Step-1

Consider the following vectors:

$\mathbf{x}=\begin{bmatrix}1\\4\\0\\2\end{bmatrix}$, and $\mathbf{y}=\begin{bmatrix}2\\-2\\1\\3\end{bmatrix}$

The objective is to find the length of each vector and check that whether they are orthogonal.

Step-2

Length of the vector $\mathbf{x}=\|\mathbf{x}\|$

$=\sqrt{\mathbf{x}^T\mathbf{x}}$

$=\sqrt{\begin{pmatrix}1 & 4 & 0 & 2\end{pmatrix}\begin{bmatrix}1\\4\\0\\2\end{bmatrix}}$

$=\sqrt{1^2+4^2+0^2+2^2}$

$=\sqrt{21}$

Step-3

Length of the vector $\mathbf{y}=\|\mathbf{y}\|$

$=\sqrt{\mathbf{y}^T\mathbf{y}}$

$=\sqrt{\begin{pmatrix}2 & -2 & 1 & 3\end{pmatrix}\begin{bmatrix}2\\-2\\1\\3\end{bmatrix}}$

$=\sqrt{2^2+(-2)^2+1^2+3^2}$

$=\sqrt{18}$

$=3\sqrt{2}$

Step-4

Inner product of the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ $= \mathbf{x}^T \mathbf{y}$

$$= \begin{pmatrix} 1 & 4 & 0 & 2 \end{pmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \\ 3 \end{bmatrix}$$

$$= 1(2) + 4(-2) + 0(1) + 2(3)$$

$$= 2 - 8 + 0 + 6$$

$$= 0$$

Therefore the given vectors are orthogonal vectors.