Step-1

Given that $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

The characteristic equation is $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix}=0$$

$$\Rightarrow \lambda^2-(a+d)\lambda+(ad-bc)=0$$

Step-2

Cayley â€“ Hamilton theorem states that every square matrix satisfies its characteristic equation.

i.e., when the eigen value λ is replaced by the matrix A in $|A-\lambda I|=0$, it has to get satisfied.

i.e., $A^2-(a+d)A+(ad-bc)I$:

$$A^2=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

Step-3

$A^2-(a+d)A+(ad-bc)I=$

$$=\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}-(a+d)\begin{bmatrix} a & b \\ c & d \end{bmatrix}+(ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} a^2+bc-a^2-ad+ad-bc & ab+bd-ab-db+0 \\ ac+cd-ac-cd+0 & bc+d^2-ad-d^2+ad-bc \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$=0$$

Therefore the matrix A satisfied its characteristic equation.

Thus, Cayley â€“ Hamilton theorem is verified.