Step-1

Let us consider the following game situation.

X holds up one hand or two, and so does *Y*.

Now, if they make the same decision, *Y* wins $10.

And if they make opposite decisions, *X* wins $10 for one hand and $70 for two:

Step-2

And the Payoff matrix is as follows

Payoff matrix (payment to *X*)

$$A=\begin{bmatrix}-10 & 70\\ 10 & -10\end{bmatrix}$$

Now the strategy used by both the plays must be a ***mixed strategy***, and the choice made by each player at every turn must be independent of the previous turn

Step-3

Now, in a mixed strategy, *X* can put up one hand with frequency x_1 and both the hands with frequency $x_2=1-x_1$

Thus, the strategy used by player *X* is as follows

$$-10x_1+70x_2=10x_1-10x_2$$
$$-10x_1+70(1-x_1)=10x_1-10(1-x_1)$$
$$100x_1=80$$
$$\boxed{x_1=\frac{4}{5}}$$

And

$$x_2=1-\frac{4}{5}$$
$$=\boxed{\frac{1}{5}}$$

Step-4

Now, in a mixed strategy, *Y* can put up one hand with frequency y_1 and both the hands with frequency $y_2=1-y_1$

Thus, the strategy used by player *Y* is as follows

$$-10y_1+10y_2=70y_1-10y_2$$

$$-10y_1+10(1-y_1)=70y_1-10(1-y_1)$$

$$100y_1=20$$

$$\boxed{y_1=\frac{1}{5}}$$

And

$$y_2=1-\frac{1}{5}$$

$$=\boxed{\frac{4}{5}}$$

Step-5

Let us calculate the average payoff

$$yAx=\begin{bmatrix}y_1 & y_2\end{bmatrix}A\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{5} & \frac{4}{5}\end{bmatrix}\begin{bmatrix}-10 & 70\\10 & -10\end{bmatrix}\begin{bmatrix}\frac{4}{5}\\\frac{1}{5}\end{bmatrix}$$

$$=\begin{bmatrix}-\frac{10}{5}+\frac{4(10)}{5} & \frac{70}{5}-\frac{4(10)}{5}\end{bmatrix}\begin{bmatrix}\frac{4}{5}\\\frac{1}{5}\end{bmatrix}$$

$$=\begin{bmatrix}6 & 6\end{bmatrix}\begin{bmatrix}\frac{4}{5}\\\frac{1}{5}\end{bmatrix}$$

$$yAx=\begin{bmatrix}6\end{bmatrix}$$

Average payoff $\boxed{yAx=6}$