Step-1

Consider any two different vectors of the same length, $\|x\| = \|y\|$.

Let

$$v = x - y$$

A Householder matrix H is given by,

$$H = I - 2\frac{vv^T}{\|v\|^2}$$

On substitution, we get,

$$H = I - 2\frac{(x-y)(x-y)^T}{\|x-y\|^2}$$

By multiplying by x, we get,

$$\begin{aligned} Hx &= Ix - 2\frac{(x-y)(x-y)^T}{\|x-y\|^2}x \\ &= x - (x-y)\frac{2(x-y)^T}{(x-y)(x-y)^T}x \\ &= x - (x-y) \\ &= y \end{aligned}$$

Therefore, we get,

$$Hx = y$$

Therefore,

$$\begin{aligned} H(Hx) &= H(y) \\ x &= Hy \end{aligned}$$

Thus, $\boxed{Hx = y}$ and $\boxed{x = Hy}$.