This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Choose the interval below that f composed with q at x = 1 is in.

$$f(x) = -3x^3 + x^2 + 4x$$
 and $g(x) = x^3 - 4x^2 + 3x$

The solution is 0.0, which is option B.

A. $(f \circ g)(1) \in [-10.36, -9.09]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-0.68, 1.48]$

* This is the correct solution

C. $(f \circ g)(1) \in [-6.93, -4.27]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-2.19, -1.23]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means f(g(x)). The order matters!

2. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = 10 and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = 3x^2 + 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(10) \in [5.53, 5.96]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(10) \in [2.49, 2.74]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(10) \in [1.91, 2.26]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(10) \in [0.81, 1.76]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

3. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = 14 and choose the interval the $f^{-1}(14)$ belongs to.

$$f(x) = \sqrt[3]{4x - 5}$$

The solution is 687.25, which is option D.

A. $f^{-1}(14) \in [678.75, 685.75]$

Distractor 1: This corresponds to

B. $f^{-1}(14) \in [-688.25, -685.25]$

This solution corresponds to distractor 2.

C. $f^{-1}(14) \in [-684.75, -681.75]$

This solution corresponds to distractor 3.

- D. $f^{-1}(14) \in [686.25, 689.25]$
 - * This is the correct solution.
- E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

4. Choose the interval below that f composed with q at x = -1 is in.

$$f(x) = 4x^3 + 2x^2 - 4x + 1$$
 and $g(x) = -x^3 - 3x^2 - 3x$

The solution is 3.0, which is option B.

A. $(f \circ g)(-1) \in [-63, -58]$

Distractor 1: Corresponds to reversing the composition.

- B. $(f \circ g)(-1) \in [-1, 8]$
 - * This is the correct solution
- C. $(f \circ g)(-1) \in [-9, -5]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-55, -50]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means f(g(x)). The order matters!

5. Determine whether the function below is 1-1.

$$f(x) = 12x^2 - 42x - 132$$

The solution is no, which is option A.

- A. No, because there is a y-value that goes to 2 different x-values.
 - * This is the solution.
- B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

6. Find the inverse of the function below. Then, evaluate the inverse at x = 10 and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = e^{x+4} - 5$$

The solution is $f^{-1}(10) = -1.292$, which is option D.

A.
$$f^{-1}(10) \in [-2.48, -2.36]$$

This solution corresponds to distractor 4.

B.
$$f^{-1}(10) \in [6.48, 6.81]$$

This solution corresponds to distractor 1.

C.
$$f^{-1}(10) \in [-3.33, -2.99]$$

This solution corresponds to distractor 3.

D.
$$f^{-1}(10) \in [-1.51, -1.22]$$

This is the solution.

E.
$$f^{-1}(10) \in [-3.43, -3.32]$$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

7. Find the inverse of the function below. Then, evaluate the inverse at x = 8 and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = e^{x+2} - 4$$

The solution is $f^{-1}(8) = 0.485$, which is option B.

A.
$$f^{-1}(8) \in [-2.56, -2.18]$$

This solution corresponds to distractor 3.

B.
$$f^{-1}(8) \in [-0.28, 0.99]$$

This is the solution.

C. $f^{-1}(8) \in [3.2, 6.23]$

This solution corresponds to distractor 1.

D. $f^{-1}(8) \in [-4.9, -2.33]$

This solution corresponds to distractor 2.

E. $f^{-1}(8) \in [-2.12, -0.75]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

8. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{5x - 25}$$
 and $g(x) = 9x^3 + 8x^2 + 7x + 8$

The solution is The domain is all Real numbers greater than or equal to x = 5.0, which is option C.

- A. The domain is all Real numbers except x = a, where $a \in [4.25, 9.25]$
- B. The domain is all Real numbers less than or equal to x = a, where $a \in [-5.33, 3.67]$
- C. The domain is all Real numbers greater than or equal to x = a, where $a \in [-1, 9]$
- D. The domain is all Real numbers except x=a and x=b, where $a\in[-8.83,-1.83]$ and $b\in[-7.25,-3.25]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

9. Determine whether the function below is 1-1.

$$f(x) = \sqrt{-4x - 15}$$

The solution is yes, which is option E.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

- E. Yes, the function is 1-1.
 - * This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

10. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 8x^2 + 5x + 4$$
 and $g(x) = \sqrt{6x - 27}$

The solution is The domain is all Real numbers greater than or equal to x = 4.5, which is option A.

- A. The domain is all Real numbers greater than or equal to x = a, where $a \in [4.5, 6.5]$
- B. The domain is all Real numbers except x = a, where $a \in [4.4, 9.4]$
- C. The domain is all Real numbers less than or equal to x = a, where $a \in [-10.75, 1.25]$
- D. The domain is all Real numbers except x = a and x = b, where $a \in [-9.17, 1.83]$ and $b \in [-6.2, 5.8]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.