Step-1

Given $W=\begin{bmatrix}2&0\\0&1\end{bmatrix}, x=(2,3), y=(1,1)$

We have to find the W-inner product of $x=(2,3)$ and $y=(1,1)$, and W-length of x.

Step-2

W-inner product of x and y

$$=(x,y)_W$$
$$=(Wy)^T(Wx)$$

Step-3

Now

$$Wx=\begin{bmatrix}2&0\\0&1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}$$
$$=\begin{bmatrix}4\\3\end{bmatrix}$$
$$Wy=\begin{bmatrix}2&0\\0&1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$
$$=\begin{bmatrix}2\\1\end{bmatrix}$$

Step-4

And

$$(Wy)^T=\begin{bmatrix}2&1\end{bmatrix}$$
$$\Rightarrow (Wy)^T(Wx)=\begin{bmatrix}2&1\end{bmatrix}\begin{bmatrix}4\\3\end{bmatrix}$$
$$=8+3$$
$$=11$$

Hence W-inner product of x with yÂ =Â 11

Step-5

The W-length of x

$$= \|x\|_W$$
$$= \|Wx\|$$

Step-6

Now

$$\|Wx\|^2 = (Wx)^T Wx$$
$$= \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$
$$= 16 + 9$$
$$= 25$$

Hence the W-length of x $\|Wx\| = 5$