Step-1

a) The system of equations is

$$\begin{aligned} ax+by&=1\\ cx+dy&=0 \end{aligned}$$

Cramerâ€™s rule:

The j Th component of $x=A^{-1}b$ is the ratio

$x_j=\dfrac{\det B_j}{\det A}$, where $B_j=\begin{bmatrix} a_{11} & a_{12} & b_1 & a_{1n}\\ . & . & . & .\\ . & . & . & .\\ . & . & . & .\\ a_{n1} & a_{n2} & b_n & a_{nn} \end{bmatrix}$ has b in column j .

Step-2

We need to solve the given system by cramerâ€™s rule

$$A=\begin{bmatrix} a & b\\ c & d \end{bmatrix} \quad x=\begin{bmatrix} x\\ y \end{bmatrix} \quad b=\begin{bmatrix} 1\\ 0 \end{bmatrix}$$

$$A_1=\begin{bmatrix} 1 & b\\ 0 & d \end{bmatrix} \quad A_2=\begin{bmatrix} a & 1\\ c & 0 \end{bmatrix}$$

Now

$$\begin{aligned} \det(A)&=\begin{vmatrix} a & b\\ c & d \end{vmatrix}\\ &=ad-bc \end{aligned}$$

Step-3

And

$$\begin{aligned} \det(A_1)&=\begin{vmatrix} 1 & b\\ 0 & d \end{vmatrix}\\ &=d \end{aligned}$$

$$\begin{aligned} \det(A_2)&=\begin{vmatrix} a & 1\\ c & 0 \end{vmatrix}\\ &=-c \end{aligned}$$

Step-4

Thus, by crammers rule, we have

$$x = \frac{\det(A_1)}{\det(A)}$$
$$= \frac{\begin{vmatrix} 1 & b \\ 0 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$
$$= \frac{d}{ad - bc}$$

Step-5

And

$$y = \frac{\det(A_2)}{\det(A)}$$
$$= \frac{\begin{vmatrix} a & 1 \\ c & 0 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$
$$= \frac{-c}{ad - bc}$$

Step-6

Thus, the solution for the given systems

$$\boxed{\begin{aligned} x &= \frac{d}{ad-bc} \\ y &= \frac{-c}{ad-bc} \end{aligned}}$$

Step-7

b) The given system is

$$\begin{aligned} x + 4y - z &= 1 \\ x + y + z &= 0 \\ 2x \qquad + 3z &= 0 \end{aligned}$$

We need to solve the given system by cramers rule

$$A-\begin{bmatrix}1 & 4 & -1\\1 & 1 & 1\\2 & 0 & 3\end{bmatrix} X=\begin{bmatrix}x\\y\\z\end{bmatrix} b=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

Step-8

Replacing the first, second and third columns of A with b we get the matrices A_1, A_2 and A_3

$$A_1=\begin{bmatrix}1 & 4 & -1\\0 & 1 & 1\\0 & 0 & 3\end{bmatrix}, A_2=\begin{bmatrix}1 & 1 & -1\\1 & 0 & 1\\2 & 0 & 3\end{bmatrix}$$

$$A_3=\begin{bmatrix}1 & 4 & 1\\1 & 1 & 0\\2 & 0 & 0\end{bmatrix}$$

Now

$$\det(A)=|A|=\begin{vmatrix}1 & 4 & -1\\1 & 1 & 1\\2 & 0 & 3\end{vmatrix}$$

$$=\begin{vmatrix}1 & 1\\0 & 3\end{vmatrix}-4\begin{vmatrix}1 & 1\\2 & 3\end{vmatrix}-\begin{vmatrix}1 & 1\\2 & 0\end{vmatrix}$$

$$=(3)-4(1)-(-2)$$

$$=1$$

Step-9

Now

$$\det(A_1)=|A_1|=\begin{vmatrix}1 & 4 & -1\\0 & 1 & 1\\0 & 0 & 3\end{vmatrix}$$

$$=\begin{vmatrix}1 & 1\\0 & 3\end{vmatrix}-4\begin{vmatrix}0 & 1\\0 & 3\end{vmatrix}-\begin{vmatrix}0 & 1\\0 & 0\end{vmatrix}$$

$= (3) - 4(0) - (0)$
$= 3$

Step-10

And

$$\det(A_2) = |A_2| = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 0 & 3 \end{vmatrix}$$

$= (0) - (1) - (0)$
$= -1$

$$\det(A_3) = |A_3| = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix}$$
$$= 1(0) - 4(0) + 1(-2)$$
$$= -2$$

Step-11

Thus, by cramers rule we have

$$x = \frac{\det(A_1)}{\det(A)}$$

$$= \frac{\begin{vmatrix} 1 & 4 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{vmatrix}}$$

$$= \frac{3}{1}$$

$= 3$

Step-12

And

$$y=\frac{\det(A_2)}{\det(A)}=\frac{\begin{vmatrix}1&1&-1\\1&0&1\\2&0&3\end{vmatrix}}{\begin{vmatrix}1&4&-1\\1&1&1\\2&0&3\end{vmatrix}}$$

$$=\frac{-1}{1}$$
$$=-1$$

Step-13

And

$$z=\frac{\det(A_3)}{\det(A)}=\frac{\begin{vmatrix}1&4&1\\1&1&0\\2&0&0\end{vmatrix}}{\begin{vmatrix}1&4&-1\\1&1&1\\2&0&3\end{vmatrix}}$$

$$=\frac{-2}{1}$$
$$=-2$$

Thus, the solution for the given system is

$$\boxed{x=3,\ y=-1,\ z=-2}$$