Step-1

Given quadratic is $f(x_1,x_2)=3(x_1+2x_2)^2+4x_2^2$

Positive definite is,

$f(x_1,x_2)=3x_1^2+16x_2^2+12x_1x_2$

$=3x_1^2+6x_1x_2+6x_1x_2+16x_2^2$

So the corresponding matrix is $A=\begin{pmatrix}3 & 6\\ 6 & 16\end{pmatrix}$.

Step-2

Now we have to write A as LDL^T.

$f(x_1,x_2)=3(x_1+2x_2)^2+4(0.x_1+1.x_2)^2$

Now the coefficients of the squares are the pivots in D and the coefficients inside the squares are columns of $L.$

Therefore,

$A=\begin{pmatrix}3 & 6\\ 6 & 16\end{pmatrix}$

$=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$

$=LDL^T$

Therefore, $\boxed{A=\begin{pmatrix}3 & 6\\ 6 & 16\end{pmatrix}=\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}}$; the coefficients of the squares are the pivots in D and the coefficients inside the squares are columns of $L.$