Step-1

Let if possible, both the alternatives hold for some A and b. Thus, we have: $Ax \geq b$ has a nonnegative solution x and there exists a vector y, such that $yA \geq 0$ and $yb < 0$.

Consider, $yAx \geq yb$.

Since $yb < 0$, it is clear that $yAx < 0$.

But we have assumed that $yA \geq 0$ and x is a nonnegative vector.

Therefore, $yAx \geq 0$.

Step-2

Thus, we have $yAx < 0$ as well as $yAx \geq 0$. This is contradiction. Thus, our assumption that both the alternatives can hold simultaneously is wrong. This shows that both the alternatives do not hold simultaneously.