Step-1

The determinant of a lower triangular matrix is nothing but the product of diagonal entries.

Given that the determinant of $L=\begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$ is not zero.

That means $acf \neq 0$

So, $a \neq 0, c \neq 0, f \neq 0$

We can simply see that the cofactors of *b, d, e* are $-\begin{vmatrix} 0 & 0 \\ e & f \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ c & a \end{vmatrix}, -\begin{vmatrix} a & 0 \\ b & 0 \end{vmatrix}$ respectively.

$L^{-1}=\dfrac{C^T}{\det L}=\dfrac{1}{acf}\begin{bmatrix} cf & 0 & 0 \\ -bf & af & 0 \\ be-cd & -ae & ac \end{bmatrix}$ is also a lower triangular matrix.

Step-2

b) $S=\begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix}$ is a symmetric matrix

The cofactor matrix of *S* is

$$C^T=\begin{bmatrix} (-1)^{1+1}\begin{vmatrix} c & e \\ e & f \end{vmatrix} & (-1)^{1+2}\begin{vmatrix} b & e \\ d & f \end{vmatrix} & (-1)^{1+3}\begin{vmatrix} b & c \\ d & e \end{vmatrix} \\ (-1)^{2+1}\begin{vmatrix} b & d \\ e & f \end{vmatrix} & (-1)^{2+2}\begin{vmatrix} a & d \\ d & f \end{vmatrix} & (-1)^{2+3}\begin{vmatrix} a & b \\ d & e \end{vmatrix} \\ (-1)^{3+1}\begin{vmatrix} b & d \\ c & e \end{vmatrix} & (-1)^{3+2}\begin{vmatrix} a & d \\ b & e \end{vmatrix} & (-1)^{3+3}\begin{vmatrix} a & b \\ b & c \end{vmatrix} \end{bmatrix}$$

$$=\begin{bmatrix} cf-e^2 & de-bf & be-cd \\ de-bf & af-d^2 & bd-ae \\ be-cd & bd-ae & ac-b^2 \end{bmatrix}$$

Observe that the ij^{th}, and ji^{th} entries of this matrix are same for every $i \neq j$. So, this matrix is symmetric.

Further, the determinant of *S* is $a\left(cf-e^2\right)+b\left(de-b^2\right)+d\left(be-cd\right)$

$=acf+2bde-ae^2-d^2\left(b+c\right)$

Consequently, $S^{-1} = \frac{1}{\det S} C^T$

$$= \frac{1}{acf + 2bde - ae^2 - d^2(b+c)} \begin{bmatrix} cf - e^2 & de - bf & be - cd \\ de - bf & af - d^2 & bd - ae \\ be - cd & bd - ae & ac - b^2 \end{bmatrix}$$ is also symmetric.