Step-1

The matrix is $U = \begin{bmatrix} 0 & 5 & 4 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

The objective is to find a basis for the column space of U by locating the pivots.

Recollect that the subspace that is spanned by the column vectors of U is called the column space of U.

The basis for the column space will be the columns that contain leading 1â€™s.

Consider the matrix,

$$U = \begin{bmatrix} 0 & 5 & 4 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Apply column operations to get pivots.

Subtract third column from second matrix and fourth column from third matrix. $(C_2 - C_3, C_3 - C_4)$

$$U \sim \begin{bmatrix} 0 & 1 & 1 & 3 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Locate the pivots in this matrix.

$$U \sim \begin{bmatrix} 0 & \boxed{1} & 1 & 3 \\ 0 & -2 & \boxed{1} & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{â€¦â€¦ (1)}$$

From the matrix, the second and third column s of U contain leading 1 and so will form a basis for the column space of U.

Therefore, the basis for the column space will be $\left\{ \begin{pmatrix} 5 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 0 \\ 0 \end{pmatrix} \right\}$.

Step-2

Express each column that is not in the basis as a combination of basic columns.

Columns 1, 4 are not in the basis.

First column can be written as,

$$\begin{pmatrix}0\\0\\0\\0\end{pmatrix}=0\begin{pmatrix}5\\0\\0\\0\end{pmatrix}+0\begin{pmatrix}4\\2\\0\\0\end{pmatrix}$$

Fourth column can be written as,

$$\begin{pmatrix}3\\1\\0\\0\end{pmatrix}=\frac{1}{5}\begin{pmatrix}5\\0\\0\\0\end{pmatrix}+\frac{1}{2}\begin{pmatrix}4\\2\\0\\0\end{pmatrix}$$

Step-3

The matrix in equation (1) represents the echelon form.

$$U\sim\begin{bmatrix}0&1&1&3\\0&-2&1&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

It is in the echelon form but with different column space.

The basis for this column space is $\left\{\begin{pmatrix}1\\-2\\0\\0\end{pmatrix},\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\right\}$

Therefore, required matrix is $\boxed{A=\begin{bmatrix}0&1&1&3\\0&-2&1&1\\0&0&0&0\\0&0&0&0\end{bmatrix}}$.