Step-1

Given m independent measurements of pulse rate are $b_1,\ldots,b_m$ are weighted by $w_1,\ldots,w_m$.

We have to find the weighted average that replaces $\bar{x}_w = \dfrac{w_1{}^2 b_1 + w_2{}^2 b_2}{w_1{}^2 + w_2{}^2}$.

Step-2

We have the weighted error is $E^2 = w_1^2\left(x-b_1\right)^2 + w_2^2\left(x-b_2\right)^2 + \ldots + w_m^2\left(x-b_m\right)^2$

And the minimizing process is $\dfrac{dE^2}{dx} = 0$

$$\Rightarrow 2w_1^2\left(x-b_1\right)+2w_2^2\left(x-b_2\right)+\ldots+2w_m^2\left(x-b_m\right)=0$$

$$\Rightarrow 2\left[w_1^2\left(x-b_1\right)+w_2^2\left(x-b_2\right)+\ldots+w_m^2\left(x-b_m\right)\right]=0$$

$$\Rightarrow w_1^2\left(x-b_1\right)+w_2^2\left(x-b_2\right)+\ldots+w_m^2\left(x-b_m\right)=0$$

Step-3

Continuation to the above

$$\Rightarrow w_1{}^2 x - w_1{}^2 b_1 + w_2{}^2 x - w_2{}^2 b_2 + \ldots + w_m{}^2 x - w_m{}^2 b_m = 0$$
$$\Rightarrow w_1{}^2 x + w_2{}^2 x + \ldots + w_m{}^2 x - \left(w_1{}^2 b_1 + w_2{}^2 b_2 + \ldots + w_m{}^2 b_m\right) = 0$$

$$\Rightarrow x\left(w_1^2 + w_2^2 + \ldots\ldots\ldots\ldots + w_m^2\right) = w_1^2 b_1 + w_2^2 b_2 + \ldots\ldots\ldots\ldots + w_m^2 b_m$$

$$\Rightarrow x = \frac{w_1^2 b_1 + w_2^2 b_2 + \ldots + w_m^2 b_m}{w_1^2 + w_2^2 + \ldots + w_m^2}$$

Hence the weighted average is $\boxed{x = \bar{x}_w = \dfrac{w_1^2 b_1 + w_2^2 b_2 + \ldots + w_m^2 b_m}{w_1^2 + w_2^2 + \ldots + w_m^2}}$.