Step-1

Consider the matrix F with 3 columns:

$$F=\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega^4\end{bmatrix}$$

Consider the matrix P as follows:

$$P=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

This gives:

$$\begin{aligned}FP&=\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega^4\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}\\ &=\begin{bmatrix}1 & 1 & 1\\ 1 & \omega^2 & \omega\\ 1 & \omega^4 & \omega^2\end{bmatrix}\\ &=\begin{bmatrix}1 & 1 & 1\\ 1 & \overline{\omega} & \overline{\omega^2}\\ 1 & \overline{\omega^2} & \overline{\omega^4}\end{bmatrix}\\ &=\overline{F}\end{aligned}$$

Step-2

The permutation matrix should be such that the first row and the first column of *FP* should be made up of 1â€™s only.

Therefore, the first row first entry column entry of *P* should be 1 and all other entries in the first row and first column should be obviously zeros.

Now the complex conjugate of *Ï‰* is *Ï‰*$^{n-1}$, The complex conjugate of *Ï‰*^{2} is *Ï‰*$^{n-2}$ and so on!

This gives us the idea about how the permutation matrix should be.

Step-3

Suppose *F* is as follows:

$$F = \begin{bmatrix} 1 & 1 & 1 & \dots & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \dots & \omega^{n-1} \\ 1 & \omega^2 & \omega^4 & \dots & \dots & \omega^{2n-2} \\ 1 & \omega^3 & \omega^6 & \dots & \dots & \omega^{3n-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \omega^{n-1} & \omega^{2n-2} & \dots & \dots & \omega^{(n-1)^2} \end{bmatrix}$$

Then we get P as follows:

$$\boxed{P = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 1 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \dots & \dots & 0 \end{bmatrix}}$$

Step-4

To obtain F^2 we can proceed as follows:

$$\begin{aligned} F\overline{F} &= nI \\ F(FP) &= nI \\ F^2 P &= nI \\ F^2 &= nIP^{-1} \end{aligned}$$

That is, $F^2 = nP^{-1}$. Similarly, $F^4 = n^2\left(P^{-1}\right)^2$.