

Step-1

Fibonacci rule is given by: $F_{k+2} = F_{k+1} + F_k$.

Let the Fibonacci matrix be as follows:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Step-2

Lucas numbers starts with L_0, L_1 is given as follows:

$$L_0 = 2$$

$$L_1 = 1$$

Lucas rule is the same as Fibonacci rule.

$$L_{k+2} = L_{k+1} + L_k$$

Step-3

Evaluate Lucas numbers L_{10} and L_1^{10} .

Step-4

First ten Lucas number is as follows:

$$\begin{aligned} L_2 &= L_1 + L_0 \\ &= 1 + 2 \\ &= 3 \end{aligned}$$

$$\begin{aligned} L_3 &= L_2 + L_1 \\ &= 3 + 1 \\ &= 4 \end{aligned}$$

Similarly, other calculated Lucas numbers are as follows:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123.

Step-5

Therefore, $L_{10} = 123$

Step-6

Eigen value of Lucas number will be:

$$\lambda_1 = \frac{1}{2}(1 + \sqrt{5})$$

$$\lambda_2 = \frac{1}{2}(1 - \sqrt{5})$$

So,

$$\begin{aligned}\lambda_1^{10} &= \left(\frac{1}{2}(1 + \sqrt{5})\right)^{10} \\ &= 122.99 \\ &\approx 123\end{aligned}$$

Step-7

Therefore, $\boxed{\lambda_1^{10} = 123}$.