Step-1

(a) Consider the set of rank 1 matrices. Let us call this set *A*. If this set *A* were a subspace of the vector space of 2 by 2 matrices, then *A* should have zero matrix.

We have $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

This matrix has rank 0 and therefore, the zero matrix does not belong to *A*.

Therefore, the set of rank 1 matrices is $\boxed{\text{not}}$ a subspace of the vector space of 2 by 2 matrices.

Step-2

(b) The two permutation matrices are $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Consider $\alpha \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \beta \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$.

Thus, the permutation matrices will span the subspace, whose each element has same entry along the diagonal and another same entry along the anti-diagonal.

Step-3

(c) Consider the set of positive matrices. Let the set be denoted by *B*. That is, in *B*, each entry in any matrix is positive.

It is clear that $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \in B$ and $\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} \in B$.

Now observe that $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

Step-4

Therefore, the span of *B* contains the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Similarly, it can be shown that the span of *B* contains the matrices $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Therefore, each matrix in the vector space of 2 by 2 matrices lies in the span of *B*.

Therefore, the $\boxed{\text{entire vector space}}$ is spanned by the set of positive matrices.

Step-5

(d) Consider the set of invertible matrices. Let the set be denoted by *C*. That is, in *C*, each matrix is invertible.

It is clear that $\begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix} \in C$ and $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \in C$.

Now observe that $\begin{bmatrix} 2 & 2 \\ 2 & 1 \end{bmatrix} + (-1)\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

Step-6

Therefore, the span of C contains the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Similarly, it can be shown that the span of C contains the matrices $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Therefore, each matrix in the vector space of 2 by 2 matrices lies in the span of C.

Therefore, the $\boxed{\text{entire vector space}}$ is spanned by the set of invertible matrices.