

Step-1

Given the quadratic equation f is $f = x^2 + 4xy + 2y^2$

We need to write the given function as a difference of two squares.

Step-2

Comparing given quadratic with $f = ax^2 + 2bxy + cy^2$.

So, $a = 1$, $2b = 4$, $c = 2$.

Now $a = 1 > 0$ and $ac - b^2 = -2 < 0$.

So origin is the saddle point.

Step-3

Now,

$$\begin{aligned} f &= x^2 + 4xy + 2y^2 \\ &= x^2 + 2(x)(2y) + 4y^2 - 2y^2 \\ &= (x + 2y)^2 - (\sqrt{2}y)^2 \\ &= (x + 2y)^2 - 2y^2 \end{aligned}$$

Therefore, $\boxed{f = (x + 2y)^2 - (\sqrt{2}y)^2}$ (difference of two squares.)