This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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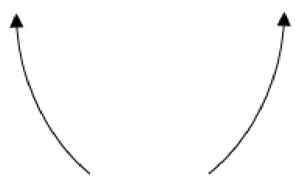
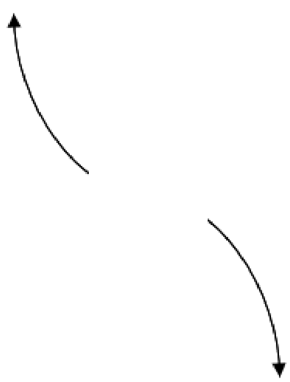
1. Describe the end behavior of the polynomial below.

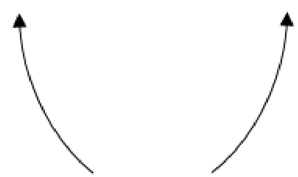
$$f(x) = 9(x-2)^2(x+2)^7(x+3)^4(x-3)^5$$

The solution is the graph below, which is option C.

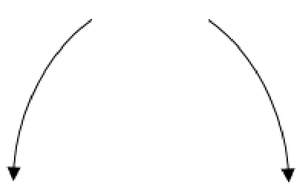




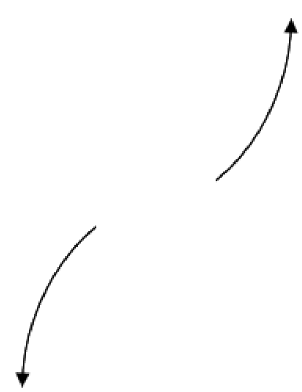
A.



C.



B.



D.

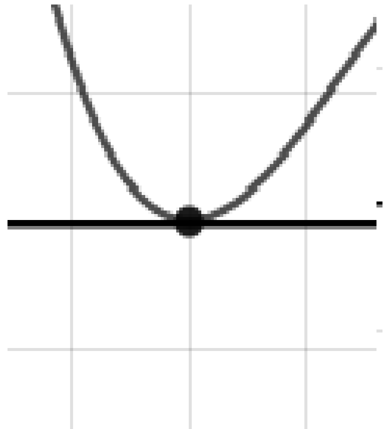
E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

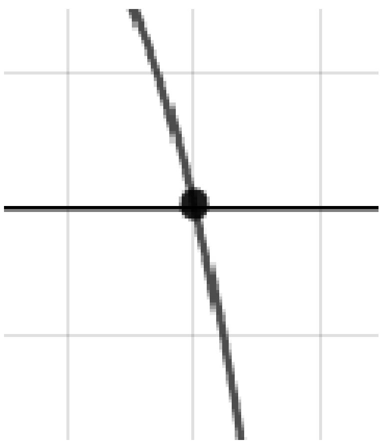
2. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = -4(x-8)^5(x+8)^4(x-3)^8(x+3)^5$$

The solution is the graph below, which is option C.

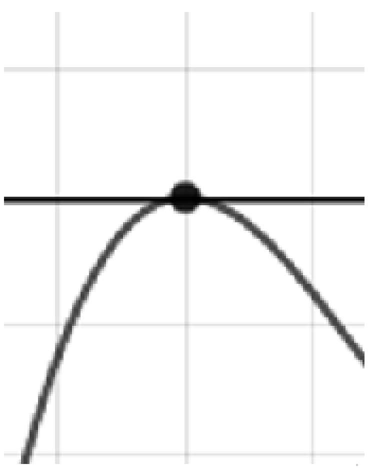


A.

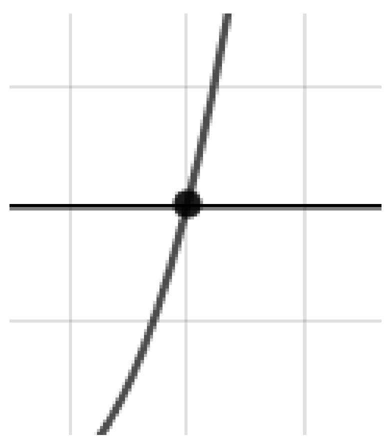


C.

B.



D.



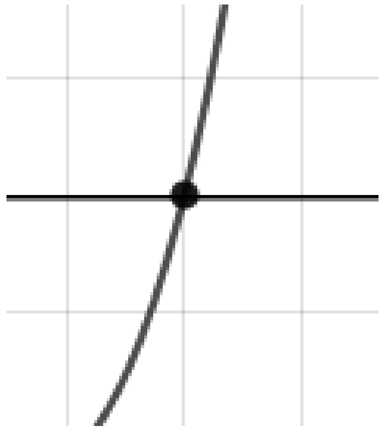
E. None of the above.

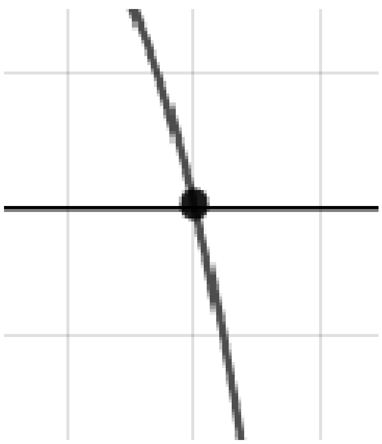
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

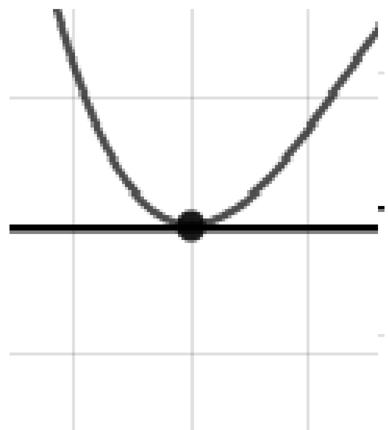
3. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

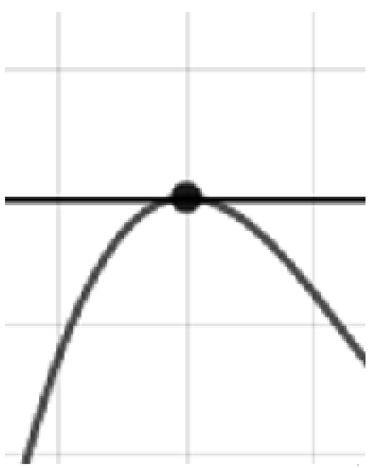
$$f(x) = 9(x+3)^5(x-3)^{10}(x-6)^8(x+6)^{11}$$

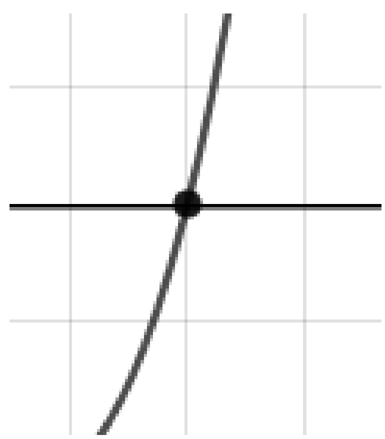
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 2i \text{ and } -1$$

The solution is $x^3 - 9x^2 + 19x + 29$, which is option C.

A. $b \in [-1, 7], c \in [0, 8]$, and $d \in [2, 6]$

$x^3 + x^2 + 3x + 2$, which corresponds to multiplying out $(x + 2)(x + 1)$.

B. $b \in [-1, 7], c \in [-9, -3]$, and $d \in [-5, -2]$

$x^3 + x^2 - 4x - 5$, which corresponds to multiplying out $(x - 5)(x + 1)$.

C. $b \in [-9, -6], c \in [15, 20]$, and $d \in [25, 36]$

* $x^3 - 9x^2 + 19x + 29$, which is the correct option.

D. $b \in [6, 16], c \in [15, 20]$, and $d \in [-30, -27]$

$x^3 + 9x^2 + 19x - 29$, which corresponds to multiplying out $(x - (5 - 2i))(x - (5 + 2i))(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 - 2i))(x - (5 + 2i))(x - (-1))$.

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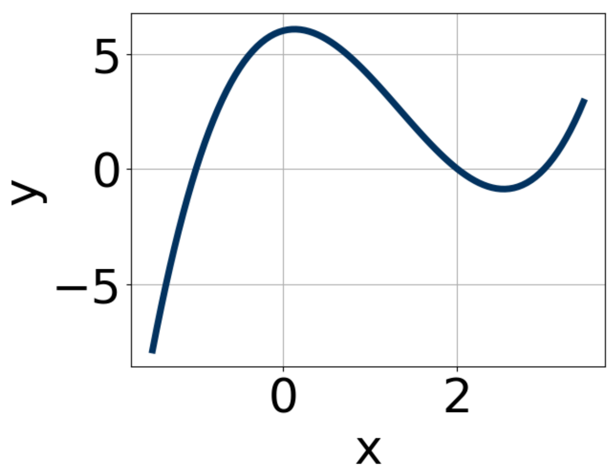
5. Which of the following equations *could* be of the graph presented below?



The solution is $10(x - 3)^7(x - 2)^7(x + 1)^{11}$, which is option E.

A. $-14(x - 3)^8(x - 2)^9(x + 1)^{11}$

The factor $(x - 3)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $18(x - 3)^6(x - 2)^7(x + 1)^7$

The factor 3 should have been an odd power.

C. $4(x-3)^4(x-2)^8(x+1)^7$

The factors 3 and 2 have have been odd power.

D. $-12(x-3)^5(x-2)^7(x+1)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $10(x-3)^7(x-2)^7(x+1)^{11}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{6}{5}, \frac{-7}{4}, \text{ and } \frac{-3}{4}$$

The solution is $80x^3 + 104x^2 - 135x - 126$, which is option E.

A. $a \in [75, 83], b \in [104, 107], c \in [-137, -125], \text{ and } d \in [122, 127]$

$80x^3 + 104x^2 - 135x + 126$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [75, 83], b \in [13, 24], c \in [-203, -197], \text{ and } d \in [-128, -119]$

$80x^3 + 16x^2 - 201x - 126$, which corresponds to multiplying out $(5x+5)(4x+4)(4x-4)$.

C. $a \in [75, 83], b \in [296, 299], c \in [341, 351], \text{ and } d \in [122, 127]$

$80x^3 + 296x^2 + 345x + 126$, which corresponds to multiplying out $(5x+5)(4x-4)(4x-4)$.

D. $a \in [75, 83], b \in [-104, -99], c \in [-137, -125], \text{ and } d \in [122, 127]$

$80x^3 - 104x^2 - 135x + 126$, which corresponds to multiplying out $(5x+6)(4x-7)(4x-3)$.

E. $a \in [75, 83], b \in [104, 107], c \in [-137, -125], \text{ and } d \in [-128, -119]$

* $80x^3 + 104x^2 - 135x - 126$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x-6)(4x+7)(4x+3)$

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 + 5i \text{ and } 4$$

The solution is $x^3 - 10x^2 + 58x - 136$, which is option C.

A. $b \in [-4, 4], c \in [-11, -8], \text{ and } d \in [19, 24]$

$x^3 + x^2 - 9x + 20$, which corresponds to multiplying out $(x-5)(x-4)$.

B. $b \in [-4, 4], c \in [-7, -4], \text{ and } d \in [6, 15]$

$x^3 + x^2 - 7x + 12$, which corresponds to multiplying out $(x-3)(x-4)$.

C. $b \in [-11, -4], c \in [54, 66], \text{ and } d \in [-137, -132]$

* $x^3 - 10x^2 + 58x - 136$, which is the correct option.

D. $b \in [10, 13], c \in [54, 66],$ and $d \in [135, 138]$

$x^3 + 10x^2 + 58x + 136$, which corresponds to multiplying out $(x - (3 + 5i))(x - (3 - 5i))(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 + 5i))(x - (3 - 5i))(x - (4))$.

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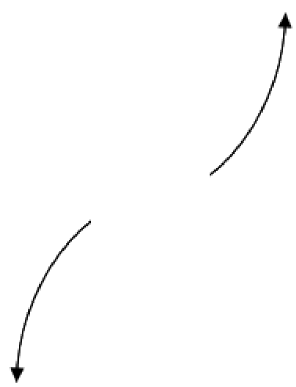
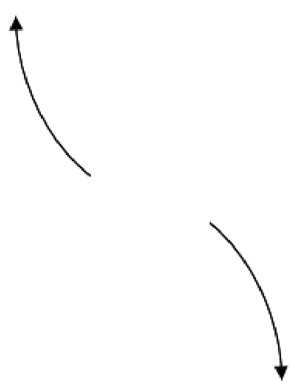
8. Describe the end behavior of the polynomial below.

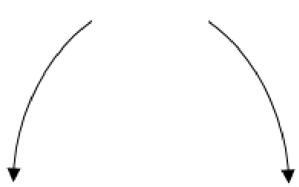
$$f(x) = 6(x + 9)^2(x - 9)^3(x + 6)^3(x - 6)^3$$

The solution is the graph below, which is option D.

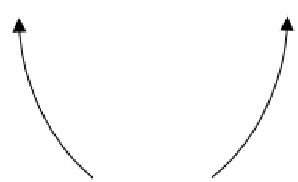




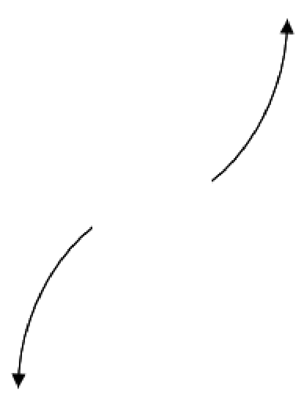
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{4}, \frac{-1}{5}, \text{ and } 5$$

The solution is $20x^3 - 91x^2 - 44x - 5$, which is option C.

A. $a \in [15, 26], b \in [-94, -89], c \in [-48, -42], \text{ and } d \in [0, 6]$

$20x^3 - 91x^2 - 44x + 5$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [15, 26], b \in [-101, -98], c \in [3, 5], \text{ and } d \in [0, 6]$

$20x^3 - 101x^2 + 4x + 5$, which corresponds to multiplying out $(4x + 4)(5x - 5)(x - 1)$.

C. $a \in [15, 26], b \in [-94, -89], c \in [-48, -42], \text{ and } d \in [-6, 2]$

* $20x^3 - 91x^2 - 44x - 5$, which is the correct option.

D. $a \in [15, 26], b \in [90, 92], c \in [-48, -42], \text{ and } d \in [0, 6]$

$20x^3 + 91x^2 - 44x + 5$, which corresponds to multiplying out $(4x - 1)(5x - 1)(x + 5)$.

E. $a \in [15, 26], b \in [-109, -104], c \in [45, 48], \text{ and } d \in [-6, 2]$

$20x^3 - 109x^2 + 46x - 5$, which corresponds to multiplying out $(4x + 4)(5x + 5)(x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 1)(5x + 1)(x - 5)$

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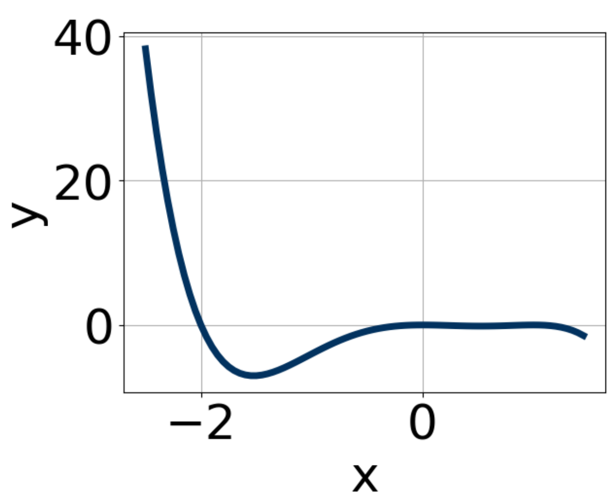
10. Which of the following equations *could* be of the graph presented below?



The solution is $-5x^{10}(x - 1)^8(x + 2)^7$, which is option A.

A. $-5x^{10}(x - 1)^8(x + 2)^7$

* This is the correct option.

B. $-9x^5(x-1)^4(x+2)^4$

The factor x should have an even power and the factor $(x+2)$ should have an odd power.

C. $-8x^7(x-1)^{10}(x+2)^{11}$

The factor x should have an even power.

D. $17x^{10}(x-1)^6(x+2)^6$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $4x^4(x-1)^{10}(x+2)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).