

Step-1

Suppose a matrix A has s linearly independent eigenvectors, then it is similar to a matrix J that is in Jordan form, with s square blocks on the diagonal:

$$J = M^{-1}AM$$

$$= \begin{bmatrix} J_1 & & \\ & \ddots & \\ & & J_s \end{bmatrix}.$$

Each block has one eigenvalue, one eigenvector, and 1s just above the diagonal:

$$J_i = \begin{bmatrix} \lambda_i & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{bmatrix}$$

Step-2

Consider permutation matrix P with 1s along the cross-diagonal (lower left to upper right).

$$P = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \ddots & & \\ 1 & & & \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \ddots & & \\ 1 & & & \end{bmatrix}$$

Suppose we have a Jordan block J_i , then we get

$$P^{-1}J_iP = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \ddots & & \\ 1 & & & \end{bmatrix} \begin{bmatrix} \lambda & 1 & & \\ & \lambda & \ddots & \\ & & \ddots & 1 \\ & & & \lambda \end{bmatrix} \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \ddots & & \\ 1 & & & \end{bmatrix}$$

$$= \begin{bmatrix} \lambda & & & \\ 1 & \lambda & & \\ & \ddots & \ddots & \\ & & 1 & \lambda \end{bmatrix}$$

$$= J_i^T$$

Therefore, Jordan block J_i is similar to J_i^T .

Step-3

Consider the Jordan form, with A similar to a matrix J , then we have

$$A = P^{-1}JP$$

By taking transpose we have:

$$\begin{aligned}(A)^T &= (P^{-1}JP)^T \\ &= (P^{-1})^T (J)^T (P)^T \\ &= (P^T)^{-1} (J)^T (P)^T\end{aligned}$$

So, from the above form $A^T = J^T$.

Step-4

We know that Jordan block J is similar to J^T .

So,

$$\begin{aligned}A^T &= J^T \\ &= J\end{aligned}$$

We have already consider A similar to a matrix J , then we have

$$\begin{aligned}A^T &= J^T \\ &= J \\ &= A\end{aligned}$$

Therefore, every matrix A is similar to its transpose A^T .