Step-1

If a_{mn} is the element in the m^{th} row and n^{th} column of the permutation matrix P, then it should be clear that

$$a_{11}(1)+a_{12}(2)+a_{13}(3)+\ldots+a_{1n}(n)=2$$

This gives us the idea that

$$a_{1j}=\begin{cases}0 \text{if } j\neq 2\\ 1 \text{if } j=2\end{cases}$$

Similarly, we should have

$$a_{21}(1)+a_{22}(2)+a_{23}(3)+\ldots+a_{2n}(n)=3$$

This gives us the idea that

$$a_{2j}=\begin{cases}0 \text{if } j\neq 3\\ 1 \text{if } j=3\end{cases}$$

Arguing similarly, finally, we get

$$a_{nj}=\begin{cases}0 \text{if } j\neq 1\\ 1 \text{if } j=1\end{cases}$$

Step-2

Therefore, the required permutation matrix P is as follows:

$$P=\begin{bmatrix}0 & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & 1 & 0 & \ldots & 0\\ 0 & 0 & 0 & 1 & \ldots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 1 & 0 & 0 & 0 & \ldots & 0\end{bmatrix}$$

Observe the following:

$$\begin{bmatrix}0 & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & 1 & 0 & \ldots & 0\\ 0 & 0 & 0 & 1 & \ldots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 1 & 0 & 0 & 0 & \ldots & 0\end{bmatrix}\begin{bmatrix}1\\ 2\\ 3\\ \vdots\\ \vdots\\ n\end{bmatrix}=\begin{bmatrix}2\\ 3\\ 4\\ \vdots\\ n\\ 1\end{bmatrix}$$

Step-3

The determinant of a Permutation matrix is either plus one or minus one. In this case, if *n* is even then the determinant is â€“1 and if *n* is odd then the determinant is +1.

To understand this, let us see how we can obtain identity matrix from such a Permutation matrix.

First we interchange the last two rows. Therefore, the new matrix has the last row coincidental with the last row of the identity matrix. Then we interchange the second last and third last rows and so on! Thus, in an $n \times n$ Permutation matrix, we have to carry out $n-1$ interchange of rows to obtain the identity matrix.

Step-4

The determinant of the identity matrix is always 1.

Note that if *n* is even, then $n-1$ is odd. Thus after odd number of row exchanges, we get the determinant equal to 1. This indicates that the determinant of the original Permutation matrix must be â€“1. Thus, when *n* is even, $\det(P) = -1$ and if *n* is odd, then $\det(P) = 1$.

Step-5

Therefore, $\boxed{\det(P) = (-1)^{n-1}}$.