Step-1

(a) A real symmetric matrix is given by

$$A^H = A$$

Consider an n by n symmetric matrix, then there are n entries on the diagonal and $(n-1)+\cdots+1$ entries above the diagonal that can be chosen arbitrary.

Since, other $1+\cdots+(n-1)$ entries below the diagonal are determined by the symmetry of the matrix.

Therefore, there are only $n+(n-1)+\cdots+1=\frac{1}{2}n(n+1)$ degrees of freedom in the selection of the n^2 entries in a n by n real symmetric matrix.

Therefore, degrees of freedom of an n by n symmetric matrix is $\boxed{\frac{1}{2}n(n+1)}$.

Step-2

Consider an n by n diagonal matrix, then there are n entries on the diagonal and it can be chosen arbitrary.

Therefore, degrees of freedom of an n by n diagonal matrix is $\boxed{n}$.

Step-3

Consider the entries of i*th* and j*th* columns of an orthogonal matrix, as a_{ki} and a_{lj} respectively.

Then by using the condition of orthogonal we have

$(a_{ki}, a_{lj}) = 0$, here $i \neq j$

This condition show a symmetric, so the degree of freedom is given by $\frac{n(n-1)}{2}$.

And

$(a_{ki}, a_{li}) = 1$, for $i = 1, \cdots, n$

This condition show the degree of freedom is given by n.

We know that an n by n matrix has n^2 degrees of freedom.

To find the total degrees of freedom of an n by orthogonal matrix, do the following calculation:

$$n^2-\frac{1}{2}(n)(n-1)-n=\frac{2n^2-n^2+n-2n}{2}$$
$$=\frac{n^2-n}{2}$$
$$=\frac{(n)(n-1)}{2}$$

Step-4

Therefore, degrees of freedom of an n by n orthogonal matrix is $\boxed{\frac{(n)(n-1)}{2}}$.

Step-5

(b) We know that Hermitian matrix A is equal to their conjugate transpose.

$$A^H=\overline{A}^T$$

Every entry in the transposed matrix is same as the complex conjugate of the corresponding entry in the original matrix.

$$\left(A^H\right)_{ij}=\overline{A_{ji}}.$$

Step-6

Consider a 3 by 3 matrix A as follows:

$$A=\begin{bmatrix} a & b+ic & e+if \\ b-ic & d & h+ik \\ e-if & h-ik & g \end{bmatrix}$$

Here a,b,c,d,e,f,g,h,k all are real numbers.

Now to find the transpose matrix, change A_{ij} by A_{ji}.

$$A^T=\begin{bmatrix} a & b-ic & e-if \\ b+ic & d & h-ik \\ e+if & h+ik & g \end{bmatrix}$$

Taking the complex conjugate of entry, we get

$$\overline{A}^T = \begin{bmatrix} a & b+ic & e+if \\ b-ic & d & h+ik \\ e-if & h-ik & g \end{bmatrix}$$

Hence we get $A = \overline{A}^T$.

Here nine real numbers determine the Hermitian matrices *A,* so the real degrees of freedom are 9.

Therefore, 3 by 3 Hermitian matrices *A* have $\boxed{9}$ real degrees of freedom.

Step-7

We know that a complex *n* by *n* matrix has $2n^2$ degrees of freedom.

Since the unitary matrix *U* satisfied that

$UU^H = I$

So, we get n^2 terms in a unitary matrix.

Hence, *n* by *n* unitary matrix has n^2 real degrees of freedom.

Therefore, 3 by 3 unitary matrices *A* have $\boxed{9}$ real degrees of freedom.