Step-1

Let the vector $x=(1,1,\ldots,1)$.

The Rayleigh quotient is given by,

$$R(x)=\frac{x^{\mathrm{T}}Ax}{x^{\mathrm{T}}x}$$

Step-2

We have,

$$\begin{aligned} x^{\mathrm{T}}x &= (1,1,\ldots,1)\begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix} \\ &= 1+1+\ldots+1 \\ &= n \end{aligned}$$

Step-3

Also, we get

$$\begin{aligned} x^{\mathrm{T}}Ax &= (1,1,\ldots,1)\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ a_{n1} & a_{n2} & \cdots & a_{nn}\end{bmatrix}\begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix} \\ &= (1,1,\ldots,1)\begin{bmatrix}a_{11}+a_{12}+\cdots+a_{1n}\\ a_{21}+a_{22}+\cdots+a_{2n}\\ \vdots\\ a_{n1}+a_{n2}+\cdots+a_{nn}\end{bmatrix} \\ &= a_{11}+a_{12}+\cdots+a_{1n}+a_{21}+a_{22}+\cdots+a_{2n}+\cdots+a_{n1}+a_{n2}+\cdots+a_{nn} \end{aligned}$$

Therefore, $x^{\mathrm{T}}Ax$ is equal to the sum of all the entries of the matrix A.

Step-4

Thus, we get the following:

$$R(x) = \frac{x^{\mathrm{T}} A x}{x^{\mathrm{T}} x}$$
$$= \frac{a_{11} + a_{12} + \cdots + a_{1n} + a_{21} + a_{22} + \cdots + a_{2n} + \cdots + a_{n1} + a_{n2} + \cdots + a_{nn}}{n}$$

We know that Rayleighâ€™s quotient always lies between the smallest eigenvalue *Î»*$_1$ and the largest eigenvalue *Î»*$_2$.

Thus,

$$\lambda_1 \le \frac{a_{11} + a_{12} + \cdots + a_{1n} + a_{21} + a_{22} + \cdots + a_{2n} + \cdots + a_{n1} + a_{n2} + \cdots + a_{nn}}{n} \le \lambda_n$$

This gives,

$$n\lambda_1 \le a_{11} + a_{12} + \cdots + a_{1n} + a_{21} + a_{22} + \cdots + a_{2n} + \cdots + a_{n1} + a_{n2} + \cdots + a_{nn} \le n\lambda_n.$$

Therefore, the relation is: *nÎ»*$_1$ ≤ sum of all the entries a_{ij} ≤ *nÎ»*$_n$.