

Step-1

The objective is to find the matrices E^2, E^8 and E^{-1} for the following matrix.

$$E = \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix}$$

Step-2

Find the matrix E^2 .

$$\begin{aligned} E^2 &= E.E \\ &= \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 6+6 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 2(6) & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 12 & 1 \end{pmatrix} \end{aligned}$$

Therefore, the matrix of E^2 is $\boxed{\begin{pmatrix} 1 & 0 \\ 12 & 1 \end{pmatrix}}$.

Step-3

Now find the value of E^8 .

$$\begin{aligned} E^3 &= E \cdot E^2 \\ &= \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 6+6 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 6+6+6 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 3(6) & 1 \end{pmatrix} \end{aligned}$$

From the above, the matrix of E^8 is,

$$E^8 = \begin{pmatrix} 1 & 0 \\ 8(6) & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 48 & 1 \end{pmatrix}$$

Hence, the matrix of E^8 is $\boxed{\begin{pmatrix} 1 & 0 \\ 48 & 1 \end{pmatrix}}$.

Step-4

Find the inverse matrix of $E = \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix}$.

The inverse matrix of matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by $\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$E^{-1} = \frac{1}{1(1)-(-6)0} \begin{pmatrix} 1 & -0 \\ -6 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}$$

Therefore, inverse of E is $\boxed{E^{-1} = \begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}}$.