

Step-1

Given that the statement is

The inverse of a 2×2 matrix seems to have determinant = 1.

$$\det A^{-1} = \det \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (1)$$

$$= \frac{ad-bc}{ad-bc} \quad (2)$$

$$= 1 \quad (3)$$

Step-2

But this is a wrong calculation since in step (2), the formula used is $\det(tA) = t \cdot \det(A)$ which is false.

We need to find correct $\det A^{-1}$

Now

$$\det A^{-1} = \det \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \left(\frac{1}{ad-bc} \right)^2 (ad-bc) \quad (\text{Here } n=2, \text{ since the size of the matrix is 2 by 2})$$

$$= \frac{1}{ad-bc} \left(\text{since } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc \right)$$

Hence the correct calculation is $\boxed{\det A^{-1} = \frac{1}{ad-bc}}$.