

Step-1

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We have to explain why the inner product of x and y equals the inner product of Px and Py .

Step-2

Let A be any matrix.

Now

$$\begin{aligned}(Px)^T A(Py) &= x^T P^T A Py \\ &= x^T (P^T AP)y \\ &= x^T Ay \quad \left(\text{Since } P^T AP = A, \text{ for any permutation matrix} \right)\end{aligned}$$

Hence the inner product of x and y equals the inner product of Px and Py .

Step-3

Given $x = (1, 2, 3)$ and $y = (1, 4, 2)$

We have to choose the permutation matrix P to show that $(Px)^T y$ is always not equal to $x^T (P^T y)$

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Let the permutation matrix P be

$$\text{Given } x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ and } y = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$\text{Then } x^T = [1 \quad 2 \quad 3] \text{ and } P^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Step-4

Now

$$\begin{aligned} Px &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 0(1)+1(2)+0(3) \\ 0(1)+0(2)+1(3) \\ 1(1)+0(2)+0(3) \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \end{aligned}$$

Therefore, $(Px)^T = [2 \ 3 \ 1]$

Step-5

Now

$$\begin{aligned} (Px)^T y &= [2 \ 3 \ 1] \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \\ &= [2(1)+3(4)+1(2)] \\ &= [2+12+2] \\ &= [16] \end{aligned}$$

Step-6

Now we find $x^T (P^T y)$

So

$$\begin{aligned}
 P^T y &= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \\
 &= \begin{bmatrix} 0(1)+0(4)+1(2) \\ 1(1)+0(4)+0(2) \\ 0(1)+1(4)+0(2) \end{bmatrix} \\
 &= \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}
 \end{aligned}$$

Step-7

Now

$$\begin{aligned}
 x^T (P^T y) &= \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \\
 &= \begin{bmatrix} 1(2)+2(1)+3(4) \end{bmatrix} \\
 &= \begin{bmatrix} 2+2+12 \end{bmatrix} \\
 &= \begin{bmatrix} 16 \end{bmatrix}
 \end{aligned}$$

Hence there exists no P such that $(Px)^T y \neq x^T (P^T y)$.