Step-1

Let P_1 = the projection matrix onto $a_1 = \frac{a_1 a_1^T}{a_1^T a_1}$

$$a_1 a_1^T = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$a_1^T a_1 = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1 + 0 = 1$$

Therefore, $P_1 = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Step-2

Let P_2 = the projection matrix onto $a_2 = \frac{a_2 a_2^T}{a_2^T a_2}$

$$a_2 a_2^T = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \end{pmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$a_2^T a_2 = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1 + 4 = 5$$

$$P_2 = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Step-3

$$P = P_1 P_2$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$P^2 = \frac{1}{25}\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}$$
$$= \frac{1}{25}\begin{bmatrix}1 & 2\\ 0 & 0\end{bmatrix}$$
$$\neq P$$

Therefore, *P*P is not a projection matrix.