**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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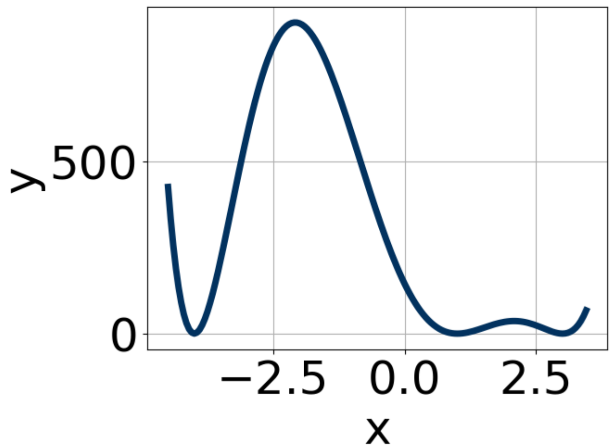
1. Which of the following equations *could* be of the graph presented below?




The solution is $7(x+4)^4(x-1)^6(x-3)^6$, which is option B.

A. $-5(x+4)^4(x-1)^4(x-3)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $7(x+4)^4(x-1)^6(x-3)^6$

* This is the correct option.

C. $2(x+4)^6(x-1)^{10}(x-3)^7$

The factor $(x-3)$ should have an even power.

D. $6(x+4)^6(x-1)^5(x-3)^{11}$

The factors $(x-1)$ and $(x-3)$ should both have even powers.

E. $-17(x+4)^{10}(x-1)^{10}(x-3)^{11}$

The factor $(x-3)$ should have an even power and the leading coefficient should be the opposite sign.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3+bx^2+cx+d$.

$$5+4i \text{ and } 2$$

The solution is $x^3-12x^2+61x-82$, which is option C.

A. $b \in [1, 8], c \in [-7.98, -6.69]$, and $d \in [9.7, 10.6]$

$x^3 + x^2 - 7x + 10$, which corresponds to multiplying out $(x - 5)(x - 2)$.

B. $b \in [1, 8], c \in [-6.23, -5.01]$, and $d \in [6.5, 9.7]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 4)(x - 2)$.

C. $b \in [-15, -10], c \in [60.98, 61.96]$, and $d \in [-84.3, -79.9]$

* $x^3 - 12x^2 + 61x - 82$, which is the correct option.

D. $b \in [8, 15], c \in [60.98, 61.96]$, and $d \in [80.1, 83.9]$

$x^3 + 12x^2 + 61x + 82$, which corresponds to multiplying out $(x - (5 + 4i))(x - (5 - 4i))(x + 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 4i))(x - (5 - 4i))(x - (2))$.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{3}, \frac{4}{5}, \text{ and } \frac{-1}{4}$$

The solution is $60x^3 + 7x^2 - 34x - 8$, which is option D.

A. $a \in [58, 61], b \in [-8, -6], c \in [-40, -31]$, and $d \in [1, 11]$

$60x^3 - 7x^2 - 34x + 8$, which corresponds to multiplying out $(3x - 2)(5x + 4)(4x - 1)$.

B. $a \in [58, 61], b \in [14, 25], c \in [-31, -29]$, and $d \in [-10, -6]$

$60x^3 + 23x^2 - 30x - 8$, which corresponds to multiplying out $(3x + 3)(5x + 5)(4x - 4)$.

C. $a \in [58, 61], b \in [-73, -63], c \in [5, 13]$, and $d \in [1, 11]$

$60x^3 - 73x^2 + 10x + 8$, which corresponds to multiplying out $(3x + 3)(5x - 5)(4x - 4)$.

D. $a \in [58, 61], b \in [-4, 10], c \in [-40, -31]$, and $d \in [-10, -6]$

* $60x^3 + 7x^2 - 34x - 8$, which is the correct option.

E. $a \in [58, 61], b \in [-4, 10], c \in [-40, -31]$, and $d \in [1, 11]$

$60x^3 + 7x^2 - 34x + 8$, which corresponds to multiplying everything correctly except the constant term.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(3x + 2)(5x - 4)(4x + 1)$

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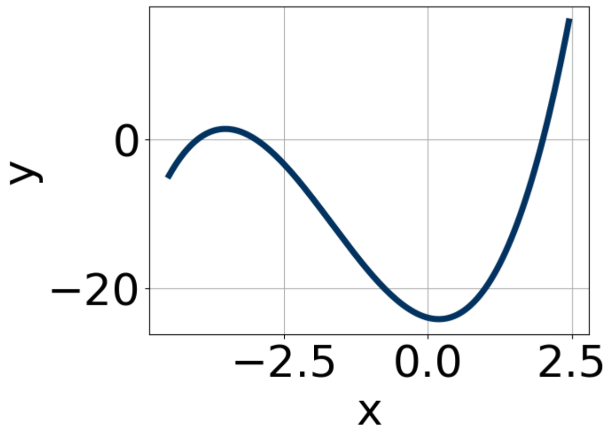
4. Which of the following equations *could* be of the graph presented below?




The solution is $16(x+3)^5(x-2)^{11}(x+4)^7$, which is option A.

A. $16(x+3)^5(x-2)^{11}(x+4)^7$

   * This is the correct option.

B. $4(x+3)^8(x-2)^{10}(x+4)^{11}$

   The factors $-3$ and $2$ have have been odd power.

C. $-16(x+3)^{11}(x-2)^5(x+4)^5$

   This corresponds to the leading coefficient being the opposite value than it should be.

D. $-20(x+3)^6(x-2)^5(x+4)^{11}$

   The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $20(x+3)^6(x-2)^7(x+4)^7$

   The factor $-3$ should have been an odd power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3+bx^2+cx+d$.

$$\frac{3}{2}, \frac{2}{5}, \text{ and } \frac{3}{4}$$

The solution is $40x^3-106x^2+81x-18$, which is option C.

A. $a \in [38, 49], b \in [38, 47], c \in [-40, -29], \text{ and } d \in [-23, -17]$

   $40x^3+46x^2-33x-18$, which corresponds to multiplying out $(2x+2)(5x+5)(4x-4)$.

B. $a \in [38, 49], b \in [13, 17], c \in [-58, -53], \text{ and } d \in [13, 20]$

   $40x^3+14x^2-57x+18$, which corresponds to multiplying out $(2x+2)(5x-5)(4x-4)$.

C. $a \in [38, 49], b \in [-108, -98], c \in [72, 89], \text{ and } d \in [-23, -17]$

   * $40x^3-106x^2+81x-18$, which is the correct option.

D. $a \in [38, 49], b \in [-108, -98], c \in [72, 89]$, and $d \in [13, 20]$

$40x^3 - 106x^2 + 81x + 18$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [38, 49], b \in [103, 112], c \in [72, 89]$, and $d \in [13, 20]$

$40x^3 + 106x^2 + 81x + 18$, which corresponds to multiplying out $(2x + 3)(5x + 2)(4x + 3)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(2x - 3)(5x - 2)(4x - 3)$

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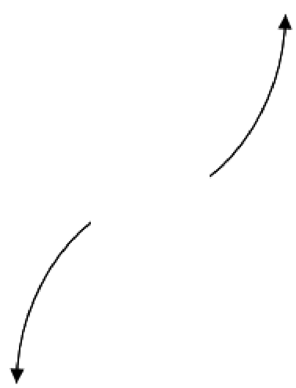
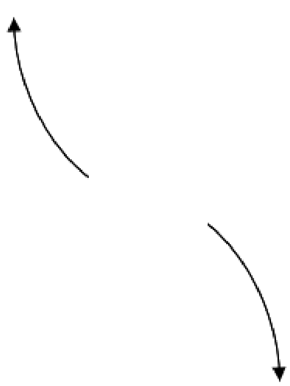
6. Describe the end behavior of the polynomial below.

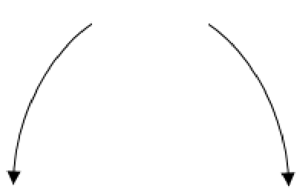
$$f(x) = 2(x + 6)^5(x - 6)^8(x - 4)^3(x + 4)^3$$

The solution is the graph below, which is option D.

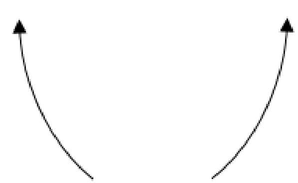




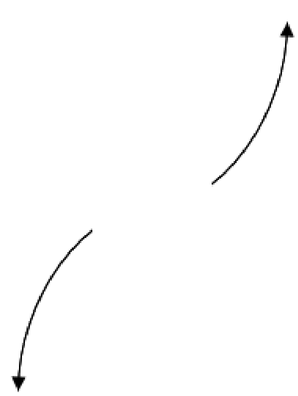
A.



B.



C.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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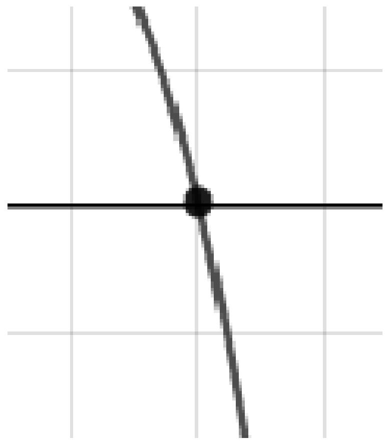
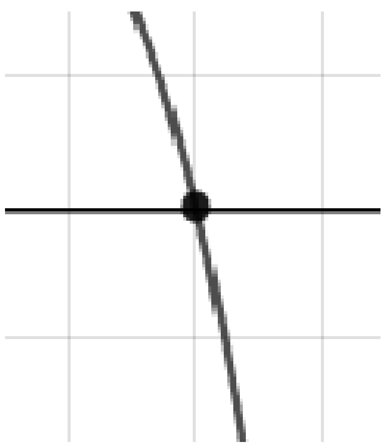
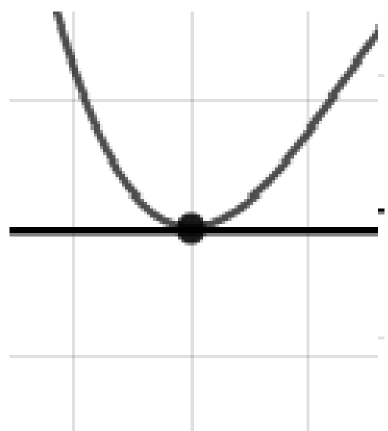
7. Describe the zero behavior of the zero $x = -7$ of the polynomial below.

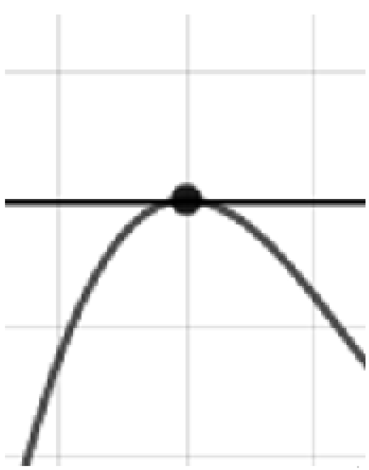
$$f(x) = 3(x-7)^4(x+7)^7(x-4)^4(x+4)^7$$

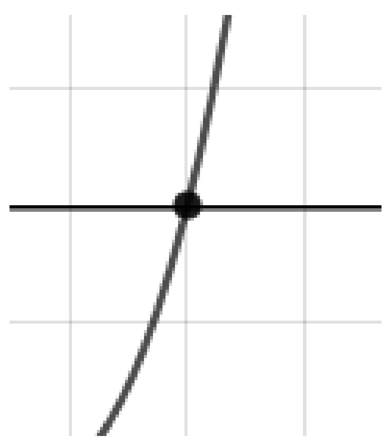
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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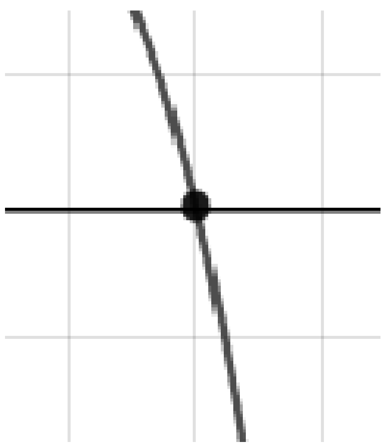
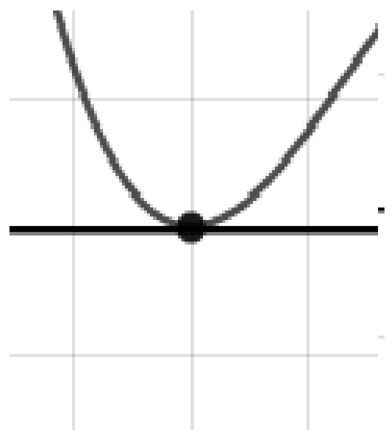
8. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = 5(x-9)^4(x+9)^3(x+3)^9(x-3)^8$$

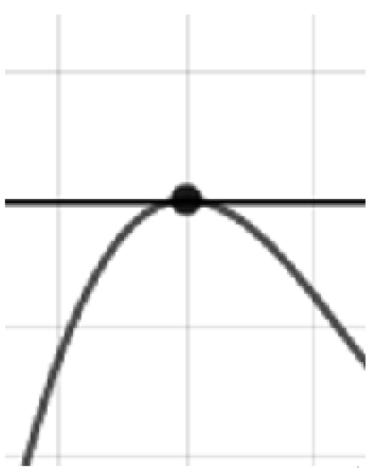
The solution is the graph below, which is option C.



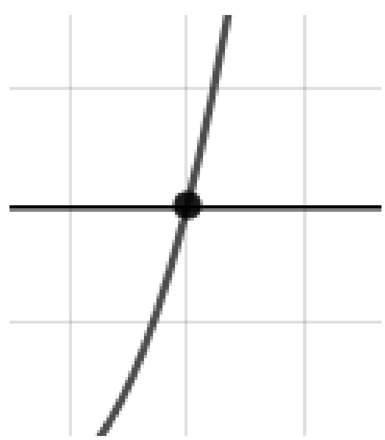
A.



C.



B.



D.

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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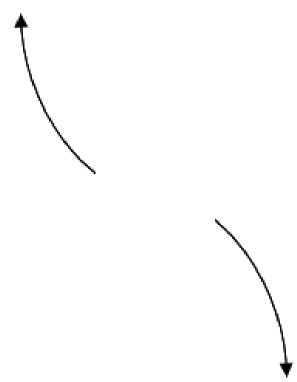
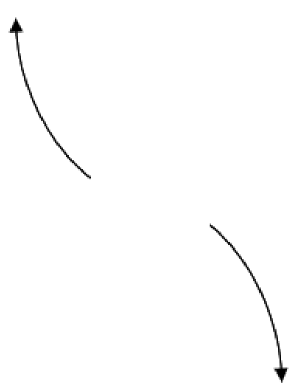
9. Describe the end behavior of the polynomial below.

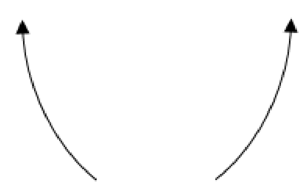
$$f(x) = -8(x+9)^2(x-9)^3(x-4)^2(x+4)^2$$

The solution is the graph below, which is option A.

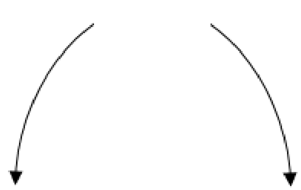




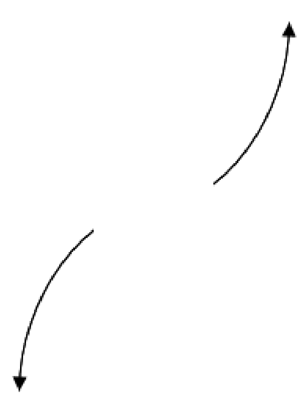
A.



C.



B.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 3i \text{ and } 1$$

The solution is $x^3 - 7x^2 + 24x - 18$, which is option A.

A. $b \in [-15, -5], c \in [18, 28],$ and $d \in [-19.7, -15.4]$

   * $x^3 - 7x^2 + 24x - 18$, which is the correct option.

B. $b \in [1, 2], c \in [-6, -1],$ and $d \in [1.5, 3.1]$

   $x^3 + x^2 - 4x + 3$, which corresponds to multiplying out $(x - 3)(x - 1)$.

C. $b \in [3, 8], c \in [18, 28],$ and $d \in [17.8, 20.6]$

   $x^3 + 7x^2 + 24x + 18$, which corresponds to multiplying out $(x - (3 - 3i))(x - (3 + 3i))(x + 1)$.

D. $b \in [1, 2], c \in [-2, 7],$ and $d \in [-3.5, 0.4]$

   $x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 - 3i))(x - (3 + 3i))(x - (1))$.