

Step-1

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$$

Let us suppose

$$\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ a & b & c - \lambda \end{vmatrix} = 0$$

is the characteristic equation

It can be simplified as
$$-\lambda \begin{vmatrix} -\lambda & 1 \\ b & c - \lambda \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ a & c - \lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda \{\lambda^2 - c\lambda - b\} + a = 0$$

$$\Rightarrow -\lambda^3 + c\lambda^2 + b\lambda + a = 0$$

Step-2

On the other hand, the given characteristic equation is $-\lambda^3 + 4\lambda^2 + 5\lambda + 6 = 0$

Comparing both the equations, we get $c = \boxed{4}, b = \boxed{5}, a = \boxed{6}$.