This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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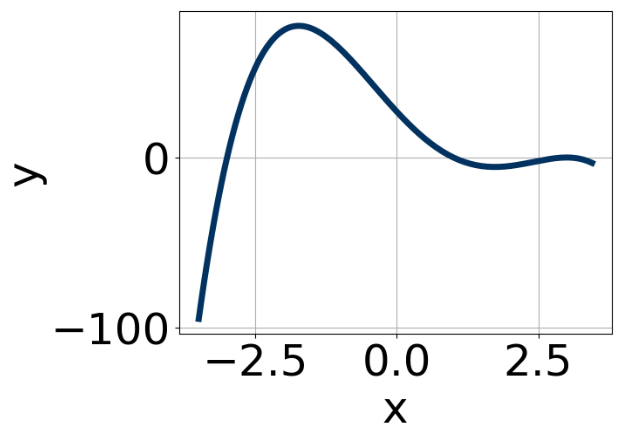
1. Which of the following equations *could* be of the graph presented below?



The solution is $-8(x-3)^4(x-1)^5(x+3)^7$, which is option C.

A. $-18(x-3)^4(x-1)^6(x+3)^9$

The factor $(x-1)$ should have an odd power.

B. $13(x-3)^8(x-1)^9(x+3)^6$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-8(x-3)^4(x-1)^5(x+3)^7$

* This is the correct option.

D. $10(x-3)^6(x-1)^9(x+3)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-19(x-3)^5(x-1)^{10}(x+3)^5$

The factor 3 should have an even power and the factor 1 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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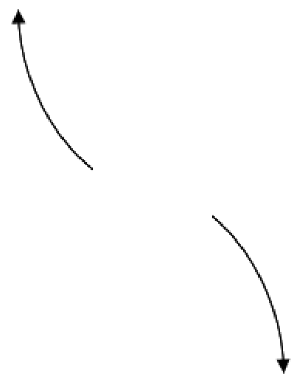
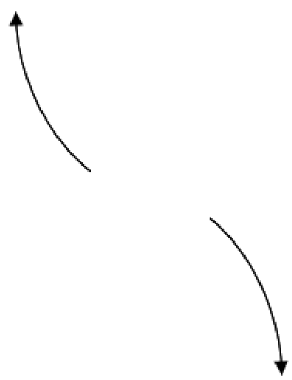
2. Describe the end behavior of the polynomial below.

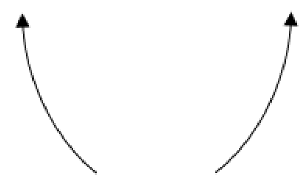
$$f(x) = -3(x-3)^5(x+3)^6(x-2)^3(x+2)^5$$

The solution is the graph below, which is option A.

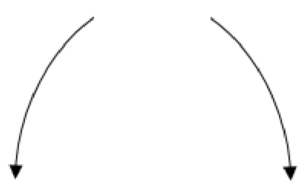




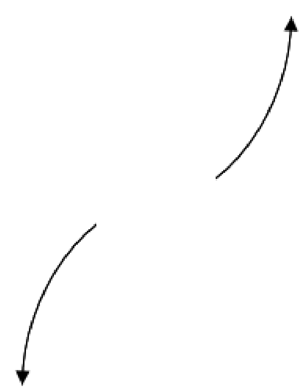
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$4, -7, \text{ and } \frac{-1}{5}$$

The solution is $5x^3 + 16x^2 - 137x - 28$, which is option C.

A. $a \in [4, 10], b \in [53.5, 57.4], c \in [150, 154], \text{ and } d \in [23, 37]$

$5x^3 + 56x^2 + 151x + 28$, which corresponds to multiplying out $(x + 4)(x + 7)(5x + 1)$.

B. $a \in [4, 10], b \in [14.4, 17.4], c \in [-139, -134],$ and $d \in [23, 37]$

$5x^3+16x^2-137x+28$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [4, 10], b \in [14.4, 17.4], c \in [-139, -134],$ and $d \in [-35, -21]$

* $5x^3 + 16x^2 - 137x - 28$, which is the correct option.

D. $a \in [4, 10], b \in [-17.7, -14.1], c \in [-139, -134],$ and $d \in [23, 37]$

$5x^3 - 16x^2 - 137x + 28$, which corresponds to multiplying out $(x + 4)(x - 7)(5x - 1)$.

E. $a \in [4, 10], b \in [-14.4, -12.2], c \in [-147, -141],$ and $d \in [-35, -21]$

$5x^3 - 14x^2 - 143x - 28$, which corresponds to multiplying out $(x + 4)(x - 7)(5x + 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-4)(x+7)(5x+1)$

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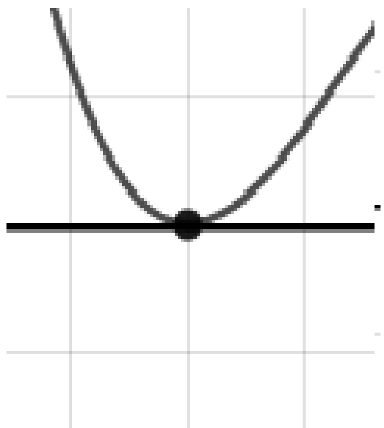
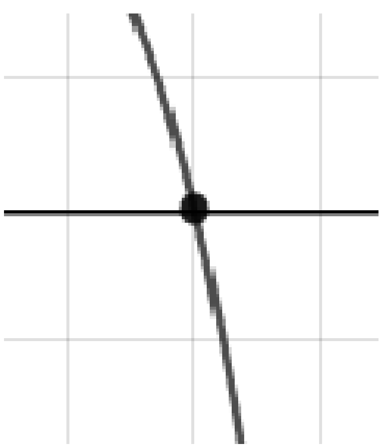
4. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

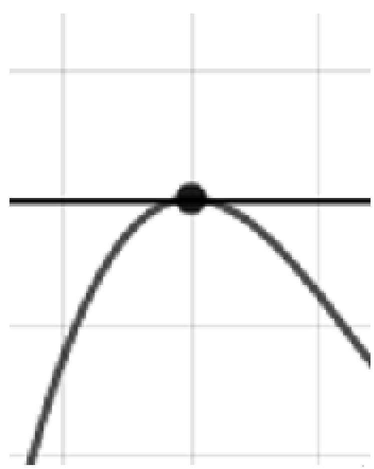
$$f(x) = 2(x - 8)^{12}(x + 8)^8(x + 9)^7(x - 9)^4$$

The solution is the graph below, which is option C.



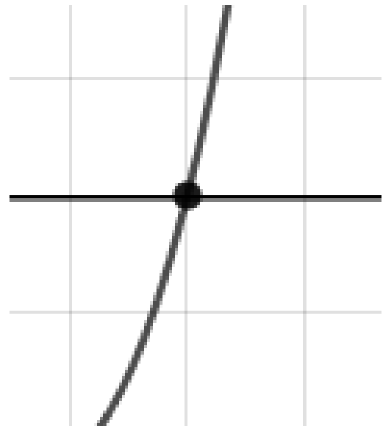


A.



B.

C.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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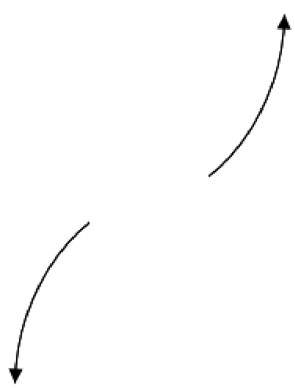
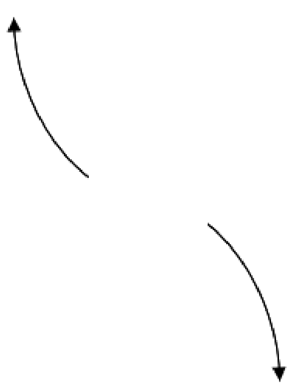
5. Describe the end behavior of the polynomial below.

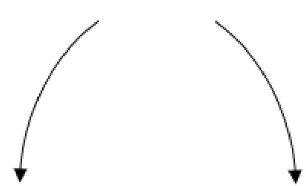
$$f(x) = 8(x+8)^2(x-8)^3(x+7)^5(x-7)^7$$

The solution is the graph below, which is option D.

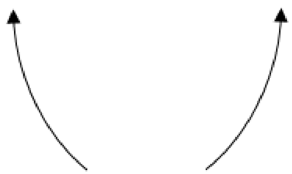


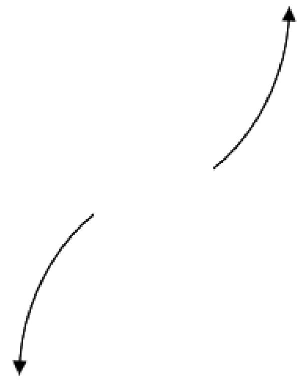


A.



B.

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 - 5i \text{ and } -1$$

The solution is $x^3 - 3x^2 + 25x + 29$, which is option C.

A. $b \in [-1.3, 1.7], c \in [5, 12], \text{ and } d \in [4, 6]$

$x^3 + x^2 + 6x + 5$, which corresponds to multiplying out $(x + 5)(x + 1)$.

B. $b \in [-1.3, 1.7], c \in [-5, 2], \text{ and } d \in [-4, 0]$

$x^3 + x^2 - x - 2$, which corresponds to multiplying out $(x - 2)(x + 1)$.

C. $b \in [-5.6, -2.4], c \in [25, 27], \text{ and } d \in [24, 31]$

* $x^3 - 3x^2 + 25x + 29$, which is the correct option.

D. $b \in [2.7, 5.9], c \in [25, 27], \text{ and } d \in [-35, -24]$

$x^3 + 3x^2 + 25x - 29$, which corresponds to multiplying out $(x - (2 - 5i))(x - (2 + 5i))(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 - 5i))(x - (2 + 5i))(x - (-1))$.

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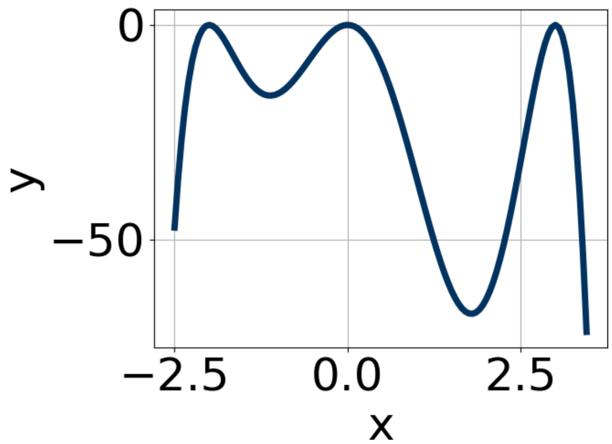
7. Which of the following equations *could* be of the graph presented below?



The solution is $-11x^4(x-3)^{10}(x+2)^6$, which is option B.

A. $20x^8(x-3)^{10}(x+2)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-11x^4(x-3)^{10}(x+2)^6$

* This is the correct option.

C. $-15x^{10}(x-3)^6(x+2)^9$

The factor $(x+2)$ should have an even power.

D. $-2x^6(x-3)^5(x+2)^{11}$

The factors $(x-3)$ and $(x+2)$ should both have even powers.

E. $17x^4(x-3)^8(x+2)^7$

The factor $(x+2)$ should have an even power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form x^3+bx^2+cx+d.

$$-4+3i \text{ and } 2$$

The solution is $x^3+6x^2+9x-50$, which is option A.

A. $b \in [2, 11], c \in [8, 12], \text{ and } d \in [-53, -49]$

* $x^3+6x^2+9x-50$, which is the correct option.

B. $b \in [-2, 4], c \in [-1, 3], \text{ and } d \in [-10, -6]$

x^3+x^2+2x-8, which corresponds to multiplying out $(x+4)(x-2)$.

C. $b \in [-2, 4], c \in [-5, -3], \text{ and } d \in [6, 8]$

x^3+x^2-5x+6, which corresponds to multiplying out $(x-3)(x-2)$.

D. $b \in [-10, -5], c \in [8, 12], \text{ and } d \in [46, 54]$

$x^3-6x^2+9x+50$, which corresponds to multiplying out $(x-(-4+3i))(x-(-4-3i))(x+2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

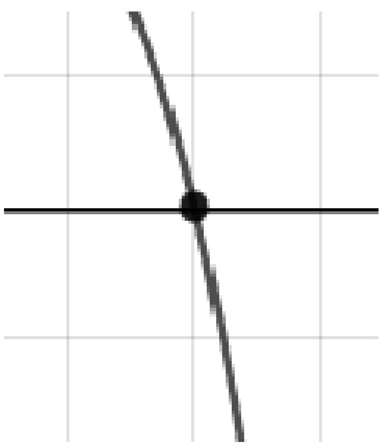
General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-4+3i))(x-(-4-3i))(x-(2))$.

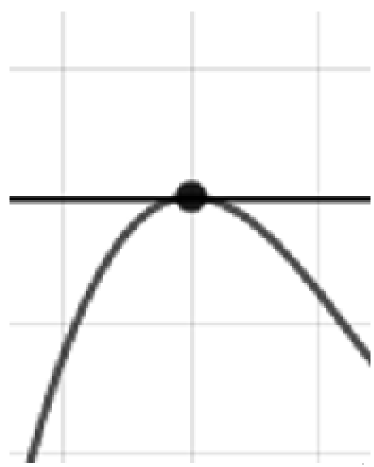
---

9. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

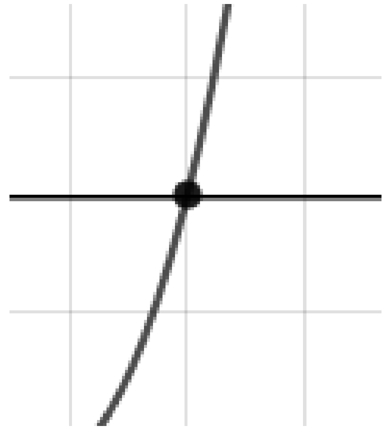
$$f(x) = 5(x+5)^7(x-5)^{10}(x+3)^5(x-3)^6$$

The solution is the graph below, which is option C.

A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$3, 6, \text{ and } \frac{-4}{3}$$

The solution is $3x^3 - 23x^2 + 18x + 72$, which is option A.

A. $a \in [2, 12], b \in [-24, -18], c \in [16, 19], \text{ and } d \in [72, 73]$

   * $3x^3 - 23x^2 + 18x + 72$, which is the correct option.

B. $a \in [2, 12], b \in [-24, -18], c \in [16, 19], \text{ and } d \in [-77, -68]$

   $3x^3 - 23x^2 + 18x - 72$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [2, 12], b \in [20, 24], c \in [16, 19], \text{ and } d \in [-77, -68]$

   $3x^3 + 23x^2 + 18x - 72$, which corresponds to multiplying out $(x + 3)(x + 6)(3x - 4)$.

D. $a \in [2, 12], b \in [-12, -2], c \in [-67, -60], \text{ and } d \in [-77, -68]$

   $3x^3 - 5x^2 - 66x - 72$, which corresponds to multiplying out $(x + 3)(x - 6)(3x + 4)$.

E. $a \in [2, 12], b \in [27, 34], c \in [90, 95], \text{ and } d \in [72, 73]$

   $3x^3 + 31x^2 + 90x + 72$, which corresponds to multiplying out $(x + 3)(x + 6)(3x + 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-3)(x-6)(3x+4)$
