

# MA215 Probability Theory

## Assignment 07

- Find the following values by using the Statistical Tables:
  - $F(-1.72)$ ,  $F(-1.723)$ ,  $F(0.48)$  and  $F(1.234)$ , where  $F(x)$  is the c.d.f. of the standard normal random variable.
  - Find  $x$  such that  $F(x) = 0.546$ , where  $F(x)$  is the c.d.f. of the standard normal random variable. Similarly find  $y$  such that  $F(y) = 0.258$ .
- Assume that heights of children in a certain age group average are normally distributed, i.e.,  $X \sim N(\mu, \sigma^2)$ , where  $\mu = 58.4$  inches and with  $\sigma = 2.9$  inches.
  - What proportion of children are between 57 and 61 inches tall?
  - What is the number  $c$  such that 90% of the children's height in a certain age group average is less than  $c$ ?
- Suppose  $X \sim N(\mu, \sigma^2)$  and let  $Y = \exp(X) = e^X$ .
  - What are all possible values of  $Y$ ?
  - Obtain the probability density function of  $Y$ .
- Suppose  $X \sim N(\mu, \sigma^2)$  and let  $Y = aX + b$  where  $a$  and  $b$  are two constants and the constant  $a$  is not zero.
  - What are all possible values of  $Y$ ?
  - Obtain the probability density function of  $Y$ .
  - Explain  $Y$  is also normally distributed. What are the parameters of  $Y$ ?
- Suppose  $X \sim N(0, 1)$  and let  $Y = X^2$ .
  - What are all possible values of  $Y$ ?
  - Obtain the probability density function of  $Y$ .
- Suppose  $Y \sim N(0, 1)$ . Let  $-\infty < a < b < +\infty$  and  $m = \frac{1}{2} \max\{a^2, b^2\}$ . Show that
$$(b - a)e^{-m} \leq \sqrt{2\pi}P\{a \leq Y \leq b\} \leq b - a.$$
- Suppose  $Y \sim N(0, 1)$ . Show that for any  $y > 0$ , we have

$$\frac{1}{y} - \frac{1}{y^3} \leq \sqrt{2\pi}e^{\frac{y^2}{2}}P\{Y \geq y\} \leq \frac{1}{y}.$$

Hint: First show that for any  $y > 0$ ,

$$e^{-\frac{y^2}{2}} \left( \frac{1}{y} - \frac{1}{y^3} \right) = \int_y^{+\infty} e^{-\frac{x^2}{2}} \left( 1 - \frac{3}{x^4} \right) dx.$$