Step-1

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Given matrix is $A=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}$

We have to factor the symmetric matrix A into $A=LDL^T$, where D is a diagonal matrix and L is a lower triangular matrix.

Step-2

We have $A=\begin{bmatrix}1 & 3\\ 3 & 2\end{bmatrix}$

Subtracting 3 times row 1 from row 2 gives

$\rightarrow\begin{bmatrix}1 & 3\\ 0 & -7\end{bmatrix}$

Step-3

Applying the same row operation reversely on the identity matrix gives L.

Adding 3 times row 1 to row 2 gives $\rightarrow\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$

Therefore, $L=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$

The transpose of the matrix L is $L^T=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$.

Step-4

Since the pivots are 1 and -7 .

So the matrix D is $D=\begin{bmatrix}1 & 0\\ 0 & -7\end{bmatrix}$

Hence the LDL^T factorization of the given matrix is $\boxed{\begin{bmatrix}1 & 3\\3 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & -7\end{bmatrix}\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}}$.

Step-5

Given matrix is $A=\begin{bmatrix}1 & b\\b & c\end{bmatrix}$.

We have to factor the symmetric matrix A into $A=LDL^T$, where D is a diagonal matrix and L is a lower triangular matrix.

Step-6

Subtracting b times row 1 from row 2 gives

$$\rightarrow\begin{bmatrix}1 & b\\0 & c-b^2\end{bmatrix}$$

Step-7

Applying the same row operations reversely on the identity matrix gives L.

Adding b times row 1 to row 2 gives

$$\rightarrow\begin{bmatrix}1 & 0\\b & 1\end{bmatrix}$$

Therefore, $L=\begin{bmatrix}1 & 0\\b & 1\end{bmatrix}$

The transpose of the matrix L is $L^T=\begin{bmatrix}1 & b\\0 & 1\end{bmatrix}$.

Step-8

Since the pivots are 1and $c-b^2$.

So the matrix D is $D=\begin{bmatrix}1 & b\\0 & c-b^2\end{bmatrix}$

Hence the LDL^T factorization of the given matrix is $\boxed{\begin{bmatrix}1 & b\\b & c\end{bmatrix}=\begin{bmatrix}1 & 0\\b & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & c-b^2\end{bmatrix}\begin{bmatrix}1 & b\\0 & 1\end{bmatrix}}$

Step-9

Given matrix is $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

We have to factor the symmetric matrix A into $A = LDL^T$, where D is a diagonal matrix and L is a lower triangular matrix.

Step-10

Adding $\frac{1}{2}$ times row 1 to row 2 gives

$$\rightarrow \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Adding row 2 to $\frac{3}{2}$ times row 3 gives

$$\rightarrow \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

Step-11

Applying the same row operations reversely on the identity matrices gives L.

Subtracting $\frac{1}{2}$ times row 1 to row 2 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Subtracting row 2 from $\frac{3}{2}$ times row 3 gives

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix}$$

Step-12

Therefore, $L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix}$

The transpose of L is $L^T = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}$

Step-13

Since the pivots are 2, $\frac{3}{2}$, and 2.

So the matrix D is $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Hence the LDL^T factorization of the given matrix is $\boxed{\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 1 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{3}{2} \end{bmatrix}}$