Step-1

Let (x,y) be the two vectors. Write down the (2×2) matrix that does the following:

(a) Reverses the direction of each vector.

Let X_1 be the required vector. Consider the following multiplication of matrices:

$$Ax = X_1$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

Above matrix multiplication reverses the direction of the vector. Therefore, matrix that reverses the direction is:

$$\boxed{A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}$$

Step-2

(b) Project every vector onto the x_2 axis.

Let X_2 be the required vector. Consider the following multiplication of matrices:

$$Ax = X_2$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$$

Above matrix multiplication project every vector onto the x_2 axis. Therefore, matrix that projects it is:

$$\boxed{A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}$$

Step-3

(c) Turn every vector clockwise through 90°.

Let X_3 be the required vector. Consider the following multiplication of matrices:

$$Ax = X_3$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix}$$

Above matrix multiplication rotates every vector by $90°$. Therefore, matrix that rotates $90°$ is:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Step-4

(d) Reflect every vector through the $45°$ line $x_1 = x_2$.

$$A = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

Above matrix reflect every vector through the $45°$.