This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 + 29x^2 - 8x - 12$$

The solution is $[-2, -0.6, 0.6666666666666666]$, which is option B.

A. $z_1 \in [-1.99, -0.93]$, $z_2 \in [1.5, 1.9]$, and $z_3 \in [1.73, 2.12]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-2.22, -1.58]$, $z_2 \in [-1.3, -0.2]$, and $z_3 \in [0.38, 0.79]$

* This is the solution!

C. $z_1 \in [-2.22, -1.58]$, $z_2 \in [-2.4, -0.7]$, and $z_3 \in [1.34, 1.69]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-1.14, -0.57]$, $z_2 \in [0.4, 0.7]$, and $z_3 \in [1.73, 2.12]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-0.38, -0.12]$, $z_2 \in [1.9, 2.8]$, and $z_3 \in [2.34, 3.32]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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2. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 + 33x^2 - 32x - 84}{x + 4}$$

The solution is $9x^2 - 3x - 20 + \frac{-4}{x+4}$, which is option E.

A. $a \in [9, 11]$, $b \in [69, 73]$, $c \in [241, 248]$, and $r \in [885, 895]$.

You divided by the opposite of the factor.

B. $a \in [9, 11]$, $b \in [-13, -9]$, $c \in [27, 29]$, and $r \in [-226, -213]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-40, -31]$, $b \in [-115, -107]$, $c \in [-478, -473]$, and $r \in [-1990, -1981]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

D. $a \in [-40, -31]$, $b \in [172, 178]$, $c \in [-743, -736]$, and $r \in [2871, 2877]$.

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [9, 11]$, $b \in [-3, 4]$, $c \in [-22, -18]$, and $r \in [-7, -1]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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3. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 3x^2 + 4x + 5$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 3}$, which is option C.

A. $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots**!

B. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 3}$

* This is the solution **since we asked for the possible Rational roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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4. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 - 59x^2 - 10x + 24$$

The solution is $[-0.6666666666666666, 0.6, 4]$, which is option A.

A. $z_1 \in [-1.14, 0.07]$, $z_2 \in [0.17, 0.63]$, and $z_3 \in [3.67, 4.06]$

* This is the solution!

B. $z_1 \in [-4.12, -3.78]$, $z_2 \in [-2, -1.4]$, and $z_3 \in [1.48, 1.6]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-4.12, -3.78]$, $z_2 \in [-0.58, -0.1]$, and $z_3 \in [1.82, 2.21]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-1.66, -1.07]$, $z_2 \in [1.16, 1.88]$, and $z_3 \in [3.67, 4.06]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-4.12, -3.78]$, $z_2 \in [-0.9, -0.39]$, and $z_3 \in [-0.04, 0.82]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 + 62x^2 - 36}{x + 4}$$

The solution is $15x^2 + 2x - 8 + \frac{-4}{x+4}$, which is option D.

A. $a \in [13, 23], b \in [-13, -8], c \in [65, 70]$, and $r \in [-361, -360]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [-65, -55], b \in [298, 305], c \in [-1214, -1206]$, and $r \in [4794, 4797]$.

You multipled by the synthetic number rather than bringing the first factor down.

C. $a \in [13, 23], b \in [120, 125], c \in [487, 489]$, and $r \in [1916, 1918]$.

You divided by the opposite of the factor.

D. $a \in [13, 23], b \in [0, 7], c \in [-16, -2]$, and $r \in [-7, 0]$.

* This is the solution!

E. $a \in [-65, -55], b \in [-181, -177], c \in [-712, -710]$, and $r \in [-2886, -2876]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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6. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 + 25x^2 - 20x - 18}{x + 2}$$

The solution is $15x^2 - 5x - 10 + \frac{2}{x+2}$, which is option E.

A. $a \in [-36, -24]$, $b \in [-38, -30]$, $c \in [-91, -88]$, and $r \in [-198, -194]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [-36, -24]$, $b \in [85, 88]$, $c \in [-192, -185]$, and $r \in [361, 363]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [15, 19]$, $b \in [-25, -15]$, $c \in [38, 45]$, and $r \in [-145, -132]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [15, 19]$, $b \in [51, 57]$, $c \in [85, 91]$, and $r \in [159, 165]$.

You divided by the opposite of the factor.

E. $a \in [15, 19]$, $b \in [-9, -2]$, $c \in [-15, -6]$, and $r \in [-2, 5]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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7. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^4 + 7x^3 + 2x^2 + 3x + 2$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$, which is option C.

A. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

D. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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8. Factor the polynomial below completely, knowing that $x - 3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 9x^4 + 27x^3 - 61x^2 - 243x - 180$$

The solution is $[-3, -1.6666666666666667, -1.3333333333333333, 3]$, which is option C.

A. $z_1 \in [-5, -2]$, $z_2 \in [1.28, 1.34]$, $z_3 \in [1.25, 2.2]$, and $z_4 \in [2, 3.1]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5, -2]$, $z_2 \in [0.53, 0.58]$, $z_3 \in [2.1, 3.67]$, and $z_4 \in [3.7, 4.4]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-5, -2]$, $z_2 \in [-1.67, -1.62]$, $z_3 \in [-1.85, -0.77]$, and $z_4 \in [2, 3.1]$

* This is the solution!

D. $z_1 \in [-5, -2]$, $z_2 \in [0.6, 0.65]$, $z_3 \in [-0.08, 1.5]$, and $z_4 \in [2, 3.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-5, -2]$, $z_2 \in [-0.76, -0.75]$, $z_3 \in [-0.84, -0.43]$, and $z_4 \in [2, 3.1]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Factor the polynomial below completely, knowing that $x+3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 - 13x^3 - 155x^2 + 117x + 180$$

The solution is $[-3, -0.8, 1.6666666666666667, 3]$, which is option E.

A. $z_1 \in [-4, 0], \;\; z_2 \in [-1.3, -1.22], z_3 \in [0.58, 0.63], \;$ and $\; z_4 \in [1, 7]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-4, 0], \;\; z_2 \in [-0.62, -0.45], z_3 \in [1.11, 1.26], \;$ and $\; z_4 \in [1, 7]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5, -4], \;\; z_2 \in [-3.13, -2.79], z_3 \in [0.04, 0.31], \;$ and $\; z_4 \in [1, 7]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-4, 0], \;\; z_2 \in [-1.77, -1.51], z_3 \in [0.76, 0.97], \;$ and $\; z_4 \in [1, 7]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-4, 0], \;\; z_2 \in [-1.12, -0.75], z_3 \in [1.44, 1.72], \;$ and $\; z_4 \in [1, 7]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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10. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 - 49x - 65}{x - 4}$$

The solution is $4x^2 + 16x + 15 + \frac{-5}{x-4}$, which is option A.

A. $a \in [3, 9], b \in [15, 17.3], c \in [12, 22], \;$ and $\; r \in [-5, -3]$.

* This is the solution!

B. $a \in [10, 24], b \in [-65.5, -61.7], c \in [206, 213], \;$ and $\; r \in [-897, -884]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [3, 9], b \in [-16.7, -14.7], c \in [12, 22], \;$ and $\; r \in [-126, -123]$.

You divided by the opposite of the factor.

D. $a \in [10, 24], b \in [62.6, 66.2], c \in [206, 213], \;$ and $\; r \in [759, 766]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [3, 9], b \in [11, 13], c \in [-17, -5], \;$ and $\; r \in [-105, -102]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.