Step-1

Given that

Polar coordinates satisfy $x = r\cos\theta$ and $y = r\sin\theta$. Polar area $Jdrd\theta$ includes J:

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

Step-2

$$\frac{\partial x}{\partial r} = \cos\theta,\ \frac{\partial x}{\partial \theta} = -r\sin\theta,\ \frac{\partial y}{\partial r} = \sin\theta,\ \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

Step-3

The two columns are orthogonal their lengths are $\boxed{\sqrt{\cos^2\theta + \sin^2\theta} = 1}$

And $\sqrt{r^2\left(\cos^2\theta + \sin^2\theta\right)} = \sqrt{r^2}$

$= r$

Thus $\boxed{J = r}$