Step-1

Given system is $2x+3y+z=8$

$4x+7y+5z=20$

$-2y+2z=0$

We have to reduce this system to upper triangular form by two row operations and then solve by back-substitution for z, y, x.

Step-2

Given system can be written as

$$\begin{pmatrix} 2 & 3 & 1 & 8 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 2 & 0 \end{pmatrix}$$

Subtract â€˜2â€™ times the row 1 from the row 2

$$\begin{pmatrix} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & -2 & 2 & 0 \end{pmatrix}$$

Subtract â€˜-2â€™ times the row 2 from the row 3.

$$\begin{pmatrix} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 8 & 8 \end{pmatrix}$$

which is upper triangular form.

The pivots are circled in $\begin{pmatrix} \boxed{2} & 3 & 1 & 8 \\ 0 & \boxed{1} & 3 & 4 \\ 0 & 0 & \boxed{8} & 8 \end{pmatrix}$

That is 2, 1, 8.

Step-3

Back ward substitution:-

We have $2x+3y+z=8$

$$y+3z=4$$
$$8z=8$$

$8z=8 \Rightarrow \boxed{z=1}$

$y+3z=4$

$\Rightarrow y+3(1)=4$

$\Rightarrow \boxed{y=1}$

And $2x+3y+z=8$

$\Rightarrow 2x+3(1)+1=8$

$\Rightarrow \boxed{x=2}$

Hence the solution is $\boxed{x=2, y=1, z=1}$