This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 6x + 3 \text{ and } g(x) = \frac{4}{4x + 17}$$

The solution is The domain is all Real numbers except $x = -4.25$, which is option A.

A. The domain is all Real numbers except $x = a,$ where $a \in [-4.25, 0.75]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-8.5, 5.5]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-8.75, -4.75]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [2.75, 8.75]$ and $b \in [-1.75, 7.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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2. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -12$ and choose the interval the $f^{-1}(-12)$ belongs to.

$$f(x) = \sqrt[3]{4x + 3}$$

The solution is -432.75, which is option A.

A. $f^{-1}(-12) \in [-433.9, -431.9]$

* This is the correct solution.

B. $f^{-1}(-12) \in [-431.6, -428.2]$

Distractor 1: This corresponds to

C. $f^{-1}(-12) \in [431.8, 433.7]$

This solution corresponds to distractor 2.

D. $f^{-1}(-12) \in [429, 432.7]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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3. Determine whether the function below is 1-1.

$$f(x) = \sqrt{-4x - 15}$$

The solution is yes, which is option D.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. Yes, the function is 1-1.

* This is the solution.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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4. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval the $f^{-1}(11)$ belongs to.

$$f(x) = \sqrt[3]{5x+3}$$

The solution is 265.6, which is option B.

A. $f^{-1}(11) \in [-265.87, -265.38]$

This solution corresponds to distractor 2.

B. $f^{-1}(11) \in [265.17, 265.77]$

* This is the correct solution.

C. $f^{-1}(11) \in [-267.63, -266.79]$

This solution corresponds to distractor 3.

D. $f^{-1}(11) \in [266.38, 267.55]$

Distractor 1: This corresponds to

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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5. Find the inverse of the function below. Then, evaluate the inverse at $x = 5$ and choose the interval that $f^{-1}(5)$ belongs to.

$$f(x) = \ln(x+2) - 3$$

The solution is $f^{-1}(5) = 2978.958$, which is option A.

A. $f^{-1}(5) \in [2975.96, 2980.96]$

This is the solution.

B. $f^{-1}(5) \in [4.39, 9.39]$

This solution corresponds to distractor 1.

C. $f^{-1}(5) \in [14.09, 20.09]$

This solution corresponds to distractor 2.

D. $f^{-1}(5) \in [1088.63, 1100.63]$

This solution corresponds to distractor 4.

E. $f^{-1}(5) \in [2979.96, 2983.96]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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6. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 2x^3 + 4x^2 + x \text{ and } g(x) = -x^3 + 2x^2 + 3x$$

The solution is 0.0, which is option D.

A. $(f \circ g)(-1) \in [8.77, 9.31]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [9.79, 11.27]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [2.48, 5.86]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [-0.31, 1.82]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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7. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 3x^3 + 3x^2 + x \text{ and } g(x) = -3x^3 - 2x^2 + 2x$$

The solution is -1.0, which is option B.

A. $(f \circ g)(-1) \in [1.2, 7.3]$

Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [-1.9, 0.2]$

* This is the correct solution

C. $(f \circ g)(-1) \in [7.5, 9.7]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-1.9, 0.2]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = \ln(x - 2) + 5$$

The solution is $f^{-1}(9) = 56.598$, which is option E.

A. $f^{-1}(9) \in [1202606.28, 1202610.28]$

This solution corresponds to distractor 1.

B. $f^{-1}(9) \in [59877.14, 59882.14]$

This solution corresponds to distractor 2.

C. $f^{-1}(9) \in [1101.63, 1103.63]$

This solution corresponds to distractor 4.

D. $f^{-1}(9) \in [49.6, 54.6]$

This solution corresponds to distractor 3.

E. $f^{-1}(9) \in [54.6, 59.6]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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9. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{4}{3x + 17} \text{ and } g(x) = \frac{4}{5x - 28}$$

The solution is The domain is all Real numbers except $x = -5.666666666666667$ and $x = 5.6$, which is option D.

A. The domain is all Real numbers except $x = a,$ where $a \in [3.67, 10.67]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [0.33, 7.33]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-5.33, -2.33]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-12.67, -3.67]$ and $b \in [4.6, 13.6]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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10. Determine whether the function below is 1-1.

$$f(x) = \sqrt{3x - 20}$$

The solution is yes, which is option C.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. Yes, the function is 1-1.

* This is the solution.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.
