Step-1

We have to show that the best least-squares fit to a set of measurements $y_1, \ldots, y_m$ by a horizontal line is their average $C = \frac{y_1 + \ldots + y_m}{m}$.

Step-2

First we write the equations that would hold if a line could go through *m* points.

Then every *C* would agree exactly with *b*,

$Ax = b$ is

$$
\begin{aligned}
C &= y_1 \\
C &= y_2 \\
&. \\
&. \\
&. \\
C &= y_m
\end{aligned}
$$

Step-3

The matrix form of the above system is

$$
\begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ . \\ . \\ 1 \end{bmatrix} [C] = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_m \end{bmatrix}
$$

Let $A = \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ . \\ . \\ 1 \end{bmatrix}, \; x = [C]$ and $b = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_m \end{bmatrix}$

Step-4

We know that the least-squares best fit is given by

$$A^T A\hat{x} = A^T b$$

$$\Rightarrow \left[1,1,1,\ldots,1(m \text{ times})\right]\begin{bmatrix}1\\1\\1\\.\\.\\.\\1\end{bmatrix}\left[\hat{C}\right] = \left[1,1,1,\ldots,1(m \text{ times})\right]\begin{bmatrix}y_1\\y_2\\y_3\\.\\.\\.\\y_m\end{bmatrix}$$

$$\Rightarrow \left[1+1+\ldots(m \text{ times})\right]\left[\hat{C}\right] = \left[y_1 + y_2 + \ldots + y_m\right]$$

$$\Rightarrow (m.1)\hat{C} = y_1 + y_2 + \ldots + y_m$$

$$\Rightarrow \hat{C} = \frac{y_1 + y_2 + \ldots + y_m}{m}$$

Hence $\boxed{C = \frac{y_1 + y_2 + \ldots + y_m}{m}}$ is the average of the measurements $y_1, y_2, \ldots, y_m$.