This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-4}{5}, 2, \text{ and } \frac{7}{5}$$

The solution is $25x^3 - 65x^2 + 2x + 56$, which is option C.

A. $a \in [24, 31], b \in [-72, -62], c \in [1, 7], \text{ and } d \in [-56, -55]$

$25x^3 - 65x^2 + 2x - 56$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [24, 31], b \in [57, 69], c \in [1, 7], \text{ and } d \in [-56, -55]$

$25x^3 + 65x^2 + 2x - 56$, which corresponds to multiplying out $(5x - 4)(x + 2)(5x + 7)$.

C. $a \in [24, 31], b \in [-72, -62], c \in [1, 7], \text{ and } d \in [53, 61]$

* $25x^3 - 65x^2 + 2x + 56$, which is the correct option.

D. $a \in [24, 31], b \in [-105, -101], c \in [138, 141], \text{ and } d \in [-56, -55]$

$25x^3 - 105x^2 + 138x - 56$, which corresponds to multiplying out $(5x - 4)(x - 2)(5x - 7)$.

E. $a \in [24, 31], b \in [-11, -3], c \in [-87, -79], \text{ and } d \in [53, 61]$

$25x^3 - 5x^2 - 82x + 56$, which corresponds to multiplying out $(5x - 4)(x + 2)(5x - 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 4)(x - 2)(5x - 7)$

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 + 4i \text{ and } -4$$

The solution is $x^3 + 4x + 80$, which is option A.

A. $b \in [-1.82, 0.38], c \in [3.17, 4.11], \text{ and } d \in [79, 85]$

* $x^3 + 4x + 80$, which is the correct option.

B. $b \in [-1.82, 0.38], c \in [3.17, 4.11], \text{ and } d \in [-80, -76]$

$x^3 + 4x - 80$, which corresponds to multiplying out $(x - (2 + 4i))(x - (2 - 4i))(x - 4)$.

C. $b \in [0.07, 1.76], c \in [1.42, 3.02], \text{ and } d \in [-10, -5]$

$x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x - 2)(x + 4)$.

D. $b \in [0.07, 1.76], c \in [-0.67, 0.1]$, and $d \in [-18, -10]$

$x^3 + x^2 - 16$, which corresponds to multiplying out $(x - 4)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 + 4i))(x - (2 - 4i))(x - (-4))$.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 + 4i \text{ and } 2$$

The solution is $x^3 - 8x^2 + 37x - 50$, which is option B.

A. $b \in [0, 2], c \in [-6.65, -5.66]$, and $d \in [8, 12]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 4)(x - 2)$.

B. $b \in [-14, -4], c \in [37, 37.08]$, and $d \in [-56, -49]$

* $x^3 - 8x^2 + 37x - 50$, which is the correct option.

C. $b \in [6, 11], c \in [37, 37.08]$, and $d \in [47, 56]$

$x^3 + 8x^2 + 37x + 50$, which corresponds to multiplying out $(x - (3 + 4i))(x - (3 - 4i))(x + 2)$.

D. $b \in [0, 2], c \in [-5.69, -4.9]$, and $d \in [3, 7]$

$x^3 + x^2 - 5x + 6$, which corresponds to multiplying out $(x - 3)(x - 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 + 4i))(x - (3 - 4i))(x - (2))$.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-6, \frac{-7}{4}, \text{ and } \frac{1}{4}$$

The solution is $16x^3 + 120x^2 + 137x - 42$, which is option A.

A. $a \in [11, 21], b \in [117, 121], c \in [134, 138]$, and $d \in [-47, -41]$

* $16x^3 + 120x^2 + 137x - 42$, which is the correct option.

B. $a \in [11, 21], b \in [-122, -115], c \in [134, 138]$, and $d \in [42, 43]$

$16x^3 - 120x^2 + 137x + 42$, which corresponds to multiplying out $(x - 6)(4x - 7)(4x + 1)$.

C. $a \in [11, 21], b \in [-72, -70], c \in [-152, -143]$, and $d \in [42, 43]$

$16x^3 - 72x^2 - 151x + 42$, which corresponds to multiplying out $(x - 6)(4x + 7)(4x - 1)$.

D. $a \in [11, 21], b \in [117, 121], c \in [134, 138]$, and $d \in [42, 43]$

$16x^3 + 120x^2 + 137x + 42$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [11, 21], b \in [-131, -122], c \in [197, 201]$, and $d \in [-47, -41]$

$16x^3 - 128x^2 + 199x - 42$, which corresponds to multiplying out $(x - 6)(4x - 7)(4x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+6)(4x+7)(4x-1)$

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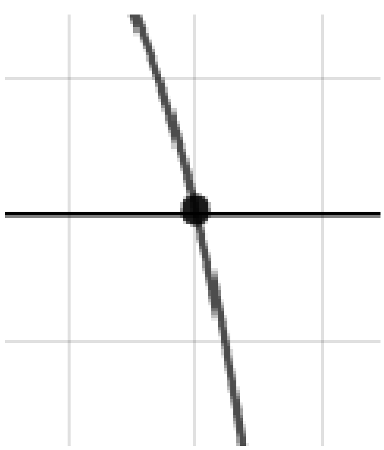
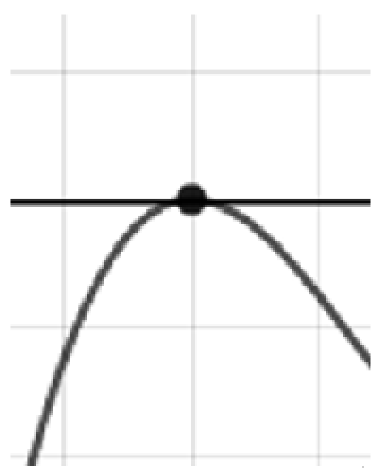
5. Describe the zero behavior of the zero $x = -7$ of the polynomial below.

$$f(x) = 9(x + 7)^8(x - 7)^9(x + 3)^4(x - 3)^5$$

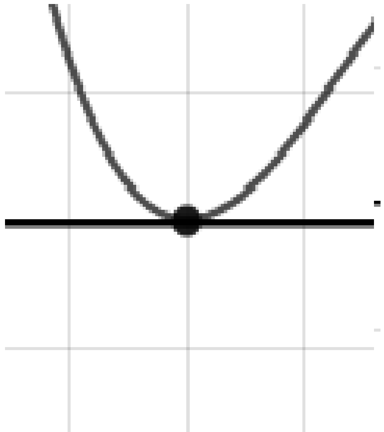
The solution is the graph below, which is option C.

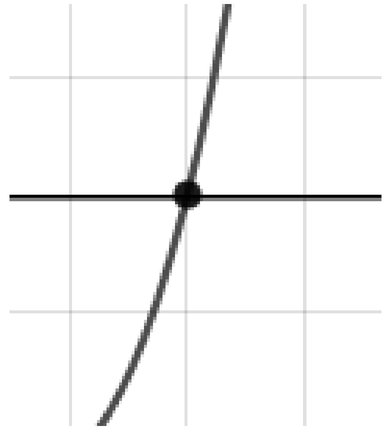


A.



B.

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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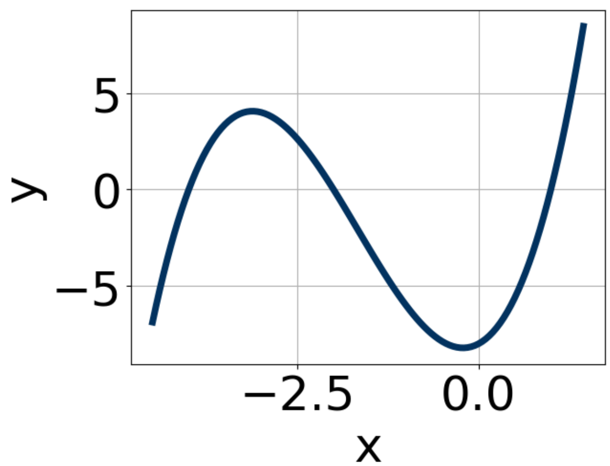
6. Which of the following equations *could* be of the graph presented below?



The solution is $12(x+4)^7(x-1)^7(x+2)^5$, which is option E.

A. $12(x+4)^{10}(x-1)^{11}(x+2)^5$

   The factor -4 should have been an odd power.

B. $14(x+4)^{10}(x-1)^6(x+2)^{11}$

   The factors -4 and 1 have have been odd power.

C. $-6(x+4)^6(x-1)^5(x+2)^9$

   The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-9(x+4)^9(x-1)^7(x+2)^5$

   This corresponds to the leading coefficient being the opposite value than it should be.

E. $12(x+4)^7(x-1)^7(x+2)^5$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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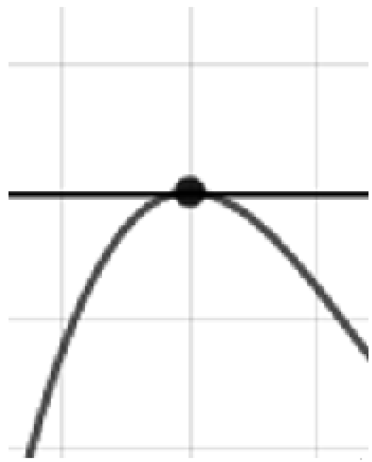
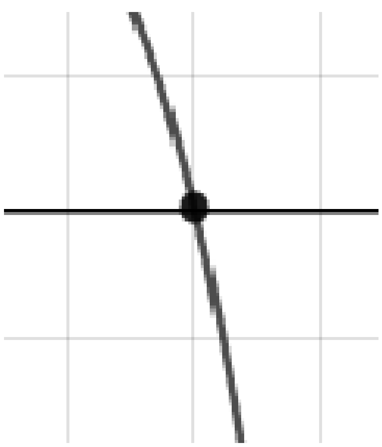
7. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

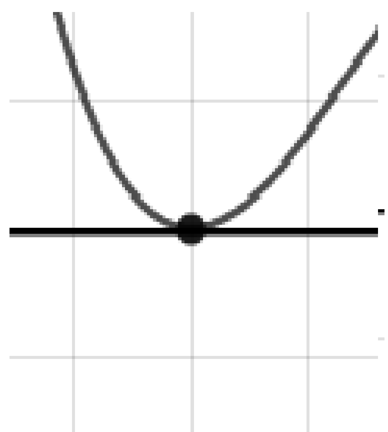
$$f(x) = -4(x+2)^5(x-2)^3(x+7)^7(x-7)^6$$

The solution is the graph below, which is option B.

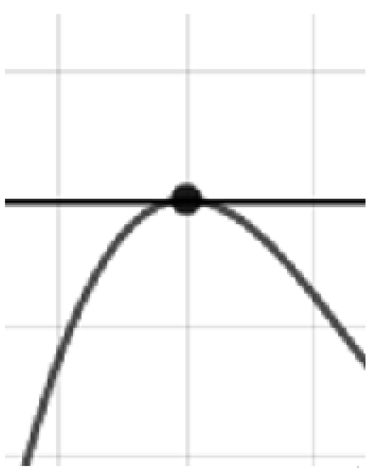




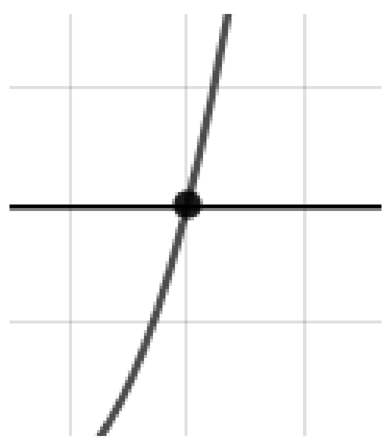
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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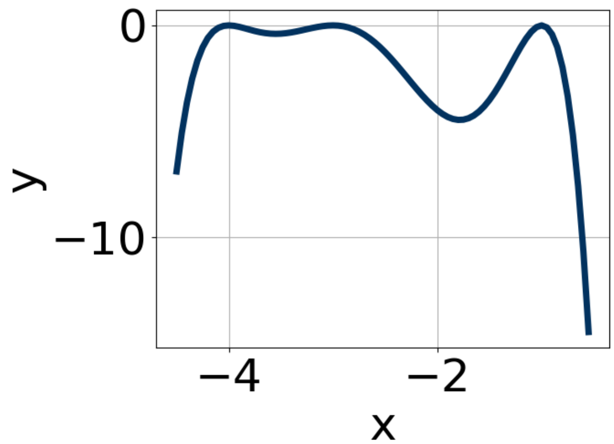
8. Which of the following equations *could* be of the graph presented below?



The solution is $-12(x+3)^{10}(x+4)^{6}(x+1)^{4}$, which is option B.

A. $10(x+3)^{10}(x+4)^{8}(x+1)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-12(x+3)^{10}(x+4)^{6}(x+1)^{4}$

* This is the correct option.

C. $7(x+3)^{8}(x+4)^{8}(x+1)^{11}$

The factor $(x+1)$ should have an even power and the leading coefficient should be the opposite sign.

D. $-6(x+3)^{4}(x+4)^{11}(x+1)^{9}$

The factors $(x+4)$ and $(x+1)$ should both have even powers.

E. $-19(x+3)^{6}(x+4)^{8}(x+1)^{7}$

The factor $(x+1)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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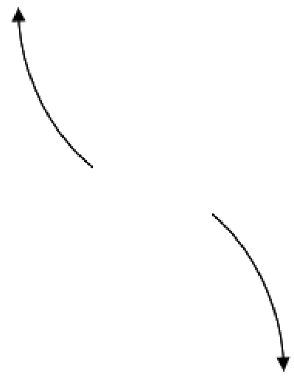
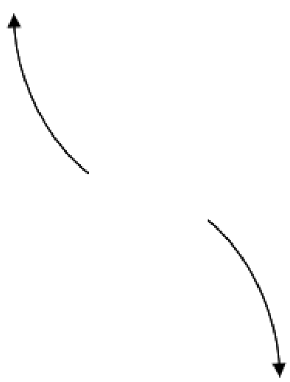
9. Describe the end behavior of the polynomial below.

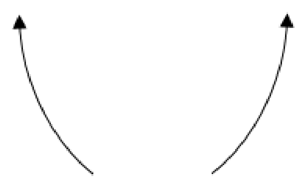
$$f(x) = -8(x+2)^{3}(x-2)^{8}(x-5)^{5}(x+5)^{5}$$

The solution is the graph below, which is option A.

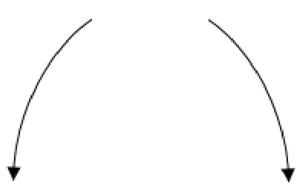




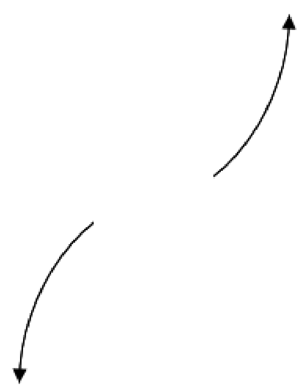
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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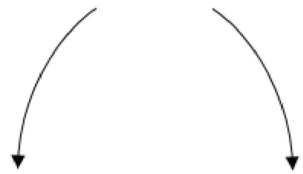
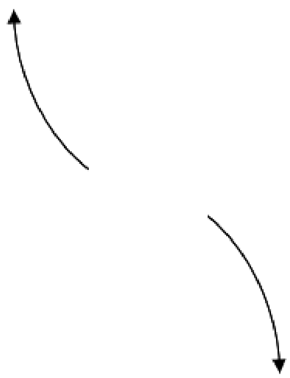
10. Describe the end behavior of the polynomial below.

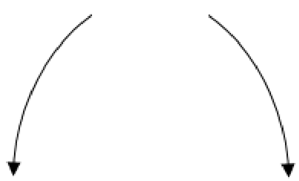
$$f(x) = -5(x-3)^4(x+3)^7(x-5)^5(x+5)^6$$

The solution is the graph below, which is option B.

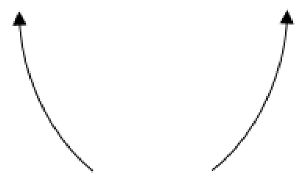




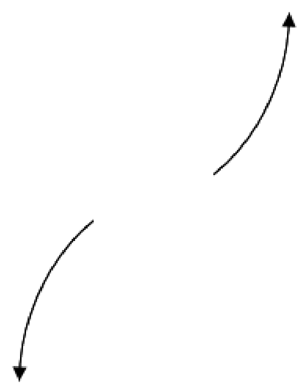
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.