

Step-1

Given lines are $3x + y = b_1$, $6x + 2y = b_2$. We have to answer the following questions:

- 1) We have to say that how the two lines are related.
- 2) When do the two lines represent the same line.
- 3) In that case we have to find a vector which is perpendicular to (b_1, b_2)
- 4) We have to find the null space of the matrix.
- 5) We have to find one particular vector in that null space.

Step-2

- 1) For the lines the ratios of the coefficients are equal.

$$\text{Since } \frac{3}{6} = \frac{1}{2}$$

Therefore $3x + y = b_1$, $6x + 2y = b_2$ are parallel.

Step-3

- 2) They are the same line if

$$\frac{3}{6} = \frac{1}{2} = \frac{b_1}{b_2}$$
$$\Rightarrow \boxed{b_2 = 2b_1}$$

Step-4

And this represents $(b_1, b_2) = (b_1, 2b_1) = b_1(1, 2)$ is perpendicular to a vector $(2, -1)$

For the finding the null space we consider the system

$$3x + y = 0$$
$$6x + 2y = 0$$

From this, the null space of the matrix is the line $3x + y = 0$

Step-5

Now

$$3x + y = 0$$

$$\Rightarrow y = -3x$$

$$\text{If } x = 1$$

$$\Rightarrow y = -3$$

One particular vector in that nullspace is $(1, -3)$.