Step-1

(a)

Consider the statement,

â€œThe vectors b that are not in the column space $C(A)$ form a subspace.â€

The objective is to determine whether the statement true or false (without a counter example if false).

The given statement is **false**.

Notice that, $C(A)$ is a vector space that contains zero vectors

The vectors b that are not in the column space do not contain zero vector.

Vectors b that not contain zero vector do not forms a vector space.

Step-2

(b)

Consider the statement,

â€œIf $C(A)$ contains only the zero vector, then A is the zero matrixâ€

The objective is to determine whether the statement true or false (without a counter example if false).

The given statement is **true**.

Column space of A consists of all linear combinations of the columns of A.

In particular, each column of A is an element of $C(A)$.

Hence, if $C(A)$ contains only the zero vector, then each column of A must be the zero vector, that means that A is the zero matrix.

Step-3

(c)

Consider the statement,

â€œThe column space of $2A$ equals the column space of Aâ€

The objective is to determine whether the statement true or false (without a counter example if false).

The given statement is **true**.

Suppose that, b is in the column space of A.

That means there exists some x such that $Ax = b$. Then

$$\begin{aligned}(2A)\left(\frac{1}{2}x\right) &= Ax \\ &= b\end{aligned}$$

So, b is in the column space of $2A$.

Hence, column space of A is contained in the column space of $2A$

Step-4

(d)

Consider the statement,

â€œThe column space of $A - I$ equals the column space of A â€

The objective is to determine whether the statement true or false (without a counter example if false).

The given statement is **false**.

If $A = I$ and A is an $n \times n$ matrix the column space of $A(= I)$ is $\mathbf{R}^n$

But the column space of $A - I =$ zero matrix contains only the zero vector.