Assignment 1 Task 2: Probability

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Question 4.1 Fruits Experiment Python Implementation

In [8]:
```
import numpy as np

def construct_boxes (boxes,card):
    temp = []
    for i in range (len(card)):
        temp.append (['apple']*card[i][0] + ['orange']*card[i][1])
    return temp

def fruits_experiment (num):
    boxes = ['red','blue','green']
    card = [[3,8],[4,4],[9,1]]
    contents = construct_boxes(boxes,card)
    rng = np.random.default_rng()
    boxes_picked = []
    fruits_picked = []
    for i in range (num):
        box_picked = rng.integers(len(boxes))
        fruit_picked = rng.integers(len(contents[box_picked]))
        boxes_picked.append(boxes[box_picked])
        fruits_picked.append(contents[box_picked][fruit_picked])
    return np.array(boxes_picked), np.array(fruits_picked)
```

In [9]:
```
print (fruits_experiment(5))
```

```
(array(['blue', 'blue', 'red', 'blue', 'green'], dtype='<U5'), array(['apple', 'or
ange', 'orange', 'apple', 'apple'], dtype='<U6'))
```

Question 4.2 Probability Calculation

To calculate the probability of picking from Yellow Box given the fruit picked was an apple, we first denote:

Y = **Picking Yellow Box**, A = **Picking an Apple**

where $P(\mathrm{Y}|\mathrm{A})$ = $\frac{P(A|Y) \cdot P(Y)}{P(A)}$ according to Bayes Rule

Firstly, we derive $P(\mathrm{A}|\mathrm{Y})$ =$\frac{9}{10}$ according to the given data and $P(\mathrm{Y}) = \frac{1}{3}$ derived from question due to uniform nature

Then, we can derive $P(A) = P(\mathrm{A}|\mathrm{Y}) \cdot P(\mathrm{Y}) + P(A|\text{not Y}) \cdot P(\text{not Y})$

$P(\mathrm{A}|\text{not Y}) = \frac{7}{16}$

$P(\text{not Y}) = \frac{2}{3}$

So $P(A) = \frac{9}{10} \cdot \frac{1}{3} + \frac{7}{16} \cdot \frac{2}{3} = \frac{71}{124}$

$P(\mathrm{A}|\mathrm{B})$ = $\frac{\frac{9}{10} \cdot \frac{1}{3}}{\frac{71}{124}}$

Final Probability of $P(\mathrm{A}|\mathrm{B}) = 0.524$

Question 5.1 Die Experiment Python Implementation

In [10]:
```
def die_experiment (reps):
    final_results = []
    for _ in range (reps):
        Z = 0
        rng = np.random.default_rng()
        X1 = rng.integers(1,7)
        X2 = rng.integers(1,7)
        Y = X1 + X2
        for _ in range (Y):
            Z += rng.integers(1,7)
        final_results.append(Z)
    return final_results
```

In [11]:
```
reps = 10000
experiment = die_experiment(10000)
```

Question 5.2 Die Experiment Expeced Value Confidence Interval Calculation

In [12]:
```
expected = np.mean(experiment) - ((1.96*np.std(experiment))/np.sqrt(reps))
print (expected)
```

```
24.403483555601007
```

Question 5.3 Derivation

For a discrete X, formula is as given below according to lecture slides

$E[Z|X_1 = x_1, X_2 = x_2] = \sum_z z \cdot P(Z = z|X_1 = x_1, X_2 = x_2)$

According to law of marginal probability, we can get $E[Z]$ from using the formular below by incorporating the conditional expectation above:

$E[Z] = \sum_{x_1,x_2} E[Z|X_1 = x_1, X_2 = x_2] \cdot P(X_1 = x_1, X_2 = x_2)$

Which is equivalent to,

$E[Z] = \sum_{x_1,x_2} \left(\sum_z z \cdot P(Z = z|X_1 = x_1, X_2 = x_2)\right) \cdot P(X_1 = x_1, X_2 = x_2)$

Thus allowing us to calculate the expected value of Z

$E[Z|X_1 = x_1, X_2 = x_2] = (x1 + x2) \cdot 3.5$ (Average value across all die rolls)

Then, $P(X_1 = x_1, X_2 = x_2) = \frac{1}{6} \cdot \frac{1}{6} $

Which means $E[Z] = \sum_{x_1,x_2} (x1 + x2) \cdot 3.5 \cdot \frac{1}{6} \cdot \frac{1}{6}$

In [13]:
```
expected_value = 0
for i in range(1, 7):
    for j in range(1, 7):
        expected_value+= (i + j) * 3.5 * (1/6) * (1/6)
print (expected_value)
```

```
24.499999999999996
```