Step-1

Given

$L=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}$, $U=\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$, and $b=\begin{pmatrix}4\\5\\6\end{pmatrix}$

We have to solve $Lc=b$ to find *c,* we have to solve $Ux=c$ to find *x,* and finally we have to find *A*

Step-2

To solve $Lc=b$, the system is

$$\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}c=\begin{pmatrix}4\\5\\6\end{pmatrix}$$

Subtracting row 1 from row 2 and row 3 gives

$$\begin{pmatrix}1&0&0\\0&1&0\\0&1&1\end{pmatrix}c=\begin{pmatrix}4\\1\\2\end{pmatrix}$$

Step-3

Subtracting row 2 from row 3 gives

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}c=\begin{pmatrix}4\\1\\1\end{pmatrix}$$

From this system, $c=\begin{pmatrix}4\\1\\1\end{pmatrix}$

Step-4

To solve $Ux=c$, the system is

$$\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}x=\begin{pmatrix}4\\1\\1\end{pmatrix}$$

Subtracting1 times row 3 from row 1 and row 2 gives

$$\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix}x=\begin{pmatrix}3\\0\\1\end{pmatrix}$$

Step-5

Subtracting1 times row 2 from row 1 gives

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}x=\begin{pmatrix}3\\0\\1\end{pmatrix}$$

From this system, $x=\begin{pmatrix}3\\0\\1\end{pmatrix}$

Step-6

$$A=LU$$
$$=\begin{pmatrix}1&0&0\\1&1&0\\1&1&1\end{pmatrix}\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}$$
$$=\begin{pmatrix}1&0&1\\1&2&2\\1&2&3\end{pmatrix}$$

$$\Rightarrow \boxed{A=\begin{pmatrix}1&0&1\\1&2&2\\1&2&3\end{pmatrix}}$$