This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 + 2i \text{ and } -3$$

The solution is $x^3 + 9x^2 + 31x + 39$, which is option B.

A. $b \in [-5, 4], c \in [5.6, 7.3], \text{ and } d \in [0, 15]$

$x^3 + x^2 + 6x + 9$, which corresponds to multiplying out $(x + 3)(x + 3)$.

B. $b \in [7, 14], c \in [30.7, 35.8], \text{ and } d \in [32, 45]$

* $x^3 + 9x^2 + 31x + 39$, which is the correct option.

C. $b \in [-10, -6], c \in [30.7, 35.8], \text{ and } d \in [-46, -38]$

$x^3 - 9x^2 + 31x - 39$, which corresponds to multiplying out $(x - (-3 + 2i))(x - (-3 - 2i))(x - 3)$.

D. $b \in [-5, 4], c \in [-1.3, 3.2], \text{ and } d \in [-6, -3]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x - 2)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 2i))(x - (-3 - 2i))(x - (-3))$.

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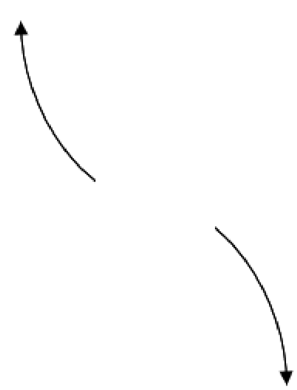
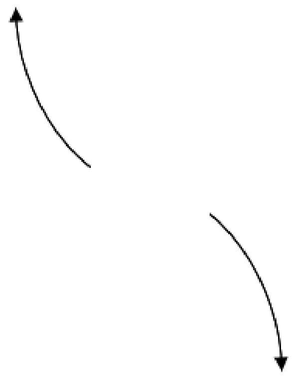
2. Describe the end behavior of the polynomial below.

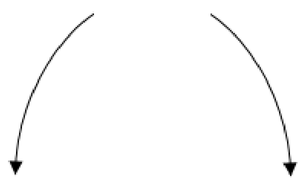
$$f(x) = -9(x + 3)^2(x - 3)^3(x + 7)^5(x - 7)^5$$

The solution is the graph below, which is option A.





A.



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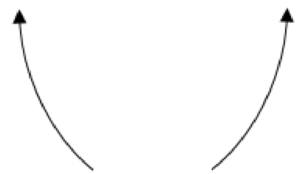
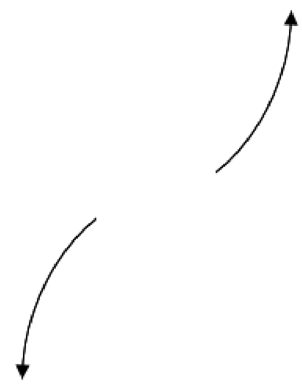


C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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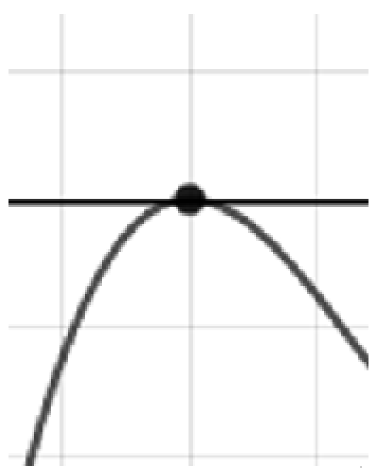
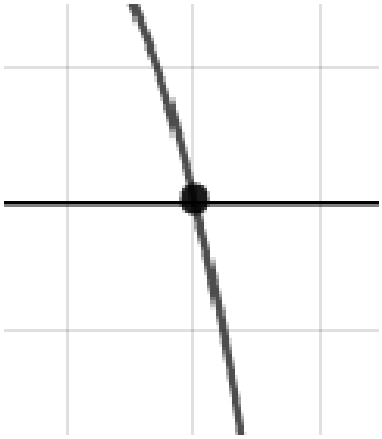
3. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

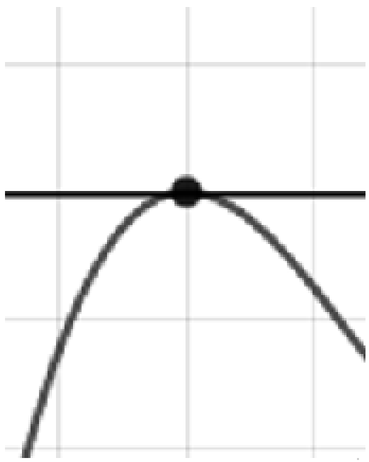
$$f(x) = -6(x-4)^7(x+4)^4(x+7)^7(x-7)^2$$

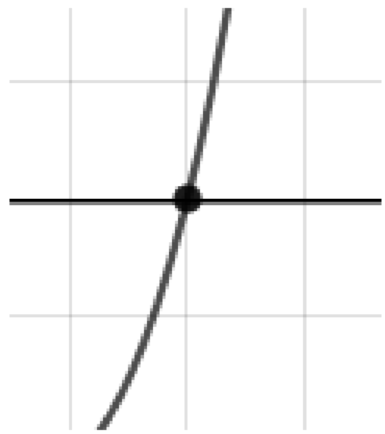
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{4}, -5, \text{ and } \frac{-4}{5}$$

The solution is $20x^3 + 91x^2 - 65x - 100$, which is option A.

A. $a \in [17, 27], b \in [86, 92], c \in [-71, -61], \text{ and } d \in [-100, -98]$

   * $20x^3 + 91x^2 - 65x - 100$, which is the correct option.

B. $a \in [17, 27], b \in [-59, -53], c \in [-192, -178], \text{ and } d \in [-100, -98]$

   $20x^3 - 59x^2 - 185x - 100$, which corresponds to multiplying out $(4x + 5)(x - 5)(5x + 4)$.

C. $a \in [17, 27], b \in [86, 92], c \in [-71, -61], \text{ and } d \in [98, 103]$

   $20x^3 + 91x^2 - 65x + 100$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [17, 27], b \in [140, 144], c \in [220, 226], \text{ and } d \in [98, 103]$

   $20x^3 + 141x^2 + 225x + 100$, which corresponds to multiplying out $(4x + 5)(x + 5)(5x + 4)$.

E. $a \in [17, 27], b \in [-98, -90], c \in [-71, -61],$ and $d \in [98, 103]$

$20x^3 - 91x^2 - 65x + 100$, which corresponds to multiplying out $(4x + 5)(x - 5)(5x - 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 5)(x + 5)(5x + 4)$

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{3}{4}, \frac{1}{2}, \text{ and } 6$$

The solution is $8x^3 - 58x^2 + 63x - 18$, which is option A.

A. $a \in [8, 11], b \in [-65, -57], c \in [61, 72],$ and $d \in [-19, -16]$

* $8x^3 - 58x^2 + 63x - 18$, which is the correct option.

B. $a \in [8, 11], b \in [-40, -35], c \in [-61, -55],$ and $d \in [-19, -16]$

$8x^3 - 38x^2 - 57x - 18$, which corresponds to multiplying out $(4x + 3)(2x + 1)(x - 6)$.

C. $a \in [8, 11], b \in [51, 60], c \in [61, 72],$ and $d \in [17, 19]$

$8x^3 + 58x^2 + 63x + 18$, which corresponds to multiplying out $(4x + 3)(2x + 1)(x + 6)$.

D. $a \in [8, 11], b \in [-65, -57], c \in [61, 72],$ and $d \in [17, 19]$

$8x^3 - 58x^2 + 63x + 18$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [8, 11], b \in [-48, -40], c \in [-17, -9],$ and $d \in [17, 19]$

$8x^3 - 46x^2 - 15x + 18$, which corresponds to multiplying out $(4x + 3)(2x - 1)(x - 6)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 3)(2x - 1)(x - 6)$

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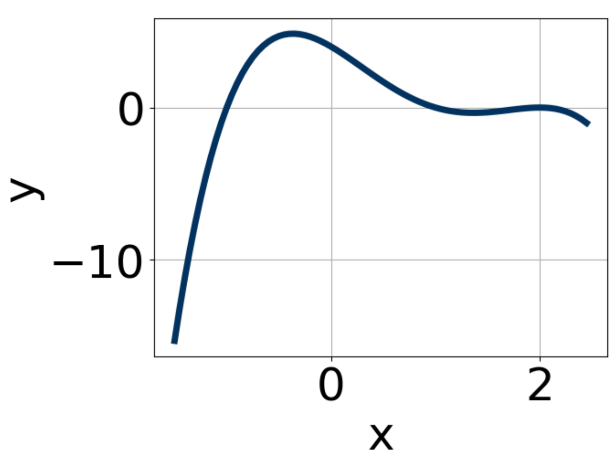
6. Which of the following equations *could* be of the graph presented below?



The solution is $-13(x - 2)^{10}(x - 1)^9(x + 1)^7$, which is option A.

A. $-13(x - 2)^{10}(x - 1)^9(x + 1)^7$

* This is the correct option.

B. $-5(x-2)^{11}(x-1)^{10}(x+1)^{5}$

The factor 2 should have an even power and the factor 1 should have an odd power.

C. $3(x-2)^{8}(x-1)^{7}(x+1)^{10}$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-10(x-2)^{6}(x-1)^{6}(x+1)^{9}$

The factor $(x-1)$ should have an odd power.

E. $6(x-2)^{10}(x-1)^{7}(x+1)^{9}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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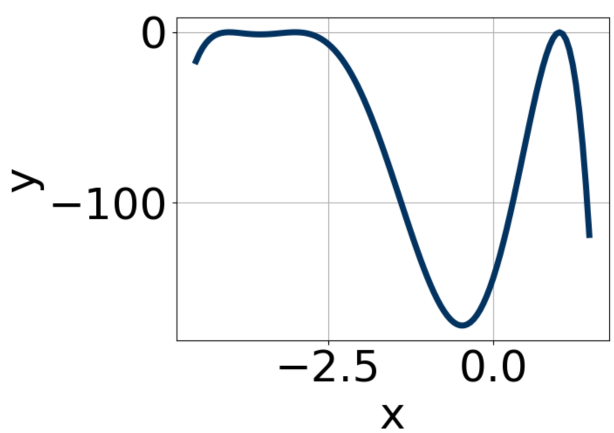
7. Which of the following equations *could* be of the graph presented below?



The solution is $-17(x+3)^{4}(x+4)^{6}(x-1)^{10}$, which is option E.

A. $-15(x+3)^{8}(x+4)^{5}(x-1)^{9}$

The factors $(x+4)$ and $(x-1)$ should both have even powers.

B. $11(x+3)^{10}(x+4)^{6}(x-1)^{9}$

The factor $(x-1)$ should have an even power and the leading coefficient should be the opposite sign.

C. $-11(x+3)^{6}(x+4)^{4}(x-1)^{11}$

The factor $(x-1)$ should have an even power.

D. $16(x+3)^{4}(x+4)^{8}(x-1)^{4}$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-17(x+3)^{4}(x+4)^{6}(x-1)^{10}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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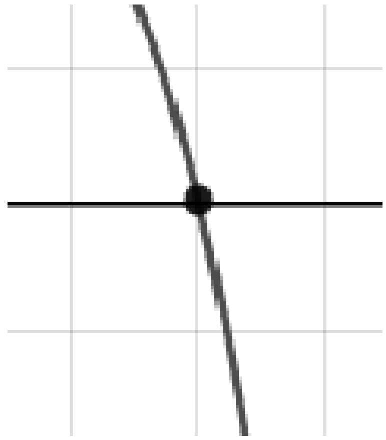
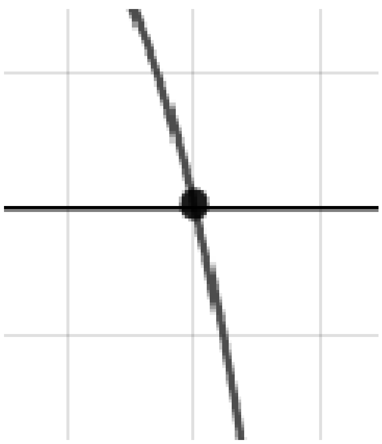
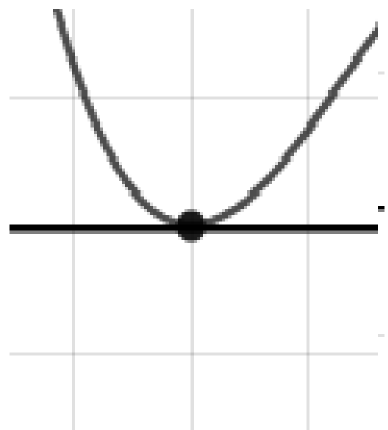
8. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

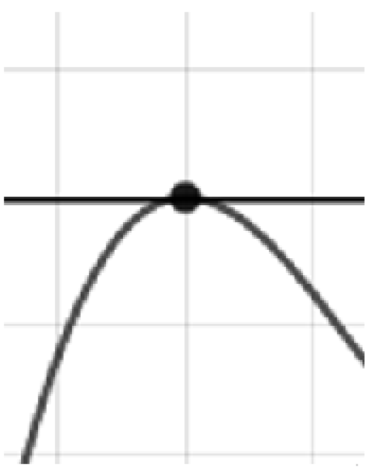
$$f(x) = -2(x-3)^4(x+3)^2(x-4)^9(x+4)^8$$

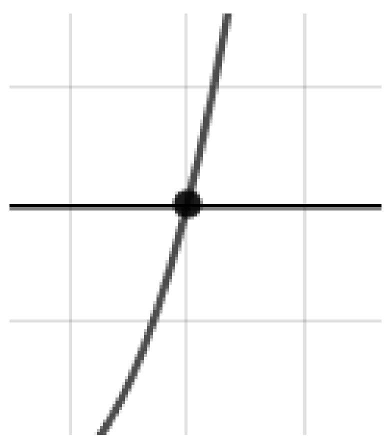
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 - 3i \text{ and } 3$$

The solution is $x^3 + 7x^2 + 4x - 102$, which is option A.

A. $b \in [6, 12], c \in [3.05, 4.75]$, and $d \in [-104, -101]$

* $x^3 + 7x^2 + 4x - 102$, which is the correct option.

B. $b \in [-11, -5], c \in [3.05, 4.75]$, and $d \in [101, 108]$

$x^3 - 7x^2 + 4x + 102$, which corresponds to multiplying out $(x - (-5 - 3i))(x - (-5 + 3i))(x + 3)$.

C. $b \in [-6, 4], c \in [-0.86, 0.21]$, and $d \in [-14, -3]$

$x^3 + x^2 + 0x - 9$, which corresponds to multiplying out $(x + 3)(x - 3)$.

D. $b \in [-6, 4], c \in [1.77, 2.48]$, and $d \in [-17, -13]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x + 5)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 - 3i))(x - (-5 + 3i))(x - (3))$.

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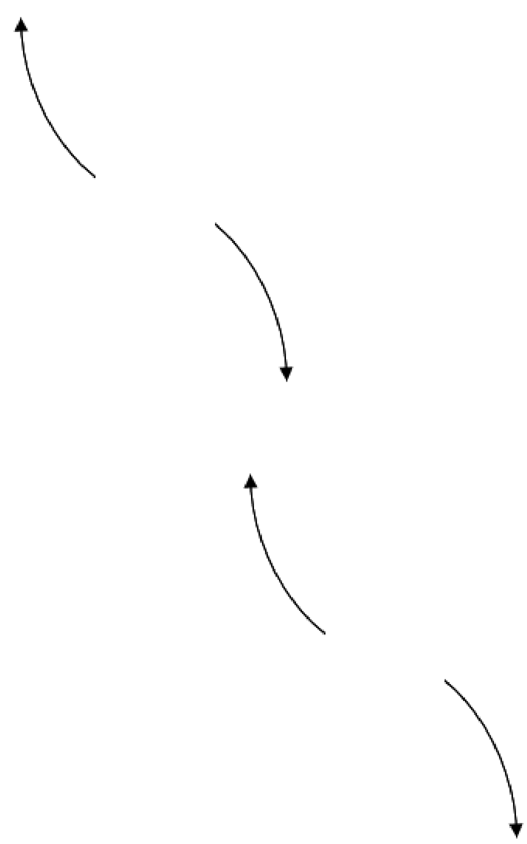
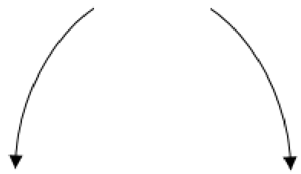
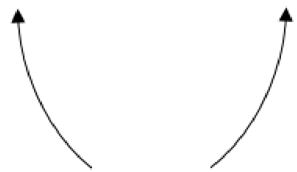
10. Describe the end behavior of the polynomial below.

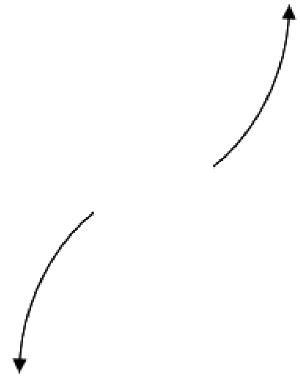
$$f(x) = -5(x - 6)^3(x + 6)^6(x + 4)^4(x - 4)^6$$

The solution is the graph below, which is option A.



B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.