This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = \ln(x - 4) + 3$$

The solution is $f^{-1}(9) = 407.429$, which is option D.

A. $f^{-1}(9) \in [162758.79, 162766.79]$

This solution corresponds to distractor 1.

B. $f^{-1}(9) \in [147.41, 155.41]$

This solution corresponds to distractor 4.

C. $f^{-1}(9) \in [442414.39, 442422.39]$

This solution corresponds to distractor 2.

D. $f^{-1}(9) \in [404.43, 409.43]$

This is the solution.

E. $f^{-1}(9) \in [398.43, 402.43]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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2. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 5x + 3 \text{ and } g(x) = 9x^2 + 3x + 8$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-4.5, -0.5]$

B. The domain is all Real numbers except $x = a$, where $a \in [2.25, 13.25]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [5.33, 10.33]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-10.25, 7.75]$ and $b \in [0.8, 7.8]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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3. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -x^3 - 2x^2 + x \text{ and } g(x) = -4x^3 + 2x^2 + x$$

The solution is -2.0, which is option D.

A. $(f \circ g)(1) \in [38, 39]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(1) \in [29, 32]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [5, 11]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-7, -1]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = 5x^2 + 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(10) \in [1.61, 1.68]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(10) \in [7.17, 7.38]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(10) \in [0.86, 1.23]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

D. $f^{-1}(10) \in [3.94, 4.3]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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5. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = x^3 + 7x^2 + 3x + 4 \text{ and } g(x) = \sqrt{-3x - 15}$$

The solution is The domain is all Real numbers less than or equal to$x = -5.0$., which is option A.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-10, 2]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-6, -3]$

C. The domain is all Real numbers except $x = a,$ where $a \in [-10.25, -3.25]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [3.33, 8.33]$ and $b \in [-10.2, -2.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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6. Determine whether the function below is 1-1.

$$f(x) = (3x - 20)^3$$

The solution is yes, which is option D.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. Yes, the function is 1-1.

* This is the solution.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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7. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 2x^3 + 2x^2 - 4x + 1 \text{ and } g(x) = -2x^3 + 4x^2 - x + 1$$

The solution is 17.0, which is option C.

A. $(f \circ g)(1) \in [-12, -5]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-2, 7]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(1) \in [16, 22]$

* This is the correct solution

D. $(f \circ g)(1) \in [11, 13]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval that $f^{-1}(11)$ belongs to.

$$f(x) = 3x^2 + 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(11) \in [6.7, 6.99]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(11) \in [1.79, 2.1]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

C. $f^{-1}(11) \in [4.6, 4.92]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(11) \in [1.71, 1.91]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Determine whether the function below is 1-1.

$$f(x) = 36x^2 + 168x + 196$$

The solution is no, which is option A.

A. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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10. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = \ln(x + 5) - 4$$

The solution is $f^{-1}(7) = 59869.142$, which is option B.

A. $f^{-1}(7) \in [59877.14, 59886.14]$

This solution corresponds to distractor 3.

B. $f^{-1}(7) \in [59865.14, 59874.14]$

This is the solution.

C. $f^{-1}(7) \in [162747.79, 162755.79]$

This solution corresponds to distractor 4.

D. $f^{-1}(7) \in [14.09, 21.09]$

This solution corresponds to distractor 1.

E. $f^{-1}(7) \in [-0.61, 4.39]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.
