Step-1

Given the quadratic form $f(x,y) = x^2 + 4xy + 3y^2$.

We need to show that f does not have a minimum at $(0,0)$ even though it has positive coefficients, need to write f as a difference of squares and find a point (x,y) where f is negative.

Step-2

The matrix of the quadratic is,

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

Comparing above two matrices,

So, $a = 1,\ b = 2,$ and $c = 3.$

Now,

$ac - b^2 = -1 < 0.$

Therefore, $\boxed{f(x,y) \text{ is not positive definite,}}$ so f $\boxed{\text{does not have a minimum at } (0,0)}$ even though it has positive coefficients,

Step-3

$f(x,y) = x^2 + 4xy + 3y^2$

$= x^2 + 2.x.(2y) + 4y^2 - y^2$

$= (x+2y)^2 - y^2$

Therefore, $\boxed{f(x,y) \text{ is a difference of squares.}}$

Step-4

For $(x,y) = (2,-1),$

$f(x,y) = f(2,-1)$

$= 0 - 1$

$= -1 < 0$

Thus at $(2,-1)$, $f(x,y)$ is negative.

Therefore, the point is $\boxed{(2,-1)}$. $f(x,y)$ is negative.