Step-1

Given that $Ax = b$ has a solution and $A^T y = 0$

Let us consider $b^T y = (Ax)^T y$

$= (x^T A^T) y$ By the properties of transposing matrices

$= x^T (A^T y)$ By the associativity of multiplication of matrices

$= x^T (0)$

$= 0$

$b^T y = 0$ means y is perpendicular to b.

Step-2

(b) given that $A^T y = c$ has a solution and $Ax = 0$

We consider $c^T x = (A^T y)^T x$

$= \left[y^T (A^T)^T \right] x$

$= (y^T A) x$

$= y^T (Ax)$

$= y^T (0)$

$= 0$

Thus, we get $c^T x = 0$ and so, x is orthogonal to c.