Step-1

We know that a matrix N is normal if $NN^H = N^H N$.

Consider N is normal matrix, then we have to show that $\|Nx\| = \|N^H x\|$ for every vector x.

If x is the vector then we know that norm is given by

$$(\|x\|)^2 = \langle x, x \rangle$$

Step-2

If we consider the vector $(\|Nx\|)^2$ then we have

$$(\|Nx\|)^2 = \langle Nx, Nx \rangle$$

Multiply the vectors by N^H, we get

$$\begin{aligned}(\|Nx\|)^2 &= \langle N^H Nx, N^H Nx \rangle \\ &= \langle N^H Nx, x \rangle\end{aligned}$$

Since $NN^H = N^H N$, so we have

$$\begin{aligned}(\|Nx\|)^2 &= \langle N^H Nx, x \rangle \\ &= \langle NN^H x, x \rangle\end{aligned}$$

Step-3

Now multiply the vectors by N^H, we get

$$\begin{aligned}(\|Nx\|)^2 &= \langle N^H NN^H x, N^H x \rangle \\ &= \langle N^H x, N^H x \rangle \\ &= (\|N^H x\|)^2\end{aligned}$$

Therefore, $(\|Nx\|)^2 = (\|N^H x\|)^2$.

Step-4

Let the i^{th} column of N be Ne_i, here $e_1,\ldots,e_n$ are the elements of a canonical basis.

Let the i^{th} row of N be $e_i^T N = \left(N^H e_i\right)^H$.

We know that the length of a matrix and its transpose are same so we have:

$$\begin{aligned}\left\|e_i^T N\right\| &= \left\|\left(N^H e_i\right)^H\right\| \\ &= \left\|\left(N^H e_i\right)\right\| \\ &= \left\|Ne_i\right\|\end{aligned}$$

Therefore, the length of i^{th} row of N is same as the i^{th} column, $\left\|e_i^T N\right\| = \left\|Ne_i\right\|$.