This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 5i \text{ and } -3$$

The solution is $x^3 + 7x^2 + 41x + 87$, which is option C.

A. $b \in [-13, -2], c \in [40, 42.8], \text{ and } d \in [-90, -77]$

$x^3 - 7x^2 + 41x - 87$, which corresponds to multiplying out $(x - (-2 - 5i))(x - (-2 + 5i))(x - 3)$.

B. $b \in [-3, 2], c \in [3.1, 6.1], \text{ and } d \in [-1, 8]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 2)(x + 3)$.

C. $b \in [6, 10], c \in [40, 42.8], \text{ and } d \in [84, 96]$

* $x^3 + 7x^2 + 41x + 87$, which is the correct option.

D. $b \in [-3, 2], c \in [5.5, 10.3], \text{ and } d \in [13, 21]$

$x^3 + x^2 + 8x + 15$, which corresponds to multiplying out $(x + 5)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 5i))(x - (-2 + 5i))(x - (-3))$.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{2}, \frac{-5}{4}, \text{ and } \frac{7}{4}$$

The solution is $32x^3 + 32x^2 - 94x - 105$, which is option B.

A. $a \in [26, 33], b \in [-38, -30], c \in [-101, -92], \text{ and } d \in [105, 110]$

$32x^3 - 32x^2 - 94x + 105$, which corresponds to multiplying out $(2x - 3)(4x - 5)(4x + 7)$.

B. $a \in [26, 33], b \in [32, 36], c \in [-101, -92], \text{ and } d \in [-111, -104]$

* $32x^3 + 32x^2 - 94x - 105$, which is the correct option.

C. $a \in [26, 33], b \in [32, 36], c \in [-101, -92], \text{ and } d \in [105, 110]$

$32x^3 + 32x^2 - 94x + 105$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [26, 33], b \in [-144, -138], c \in [207, 217],$ and $d \in [-111, -104]$

$32x^3 - 144x^2 + 214x - 105$, which corresponds to multiplying out $(2x - 3)(4x - 5)(4x - 7)$.

E. $a \in [26, 33], b \in [-72, -58], c \in [-56, -45],$ and $d \in [105, 110]$

$32x^3 - 64x^2 - 46x + 105$, which corresponds to multiplying out $(2x - 3)(4x + 5)(4x - 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 3)(4x + 5)(4x - 7)$

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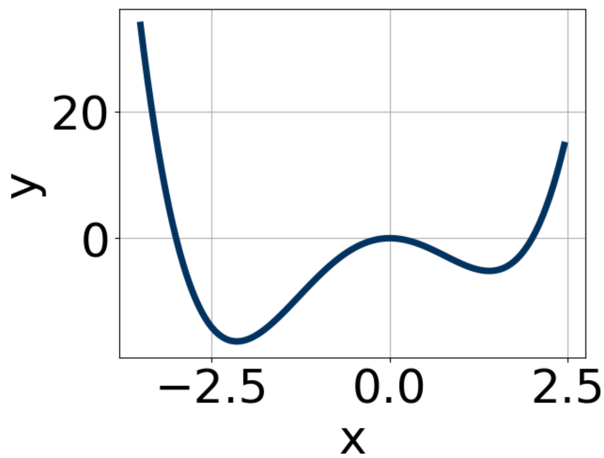
3. Which of the following equations *could* be of the graph presented below?



The solution is $12x^{10}(x + 3)^9(x - 2)^{11}$, which is option B.

A. $-13x^6(x + 3)^7(x - 2)^{10}$

The factor $(x - 2)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $12x^{10}(x + 3)^9(x - 2)^{11}$

* This is the correct option.

C. $-3x^6(x + 3)^5(x - 2)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $15x^7(x + 3)^{10}(x - 2)^5$

The factor 0 should have an even power and the factor -3 should have an odd power.

E. $19x^{10}(x + 3)^8(x - 2)^9$

The factor $(x + 3)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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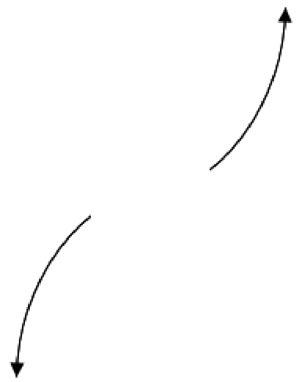
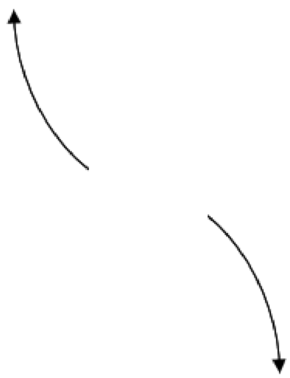
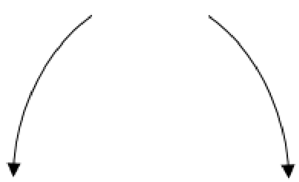
4. Describe the end behavior of the polynomial below.

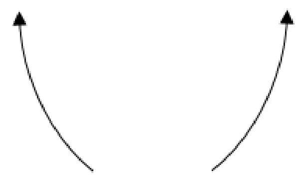
$$f(x) = 7(x - 9)^5(x + 9)^8(x + 2)^4(x - 2)^4$$

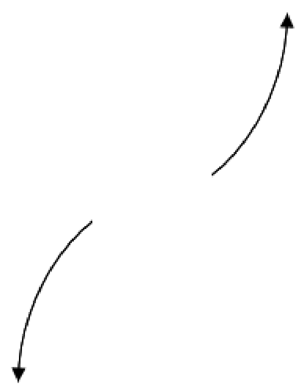
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-3, \frac{2}{3}, \text{ and } -7$$

The solution is $3x^3 + 28x^2 + 43x - 42$, which is option D.

A. $a \in [3, 5], b \in [-28.7, -25.9], c \in [42, 48], \text{ and } d \in [41, 44]$

$3x^3 - 28x^2 + 43x + 42$, which corresponds to multiplying out $(x - 3)(3x + 2)(x - 7)$.

B. $a \in [3, 5], b \in [13.4, 15.1], c \in [-56, -54], \text{ and } d \in [-42, -41]$

$3x^3 + 14x^2 - 55x - 42$, which corresponds to multiplying out $(x - 3)(3x + 2)(x + 7)$.

C. $a \in [3, 5], b \in [6.1, 12.1], c \in [-75, -70], \text{ and } d \in [41, 44]$

$3x^3 + 10x^2 - 71x + 42$, which corresponds to multiplying out $(x - 3)(3x - 2)(x + 7)$.

D. $a \in [3, 5], b \in [27.6, 30.4], c \in [42, 48], \text{ and } d \in [-42, -41]$

* $3x^3 + 28x^2 + 43x - 42$, which is the correct option.

E. $a \in [3, 5], b \in [27.6, 30.4], c \in [42, 48], \text{ and } d \in [41, 44]$

$3x^3 + 28x^2 + 43x + 42$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+3)(3x-2)(x+7)$

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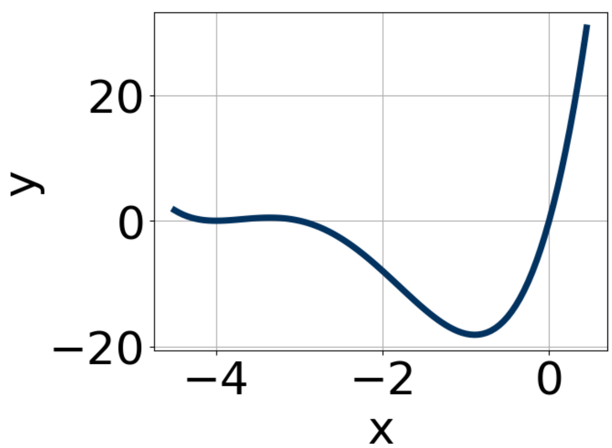
6. Which of the following equations *could* be of the graph presented below?



The solution is $18x^{11}(x + 4)^4(x + 3)^7$, which is option C.

A. $-14x^5(x + 4)^{10}(x + 3)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-15x^6(x + 4)^8(x + 3)^5$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

C. $18x^{11}(x + 4)^4(x + 3)^7$

* This is the correct option.

D. $12x^{11}(x + 4)^{10}(x + 3)^{10}$

The factor $(x + 3)$ should have an odd power.

E. $5x^{11}(x + 4)^9(x + 3)^4$

The factor -4 should have an even power and the factor -3 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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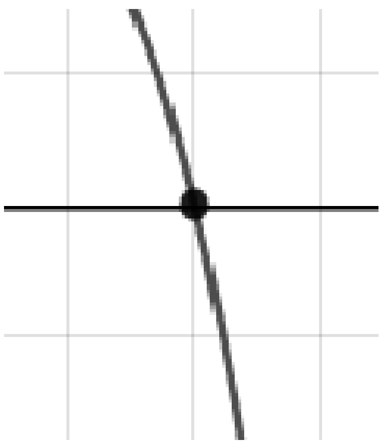
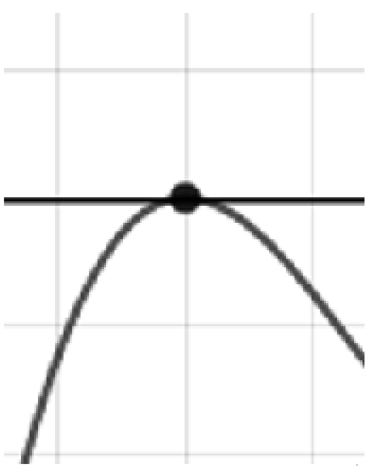
7. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

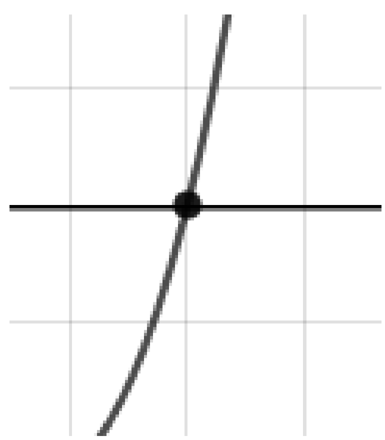
$$f(x) = 3(x-7)^8(x+7)^{11}(x+5)^3(x-5)^4$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 3i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 33x - 25$, which is option C.

A. $b \in [-4, 3], c \in [1, 7], \text{ and } d \in [-6, -1]$

$x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

B. $b \in [-4, 3], c \in [-11, -3], \text{ and } d \in [1, 6]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

C. $b \in [-10, -7], c \in [31, 38], \text{ and } d \in [-25, -20]$

* $x^3 - 9x^2 + 33x - 25$, which is the correct option.

D. $b \in [3, 13], c \in [31, 38], \text{ and } d \in [14, 32]$

$x^3 + 9x^2 + 33x + 25$, which corresponds to multiplying out $(x - (4 - 3i))(x - (4 + 3i))(x + 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 3i))(x - (4 + 3i))(x - (1))$.

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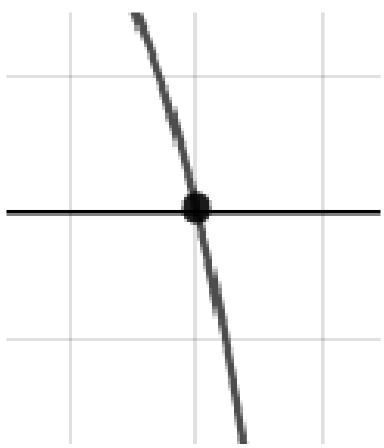
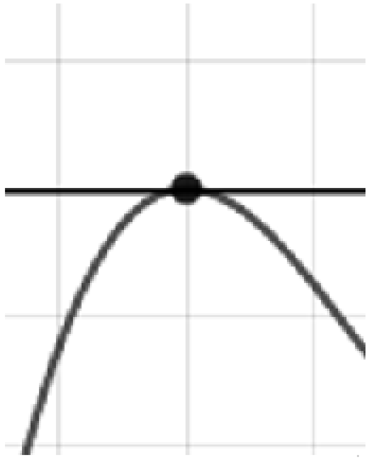
9. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

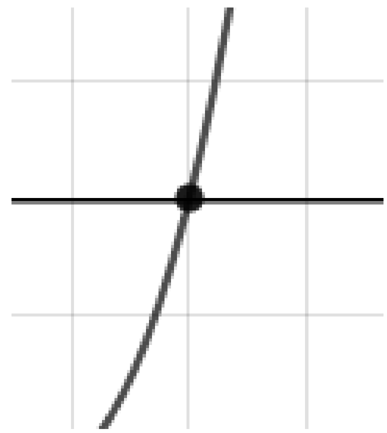
$$f(x) = -8(x - 5)^{10}(x + 5)^{8}(x - 7)^{11}(x + 7)^{6}$$

The solution is the graph below, which is option A.



B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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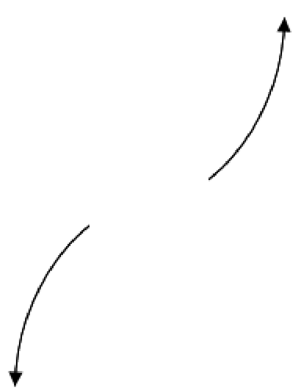
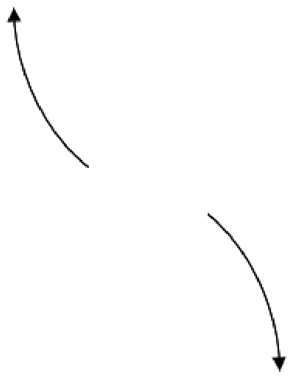
10. Describe the end behavior of the polynomial below.

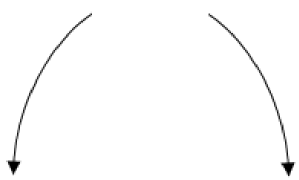
$$f(x) = 8(x+6)^5(x-6)^{10}(x+8)^2(x-8)^2$$

The solution is the graph below, which is option D.

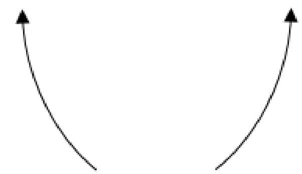




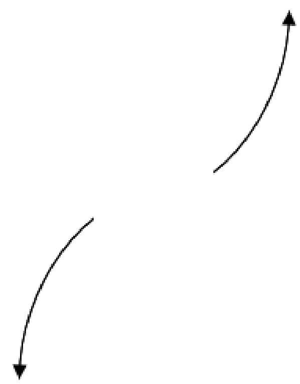
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.