Step-1

Given $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix}$

We have to find that the two elimination matrices E_{21}, E_{32} to put A into upper triangular form $E_{32}E_{21}A = U$ and we have to multiply by E_{32}^{-1}, E_{21}^{-1} to factor A into $LU = E_{21}^{-1}E_{32}^{-1}U$

Step-2

Subtracting 2 times row 1 from row 2 gives

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 4 & 0 \end{pmatrix}$$

Subtracting 2 times row 2 from row 3 gives

$$U = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix}$$

Step-3

To find L reversing the operations on identity matrix;

Adding 2 times row 1 to row 2 and adding 2 times row 2 to row 3 gives

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\begin{aligned} LU &= \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix} \end{aligned}$$

Step-4

Applying elementary operation on identity matrix that is

Subtracting 2 times of row 1 from row 2 gets E_{21}

$$E_{21} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Subtracting 2 times of row 2 from row 3 gets E_{32}

$$E_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Step-5

Now

$$\begin{aligned} E_{32}E_{21}A &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix} = U \end{aligned}$$

This is the required upper triangular form.

Step-6

By reversing the operations held on E_{21} gets E_{21}^{-1}

$$E_{32}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

By reversing the operations held on E_{32} gets E_{32}^{-1}

$$E_{21}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Step-7

Therefore

$$E_{21}^{-1}E_{32}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\Rightarrow E_{21}^{-1}E_{32}^{-1}U = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & -6 \end{pmatrix}$$

$$= LU$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ 0 & 4 & 0 \end{pmatrix}$$

$$= A$$

So this is the required factor form A.