

Step-1

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Step-2

Given matrix is $A = \begin{pmatrix} 0.001 & 0 \\ 1 & 1000 \end{pmatrix}$

We write this in the form LDU .

$$L = \begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}, DU = \begin{bmatrix} x_{11} & x_{12} \\ 0 & x_{22} \end{bmatrix}$$

$$LDU = \begin{bmatrix} x_{11} & x_{12} \\ l_{21}x_{11} & l_{21}x_{12} + x_{22} \end{bmatrix} = \begin{bmatrix} 0.001 & 0 \\ 1 & 1000 \end{bmatrix}$$

$$\Rightarrow x_{11} = 0.001, x_{12} = 0,$$

$$l_{21}x_{11} = 1 \Rightarrow l_{21} = 1000, x_{22} = 1000$$

Step-3

$$LDU = \begin{bmatrix} 1 & 0 \\ 1000 & 1 \end{bmatrix} \begin{bmatrix} 0.001 & 0 \\ 0 & 1000 \end{bmatrix}$$

$$DU = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.001 & 0 \\ 0 & 1000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} d_1 & d_1 u_{12} \\ 0 & d_2 \end{bmatrix} = \begin{bmatrix} 0.001 & 0 \\ 0 & 1000 \end{bmatrix}$$

$$\Rightarrow d_1 = 0.001, u_{12} = 0, d_2 = 1000$$

$$D = \begin{bmatrix} 0.001 & 0 \\ 0 & 1000 \end{bmatrix}, U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The pivots are 0.001 and 1000. $\in \mathbb{R}^+(1)$

Step-4

On the other hand, we apply row operations on the pre multiple and column operations on post multiple identity matrices that were applied first upon A .

For suppose $A = IAI$

$$A = \begin{bmatrix} 0.001 & 0 \\ 1 & 1000 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 1000R_1 \Rightarrow \begin{bmatrix} 0.001 & 0 \\ 0 & 1000 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1000 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Observe that the given matrix is reduced to the diagonal matrix on the left hand side and so, the pivots are 0.001 and 1000. $\hat{A} \in \mathbb{R}^n$ (2)

Comparing (1) and (2), we say that on partial pivoting in two ways by direct elimination, the pivots remain the same.