Step-1

In the Gram-Schmidt formula $C = c - (q_1^T c)q_1 - (q_2^T c)q_2$, we have to verify that C is orthogonal to q_1 and q_2.

Now consider

$$\begin{aligned} C^T q_1 &= \left[c - (q_1^T c)q_1 - (q_2^T c)q_2\right]^T q_1 \\ &= \left\{c^T - \left[(q_1^T c)q_1\right]^T - \left[(q_2^T c)q_2\right]^T\right\} q_1 \\ &= \left[c^T - q_1^T (q_1^T c)^T - q_2^T (q_2^T c)^T\right] q_1 \\ &= \left[c^T - q_1^T c^T q_1 - q_2^T c^T q_2\right] q_1 \end{aligned}$$

Step-2

$$= c^T q_1 - q_1^T q_1 (c^T q_1) - q_2^T (c^T q_2) q_1$$

$$= c^T q_1 - q_1^T q_1 (c^T q_1) - q_2^T c^T q_2 q_1$$

As q_1, q_2 are orthonormal columns, $q_1 q_2 = 0$ and $q_1^T q_1 = 1$

$$\begin{aligned} C^T q_1 &= c^T q_1 - 1(c^T q_1) - 0 \\ &= 0 \end{aligned}$$

Therefore $C^T q_1 = 0$

Step-3

Similarly

$$\begin{aligned} C^T q_2 &= c^T q_2 - q_1^T (c^T q_1) q_2 - q_2^T (c^T q_2) q_2 \\ &= c^T q_2 - q_1^T c^T q_1 q_2 - 1 c^T q_2 \\ &= 1(c^T q_1) - q_2^T c^T q_2 q_1 \\ &= c^T q_2 - 0 - 1(c^T q_2) \\ &= 0 \end{aligned}$$

Therefore $C^T q_2 = 0$

Hence C is orthogonal to q_1 and q_2