Step-1

Thus, we get

$$Ax = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}x_1 + \ldots + a_{1n}x_n \\ a_{21}x_1 + \ldots + a_{2n}x_n \\ \vdots \\ a_{n1}x_1 + \ldots + a_{nn}x_n \end{bmatrix}$$

Thus, in the right hand side of Ax, we see an n by 1 matrix, whose each entry has n products.

Therefore, to carry out Ax, we have to do n^2 number of products.

Step-2

Now consider a circulant matrix $C = F\Lambda F^{-1}$.

From the Fast Fourier Transform, we know that to multiply x by F and F^{-1}, we need only $n \log n$ multiplications. Also, for multiplying by Î›, we need only additional n multiplications. The sum of all these number of multiplications is less than n2.

Therefore, to obtain Cx, it is easier to multiply first with F^{-1}, then Î› and then by F than multiplying directly by C.