Step-1

When we consider the block multiplication, suppose $A = LU$ has $A_k = L_k U_k$ in the upper left corner, as follows:

$$\begin{aligned} A &= \begin{bmatrix} A_k & * \\ * & * \end{bmatrix} \\ &= \begin{bmatrix} L_k & 0 \\ * & * \end{bmatrix} \begin{bmatrix} U_k & * \\ 0 & * \end{bmatrix} \end{aligned}$$

Step-2

(a) Suppose the first three pivots of A are 2, 3, and â€“1. The lower triangular matrices have 1â€™s along the diagonals and therefore, the determinant of L_K is equal to 1, for any K.

Thus, we have

$\boxed{L_1 = 1, L_2 = 1, L_3 = 1}$

Step-3

It is clear that the determinant of U_K is equal to the determinant of A_K, for each K.

The pivot entries of A are 2, 3, and â€“1.

Therefore, the determinant of A_1 is 2.

The determinant of A_2 is $2 \times 3 = 6$.

The determinant of A_3 is $2 \times 3 \times -1 = -6$.

Step-4

Thus, we have the following: $\boxed{A_1 = 2}$, $\boxed{U_1 = 2}$, $\boxed{A_2 = 6}$, $\boxed{U_2 = 6}$, $\boxed{A_3 = -6}$, $\boxed{U_3 = -6}$.

Step-5

(b) Let us suppose that the determinants of A_1, A_2, and A_3 are 5, 6, and 7 respectively. We need to find the pivot entries of A.

Since, determinant of A_1 is equal to 5, the first pivot of A must be $\boxed{5}$.

Step-6

Now, the determinant of A_2 is 6.

Therefore, the second pivot of A must be such that the product its first pivot and second pivot must be 6.

The first pivot is 5.

Therefore, the second pivot must be $\boxed{\frac{6}{5}}$.

Step-7

Now, the determinant of A_3 is 7.

Therefore, the third pivot of A must be such that the product its three pivots must be 7.

The first pivot is 5. The second pivot is $\frac{6}{5}$.

Therefore, the third pivot must be $\frac{7}{6}$, since $5\times\frac{6}{5}\times\frac{7}{6}=7$.

Therefore, the third pivot must be $\boxed{\frac{7}{6}}$.