This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 - 6x^2 - 51x - 39}{x - 3}$$

The solution is $9x^2 + 21x + 12 + \frac{-3}{x-3}$, which is option A.

A. $a \in [7, 16], \; b \in [20, 28], \; c \in [7, 14], \text{ and } r \in [-5, 1].$

* This is the solution!

B. $a \in [7, 16], \; b \in [12, 13], \; c \in [-29, -21], \text{ and } r \in [-93, -91].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [22, 30], \; b \in [74, 79], \; c \in [174, 179], \text{ and } r \in [482, 484].$

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [7, 16], \; b \in [-41, -28], \; c \in [47, 55], \text{ and } r \in [-184, -176].$

You divided by the opposite of the factor.

E. $a \in [22, 30], \; b \in [-91, -84], \; c \in [209, 214], \text{ and } r \in [-673, -663].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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2. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 2x^4 + 3x^3 + 6x^2 + 5x + 6$$

The solution is $\pm 1, \pm 2, \pm 3, \pm 6$, which is option B.

A. All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 2, \pm 3, \pm 6$

* This is the solution **since we asked for the possible Integer roots**!

C. $\pm 1, \pm 2$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Factor the polynomial below completely, knowing that $x-2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 + 35x^3 - 23x^2 - 140x - 100$$

The solution is $[-2, -1.6666666666666667, -1.25, 2]$, which is option E.

A. $z_1 \in [-6, -1], \ z_2 \in [0.2, 0.57], z_3 \in [1.92, 2.12], \text{ and } z_4 \in [4, 9]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-6, -1], \ z_2 \in [1.22, 1.29], z_3 \in [1.54, 1.77], \text{ and } z_4 \in [0, 4]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-6, -1], \ z_2 \in [0.59, 0.96], z_3 \in [0.61, 0.88], \text{ and } z_4 \in [0, 4]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-6, -1], \ z_2 \in [-1.42, -0.64], z_3 \in [-0.94, -0.45], \text{ and } z_4 \in [0, 4]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-6, -1], \ z_2 \in [-1.72, -1.33], z_3 \in [-1.28, -1.01], \text{ and } z_4 \in [0, 4]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 + 66x^2 + 15x - 31}{x + 4}$$

The solution is $15x^2 + 6x - 9 + \dfrac{5}{x+4}$, which is option C.

A. $a \in [-61, -59], \ b \in [302, 308], \ c \in [-1214, -1208], \text{ and } r \in [4805, 4810].$

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [14, 17], \ b \in [120, 132], \ c \in [519, 521], \text{ and } r \in [2043, 2046].$

You divided by the opposite of the factor.

C. $a \in [14, 17], \ b \in [-1, 8], \ c \in [-10, -4], \text{ and } r \in [0, 10].$

* This is the solution!

D. $a \in [-61, -59], \; b \in [-174, -173], \; c \in [-681, -677], \text{ and } r \in [-2762, -2750].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [14, 17], \; b \in [-15, -8], \; c \in [60, 62], \text{ and } r \in [-333, -330].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 + 12x^2 - 11}{x + 2}$$

The solution is $4x^2 + 4x - 8 + \dfrac{5}{x+2}$, which is option A.

A. $a \in [2, 7], b \in [3.6, 4.5], c \in [-10, -6], \text{ and } r \in [3, 11].$

* This is the solution!

B. $a \in [2, 7], b \in [-2, 1.1], c \in [-3, 3], \text{ and } r \in [-11, -9].$

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-13, -7], b \in [26.7, 30.3], c \in [-56, -51], \text{ and } r \in [98, 104].$

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [-13, -7], b \in [-7.6, -3.8], c \in [-10, -6], \text{ and } r \in [-27, -25].$

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [2, 7], b \in [15.7, 20.2], c \in [39, 41], \text{ and } r \in [68, 73].$

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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6. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 2x^2 + 3x + 7$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2}$, which is option C.

A. $\pm 1, \pm 7$

This would have been the solution **if asked for the possible Integer roots**!

B. $\pm 1, \pm 2$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2}$

* This is the solution **since we asked for the possible Rational roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 + 65x^2 - 122}{x + 5}$$

The solution is $12x^2 + 5x - 25 + \dfrac{3}{x+5}$, which is option A.

A. $a \in [10, 14], b \in [5, 9], c \in [-28, -24],$ and $r \in [2, 5]$.

* This is the solution!

B. $a \in [-63, -57], b \in [363, 367], c \in [-1829, -1822],$ and $r \in [9003, 9008]$.

You multipled by the synthetic number rather than bringing the first factor down.

C. $a \in [10, 14], b \in [115, 126], c \in [624, 629],$ and $r \in [2998, 3004]$.

You divided by the opposite of the factor.

D. $a \in [-63, -57], b \in [-238, -228], c \in [-1181, -1172],$ and $r \in [-5999, -5996]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [10, 14], b \in [-9, -6], c \in [40, 45],$ and $r \in [-378, -373]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 + 31x^2 - 50x - 24$$

The solution is $[-3, -0.4, 1.3333333333333333]$, which is option E.

A. $z_1 \in [-1.8, -1.17], \ z_2 \in [0.16, 0.41],$ and $z_3 \in [2.57, 3.15]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-1.24, -0.6], \ z_2 \in [2.46, 2.75],$ and $z_3 \in [2.57, 3.15]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-3.68, -2.89], \ z_2 \in [-3.01, -2.32],$ and $z_3 \in [0.45, 1.14]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-4.14, -3.63]$, $z_2 \in [-0.24, 0.29]$, and $z_3 \in [2.57, 3.15]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-3.68, -2.89]$, $z_2 \in [-0.43, -0.33]$, and $z_3 \in [1.27, 1.74]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 - 64x^2 + 12x + 16$$

The solution is $[-0.4, 0.6666666666666666, 4]$, which is option E.

A. $z_1 \in [-5, -3]$, $z_2 \in [-0.78, -0.27]$, and $z_3 \in [0.22, 0.71]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5, -3]$, $z_2 \in [-1.61, -0.92]$, and $z_3 \in [2.35, 2.71]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5, -3]$, $z_2 \in [-2.24, -1.98]$, and $z_3 \in [0.04, 0.16]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-2.5, -1.5]$, $z_2 \in [1.14, 1.56]$, and $z_3 \in [3.73, 4.19]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-1.4, 1.6]$, $z_2 \in [0.54, 1.28]$, and $z_3 \in [3.73, 4.19]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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10. Factor the polynomial below completely, knowing that $x+2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^4 + 129x^3 + 194x^2 - 48x - 160$$

The solution is $[-4, -2, -1.25, 0.8]$, which is option C.

A. $z_1 \in [-4.33, -3.88]$, $z_2 \in [0.19, 0.53], z_3 \in [1.37, 2.44]$, and $z_4 \in [3.2, 5.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-4.33, -3.88]$, $z_2 \in [-2.12, -1.88], z_3 \in [-1.01, -0.29]$, and $z_4 \in [1, 1.5]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-4.33, -3.88]$, $z_2 \in [-2.12, -1.88], z_3 \in [-1.98, -1.21]$, and $z_4 \in [0.2, 0.9]$

* This is the solution!

D. $z_1 \in [-0.99, -0.08]$, $z_2 \in [1.24, 1.6], z_3 \in [1.37, 2.44]$, and $z_4 \in [3.2, 5.3]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-1.61, -1.07]$, $z_2 \in [0.76, 1.2], z_3 \in [1.37, 2.44]$, and $z_4 \in [3.2, 5.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.