**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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66. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder $r$.

$$\frac{4x^3 - 12x + 5}{x + 2}$$

The solution is $4x^2 - 8x + 4 + \frac{-3}{x+2}$

A. $a \in [0, 5], b \in [3, 9], c \in [1, 7], \text{ and } r \in [11, 17]$.

You divided by the opposite of the factor.

B. $a \in [-14, -6], b \in [15, 20], c \in [-48, -39], \text{ and } r \in [91, 94]$.

You multipled by the synthetic number rather than bringing the first factor down.

C. $a \in [0, 5], b \in [-10, -2], c \in [1, 7], \text{ and } r \in [-7, -1]$.

* This is the solution!

D. $a \in [-14, -6], b \in [-18, -13], c \in [-48, -39], \text{ and } r \in [-84, -78]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [0, 5], b \in [-13, -10], c \in [23, 26], \text{ and } r \in [-69, -66]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comments: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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67. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 - 5x^2 - 22x + 24$$

The solution is $[-2, 1.3333333333333333, 1.5]$

A. $z_1 \in [-2.82, -1.65], \; z_2 \in [-0.5, 0.9], \text{ and } z_3 \in [0.25, 1.03]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-3.34, -2.54], \; z_2 \in [-1, -0.3], \text{ and } z_3 \in [1.94, 2.41]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-2.82, -1.65], \; z_2 \in [1.2, 2.2], \text{ and } z_3 \in [1.45, 1.52]$

* This is the solution!

D. $z_1 \in [-1.57, -1.04], \; z_2 \in [-1.9, -1.2], \text{ and } z_3 \in [1.94, 2.41]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-0.93, -0.6]$, $z_2 \in [-1, -0.3]$, and $z_3 \in [1.94, 2.41]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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68. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder $r$.

$$\frac{6x^3 + 43x^2 + 86x + 38}{x + 4}$$

The solution is $6x^2 + 19x + 10 + \dfrac{-2}{x+4}$

A. $a \in [2, 10]$, $b \in [17, 21]$, $c \in [8, 17]$, and $r \in [-3, 0]$.

* This is the solution!

B. $a \in [-33, -18]$, $b \in [134, 145]$, $c \in [-473, -467]$, and $r \in [1917, 1919]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [2, 10]$, $b \in [65, 70]$, $c \in [349, 360]$, and $r \in [1451, 1462]$.

You divided by the opposite of the factor.

D. $a \in [2, 10]$, $b \in [11, 15]$, $c \in [19, 25]$, and $r \in [-75, -65]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [-33, -18]$, $b \in [-57, -51]$, $c \in [-131, -118]$, and $r \in [-469, -465]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comments: Be sure to synthetically divide by the zero of the denominator!

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69. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 2x^4 + 4x^3 + 3x^2 + 7x + 7$$

The solution is $\pm 1, \pm 7$

A. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2}$

This would have been the solution **if asked for the possible Rational roots**!

B. $\pm 1, \pm 2$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comments: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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70. Factor the polynomial below completely, knowing that $x+5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 + 33x^3 - 165x^2 - 448x - 240$$

The solution is $[-5, -1.5, -0.8, 4]$

A. $z_1 \in [-4.2, -1.2]$, $z_2 \in [0.22, 0.43]$, $z_3 \in [2.96, 3.09]$, and $z_4 \in [4.04, 5.73]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-7.3, -4.2]$, $z_2 \in [-1.53, -1.46]$, $z_3 \in [-0.94, -0.78]$, and $z_4 \in [3.87, 4.67]$

* This is the solution!

C. $z_1 \in [-4.2, -1.2]$, $z_2 \in [0.47, 0.77]$, $z_3 \in [0.9, 1.27]$, and $z_4 \in [4.04, 5.73]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-7.3, -4.2]$, $z_2 \in [-1.39, -1.22]$, $z_3 \in [-0.72, -0.65]$, and $z_4 \in [3.87, 4.67]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-4.2, -1.2]$, $z_2 \in [0.74, 1]$, $z_3 \in [1.36, 1.51]$, and $z_4 \in [4.04, 5.73]$

Distractor 1: Corresponds to negatives of all zeros.

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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