Step-1

We know that $Ax = b$ has a nonnegative solution or there exists a vector y, such that $yA \geq 0$ and $yb < 0$.

Let $y = (1, -1)$.

Therefore,

$$yA = (1, -1)\begin{bmatrix} 1 & 3 & -5 \\ 1 & -4 & -7 \end{bmatrix}$$
$$= (1 \times 1 + (-1) \times 1, 1 \times 3 + (-1) \times (-4), 1 \times (-5) + (-1) \times (-7))$$
$$= (0, 7, 2)$$

Also,

$$yb = (1, -1)\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$
$$= 1 \times 2 + (-1) \times 3$$
$$= -1$$
$$< 0$$

Step-2

Thus, we have produced a vector $y = (1, -1)$ such that $yA \geq 0$ and $yb < 0$. Therefore, for $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & -4 & -7 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, the equation $Ax = b$ has no nonnegative solution.