Step-1

The objective is to find eigenvalues and eigenvectors of matrix A and sketch the ellipse.

Step-2

Consider an ellipse $u^2+4v^2=1$, which corresponds to the matrix $A=\begin{pmatrix}1 & 0\\ 0 & 4\end{pmatrix}$.

The characteristic equation for the matrix is,

$$\begin{aligned}|A-\lambda I|&=0\\ \begin{vmatrix}1-\lambda & 0\\ 0 & 4-\lambda\end{vmatrix}&=0\\ (\lambda-1)(\lambda-4)&=0\end{aligned}$$

So, the Eigen values are $\lambda_1=1$ and $\lambda_1=4$.

The corresponding vectors are,

$$\begin{aligned}(A-\lambda_1 I)s_1&=\begin{bmatrix}1-1 & 0\\ 0 & 4-1\end{bmatrix}s_1\\ &=\begin{bmatrix}0 & 0\\ 0 & 3\end{bmatrix}s_1\end{aligned}$$

A eigenvectors is $s_1=(0,0)^T$.

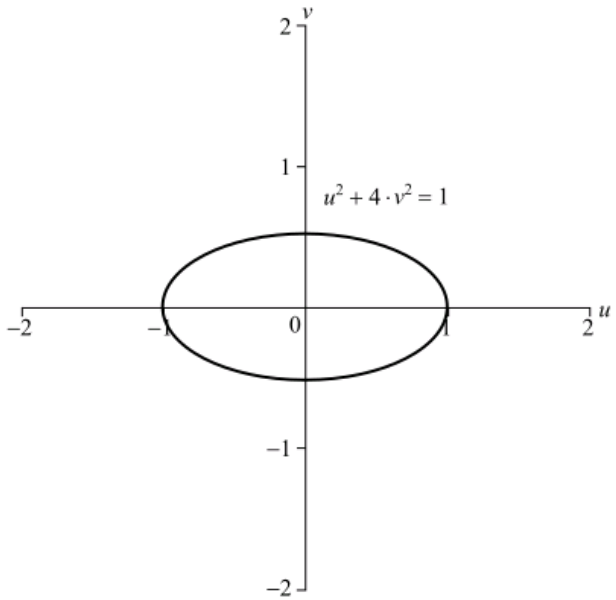
$$\begin{aligned}(A-\lambda_2 I)s_1&=\begin{bmatrix}1-4 & 0\\ 0 & 4-4\end{bmatrix}s_1\\ &=\begin{bmatrix}-3 & 0\\ 0 & 0\end{bmatrix}s_1\end{aligned}$$

A eigenvectors is $s_2=(0,0)^T$.

The given ellipse equation is centered at origin, its major axis has half-length of, $\frac{1}{\sqrt{1}}=1$ and points in the direction of s_1 while its major axis has half-length of $\frac{1}{\sqrt{4}}=\frac{1}{2}=0.5$ and points in the direction of s_2.

Step-3

The graph of ellipse is as shown below.


v
2
1
$u^2 + 4 \cdot v^2 = 1$
−2
−1
0
1
2
u
−1
−2