

MA215 Probability Theory

Assignment 03

1. Show that if the conditional probabilities exist, then

$$\begin{aligned} P(A_1 \cap A_2 \cap \cdots \cap A_n) \\ = P(A_1)P(A_2 | A_1)P(A_3 | A_1 \cap A_2) \cdots P(A_n | A_1 \cap A_2 \cap \cdots \cap A_{n-1}). \end{aligned}$$

2. Urn A has 3 red balls and 2 white balls, and urn B has 2 red balls and 5 white balls. A fair coin is tossed; if it lands heads up, a ball is drawn from urn A and otherwise a ball is drawn from urn B .

- (a) What is the probability that a red ball is drawn?
(b) If a red ball is drawn, what is the probability that the coin landed heads up?

3. Urn A has 4 red, 3 blue and 2 green balls. Urn B has 2 red, 3 blue and 4 green balls. A ball is drawn from urn A and put into urn B and then a ball is drawn from urn B .

- (a) What is the probability that a red ball is drawn from urn B ?
(b) If a red ball is drawn from urn B , what is the probability that a red ball was drawn from urn A ?

4. There are 3 cabinets A , B , and C , each of which has 2 drawers. Each drawer contains 1 coin; A has 2 gold coins, B has 2 silver coins and C has 1 gold and 1 silver coin. Take an experiment as a cabinet is chosen at random, one drawer is opened and a silver coin has found. What is the probability that the other drawer in that cabinet contains a silver coin?

5. If B is an event with $P(B) > 0$, show that the set function $Q(A) = P(A | B)$ is a probability measure. Thus, we can use the following formulas in lectures

$$\begin{aligned} P(A \cup C | B) &= P(A | B) + P(C | B) - P(A \cap C | B), \\ P(A^c | B) &= 1 - P(A | B). \end{aligned}$$

6. Show that if A , B , and C are mutually independent, then $A \setminus B$ and C are independent and $A \cup B$ and C are independent.
7. The probability of the closing of the i th relay in the circuits shown is given by p_i ; $i = 1, 2, 3, 4, 5$. If all relays function independently, what is the probability that a current flows between A and B for the respective circuits? (see the next page for the two cases of circuit)

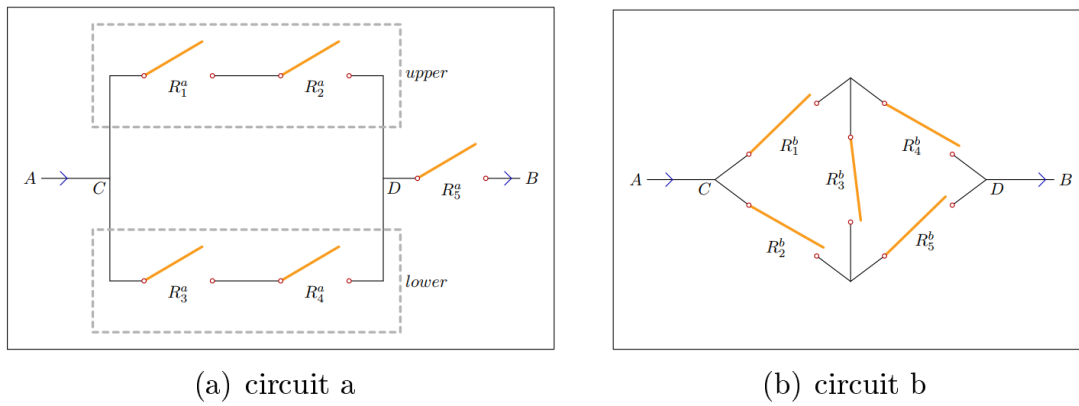


Figure 1: Figure of Problem 7