

## Step-1

Given that

Rule (2): The determinant changes sign when two rows are exchanged.

Rule (3): The determinant depends linearly on the first row.

Rule (6): If  $A$  has a row of zeros, then  $\det(A) = 0$

We have to show that how rule (6) comes directly from rules 2 and 3.

## Step-2

Suppose a square matrix has a row of zeros, say the  $k^{th}$  row  $R_k$  consist of all zeros. It looks as below:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

## Step-3

Then by rule 3,  $t|A| = |B|$  where  $B$  is obtained by multiplying a sample row in  $A$ ,

Now

$$2|A| = |A| \text{ (multiplying } k\text{th row by } 2A \text{ is unaltered)}$$

$$\Rightarrow 2|A| - |A| = 0$$

$$\Rightarrow |A| = 0$$

Therefore rule (6) comes directly from rules 2 and 3.