This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$6, \frac{2}{3}, \text{ and } \frac{-3}{5}$$

The solution is $15x^3 - 91x^2 + 36$, which is option B.

A. $a \in [13, 17], b \in [87.9, 89.1], c \in [-13, -6], \text{ and } d \in [-42, -31]$

$15x^3 + 89x^2 - 12x - 36$, which corresponds to multiplying out $(x + 6)(3x - 2)(5x + 3)$.

B. $a \in [13, 17], b \in [-93.3, -90.6], c \in [-2, 4], \text{ and } d \in [35, 42]$

* $15x^3 - 91x^2 + 36$, which is the correct option.

C. $a \in [13, 17], b \in [89.6, 91.6], c \in [-2, 4], \text{ and } d \in [-42, -31]$

$15x^3 + 91x^2 - 36$, which corresponds to multiplying out $(x + 6)(3x + 2)(5x - 3)$.

D. $a \in [13, 17], b \in [106.9, 113.9], c \in [118, 124], \text{ and } d \in [35, 42]$

$15x^3 + 109x^2 + 120x + 36$, which corresponds to multiplying out $(x + 6)(3x + 2)(5x + 3)$.

E. $a \in [13, 17], b \in [-93.3, -90.6], c \in [-2, 4], \text{ and } d \in [-42, -31]$

$15x^3 - 91x^2 - 36$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-6)(3x-2)(5x+3)$

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 2i \text{ and } 3$$

The solution is $x^3 + 7x^2 - x - 87$, which is option C.

A. $b \in [-5, 4], c \in [-5.1, -3.5], \text{ and } d \in [4, 9]$

$x^3 + x^2 - 5x + 6$, which corresponds to multiplying out $(x - 2)(x - 3)$.

B. $b \in [-10, -3], c \in [-1.4, -0.2], \text{ and } d \in [84, 89]$

$x^3 - 7x^2 - x + 87$, which corresponds to multiplying out $(x - (-5 + 2i))(x - (-5 - 2i))(x + 3)$.

C. $b \in [4, 16], c \in [-1.4, -0.2], \text{ and } d \in [-90, -84]$

* $x^3 + 7x^2 - x - 87$, which is the correct option.

D. $b \in [-5, 4], c \in [1, 2.4], \text{ and } d \in [-21, -11]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x + 5)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 2i))(x - (-5 - 2i))(x - (3))$.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 4i \text{ and } -1$$

The solution is $x^3 - 7x^2 + 24x + 32$, which is option D.

A. $b \in [1, 5], c \in [0, 10],$ and $d \in [0, 9]$

$x^3 + x^2 + 5x + 4$, which corresponds to multiplying out $(x + 4)(x + 1)$.

B. $b \in [1, 5], c \in [-10, 2],$ and $d \in [-10, 3]$

$x^3 + x^2 - 3x - 4$, which corresponds to multiplying out $(x - 4)(x + 1)$.

C. $b \in [5, 9], c \in [17, 29],$ and $d \in [-32, -27]$

$x^3 + 7x^2 + 24x - 32$, which corresponds to multiplying out $(x - (4 - 4i))(x - (4 + 4i))(x - 1)$.

D. $b \in [-12, -3], c \in [17, 29],$ and $d \in [32, 35]$

* $x^3 - 7x^2 + 24x + 32$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 4i))(x - (4 + 4i))(x - (-1))$.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{4}, \frac{7}{3}, \text{ and } -1$$

The solution is $12x^3 - 37x^2 + 49$, which is option D.

A. $a \in [5, 19], b \in [-39, -36], c \in [-4, 2],$ and $d \in [-52, -46]$

$12x^3 - 37x^2 - 49$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [5, 19], b \in [56, 69], c \in [95, 101],$ and $d \in [47, 55]$

$12x^3 + 61x^2 + 98x + 49$, which corresponds to multiplying out $(4x + 7)(3x + 7)(x + 1)$.

C. $a \in [5, 19], b \in [2, 7], c \in [-61, -52],$ and $d \in [-52, -46]$

$12x^3 + 5x^2 - 56x - 49$, which corresponds to multiplying out $(4x + 7)(3x - 7)(x + 1)$.

D. $a \in [5, 19], b \in [-39, -36], c \in [-4, 2],$ and $d \in [47, 55]$

* $12x^3 - 37x^2 + 49$, which is the correct option.

E. $a \in [5, 19], b \in [36, 41], c \in [-4, 2],$ and $d \in [-52, -46]$

$12x^3 + 37x^2 - 49$, which corresponds to multiplying out $(4x + 7)(3x + 7)(x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 7)(3x - 7)(x + 1)$

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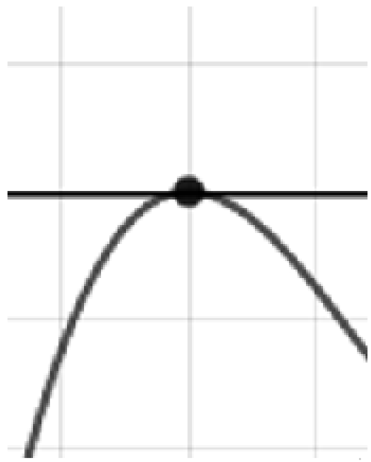
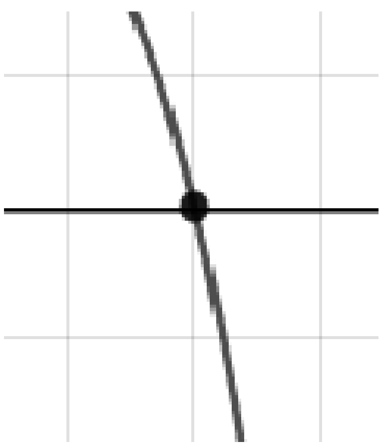
5. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

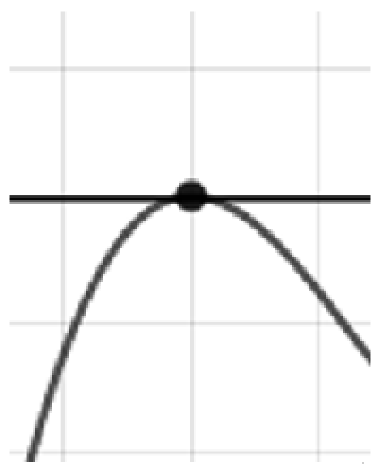
$$f(x) = -5(x - 4)^4(x + 4)^9(x - 2)^3(x + 2)^7$$

The solution is the graph below, which is option B.

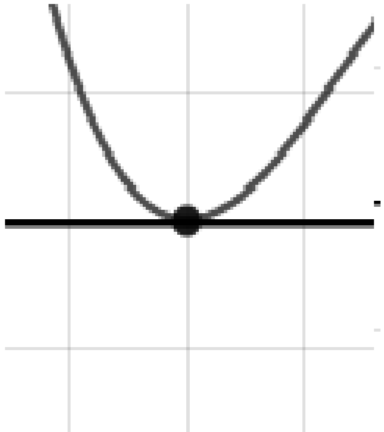


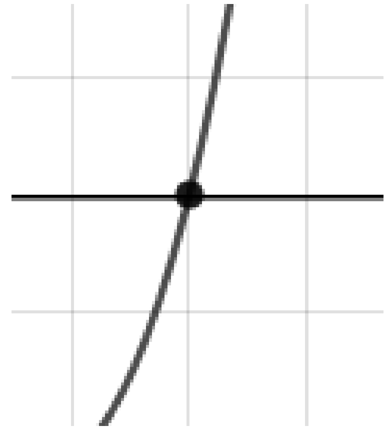


A.



B.

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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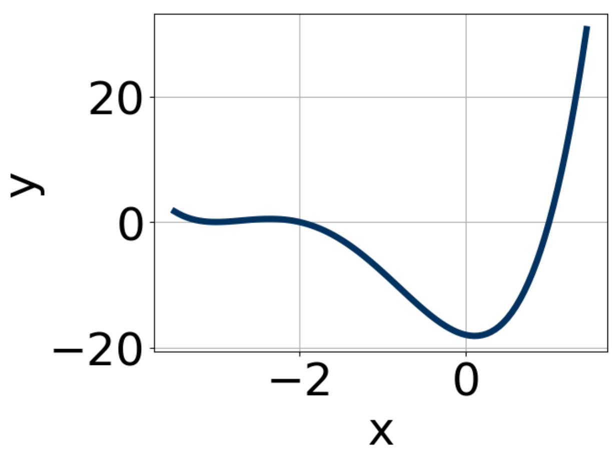
6. Which of the following equations *could* be of the graph presented below?



The solution is $8(x+3)^4(x+2)^5(x-1)^9$, which is option B.

A. $17(x+3)^9(x+2)^4(x-1)^9$

The factor -3 should have an even power and the factor -2 should have an odd power.

B. $8(x+3)^4(x+2)^5(x-1)^9$

* This is the correct option.

C. $11(x+3)^{10}(x+2)^{10}(x-1)^7$

The factor $(x+2)$ should have an odd power.

D. $-3(x+3)^8(x+2)^5(x-1)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-3(x+3)^4(x+2)^9(x-1)^8$

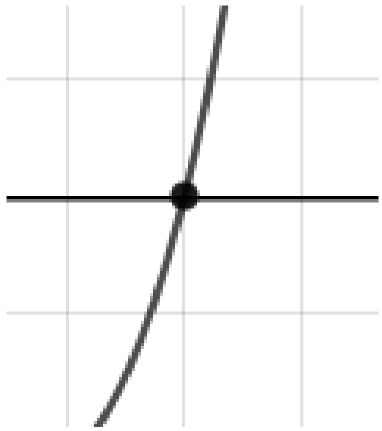
The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

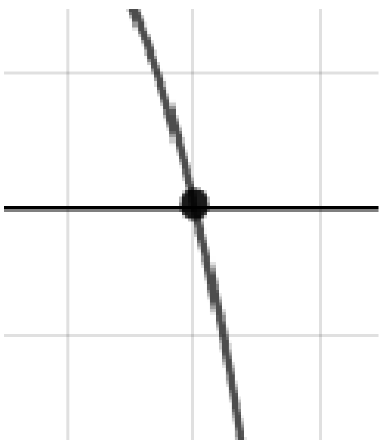
7. Describe the zero behavior of the zero $x = 6$ of the polynomial below.

$$f(x) = 5(x+3)^8(x-3)^4(x-6)^{13}(x+6)^8$$

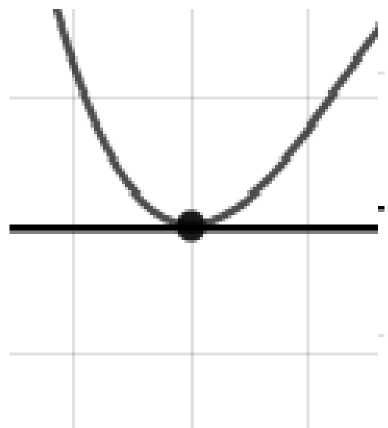
The solution is the graph below, which is option D.



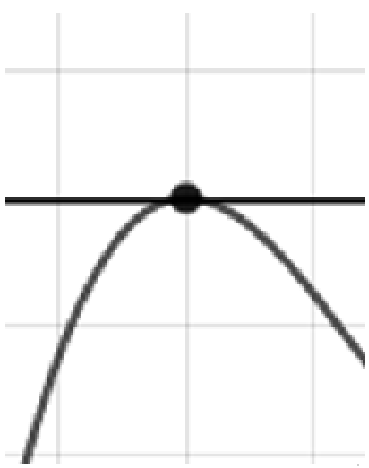
A.



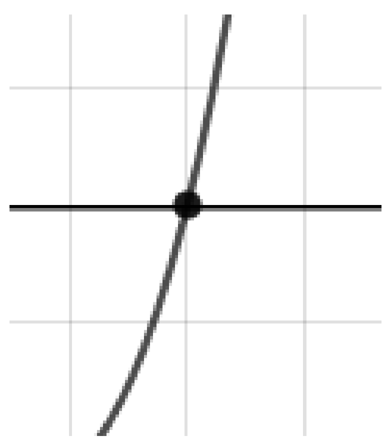
C.



B.



D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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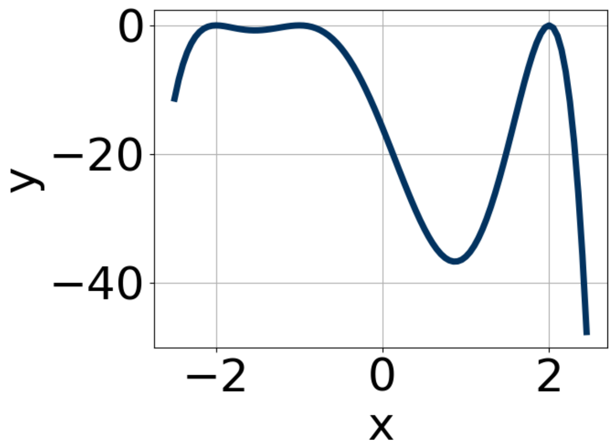
8. Which of the following equations *could* be of the graph presented below?



The solution is $-19(x-2)^8(x+2)^4(x+1)^4$, which is option D.

A. $7(x-2)^4(x+2)^6(x+1)^9$

The factor $(x+1)$ should have an even power and the leading coefficient should be the opposite sign.

B. $-4(x-2)^8(x+2)^{11}(x+1)^9$

The factors $(x+2)$ and $(x+1)$ should both have even powers.

C. $13(x-2)^4(x+2)^8(x+1)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-19(x-2)^8(x+2)^4(x+1)^4$

* This is the correct option.

E. $-16(x-2)^6(x+2)^4(x+1)^{11}$

The factor $(x+1)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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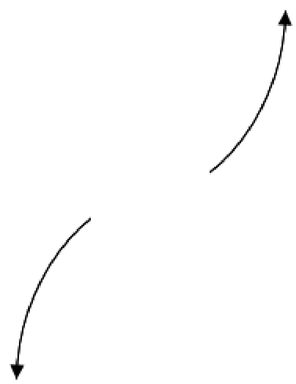
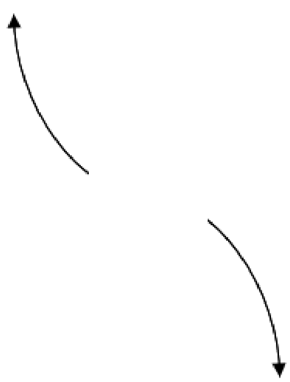
9. Describe the end behavior of the polynomial below.

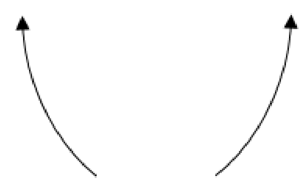
$$f(x) = 4(x+5)^5(x-5)^{10}(x-2)^3(x+2)^5$$

The solution is the graph below, which is option D.

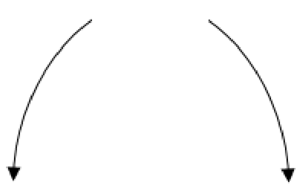




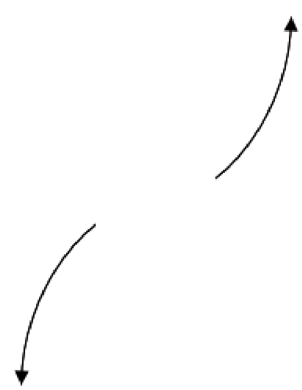
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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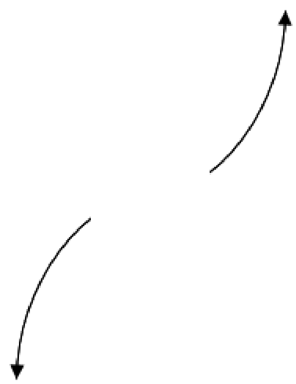
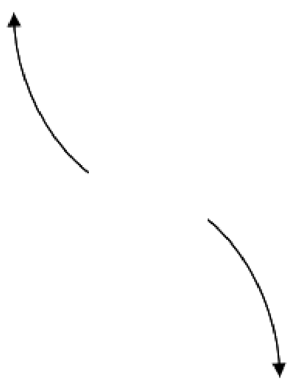
10. Describe the end behavior of the polynomial below.

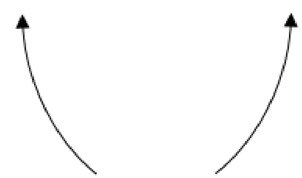
$$f(x) = 4(x+6)^2(x-6)^5(x+4)^2(x-4)^2$$

The solution is the graph below, which is option D.

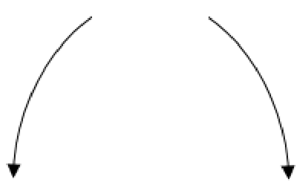




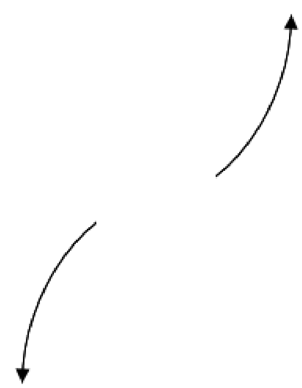
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.