Step-1

Given that if all r pivot variables come last; we have to describe the four blocks in the m by n reduced echelon firm (the block B should be r by r):

$$R = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

We have to find that the null space matrix N of special solutions and its shape.

Step-2

Suppose the pivot variables are last variables then B has the r – pivot columns. Remaining are all free variables.

Then

$$D = 0$$
$$C = 0$$

Therefore $$R = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}$$

B is $r \times r$ identity matrix.

Step-3

$$R = \begin{bmatrix} A & I \\ 0 & 0 \end{bmatrix}$$, if $RX = 0$ then the null space is as below:

Therefore the null space is in the form

$$N = \begin{bmatrix} I \\ -A \end{bmatrix}$$

$$= \begin{bmatrix} B \\ -A \end{bmatrix}$$

It is shape is n by r.