Step-1

(a)

Given vectors are $b=\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $a=\begin{bmatrix}1\\1\\1\end{bmatrix}$

The projection of b onto a is $\hat{x}=\left(\dfrac{a^Tb}{a^Ta}\right)a$

$$\begin{aligned}a^Tb&=\begin{bmatrix}1&1&1\end{bmatrix}\begin{bmatrix}1\\2\\2\end{bmatrix}\\&=1+2+2\\&=5\end{aligned}$$

Also,

$$\begin{aligned}a^Ta&=\begin{bmatrix}1&1&1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}\\&=1+1+1\\&=3\end{aligned}$$

Step-2

So, put values in above formula;

$$\hat{x}=\frac{5}{3}$$

Therefore,

$$\begin{aligned}P&=\hat{x}a\\&=\frac{5}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}\\&=\begin{bmatrix}5/3\\5/3\\5/3\end{bmatrix}\end{aligned}$$

$$e = b - P$$
$$= \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 5/3 \\ 5/3 \\ 5/3 \end{bmatrix}$$
$$= \begin{bmatrix} -2/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

Now, verify e is perpendicular to a or not.

$$e^T = \begin{pmatrix} -2/3 & 1/3 & 1/3 \end{pmatrix}$$
$$e^T a = \begin{pmatrix} -2/3 & 1/3 & 1/3 \end{pmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
$$= \frac{-2}{3} + \frac{1}{3} + \frac{1}{3}$$
$$= 0$$

Therefore, e is perpendicular to a.

Step-3

(b)

Given vectors are $b = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $a = \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$

The projection of b onto a = $\hat{x} = \dfrac{a^T b}{a^T a}$

$$a^T b = \begin{bmatrix} -1 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$
$$= -1 - 9 - 1$$
$$= -11$$

$$a^T a = \begin{bmatrix} -1 & -3 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}$$
$$= 1 + 9 + 1$$
$$= 11$$

Step-4

Put values in formula, and obtain;

$$\begin{aligned}\hat{x} &= \frac{-11}{11} \\ &= -1\end{aligned}$$

Therefore,

Step-5

$$\begin{aligned}P &= \hat{x}a \\ &= -1\begin{bmatrix}-1\\-3\\-1\end{bmatrix} \\ &= \begin{bmatrix}1\\3\\1\end{bmatrix} \\ &= b\end{aligned}$$

$$\begin{aligned}e &= b - P \\ &= \begin{bmatrix}1\\3\\1\end{bmatrix} - \begin{bmatrix}1\\3\\1\end{bmatrix} \\ &= \begin{bmatrix}0\\0\\0\end{bmatrix}\end{aligned}$$

Observe that the zero vectors are perpendicular to every vector.

So, $\boxed{e^T a = 0}$ verifies the projection of b upon a.