Step-1

Consider matrices of (2×2) with 1â€™s and 0â€™s. Determine how many are invertible.

Recall that a matrix is invertible if its determinant is non-zero.

Step-2

Determinants of all the matrices (2×2) containing 1â€™s and 0â€™s are as follows:

Consider the matrices with only one 1â€™s.

$$\begin{bmatrix}1&0\\0&0\end{bmatrix},\begin{bmatrix}0&1\\0&0\end{bmatrix},\begin{bmatrix}0&0\\1&0\end{bmatrix},\begin{bmatrix}0&0\\0&1\end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

Step-3

Consider the matrices with two 1â€™s.

$$\begin{bmatrix}1&1\\0&0\end{bmatrix},\begin{bmatrix}0&1\\0&1\end{bmatrix},\begin{bmatrix}1&0\\1&0\end{bmatrix},\begin{bmatrix}0&0\\1&1\end{bmatrix}$$

Determinant of all these matrices are zero. So, none of them are invertible.

Step-4

Consider the matrices with three 1â€™s.

$$\begin{bmatrix}1&1\\1&0\end{bmatrix},\begin{bmatrix}0&1\\1&1\end{bmatrix},\begin{bmatrix}1&0\\1&1\end{bmatrix},\begin{bmatrix}1&1\\0&1\end{bmatrix}$$

Determinant of all these matrices are nonzero. So, each one of them are invertible.

Step-5

Consider the matrix with 1â€™s positioned on the diagonal and anti-diagonal.

$$\begin{bmatrix}1&0\\0&1\end{bmatrix},\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

Determinant of all these matrices are nonzero. So, each one of them are invertible.

Step-6

Consider the matrix with all 1â€™s and 0â€™s.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Determinant of all these matrices are zero. So, none of them are invertible.

Step-7

Therefore, from sixteen (2×2) matrices with 0â€™s and 1â€™s only 6 are invertible.