Step-1

Factorization of A into triangular times symmetric:

Let A be factorized into LDU.

$$A = LDU$$

Then matrix A can be written as $L(U^T)^{-1}$ times $U^T DU$. Here, $L(U^T)^{-1}$ is a matrix with diagonal all 1â€™s. Explain why $L(U^T)^{-1}$ is triangular and why $U^T DU$ is symmetric.

Step-2

Recall that if matrix A is factored into LDU with no row exchanges then matrix U is exactly L^T.

Now consider the product of lower triangular matrix and inverse transpose of upper triangular matrix:

$$\begin{aligned} L(U^T)^{-1} &= L\left((L^T)^T\right)^{-1} \\ &= L(L)^{-1} \\ &= I \end{aligned}$$

In other way it can also be said that product of two triangular matrices results into a triangular matrix. As here L and U are both triangular matrices.

Therefore, $L(U^T)^{-1}$ is triangular.

Step-3

Recall that $A = A^T$ then matrix A is symmetric. Now, do the following calculations:

$$\begin{aligned} (U^T DU)^T &= U^T D^T (U^T)^T \\ &= U^T D^T U \\ &= U^T DU \end{aligned}$$

Here, $D = D^T$ is taken as it is a pivot matrix. Therefore, above calculations shows that $U^T DU$ is symmetric.