This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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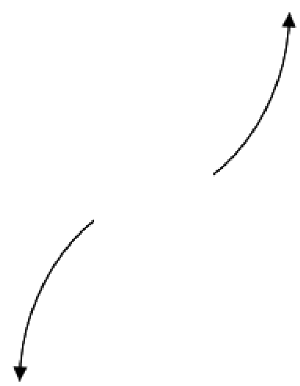
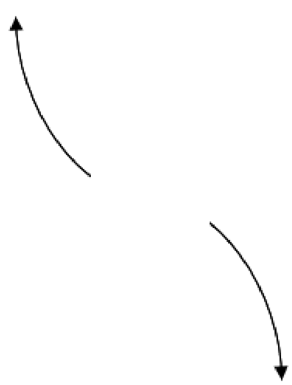
1. Describe the end behavior of the polynomial below.

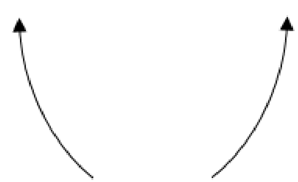
$$f(x) = 9(x-8)^3(x+8)^4(x-3)^5(x+3)^7$$

The solution is the graph below, which is option D.

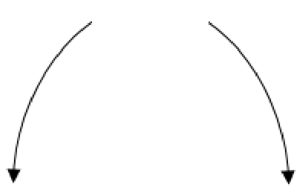




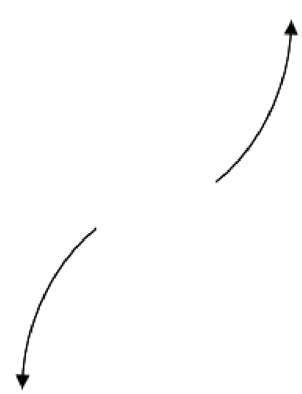
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 3i \text{ and } 2$$

The solution is $x^3 + 2x^2 + 5x - 26$, which is option C.

A. $b \in [0.54, 1.25], c \in [-9, -2], \text{ and } d \in [1, 8]$

$x^3 + x^2 - 5x + 6$, which corresponds to multiplying out $(x - 3)(x - 2)$.

B. $b \in [0.54, 1.25], c \in [-2, 1], \text{ and } d \in [-4, 0]$

$x^3 + x^2 - 4$, which corresponds to multiplying out $(x + 2)(x - 2)$.

C. $b \in [1.7, 2.1], c \in [4, 7], \text{ and } d \in [-29, -22]$

* $x^3 + 2x^2 + 5x - 26$, which is the correct option.

D. $b \in [-2.25, -0.75], c \in [4, 7], \text{ and } d \in [25, 29]$

$x^3 - 2x^2 + 5x + 26$, which corresponds to multiplying out $(x - (-2 + 3i))(x - (-2 - 3i))(x + 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 3i))(x - (-2 - 3i))(x - (2))$.

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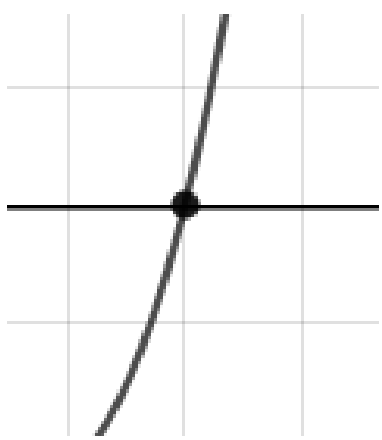
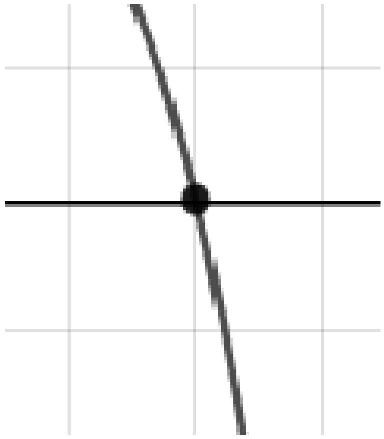
3. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

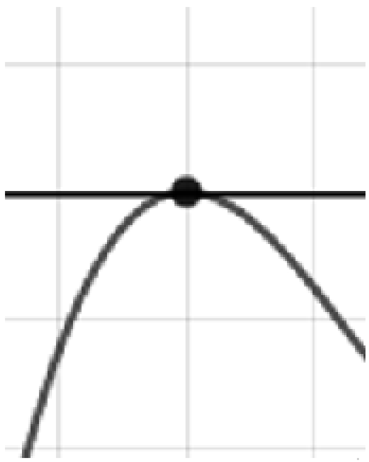
$$f(x) = -8(x - 2)^9(x + 2)^{12}(x + 6)^7(x - 6)^9$$

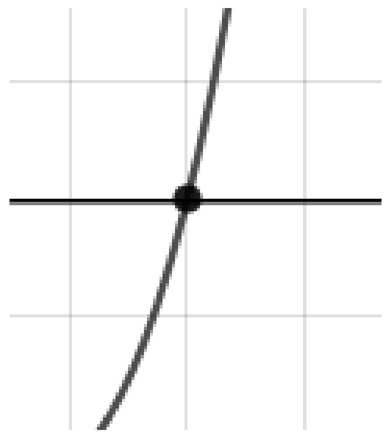
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{6}{5}, \frac{-1}{3}, \text{ and } \frac{2}{5}$$

The solution is $75x^3 - 95x^2 - 4x + 12$, which is option B.

A. $a \in [70, 82], b \in [34, 42], c \in [-60, -51], \text{ and } d \in [10, 14]$

   $75x^3 + 35x^2 - 56x + 12$, which corresponds to multiplying out $(5x + 6)(3x - 1)(5x - 2)$.

B. $a \in [70, 82], b \in [-103, -90], c \in [-5, -2], \text{ and } d \in [10, 14]$

   * $75x^3 - 95x^2 - 4x + 12$, which is the correct option.

C. $a \in [70, 82], b \in [91, 96], c \in [-5, -2], \text{ and } d \in [-15, -9]$

   $75x^3 + 95x^2 - 4x - 12$, which corresponds to multiplying out $(5x + 6)(3x - 1)(5x + 2)$.

D. $a \in [70, 82], b \in [-103, -90], c \in [-5, -2], \text{ and } d \in [-15, -9]$

$75x^3 - 95x^2 - 4x - 12$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [70, 82], b \in [78, 91], c \in [-17, -13],$ and $d \in [-15, -9]$

$75x^3 + 85x^2 - 16x - 12$, which corresponds to multiplying out $(5x + 6)(3x + 1)(5x - 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 6)(3x + 1)(5x - 2)$

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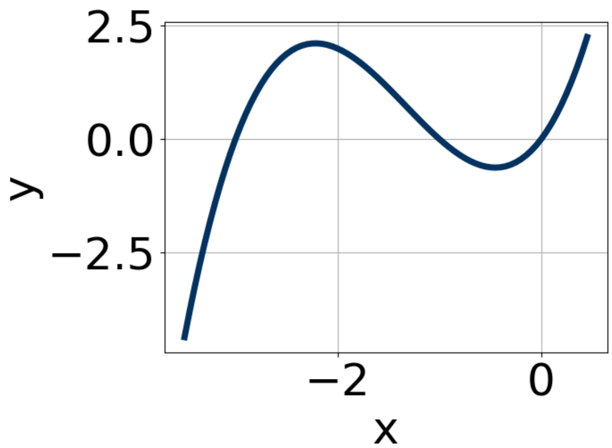
5. Which of the following equations *could* be of the graph presented below?



The solution is $15x^5(x + 3)^5(x + 1)^5$, which is option A.

A. $15x^5(x + 3)^5(x + 1)^5$

* This is the correct option.

B. $11x^9(x + 3)^4(x + 1)^9$

The factor -3 should have been an odd power.

C. $12x^7(x + 3)^4(x + 1)^6$

The factors -3 and -1 have have been odd power.

D. $-19x^5(x + 3)^{10}(x + 1)^9$

The factor $(x + 3)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-9x^7(x + 3)^{11}(x + 1)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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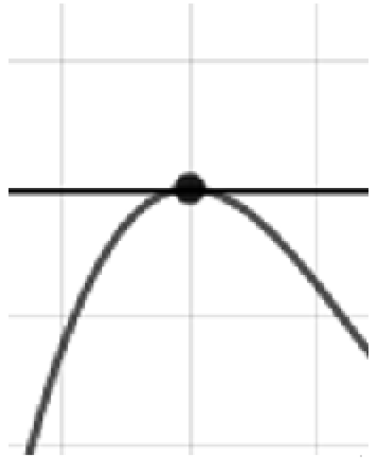
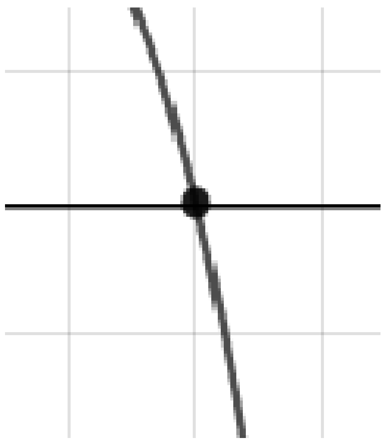
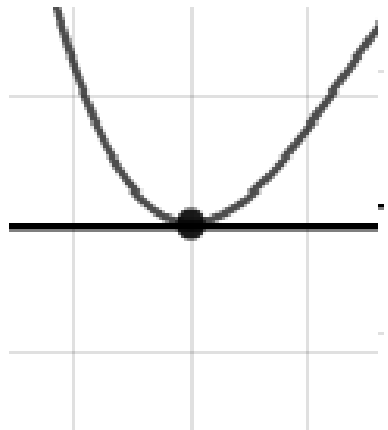
6. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

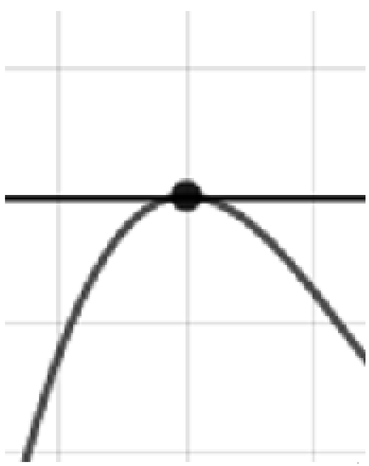
$$f(x) = 4(x + 4)^8(x - 4)^{13}(x + 9)^7(x - 9)^8$$

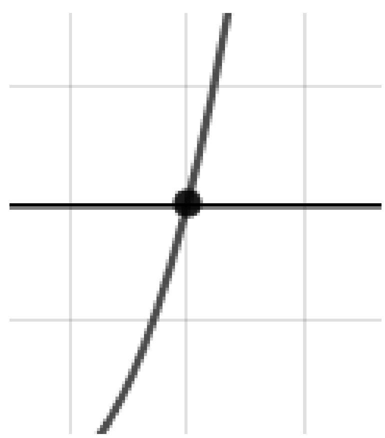
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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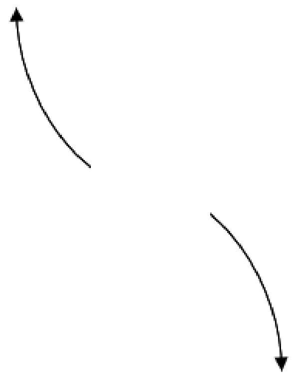
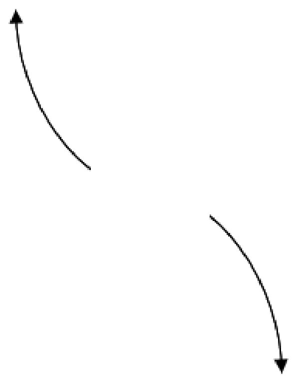
7. Describe the end behavior of the polynomial below.

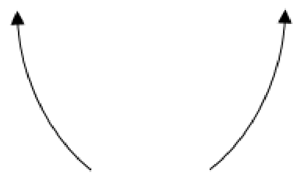
$$f(x) = -7(x+2)^4(x-2)^7(x+9)^4(x-9)^6$$

The solution is the graph below, which is option A.

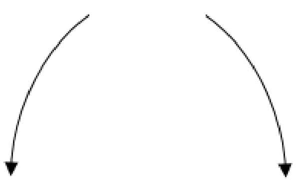




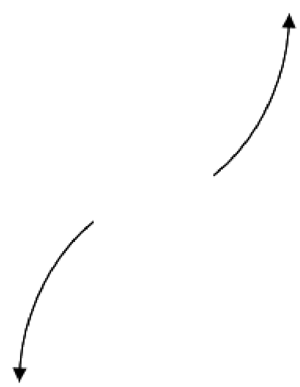
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 + 5i \text{ and } -3$$

The solution is $x^3 - 3x^2 + 16x + 102$, which is option C.

A. $b \in [2.51, 4.34], c \in [15.82, 16.89],$ and $d \in [-103.2, -101.1]$

$x^3 + 3x^2 + 16x - 102$, which corresponds to multiplying out $(x - (3 + 5i))(x - (3 - 5i))(x - 3)$.

B. $b \in [-1.5, 2.17], c \in [-1.04, 0.81],$ and $d \in [-9.6, -7.5]$

$x^3 + x^2 - 9$, which corresponds to multiplying out $(x - 3)(x + 3)$.

C. $b \in [-4.81, -1.83], c \in [15.82, 16.89],$ and $d \in [99.1, 105]$

* $x^3 - 3x^2 + 16x + 102$, which is the correct option.

D. $b \in [-1.5, 2.17], c \in [-2.18, -1.34],$ and $d \in [-19.2, -13.9]$

$x^3 + x^2 - 2x - 15$, which corresponds to multiplying out $(x - 5)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 + 5i))(x - (3 - 5i))(x - (-3))$.

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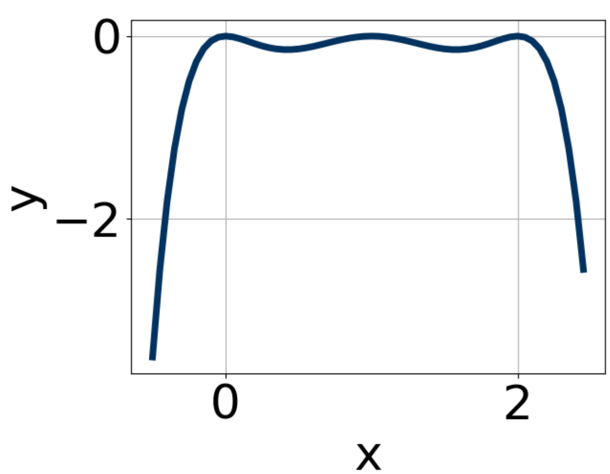
9. Which of the following equations *could* be of the graph presented below?



The solution is $-17x^6(x - 2)^8(x - 1)^4$, which is option A.

A. $-17x^6(x - 2)^8(x - 1)^4$

* This is the correct option.

B. $-16x^8(x - 2)^4(x - 1)^{11}$

The factor $(x - 1)$ should have an even power.

C. $13x^8(x - 2)^8(x - 1)^{11}$

The factor $(x - 1)$ should have an even power and the leading coefficient should be the opposite sign.

D. $18x^8(x - 2)^6(x - 1)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-2x^8(x - 2)^5(x - 1)^7$

The factors $(x - 2)$ and $(x - 1)$ should both have even powers.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-4, \frac{3}{5}, \text{ and } \frac{5}{3}$$

The solution is $15x^3 + 26x^2 - 121x + 60$, which is option B.

A. $a \in [14, 16], b \in [-94, -92], c \in [149, 155]$, and $d \in [-66, -56]$

$15x^3 - 94x^2 + 151x - 60$, which corresponds to multiplying out $(x - 4)(5x - 3)(3x - 5)$.

B. $a \in [14, 16], b \in [20, 30], c \in [-121, -118]$, and $d \in [53, 62]$

* $15x^3 + 26x^2 - 121x + 60$, which is the correct option.

C. $a \in [14, 16], b \in [20, 30], c \in [-121, -118]$, and $d \in [-66, -56]$

$15x^3 + 26x^2 - 121x - 60$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [14, 16], b \in [-83, -73], c \in [46, 50]$, and $d \in [53, 62]$

$15x^3 - 76x^2 + 49x + 60$, which corresponds to multiplying out $(x - 4)(5x + 3)(3x - 5)$.

E. $a \in [14, 16], b \in [-29, -22], c \in [-121, -118]$, and $d \in [-66, -56]$

$15x^3 - 26x^2 - 121x - 60$, which corresponds to multiplying out $(x - 4)(5x + 3)(3x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+4)(5x-3)(3x-5)$
