

Step-1

a) A is a square matrix, let A be an n by n matrix.

Suppose $x \in N(A)$

Then we follow that $Ax = 0$

Applying A on both sides, we get $A^2x = 0$

So, it follows that x is in the null space of A^2

Therefore, every vector in the null space of A is also in the null space of A^2

In other words, null space of A^2 contains the null space of A .

Step-2

b) Suppose x is in the column space of A^2

Then we follow that there exists a non zero vector b such that $A^2x = b$

Applying A^{-1} on both sides, we get $Ax = A^{-1}b$

Writing $A^{-1}b = c$, we are left with $Ax = c$ where c is a non zero vector.

From this, we can say that x is in the column space of A .

Therefore, every vector in the column space of A^2 is also in the column space of A .

In other words, the column space of A^2 is contained in the column space of A .