Step-1

Given A is an m by n matrix of rank r. Suppose there are right â€“ hand sides b for which $Ax = b$ has no solution.

(a)

We need find at what inequalities must be true between $m, n,$ and r.

A is an m by n matrix of rank r.

$AX = b$ has no solution means that $r < m$

Since $\dim(C(A)) + \dim(N(A)) =$ number of columns of A. So, $r \le n$.

We cannot compare m and n.

Step-2

(b)

We need to explain about $A^T y = 0$ has a nonzero solution.

If $m - r > 0$, i.e. $\dim(N(A^T)) > 0$ than $N(A^T)$ has a non zero vector.

Therefore, $A^T y = 0$ has a non zero solution.