

Step-1

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$$

Consider the factored form

The objective is to find what multiple l_{32} of row2 of A will elimination subtract from row 3 of A and to find the pivots and to check whether the row exchange is required or not.

Step-2

Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$

Compare with $A = lu$ where

$$l = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \text{ and } u = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$$

$$l = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & \boxed{l_{32}} & l_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & \boxed{4} & 1 \end{bmatrix}$$

Consider l matrix that is

Then $l_{32} = 4$

Hence elimination will subtract 4 times row2 from row 3

Step-3

Consider u matrix to find the pivots

Then $u = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix} = \begin{bmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$

The pivots are along the diagonal of the matrix that is $\begin{bmatrix} \boxed{5} & 7 & 8 \\ 0 & \boxed{2} & 3 \\ 0 & 0 & \boxed{6} \end{bmatrix}$

Hence $\boxed{5}$, $\boxed{2}$ and $\boxed{6}$ are the pivots of the matrix

Step-4

To find out whether a row exchange will be needed or not, first determine A

Consider

$$A = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & 7 & 8 \\ 2 & 1 & 0 & 0 & 2 & 3 \\ 1 & 4 & 1 & 0 & 0 & 6 \end{array} \right]$$
$$= \left[\begin{array}{ccc} 5 & 7 & 8 \\ 10 & 16 & 19 \\ 5 & 15 & 26 \end{array} \right]$$

Carry the first elimination

$$A = \left[\begin{array}{ccc} 5 & 7 & 8 \\ 10 & 16 & 19 \\ 5 & 15 & 26 \end{array} \right] \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

Carry the Second elimination

$$A = \left[\begin{array}{ccc} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 8 & 18 \end{array} \right] R_3 \rightarrow R_3 - 4R_2$$
$$A = \left[\begin{array}{ccc} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{array} \right] = u$$

Step-5

Hence, $\boxed{\text{there would not be a need for a row exchange}}$