Step-1

Consider the following permutation matrix:

$$P=\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$$

Then

$$\det(P)=\begin{vmatrix}0&0&1\\0&1&0\\1&0&0\end{vmatrix}$$
$$=-1$$

Since $\det(P)\neq 0$, so the matrix P is non-singular.

Thus, the matrix P has inverse.

The object is to find the inverse of the given matrix using Gauss-Jordan method.

Step-2

Using the Gauss-Jordan Method to Find P^{-1}.

$$\begin{bmatrix}P & I\end{bmatrix}=\begin{bmatrix}0&0&1&1&0&0\\0&1&0&0&1&0\\1&0&0&0&0&1\end{bmatrix}$$

$$\sim\begin{bmatrix}1&0&0&0&0&1\\0&1&0&0&1&0\\0&0&1&1&0&0\end{bmatrix}\begin{matrix}\\ \\ R_3\leftrightarrow R_1\end{matrix}$$

$$=\begin{bmatrix}I & P^{-1}\end{bmatrix}$$

Therefore, the inverse of the matrix P is,

$$P^{-1}=\boxed{\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}}.$$

Step-3

Consider the following permutation matrix:

$$P=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

Then

$$\begin{aligned}\det(P)&=\begin{vmatrix}0&0&1\\1&0&0\\0&1&0\end{vmatrix}\\&=1\end{aligned}$$

Since $\det(P)\neq 0,$ so the matrix P is non-singular.

Thus, the matrix P has inverse.

The object is to find the inverse of the given matrix using Gauss-Jordan method.

Step-4

Using the Gauss-Jordan Method to Find P^{-1}.

$$\begin{aligned}\begin{bmatrix}P & I\end{bmatrix}&=\begin{bmatrix}0&0&1&1&0&0\\1&0&0&0&1&0\\0&1&0&0&0&1\end{bmatrix}\\&\sim\begin{bmatrix}1&0&0&0&1&0\\0&0&1&1&0&0\\0&1&0&0&0&1\end{bmatrix}\ R_2\leftrightarrow R_1\\&\sim\begin{bmatrix}1&0&0&0&1&0\\0&1&0&0&0&1\\0&0&1&1&0&0\end{bmatrix}\ R_3\leftrightarrow R_2\\&=\begin{bmatrix}I & P^{-1}\end{bmatrix}\end{aligned}$$

Therefore, the inverse of the matrix P is,

$$P^{-1}=\boxed{\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}}.$$

Step-5

(b)

The transpose of $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ is,

$$P^T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{or} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Since $P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, so $P^{-1} = P^T$.

Therefore, P^{-1} is always the same as P^T.

Step-6

To show that the 1s are in the right places to give $PP^T = I$.

Consider the arbitrary matrix is,

$$P = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

Then

$$PP^T = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+c^2 & ad+be+cf & ag+bh+ci \\ ad+be+cf & d^2+e^2+f^2 & dg+eh+fi \\ ag+bh+ci & dg+eh+fi & g^2+h^2+i^2 \end{bmatrix}$$ â€¦â€¦ (1)

Step-7

From this you can see if you have a permutation matrix that there can only be one 1s in each row and column.

So, a,b,c cannot be equal to zero at the same time.

The same is true for d,e,f and $g,h,i.$

Thus from (1), see that the entries along the diagonal is always $a^2+b^2+c^2=1,\ d^2+e^2+f^2=1,$ and $g^2+h^2+i^2=1.$

Now looking at the other entries (not along the diagonal) they are always zero.

Adopting this line of thought we can conclude that every entry, save the entries along the diagonal, is zero.

Therefore, every permutation matrix multiplied with its transposed yields a matrix in which only the entries along the diagonal are 1, which implies that this matrix is the identity matrix.

Hence, the 1s are in the right places to give $PP^T=I.$