This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{5}, -6, \text{ and } \frac{4}{3}$$

The solution is $15x^3 + 76x^2 - 92x - 48$, which is option B.

A. $a \in [12, 19], b \in [72, 81], c \in [-96, -91], \text{ and } d \in [47, 50]$

$15x^3 + 76x^2 - 92x + 48$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [12, 19], b \in [72, 81], c \in [-96, -91], \text{ and } d \in [-54, -45]$

* $15x^3 + 76x^2 - 92x - 48$, which is the correct option.

C. $a \in [12, 19], b \in [58, 66], c \in [-152, -146], \text{ and } d \in [47, 50]$

$15x^3 + 64x^2 - 148x + 48$, which corresponds to multiplying out $(5x - 2)(x + 6)(3x - 4)$.

D. $a \in [12, 19], b \in [-79, -73], c \in [-96, -91], \text{ and } d \in [47, 50]$

$15x^3 - 76x^2 - 92x + 48$, which corresponds to multiplying out $(5x - 2)(x - 6)(3x + 4)$.

E. $a \in [12, 19], b \in [-124, -115], c \in [159, 171], \text{ and } d \in [-54, -45]$

$15x^3 - 116x^2 + 164x - 48$, which corresponds to multiplying out $(5x - 2)(x - 6)(3x - 4)$.

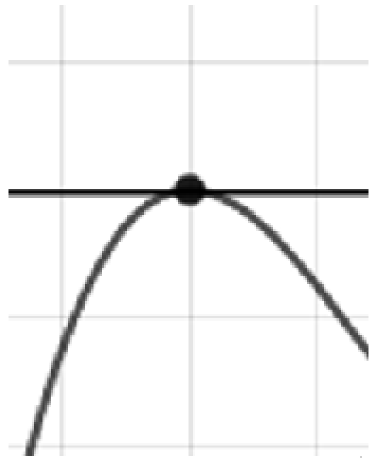
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 2)(x + 6)(3x - 4)$

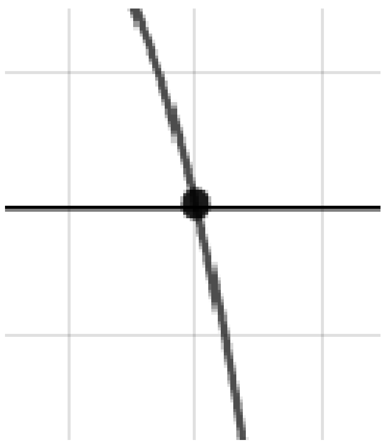
---

2. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

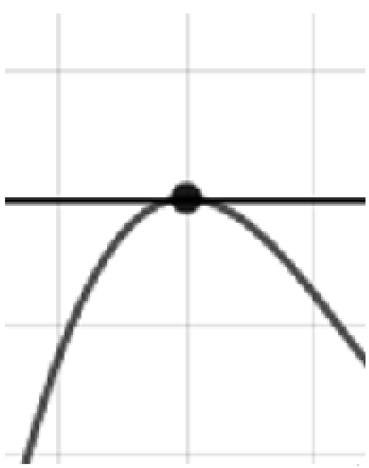
$$f(x) = -6(x + 3)^{11}(x - 3)^7(x + 4)^9(x - 4)^4$$

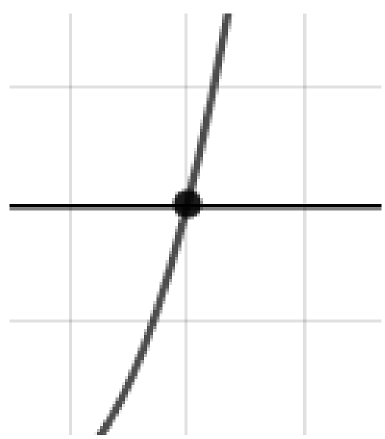
The solution is the graph below, which is option B.



A. 

C. 

B. 

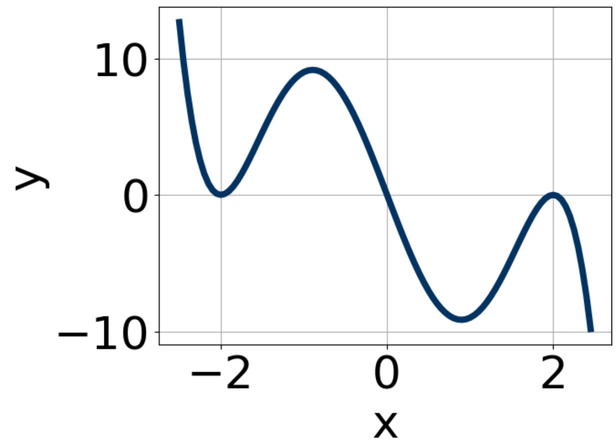
D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

3. Which of the following equations *could* be of the graph presented below?



The solution is $-18x^5(x+2)^8(x-2)^8$, which is option C.

A. $11x^{11}(x+2)^6(x-2)^4$

   This corresponds to the leading coefficient being the opposite value than it should be.

B. $-7x^9(x+2)^4(x-2)^9$

   The factor $(x-2)$ should have an even power.

C. $-18x^5(x+2)^8(x-2)^8$

   * This is the correct option.

D. $17x^{10}(x+2)^6(x-2)^6$

   The factor x should have an odd power and the leading coefficient should be the opposite sign.

E. $-12x^6(x+2)^{10}(x-2)^7$

   The factor $(x-2)$ should have an even power and the factor x should have an odd power.

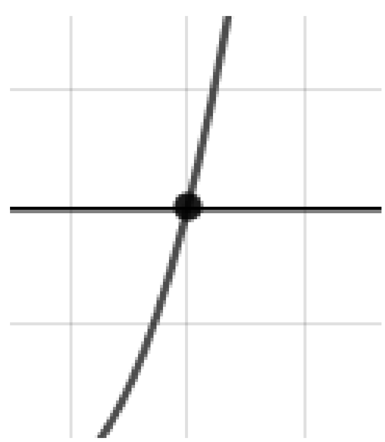
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

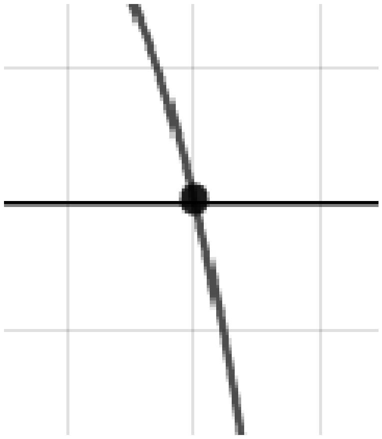
---

4. Describe the zero behavior of the zero $x=-3$ of the polynomial below.

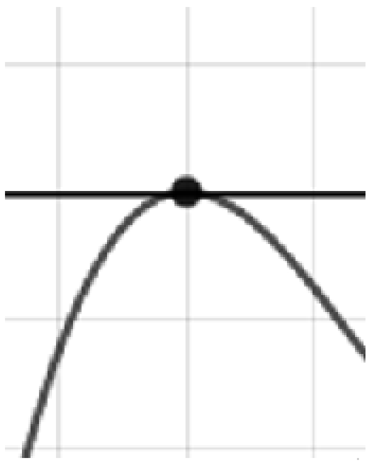
$$f(x) = -6(x-3)^8(x+3)^9(x+9)^3(x-9)^5$$

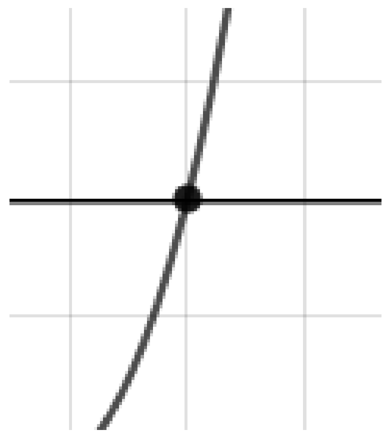
The solution is the graph below, which is option D.



A. 

C. 

B. 

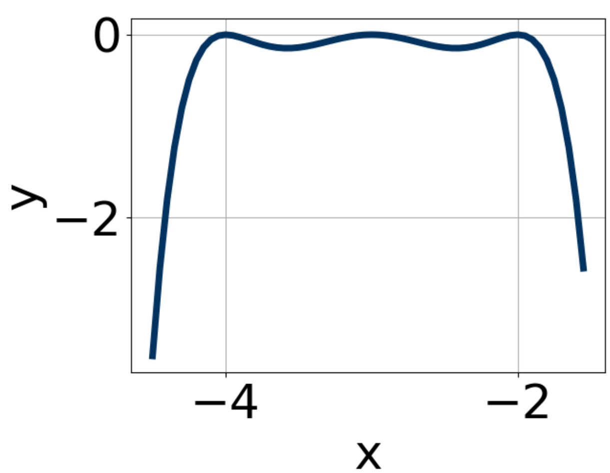
D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

5. Which of the following equations *could* be of the graph presented below?



The solution is $-19(x+3)^8(x+4)^{10}(x+2)^6$, which is option B.

A. $-4(x+3)^8(x+4)^{11}(x+2)^{11}$

The factors $(x+4)$ and $(x+2)$ should both have even powers.

B. $-19(x+3)^8(x+4)^{10}(x+2)^6$

* This is the correct option.

C. $-4(x+3)^4(x+4)^8(x+2)^9$

The factor $(x+2)$ should have an even power.

D. $18(x+3)^{10}(x+4)^6(x+2)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $2(x+3)^4(x+4)^4(x+2)^{11}$

The factor $(x+2)$ should have an even power and the leading coefficient should be the opposite sign.

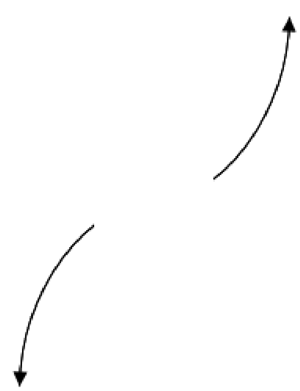
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

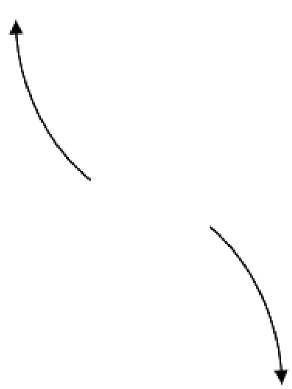
---

6. Describe the end behavior of the polynomial below.

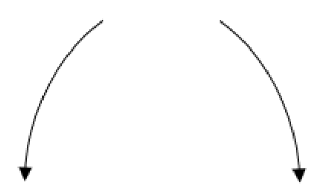
$$f(x) = 3(x-2)^3(x+2)^6(x-3)^3(x+3)^5$$

The solution is the graph below, which is option D.

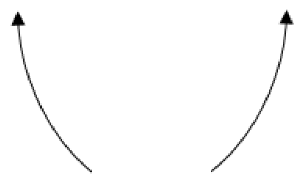


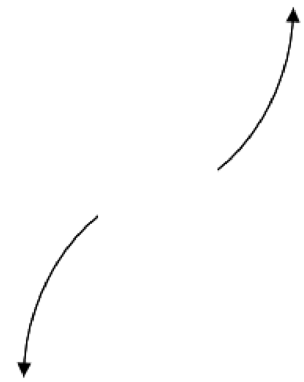


A.



B.

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 5i \text{ and } -2$$

The solution is $x^3 - 6x^2 + 25x + 82$, which is option A.

A. $b \in [-7, 0], c \in [24.91, 26.45]$, and $d \in [80.64, 83.45]$

   * $x^3 - 6x^2 + 25x + 82$, which is the correct option.

B. $b \in [-1, 2], c \in [-3.56, -2.79]$, and $d \in [-10.45, -8.73]$

   $x^3 + x^2 - 3x - 10$, which corresponds to multiplying out $(x - 5)(x + 2)$.

C. $b \in [2, 10], c \in [24.91, 26.45]$, and $d \in [-84.05, -81.44]$

   $x^3 + 6x^2 + 25x - 82$, which corresponds to multiplying out $(x - (4 + 5i))(x - (4 - 5i))(x - 2)$.

D. $b \in [-1, 2], c \in [-2.86, -1.28]$, and $d \in [-9.04, -7.02]$

   $x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x - 4)(x + 2)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

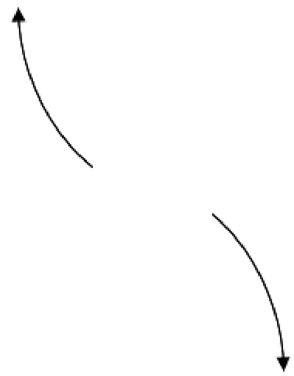
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 + 5i))(x - (4 - 5i))(x - (-2))$.

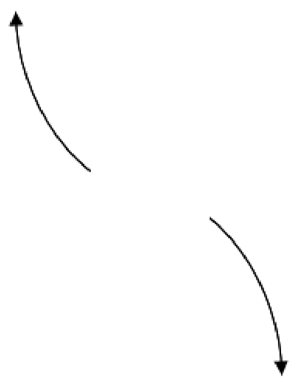
---

8. Describe the end behavior of the polynomial below.

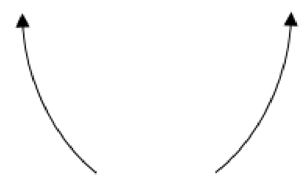
$$f(x) = -4(x + 2)^3(x - 2)^4(x - 7)^4(x + 7)^6$$

The solution is the graph below, which is option A.

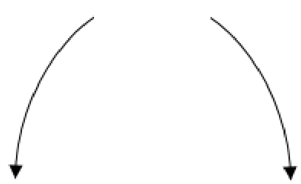




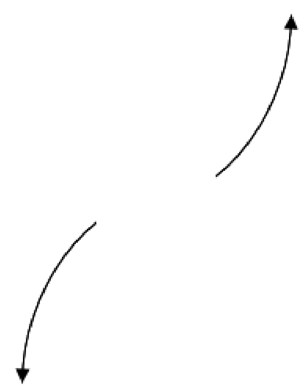
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 2i \text{ and } -4$$

The solution is $x^3 - 4x^2 - 12x + 80$, which is option A.

A. $b \in [-4.2, -2.5], c \in [-14.9, -10.4]$, and $d \in [75, 81]$

   * $x^3 - 4x^2 - 12x + 80$, which is the correct option.

B. $b \in [-1.7, 2.1], c \in [0.3, 5.3]$, and $d \in [-10, -2]$

$x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x-2)(x+4)$.

C. $b \in [2.9, 6.9], c \in [-14.9, -10.4]$, and $d \in [-82, -79]$

$x^3 + 4x^2 - 12x - 80$, which corresponds to multiplying out $(x-(4+2i))(x-(4-2i))(x-4)$.

D. $b \in [-1.7, 2.1], c \in [-0.5, 0.9]$, and $d \in [-20, -13]$

$x^3 + x^2 - 16$, which corresponds to multiplying out $(x-4)(x+4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(4+2i))(x-(4-2i))(x-(-4))$.

---

10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$1, -5, \text{ and } \frac{-1}{3}$$

The solution is $3x^3 + 13x^2 - 11x - 5$, which is option B.

A. $a \in [-4, 5], b \in [18, 24], c \in [19, 24]$, and $d \in [3, 14]$

$3x^3 + 19x^2 + 21x + 5$, which corresponds to multiplying out $(x+1)(x+5)(3x+1)$.

B. $a \in [-4, 5], b \in [11, 14], c \in [-15, -5]$, and $d \in [-9, 2]$

* $3x^3 + 13x^2 - 11x - 5$, which is the correct option.

C. $a \in [-4, 5], b \in [-15, -12], c \in [-15, -5]$, and $d \in [3, 14]$

$3x^3 - 13x^2 - 11x + 5$, which corresponds to multiplying out $(x+1)(x-5)(3x-1)$.

D. $a \in [-4, 5], b \in [-12, -6], c \in [-19, -14]$, and $d \in [-9, 2]$

$3x^3 - 11x^2 - 19x - 5$, which corresponds to multiplying out $(x+1)(x-5)(3x+1)$.

E. $a \in [-4, 5], b \in [11, 14], c \in [-15, -5]$, and $d \in [3, 14]$

$3x^3 + 13x^2 - 11x + 5$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-1)(x+5)(3x+1)$
