Step-1

Let $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

Then we know that the pivots of A are 2, $\frac{3}{2}$, $\frac{4}{3}$.

Thus, every pivot of A is greater than 1.

Step-2

To obtain the eigenvalues of A, solve $\det(A-\lambda I)=0$.

This gives,

$$\begin{aligned} 0 &= \begin{vmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \\ &= (2-\lambda)^3 - (2-\lambda) - (2-\lambda) \\ &= \left(8-12\lambda+6\lambda^2-\lambda^3\right) - 4 + 2\lambda \\ &= -\lambda^3 + 6\lambda^2 - 10\lambda + 4 \end{aligned}$$

Step-3

The roots of the equation $\lambda^3 - 6\lambda^2 + 10\lambda - 4 = 0$ are calculated by using online eigenvalue calculator. See the screenshot below:

Step-4

The three roots are 2, 3.414, and 0.585.

Step-5

Thus, the three eigenvalues of A are 2, 3.414, and 0.585. Thus, one of the eigenvalues is less than 1.

Thus, a matrix may have all pivots greater than 1 and yet some of its eigenvalues may be less than 1.