Step-1

Since a one dimensional subspace in $\mathbb{R}^3$ is a straight line and a two dimensional subspace is a plane.

Now to find a two dimensional subspace of $\mathbb{R}^3$ that do not contain $(1,0,0),(0,1,0)$ and $(0,0,1)$

Geometrically this can be interpreted as a plane in $\mathbb{R}^3$ that does not contain the points

$(1,0,0),(0,1,0)$ and $(0,0,1)$

For, this consider the position vectors $(4,1,0),(2,5,2)$

Clearly, these are linearly independent.

Thus $\{(4,1,0)(2,5,2)\}$ is a basis for *S*.

Now, this matrix $\begin{bmatrix} 4 & 1 & 0 \\ 2 & 5 & 2 \\ 1 & 0 & 0 \end{bmatrix}$ has the determinant 2 not zero.

That means the vectors $(4,1,0),(2,5,2),(1,0,0)$ are linearly independent and so, they does not lie on the same plane.

In other words, $(1,\ 0,\ 0)$ lie away from the plane generated by $(4,1,0),(2,5,2)$.

Step-2

Similarly, the determinant of $\begin{bmatrix} 4 & 1 & 0 \\ 2 & 5 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ is 8 and so, (0, 1, 0) is not on the plane spanned by $(4,1,0),(2,5,2)$.

The determinant of $\begin{bmatrix} 4 & 1 & 0 \\ 2 & 5 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is 18 and so, (0, 0, 1) is not on the plane spanned by $(4,1,0),(2,5,2)$.

Hence, $\boxed{\{(4,1,0)(2,5,2)\}}$ is a two-dimensional subspace of $\mathbb{R}^3$ that contains none of the coordinate vectors.