Step-1

The objective is to determine a vector perpendicular to **P**, matrix that has the plane **P** as its null space, and what matrix has **P** as its row.

Step-2

Given plane is $\mathbf{P} = x + 2y - z = 0$.

Put, $y = k$ and $z = m$.

Parameters, write this as $x = m - 2k$

So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = k \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + m \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

In other words, the plane is the subspace of $\mathbf{R}^3$ spanned by $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$.

Clearly, this plane is a 2 â€“ dimensional subspace of $\mathbf{R}^3$

Step-3

Any subspace **Q** perpendicular to the plane **P** is perpendicular to both the vectors span **P**.

Suppose (x, y, z) is a vector perpendicular to **P**.

Then,

$$\begin{aligned} 1x + 0y + 1z &= 0 \\ -2x + 1y + 0z &= 0 \end{aligned}$$

Consider the coefficients as a matrix and reduce it using the row operations, and get;

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$
$$R_2 \to R_2 + 2R_1$$
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

This is the row reduced form.

So, rewrite the homogeneous equations from this.

$$x+z=0$$
$$y+2z=0$$

So,

$$x=-z$$
$$y=-2z$$

So, the solution set is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ by putting $t=-z$ a parameter.

Thus, the orthogonal complement of the given plane is the straight line spanned by $\boxed{(1,2,-1)}$

Step-4

The vector spanning the null space of the matrix is $(1,\ 2,\ -1)$ and so, is perpendicular to the given plane **P**.

The matrix reduced is $\boxed{A=\begin{bmatrix} 1 & 2 & -1 \end{bmatrix}}$

Step-5

Now, to find the matrix *B* that has the plane **P** as its row space as,

$$P = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + 2y - z = 0 \right\}$$

$$P = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x = -2y + z \right\}$$

$$P = \left\{ \begin{bmatrix} -2y \\ y \\ 0 \end{bmatrix} + \begin{bmatrix} z \\ 0 \\ z \end{bmatrix} : y, z \in \mathbb{R} \right\}$$

$$P = \left\{ y \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} : y, z \in \mathbb{R} \right\}$$

$$= span \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Therefore, matrix *B* that has P as its row space, is,

$$\boxed{B = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}$$