Step-1

Since with every ordering of the rows of A leaves at least one zero on the diagonal, there must be a column of all zeros in the original square matrix A.

Let i^{th} column be the column of zeros in the matrix A.

Consider again, $\det A = \sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$

Step-2

While carrying out the summation in the right hand side of $\det A = \sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P$, every term contains one and only one term from the i^{th} column.

That is, every term in the summation must be equal to zero, since zero multiplied by any number is zero.

Therefore, $\sum_{\text{all } P's} \left(a_{1\alpha} a_{2\beta} \ldots a_{n\nu}\right) \det P = 0$

Step-3

This implies that $\det A = 0$. But this is a contradiction, since we have assumed that $\det A \neq 0$.

Therefore, our assumption that every ordering of A leaves at least one zero on the diagonal is wrong.

Thus, there exists some ordering of A, which leaves no zero on the diagonal.