

Step-1

Consider the following proof:

A projection matrix P is given by $P = A(A^T A)^{-1} A^T$

Therefore, $\det(P) = \det\left(A(A^T A)^{-1} A^T\right)$.

This is same as $|A| \frac{1}{|A^T| |A|} |A^T|$.

Now, cancel $|A|$ and $|A^T|$ and thus get $\det P = 1$.

Step-2

Although this proof seems to be correct, the matrix A may be rectangular. In such case A^T is also rectangular.

Determinant can be defined of a square matrix only. Determinant of a rectangular matrix is not defined.

So, if A is rectangular then, $\boxed{\det(A^T A) \neq \det(A^T) \det(A)}$

Thus, the expression $|A| \frac{1}{|A^T| |A|} |A^T|$ is meaningless.

This is the mistake in the above proof.