Step-1

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(a) Let $L_1D_1U_1 = L_2D_2U_2$

Multiplying left sides with L_1^{-1} both sides gives

$$L^{-1}{}_1L_1D_1U_1 = L_1^{-1}L_2D_2U_2$$
$$ID_1U_1 = L_1^{-1}L_2D_2U_2$$

Since $L^{-1}{}_1L_1 = I, ID_1U_1 = D_1U_1$

$$D_1U_1 = L_1^{-1}L_2D_2U_2$$

Also, by multiplying right side with U_2^{-1}, we get $D_1U_1 = L_1^{-1}L_2D_2U_2$

Step-2

$$D_1U_1U_2^{-1} = L_1^{-1}L_2D_2U_2U_2^{-1}$$
$$D_1U_1U_2^{-1} = L_1^{-1}L_2D_2I$$

Since $U^{-1}{}_1U_2^{-1} = I, ID_2 = D_2$

$$D_1U_1U_2^{-1} = L_1^{-1}L_2D_2$$

We know that the inverse of an upper triangular matrix is upper triangular and the inverse of the lower triangular matrix lower, product of lower triangular matrices is a lower triangular matrix and the product of upper triangular matrices is an upper triangular matrix.

Therefore $\boxed{L_1^{-1}L_2D = DU_1U_2^{-1}}$

Step-3

(b) $D_1U_1U_2^{-1} = L_1^{-1}L_2D_2$ is possible if and only if $L_1^{-1}L_2 = U_1U_2^{-1} = I$

Consequently, $D_1 = D_2$

Also, $L_1^{-1}L_2 = I \Rightarrow L_1 = L_2$ and $U_1U_2^{-1} = I \Rightarrow U_1 = U_2$

Therefore, the *LDU* factorization is unique for every invertible matrix *A*.