

Step-1

Given that
$$[A \ I] = \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

We have to eliminate above and below the pivots to reduce $[A \ I]$ to $[I \ A^{-1}]$.

Step-2

Exchanging row 1 and row 2 gives

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Subtracting 2 times row 1 from row 2

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Step-3

Subtracting 2 times row 3 from row 1 and adding 3 times row 3 to row 2

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 0 & 4 & 1 & -2 & 3 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Dividing row 2 by 4 gives

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 1 & -2 \\ 0 & 0 & 1 & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

Step-4

Adding 3 times row 2 to row 3, subtracting 2 times row 2 from row 3

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{2} & -\frac{1}{2} & \mathbf{1} & -\frac{1}{2} \end{bmatrix}$$

Exchanging row 2 and row 3

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & -\frac{1}{2} & \mathbf{1} & -\frac{1}{2} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix} = \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

Hence $\begin{bmatrix} I & A^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & -\frac{1}{2} & \mathbf{1} & -\frac{1}{2} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$