

## Step-1

(a)

To find the first column of  $3 \times 3$  matrix, consider the following vector:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Project the vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  onto  $x$ - $y$  plane, and get the vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ .

## Step-2

To find the second and third column of  $3 \times 3$  matrix, consider the following vectors:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ And } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Project the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  onto  $x$ - $y$  plane, and get the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ .

Now, project the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  onto  $x$ - $y$  plane, and get the vector  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .

## Step-3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Therefore, the matrix transformation by projecting every vector onto the  $x$ - $y$  plane is given by  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ .

## Step-4

(b)

To find the first column of  $3 \times 3$  matrix, consider the following vector:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Reflect the vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  through the  $x$ - $y$  plane, we get the vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ .

## Step-5

To find the second and third column of  $3 \times 3$  matrix, consider the following vectors:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ And } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Reflect the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  through the  $xy$ -plane, and get the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ .

Now, reflect the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  through the  $xy$ -plane, and get the vector  $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ .

## Step-6

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Therefore, the matrix of transformation by reflecting every vector through the  $x$ - $y$  plane is given by  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ .

## Step-7

(c)

To find the first column of  $3 \times 3$  matrix, consider the following vector:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Rotate the vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  in the  $x$ - $y$  plane by  $90^\circ$ , and get the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ .

To find the second and third column of  $3 \times 3$  matrix, consider the following vectors:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ And } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Rotate the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  in the  $x$ - $y$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  in the  $x$ - $y$  plane by  $90^\circ$ , and get the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ .

## Step-8

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, the matrix of transformation by rotating the  $x$ - $y$  plane through  $90^\circ$ , leaving the  $z$ -axis alone is given by  $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .

## Step-9

(d)

To find the first column of  $3 \times 3$  matrix, consider the following vector:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Rotate the vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  in the  $x$ - $y$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  in  $x$ - $z$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  in  $y$ - $z$  plane by  $90^\circ$ , we get vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ .

## Step-10

To find the second column of  $3 \times 3$  matrix, consider the following vectors:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Rotate the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  in the  $x$ - $y$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$  in  $x$ - $z$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$  in  $y$ - $z$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ .

## Step-11

To find the third column of  $3 \times 3$  matrix, consider the following vector:

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Rotate the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  in the  $x$ - $y$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  in  $x$ - $z$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$  in  $y$ - $z$  plane by  $90^\circ$ , and get vector  $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ .

Therefore, the matrix transformation by rotating the  $x$ - $y$  plane, then  $x$ - $z$  plane and then  $y$ - $z$  through  $90^\circ$ , is given by  $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ .

## Step-12

(e)

To find the first column of  $3 \times 3$  matrix, consider the following vector:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

## Step-13

Rotate the vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  in the  $x$ - $y$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$  in  $x$ - $z$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  in  $y$ - $z$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ .

## Step-14

To find the second column of  $3 \times 3$  matrix, consider the following vector:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Rotate the vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  in the  $x$ - $y$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$  in  $x$ - $z$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$  in  $y$ - $z$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ .

## Step-15

To find the third column of  $3 \times 3$  matrix, consider the following vector:

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Rotate the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  in the  $x$ - $y$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  in  $x$ - $z$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ .

Rotate the vector  $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$  in  $y$ - $z$  plane by  $180^\circ$ , and get vector  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ .

## Step-16

Therefore, the matrix of transformation by rotating the  $x$ - $y$  plane, then  $x$ - $z$  plane and then  $y$ - $z$  by  $180^\circ$ , is given by  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .