Step-1

Given the matrix $A=\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, with Eigen values $\lambda_1 = 1$ and $\lambda_2 = 3$. The initial guess is $u_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

We need to apply the power method three times to the initial guess and we need to find the limiting vector.

The power method is $u_{k+1} = Au_k$,

When $k=0$ then,

$$\begin{aligned} u_1 &= Au_0 \\ &= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ -1 \end{bmatrix} \end{aligned}$$

Therefore, $\boxed{u_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}}$

Step-2

When $k=1$ then,

$$\begin{aligned} u_2 &= Au_1 \\ &= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ -4 \end{bmatrix} \end{aligned}$$

Therefore, $\boxed{u_2 = \begin{bmatrix} 5 \\ -4 \end{bmatrix}}$

Step-3

When $k=1$ then,

$$u_3 = Au_2$$
$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$
$$= \begin{bmatrix} 14 \\ -13 \end{bmatrix}$$

Therefore, $\boxed{u_3 = \begin{bmatrix} 14 \\ -13 \end{bmatrix}}$.

Step-4

We observe that the ratio between (1, 1) entry and (1, 2) entry of u_k is becoming -1 as k is going alarmingly high.

i.e., $\lim_{k \to \infty} u_{k+1} = \begin{bmatrix} t \\ -t \end{bmatrix}$

The unit vector along this is,

$$\frac{1}{\sqrt{t^2 + t^2}} \begin{bmatrix} t \\ -t \end{bmatrix} = \frac{1}{t\sqrt{2}} \begin{bmatrix} t \\ -t \end{bmatrix}$$
$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Therefore, the limiting vector is $\boxed{u_\infty = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$.