

Step-1

A graph consists of a set of vertices or nodes, and a set of edges that connect them. The edge goes from node j to node k , then that row has -1 in column j and +1 in column k .

The second graph represents the six games between four teams. So, the incidence matrix A for the second graph in the figure is,

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Step-2

The score differences are b_1, b_2, \dots, b_6 . We need to find the conditions, so that we can assign potentials x_1, \dots, x_4 so that the potential differences agrees with the b â€™s.

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

Let

We need set the incidence matrix as $\begin{bmatrix} A & b \end{bmatrix}$. So,

$$\begin{bmatrix} A & b \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 & 0 & b_1 \\ -1 & 0 & 1 & 0 & b_2 \\ 0 & -1 & 1 & 0 & b_3 \\ 0 & -1 & 0 & 1 & b_4 \\ -1 & 0 & 0 & 1 & b_5 \\ 0 & 0 & -1 & 1 & b_6 \end{bmatrix}$$

Apply $R_1 \rightarrow R_1 + R_4$, $R_2 \rightarrow R_2 - R_5$ and $R_4 \rightarrow R_4 - R_6$

$$= \begin{bmatrix} -1 & 0 & 0 & 1 & b_1 + b_4 \\ 0 & 0 & 1 & -1 & b_2 - b_5 \\ 0 & -1 & 1 & 0 & b_3 \\ 0 & -1 & 1 & 0 & b_4 - b_6 \\ -1 & 0 & 0 & 1 & b_5 \\ 0 & 0 & -1 & 1 & b_6 \end{bmatrix}$$

Step-3

Apply $R_1 \rightarrow R_1 - R_5$ and $R_3 \rightarrow R_3 - R_4$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & b_1 + b_4 - b_5 \\ 0 & 0 & 1 & -1 & b_2 - b_5 \\ 0 & 0 & 0 & 0 & b_3 - b_4 + b_6 \\ 0 & -1 & 1 & 0 & b_4 - b_6 \\ -1 & 0 & 0 & 1 & b_5 \\ 0 & 0 & -1 & 1 & b_6 \end{bmatrix}$$

Apply $R_2 \rightarrow R_2 + R_6$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & b_1 + b_4 - b_5 \\ 0 & 0 & 0 & 0 & b_2 - b_5 + b_6 \\ 0 & 0 & 0 & 0 & b_3 - b_4 + b_6 \\ 0 & -1 & 1 & 0 & b_4 - b_6 \\ -1 & 0 & 0 & 1 & b_5 \\ 0 & 0 & -1 & 1 & b_6 \end{bmatrix}$$

Therefore, $AX = b$ is solvable if

$$b_1 + b_4 - b_5 = 0$$

$$b_3 - b_4 + b_6 = 0$$

$$b_2 - b_5 + b_6 = 0$$

Therefore, it is possible at $\boxed{b_1 + b_4 - b_5 = 0, b_3 - b_4 + b_6 = 0 \text{ and } b_2 - b_5 + b_6 = 0}$