

## Step-1

Given that  $A = \begin{pmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{pmatrix}, x = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

We have to find  $Ax$  to find the solution  $x$  to the system  $Ax = O$  and try to find more solutions to this system.

## Step-2

$$\begin{aligned} Ax &= \begin{pmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \\ &= 2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} -6 \\ 2 \\ -1 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\ &= O \end{aligned}$$

Therefore  $Ax = O$

## Step-3

Hence  $x = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$  is the solution of the equation  $Ax = O$

And the remaining solutions are  $cx = c \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$  where  $c \in \mathbb{R}$  is arbitrary constant.