Step-1

Given Fibonacciâ€™s matrix is $A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$.

We have to verify the Cayley-Hamilton Theorem on the given Fibonacciâ€™s matrix.

Step-2

The characteristic equation of A is

$$|A-\lambda I|=0$$

$$\Rightarrow\begin{vmatrix}1-\lambda & 1\\ 1 & -\lambda\end{vmatrix}=0$$

$$\Rightarrow-\lambda(1-\lambda)-1=0$$
$$\Rightarrow\lambda^2-\lambda-1=0$$

Therefore $\det(A-\lambda \mathrm{I})=\lambda^2-\lambda-1$

Step-3

By Cayley â€“ theorem, every square matrix satisfies its characteristic equation.

Therefore, $A^2-A-I=0$

To prove this $A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$

$$\begin{aligned}A^2&=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\\ &=\begin{bmatrix}1(1)+1(1) & 1(1)+1(0)\\ 1(1)+0(1) & 1(1)+0(0)\end{bmatrix}\\ &=\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}\end{aligned}$$

Step-4

Therefore,

$$
\begin{aligned}
A^2 - A - I &= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 2-2-1 & 1-1-0 \\ 1-1-0 & 1-0-1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
&= 0
\end{aligned}
$$

Hence the given Fibonacci matrix satisfies the Cayley-Hamilton theorem.