This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{2}, \frac{-6}{5}, \text{ and } \frac{7}{4}$$

The solution is $40x^3 + 38x^2 - 117x - 126$, which is option A.

A. $a \in [40, 41], b \in [37, 46], c \in [-118, -115], \text{ and } d \in [-127, -124]$

* $40x^3 + 38x^2 - 117x - 126$, which is the correct option.

B. $a \in [40, 41], b \in [-178, -176], c \in [250, 263], \text{ and } d \in [-127, -124]$

$40x^3 - 178x^2 + 261x - 126$, which corresponds to multiplying out $(2x - 3)(5x - 6)(4x - 7)$.

C. $a \in [40, 41], b \in [37, 46], c \in [-118, -115], \text{ and } d \in [126, 127]$

$40x^3 + 38x^2 - 117x + 126$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [40, 41], b \in [-47, -32], c \in [-118, -115], \text{ and } d \in [126, 127]$

$40x^3 - 38x^2 - 117x + 126$, which corresponds to multiplying out $(2x - 3)(5x - 6)(4x + 7)$.

E. $a \in [40, 41], b \in [-82, -80], c \in [-56, -48], \text{ and } d \in [126, 127]$

$40x^3 - 82x^2 - 51x + 126$, which corresponds to multiplying out $(2x - 3)(5x + 6)(4x - 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 3)(5x + 6)(4x - 7)$

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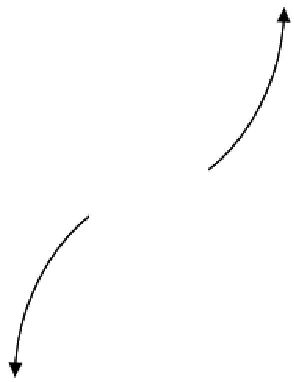
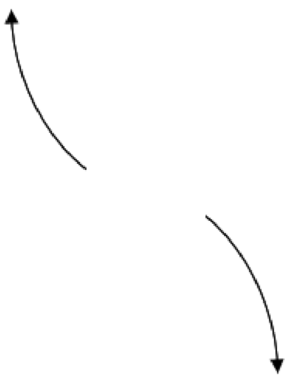
2. Describe the end behavior of the polynomial below.

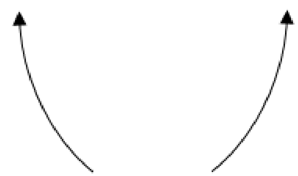
$$f(x) = 6(x - 5)^4(x + 5)^7(x + 9)^3(x - 9)^5$$

The solution is the graph below, which is option D.

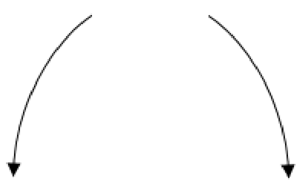




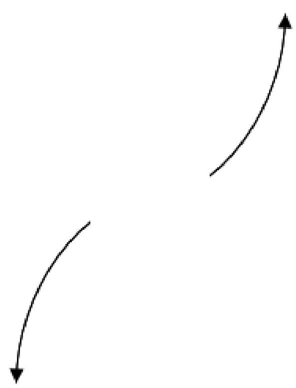
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 2i \text{ and } 3$$

The solution is $x^3 + x^2 - 4x - 24$, which is option A.

A. $b \in [0, 1.7], c \in [-4.1, -3.6],$ and $d \in [-27, -20]$

   * $x^3 + x^2 - 4x - 24$, which is the correct option.

B. $b \in [-3.8, 0.8], c \in [-4.1, -3.6]$, and $d \in [21, 27]$

$x^3 - 1x^2 - 4x + 24$, which corresponds to multiplying out $(x - (-2 + 2i))(x - (-2 - 2i))(x + 3)$.

C. $b \in [0, 1.7], c \in [-5.5, -4.1]$, and $d \in [6, 7]$

$x^3 + x^2 - 5x + 6$, which corresponds to multiplying out $(x - 2)(x - 3)$.

D. $b \in [0, 1.7], c \in [-3.9, 3.7]$, and $d \in [-9, -3]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x + 2)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 2i))(x - (-2 - 2i))(x - (3))$.

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{4}, \frac{1}{4}, \text{ and } \frac{2}{3}$$

The solution is $48x^3 - 128x^2 + 85x - 14$, which is option D.

A. $a \in [44, 56], b \in [126, 137], c \in [81, 90]$, and $d \in [11, 17]$

$48x^3 + 128x^2 + 85x + 14$, which corresponds to multiplying out $(4x + 7)(4x + 1)(3x + 2)$.

B. $a \in [44, 56], b \in [-135, -118], c \in [81, 90]$, and $d \in [11, 17]$

$48x^3 - 128x^2 + 85x + 14$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [44, 56], b \in [63, 68], c \in [-46, -41]$, and $d \in [-14, -7]$

$48x^3 + 64x^2 - 43x - 14$, which corresponds to multiplying out $(4x + 7)(4x + 1)(3x - 2)$.

D. $a \in [44, 56], b \in [-135, -118], c \in [81, 90]$, and $d \in [-14, -7]$

* $48x^3 - 128x^2 + 85x - 14$, which is the correct option.

E. $a \in [44, 56], b \in [39, 42], c \in [-69, -65]$, and $d \in [11, 17]$

$48x^3 + 40x^2 - 69x + 14$, which corresponds to multiplying out $(4x + 7)(4x - 1)(3x - 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 7)(4x - 1)(3x - 2)$

---

5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 - 3i \text{ and } -2$$

The solution is $x^3 - 2x^2 + 5x + 26$, which is option D.

A. $b \in [0.54, 1.9], c \in [-1, 4]$, and $d \in [-4, -2]$

$x^3 + x^2 - 4$, which corresponds to multiplying out $(x - 2)(x + 2)$.

B. $b \in [1.49, 3.68], c \in [5, 7],$ and $d \in [-31, -17]$

$x^3 + 2x^2 + 5x - 26$, which corresponds to multiplying out $(x - (2 - 3i))(x - (2 + 3i))(x - 2)$.

C. $b \in [0.54, 1.9], c \in [5, 7],$ and $d \in [4, 8]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 3)(x + 2)$.

D. $b \in [-2.47, -1.16], c \in [5, 7],$ and $d \in [19, 30]$

* $x^3 - 2x^2 + 5x + 26$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 - 3i))(x - (2 + 3i))(x - (-2))$.

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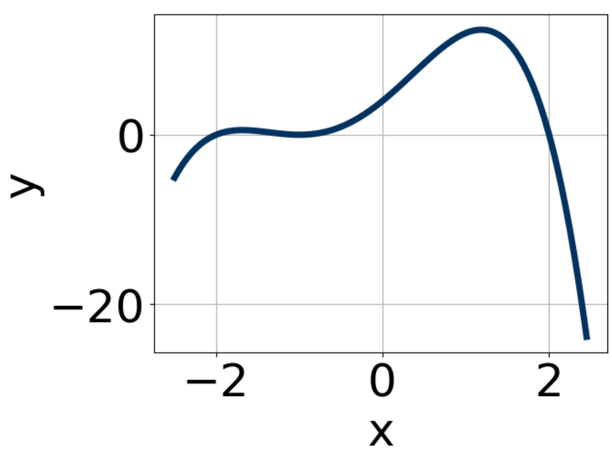
6. Which of the following equations *could* be of the graph presented below?



The solution is $-9(x + 1)^6(x + 2)^{11}(x - 2)^5$, which is option B.

A. $-11(x + 1)^{10}(x + 2)^{10}(x - 2)^7$

The factor $(x + 2)$ should have an odd power.

B. $-9(x + 1)^6(x + 2)^{11}(x - 2)^5$

* This is the correct option.

C. $4(x + 1)^8(x + 2)^5(x - 2)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-14(x + 1)^5(x + 2)^6(x - 2)^5$

The factor -1 should have an even power and the factor -2 should have an odd power.

E. $10(x + 1)^8(x + 2)^{11}(x - 2)^8$

The factor $(x - 2)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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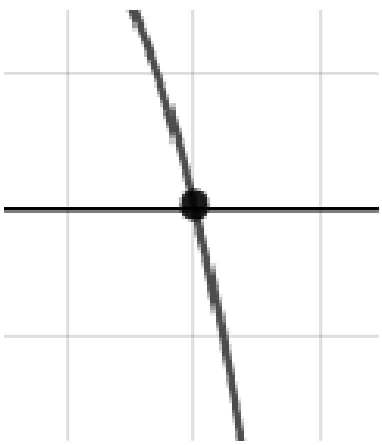
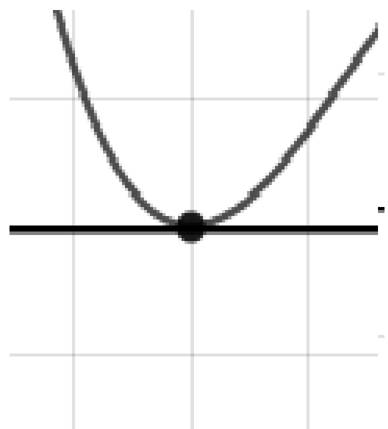
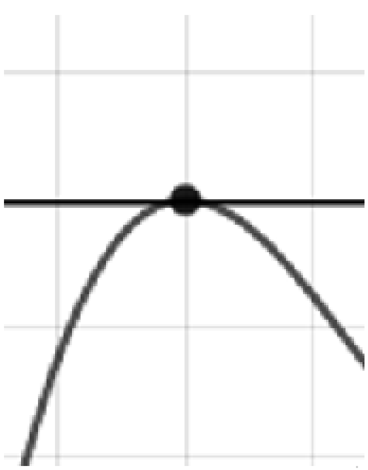
7. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

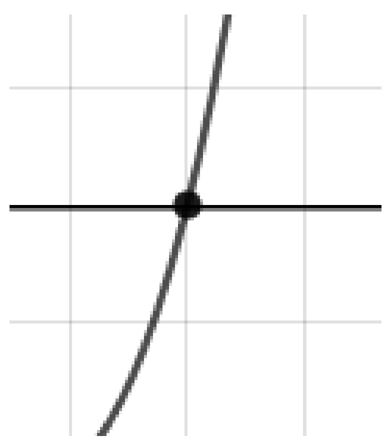
$$f(x) = 5(x+5)^{12}(x-5)^{8}(x-7)^{10}(x+7)^{5}$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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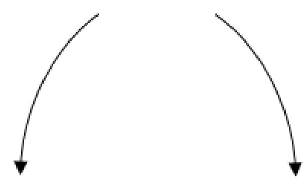
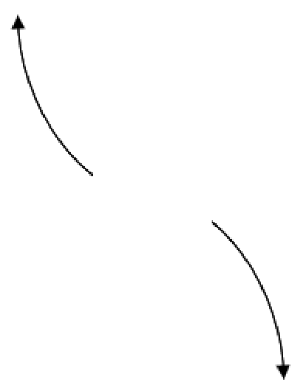
8. Describe the end behavior of the polynomial below.

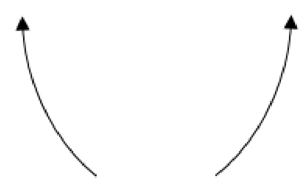
$$f(x) = -8(x+7)^2(x-7)^3(x+5)^2(x-5)^3$$

The solution is the graph below, which is option B.

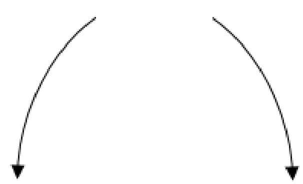




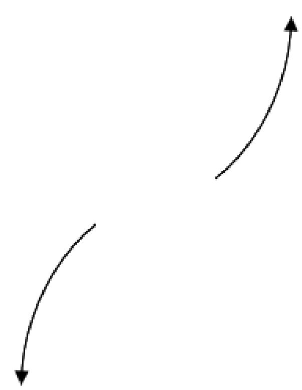
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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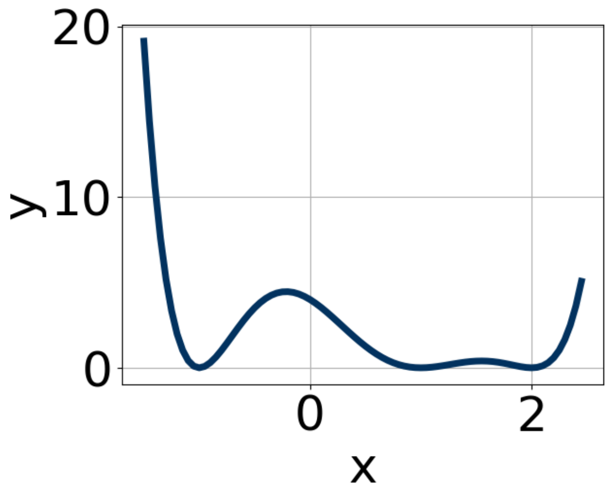
9. Which of the following equations *could* be of the graph presented below?



The solution is $2(x-2)^4(x-1)^4(x+1)^6$, which is option D.

A. $14(x-2)^8(x-1)^7(x+1)^5$

   The factors $(x-1)$ and $(x+1)$ should both have even powers.

B. $12(x-2)^4(x-1)^{10}(x+1)^9$

   The factor $(x+1)$ should have an even power.

C. $-3(x-2)^6(x-1)^{10}(x+1)^8$

   This corresponds to the leading coefficient being the opposite value than it should be.

D. $2(x-2)^4(x-1)^4(x+1)^6$

   * This is the correct option.

E. $-6(x-2)^{10}(x-1)^4(x+1)^9$

   The factor $(x+1)$ should have an even power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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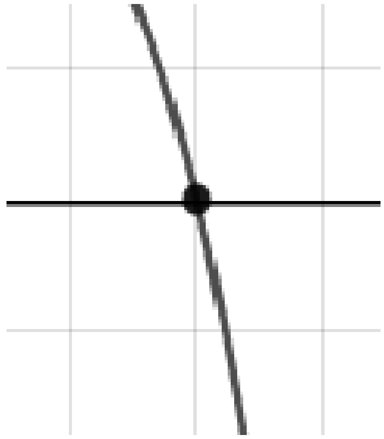
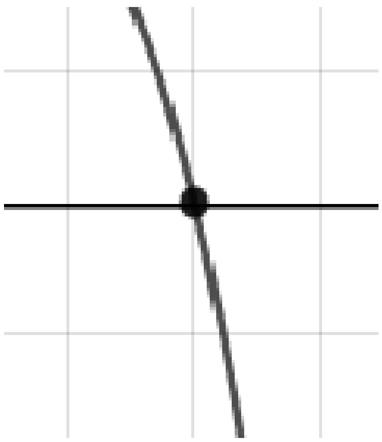
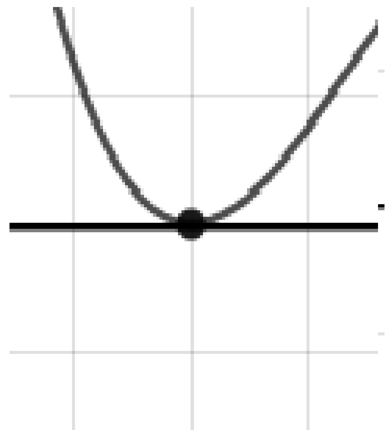
10. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

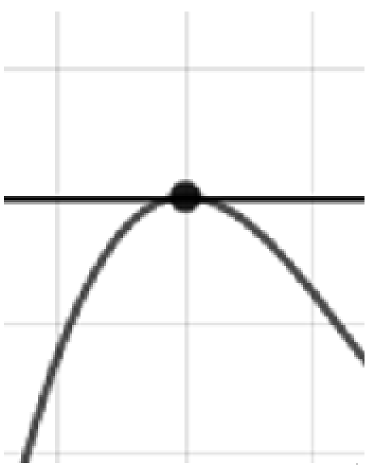
$$f(x) = -5(x+3)^7(x-3)^5(x+8)^6(x-8)^3$$

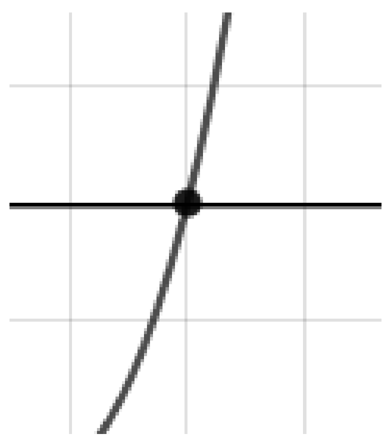
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.