

## Step-1

Given that the sum of the vectors  $f(x)$  and  $g(x)$  in  $\mathbf{F}$  is defined to be  $f(g(x))$ .

Then the zero vector is  $g(x) = x$

Keeping the usual scalar multiplication,

Then  $(f+g)(x)$  is the usual  $f(g(x))$

And  $(g+f)(x)$  is  $g(f(x))$

But  $g(f(x)) \neq f(g(x))$

For example  $f(x) = x^2$ ,  $g(x) = x+3$

## Step-2

Now

$$\begin{aligned}f(g(x)) &= f(x+3) \\&= (x+3)^2 \\&= x^2 + 3x + 9\end{aligned}$$

$$\begin{aligned}g(f(x)) &= g(x^2) \\&= x^2 + 3\end{aligned}$$

Therefore,  $f(g(x)) \neq g(f(x))$

So the rule  $f+g = g+f$  is broken.

Rule 4 is also broken, because there must be no inverse function  $f^{-1}(x)$  such that  $f(f^{-1}(x)) = x$ .

If the inverse function exists, it will be the vector  $-f$

For example:

Suppose  $f(x) = x^2 + 3$ , there is no function  $f^{-1}$

$$f(f^{-1}(x)) = x$$

Therefore rule 4 is also broken.