Step-1

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Given matrix is $A=\begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$

a) We have to factor A into LU, assuming $v_2 \neq 0$

We have $A=\begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$

Dividing row 2 by $\left(\frac{1}{v_2}\right)$ gives

$$\rightarrow \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$$

Step-2

Subtracting v_3 times row 2 from row 3, v_4 times row 2 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Subtracting v_1 times row 2 from row 1 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, the upper triangular matrix is $U=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$.

Step-3

To get the matrix L, we have to do the reverse operations on I .

That is, multiplying row 2 by v_2 gives

$$\begin{bmatrix}1&0&0&0\\0&v_2&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Adding v_3 times row 2 to row 3, v_4 times row 2 to row 4, $v_1$times row 2 to row 3 gives

$$\begin{bmatrix}1&v_1&0&0\\0&v_2&0&0\\0&v_3&1&0\\0&v_4&0&1\end{bmatrix}$$

Step-4

Therefore, the lower triangular matrix is $L=\begin{bmatrix}1&v_1&0&0\\0&v_2&0&0\\0&v_3&1&0\\0&v_4&0&1\end{bmatrix}$

Hence the LU factorization of the given matrix A is $\boxed{A=\begin{bmatrix}1&v_1&0&0\\0&v_2&0&0\\0&v_3&1&0\\0&v_4&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}}$

Step-5

b) We have to find A^{-1}, which has the same form as A.

Consider

$$[A \quad I] = \begin{bmatrix} 1 & v_1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & v_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & v_3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & v_4 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Dividing row 2 by $\left(\frac{1}{v_2}\right)$ gives

$$\rightarrow \begin{bmatrix} 1 & v_1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{v_2} & 0 & 0 \\ 0 & v_3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & v_4 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-6

Subtracting v_3 times row 2 from row 3, v_4 times row 2 from row 4 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -\frac{v_1}{v_2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{v_2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{v_3}{v_2} & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{v_4}{v_2} & 0 & 1 \end{bmatrix}$$

By gauss elimination method

$$\boxed{A^{-1} = \begin{bmatrix} 1 & -\frac{v_1}{v_2} & 0 & 0 \\ 0 & \frac{1}{v_2} & 0 & 0 \\ 0 & -\frac{v_3}{v_2} & 1 & 0 \\ 0 & -\frac{v_4}{v_2} & 0 & 1 \end{bmatrix}}$$