Step-1

Let B be the following matrix:

$$B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

Diagonalize B and compute $S\Lambda^k S^{-1}$ to get following formula for B^k:

$$B^k = \frac{1}{2}\begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}$$

Step-2

To diagonalize the matrix B follow the following steps:

$$B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$
$$B - \lambda I = \begin{bmatrix} 3-\lambda & 1 \\ 0 & 2-\lambda \end{bmatrix}$$
$$\begin{aligned} \det(B - \lambda I) &= (3-\lambda)(2-\lambda) \\ &= \lambda^2 - 5\lambda + 6 \end{aligned}$$

Put the determinant value equal to zero, to get following roots as Eigen values:

$$\lambda_1 = 2$$
$$\lambda_2 = 3$$

Step-3

Eigen vectors corresponding to the Eigen values are calculated as follows:

For $\lambda_1 = 2$

$$(B - \lambda_1 I)x_1 = 0$$
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{aligned} x_1 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{aligned}$$

Step-4

For $\lambda_2 = 3$

$$(B - \lambda_2 I)x_2 = 0$$

$$\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} x_2 &= \begin{bmatrix} y \\ z \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{aligned}$$

Step-5

Thus, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$$

Step-6

Therefore, diagonalisation of matrix *B* is as follows:

$$\begin{aligned} B &= S\Lambda S^{-1} \\ &= \boxed{\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}} \end{aligned}$$

Step-7

Now, do the following calculations to get B^k:

$$\begin{aligned} B^k &= S\Lambda^k S^{-1} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3^k & 0 \\ 0 & 2^k \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix} \end{aligned}$$

Step-8

Therefore,

$$\boxed{B^k = \frac{1}{2}\begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}}$$