Step-1

Let the matrix be $A=\begin{bmatrix}1 & 2 & 3 & 4\\ 0 & 1 & 2 & 3\\ 0 & 0 & 1 & 2\end{bmatrix}$

Basis for column space of A is $\{(1,0,0),(0,1,0),(0,0,1)\}$

Basis for row space of $A=\{(1,2,3,4),(0,1,2,3),(0,0,1,2)\}$

Basis for null space $=(0,1,-2,1)$

Basis for left null space is empty.

Step-2

We need to interchange the first two rows, and then the matrix is $A=\begin{bmatrix}0 & 1 & 2 & 3\\ 1 & 2 & 3 & 4\\ 0 & 0 & 1 & 2\end{bmatrix}$

Basis for column space of A is $\{(0,1,0),(1,0,0),(0,0,1)\}$

Step-3

And, row space of $A=$ row space of C

Basis for row space of $A=\{(0,1,2,3),(1,2,3,4),(0,0,1,2)\}$

In order to find the null space we need to set $Ax=0$

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_2 + 2x_3 + 3x_4 = 0$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 0$$

$$x_3 + 2x_4 = 0$$

$$\Rightarrow x_3 = -2x_4$$

$$\begin{aligned} x_2 &= -2x_3 - 3x_4 \\ &= 4x_4 - 3x_4 \\ &= x_4 \end{aligned}$$

$$\begin{aligned} x_1 &= -2x_2 - 3x_3 - 4x_4 \\ &= -2x_4 + 6x_4 - 4x_4 \\ &= 0 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ x_4 \\ -2x_4 \\ x_4 \end{bmatrix}$$

$$= x_4 \begin{bmatrix} 0 \\ 1 \\ -2 \\ 1 \end{bmatrix}$$

So, basis for null space $= (0,1,-2,1)$

Step-4

In order to find the left null space, we need to set $A^T x = 0$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_2 = 0$$

$$x_1 + 2x_2 = 0$$

$$2x_1 + 3x_2 + x_3 = 0$$

$$3x_1 + 4x_2 + 2x_3 = 0$$

$$\Rightarrow x_1 = x_2 = x_3 = 0$$

So, basis for left null space is empty.

Therefore, $\boxed{\text{basis for row space and null space are the same.}}$

Step-5

If $y=(1,2,3,4)$ is in the left null space of A

If first two rows are changed then $y=(2,1,3,4)$ is in the left null space of the new matrix.

Therefore, the vector in the left null space of the new matrix is $\boxed{y=(2,1,3,4)}$.