## Step-1

The product rule of determinant says that if $A$ and $B$ are square matrices of the same order, then $\det(AB) = (\det A)(\det B)$.

It is also known that $\det I = 1$.

Therefore, we get

$$\begin{aligned}1 &= \det I \\ &= \det\left(Q^{\mathrm{T}}Q\right) \\ &= \det\left(Q^{\mathrm{T}}\right)\det(Q)\end{aligned}$$

## Step-2

We also know that determinant of a matrix $A$ is equal to the determinant of its transpose, $A^{\mathrm{T}}$. Therefore, $\det(Q) = \det\left(Q^{\mathrm{T}}\right)$.

Thus, $\left(\det(Q)\right)^2 = 1$

Therefore, $\boxed{\det(Q) = \pm 1}$.

By the same logic, $\det\left(Q^{\mathrm{T}}\right) = \pm 1$.

## Step-3

Now we show that $Q^2$ is also an Orthogonal matrix. Thus, we need to show that $\left(Q^2\right)^{-1} = \left(Q^2\right)^{\mathrm{T}}$.

$$\begin{aligned}Q^2\left(Q^2\right)^{\mathrm{T}} &= (Q\cdot Q)\left(Q^{\mathrm{T}}\cdot Q^{\mathrm{T}}\right) \\ &= Q\left(Q\cdot Q^{\mathrm{T}}\right)Q^{\mathrm{T}} \\ &= Q\cdot Q^{\mathrm{T}} \\ &= I\end{aligned}$$

Thus, $Q^2$ is an Orthogonal matrix. Similarly, it can be shown that $Q^{n}$ is also an Orthogonal matrix.

## Step-4

If $\det Q$ is not equal to $\pm 1$, then $\det\left(Q^n\right)$ would either tend to zero or would tend to plus or minus infinity.

But, $Q^{n}$ remains an Orthogonal matrix.

This also shows that $\boxed{\det Q = \pm 1}$.