Step-1

Given that A is an n by n matrix and is invertible, that is, $AA^{-1} = I$. Then we have to find that the first column of A^{-1} is orthogonal to the spanned by which rows of A.

Step-2

Let $A = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix}, A^{-1} = \begin{bmatrix} C_1 & C_2 & \cdots & C_n \end{bmatrix}$

Where $R_1, R_2, ..., R_n$ are n rows and $C_1, C_2, ..., C_n$ are n columns of A

Step-3

Then

$$AA^{-1} = I$$

$$\Rightarrow \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} \begin{bmatrix} C_1 & C_2 & \cdots & C_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} R_1C_1 & R_1C_2 & R_1C_3 & \cdots & R_1C_n \\ R_2C_1 & R_2C_2 & R_2C_3 & \cdots & R_2C_n \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ R_nC_1 & R_nC_2 & R_nC_3 & \cdots & R_nC_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

By equating the first columns of the matrices, we get $R_1C_1 = 1, R_2C_1 = 0, R_3C_1 = 0, ..., R_nC_1 = 0$

From the second equation onwards, we have the column 1 of A^{-1} is orthogonal to the space spanned by 2^{nd}, 3^{rd}â€¦ nth rows of A.