FIT5201 Assignment 2 Task 3: Covariances

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Question 4: Initialising given information

In [50]:
```
import numpy as np
mean = np.array([100,100,50,50])
sd = np.array([20,22,5,5])
correlation = np.array([[1.0,0.90,0.0,0.0],[0.9,1.0,-0.9,-0.9],[0.0,-0.9,1.0,0.5
covariance = np.empty([4, 4])
for i in range (4):
    for j in range (4):
        covariance[i,j] = (correlation[i,j])*sd[i]*sd[j]
variance = np.empty([4])
for i in range (4):
    variance[i] = covariance[i,i]
```

Question 4: Calculate expected value of profit

In [51]:
```
portfolio_1 = np.array([50,0,25,25])
portfolio_2 = np.array([0,50,50,0])
portfolio_3 = np.array([0,50,25,25])
portfolio_4 = np.array([25,25,25,25])
# Expected Value
p1_exp = mean.dot(portfolio_1.T)
p2_exp = mean.dot(portfolio_2.T)
p3_exp = mean.dot(portfolio_3.T)
p4_exp = mean.dot(portfolio_4.T)
var_p1 = 0
for i in range (4):
    for j in range (4):
        var_p1 += portfolio_1[i]*portfolio_1[j]*covariance[i,j]
portfolio1_std_devs = np.sqrt(np.dot(np.dot(portfolio_1, covariance), portfolio_
portfolio2_std_devs = np.sqrt(np.dot(np.dot(portfolio_2, covariance), portfolio_
portfolio3_std_devs = np.sqrt(np.dot(np.dot(portfolio_3, covariance), portfolio_
portfolio4_std_devs = np.sqrt(np.dot(np.dot(portfolio_4, covariance), portfolio_

print ("Portfolio 1 Expected Value:",p1_exp,", Standard Deviation:",portfolio1_s
print ("Portfolio 2 Expected Value:",p2_exp,", Standard Deviation:",portfolio2_s
print ("Portfolio 3 Expected Value:",p3_exp,", Standard Deviation:",portfolio3_s
print ("Portfolio 4 Expected Value:",p4_exp,", Standard Deviation:",portfolio4_s
```

```
Portfolio 1 Expected Value: 7500 , Standard Deviation: 1023.1690964840562
Portfolio 2 Expected Value: 7500 , Standard Deviation: 881.7596044274198
Portfolio 3 Expected Value: 7500 , Standard Deviation: 872.8545125048046
Portfolio 4 Expected Value: 7500 , Standard Deviation: 920.2581159652981
```

Question 4: Analysis for conservative investor

After looking at the values above, it seems like the expected values for all of the portfolios are the same, which means that for a conservative investor, I would suggest them to take **Portfolio 3** due to its lowest standard deviation (which in financial terms mean less volatility) in their investments

Question 4: Mathematical background of calculations

To calculate the Covariance Matrix, I first used the formula $Cov(X_i, X_j) = \rho(X_i, X_j) * Std(X_i)Std(X_j)$ which can be derived from the formula given in the assignment sheet. Then, to calculate the Expected Value, I used the formula $E(Y) = E(X) \cdot P$ where **P** represents the portfolio's weights. This is similar to the $E(X+Y) = E(X) + E(Y)$ formula in the lectures but I used dot product instead for efficient computation.
Then lastly, we can derive the Variance by utilising the Covariance between linear combinations formula given in the lectures. The derivation can be seen as belows:

$$\begin{aligned}\mathrm{Cov}\left(\boldsymbol{a}^T\boldsymbol{x}, \boldsymbol{b}^T\boldsymbol{x}\right) &= \sum_{i=1}^{d}\sum_{j=1}^{d} a_i b_j \sigma_{i,j} \\ &= \sum_{i=1}^{d} a_i b_i \sigma_i^2 + 2\sum_{i=1}^{d}\sum_{j=i+1}^{d} a_i b_j \sigma_{i,j} \\ &= \boldsymbol{a}^T \boldsymbol{\Sigma} \boldsymbol{b}\end{aligned}$$

Special case: $\mathrm{Var}\left(\boldsymbol{a}^T\boldsymbol{x}\right) = \boldsymbol{a}^T\boldsymbol{\Sigma}\boldsymbol{a}$

We use the special case here which is possible due to the generalised Variance Rule $Var(X) = Cov(X, X)$

In [ ]: