Step-1

We have to find the matrix that projects every point in the plane onto the line $x + 2y = 0$.

Given line is $x + 2y = 0$

Put

$y = k$

$\Rightarrow x = -2k$

Step-2

Therefore any point on the line is $a = (-2k, k)$

$$aa^T = \begin{bmatrix} -2k \\ k \end{bmatrix} \begin{bmatrix} -2k & k \end{bmatrix}$$

$$= \begin{bmatrix} 4k^2 & -2k^2 \\ -2k^2 & k^2 \end{bmatrix}$$

$$a^T a = \begin{bmatrix} -2k & k \end{bmatrix} \begin{bmatrix} -2k \\ k \end{bmatrix}$$

$$= 5k^2$$

Step-3

The projection matrix

$$P = \frac{aa^T}{a^T a}$$

$$= \frac{1}{5k^2} \begin{bmatrix} 4k^2 & -2k^2 \\ -2k^2 & k^2 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$$

So the required matrix is $\dfrac{1}{5} \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$