Step-1

The objective is to find the inverse of the matrices.

The matrix is $A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}$.

Use gauss-Jordan elimination process to find the inverse of the matrices.

Write the echelon form of the matrix as:

$$[A_1 \quad e_1 \quad e_2 \quad e_3 \quad e_4] = \left[\begin{array}{cccc|cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} & 0 & 0 & 1 & 0 \\ \mathbf{4} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 & 1 \end{array}\right].$$

Step-2

Swap rows 1 and 4.

$$\left[\begin{array}{cccc|cccc} \mathbf{4} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} & 0 & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 & 0 \end{array}\right].$$

Divide row 1 by 4.

$$\left[\begin{array}{cccc|cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 & \frac{1}{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} & 0 & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 & 0 \end{array}\right].$$

Step-3

Swap rows 1 and 4.

$$\left[\begin{array}{cccc|cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 & \frac{1}{4} \\ \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} & 0 & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{2} & \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 & 0 \end{array}\right].$$

Divide row 2 by 3 and row 3 by 2.

$$\left[\begin{array}{cccc|cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 & 0 & 0 & \frac{1}{4} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & 0 & 0 & \frac{1}{3} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & 0 & \frac{1}{2} & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & 1 & 0 & 0 & 0 \end{array}\right].$$

Therefore, the inverse of A_1 is $\boxed{A_1^{-1} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}.$

Step-4

The matrix is $A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 \end{bmatrix}.$

Use gauss-Jordan elimination process to find the inverse of the matrices.

Write the echelon form of the matrix as:

$$[A_2 \quad e_1 \quad e_2 \quad e_3 \quad e_4] = \left[\begin{array}{cccc|cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ -\mathbf{\frac{1}{2}} & \mathbf{1} & \mathbf{0} & \mathbf{0} & 0 & 1 & 0 & 0 \\ \mathbf{0} & -\mathbf{\frac{2}{3}} & \mathbf{1} & \mathbf{0} & 0 & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & -\mathbf{\frac{3}{4}} & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[U \quad L^{-1}\right]$$

Step-5

The upper triangular U appears in the first three columns. The other three columns are the same as L^{-1}.

The second half will go from U to I.

That takes L^{-1} to $U^{-1}L^{-1}$ which is A^{-1}.

Step-6

Add $\frac{1}{2}$ times row 1 to row 2.

$$\left[\begin{array}{cccc|cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{1}{2} & 1 & 0 & 0 \\ \mathbf{0} & -\mathbf{\frac{2}{3}} & \mathbf{1} & \mathbf{0} & 0 & 0 & 1 & 0 \\ \mathbf{0} & \mathbf{0} & -\mathbf{\frac{3}{4}} & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Add $\frac{2}{3}$ times row 2 to row 3.

$$\left[\begin{array}{cccc|cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{1}{2} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \mathbf{0} & \mathbf{0} & -\mathbf{\frac{3}{4}} & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Step-7

Add $\frac{3}{4}$ times row 3 to row 4.

$$\left[\begin{array}{cccc|cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{1}{2} & 1 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{array}\right]$$

Therefore, the inverse of A_2 is $\boxed{A_2^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & 1 \end{bmatrix}}.$

Step-8

The matrix is $A_3 = \begin{bmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{bmatrix}.$

Use gauss-Jordan elimination process to find the inverse of the matrices.

Write the echelon form of the matrix as:

$$\begin{bmatrix} A_3 & e_1 & e_2 & e_3 & e_4 \end{bmatrix} = \left[\begin{array}{cccc|cccc} a & b & 0 & 0 & 1 & 0 & 0 & 0 \\ c & d & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array}\right].$$

Multiply row 1 by $\frac{1}{a}$.

$$\left[\begin{array}{cccc|cccc} 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 \\ c & d & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array}\right]$$

Step-9

Add $-c$ times row 1 to row 2.

$$\left[\begin{array}{cccc|cccc} 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 \\ 0 & \frac{da-cb}{a} & 0 & 0 & -\frac{c}{a} & 1 & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 2 by $\frac{a}{ad-bc}$.

$$\left[\begin{array}{cccc|cccc} 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array}\right]$$

Step-10

Add $-\frac{1}{ab}$ times row 2 to row 1.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 & 1 & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array}\right]$$

Multiply row 3 by $\frac{1}{a}$.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ 0 & 0 & c & d & 0 & 0 & 0 & 1 \end{array}\right]$$

Step-11

Add $-c$ times row 3 to row 4.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ 0 & 0 & 0 & \frac{ad-bc}{a} & 0 & 0 & -\frac{c}{a} & 1 \end{array}\right]$$

Multiply row 4 by $\frac{a}{ad-bc}$.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & 1 & \frac{b}{a} & 0 & 0 & \frac{1}{a} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

Step-12

Add $-\frac{1}{ab}$ times row 4 to row 3.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{array}\right]$$

Therefore, the inverse of A_3 is

$$A_3^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & -\frac{b}{ad-bc} & 0 & 0 \\ -\frac{c}{ad-bc} & \frac{a}{ad-bc} & 0 & 0 \\ 0 & 0 & \frac{d}{ad-bc} & -\frac{b}{ad-bc} \\ 0 & 0 & -\frac{c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}.$$