

Step-1

Given $a = (2, -2, 1)$. We have to find the length of a and write two independent vectors that are perpendicular to a .

Step-2

$$\begin{aligned}\|a\|^2 &= a^T a \\ &= (2, -2, 1) \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \\ &= 4 + 4 + 1 = 9\end{aligned}$$

The length of a is $\|a\| = \sqrt{9}$

$$= \boxed{3}$$

Step-3

Let $\alpha = (x, y, z)$ is perpendicular to a

Therefore $a^T \alpha = 0$

$$\begin{aligned}\Rightarrow (2, -2, 1) \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= 0 \\ \Rightarrow 2x - 2y + z &= 0\end{aligned}$$

Putting $x = k_1, y = k_2$

$$\Rightarrow z = -2k_1 + 2k_2$$

Step-4

$$\text{Therefore } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} k_1 \\ k_2 \\ -2k_1 + 2k_2 \end{pmatrix}$$

$$= k_1 \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + k_2 \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

Hence $(1, 0, -2), (0, 1, 2)$ are two independent vectors perpendicular to α .