

## Step-1

Given data is  $b = 0$  at  $t = 0$

$b = 0$  at  $t = 1$

$b = 12$  at  $t = 3$

We have to best fit this data with a straight line.

## Step-2

Let  $C + Dt = b$  be the required straight line that best fits the given data, then

$$C + 0.D = 0$$

$$C + 1.D = 0$$

$$C + 3.D = 12$$

In matrix form,

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$\Rightarrow Ax = b$$

$$\text{where } A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix}, \quad x = \begin{bmatrix} C \\ D \end{bmatrix} \text{ and } b = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

## Step-3

$$\begin{aligned} A^T A &= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \end{pmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1+1+1 & 0+1+3 \\ 0+1+3 & 0+1+9 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 4 \\ 4 & 10 \end{bmatrix} \end{aligned}$$

$$(A^T A)^{-1} = \frac{1}{14} \begin{bmatrix} 10 & -4 \\ -4 & 3 \end{bmatrix}$$

$$\begin{aligned} (A^T A)^{-1} A^T &= \frac{1}{14} \begin{bmatrix} 10 & -4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix} \\ &= \frac{1}{14} \begin{bmatrix} 10 & 6 & -2 \\ -4 & -1 & 5 \end{bmatrix} \end{aligned}$$

#### Step-4

$$\hat{x} = \begin{bmatrix} \hat{C} \\ \hat{D} \end{bmatrix}$$

Let

$$\hat{x} = (A^T A)^{-1} A^T b$$

$$= \frac{1}{14} \begin{bmatrix} 10 & 6 & -2 \\ -4 & -1 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} -24 \\ 60 \end{bmatrix}$$

$$= \begin{bmatrix} -12/7 \\ 30/7 \end{bmatrix}$$

Therefore  $\hat{C} = \frac{-12}{7}, \hat{D} = \frac{30}{7}$      $\hat{A} \hat{A} \hat{A}$

Hence required straight line is  $\boxed{b = \frac{-12}{7} + \left(\frac{30}{7}\right)t}$