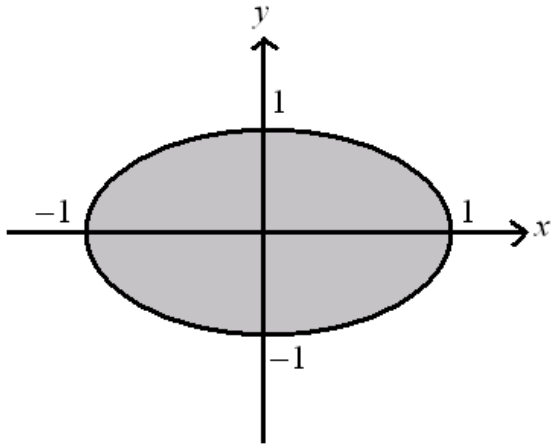# Step-1

The graph of tilted ellipse $x^2 + xy + y^2 = 1$ is as shown below.

Step-2

$$A = \begin{pmatrix} 1 & \dfrac{1}{2} \\ \dfrac{1}{2} & 1 \end{pmatrix}$$

The corresponding matrix A is

Eigen values of A are,

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & \dfrac{1}{2} \\ \dfrac{1}{2} & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)^2 - \frac{1}{4} = 0$$

$$\Rightarrow \left(1 - \lambda + \frac{1}{2}\right)\left(1 - \lambda - \frac{1}{2}\right) = 0$$

$$\Rightarrow \lambda_1 = \frac{1}{2}, \ \lambda_1 = \frac{3}{2}.$$

Step-3

We know that the half lengths of its axes are $a = \dfrac{1}{\sqrt{\lambda_1}}$ and $b = \dfrac{1}{\sqrt{\lambda_2}}$.

$\Rightarrow \lambda_1 = \dfrac{1}{2}$ and $\lambda_1 = \dfrac{3}{2}$.

$\Rightarrow a = \sqrt{2}$ and $b = \sqrt{\dfrac{2}{3}}$

Therefore $\boxed{a = \sqrt{2}, \ b = \sqrt{\dfrac{2}{3}}}$.