

Step-1

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$$

Compute A^2 and then fill the following blank:

$$\begin{aligned} e^{At} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} t & 3t \\ 0 & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} & \\ & \end{bmatrix} \\ &= \begin{bmatrix} e^t & \\ 0 & \end{bmatrix} \end{aligned}$$

Step-2

Calculate the following:

$$\begin{aligned} A \cdot A &= \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \\ A^2 &= \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Therefore,

$$\boxed{A^2 = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}}$$

Step-3

Next put the value of A^2 in the expansion of e^{At} .

$$\begin{aligned} e^{At} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} t & 3t \\ 0 & 0 \end{bmatrix} + \frac{1}{2!} \begin{bmatrix} t^2 & 3t^2 \\ 0 & 0 \end{bmatrix} + \dots \\ &= \begin{bmatrix} e^t & 3(e^t - 1) \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Therefore,

$$\boxed{e^{At} = \begin{bmatrix} e^t & 3(e^t - 1) \\ 0 & 1 \end{bmatrix}}$$