## Step-1

Addition rule: let $T, W$ are linear transformations from $\mathrm{R}^n$ to $\mathrm{R}^n$

We define $(T+W):\mathrm{R}^n \to \mathrm{R}^n$

By $(T+W)(x)=T(x)+W(x)$, for all $x \in \mathrm{R}^n$ and $c$ is a scalar in $\mathrm{R}$,

$c\mathrm{T}:\mathrm{R}^n \to \mathrm{R}^n$

By $(cT)(x)=c(T(x))$ for all $x \in \mathrm{R}^n$

Thus the set of all $S=\left\{T / T:\mathrm{R}^n \to \mathrm{R}^n \text{ is a linear transformation}\right\}$ is a vector space over the field $\mathrm{R}$

$\dim S = n^2$

## Step-2

Basis for this $S=\left\{T_{ij} / T_{ij}:\mathrm{R}^n \to \mathrm{R}^n, 1 \le i, j \le n\right\}$

When $T_{ij}(\alpha_k)=\begin{cases} 0 & \text{if } k \ne j \\ \alpha_k & \text{if } k = j \end{cases}$

Hence $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is a basis for $\mathrm{R}^n$