Step-1

Given inequality is $\|A\|^2 = \max_{x \neq 0} \frac{\|Ax\|^2}{\|x\|^2}$

We have to prove the given inequality.

Step-2

We know that the norm of A is the number $\|A\|$ is $\|A\|^2 = \max_{x \neq 0} \frac{\|Ax\|^2}{\|x\|^2}$ â€¦â€¦ (1)

Let A, B are square matrices and x is any column matrix.

Then

$\|(A+B)x\| = \|Ax + Bx\|$ (Since matrix multiplication is associative)

$\leq \|Ax\| + \|Bx\|$ (By the triangular inequality)

Dividing throughout by $\|x\|$ and applying the maximum limit, we get

$$\max_{x \neq 0} \frac{\|(A+B)x\|}{\|x\|} \leq \max_{x \neq 0} \frac{\|Ax\|}{\|x\|} + \max_{x \neq 0} \frac{\|Bx\|}{\|x\|}$$

$\Rightarrow \|A+B\| \leq \|A\| + \|B\|$ (Since by (1))

Hence $\boxed{\|A+B\| \leq \|A\| + \|B\|}$