Step-1

Given matrix is $A=\begin{bmatrix}1 & 1\\ 3 & 3\end{bmatrix}$

We have to show that A has no inverse by solving $Ax=0$.

Step-2

The matrix form of $Ax=0$ is

$$\begin{bmatrix}1 & 1\\ 3 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

Subtracting 3 times row 1 from row 3 gives

$$\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

This shows that $x+y=0$ which has no solution

Hence the given matrix A has no inverse.

Step-3

Now we have to show that A has no inverse by solving $\begin{bmatrix}1 & 1\\ 3 & 3\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Given system is

$$\begin{bmatrix}1 & 1\\ 3 & 3\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

By matrix multiplication

$$\begin{bmatrix}a+c & b+d\\ 3a+3c & 3b+3d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Equating the corresponding elements

$$\begin{aligned} a+c&=1 & b+d&=0\\ 3a+3c&=0 & 3b+3d&=1\end{aligned}$$

But by try to solve $a+c=1, 3a+3b=0$, it has no solution

Because they are parallel lines and leads to $0 = 1$ case.

Similarly, the other two equations also lead to $0 = 1$ condition.

Hence the given matrix A has no solution.