Step-1

Consider two matrices *A* and *B*. If these have same Eigen values and same Eigen vectors then are they considered as equal matrices?

Step-2

The answer is no. Consider the following two matrices:

$$A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

$$B=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

Both matrices are upper triangular matrices. So, Eigen values of both matrices are $\lambda=(1,1)$

Step-3

To calculate Eigen vectors do the following calculations:

$$(A-\lambda_1 I)x=0$$

$$\begin{bmatrix}1-\lambda & 1\\ 0 & 1-\lambda\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

On solving values of *y* and *z* corresponding to $\lambda=1$ are as follows:

$$x_1=\begin{bmatrix}y\\ z\end{bmatrix}$$

$$=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$x_2=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Step-4

Similarly, Eigen values of other matrix can be calculated. These are given as below:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step-5

Eigen vectors of both matrices are also same. However matrices are not equal.

Step-6

Therefore, if two matrices *A* and *B* have same Eigen values and same Eigen vectors then they can not be considered as equal matrices.

Step-7

Find matrices *A* and *B* such that $A \neq B$ also Eigen values and Eigen vectors should be as follows:

$\lambda = (0,0)$
$v_1 = (x_1, 0)$

Step-8

Consider the following matrices:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$$

Both matrices are upper triangular matrices. So, Eigen values of both matrices are $\lambda = (0,0)$

Step-9

To calculate Eigen vectors do the following calculations:

$$(A-\lambda_1 I)x=0$$

$$\begin{bmatrix} 0-\lambda & 1 \\ 0 & 0-\lambda \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step-10

On solving values of *y* and *z* corresponding to $\lambda=0$ are as follows:

$$v_1=\begin{bmatrix} y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_2=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step-11

Similarly, Eigen values of other matrix can be calculated. These are given as below:

$$v_1=\begin{bmatrix} y \\ z \end{bmatrix}$$

$$=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$v_2=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step-12

Eigen vectors of both matrices *A* and *B* are also same. However matrices are not equal. Therefore,

$$A=\boxed{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}$$

$$B=\boxed{\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}}$$