Step-1

a)

Consider that A is invertible and $AB = AC$.

The objective is to prove that $B = C$.

Since A is invertible, so A^{-1} exists and $AA^{-1} = A^{-1}A = I$.

Step-2

Multiply both sides of the equation $AB = AC$ with A^{-1}.

$$\begin{aligned} AB &= AC \\ A^{-1}(AB) &= A^{-1}(AC) \\ A^{-1}AB &= A^{-1}AC \\ (A^{-1}A)B &= (A^{-1}A)C \qquad \text{Use } A^{-1}A = I \\ IB &= IC \\ B &= C \end{aligned}$$

Therefore, $B = C$.

Step-3

b)

Consider the matrix,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

The objective is to find the matrices B and C such that $AB = AC$ and $B \neq C$.

Step-4

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $C = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$.

Find the products AB and AC.

$$
\begin{aligned}
AB &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \\
&= \begin{bmatrix} 1(a)+0(c) & 1(b)+0(d) \\ 0(a)+0(c) & 0(b)+0(d) \end{bmatrix} \\
&= \begin{bmatrix} a+0 & b+0 \\ 0+0 & 0+0 \end{bmatrix} \\
&= \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
AC &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix} \\
&= \begin{bmatrix} 1(x)+0(z) & 1(y)+0(w) \\ 0(x)+0(z) & 0(y)+0(w) \end{bmatrix} \\
&= \begin{bmatrix} x+0 & y+0 \\ 0+0 & 0+0 \end{bmatrix} \\
&= \begin{bmatrix} x & y \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

Step-5

Since $AB = AC$, so equate the matrices AB and AC.

$$
\begin{aligned}
AB &= AC \\
\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} x & y \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

Equate the corresponding elements.

$a = x$ and $b = y$.

Thus, the required matrices B and C that satisfy $AB = AC$ and $B \neq C$ are

$$\boxed{B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, C = \begin{bmatrix} a & b \\ z & w \end{bmatrix} \text{ and } c \neq z,\ d \neq w}.$$