

Step-1

Let A is symmetric positive definite and Q is an orthogonal matrix.

If Q is an orthogonal matrix, therefore, Q must contain eigenvectors of A .

Hence, $Q^T A Q$ is not a diagonal matrix.

(a) ☐ false

Step-2

(b)

The matrix Q is an orthogonal,

Implies;

$$\begin{aligned} Q^T Q &= Q Q^T \\ &= I \end{aligned}$$

Therefore,

$$Q^T = Q^{-1}$$

Thus,

$$\begin{aligned} Q^T A Q &= A \\ Q^{-1} A Q &= A \end{aligned}$$

Therefore, the eigenvalues of A are equal to eigenvalues of $Q^T A Q$.

Since, A is symmetric positive definite implies $Q^T A Q$ is also symmetric positive definite

Thus, ☐ (b) True

Step-3

(c)

As, the eigenvalues of A are equal to eigenvalues of $Q^T A Q$

Hence, statement is ☐ true

Step-4

(d)

If λ is the Eigen value of A

Then $e^{-\lambda}$ is the Eigen value of e^{-A}

And it is known that $e^{-\lambda} \geq 0$ for any value of λ

Thus,

e^{-A} is symmetric positive definite;

Hence, statement is ☐ true

☐ (a) False ☐ (b) True ☐ (c) True ☐ (d) True