Step-1

Consider equation of a line is x + 4y = 7

The objective is to determine the equation for the parallel line through the point x = 0, y = 0.

Also, determine the equation of the line that intersects the line x + 4y = 7 at the point x = 3, y = 1.

Step-2

Assume the equation of the line is x + 4y = 7.

It can be rewritten as,

$$4y = -x + 7$$

$$y = -\frac{1}{4}x + \frac{7}{4}$$

For an equation y = mx + c, $m = \hat{a} \in [\hat{a} \in (1)]$ represents the slope of the line.

Compare $y = -\frac{1}{4}x + \frac{7}{4}$ with equation (1).

m = -1

Recall that any parallel lines have the same slope.

Therefore, the line y = ax + b that is, parallel to x + 4y = 7 has slope $a = -\frac{1}{4}$.

The equation of line is rewritten as,

$$y = -\frac{1}{4}x + b$$

Step-3

Now, calculate the unknown value b.

The line $y = -\frac{1}{4}x + b$ passes through (0,0).

Substitute x = 0, y = 0 to get,

$$y = -\frac{1}{4}x + b$$

$$0 = 0 + b$$

$$b = 0$$

Thus,
$$b = 0$$

Hence, a line that passes through (0,0) and is parallel to x+4y=7 is given by,

$$y = -\frac{1}{4}x$$

Step-4

Calculate the equation of line that intersects line x + 4y = 7 at the point (3,1).

Assume the line is perpendicular to line x + 4y = 7 (There are other ways of intersection of line but here it is assumed that lines intersect with each other perpendicularly.)

Suppose the equation of line is $y = m_1 x + b$

Recall that for two perpendicular lines the slope m, m_1 of the two lines satisfies the relation $m \cdot m_1 = -1$.

The slope of the line x + 4y = 7 is given as $m = -\frac{1}{4}$.

Calculate the value of unknown slope m_1

$$m \cdot m_1 = -1$$

$$-\frac{1}{4} \cdot m_1 = -1$$

$$m_1 = 4$$

The equation of line is given as, y = 4x + b.

Put x = 3, y = 1 in the above equation to get,

$$1 = 4(3) + b$$

$$1-12 = b$$

$$b = -11$$

Thus,
$$b = -11$$

Substitute the value of $m_1 = 4, b = -11$ to get, y = 4x - 11

Hence, the equation of the line is y = 4x - 11.