Step-1

Note: Real parts of the Eigen values govern the stability. The differential equation $du/dt = Au$ is:

Stable: If $\mathrm{Re}(\lambda_i) < 0$

Neutrally stable: If all $\mathrm{Re}(\lambda_i) \le 0$ and $\lambda_1 = 0$.

Unstable: If any Eigen value has $\mathrm{Re}(\lambda_i) > 0$.

Step-2

(a) Consider the following matrix.

$$A = \begin{bmatrix} a & b+c \\ b-c & -a \end{bmatrix}$$

Here, trace of matrix *A* is zero. To show that Eigen values are real exactly when $a^2 + b^2 \ge c^2$.

Step-3

First step is to find the Eigen values of matrix *A*. Do the following calculations;

$$A - \lambda I = \begin{bmatrix} a-\lambda & b+c \\ b-c & -a-\lambda \end{bmatrix}$$
$$\det(A - \lambda I) = 0$$
$$(a-\lambda)(-a-\lambda) - (b^2 - c^2) = 0$$
$$\lambda^2 - a^2 = (b^2 - c^2)$$

To get the real Eigen values following must be true:

$$\lambda^2 > 0$$
$$a^2 + b^2 - c^2 > 0$$
$$a^2 + b^2 > c^2$$

Step-4

Therefore, Eigen values are real exactly when $\boxed{a^2 + b^2 \ge c^2}$.