

Step-1

Step-2

(a) The projection matrix $P_1 = \frac{aa^T}{a^T a}$

$$\begin{aligned} aa^T &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{aligned}$$

Also,

$$\begin{aligned} a^T a &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\ &= 1+1+1 \\ &= 3 \end{aligned}$$

Step-3

$$P_1 = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} P_1^2 &= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \\ &= \frac{1}{9} \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{aligned}$$

Therefore $P_1^2 = P$

Step-4

$$P_1 b = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$
$$= \frac{1}{3} \begin{bmatrix} 1+2+2 \\ 1+2+2 \\ 1+2+2 \end{bmatrix}$$

$$P_1 b = \frac{1}{3} \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$$

Therefore

Step-5

(b) The projection matrix $P_2 = \frac{aa^T}{a^T a}$

$$aa^T = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix} \begin{bmatrix} -1 & -3 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

$$a^T a = \begin{bmatrix} -1 & -3 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$$
$$= 1+9+1$$
$$= 11$$

Step-6

$$P_2 = \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\begin{aligned}
P_2^2 &= \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix} \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix} \\
&= \frac{1}{121} \begin{bmatrix} 11 & 33 & 11 \\ 33 & 99 & 33 \\ 11 & 33 & 11 \end{bmatrix} \\
&= \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix}
\end{aligned}$$

Therefore $P_2^2 = P$

$$\begin{aligned}
P_2 b &= \frac{1}{11} \begin{bmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \\
&= \frac{1}{11} \begin{bmatrix} 11 \\ 33 \\ 11 \end{bmatrix}
\end{aligned}$$

$$P_2 b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Therefore