

Step-1

Given that P is a projection onto the column space of A .

The objective is to find the projection onto the left null space of A .

Step-2

If P is the projection onto the column space of A

It is given a matrix formula for splitting any b into two perpendicular components.

Pb is in the column space $C(A)$ and the other component $(I - P)b$ is in the left null space $N(A^T)$, which is orthogonal to the column space.

Hence, $(I - P)$ is the projection onto the left null space.