

Step-1

Given parabola $y = a + bx + cx^2$

And the points are $(1, 4), (2, 8)$ and $(3, 14)$

Parabola at $(1, 4)$ is $a + b + c = 14$

Parabola at $(2, 8)$ is $a + 2b + 4c = 8$

Parabola at $(3, 14)$ is $a + 3b + 9c = 14$

Step-2

The above system in matrix form is

$$\begin{pmatrix} 1 & 1 & 1 & 14 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 4 \end{pmatrix}$$

apply $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 14 \\ 0 & 1 & 3 & -6 \\ 0 & 2 & 8 & -10 \end{pmatrix}$$

apply $R_3 \rightarrow \frac{R_3}{2}$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 14 \\ 0 & 1 & 3 & -6 \\ 0 & 1 & 4 & -5 \end{pmatrix}$$

apply $R_3 \rightarrow R_3 - R_2$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 14 \\ 0 & 1 & 3 & -6 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

which is upper triangular matrix.

Step-3

$$a+b+c=14$$

$$b+3c=-6$$

That is $c=1$

As $c=1$,

We get,

$$b+3=-6$$

$$b=-9$$

And,

$$a-9+1=14$$

$$a=22$$

Hence the solutions are $\boxed{a=22, b=-9, c=1}$.