This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -4x^3 - 3x^2 + 3x \text{ and } g(x) = 2x^3 - 2x^2 - 2x + 2$$

The solution is 0.0, which is option A.

A. $(f \circ g)(-1) \in [-1, 1]$

* This is the correct solution

B. $(f \circ g)(-1) \in [-32, -25]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [-15, -4]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-25, -12]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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2. Determine whether the function below is 1-1.

$$f(x) = (4x - 26)^3$$

The solution is yes, which is option E.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. Yes, the function is 1-1.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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3. Determine whether the function below is 1-1.

$$f(x) = -30x^2 - 237x - 405$$

The solution is no, which is option D.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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4. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{5}{4x - 25} \text{ and } g(x) = 7x^2 + 7x + 3$$

The solution is The domain is all Real numbers except $x = 6.25$, which is option A.

A. The domain is all Real numbers except $x = a,$ where $a \in [6.25, 11.25]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-1, 8]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-10.5, -2.5]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [1.33, 8.33]$ and $b \in [-4.4, 4.6]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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5. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = \ln(x - 5) + 4$$

The solution is $f^{-1}(10) = 408.429$, which is option B.

A. $f^{-1}(10) \in [149.41, 158.41]$

This solution corresponds to distractor 4.

B. $f^{-1}(10) \in [407.43, 410.43]$

This is the solution.

C. $f^{-1}(10) \in [3269020.37, 3269025.37]$

This solution corresponds to distractor 2.

D. $f^{-1}(10) \in [392.43, 401.43]$

This solution corresponds to distractor 3.

E. $f^{-1}(10) \in [1202608.28, 1202615.28]$

This solution corresponds to distractor 1.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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6. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 2x^3 + 3x^2 + 2x \text{ and } g(x) = 4x^3 - 2x^2 - 4x$$

The solution is -8.0, which is option A.

A. $(f \circ g)(-1) \in [-10, -4]$

* This is the correct solution

B. $(f \circ g)(-1) \in [-3, 1]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [-22, -14]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [2, 12]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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7. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = \ln(x + 5) - 3$$

The solution is $f^{-1}(10) = 442408.392$, which is option E.

A. $f^{-1}(10) \in [1091.63, 1092.63]$

This solution corresponds to distractor 1.

B. $f^{-1}(10) \in [3269014.37, 3269017.37]$

This solution corresponds to distractor 4.

C. $f^{-1}(10) \in [442413.39, 442420.39]$

This solution corresponds to distractor 3.

D. $f^{-1}(10) \in [142.41, 152.41]$

This solution corresponds to distractor 2.

E. $f^{-1}(10) \in [442404.39, 442411.39]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -12$ and choose the interval that $f^{-1}(-12)$ belongs to.

$$f(x) = 5x^2 + 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-12) \in [4.65, 4.78]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(-12) \in [1, 1.65]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

C. $f^{-1}(-12) \in [3.55, 3.95]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(-12) \in [1.67, 1.76]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval the $f^{-1}(-15)$ belongs to.

$$f(x) = \sqrt[3]{4x + 3}$$

The solution is -844.5, which is option B.

A. $f^{-1}(-15) \in [844.42, 845.34]$

This solution corresponds to distractor 2.

B. $f^{-1}(-15) \in [-845.53, -843.31]$

* This is the correct solution.

C. $f^{-1}(-15) \in [-843.52, -842.62]$

Distractor 1: This corresponds to

D. $f^{-1}(-15) \in [842.23, 844.38]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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10. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{6x + 24} \text{ and } g(x) = x^4 + 9x^3 + 8x^2 + x + 6$$

The solution is The domain is all Real numbers greater than or equal to$x = -4.0$., which is option B.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [0.75, 6.75]$

B. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-7, -3]$

C. The domain is all Real numbers except $x = a$, where $a \in [3.25, 5.25]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-12.25, -5.25]$ and $b \in [-6.4, -4.4]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.
