

## Step-1

Let  $A$  be an  $n$  by  $n$  matrix such that  $A^2 = A$  and  $\text{rank } A = n$

$\text{rank } A = n$  says  $A$  is an invertible matrix

Given that  $A^2 = A$

Multiplying  $A^{-1}$  on both sides, we get

$$A^{-1}A^2 = A^{-1}A$$

By the associativity of multiplication of matrices, we get

$$(A^{-1}A)A = A^{-1}A$$

$$\Rightarrow IA = I \quad (I \text{ is } n \text{ by } n \text{ identity matrix})$$

$$\Rightarrow A = I$$