

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

If you have a suggestion to make the keys better, please fill out the short survey [here](#).

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

1. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^4 + 6x^3 + 4x^2 + 4x + 5$$

The solution is All combinations of:  $\frac{\pm 1, \pm 5}{\pm 1, \pm 7}$ , which is option C.

A.  $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots!**

B. All combinations of:  $\frac{\pm 1, \pm 7}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of:  $\frac{\pm 1, \pm 5}{\pm 1, \pm 7}$

\* This is the solution **since we asked for the possible Rational roots!**

D.  $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

**General Comment:** We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

2. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where  $z_1 \leq z_2 \leq z_3$ . *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 - 49x^2 + 44x - 12$$

The solution is  $[0.6, 0.6666666666666666, 2]$ , which is option B.

A.  $z_1 \in [-2.1, -1]$ ,  $z_2 \in [-1.14, -0.19]$ , and  $z_3 \in [-1.08, -0.56]$

Distractor 1: Corresponds to negatives of all zeros.

B.  $z_1 \in [-0.1, 0.9]$ ,  $z_2 \in [0.07, 0.97]$ , and  $z_3 \in [1.54, 2.39]$

\* This is the solution!

C.  $z_1 \in [-2.1, -1]$ ,  $z_2 \in [-2.02, -1.81]$ , and  $z_3 \in [-0.39, -0.08]$

Distractor 4: Corresponds to moving factors from one rational to another.

D.  $z_1 \in [-2.1, -1]$ ,  $z_2 \in [-1.73, -1.51]$ , and  $z_3 \in [-1.6, -1.14]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E.  $z_1 \in [1, 2.5]$ ,  $z_2 \in [1.49, 1.7]$ , and  $z_3 \in [1.54, 2.39]$

Distractor 2: Corresponds to inversing rational roots.

**General Comment:** Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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3. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 3x^4 + 7x^3 + 2x^2 + 5x + 2$$

The solution is All combinations of:  $\frac{\pm 1, \pm 2}{\pm 1, \pm 3}$ , which is option C.

A.  $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots!**

B. All combinations of:  $\frac{\pm 1, \pm 3}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of:  $\frac{\pm 1, \pm 2}{\pm 1, \pm 3}$

\* This is the solution **since we asked for the possible Rational roots!**

D.  $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

**General Comment:** We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form  $ax^2 + bx + c$  and remainder  $r$ .

$$\frac{16x^3 + 32x^2 - 4x - 12}{x + 2}$$

The solution is  $16x^2 - 4 + \frac{-4}{x + 2}$ , which is option E.

A.  $a \in [-32, -31]$ ,  $b \in [-32, -26]$ ,  $c \in [-72, -63]$ , and  $r \in [-151, -145]$ .

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B.  $a \in [-32, -31]$ ,  $b \in [95, 101]$ ,  $c \in [-200, -193]$ , and  $r \in [376, 387]$ .

You multiplied by the synthetic number rather than bringing the first factor down.

C.  $a \in [16, 18]$ ,  $b \in [-17, -15]$ ,  $c \in [43, 45]$ , and  $r \in [-147, -143]$ .

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D.  $a \in [16, 18]$ ,  $b \in [60, 66]$ ,  $c \in [124, 131]$ , and  $r \in [235, 239]$ .

You divided by the opposite of the factor.

E.  $a \in [16, 18]$ ,  $b \in [-1, 3]$ ,  $c \in [-15, 2]$ , and  $r \in [-6, -1]$ .

\* This is the solution!

**General Comment:** Be sure to synthetically divide by the zero of the denominator!

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form  $ax^2 + bx + c$  and remainder  $r$ .

$$\frac{4x^3 - 10x^2 - 16x + 38}{x - 2}$$

The solution is  $4x^2 - 2x - 20 + \frac{-2}{x - 2}$ , which is option A.

A.  $a \in [2.9, 4.7]$ ,  $b \in [-5.6, 0.9]$ ,  $c \in [-21.9, -18.1]$ , and  $r \in [-3, 0]$ .

\* This is the solution!

B.  $a \in [2.9, 4.7]$ ,  $b \in [-7, -4.7]$ ,  $c \in [-22.3, -20.4]$ , and  $r \in [12, 26]$ .

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C.  $a \in [4.3, 9.4]$ ,  $b \in [5.5, 7.2]$ ,  $c \in [-4.9, -1.2]$ , and  $r \in [25, 34]$ .

You multiplied by the synthetic number rather than bringing the first factor down.

D.  $a \in [4.3, 9.4]$ ,  $b \in [-29, -25.9]$ ,  $c \in [35.5, 41]$ , and  $r \in [-35, -32]$ .

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E.  $a \in [2.9, 4.7]$ ,  $b \in [-18.6, -17.5]$ ,  $c \in [18.1, 23.6]$ , and  $r \in [-3, 0]$ .

You divided by the opposite of the factor.

**General Comment:** Be sure to synthetically divide by the zero of the denominator!

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6. Perform the division below. Then, find the intervals that correspond to the quotient in the form  $ax^2 + bx + c$  and remainder  $r$ .

$$\frac{10x^3 - 70x - 63}{x - 3}$$

The solution is  $10x^2 + 30x + 20 + \frac{-3}{x - 3}$ , which is option D.

A.  $a \in [9, 11]$ ,  $b \in [12, 24]$ ,  $c \in [-32, -28]$ , and  $r \in [-125, -117]$ .

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

B.  $a \in [9, 11]$ ,  $b \in [-31, -28]$ ,  $c \in [18, 24]$ , and  $r \in [-125, -117]$ .

You divided by the opposite of the factor.

C.  $a \in [30, 35]$ ,  $b \in [88, 93]$ ,  $c \in [198, 202]$ , and  $r \in [537, 540]$ .

You multiplied by the synthetic number rather than bringing the first factor down.

D.  $a \in [9, 11], b \in [28, 32], c \in [18, 24]$ , and  $r \in [-7, -2]$ .

\* This is the solution!

E.  $a \in [30, 35], b \in [-93, -88], c \in [198, 202]$ , and  $r \in [-665, -660]$ .

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

**General Comment:** Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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7. Factor the polynomial below completely, knowing that  $x - 2$  is a factor. Then, choose the intervals the zeros of the polynomial belong to, where  $z_1 \leq z_2 \leq z_3 \leq z_4$ . *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 + 25x^3 - 114x^2 - 48x + 160$$

The solution is  $[-4, -1.3333333333333333, 1.25, 2]$ , which is option D.

A.  $z_1 \in [-3.5, -1.7], z_2 \in [-0.43, -0.4], z_3 \in [3.99, 4.05]$ , and  $z_4 \in [4, 9]$

Distractor 4: Corresponds to moving factors from one rational to another.

B.  $z_1 \in [-5.9, -2.3], z_2 \in [-0.78, -0.74], z_3 \in [0.79, 0.85]$ , and  $z_4 \in [1, 3]$

Distractor 2: Corresponds to inverting rational roots.

C.  $z_1 \in [-3.5, -1.7], z_2 \in [-0.82, -0.79], z_3 \in [0.72, 0.76]$ , and  $z_4 \in [4, 9]$

Distractor 3: Corresponds to negatives of all zeros AND inverting rational roots.

D.  $z_1 \in [-5.9, -2.3], z_2 \in [-1.38, -1.3], z_3 \in [1.23, 1.25]$ , and  $z_4 \in [1, 3]$

\* This is the solution!

E.  $z_1 \in [-3.5, -1.7], z_2 \in [-1.32, -1.23], z_3 \in [1.33, 1.38]$ , and  $z_4 \in [4, 9]$

Distractor 1: Corresponds to negatives of all zeros.

**General Comment:** Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where  $z_1 \leq z_2 \leq z_3$ . *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 + 41x^2 + 45x - 50$$

The solution is  $[-5, -2.5, 0.6666666666666666]$ , which is option C.

A.  $z_1 \in [-5.23, -4.91], z_2 \in [-0.6, -0.1]$ , and  $z_3 \in [1.2, 1.6]$

Distractor 2: Corresponds to inverting rational roots.

B.  $z_1 \in [-0.68, -0.37], z_2 \in [1.92, 2.62]$ , and  $z_3 \in [3.9, 6.2]$

Distractor 1: Corresponds to negatives of all zeros.

C.  $z_1 \in [-5.23, -4.91], z_2 \in [-3.38, -2.35]$ , and  $z_3 \in [0.4, 1.4]$

\* This is the solution!

D.  $z_1 \in [-1.59, -1.34], z_2 \in [0.25, 0.49]$ , and  $z_3 \in [3.9, 6.2]$

Distractor 3: Corresponds to negatives of all zeros AND inverting rational roots.

E.  $z_1 \in [-0.49, -0.12]$ ,  $z_2 \in [4.57, 5.85]$ , and  $z_3 \in [3.9, 6.2]$

Distractor 4: Corresponds to moving factors from one rational to another.

**General Comment:** Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Perform the division below. Then, find the intervals that correspond to the quotient in the form  $ax^2 + bx + c$  and remainder  $r$ .

$$\frac{20x^3 - 84x^2 + 62}{x - 4}$$

The solution is  $20x^2 - 4x - 16 + \frac{-2}{x - 4}$ , which is option C.

A.  $a \in [79, 83]$ ,  $b \in [-406, -397]$ ,  $c \in [1615, 1618]$ , and  $r \in [-6403, -6399]$ .

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B.  $a \in [79, 83]$ ,  $b \in [225, 245]$ ,  $c \in [942, 956]$ , and  $r \in [3838, 3846]$ .

You multiplied by the synthetic number rather than bringing the first factor down.

C.  $a \in [20, 27]$ ,  $b \in [-11, 3]$ ,  $c \in [-20, -12]$ , and  $r \in [-2, 1]$ .

\* This is the solution!

D.  $a \in [20, 27]$ ,  $b \in [-28, -17]$ ,  $c \in [-75, -71]$ , and  $r \in [-155, -152]$ .

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E.  $a \in [20, 27]$ ,  $b \in [-167, -161]$ ,  $c \in [651, 660]$ , and  $r \in [-2562, -2555]$ .

You divided by the opposite of the factor.

**General Comment:** Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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10. Factor the polynomial below completely, knowing that  $x - 3$  is a factor. Then, choose the intervals the zeros of the polynomial belong to, where  $z_1 \leq z_2 \leq z_3 \leq z_4$ . *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 - 89x^3 + 238x^2 - 123x - 180$$

The solution is  $[-0.6, 2.5, 3, 4]$ , which is option A.

A.  $z_1 \in [-0.6, 0.4]$ ,  $z_2 \in [1.1, 3.5]$ ,  $z_3 \in [2.77, 3.1]$ , and  $z_4 \in [3.12, 4.1]$

\* This is the solution!

B.  $z_1 \in [-1.67, -0.67]$ ,  $z_2 \in [-0.8, 0.9]$ ,  $z_3 \in [2.77, 3.1]$ , and  $z_4 \in [3.12, 4.1]$

Distractor 2: Corresponds to inverting rational roots.

C.  $z_1 \in [-6, -2]$ ,  $z_2 \in [-4.3, -1.2]$ ,  $z_3 \in [-0.41, -0.36]$ , and  $z_4 \in [1.43, 2.5]$

Distractor 3: Corresponds to negatives of all zeros AND inverting rational roots.

D.  $z_1 \in [-6, -2]$ ,  $z_2 \in [-4.3, -1.2]$ ,  $z_3 \in [-3.11, -2.35]$ , and  $z_4 \in [-0.23, 1.2]$

Distractor 1: Corresponds to negatives of all zeros.

E.  $z_1 \in [-6, -2]$ ,  $z_2 \in [-4.3, -1.2]$ ,  $z_3 \in [-0.85, -0.44]$ , and  $z_4 \in [2.73, 3.26]$

Distractor 4: Corresponds to moving factors from one rational to another.

**General Comment:** Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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