

Step-1

Complex inner product: $u^H v = \overline{u_1} v_1 + \cdots + \overline{u_n} v_n$.

Orthogonality: $u^H v = 0$

Step-2

Consider the following vectors:

$$v = (1, i, 1)$$

$$w = (i, 1, 0)$$

$$z = (a, b, c)$$

These vectors form an orthogonal basis. Find vector z .

Step-3

If these vectors form an orthogonal basis then their inner product will be zero. Calculate the inner product of these vectors:

$$\begin{aligned} v \cdot w &= 1 \cdot i + (-i) \cdot 1 + 1 \cdot 0 \\ &= i - i + 0 \\ &= 0 \end{aligned}$$

Step-4

Similarly, inner product of other two vectors must also be zero.

$$w \cdot z = 0$$

$$v \cdot z = 0$$

Calculate the following:

$$\begin{aligned} w \cdot z &= (-i) \cdot a + 1 \cdot b + 0 \cdot c \\ &= -ia + b \\ v \cdot z &= 1 \cdot a + (-i) \cdot b + 1 \cdot c \\ &= a - ib + c \end{aligned}$$

Step-5

On solving following results are obtained:

$$w \cdot z = 0$$

$$-ia + b = 0$$

$$b = ia$$

Similarly,

$$v \cdot z = 0$$

$$a - ib + c = 0$$

$$a + a + c = 0$$

$$2a + c = 0$$

Put $a = 1$ in the above results. Following values are obtained:

$$b = i$$

$$c = -2$$

Step-6

Therefore, vector is $\boxed{z = (1, i, -2)}$. All these three vectors v , w , and z are an orthogonal basis for $\boxed{\mathbb{C}^3}$.