

Step-1

(a)

A is a matrix of m rows and n columns

It is known that the maximum number of linearly independent rows = maximum number of linearly independent columns = $\min \{m, n\}$.

Given that the rows of A are linearly independent. So, rank is m .

Further, it follows that for $m \leq n$.

The rank of column space is also m .

Consequently, the column space is \mathbb{R}^m

The rank of the left null space is $n - m$.

Therefore, the left null space is \mathbb{R}^{n-m}

Step-2

(b)

Given that A is a matrix of size 8×10

That means the number of rows is 8 and columns is 10

Given that the null space of A is of dimension 2

So, the dimension of range space of A is $10 - 2 = 8$

Let, consider $Ax = b$

Then x is of size 10×1 and b is of size 8×1

Since the rank of A is 8, it can be written the two variables of x as the linear combinations of remaining 8 variables.

Therefore, the solution set has two linearly independent columns and each column has 10 entries in which the bottom two entries are zero denoting x_9, x_{10} .

Let them be b_1, b_2

Step-3

Suppose $b(8 \times 1)$ is any non zero vector.

Then it can be seen that the matrix $[b_1 \ b_2 \ b]$ is of size 10×3 and bottom two rows are necessarily zero.

So, the non zero rows of this matrix are 8 and columns are 3

By the properties of matrices, it can be said that the number of linearly dependent rows is less than 3

In other words, it is not necessary that one column is dependent of other four columns for the infinite possible columns b .

More precisely, every column b cannot necessarily be written as a linear combination of the solutions of $Ax = b$.

Therefore, every vector b cannot be in the form Ax .

Therefore, the given statement is **false**.