Step-1

Suppose $x = (x, y, t)$ is any point in $\mathbf{R}^3$

We find the orthogonal projection p of a onto the line of intersection of the planes $x+y+t=0$ and $x-t=0$

The orthogonal projection is nothing but the null space of the matrix A whose rows are the coefficients of the planes such that

The matrix form of above equations is $Ax = 0$

So, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \\ t \end{bmatrix}$

Step-2

Applying $R_2 \to R_2 - R_1$ upon this, we get $\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix}$

$R_2(-1) \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

This is the row reduced form and so, we rewrite the equations from this.

$$\begin{aligned} y + 2t &= 0 \\ x + y + t &= 0 \end{aligned}$$

1st equation gives $y = -2t$ and so, $x = t$

Therefore, $\begin{bmatrix} x \\ y \\ t \end{bmatrix} = k \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ where k = t is the parameter.

Putting $k = 1$, we get $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ is the required orthogonal projection p

Step-3

The required projection matrix is $P = \dfrac{pp^T}{p^T p}$

$$= \frac{\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}(1,-2,1)}{(1,-2,1)\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}$$

$$= \boxed{\frac{1}{6}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}}$$