## Step-1

a) Given system is

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

We have to find the values of  $b_1$ ,  $b_2$  and  $b_3$  or the condition on  $b_1$ ,  $b_2$  and  $b_3$  for which the given system is solvable.

## Step-2

Consider the matrix [A,B] as follows:

$$\begin{bmatrix} 1 & 4 & 2 & b_1 \\ 2 & 8 & 4 & b_2 \\ -1 & -4 & -2 & b_3 \end{bmatrix}$$

$$\begin{array}{ccccc}
R_2 - 2R_1 & 1 & 4 & 2 & b_1 \\
0 & 0 & 0 & b_2 - 2b_1 \\
0 & 0 & 0 & b_3 + b_1
\end{array}$$

Therefore the above system is solvable if  $b_3 + b_1 = 0$  and  $b_2 - 2b_1 = 0$ 

## Step-3

b) Given system is

$$\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

We have to find the values of  $b_1$ ,  $b_2$  and  $b_3$  or the condition  $b_1$ ,  $b_2$  and  $b_3$  for which the given system is solvable.

## Step-4

Consider the following matrix

$$[A,B] = \begin{bmatrix} 1 & 4 & b_1 \\ 2 & 9 & b_2 \\ -1 & -4 & b_3 \end{bmatrix}$$

$$\begin{array}{cccc} R_2 - 2R_1 & 1 & 4 & b_1 \\ R_3 + R_1 & 0 & 1 & b_2 - 2b_1 \\ 0 & 0 & b_3 + b_1 \end{array}$$

Therefore the given system is solvable if  $b_3 + b_1 = 0$