Step-1

Given

$$A=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix},\quad B=\begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}\text{ and }\quad C=\begin{bmatrix}2&2&2\\2&2&2\\2&2&2\end{bmatrix}$$

Now, $A=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}$

$$\begin{aligned}A-\lambda I&=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\\&=\begin{bmatrix}1&2&3\\0&4&5\\0&0&6\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}\\&=\begin{bmatrix}1-\lambda&2&3\\0&4-\lambda&5\\0&0&6-\lambda\end{bmatrix}\end{aligned}$$

Step-2

Then,

$$A-\lambda I=\begin{vmatrix}1-\lambda&2&3\\0&4-\lambda&5\\0&0&6-\lambda\end{vmatrix}$$

$$\begin{aligned}(1-\lambda)[(4-\lambda)(6-\lambda)]&=(1-\lambda)[24-4\lambda-6\lambda+\lambda^2]\\&=(1-\lambda)[\lambda^2-10\lambda+24]\\&=\lambda^2-10\lambda+24-\lambda^3+10\lambda^2-24\lambda\\&=-\lambda^3+11\lambda^2-34\lambda+24\end{aligned}$$

Step-3

$$A-\lambda I=0$$

Now, $\lambda^3-11\lambda^2+34\lambda-24=0$

$$\begin{array}{r|rrrr} 1 & 1 & -11 & 34 & -24 \\ & 0 & 1 & -10 & 24 \\ \hline 4 & 1 & -10 & 24 & 0 \\ & 0 & 4 & -24 & 0 \\ \hline & 1 & -6 & 0 & \end{array}$$

The eigenvalues are

$$(\lambda-1)(\lambda-4)(\lambda-6)=0$$

Therefore the eigenvalues of A are $\boxed{1,\ 4,\ \text{and } 6}$

Step-4

Now, consider $B=\begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}$

$$\begin{aligned} B-\lambda I &= \begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}-\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \\ &= \begin{bmatrix}0&0&1\\0&2&0\\3&0&0\end{bmatrix}-\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix} \\ &= \begin{bmatrix}-\lambda&0&1\\0&2-\lambda&0\\3&0&-\lambda\end{bmatrix} \end{aligned}$$

Step-5

Now find eigenvalues of B

$$|B-\lambda I|=\begin{vmatrix}-\lambda&0&1\\0&2-\lambda&0\\3&0&-\lambda\end{vmatrix}$$

This implies;

$= (-\lambda)(2-\lambda)(-\lambda) + 1(-(2-\lambda)3)$
$= \lambda^2(2-\lambda) - 6 + 3\lambda$
$= 2\lambda^2 - \lambda^3 - 6 + 3\lambda$
$= -\lambda^3 + 2\lambda^2 + 3\lambda - 6$

Step-6

Now,

$B - \lambda I = 0$

This implies;

$$-\lambda^3 + 2\lambda^2 + 3\lambda - 6 = 0$$

$$\begin{array}{c|cccc} 2 & 1 & -2 & -3 & 6 \\ & 0 & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$$(\lambda - 2)(\lambda^2 - 3) = 0$$

Step-7

The Eigen values are $2, \pm\sqrt{3}$

Therefore the eigenvalues of B are $\boxed{2, \pm\sqrt{3}}$

Step-8

Now, consider

$$C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

$$|C-\lambda I| = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 2 & 2 \\ 2 & 2-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{bmatrix}$$

Step-9

Now,

$$|C-\lambda I| = \begin{vmatrix} 2-\lambda & 2 & 2 \\ 2 & 2-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix}$$

This implies;

$$\begin{aligned}
&= (2-\lambda)[(2-\lambda)(2-\lambda)-4]-2[2(2-\lambda)-4]+2[4-2(2-\lambda)] \\
&= (2-\lambda)[4-2\lambda-2\lambda+\lambda^2-4]-2[4-2\lambda-4]+2[4-4+2\lambda] \\
&= (2-\lambda)[\lambda^2-4\lambda]-2[-2\lambda]+2[2\lambda] \\
&= 2\lambda^2-8\lambda-\lambda^3+4\lambda^2+4\lambda+4\lambda
\end{aligned}$$

Thus,

$$|C-\lambda I| = -\lambda^3+6\lambda^2$$

Step-10

Then,

$$\begin{aligned}
\lambda^3-6\lambda^2 &= 0 \\
\lambda(\lambda^2-6\lambda) &= 0 \\
\lambda &= 0 \\
\lambda^2-6\lambda &= 0
\end{aligned}$$

$$\begin{aligned}
\lambda(\lambda-6) &= 0 \\
\lambda &= 0, 6
\end{aligned}$$

The Eigen values are 0, 0, and 6

Therefore the eigenvalues of C are $\boxed{0,\ 0,\ \text{and } 6}$