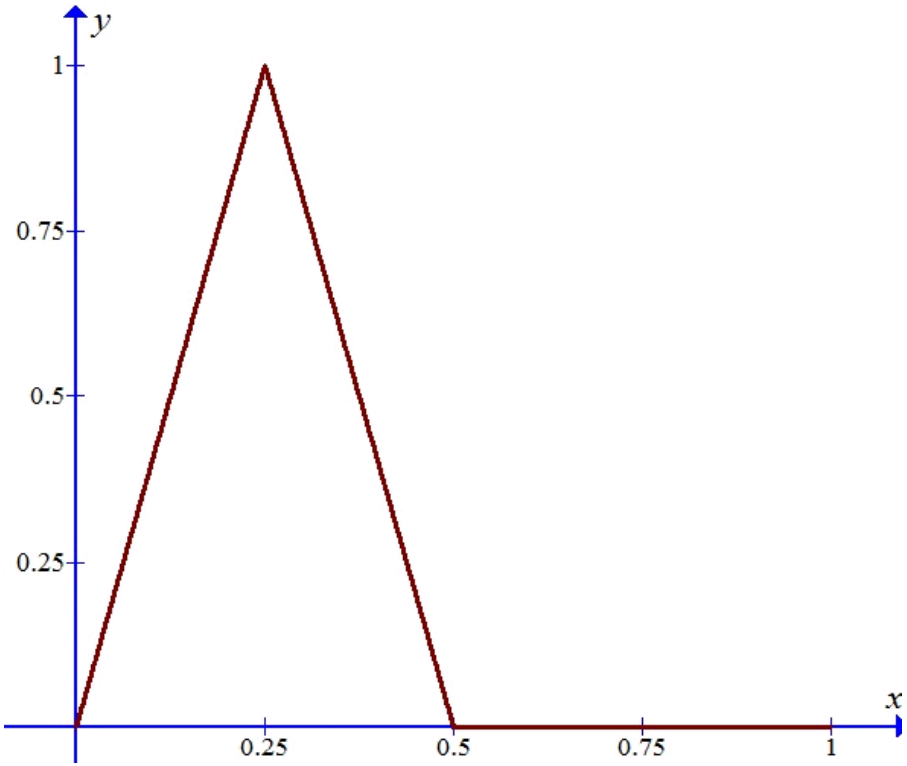
Step-1

The graph of the hat function is as drawn below:



Step-2

Since $h=\frac{1}{4}$, we have the matrix A as follows:

$$A=4\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$$

Clearly, $$b=\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}$$

Step-3

Solve the system $Ay = b$.

We have,

$$4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 8y_1 - 4y_2 \\ -4y_1 + 8y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

Step-4

Consider the following:

$$8y_1 - 4y_2 = \frac{1}{2}$$

$$-4y_1 + 8y_2 = \frac{1}{2}$$

Subtracting the second equation from the first equation, we get

$$12y_1 - 12y_2 = 0$$

Step-5

Thus, $y_1 = y_2$.

Therefore,

$$8y_1 - 4y_1 = \frac{1}{2}$$

$$4y_1 = \frac{1}{2}$$

$$y_1 = \frac{1}{8}$$

Thus, $y = \begin{bmatrix} \frac{1}{8} \\ \frac{1}{8} \end{bmatrix}$.

As we have considered only one hat function, $U(x) = \frac{1}{8} V_1$. This function has the value $\frac{1}{8}$ at the mesh point. This is same as $\frac{2}{16}$.

Now, at $x = \frac{1}{4}$, we have

$$\begin{aligned} x - x^2 &= \frac{1}{4} - \frac{1}{16} \\ &= \frac{4-1}{16} \\ &= \frac{3}{16} \end{aligned}$$

Step-6

Thus, the actual value at the mesh point is $\boxed{\frac{3}{16}}$ and the approximate value is $\boxed{\frac{2}{16}}$. The difference between the values is very less.