This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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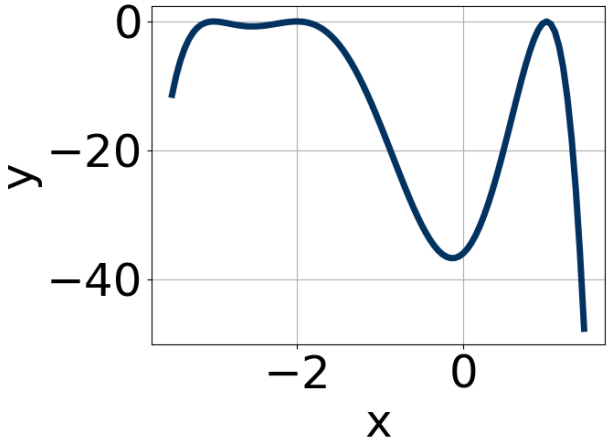
1. Which of the following equations *could* be of the graph presented below?



The solution is $-5(x+2)^6(x-1)^4(x+3)^{10}$, which is option E.

A. $-5(x+2)^6(x-1)^8(x+3)^{11}$

The factor $(x+3)$ should have an even power.

B. $12(x+2)^6(x-1)^{10}(x+3)^5$

The factor $(x+3)$ should have an even power and the leading coefficient should be the opposite sign.

C. $7(x+2)^{10}(x-1)^{10}(x+3)^8$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-7(x+2)^6(x-1)^7(x+3)^7$

The factors $(x-1)$ and $(x+3)$ should both have even powers.

E. $-5(x+2)^6(x-1)^4(x+3)^{10}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 + 5i \text{ and } -4$$

The solution is $x^3 + 13x + 116$, which is option C.

A. $b \in [0.16, 1.34], c \in [0, 5],$ and $d \in [-9, -3]$

$x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x-2)(x+4)$.

B. $b \in [-1.51, 0.14], c \in [4, 21],$ and $d \in [-119, -107]$

$x^3 + 13x - 116$, which corresponds to multiplying out $(x-(2+5i))(x-(2-5i))(x-4)$.

C. $b \in [-1.51, 0.14], c \in [4, 21],$ and $d \in [112, 121]$

* $x^3 + 13x + 116$, which is the correct option.

D. $b \in [0.16, 1.34], c \in [-2, 0],$ and $d \in [-20, -14]$

$x^3 + x^2 - x - 20$, which corresponds to multiplying out $(x-5)(x+4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(2+5i))(x-(2-5i))(x-(-4))$.

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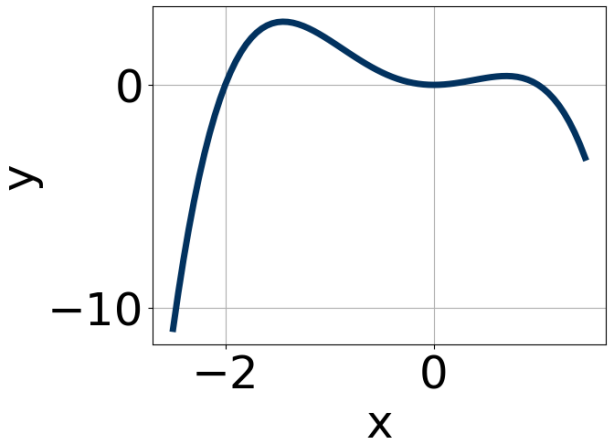
3. Which of the following equations *could* be of the graph presented below?



The solution is $-16x^{10}(x+2)^5(x-1)^5$, which is option A.

A. $-16x^{10}(x+2)^5(x-1)^5$

* This is the correct option.

B. $-14x^{10}(x+2)^8(x-1)^7$

The factor $(x+2)$ should have an odd power.

C. $-15x^5(x+2)^4(x-1)^9$

The factor 0 should have an even power and the factor -2 should have an odd power.

D. $8x^4(x+2)^{11}(x-1)^8$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $2x^4(x+2)^5(x-1)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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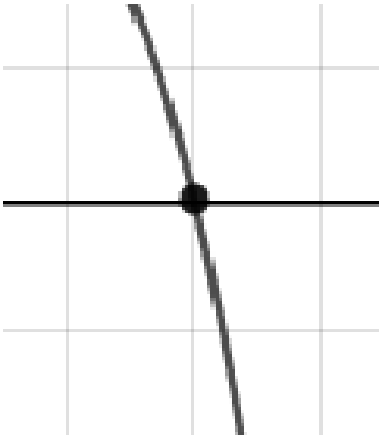
4. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

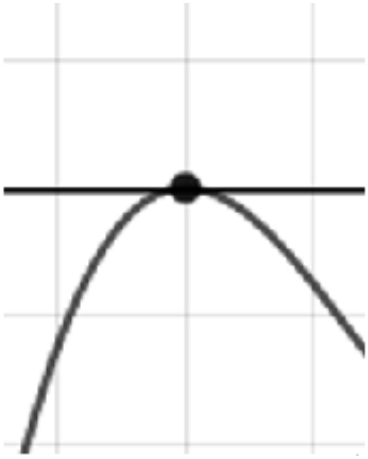
$$f(x) = -3(x+3)^2(x-3)^5(x+4)^9(x-4)^{10}$$

The solution is the graph below, which is option C.

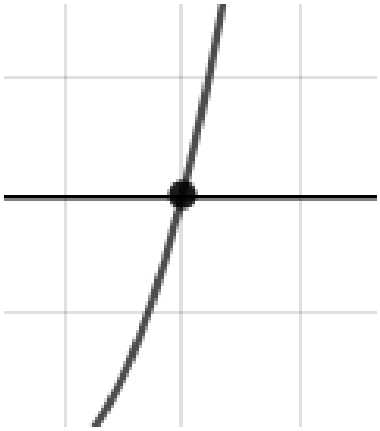
A.



B.



C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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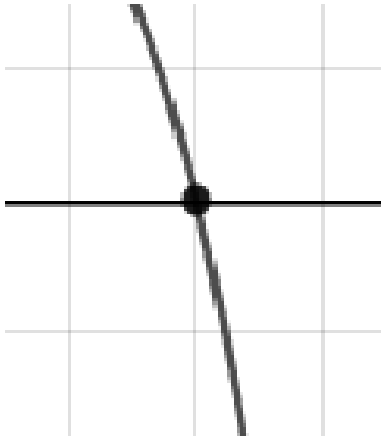
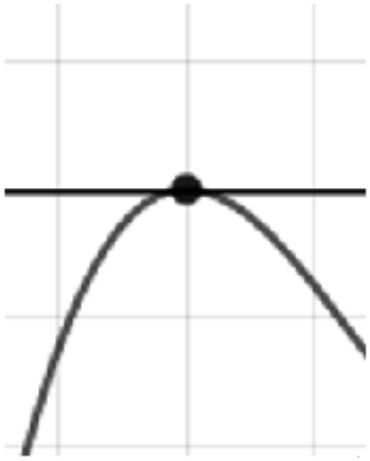
5. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

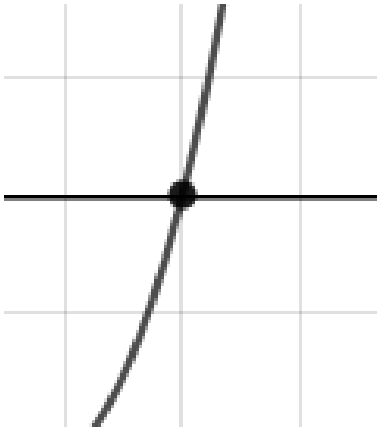
$$f(x) = 9(x+5)^5(x-5)^{10}(x+7)^9(x-7)^{12}$$

The solution is the graph below, which is option C.

A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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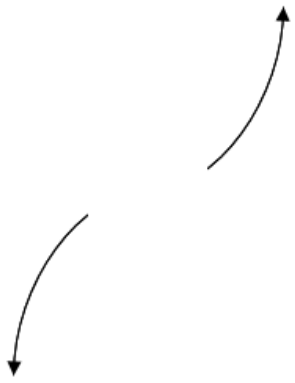
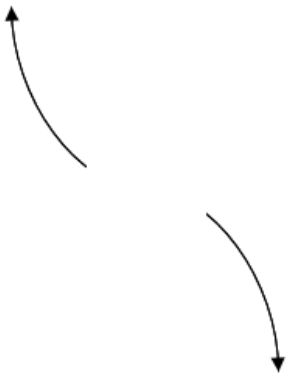
6. Describe the end behavior of the polynomial below.

$$f(x) = 9(x-4)^4(x+4)^9(x-6)^4(x+6)^4$$

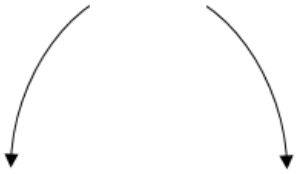
The solution is the graph below, which is option D.



A.

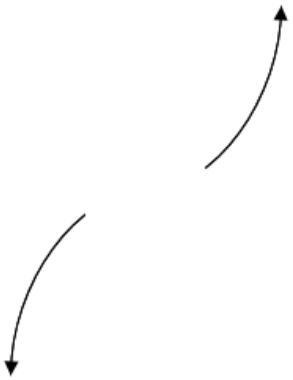


B.



C.

D.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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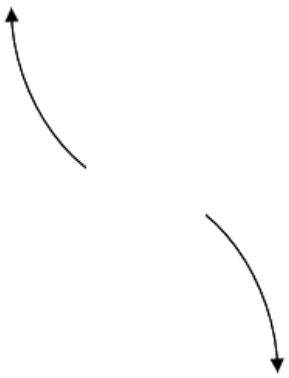
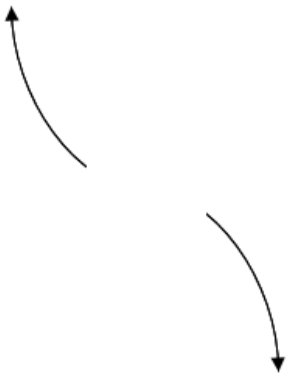
7. Describe the end behavior of the polynomial below.

$$f(x) = -9(x-9)^2(x+9)^3(x-4)^2(x+4)^4$$

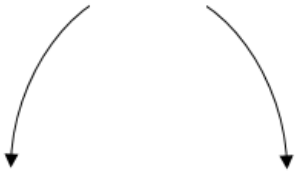
The solution is the graph below, which is option A.



A.

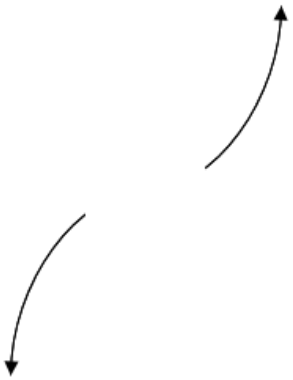


B.



C.

D.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{5}, \frac{3}{5}, \text{ and } \frac{2}{3}$$

The solution is $75x^3 - 65x^2 - 8x + 12$, which is option C.

A. $a \in [74, 76], b \in [58, 66], c \in [-12, -2],$ and $d \in [-18, -4]$

$75x^3 + 65x^2 - 8x - 12$, which corresponds to multiplying out $(5x - 2)(5x + 3)(3x + 2)$.

B. $a \in [74, 76], b \in [-36, -34], c \in [-30, -26],$ and $d \in [12, 17]$

$75x^3 - 35x^2 - 28x + 12$, which corresponds to multiplying out $(5x + 5)(5x + 5)(3x - 3)$.

C. $a \in [74, 76], b \in [-70, -60], c \in [-12, -2],$ and $d \in [12, 17]$

* $75x^3 - 65x^2 - 8x + 12$, which is the correct option.

D. $a \in [74, 76], b \in [-129, -119], c \in [61, 70],$ and $d \in [-18, -4]$

$75x^3 - 125x^2 + 68x - 12$, which corresponds to multiplying out $(5x + 5)(5x - 5)(3x - 3)$.

E. $a \in [74, 76], b \in [-70, -60], c \in [-12, -2],$ and $d \in [-18, -4]$

$75x^3 - 65x^2 - 8x - 12$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 2)(5x - 3)(3x - 2)$

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{5}, \frac{3}{4}, \text{ and } \frac{-3}{2}$$

The solution is $40x^3 + 86x^2 - 3x - 63$, which is option B.

A. $a \in [40, 44], b \in [-92, -83], c \in [-5, 1],$ and $d \in [59, 64]$

$40x^3 - 86x^2 - 3x + 63$, which corresponds to multiplying out $(5x - 7)(4x + 3)(2x - 3)$.

B. $a \in [40, 44], b \in [85, 87], c \in [-5, 1],$ and $d \in [-67, -58]$

* $40x^3 + 86x^2 - 3x - 63$, which is the correct option.

C. $a \in [40, 44], b \in [30, 40], c \in [-84, -78],$ and $d \in [-67, -58]$

$40x^3 + 34x^2 - 81x - 63$, which corresponds to multiplying out $(5x + 5)(4x + 4)(2x - 2)$.

D. $a \in [40, 44], b \in [-27, -22], c \in [-87, -82],$ and $d \in [59, 64]$

$40x^3 - 26x^2 - 87x + 63$, which corresponds to multiplying out $(5x + 5)(4x - 4)(2x - 2)$.

E. $a \in [40, 44], b \in [85, 87], c \in [-5, 1],$ and $d \in [59, 64]$

$40x^3 + 86x^2 - 3x + 63$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 7)(4x - 3)(2x + 3)$

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 5i \text{ and } 3$$

The solution is $x^3 - 9x^2 + 52x - 102$, which is option A.

A. $b \in [-14, -5], c \in [51, 58],$ and $d \in [-109, -97]$

* $x^3 - 9x^2 + 52x - 102$, which is the correct option.

B. $b \in [4, 16], c \in [51, 58],$ and $d \in [99, 103]$

$x^3 + 9x^2 + 52x + 102$, which corresponds to multiplying out $(x - (3 - 5i))(x - (3 + 5i))(x + 3)$.

C. $b \in [-1, 4], c \in [1, 3],$ and $d \in [-17, -10]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x + 5)(x - 3)$.

D. $b \in [-1, 4], c \in [-19, -3],$ and $d \in [5, 11]$

$x^3 + x^2 - 6x + 9$, which corresponds to multiplying out $(x - 3)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 - 5i))(x - (3 + 5i))(x - (3))$.