

Step-1

$$\text{Suppose the given system is } \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

It can be written as $Ax = b$.

$$\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{b}$$

We can write this as where \mathbf{a}_i is the column of A . \mathbf{b} is the column vector on the right of the given system.

If $\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3 = \mathbf{b}$ goes into the 3rd column to produce B_3 , then we can write

$$\begin{aligned} |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{b}| &= |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3| \\ &= |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_1 x_1| + |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_2 x_2| + |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 x_3| \quad \text{By properties of determinants} \\ &= x_1 |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_1| + x_2 |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_2| + x_3 |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3| \quad \text{By the properties of determinants} \\ &= x_1 (0) + x_2 (0) + x_3 |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3| \quad \text{If two columns are identical, then the determinant is zero.} \\ &= x_3 |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3| \end{aligned}$$

Step-2

$$\text{So, } |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{b}| = x_3 |\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3|$$

$$\text{Or, } x_3 = \frac{|\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{b}|}{|\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3|}$$

$$\text{Similarly, } x_2 = \frac{|\mathbf{a}_1 \quad \mathbf{b} \quad \mathbf{a}_3|}{|\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3|}$$