Step-1

For the following matrices we have to find that which numbers a, b, c lead to row exchange, and which make the matrix singular.

$$A = \begin{pmatrix} 1 & 2 & 0 \\ a & 8 & 3 \\ 0 & b & 5 \end{pmatrix}, \text{ and } A = \begin{pmatrix} c & 2 \\ 6 & 4 \end{pmatrix}$$

Step-2

Fist we consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 \\ a & 8 & 3 \\ 0 & b & 5 \end{pmatrix}$$

If $a = 4$ then the matrix is

$$\begin{pmatrix} 1 & 2 & 0 \\ 4 & 8 & 3 \\ 0 & b & 5 \end{pmatrix}$$

And subtracting 4 times row 1 from row 2 gives

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & b & 5 \end{pmatrix}$$

There is 0 in the position of second pivot.

So the matrix needs row exchange when $a = 4$

Step-3

The matrix is singular if and only if the determinant is zero so

By finding the determinant *A,* we have

$$40 - 3b - 10a = 0$$
$$\Rightarrow 10a + 3b = 40$$

So the matrix is singular if $10a + 3b = 40$

Step-4

Now consider the second matrix

$$A = \begin{pmatrix} c & 2 \\ 6 & 4 \end{pmatrix}$$

If $c = 0$ then it leads to row exchange because if $c = 0$ then the matrix is

$$\begin{pmatrix} 0 & 2 \\ 6 & 4 \end{pmatrix}$$

Here the first coefficient is zero so it needs row exchange.

Hence A needs row exchange if $c = 0$

Step-5

The matrix is singular when $c = 3$

Since if $c = 3$, its determinant becomes zero so it is singular.

Hence the matrix A is singular if $c = 3$