This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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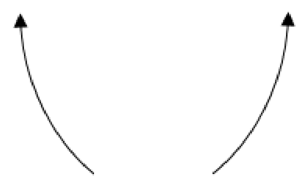
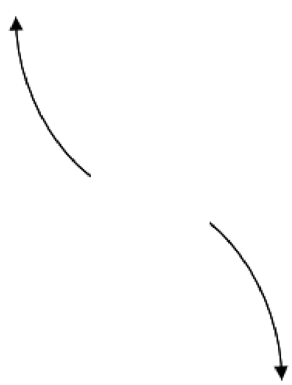
1. Describe the end behavior of the polynomial below.

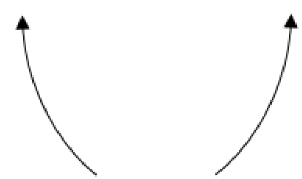
$$f(x) = 6(x+6)^5(x-6)^6(x+2)^2(x-2)^3$$

The solution is the graph below, which is option C.

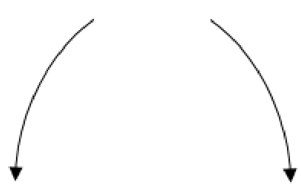




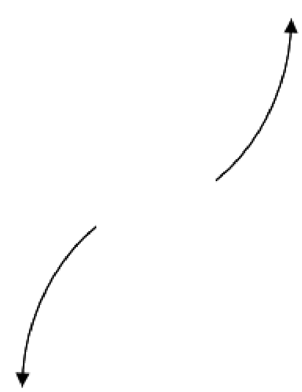
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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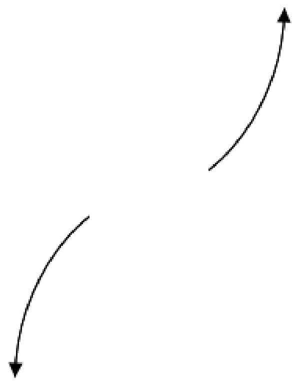
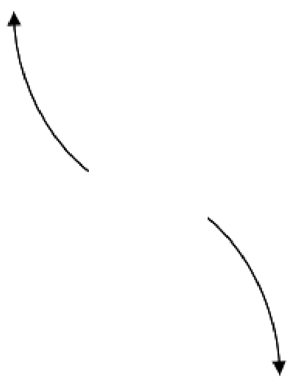
2. Describe the end behavior of the polynomial below.

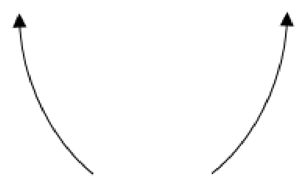
$$f(x) = 2(x-7)^3(x+7)^6(x+3)^3(x-3)^3$$

The solution is the graph below, which is option D.

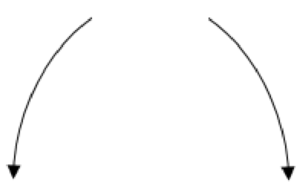




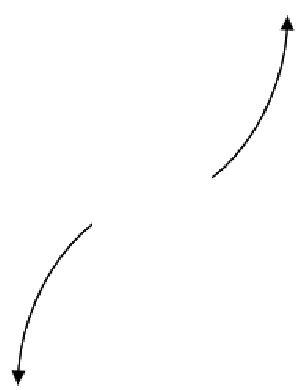
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{4}, 3, \text{ and } \frac{-1}{3}$$

The solution is $12x^3 - 11x^2 - 68x - 21$, which is option A.

A. $a \in [11, 19], b \in [-11, -6], c \in [-71, -67],$ and $d \in [-22, -17]$

* $12x^3 - 11x^2 - 68x - 21$, which is the correct option.

B. $a \in [11, 19], b \in [-63, -49], c \in [42, 47],$ and $d \in [9, 23]$

$12x^3 - 53x^2 + 44x + 21$, which corresponds to multiplying out $(4x + 4)(x - 1)(3x - 3)$.

C. $a \in [11, 19], b \in [18, 26], c \in [-61, -54],$ and $d \in [-22, -17]$

$12x^3 + 19x^2 - 58x - 21$, which corresponds to multiplying out $(4x + 4)(x + 1)(3x - 3)$.

D. $a \in [11, 19], b \in [-11, -6], c \in [-71, -67],$ and $d \in [9, 23]$

$12x^3 - 11x^2 - 68x + 21$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [11, 19], b \in [7, 17], c \in [-71, -67],$ and $d \in [9, 23]$

$12x^3 + 11x^2 - 68x + 21$, which corresponds to multiplying out $(4x - 7)(x + 3)(3x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 7)(x - 3)(3x + 1)$

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 2i \text{ and } -4$$

The solution is $x^3 + 10x^2 + 37x + 52$, which is option A.

A. $b \in [6, 17], c \in [35.32, 37.41],$ and $d \in [48.7, 52.2]$

* $x^3 + 10x^2 + 37x + 52$, which is the correct option.

B. $b \in [-6, 4], c \in [6.42, 8.44],$ and $d \in [10.6, 15.5]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 3)(x + 4)$.

C. $b \in [-11, -5], c \in [35.32, 37.41],$ and $d \in [-52.5, -50.6]$

$x^3 - 10x^2 + 37x - 52$, which corresponds to multiplying out $(x - (-3 - 2i))(x - (-3 + 2i))(x - 4)$.

D. $b \in [-6, 4], c \in [5.93, 6.06],$ and $d \in [6.6, 10.2]$

$x^3 + x^2 + 6x + 8$, which corresponds to multiplying out $(x + 2)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 2i))(x - (-3 + 2i))(x - (-4))$.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, 4, \text{ and } 7$$

The solution is $4x^3 - 45x^2 + 123x - 28$, which is option C.

A. $a \in [2, 5], b \in [-47.3, -43.3], c \in [123, 132],$ and $d \in [23, 29]$

$4x^3 - 45x^2 + 123x + 28$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [2, 5], b \in [-12.4, -9.4], c \in [-115, -111],$ and $d \in [-31, -19]$

$4x^3 - 11x^2 - 115x - 28$, which corresponds to multiplying out $(4x + 4)(x + 1)(x - 1)$.

C. $a \in [2, 5], b \in [-47.3, -43.3], c \in [123, 132],$ and $d \in [-31, -19]$

* $4x^3 - 45x^2 + 123x - 28$, which is the correct option.

D. $a \in [2, 5], b \in [43.4, 45.2], c \in [123, 132],$ and $d \in [23, 29]$

$4x^3 + 45x^2 + 123x + 28$, which corresponds to multiplying out $(4x + 1)(x + 4)(x + 7)$.

E. $a \in [2, 5], b \in [-43.9, -40.8], c \in [100, 102],$ and $d \in [23, 29]$

$4x^3 - 43x^2 + 101x + 28$, which corresponds to multiplying out $(4x + 4)(x - 1)(x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(x - 4)(x - 7)$

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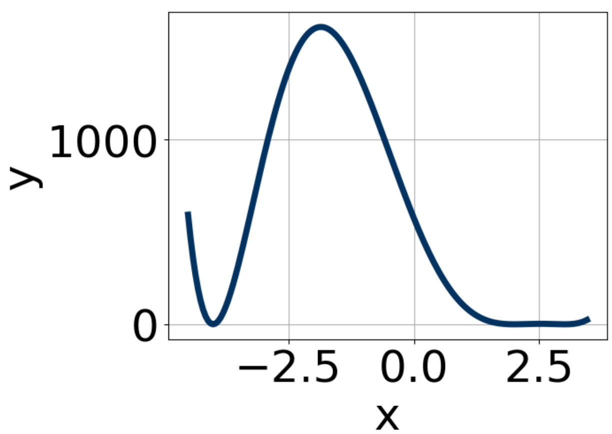
6. Which of the following equations *could* be of the graph presented below?



The solution is $4(x - 2)^6(x + 4)^4(x - 3)^6$, which is option A.

A. $4(x - 2)^6(x + 4)^4(x - 3)^6$

* This is the correct option.

B. $-12(x - 2)^8(x + 4)^6(x - 3)^7$

The factor $(x - 3)$ should have an even power and the leading coefficient should be the opposite sign.

C. $14(x - 2)^8(x + 4)^8(x - 3)^{11}$

The factor $(x - 3)$ should have an even power.

D. $-4(x - 2)^4(x + 4)^4(x - 3)^8$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $18(x - 2)^6(x + 4)^{11}(x - 3)^9$

The factors $(x + 4)$ and $(x - 3)$ should both have even powers.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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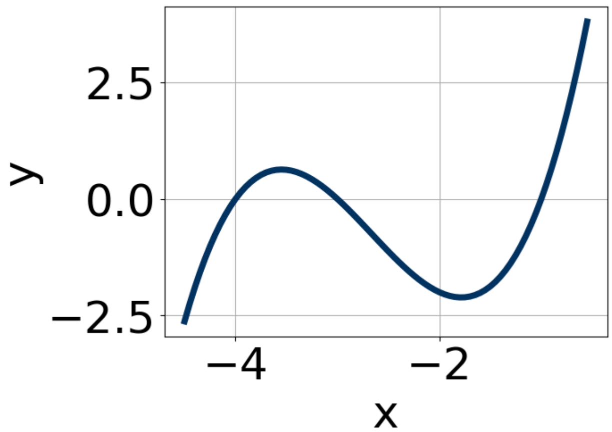
7. Which of the following equations *could* be of the graph presented below?



The solution is $20(x+4)^{11}(x+1)^{11}(x+3)^{11}$, which is option B.

A. $-17(x+4)^{9}(x+1)^{11}(x+3)^{5}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $20(x+4)^{11}(x+1)^{11}(x+3)^{11}$

* This is the correct option.

C. $16(x+4)^{10}(x+1)^{4}(x+3)^{7}$

The factors -4 and -1 have have been odd power.

D. $2(x+4)^{6}(x+1)^{5}(x+3)^{9}$

The factor -4 should have been an odd power.

E. $-5(x+4)^{4}(x+1)^{5}(x+3)^{11}$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 3i \text{ and } -1$$

The solution is $x^3 - 9x^2 + 24x + 34$, which is option D.

A. $b \in [-6, 5], c \in [-9, -3], \text{ and } d \in [-8, -4.4]$

$x^3 + x^2 - 4x - 5$, which corresponds to multiplying out $(x-5)(x+1)$.

B. $b \in [-6, 5], c \in [-3, 5], \text{ and } d \in [-4.4, -2.5]$

$x^3 + x^2 - 2x - 3$, which corresponds to multiplying out $(x-3)(x+1)$.

C. $b \in [8, 12], c \in [20, 27],$ and $d \in [-36.8, -29.5]$

$x^3 + 9x^2 + 24x - 34$, which corresponds to multiplying out $(x - (5 + 3i))(x - (5 - 3i))(x - 1)$.

D. $b \in [-16, -5], c \in [20, 27],$ and $d \in [31.3, 34.5]$

* $x^3 - 9x^2 + 24x + 34$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 3i))(x - (5 - 3i))(x - (-1))$.

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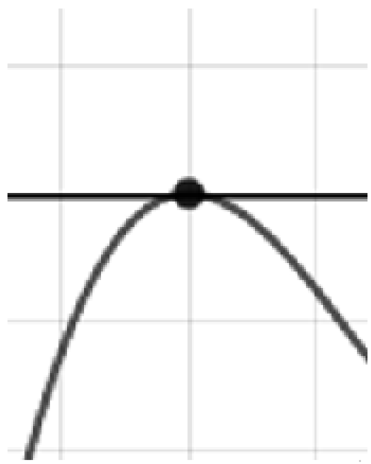
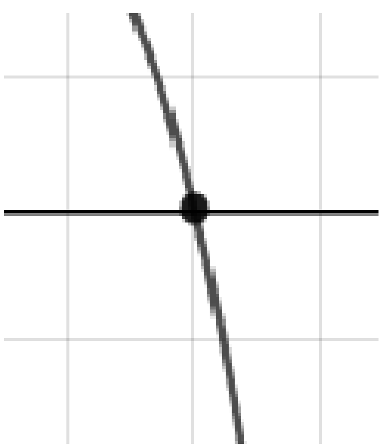
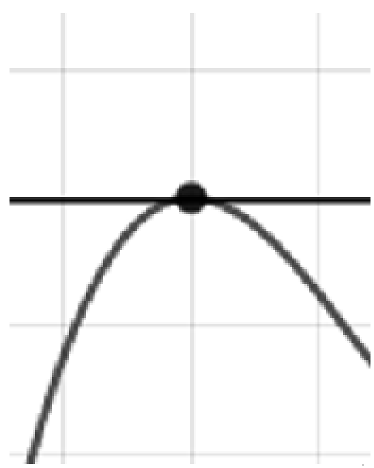
9. Describe the zero behavior of the zero $x = -7$ of the polynomial below.

$$f(x) = 7(x + 7)^4(x - 7)^7(x + 3)^5(x - 3)^7$$

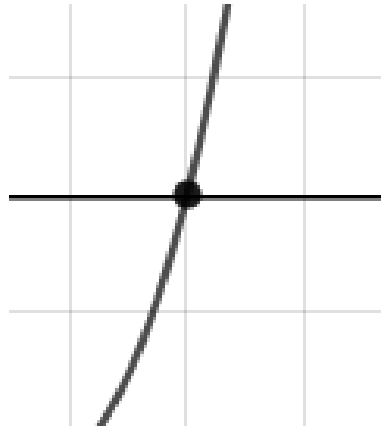
The solution is the graph below, which is option B.



A. 

B. 

C.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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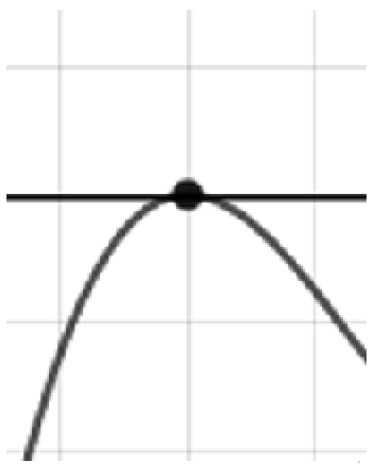
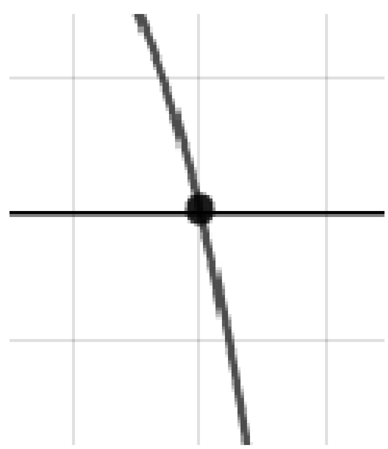
10. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

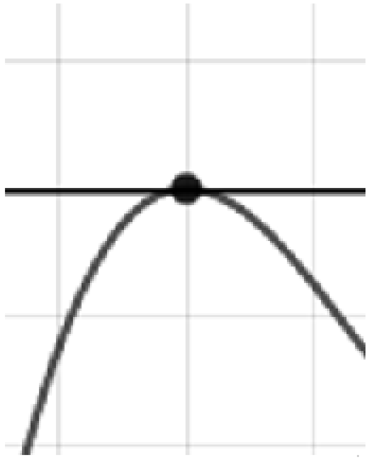
$$f(x) = 5(x-3)^7(x+3)^{10}(x+6)^4(x-6)^8$$

The solution is the graph below, which is option B.

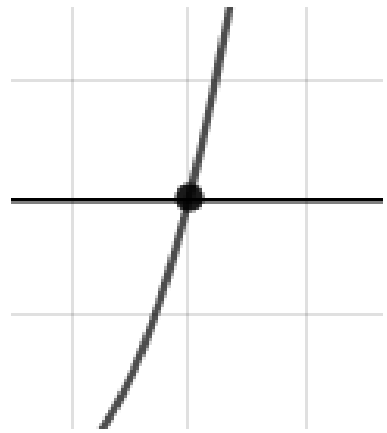




A.

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.