

## Step-1

Given that  $A$  and  $B$  are of  $n$  by  $n$  matrices with all entries equals to 1 and  $C$  is  $n$  by  $n$  matrix with entries  $c_{jl} = 2$ . We have to find  $(AB)_{ij}$  and  $(AB)C, A(BC)$ .

## Step-2

By definition of product of matrices

$$(AB)_{ij} = \sum_k a_{ik} b_{kj}$$

The elements of  $AB$  are the sum of products of corresponding elements of rows and columns so

$$\begin{aligned}\sum_k a_{ik} b_{kj} &= 1.1 + 1.1 + 1.1 + \dots + 1.1 \\ &= 1 + 1 + 1 + \dots + 1 \text{ (n times)} \\ &= n\end{aligned}$$

Hence the elements of  $AB$  of order  $n$  by  $n$  consists all elements as  $n$ .

## Step-3

Since  $C$  is  $n$  by  $n$  matrix with entries  $c_{jl} = 2$ , by definition

$$\begin{aligned}\sum_k b_{kj} c_{jl} &= 1(2) + 1(2) + 1(2) + \dots + 1(2) \\ &= 2 + 2 + 2 + \dots + 2 \text{ (n times)} \\ &= 2n\end{aligned}$$

$$\begin{aligned}\sum_k a_{ik} \left( \sum_l b_{kj} c_{jl} \right) &= 1.2n + 1.2n + 1.2n + \dots + 1.2n \\ &= 2n + 2n + 2n + \dots + 2n \text{ (n times)} \\ &= n.(2n) \\ &= 2n^2\end{aligned}$$

Hence all the elements of  $A(BC)$  are  $2n^2$ .

## Step-4

We have  $\sum_k a_{ik} b_{kj} = n$  and  $c_{jl} = 2$

$$\begin{aligned}
\text{Now } \sum_j \left( \sum_k a_{ik} b_{kj} \right) c_{jl} &= n.2 + n.2 + \dots + n.2 \\
&= 2n + 2n + 2n + \dots + 2n \text{ (n times)} \\
&= n.(2n) \\
&= 2n^2
\end{aligned}$$

Hence all the elements of  $(AB)C$  are  $2n^2$ .