

## Step-1

If  $Q$  is an orthogonal matrix, so that  $Q^T Q = I$ , we have to prove that  $\det Q$  equals to  $+1$  or  $-1$  and we have to find that what kind of box is formed from the rows (or columns) of  $Q$ .

## Step-2

We have  $Q^T A = I$  (since  $Q$  is an orthogonal matrix)

$$\Rightarrow \det(Q^T Q) = \det I$$

$$\Rightarrow \det Q^T \cdot \det Q = 1 \quad (\text{since } \det I = 1)$$

(since  $\det(AB) = \det A \det B$  for any 2 matrices A,B)

## Step-3

$$\Rightarrow \det Q \cdot \det Q = 1$$

(since  $\det(A^T) = \det A$  for any matrix  $A$ )

$$\Rightarrow (\det Q)^2 = 1$$

$$\Rightarrow \det Q = \sqrt{1} = \boxed{\pm 1}$$

Thus,  $\det Q = \pm 1$ , where  $Q$  is an orthogonal matrix.

A box of volume 1 is formed from rows of  $Q$ .