Step-1

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Given that R is a rectangular m by n matrix and A is m by m symmetric matrix.

(a) We have to show that $R^T AR$ is symmeric and find the shape of the matrix.

Now

$$\begin{aligned} \left(R^T AR\right)^T &= R^T A\left(R^T\right)^T && \left(\text{Since } \left(AB\right)^T = B^T A^T\right) \\ &= R^T AR && \left(\text{Since } \left(R^T\right)^T = R\right) \end{aligned}$$

Since $\left(R^T AR\right)^T = R^T AR$

So $R^T AR$ is symmetric.

Step-2

Now we have to fins the shape of the matrix.

The transpose of $R^T AR$ is $\left(R^T AR\right)^T = R^T A^T (R^T)^T = R^T A^T R$ = n by n order.

Since R is m by n

So R^T is n by m

Now R^T is n by m, A is m by m and A^T is m by m.

So R^T times A^T is of order n by m

And $R^T A^T$ times R is of order n by n.

Hence the shape of $R^T AR$ is n by n.

Step-3

(b) We have to show why $R^T R$ has no negative numbers on its diagonal.

We have

$$\left(R^T R\right)_{ij} = \left(column\ j\ of\ R\right).\left(column\ j\ of\ R\right)$$
$$= square\ of\ \ column\ \ j$$

In R^T and R diagonal elements becomes squares of column elements and squares are always positive.

Hence the numbers on the diagonal of $R^T R$ are not negative.