

Step-1

The objective is to find matrix $A^T B, B^T A, AB^T$, and BA^T if $A = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

Step-2

Consider;

$$A = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

So,

$$A = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$A^T = [1 \ 3]$$

And

$$B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$B^T = [2 \ 2]$$

Step-3

Now, compute;

$$\begin{aligned} A^T . B &= [1 \ 3]_{1 \times 2} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix}_{2 \times 1} \\ &= (2 + 6) \\ &= (8)_{1 \times 1} \end{aligned}$$

Thus, $\boxed{A^T B = 8}$

Step-4

$$\begin{aligned}
 B^T.A &= \begin{bmatrix} 2 & 2 \end{bmatrix}_{1 \times 2} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix}_{2 \times 1} \\
 &= (6+2) \\
 &= (8)_{1 \times 1}
 \end{aligned}$$

Thus, $\boxed{B^T A = 8}$

Step-5

Now, compute;

$$A.B^T = \begin{bmatrix} 3 \\ 1 \end{bmatrix}_{2 \times 1} \cdot \begin{bmatrix} 2 & 2 \end{bmatrix}_{1 \times 2}$$

Thus, $\boxed{A.B^T = \begin{bmatrix} 6 & 6 \\ 2 & 2 \end{bmatrix}_{2 \times 2}}$

Step-6

$$B.A^T = \begin{bmatrix} 2 \\ 2 \end{bmatrix}_{2 \times 1} \cdot \begin{bmatrix} 1 & 3 \end{bmatrix}_{1 \times 2}$$

Thus, $\boxed{B.A^T = \begin{bmatrix} 6 & 6 \\ 2 & 2 \end{bmatrix}_{2 \times 2}}$