Step-1

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Let $A=\begin{bmatrix} n & -1 & \dots & -1 \\ -1 & n & \dots & -1 \\ \vdots & \vdots & \vdots & -1 \\ -1 & -1 & -1 & n \end{bmatrix}$

Then $A^{-1}=\dfrac{1}{n+1}\begin{bmatrix} c & 1 & \dots & 1 \\ 1 & c & \dots & 1 \\ \vdots & \vdots & \vdots & 1 \\ 1 & 1 & 1 & c \end{bmatrix}$

We have to find the value of c.

Step-2

We know that $A^{-1}A=AA^{-1}=I$

So

$$\frac{1}{n+1}\begin{bmatrix} c & 1 & \dots & 1 \\ 1 & c & \dots & 1 \\ \vdots & \vdots & \vdots & 1 \\ 1 & 1 & 1 & c \end{bmatrix}\begin{bmatrix} n & -1 & \dots & -1 \\ -1 & n & \dots & -1 \\ \vdots & \vdots & \vdots & -1 \\ -1 & -1 & -1 & n \end{bmatrix}$$
$$=\begin{bmatrix} n & -1 & \dots & -1 \\ -1 & n & \dots & -1 \\ \vdots & \vdots & \vdots & -1 \\ -1 & -1 & -1 & n \end{bmatrix}\frac{1}{n+1}\begin{bmatrix} c & 1 & \dots & 1 \\ 1 & c & \dots & 1 \\ \vdots & \vdots & \vdots & 1 \\ 1 & 1 & 1 & c \end{bmatrix}=\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-3

By equating first row first element in the product of $A^{-1}A=I$, we get

$$\frac{1}{n+1}\left[cn-1-1-1....-1\right]=1$$
$$\frac{1}{n+1}\left[cn-(n-1)\right]=1$$

Since there are $(n-1)$ entries of (-1) in the first row after the first entry (cn).

So their sum is $(n-1)$.

Therefore,

$$
\begin{aligned}
\frac{1}{n+1}\left[cn-n+1\right]&=1\\
cn-n+1&=n+1\\
cn&=2n\\
\Rightarrow\ c&=2
\end{aligned}
$$

Hence the value of c in A^{-1} is $\boxed{c=2}$.