Step-1

P_1 = The projection matrix onto the line through $a_1 = \dfrac{a_1 a_1^T}{a_1^T a_1}$

$$a_1 a_1^T = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} (-1, 2, 2)$$
$$= \begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$$

$$a_1^T a_1 = \begin{bmatrix} -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$
$$= 1 + 4 + 4$$
$$= 9$$

$$P_1 = \frac{1}{9} \begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$$

Step-2

P_2 = The projection matrix onto the line through $a_2 = \dfrac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} (2, 2, -1)$$
$$= \begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$a_2^T a_2 = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$
$$= 4 + 4 + 1$$
$$= 9$$

$$P_2 = \frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

Step-3

$$\begin{aligned} P_1P_2 &= \frac{1}{9}\begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}\frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix} \\ &= \frac{1}{81}\begin{bmatrix} 4-8+4 & 4-8+4 & -2+4-2 \\ -8+16-8 & -8+16-8 & 4-8+4 \\ -8+16-8 & -8+16-8 & 4-8+4 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

Therefore P_1P_2 is a zero matrix because $a_1 \perp a_2$

This result is verified with $a_1^T a_2 = (-1,2,2)\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

$$\begin{aligned} &= -2+4-2 \\ &= 0 \end{aligned}$$