

Step-1

4764-1.5-16E AID: 124

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A permutation matrix is a square matrix that has exactly one 1 in each row or column and the rest of the entries are zero.

We write a 4 by 4 permutation matrix that becomes the identity matrix with just three row exchanges.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Let us consider

1st exchange:

$$R_4 \leftrightarrow R_2 \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

2nd exchange:

$$R_1 \leftrightarrow R_3 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

3rd exchange:

$$R_3 \leftrightarrow R_4 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$