## Step-1

Let $x_1$, $y_1$, and $z_1$ be the eigenvectors of $A$, $B$, and $A+B$ respectively, corresponding to the smallest eigenvalues.

Thus, we have

$$Ax_1 = \lambda_1 x_1$$
$$By_1 = \mu_1 y_1$$
$$Cz_1 = \theta_1 z_1$$

## Step-2

Suppose $w$ is any vector. From the properties of Rayleigh quotient, we can say the following:

$$\frac{w^{\mathrm{T}} Aw}{w^{\mathrm{T}} w} \geq \lambda_1$$
$$\frac{w^{\mathrm{T}} Bw}{w^{\mathrm{T}} w} \geq \mu_1$$

## Step-3

Now consider the following;

$$\begin{aligned}\theta_1 &= R(z_1) \\ &= \frac{z_1^{\mathrm{T}}(A+B)z_1}{z_1^{\mathrm{T}} z_1} \\ &= \frac{z_1^{\mathrm{T}} Az_1 + z_1^{\mathrm{T}} Bz_1}{z_1^{\mathrm{T}} z_1}\end{aligned}$$

$$\begin{aligned}\theta_1 &= \frac{z_1^{\mathrm{T}} Az_1}{z_1^{\mathrm{T}} z_1} + \frac{z_1^{\mathrm{T}} Bz_1}{z_1^{\mathrm{T}} z_1} \\ &\geq \lambda_1 + \mu_1\end{aligned}$$

## Step-4

Thus, we have shown that $\boxed{\theta_1 \geq \lambda_1 + \mu_1}$.