Step-1

Given that $a+b=c+d$

$Ax=\lambda x$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}=\lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} a+b \\ c+d \end{bmatrix}=\lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} a+b \\ a+b \end{bmatrix}=\lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\Rightarrow (a+b)\begin{bmatrix} 1 \\ 1 \end{bmatrix}=\lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Therefore $\lambda=a+b$ is one eigen value.

Trace of $A=\lambda_1+\lambda_2$

$a+d=(a+b)+\lambda_2$

$$\begin{aligned}\lambda_2 &= a+d-a-b \\ &= d-b\end{aligned}$$

Therefore the eigen values are $\lambda_1=a+b, \lambda_2=d-b$

Step-2

To find eigen vector for $\lambda_1=a+b$, we solve $(A-\lambda_1 I)x=0$

$$\begin{bmatrix} a-(a+b) & b \\ c & d-(a+b) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} -b & b \\ c & d-(c+d) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ by replacing *a* +*b* with *c*+ *d*.

$$\begin{bmatrix} -b & b \\ c & -c \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Applying the row operations on the coefficient matrix of this homogeneous system, we get

$R_2 \to bR_2 + cR_1, R_1 / -b \Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$x_1 - x_2 = 0$

Putting $x_1 = 1$, the solution set $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to $\lambda_1 = a + b$

Step-3

Similarly, To find eigen vector for $\lambda_2 = d - b$, we solve $(A - \lambda_2 I)x = 0$

$\begin{bmatrix} a-(d-b) & b \\ c & d-(d-b) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} (a+b)-d & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} (c+d)-d & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ by replacing $c+d$ with $a+b$.

$\begin{bmatrix} c & b \\ c & b \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Applying the row operations on the coefficient matrix of this homogeneous system, we get

$R_2 \to R_2 - R_1 \Rightarrow \begin{bmatrix} c & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

This is the reduced matrix and so, we rewrite the homogeneous system from this.

$cx_1 + bx_2 = 0$

Putting $x_1 = 1$, the solution set $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -b/c \end{bmatrix}$ is the eigen vector corresponding to $\lambda_2 = d - b$

Step-4

Thus the eigen values are $\lambda_1 = a+b$ and $\lambda_2 = d-b$ while the respective eigen vectors are $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\-b/c\end{bmatrix}$