Step-1

Consider the following:

$$
\begin{aligned}
Ax_k &= \begin{bmatrix} 2 & -1 & & \\ -1 & . & . & \\ & . & . & -1 \\ & & -1 & 2 \end{bmatrix} \begin{bmatrix} \sin k\pi h \\ \sin 2k\pi h \\ \vdots \\ \sin nk\pi h \end{bmatrix} \\
&= \begin{bmatrix} 2\sin k\pi h - \sin 2k\pi h \\ -\sin k\pi h + 2\sin 2k\pi h - \sin 3k\pi h \\ \vdots \\ -\sin(n-1)k\pi h + 2\sin nk\pi h \end{bmatrix}
\end{aligned}
$$

The i^{th} entry in the product is given by $-\sin(i-1)k\pi h + 2\sin ik\pi h - \sin(i+1)k\pi h$.

Step-2

By Trigonometry, we know that $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$.

Therefore, we get

$$
\begin{aligned}
-\sin(i-1)k\pi h + 2\sin ik\pi h - \sin(i+1)k\pi h &= 2\sin ik\pi h - \left(\sin(i-1)k\pi h + \sin(i+1)k\pi h\right) \\
&= 2\sin ik\pi h - \left(2\sin ik\pi h\cos k\pi h\right) \\
&= 2\sin ik\pi h\left(1 - \cos k\pi h\right) \\
&= \left(2 - 2\cos k\pi h\right)\sin ik\pi h
\end{aligned}
$$

This is same as the i^{th} entry in the product $(2 - 2\cos k\pi h)x_k$.

Therefore, $\boxed{\lambda_k = (2 - 2\cos k\pi h)}$ is the eigenvalue of the matrix A.

Step-3

Now let A be the same matrix of the order 3 by 3.

That is, $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

Therefore,

$$h = \frac{1}{n+1}$$
$$= \frac{1}{4}$$

Step-4

Consider the 3 eigenvalues of A.

$$\lambda_1 = 2 - 2\cos\frac{\pi}{4}$$
$$= 2 - 2\frac{1}{\sqrt{2}}$$
$$= 2 - \sqrt{2}$$

$$\lambda_2 = 2 - 2\cos\frac{2\pi}{4}$$
$$= 2 - 2\cos\frac{\pi}{2}$$
$$= 2$$

$$\lambda_3 = 2 - 2\cos\frac{3\pi}{4}$$
$$= 2 - 2\left(-\frac{1}{\sqrt{2}}\right)$$
$$= 2 + \sqrt{2}$$

Step-5

Thus, the three eigenvalues of A are $\boxed{2-\sqrt{2}}$, $\boxed{2}$, and $\boxed{2+\sqrt{2}}$.