Step-1

Consider the following systems

$$\begin{cases} 2x-y+z=0 \\ 2x-y+z=0 \\ 4x+y+z=2 \end{cases}$$ â€¦â€¦ (1)

$$\begin{cases} 2x+2y+z=0 \\ 4x+4y+z=0 \\ 6x+6y+z=0 \end{cases}$$ â€¦â€¦ (2)

Step-2

Write the system (1) in matrix notation:

$$\underbrace{\begin{bmatrix} 2 & -1 & 1 \\ 2 & -1 & 1 \\ 4 & 1 & 1 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X}=\underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{\mathbf{b}}$$

Notice that, in the above notation row 1 and row 2 of the matrix A are same.

Step-3

Augmented matrix associated with the above one is,

$$[A \mid \mathbf{b}] = \left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 2 & -1 & 1 & 0 \\ 4 & 1 & 1 & 2 \end{array}\right]$$

$$R_2 \to R_2 - R_1$$

$$\approx \left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 2 \end{array}\right]$$

$$R_2 \leftrightarrow R_3$$

$$\approx \left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 4 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_2 \to R_2 - 2R_1$$

$$\approx \left[\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 0 & 3 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Step-4

From the calculation of $[A \mid \mathbf{b}]$, notice that, if row 1 of a matrix is equal to row 2, then row 2 is equal to zero by the operation $R_2 \to R_2 - R_1$ and then exchange zero row with row 3 by the operation $R_2 \leftrightarrow R_3$, finally observe that there is **no third pivot**.[There is no non-zero element in third row and third column.]

Step-5

Write the system (2) in matrix notation:

$$\underbrace{\begin{bmatrix} 2 & 2 & 1 \\ 4 & 4 & 1 \\ 6 & 6 & 1 \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{\mathbf{b}_1}$$

Notice that, in the above notation columns 1 and 2 of the matrix A_1 are same.

Step-6

Augmented matrix associated with the above one is,

$$[A_1 \mid \mathbf{b}_1] = \left[\begin{array}{ccc|c} 2 & 2 & 1 & 0 \\ 4 & 4 & 1 & 0 \\ 6 & 6 & 1 & 2 \end{array}\right]$$

$$R_2 \to R_2 - 2R_1$$
$$R_3 \to R_3 - 3R_1$$

$$\approx \left[\begin{array}{ccc|c} 2 & 2 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -2 & 2 \end{array}\right]$$

$$R_2 \leftrightarrow R_3$$

$$\approx \left[\begin{array}{ccc|c} 2 & 2 & 1 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

$$R_2 \to R_2 - 2R_3$$

$$\approx \left[\begin{array}{ccc|c} 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

From the calculation of $[A_1 \mid \mathbf{b}_1]$, notice that, if column 1 of a matrix is equal to column 2, there is **no second pivot**.[There is no non-zero element in second row and second column.]