Step-1

a) Given that x is a null space of A

$\Rightarrow Ax = 0$

Let us consider $\left(M^{-1}AM\right)\left(M^{-1}x\right)$

$$= M^{-1}A\left(MM^{-1}\right)x$$
$$= M^{-1}\left(AI\right)x$$
$$= M^{-1}\left(Ax\right)$$
$$= M^{-1}\left(0\right)$$
$$= 0$$

$\left(M^{-1}AM\right)\left(M^{-1}x\right) = 0$ confirms that $M^{-1}x$ is in the null space of $M^{-1}AM$.

Step-2

b) We have that similar matrix $B = M^{-1}AM$ is closely connected to A.

Every linear transformation is represented by a matrix.

The matrix depends on the choice of basis.

If we change the basis to M, we change the matrix A to a similar matrix B.

Similar matrices represent the same transformation T with respect to different bases.

But this confirms that the dimension of B is equal to the dimension of A.

The set of linearly independent vectors in A are reduced to row echelon form by the multiplication of M and its inverse to get B. Thus, the basis of A and that of B are same.

Putting these things together, we get $M^{-1}AM$ and A have the same vectors, bases and dimension.