## Step-1

Let $z$ be half way between $x$ and $y$.

We have to show that $Az$ is halfway between $Ax$ and $Ay$.

## Step-2

Given that every straight line remains straight after a linear transformation.

Let $A$ be a linear major matrix.

Since $z$ is the halfway between $x$ and $y$.

So

$$z = \frac{x+y}{2}$$
$$= \frac{x}{2} + \frac{y}{2}$$

## Step-3

Now

$$Az = A\left(\frac{x+y}{2}\right)$$
$$= A\left(\frac{x}{2} + \frac{y}{2}\right)$$

$$= A\left(\frac{x}{2}\right) + A\left(\frac{y}{2}\right) \quad (\text{since } A \text{ linear transformation})$$

$$= \frac{1}{2}(Ax + Ay) \quad (\text{since } A \text{ linear transformation})$$

Hence $Az$ is halfway between $Ax$ and $Ay$.