Step-1

Given that $(1,0)$ and $(0,1)$ transformed to $(1,4)$ and $(1,5)$ by a matrix.

We have to find the matrix that transforms $(1,0)$ and $(0,1)$ to $(1,4)$ and $(1,5)$.

Step-2

We have

$T(1,0)=(1,4)$
$T(0,1)=(1,5)$

Therefore, the matrix M of linear transformation T under the basis $(1,0)$ and $(0,1)$

is $\boxed{M=\begin{bmatrix}1 & 1\\ 4 & 5\end{bmatrix}}$.

Step-3

Given that $a(1,4)+b(1,5)=(1,0)$

We have to find a and b

Now,

$a(1,4)+b(1,5)=(1,0)$
$\Rightarrow (a+b,4a+5b)=(1,0)$
$\Rightarrow a+b=1\ ,4a+5b=0$

Solving these equations, we get

$b=-4, a=5$

Hence $\boxed{(a,b)=(5,-4)}$

Step-4

We have to find how the new coordinates $(5,-4)$ of $(1,0)$ is related to M or M^{-1}.

The new coordinates of $(1,0)$ are related to M^{-1} because $(1,0)=5(1,4)+(-4)(1,5)$.