This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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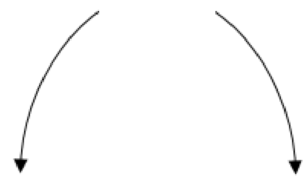
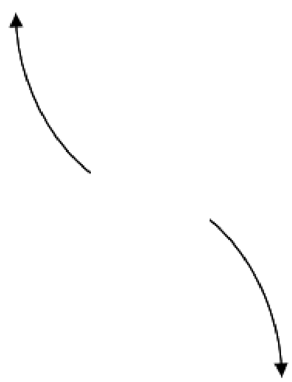
1. Describe the end behavior of the polynomial below.

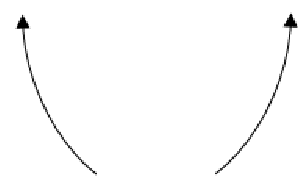
$$f(x) = -8(x-4)^4(x+4)^5(x+9)^3(x-9)^4$$

The solution is the graph below, which is option B.

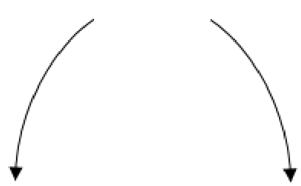




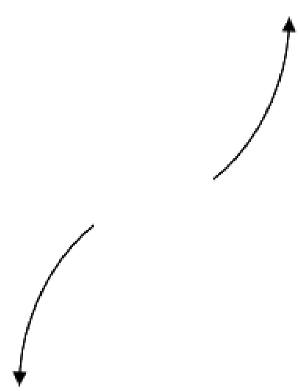
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 4i \text{ and } 3$$

The solution is $x^3 + 3x^2 + 7x - 75$, which is option D.

A. $b \in [-1.9, 1.65], c \in [0.21, 3.58], \text{ and } d \in [-17, -10]$

$x^3 + x^2 + x - 12$, which corresponds to multiplying out $(x + 4)(x - 3)$.

B. $b \in [-1.9, 1.65], c \in [-0.45, 0.03], \text{ and } d \in [-10, -4]$

$x^3 + x^2 - 9$, which corresponds to multiplying out $(x + 3)(x - 3)$.

C. $b \in [-3.44, -2.63], c \in [6.39, 7.85], \text{ and } d \in [73, 79]$

$x^3 - 3x^2 + 7x + 75$, which corresponds to multiplying out $(x - (-3 - 4i))(x - (-3 + 4i))(x + 3)$.

D. $b \in [2.57, 3.6], c \in [6.39, 7.85], \text{ and } d \in [-75, -74]$

* $x^3 + 3x^2 + 7x - 75$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 4i))(x - (-3 + 4i))(x - (3))$.

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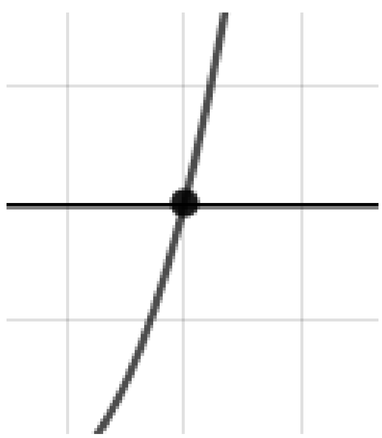
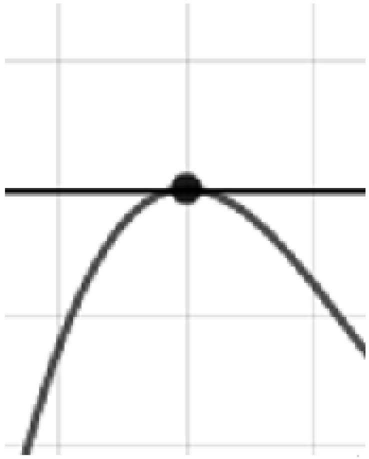
3. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

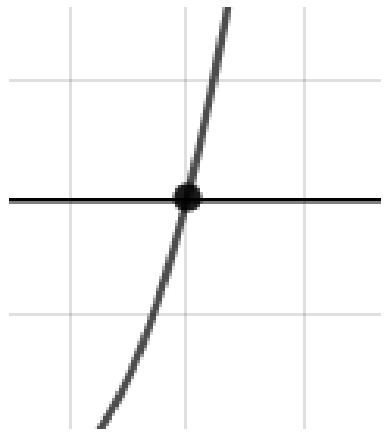
$$f(x) = -9(x + 5)^3(x - 5)^2(x - 8)^6(x + 8)^3$$

The solution is the graph below, which is option D.



B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{5}, 5, \text{ and } \frac{3}{2}$$

The solution is $10x^3 - 79x^2 + 166x - 105$, which is option D.

A. $a \in [6, 13], b \in [-51, -48], c \in [-22, -13]$, and $d \in [97, 108]$

$10x^3 - 51x^2 - 16x + 105$, which corresponds to multiplying out $(5x + 7)(x - 5)(2x - 3)$.

B. $a \in [6, 13], b \in [43, 51], c \in [-30, -22]$, and $d \in [-111, -103]$

$10x^3 + 49x^2 - 26x - 105$, which corresponds to multiplying out $(5x + 7)(x + 5)(2x - 3)$.

C. $a \in [6, 13], b \in [-80, -75], c \in [161, 167]$, and $d \in [97, 108]$

$10x^3 - 79x^2 + 166x + 105$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [6, 13], b \in [-80, -75], c \in [161, 167]$, and $d \in [-111, -103]$

* $10x^3 - 79x^2 + 166x - 105$, which is the correct option.

E. $a \in [6, 13], b \in [70, 80], c \in [161, 167],$ and $d \in [97, 108]$

$10x^3 + 79x^2 + 166x + 105$, which corresponds to multiplying out $(5x + 7)(x + 5)(2x + 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 7)(x - 5)(2x - 3)$

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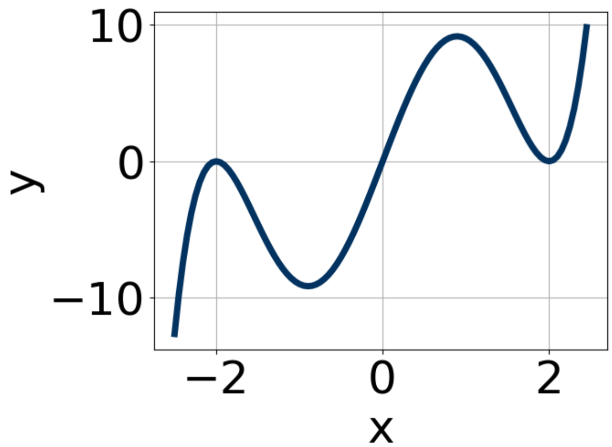
5. Which of the following equations *could* be of the graph presented below?



The solution is $17x^7(x - 2)^6(x + 2)^4$, which is option B.

A. $18x^{10}(x - 2)^6(x + 2)^9$

The factor $(x + 2)$ should have an even power and the factor x should have an odd power.

B. $17x^7(x - 2)^6(x + 2)^4$

* This is the correct option.

C. $-9x^8(x - 2)^8(x + 2)^6$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

D. $16x^7(x - 2)^8(x + 2)^7$

The factor $(x + 2)$ should have an even power.

E. $-14x^9(x - 2)^8(x + 2)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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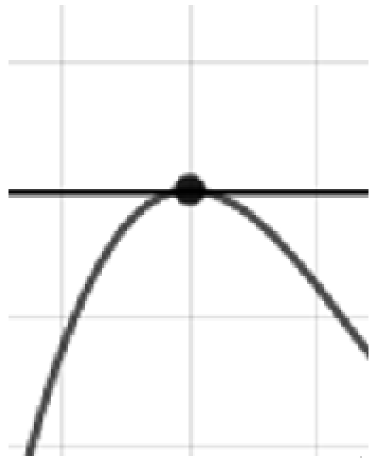
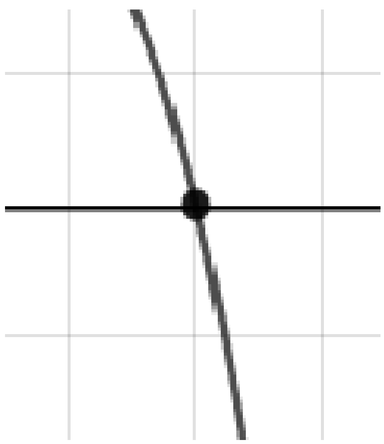
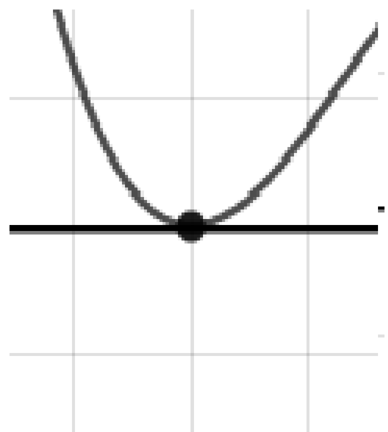
6. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

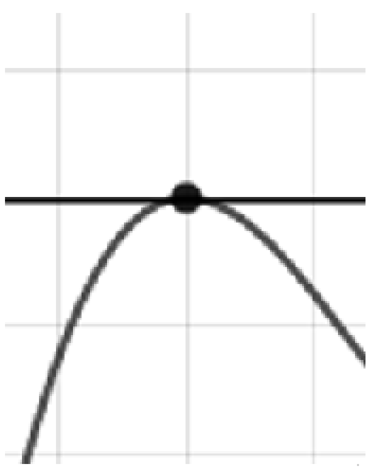
$$f(x) = 7(x - 3)^9(x + 3)^{10}(x - 8)^6(x + 8)^7$$

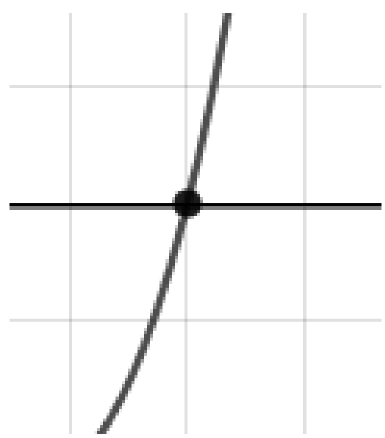
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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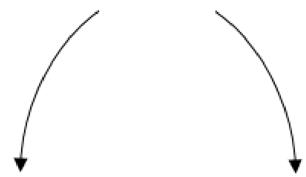
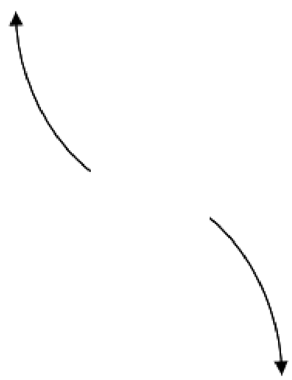
7. Describe the end behavior of the polynomial below.

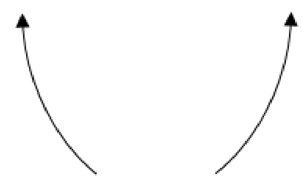
$$f(x) = -9(x+3)^2(x-3)^3(x-8)^3(x+8)^4$$

The solution is the graph below, which is option B.

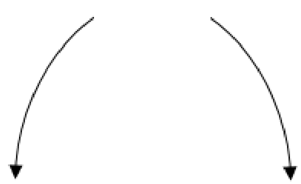




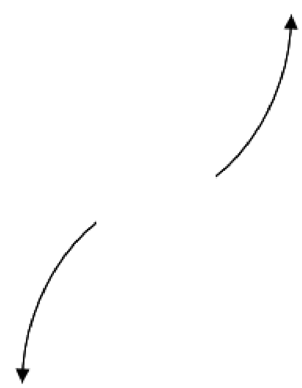
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 5i \text{ and } -1$$

The solution is $x^3 + 5x^2 + 33x + 29$, which is option C.

A. $b \in [-6.2, -3.4], c \in [31, 34.2],$ and $d \in [-29.2, -25]$

$x^3 - 5x^2 + 33x - 29$, which corresponds to multiplying out $(x - (-2 - 5i))(x - (-2 + 5i))(x - 1)$.

B. $b \in [-3.3, 2.4], c \in [5.7, 6.4],$ and $d \in [2.1, 6.8]$

$x^3 + x^2 + 6x + 5$, which corresponds to multiplying out $(x + 5)(x + 1)$.

C. $b \in [1.6, 5.7], c \in [31, 34.2],$ and $d \in [27.5, 30.4]$

* $x^3 + 5x^2 + 33x + 29$, which is the correct option.

D. $b \in [-3.3, 2.4], c \in [-1.2, 4.8],$ and $d \in [0, 3.6]$

$x^3 + x^2 + 3x + 2$, which corresponds to multiplying out $(x + 2)(x + 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 5i))(x - (-2 + 5i))(x - (-1))$.

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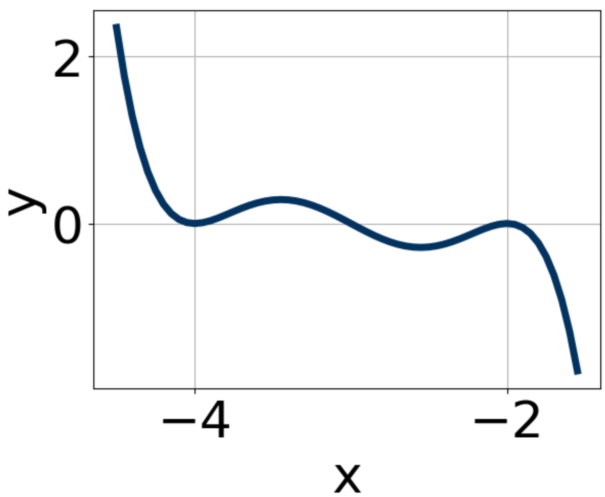
9. Which of the following equations *could* be of the graph presented below?



The solution is $-9(x + 2)^6(x + 4)^8(x + 3)^{11}$, which is option D.

A. $-7(x + 2)^4(x + 4)^7(x + 3)^{10}$

The factor $(x + 4)$ should have an even power and the factor $(x + 3)$ should have an odd power.

B. $2(x + 2)^6(x + 4)^6(x + 3)^6$

The factor $(x + 3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $20(x + 2)^6(x + 4)^6(x + 3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-9(x + 2)^6(x + 4)^8(x + 3)^{11}$

* This is the correct option.

E. $-15(x + 2)^8(x + 4)^5(x + 3)^5$

The factor $(x + 4)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{5}, \frac{-3}{4}, \text{ and } \frac{5}{3}$$

The solution is $60x^3 - 139x^2 + 2x + 105$, which is option A.

A. $a \in [60, 69], b \in [-143, -136], c \in [2, 9], \text{ and } d \in [100, 106]$

   * $60x^3 - 139x^2 + 2x + 105$, which is the correct option.

B. $a \in [60, 69], b \in [-69, -57], c \in [-129, -122], \text{ and } d \in [100, 106]$

   $60x^3 - 61x^2 - 128x + 105$, which corresponds to multiplying out $(5x + 7)(4x - 3)(3x - 5)$.

C. $a \in [60, 69], b \in [-143, -136], c \in [2, 9], \text{ and } d \in [-109, -102]$

   $60x^3 - 139x^2 + 2x - 105$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [60, 69], b \in [24, 32], c \in [-153, -147], \text{ and } d \in [-109, -102]$

   $60x^3 + 29x^2 - 152x - 105$, which corresponds to multiplying out $(5x + 7)(4x + 3)(3x - 5)$.

E. $a \in [60, 69], b \in [138, 144], c \in [2, 9], \text{ and } d \in [-109, -102]$

   $60x^3 + 139x^2 + 2x - 105$, which corresponds to multiplying out $(5x + 7)(4x - 3)(3x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 7)(4x + 3)(3x - 5)$