Step-1

Given system is $u + v + w = 2$

$u + 3v + 3w = 0$

$u + 3v + 5w = 2$

We have to find the triangular system after forward elimination and the solution.

Step-2

Given system can be written as

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 3 & 3 & 0 \\ 1 & 3 & 5 & 2 \end{pmatrix}$$

Subtract '1' time the row 1 from the row 2

Subtract '1' time the row 1 from the row 3

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & -2 \\ 0 & 2 & 4 & 0 \end{pmatrix}$$

Step-3

Subtract '1' times the row 2 from the row 3.

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{pmatrix}$$

which is upper triangular form.

Hence the triangular system after forward elimination

$$\begin{array}{r} u + v + w = 2 \\ 2v + 2w = -2 \\ 2w = 2 \end{array}$$

Step-4

From above upper triangular form, we have

$2w = 2$

$\Rightarrow w = 1$

$2v + 2w = -2$

$\Rightarrow 2v + 2(1) = -2$

$\Rightarrow v = -2$

$u + v + w = 2$

$\Rightarrow u - 2 + 1 = 2$

$\Rightarrow u = 3$

Therefore the solution is $\boxed{u = 3, v = -2, w = 1}$