Step-1

Given that addition and multiplication are required to satisfy the following eight rules.

1. $x + y = y + x$

2. $x + (y + z) = (x + y) + z$

3. There is a unique â€œzero vectorâ€ such that $x + 0 = x$ for all *x*.

4. For each x there is a unique number $-x$ such that $x + (-x) = 0$.

5. $1x = x$

6. $(c_1 c_2)x = c_1(c_2 x)$

7. $c(x + y) = cx + cy$

8. $(c_1 + c_2)x = c_1 x + c_2 x$

Step-2

(a)

Suppose addition in $\mathbf{R}^2$ adds an extra 1 to each component,

So that $(3,1) + (5,0) = (9,2)$ instead of $(8,1)$.

The objective is to verify with scalar multiplication unchanged, which rules are broken.

Rule 7 is broken since

If $c = 3$, $x = (3,1)$, $y = (5,0)$

Then,

$$\begin{aligned} 3(x + y) &= 3((3,1) + (5,0)) \\ &= 3(9,2) \\ &= (27,6) \\ 3x + 3y &= (9,3) + (15,0) \\ &= (25,4) \end{aligned}$$

Therefore, $3(x+y) \neq 3x+3y$

Rule 8 is also broken.

$$\begin{aligned} &c_1 = 2, c_2 = 3 \\ &(c_1+c_2)x = 5(3,1) = (15,5) \\ &c_1x + c_2x = 2(3,1) + 3(3,1) \\ &\qquad = (6,2) + (9,3) \\ &\qquad = (16,6) \end{aligned}$$

Therefore, $(c_1+c_2)x \neq c_1x + c_2x$

Thus, the rules $\boxed{\text{7 and 8}}$ are broken.

Step-3

(b)

The objective is to show that the set of all positive real numbers, with $x+y$ and cx redefined to equal the usual xy and x^c is a vector space.

Rules 1 and 2 are not satisfied.

One is a zero vector since $x+1 = x1 = x$.

Therefore, the required zero vector is, $\boxed{1}$

For each x position, real number $\frac{1}{x}$ is the $(-x)$ for x.

$$\begin{aligned} 4.\ x+(-x) &= x \cdot \frac{1}{x} \\ &= 1 \end{aligned}$$

$$\begin{aligned} 6.\ (c_1c_2)x &= x^{c_1c_2} \\ &= \left(x^{c_2}\right)^{c_1} \\ &= (c_2x)^{c_1} \\ &= c_1(c_2x) \end{aligned}$$

7. $(c_1+c_2)=x^{c_1+c_2}$

$$=x^{c_1}\cdot x^{c_2}$$
$$=x^{c_1}+x^{c_2}$$
$$=c_1x+c_2x$$

8. $c(x+y)=(x+y)^c$

$$=(xy)^c$$
$$=x^c y^c$$
$$=x^c+y^c$$
$$=cx+cy$$

For the above defined vector addition and scalar multiplication, the set of positive real numbers is a $\boxed{\text{vector space}}$ with 1 as a $\boxed{\text{zero vector}}$.

Step-4

(c)

Suppose $(x_1,x_2)+(y_1,y_2)$ is defined as, (x_1+y_2,x_2+y_1).

The objective is to verify that which of the eight conditions are not satisfied with the usual scalar multiplication defined as, $cx=(cx_1,cx_2)$.

Step-5

Rule 1 is not satisfied since

$$(1,2)+(3,3)=(1+3,2+3)$$
$$=(4,5)$$
$$(3,3)+(1,2)=(3+2,3+1)$$
$$=(5,4)$$

Therefore, $(1,2)+(3,3)\neq(3,3)+(1,2)$

Similarly, Rule 2 is not satisfied.

$$x+(y+z)\neq(x+y)+z$$

For,

$$\left((1,1)+(2,3)\right)+(4,5)=(4,3)+(4,5)$$
$$=(9,7)$$
$$(1,1)+\left((2,3)+(4,5)\right)=(1,1)+(7,7)$$
$$=(8,8)$$

Therefore, Rule (2) is also not satisfied.

Step-6

In the same way, Rule 8 is not satisfied since

$$(2+3)(1,2)=(5,10)$$
$$2(1,2)+3(1,2)=(2,4)+(3,6)$$
$$=(2+6,4+3)$$
$$=(8,7)$$

$$(2+3)(1,2)\neq 2(1,2)+3(1,2)$$

Thus, the rules $1,2$ and 8 are $\boxed{\text{not satisfied}}$.