This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$-7, \frac{7}{2}, \text{ and } \frac{1}{2}$$

The solution is $4x^3+12x^2-105x+49$, which is option D.

A. $a \in [-3, 6], b \in [-47, -42], c \in [117, 124], \text{ and } d \in [-51, -39]$

$4x^3-44x^2+119x-49$, which corresponds to multiplying out $(x-7)(2x-7)(2x-1)$.

B. $a \in [-3, 6], b \in [-18, -15], c \in [-100, -89], \text{ and } d \in [48, 54]$

$4x^3-16x^2-91x+49$, which corresponds to multiplying out $(x-7)(2x+7)(2x-1)$.

C. $a \in [-3, 6], b \in [-15, -8], c \in [-109, -101], \text{ and } d \in [-51, -39]$

$4x^3-12x^2-105x-49$, which corresponds to multiplying out $(x-7)(2x+7)(2x+1)$.

D. $a \in [-3, 6], b \in [12, 17], c \in [-109, -101], \text{ and } d \in [48, 54]$

* $4x^3+12x^2-105x+49$, which is the correct option.

E. $a \in [-3, 6], b \in [12, 17], c \in [-109, -101], \text{ and } d \in [-51, -39]$

$4x^3+12x^2-105x-49$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+7)(2x-7)(2x-1)$

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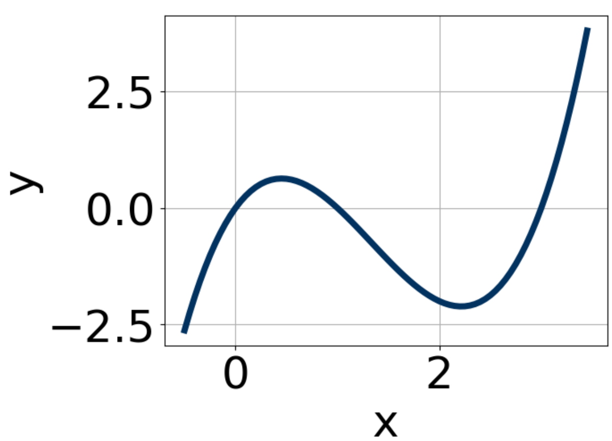
2. Which of the following equations *could* be of the graph presented below?



The solution is $13x^7(x-1)^7(x-3)^{11}$, which is option A.

A. $13x^7(x-1)^7(x-3)^{11}$

  * This is the correct option.

B. $-18x^9(x-1)^{10}(x-3)^9$

  The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-15x^5(x-1)^{11}(x-3)^{11}$

  This corresponds to the leading coefficient being the opposite value than it should be.

D. $8x^9(x-1)^8(x-3)^5$

  The factor 1 should have been an odd power.

E. $3x^7(x-1)^4(x-3)^{10}$

  The factors 1 and 3 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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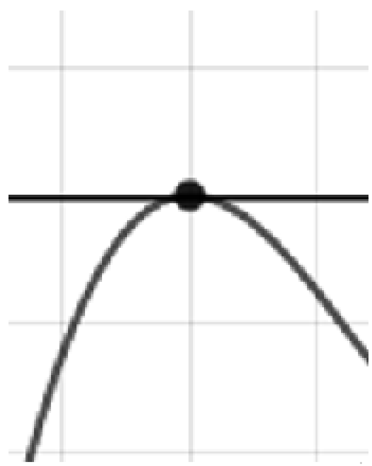
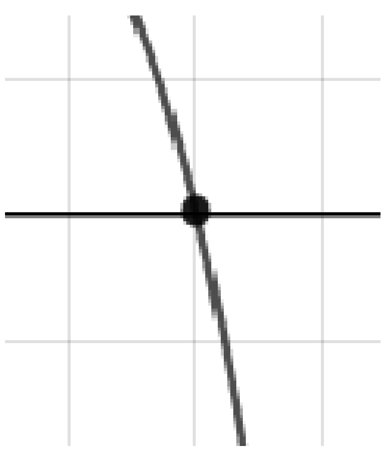
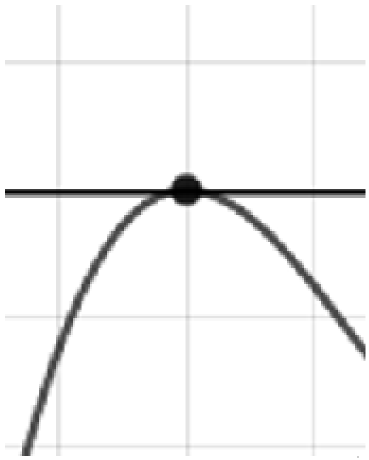
3. Describe the zero behavior of the zero $x = -9$ of the polynomial below.

$$f(x) = 8(x-9)^5(x+9)^8(x-8)^6(x+8)^8$$

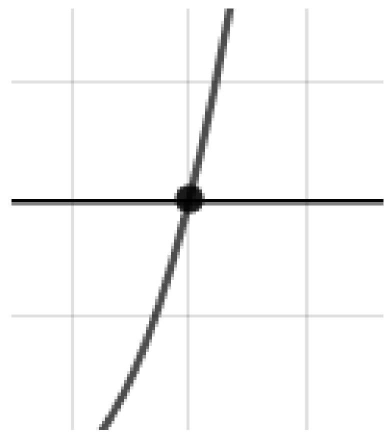
The solution is the graph below, which is option B.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 3i \text{ and } -3$$

The solution is $x^3 + 7x^2 + 25x + 39$, which is option D.

A. $b \in [1, 4], c \in [5.44, 8.1], \text{ and } d \in [8.6, 9.7]$

$x^3 + x^2 + 6x + 9$, which corresponds to multiplying out $(x + 3)(x + 3)$.

B. $b \in [-13, -5], c \in [23.8, 26.58], \text{ and } d \in [-41.4, -37.8]$

$x^3 - 7x^2 + 25x - 39$, which corresponds to multiplying out $(x - (-2 - 3i))(x - (-2 + 3i))(x - 3)$.

C. $b \in [1, 4], c \in [3.4, 5.01], \text{ and } d \in [5.2, 6.9]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 2)(x + 3)$.

D. $b \in [2, 15], c \in [23.8, 26.58], \text{ and } d \in [37.2, 41.1]$

* $x^3 + 7x^2 + 25x + 39$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 3i))(x - (-2 + 3i))(x - (-3))$.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-5, -2, \text{ and } 3$$

The solution is $x^3 + 4x^2 - 11x - 30$, which is option E.

A. $a \in [-5, 6], b \in [-4.3, -3.7], c \in [-13, -6], \text{ and } d \in [25, 37]$

$x^3 - 4x^2 - 11x + 30$, which corresponds to multiplying out $(x - 5)(x - 2)(x + 3)$.

B. $a \in [-5, 6], b \in [1.6, 4.9], c \in [-13, -6], \text{ and } d \in [25, 37]$

$x^3 + 4x^2 - 11x + 30$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [-5, 6], b \in [-6.1, -5.4], c \in [-9, 1], \text{ and } d \in [25, 37]$

$x^3 - 6x^2 - x + 30$, which corresponds to multiplying out $(x - 5)(x + 2)(x - 3)$.

D. $a \in [-5, 6], b \in [-10.9, -9.5], c \in [29, 36], \text{ and } d \in [-30, -23]$

$x^3 - 10x^2 + 31x - 30$, which corresponds to multiplying out $(x - 5)(x - 2)(x - 3)$.

E. $a \in [-5, 6], b \in [1.6, 4.9], c \in [-13, -6], \text{ and } d \in [-30, -23]$

* $x^3 + 4x^2 - 11x - 30$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+5)(x+2)(x-3)$

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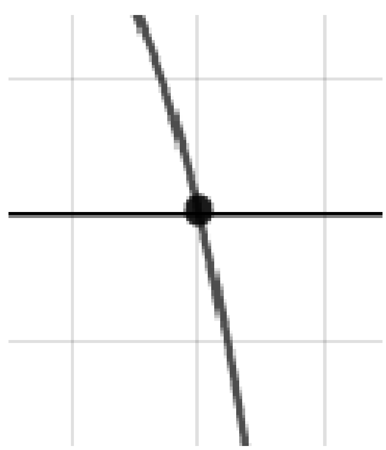
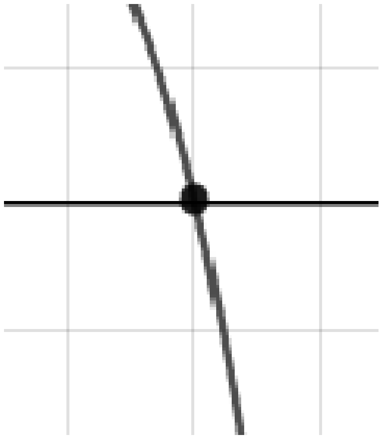
6. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

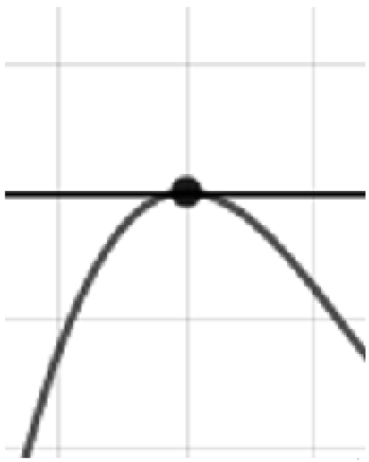
$$f(x) = 8(x - 4)^5(x + 4)^8(x - 8)^3(x + 8)^5$$

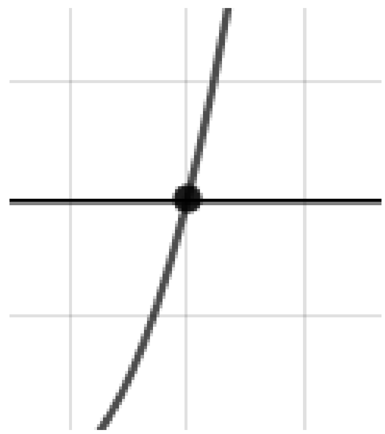
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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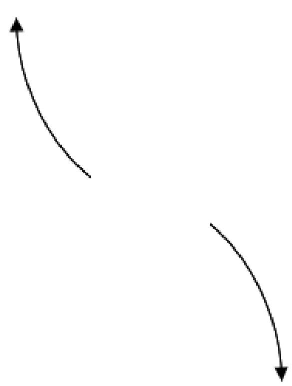
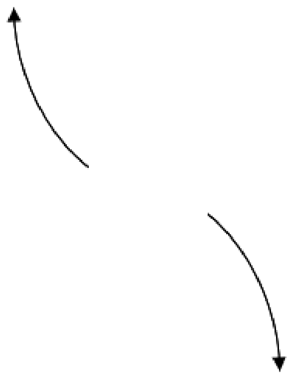
7. Describe the end behavior of the polynomial below.

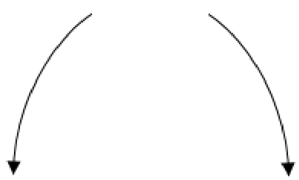
$$f(x) = -7(x+2)^3(x-2)^6(x-3)^5(x+3)^7$$

The solution is the graph below, which is option A.





A.



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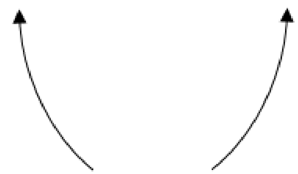
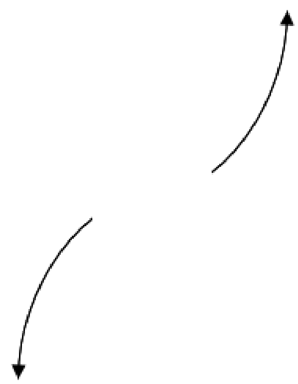


C.



D.

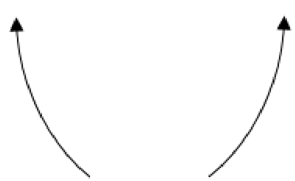
E. None of the above.

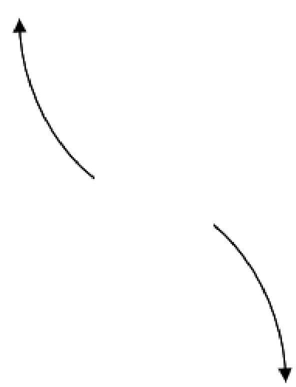
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

8. Describe the end behavior of the polynomial below.

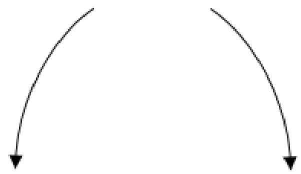
$$f(x) = 9(x-6)^3(x+6)^8(x-7)^3(x+7)^4$$

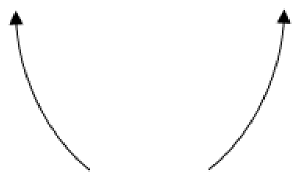
The solution is the graph below, which is option C.

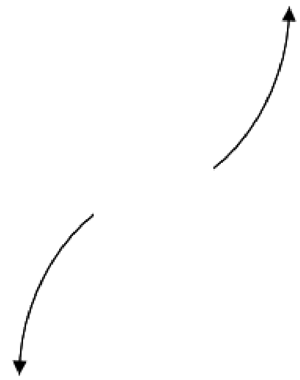




A.

B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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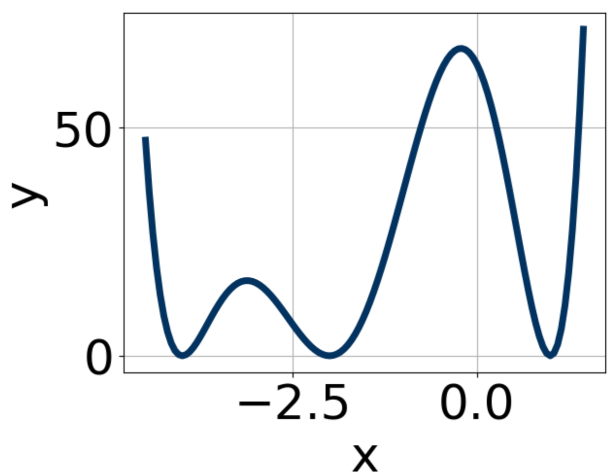
9. Which of the following equations *could* be of the graph presented below?



The solution is $20(x+4)^{10}(x+2)^{8}(x-1)^{8}$, which is option E.

A. $-8(x+4)^{10}(x+2)^{10}(x-1)^{8}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-15(x+4)^{4}(x+2)^{10}(x-1)^{7}$

The factor $(x-1)$ should have an even power and the leading coefficient should be the opposite sign.

C. $16(x+4)^{10}(x+2)^{8}(x-1)^{9}$

The factor $(x-1)$ should have an even power.

D. $3(x+4)^{4}(x+2)^{5}(x-1)^{7}$

The factors $(x+2)$ and $(x-1)$ should both have even powers.

E. $20(x+4)^{10}(x+2)^8(x-1)^8$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 - 4i \text{ and } -3$$

The solution is $x^3 - 1x^2 + 8x + 60$, which is option B.

A. $b \in [-0.9, 1.3], c \in [7.04, 8.22], \text{ and } d \in [-63, -58]$

$x^3 + x^2 + 8x - 60$, which corresponds to multiplying out $(x-(2-4i))(x-(2+4i))(x-3)$.

B. $b \in [-1.3, 0.3], c \in [7.04, 8.22], \text{ and } d \in [58, 61]$

* $x^3 - 1x^2 + 8x + 60$, which is the correct option.

C. $b \in [-0.9, 1.3], c \in [0.36, 1.21], \text{ and } d \in [-9, -1]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x-2)(x+3)$.

D. $b \in [-0.9, 1.3], c \in [6.7, 7.73], \text{ and } d \in [9, 17]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x+4)(x+3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(2-4i))(x-(2+4i))(x-(-3))$.
