Step-1

The objective is to reduce $Ux=c$ and then $Rx=d$ by using Gaussian Elimination method:

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 1 & 3 & 2 & 0 \\ 2 & 0 & 4 & 9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} 2 \\ 5 \\ 10 \end{bmatrix}$$
$$=b$$

And also find a particular solution x_p and all null space solutions x_n.

Step-2

Given system $Ax=B$ is in the form;

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 1 & 3 & 2 & 0 \\ 2 & 0 & 4 & 9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} 2 \\ 5 \\ 10 \end{bmatrix}$$
$$=b$$

$$\xrightarrow[R_3-2R_1]{R_2-R_1,}\begin{bmatrix} 1 & 0 & 2 & 3 \\ 1 & 3 & 0 & -3 \\ 0 & 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_3 \end{bmatrix}=\begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2}\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_3 \end{bmatrix}=\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

Therefore 1 and 2 are pivot columns, x_1, x_2 are pivot variables and x_3, x_4 are free variables.

$$\begin{aligned} x_1+2x_3+3x_4 &= 2 \\ x_2-x_4 &= 1 \\ 3x_4 &= 6 \end{aligned}$$

$$\begin{aligned} x_4 &= 2 \\ x_2 &= 1+x_4 \\ &= 1+2 \\ &= 3 \end{aligned}$$

$$
\begin{aligned}
x_1 &= 2-2x_3-3x_4 \\
&= 2-2x_3-6 \\
&= -3-2x_3
\end{aligned}
$$

Therefore,

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3-2x_3 \\ 3 \\ x_3 \\ 2 \end{bmatrix}
$$

$$
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}
$$

The particular solution of given system, $x_p = \begin{bmatrix} -3 \\ 3 \\ 0 \\ 2 \end{bmatrix}$.

Step-3

The null space by $Rx = 0$

$$
\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
\begin{aligned}
x_2 - x_4 &= 0 \\
x_2 &= x_4 \\
x_4 + 2x_3 + 3x_4 &= 0 \\
x_1 &= -2x_3
\end{aligned}
$$

Therefore ,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ 0 \\ x_3 \\ 0 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Hence the null space $\boxed{x_n = \left\{ x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} : x_3 \in R \right\}}$.