Step-1

Given that 3 by 3 matrix A is invertible.

The matrix A is invertible, which means that the matrix A is non â€“ singular. So, the rank of A is 3.

The dimension of row space and column space is,

$$\dim\left(C(A)^T\right)=3 \text{ and } \dim\left(C(A)\right)=3$$

Row basis is $\{(1,0,0),(0,1,0),(0,0,1)\}$ and its dimension is 3.

Column basis is $\{(1,0,0),(0,1,0),(0,0,1)\}$ and its dimension is 3.

Step-2

Null space basis is,

Letâ€™s write in $AX=0$,

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=0$$
$$\begin{aligned}\Rightarrow x_1&=0\\x_2&=0\\x_3&=0\end{aligned}$$

Null basis is empty and its dimension is 0.

$$\begin{aligned}&\text{i.e.}\dim\left(C(B)\right)+\dim\left(N(B)\right)=3\\&\Rightarrow\dim\left(N(B)\right)=3-3\\&\Rightarrow\dim\left(N(B)\right)=0\end{aligned}$$

Step-3

In order to find the left null space basis, we need to find the transpose of the matrix is,

The transpose of the matrix is $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$.

Letâ€™s write in $A^TX=0$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{aligned} \Rightarrow x_1 &= 0 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned}$$

Left null basis is empty and its dimension is 0.

$$\begin{aligned} &\text{i.e.} \dim\left(C\left(B^T\right)\right) + \dim\left(N\left(B^T\right)\right) = 3 \\ &\Rightarrow \dim\left(N\left(B^T\right)\right) = 3 - 3 \\ &\Rightarrow \dim\left(N\left(B^T\right)\right) = 0 \end{aligned}$$

Therefore, row space basis and column space is $\boxed{\{(1,0,0),(0,1,0),(0,0,1)\}}$ and its dimension is $\boxed{3}$. Null space and left null space dimension is $\boxed{0}$.

Step-4

The basis for 3 by 6 matrix is,

$$\begin{aligned} B &= \begin{bmatrix} A \\ A \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Row space basis is $\{(1,0,0,1,0,0),(0,1,0,0,1,0),(0,0,1,0,0,1)\}$ and its dimension is 3.

Column space basis is $\{(1,0,0),(0,1,0),(0,0,1)\}$ and its dimension is 3.

Step-5

Null space basis is,

Letâ€™s write in $AX = 0$,

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = 0$$

$$\begin{aligned} \Rightarrow\, & x_1 + x_4 = 0 \\ & x_2 + x_5 = 0 \\ & x_3 + x_6 = 0 \\ \Rightarrow\, & x_1 = -x_4 \\ & x_2 = -x_5 \\ & x_3 = -x_6 \end{aligned}$$

So,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -x_4 \\ -x_5 \\ -x_6 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$$= x_4 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

The null space basis is $\{(-1,0,0,1,0,0),(0,-1,0,0,1,0),(0,0,-1,0,0,1)\}$ and its dimension 0.

$$\begin{aligned} & \text{i.e.}\dim(C(B)) + \dim(N(B)) = 6 \\ & \Rightarrow \dim(N(B)) = 6 - 3 \\ & \Rightarrow \dim(N(B)) = 3 \end{aligned}$$

Step-6

In order to find the left null space basis, we need to find the transpose of the matrix is, The transpose of the matrix is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Letâ€™s write in $A^T X = 0$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow x_1 = 0$$
$$x_2 = 0$$
$$x_3 = 0$$

The left null space basis is empty and its dimension 0.

$$\text{i.e.} \dim\left(C\left(B^T\right)\right) + \dim\left(N\left(B^T\right)\right) = 6$$
$$\Rightarrow \dim\left(N\left(B^T\right)\right) = 6 - 3$$
$$\Rightarrow \dim\left(N\left(B^T\right)\right) = 3$$

Therefore, row space basis is $\boxed{\{(1,0,0,1,0,0),(0,1,0,0,1,0),(0,0,1,0,0,1)\}}$ and its dimension is $\boxed{3}$, column space basis is $\boxed{\{(1,0,0),(0,1,0),(0,0,1)\}}$ and its dimension is $\boxed{3}$ and null space basis is $\boxed{\{(-1,0,0,1,0,0),(0,-1,0,0,1,0),(0,0,-1,0,0,1)\}}$ and its dimension is $\boxed{3}$ and left null space basis is empty and its dimension is $\boxed{3}$