Step-1

(a) To decide the dimension of the subspace, we are required to have two properties.

(1) Spanning

(2) Linearly independence.

If m vectors satisfy both the conditions, then only they form a basis to the subspace and thus, the dimension of subspace will be m.

But in our case, we are given with spanning only and no linear independence.

So, we cannot judge the dimension of the subspace.

Therefore, the given statement is false

Step-2

(b) A subset S in a vector space V can be a subspace if at least zero vector is present in S.

It follows that every subspace has zero vector.

So, the intersection of subspaces also has zero vector.

Therefore, the intersection of subspaces is non empty.

Step-3

(c) False

To establish this, we take the example

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, y = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$AX = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$AY = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$AX = AY$ but $X \neq Y$

Step-4

(d) The row space of a matrix A is spanned by the set of linearly independent rows of A.

By reducing a matrix to its echelon form, we can know the rank of the matrix.

The rank of the matrix is nothing but the dimension of the row space.

If r is the rank of A, then we have to understand that any set of r linearly independent rows of A form the basis of the row space and not the specific set of rows.

Therefore, the given statement that the row space has unique basis is false.

Step-5

(e) Suppose A is a matrix of order $n \times n$ with all the columns linearly independent.

So, column rank A is n.

But column rank and row rank of a matrix are equal and so, the matrix is a non singular matrix.

More precisely, $|A| \neq 0$

Consequently, $|A||A| \neq 0$

By the properties of determinants, we can write $|A^2| \neq 0$

So, rank A^2 is n

Rank of A^2 = dimension of A^2

The columns of A^2 are linearly independent.

Therefore, the given statement is true