MA215 Probability Theory

Assignment 11

1. Let $Y = e^X$ with $X \sim N(\mu, \sigma^2)$. Use the following two methods to obtain $E(Y)$.

   (a) First obtain the p.d.f. $f_Y(y)$ of Y, and then find $E(Y)$ by using $f_Y(y)$.

   (b) Find $E(Y)$ directly by viewing Y as a function of X and then using the formula of getting the expected value of a function of the random variable X.

2. (a) Suppose the random variable X obeys the uniformly distribution over interval $[a, b]$. Find $E(X^2)$ and $E(X^2) - [E(X)]^2$.

   (b) Suppose $X \sim N(\mu, \sigma^2)$. Find $E(X^2)$ and $E(X^2) - [E(X)]^2$.

3. (a) The p.d.f. of X is given by

$$f_X(x) = \begin{cases} \frac{1}{x(\ln 3)}, & 1 < x < 3, \\ 0, & \text{otherwise.} \end{cases}$$

   Find $E(X)$, $E(X^2)$, and $E(X^3)$.

   (b) Use the results of part (a) to determine $E(X^3 + 2X^2 - 3X + 1)$.

4. The p.d.f. X is given by

$$f_X(x) = \begin{cases} \frac{x}{2}, & 0 < x \leqslant 1, \\ \frac{1}{2}, & 1 < x \leqslant 2, \\ \frac{3-x}{2}, & 2 < x \leqslant 3, \\ 0, & \text{otherwise.} \end{cases}$$

   Find the expectation of $Y = X^2 - 5X + 3$.

5. The two continuous random variables X and Y have joint p.d.f.

$$f(x, y) = \begin{cases} x + y, & 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1, \\ 0, & \text{otherwise.} \end{cases}$$

   Find $E[(X + Y)^2]$.