Step-1

Yes, there exist a real 2×2 matrix other than I satisfies the given conditions.

Let us take $A=\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}$ is a real 2×2 matrix.

$$\begin{aligned}A^2 &= \begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}\\ &= \begin{bmatrix}0 & -1\\ 1 & -1\end{bmatrix}\end{aligned}$$

$$\begin{aligned}A^3 &= \begin{bmatrix}0 & -1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & 0\end{bmatrix}\\ &= \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = I\end{aligned}$$

Therefore $A^3 = I$

To find the eigen values let us take

$$|A-\lambda I| = 0$$

$$\begin{aligned}&\Rightarrow \begin{vmatrix}-1-\lambda & 1\\ -1 & -\lambda\end{vmatrix} = 0\\ &\Rightarrow \left(-(1-\lambda)\right)(-\lambda)+1=0\\ &\Rightarrow \lambda+\lambda^2+1=0\\ &\Rightarrow \lambda^2+\lambda+1=0\\ &\Rightarrow \lambda = \frac{-1\pm\sqrt{1-4}}{2}\\ &\quad = \frac{-1\pm i\sqrt{3}}{2}\end{aligned}$$

These are cube root of unity therefore

$$\lambda^3 = 1$$

$$\begin{aligned}\lambda_1 &= \frac{-1+i\sqrt{3}}{2}, \lambda_2 = \frac{-1-i\sqrt{3}}{2}\\ &= e^{\frac{2\pi i}{3}}, e^{\frac{-2\pi i}{3}}\end{aligned}$$

Step-2

The trace of $A = \lambda_1 + \lambda_2$

$$
\begin{aligned}
&= e^{\frac{2\pi i}{3}} + e^{\frac{-2\pi i}{3}} \\
&= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) + \left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right) \\
&= 2\cos\frac{2\pi}{3} \\
&= 2\left(\frac{-1}{2}\right) \\
&= -1
\end{aligned}
$$

The determinant of $A = \lambda_1\lambda_2$

$$
\begin{aligned}
&= e^{\frac{2\pi i}{3}} . e^{\frac{-2\pi i}{3}} \\
&= e^{0} \\
&= \boxed{1}
\end{aligned}
$$