Step-1

Suppose that, the values $b_1 = 1$ and $b_2 = 7$ at times $t_1 = 1$ and $t_2 = 2$ are fitted by a line b = Dt through the origin.

Its need to solve D = 1 and 2D = 7 by least-squares

The least square solution to the equation Ax = b is $A^T A \hat{x} = A^T b$

We have D = 1 and 2D = 7

$$\Rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} D = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

Take
$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, x = D, b = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

Step-2

Now $A^T A \widehat{D} = A^T b$

First calculate $A^T A$

$$A^{T}A = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

= 1(1)+2(2)
= 1+4
= 5

And

$$A^{T}b = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \end{bmatrix}$$
$$= 1(1) + 2(7)$$
$$= 1 + 14$$
$$= 15$$

Use the values $A^T A = 5$ and $A^T b = 15$ in $A^T A \widehat{D} = A^T b$, then we get

$$5\widehat{D} = 15$$

$$\Rightarrow \widehat{D} = 3$$

Therefore, the least-square solution to D = 1 and 2D = 7 is $\widehat{D} = 3$.

Hence, the best line is b = 3t

Step-3

The sketch of the best line b = 3t is as shown below.

