Step-1

We have to prove that a matrix with a column of zeros cannot have an inverse.

Step-2

Let $A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n-1} & 0 \\ a_{21} & a_{22} & \dots & a_{2n-1} & 0 \\ \vdots & \ddots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn-1} & 0 \end{bmatrix}$ be a matrix with a zero column.

We know that if a matrix A is invertible, then A has n pivot positions.

Since the given $n \times n$ matrix A has no pivot position in the last column.

So the matrix A is not invertible.

Hence a matrix with zero columns is not invertible.