Step-1

Given that $Ax = \lambda Mx$

$$\Rightarrow \begin{pmatrix} 6 & -3 \\ -3 & 6 \end{pmatrix} x = \frac{\lambda}{18} \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} x$$

$$\Rightarrow \begin{pmatrix} 6-\frac{4\lambda}{18} & -3-\frac{\lambda}{18} \\ -3-\frac{\lambda}{18} & 6-\frac{4\lambda}{18} \end{pmatrix} x = 0$$

For Eigen values,

$$\begin{vmatrix} 6-\frac{4\lambda}{18} & -3-\frac{\lambda}{18} \\ -3-\frac{\lambda}{18} & 6-\frac{4\lambda}{18} \end{vmatrix} = 0$$

$$\Rightarrow \left(6-\frac{4\lambda}{18}-3-\frac{\lambda}{18}\right)\left(6-\frac{4\lambda}{18}+3+\frac{\lambda}{18}\right) = 0$$

$$\Rightarrow \left(3-\frac{5\lambda}{18}\right)\left(9-\frac{3\lambda}{18}\right) = 0$$

$$\Rightarrow \lambda_1 = \frac{54}{5},\ \lambda_2 = 54$$

Therefore, the Eigen values are $\boxed{\lambda_1 = \frac{54}{5} \text{ and } \lambda_2 = 54}$.

Step-2

Let $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be the Eigen vector corresponding to $\lambda_1 = \frac{54}{5}$.

Now,

$$\Rightarrow \begin{pmatrix} 6-\frac{4(54)}{18} & -3-\frac{54}{18} \\ -3-\frac{54}{18} & 6-\frac{4(54)}{18} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -6 & -6 \\ -6 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_1 + x_2 = 0$$

Thus $\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$ is the Eigen vector corresponding to $\lambda_2 = 54.$

Step-3

Let $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be the Eigen vector corresponding to $\lambda_1 = \frac{54}{5}$.

$$\Rightarrow \begin{pmatrix} 6 - \frac{4}{18}.\frac{54}{5} & -3 - \frac{1}{18}.\frac{54}{5} \\ -3 - \frac{1}{18}.\frac{54}{5} & 6 - \frac{4}{18}.\frac{54}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \frac{18}{5} & -\frac{18}{5} \\ -\frac{18}{5} & \frac{18}{5} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow x_1 - x_2 = 0$$

Thus $\boxed{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$ is the Eigen vector corresponding to $\lambda_2 = \frac{54}{5}.$

Therefore, the Eigen vectors are $\boxed{\begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{and} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$.