

Step-1

(a) The set $\{(1, 2, 0), (0, 1, -1)\}$ is linearly independent as one vector is not a scalar multiple of the other.

But the dimension of R^3 is 3 while there are only two vectors present in the given set.

So, these vectors cannot span R^3

In other words, there are vectors in R^3 which cannot be written as linear combinations of $\{(1, 2, 0), (0, 1, -1)\}$

For example, $(1, 2, 3) = a(1, 2, 0) + b(0, 1, -1)$ is not satisfied by any scalars a and b .

Therefore the set is not a basis for R^3

Step-2

(b) We know that the maximum number of linearly independent vectors that can span R^3 is 3 and forms a basis.

But in our case, there are four vectors in the set $\{(1, 1, -1), (2, 3, 4), (-4, 1, -1), (0, 1, -1)\}$

So, this is not linearly independent and thus cannot form a basis to R^3

Step-3

(c) Let $a(1, 2, 2) + b(-1, 2, 1) + c(0, 8, 0) = (0, 0, 0)$

$$a - b = 0 \quad \text{--- (1)}$$

$$2a + 2b + 8c = 0 \quad \text{--- (2)}$$

$$2a + b = 0 \quad \text{--- (3)}$$

By (1) + (3), we have $3a = 0$

$$\Rightarrow a = 0$$

Using this in (1), we get $b = 0$

Using these in (2), we get $c = 0$

Hence the vector $\{(1, 2, 2), (-1, 2, 1), (0, 8, 0)\}$ is a linearly independent set.

Also, the number of vectors in this set = 3 = dimension of R^3

So, these vectors can span R^3

Therefore, $\{(1, 2, 2), (-1, 2, 1), (0, 8, 0)\}$ forms a basis to R^3

Step-4

(d) Let $a(1, 2, 2) + b(-1, 2, 1) + c(0, 8, 6) = 0$

$$\Rightarrow a - b = 0$$

$$2a + 2b + 8c = 0$$

$$2a + b + 6c = 0$$

$$\Rightarrow a - b = 0 \quad \text{--- (1)}$$

$$a + b + 4c = 0 \quad \text{--- (2)}$$

$$2a + b + 6c = 0 \quad \text{--- (3)}$$

Using $a = b$ from (1) in (2) and (3), we get $2b + 4c = 0$

Consequently, $a = b = -2c$ such that the equation is satisfied.

That means one vector is linearly dependent of the other two.

Therefore, they cannot form a basis to R^3