

## Step-1

Given that  $A$  is an  $m$  by  $n$  matrix of rank  $r$  and its reduced echelon form is  $R$ .

We have to describe exactly the reduced row echelon form of  $R^T$  (not  $A^T$ ).

## Step-2

Given that  $A$  is an  $m$  by  $n$  matrix of rank  $r$  and its reduced echelon form is  $R$ .

Therefore  $R$  has  $r$  independent rows (or  $R$  has  $r$  non zero rows). Then  $R^T$  has  $r$  independent columns. That is  $R^T$  has  $r$  columns which are pivot columns.

Therefore  $R^T$  has  $r$  non-zero columns and  $m-r$  non-zero rows.

Thus  $R^T R$  is in the form

$$R^T R = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \text{ where } I, \text{ is the identity matrix of } r \text{ by } r.$$