Step-1

The matrix is $A = \begin{pmatrix} 1 & c & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{pmatrix}$

The objective is to find number c that leads to zero in the second pivot position and to find number c that produces 0 in third pivot position.

Recollect that if a matrix is in row echelon form, then first non- zero entry of each row is called a pivot.

First, substitute $c = 2$ in matrix A then the matrix is,

$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{pmatrix}$

Step-2

Apply elementary row operations.

Subtract 2 times row 1 from row 2.

$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 3 & 5 & 1 \end{pmatrix}$

If c is 2 then the second pivot becomes zero.

Therefore, the value $\boxed{c = 2}$ leads to zero in second pivot.

Step-3

Next, substitute $c = 1$ in matrix, A then the matrix is,

$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{pmatrix}$

Apply elementary row operations.

Subtract 3 times row 1 from row 3 .

$$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 4 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Subtract 2 times row 1 from row 2 .

$$R_2 \rightarrow R_2 - R_1 \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Subtract second row from third row.

$$R_3 \rightarrow R_3 - R_2 \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

That is the third pivot (third diagonal element) is zero.

Therefore, the value $\boxed{c = 1}$ leads to 0 in third pivot position.