This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Choose the interval below that f composed with g at x = -1 is in.

$$f(x) = -2x^3 + 2x^2 + x$$
 and  $g(x) = 2x^3 - 2x^2 - 3x - 2$ 

The solution is 69.0, which is option C.

A.  $(f \circ g)(-1) \in [21, 26]$ 

Distractor 1: Corresponds to reversing the composition.

B.  $(f \circ g)(-1) \in [71, 79]$ 

Distractor 2: Corresponds to being slightly off from the solution.

C.  $(f \circ g)(-1) \in [66, 70]$ 

\* This is the correct solution

D.  $(f \circ q)(-1) \in [13, 19]$ 

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** f composed with g at x means f(g(x)). The order matters!

2. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 7x + 6$$
 and  $g(x) = \frac{3}{3x - 14}$ 

The solution is The domain is all Real numbers except x = 4.66666666666667, which is option C.

- A. The domain is all Real numbers less than or equal to x = a, where  $a \in [1.5, 7.5]$
- B. The domain is all Real numbers greater than or equal to x = a, where  $a \in [3, 8]$
- C. The domain is all Real numbers except x = a, where  $a \in [-3.33, 10.67]$
- D. The domain is all Real numbers except x = a and x = b, where  $a \in [-5.8, -4.8]$  and  $b \in [-0.33, 6.67]$
- E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

3. Find the inverse of the function below. Then, evaluate the inverse at x = 7 and choose the interval that  $f^{-1}(7)$  belongs to.

$$f(x) = e^{x+3} + 3$$

The solution is  $f^{-1}(7) = -1.614$ , which is option B.

A. 
$$f^{-1}(7) \in [3.79, 5.03]$$

This solution corresponds to distractor 3.

B. 
$$f^{-1}(7) \in [-1.91, -0.72]$$

This is the solution.

C. 
$$f^{-1}(7) \in [5.14, 5.92]$$

This solution corresponds to distractor 4.

D. 
$$f^{-1}(7) \in [3.79, 5.03]$$

This solution corresponds to distractor 1.

E. 
$$f^{-1}(7) \in [5.14, 5.92]$$

This solution corresponds to distractor 2.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

4. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -11 and choose the interval the  $f^{-1}(-11)$  belongs to.

$$f(x) = \sqrt[3]{3x - 5}$$

The solution is -442.0, which is option D.

A. 
$$f^{-1}(-11) \in [439.9, 442.5]$$

This solution corresponds to distractor 2.

B. 
$$f^{-1}(-11) \in [443.6, 445.4]$$

This solution corresponds to distractor 3.

C. 
$$f^{-1}(-11) \in [-446.5, -444.3]$$

Distractor 1: This corresponds to

D. 
$$f^{-1}(-11) \in [-442.2, -441.3]$$

\* This is the correct solution.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

5. Determine whether the function below is 1-1.

$$f(x) = -30x^2 - 251x - 494$$

The solution is no, which is option A.

A. No, because there is a y-value that goes to 2 different x-values.

\* This is the solution.

B. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

C. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

6. Determine whether the function below is 1-1.

$$f(x) = (6x + 30)^3$$

The solution is yes, which is option C.

A. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

- C. Yes, the function is 1-1.
  - \* This is the solution.
- D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

7. Find the inverse of the function below. Then, evaluate the inverse at x = 8 and choose the interval that  $f^{-1}(8)$  belongs to.

$$f(x) = e^{x-3} - 3$$

The solution is  $f^{-1}(8) = 5.398$ , which is option D.

A. 
$$f^{-1}(8) \in [-1.9, -0.73]$$

This solution corresponds to distractor 4.

B. 
$$f^{-1}(8) \in [-0.66, 0.08]$$

This solution corresponds to distractor 1.

C. 
$$f^{-1}(8) \in [-0.66, 0.08]$$

This solution corresponds to distractor 3.

D. 
$$f^{-1}(8) \in [5.18, 5.8]$$

This is the solution.

E. 
$$f^{-1}(8) \in [-1.9, -0.73]$$

This solution corresponds to distractor 2.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -11 and choose the interval the  $f^{-1}(-11)$  belongs to.

$$f(x) = \sqrt[3]{5x - 3}$$

The solution is -265.6, which is option B.

A. 
$$f^{-1}(-11) \in [-267.24, -266.28]$$

Distractor 1: This corresponds to

B. 
$$f^{-1}(-11) \in [-266.11, -263.86]$$

\* This is the correct solution.

C. 
$$f^{-1}(-11) \in [266.35, 267.67]$$

This solution corresponds to distractor 3.

D. 
$$f^{-1}(-11) \in [265.37, 266.28]$$

This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

9. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{3}{5x - 31}$$
 and  $g(x) = 9x^3 + 6x^2 + 8x + 2$ 

The solution is The domain is all Real numbers except x = 6.2, which is option C.

- A. The domain is all Real numbers less than or equal to x = a, where  $a \in [-6.2, -3.2]$
- B. The domain is all Real numbers greater than or equal to x = a, where  $a \in [4.25, 9.25]$
- C. The domain is all Real numbers except x = a, where  $a \in [4.2, 10.2]$
- D. The domain is all Real numbers except x = a and x = b, where  $a \in [-0.4, 12.6]$  and  $b \in [4.4, 6.4]$
- E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

10. Choose the interval below that f composed with g at x = 1 is in.

$$f(x) = 3x^3 - 2x^2 + x$$
 and  $g(x) = 3x^3 + 2x^2 - 4x + 1$ 

The solution is 18.0, which is option A.

A. 
$$(f \circ g)(1) \in [17, 19.3]$$

<sup>\*</sup> This is the correct solution

B.  $(f \circ g)(1) \in [22.9, 26.7]$ 

Distractor 1: Corresponds to reversing the composition.

C.  $(f \circ g)(1) \in [22.9, 26.7]$ 

Distractor 2: Corresponds to being slightly off from the solution.

D.  $(f \circ g)(1) \in [31.1, 32.9]$ 

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** f composed with g at x means f(g(x)). The order matters!