**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

1. Determine the domain of the function below.

$$f(x) = \frac{6}{15x^2 + 27x + 12}$$

The solution is All Real numbers except $x = -1.000$ and $x = -0.800$., which is option A.

A. All Real numbers except $x = a$ and $x = b$, where $a \in [-1.16, -0.98]$ and $b \in [-0.99, -0.75]$

All Real numbers except $x = -1.000$ and $x = -0.800$, which is the correct option.

B. All Real numbers except $x = a$, where $a \in [-20.05, -19.94]$

All Real numbers except $x = -20.000$, which corresponds to removing a distractor value from the denominator.

C. All Real numbers.

This corresponds to thinking the denominator has complex roots or that rational functions have a domain of all Real numbers.

D. All Real numbers except $x = a$, where $a \in [-1.16, -0.98]$

All Real numbers except $x = -1.000$, which corresponds to removing only 1 value from the denominator.

E. All Real numbers except $x = a$ and $x = b$, where $a \in [-20.05, -19.94]$ and $b \in [-9.06, -9]$

All Real numbers except $x = -20.000$ and $x = -9.000$, which corresponds to not factoring the denominator correctly.
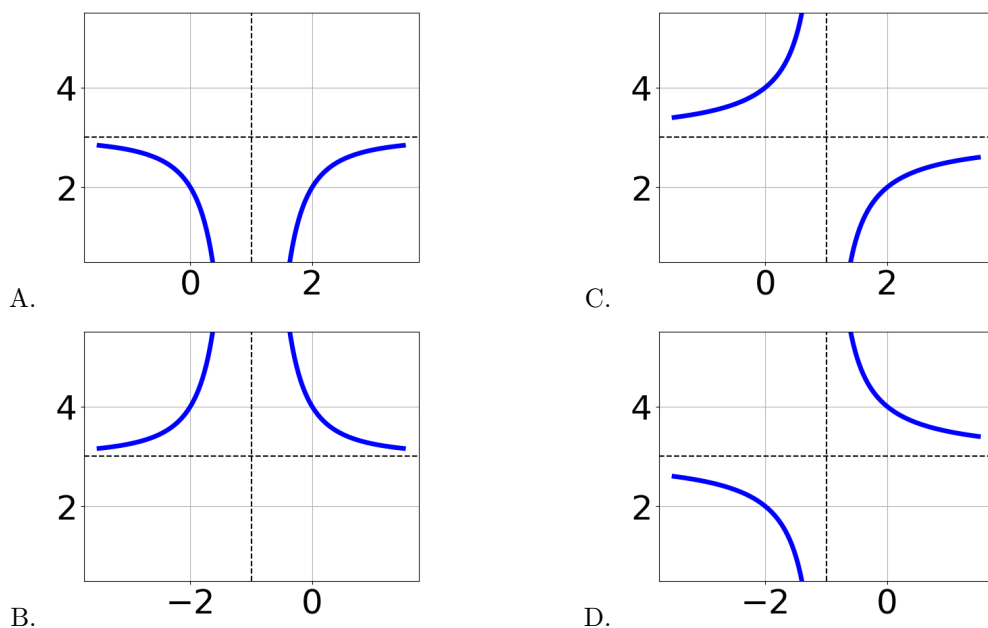
**General Comment:** Recall that dividing by zero is not a real number. Therefore the domain is all real numbers **except** those that make the denominator 0.

2. Choose the graph of the equation below.

$$f(x) = \frac{1}{x + 1} + 3$$

The solution is the graph below, which is option D.
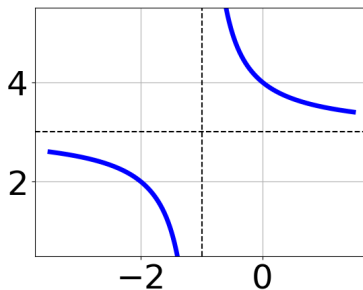
A.

C.

B.

D.

E. None of the above.

**General Comment:** Remember that the general form of a basic rational equation is $f(x) = \dfrac{a}{(x - h)^n} + k$, where $a$ is the leading coefficient (and in this case, we assume is either 1 or $-1$), $n$ is the degree (in this case, either 1 or 2), and $(h, k)$ is the intersection of the asymptotes.

3. Choose the equation of the function graphed below.
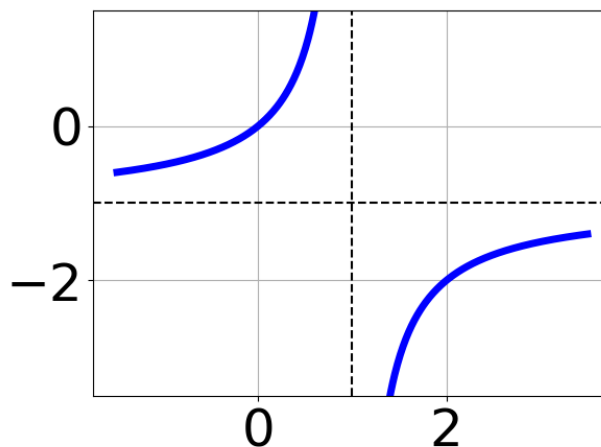
The solution is None of the above as it should be $f(x) = \dfrac{-1}{x - 1} - 1$, which is option E.

A. $f(x) = \dfrac{1}{x + 1} + 3$

Corresponds to using the general form $f(x) = \dfrac{a}{x + h} + k$, the opposite leading coefficient AND not noticing the $y$-value was wrong.

B. $f(x) = \dfrac{-1}{x - 1} + 3$

The $y$-value of the equation does not match the graph.

C. $f(x) = \dfrac{1}{(x+1)^2} + 3$

Corresponds to thinking the graph was a shifted version of $\dfrac{1}{x^2}$, using the general form $f(x) = \dfrac{a}{x+h} + k$, the opposite leading coefficient, AND not noticing the $y$-value was wrong.

D. $f(x) = \dfrac{-1}{(x-1)^2} + 3$

Corresponds to thinking the graph was a shifted version of $\dfrac{1}{x^2}$ not noticing the $y$-value was wrong.
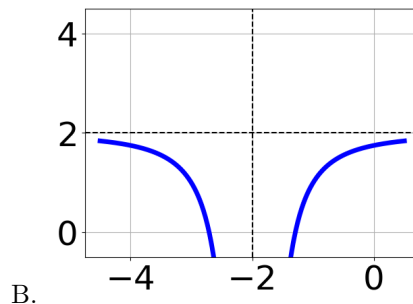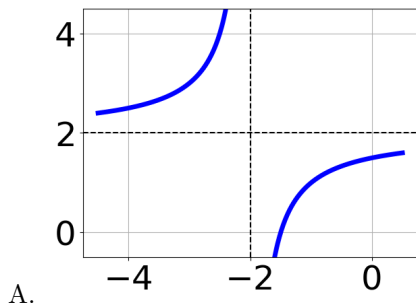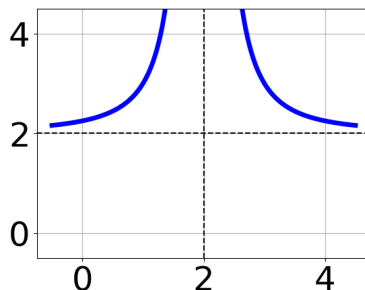
E. None of the above

None of the equation options were the correct equation.

**General Comment:** Remember that the general form of a basic rational equation is $f(x) = \dfrac{a}{(x-h)^n} + k$, where $a$ is the leading coefficient (and in this case, we assume is either 1 or $-1$), $n$ is the degree (in this case, either 1 or 2), and $(h, k)$ is the intersection of the asymptotes.

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4. Choose the graph of the equation below.

$$f(x) = \frac{1}{(x-2)^2} + 2$$

The solution is the graph below, which is option D.





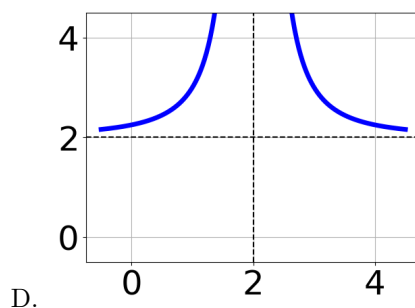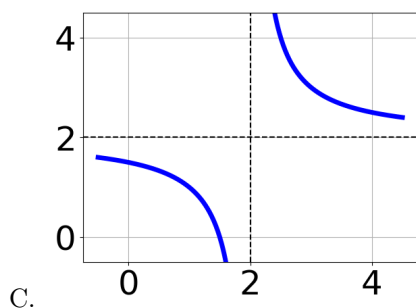A.



B.

C.



D.

E. None of the above.

**General Comment:** Remember that the general form of a basic rational equation is $f(x) = \dfrac{a}{(x-h)^n} + k$, where $a$ is the leading coefficient (and in this case, we assume is either $1$ or $-1$), $n$ is the degree (in this case, either $1$ or $2$), and $(h, k)$ is the intersection of the asymptotes.

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5. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{88}{44x - 22} + 1 = \frac{88}{44x - 22}$$

The solution is all solutions are invalid or lead to complex values in the equation., which is option D.

A. $x \in [-1.3, -0.4]$

   $x = -0.500$, which corresponds to not distributing the factor $44x - 22$ correctly when trying to eliminate the fraction.

B. $x_1 \in [-1.3, -0.4]$ and $x_2 \in [0.5, 1.5]$

   $x = -0.500$ and $x = 0.500$, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

C. $x \in [0.5, 1.5]$

   $x = 0.500$, which corresponds to not checking if this value leads to dividing by $0$ in the original equation and thus is not a valid solution.

D. All solutions lead to invalid or complex values in the equation.

   *$x = 0.500$ leads to dividing by $0$ in the original equation and thus is not a valid solution, which is the correct option.

E. $x_1 \in [0.3, 1.5]$ and $x_2 \in [0.5, 1.5]$

   $x = 0.500$ and $x = 0.500$, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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6. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{-6x}{-7x - 7} + \frac{-2x^2}{14x^2 + 49x + 35} = \frac{-3}{-2x - 5}$$

The solution is There are two solutions: $x = 1.067$ and $x = -1.967$, which is option A.

A. $x_1 \in [0.96, 1.29]$ and $x_2 \in [-3.59, -1.43]$

   * $x = 1.067$ and $x = -1.967$, which is the correct option.

B. All solutions lead to invalid or complex values in the equation.

C. $x_1 \in [0.96, 1.29]$ and $x_2 \in [-1.5, -0.92]$

D. $x \in [-3.1, -2.03]$

E. $x \in [-2.28, -1.93]$

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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7. Determine the domain of the function below.

$$f(x) = \frac{5}{24x^2 - 14x - 20}$$

The solution is All Real numbers except $x = -0.667$ and $x = 1.250$., which is option A.

A. All Real numbers except $x = a$ and $x = b$, where $a \in [-1.9, -0.1]$ and $b \in [0.6, 2.8]$

   All Real numbers except $x = -0.667$ and $x = 1.250$, which is the correct option.

B. All Real numbers except $x = a$ and $x = b$, where $a \in [-25.1, -23.2]$ and $b \in [19.1, 20.6]$

   All Real numbers except $x = -24.000$ and $x = 20.000$, which corresponds to not factoring the denominator correctly.

C. All Real numbers except $x = a$, where $a \in [-25.1, -23.2]$

   All Real numbers except $x = -24.000$, which corresponds to removing a distractor value from the denominator.

D. All Real numbers except $x = a$, where $a \in [-1.9, -0.1]$

   All Real numbers except $x = -0.667$, which corresponds to removing only 1 value from the denominator.

E. All Real numbers.

   This corresponds to thinking the denominator has complex roots or that rational functions have a domain of all Real numbers.

**General Comment:** Recall that dividing by zero is not a real number. Therefore the domain is all real numbers **except** those that make the denominator 0.

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8. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{3}{3x - 2} + 6 = \frac{-2}{27x - 18}$$

The solution is $x = 0.488$, which is option D.

A. All solutions lead to invalid or complex values in the equation.

   This corresponds to thinking $x = 0.488$ leads to dividing by zero in the original equation, which it does not.

B. $x_1 \in [0.26, 0.47]$ and $x_2 \in [-0.51, 3.49]$

   $x = 0.389$ and $x = 0.488$, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

C. $x \in [-0.89, -0.84]$

$x = -0.846$, which corresponds to not distributing the factor $3x - 2$ correctly when trying to eliminate the fraction.

D. $x \in [0.49, 2.49]$

\* $x = 0.488$, which is the correct option.

E. $x_1 \in [-0.89, -0.84]$ and $x_2 \in [-0.51, 3.49]$

$x = -0.846$ and $x = 0.488$, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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9. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{-4x}{5x - 5} + \frac{-7x^2}{-35x^2 + 35} = \frac{-5}{-7x - 7}$$

The solution is There are two solutions: $x = 0.406$ and $x = -2.930$, which is option C.

A. $x \in [-2.4, -0.1]$

B. All solutions lead to invalid or complex values in the equation.
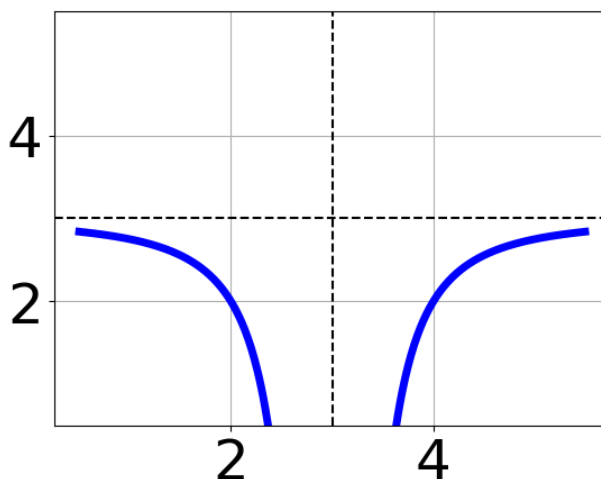
C. $x_1 \in [-0.4, 0.7]$ and $x_2 \in [-6.93, -0.93]$

\* $x = 0.406$ and $x = -2.930$, which is the correct option.

D. $x_1 \in [-0.4, 0.7]$ and $x_2 \in [0, 11]$

E. $x \in [-5.1, -1.5]$

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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10. Choose the equation of the function graphed below.



The solution is None of the above as it should be $f(x) = \frac{-1}{(x-3)^2} + 3$, which is option E.

A. $f(x) = \dfrac{-1}{x+3} + 1$

Corresponds to thinking the graph was a shifted version of $\dfrac{1}{x}$ AND not noticing the $y$-value was wrong.

B. $f(x) = \dfrac{1}{x-3} + 1$

Corresponds to thinking the graph was a shifted version of $\dfrac{1}{x}$, using the general form $f(x) = \dfrac{a}{(x-h)^2} + k$, the opposite leading coefficient, AND not noticing the $y$-value was wrong.

C. $f(x) = \dfrac{1}{(x-3)^2} + 1$

Corresponds to using the general form $f(x) = \dfrac{a}{(x-h)^2} + k$, the opposite leading coefficient, AND not noticing the $y$-value was wrong.

D. $f(x) = \dfrac{-1}{(x+3)^2} + 1$

The $x$- and $y$-value of the equation does not match the graph.

E. None of the above

None of the equation options were the correct equation.

**General Comment:** Remember that the general form of a basic rational equation is $f(x) = \dfrac{a}{(x-h)^n} + k$, where $a$ is the leading coefficient (and in this case, we assume is either 1 or $-1$), $n$ is the degree (in this case, either 1 or 2), and $(h, k)$ is the intersection of the asymptotes.