Step-1

a) We have to find a matrix that transforms $(1,0)$ and $(0,1)$ to (r,t) and (s,u).

We have

$$T(1,0)=(r,t)$$
$$T(0,1)=(s,u)$$

Therefore, the matrix M of the linear transformation T under the basis $(1,0),(0,1)$ is $\boxed{\begin{bmatrix} r & s \\ t & u \end{bmatrix}}$

Step-2

b) We have to find a matrix that transforms (a,c) and (b,d) to $(1,0)$ and $(0,1)$.

We have

$$T(a,c)=(1,0)$$
$$T(b,d)=(0,1)$$

The matrix of the transform T under the elements (a,c) and (b,d) (if $\{(a,c),(b,d)\}$ is a basis of R^2) is $\boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$.

Step-3

c) We have to find the condition on a,b,c,d that will make part (b) impossible.

If the vectors $(a,b),(c,d)$ are linearly dependent,

That is there exist not all zero scalars x,y such that $x(a,b)+y(c,d)=0$

Then part (b) is impossible.

Hence part (b) is impossible if the vectors $(a,b),(c,d)$ are linearly dependent.