

Step-1

(a).

Given function is $F(x, y) = -1 + 4(e^x - x) - 5x \sin y + 6y^2$ and given stationary point is $(x, y) = (0, 0)$.

We need to decide between a minimum, maximum, or saddle point.

We need to find the first and second derivatives now.

$$F_x = 4e^x - 4 - 5 \sin y.$$

$$F_{xx} = 4e^x$$

$$\Rightarrow (F_{xx})_{(0,0)}$$

$$\Rightarrow 4 > 0$$

$$F_y = -5x \cos y + 12y$$

$$F_{yy} = 5x \sin y + 12$$

$$F_{xy} = -5 \cos y$$

Step-2

Given the stationary point is $(x, y) = (0, 0)$.

$$\text{Now } (F_{xy})_{(0,0)} = -5$$

$$\text{And } (F_{yy})_{(0,0)} = 12$$

And,

$$(F_{xx})(F_{yy}) = 4(12) \\ = 48$$

$$(F_{xy})^2 = 25$$

$$\text{Clearly } (F_{xx})(F_{yy}) > (F_{xy})^2$$

So $F(x, y)$ has minimum at $(0, 0)$.

Therefore, $F(x, y)$ has minimum at $(0, 0)$.

Step-3

(b).

Given function is $F(x, y) = (x^2 - 2x)\cos y$.

We need to decide between a minimum, maximum, or saddle point.

We need to find the first and second derivatives now.

$$F_x = (2x - 2)\cos y$$

$$F_{xx} = 2\cos y$$

$$F_y = -(x^2 - 2x)\sin y$$

$$F_{yy} = -(x^2 - 2x)\cos y$$

$$F_{xy} = -(2x - 2)\sin y$$

Step-4

Given stationary point is $(x, y) = (1, \pi)$.

Now,

$$\begin{aligned}(F_{xx})_{(1, \pi)} &= 2\cos \pi \\ &= -2 < 0\end{aligned}$$

And,

$$\begin{aligned}(F_{xx})(F_{yy}) &= (-2)(-1) \\ &= 2\end{aligned}$$

$$(F_{xy})^2 = 0$$

Clearly $(F_{xx})(F_{yy}) > (F_{xy})^2$

So $F(x, y)$ has maximum at $(1, \pi)$.

Therefore, $F(x, y)$ has maximum at $(1, \pi)$.