Step-1

A linear transformation from V to W has an inverse from W to V when the range is all of W and the kernel contains only $V = 0$.

We have to verify which of these transformations are invertible.

Step-2

(a) Given transformation is $T(v_1, v_2) = (v_2, v_2)$

Now

$$\begin{aligned}\ker T &= \{(v_1, v_2) / T(v_1, v_2) = 0\} \\ &= \{(v_1, v_2) / (v_2, v_2) = 0\} \\ &= \{(v_1, v_2) / v_2 = 0\} \\ &= \{(v_1, 0) / v_1 \in \mathbf{R}\}\end{aligned}$$

Since $\text{kernel of } T \neq 0$

So T is not invertible.

Hence the given transformation is not invertible.

Step-3

(b) Given transformation is $T : \mathbf{R}^2 \to \mathbf{R}^3$ by $T(v_1, v_2) = (v_1, v_2, v_1 + v_2)$

Consider $(1,1,5) \in \mathbf{R}^3$

But there is no $(v_1, v_2) \in \mathbf{R}^2$ such that $T(v_1, v_2) = (1,1,5)$

Since $(v_1, v_2, v_1 + v_2) = (1,1,5)$

$v_1 = 1,\ v_2 = 1,\ v_1 + v_2 = 5$

Which is impossible

Thus range is not all of $\mathbf{R}^3$

Hence the given transformation T is not invertible.

Step-4

(c) Given transformation is $T(v_1, v_2) = v_1$

Now

$$\begin{aligned}\ker T &= \{(v_1, v_2) / T(v_1, v_2) = 0\} \\ &= \{(v_1, v_2) / v_2 = 0\} \\ &= \{(0, v_2) / v_2 \in \mathbf{R}\}\end{aligned}$$

$T(v_1, v_2) = 0$

$\Rightarrow v_1 = 0$

Since $\text{kernel of } T \neq 0$

So T is not invertible.

Hence the given transformation is not invertible.