

## Step-1

Let us consider the following matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Now, in a mixed strategy, for columns, use the frequency  $x_1$  and  $x_2 = 1 - x_1$

Thus, we have the following expression

$$ax_1 + b(1 - x_1) = cx_1 + d(1 - x_1)$$

$$x_1(a - b - c + d) = d - b$$

$$x_1 = \frac{d - b}{a - b - c + d} \quad (1)$$

And

$$\begin{aligned} 1 - x_1 &= 1 - \frac{d - b}{a - b - c + d} \\ &= \frac{a - b - c + d - d + b}{a - b - c + d} \end{aligned}$$

$$1 - x_1 = \frac{a - c}{a - b - c + d} \quad (2)$$

## Step-2

Similarly, in a mixed strategy, for rows, use the frequency  $y_1$  and  $y_2 = 1 - y_1$

Thus, we have the following expression

$$ay_1 + c(1 - y_1) = by_1 + d(1 - y_1)$$

$$y_1(a - c - b + d) = d - c$$

$$y_1 = \frac{d - c}{a - c - b + d} \quad (3)$$

And

$$1 - y_1 = 1 - \frac{d - c}{a - c - b + d}$$

$$= \frac{a - c - b + d - d + c}{a - c - b + d}$$

$$1 - y_1 = \frac{a - b}{a - c - b + d} \quad (4)$$

But, the expression for  $u$  is as follows

$$u = ax_1 + b(1 - x_1)$$

$$u = a \left( \frac{d - b}{a - b - c + d} \right) + b \left( \frac{a - c}{a - b - c + d} \right) \quad (\text{From equation (1) and equation (2)})$$

$$u = \frac{ad - ab + ab - bc}{a - b - c + d}$$

$$u = \frac{ad - bc}{a - b - c + d} \quad (5)$$

### Step-3

Similarly, the expression for  $v$  is as follows

$$v = ay_1 + c(1 - y_1)$$

$$v = a \left( \frac{d - c}{a - c - b + d} \right) + c \left( \frac{a - b}{a - c - b + d} \right) \quad (\text{From equation (3) and equation (4)})$$

$$v = \frac{ad - ac + ac - bc}{a - c - b + d}$$

$$v = \frac{ad - bc}{a - b - c + d} \quad (6)$$

Thus, from expressions (5), and (6), it is observed that  $u = v$