Step-1

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Let the matrix A be such that row 1 + row 2 = row 3.

That is

$$A=\begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$$

We have to show that A has no inverse.

Step-2

We have $A=\begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$

Subtracting row 1 from row 3 gives

$$A\approx\begin{bmatrix} a & b & c \\ d & e & f \\ d & e & f \end{bmatrix}$$

Subtracting row 2 from row 3 gives

$$A\approx\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}$$

The last row of A is zero.

Therefore, its determinant becomes zero.

So that the matrix A does not possesses inverse.

Step-3

(a) The given system is $Ax=(1,0,0)$.

We have to show that the given system has no solution.

Now the system becomes

$$\begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Now performing (row 1 + row 2) â€“row3 gives

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

The last row shows that $0=-1$ condition.

So the system $Ax=(1,0,0)$ has no solution.

Step-4

(b) We have to explain which right hand sides (b_1,b_2,b_3) might allow a solution to $Ax=b$.

The right hand sided must satisfy $b_1+b_2=b_3$

Then the system $Ax=b$ has a solution.

Step-5

(c) We have to tell what happens to row 3 of A in elimination.

We have $A=\begin{bmatrix} a & b & c \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$

Subtracting row 1 from row 3 gives

$$A\approx\begin{bmatrix} a & b & c \\ d & e & f \\ d & e & f \end{bmatrix}$$

Subtracting row 2 from row 3 gives

$$A\approx\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 0 \end{bmatrix}$$

Row 3 becomes a row of zeros and that has no third pivot.

Hence the row 3 becomes a zero row in elimination.