Step-1

A is set of 4×4 matrix diagonalized by the eigen vector matrix *S*.

$\Rightarrow A_i = S\Lambda S^{-1}$ for every A_i matrix in *A*.

Let $A_1, A_2 \in A$

Then we have $A_1 = S\Lambda_1 S^{-1}$ and $A_2 = S\Lambda_2 S^{-1}$

$$\begin{aligned} A_1 + A_2 &= S\Lambda_1 S^{-1} + S\Lambda_2 S^{-1} \\ &= S\left(\Lambda_1 + \Lambda_2\right)S^{-1} \end{aligned}$$

While the sum of diagonal matrices is diagonal, we have $\Lambda_1 + \Lambda_2$ is a diagonal matrix and so, $A_1 + A_2$ is diagonalized by *S*.

This confirms that $A_1 + A_2$ is in *A*. â€¦â€¦ (1)

Step-2

Suppose *a* is any scalar and A_i is any member of *A*.

Then we have $a A_i = a S\Lambda S^{-1}$

We know that the scalar *a* commutes with the product of matrices and so, this equation can be written as $aA_i = S\left(a\Lambda\right)S^{-1}$

$a\Lambda$ is the product of *a* with the diagonal entries and allows the resultant matrix is also a diagonal matrix.

So, *S* diagonalizes aA_i.

In other words, aA_i is also a member of *A*. â€¦â€¦ (2)

(1), (2) confirms that *A* is a subspace of all 2×2 matrices.

Step-3

If *I* diagonalizes A_i, then we write $A_i = IA_iI^{-1} = IA_iI$

But we know that every matrix A_i can be written like this regardless of whether A_i is diagonalizable or not.

In other words, *I* cannot diagonalize any matrix.

Or, *S* cannot be replaced by *I*.

In other words, A_i is a matrix spanned by all the four standard basis matrices.

Thus, the dimension of A_i is 4.