Step-1

If we consider a Pascal matrix, its determinant is always 1.

Consider

$$\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1$$
$$= 1$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{vmatrix} = 1\begin{vmatrix} 2 & 3 \\ 3 & 6 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$
$$= 1(12-9) - 1(6-3) + 1(3-2)$$
$$= 3-3+1$$
$$= 1$$

Step-2

In any $n \times n$ Pascal matrix, the largest entry is a_{nn}. If we remove the n^{th} column and n^{th} row from an $n \times n$ Pascal matrix, the remaining matrix is also Pascal matrix of order $n-1 \times n-1$. Therefore, the determinant of the matrix, obtained by removing the n^{th} row and column from the $n \times n$ Pascal matrix is also 1.

Let the element in the last row be denoted by a_{n1}, a_{n2}, ..., a_{nn}. The cofactors of these elements be denoted by A_{n1}, A_{n2}, ..., A_{nn}.

Note that $\det(A_{nn}) = 1$.

Step-3

If A is the Pascal matrix of order n by n, we get

$$\begin{aligned} 1 &= \det(A) \\ &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + a_{nn}A_{nn} \\ &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1 + 1)A_{nn} \\ &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + A_{nn} \end{aligned}$$

But then $A_{nn} = 1$. Therefore,

$$\begin{aligned} 1 &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + A_{nn} \\ &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} + 1 \\ 0 &= a_{n1}A_{n1} + a_{n2}A_{n2} + \ldots + (a_{nn} - 1)A_{nn} \end{aligned}$$

Step-4

Thus, if we make decrease the largest entry of the Pascal matrix by 1, the resultant matrix always has the determinant zero.