Step-1

Suppose U is unitary and Q is a real orthogonal matrix.

We have to show that U^{-1} and UQ are unitary.

Step-2

Since U is unitary matrix.

So $U^H U = I$

Since Q is real orthogonal matrix.

So $Q^T Q = I$

Now

$$\left(U^{-1}\right)^H .U^{-1} = \left(U^{-1}\right)^H U^H \quad \left(\text{Since } U^H = U^{-1}\right)$$

$$= \left(U^H\right)^H U^H$$
$$= UU^H \qquad \left(\text{Since } \left(U^H\right)^H = U\right)$$
$$= UU^{-1}$$
$$= I$$

Since $\left(U^{-1}\right)^H U^{-1} = I$

Therefore U^{-1} is also unitary.

Step-3

Now

$$\begin{aligned}(UQ)^H (UQ) &= \left(Q^H U^H\right)(UQ)\\ &= Q^H \left(U^H U\right) Q\\ &= Q^H (IQ)\end{aligned}$$

$$= Q^H Q$$

Since Q is real orthogonal.

So $Q^T = Q^H$

$$= Q^T Q$$
$$= I$$

Therefore, $(UQ)^H (UQ) = I$

Hence UQ is also unitary.