Step-1

We have to check whether the following statements in (a) and (b) are true or false with an example.

(a) Q^{-1} is an orthogonal matrix when Q is an orthogonal matrix.

For example, suppose $Q=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

Step-2

$$QQ^T=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=I$$

Therefore Q is an orthogonal matrix.

Step-3

$$Q^{-1}=\frac{1}{0-1}\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$\left(Q^{-1}\right)^T Q^{-1}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=I$$

Hence Q^{-1} is also an orthogonal matrix.

Therefore the given statement is $\boxed{\text{true}}$.

(b) If Q (3 by 2) has orthonormal columns then $\|Qx\|=\|x\|$

For example, suppose $Q=\begin{bmatrix}q_1 & q_2\end{bmatrix}$ is a 3×2 matrix, where q_1,q_2 are orthonormal columns of Q

That is,

$q_1^T q_1 = 1, q_2^T q_2 = 1$, and $q_1^T q_2 = 0, q_2^T q_1 = 0$ Â â€¦â€¦ (1)

Step-4

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, then

$$\begin{aligned}
\|Qx\|^2 &= (Qx)^T (Qx) = (q_1x_1 + q_2x_2)^T (q_1x_1 + q_2x_2) \\
&= \left[(q_1x_1)^T + (q_2x_2)^T\right](q_1x_1 + q_2x_2) \\
&= \left(x_1^T q_1^T + x_2^T q_2^T\right)(q_1x_1 + q_2x_2) \\
&= x_1q_1^T q_1x_1 + x_2q_2^T q_1x_1 + x_1q_1^T q_2x_2 + x_2q_2^T q_2x_2
\end{aligned}$$

$\left(\text{Since are real numbers}, x_1^T = x_1, x_2^T = x_2\right)$

Step-5

$$= x_1 1 x_1 + x_2 .0 x_1 + x_1 .0 x_2 + x_2 1 x_2$$

$$= x_1^2 + x_2^2$$

Step-6

Therefore

$$\begin{aligned}
\|Qx\|^2 &= x_1^2 + x_2^2 \\
&= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\
&= x^T x \\
&= \|x\|^2
\end{aligned}$$

Â Â Â

$\Rightarrow \|Qx\| = \|x\|$ Â

Therefore the given statement is $\boxed{\text{true}}$.Â

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