Step-1

Consider that $A = S\begin{bmatrix} \lambda_1 & 2 \\ 0 & \lambda_2 \end{bmatrix}S^{-1}$.

The objective is to find the determinant of A and A^{-1}.

Step-2

Find the determinant of matrix A as,

Recall: $\det(A_1 \cdot A_2 \cdots A_n) = \det(A_1)\cdot\det(A_2)...\det(A_n)$

Therefore,

$$\begin{aligned}
\det(A) &= \det\left(S\begin{bmatrix} \lambda_1 & 2 \\ 0 & \lambda_2 \end{bmatrix}S^{-1}\right) \\
&= \det(S)\cdot\det\left(\begin{bmatrix} \lambda_1 & 2 \\ 0 & \lambda_2 \end{bmatrix}\right)\cdot\det(S^{-1}) \\
&= \det(S)\cdot\det\left(\begin{bmatrix} \lambda_1 & 2 \\ 0 & \lambda_2 \end{bmatrix}\right)\cdot\left(\frac{1}{\det(S)}\right) \qquad \text{Since } \det(A^{-1}) = \frac{1}{\det(A)} \\
&= \left(\cancel{\det(S)}\cdot\frac{1}{\cancel{\det(S)}}\right)(\lambda_1\cdot\lambda_2 - 0) \\
&= \lambda_1\lambda_2
\end{aligned}$$

Therefore, the value of $\det(A) = \lambda_1\lambda_2$ â€¦â€¦ (1)

Step-3

Find the determinant of matrix A^{-1} as,

$$\begin{aligned}
\det(A^{-1}) &= \frac{1}{\det(A)} \\
&= \frac{1}{\lambda_1\lambda_2} \qquad \text{From equation (1)}
\end{aligned}$$

Therefore, the value of $\det(A^{-1}) = \frac{1}{\lambda_1\lambda_2}$.