Step-1

Thus, if we let $A=\begin{bmatrix}0.9 & 0.3\\ 0.1 & 0.7\end{bmatrix}$, then A^{-1} has the eigenvalues as 1 and $\frac{1}{0.6}$. From the equation $\frac{1}{\lambda}x=A^{-1}x$, it is clear that the eigenvectors of A are same as that of A^{-1}.

Let us obtain the eigenvectors of A.

Step-2

Write $Ax=\lambda x$, where $\lambda=1$. This gives

$$\begin{bmatrix}0.9 & 0.3\\ 0.1 & 0.7\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}x\\ y\end{bmatrix}$$
$$0.9x+0.3y=x$$
$$0.1x+0.7y=y$$

Therefore,

$$-0.1x+0.3y=0$$
$$0.1x-0.3y=0$$

Thus, if (x,y) is an eigenvector of A, then $0.1x=0.3y$. Therefore, $(3,1)$ is an eigenvector of A.

Step-3

Write $Ax=\lambda x$, where $\lambda=0.6$. This gives

$$\begin{bmatrix}0.9 & 0.3\\ 0.1 & 0.7\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=0.6\begin{bmatrix}x\\ y\end{bmatrix}$$
$$0.9x+0.3y=0.6x$$
$$0.1x+0.7y=0.6y$$

Therefore,

$$0.3x+0.3y=0$$
$$0.1x+0.1y=0$$

Thus, if (x,y) is an eigenvector of A, then $0.1x=-0.1y$. Therefore, $(1,-1)$ is an eigenvector of A.

Step-4

The eigenvectors of the matrix A are $(3,1)$ and $(1,-1)$. Therefore, the eigenvectors of A^{-1} are $\boxed{(3,1)\text{and}(1,-1)}$.

Suppose we consider the inverse power method and start with $u_{-k} = A^{-k} u_0$. This is same as $A^k u_{-k} = u_0$.

In this case, the method converges to the smallest eigenvalue and its corresponding eigenvector. Out of the two eigenvalues, the smallest one is 1. Therefore, the method converges to the eigenvector $\boxed{(3,1)}$.