Step-1

We know that $Ax = b$ has a solution or there exists a vector y, such that $yA = 0$ and $yb \neq 0$.

Let $y = (2, -1)$.

Therefore,

$$\begin{aligned} yA &= (2,-1)\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \\ &= (2\times 2 + (-1)\times 4, 2\times 2 + (-1)\times 4) \\ &= (0,0) \end{aligned}$$

Also,

$$\begin{aligned} yb &= (2,-1)\begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= 2\times 1 + (-1)\times 1 \\ &= 1 \\ &\neq 0 \end{aligned}$$

Step-2

Thus, we have produced a vector $y = (2, -1)$ such that $yA = 0$ and $yb \neq 0$. Therefore, for $A = \begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, the equation $Ax = b$ has no solution.