**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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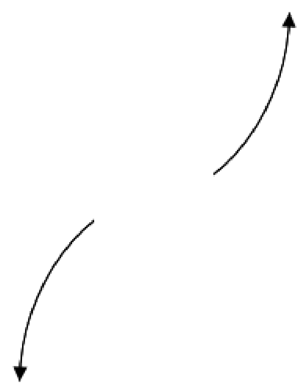
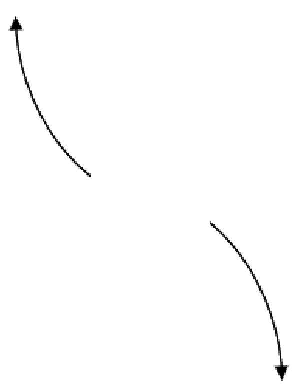
1. Describe the end behavior of the polynomial below.

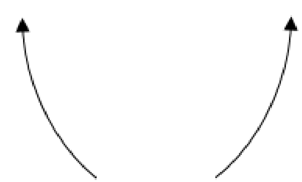
$$f(x) = 7(x+2)^2(x-2)^3(x+8)^2(x-8)^4$$

The solution is the graph below, which is option D.

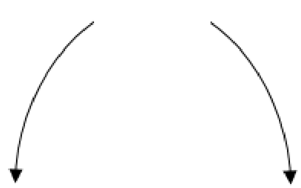




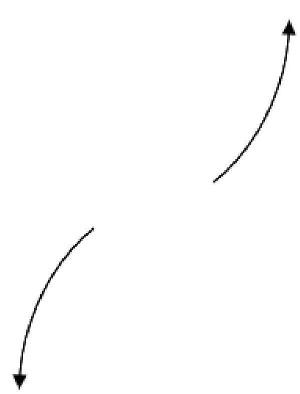
A.



C.



B.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{5}, \frac{-4}{5}, \text{ and } 5$$

The solution is $25x^3 - 70x^2 - 247x - 140$, which is option A.

A. $a \in [19, 30], b \in [-70, -63], c \in [-249, -243], \text{ and } d \in [-143, -137]$

* $25x^3 - 70x^2 - 247x - 140$, which is the correct option.

B. $a \in [19, 30], b \in [-187, -178], c \in [302, 305], \text{ and } d \in [-143, -137]$

$25x^3 - 180x^2 + 303x - 140$, which corresponds to multiplying out $(5x + 5)(5x + 5)(x - 1)$.

C. $a \in [19, 30], b \in [62, 76], c \in [-249, -243], \text{ and } d \in [140, 144]$

$25x^3 + 70x^2 - 247x + 140$, which corresponds to multiplying out $(5x - 7)(5x - 4)(x + 5)$.

D. $a \in [19, 30], b \in [-143, -137], c \in [45, 48], \text{ and } d \in [140, 144]$

$25x^3 - 140x^2 + 47x + 140$, which corresponds to multiplying out $(5x + 5)(5x - 5)(x - 1)$.

E. $a \in [19, 30], b \in [-70, -63], c \in [-249, -243], \text{ and } d \in [140, 144]$

$25x^3 - 70x^2 - 247x + 140$, which corresponds to multiplying everything correctly except the constant term.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(5x + 7)(5x + 4)(x - 5)$

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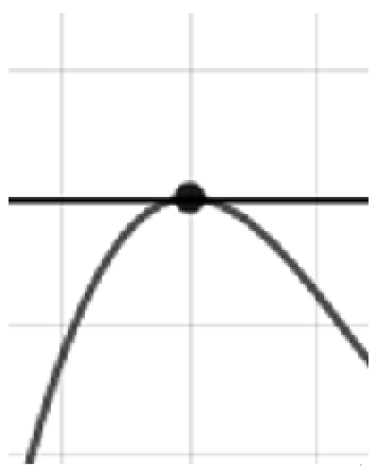
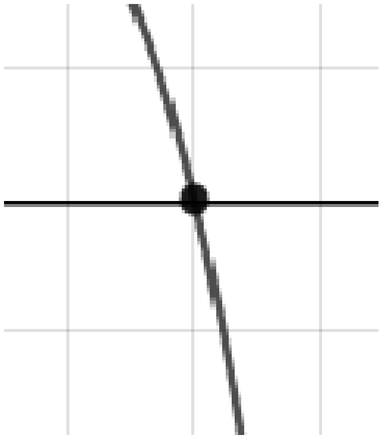
3. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

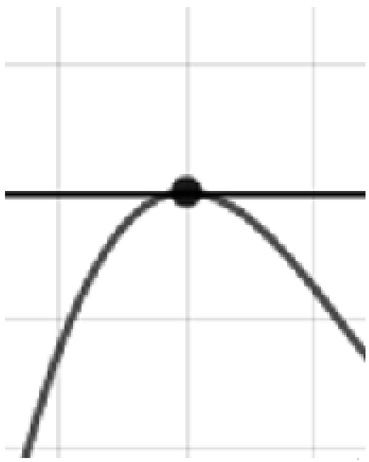
$$f(x) = 4(x - 6)^5(x + 6)^{10}(x + 3)^7(x - 3)^9$$

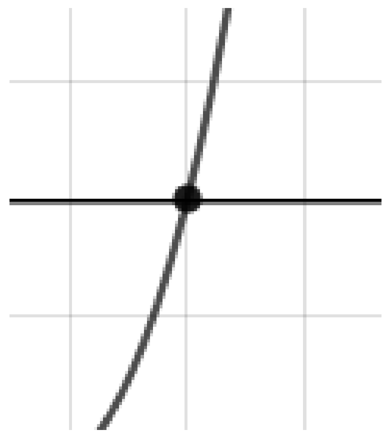
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 5i \text{ and } 2$$

The solution is $x^3 - 12x^2 + 70x - 100$, which is option C.

A. $b \in [1, 7], c \in [0, 9], \text{ and } d \in [-11, -9]$

$x^3 + x^2 + 3x - 10$, which corresponds to multiplying out $(x + 5)(x - 2)$.

B. $b \in [1, 7], c \in [-14, -5], \text{ and } d \in [8, 12]$

$x^3 + x^2 - 7x + 10$, which corresponds to multiplying out $(x - 5)(x - 2)$.

C. $b \in [-20, -8], c \in [63, 71], \text{ and } d \in [-104, -92]$

* $x^3 - 12x^2 + 70x - 100$, which is the correct option.

D. $b \in [11, 17], c \in [63, 71], \text{ and } d \in [98, 106]$

$x^3 + 12x^2 + 70x + 100$, which corresponds to multiplying out $(x - (5 - 5i))(x - (5 + 5i))(x + 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(5-5i))(x-(5+5i))(x-(2))$.

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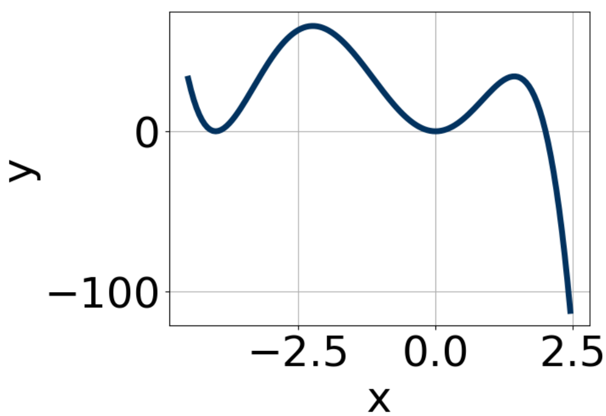
5. Which of the following equations *could* be of the graph presented below?




The solution is $-12x^4(x+4)^8(x-2)^7$, which is option A.

A. $-12x^4(x+4)^8(x-2)^7$

* This is the correct option.

B. $-11x^9(x+4)^{10}(x-2)^9$

The factor $x$ should have an even power.

C. $4x^4(x+4)^8(x-2)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $11x^8(x+4)^4(x-2)^8$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-12x^9(x+4)^8(x-2)^4$

The factor $x$ should have an even power and the factor $(x-2)$ should have an odd power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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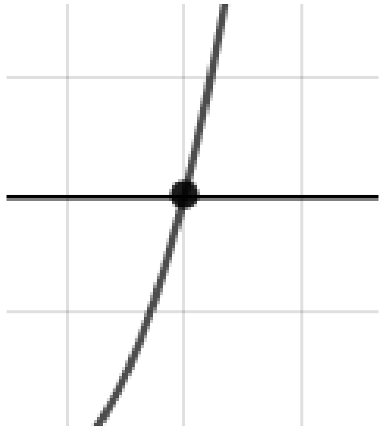
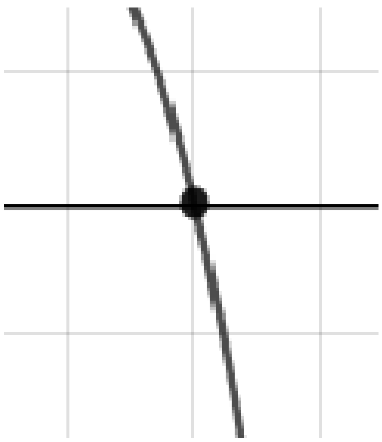
6. Describe the zero behavior of the zero $x=4$ of the polynomial below.

$$f(x) = 3(x-3)^8(x+3)^4(x+4)^8(x-4)^5$$

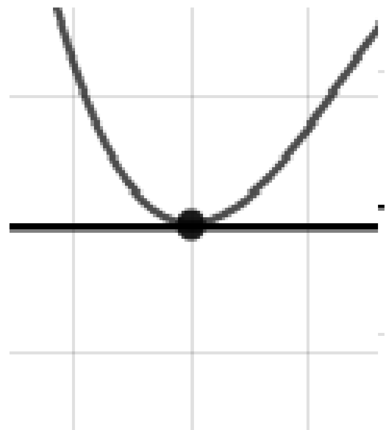
The solution is the graph below, which is option D.



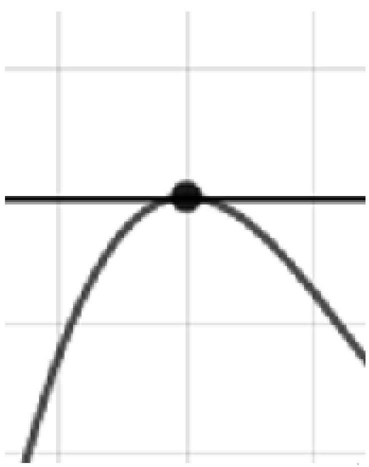
A.



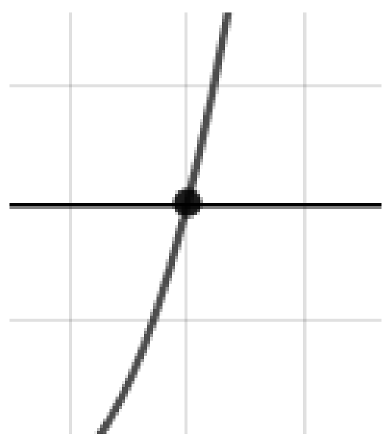
C.



B.



D.



E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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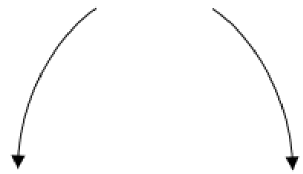
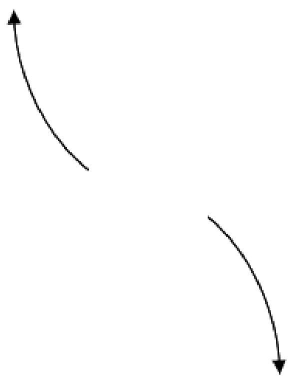
7. Describe the end behavior of the polynomial below.

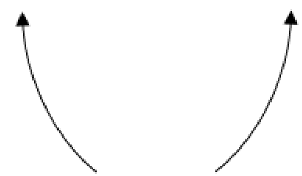
$$f(x) = -8(x+9)^2(x-9)^7(x+5)^5(x-5)^6$$

The solution is the graph below, which is option B.

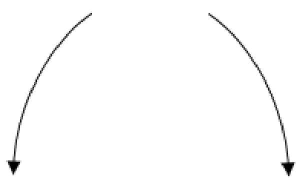




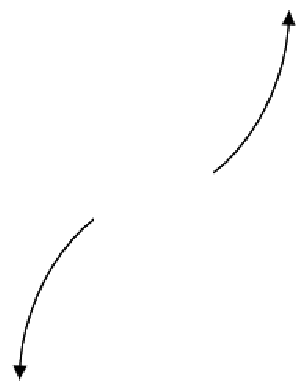
A.



C.



B.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 3i \text{ and } 4$$

The solution is $x^3 + 6x^2 - 6x - 136$, which is option C.

A. $b \in [0, 4], c \in [-2.1, 2],$ and $d \in [-25, -13]$

$x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

B. $b \in [0, 4], c \in [-7.1, -6.5],$ and $d \in [10, 14]$

$x^3 + x^2 - 7x + 12$, which corresponds to multiplying out $(x - 3)(x - 4)$.

C. $b \in [4, 15], c \in [-6.9, -5.4],$ and $d \in [-140, -129]$

* $x^3 + 6x^2 - 6x - 136$, which is the correct option.

D. $b \in [-11, -4], c \in [-6.9, -5.4],$ and $d \in [136, 144]$

$x^3 - 6x^2 - 6x + 136$, which corresponds to multiplying out $(x - (-5 + 3i))(x - (-5 - 3i))(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 3i))(x - (-5 - 3i))(x - (4))$.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{3}, \frac{-6}{5}, \text{ and } \frac{4}{3}$$

The solution is $45x^3 + 9x^2 - 74x - 24$, which is option B.

A. $a \in [45, 55], b \in [9, 16], c \in [-77.5, -73.9],$ and $d \in [21, 28]$

$45x^3 + 9x^2 - 74x + 24$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [45, 55], b \in [9, 16], c \in [-77.5, -73.9],$ and $d \in [-24, -21]$

* $45x^3 + 9x^2 - 74x - 24$, which is the correct option.

C. $a \in [45, 55], b \in [-13, -5], c \in [-77.5, -73.9],$ and $d \in [21, 28]$

$45x^3 - 9x^2 - 74x + 24$, which corresponds to multiplying out $(3x - 1)(5x - 6)(3x + 4)$.

D. $a \in [45, 55], b \in [-24, -13], c \in [-73.1, -67.6],$ and $d \in [21, 28]$

$45x^3 - 21x^2 - 70x + 24$, which corresponds to multiplying out $(3x + 3)(5x - 5)(3x - 3)$.

E. $a \in [45, 55], b \in [-130, -126], c \in [109, 113],$ and $d \in [-24, -21]$

$45x^3 - 129x^2 + 110x - 24$, which corresponds to multiplying out $(3x + 3)(5x + 5)(3x - 3)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(3x + 1)(5x + 6)(3x - 4)$

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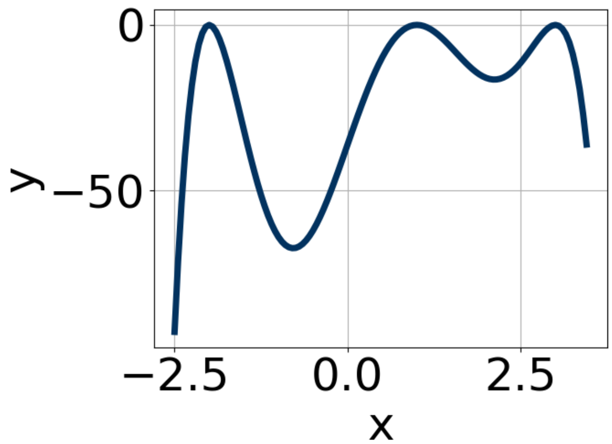
10. Which of the following equations *could* be of the graph presented below?




The solution is $-11(x-1)^6(x+2)^{10}(x-3)^4$, which is option B.

A. $13(x-1)^4(x+2)^4(x-3)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-11(x-1)^6(x+2)^{10}(x-3)^4$

* This is the correct option.

C. $-13(x-1)^{10}(x+2)^4(x-3)^7$

The factor $(x-3)$ should have an even power.

D. $9(x-1)^{10}(x+2)^{10}(x-3)^9$

The factor $(x-3)$ should have an even power and the leading coefficient should be the opposite sign.

E. $-13(x-1)^6(x+2)^5(x-3)^{11}$

The factors $(x+2)$ and $(x-3)$ should both have even powers.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).