Step-1

Given that to solve a rectangular system $Ax = b$, we replace A^{-1}(which does not exist) by $\left(A^T A\right)^{-1} A^T$ (which exists if *A* has independent columns). We have to show that this is a left inverse of *A* but not right inverse. On the left of *A* it gives the identity; on the right it gives the projection *P*.

Step-2

Since *A* has independent columns, that is, $A^T A = I$, we have

$\left(A^T A\right)^{-1} A^T A$

$= I^{-1} I$

$= I$ â€¦â€¦ (1)

Step-3

And

$A\left(A^T A\right)^{-1} A^T$

$= AA^{-1}\left(A^T\right)^{-1} A^T$

$\neq I$ â€¦â€¦ (2)

Because A^{-1}does not exist, that is, $AA^{-1} \neq I$

Step-4

Hence from (1) and (2), $\left(A^T A\right)^{-1} A^T$ is a left inverse of *A* but not right inverse.

Since $A\left(A^T A\right)^{-1} A^T$ is a projection matrix, by (2), on the right of *A*, $\left(A^T A\right)^{-1} A^T$ gives the projection *P*.

Hence the required result is proved.