

## Step-1

Let us consider the following vectors

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \text{and} \quad c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Therefore, the corresponding primal of the LPP is as follows

Minimize:  $x_1 + x_2$

Subject to following constraints, along with non-negativity constraints

$$x_1 \geq 1$$

$$x_2 \geq -1$$

## Step-2

Solving the above constraints, we get the following vector

$$x_1 = 1$$

$$x_2 = 0$$

Therefore, the feasible vector is

$$\boxed{x^* = [1 \quad 0]^T}$$

## Step-3

Now, the corresponding dual of the LPP is as follows

Maximize:  $y_1 - y_2$

Subject to following constraints, along with non-negativity constraints

$$y_1 \leq 1$$

$$y_2 \leq 1$$

## Step-4

Solving the above constraints, we get the following vector

$$y_1 = 1$$

$$y_2 = 0$$

Therefore, the feasible vector is

$$\mathbf{y}^* = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

## Step-5

Let us calculate the following terms,

$$\begin{aligned} \mathbf{c}^T \mathbf{x} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \\ &= 1 \end{aligned}$$

And

$$\begin{aligned} \mathbf{y}^T \mathbf{b} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ &= 1 - 0 \\ &= 1 \end{aligned}$$

Now,  $\mathbf{c}^T \mathbf{x} = \mathbf{y}^T \mathbf{b}$ , thus the value of vectors  $x$  and  $y$  are optimal.

Since, the second inequality in both  $\mathbf{A} \mathbf{x}^* \geq \mathbf{b}$  and  $\mathbf{y}^* \mathbf{A} \leq \mathbf{c}$  are strict, so the second components of  $\mathbf{y}^*$  and  $\mathbf{x}^*$  are zero