Step-1

The objective is to find the submatrix S from the pivot rows and pivot columns of each A.

The first matrix is $A=\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 4\end{bmatrix}$.

Subtract row 1 from row 2 to get the reduced matrix.

$\begin{bmatrix}1 & 2 & 3\\ 0 & 0 & 1\end{bmatrix}$.

From the reduced form, observe that the pivot columns are the first and third columns and the pivot rows are the first and second rows.

The rank of A is 2.

Thus, the invertible 2×2 submatrix of A is $\boxed{\begin{bmatrix}1 & 3\\ 1 & 4\end{bmatrix}}$.

Step-2

The second matrix is $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\end{bmatrix}$.

Subtract 2 times of row 1 from row 2 to get the reduced matrix.

$\begin{bmatrix}1 & 2 & 3\\ 0 & 0 & 0\end{bmatrix}$.

From the reduced form, observe that the first column is the only pivot column and the first row is the only pivot row.

Thus, the rank of A is 1 and the invertible 1×1 submatrix consists of the first column of first row.

That is, $\boxed{[1]}$.

Step-3

The third matrix is $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$.

The pivot columns of A are the second and third columns and the pivot rows of A are the first and third rows.

Thus, the rank of A is 2.

The invertible 2×2 submatrix consists of the second and third columns of the first and third rows.

That is, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.