

## Step-1

We have to explain that why the pivot rows and pivot columns of  $A$  (not  $R$ ) always give an  $r$  by  $r$  invertible submatrix of  $A$ .

## Step-2

Let  $A$  be an  $m$  by  $n$  matrix and suppose  $A$  has  $r$  pivot columns.

Then the  $r$  pivot columns of  $A$  form a  $m$  by  $r$  submatrix of  $A$ .

Then  $A$  has  $r$  pivot rows.

## Step-3

Then the  $\hat{r}$  pivot rows of  $A$  forms an  $r$  by  $n$  submatrix of  $A$

Therefore these  $r$  pivot rows and  $r$  pivot columns forms an  $r$  by  $r$  matrix, which is sub matrix of  $A$  and the determinant is not equal to zero. Hence this  $r$  by  $r$  matrix is an invertible matrix, which is the submatrix of  $A$ .