

## Step-1

To prove that  $\mathbf{V}^\perp \subset \mathbf{S}^\perp$  if  $\mathbf{S} \subset \mathbf{V}$

Orthogonal complement of  $\mathbf{S}$  is defined as;

$$\mathbf{S}^\perp = \{ \alpha \in \mathbf{V} / \alpha^T \beta = 0, \forall \beta \in \mathbf{S} \}$$

In the same way, orthogonal complement of  $\mathbf{V}$  is defined as;

$$\mathbf{V}^\perp = \{ \alpha \in \mathbf{V} / \alpha^T \beta = 0, \forall \beta \in \mathbf{V} \}$$

## Step-2

Suppose,

$$\alpha \in \mathbf{V}^\perp$$

this implies  $\alpha^T \beta = 0, \forall \beta \in \mathbf{V}$

this implies  $\alpha^T \beta = 0, \forall \beta \in \mathbf{S}$  (since  $\mathbf{S} \subset \mathbf{V}$  )

this implies  $\alpha \in \mathbf{S}^\perp$

Therefore,  $\boxed{\mathbf{V}^\perp \subset \mathbf{S}^\perp}$