Step-1

The first column of A is the vector $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Its length is $\sqrt{5}$.

Therefore, the first column of the matrix Q is $\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{bmatrix}$. The second column of the matrix Q must be of the length 1 and it should be orthogonal to the first column. Thus, the second column of Q must be $\begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$.

Thus, $Q = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix}$.

Step-2

If we assume that the column of the matrix A are a and b, then the matrix R is given by $R = \begin{bmatrix} q_1^{\mathrm{T}} a & q_1^{\mathrm{T}} b \\ 0 & q_2^{\mathrm{T}} b \end{bmatrix}$.

Thus, we get

$$
\begin{aligned}
R &= \begin{bmatrix} 2 \times \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} - \frac{2}{\sqrt{5}} \\ 0 & \frac{-1}{\sqrt{5}} + \frac{4}{\sqrt{5}} \end{bmatrix} \\
&= \begin{bmatrix} \sqrt{5} & \frac{-4}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{bmatrix}
\end{aligned}
$$

Step-3

Thus, we have

$$
\begin{aligned}
A &= Q_0 R_0 \\
&= \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \sqrt{5} & \frac{-4}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{bmatrix}
\end{aligned}
$$

Step-4

Let $A_1 = R_0 Q_0$

Thus, we get

$$
\begin{aligned}
A_1 &= R_0 Q_0 \\
&= \begin{bmatrix} \sqrt{5} & \frac{-4}{\sqrt{5}} \\ 0 & \frac{3}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \\
&= \begin{bmatrix} 2+\frac{4}{5} & 1-\frac{8}{5} \\ \frac{-3}{5} & \frac{6}{5} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= \begin{bmatrix} \frac{14}{5} & \frac{-3}{5} \\ \frac{-3}{5} & \frac{6}{5} \end{bmatrix} \\
&= \frac{1}{5}\begin{bmatrix} 14 & -3 \\ -3 & 6 \end{bmatrix}
\end{aligned}
$$

Step-5

Thus, we have shown that $\boxed{A_1 = \frac{1}{5}\begin{bmatrix} 14 & -3 \\ -3 & 6 \end{bmatrix}}$.