Step-1

Consider a general 2 by 2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Thus, matrix D is given by,

$$D = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

We know that,

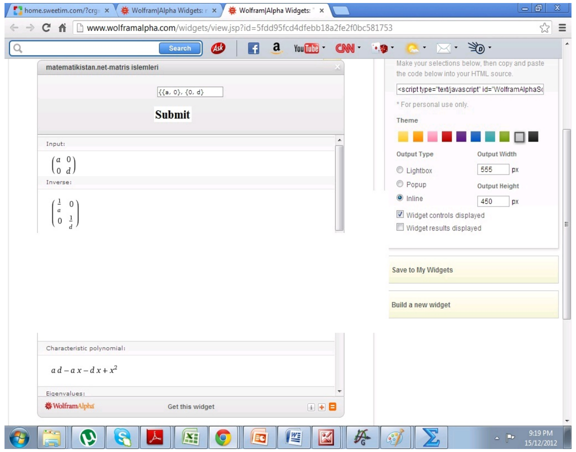
$$A = D + L + U$$

Therefore, we get,

$$\begin{aligned} L + U &= A - D \\ &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \\ &= \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix} \end{aligned}$$

Step-2

By using matrix calculator (the screenshot is given below), the inverse of D is given by,



Therefore,

$$D^{-1} = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{d} \end{bmatrix}$$

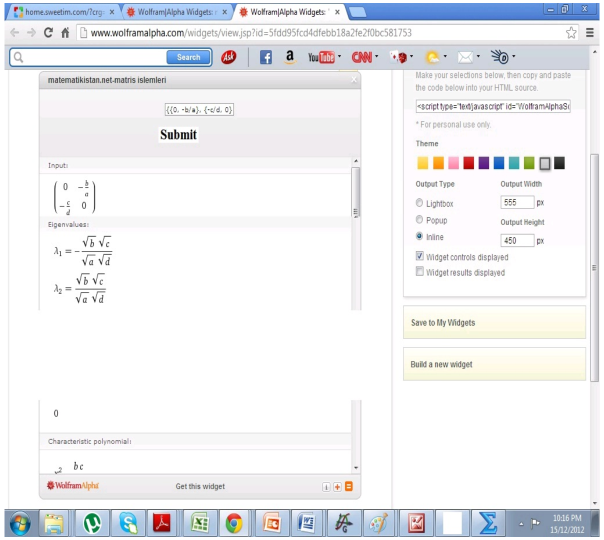
Step-3

By multiplying D^{-1} and $L + U$, the Jacobi iteration matrix $S^{-1}T$ is given by,

$$\begin{aligned} S^{-1}T &= -D^{-1}(L+U) \\ &= -\begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{d} \end{bmatrix} \begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -\frac{b}{a} \\ -\frac{c}{d} & 0 \end{bmatrix} \end{aligned}$$

Step-4

By using matrix calculator (the screenshot is given below), the eigenvalues of $S^{-1}T$ are given by,



Step-5

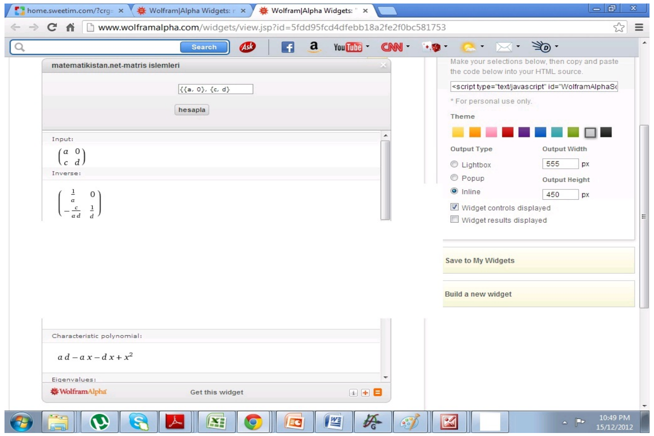
Therefore, we get,

$$\mu_{\max} = \frac{\sqrt{bc}}{\sqrt{ad}}$$

Similarly,

$$\begin{aligned}(D+L) &= \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix} \\ &= \begin{bmatrix} a & 0 \\ c & d \end{bmatrix}\end{aligned}$$

By using matrix calculator (the screenshot is given below), the inverse of $D+L$ is given by,



Therefore,

$$(D+L)^{-1} = \begin{bmatrix} \frac{1}{a} & 0 \\ -\frac{c}{ad} & \frac{1}{d} \end{bmatrix}$$

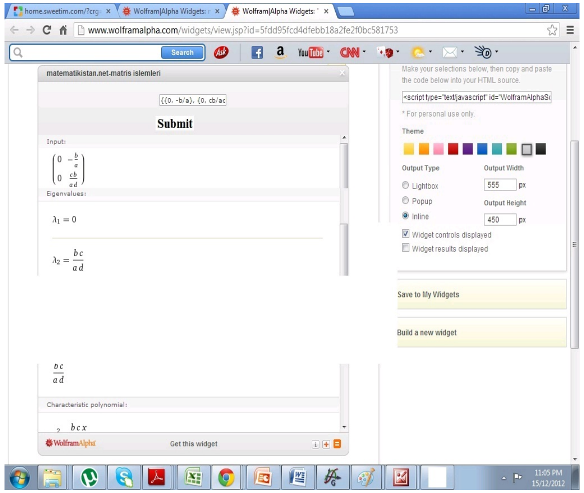
Step-6

By multiplying $(D+L)^{-1}$ and U, the Gauss-Seidel matrix $-(D+L)^{-1}U$ is given by,

$$-(D+L)^{-1}U = -\begin{bmatrix} \frac{1}{a} & 0 \\ -\frac{c}{ad} & \frac{1}{d} \end{bmatrix}\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\frac{b}{a} \\ 0 & \frac{cb}{ad} \end{bmatrix}$$

By using matrix calculator (the screenshot is given below), the eigenvalues of $-(D+L)^{-1}U$ are given by,



Therefore, we get,

$$\lambda_{\max} = \frac{bc}{ad}$$

From $\lambda_{\max} = \frac{bc}{ad}$ and $\mu_{\max} = \frac{\sqrt{bc}}{\sqrt{ad}}$, we get,

$$\lambda_{\max} = \mu_{\max}^2$$

Thus, the Jacobi iteration matrix $S^{-1}T$ is $\boxed{-D^{-1}(L+U) = \begin{bmatrix} 0 & -\frac{b}{a} \\ -\frac{c}{d} & 0 \end{bmatrix}}$, the Gauss-Seidel matrix $-(D+L)^{-1}U$ is $\boxed{-(D+L)^{-1}U = \begin{bmatrix} 0 & -\frac{b}{a} \\ 0 & \frac{cb}{ad} \end{bmatrix}}$, and $\boxed{\lambda_{\max} = \mu_{\max}^2}$.