Step-1

a) From the property of Eigen values, the product of Eigen values of the matrix is equal to the determinant of that matrix.

So, the determinant of the matrix B is the product of Eigen values of the matrix B.

The Eigen values of the matrix B are 0, 1, 2.

Then, the determinant of the matrix B is $(0)(1)(2)=0$.

So, the matrix B is a singular matrix and its rank is less than 3.

Further, it has two non-zero Eigen values and so, the respective Eigen vectors are linearly independent, then the rank of the matrix B is 2.

Step-2

b) Determinant of $B^T B = |B^T B|$

$$\begin{aligned}&= |B^T|\cdot|B|\\ &= |B^T|\cdot(0)\\ &= 0\end{aligned}$$

Step-3

c) The Eigen values of B and that of B^T are one and the same.

Also, the Eigen value of B^n is λ^n when the Eigen value of B is λ.

But nothing to confirm the Eigen values of $B^T B$.

So, the data is insufficient.

Step-4

d) The roots of $|B-\lambda I|=0$ are $\lambda = 0,1,2$.

So, the Eigen values of $B+I$ are the roots of $|B-(1+\lambda)I|=0$ are $1+\lambda = 1,2,3$.

Consequently, the Eigen values of $(B+I)^{-1}$ are $1, \frac{1}{2},$ and $\frac{1}{3}$.