

Step-1

A) determinant of 4 by 4 matrix whose entries are a_{ij} = smaller of i and j

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

And

$$A = LU \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The pivots are $\boxed{1, 1, 1, 1}$ and $\det A = \boxed{1}$

Step-2

b) Given $n_1 = 2, n_2 = 6, n_3 = 8, n_4 = 10$

a_{ij} = smaller of n_i and n_j

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 2 & 6 & 8 & 8 \\ 2 & 6 & 8 & 10 \end{bmatrix}$$

Here

$$\det A = \begin{vmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 2 & 6 & 8 & 8 \\ 2 & 6 & 8 & 10 \end{vmatrix}$$

Step-3

Now

$$= \begin{vmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 2 & 6 & 8 & 8 \\ 0 & 0 & 0 & 2 \end{vmatrix} \text{ adding } -1 \text{ time the third row to the fourth}$$

$$= \begin{vmatrix} 2 & 2 & 2 & 2 \\ 2 & 6 & 6 & 6 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix} \text{ adding } -1 \text{ time the second row to the third}$$

Step-4

Then

$$= \begin{vmatrix} 2 & 2 & 2 & 2 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 \end{vmatrix} \text{ adding } -1 \text{ time the first row to the second}$$

$$= (2)(4)(2)(2)$$

$$= \boxed{32}$$

Step-5

We can give a general rule for any $n_1 \leq n_2 \leq n_3 \leq n_4$ as $\det A$ for A defined as above is given by

$$\det A = n_1 (n_2 - n_1)(n_3 - n_2)(n_4 - n_3)$$