Step-1

(a) Given vectors are $b=\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}$ and $a=\begin{bmatrix}1 \\ 0\end{bmatrix}$

The projection of b onto a = $\hat{x}a=\frac{a^T b}{a^T a}a$ â€¦â€¦ (1)

$$a^T b=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\cos\theta \\ \sin\theta\end{bmatrix}$$
$$=\cos\theta$$

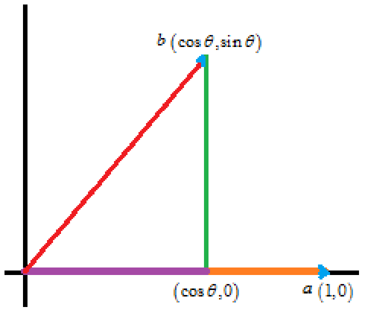
$$a^T a=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}$$
$$=1+0$$
$$=1$$

So, $\hat{x}=\frac{\cos\theta}{1}=\cos\theta$ â€¦â€¦ (2)

Step-2

Using (2) in (1), we get $P=\hat{x}\,a=\cos\theta\begin{bmatrix}1 \\ 0\end{bmatrix}$

Therefore, the required projection matrix is $P=\begin{bmatrix}\cos\theta \\ 0\end{bmatrix}$



Step-3

(b) Given vectors are $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $a = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The projection of b on to a = $\hat{x} a = \frac{a^T b}{a^T a} a$

$$a^T b = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 - 1 = 0$$

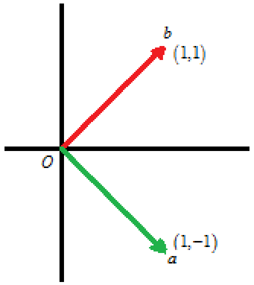
And $a^T a = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1 + 1 = 2$

So $\hat{x} = \frac{0}{2} = 0$

Therefore $P = \hat{x} a = 0 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Hence $P = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Step-4



Observe that the vectors (1, 1) and (1, -1) are perpendicular which meet at the origin O and so, the projection of b upon a is the footsteps of b nothing but the origin (0, 0).