Sequence Dependent Scheduling 1

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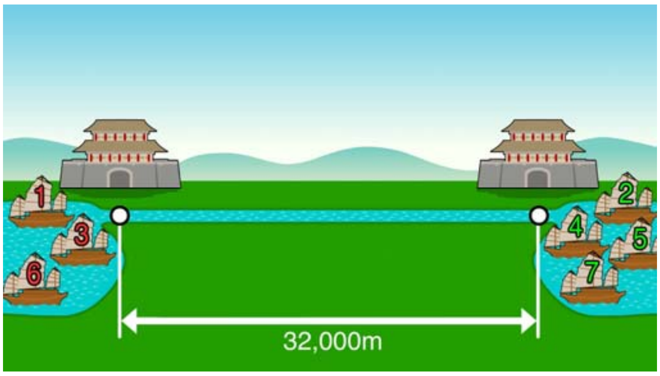
Sailing in a Narrow Channel



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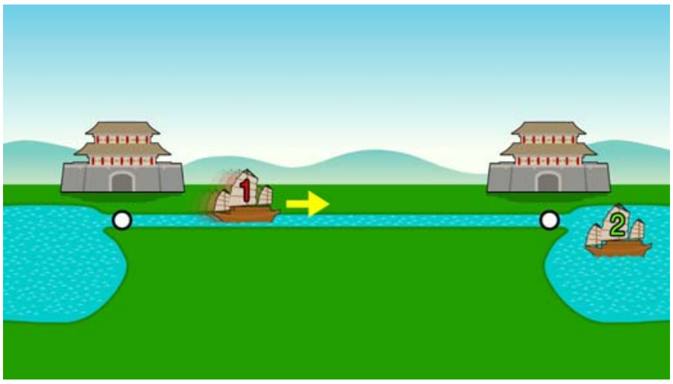
No ships in Opposite Directions



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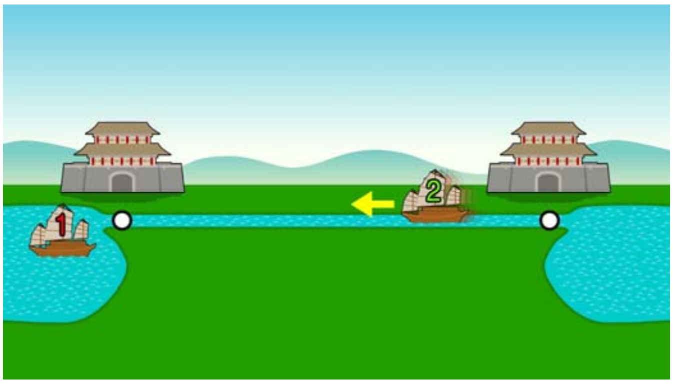

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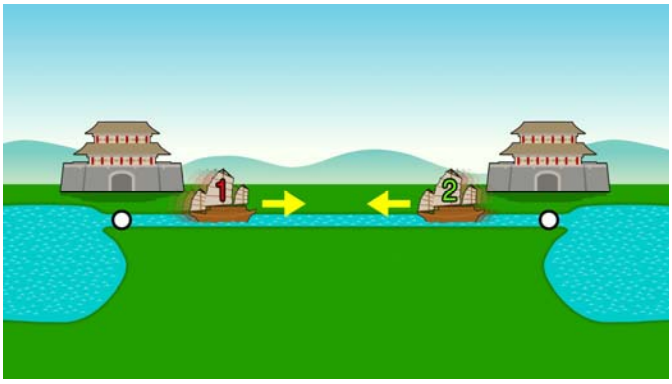

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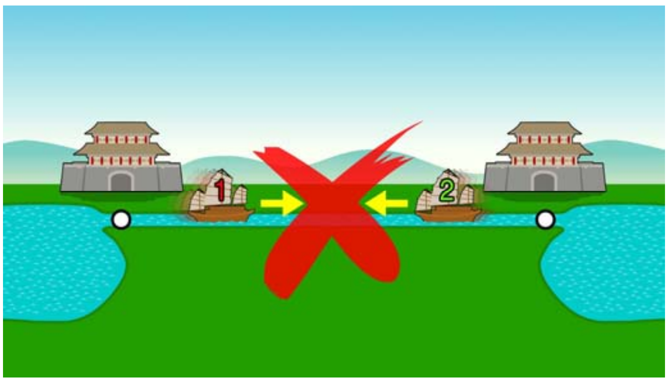

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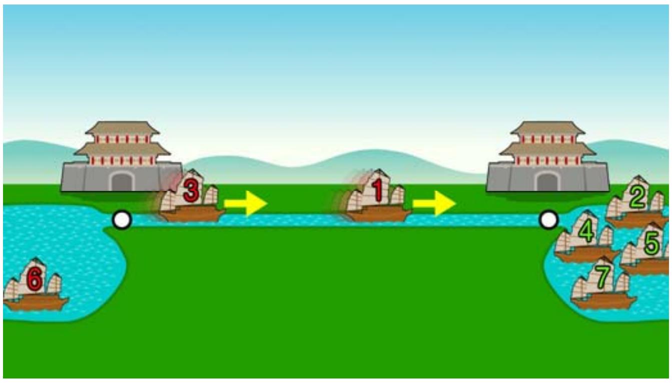
Leeway—Safety Distance



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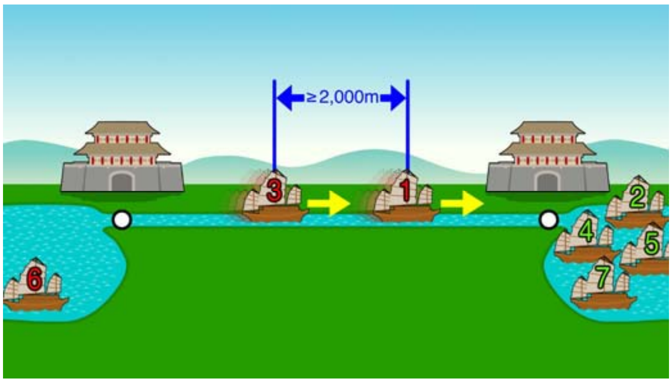

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No Earlier Than Desired Time

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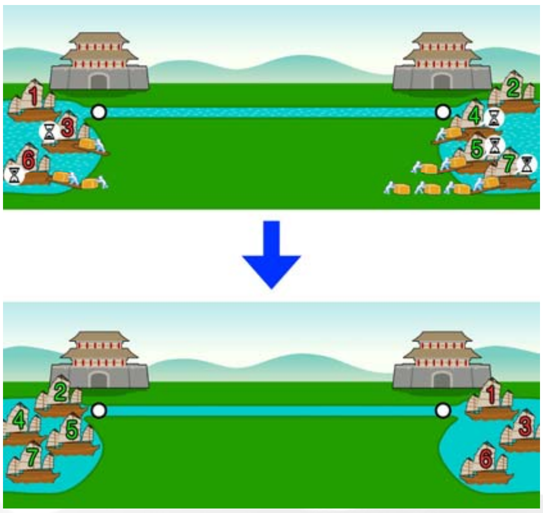
Optimizing the Makespan



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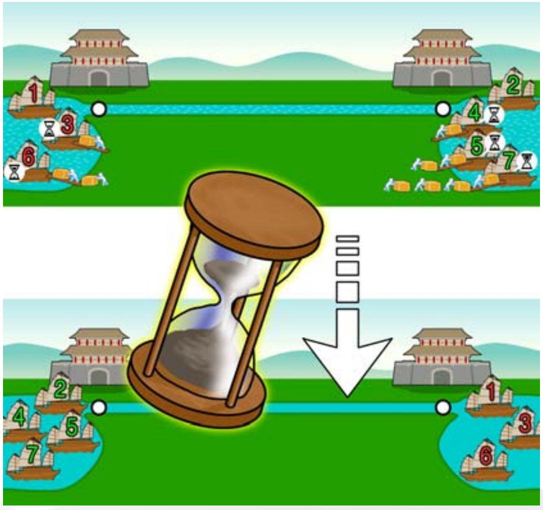
Optimizing the Makespan



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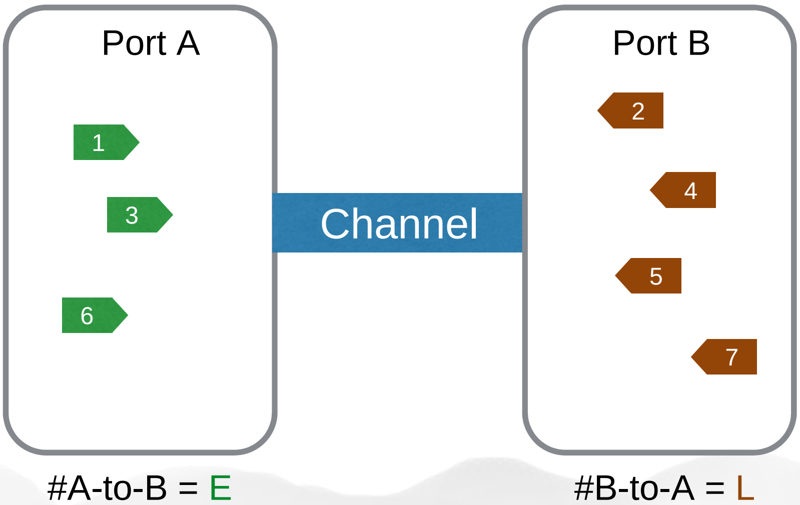
Single Channel Problem



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Single Channel Problem

- Given ports *A* and *B* connected by a channel. Consider a set of *E* ships going from port *A* to *B*, and *L* ships going from port *B* to *A*. We need to choose when the ships should enter the channel
  - Each ship has a specific speed and can leave no earlier than a desired time for that ship
  - The channel is 32000m long
  - A ship can enter only if the channel is **clear**, i.e. no ships sail in opposite directions simultaneously
  - Two ships cannot be closer than 2000m
  - Minimize the time to move all the ships

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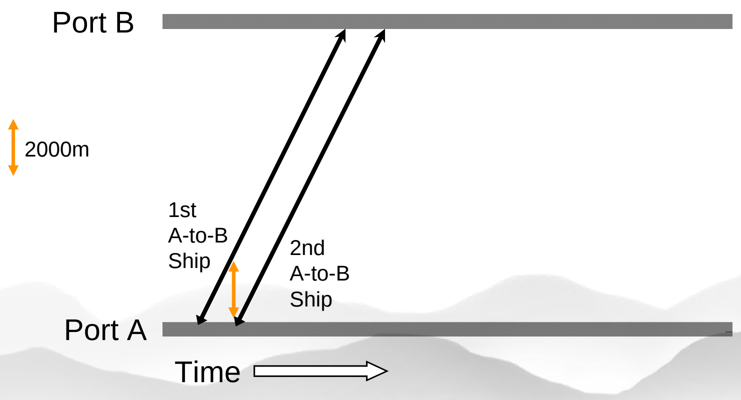
Leeway Constraint

- A channel is a complex unary resource
- The time distance between ships is dependent on the relative directions and relative speeds



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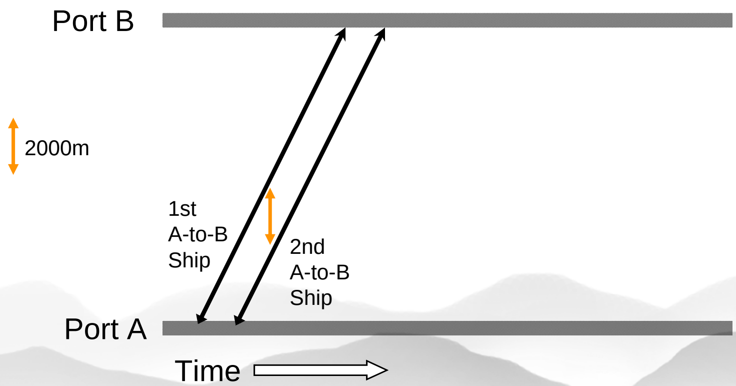

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Leeway Constraint

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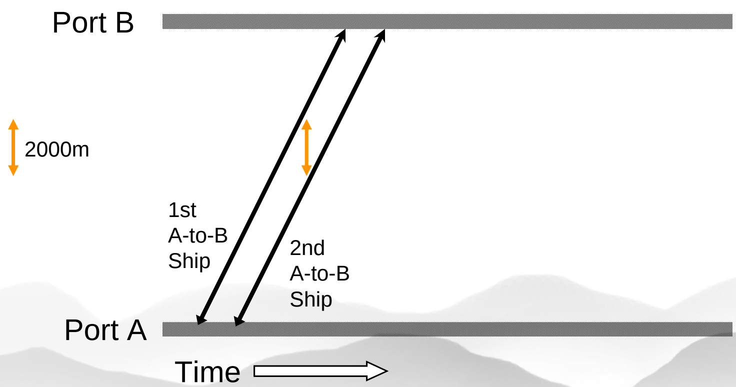

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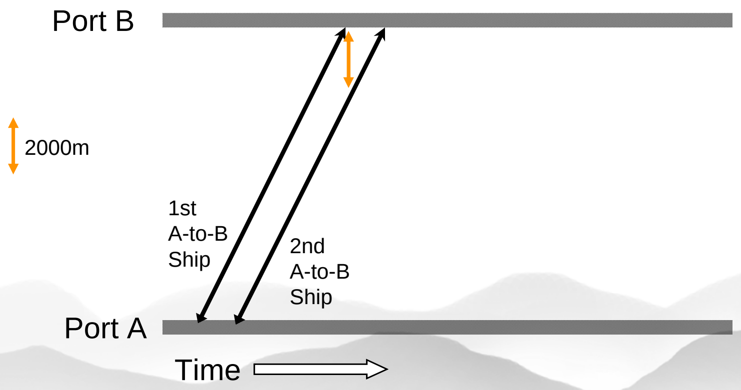

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Leeway Constraint

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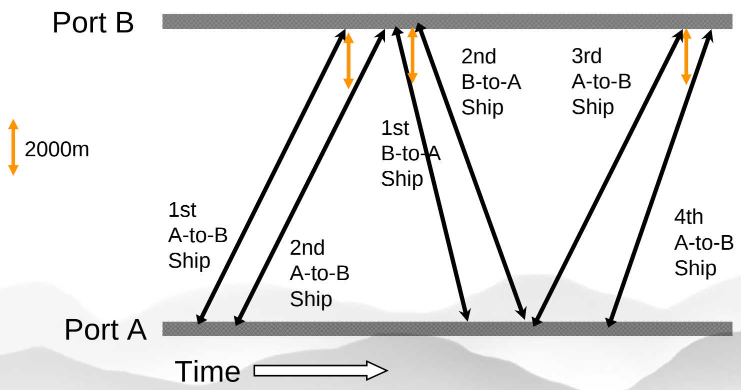

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Sequence Dependence

- Particularly for unary resources
- The schedule may depend on which tasks precedes another on that resource
- Examples
  - smelting: machinery must cool to perform “cold” task after “hot task”
  - embroidery: colors of thread may need changing between tasks
  - single channel: ships traveling in different directions need to wait until channel is clear
- Effectively the start time of the next task is delayed depending on the previous task

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Single Channel (singleChannel.mzn)

- Data

```
int: len;

int: nS; % number of ships
set of int: SHIP = 1..nS;
array[SHIP] of int: speed;   % 1000m time
array[SHIP] of int: desired; % desired time
int: atob = 1; int: btoa = 2;
array[SHIP] of atob..btoa: dirn;

int: leeway;  % leeway between 2 ships
int: maxt;        % maximum time
set of int: TIME = 0..maxt;
```

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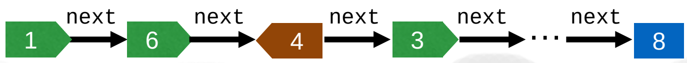
Modeling Activity Sequences

- In order to make decision about the next activity (ship), we need to know what the current activity (ship) is

- But the **last** ship does not have a next ship!

- Introduce a dummy ship at the end



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Single Channel (singleChannel.mzn)

- Decisions
  - add a dummy ship as last ship in the channel
  - so each ship will have a next ship

```
set of int: SHIPE = 1..nS+1; % add dummy
int: dummy = 3;
array[SHIPE] of atob..dummy: kind
           = dirn ++ [ dummy ];
array[SHIPE] of int: speede = speed++[0];
array[SHIPE] of var TIME: start;
array[SHIPE] of var TIME: end;

array[SHIP] of var SHIPE: next; % next ship
```

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Single Channel Constraints (singleChannel.mzn)

- Dummy ship is last

```
start[nS+1] = maxt;
end[nS+1] = maxt;
```

- Relationship between start and end

```
forall(s in SHIP)
   (end[s] = start[s] + len*speed[s]);
```

- The next ships are all different

```
alldifferent(next);
```

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Single Channel

- Reasoning about the channel
  - once we know the next ship, it's reasonably simple
- If the next ship is in the opposite direction
  - it can only start once we end
- If the next ship is in the same direction
  - it must start after we travel 2000m
- Is that enough?
  - NO, a ship is **not** allowed to "catch up"
  - it must still be at least 2000m apart from us when we reach the destination

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Single Channel (singleChannel.mzn)

- Relationship between a ship and its next ship
  - the start and end time are constrained

```
forall(s in SHIP)
          % ships of opposite dirn
      (if kind[s] + kind[next[s]] = 3 then
          end[s] <= start[next[s]]
       else % same dirn
start[s]+speed[s]*leeway <= start[next[s]] /\
  end[s]+speede[next[s]]*leeway <= end[next[s]]
       endif);
```

- Cannot leave before desired time

```
forall(s in SHIP)(start[s] >= desired[s]);
```

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Single Channel

- Objective

```
solve minimize max(s in SHIP)(end[s]);
```

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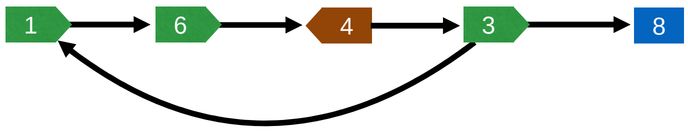
Subtleties of the Model

- Is the `alldifferent` constraint enough
  - for example this satisfies the constraint



- ***BUT*** start times of the ships increase

- But for other similar problems we will need
  - the `circuit` global constraint

Solving the Model

```
start = [271, 43, 385, 33, 115, 281, 175, 600];
end = [431, 235, 513, 193, 243, 505, 271, 600];
next = [6, 5, 8, 2, 7, 3, 1];
----------
==========
```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