This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{3}{2}, -6, \text{ and } \frac{5}{3}$$

The solution is $6x^3 + 17x^2 - 99x + 90$, which is option E.

A. $a \in [5, 11], b \in [34, 42], c \in [-24, -15],$ and $d \in [-99, -85]$

$6x^3 + 35x^2 - 21x - 90$, which corresponds to multiplying out $(2x + 2)(x - 1)(3x - 3)$.

B. $a \in [5, 11], b \in [14, 18], c \in [-103, -96],$ and $d \in [-99, -85]$

$6x^3 + 17x^2 - 99x - 90$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [5, 11], b \in [-22, -16], c \in [-103, -96],$ and $d \in [-99, -85]$

$6x^3 - 17x^2 - 99x - 90$, which corresponds to multiplying out $(2x + 3)(x - 6)(3x + 5)$.

D. $a \in [5, 11], b \in [-41, -32], c \in [-11, -3],$ and $d \in [87, 95]$

$6x^3 - 37x^2 - 9x + 90$, which corresponds to multiplying out $(2x + 2)(x + 1)(3x - 3)$.

E. $a \in [5, 11], b \in [14, 18], c \in [-103, -96],$ and $d \in [87, 95]$

* $6x^3 + 17x^2 - 99x + 90$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 3)(x + 6)(3x - 5)$

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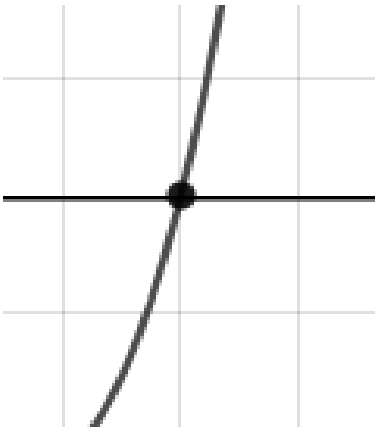
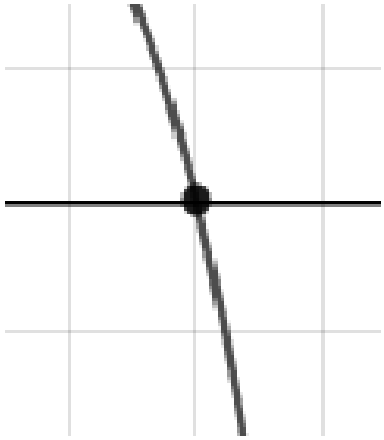
2. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

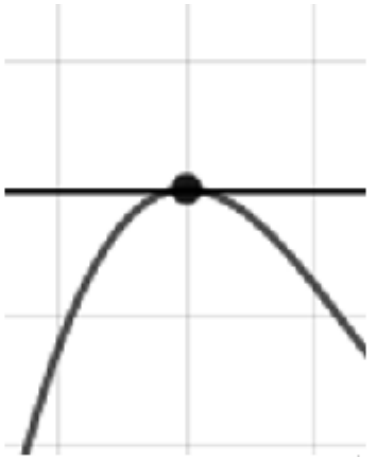
$$f(x) = 6(x - 6)^{10}(x + 6)^9(x + 3)^{12}(x - 3)^7$$

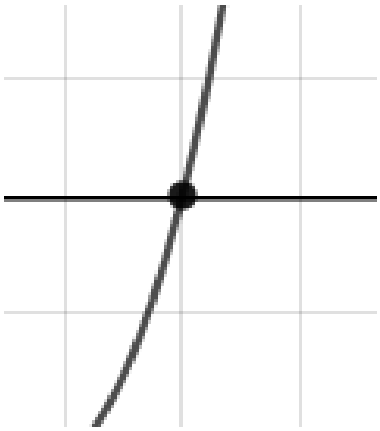
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$6, 4, \text{ and } \frac{5}{4}$$

The solution is $4x^3 - 45x^2 + 146x - 120$, which is option A.

A. $a \in [4, 8], b \in [-48, -38], c \in [139, 147],$ and $d \in [-122, -114]$

* $4x^3 - 45x^2 + 146x - 120$, which is the correct option.

B. $a \in [4, 8], b \in [-1, 4], c \in [-110, -99],$ and $d \in [117, 124]$

$4x^3 + 3x^2 - 106x + 120$, which corresponds to multiplying out $(x + 1)(x - 1)(4x - 4)$.

C. $a \in [4, 8], b \in [-48, -38], c \in [139, 147],$ and $d \in [117, 124]$

$4x^3 - 45x^2 + 146x + 120$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [4, 8], b \in [45, 46], c \in [139, 147],$ and $d \in [117, 124]$

$4x^3 + 45x^2 + 146x + 120$, which corresponds to multiplying out $(x + 6)(x + 4)(4x + 5)$.

E. $a \in [4, 8], b \in [29, 37], c \in [42, 52],$ and $d \in [-122, -114]$

$4x^3 + 35x^2 + 46x - 120$, which corresponds to multiplying out $(x + 1)(x + 1)(4x - 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-6)(x-4)(4x - 5)$

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 4i \text{ and } 2$$

The solution is $x^3 + 6x^2 + 16x - 64$, which is option D.

A. $b \in [0, 1.3], c \in [-8, -4],$ and $d \in [1, 15]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 4)(x - 2)$.

B. $b \in [0, 1.3], c \in [0, 4],$ and $d \in [-15, -3]$

$x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x + 4)(x - 2)$.

C. $b \in [-7.3, -2], c \in [12, 25],$ and $d \in [64, 74]$

$x^3 - 6x^2 + 16x + 64$, which corresponds to multiplying out $(x - (-4 + 4i))(x - (-4 - 4i))(x + 2)$.

D. $b \in [4.2, 7.3], c \in [12, 25],$ and $d \in [-64, -61]$

* $x^3 + 6x^2 + 16x - 64$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 4i))(x - (-4 - 4i))(x - (2))$.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 5i \text{ and } 4$$

The solution is $x^3 + 13x - 116$, which is option C.

A. $b \in [-1.34, 0.32], c \in [12.8, 14.7],$ and $d \in [113, 118]$

$x^3 + 13x + 116$, which corresponds to multiplying out $(x - (-2 - 5i))(x - (-2 + 5i))(x + 4)$.

B. $b \in [0.71, 1.99], c \in [-2.2, -0.1],$ and $d \in [-13, -3]$

$x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x + 2)(x - 4)$.

C. $b \in [-1.34, 0.32], c \in [12.8, 14.7],$ and $d \in [-121, -114]$

* $x^3 + 13x - 116$, which is the correct option.

D. $b \in [0.71, 1.99], c \in [-1.7, 1.2],$ and $d \in [-24, -19]$

$x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-2-5i))(x-(-2+5i))(x-(4))$.

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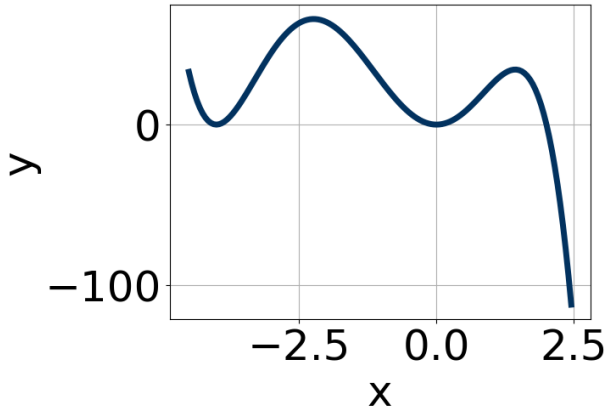
6. Which of the following equations *could* be of the graph presented below?



The solution is $-17x^{10}(x+4)^8(x-2)^9$, which is option B.

A. $9x^6(x+4)^4(x-2)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-17x^{10}(x+4)^8(x-2)^9$

* This is the correct option.

C. $-9x^{10}(x+4)^7(x-2)^4$

The factor $(x+4)$ should have an even power and the factor $(x-2)$ should have an odd power.

D. $5x^{10}(x+4)^8(x-2)^6$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-10x^{10}(x+4)^7(x-2)^9$

The factor $(x+4)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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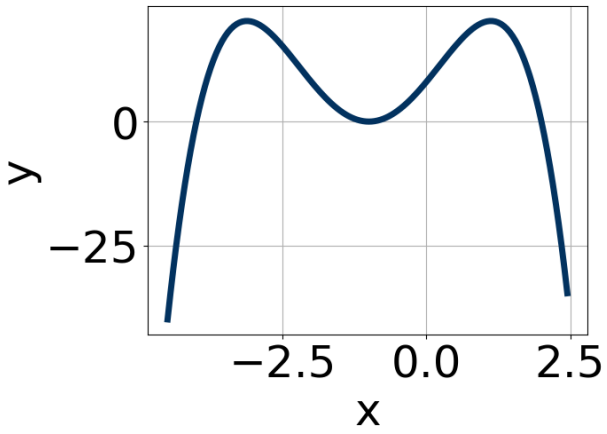
7. Which of the following equations *could* be of the graph presented below?



The solution is $-16(x+1)^4(x+4)^9(x-2)^9$, which is option E.

A. $-10(x+1)^9(x+4)^8(x-2)^9$

The factor -1 should have an even power and the factor -4 should have an odd power.

B. $-16(x+1)^6(x+4)^6(x-2)^5$

The factor $(x+4)$ should have an odd power.

C. $12(x+1)^4(x+4)^9(x-2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $5(x+1)^8(x+4)^5(x-2)^{10}$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-16(x+1)^4(x+4)^9(x-2)^9$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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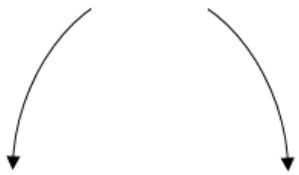
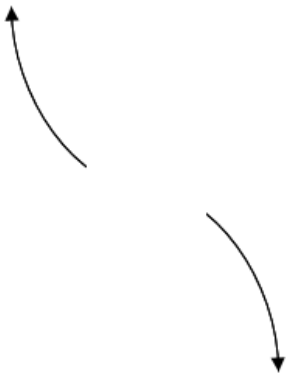
8. Describe the end behavior of the polynomial below.

$$f(x) = -4(x-4)^2(x+4)^7(x-7)^3(x+7)^4$$

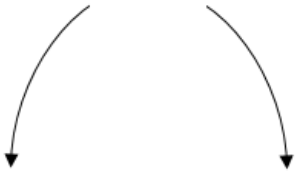
The solution is the graph below, which is option B.



A.

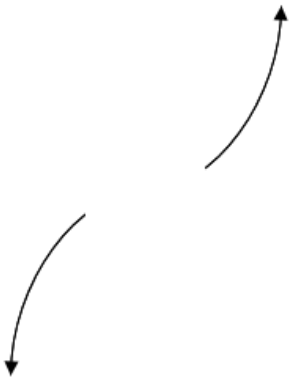


B.



C.

D.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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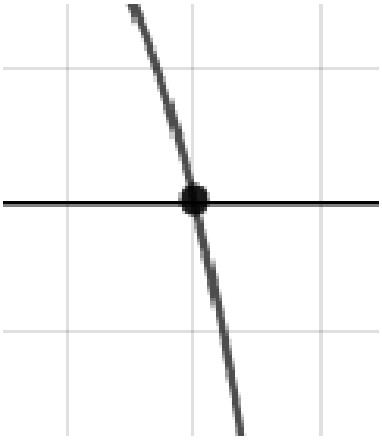
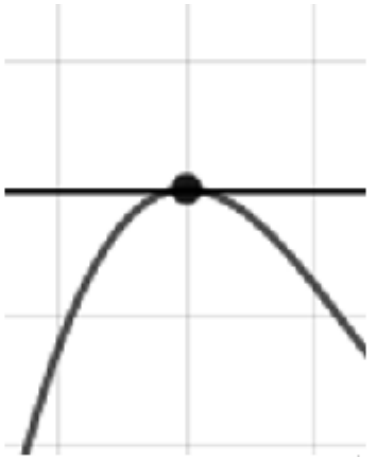
9. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

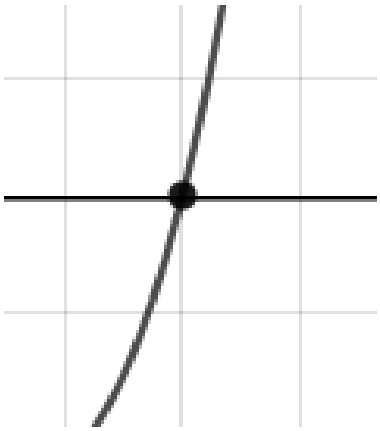
$$f(x) = 7(x-3)^9(x+3)^{10}(x+4)^9(x-4)^{13}$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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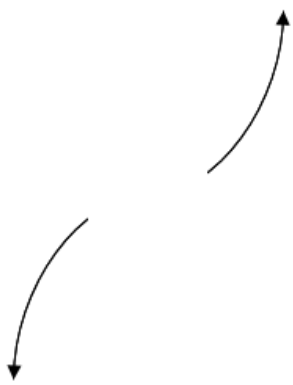
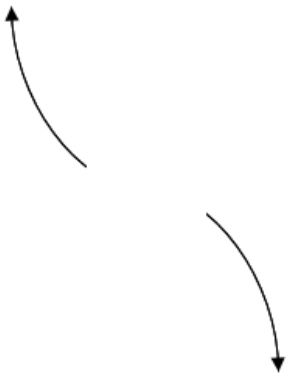
10. Describe the end behavior of the polynomial below.

$$f(x) = 2(x+4)^2(x-4)^5(x+2)^3(x-2)^5$$

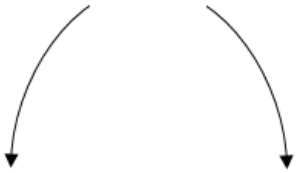
The solution is the graph below, which is option D.



A.

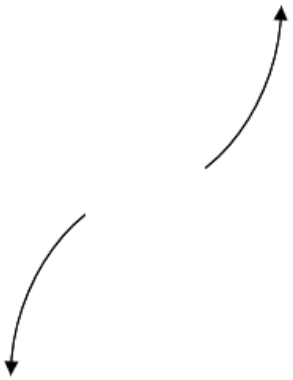


B.



C.

D.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.
