Step-1

Trace: Sum along the main diagonal is called the trace of the matrix.

Let following be the permutation matrix *P*:

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

There can be six 3 by 3 permutation matrices of *P*. Let $(\lambda_1, \lambda_2, \lambda_3)$ be the eigen values.

Step-2

Recall that characteristic equation is always put equal to zero to get the eigen values. This means that determinant value of $P - \lambda I$ must be equal to zero.

$$\det(P - \lambda I) = 0$$

This gives the equation:

$$\lambda^3 = 1$$

The solutions of this equation are as follows:

$$\lambda_1 = 1$$
$$\lambda_2 = e^{\frac{2\pi i}{3}}$$
$$\lambda_3 = e^{\frac{-2\pi i}{3}}$$

Here, only one value is real and others are complex conjugates.

Step-3

The determinants of *P* will be product of the eigen values:

$$\begin{aligned} \lambda_1\lambda_2\lambda_3 &= 1 \cdot e^{\frac{2\pi i}{3}} \cdot e^{\frac{-2\pi i}{3}} \\ &= 1 \end{aligned}$$

Therefore, determinant of *P* is $\boxed{1}$.

Step-4

Recall that starting element of a matrix to do certain calculations is called as pivot element.

Therefore, pivot element of P will be $\boxed{1}$.

Step-5

Trace of P will be the sum of all the eigen values:

$$\begin{aligned}(\lambda_1+\lambda_2+\lambda_3) &= 1+e^{\frac{2\pi i}{3}}+e^{\frac{-2\pi i}{3}} \\ &= 1+\cos\left(\frac{2\pi i}{3}\right)+i\sin\left(\frac{2\pi i}{3}\right)+\cos\left(\frac{2\pi i}{3}\right)-i\sin\left(\frac{2\pi i}{3}\right) \\ &= 0\end{aligned}$$

Therefore, trace of P is $\boxed{0}$.

Step-6

Three numbers that can be eigen values of P will be as follows:

$$\begin{aligned}\lambda_1 &= 1 \\ \lambda_2 &= e^{\frac{2\pi i}{3}} \\ \lambda_3 &= e^{\frac{-2\pi i}{3}}\end{aligned}$$

Therefore, eigen values will be $\boxed{\left(1, e^{\frac{2\pi i}{3}}, e^{\frac{-2\pi i}{3}}\right)}$.