**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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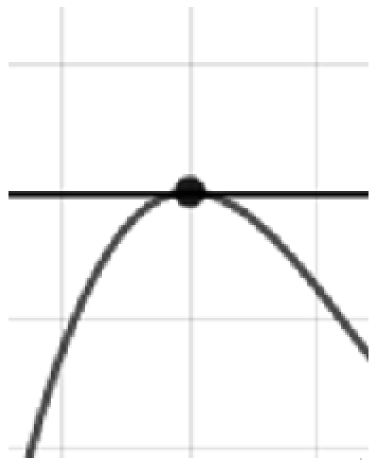
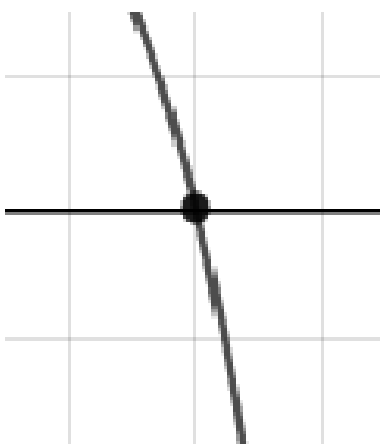
1. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

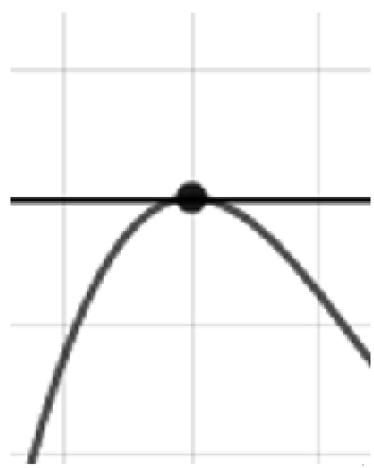
$$f(x) = -6(x-6)^9(x+6)^{14}(x+3)^6(x-3)^9$$

The solution is the graph below, which is option B.

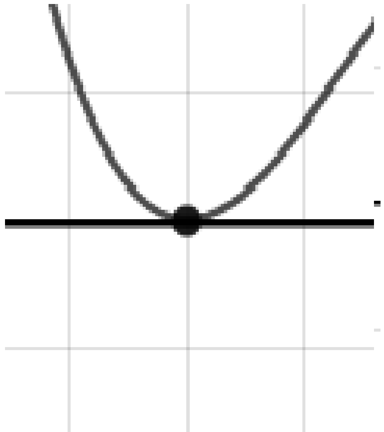


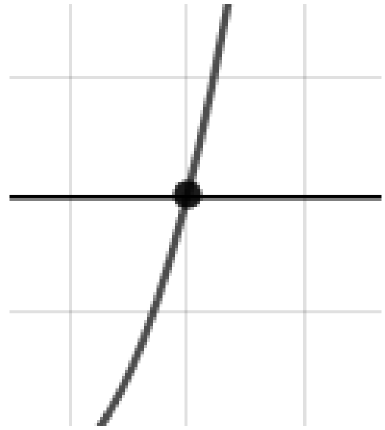


A.



B.

C. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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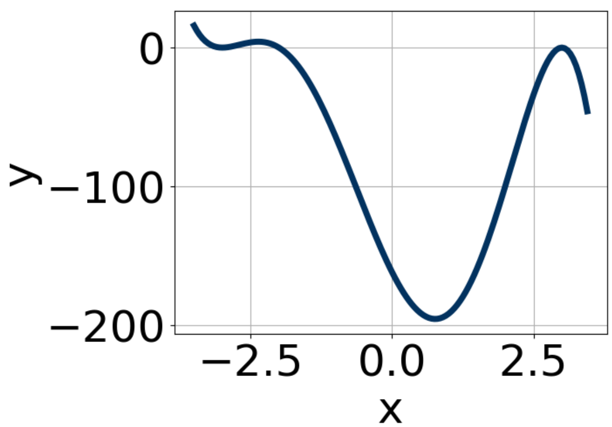
2. Which of the following equations *could* be of the graph presented below?




The solution is $-15(x+3)^{10}(x-3)^{6}(x+2)^{11}$, which is option D.

A. $17(x+3)^{10}(x-3)^{8}(x+2)^{7}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-10(x+3)^{6}(x-3)^{7}(x+2)^{7}$

The factor $(x-3)$ should have an even power.

C. $-8(x+3)^{6}(x-3)^{9}(x+2)^{8}$

The factor $(x-3)$ should have an even power and the factor $(x+2)$ should have an odd power.

D. $-15(x+3)^{10}(x-3)^{6}(x+2)^{11}$

* This is the correct option.

E. $19(x+3)^{4}(x-3)^{6}(x+2)^{10}$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

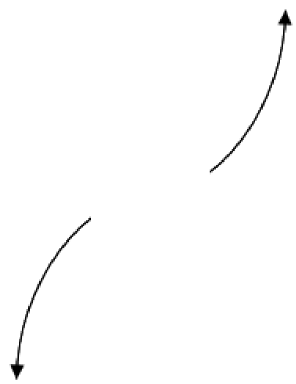
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

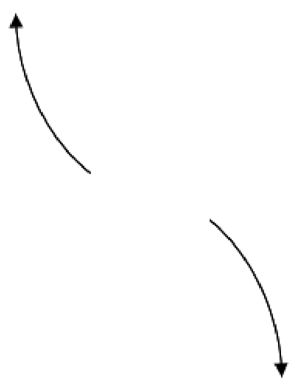
---

3. Describe the end behavior of the polynomial below.

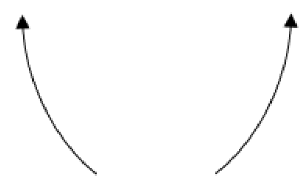
$$f(x) = 2(x-9)^2(x+9)^3(x-2)^4(x+2)^6$$

The solution is the graph below, which is option D.

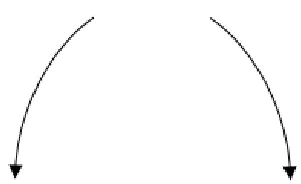




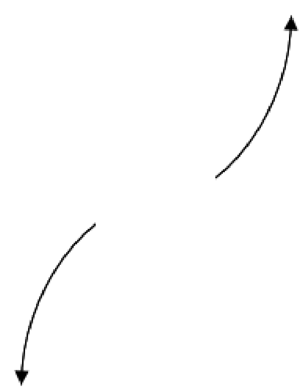
A.



C.



B.



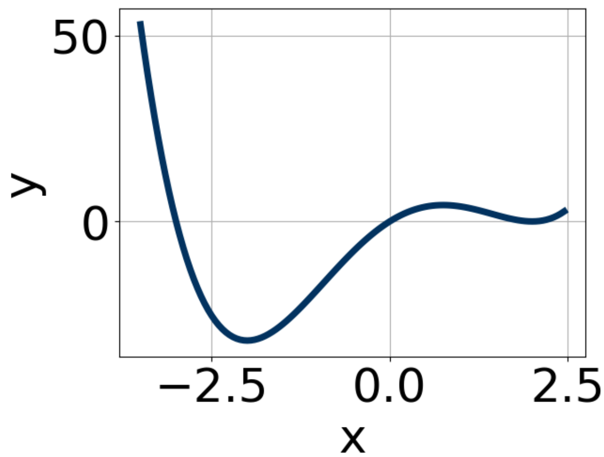
D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

4. Which of the following equations *could* be of the graph presented below?




The solution is $14x^5(x-2)^{10}(x+3)^{11}$, which is option C.

A. $19x^5(x-2)^6(x+3)^8$

The factor $(x+3)$ should have an odd power.

B. $-15x^4(x-2)^4(x+3)^{11}$

The factor $x$ should have an odd power and the leading coefficient should be the opposite sign.

C. $14x^5(x-2)^{10}(x+3)^{11}$

* This is the correct option.

D. $18x^5(x-2)^9(x+3)^4$

The factor 2 should have an even power and the factor $-3$ should have an odd power.

E. $-15x^{11}(x-2)^6(x+3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

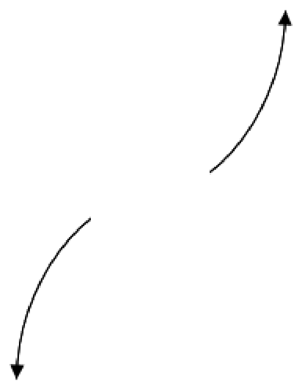
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

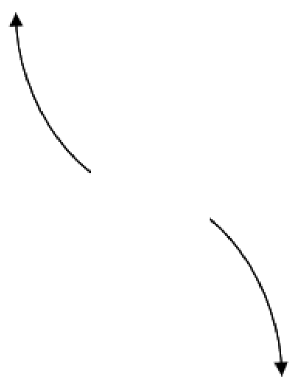
---

5. Describe the end behavior of the polynomial below.

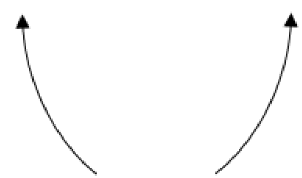
$$f(x) = 6(x-2)^4(x+2)^5(x+3)^4(x-3)^4$$

The solution is the graph below, which is option D.

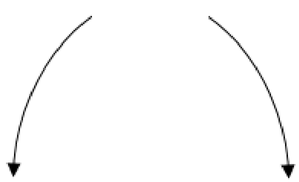




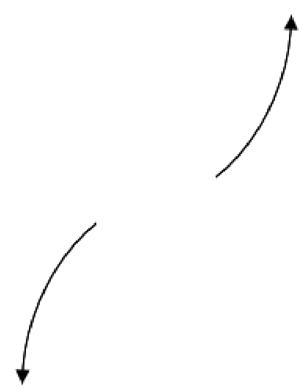
A.



C.



B.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-7, \frac{2}{3}, \text{ and } -2$$

The solution is $3x^3 + 25x^2 + 24x - 28$, which is option D.

A. $a \in [-1, 5], b \in [-17.9, -15.3], c \in [-36, -27], \text{ and } d \in [27, 36]$

$3x^3 - 17x^2 - 32x + 28$, which corresponds to multiplying out $(x + 1)(3x - 3)(x - 1)$.

B. $a \in [-1, 5], b \in [23.4, 27.8], c \in [22, 26],$ and $d \in [27, 36]$

$3x^3 + 25x^2 + 24x + 28$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [-1, 5], b \in [-13.7, -11.2], c \in [-60, -51],$ and $d \in [-30, -25]$

$3x^3 - 13x^2 - 52x - 28$, which corresponds to multiplying out $(x + 1)(3x + 3)(x - 1)$.

D. $a \in [-1, 5], b \in [23.4, 27.8], c \in [22, 26],$ and $d \in [-30, -25]$

* $3x^3 + 25x^2 + 24x - 28$, which is the correct option.

E. $a \in [-1, 5], b \in [-27.3, -24], c \in [22, 26],$ and $d \in [27, 36]$

$3x^3 - 25x^2 + 24x + 28$, which corresponds to multiplying out $(x - 7)(3x + 2)(x - 2)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(x+7)(3x-2)(x+2)$

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{2}, -3, \text{ and } \frac{1}{3}$$

The solution is $6x^3 + 25x^2 + 18x - 9$, which is option A.

A. $a \in [2, 7], b \in [24, 32], c \in [16, 21],$ and $d \in [-16, -2]$

* $6x^3 + 25x^2 + 18x - 9$, which is the correct option.

B. $a \in [2, 7], b \in [-25, -22], c \in [16, 21],$ and $d \in [6, 10]$

$6x^3 - 25x^2 + 18x + 9$, which corresponds to multiplying out $(2x - 3)(x - 3)(3x + 1)$.

C. $a \in [2, 7], b \in [-32, -27], c \in [32, 43],$ and $d \in [-16, -2]$

$6x^3 - 29x^2 + 36x - 9$, which corresponds to multiplying out $(2x + 2)(x + 1)(3x - 3)$.

D. $a \in [2, 7], b \in [24, 32], c \in [16, 21],$ and $d \in [6, 10]$

$6x^3 + 25x^2 + 18x + 9$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [2, 7], b \in [6, 8], c \in [-30, -29],$ and $d \in [6, 10]$

$6x^3 + 7x^2 - 30x + 9$, which corresponds to multiplying out $(2x + 2)(x - 1)(3x - 3)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(2x + 3)(x + 3)(3x - 1)$

---

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 2i \text{ and } -4$$

The solution is $x^3 + 14x^2 + 69x + 116$, which is option B.

A. $b \in [-10, 3], c \in [8, 16],$ and $d \in [14, 22]$

$x^3 + x^2 + 9x + 20$, which corresponds to multiplying out $(x + 5)(x + 4)$.

B. $b \in [8, 18], c \in [62, 78]$, and $d \in [116, 124]$

* $x^3 + 14x^2 + 69x + 116$, which is the correct option.

C. $b \in [-22, -9], c \in [62, 78]$, and $d \in [-118, -111]$

$x^3 - 14x^2 + 69x - 116$, which corresponds to multiplying out $(x - (-5 + 2i))(x - (-5 - 2i))(x - 4)$.

D. $b \in [-10, 3], c \in [-2, 3]$, and $d \in [-14, -5]$

$x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x - 2)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

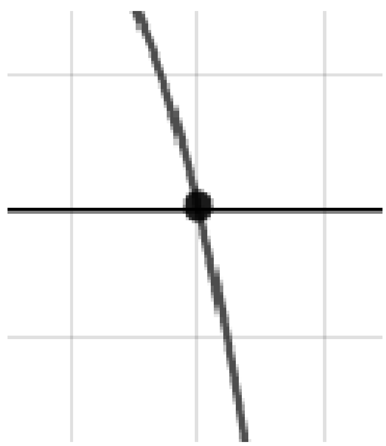
**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 2i))(x - (-5 - 2i))(x - (-4))$.

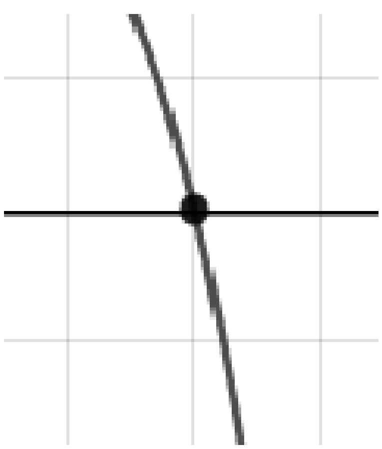
---

9. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

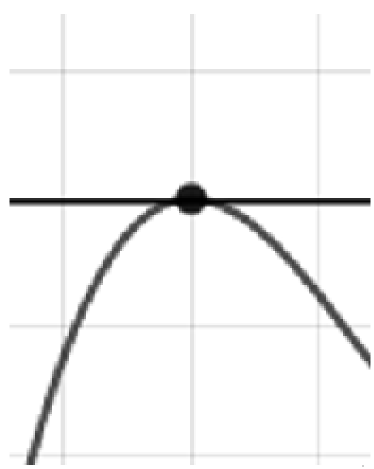
$$f(x) = -8(x - 4)^8(x + 4)^{11}(x + 9)^3(x - 9)^4$$

The solution is the graph below, which is option A.



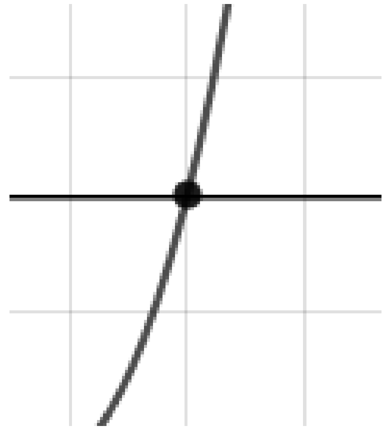


A.



B.

C. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 5i \text{ and } -1$$

The solution is $x^3 + 11x^2 + 60x + 50$, which is option A.

A. $b \in [7, 17], c \in [58, 67], \text{ and } d \in [46, 54]$

   * $x^3 + 11x^2 + 60x + 50$, which is the correct option.

B. $b \in [-15, -6], c \in [58, 67], \text{ and } d \in [-50, -44]$

   $x^3 - 11x^2 + 60x - 50$, which corresponds to multiplying out $(x - (-5 + 5i))(x - (-5 - 5i))(x - 1)$.

C. $b \in [-4, 5], c \in [4, 7], \text{ and } d \in [1, 6]$

   $x^3 + x^2 + 6x + 5$, which corresponds to multiplying out $(x + 5)(x + 1)$.

D. $b \in [-4, 5], c \in [-10, -3], \text{ and } d \in [-16, 1]$

   $x^3 + x^2 - 4x - 5$, which corresponds to multiplying out $(x - 5)(x + 1)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 5i))(x - (-5 - 5i))(x - (-1))$.

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