Step-1

Given that $\mathbf{P}_0$ is the plane through $(0,0,0)$ parallel to the plane **P** where the equation for **P** is $x+y-2z=4$.

We have to find the equation for $\mathbf{P}_0$ and two vectors in $\mathbf{P}_0$ such that their sum is in $\mathbf{P}_0$.

Step-2

We have **P** is the plane equation such that $\vec{r}\cdot\vec{n}=\mathbf{P}$, where **P** is the perpendicular distance from origin to plane $\vec{n}=\vec{i}+\vec{j}-2\vec{k}$

Now $\mathbf{P}_0$ is the plane through $(0,0,0)$ parallel to the plane **P**

Therefore $\vec{r}\cdot\vec{n}=0$ where $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ $(x,y,z)\in\mathbf{P}_0$

Therefore, $\left(x\vec{i}+y\vec{j}+z\vec{k}\right)\cdot\left(\vec{i}+\vec{j}-2\vec{k}\right)=0$

$\Rightarrow x+y-2z=0$

Therefore, the equation of the plane $\mathbf{P}_0$ is $x+y-2z=0$

Step-3

Let $(x_1,y_1,z_1),(x_2,y_2,z_2)\in\mathbf{P}_0$

Now $(x_1,y_1,z_1)+(x_2,y_2,z_2)=(x_1+x_2,y_1+y_2,z_1+z_2)$

Already we have,

$$x_1+y_1-2z_1=0$$
$$x_2+y_2-2z_2=0$$

$$\Rightarrow x_1+y_1-2z_1+x_2+y_2-2z_2=0$$
$$\Rightarrow (x_1+x_2)+(y_1+y_2)-2(z_1+z_2)=0$$

Therefore, $(x_1+y_1,x_2+y_2,x_3+y_3)\in\mathbf{P}_0$

Step-4

Hence the sum of the vectors in $\mathbf{P}_0$ is also belongs to $\mathbf{P}_0$

Now $(1,1,1),(2,0,1) \in \mathbf{P}_0$ and then sum is

$(1,1,1)+(2,0,1)=(3,1,2) \in \mathbf{P}_0$

Also $k(x_1, y_1, z_1)=(kx_1, ky_1, kz_1) \in \mathbf{P}_0$, where $(x_1, y_1, z_1) \in \mathbf{P}_0$

Therefore, $\mathbf{P}_0$ is a vector space