

Step-1

Now consider the following:

$$F_{1024} = \begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix} \begin{bmatrix} F_{512} \\ F_{512} \end{bmatrix} \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix}$$

By using the above rule, we get

$$\begin{aligned} F_{1024}^{-1} &= \left(\begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix} \begin{bmatrix} F_{512} \\ F_{512} \end{bmatrix} \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix} \right)^{-1} \\ &= \begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix}^{-1} \begin{bmatrix} F_{512} \\ F_{512} \end{bmatrix}^{-1} \begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix}^{-1} \end{aligned}$$

Note that the inverse of an even permutation is another even permutation. Similarly, inverse of an odd permutation is another odd permutation. Also, these inverses are actually the transposes of the original permutations. The inverse of an Identity matrix is the same Identity matrix only.

Thus, we get

$$F_{1024}^{-1} = \boxed{\begin{bmatrix} \text{even-odd} \\ \text{permutation} \end{bmatrix}^T \begin{bmatrix} F_{512}^{-1} & \\ & F_{512}^{-1} \end{bmatrix} \begin{bmatrix} I_{512} & D_{512}^{-1} \\ I_{512} & -D_{512}^{-1} \end{bmatrix}}$$