This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = x^4 + 8x^3 + 3x^2 + 5x + 1 \text{ and } g(x) = 7x^4 + 7x^3 + 5x^2 + 3$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-10, 1]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-7.67, 1.33]$

C. The domain is all Real numbers except $x = a,$ where $a \in [-11.2, -2.2]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-7.6, -0.6]$ and $b \in [-10.67, 2.33]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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2. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 12$ and choose the interval the $f^{-1}(12)$ belongs to.

$$f(x) = \sqrt[3]{4x + 3}$$

The solution is 431.25, which is option D.

A. $f^{-1}(12) \in [-431.95, -429.73]$

This solution corresponds to distractor 2.

B. $f^{-1}(12) \in [431.69, 433.5]$

Distractor 1: This corresponds to

C. $f^{-1}(12) \in [-434.68, -431.75]$

This solution corresponds to distractor 3.

D. $f^{-1}(12) \in [430.68, 431.77]$

* This is the correct solution.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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3. Determine whether the function below is 1-1.

$$f(x) = 36x^2 - 252x + 441$$

The solution is no, which is option C.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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4. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 14$ and choose the interval that $f^{-1}(14)$ belongs to.

$$f(x) = 4x^2 + 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(14) \in [1.54, 1.89]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(14) \in [1.9, 2.03]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

C. $f^{-1}(14) \in [5.68, 5.84]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(14) \in [3.34, 3.92]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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5. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = e^{x+5} + 5$$

The solution is $f^{-1}(9) = -3.614$, which is option E.

A. $f^{-1}(9) \in [5.8, 7]$

This solution corresponds to distractor 3.

B. $f^{-1}(9) \in [6.9, 9.7]$

This solution corresponds to distractor 2.

C. $f^{-1}(9) \in [5.8, 7]$

This solution corresponds to distractor 1.

D. $f^{-1}(9) \in [6.9, 9.7]$

This solution corresponds to distractor 4.

E. $f^{-1}(9) \in [-4.6, -3.5]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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6. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 3x^3 + 2x^2 - 4x + 1 \text{ and } g(x) = 3x^3 - 4x^2 + 4x - 4$$

The solution is 4.0, which is option C.

A. $(f \circ g)(1) \in [14, 21]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [12, 13]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [4, 9]$

* This is the correct solution

D. $(f \circ g)(1) \in [12, 13]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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7. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -3x^3 + 4x^2 + 4x \text{ and } g(x) = -x^3 - 3x^2 - 3x - 2$$

The solution is 3.0, which is option A.

A. $(f \circ g)(-1) \in [2, 5]$

* This is the correct solution

B. $(f \circ g)(-1) \in [-67, -58]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [-76, -70]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [4, 13]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = \ln(x - 5) - 5$$

The solution is $f^{-1}(7) = 162759.791$, which is option C.

A. $f^{-1}(7) \in [162747.79, 162753.79]$

This solution corresponds to distractor 2.

B. $f^{-1}(7) \in [10.39, 14.39]$

This solution corresponds to distractor 1.

C. $f^{-1}(7) \in [162757.79, 162762.79]$

This is the solution.

D. $f^{-1}(7) \in [-1.61, 3.39]$

This solution corresponds to distractor 4.

E. $f^{-1}(7) \in [162747.79, 162753.79]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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9. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{2}{5x + 31} \text{ and } g(x) = 2x^4 + 8x^2 + 8x + 7$$

The solution is The domain is all Real numbers except $x = -6.2$, which is option B.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-5.4, 0.6]$

B. The domain is all Real numbers except $x = a$, where $a \in [-9.2, -5.2]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [2.5, 8.5]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-10.2, -2.2]$ and $b \in [-5.8, -4.8]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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10. Determine whether the function below is 1-1.

$$f(x) = \sqrt{-5x + 19}$$

The solution is yes, which is option D.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. Yes, the function is 1-1.

* This is the solution.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.
