Step-1

Given that $Ax = \lambda x$

a) Multiply with A on both sides.

$$A(Ax) = A(\lambda x)$$

But λ is a scalar and so, it commutes with the matrix A.

So, the above equation becomes as,

$$\begin{aligned} A(Ax) &= \lambda(Ax) \\ A^2x &= \lambda(\lambda x) \quad \text{Since } Ax = \lambda x \\ &= \lambda^2 x \end{aligned}$$

Hence, λ^2 is the Eigen value of A^2.

Step-2

b) Multiply with A^{-1} on both sides.

$$\begin{aligned} Ax &= \lambda x \\ A^{-1}(Ax) &= A^{-1}(\lambda x) \\ (A^{-1}A)x &= \lambda(A^{-1}x) \quad \text{Since commutativity of } \lambda \end{aligned}$$

$$\begin{aligned} Ix &= \lambda(A^{-1}x) \\ x &= \lambda(A^{-1}x) \\ A^{-1}x &= \frac{1}{\lambda}x \end{aligned}$$

Therefore, $\frac{1}{\lambda}$ is the Eigen value of A^{-1}.

Step-3

c) From the data, $Ax = \lambda x$.

When x is a matrix of column of size $n \times 1$, then it can be written as $x = Ix$.

where I is the identity matrix of size $n \times n$.

So, add $x = Ix$ on the respective sides of the above equation.

$$
\begin{aligned}
Ax &= \lambda x \\
Ax + Ix &= \lambda x + Ix \\
Ax + Ix &= \lambda x + x \\
(A + I)x &= (\lambda + 1)x
\end{aligned}
$$

Therefore $\lambda + 1$ is the Eigen value of $A + I$.