

Step-1

Consider the equation:

$$Ax = b$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

The objective is to find the minimum length least squares solution $x^+ = A^+b$ to the above equation.

Step-2

First find the general solution to $A^T A \hat{x} = A^T b$.

Obtain, $A^T A$ as follows:

$$A^T A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Step-3

Assume that,

$$\hat{x} = (C, D, E)$$

So,

$$A^T A \hat{x} = A^T b$$
$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$
$$\begin{bmatrix} 3C + D + E \\ C + D + E \\ C + D + E \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$$

Step-4

Consider the following three equations:

$$3C + D + E = 4$$

$$C + D + E = 2$$

$$C + D + E = 2$$

Subtracting second (or third) equation from the first equation, obtained as $2C = 2$.

This gives $C = 1$.

Substituting this value in the second (or third) equation, obtained as, $D + E = 1$.

That is, $E = 1 - D$.

Thus, the general solution is, $\hat{x} = (1, D, 1 - D)$.

Step-5

Assume that, $\hat{x} = (1, D, 1 - D)$ lies in the row space of the matrix A .

So, \hat{x} can be expressed as a linear combination of the rows of the matrix A

Consider,

$$\begin{aligned}(1, D, 1 - D) &= a(1, 0, 0) + b(1, 0, 0) + c(1, 1, 1) \\ &= (a + b + c, c, c)\end{aligned}$$

Thus, $D = 1 - D$

$$D = \frac{1}{2}.$$

This gives, $\boxed{\hat{x} = \left(1, \frac{1}{2}, \frac{1}{2}\right)}.$

This solution fits for the plane $C + Dt + Ez = 2$ for $t = z = 1$.