Step-1

Let A be the following matrix:

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

Diagonalize A and compute $S\Lambda^k S^{-1}$ to get following formula for A^k:

$$A^k = \frac{1}{2}\begin{bmatrix} 5^k+1 & 5^k-1 \\ 5^k-1 & 5^k+1 \end{bmatrix}$$

Step-2

To diagonalize the matrix A follow the following steps:

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$
$$A - \lambda I = \begin{bmatrix} 3-\lambda & 2 \\ 2 & 3-\lambda \end{bmatrix}$$
$$\det(A - \lambda I) = (3-\lambda)^2 - 4$$
$$= \lambda^2 - 6\lambda + 5$$

Put the determinant value equal to zero, to get following roots as Eigen values:

$$\lambda_1 = 5$$
$$\lambda_2 = 1$$

Step-3

Eigen vectors corresponding to the Eigen values are calculated as follows:

For $\lambda_1 = 5$

$$(A - \lambda I)x_1 = 0$$
$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$x_1 = \begin{bmatrix} y \\ z \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Step-4

For $\lambda_2 = 1$

$$(A - \lambda_2 I)x_2 = 0$$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Step-5

Thus, Eigen vector matrix is as follows:

$$S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$S^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Step-6

Therefore, diagonalisation of matrix A is as follows:

$$A = S\Lambda S^{-1}$$

$$= \boxed{\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}$$

Step-7

Now, do the following calculations to get A^k:

$$A^k = S\Lambda^k S^{-1}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5^k & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 5^k + 1 & 5^k - 1 \\ 5^k - 1 & 5^k + 1 \end{bmatrix}$$

Step-8

Therefore,

$$\boxed{A^k = \frac{1}{2}\begin{bmatrix} 5^k + 1 & 5^k - 1 \\ 5^k - 1 & 5^k + 1 \end{bmatrix}}$$