Step-1

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Given block matrices are $\begin{bmatrix} I & 0 \\ C & I \end{bmatrix}, \begin{bmatrix} A & 0 \\ C & D \end{bmatrix}, \begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$.

We have to find and check the inverses of these block matrices.

Step-2

Assuming that the inverses of I, A, C and D exists and they be I, A^{-1}, C^{-1} and D^{-1} respectively.

Let $M = \begin{bmatrix} I & 0 \\ C & I \end{bmatrix}$

For a 2 by 2 matrix, the inverse is $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ â€¦â€¦ (1)

Step-3

By (1), the inverse of $\begin{pmatrix} I & 0 \\ C & I \end{pmatrix}$ is

$$M^{-1} = \frac{1}{I.I-0.C}\begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}$$

$$= \frac{1}{I^2}\begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}$$

$$= \begin{bmatrix} I & 0 \\ -C & I \end{bmatrix} \qquad \left(\text{Since } I^2 = I\right)$$

Therefore, the inverse of $\begin{bmatrix} I & 0 \\ C & I \end{bmatrix}$ is $\boxed{\begin{bmatrix} I & 0 \\ -C & I \end{bmatrix}}$.

Step-4

Let $N = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$

By (1), the inverse of $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ is

$$N^{-1} = \frac{1}{A.D - 0.C}\begin{bmatrix} D & 0 \\ -C & A \end{bmatrix}$$

$$= \frac{1}{AD}\begin{bmatrix} D & 0 \\ -C & A \end{bmatrix}$$

$$= (AD)^{-1}\begin{bmatrix} D & 0 \\ -C & A \end{bmatrix} \qquad \left(\text{Since } \frac{1}{AD} = (AD)^{-1}\right)$$

$$= D^{-1}A^{-1}\begin{bmatrix} D & 0 \\ -C & A \end{bmatrix} \qquad \left(\text{Since } (AD)^{-1} = D^{-1}A^{-1}\right)$$

Step-5

Continuation to the above

$$= \begin{bmatrix} D^{-1}A^{-1}D & 0 \\ -D^{-1}A^{-1}C & D^{-1}A^{-1}A \end{bmatrix}$$

$$= \begin{bmatrix} D^{-1}DA^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1}A^{-1}A \end{bmatrix}$$

$$= \begin{bmatrix} A^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1} \end{bmatrix} \qquad \begin{pmatrix} \text{Since } D^{-1}D = I \\ \text{and } A^{-1}A = I \end{pmatrix}$$

Hence the inverse of the block matrix $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ is $\boxed{\begin{bmatrix} A^{-1} & 0 \\ -D^{-1}A^{-1}C & D^{-1} \end{bmatrix}}$.

Step-6

Let $P = \begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$

By (1), the inverse of $\begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$ is

$$P^{-1} = \frac{1}{0.D - I.I}\begin{bmatrix} D & -I \\ -I & 0 \end{bmatrix}$$

$$= \frac{1}{-I^2}\begin{bmatrix} D & -I \\ -I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -D & I \\ I & 0 \end{bmatrix}$$

Therefore, the inverse of $\begin{bmatrix} 0 & I \\ I & D \end{bmatrix}$ is $\boxed{\begin{bmatrix} -D & I \\ I & 0 \end{bmatrix}}$.