Step-1

Given system is $2x + 3y = 1$

$10x + 9y = 11$

Given system can be written matrix form as

$$\begin{pmatrix} 2 & 3 \\ 10 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 11 \end{pmatrix}$$

The augmented matrix is

$$\begin{pmatrix} 2 & 3 & 1 \\ 10 & 9 & 11 \end{pmatrix}$$

Step-2

Subtract $\frac{10}{2} = 5$ times the first row from the second row to get $\begin{pmatrix} \boxed{2} & 3 & 1 \\ 0 & \boxed{-6} & 6 \end{pmatrix}$ which is an upper triangular system

$2x + 3y = 1$

$-6y = 6$

By back-substitution $-6y = 6$

$\Rightarrow y = -1$

And $2x + 3(-1) = 1$

$\Rightarrow x = 2$

Hence the solution is $\boxed{(2, -1)}$

Step-3

Verification:-

Put $x = 2, y = -1$ in the given system

$2x + 3y = 2(2) + 3(-1)$

$= 1$

$$\begin{aligned}10x+9y&=10(2)\ +\ 9(-1)\\&=11\end{aligned}$$

Hence x times $(2,10)$ plus y times $(3,9)$ equals $(1,11)$.

Step-4

If right-hand side changes to $(4,44)$,then the augmented matrix is

$$\begin{pmatrix}2 & 3 & 4\\10 & 9 & 44\end{pmatrix}$$

Subtract â€˜5â€™ times the first row from the second row

$$\begin{pmatrix}\boxed{2} & 3 & 4\\0 & \boxed{-6} & 24\end{pmatrix}$$

which is upper triangular system $2x+3y=4$

$-6y=24$

By back-substitution, we have $-6y=24$

$\Rightarrow y=-4$

And $2x+3(-4)=4$

$\Rightarrow x=8$

Hence the solution is $\boxed{(8,-4)}$