MA202 Complex Analysis, Midterm Exam

Name:

ID:

Problem 1. [15 pts]

(i) What is the definition of a holomorphic function on a domain $\Omega \subseteq \mathbb{C}$?

(ii) Determine whether $z^3, |z|^3$ are holomorphic functions on $\mathbb{C}$.

Problem 2. [15 pts] Let f be a holomorphic function on a connected open set $\Omega \subseteq \mathbb{C}$. Show that if $|f|$ is constant, then f is constant.

Problem 3. [20 pts] Show that for all $w \in \mathbb{C}$, there is

$$\int_{-\infty}^{\infty} e^{-\pi x^2} e^{2\pi i x w} \mathrm{d}x = e^{-\pi w^2}.$$

Problem 4. [20 pts] Let n be an integer $\geqslant 2$ and α a real number such that $n > 1 + \alpha > 0$. Evaluate the integral

$$\int_0^{\infty} \frac{x^\alpha}{1 + x^n} \mathrm{d}x.$$

Problem 5. [15 pts] Find the number of zeros, counting multiplicity, of the polynomial $z^8 - 7z^3 + 2z + 1$ in the annulus $1 < |z| < 2$.

Problem 6. [15 pts] Let $f(z)$ be a holomorphic function on the annulus

$$\Omega := \{z \in \mathbb{C} : r_1 < |z| < r_2\}.$$

Show that there exists complex numbers $\{a_n\}_{n \in \mathbb{Z}}$ such that

$$f(z) = \sum_{n \in \mathbb{Z}} a_n z^n,$$

where the right hand side is absolutely and uniformly convergent on any closed annulus $\Omega' := \{z \in \mathbb{C} : r_1' \leqslant |z| \leqslant r_2'\}$ contained in Ω.