Step-1

Suppose $T(M)=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}[M]\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$

We have to find a matrix with $T(M)\neq 0$.

Step-2

Let $M=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

Then

$$T(M)=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & a\\ 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0 & a\\ 0 & 0\end{bmatrix}$$

Therefore, $T(M)\neq 0$ implies $b\neq 0$

Step-3

Thus range of *T* is

$$\{T(M)/M\in M_{2\times 2}\}$$
$$=\left\{\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}[M]\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}/M\in M_{2\times 2}\right\}$$
$$=\left\{\begin{bmatrix}0 & a\\ 0 & 0\end{bmatrix}/a\in \mathrm{R}\right\}$$

Step-4

Now the Kernel of *T* is

$$\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} / T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = 0\right\}$$

$$= \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} / \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = 0\right\}$$

$$= \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} / \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} = 0\right\}$$

Step-5

Continuation to the above

$$= \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} / a = 0\right\}$$

$$= \left\{\begin{bmatrix} 0 & b \\ c & d \end{bmatrix} / b, c, d \in \mathbf{R}\right\}$$

Hence the kernel of T is $\boxed{\left\{\begin{bmatrix} 0 & b \\ c & d \end{bmatrix} / b, c, d \in \mathbf{R}\right\}}$

Hence if $M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then M is nonzero matrix $\ni T(M) \neq 0$