Step-1

Multiply $Aq_j = b_{j-1}q_{j-1} + a_jq_j + b_jq_{j+1}$ throughout by q_j^{T}. Since, the vectors q_i are orthogonal,

$$q_i^{\mathrm{T}}q_j = 1 \text{ if } i = j$$
$$q_i^{\mathrm{T}}q_j = 0 \text{ if } i \neq j$$

Thus, we get

$$\begin{aligned} q_j^{\mathrm{T}}\left(Aq_j\right) &= q_j^{\mathrm{T}}\left(b_{j-1}q_{j-1} + a_jq_j + b_jq_{j+1}\right) \\ &= q_j^{\mathrm{T}}b_{j-1}q_{j-1} + q_j^{\mathrm{T}}a_jq_j + q_j^{\mathrm{T}}b_jq_{j+1} \\ &= b_{j-1}q_j^{\mathrm{T}}q_{j-1} + a_jq_j^{\mathrm{T}}q_j + b_jq_j^{\mathrm{T}}q_{j+1} \\ &= a_j \end{aligned}$$

Therefore, the formula for a_j is $\boxed{a_j = q_j^{\mathrm{T}}Aq_j}$.

Step-2

Consider again $Aq_j = b_{j-1}q_{j-1} + a_jq_j + b_jq_{j+1}$. Let us express this equation in the matrix form. As $q_0 = 0$, we need not consider a_0 and b_0.

We get

$$A\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & 0 & 0 & \ldots & 0 \\ b_1 & a_2 & b_2 & 0 & \ldots & 0 \\ 0 & b_2 & a_3 & b_3 & \ldots & 0 \\ 0 & 0 & b_3 & . & . & . \\ 0 & 0 & . & . & . & . \\ 0 & 0 & . & . & . & a_n \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_n \end{bmatrix}$$

Step-3

If we write $T = \begin{bmatrix} a_1 & b_1 & 0 & 0 & \ldots & 0 \\ b_1 & a_2 & b_2 & 0 & \ldots & 0 \\ 0 & b_2 & a_3 & b_3 & \ldots & 0 \\ 0 & 0 & b_3 & . & . & . \\ 0 & 0 & . & . & . & . \\ 0 & 0 & . & . & . & a_n \end{bmatrix}$, then we get the following:

The equation $Aq_j = b_{j-1}q_{j-1} + a_jq_j + b_jq_{j+1}$ can be expressed as $AQ = QT$, where T is a symmetric matrix.