Step-1

Suppose that S and T are subspaces of R^{13}.

Consider the dimensions of subspaces:

$\dim S = 7$ and $\dim T = 8$.

Hence, S is subset of T.

$S \subset T$.

Step-2

(a)

Objective is to find the largest possible dimension of $S \cap T$.

$\dim S = 7$

< 8

$= \dim T$

Hence, $\dim S < \dim T$.

The largest possible dimension of $S \cap T$ is shown below:

$\dim(S \cap T) = \dim S$ Since $S \subset T$.

$= \boxed{7}$.

Step-3

(b)

Objective is to find the smallest possible dimension of $S \cap T$.

Here S and T are subspaces of R^{13}.

Hence, $\dim(S+T) = 13$

To find the $\dim(S \cap T)$, use the dimension formula:

$\dim(S+T) + \dim(S \cap T) = \dim(S) + \dim(T)$

Substitute the values of $\dim(S+T)=13$, $\dim(S)=7$ and $\dim(T)=8$ in the above formula.

$13+\dim(S\cap T)=7+8$

$13+\dim(S\cap T)=15$

$\dim(S\cap T)=2$

Hence, the smallest dimension of $S\cap T$ is $\dim(S\cap T)=\boxed{2}$.

Step-4

(c)

Objective is to find the smallest possible dimension of $(S+T)$.

The smallest possible dimension of $(S+T)$ is shown below:

$$\begin{aligned}\dim(\mathbf{S}+\mathbf{T}) &= \text{Maximum of }\{\dim\mathbf{S},\dim\mathbf{T}\}\\ &= \boxed{8}. \qquad \text{Since } S\subset T.\end{aligned}$$

Hence, the smallest dimension of $(S+T)$ is $\boxed{8}$.

Step-5

(d)

Objective is to find the largest possible dimension of $(S+T)$.

The largest possible dimension of $(S+T)$ is shown below:

$$\begin{aligned}\dim(\mathbf{S}+\mathbf{T}) &= \dim\mathbf{R}\\ &= \boxed{13}.\end{aligned}$$

Hence, the largest possible dimension of $(S+T)$ is $\boxed{13}$.

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