

Step-1

The eigenvalues of either the upper or the lower triangular matrix are nothing but the diagonal entries.

Consider a matrix $A = \begin{bmatrix} B & C \\ 0 & D \end{bmatrix}$ where B has eigenvalues $1, 2, 3$, C has eigenvalues $4, 5, 6$ and D has eigenvalues $7, 8, 9$.

Step-2

The objective is to find the eigenvalues of matrix A .

It is known that product of the eigenvalues of the matrix is determinant of the matrix so determinant of the matrices B and D are $(1)(2)(3)$ and $(7)(8)(9)$ respectively.

The determinant of the matrix A is the product of determinants of the matrices B and D .

So determinant of the matrix is $(1)(2)(3)(7)(8)(9)$.

Since determinant is the product of the eigenvalues so it can be said that the numbers $1, 2, 3, 7, 8, 9$ are eigenvalues of the matrix A .

Hence, the eigenvalues of A are 1, 2, 3, 7, 8 and 9.