This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Choose the model type that would best describe the scenario below.

   Social distancing is a common tactic to counter potential epidemics. This is due to the exponential increase in number of people infected as the density of people living in an area increases.

   The solution is None of the above, which is option D.

   A. Indirect variation

   B. Joint variation

   C. Direct variation

   D. None of the above

   General Comment: This is an exponential variation, which grows significantly faster than any power function.

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2. For the scenario below, model the rate of vibration (cm/s) of the string in terms of the length of the string. Then determine the variation constant k of the model (if possible). The constant should be in terms of cm and s.

   The rate of vibration of a string under constant tension varies based on the type of string and the length of the string. The rate of vibration of string ω decreases as the square length of the string increases. For example, when string ω is 4 mm long, the rate of vibration is 32 cm/s.

   The solution is $k = 5.12$, which is option C.

   A. $k = 200.00$

   This option uses the model $R = kl^2$ as if this is a direct variation.

   B. $k = 2.00$

   This option uses the model $R = kl^2$ as if this is a direct variation AND does not convert from mm to cm so that the units match.

   C. $k = 5.12$

   * This is the correct option, which corresponds to the model $R = \frac{k}{l^2}$ AND converts from mm to cm.

   D. $k = 512.00$

   This option uses the correct model, $R = \frac{k}{l^2}$, but does not convert from mm to cm so that the units match.

   E. None of the above.

   Talk with the coordinator if you chose this option.

General Comment: The most common mistake on this question is to not convert mm to cm! When modeling, you need to make sure all of the units for your variables are compatible.

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3. For the scenario below, use the model for the volume of a cylinder as $V = \pi r^2 h$ to find the coefficient for the model of the new volume $V_{extnew} = kr^2 h$.

Pepsi wants to increase the volume of soda in their cans. They've decided to decrease the radius by 15 percent and increase the height by 20 percent. They want to model the new volume based on the radius and height of the original cans.

The solution is $k = 2.72376$, which is option C.

A. $k = 0.00450$

This corresponds to the model: $V = (0.15r)^2(0.20h)$.

B. $k = 0.01414$

This corresponds to the model: $V = \pi(0.15r)^2(0.20h)$.

C. $k = 2.72376$

* This is the correct option and corresponds to the model: $V = \pi(0.85r)^2(1.20h)$.

D. $k = 0.86700$

This corresponds to the model: $V = (0.85r)^2(1.20h)$.

E. None of the above.

If you chose this, please talk with the coordinator to discuss why you believe none of the options are correct.

General Comment: When calculating the new dimensions, you need to add/subtract from 100%. For example, a 10% increase in height would result in 110% of the original height: $1.1h_{old} = h_{new}$.

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4. For the scenario below, find the variation constant k of the model (if possible).

In an alternative galaxy, the square of the time, T (Earth years), required for a planet to orbit Sun χ increases as the quartic of the distance, d (AUs), that the planet is from Sun χ increases. For example, when Ea's average distance from Sun χ is 3, it takes 50 Earth days to complete an orbit.

The solution is $k = 30.864$, which is option C.

A. $k = 5.373$

This corresponds to the model $T^{1/2} = kd^{1/4}$.

B. $k = 202500.000$

This corresponds to the model $T^2 = \frac{k}{d^4}$.

C. $k = 30.864$

* This is the correct option corresponding to the model $T^2 = kd^4$.

D. $k = 4.028$

This copies the constant used in the homework.

E. Unable to compute the constant based on the information given.

This corresponds to believing you cannot determine the type of model from the information given.

General Comment: Since T increases proportionally as d increases, we know this is a direct variation model.

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5. A town has an initial population of 50000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

Year	1	2	3	4	5	6	7	8	9
Pop.	49970	49940	49910	49880	49850	49820	49790	49760	49730

The solution is Linear, which is option C.

A. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

B. Exponential

This suggests the fastest of growths that we know.

C. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

D. Logarithmic

This suggests the slowest of growths that we know.

E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

General Comment: We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

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6. A town has an initial population of 60000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

Year	1	2	3	4	5	6	7	8	9
Pop.	59800	59200	56800	47200	8800	0	0	0	0

The solution is Exponential, which is option C.

A. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

B. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

C. Exponential

This suggests the fastest of growths that we know.

D. Logarithmic

This suggests the slowest of growths that we know.

E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

General Comment: We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

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7. For the scenario below, find the variation constant k of the model (if possible).

In an alternative galaxy, the square of the time, T (Earth years), required for a planet to orbit Sun χ decreases as the square of the distance, d (AUs), that the planet is from Sun χ decreases. For example, when Ea's average distance from Sun χ is 9, it takes 78 Earth days to complete an orbit.

The solution is $k = 75.111$, which is option C.

A. $k = 492804.000$

This corresponds to the model $T^2 = \dfrac{k}{d^2}$.

B. $k = 2.944$

This corresponds to the model $T^{1/2} = kd^{1/2}$.

C. $k = 75.111$

* This is the correct option corresponding to the model $T^2 = kd^2$.

D. $k = 4.028$

This copies the constant used in the homework.

E. Unable to compute the constant based on the information given.

This corresponds to believing you cannot determine the type of model from the information given.

General Comment: Since T decreases proportionally as d decreases, we know this is a direct variation model.

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8. For the scenario below, use the model for the volume of a cylinder as $V = \pi r^2 h$ to find the coefficient for the model of the new volume $V_{extnew} = kr^2h$.

Pepsi wants to increase the volume of soda in their cans. They've decided to increase the radius by 13 percent and increase the height by 19 percent. They want to model the new volume based on the radius and height of the original cans.

The solution is $k = 4.77368$, which is option D.

A. $k = 1.51951$

This corresponds to the model: $V = (1.13r)^2(1.19h)$.

B. $k = 0.00321$

This corresponds to the model: $V = (0.13r)^2(0.19h)$.

C. $k = 0.01009$

This corresponds to the model: $V = \pi(0.13r)^2(0.19h)$.

D. $k = 4.77368$

* This is the correct option and corresponds to the model: $V = \pi(1.13r)^2(1.19h)$.

E. None of the above.

If you chose this, please talk with the coordinator to discuss why you believe none of the options are correct.

General Comment: When calculating the new dimensions, you need to add/subtract from 100%. For example, a 10% increase in height would result in 110% of the original height: $1.1h_{old} = h_{new}$.

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9. Choose the model type that would best describe the scenario below.

In economics, there are two common equations to model interest earned. The compound interest formula is $A = P(1 + \frac{r}{n})^{nt}$, where A is the amount of money you end up with, P is your starting money, r is the interest rate, n is the number of times compounded in a year, and t is the total number of years. For example, if you were a parent and wanted to save $10,000 in 3 years-time at 3.5% interest compounded monthly, you would need to invest about $9,000.

The solution is None of the above, which is option D.

A. Indirect variation

B. Joint variation

C. Direct variation

D. None of the above

General Comment: When thinking about power functions, we want the exponent to be constant and the base to be a variable (or variables). In this case, we see variables in the exponent, which tips us off that this is not a power variation.

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10. For the scenario below, model the rate of vibration (cm/s) of the string in terms of the length of the string. Then determine the variation constant k of the model (if possible). The constant should be in terms of cm and s.

The rate of vibration of a string under constant tension varies based on the type of string and the length of the string. The rate of vibration of string ω increases as the cube length of the string decreases. For example, when string ω is 3 mm long, the rate of vibration is 30 cm/s.

The solution is $k = 0.81$, which is option D.

A. $k = 810.00$

This option uses the correct model, $R = \frac{k}{l^3}$, but does not convert from mm to cm so that the units match.

B. $k = 1.11$

This option uses the model $R = kl^3$ as if this is a direct variation AND does not convert from mm to cm so that the units match.

C. $k = 1111.11$

This option uses the model $R = kl^3$ as if this is a direct variation.

D. $k = 0.81$

* This is the correct option, which corresponds to the model $R = \frac{k}{l^3}$ AND converts from mm to cm.

E. None of the above.

Talk with the coordinator if you chose this option.

General Comment: The most common mistake on this question is to not convert mm to cm! When modeling, you need to make sure all of the units for your variables are compatible.