

## Step-1

Given system is  $2u - v = 0$

$$-u + 2v - w = 0$$

$$-v + 2w - z = 0$$

$$-w + 2z = 5$$

We have to find the pivots and solve this system.

## Step-2

Given system can be written as

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix}$$

apply  $R_2 \rightarrow 2R_2 + R_1$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix}$$

apply  $R_3 \rightarrow 3R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 4 & -3 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{bmatrix}$$

## Step-3

apply  $R_4 \rightarrow 4R_4 + R_3$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 4 & -3 & 0 \\ 0 & 0 & 0 & 5 & 20 \end{bmatrix}$$

which is upper triangular form.

$$\begin{bmatrix} \boxed{2} & -1 & 0 & 0 & 0 \\ 0 & \boxed{3} & -2 & 0 & 0 \\ 0 & 0 & \boxed{4} & -3 & 0 \\ 0 & 0 & 0 & \boxed{5} & 20 \end{bmatrix}$$

The pivots are circled in

That is  $\boxed{2, 3, 4, 5}$ .

## Step-4

From above upper triangular form, we have

$$\begin{aligned} 2u - v &= 0 \\ 3v - 2w &= 0 \\ 4w - 3z &= 0 \\ 5z &= 20 \end{aligned}$$

By back ward substitution,

$$\begin{aligned} 5z &= 20 \\ \Rightarrow \boxed{z = 4} \\ 4w - 3z &= 0 \\ \Rightarrow 4w - 3(4) &= 0 \end{aligned}$$

$$\Rightarrow \boxed{w = 3}$$

$$\begin{aligned} 3v - 2w &= 0 \\ \Rightarrow 3v - 2(3) &= 0 \end{aligned}$$

$$\Rightarrow \boxed{v = 2}$$

$$\begin{aligned} 2u - v &= 0 \\ \Rightarrow 2u - 2 &= 0 \end{aligned}$$

$$\Rightarrow \boxed{u = 1}$$

Solutions are  $\boxed{u = 1, v = 2, w = 3, z = 4}$