

## Step-1

Given that if  $A$  has independent columns, and then  $A^T A$  is square and symmetric and invertible.

So,

$$\begin{aligned} x^T A^T A x &= (Ax)^T (Ax) \\ &= \text{length squared} \end{aligned}$$

So,  $x^T A^T A x = 0$  only if  $Ax = 0$

## Step-2

Given that  $A$  have independent columns

So,  $Ax = 0$  only when  $x = 0$

Therefore,  $A^T A$  is positive definite.