This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -x^3 - 3x^2 - 3x \text{ and } g(x) = -x^3 + 2x^2 + 4x$$

The solution is 1.0, which is option C.

A. $(f \circ g)(-1) \in [4.87, 5.74]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(-1) \in [5.45, 7.54]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [0.7, 2.2]$

* This is the correct solution

D. $(f \circ g)(-1) \in [14.76, 15.37]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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2. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 - 1x^2 - 2x + 4 \text{ and } g(x) = -2x^3 - 3x^2 + 3x - 1$$

The solution is 55.0, which is option B.

A. $(f \circ g)(1) \in [-15, -13]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [54, 58]$

* This is the correct solution

C. $(f \circ g)(1) \in [-7, -3]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(1) \in [49, 54]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 2x^4 + 5x^3 + 2x^2 + 5x + 2 \text{ and } g(x) = \sqrt{-3x + 4}$$

The solution is The domain is all Real numbers less than or equal to$x = 1.3333333333333333$., which is option C.

A. The domain is all Real numbers except $x = a,$ where $a \in [-7.33, -1.33]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [0.33, 12.33]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-1.67, 2.33]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-6.6, -2.6]$ and $b \in [-7.4, -2.4]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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4. Determine whether the function below is 1-1.

$$f(x) = 36x^2 - 312x + 676$$

The solution is no, which is option E.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval that $f^{-1}(11)$ belongs to.

$$f(x) = 3x^2 - 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(11) \in [4.88, 5.45]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B. $f^{-1}(11) \in [8.06, 10.23]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(11) \in [1.93, 2.97]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

D. $f^{-1}(11) \in [1.57, 2.01]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -10$ and choose the interval the $f^{-1}(-10)$ belongs to.

$$f(x) = \sqrt[3]{5x + 4}$$

The solution is -200.8, which is option B.

A. $f^{-1}(-10) \in [-199.57, -198.15]$

Distractor 1: This corresponds to

B. $f^{-1}(-10) \in [-201.01, -200.17]$

* This is the correct solution.

C. $f^{-1}(-10) \in [200.09, 202.09]$

This solution corresponds to distractor 2.

D. $f^{-1}(-10) \in [198.75, 199.54]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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7. Determine whether the function below is 1-1.

$$f(x) = 36x^2 + 456x + 1444$$

The solution is no, which is option C.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

C. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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8. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = e^{x+3} + 5$$

The solution is $f^{-1}(7) = -2.307$, which is option B.

A. $f^{-1}(7) \in [6.2, 6.67]$

This solution corresponds to distractor 3.

B. $f^{-1}(7) \in [-2.33, -2.09]$

This is the solution.

C. $f^{-1}(7) \in [7.4, 7.53]$

This solution corresponds to distractor 2.

D. $f^{-1}(7) \in [7.16, 7.41]$

This solution corresponds to distractor 4.

E. $f^{-1}(7) \in [3.55, 3.84]$

This solution corresponds to distractor 1.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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9. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{4x - 30} \text{ and } g(x) = 5x^2 + 3x + 7$$

The solution is The domain is all Real numbers greater than or equal to$x = 7.5.$, which is option A.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [6.5, 12.5]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-9.75, -0.75]$

C. The domain is all Real numbers except $x = a,$ where $a \in [-0.6, 8.4]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [4.33, 14.33]$ and $b \in [-8.67, -2.67]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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10. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = e^{x-2} + 4$$

The solution is $f^{-1}(9) = 3.609$, which is option E.

A. $f^{-1}(9) \in [5.76, 6.31]$

This solution corresponds to distractor 4.

B. $f^{-1}(9) \in [6.42, 7.33]$

This solution corresponds to distractor 2.

C. $f^{-1}(9) \in [-0.46, 0.18]$

This solution corresponds to distractor 1.

D. $f^{-1}(9) \in [6.23, 6.49]$

This solution corresponds to distractor 3.

E. $f^{-1}(9) \in [2.99, 4.53]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.
