Step-1

This series $\left(I+A+A^2+\ldots\right)$ represent $\left(I-A\right)^{-1}$.

Check the following:

$$\left(I-A\right)\left(I+A+A^2+\ldots\right)=I$$

Solve the left hand side.

$$\begin{aligned}\left(I-A\right)\left(I+A+A^2+\ldots\right)&=I\left(I+A+A^2+\ldots\right)-A\left(I+A+A^2+\ldots\right)\\&=\left(I+A+A^2+\ldots\right)-\left(A+A^2+A^3+\ldots\right)\\&=I\end{aligned}$$

Therefore, left hand side comes out to be equal to right hand side.

Step-2

Consider the following matrix which has $\lambda_{\max}=0$.

$$A=\begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix}$$

Step-3

Find the powers of matrix A as follows:

$$\begin{aligned}A\cdot A&=\begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix}\\A^2&=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}\\A^2\cdot A&=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}0&1&1\\0&0&1\\0&0&0\end{bmatrix}\\A^3&=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\end{aligned}$$

Step-4

Here, A^3 results into zero matrix. so other higher powers of matrix A will also be zero.

Step-5

Substitute the values into the following series:

$$(I-A)^{-1} = \left(I + A + A^2\right)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Step-6

Therefore, above result shows the following:

$$\boxed{(I-A)^{-1} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$$