Step-1

The objective is to determine the condition for b_1 and b_2, for that $Ax = b$ have a solution, also find two vector in the nullspace of A and the complete solution to $Ax = b$.

Provided matrix:

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & 7 \end{bmatrix},$$
$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Step-2

Consider the provided matrix,

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & 7 \end{bmatrix},$$
$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Add column in the matrix A at the end and perform the multiplication by 2 with 1st row and then subtract it from 2nd row as follows:

$$\begin{bmatrix} 1\,2\,0\,3\,b_1 \\ 2\,4\,0\,7\,b_2 \end{bmatrix} \rightarrow \begin{bmatrix} 1\,2\,0\,3\,b_1 \\ 0\,0\,0\,1\,b_2 - 2b_1 \end{bmatrix}$$

Again perform the multiplication by 3 with 2nd row and then subtract it from 1st row as follows:

$$\begin{bmatrix} 1\,2\,0\,3\,b_1 \\ 0\,0\,0\,1\,b_2 - 2b_1 \end{bmatrix} \rightarrow \begin{bmatrix} 1\,2\,0\,0\,7b_1 - 3b_2 \\ 0\,0\,0\,1\,b_2 - 2b_1 \end{bmatrix}$$

No condition, **there is a solution for any choice of** b_1 **and** b_2. Both the below matrices satisfy $Ax = 0$,

$$\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

The general solution is,

$$\boxed{x = \begin{bmatrix} 7b_1 - 3b_2 \\ 0 \\ 0 \\ b_2 - 2b_1 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}$$