Step-1

(a)

The objective is to find the dimension of the column space of matrix A.

$$A=\begin{bmatrix}1 & 1 & 0\\ 1 & 3 & 1\\ 3 & 1 & -1\end{bmatrix}$$

Step-2

Consider the matrix

$$A=\begin{bmatrix}1 & 1 & 0\\ 1 & 3 & 1\\ 3 & 1 & -1\end{bmatrix}$$

Let,

$$v_1=\begin{bmatrix}1\\ 1\\ 3\end{bmatrix}$$
$$v_2=\begin{bmatrix}1\\ 3\\ 1\end{bmatrix}$$
$$v_3=\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}$$

Let $c_1v_1+c_2v_2+c_3v_3=0$

$$c_1\begin{bmatrix}1\\ 1\\ 3\end{bmatrix}+c_2\begin{bmatrix}1\\ 3\\ 1\end{bmatrix}+c_3\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

That is;

$$\begin{aligned}c_1+c_2&=0\\ c_1+3c_2+c_3&=0\\ 3c_1+c_2-c_3&=0\end{aligned}$$

From the above system of equations,

$$\begin{aligned}(3c_1+c_2-c_3)+(c_1+3c_2+c_3)&=0\\4c_1+4c_2&=0\\c_1&=-c_2\end{aligned}$$

Now,

$$\begin{aligned}c_3&=-c_1-3c_2\\&\quad(\text{Since}\,c_1=-c_2)\\&=c_2-3c_2\\&=-2c_2\end{aligned}$$

Therefore $-c_2v_1+c_2v_2-2c_2v_2=0$

If $c_2=1,-v_1+v_2-2v_3=0$

Therefore v_1,v_2,v_3 $\boxed{\text{are linearly dependent.}}$

If $c_1v_1+c_2v_2=0$

$$c_1\begin{bmatrix}1\\1\\3\end{bmatrix}+c_2\begin{bmatrix}1\\3\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

That is;

$c_1+c_2=0$ â€¦â€¦ (1)

$c_1+3c_2=0$ â€¦â€¦ (2)

$3c_1+c_2=0$ â€¦â€¦ (3)

Apply $(3\times(1))-(2)$;

This implies;

$$\begin{aligned}2c_2&=0\\c_2&=0\end{aligned}$$

Plug this value in equation (1)

So, $c_2=0$

Therefore, v_1,v_2 are linearly independent and $\{v_1\ \ ,v_2\}$ spans columns space.

Therefore dimension of column space $\boxed{A=2}$.

Step-3

(b)

The objective is to find the column space of matrix *U*.

$$U=\begin{bmatrix}1&1&0\\0&2&1\\0&0&0\end{bmatrix}$$

Step-4

Consider the matrix;

$$U=\begin{bmatrix}1&1&0\\0&2&1\\0&0&0\end{bmatrix}$$

Step-5

Let

$$v_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$$
$$v_2=\begin{bmatrix}1\\2\\0\end{bmatrix}$$
$$v_3=\begin{bmatrix}0\\1\\0\end{bmatrix}$$

Clearly,

$$v_1+2v_3=v_2$$
$$v_1-v_2+2v_3=0$$

Therefore v_1,v_2,v_3 are dependent and $c_1v_1+c_2v_2=0$

$$c_1 + c_2 = 0$$
$$2c_2 = 0$$

Thus,

$$c_1 = 0$$
$$c_2 = 0$$

Hence v_1, v_2 are linearly independent and $\{v_1, v_2\}$ spans column space of U .

Therefore, the dimension of column space of $\boxed{U = 2}$.

Step-6

(c)

The objective is to find the dimension of the row space of matrix *A*.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}$$

Step-7

Let

$$v_1 = (1,1,0)$$
$$v_2 = (1,3,1)$$
$$v_3 = (3,1,-1)$$

Now,

$$\begin{aligned} c_1v_1 + c_2v_2 + c_3v_3 &= 0 \\ c_1(1,1,0) + c_2(1,3,1) + c_3(3,1,-1) &= 0 \\ c_1 + c_2 + 3c_3 &= 0 \\ c_1 + 3c_2 + c_3 &= 0 \end{aligned}$$

Solve these two equations.

So,

$c_2 = c_3$ and
$c_1 = -c_2 - 3c_3$

This implies;

$$c_2 = c_3$$
$$c_1 = -4c_3$$

So,

$$c_1v_1 + c_2v_2 + c_3v_3 = 0$$
$$-4c_3v_1 + c_3v_2 + c_3v_3 = 0$$

If $c_3 = 1$, then;

$$-4v_1 + v_2 + v_3 = 0$$

Therefore, $\{v_1, v_2, v_3\}$ are independent.

But, $\{v_1, v_2\}$ are independent and $\{v_1, v_2\}$ spans row spaces row space of A.

Therefore, the dimension of row space $\boxed{A = 2}$.

Step-8

(d)

The objective is to find the row space of matrix *U*.

$$U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Step-9

The row space of U = spanned by $\{(1,1,0),(0,2,1)\}$

If $(1,3,1) = (1,1,0) + (0,2,1)$ and $(1,1,0),(1,3,1)$ are linearly independent $\{(1,1,0),(1,3,1)\}$ space the row space of U.

Therefore, the row space of matrix *U* is $\boxed{U = 2}$

Hence, $\boxed{\text{row space of } U \ = \ \text{Row space of } A}$.