Step-1

Given that $b = 0,8,8,20$ at $t = 0,1,3,4$

Write an equation for the closest parabola $b = C + Dt + Et^2$

Step-2

First write the equations that would hold if a line could go through all four points.

Then, every $C + Dt + Et^2$ agree exactly with b.

Now $Ax = b$ is;

$$C + D(0) + E(0)^2 = 0$$
$$C + D(1) + E(1)^2 = 8$$
$$C + D(3) + E(3)^2 = 8$$
$$C + D(4) + E(4)^2 = 20$$

Step-3

The matrix form of the above system is;

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}$$
$$x = \begin{bmatrix} C \\ D \\ E \end{bmatrix}$$
$$b = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

Therefore, the closest parabola is $\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}\begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}}$.

Step-4

Now, to find the normal equations $A^T A\hat{x} = A^T b$

Now,

$$A^T A\hat{x} = A^T b$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 1 & 9 & 16 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix}\begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 1 & 9 & 16 \end{bmatrix}\begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$$

Step-5

Continuation to the above

$$\begin{bmatrix} \begin{pmatrix} 1(1)+1(1) \\ +1(1)+1(1) \end{pmatrix} & \begin{pmatrix} 1(0)+1(1) \\ +1(3)+1(4) \end{pmatrix} & \begin{pmatrix} 1(0)+1(1) \\ +1(9)+1(16) \end{pmatrix} \\ \begin{pmatrix} 0(1)+1(1) \\ +3(1)+4(1) \end{pmatrix} & \begin{pmatrix} 0(0)+1(1) \\ +3(3)+4(4) \end{pmatrix} & \begin{pmatrix} 0(0)+1(1) \\ +3(9)+4(16) \end{pmatrix} \\ \begin{pmatrix} 0(1)+1(1) \\ +9(1)+16(1) \end{pmatrix} & \begin{pmatrix} 0(0)+1(1) \\ +9(3)+16(4) \end{pmatrix} & \begin{pmatrix} 0(0)+1(1) \\ +9(9)+16(16) \end{pmatrix} \end{bmatrix}\begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{pmatrix} 1(0)+1(8) \\ +1(8)+1(20) \end{pmatrix} \\ \begin{pmatrix} 0(0)+1(8) \\ +3(8)+4(20) \end{pmatrix} \\ \begin{pmatrix} 0(0)+1(8) \\ +9(8)+16(20) \end{pmatrix} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 8 & 26 \\ 8 & 26 & 92 \\ 26 & 92 & 338 \end{bmatrix}\begin{bmatrix} \hat{C} \\ \hat{D} \\ \hat{E} \end{bmatrix} = \begin{bmatrix} 36 \\ 112 \\ 400 \end{bmatrix}$$

Hence the three normal equations are;

$$\boxed{\begin{aligned} &4\hat{C}+8\hat{D}+26\hat{E}=36 \\ &8\hat{C}+26\hat{D}+92\hat{E}=112 \\ &26\hat{C}+92\hat{D}+338\hat{E}=400 \end{aligned}}$$