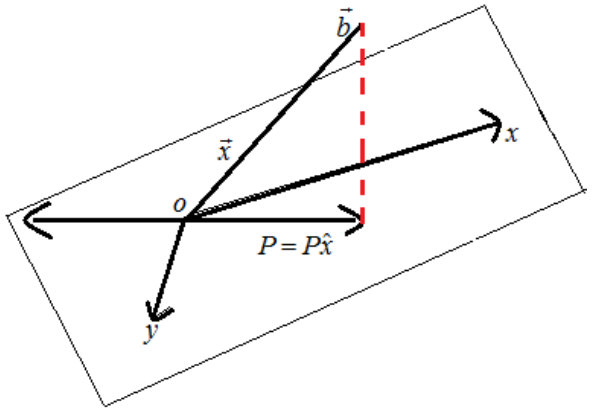
Step-1

Given that P is the projection matrix onto a line in the $x-y$ plane.

The geometrical explain is as follows,



$P = P\vec{x}$

$\vec{b} = x - P$

$= (I - P)x$

Step-2

The projection matrix P has two basic properties.

(i) $P^2 = P$

(ii) $P^T = P$

Conversely, any symmetric matrix with $P^2 = P$ represents a projection.

So,

$H^2 = (I - 2P)(I - 2P)$

$= I^2 - 2IP - 2PI + 4PP$

$= I - 2P - 2P + 4P^2 \quad (\text{Since } I^2 = I)$

$= I - 4P + 4P \qquad (\text{From the properties})$

Â Â Â Â Â Â = I Â Â Â Â Â Â Â Â Â Â Â Â

Therefore, $\boxed{H^2 = I}$ and $\boxed{\text{two reflections (i.e. reflection of the same reflection) gives } I}$.

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