Step-1

Consider the equation $Ax = \lambda Mx$. We need to show that if the matrices A and M are symmetric indefinite, then the eigenvalues might not be real.

Consider the following:

$$\begin{aligned} 0 &= \det\left(A - \lambda M\right) \\ &= \det\left(\begin{bmatrix} a & b \\ b & d \end{bmatrix} - \lambda \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\right) \\ &= \det\left(\begin{bmatrix} a - \lambda m_1 & b \\ b & d - \lambda m_2 \end{bmatrix}\right) \end{aligned}$$

$$\begin{aligned} 0 &= \left(a - \lambda m_1\right)\left(d - \lambda m_2\right) - b^2 \\ &= ad - \left(am_2 + dm_1\right)\lambda + m_1 m_2 \lambda^2 - b^2 \\ &= m_1 m_2 \lambda^2 - \left(am_2 + dm_1\right)\lambda + \left(ad - b^2\right) \end{aligned}$$

Step-2

Consider the quadratic equation $m_1 m_2 \lambda^2 - \left(am_2 + dm_1\right)\lambda + \left(ad - b^2\right) = 0$. Since, we want complex eigenvalues, the roots of this equation should be complex numbers. We know that a quadratic equation $ax^2 + bx + c = 0$ doesnâ€™t have real roots if $b^2 - 4ac < 0$, that is, if $b^2 < 4ac$.

Consider the following:

$$\begin{aligned} \left(am_2 + dm_1\right)^2 &< 4m_1 m_2 \left(ad - b^2\right) \\ a^2 m_2{}^2 + 2adm_1 m_2 + d^2 m_1{}^2 &< 4m_1 m_2 ad - 4m_1 m_2 b^2 \\ a^2 m_2{}^2 - 2adm_1 m_2 + d^2 m_1{}^2 &< -4m_1 m_2 b^2 \\ \left(am_2 - dm_1\right)^2 &< -4m_1 m_2 b^2 \end{aligned}$$

Step-3

Therefore, if we have $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$ and $M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ such that $\left(am_2 - dm_1\right)^2 < -4m_1 m_2 b^2$, then the equation $Ax = \lambda Mx$ will not have real eigenvalues. Consider the following example $A = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$, $M = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$.

Here we get

$$\begin{aligned} \left(am_2 - dm_1\right)^2 &= \left(3 \times 2 - (-3) \times (-2)\right)^2 \\ &= \left(6 - 6\right)^2 \\ &= 0 \end{aligned}$$

$$-4m_1m_2b^2 = -4(-3)(2)(1)^2$$
$$= 24$$

Note that $(am_2 - dm_1)^2 < -4m_1m_2b^2$.

Thus, a required example is $\boxed{A = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}}$ and $\boxed{M = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}}$. Of course, we can create any number of such examples.