Step-1

From the relation $A = R^{\mathrm{T}} R$, we can write down the following:

$$\begin{aligned}\det(A) &= \det\left(R^{\mathrm{T}} R\right)\\ &= \det\left(R^{\mathrm{T}}\right)\det(R)\\ &= \det(R)\det(R)\\ &= \left(\det(R)\right)^2\end{aligned}$$

Step-2

Now $\left(\det(R)\right)^2$ is equal to the square of the R parallelepiped.

Note that, a_{jj} is equal to the product of the j^{th} row of R^{T} and the j^{th} column of R. Therefore, a_{jj} is equal to the length squared of the j^{th} column of R.

It is obvious that the volume of the R parallelepiped cannot be greater than the product of the length squared columns of R.

Step-3

Therefore, volume of the R parallelepiped is less than or equal to $a_{11}a_{22}\cdots a_{nn}$.

Therefore, $\boxed{\det A \le a_{11}a_{22}\cdots a_{nn}}$.