Step-1

Let A, B, and C be matrices with respect to the vector subspaces $\mathbf{V}$, $\mathbf{W}$, and $\mathbf{V} \cap \mathbf{W}$ respectively.

Therefore,

$$\operatorname{rank}(A+B)+\operatorname{rank} C=\operatorname{rank} A+\operatorname{rank} B$$

Step-2

The rank of a matrix cannot be negative.

Therefore, $\operatorname{rank} C \geq 0$.

Thus, $\operatorname{rank}(A+B) \leq \operatorname{rank} A+\operatorname{rank} B$

Hence proved