Step-1

Given $\mathbf{V}$ has dimension k .

(a) Suppose that there are k independent vectors in a set which is not a basis for $\mathbf{V}$.

Therefore there exists one more vector such that the set of all $k+1$ vectors are independent.

But dimension of $\mathbf{V} > k+1$

$\Rightarrow k > k+1$

This is a contradiction.

Therefore our assumption is wrong.

Therefore set of k independent vectors is a basis for $\mathbf{V}$.

Step-2

(b) Let k vectors span $\mathbf{V}$.

Suppose these k vectors are linearly dependent therefore one of the vectors in the k vectors is linear combination of the other vectors.

Therefore $k-1$ vectors span $\mathbf{V}$

We know that minimal spanning set is a basis for V.

$\Rightarrow k \le k-1$

This is a contradiction.

Therefore our assumption is wrong.

They any k Vectors that span $\mathbf{V}$ form a basis.