Step-1

Consider the following unitary matrix:

$$V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$$

Diagonalize it to reach $V = U\Lambda U^H$.

Step-2

First step is to find the Eigen values and Eigen vectors of matrix V. To calculate the Eigen values do the following calculations;

$$V - \lambda I = \frac{1}{\sqrt{3}}\begin{bmatrix} 1-\lambda & 1-i \\ 1+i & -1-\lambda \end{bmatrix}$$

$$\det(V - \lambda I) = 0$$

$$\frac{1}{\sqrt{3}}\left((-1-\lambda)(1-\lambda) - (1+i)(1-i)\right) = 0$$

$$\frac{1}{\sqrt{3}}\left(\lambda^2 - 3\right) = 0$$

After solving following values are obtained:

$$\lambda_1 = 1$$
$$\lambda_2 = -1$$

Step-3

To calculate Eigen vectors do the following calculations:

$$(V - \lambda I)x = 0$$

$$\begin{bmatrix} \frac{1}{\sqrt{3}} - 1 & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} - 1 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1-\sqrt{3}}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & \frac{-1-\sqrt{3}}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving, values of y and z corresponding to $\lambda = 1$ are as follows:

$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} \left(1+\sqrt{3}\right) \\ 1+i \end{bmatrix}$$

Step-4

Similarly, Eigen vectors corresponding to Eigen value $\lambda = -1$ is as follows:

$$(V - \lambda I)x = 0$$
$$\begin{bmatrix} \frac{1}{\sqrt{3}}+1 & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & -\frac{1}{\sqrt{3}}+1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} \frac{1+\sqrt{3}}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \\ \frac{1+i}{\sqrt{3}} & \frac{-1+\sqrt{3}}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

On solving values of y and z are as follows:

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} -1+i \\ \left(1+\sqrt{3}\right) \end{bmatrix}$$

Step-5

To put the Eigen vectors in unitary matrix make them orthonormal by dividing the length of the vector.

$$\|x\|^2 = \left|\left(1+\sqrt{3}\right)^2\right| + \left|(1+i)^2\right|$$
$$= \left|1+3+2\sqrt{3}\right| + \left|1+i^2+2i\right|$$
$$= \left|4+2\sqrt{3}\right| + |2i|$$
$$= 6+2\sqrt{3}$$

Let the length be L. So $L^2 = 6+2\sqrt{3}$.

Step-6

Now, the diagonalization of the matrix can be written as follows:

$$
\begin{aligned}
V &= U\Lambda U^{H} \\
&= \frac{1}{L}\begin{bmatrix} 1+\sqrt{3} & -1+i \\ 1+i & 1+\sqrt{3} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\frac{1}{L}\begin{bmatrix} 1+\sqrt{3} & 1-i \\ -1-i & 1+\sqrt{3} \end{bmatrix}
\end{aligned}
$$

Here, $L^2 = 6+2\sqrt{3}$

Step-7

Therefore, unitary matrix V is diagonalize to reach $\boxed{V = U\Lambda U^{H}}$. All Eigen values of unitary matrix is $|\lambda| = 1$.