Step-1

4764-1-25RE AID: 124

RID: 175 | 3/13/12

The matrix A is written as the product of the upper triangular matrix and its transpose.

$$A = LL^T$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying the determinant on both sides, we get $|A| = |L||L^T|$ by the properties of determinants.

While L and its transpose are triangular matrices, their respective determinants are nothing but the product of the respective diagonals.

So, $|A| = 1 \times 1$

While the determinant of A is not zero, we can say that A is non singular and so invertible. â€¦â€¦ (1)

Step-2

$$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 5 \\ 0 & 12 & 26 \end{bmatrix}$$

Observe that A is not symmetric while the 2, 3 entry is different from 3, 2 entry.

â€¦â€¦ (2)

The matrix A is tridiagonal if all non zero entries lie on the main diagonal and the two adjacent diagonals. Outside this band all entries are zero.

In our case, the diagonal entries are 1, 5, and 26 which are all not zero and so condition one satisfied.

The adjacent diagonals have the entries 2, 5 and 2, 12 which are also not zero.

Condition two is satisfied.

Other entries are in the places 1, 3 and 3, 1 which are both zeroes.

So, condition three is satisfied.

Therefore, the matrix A is tridiagonal. â€¦â€¦ (3)

Step-3

The pivot entries are obtained by reducing the matrix to lower triangular matrix using row operations.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 5 \\ 0 & 12 & 26 \end{bmatrix}$$

$$R_2 \to R_2 - 2R_1 \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 5 \\ 0 & 12 & 26 \end{bmatrix}$$

$$R_3 \to R_3 - 12R_2 \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & -34 \end{bmatrix}$$

$$\frac{R_3}{-34} \Rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, the pivot entries are 1, 1, and 1. â€¦â€¦ (4)

In the above course of reduction, we noted that 12 times the row 2 is subtracted from row 3 to make the 3, 2 entry zero. â€¦â€¦ (5)