Step-1

Consider the following matrix:

$$A = \begin{bmatrix} a & -0.8 \\ 0.8 & 0.2 \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(A - \lambda I) = 0$. This gives,

$$0 = \begin{vmatrix} a - \lambda & -0.8 \\ 0.8 & 0.2 - \lambda \end{vmatrix}$$
$$= (a - \lambda)(0.2 - \lambda) + 0.64$$
$$= \lambda^2 - (0.2 + a)\lambda + 0.2a + 0.64$$

Step-2

Consider the equation, $\lambda^2 - (0.2 + a)\lambda + 0.2a + 0.64 = 0$.

Thus,

$$\lambda = \frac{(0.2+a) \pm \sqrt{(0.2+a)^2 - 4(0.2a+0.64)}}{2}$$

$$= \frac{(0.2+a) \pm \sqrt{(0.04+0.4a+a^2) - (0.8a+2.56)}}{2}$$

$$= \frac{(0.2+a) \pm \sqrt{a^2 - 0.4a - 2.52}}{2}$$

Step-3

Since $|\lambda_i| < 1$, we get the following:

$$-2 < (0.2+a) \pm \sqrt{a^2 - 0.4a - 2.52} < 2$$

$$-2.2 < a \pm \sqrt{a^2 - 0.4a - 2.52} < 1.98$$

$$-2.2 - a < \pm \sqrt{a^2 - 0.4a - 2.52} < 1.98 - a$$

$$(-2.2-a)^2 < a^2 - 0.4a - 2.52 < (1.98-a)^2$$

Simplifying, we get

$$a^2 + 4.4a + 4.84 < a^2 - 0.4a - 2.52 < a^2 - 3.96a + 3.9204$$

 $4.4a + 4.84 < -0.4a - 2.52 < -3.96a + 3.9204$
 $4.4a + 7.36 < -0.4a < -3.96a + 6.4404$
 $-11a - 18.4 > a > 9.9a - 16.101$

Step-4

Consider the two inequalities separately.

$$-11a-18.4 > a$$

$$-18.4 > 12a$$

$$\frac{-18.4}{12} > a$$

$$-1.533 > a$$

$$a > 9.9a-16.101$$

$$-8.9a > -16.101$$

$$a < \frac{16.101}{8.9}$$

$$a < 1.809$$

Step-5

Thus, if matrix A has to be stable, a < -1.533.

If the matrix A has to be neutrally stable, some of its eigenvalues satisfy $|\lambda_i| = 1$ and other eigenvalues satisfy $|\lambda_i| < 1$.

This gives, a = 1.809 or a = -1.533.

Consider the following matrix:

$$B = \begin{bmatrix} b & 0.8 \\ 0 & 0.2 \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(B - \lambda I) = 0$. This gives,

$$0 = \begin{vmatrix} b - \lambda & 0.8 \\ 0 & 0.2 - \lambda \end{vmatrix}$$
$$= (b - \lambda)(0.2 - \lambda) + 0$$
$$= \lambda^2 - (0.2 + b)\lambda + 0.2b$$

Step-6

Consider the equation $\lambda^2 - (0.2 + b)\lambda + 0.2b = 0$.

Thus,

$$\lambda = \frac{(0.2+b) \pm \sqrt{(0.2+b)^2 - 0.8b}}{2}$$

$$= \frac{(0.2+b) \pm \sqrt{b^2 + 0.4b + 0.04 - 0.8b}}{2}$$

$$= \frac{(0.2+b) \pm \sqrt{b^2 - 0.4b + 0.04}}{2}$$

$$\lambda = \frac{(0.2+b) \pm \sqrt{(b-0.2)^2}}{2}$$

$$= \frac{(0.2+b) \pm (b-0.2)}{2}$$

Step-7

= b or 0.2

For the matrix *B* to be stable, we want $|\lambda_i| < 1$. Therefore, [b < 1].

For the matrix *B* to be neutrally stable, we want some eigenvalue \hat{I} such that $|\hat{\lambda}_i| = 1$ and other \hat{I} such that $|\hat{\lambda}_i| < 1$. Therefore, [b=1].

Step-8

Consider the following matrix:

$$C = \begin{bmatrix} c & 0.8 \\ 0.2 & c \end{bmatrix}$$

To obtain its eigenvalues, solve $\det(C - \lambda I) = 0$. This gives,

$$0 = \begin{vmatrix} c - \lambda & 0.8 \\ 0.2 & c - \lambda \end{vmatrix}$$
$$= (c - \lambda)^2 - 0.16$$
$$= \lambda^2 - 2c\lambda + (c^2 - 0.16)$$

Step-9

Consider the equation $\lambda^2 - 2c\lambda + (c^2 - 0.16) = 0$

Thus,

$$\lambda = \frac{2c \pm \sqrt{4c^2 - 4(c^2 - 0.16)}}{2}$$
$$= \frac{2c \pm \sqrt{0.64}}{2}$$
$$= \frac{2c \pm 0.8}{2}$$
$$= c \pm 0.4$$

Step-10

For the matrix *C* to be stable, we want $|\lambda_i| < 1$. Therefore, $\boxed{-0.6 < c < 0.6}$.

For the matrix *C* to be neutrally stable, we want some eigenvalue \hat{I} such that $|\lambda_i| = 1$ and other \hat{I} such that $|\lambda_i| < 1$. Therefore, $\boxed{-0.6 \le c \le 0.6}$.