Step-1

a) We first find the area of the parallelogram.

Suppose $OP = (x_1, y_1),\ OQ = (x_2, y_2)$

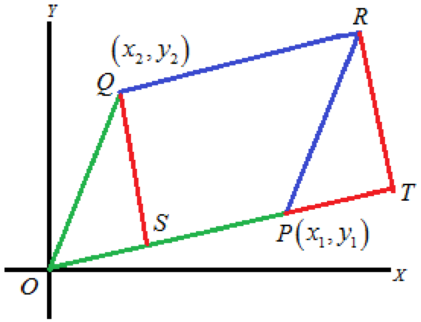
Extend the line *PR* such that it is parallel to *OQ* and is of length *OQ*.

Join *QR* to complete the parallelogram.

Suppose *S* is the point on *OP* such that *QS* is perpendicular to *OP*.

So, *OQS* form a right triangle.

From this, *QS* is the height and *OS* is the base.



Step-2

Then the projection of *OQ* on *OP* is $OS = \dfrac{OQ \cdot OP}{|OP|} = \dfrac{(x_2, y_2)\cdot(x_1, y_1)}{\sqrt{x_1^2 + y_1^2}}$

$$= \frac{x_1x_2 - y_1y_2}{\sqrt{x_1^2 + y_1^2}}$$

So, height of the right triangle is $QS = \sqrt{hypotenuse^2 - base^2}$

$$= \sqrt{\left(x_2^2 + x_1^2\right) - \left(\frac{x_1x_2 - y_1y_2}{\sqrt{x_1^2 + y_1^2}}\right)^2}$$

$$= \sqrt{\frac{\left(x_1^2x_2^2 + y_1^2y_2^2 + x_1^2y_2^2 + x_2^2y_1^2\right) - \left(x_1^2x_2^2 + y_1^2y_2^2 - 2x_1x_2y_1y_2\right)}{x_1^2 + y_1^2}}$$

$$= \sqrt{\frac{\left(x_1 y_2 - x_2 y_1\right)^2}{{x_1}^2 + {y_1}^2}}$$

$= \dfrac{x_1 y_2 - x_2 y_1}{|OP|}$ â€¦â€¦ (1)

Step-3

Further, we extend the vector *OP* to *PT* such that *QSTR* is the rectangle.

Obviously, the right triangle *OQS* is identical to triangle *PRT*.

Consequently, the base of rectangle = |*ST*| = |*OP*| â€¦â€¦ (2)

Area of the rectangle is base $\times$ height = (1) $\times$ (2) $= x_1 y_2 - x_2 y_1$

$$= \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

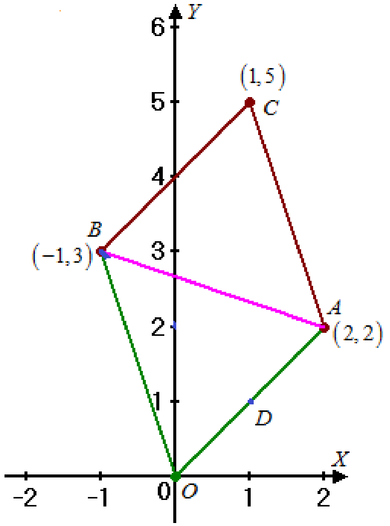
We easily see that by removing the triangle *PRT* from the rectangle *QSRT* and adding the triangle *OQS*, we get the parallelogram.

While the triangles *PRT* and *OQS* are identical, it follows that area of the rectangle *OSRT* is equal to the area of the parallelogram *OPQR*.

Thus, the area of the parallelogram $= \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$ with the adjoining sides $OP = \left(x_1, y_1\right)$, and $OQ = \left(x_2, y_2\right)$.

Step-4

Using the above discussion, we follow that *OAB* triangle is half *OABC* parallelogram.



Step-5

The area of parallelogram $OABC = \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix}$

So, area of the triangle *OAB* is $\frac{1}{2}\begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix}$

Step-6

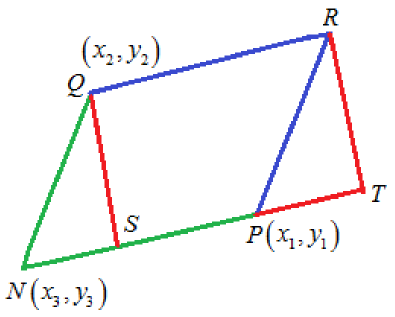
(b) Suppose $N(x_3, y_3), P(x_1, y_1), Q(x_2, y_2)$ are three vertices

Step-7

Then $NP = (x_1 - x_3, y_1 - y_3), NQ = (x_2 - x_3, y_2 - y_3)$ are the adjoining sides of the parallelogram and proceeding as in the above case, we get NS is the projection of *NQ* upon *NP* given by $NS = \frac{NQ \cdot NP}{|NP|}$

$= \frac{(x_1 - x_3, y_1 - y_3)\cdot(x_2 - x_3, y_2 - y_3)}{\sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}}$ â€¦â€¦ (3)

Step-8



Step-9

Consequently, the height of the right triangle NQS is QS

$$= \sqrt{\left\{(x_1 - x_3)^2 + (y_1 - y_3)^2\right\} - \left\{\frac{(x_1 - x_3, y_1 - y_3)\cdot(x_2 - x_3, y_2 - y_3)}{\sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}}\right\}^2}$$

$$= \frac{(x_2 - x_3)(y_1 - y_3) - (x_1 - x_3)(y_2 - y_3)}{|NP|}$$ â€¦â€¦ (4)

Proceeding to construct the rectangle in the above case, the base is *NP* and so, the area of the rectangle is $|NP| \times \frac{(x_2 - x_3)(y_1 - y_3) - (x_1 - x_3)(y_2 - y_3)}{|NP|}$

$$= (x_2 - x_3)(y_1 - y_3) - (x_1 - x_3)(y_2 - y_3)$$

Step-10

So, the area of the parallelogram NPQR is $(x_2 - x_3)(y_1 - y_3) - (x_1 - x_3)(y_2 - y_3)$

$$= x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Therefore, the area of the triangle *NPQ* $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

In our case, replacing NPQ by $C(1,-4), A(2,2), B(-1,3)$, the area of the required triangle $= \left|\frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -1 & 3 & 1 \\ 1 & -4 & 1 \end{vmatrix}\right|$