

## Step-1

We have to find the rank of the following matrices, and we have to express  $A$  as  $A = uv^T$  :

$$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix}, \text{ and } A = \begin{bmatrix} 2 & -2 \\ 6 & -6 \end{bmatrix}$$

## Step-2

Consider the first matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_1} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

## Step-3

Therefore there is only one pivot column (first column)

So rank of  $A = 1$ .

## Step-4

Now consider the product of the first column which is a linear combination the remaining columns and the first row which also a linear combination of the remaining rows then we have

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 6 \end{bmatrix} = A$$

## Step-5

Therefore

$$A = uv^T$$

$$\Rightarrow A = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

Here

## Step-6

Next we consider the second matrix

$$A = \begin{bmatrix} 2 & -2 \\ 6 & -6 \end{bmatrix}$$

$$\underline{R_2 - 3R_1, \frac{1}{2}R_1} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Therefore  $A$  has one pivot column (first column)

$$\boxed{\text{So rank } A = 1}$$

## Step-7

And consider the product of the first column which is a linear combination the remaining columns and the first row which also a linear combination of the remaining rows then we have

$$\begin{bmatrix} 2 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 6 & -6 \end{bmatrix} = A$$

## Step-8

Therefore

$$A = uv^T$$

$$\Rightarrow A = \begin{bmatrix} 2 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$u = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Here