Step-1

This series $\left(I + A + A^2 + \ldots\right)$ represent $\left(I - A\right)^{-1}$.

Consider the following matrix which has $\lambda_{\max} = 0$.

$$A = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix}$$

Step-2

Find the powers of matrix A and show that it equals to $\left(I - A\right)^{-1}$.

Step-3

Find the powers of matrix A as follows:

$$A \cdot A = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.25 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.25 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0.05 \\ 0 & 0.125 \end{bmatrix}$$

Step-4

Similarly,

$$A^3 \cdot A = \begin{bmatrix} 0 & 0.05 \\ 0 & 0.125 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 0.025 \\ 0 & 0.0625 \end{bmatrix}$$

$$A^4 \cdot A = \begin{bmatrix} 0 & 0.025 \\ 0 & 0.0625 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & 0.0125 \\ 0 & 0.03125 \end{bmatrix}$$

$$A^5 \cdot A = \begin{bmatrix} 0 & 0.0125 \\ 0 & 0.03125 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} 0 & 0.00625 \\ 0 & 0.015625 \end{bmatrix}$$

Step-5

Here, A^6 matrix elements are very small and reduce further in other higher powers of matrix A.

Step-6

Substitute the values into the following series:

$$\begin{aligned}(I - A)^{-1} &= \left(I + A + A^2 + A^3 + A^4 + A^5 + A^6\right) \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix} + \begin{bmatrix} 0 & 0.1 \\ 0 & 0.25 \end{bmatrix} + \begin{bmatrix} 0 & 0.05 \\ 0 & 0.125 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0.025 \\ 0 & 0.0625 \end{bmatrix} + \begin{bmatrix} 0 & 0.0125 \\ 0 & 0.03125 \end{bmatrix} + \begin{bmatrix} 0 & 0.00625 \\ 0 & 0.015625 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0.39375 \\ 0 & 1.98437 \end{bmatrix} \\ &\approx \begin{bmatrix} 1 & 0.4 \\ 0 & 2 \end{bmatrix}\end{aligned}$$

Step-7

Now, calculate the following:

$$\begin{aligned}(I - A)^{-1} &= \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.2 \\ 0 & 0.5 \end{bmatrix}\right\}^{-1} \\ &= \begin{bmatrix} 1 & 0.4 \\ 0 & 2 \end{bmatrix}\end{aligned}$$

Step-8

Therefore, above result shows the following:

$$\boxed{(I - A)^{-1} = \left(I + A + A^2 + \ldots\right)}$$