Step-1

Let A be a square matrix such that $A^2 = A$.

Such a matrix is called Idempotent Matrix.

Suppose A has full set of eigenvectors and thus, A can be diagonalized. Therefore, the rank of the matrix A is n, where A is n by n matrix.

Step-2

Let us consider x to be an eigenvector with respect to the eigenvalue $\lambda = 1$.

Thus, we have $Ax = x$.

But, $A^2 = A$.

Thus, we get

$$\begin{aligned} A(Ax) &= Ax \\ A^2x &= x \end{aligned}$$

This clearly indicates that the vector x lies in the row space of the matrix A.

Step-3

Let us consider y to be an eigenvector with respect to the eigenvalue $\lambda = 1$.

Thus, we have

$$\begin{aligned} Ay &= 0y \\ &= 0 \end{aligned}$$

This indicates that y must be in the nullspace of the matrix A.