

Step-1

Given that $u = (1+i, 1-i, 1+2i)$ and $v = (i, i, i)$

We have to find the lengths of u and v .

Step-2

We know that if $x = (x_1, \dots, x_n)$ then the length of x is $\|x\| = \sqrt{x_1^2 + \dots + x_n^2}$

Now the length of u is

$$\begin{aligned}\|u\|^2 &= |1+i|^2 + |1-i|^2 + |1+2i|^2 \\ &= (1^2 + 1^2) + (1^2 + (-1)^2) + (1^2 + 2^2) \\ &= 2 + 2 + 5 \\ &= 9 \\ \Rightarrow \|u\| &= \sqrt{9} = 3\end{aligned}$$

Hence $\boxed{\|u\| = 3}$

Step-3

Now the length of v is

$$\begin{aligned}\|v\|^2 &= |i|^2 + |i|^2 + |i|^2 \\ &= 1^2 + 1^2 + 1^2 \\ &= 3 \\ \Rightarrow \|v\| &= \sqrt{3}\end{aligned}$$

Hence $\boxed{\|v\| = \sqrt{3}}$

Step-4

Since $u = (1+i, 1-i, 1+2i)$

So
$$u = \begin{bmatrix} 1+i \\ 1-i \\ 1+2i \end{bmatrix}$$

Then $U^H = [1-i \ 1+i \ 1-2i]$

And $v = (i, i, i)$

Then $v = \begin{bmatrix} i \\ i \\ i \end{bmatrix}$ and $v^H = [-i \ -i \ -i]$

Step-5

Now we have to find $u^H v$

Now

$$\begin{aligned} u^H v &= [1-i \ 1+i \ 1-2i] \begin{bmatrix} i \\ i \\ i \end{bmatrix} \\ &= [i(1-i) + i(1+i) + i(1-2i)] \\ &= [i - i^2 + i + i^2 i - 2i^2] \\ &= [3i + 2] \\ &= [2 + 3i] \end{aligned}$$

Therefore, $\boxed{u^H v = [2 + 3i]}$

Step-6

Now we have to find $v^H u$

Now

$$\begin{aligned} v^H u &= [-i \ -i \ -i] \begin{bmatrix} 1+i \\ 1-i \\ 1+2i \end{bmatrix} \\ &= [-i(1+i) - i(1-i) - i(1+2i)] \\ &= [-i - i^2 - i - i^2 - i - 2i^2] \\ &= [-3i - 2i^2] \\ &= [2 - 3i] \end{aligned}$$

Therefore, $v''u = [2 - 3i]$