

Step-1

Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be Eigen values of A .

It is given that A Is positive definite.

This implies;

$$\lambda_1 > 0, \lambda_2 > 0, \dots, \lambda_n > 0.$$

It is known that if $\lambda_1, \lambda_2, \dots, \lambda_n$ are Eigen values of A then $\lambda_1^2, \lambda_2^2, \dots, \lambda_n^2$ are Eigen values of A^2 .

Clearly $\lambda_1^2 > 0, \lambda_2^2 > 0, \dots, \lambda_n^2 > 0$.

So the Eigen values of A^2 are all positive.

Thus, A^2 is also positive definite.

Step-2

It is known that if $\lambda_1, \lambda_2, \dots, \lambda_n$ are Eigen values of A , then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_n}$ are Eigen values of A^{-1} .

Clearly, $\frac{1}{\lambda_1} > 0, \frac{1}{\lambda_2} > 0, \dots, \frac{1}{\lambda_n} > 0$.

So the Eigen values of A^{-1} are also positive.

Hence, the matrix A^{-1} is also positive definite.