This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 5i \text{ and } -3$$

The solution is $x^3 + 11x^2 + 65x + 123$, which is option B.

A. $b \in [-14, -10], c \in [63, 72]$, and $d \in [-129, -116]$

$x^3 - 11x^2 + 65x - 123$, which corresponds to multiplying out $(x - (-4 + 5i))(x - (-4 - 5i))(x - 3)$.

B. $b \in [7, 14], c \in [63, 72]$, and $d \in [119, 125]$

* $x^3 + 11x^2 + 65x + 123$, which is the correct option.

C. $b \in [1, 9], c \in [-5, 1]$, and $d \in [-24, -14]$

$x^3 + x^2 - 2x - 15$, which corresponds to multiplying out $(x - 5)(x + 3)$.

D. $b \in [1, 9], c \in [4, 13]$, and $d \in [11, 20]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 4)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 5i))(x - (-4 - 5i))(x - (-3))$.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-5}{2}, \frac{-7}{4}, \text{ and } -2$$

The solution is $8x^3 + 50x^2 + 103x + 70$, which is option B.

A. $a \in [3, 9], b \in [3, 13], c \in [-51, -46]$, and $d \in [-70, -67]$

$8x^3 + 10x^2 - 47x - 70$, which corresponds to multiplying out $(2x - 5)(4x + 7)(x + 2)$.

B. $a \in [3, 9], b \in [48, 52], c \in [97, 108]$, and $d \in [68, 74]$

* $8x^3 + 50x^2 + 103x + 70$, which is the correct option.

C. $a \in [3, 9], b \in [48, 52], c \in [97, 108]$, and $d \in [-70, -67]$

$8x^3 + 50x^2 + 103x - 70$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [3, 9], b \in [-60, -47], c \in [97, 108],$ and $d \in [-70, -67]$

$8x^3 - 50x^2 + 103x - 70$, which corresponds to multiplying out $(2x - 5)(4x - 7)(x - 2)$.

E. $a \in [3, 9], b \in [-23, -17], c \in [-33, -29],$ and $d \in [68, 74]$

$8x^3 - 18x^2 - 33x + 70$, which corresponds to multiplying out $(2x - 5)(4x - 7)(x + 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 5)(4x + 7)(x + 2)$

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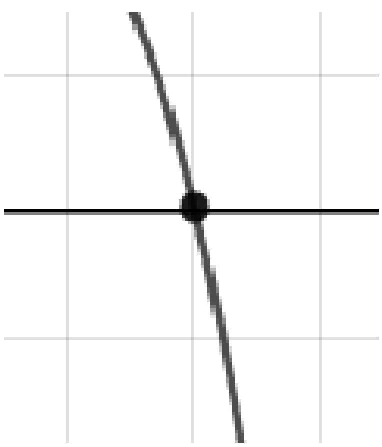
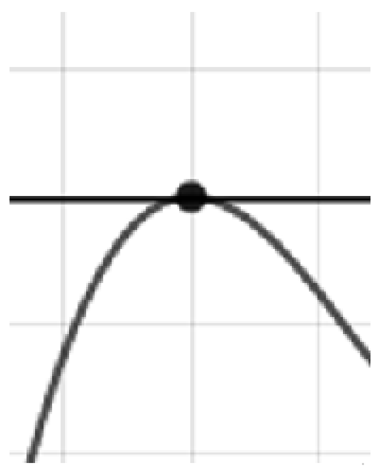
3. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

$$f(x) = 2(x - 7)^{10}(x + 7)^8(x - 5)^8(x + 5)^3$$

The solution is the graph below, which is option C.

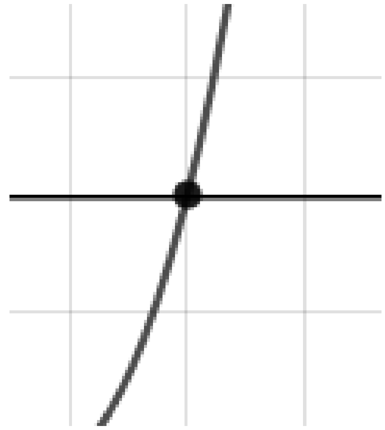


A.



B.

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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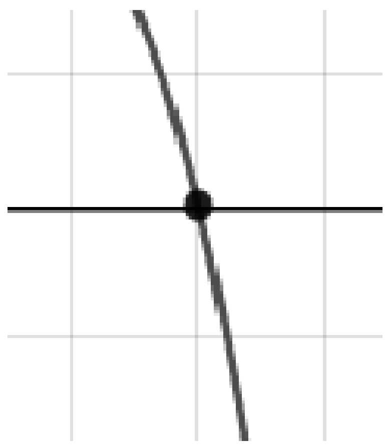
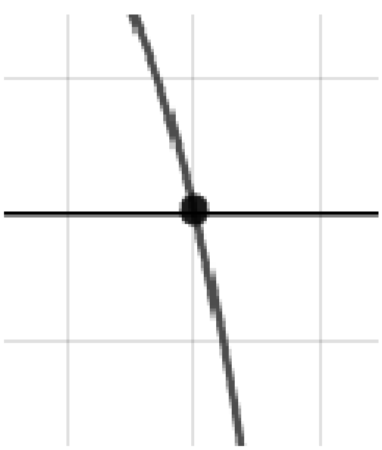
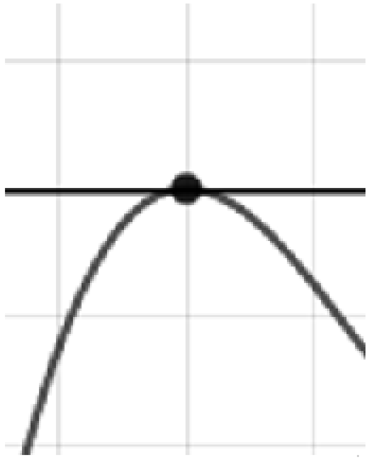
4. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

$$f(x) = -4(x+4)^8(x-4)^7(x-5)^9(x+5)^4$$

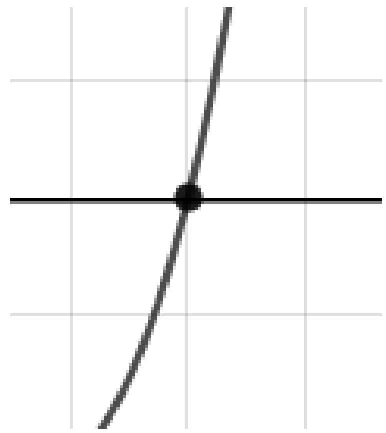
The solution is the graph below, which is option A.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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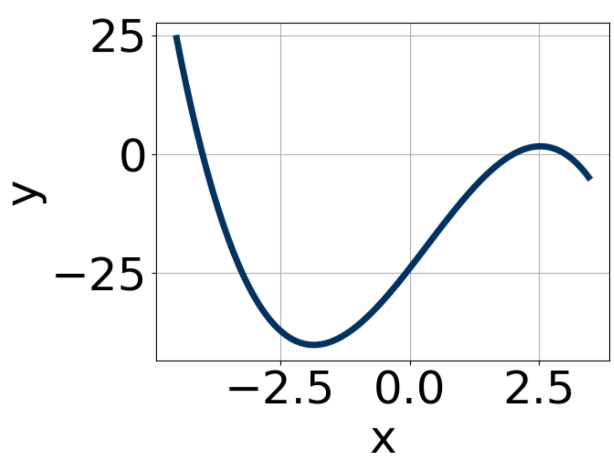
5. Which of the following equations *could* be of the graph presented below?



The solution is $-11(x-3)^{11}(x-2)^{5}(x+4)^{9}$, which is option C.

A. $6(x-3)^{8}(x-2)^{7}(x+4)^{5}$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-9(x-3)^8(x-2)^{10}(x+4)^7$

The factors 3 and 2 have have been odd power.

C. $-11(x-3)^{11}(x-2)^5(x+4)^9$

* This is the correct option.

D. $-5(x-3)^{10}(x-2)^7(x+4)^7$

The factor 3 should have been an odd power.

E. $19(x-3)^7(x-2)^9(x+4)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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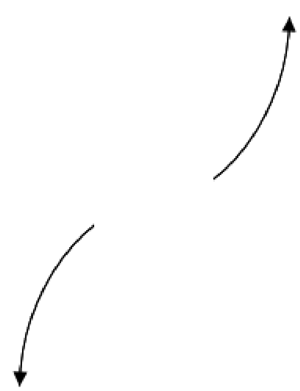
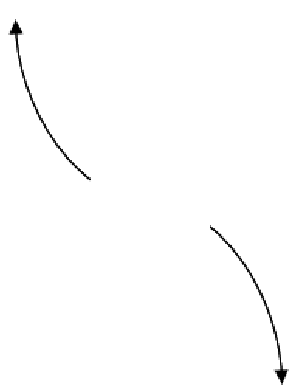
6. Describe the end behavior of the polynomial below.

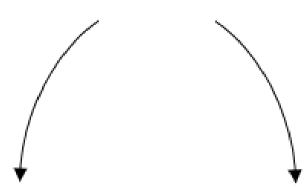
$$f(x) = 8(x+8)^2(x-8)^3(x+4)^5(x-4)^5$$

The solution is the graph below, which is option D.

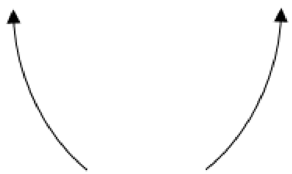


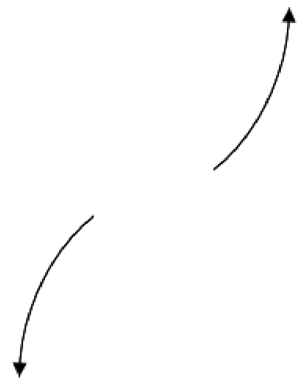


A.



B.

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 5i \text{ and } -4$$

The solution is $x^3 - 2x^2 + 10x + 136$, which is option B.

A. $b \in [0.96, 1.62], c \in [7.42, 9.73]$, and $d \in [17, 22]$

   $x^3 + x^2 + 9x + 20$, which corresponds to multiplying out $(x+5)(x+4)$.

B. $b \in [-2.98, -1.83], c \in [9.75, 11.93]$, and $d \in [132, 142]$

   * $x^3 - 2x^2 + 10x + 136$, which is the correct option.

C. $b \in [1.06, 2.5], c \in [9.75, 11.93]$, and $d \in [-137, -131]$

   $x^3 + 2x^2 + 10x - 136$, which corresponds to multiplying out $(x-(3-5i))(x-(3+5i))(x-4)$.

D. $b \in [0.96, 1.62], c \in [0.77, 1.03]$, and $d \in [-18, -5]$

   $x^3 + x^2 + x - 12$, which corresponds to multiplying out $(x-3)(x+4)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x-(3-5i))(x-(3+5i))(x-(-4))$.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$2, \frac{3}{5}, \text{ and } 4$$

The solution is $5x^3 - 33x^2 + 58x - 24$, which is option B.

A. $a \in [2, 7], b \in [-10, -1], c \in [-54, -35],$ and $d \in [-28, -21]$

$5x^3 - 7x^2 - 46x - 24$, which corresponds to multiplying out $(x + 2)(5x + 3)(x - 4)$.

B. $a \in [2, 7], b \in [-38, -29], c \in [54, 63],$ and $d \in [-28, -21]$

* $5x^3 - 33x^2 + 58x - 24$, which is the correct option.

C. $a \in [2, 7], b \in [-38, -29], c \in [54, 63],$ and $d \in [19, 31]$

$5x^3 - 33x^2 + 58x + 24$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [2, 7], b \in [32, 34], c \in [54, 63],$ and $d \in [19, 31]$

$5x^3 + 33x^2 + 58x + 24$, which corresponds to multiplying out $(x + 2)(5x + 3)(x + 4)$.

E. $a \in [2, 7], b \in [-18, -8], c \in [-41, -31],$ and $d \in [19, 31]$

$5x^3 - 13x^2 - 34x + 24$, which corresponds to multiplying out $(x + 2)(5x - 3)(x - 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-2)(5x-3)(x-4)$

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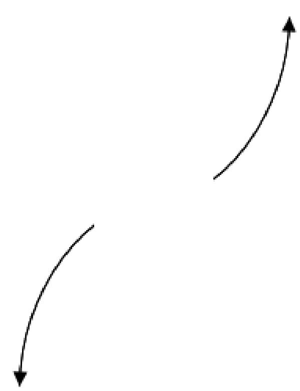
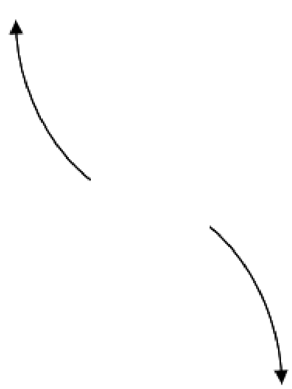
9. Describe the end behavior of the polynomial below.

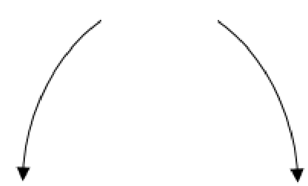
$$f(x) = 4(x - 6)^4(x + 6)^7(x + 8)^5(x - 8)^5$$

The solution is the graph below, which is option D.

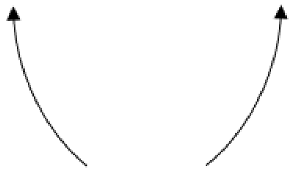




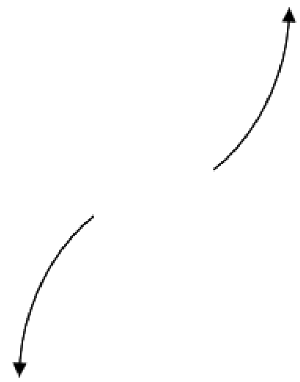
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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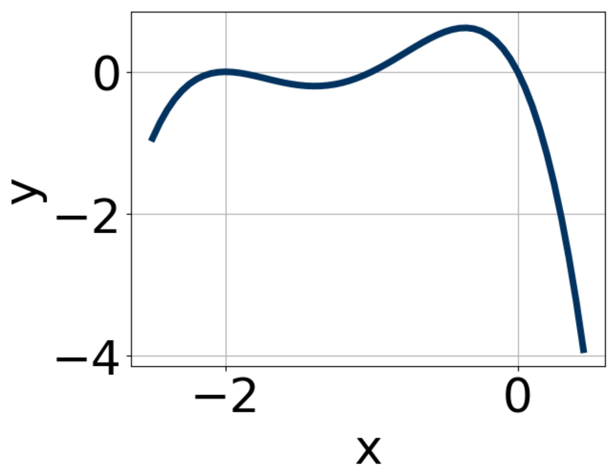
10. Which of the following equations *could* be of the graph presented below?



The solution is $-19x^9(x+2)^4(x+1)^7$, which is option C.

A. $-20x^9(x+2)^7(x+1)^{10}$

The factor -2 should have an even power and the factor -1 should have an odd power.

B. $14x^8(x+2)^8(x+1)^9$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

C. $-19x^9(x+2)^4(x+1)^7$

* This is the correct option.

D. $19x^5(x+2)^{10}(x+1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-18x^7(x+2)^4(x+1)^{10}$

The factor $(x+1)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).