

Step-1

Let the matrix be $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 9 \end{bmatrix}$ and its rank is 1.

In order to find the null space basis, we need to set it as homogenous equation.

$$Ax = 0$$

Apply $R_2 \rightarrow R_2 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 + 2x_2 + 3x_3 = 0$$

$$\Rightarrow x_1 = -2x_2 - 3x_3$$

$$\begin{aligned} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} -2x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix} \\ &= x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \end{aligned}$$

Therefore, $(1, 0)$ is in column space.

The basis for $C(A)$ is $\{(1, 0)\}$.

The basis for null space is $\{(-2, 1, 0), (-3, 0, 1)\}$.

Step-2

The row space contains all multiples of $(1, 2, 3)$. Therefore, the basis for row space is $\{(1, 2, 3)\}$.

The left null space basis is,

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 6 \\ 3 & 9 \end{bmatrix}$$

The transpose of the matrix is

The homogeneous equations using this.

$$A^T x = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 6 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Apply $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 + 3x_2 = 0$$

$$\Rightarrow x_1 = -3x_2$$

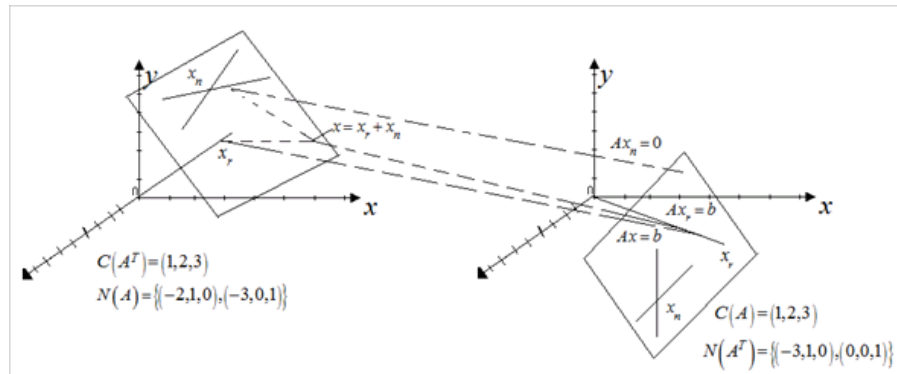
$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3x_2 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The basis for null space is $\{(-3, 1, 0), (0, 0, 1)\}$.

Step-3

The figure is as shown below.



Therefore, no two vectors are orthogonal.