Step-1

Let A be n by n matrix.

The null space of A contain $x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

It gives $\boxed{Ax = 0}$ then $\boxed{A^T Ax = 0}$

Therefore, the null space of $A^T A$ contains $(1,1,1,1)$.

Step-2

So, the dimensions of null space of $A^T A$ is,

$$\dim\left(N\left(A^T A\right)\right) = 1$$

The rank of $A^T A$ is,

$$\begin{aligned} \text{Rank}\left(A^T A\right) &\neq 1 = n \\ &= n - 1 \end{aligned}$$

Therefore, the rank of $A^T A$ is $\boxed{n-1}$.