Step-1

Given that $M = \text{all } 3\times3 \text{ matrices}$

We have to find that whether the following statements are true or false.

Step-2

a) The Skew-symmetric matrices in **M** $\left(\text{with } A^T = -A\right)$ form a subspace

Let S = set of all skew symmetric matrices of **M**.

Let $A, B \in S$

$$\begin{aligned}(A+B)^T &= A^T + B^T \\ &= (-A)+(-B) \\ &= -(A+B)\end{aligned}$$

Therefore $A+B \in S$

Step-3

Let $c \in R, A \in S$

$$\begin{aligned}(cA)^T &= c\left(A^T\right) \\ &= c(-A) \\ &= -(cA)\end{aligned}$$

Therefore $cA \in S$

Hence S is a subspace of **M**.

So the given statement is $\boxed{\text{true}}$

Step-4

b) The set of all unsymmetric matrices in **M** does not form a subspace. $\left(\text{with } A^T \neq A\right)$

Let $U = \text{set of all matrices } A \text{ with } A^T \neq A$

Let $B=\begin{bmatrix}0&2&0\\0&0&0\\0&0&0\end{bmatrix}, C=\begin{bmatrix}0&0&0\\2&0&0\\0&0&0\end{bmatrix}$

$B^T \neq B, C^T \neq C,$ threfore $B, C \in U$

Step-5

But $B+C=\begin{bmatrix}0&2&0\\2&0&0\\0&0&0\end{bmatrix}$

$$(B+C)^T=\begin{bmatrix}0&2&0\\2&0&0\\0&0&0\end{bmatrix}$$

B+C does not belongs to *U*.

Therefore *U* is not a subspace of ***M***.

Hence, the given statement is $\boxed{\text{false}}$.

Step-6

c) The set all matrices that have $(1,1,1)$ in their nullspace form a subspace.

Let *D* be the set all matrices that have $(1,1,1)$ in their nullspace.

Let $A=\begin{bmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{bmatrix} B=\begin{bmatrix}d_1&d_2&d_3\\e_1&e_2&e_3\\f_1&f_2&f_3\end{bmatrix}$ belongs to *D*

$$\Rightarrow A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}, B\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Step-7

Now $(A+B)\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}a_1+d_1&a_2+d_2&a_3+d_2\\b_1+e_1&b_2+e_2&b_3+c_3\\c_1+f_1&c_2+f_2&c_3+f_3\end{bmatrix}$

$$=\begin{bmatrix} a_1+d_1+a_2+d_2+a_3+d_2 \\ b_1+e_1+b_2+e_2+b_3+e_3 \\ c_1+f_1+c_2+f_2+c_3+f_3 \end{bmatrix}$$

$$=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left(\text{since } A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } B=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right)$$

Thus $A+B\in D$

$$(cA)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$=c\left(A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

$$=c\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore $cA\in D$

Here D is a subspace of **M**.

So the given statement is $\boxed{\text{true}}$.