

Step-1

Given

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}, A^{-1} = \frac{1}{10} \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \text{ and } A - \lambda I = \begin{bmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix}.$$

We need to find the determinants of A , A^{-1} and $A - \lambda I$.

And also we need to find the value of λ for which $A - \lambda I$ is a singular matrix.

Step-2

$$\text{a) } \det(A) = \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix}$$

$$= (4)(3) - (1)(2) \left[\text{since } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \right]$$

$$= \boxed{10}$$

$$\text{Hence } \boxed{\det \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} = 10}$$

Step-3

$$\text{b) We have to find the determinant of } A^{-1} = \frac{1}{10} \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}.$$

We know that $\det(tA) = t^n \det A$, where A is an $n \times n$ matrix.

Now

$$\det A^{-1} = \det \left(\frac{1}{10} \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \right)$$

$$= \left(\frac{1}{10} \right)^2 \det \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$$

$$= \left(\frac{1}{10}\right)^2 [(3)(4) - (-1)(-2)]$$

$$= \frac{1}{100}(10)$$

$$\boxed{= \frac{1}{10}}$$

Thus, $\boxed{\det A^{-1} = \frac{1}{10}}$

Step-4

c) We have to find the determinant of $A - \lambda I = \begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix}$.

Now

$$\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix}$$

$$= (4-\lambda)(3-\lambda) - (2)(1)$$

$$= 12 - 4\lambda - 3\lambda + \lambda^2 - 2$$

$$= \lambda^2 - 7\lambda + 10$$

Step-5

We need to find the values of λ for which $A - \lambda I$ is a singular matrix.

We know that the determinant of a singular matrix is 0.

$$\det(A - \lambda I) = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 10 = 0$$

$$\Rightarrow (\lambda - 2)(\lambda - 5) = 0$$

$$\Rightarrow \lambda = 2 \text{ or } 5$$

Thus, $A - \lambda I$ is a singular matrix if $\boxed{\lambda = 2 \text{ or } 5}$