

Step-1

AB can be expressed as

$$\begin{aligned}AB &= I(AB) \\&= (B^{-1}B)(AB) \\&= B^{-1}(BA)B\end{aligned}$$

Therefore AB is similar to BA .

Similar matrices have the same eigen values.

Step-2

Through another view, suppose x is the eigen vector corresponding to the eigen value λ of AB .

Let us take $(AB)x = \lambda x$

$$\Rightarrow (B^{-1}(BA)B)x = \lambda x$$

$$\Rightarrow (BA)Bx = \lambda(Bx)$$

The eigen value of BA is also λ and the respective eigen vector is Bx .

Therefore AB and BA have the same eigen values.