

## Step-1

We have to find the examples of 2 by 2 matrices such that the given condition is obeyed by trial and error method.

## Step-2

(a)  $A^2 = -I$ ,  $A$  having only real values.

$$\text{Let } A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{aligned} A^2 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \\ &= -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= -I \end{aligned}$$

## Step-3

(b)  $B^2 = 0$ , although  $B \neq 0$

$$\text{Let } B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\begin{aligned} B^2 &= \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ &= \text{null matrix} \end{aligned}$$

## Step-4

(c)  $CD = -DC$ , not allowing the case  $CD = 0$

$$\text{Let } C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } D = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$CD = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$DC = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ = - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ = -CD$$

## Step-5

(d)  $EF = 0$ , although  $E$  or  $F$  are zero

$$\text{Let } E = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \text{ and } F = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$EF = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \\ = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ = \text{null matrix.}$$