

## Step-1

a)

Consider that a statement is "If  $A$  and  $B$  are identical except that  $b_{11} = 2a_{11}$ , then  $\det B = 2 \det A$ ".

The objective is to find that whether the statement is true or false.

## Step-2

Clearly the above statement is **false**.

To prove this consider an example as,

**Example:**

$$\text{Let } A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \text{ where } b_{11} = 2a_{11}.$$

Therefore,

$$\begin{aligned} \det A &= (1)(1) - (1)(1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \det B &= (2)(1) - (1)(1) \\ &= 1 \end{aligned}$$

Clearly  $\det B \neq 2 \det A$

Therefore, the given statement is false.

## Step-3

b)

Consider that a statement is "The determinant is the product of pivots".

The objective is to find that whether the statement is true or false.

## Step-4

Clearly the above statement is **false**.

To prove this, consider an example as,

**Example:**

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Three row exchanges (Row 3, Row 4) and (Row 2, Row3) result in identity matrix which is upper triangular here product of pivots is 1 but  $\det A = (-1)^3 \cdot 1 = -1$

The given statement is false

## Step-5

c)

The given statement "If  $A$  is invertible  $B$  is singular then  $A + B$  is invertible" is **False**.

The objective is to find that whether the statement is true or false.

## Step-6

Clearly the above statement is **false**.

To prove this, consider an example as,

**Example:**

Consider

$$A = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Now

$$\begin{aligned} \det A &= -2 + 1 \\ &= -1 \\ &\neq 0 \end{aligned}$$

And

$$\begin{aligned} \det B &= 1 - 1 \\ &= 0 \end{aligned}$$

So that  $A$  is invertible and  $B$  is singular, also

$$A + B = \begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix}$$

This is singular, as  $\det(A+B)=0$  and is not invertible.

Hence, the given statement is false.

## Step-7

d)

The given statement "If  $A$  is invertible and  $B$  is singular, then  $AB$  is singular".

The objective is to find that whether the statement is true or false

## Step-8

If  $A$  is invertible and  $B$  is singular then  $\det(A) \neq 0, \det(B) = 0$

We know that  $\det(AB) = \det A \cdot \det B$

Consider,

$$\begin{aligned}\det(AB) &= \det A \cdot \det B \\ &= (\det A)(0) \quad \text{since } \det B = 0 \\ &= 0\end{aligned}$$

Since  $\det(AB)$  then the matrix  $AB$  is singular

Hence, the given statement is true.

## Step-9

e)

The given statement "The determinant of  $AB - BA$  is zero".

The objective is to find that whether the statement is true or false.

## Step-10

Clearly the above statement is **false**.

To prove this, consider an example as,

**Example:**

Consider

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

Then

$$AB = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}, BA = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

## Step-11

Now,

$$AB - BA = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$$

$$\begin{aligned} \det(AB - BA) &= -1 + 2 \\ &= 1 \\ &\neq 0 \end{aligned}$$

Hence, the given statement is false.