Step-1

Let P_1 = the projection matrix onto the line through $a_1 = \frac{a_1 a_1^T}{a_1^T a_1}$ â€¦â€¦ (1)

$$a_1 a_1^T = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} (-1, 2, 2)$$
$$= \begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$$

$$a^T a = \begin{bmatrix} -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$
$$= 1 + 4 + 4$$
$$= 9$$

In view of (1), we get $P_1 = \frac{1}{9} \begin{bmatrix} 1 & -2 & -2 \\ -2 & 4 & 4 \\ -2 & 4 & 4 \end{bmatrix}$

Step-2

Let P_2 = the projection matrix onto the line through $a_2 = \frac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} (2, 2, -1)$$
$$= \begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$a_2^T a_2 = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$
$$= 4 + 4 + 1$$
$$= 9$$

$$P_2 = \frac{1}{9}\begin{bmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

Step-3

Let P_3 = the projection matrix onto the line through $a_3 = \frac{a_3 a_3^T}{a_3^T a_3}$

$$a_3 a_3^T = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}(2,-1,2)$$
$$= \begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix}$$

$$a_3^T a_3 = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$
$$= 4+1+4$$
$$= 9$$

$$P_3 = \frac{1}{9}\begin{bmatrix} 4 & -2 & 4 \\ -2 & 1 & -2 \\ 4 & -2 & 4 \end{bmatrix}$$

Step-4

$$P_1+P_2+P_3=\frac{1}{9}\begin{bmatrix}1 & -2 & -2\\ -2 & 4 & 4\\ -2 & 4 & 4\end{bmatrix}+\frac{1}{9}\begin{bmatrix}4 & 4 & -2\\ 4 & 4 & -2\\ -2 & -2 & 1\end{bmatrix}+\frac{1}{9}\begin{bmatrix}4 & -2 & 4\\ -2 & 1 & -2\\ 4 & -2 & 4\end{bmatrix}$$
$$=\frac{1}{9}\begin{bmatrix}1+4+4 & -2+4-2 & -2-2+4\\ -2+4-2 & 4+4+1 & 4-2-2\\ -2-2+4 & 4-2-2 & 4+1+4\end{bmatrix}$$
$$=\frac{1}{9}\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Step-5

We easily see that $a_1^T a_2=(-1,2,2)\begin{pmatrix}2\\2\\-1\end{pmatrix}$

Step-6

$$=-2+4-2$$
$$=0$$

$$a_2^T a_3=(2,2,-1)\begin{pmatrix}2\\-1\\2\end{pmatrix}$$

$$=4-2-2$$
$$=0$$

$$a_3^T a_1=(2,-1,2)\begin{pmatrix}2\\-1\\2\end{pmatrix}$$

$$=4-2-2$$
$$=0$$

That means $\{a_1,a_2,a_3\}$ is an orthogonal set of vectors and so, are linearly independent, the dimension of each vector is 3 and thus forms a basis to $\mathbf{R}^3$