## Step-1

We know that the determinant is defined only for a square matrix.

So, from the given details, we follow that $Ax = b$ is a non homogeneous system of three linear equations in three variables seen as

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

This can otherwise be written as $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \mathbf{b}$ where $\mathbf{a}_i$ is the $i^{\text{th}}$ column of the coefficient matrix $A$.

This equation can simply be written as $\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3 = \mathbf{b}$

## Step-2

Let us consider $|\mathbf{b}\ \mathbf{a}_2\ \mathbf{a}_3| = |\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3|$

$= |\mathbf{a}_1 x_1\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + |\mathbf{a}_2 x_2\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + |\mathbf{a}_3 x_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3|$ By the properties of determinants.

$$\begin{aligned} &= x_1 |\mathbf{a}_1\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + x_2 |\mathbf{a}_2\ \ \mathbf{a}_2\ \ \mathbf{a}_3| + x_3 |\mathbf{a}_3\ \ \mathbf{a}_2\ \ \mathbf{a}_3| \\ &= x_1 |A| + x_2 (0) + x_3 (0) \\ &= x_1 |A| \end{aligned}$$

Consequently, $x_1 = \dfrac{|\mathbf{b}\ \mathbf{a}_2\ \mathbf{a}_3|}{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|} = \dfrac{B_1}{|A|}$

## Step-3

(b) Proceeding as above by using $\mathbf{a}_1 x_1 + \mathbf{a}_2 x_2 + \mathbf{a}_3 x_3 = \mathbf{b}$ in the middle place of $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$, we get $x_2 = \dfrac{|\mathbf{a}_1\ \mathbf{b}\ \mathbf{a}_3|}{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|} = \dfrac{B_2}{|A|}$,

Similarly, $x_3 = \dfrac{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{b}|}{|\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3|} = \dfrac{B_3}{|A|}$