Step-1

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The given system is $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \frac{\pi^2}{4}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

This can be followed as $Au = b$ where A is the coefficients matrix, u is the variable matrix and b is the constant matrix.

We apply row operations on the augmented matrix [A: b] to reduce it to echelon form and then get the possible solutions for the system.

$$[A:b] = \left[\begin{array}{ccc|c} 2 & -1 & 0 & \pi^2/4 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -\pi^2/4 \end{array}\right]$$

$$R_2 \to 2R_2 + R_1 \Rightarrow \left[\begin{array}{ccc|c} 2 & -1 & 0 & \pi^2/4 \\ 0 & 3 & -2 & \pi^2/4 \\ 0 & -1 & 2 & -\pi^2/4 \end{array}\right]$$

$$R_3 \to 3R_3 + R_2 \Rightarrow \left[\begin{array}{ccc|c} 2 & -1 & 0 & \pi^2/4 \\ 0 & 3 & -2 & \pi^2/4 \\ 0 & 0 & 4 & -\pi^2/2 \end{array}\right]$$

Step-2

We are required to make the diagonal entries 1â€™s or 0 to call this form as the echelon form.

But that complicates the rewriting of non homogeneous equations and then finding solutions.

So, we stop the procedure here and rewrite the non homogeneous equations from below.

$$\begin{aligned} 4u_3 &= \frac{-\pi^2}{2} \\ 3u_2 - 2u_3 &= \frac{\pi^2}{4} \\ 2u_1 - u_2 &= \frac{\pi^2}{4} \end{aligned}$$

Step-3

Consequently, $u_3 = \frac{-\pi^2}{8}$, using this in the 2nd equation, we get $u_2 = 0$ and 3rd equation gives $u_1 = \frac{\pi^2}{8}$ â€¦â€¦ (1)

On the other hand, for the function $f(x) = 4\pi^2 \sin 2\pi x$ and $h = \frac{1}{4}$, we have $u = \sin 2\pi x$

$$u_1 = \sin 2\pi\left(\frac{1}{4}\right)$$
$$= 1$$

Similarly,

$$u_2 = \sin 2\pi\left(\frac{1}{2}\right)$$
$$= 0$$

$$u_3 = \sin 2\pi\left(\frac{3}{4}\right)$$
$$= -1$$ â€¦â€¦ (2)

Comparing (1) and (2), the true solution $(u_1, u_2, u_3) = (1, 0, -1)$ is replaced by $\left(\frac{\pi^2}{8}, 0, \frac{-\pi^2}{8}\right)$