Step-1

Consider the subspace **V** + **W**. Obviously, it is spanned by the vectors v_1, v_2, w_1, and w_2.

Note the following:

$$\begin{aligned} v_2 - v_1 + w_1 &= (1,0,1,0) - (1,1,0,0) + (0,1,0,1) \\ &= (0,-1,1,0) + (0,1,0,1) \\ &= (0,0,1,1) \\ &= w_2 \end{aligned}$$

Step-2

Thus, w_2 is a linear combination of v_1, v_2, and w_1. Also, these three vectors are linearly independent.

The space **V** + **W** is spanned by v_1, v_2, w_1, and w_2. Out of these four vectors, w_2 is dependent on the remaining three vectors.

Therefore, one of the bases of **V** + **W** is $\boxed{\{v_1, v_2, w_1\}}$.

Step-3

The space $\mathbf{V} \cap \mathbf{W}$ is the space of all vectors, which are common to **V** and **W**.

Note the following:

$$\begin{aligned} v_1 - v_2 &= (1,1,0,0) - (1,0,1,0) \\ &= (0,1,-1,0) \end{aligned}$$

Also, note the following:

$$\begin{aligned} w_1 - w_2 &= (0,1,0,1) - (0,0,1,1) \\ &= (0,1,-1,0) \end{aligned}$$

Step-4

Thus, any vector, which is a multiple of $(0,1,-1,0)$ lies in the space $\mathbf{V} \cap \mathbf{W}$.

Thus, a basis of $\mathbf{V} \cap \mathbf{W}$ is $\boxed{(0,1,-1,0)}$ and dimension of $\mathbf{V} \cap \mathbf{W} = 1$.