Step-1

Given matrix is $A=\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}.$

So,

$$\begin{aligned}x^T Ax &= \begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix}\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}\begin{pmatrix}x\\ x_2\\ x_3\end{pmatrix}\\ &= 2x_1^2+2x_2^2+2x_3^2-2x_1x_2-2x_2x_3-2x_3x_1\\ &= 2\left(x_1^2+x_2^2+x_3^2-x_1x_2-x_2x_3-x_3x_1\right)\end{aligned}$$

Step-2

$$A=\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}.$$

Apply $R_2 \to R_2+\frac{1}{2}R_1$ and $R_3 \to R_3+\frac{1}{2}R_1$

$$=\begin{pmatrix}2 & -1 & -1\\ 0 & \frac{3}{2} & -\frac{3}{2}\\ 0 & -\frac{3}{2} & \frac{3}{2}\end{pmatrix}$$

Apply $R_3 \to R_3+R_2$

$$=\begin{pmatrix}2 & -1 & -1\\ 0 & \frac{3}{2} & -\frac{3}{2}\\ 0 & 0 & 0\end{pmatrix}$$

So we can write it as $x^T Ax = 2\left(x_1-\frac{x_2}{2}-\frac{x_3}{2}\right)^2+\frac{3}{2}\left(x_2-x_3\right)^2$

Therefore, $\boxed{x^T Ax = 2\left(x_1 - \frac{x_2}{2} - \frac{x_3}{2}\right)^2 + \frac{3}{2}\left(x_2 - x_3\right)^2}$.

Thus, $x^T Ax$ is sum of two squares and A has two pivots.

Step-3

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} x^T Bx &= \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_3x_1 \\ &= \left(x_1 + x_2 + x_3\right)^2 \end{aligned}$$

Therefore, $\boxed{x^T Bx = \left(x_1 + x_2 + x_3\right)^2}$.

Thus, $x^T Bx$ can be written as one square and B has one pivot.