Tutorial Class 8 Midterm 2023-04-13

南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目： 概率论与数理统计 开课单位： 数学系

考试时长： 2023/4/8 14:00–16:00 命题教师： 概率统计教学组

题号	1	2	3	4	5	6	7	8
分值	20 分	20 分	10 分	10 分	10 分	10 分	10 分	10 分

本试卷共三大部分，满分（100）分

（考试结束后请将试卷、答题卡一起交给监考老师）

第一部分 选择题（每题4分，总共20分）

Part One – Single Choice (4 marks each question, 20 marks in total)

1. 已知事件A与B独立，B与C独立，A与C不交，且$P(A\cup B)=2/3; P(B\cup C)=3/4; P(A\cup B\cup C)=11/12$，则概率$P(B)$等于(　　).

   For three events A, B and C: A and B are independent, B and C are independent and A and C are mutually disjoint. Also, $P(A\cup B)=2/3; P(B\cup C)=3/4; P(A\cup B\cup C)=11/12$. Then the value of $P(B)$ is:

   A. $\frac{5}{12}$　B. $\frac{1}{3}$　C. $\frac{1}{4}$　D. $\frac{1}{2}$

2. 从6名男生和4名女生中随机抽取3名男生和2名女生组成学生会。如果其中一名男生和一名女生拒绝同时为学生会服务，则能够成功组成学生会的概率是？

   From a group of 6 men and 4 women a student union consisting of 3 men and 2 women is to be formed. What is the probability of successfully forming a student union if 1 man and 1 woman refuse to serve together?

   A. $\frac{3}{4}$　B. $\frac{5}{12}$　C. $\frac{1}{4}$　D. $\frac{5}{8}$

3. 已知$X\sim U(0,4)$，$Y\sim Exp(\lambda)$，若$P(X<3)=P(Y<1)$，则λ等于(　　).

   Let $X\sim U(0,4)$ and $Y\sim Exp(\lambda)$. Suppose $P(X<3)=P(Y<1)$, find λ.

   A. $\ln(3)$　B. $\ln(4)$　C. $-\ln(3)$　D. $\ln(0.25)$

4. 已知连续型随机变量X的密度函数为$f(x) = \begin{cases} 4(1-x)^3 & \text{for} \quad 0 < x < 1 \\ 0 & \text{其他.} \end{cases}$ 求a的值使得$P(X \le a) = \frac{15}{16}$.

   Suppose X follows a distribution which has density $f(x) = \begin{cases} 4(1-x)^3 & \text{for} \quad 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

   Find the value of x such that $P(X \le x) = \frac{15}{16}$.

   A. $\frac{1}{\sqrt{2}}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$

5. 设(X, Y)在G内服从均匀分布, 其中G是由x轴, y轴以及直线$2x + y - 2 = 0$所围成的三角形区域. 则以下错误的是(   )

   Let (X, Y) follow a uniform distribution within G, where G is a triangular region enclosed by a x axis, a y axis, and a straight line $2x + y - 2 = 0$. Then, which of the following items is wrong?

   A. $f_X(x) = \begin{cases} 2-2x, & 0 < x < 1, \\ 0, & \text{others.} \end{cases}$ B. $f_Y(y) = \begin{cases} \frac{2-y}{2}, & 0 < y < 2, \\ 0, & \text{others.} \end{cases}$

   C. When $0 < x < 1$, we have $f_{Y|X}(y|x) = \begin{cases} \frac{1}{2-2x}, & 0 < y < 2-2x, \\ 0, & \text{others.} \end{cases}$

   D. When $0 < y < 2$, we have $f_{X|Y}(x|y) = \begin{cases} \frac{2}{2-y}, & 0 < x < 2-y, \\ 0, & \text{others.} \end{cases}$

第二部分 填空题（每空2分，总共20分）

Part Two – Blank Filling (2 marks each blank, 20 marks in total)

1. 有一个盒子装有3枚硬币，其中两枚硬币质地均匀，第三枚有3/4的可能性正面朝上。从盒中随机选取一枚硬币并抛掷3次，则3次均出现正面的可能性为__________.

   A box contains 3 coins, two of which are fair and the third has probability 3/4 of coming up heads. A coin is chosen randomly from the box and tossed 3 times. The probability that all 3 tosses are heads is __________.

2. 事件A和B独立，且$P(A)=0.4, P(A\cap B)=0.24$, 则$P(\bar{A}B\cup A\bar{B})=$ ________.

   Let A and B be two independent events and $P(A)=0.4, P(A\cap B)=0.24$, then $P(\bar{A}B\cup A\bar{B})=$ ________.

3. 设$P(A)=0.7, P(B)=0.5, P(A\cup B)=0.9$, 则$P(A|\bar{A}\cup B)=$ ________.

   Suppose $P(A)=0.7, P(B)=0.5, P(A\cup B)=0.9$, then $P(A|\bar{A}\cup B)=$ ________.

4. 在一项全是选择题的考试中，共有6道题目。每道题目有3个选项，只有一个是正确答案。假设你随机地回答每一道题。通过此考试需要答对5或者6道题目。已知你已经通过了考试，你答对5道题目的概率是__________.

   An examination consists of six single choices and each question has 3 choices with only one being correct. Now, you make a random choice for each question, and you need 5 or 6 correct answers to pass. Given that you have passed, what is the probability that you got 5 correct answers?________.

5. 某家庭有n个孩子，其中至少有$n-1$个男孩，则全部是男孩的概率为__________.

   A family has n children, at least $n-1$ of which are boys, the probability that all n children are boys is ________.

6. 设X服从标准正态分布，其累积分布函数为$\Phi(x)$，则$P\{|2\Phi(X)-1|>\frac{1}{3}\}=$ ________.

   Let X have the standard normal distribution with the cumulative distribution function $\Phi(x)$. Then $P\{|2\Phi(X)-1|>\frac{1}{3}\}=$ ________.

7. 设随机变量X的概率密度函数$f(x)$满足$f(1-x)=f(1+x), \forall x\in\mathbb{R}$，且$\int_0^2 f(x)dx=0.5$, 则$P\{X<0\}=$ __________.

   Let $f(x)$ be the probability density function of the random variable X. If $f(1-x)=f(1+x), \forall x\in\mathbb{R}$ and $\int_0^2 f(x)dx=0.5$, then $P\{X<0\}=$ ________.

8. 设X服从参数为λ的泊松分布，且$P\{X=1\}=P\{X=2\}$，则$P\{0<X^2<2\}=$ ________.

   Suppose that X follows a Poisson distribution with parameter λ and $P\{X=1\}=P\{X=2\}$, then $P\{0<X^2<2\}=$ ________.

9. 设随机变量(X,Y)服从二维正态分布$N(-1,2,2^2,3^2,0.5)$，则$2X+1$服从分布__________.

   Suppose that (X,Y) has a bivariate normal distribution $N(-1,2,2^2,3^2,0.5)$, then $2X+1$ has a distribution of ________.

10. 设随机变量X和Y互相独立, 且都服从区间$(0,1)$上的均匀分布, 则$P\{X^2+Y^2 \leqslant 1\}=$ __________.

Suppose the r.v. X and Y are independent with $X \sim U(0,1)$ and $Y \sim U(0,1)$, then $P\{X^2+Y^2 \leqslant 1\}=$ __________.

第三部分 解答题（每题10分，总共60分）

Part Three–Question Answering (10 marks each question, 60 marks in total)

1. 投掷两颗骰子, 求下列事件的概率:

(1) 点数之和为7;

(2) 点数之和大于5;

(3) 两个点数中一个恰好是另一个的两倍.

Throwing two dices at the same time, and calculate the probability of the following events:

(1) The sum of the two points is 7;

(2) The sum of the two points is greater than 5;

(3) One of the two points is exactly twice that of the other one.

2. 随着我校成功入选“双一流”，我校的研究生生源也越来越好. 假设在近期研究生复试中的三个候考教室里分别有10、15、25名考生，其中女生分别为3人、7人、5人. 现随机选取一个候考教室，然后从中随机地先后抽出共两名考生.

(1) 求先抽到的考生是女生的概率p;

(2) 已知后抽到的考生是男生, 求先抽到的考生是女生的概率q.

The quality of the recruited students is getting better and better, due to SUSTech being successfully selected into “double first-class”. Suppose there are 10, 15, and 25 candidates in the three waiting classrooms in the recent postgraduate reexamination, which includes 3, 7, and 5 girls, respectively. Now firstly select a waiting classroom randomly, and then randomly select two candidates in turn.

(1) Calculate the probability p that the first candidate is a girl;

(2) If the second candidate is a boy, calculate the probability q that the first candidate is a girl.

3. 设在15只同类型产品中有2只为次品，现从中取3次，每次任取1只，作不放回抽样，以X表示取出的次品个数，求：

(1) X的频率函数;

(2) X的分布函数;

(3) 令$Y = X^2 + 1$, 求$P(1 < Y \leqslant 2)$.

Suppose there are 2 defectives among 15 products. Now select 3 products one by one without replacement. Let X be the number of defective products.

(1) Find the frequency function of r.v. X;

(2) Find the distribution function of r.v. X;

(3) Let $Y = X^2 + 1$ and calculate $P(1 < Y \leqslant 2)$.

4. 设随机变量X的密度函数为$f(x) = \begin{cases} \frac{1}{9}x^2, & 0 < x < 3, \\ 0, & 其他. \end{cases}$ 令$Y = \begin{cases} 2, & X \leqslant 1, \\ X, & 1 < X < 2, \\ 1, & X \geqslant 2. \end{cases}$

(1) 求Y的分布函数;

(2) 求概率$P\{X \leqslant Y\}$.

The probability density function of r.v. X is $f(x) = \begin{cases} \frac{1}{9}x^2, & 0 < x < 3, \\ 0, & \text{others}. \end{cases}$

Let r.v. $Y = \begin{cases} 2, & X \leqslant 1, \\ X, & 1 < X < 2, \\ 1, & X \geqslant 2. \end{cases}$

(1) Find the distribution function of Y;

(2) Calculate the probability $P\{X \leqslant Y\}$.

5. 随机变量X在$1,2,3$三个数字中等可能地取值，随机变量Y在$1\sim X$中等可能的取一整数值,求:

(1) X和Y的联合频率函数和边际频率函数;

(2) X和Y是否独立;

(3) 在$Y=2$的条件下X的条件频率函数.

The random variable X takes moderately possible values in $1,2,3$ and the random variable Y takes moderately possible integer values in $1\sim X$, find:

(1) The joint frequency function and marginal frequency function of X and Y;

(2) Whether X and Y are independent;

(3) Given $Y=2$，find the conditional frequency function of X.

6. 设(X,Y)的联合概率密度为:

$$f(x,y)=(A+\sin x\sin y)\,\varphi(x)\,\varphi(y)$$

其中$\varphi(\cdot)$是标准正态分布的密度函数,$-\infty<x<\infty,-\infty<y<\infty$

(1) 求A的值;

(2) 求边缘密度函数$f_X(x)$和$f_Y(y)$, 并且判断X与Y是否独立;

(3) 求$Z=2X^2+1$的概率密度函数。

The joint pdf of (X,Y) is:

$$f(x,y)=(A+\sin x\sin y)\,\varphi(x)\,\varphi(y)$$

where $\varphi(\cdot)$ is the pdf of standard normal distribution,$-\infty<x<\infty,-\infty<y<\infty$

(1) Find A;

(2) Find Marginal density function $f_X(x)$ and $f_Y(y)$, and determine whether X and Y are independent;

(3) Find the probability density function of $Z=2X^2+1$.