

Step-1

Given that the equations of three planes:

$$x + y + z = 2$$

$$x + 2y + z = 3$$

$$2x + 3y + 2z = 5$$

We have to find that why the three columns form singular case, and we have to find two combinations of the columns that give $b = (2, 3, 5)$, and finally we have to find the value of c such that $b = (4, 6, c)$

Step-2

The three columns of the above equations are

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

The first and third columns are equal, so on column operation $C_3 \rightarrow C_3 - C_1$, we will get a zero column, and hence three columns will form the singular case.

Step-3

The given three planes can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Apply $R_3 \rightarrow R_3 - (R_1 + R_2)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

Step-4

Apply $R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Apply $R_1 \rightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Step-5

$$\Rightarrow x + z = 1$$

$$y = 1$$

Put $z = k$

$$\Rightarrow x = 1 - k$$

Step-6

Therefore the solution is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-k \\ 1 \\ k \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

From the above, the two combinations of the columns that give $b = (2, 3, 5)$ are

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, k \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \text{ where } k \text{ is any real number}$$

Step-7

If we multiply $b = (2, 3, 5)$ with 2, then we get $(4, 6, 10)$, compare this vector with $b = (4, 6, c)$, we get $\boxed{c=10}$