

## Step-1

Given that adding row 1 of  $A$  to row 2 produces  $B$ . Adding column 1 to column 2 produces  $C$ .

So the matrices are  $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ ,  $C = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ .

A combination of columns of matrix  $C$  is also a combination of the columns of  $A$ .

We have to find which two matrices have the same column space.

## Step-2

Let  $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbf{C}(A)$

Then

$$\begin{aligned} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} &= c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ &= c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} - c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ &= [c_1 - c_2] \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 6 \end{bmatrix} \end{aligned}$$

Therefore  $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$  is linear combination of  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ,  $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$ , the column of  $C$ .

## Step-3

If  $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbf{C}(C)$

Then  $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = d_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = d_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \left[ \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right]$$

$$= d_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= (d_1 + d_2) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + d_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Therefore  $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbf{C}(A)$

Hence the matrices  $\boxed{A \text{ and } C}$  have same column spaces.