



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目: 高等数学(上) A

开课单位: 数学系

考试时长: 120 分钟

命题教师: _____

题号	1	2	3	4	5	6	7	8	9
分值	15 分	15 分	10 分	10 分	10 分	10 分	10 分	10 分	10 分

本试卷共 9 道大题, 满分 100 分. (考试结束后请将试卷、答题本、草稿纸一起交给监考老师)

注意: 本试卷里的中文为直译(即完全按英文字面意思直接翻译), 所有数学词汇的定义请参照教材(Thomas' Calculus, 13th Edition)中的定义。如果其中有些数学词汇的定义不同于中文书籍(比方说同济大学的高等数学教材)里的定义, 以教材(Thomas' Calculus, 13th Edition)中的定义为准。

1. (15 pts) Multiple Choice Questions: (only one correct answer for each of the following questions.)

(1) Let $f(x) = |x| \sin x$. The greatest value of n , for which $f^{(n)}(0)$ exists, is _____.

- (A) 0 (B) 1 (C) 2 (D) 3

(2) If $\lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x+1} - ax - b \right) = \frac{1}{2}$, then the values of a, b are _____.

- (A) $a = 1, b = -\frac{3}{2}$ (B) $a = -1, b = \frac{3}{2}$
(C) $a = -1, b = 1$ (D) $a = 1, b = -1$.

(3) The average value of function $g(x) = x^2 + 6$, for $0 \leq x \leq 6$ is _____.

- (A) 12 (B) 18 (C) 16 (D) 10

(4) Which one of the following functions is not differentiable at $x = 0$?

- (A) $f(x) = |x| \sin |x|$
(B) $f(x) = |x| \sin \sqrt{|x|}$
(C) $f(x) = \cos |x|$
(D) $f(x) = \cos \sqrt{|x|}$

(5) What is the derivative of $f(x) = \frac{1-\sin x}{1+\sin x}$ at $x = \pi/6$?

- (A) $\frac{4\sqrt{3}}{9}$ (B) $-\frac{\sqrt{3}}{3}$ (C) $-\frac{4}{3\sqrt{3}}$ (D) $\frac{1}{3}$

2. (15 pts) Please fill in the blank for the questions below.

(1) The integration $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^3 x dx$ equals _____.

(2) If f is continuous and $\int_0^{x^3-1} f(t) dt = x$, then $f(7) =$ _____.

(3) If $f'(x) = \left(x + \frac{1}{x}\right)^2$ and $f(1) = 1$, then $f(x) =$ _____.

- (4) A particle is moving on the sphere $x^2 + y^2 + z^2 = 13^2$. While $t = t_0$, $x(t_0) = 3$, $y(t_0) = 4$, $z(t_0) = 12$, $x'(t_0) = 4$, $y'(t_0) = 3$, then $z'(t_0) = \underline{\hspace{2cm}}$.

(5) $\lim_{s \rightarrow a} \frac{\sqrt{s^2+1} - \sqrt{a^2+1}}{s-a} = \underline{\hspace{2cm}}$.

3. (10 pts) Find the limits (DO NOT apply l'Hôpital's Rule).

(1) $\lim_{x \rightarrow \infty} \frac{\sqrt{3+x} - \sqrt{x+1}}{x^2 + x - 2}$.

(2) $\lim_{x \rightarrow 0} \frac{\cos x - \sec^2 x}{x \sin x}$.

4. (10 pts) Evaluate the integral.

(1) $\int_0^{2\pi} |\sin^2 x - \cos^2 x| dx$.

(2) $\int_0^1 (x+2)\sqrt{1-x^2} dx$.

5. (10 pts) Let $f(x) = \frac{x^3+x-2}{x-x^2}$.

- (1) Identify the inflection points and local maxima and minima of the function that may exist.
- (2) Identify the horizontal, vertical, and oblique asymptotes that may exist.
- (3) Graph the function.

6. (10 pts)

- (1) Find $\frac{dy}{dx}$ if

$$y(x) = \int_1^{1+2x} \sqrt{t^2 - 1} dt, \quad x > 0.$$

- (2) Find the equation of the line that is tangent to the curve $x^2 - y^2 = 9$ at point $(5, -4)$.

7. (10 pts) Find the area of the region bounded by curves $y = x^2$ and $y = 2x - x^2$.

8. (10 pts) Find the volume of the solid generated by revolving the region bounded by $y = 2x - 1$, $y = \sqrt{x}$ and y -axis about the line $x = -1$.

9. (10 pts) Assume that f is continuous on $[0, 1]$. Show that there exists a number $c \in (0, 1)$ such that $f(c) = \int_0^1 f(x) dx$.

一、(15分) 单项选择题: (每题只有一个正确答案.)

(1) 令 $f(x) = |x| \sin x$. 使得 $f^{(n)}(0)$ 存在的 n 的最大值为_____.

- (A) 0 (B) 1 (C) 2 (D) 3

(2) 若 $\lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x+1} - ax - b \right) = \frac{1}{2}$, 则 a, b 的值为_____.

- (A) $a = 1, b = -\frac{3}{2}$ (B) $a = -1, b = \frac{3}{2}$
(C) $a = -1, b = 1$ (D) $a = 1, b = -1$

(3) 函数 $g(x) = x^2 + 6$ 在 $0 \leq x \leq 6$ 上的平均值为_____.

- (A) 12 (B) 18 (C) 16 (D) 10

(4) 下列哪个函数在 $x = 0$ 处不可微?

- (A) $f(x) = |x| \sin |x|$
(B) $f(x) = |x| \sin \sqrt{|x|}$
(C) $f(x) = \cos |x|$
(D) $f(x) = \cos \sqrt{|x|}$

(5) 函数 $f(x) = \frac{1-\sin x}{1+\sin x}$ 在 $x = \pi/6$ 的导数是_____.

- (A) $\frac{4\sqrt{3}}{9}$ (B) $-\frac{\sqrt{3}}{3}$ (C) $-\frac{4}{3\sqrt{3}}$ (D) $\frac{1}{3}$

二、(15分) 填空题.

(1) 定积分 $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^3 x dx$ 的值为_____.

(2) 如果 f 连续且 $\int_0^{x^3-1} f(t) dt = x$, 则 $f(7) =$ _____.

(3) 如果 $f'(x) = (x + \frac{1}{x})^2$ 且 $f(1) = 1$, 则 $f(x) =$ _____.

(4) 一个质点在球面 $x^2 + y^2 + z^2 = 13^2$ 上运动. 在 $t = t_0$ 时, $x(t_0) = 3, y(t_0) = 4, z(t_0) = 12, x'(t_0) = 4, y'(t_0) = 3$, 则 $z'(t_0) =$ _____.

(5) $\lim_{s \rightarrow a} \frac{\sqrt{s^2+1}-\sqrt{a^2+1}}{s-a} =$ _____.

三、(10分) 求下列极限 (不要使用洛必达法则).

(1) $\lim_{x \rightarrow \infty} \frac{\sqrt{3+x} - \sqrt{x+1}}{x^2 + x - 2}$.

(2) $\lim_{x \rightarrow 0} \frac{\cos x - \sec^2 x}{x \sin x}$.

四、(10分) 计算积分.

(1) $\int_0^{2\pi} |\sin^2 x - \cos^2 x| dx$.

(2) $\int_0^1 (x+2)\sqrt{1-x^2} dx$.

五、(10分) 考虑函数 $f(x) = \frac{x^3+x-2}{x-x^2}$.

(1) 求所有 (局部) 极值点和拐点.

(2) 求所有水平渐近线、垂直渐近线和斜渐近线.

(3) 作出函数 $f(x)$ 的简略图.

六、 (10分)

(1) 求 $\frac{dy}{dx}$, 这里

$$y(x) = \int_1^{1+2x} \sqrt{t^2 - 1} dt, \quad x > 0.$$

(2) 求曲线 $x^2 - y^2 = 9$ 在点 $(5, -4)$ 处的切线.

七、 (10分) 求曲线 $y = x^2$ 和 $y = 2x - x^2$ 所围成的区域的面积.

八、 (10分) 曲线 $y = 2x - 1$, $y = \sqrt{x}$ 和 y 轴围成一个区域. 把这个区域绕直线 $x = -1$ 旋转可得一个旋转体, 求此旋转体的体积.

九、 (10分) 设函数 f 在区间 $[0, 1]$ 上连续. 证明: 至少存在一点 $c \in (0, 1)$ 使得 $f(c) = \int_0^1 f(x) dx$.