

Step-1

Given

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-2

First let us find the cofactors of 1, the elements in first row of A , that is

$$a_{11} = A_{11} = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

Cofactor of

$$= 0$$

$$a_{12} = A_{12} = - \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

Cofactor of

$$= -(0-1)$$

$$= 1$$

Step-3

$$a_{13} = A_{13} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix}$$

Cofactor of

$$= 0$$

$$a_{14} = A_{14} = - \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Cofactor of

$$= \boxed{-1}$$

Step-4

Now determinant of A is

$$\begin{aligned}\det A &= a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} + a_{14}A_{14} \\ &= 0(0) + 1(1) + 0(0) + 0(-1) \\ &= \boxed{1}\end{aligned}$$

Step-5

(B)

The elements in first row of B ,

$$b_{11} = B_{11} = \begin{vmatrix} 3 & 4 & 5 \\ 7 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$

Cofactor of

$$\begin{aligned}&= 3(8) - 4(7) + 5(0) \\ &= 24 - 28 \\ &= -4\end{aligned}$$

$$b_{12} = B_{12} = - \begin{vmatrix} 0 & 4 & 5 \\ 6 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$

Cofactor of

$$\begin{aligned}&= -[0(8-0) - 4(6-0) + 5(0)] \\ &= 24\end{aligned}$$

Step-6

$$b_{13} = B_{13} = \begin{vmatrix} 0 & 3 & 5 \\ 6 & 7 & 9 \\ 0 & 0 & 1 \end{vmatrix}$$

Cofactor of

$$\begin{aligned}&= 0(7-0) - 3(6-0) + 5(0-0) \\ &= -18\end{aligned}$$

$$b_{14} = B_{14} = - \begin{vmatrix} 0 & 3 & 4 \\ 6 & 7 & 8 \\ 0 & 0 & 0 \end{vmatrix}$$

Cofactor of

$$= 0$$

Step-7

Now, determinant of B is

$$\det B = b_{11}B_{11} + b_{12}B_{12} + b_{13}B_{13} + b_{14}B_{14}$$

$$= 0(-4) + 0(24) + 1(-18) + 2(0)$$

$$= \boxed{-18}$$