Step-1

Consider the following differential equation:

$$\frac{du}{dt} = Ju$$

Here, J is a 2 by 2 Jordan block defined as below:

$$J = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$$

Whereas,

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Step-2

Initial value:

Step-3

$$u(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Step-4

Solve by back substitution method.

Step-5

Above differential equation can be written as follows:

$$\frac{du_1}{dt} = 5u_1 + u_2$$
$$\frac{du_2}{dt} = 5u_2$$

Step-6

The system is triangular. So, solve the single variable equation and move upwards in further equation by back substitution method:

$$\frac{du_2}{dt} = 5u_2$$

Recall that solution of the differential equation is given as follows:

$$\frac{du}{dt} = au$$

Solution:

$$u(t) = e^{at}u(0)$$

So, the solution of the differential equation starting with initial values is:

$$\begin{aligned} u_2 &= u_2(0)e^{5t} \\ &= 2e^{5t} \end{aligned}$$

Step-7

Solve the next equation having variable u_2 and substitute the values:

$$\begin{aligned} \frac{du_1}{dt} &= 5u_1 + u_2 \\ u_1 &= \left(u_1(0) + tu_2(0)\right)e^{5t} \\ &= (1+2t)e^{5t} \end{aligned}$$

Step-8

Therefore, the solution is:

$$\boxed{\begin{aligned} u_1 &= (1+2t)e^{5t} \\ u_2 &= 2e^{5t} \end{aligned}}$$