

Step-1

Given that

$$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \text{ And}$$

$$v_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\text{The vector } \begin{bmatrix} 3 \\ 9 \end{bmatrix} = C_1 V_1 + C_2 V_2$$

$$= C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 + C_2 \\ C_1 + 4C_2 \end{bmatrix}$$

This implies;

$$C_1 + C_2 = 3$$

$$C_1 + 4C_2 = 9$$

Step-2

Solving these and get

$$C_1 = 1, C_2 = 2$$

$$\text{Also, } \begin{bmatrix} 3 \\ 9 \end{bmatrix} = d_1 v_1 + d_2 v_2$$

$$= d_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + d_2 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2d_1 + d_2 \\ 3d_1 + 4d_2 \end{bmatrix}$$

This implies;

$$2d_1 + d_2 = 3$$

$$5d_1 + 4d_2 = 9$$

Step-3

Solving these and get $d_1 = 1, d_2 = 1$

The column of M comes from V_1, V_2 as;

$$\sum m_{ij} v_i$$

This implies;

$$V_1 = m_{11}v_1 + m_{21}v_2$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = m_{11} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + m_{21} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

This implies;

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2m_{11} + m_{21} \\ 5m_{11} + 4m_{21} \end{bmatrix}$$

$$2m_{11} + m_{21} = 1$$

$$5m_{11} + 4m_{21} = 1$$

Step-4

Solving these and get;

$$m_{11} = 1$$

$$m_{21} = -1$$

$$V_2 = m_{12}v_1 + m_{22}v_2$$

$$\begin{bmatrix} 1 \\ 4 \end{bmatrix} = m_{12} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + m_{22} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2m_{12} + m_{22} \\ 5m_{12} + 4m_{22} \end{bmatrix}$$

This implies;

$$2m_{12} + m_{22} = 1$$

$$5m_{12} + 4m_{22} = 4$$

Solving these and get;

$$m_{12} = 0$$

$$m_{22} = 1$$

Step-5

Hence,

$$M = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\begin{aligned} MC &= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= d \end{aligned}$$

Hence, M connects c to d