Step-1

(i) For the matrix $A = \begin{bmatrix} 3 & 5 \\ 6 & 9 \end{bmatrix}$

Cofactor of 3 = 9

Cofactor of 5 = â€“6

Cofactor of $6 = -5$

Cofactor of 9 = 3

So that the matrix of cofactors is

$$C_A = \begin{bmatrix} 9 & -6 \\ -5 & 3 \end{bmatrix}$$

Step-2

And

$$\det A = 27 - 30$$
$$= -3$$

$$\neq 0$$

Step-3

Hence inverse of A exists and

$$A^{-1} = \frac{1}{\det A}.C_A{}^T$$

$$= \frac{1}{-3}\begin{bmatrix} 9 & -5 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & \frac{5}{3} \\ 2 & -1 \end{bmatrix}$$

And null space of $A = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

Step-4

(ii) For the matrix

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Cofactor of $\cos\theta = \cos\theta$

Cofactor of $-\sin\theta = -\sin\theta$

Cofactor of $\sin\theta = \sin\theta$

Cofactor of $\cos\theta = \cos\theta$

Step-5

So that the matrix of cofactors is

$$C_A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

And

$$\begin{aligned} \det A &= \cos^2\theta + \sin^2\theta \\ &= 1 \\ &\neq 0 \end{aligned}$$

Step-6

Hence inverse of A exists and

$$A^{-1} = \frac{1}{\det A} C_A^T$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

<u>Note</u>: observe that A^{-1} is the transpose of A.

Null space of $A = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

Step-7

(iii) $A = \begin{bmatrix} a & b \\ a & b \end{bmatrix}$

<u>Row1:</u> cofactor of $a = b$

Cofactor of $b =$ â€“a

<u>Row 2:</u> cofactor of $a = -b$

Cofactor of $b = a$

Step-8

So that the matrix of cofactors is

$$C_A = \begin{bmatrix} b & -a \\ -b & a \end{bmatrix}$$

Here $\det A = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} / ax + by = 0 \right\}$

Here A is singular. So, A^{-1} do not exist.