MA215 Probability Theory

Assignment 10

1. Suppose a player plays the following gambling games which is known as the wheel of fortune. The player bets on one of the numbers 1 through 6. Three dice are then rolled, and if the number bet by the player appears i times, $i = 1, 2, 3$; then the player wins i units; on the other hand, if the number bet by the player does not appear on any of the dies, then the player loses 1 unit. Is this game fair to the player?

2. Suppose the r.v. X takes non-negative integer values only. Show that

$$E(X) = \sum_{n=0}^{\infty} P(X > n) = \sum_{n=1}^{\infty} P(X \geqslant n).$$

3. (a) Suppose the r.v. X obeys the uniformly distribution over $[a, b]$. Find $E(X)$.

   (b) Suppose the r.v. X obeys the general $\varGamma$ distribution with parameters λ and α where $\lambda > 0, \alpha > 0$. Write down the p.d.f. of this general $\varGamma$ random variable and the analytic form of the $\varGamma$ function $\varGamma(\alpha)$ for $\alpha > 0$ and hence find the $E(X)$ of this general $\varGamma$ random variable.

   (c) Suppose $Y = X^2$ where X is normally distributed with parameters μ and σ^2. Obtain the p.d.f. of Y and then find $E(Y)$.

4. (a) Suppose that the two discrete r.v.s X and Y have joint p.m.f. given by

X	$Y=1$	$Y=2$	$Y=3$	$Y=4$
$X=1$	2/32	3/32	4/32	5/32
$X=2$	3/32	4/32	5/32	6/32

   Obtain $E(X)$ and $E(Y)$.

   (b) Suppose that the two continuous r.v.s X and Y have joint p.d.f.

$$f(x, y) = \begin{cases} x + y, & 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1, \\ 0, & \text{otherwise.} \end{cases}$$

   Find $E(X)$ and $E(Y)$.