Step-1

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Given matrix is $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix}$

We have to compute the symmetric LDL^T factorization of A.

Step-2

We have $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix}$

Subtracting 3 times row 1 from row 2 and 5 times row 1 from row 3 gives

$$A \approx \begin{bmatrix} 1 & 3 & 5 \\ 0 & 3 & 3 \\ 0 & 3 & 5 \end{bmatrix}$$

Subtracting row 2 from row 3 gives

$$A \approx \begin{bmatrix} 1 & 3 & 5 \\ 0 & 3 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

Step-3

Here the pivot positions are $1,3,2$.

Applying the same row operations reversely on the identity matrix gives L.

Adding 3 times row 1 to row 2 and 5 times row 1 to row 3 gives $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$

Adding row 2 to row 3 gives $L \approx \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}$

Now the transpose of L is $L^T = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Step-4

Since the pivots are 1, 3, and 2.

So the matrix D is $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Hence the LDL^T factorization of A is $\boxed{A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$.

Step-5

Given matrix is $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Subtracting $\left(\frac{c}{a}\right)$ times row 1 from row 2 gives $A \approx \begin{bmatrix} a & b \\ 0 & d - \frac{bc}{a} \end{bmatrix}$

Step-6

Applying the same row operations reversely on the identity matrices gives L.

Adding $\left(\frac{b}{a}\right)$ times row 1 to row 2 gives $L = \begin{bmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{bmatrix}$

Step-7

The transpose of the matrix L is $L^T = \begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix}$

Step-8

The pivots are a and $d - \frac{bc}{a}$

So the matrix D is $D = \begin{bmatrix} a & 0 \\ 0 & d - \frac{bc}{a} \end{bmatrix}$

Hence the LDL^T factorization of A is $\boxed{A = \begin{bmatrix} 1 & 0 \\ \frac{b}{a} & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & d - \frac{cb}{a} \end{bmatrix} \begin{bmatrix} 1 & \frac{b}{a} \\ 0 & 1 \end{bmatrix}}$