightarrow \begin{vmatrix} 2 & -1 & 0 & -1 \\ 0 & 3 & -2 & -1 \\ 0 & 0 & 4 & -4 \\ 0 & 0 & -8 & 8 \end{vmatrix}$$

$$R_4 \to 2R_4 + R_3 \Rightarrow \begin{vmatrix} 2 & -1 & 0 & -1 \\ 0 & 3 & -2 & -1 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Step-6

By property 3, this determinant is zero.

Therefore, $\det\begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} = 0$