

Step-1

We have to verify that if Q is orthogonal, is the same true of Q^3 .

We know that if Q is orthogonal then $Q^T = Q^{-1}$

If Q is square then Q^3 is square matrix.

Step-2

Now consider

$$\begin{aligned}(Q^3)^T Q^3 &= (Q^T)^3 Q^3 \\ &= (Q^{-1})^3 Q^3 \\ &= (Q^3)^{-1} Q^3 \\ &= I\end{aligned}$$

Therefore Q^3 is invertible and $(Q^3)^{-1} = (Q^3)^T$

Hence Q^3 is also orthogonal.

Therefore if Q is orthogonal, then Q^3 is also orthogonal.