

## Step-1

$$A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

Given matrix,

$$|A_{11}| = 2 > 0$$

$$|A_{22}| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 4 - 1$$

$$= 3 > 0$$

$$|A_{33}| = |A|$$

$$= 2(4 - 1) + 1(-2 - 1) - (1 + 2)$$

$$= 6 - 3 - 3$$

$$= 0$$

So  $A$  is not positive definite.

## Step-2

$$B = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$$

Given matrix,

$$|B_{11}| = 2 > 0$$

$$|B_{22}| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 4 - 1$$

$$= 3 > 0$$

$$|B_{33}| = |B|$$

$$= 2(4 - 1) + 1(-2 + 1) - (-1 + 2)$$

$$= 6 - 1 - 1$$

$$= 4 > 0$$

Thus  $|B_{11}| > 0$ ,  $|B_{22}| > 0$ ,  $|B_{33}| > 0$ .

Therefore,  $B$  is positive definite.

### Step-3

$$\text{Given matrix, } C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}^2$$

$$C = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 5 \end{pmatrix}$$

### Step-4

Now calculate determinant of left sub matrices

$$|C_{11}| = 5 > 0$$

$$\begin{aligned} |C_{22}| &= \begin{vmatrix} 5 & 2 \\ 2 & 2 \end{vmatrix} \\ &= 10 - 4 \\ &= 6 > 0 \end{aligned}$$

$$\begin{aligned} |C_{33}| &= |C| \\ &= 5(6) - 2(8) + 1(2) \\ &= 30 - 16 + 2 \\ &= 16 > 0 \end{aligned}$$

Thus  $|C_{11}| > 0$ ,  $|C_{22}| > 0$ ,  $|C_{33}| > 0$ .

Therefore  $C$  is positive definite.