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Multiple Modeling

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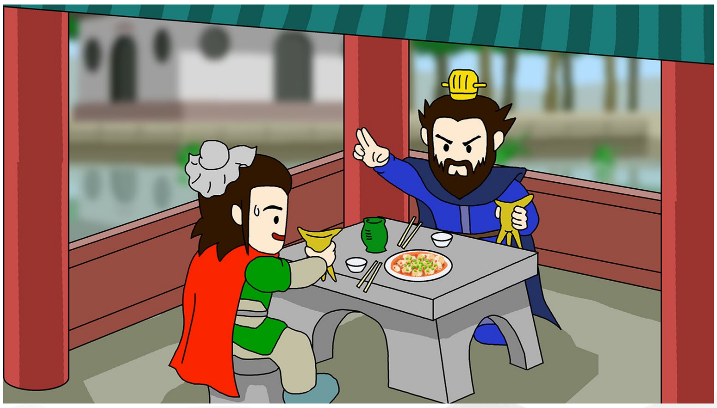
Judging Heroes Cooking Wine



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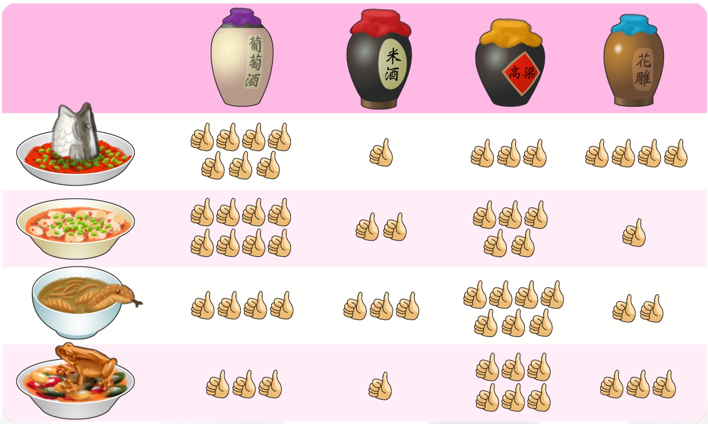
The Joy of Food-Wine Pairing



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Another Pure Assignment Problem

- To determine a function f: DOM ➔ COD again
  - DOM = food and COD = wine
- Constraint: Pair up each dish with a different drink
- Objective: maximize the joyfulness of the culinary (and political) occasion

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The Cooking Wine Problem (foodToWine.mzn)

```
include "globals.mzn";

enum FOOD;
enum WINE;
array[FOOD, WINE] of int: joy;

array[FOOD] of var WINE: drink;

constraint alldifferent(drink);

solve maximize sum(f in FOOD)(joy[f, drink[f]]);
```

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Solving the Food-Wine Model

```
Pair Food CHILIFISHHEAD with Wine HUADIAO
Pair Food MAPOTOFU with Wine GRAPE
Pair Food SNAKESOUP with Wine RICE
Pair Food GONGBAOFROG with Wine GAOLIANG


Joy: 21
----------
==========
```

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Multiple Models

- Discrete optimization problems often have
  - multiple viewpoints on the same problem
- We can build two (or more) completely distinct models to solve the same problem
- We can also combine them

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Viewpoints

- Function f: DOM ➔ COD is special when
  - |DOM| = |COD|
  - Function f is a bijection
- A viewpoint looks at the decisions of the problem from a specific angle
- This is a complete matching:
  - match each *d* in DOM with a different *c* in COD
  - or, equivalently, match each *c* in COD with a different *d* in DOM
  - two different **viewpoints** ==> two complementary models

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Complete Matching

- A bijective function has two viewpoints
- The (usual) function

```
array[DOM] of var COD: f;
```

- And the inverse function

```
array[COD] of var DOM: finv;
```

- As much as we can pair drinks to food, we can also pair food to drinks
- In the Cooking Wine problem, the inverse function is

```
array[WINE] of var FOOD: eat;
```

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The Cooking Wine Problem (wineToFood.mzn)

```
include "globals.mzn";

enum FOOD;
enum WINE;
array[FOOD, WINE] of int: joy;
```

```
array[WINE] of var FOOD: eat;

constraint alldifferent(eat);

solve maximize sum(w in WINE)(joy[eat[w], w]);
```

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Solving the Inverse Wine-Food Model

```
Pair Food MAPOTOFU with Wine GRAPE
Pair Food SNAKESOUP with Wine RICE
Pair Food GONGBAOFROG with Wine GAOLIANG
Pair Food CHILIFISHHEAD with Wine HUADIAO


Joy: 21
----------
==========
```

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Matching Food and Wine

- Which model is likely better?
  - Original

```
alldifferent(drink);
maximize sum(f in FOOD)(joy[f, drink[f]]);
```

  - Inverse

```
alldifferent(eat);
maximize sum(w in WINE)(joy[eat[w], w]);
```

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Channeling Constraints

- Some constraints of the problem may be easier to express using the function, or its inverse
- Why not use both!?

- We can combine the two models
- We need to make the two functions agree by using channeling constraints

```
forall(w in WINE, f in FOOD)
      (eat[w] = f <->
       drink[f] = w);
```

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Combining Models Using inverse

- This channeling constraint can also be captured by the global constraint
  - `inverse(eat, drink)`
  - or `inverse(drink, eat)`
  - Note we can **remove** the `alldifferent` constraints, made redundant by `inverse`

- Why would we combine models?

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The Cooking Wine Problem (combined.mzn)

```
include "globals.mzn";

enum FOOD;
enum WINE;
array[FOOD, WINE] of int: joy;

array[FOOD] of var WINE: drink;
array[WINE] of var FOOD: eat;

constraint inverse(eat, drink);

solve maximize sum(f in FOOD)(joy[f, drink[f]]);
% solve maximize sum(w in WINE)(joy[eat[w], w]);
```

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The Cooking Wine Problem (combined.mzn)

```
include "globals.mzn";

enum FOOD;
enum WINE;
array[FOOD, WINE] of int: joy;

array[FOOD] of var WINE: drink;
array[WINE] of var FOOD: eat;

constraint inverse(eat, drink);

solve maximize sum(f in FOOD)(joy[f, drink[f]]);
% solve maximize sum(w in WINE)(joy[eat[w], w]);
```

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Why Combining Models?

- CP-based solvers can benefit from combining models to improve solving efficiency if done properly
- **Ease** of expression of constraints.
  - The Cooking Wine problem, as is, is a pure assignment. **NO!**
  - But what about side constraints?

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Example Side Constraints

- Richness in taste

3 2 1 4

```
array[FOOD] of int: taste;
```

- The dish paired with GAOLIANG should be richer in taste than that paired with GRAPE wine

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Example Side Constraints

- Richness in taste

3 2 1 4

```
array[FOOD] of int: taste;
```

- The dish paired with GAOLIANG should be richer in taste than that paired with GRAPE wine

```
taste[eat[GRAPE]] < taste[eat[GAOLIANG]];
```

- This constraint is difficult, if not impossible, to express in the food-wine model

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Example Side Constraints

- Alcohol contents

1 3 4 2

```
array[WINE] of int: alcohol;
```

- The drink paired with GONGBAOFROG should be stronger in alcohol than that paired with SNAKESOUP

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Example Side Constraints

- Alcohol contents

1 3 4 2

```
array[WINE] of int: alcohol;
```

- The drink paired with GONGBAOFROG should be stronger in alcohol than that paired with SNAKESOUP

```
alcohol[drink[SNAKESOUP]] <
    alcohol[drink[GONGBAOFROG]];
```

- This constraint is difficult, if not impossible, to express in the wine-food model

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Example Side Constraints

- MAPOTOFU is paired with RICE wine if and only if SNAKESOUP is paired with HUADIAO wine

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Example Side Constraints

- MAPOTOFU is paired with RICE wine if and only if SNAKESOUP is paired with HUADIAO wine

```
eat[RICE] = MAPOTOFU <->
   eat[HUADIAO] = SNAKESOUP;
```

OR

```
drink[MAPOTOFU] = RICE <->
   drink[SNAKESOUP] = HUADIAO;
```

OR

```
eat[RICE] = MAPOTOFU <->
   drink[SNAKESOUP] = HUADIAO;
```

OR

```
drink[MAPOTOFU] = RICE <->
   eat[HUADIAO] = SNAKESOUP;
```

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Summary

- Multiple viewpoints of the problem leads to
  - multiple models
- Different viewpoints can express different constraints
  - more naturally and succinctly
  - better for the solvers (usually succinct is better)
- Channeling constraints make the viewpoints agree and unite them
- Combining the models can sometimes improve solving efficiency on either single model

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