Answer

2021-2022 Calculus I Mid

1. Multiple Choice Questions:
   (1).B
   (2).A
   (3).C
   (4).A
   (5).C
2. Fill in the blanks:
   (1).3
   (2).$n!$
   (3).$\frac{2sinx+xcosx}{4\sqrt{xsinx\sqrt{sinx}}}$
   (4).$\frac{1}{6}$
   (5).$y=x+1$
3. $S_{\Delta}\le 100$,the equal sign holds when $a=b=10\sqrt{2}$.
4. $\frac{dy}{dx}|_{x=0}=0$, $\frac{d^2y}{dx^2}|_{x=0}=-\frac{1}{2}$.
5. $2\pi-\frac{(\pi)^2}{2}$
6. (1)$\frac{4}{3}$.
   (2)$\frac{11}{3}$.
7. (1)$\frac{1}{24}$.
   (2)$\frac{1}{2}$.
8. $y=2x+1$.
9. when $x=\frac{\pi}{4}$, $f(x)$ takes its minimum and $f(\frac{\pi}{4})=2\sqrt{2}-1$.

2020-2021 Calculus I Mid

1. Multiple Choice Questions:
   (1).B
   (2).A
   (3).B
   (4).D
   (5).C
2. Fill in the blanks:
   (1).0
   (2).$\frac{1}{12}$

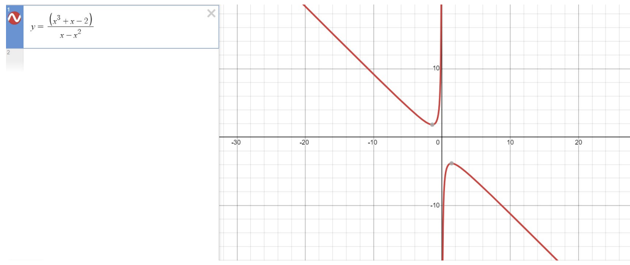
(3).$\frac{1}{3}x^3+2x-\frac{1}{x}-\frac{1}{3}$

(4).-2

(5).$\frac{a}{\sqrt{a^2+1}}$

3. (1)0 . (2)$-\frac{3}{2}$.
4. (1)4 . (2)$\frac{1}{3}+\frac{\pi}{2}$.
5. (1)local maxima at $x=\sqrt{2}$,local minima at $x=-\sqrt{2}$.

   (2)horizontal asymptote: not exist; vertical asymptote: x=0; oblique asymptotes: $y=-x-1$.



6. (1)$\frac{dy}{dx}=4\sqrt{x^2+x}$.

   (2)$y=-\frac{5}{4}(x-5)+4$.
7. The area is $\frac{1}{3}$.
8. The volume is $\frac{9\pi}{5}$.
9. You can easily show the conclusion by the mean value theorem of integration for continuous functions.

2019-2020 Calculus I Mid

1. True or False:

   (1).TRUE

   (2).FALSE

   (3).TRUE

   (4).TRUE

   (5).FALSE

   (counter-example: f(x)=1 can be considered as peridic function with period 1 ,then g(x)=x is not a periodic function with f(x)&g(x) satisying such condition)
2. Multiole Choice Questions:

   (1).D

   (2).B

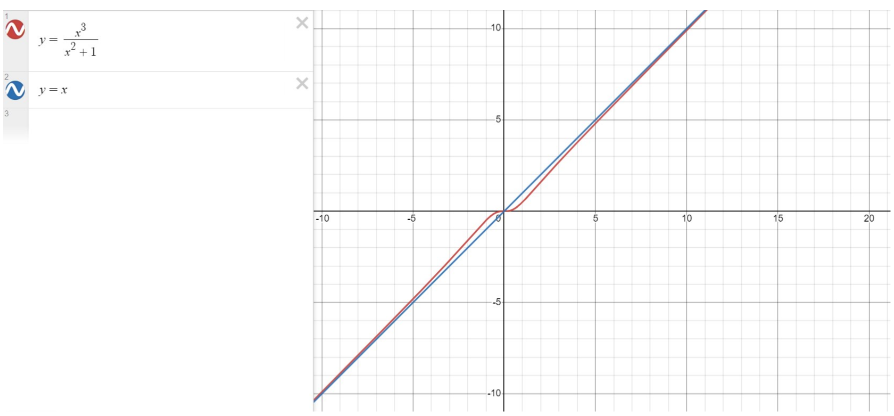
   (3).C

   (4).D (keep ur eyes open: B is a Common Mistake)

   (5).B
3. (1)Deravatives is nonnegative,and does not exist zero point which changes sign,so local maxima and local minima do not exist. inflection point: x=0

   (2)horizontal asymptote: not exist; vertical asymptote: NE; oblique asymptotes: $y=x$.

   (3)



4. (1)$\frac{5}{4}+sin5$.
   (2)$\frac{\pi}{4}$
5. (1)$\frac{1}{6}a^4+\frac{1}{2}$
   (2)$\frac{4\sqrt{2}}{3}$
6. Write the explicit function respectively as $r_1(x)$ (y from $\frac{2}{3}to1$) and r_2 (y from $0to\frac{2}{3}$) on $[0,\frac{16}{9}]$ .Obviously,volume is $\int_0^{\frac{16}{9}}(\pi(2-r_1(x))^2)dx-\int_0^{\frac{16}{9}}(\pi(2-r_2(x))^2)dx=\ldots\ldots$

   此类题不考，无需担心。
7. $f(\frac{11\pi}{60})=\frac{\pi}{45}+\frac{\sqrt{3}}{3}$.
8. The area is $\int_0^2(3-x-\frac{1}{4}x^2)dx-\int_0^1(3-x-(\sqrt{x}+1))dx=\frac{5}{2}$
9. All critical points on $\mathbb{R}$ can be found,x=0/$\pm(\frac{\pi}{4}+\frac{k\pi}{2})^{\frac{1}{4}}$,where k=0,1,2....
   In the interval [-1,1],x=0/$\pm(\frac{\pi}{4})^{\frac{1}{4}}$
10. Structure $\phi(x)=f(x)-\alpha x-\beta$,then let $\alpha=\frac{f(b)-f(a)}{b-a}$,hence,by Rolle's theorem,qed.

2022-2023 Calculus I Mid

1. Multiple Choice Questions:

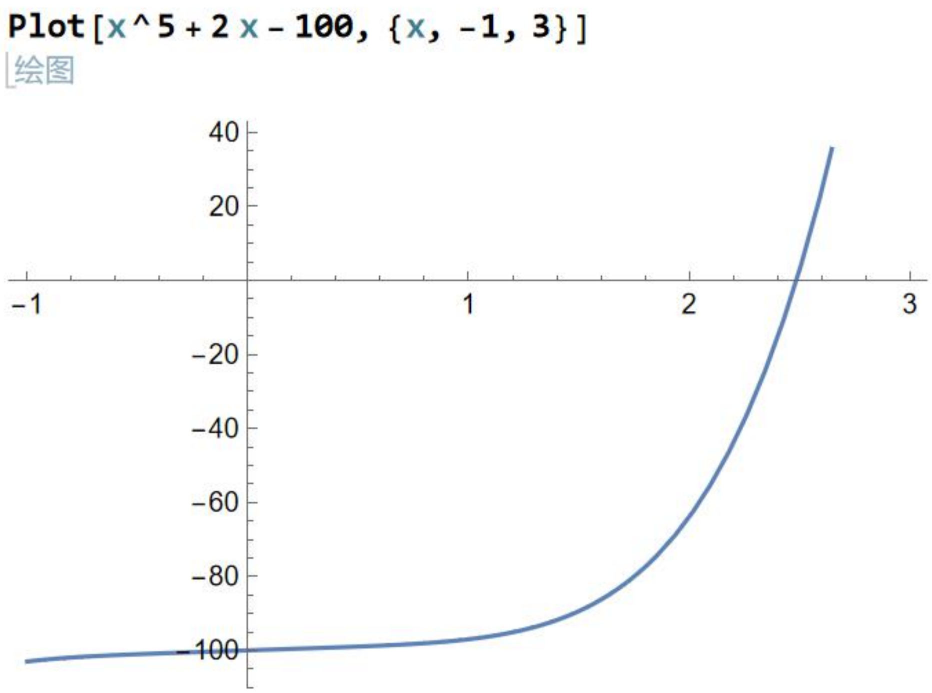
(1).C
(2).B
(3).A
(4).C
(5).A

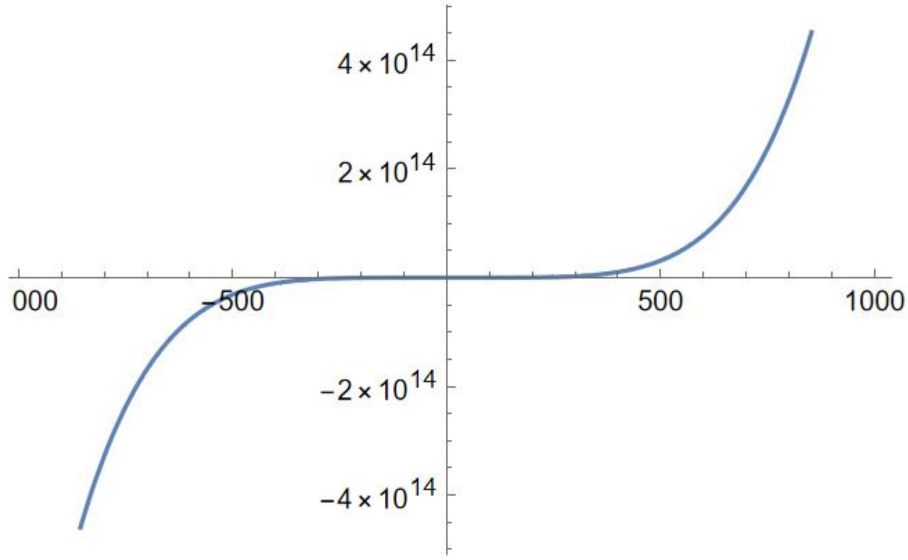
2. Fill in the blanks:

(1).$\frac{2}{\pi}$
(2).$16 or -16$
(3).more than 2 methods， $\frac{11}{2^{15/4}}$
(4).2
(5).0

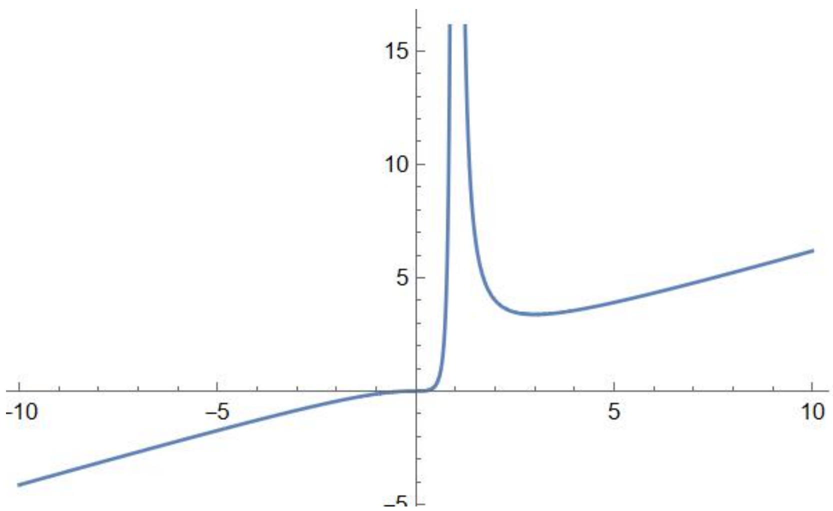
3. **Proof.** recall how we show the existence of root. And we assume there only 1 root and deduce a contradiction algebraically.

I draw 2 figures for illustration





4. $\frac{37}{168}$ substitution: $x \to tan\theta$
5. we know linear approximate part $\mathcal{L}(x) = f(x_0) + f'(x_0) \cdot (x - x_0)$ answer is $\frac{3}{2}x + 3$
6. hint: rewrite $2(x-1) + 3 + \frac{b+1}{x-1}$, b=-1.a=3
7. nothing to write, you just take the derivative of that, and then solve the equation



8. $y = \frac{1}{7}(x-1) + 1$,hint: $2y^3 - y^2 + 3y - 4 = 0 \to (y-1)(\dots)$
9. $$F(x) = \int_0^x xtf(x^2 - t^2)dt = \dots = \dots = \dots = -\frac{1}{2}x \int_0^x f(-t^2 + x^2)d(-t^2 + x^2)$$

   then you can compute it...