This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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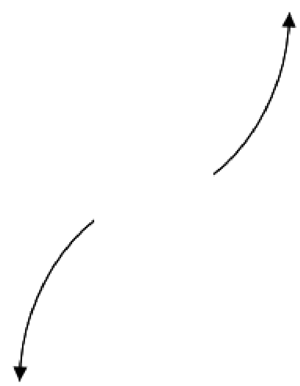
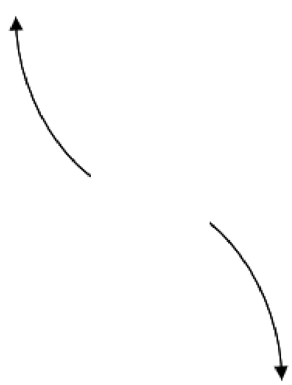
1. Describe the end behavior of the polynomial below.

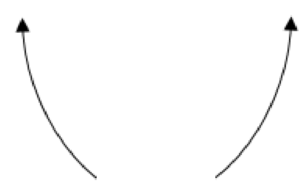
$$f(x) = 5(x+8)^3(x-8)^8(x+6)^5(x-6)^5$$

The solution is the graph below, which is option D.

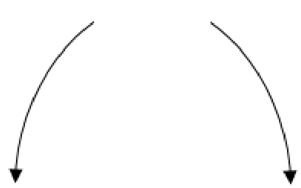




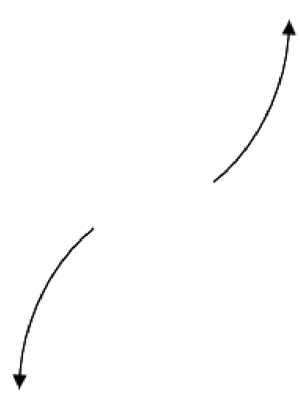
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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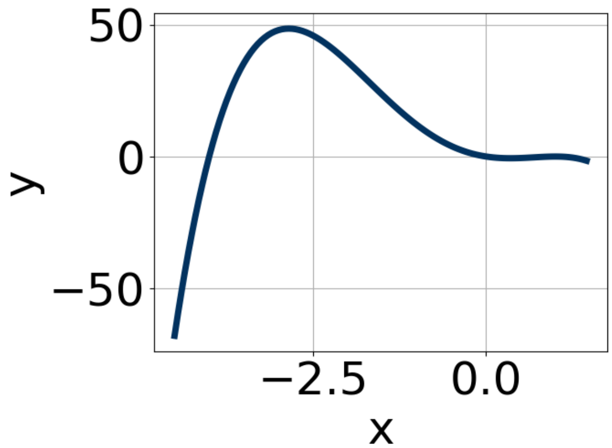
2. Which of the following equations *could* be of the graph presented below?



The solution is $-3x^5(x-1)^6(x+4)^{11}$, which is option D.

A. $-20x^8(x-1)^4(x+4)^5$

The factor x should have an odd power.

B. $14x^5(x-1)^4(x+4)^4$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-7x^6(x-1)^5(x+4)^9$

The factor 1 should have an even power and the factor 0 should have an odd power.

D. $-3x^5(x-1)^6(x+4)^{11}$

* This is the correct option.

E. $20x^7(x-1)^{10}(x+4)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{2}, \frac{-4}{5}, \text{ and } -1$$

The solution is $10x^3 - 17x^2 - 55x - 28$, which is option E.

A. $a \in [8, 21], b \in [14, 22], c \in [-56, -48], \text{ and } d \in [21, 32]$

$10x^3 + 17x^2 - 55x + 28$, which corresponds to multiplying out $(2x+7)(5x-4)(x-1)$.

B. $a \in [8, 21], b \in [32, 42], c \in [-1, 9]$, and $d \in [-30, -24]$

$10x^3 + 37x^2 - x - 28$, which corresponds to multiplying out $(2x + 7)(5x - 4)(x + 1)$.

C. $a \in [8, 21], b \in [45, 61], c \in [65, 76]$, and $d \in [21, 32]$

$10x^3 + 53x^2 + 71x + 28$, which corresponds to multiplying out $(2x + 7)(5x + 4)(x + 1)$.

D. $a \in [8, 21], b \in [-17, -10], c \in [-56, -48]$, and $d \in [21, 32]$

$10x^3 - 17x^2 - 55x + 28$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [8, 21], b \in [-17, -10], c \in [-56, -48]$, and $d \in [-30, -24]$

* $10x^3 - 17x^2 - 55x - 28$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 7)(5x + 4)(x + 1)$

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{4}{3}, \frac{-4}{5}, \text{ and } \frac{-5}{3}$$

The solution is $45x^3 + 51x^2 - 88x - 80$, which is option B.

A. $a \in [38, 47], b \in [-53, -46], c \in [-95, -81]$, and $d \in [75, 85]$

$45x^3 - 51x^2 - 88x + 80$, which corresponds to multiplying out $(3x + 4)(5x - 4)(3x - 5)$.

B. $a \in [38, 47], b \in [46, 59], c \in [-95, -81]$, and $d \in [-82, -72]$

* $45x^3 + 51x^2 - 88x - 80$, which is the correct option.

C. $a \in [38, 47], b \in [46, 59], c \in [-95, -81]$, and $d \in [75, 85]$

$45x^3 + 51x^2 - 88x + 80$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [38, 47], b \in [96, 104], c \in [-8, 0]$, and $d \in [-82, -72]$

$45x^3 + 99x^2 - 8x - 80$, which corresponds to multiplying out $(3x + 4)(5x - 4)(3x + 5)$.

E. $a \in [38, 47], b \in [170, 174], c \in [208, 212]$, and $d \in [75, 85]$

$45x^3 + 171x^2 + 208x + 80$, which corresponds to multiplying out $(3x + 4)(5x + 4)(3x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 4)(5x + 4)(3x + 5)$

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 + 2i \text{ and } 4$$

The solution is $x^3 - 10x^2 + 37x - 52$, which is option B.

A. $b \in [-3, 3], c \in [-7.24, -6.71]$, and $d \in [12, 18]$

$x^3 + x^2 - 7x + 12$, which corresponds to multiplying out $(x - 3)(x - 4)$.

B. $b \in [-17, -6], c \in [35.65, 38.84]$, and $d \in [-55, -51]$

* $x^3 - 10x^2 + 37x - 52$, which is the correct option.

C. $b \in [-3, 3], c \in [-6.62, -5.73]$, and $d \in [0, 11]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 2)(x - 4)$.

D. $b \in [9, 11], c \in [35.65, 38.84]$, and $d \in [47, 55]$

$x^3 + 10x^2 + 37x + 52$, which corresponds to multiplying out $(x - (3 + 2i))(x - (3 - 2i))(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 + 2i))(x - (3 - 2i))(x - (4))$.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 2i \text{ and } -1$$

The solution is $x^3 + 7x^2 + 19x + 13$, which is option D.

A. $b \in [-1.4, 1.6], c \in [2.69, 3.9]$, and $d \in [0.87, 2.86]$

$x^3 + x^2 + 3x + 2$, which corresponds to multiplying out $(x + 2)(x + 1)$.

B. $b \in [-1.4, 1.6], c \in [3.53, 4.78]$, and $d \in [2.66, 3.65]$

$x^3 + x^2 + 4x + 3$, which corresponds to multiplying out $(x + 3)(x + 1)$.

C. $b \in [-7.8, -5.8], c \in [18.72, 19.42]$, and $d \in [-13.71, -12.62]$

$x^3 - 7x^2 + 19x - 13$, which corresponds to multiplying out $(x - (-3 - 2i))(x - (-3 + 2i))(x - 1)$.

D. $b \in [3.5, 10.6], c \in [18.72, 19.42]$, and $d \in [10, 13.23]$

* $x^3 + 7x^2 + 19x + 13$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 2i))(x - (-3 + 2i))(x - (-1))$.

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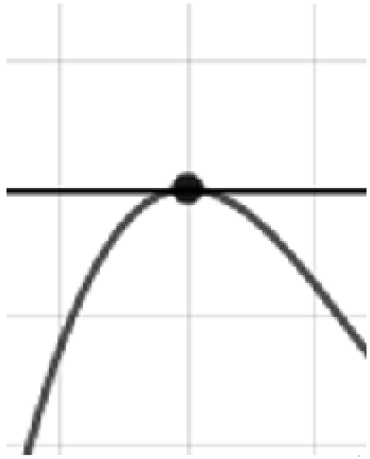
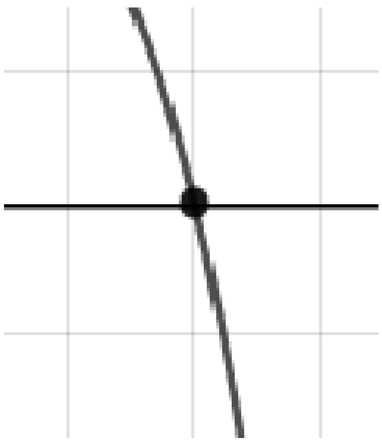
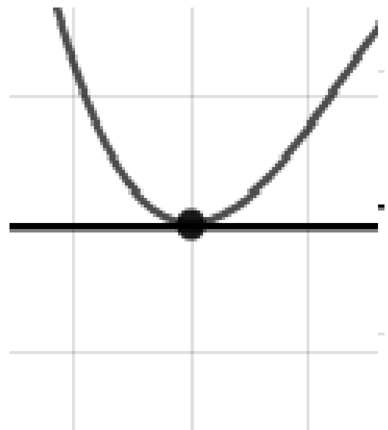
7. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

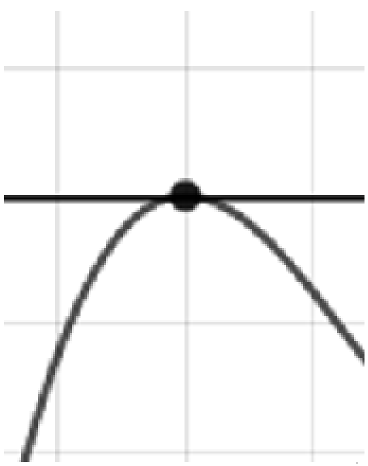
$$f(x) = 4(x + 5)^6(x - 5)^{11}(x - 6)^6(x + 6)^9$$

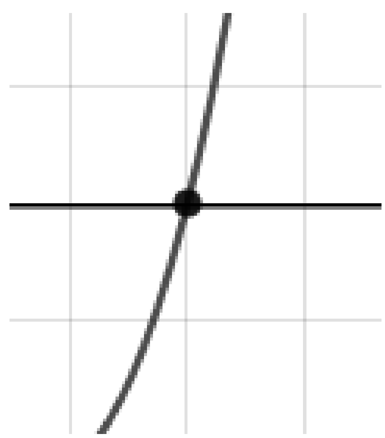
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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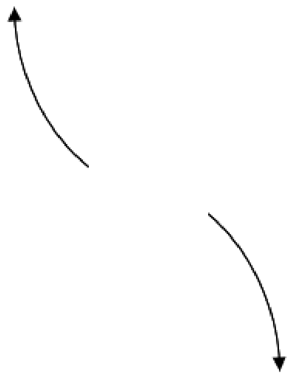
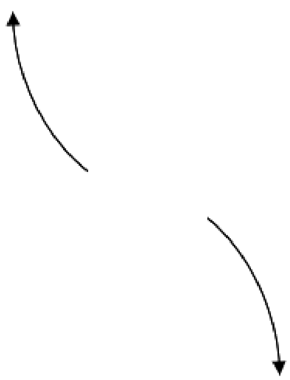
8. Describe the end behavior of the polynomial below.

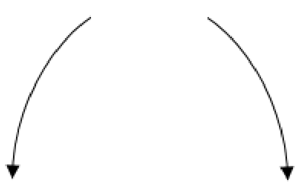
$$f(x) = -3(x-9)^5(x+9)^6(x-3)^4(x+3)^6$$

The solution is the graph below, which is option A.

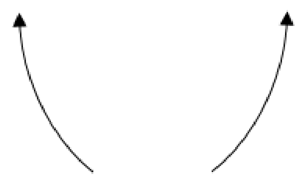




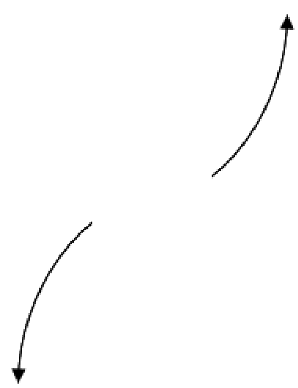
A.



B.



C.

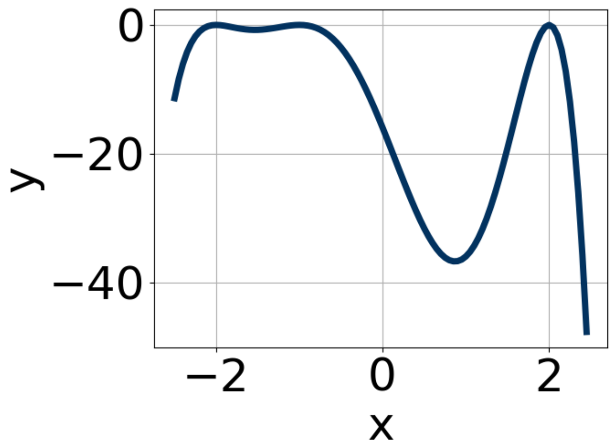


D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

9. Which of the following equations *could* be of the graph presented below?



The solution is $-5(x+1)^4(x+2)^6(x-2)^4$, which is option D.

A. $-3(x+1)^{10}(x+2)^8(x-2)^{11}$

The factor $(x-2)$ should have an even power.

B. $-19(x+1)^4(x+2)^9(x-2)^9$

The factors $(x+2)$ and $(x-2)$ should both have even powers.

C. $15(x+1)^4(x+2)^{10}(x-2)^9$

The factor $(x-2)$ should have an even power and the leading coefficient should be the opposite sign.

D. $-5(x+1)^4(x+2)^6(x-2)^4$

* This is the correct option.

E. $15(x+1)^4(x+2)^8(x-2)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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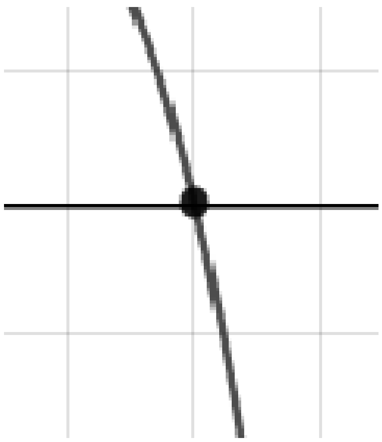
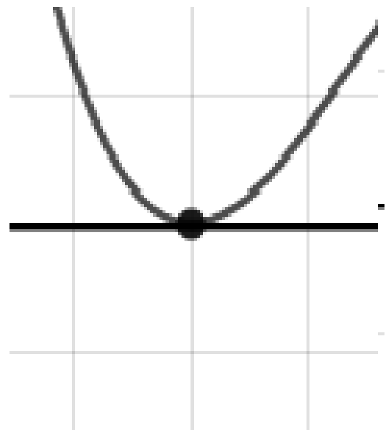
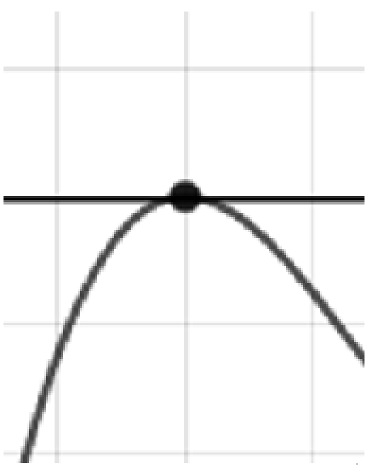
10. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

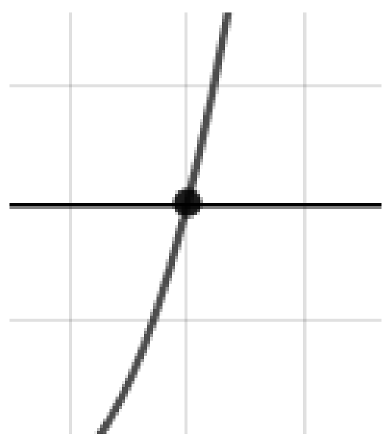
$$f(x) = 8(x-7)^6(x+7)^3(x-2)^6(x+2)^3$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.