Step-1

Let $T:\mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation.

We have to prove that T^2 is also a linear transformation.

Step-2

We know that a transformation T is said to be a linear transformation if $T(ax+by)=aT(x)+bT(y)$, where x, y are vectors and a, b are scalars.

Let $x, y \in \mathbf{R}^3$

Now

$T^2(x+y)=T(T(x+y))$

$=T(T(x)+T(y)) \quad$ (since T is linear)

$=T(T(x))+T(T(y)) \quad$ (since T is linear and $T(x), T(y) \in R^3$)

$=T^2(x)+T^2(y)$

Step-3

And let $x \in \mathbf{R}^3, a \in \mathbf{R}$

$T^2(ax)=T(T(ax))$

$=T(aT(x)) \quad$ (since T is linear)

$=aT(T(x)) \quad$ (since T is linear & $T(x) \in \mathbf{R}^3$)

$=aT^2(x)$

Hence T^2 is also a linear transformation.