This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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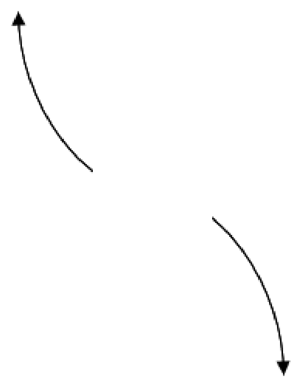
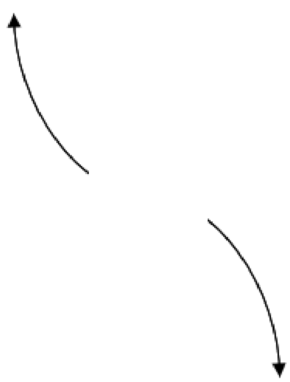
1. Describe the end behavior of the polynomial below.

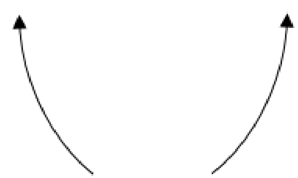
$$f(x) = -3(x-8)^5(x+8)^{10}(x-6)^3(x+6)^5$$

The solution is the graph below, which is option A.

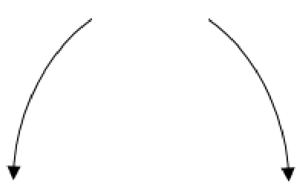




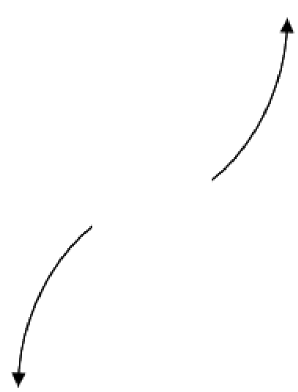
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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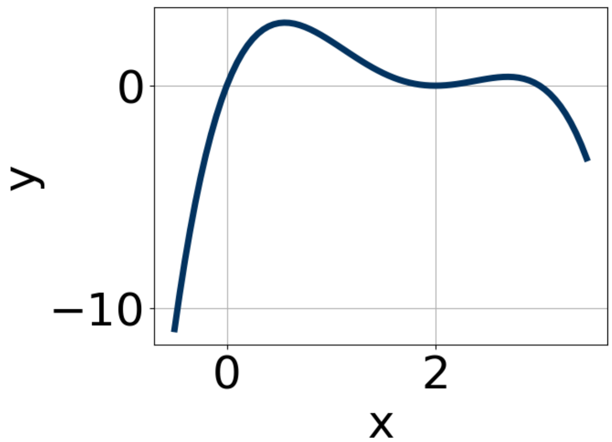
2. Which of the following equations *could* be of the graph presented below?



The solution is $-13x^5(x-2)^8(x-3)^9$, which is option E.

A. $20x^9(x-2)^6(x-3)^6$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-3x^6(x-2)^{11}(x-3)^{11}$

The factor 2 should have an even power and the factor 0 should have an odd power.

C. $14x^9(x-2)^6(x-3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-19x^6(x-2)^{10}(x-3)^9$

The factor x should have an odd power.

E. $-13x^5(x-2)^8(x-3)^9$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$7, \frac{-4}{5}, \text{ and } \frac{-3}{2}$$

The solution is $10x^3 - 47x^2 - 149x - 84$, which is option B.

A. $a \in [8, 12], b \in [88, 101], c \in [170, 175], \text{ and } d \in [81, 90]$

$10x^3 + 93x^2 + 173x + 84$, which corresponds to multiplying out $(x+7)(5x+4)(2x+3)$.

B. $a \in [8, 12], b \in [-49, -45], c \in [-149, -146], \text{ and } d \in [-84, -79]$

* $10x^3 - 47x^2 - 149x - 84$, which is the correct option.

C. $a \in [8, 12], b \in [44, 52], c \in [-149, -146],$ and $d \in [81, 90]$

$10x^3 + 47x^2 - 149x + 84$, which corresponds to multiplying out $(x + 7)(5x - 4)(2x - 3)$.

D. $a \in [8, 12], b \in [76, 78], c \in [36, 43],$ and $d \in [-84, -79]$

$10x^3 + 77x^2 + 37x - 84$, which corresponds to multiplying out $(x + 7)(5x - 4)(2x + 3)$.

E. $a \in [8, 12], b \in [-49, -45], c \in [-149, -146],$ and $d \in [81, 90]$

$10x^3 - 47x^2 - 149x + 84$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-7)(5x+4)(2x+3)$

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 2i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 28x - 20$, which is option D.

A. $b \in [-5, 6], c \in [-5, -1],$ and $d \in [-1, 8]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

B. $b \in [7, 10], c \in [25, 33],$ and $d \in [17, 23]$

$x^3 + 9x^2 + 28x + 20$, which corresponds to multiplying out $(x - (4 - 2i))(x - (4 + 2i))(x + 1)$.

C. $b \in [-5, 6], c \in [-4, 3],$ and $d \in [-4, -1]$

$x^3 + x^2 + x - 2$, which corresponds to multiplying out $(x + 2)(x - 1)$.

D. $b \in [-12, -4], c \in [25, 33],$ and $d \in [-21, -15]$

* $x^3 - 9x^2 + 28x - 20$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 2i))(x - (4 + 2i))(x - (1))$.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{3}{5}, \frac{7}{2}, \text{ and } 3$$

The solution is $10x^3 - 71x^2 + 144x - 63$, which is option E.

A. $a \in [9, 13], b \in [-61, -58], c \in [60, 69],$ and $d \in [58, 67]$

$10x^3 - 59x^2 + 66x + 63$, which corresponds to multiplying out $(5x + 3)(2x - 7)(x - 3)$.

B. $a \in [9, 13], b \in [65, 75], c \in [143, 149],$ and $d \in [58, 67]$

$10x^3 + 71x^2 + 144x + 63$, which corresponds to multiplying out $(5x + 3)(2x + 7)(x + 3)$.

C. $a \in [9, 13], b \in [11, 13], c \in [-103, -100],$ and $d \in [-72, -62]$

$10x^3 + 11x^2 - 102x - 63$, which corresponds to multiplying out $(5x + 3)(2x + 7)(x - 3)$.

D. $a \in [9, 13], b \in [-72, -62], c \in [143, 149],$ and $d \in [58, 67]$

$10x^3 - 71x^2 + 144x + 63$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [9, 13], b \in [-72, -62], c \in [143, 149],$ and $d \in [-72, -62]$

* $10x^3 - 71x^2 + 144x - 63$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 3)(2x - 7)(x - 3)$

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 5i \text{ and } -2$$

The solution is $x^3 + 8x^2 + 46x + 68$, which is option D.

A. $b \in [-6, 3], c \in [4.7, 6.1],$ and $d \in [5.4, 9.1]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 3)(x + 2)$.

B. $b \in [-6, 3], c \in [5.7, 8.3],$ and $d \in [8.4, 11.7]$

$x^3 + x^2 + 7x + 10$, which corresponds to multiplying out $(x + 5)(x + 2)$.

C. $b \in [-8, -2], c \in [44, 46.1],$ and $d \in [-69.7, -66.9]$

$x^3 - 8x^2 + 46x - 68$, which corresponds to multiplying out $(x - (-3 - 5i))(x - (-3 + 5i))(x - 2)$.

D. $b \in [7, 15], c \in [44, 46.1],$ and $d \in [63.1, 71.7]$

* $x^3 + 8x^2 + 46x + 68$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 5i))(x - (-3 + 5i))(x - (-2))$.

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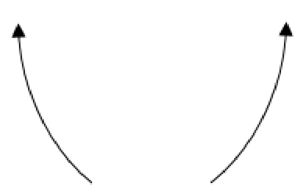
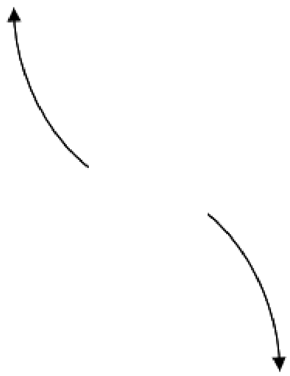
7. Describe the end behavior of the polynomial below.

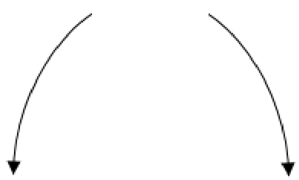
$$f(x) = 4(x + 2)^4(x - 2)^9(x + 9)^3(x - 9)^4$$

The solution is the graph below, which is option C.

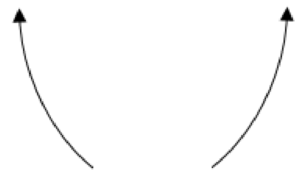




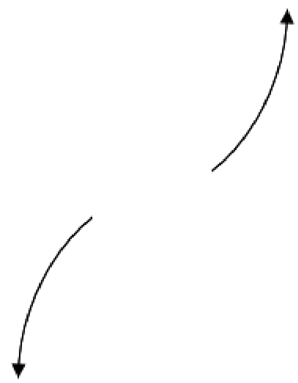
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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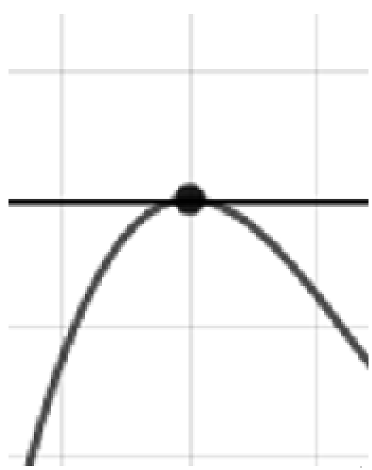
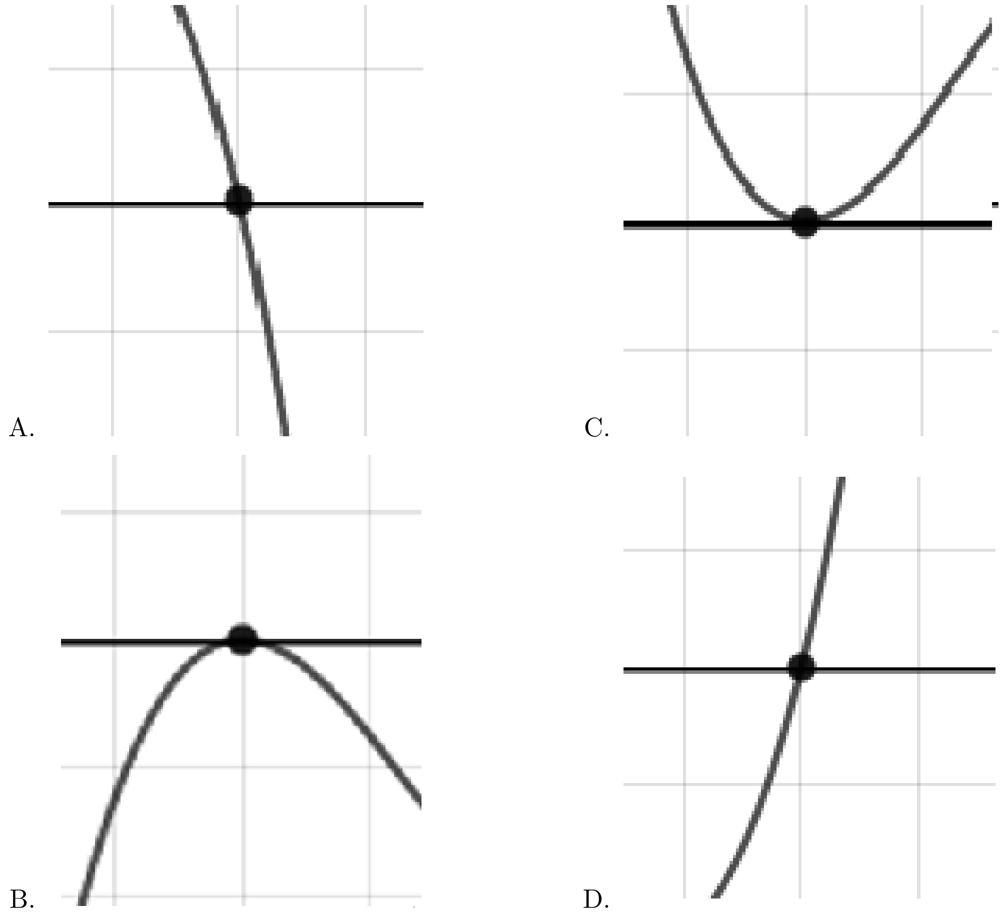
8. Describe the zero behavior of the zero $x = -2$ of the polynomial below.

$$f(x) = -8(x-6)^{11}(x+6)^{9}(x-2)^{5}(x+2)^{4}$$

The solution is the graph below, which is option B.





A. B. C. D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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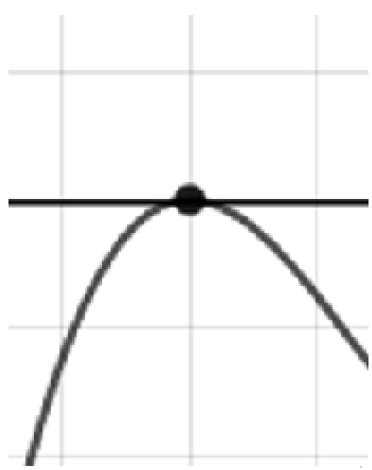
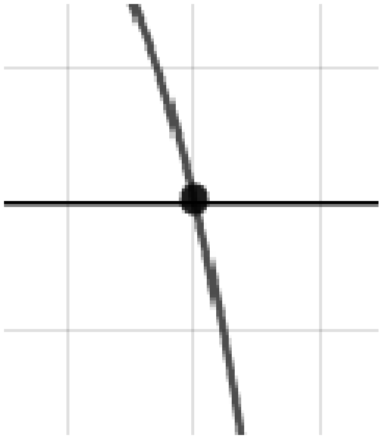
9. Describe the zero behavior of the zero $x = -2$ of the polynomial below.

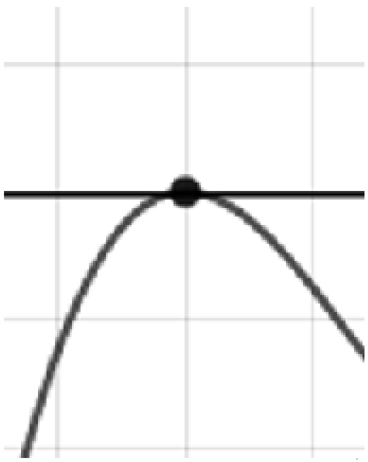
$$f(x) = -4(x+5)^9(x-5)^5(x+2)^{10}(x-2)^9$$

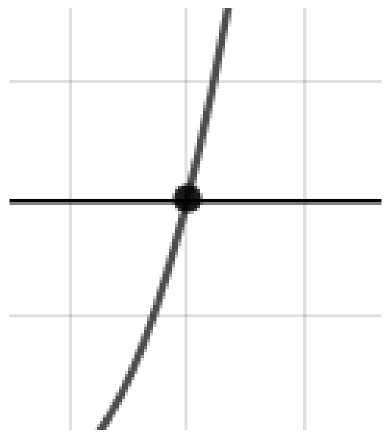
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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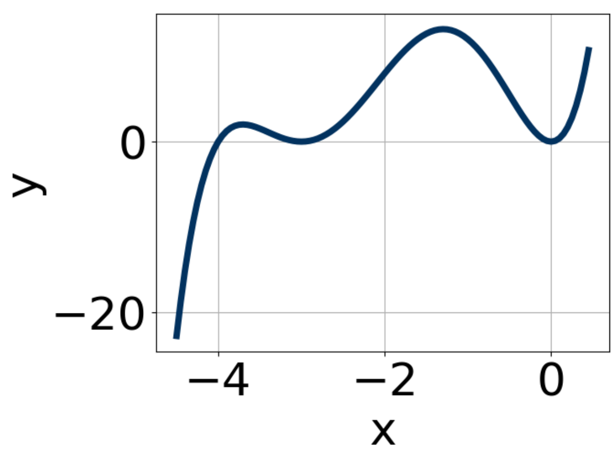
10. Which of the following equations *could* be of the graph presented below?



The solution is $19x^6(x+3)^8(x+4)^{11}$, which is option E.

A. $-18x^{10}(x+3)^6(x+4)^8$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $7x^5(x+3)^6(x+4)^6$

The factor x should have an even power and the factor $(x+4)$ should have an odd power.

C. $-14x^8(x+3)^{10}(x+4)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $2x^5(x+3)^{10}(x+4)^7$

The factor x should have an even power.

E. $19x^6(x+3)^8(x+4)^{11}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).