Step-1

Since **V** is a subspace, it is a vector space in its own right. Therefore, it has a basis. Let the dimension of **V** be m. Note that $m < n$.

Let $v_1, v_2, \ldots, v_m$ be a basis of **V**.

Step-2

Form a matrix A of order n by n, such that its first m rows are the vectors $v_1, v_2, \ldots, v_m$. Put zero in each entry of the last $n-m$ rows of A.

Let us show that the matrix A has row space = **V** and null space = **W**.

Step-3

Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ be any vector in $\mathbf{R}^n$.

Consider the following:

$$A\alpha = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \\ 0\ 0\ 0 \ldots 0 \\ \vdots \\ 0\ 0\ 0 \ldots 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$$

It is obvious that $A\alpha$ will have all zero entries in the last $n-m$ rows. Therefore, for the matrix A, its row space is **V**.

Step-4

If we consider a vector Î² from **W**, then its initial m entries will be zeros. The nonzero entries will be from $m+1$ to n.

Therefore, $A\beta = 0$.

Thus, null space of the matrix A is **W**.

Step-5

As an example, consider the vector space $\mathbf{R}^3$. Let **V** be the xy-plane and let **W** be the z-axis.

Then $\{(1,0,0),(0,1,0)\}$ is a basis for **V**.

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Step-6

Let $\alpha = (p, q, r)$ be any vector.

Then

$$A\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$
$$= \begin{bmatrix} p \\ q \\ 0 \end{bmatrix}$$

Also, let Î² be any vector along the *z*-axis. Then $\beta = (0, 0, s)$.

Therefore,

$$A\beta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ s \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$