Step-1

The objective is to find the vectors (b_1,b_2,b_3) that are in the column space of A that is given below.

Further objective is to find combinations of the rows of A that give zero.

(a)

The column space contains all linear combinations of the columns of A.

So, $(b_1,b_2,b_3) \in$ Column space of A if,

$$\begin{aligned}(b_1,b_2,b_3) &= c_1(1,2,0)+c_2(2,6,2)+c_3(1,3,5)\\ &=(c_1,2c_1,0)+(2c_2,6c_2,2c_2)+(c_3,3c_3,5c_3)\\ &=(c_1+2c_2+c_3,2c_1+6c_2+3c_3,2c_2+5c_3)\end{aligned}$$

Step-2

Assume that,

$$\begin{aligned}a(1,2,1)+b(2,6,3)+c(0,2,5)&=(0,0,0)\\ (a+2b,2a+6b+2c,a+3b+5c)&=(0,0,0)\end{aligned}$$

Rewrite into equation from as follows:

$$\begin{aligned}a+2b&=0\\ 2a+6b+2c&=0\\ a+3b+5c&=0\end{aligned}$$

$a+2b=0$ implies, $a=-2b$.

Substitute $a=-2b$ into $2a+6b+2c=0$ as follows:

$$\begin{aligned}2a+6b+2c&=0\\ 2b+2c&=0\end{aligned}$$

Substitute $a=-2b$ into $a+3b+5c=0$ as follows:

$$\begin{aligned}a+3b+5c&=0\\ b+5c&=0\end{aligned}$$

Solve the equation $2b+2c=0$ and $b+5c=0$ to get $b=c=0$.

Hence, $0(1,2,1)+0(2,6,3)+0(0,2,5)=(0,0,0)$.

Therefore, the rows of A are linearly independent.

Step-3

(b)

The column space contains all linear combinations of the columns of A.

So, $(b_1, b_2, b_3) \in$ Column space of A if,

$$\begin{aligned}(b_1, b_2, b_3) &= c_1(1,1,2) + c_2(1,2,4) + c_3(1,4,8) \\ &= (c_1, c_1, 2c_1) + (c_2, 2c_2, 4c_2) + (c_3, 4c_3, 8c_3) \\ &= (c_1 + 2c_2 + c_3, c_1 + 2c_2 + 4c_3, 2c_1 + 4c_2 + 8c_3)\end{aligned}$$

Step-4

Assume that,

$$a(1,1,1) + b(1,2,4) + c(2,4,8) = (0,0,0)$$
$$(a + b + 2c, a + 2b + 4c, a + 4b + 8c) = (0,0,0)$$

Rewrite into equation from as follows:

$$\begin{aligned} a + b + 2c &= 0 \\ a + 2b + 4c &= 0 \\ a + 4b + 8c &= 0 \end{aligned}$$

Solving $a + 2b + 4c = 0$ and $a + 4b + 8c = 0$ to get $2b + 4c = 0$.

$2b + 4c = 0$ implies $b = -2c$.

Substitute $b = -2c$ in $a + b + 2c = 0$ to get $a = 0$.

For $b = -2c$, take $c = 1$.

This implies, $b = -2$.

Hence, $0(1,1,1) + (-2)(1,2,4) + 1(2,4,8) = (0,0,0)$ is the required linear combination.