Step-1

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By partitioning a square matrix as two sub square matrices $A_{r\times r}$, $C_{n-r\times n-r}$ on the north west and south east corners, $D_{r\times n-r}, E_{n-r\times r}$ matrices along the anti â€“ diagonal are such that $d_{ij} = e_{ji}$ for every i^{th} row and j^{th} column.

In the given case, we have $C_{n-r\times n-r}$ is a zero matrix.

So, the given matrix looks like $B = \begin{bmatrix} A & D \\ E & C \end{bmatrix}$

We follow that $B^T = \begin{bmatrix} A^T & E^T \\ D^T & C^T \end{bmatrix}$

While C is a zero matrix, we get C^T is also a zero matrix.

Further, we have noted that $D^T = E, E^T = D$

Therefore, B^T is a North West matrix.

Step-2

$$B^2 = \begin{bmatrix} A^2 + DE & AD + DC \\ EA + CE & ED + C^2 \end{bmatrix}$$

We observe that the South West matrix in this square is $ED + C^2$ not necessarily zero while E and D are not known to be the zero matrices.

Therefore, B^2 is not a North West matrix

Step-3

$$B^{-1} = \frac{1}{|AC| - |DE|} \begin{bmatrix} C & -D \\ -E & A \end{bmatrix}$$

We follow that the North west corner is the zero matrix and the South East is a non zero matrix.

Therefore, the inverse of a North West matrix is a South East matrix.

Step-4

Suppose $B = \begin{bmatrix} X & Y \\ Z & W \end{bmatrix}$ is a North West matrix.

Then by definition, we have W is a zero square sub matrix of B.

Suppose $C = \begin{bmatrix} P & Q \\ R & S \end{bmatrix}$ is a South East matrix.

Then P is a zero matrix and the respective sizes of the sub matrices in both B and C are multipliable.

$$BC = \begin{bmatrix} XP + YR & XQ + YS \\ ZP + WR & ZQ + WS \end{bmatrix}$$

Using $W = 0$ and $P = 0$ in this, we get $BC = \begin{bmatrix} YR & XQ + YS \\ 0 & ZQ \end{bmatrix}$

The product of matrix is neither North West nor South East.