This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 7x^2 + 3x + 1 \text{ and } g(x) = 6x + 4$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers except $x = a,$ where $a \in [2.6, 10.6]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-8.6, 0.4]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-7, -4]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-9.83, -4.83]$ and $b \in [-6.17, 1.83]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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2. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = e^{x+2} - 5$$

The solution is $f^{-1}(8) = 0.565$, which is option C.

A. $f^{-1}(8) \in [3.96, 4.92]$

This solution corresponds to distractor 1.

B. $f^{-1}(8) \in [-4.14, -3.73]$

This solution corresponds to distractor 2.

C. $f^{-1}(8) \in [0.09, 0.79]$

This is the solution.

D. $f^{-1}(8) \in [-3.63, -2.86]$

This solution corresponds to distractor 3.

E. $f^{-1}(8) \in [-3.11, -2.22]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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3. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 - 1x^2 + 4x - 4 \text{ and } g(x) = -x^3 + 3x^2 - x - 3$$

The solution is 0.0, which is option B.

A. $(f \circ g)(1) \in [53, 61]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(1) \in [0, 12]$

* This is the correct solution

C. $(f \circ g)(1) \in [40, 46]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-11, -7]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{6x - 22} \text{ and } g(x) = 3x^4 + 3x^3 + 5x^2 + 3x + 3$$

The solution is The domain is all Real numbers greater than or equal to$x = 3.6666666666666665.$, which is option A.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-2.33, 6.67]$

B. The domain is all Real numbers except $x = a,$ where $a \in [3.2, 12.2]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [4.83, 11.83]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-0.17, 8.83]$ and $b \in [1.6, 6.6]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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5. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = e^{x+3} - 2$$

The solution is $f^{-1}(7) = -0.803$, which is option E.

A. $f^{-1}(7) \in [4.72, 5.49]$

This solution corresponds to distractor 1.

B. $f^{-1}(7) \in [0.18, 0.6]$

This solution corresponds to distractor 4.

C. $f^{-1}(7) \in [-0.62, -0.49]$

This solution corresponds to distractor 3.

D. $f^{-1}(7) \in [-0.47, -0.2]$

This solution corresponds to distractor 2.

E. $f^{-1}(7) \in [-1.09, -0.71]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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6. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 2x^3 - 4x^2 + x \text{ and } g(x) = 4x^3 - 2x^2 + x$$

The solution is 21.0, which is option A.

A. $(f \circ g)(1) \in [20.5, 25.1]$

* This is the correct solution

B. $(f \circ g)(1) \in [24.8, 28.5]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-14.9, -11.7]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-9.6, -6.6]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval the $f^{-1}(-15)$ belongs to.

$$f(x) = \sqrt[3]{5x + 4}$$

The solution is -675.8, which is option C.

A. $f^{-1}(-15) \in [675.15, 675.9]$

This solution corresponds to distractor 2.

B. $f^{-1}(-15) \in [674.07, 674.56]$

This solution corresponds to distractor 3.

C. $f^{-1}(-15) \in [-676.04, -675.8]$

* This is the correct solution.

D. $f^{-1}(-15) \in [-674.37, -673.71]$

Distractor 1: This corresponds to

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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8. Determine whether the function below is 1-1.

$$f(x) = (6x - 29)^3$$

The solution is yes, which is option D.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. Yes, the function is 1-1.

* This is the solution.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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9. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = 2x^2 + 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(10) \in [2.77, 3.36]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B. $f^{-1}(10) \in [1.73, 2.01]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C. $f^{-1}(10) \in [2.51, 2.65]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(10) \in [4.62, 4.91]$

Distractor 4: This corresponds to both distractors 2 and 3.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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10. Determine whether the function below is 1-1.

$$f(x) = 20x^2 + 14x - 528$$

The solution is no, which is option D.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.
