Step-1

Suppose that, the values $b_1 = 1$ and $b_2 = 7$ at times $t_1 = 1$ and $t_2 = 2$ are fitted by a line $b = Dt$ through the origin.

Its need to solve $D = 1$ and $2D = 7$ by least-squares

The least square solution to the equation $Ax = b$ is $A^T A\hat{x} = A^T b$

We have $D = 1$ and $2D = 7$

$$\Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} D = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

Take $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, x = D, b = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$

Step-2

Now $A^T A\widehat{D} = A^T b$

First calculate $A^T A$

$$\begin{aligned} A^T A &= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &= 1(1) + 2(2) \\ &= 1 + 4 \\ &= 5 \end{aligned}$$

And

$$\begin{aligned} A^T b &= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \end{bmatrix} \\ &= 1(1) + 2(7) \\ &= 1 + 14 \\ &= 15 \end{aligned}$$

Use the values $A^T A = 5$ and $A^T b = 15$ in $A^T A\widehat{D} = A^T b$, then we get

$$5\widehat{D} = 15$$

$$\Rightarrow \widehat{D} = 3$$

Therefore, the least-square solution to $D = 1$ and $2D = 7$ is $\widehat{D} = 3$.

Hence, the best line is $\boxed{b = 3t}$

Step-3

The sketch of the best line $b = 3t$ is as shown below.