

Step-1

We have to find three values of a for which elimination breaks down, temporarily or permanently, in

$$au + v = 1$$

$$4u + av = 2$$

Breakdown at the first step and can be fixed by exchanging rows-but not breakdown at the last step.

Step-2

If $a = 0$ then the equations are

$$v = 1$$

$$4u = 2$$

Now they require row exchange and the system is non singular.

Step-3

If $a = 2$ then the equations are

$$2u + v = 1$$

$$4u + 2v = 2$$

It is singular system with 2 pivots by applying row 2 \leftarrow 2times of row1 becomes

$$2u + v = 1$$

Now this has infinite solutions.

Step-4

If $a = -2$ then the equations are

$$-2u + v = 1$$

$$4u - 2v = 2$$

It is singular system with no solution because after elimination of first step it becomes $0 = 1$ position.