Step-1

a) Given that λ is an Eigen value of the matrix A and also its Eigen vector is x.

That is $Ax = \lambda x$.

Now,

$$\begin{aligned} Ax &= \lambda x \\ Ax - 7x &= \lambda x - 7x && \text{Add } -7x \text{ on both sides} \\ Ax - 7Ix &= \lambda x - 7x \\ (A - 7I)x &= (\lambda - 7)x && \text{Take } x \text{ as common on both sides} \\ Bx &= (\lambda - 7)x && \text{Since } A - 7I = B \end{aligned}$$

Therefore x is an Eigen vector of $B = A - 7I$, and also $\lambda - 7$ is also an Eigen value.

Here, observe that Eigen values are reduced by 7 to λ with unchanged Eigen vectors.

Step-2

b) Assume that $\lambda \neq 0$, and also $Ax = \lambda x$.

Multiply both sides with A^{-1}.

$$\begin{aligned} A^{-1}(Ax) &= A^{-1}(\lambda x) \\ (A^{-1}A)x &= \lambda(A^{-1}x) \\ Ix &= \lambda(A^{-1}x) && \text{Since } A^{-1}A = I \end{aligned}$$

$$\begin{aligned} x &= \lambda A^{-1}x \\ A^{-1}x &= \frac{1}{\lambda}x \\ A^{-1}x &= \left(\frac{1}{\lambda}\right)x \end{aligned}$$

Observe that the vector x satisfies $A^{-1}x = \left(\frac{1}{\lambda}x\right)$.

Hence, the Eigen value of A^{-1} is $\frac{1}{\lambda}$ and the Eigen vector does not change.

Thus, conclude that the Eigen value of A^{-1} is the reciprocal of the Eigen value of A and the respective Eigen vectors are one and the same.