Step-1

Let the third column of Q is $u_3 = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$

Then $Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{14} & \alpha \\ 1/\sqrt{3} & 2/\sqrt{14} & \beta \\ 1/\sqrt{3} & -3/\sqrt{14} & \lambda \end{bmatrix}$

$Q^T Q = I$

$$\begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{14} & 2/\sqrt{14} & -3/\sqrt{14} \\ \alpha & \beta & \gamma \end{bmatrix}\begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{14} & \alpha \\ 1/\sqrt{3} & 2/\sqrt{14} & \beta \\ 1/\sqrt{3} & -3/\sqrt{14} & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step-2

$$\Rightarrow \begin{bmatrix} 1 & 0 & \dfrac{\alpha+\beta+\gamma}{\sqrt{3}} \\ 0 & 1 & \dfrac{\alpha+2\beta-3\gamma}{\sqrt{14}} \\ \dfrac{\alpha+\beta+\gamma}{\sqrt{3}} & \dfrac{\alpha+2\beta-3\gamma}{\sqrt{14}} & \alpha^2+\beta^2+\gamma^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If two matrices are equal then its corresponding entries are equal

Therefore $\dfrac{\alpha+\beta+\gamma}{\sqrt{3}} = 0, \dfrac{\alpha+2\beta-3\gamma}{\sqrt{14}} = 0$ and $\alpha^2+\beta^2+\gamma^2 = 1$

By the consequence of the 2nd equation, we have $\alpha = 3\gamma - 2\beta$

Using this in (1), we get $4\gamma = \beta$

So, $\alpha = -5\gamma$

Using these in the 3rd equation, we get $\gamma = \dfrac{1}{\sqrt{42}}$

From this, we get $\beta = \dfrac{4}{\sqrt{42}}$ and $\alpha = \dfrac{-5}{\sqrt{42}}$

Therefore, the required third column is $u_3 = \begin{bmatrix} -5/\sqrt{42} \\ 4/\sqrt{42} \\ 1/\sqrt{42} \end{bmatrix}$

Step-3

To verify the orthogonality of the columns of the matrix, let us consider $u_1 = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \ u_2 = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ -3/\sqrt{14} \end{bmatrix}, \ u_3 = \begin{bmatrix} -5/\sqrt{42} \\ 4/\sqrt{42} \\ 1/\sqrt{42} \end{bmatrix}$

$$u_1^T u_3 = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} -5/\sqrt{42} \\ 4/\sqrt{42} \\ 1/\sqrt{42} \end{bmatrix}$$

$$= \frac{-5+4+1}{\sqrt{126}}$$

$$= 0$$

$$u_2^T u_3 = \begin{bmatrix} 1/\sqrt{14} & 2/\sqrt{14} & -3/\sqrt{14} \end{bmatrix} \begin{bmatrix} -5/\sqrt{42} \\ 4/\sqrt{42} \\ 1/\sqrt{42} \end{bmatrix}$$

$$= \frac{-5+8-3}{\sqrt{588}}$$

$$= 0$$

Also, $\|u_3\| = \sqrt{\left(\frac{-5}{\sqrt{42}}\right)^2 + \left(\frac{4}{\sqrt{42}}\right)^2 + \left(\frac{1}{\sqrt{42}}\right)^2}$

$$= \sqrt{\frac{25+16+1}{42}}$$

$$= 1$$

Therefore, the third column u_3 is a unit vector and that is orthogonal to the other columns u_1, u_2

Step-4

On the other hand, we consider the rows of the matrix as

$$v_1 = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{14} & -5/\sqrt{42} \end{bmatrix}$$
$$v_2 = \begin{bmatrix} 1/\sqrt{3} & 2/\sqrt{14} & 4/\sqrt{42} \end{bmatrix}$$
$$v_3 = \begin{bmatrix} 1/\sqrt{3} & -3/\sqrt{14} & 1/\sqrt{42} \end{bmatrix}$$

$$\begin{aligned} v_1^T v_3 &= \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{14} & -5/\sqrt{42} \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} \\ -3/\sqrt{14} \\ 1/\sqrt{42} \end{bmatrix} \\ &= \frac{1}{3} - \frac{3}{14} - \frac{5}{42} \\ &= \frac{14-9-5}{42} \\ &= 0 \end{aligned}$$

Step-5

$$\begin{aligned} v_2^T v_3 &= \begin{bmatrix} 1/\sqrt{3} & 2/\sqrt{14} & 4/\sqrt{42} \end{bmatrix} \begin{bmatrix} 1/\sqrt{3} \\ -3/\sqrt{14} \\ 1/\sqrt{42} \end{bmatrix} \\ &= \frac{1}{3} - \frac{6}{14} + \frac{4}{42} \\ &= \frac{14-18+4}{42} \\ &= 0 \end{aligned}$$

$$\begin{aligned} \|v_1\| &= \sqrt{\frac{1}{3} + \frac{1}{14} + \frac{25}{42}} \\ &= \frac{14+3+25}{42} \\ &= 1 \end{aligned}$$

Step-6

$$
\begin{aligned}
\|v_2\| &= \sqrt{\frac{1}{3}+\frac{4}{14}+\frac{16}{42}} \\
&= \frac{14+12+16}{42} \\
&= 1 \\
\|v_3\| &= \sqrt{\frac{1}{3}+\frac{9}{14}+\frac{1}{42}} \\
&= \frac{14+27+1}{42} \\
&= 1
\end{aligned}
$$

Therefore, the rows of the matrix are mutually orthogonal and are unit vectors.