

Step-1

If A is positive definite then,

$$\Rightarrow x^T A x > 0 \text{ for any } x \neq 0 \quad (1)$$

If B is positive definite then,

$$\Rightarrow x^T B x > 0 \text{ for any } x \neq 0 \quad (2)$$

Step-2

Now for $x \neq 0$,

$$\begin{aligned} x^T (A + B) x &= x^T A x + x^T B x \\ &> 0 \quad (\text{using (1) and (2)}) \end{aligned}$$

Thus $x^T (A + B) x > 0$ for $x \neq 0$.

Therefore, $A + B$ is also positive definite.