This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -12 and choose the interval that  $f^{-1}(-12)$  belongs to.

$$f(x) = 4x^2 - 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A.  $f^{-1}(-12) \in [2.15, 3.38]$ 

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B.  $f^{-1}(-12) \in [1.31, 1.64]$ 

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C.  $f^{-1}(-12) \in [5.6, 7.09]$ 

Distractor 4: This corresponds to both distractors 2 and 3.

D.  $f^{-1}(-12) \in [1.78, 2.28]$ 

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

- E. The function is not invertible for all Real numbers.
  - \* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

2. Choose the interval below that f composed with q at x = -1 is in.

$$f(x) = -2x^3 - 4x^2 - 4x$$
 and  $g(x) = 2x^3 - 3x^2 - 4x$ 

The solution is 2.0, which is option A.

- A.  $(f \circ g)(-1) \in [0, 4]$ 
  - \* This is the correct solution
- B.  $(f \circ g)(-1) \in [-4, 1]$

Distractor 2: Corresponds to being slightly off from the solution.

C.  $(f \circ q)(-1) \in [-4, 1]$ 

Distractor 1: Corresponds to reversing the composition.

D.  $(f \circ g)(-1) \in [-13, -9]$ 

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Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** f composed with g at x means f(g(x)). The order matters!

3. Find the inverse of the function below. Then, evaluate the inverse at x = 8 and choose the interval that  $f^{-1}(8)$  belongs to.

$$f(x) = e^{x+3} + 2$$

The solution is  $f^{-1}(8) = -1.208$ , which is option E.

A.  $f^{-1}(8) \in [4.35, 4.46]$ 

This solution corresponds to distractor 4.

B.  $f^{-1}(8) \in [4.53, 4.96]$ 

This solution corresponds to distractor 1.

C.  $f^{-1}(8) \in [3.57, 3.73]$ 

This solution corresponds to distractor 3.

D.  $f^{-1}(8) \in [4.25, 4.31]$ 

This solution corresponds to distractor 2.

E.  $f^{-1}(8) \in [-1.3, -1.13]$ 

This is the solution.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

4. Determine whether the function below is 1-1.

$$f(x) = \sqrt{4x - 16}$$

The solution is yes, which is option B.

A. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

- B. Yes, the function is 1-1.
  - \* This is the solution.
- C. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

5. Determine whether the function below is 1-1.

$$f(x) = -12x^2 + 11x + 56$$

The solution is no, which is option D.

A. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because there is a y-value that goes to 2 different x-values.

\* This is the solution.

E. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

6. Choose the interval below that f composed with g at x = -1 is in.

$$f(x) = -2x^3 + 3x^2 + x - 4$$
 and  $g(x) = 2x^3 + 2x^2 + 2x$ 

The solution is 22.0, which is option B.

A.  $(f \circ g)(-1) \in [31, 36]$ 

Distractor 2: Corresponds to being slightly off from the solution.

- B.  $(f \circ g)(-1) \in [21, 27]$ 
  - \* This is the correct solution
- C.  $(f \circ g)(-1) \in [-8, -1]$

Distractor 3: Corresponds to being slightly off from the solution.

D.  $(f \circ g)(-1) \in [-2, 5]$ 

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

**General Comment:** f composed with g at x means f(g(x)). The order matters!

7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -15 and choose the interval the  $f^{-1}(-15)$  belongs to.

$$f(x) = \sqrt[3]{5x - 4}$$

The solution is -674.2, which is option A.

A. 
$$f^{-1}(-15) \in [-674.97, -673.41]$$

<sup>\*</sup> This is the correct solution.

B.  $f^{-1}(-15) \in [673.81, 675.29]$ 

This solution corresponds to distractor 2.

C.  $f^{-1}(-15) \in [-677.11, -675.68]$ 

Distractor 1: This corresponds to

D.  $f^{-1}(-15) \in [675.31, 676.63]$ 

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

8. Find the inverse of the function below. Then, evaluate the inverse at x = 9 and choose the interval that  $f^{-1}(9)$  belongs to.

$$f(x) = \ln(x - 4) + 2$$

The solution is  $f^{-1}(9) = 1100.633$ , which is option C.

A.  $f^{-1}(9) \in [59875.14, 59882.14]$ 

This solution corresponds to distractor 1.

B.  $f^{-1}(9) \in [442414.39, 442417.39]$ 

This solution corresponds to distractor 2.

C.  $f^{-1}(9) \in [1096.63, 1103.63]$ 

This is the solution.

D.  $f^{-1}(9) \in [144.41, 155.41]$ 

This solution corresponds to distractor 4.

E.  $f^{-1}(9) \in [1087.63, 1093.63]$ 

This solution corresponds to distractor 3.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

9. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 9x^3 + x^2 + 5x + 7$$
 and  $g(x) = \sqrt{6x + 36}$ 

The solution is The domain is all Real numbers greater than or equal to x = -6.0, which is option C.

- A. The domain is all Real numbers less than or equal to x = a, where  $a \in [1, 4]$
- B. The domain is all Real numbers except x = a, where  $a \in [1.8, 7.8]$
- C. The domain is all Real numbers greater than or equal to x = a, where  $a \in [-6, -4]$
- D. The domain is all Real numbers except x = a and x = b, where  $a \in [-0.4, 6.6]$  and  $b \in [4.33, 9.33]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

10. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 6x^3 + 6x + 9$$
 and  $g(x) = 8x^2 + 5x + 3$ 

The solution is  $(-\infty, \infty)$ , which is option E.

- A. The domain is all Real numbers except x = a, where  $a \in [1.33, 6.33]$
- B. The domain is all Real numbers less than or equal to x = a, where  $a \in [-7.2, -3.2]$
- C. The domain is all Real numbers greater than or equal to x = a, where  $a \in [7.33, 9.33]$
- D. The domain is all Real numbers except x = a and x = b, where  $a \in [-6.25, 5.75]$  and  $b \in [-11.8, -1.8]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.