This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

26. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$4-5i$$
 and  $4$ 

The solution is  $x^3 - 12x^2 + 73x - 164$ 

- A.  $b \in [-17, -8], c \in [72, 75], \text{ and } d \in [-165, -158]$ \*  $x^3 - 12x^2 + 73x - 164$ , which is the correct option.
- B.  $b \in [0, 3], c \in [-7, 4]$ , and  $d \in [-27, -16]$  $x^3 + x^2 + x - 20$ , which corresponds to multiplying out (x + 5)(x - 4).
- C.  $b \in [9, 17], c \in [72, 75]$ , and  $d \in [162, 169]$  $x^3 + 12x^2 + 73x + 164$ , which corresponds to multiplying out (x - (4 - 5i))(x - (4 + 5i))(x + 4).
- D.  $b \in [0,3], c \in [-9,-4]$ , and  $d \in [10,22]$  $x^3 + x^2 - 8x + 16$ , which corresponds to multiplying out (x-4)(x-4).
- E. None of the above.

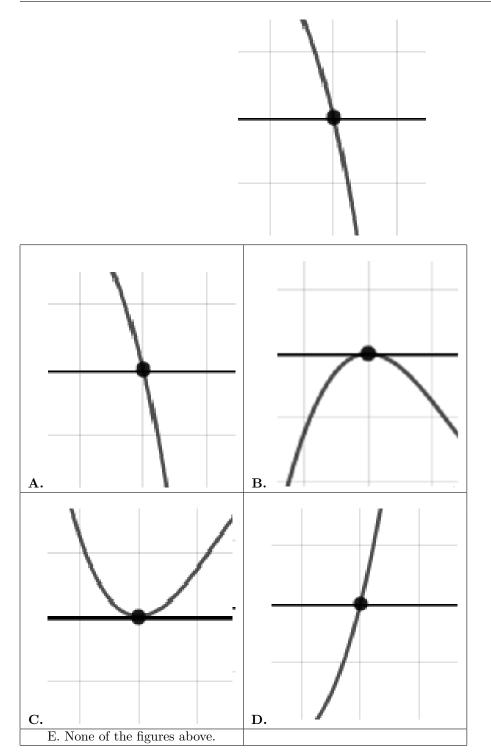
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comments: Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (4 - 5i))(x - (4 + 5i))(x - (4)).

27. Describe the zero behavior of the zero x = 9 of the polynomial below.

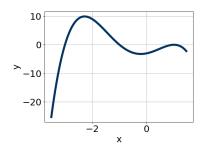
$$f(x) = -7(x-3)^{10}(x+3)^8(x-9)^5(x+9)^2$$

The solution is



**General Comments:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

<sup>28.</sup> Which of the following equations *could* be of the graph presented below?



The solution is  $-6(x-1)^6(x+3)^7(x+1)^7$ 

A. 
$$-6(x-1)^{10}(x+3)^8(x+1)^9$$

The factor (x+3) should have an odd power.

B. 
$$19(x-1)^{10}(x+3)^{11}(x+1)^7$$

This corresponds to the leading coefficient being the opposite value than it should be.

C. 
$$-6(x-1)^6(x+3)^7(x+1)^7$$

\* This is the correct option.

D. 
$$-2(x-1)^9(x+3)^{10}(x+1)^{11}$$

The factor 1 should have an even power and the factor -3 should have an odd power.

E. 
$$4(x-1)^{10}(x+3)^9(x+1)^6$$

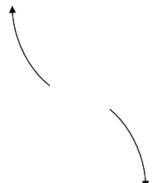
The factor (x + 1) should have an odd power and the leading coefficient should be the opposite sign.

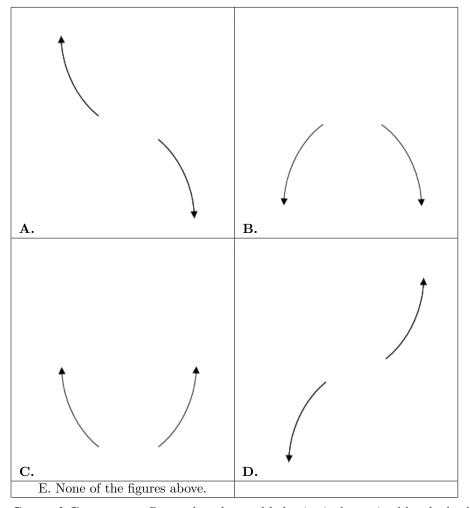
General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

29. Describe the end behavior of the polynomial below.

$$f(x) = -3(x-7)^5(x+7)^{10}(x+4)^2(x-4)^4$$

The solution is





**General Comments:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

30. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{1}{2}, \frac{6}{5}$$
, and  $\frac{5}{3}$ 

The solution is  $30x^3 - 101x^2 + 103x - 30$ 

A.  $a \in [29, 35], b \in [-102, -98], c \in [102, 106], \text{ and } d \in [-36, -28]$ \*  $30x^3 - 101x^2 + 103x - 30$ , which is the correct option.

B. 
$$a \in [29, 35], b \in [100, 103], c \in [102, 106], \text{ and } d \in [25, 33]$$
  
  $30x^3 + 101x^2 + 103x + 30, \text{ which corresponds to multiplying out } (2x + 1)(5x + 6)(3x + 5).$ 

C.  $a \in [29, 35], b \in [-102, -98], c \in [102, 106]$ , and  $d \in [25, 33]$  $30x^3 - 101x^2 + 103x + 30$ , which corresponds to multiplying everything correctly except the constant term.

D. 
$$a \in [29, 35], b \in [-82, -69], c \in [15, 19], \text{ and } d \in [25, 33]$$
  
  $30x^3 - 71x^2 + 17x + 30$ , which corresponds to multiplying out  $(2x + 2)(5x - 5)(3x - 3)$ .

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E. a \in [29, 35], b \in [-3, 2], c \in [-71, -61], \text{ and } d \in [-36, -28]
30x^3 + x^2 - 67x - 30, which corresponds to multiplying out (2x + 2)(5x + 5)(3x - 3).
```

General Comments: To construct the lowest-degree polynomial, you want to multiply out (2x - 1)(5x - 6)(3x - 5)

 $\operatorname{Summer} \operatorname{C} 2020$