Step-1

Given that the solutions to the linear differential equation $\frac{d^2u}{dt^2} = u$ form a vector space.

We have to find two independent solutions to give a basis for that solution space.

Step-2

We have $\frac{d^2u}{dt^2} = u$

$\Rightarrow u'' - u = 0$

The auxiliary equation is $m^2 - 1 = 0$

$\Rightarrow m = \pm 1$

Therefore, $u = e^t, u = e^{-t}$ are solutions of $\frac{d^2u}{dt^2} = u$.

And e^t, e^{-t} are independent solutions of the basis for the solution space is $\boxed{\{e^t, e^{-t}\}}$.

The general solution of the given differential equation is $u = c_1 e^t + c_2 e^{-t}$, where c_1, c_2 are constants.