Step-1

Let $A_n = \left(a_{ij}\right)_{n\times n}$ where

$$\begin{aligned} A_1 &= (1+1) \\ &= (2) \end{aligned}$$

$\Rightarrow \det A_1 = 2$

Step-2

$$A_2 = \begin{pmatrix} 1+1 & 1+2 \\ 2+1 & 2+2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \det A_2 &= 8-9 \\ &= -1 \end{aligned}$$

Step-3

$$A_3 = \begin{pmatrix} 1+1 & 1+2 & 1+3 \\ 2+1 & 2+2 & 2+3 \\ 3+1 & 3+2 & 3+3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix}$$

Step-4

$$\Rightarrow \det A_3 = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$$

$= \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$ Adding â€“1 time the second row to the third and â€“1 time the first row to the second.

=0 (since two rows are qual.)

Step-5

For any $n \geq 3$

$$A_n = \begin{pmatrix} 1 & 3 & 4 \ldots n+1 \\ 3 & 4 & 5 \ldots\ n+2 \\ 4 & 5 & 6 \ldots n+3 \\ n+1 & n+2 & n+3 \ldots 2n \end{pmatrix}$$

Cleary subtracting 1st row from 2nd row and 2nd row from third two result in a matrix of two identical rows containing all entries equal to 1 and hence $\det A_n = 0$ for n ≥ 3

Thus, if a_{ij} is $i+j$, we have $\det A = 0$ (exception when $n = 1$ or 2)