This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = e^{x-5} - 3$$

The solution is $f^{-1}(10) = 7.565$, which is option E.

A. $f^{-1}(10) \in [-2.06, -1.26]$

This solution corresponds to distractor 4.

B. $f^{-1}(10) \in [-0.43, 0.04]$

This solution corresponds to distractor 3.

C. $f^{-1}(10) \in [-2.62, -2.38]$

This solution corresponds to distractor 1.

D. $f^{-1}(10) \in [-1.11, -0.53]$

This solution corresponds to distractor 2.

E. $f^{-1}(10) \in [7.54, 8.2]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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2. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 5x^4 + 4x^3 + 4x^2 + 3x + 2 \text{ and } g(x) = \frac{4}{5x - 33}$$

The solution is The domain is all Real numbers except $x = 6.6$, which is option A.

A. The domain is all Real numbers except $x = a,$ where $a \in [4.6, 10.6]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [4.5, 5.5]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-6, -4]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [5.25, 6.25]$ and $b \in [4.6, 9.6]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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3. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -x^3 - 3x^2 + 3x + 2 \text{ and } g(x) = -x^3 + 3x^2 + x$$

The solution is -43.0, which is option C.

A. $(f \circ g)(1) \in [9, 13]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-55, -49]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-43, -40]$

* This is the correct solution

D. $(f \circ g)(1) \in [1, 7]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{1}{4x - 25} \text{ and } g(x) = \frac{5}{5x + 26}$$

The solution is The domain is all Real numbers except $x = 6.25$ and $x = -5.2$, which is option D.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-4.33, 1.67]$

B. The domain is all Real numbers except $x = a$, where $a \in [-7.6, 0.4]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-15, -5]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [5.25, 10.25]$ and $b \in [-5.2, -2.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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5. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = x^3 - 2x^2 + x \text{ and } g(x) = -x^3 - 4x^2 + 4x$$

The solution is -4.0, which is option B.

A. $(f \circ g)(1) \in [-8.6, -4.2]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-4.6, -3.6]$

* This is the correct solution

C. $(f \circ g)(1) \in [-3.7, 0.7]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(1) \in [-15.4, -12.3]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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6. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^{-1}(-15)$ belongs to.

$$f(x) = 5x^2 + 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [4.73, 5.5]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(-15) \in [2.5, 3.56]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(-15) \in [1.7, 2.39]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

D. $f^{-1}(-15) \in [1.43, 1.8]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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7. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = e^{x+2} + 5$$

The solution is $f^{-1}(8) = -0.901$, which is option E.

A. $f^{-1}(8) \in [2.94, 3.21]$

This solution corresponds to distractor 1.

B. $f^{-1}(8) \in [7.13, 7.36]$

This solution corresponds to distractor 4.

C. $f^{-1}(8) \in [6.4, 6.84]$

This solution corresponds to distractor 3.

D. $f^{-1}(8) \in [7.52, 7.9]$

This solution corresponds to distractor 2.

E. $f^{-1}(8) \in [-1.07, -0.64]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = 5x^2 + 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(10) \in [0.41, 1.38]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(10) \in [6.96, 7.59]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(10) \in [1.26, 2.04]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(10) \in [3.23, 4.28]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Determine whether the function below is 1-1.

$$f(x) = 16x^2 + 176x + 484$$

The solution is no, which is option B.

A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

B. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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10. Determine whether the function below is 1-1.

$$f(x) = (3x - 20)^3$$

The solution is yes, which is option D.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

D. Yes, the function is 1-1.

* This is the solution.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.
