This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-1, -6, \text{ and } \frac{5}{2}$$

The solution is $2x^3 + 9x^2 - 23x - 30$, which is option A.

A. $a \in [1, 4], b \in [8, 10.2], c \in [-27, -22], \text{ and } d \in [-36, -26]$

* $2x^3 + 9x^2 - 23x - 30$, which is the correct option.

B. $a \in [1, 4], b \in [8, 10.2], c \in [-27, -22], \text{ and } d \in [23, 31]$

$2x^3 + 9x^2 - 23x + 30$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [1, 4], b \in [-19.9, -15.6], c \in [42, 56], \text{ and } d \in [-36, -26]$

$2x^3 - 19x^2 + 47x - 30$, which corresponds to multiplying out $(x - 1)(x - 6)(2x - 5)$.

D. $a \in [1, 4], b \in [-12.1, -5.2], c \in [-27, -22], \text{ and } d \in [23, 31]$

$2x^3 - 9x^2 - 23x + 30$, which corresponds to multiplying out $(x - 1)(x - 6)(2x + 5)$.

E. $a \in [1, 4], b \in [4.5, 8.8], c \in [-39, -32], \text{ and } d \in [23, 31]$

$2x^3 + 5x^2 - 37x + 30$, which corresponds to multiplying out $(x - 1)(x + 6)(2x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+1)(x+6)(2x-5)$

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 3i \text{ and } -2$$

The solution is $x^3 - 6x^2 + 9x + 50$, which is option C.

A. $b \in [5, 17], c \in [6, 9.1], \text{ and } d \in [-53.4, -49.1]$

$x^3 + 6x^2 + 9x - 50$, which corresponds to multiplying out $(x - (4 + 3i))(x - (4 - 3i))(x - 2)$.

B. $b \in [0, 4], c \in [-3.7, -1.4], \text{ and } d \in [-11, -7.5]$

$x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x - 4)(x + 2)$.

C. $b \in [-14, 0], c \in [6, 9.1], \text{ and } d \in [49.6, 50.9]$

* $x^3 - 6x^2 + 9x + 50$, which is the correct option.

D. $b \in [0, 4], c \in [-1.8, 0.5],$ and $d \in [-6.5, -1.6]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x - 3)(x + 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 + 3i))(x - (4 - 3i))(x - (-2))$.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 3i \text{ and } -1$$

The solution is $x^3 + 11x^2 + 44x + 34$, which is option C.

A. $b \in [-9, 7], c \in [-3, 0],$ and $d \in [-5, 0]$

$x^3 + x^2 - 2x - 3$, which corresponds to multiplying out $(x - 3)(x + 1)$.

B. $b \in [-9, 7], c \in [0, 12],$ and $d \in [1, 7]$

$x^3 + x^2 + 6x + 5$, which corresponds to multiplying out $(x + 5)(x + 1)$.

C. $b \in [8, 18], c \in [40, 48],$ and $d \in [32, 38]$

* $x^3 + 11x^2 + 44x + 34$, which is the correct option.

D. $b \in [-13, -7], c \in [40, 48],$ and $d \in [-34, -25]$

$x^3 - 11x^2 + 44x - 34$, which corresponds to multiplying out $(x - (-5 + 3i))(x - (-5 - 3i))(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 3i))(x - (-5 - 3i))(x - (-1))$.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$3, \frac{5}{2}, \text{ and } \frac{3}{5}$$

The solution is $10x^3 - 61x^2 + 108x - 45$, which is option D.

A. $a \in [6, 17], b \in [61, 69], c \in [102, 110],$ and $d \in [44, 50]$

$10x^3 + 61x^2 + 108x + 45$, which corresponds to multiplying out $(x + 3)(2x + 5)(5x + 3)$.

B. $a \in [6, 17], b \in [-3, 3], c \in [-79, -76],$ and $d \in [44, 50]$

$10x^3 - 1x^2 - 78x + 45$, which corresponds to multiplying out $(x + 3)(2x - 5)(5x - 3)$.

C. $a \in [6, 17], b \in [46, 56], c \in [41, 44],$ and $d \in [-48, -41]$

$10x^3 + 49x^2 + 42x - 45$, which corresponds to multiplying out $(x + 3)(2x + 5)(5x - 3)$.

D. $a \in [6, 17], b \in [-61, -59], c \in [102, 110],$ and $d \in [-48, -41]$

* $10x^3 - 61x^2 + 108x - 45$, which is the correct option.

E. $a \in [6, 17], b \in [-61, -59], c \in [102, 110],$ and $d \in [44, 50]$

$10x^3 - 61x^2 + 108x + 45$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-3)(2x-5)(5x-3)$

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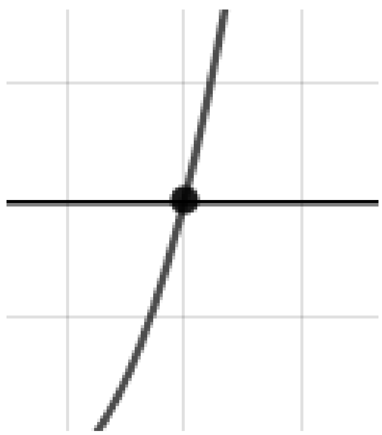
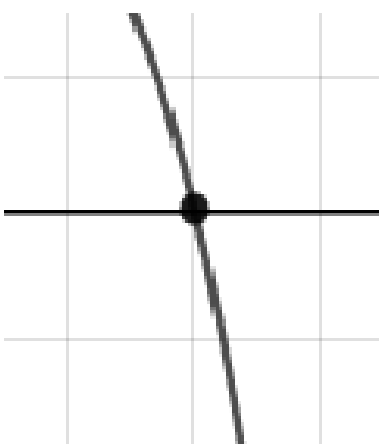
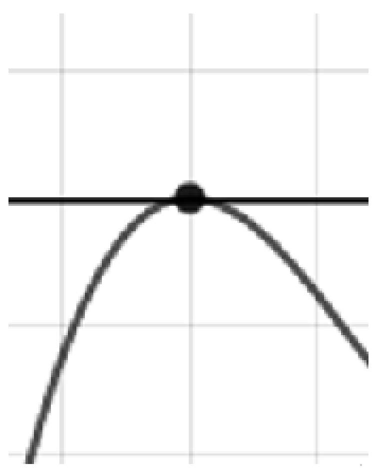
5. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

$$f(x) = 7(x + 7)^{11}(x - 7)^8(x + 5)^3(x - 5)^2$$

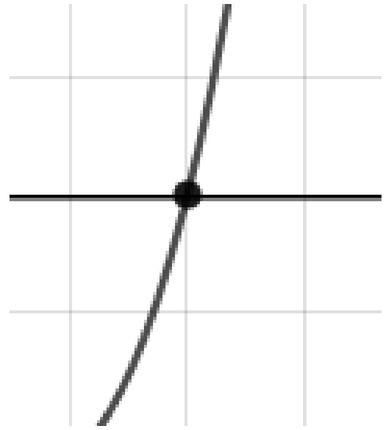
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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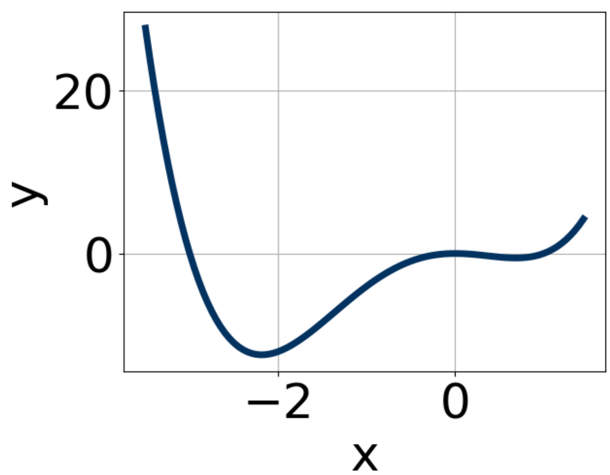
6. Which of the following equations *could* be of the graph presented below?



The solution is $7x^6(x-1)^5(x+3)^9$, which is option B.

A. $4x^8(x-1)^6(x+3)^9$

   The factor $(x-1)$ should have an odd power.

B. $7x^6(x-1)^5(x+3)^9$

   * This is the correct option.

C. $-19x^8(x-1)^7(x+3)^7$

   This corresponds to the leading coefficient being the opposite value than it should be.

D. $-10x^{10}(x-1)^{11}(x+3)^{10}$

   The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $9x^{11}(x-1)^{10}(x+3)^9$

The factor 0 should have an even power and the factor 1 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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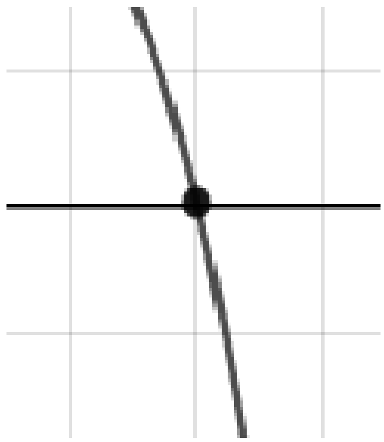
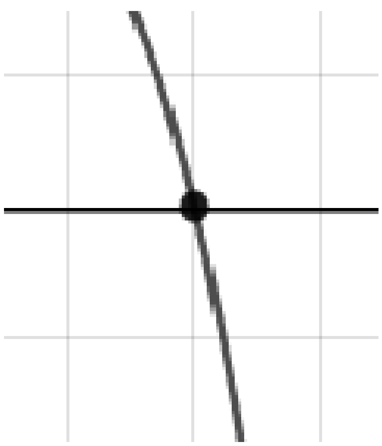
7. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

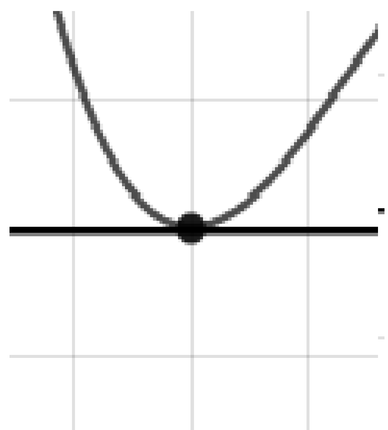
$$f(x) = 9(x+3)^3(x-3)^8(x+2)^4(x-2)^5$$

The solution is the graph below, which is option A.

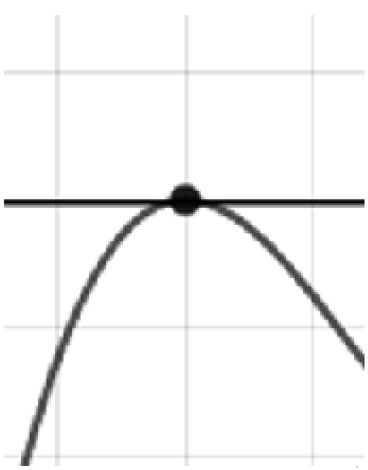




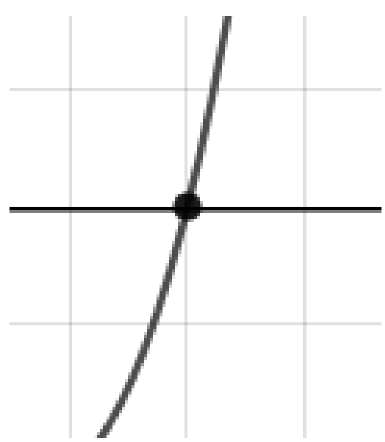
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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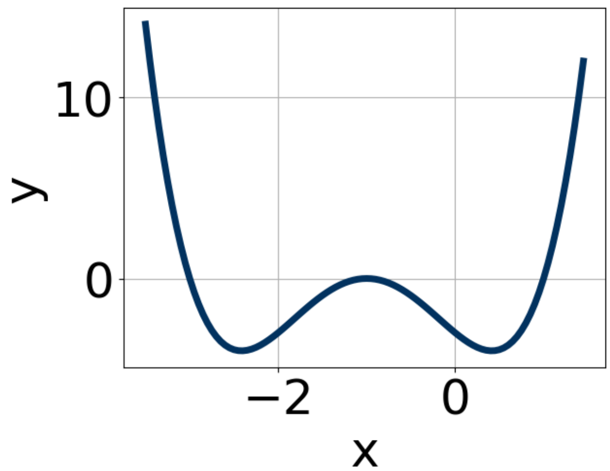
8. Which of the following equations *could* be of the graph presented below?



The solution is $17(x+1)^8(x+3)^{11}(x-1)^{11}$, which is option A.

A. $17(x+1)^8(x+3)^{11}(x-1)^{11}$

   * This is the correct option.

B. $-18(x+1)^{10}(x+3)^5(x-1)^8$

   The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $11(x+1)^8(x+3)^8(x-1)^{11}$

   The factor $(x+3)$ should have an odd power.

D. $-10(x+1)^6(x+3)^{11}(x-1)^9$

   This corresponds to the leading coefficient being the opposite value than it should be.

E. $11(x+1)^5(x+3)^6(x-1)^9$

   The factor -1 should have an even power and the factor -3 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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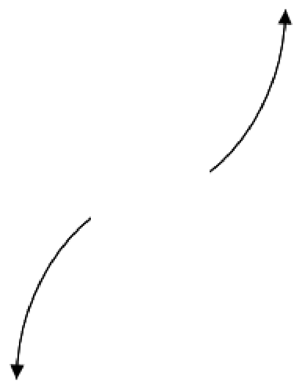
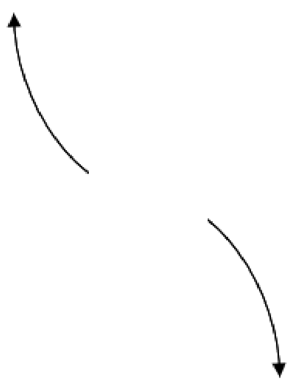
9. Describe the end behavior of the polynomial below.

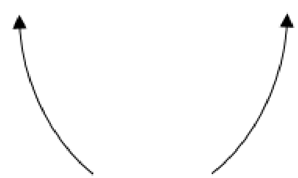
$$f(x) = 7(x+6)^4(x-6)^5(x-8)^5(x+8)^5$$

The solution is the graph below, which is option D.

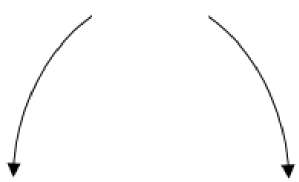




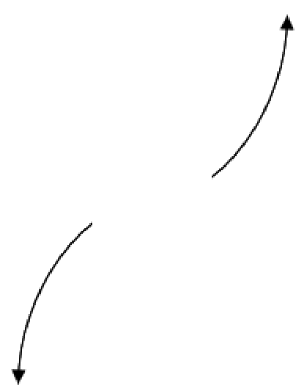
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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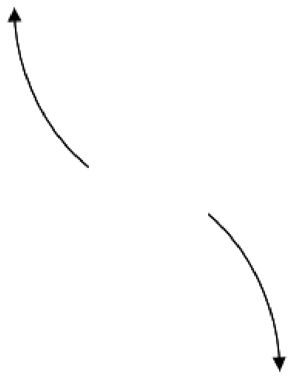
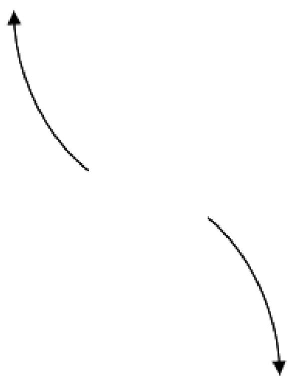
10. Describe the end behavior of the polynomial below.

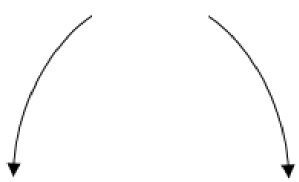
$$f(x) = -5(x+7)^2(x-7)^3(x+3)^2(x-3)^4$$

The solution is the graph below, which is option A.

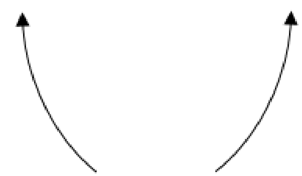




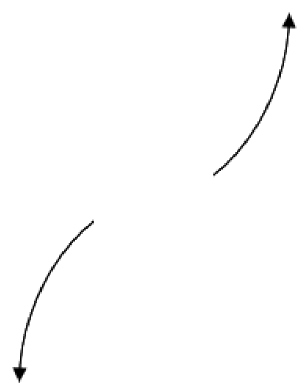
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.