Step-1

Given that $A=\begin{pmatrix}9&0&0\\0&1&2\\0&2&8\end{pmatrix}$.

In the Cholesky factorization, $A=CC^T$, with $C=L\sqrt{D}$, the square roots of the pivots are on the diagonal of C.

$$A=\begin{pmatrix}9&0&0\\0&1&2\\0&2&8\end{pmatrix}$$

Apply $R_3\to R_3+(-)R_2$

$$=\begin{pmatrix}9&0&0\\0&1&2\\0&0&4\end{pmatrix}$$

Thus, the pivots are $\boxed{9,1,4}$.

Step-2

Now,

$$\begin{aligned}A&=LDL^T\\&=\left(L\sqrt{D}\right)\left(\sqrt{D}L^T\right)\\&=\begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix}\begin{pmatrix}9&0&0\\0&1&0\\0&0&4\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&2\\0&0&1\end{pmatrix}\end{aligned}$$

Here,

$L=\begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix}$, $D=\begin{pmatrix}9&0&0\\0&1&0\\0&0&4\end{pmatrix}$ and $\sqrt{D}=\begin{pmatrix}3&0&0\\0&1&0\\0&0&2\end{pmatrix}$

Step-3

So,

$$C = L\sqrt{D}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 2 \end{pmatrix}$$

Therefore, the matrix $\boxed{C = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 2 \end{pmatrix}}$

Step-4

Given matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 7 \end{pmatrix}$

Apply operations $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 6 \end{pmatrix}$$

Apply $R_3 \to R_3 - R_2$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{pmatrix}$$

Thus, the pivots are $\boxed{1,1,5}$.

Step-5

Now,

$$A = LDL^T$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Here,

$$L=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}, D=\begin{pmatrix}1&0&0\\0&1&0\\0&0&5\end{pmatrix} \text{ and } \sqrt{D}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&\sqrt{5}\end{pmatrix}$$

Step-6

Now,

$C = L\sqrt{D}$

$$=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&\sqrt{5}\end{pmatrix}$$

$$=\begin{pmatrix}1&0&0\\1&1&0\\1&1&\sqrt{5}\end{pmatrix}.$$

Step-7

Therefore, the matrix $\boxed{C=\begin{pmatrix}1&0&0\\1&1&0\\1&1&\sqrt{5}\end{pmatrix}}$