

## Step-1

Given that  $A$  is 3 by 4 matrix and  $B$  is 4 by 5 matrix and  $AB = 0$ . We have to prove that

$$\text{rank}(A) + \text{rank}(B) \leq 4$$

Sylvester's rank inequality:

If  $A$  is an  $m$  by  $n$  matrix and  $B$  is an  $n$  by  $k$  matrix then

$$\text{rank}(A) + \text{rank}(B) - n \leq \text{rank}(AB) \quad (1)$$

## Step-2

Given  $A$  is an 3 by 4 matrix and  $B$  is an 4 by 5 matrix and  $AB = 0$

Therefore  $n = 4$  and  $\text{rank}(AB) = 0$

By (1),  $\text{rank}(A) + \text{rank}(B) - 4 \leq 0$

Hence  $\text{rank}(A) + \text{rank}(B) \leq 4$