This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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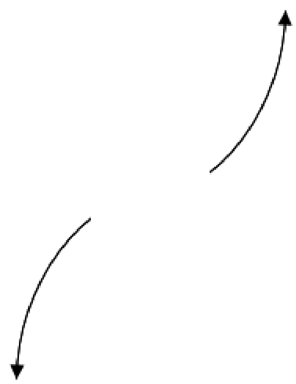
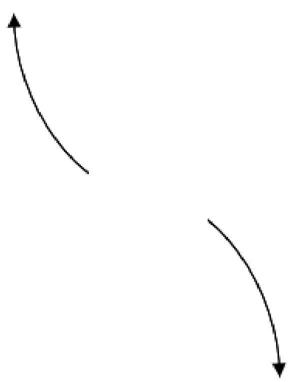
1. Describe the end behavior of the polynomial below.

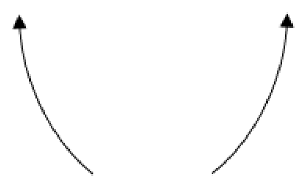
$$f(x) = 3(x-5)^5(x+5)^{10}(x+2)^2(x-2)^4$$

The solution is the graph below, which is option D.

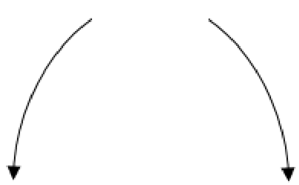




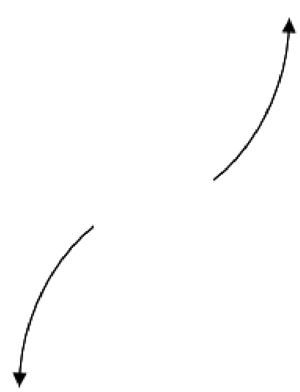
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$2, \frac{7}{5}, \text{ and } \frac{-7}{4}$$

The solution is $20x^3 - 33x^2 - 63x + 98$, which is option D.

A. $a \in [16, 26], b \in [99, 104], c \in [169, 183], \text{ and } d \in [92, 109]$

$20x^3 + 103x^2 + 175x + 98$, which corresponds to multiplying out $(x + 1)(5x + 5)(4x - 4)$.

B. $a \in [16, 26], b \in [30, 36], c \in [-66, -61], \text{ and } d \in [-98, -94]$

$20x^3 + 33x^2 - 63x - 98$, which corresponds to multiplying out $(x + 2)(5x + 7)(4x - 7)$.

C. $a \in [16, 26], b \in [43, 48], c \in [-40, -34], \text{ and } d \in [-98, -94]$

$20x^3 + 47x^2 - 35x - 98$, which corresponds to multiplying out $(x + 1)(5x - 5)(4x - 4)$.

D. $a \in [16, 26], b \in [-44, -31], c \in [-66, -61], \text{ and } d \in [92, 109]$

* $20x^3 - 33x^2 - 63x + 98$, which is the correct option.

E. $a \in [16, 26], b \in [-44, -31], c \in [-66, -61], \text{ and } d \in [-98, -94]$

$20x^3 - 33x^2 - 63x - 98$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-2)(5x-7)(4x+7)$

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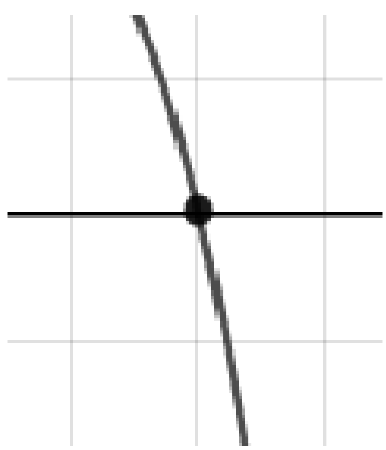
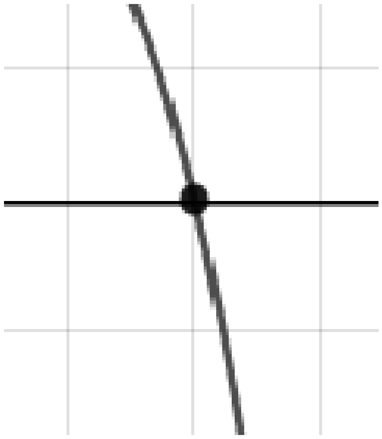
3. Describe the zero behavior of the zero $x = -2$ of the polynomial below.

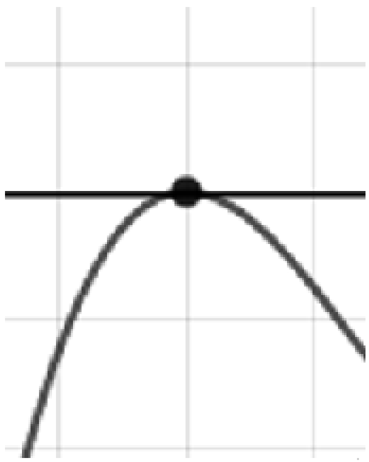
$$f(x) = 7(x - 6)^{11}(x + 6)^8(x - 2)^4(x + 2)^3$$

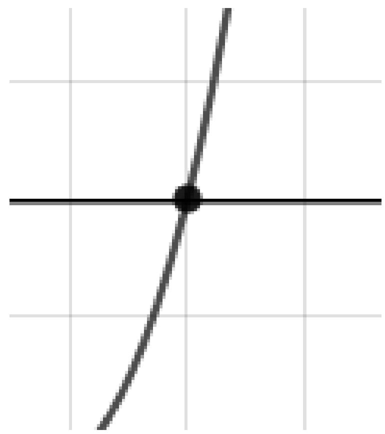
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 5i \text{ and } 2$$

The solution is $x^3 + 4x^2 + 22x - 68$, which is option D.

A. $b \in [0.5, 3.6], c \in [1.5, 3.3]$, and $d \in [-13, -9]$

$x^3 + x^2 + 3x - 10$, which corresponds to multiplying out $(x + 5)(x - 2)$.

B. $b \in [-4.5, -1.5], c \in [17.2, 24.6]$, and $d \in [67, 70]$

$x^3 - 4x^2 + 22x + 68$, which corresponds to multiplying out $(x - (-3 - 5i))(x - (-3 + 5i))(x + 2)$.

C. $b \in [0.5, 3.6], c \in [-0.3, 2.4]$, and $d \in [-8, 2]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x + 3)(x - 2)$.

D. $b \in [3.9, 4.8], c \in [17.2, 24.6]$, and $d \in [-74, -62]$

* $x^3 + 4x^2 + 22x - 68$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-3-5i))(x-(-3+5i))(x-(2))$.

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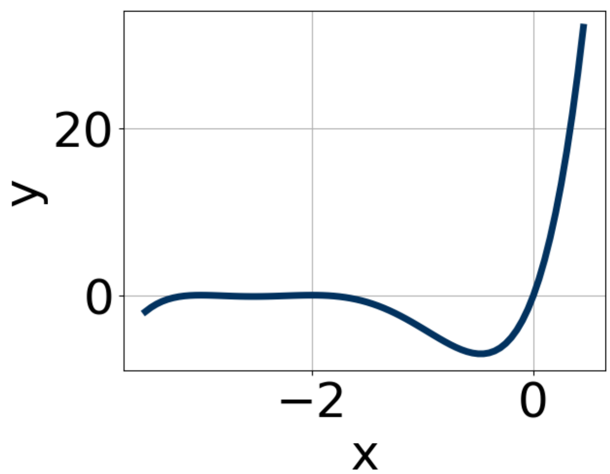
5. Which of the following equations *could* be of the graph presented below?



The solution is $7x^9(x+2)^{10}(x+3)^{10}$, which is option C.

A. $-15x^8(x+2)^6(x+3)^6$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

B. $-16x^7(x+2)^6(x+3)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $7x^9(x+2)^{10}(x+3)^{10}$

* This is the correct option.

D. $9x^8(x+2)^6(x+3)^9$

The factor $(x+3)$ should have an even power and the factor x should have an odd power.

E. $15x^9(x+2)^6(x+3)^5$

The factor $(x+3)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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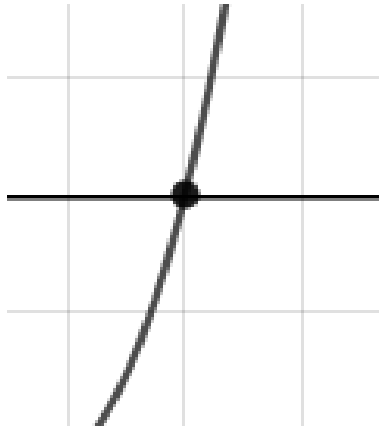
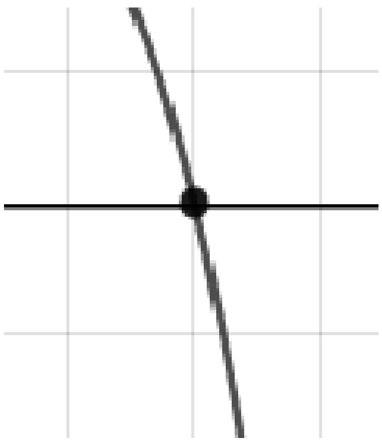
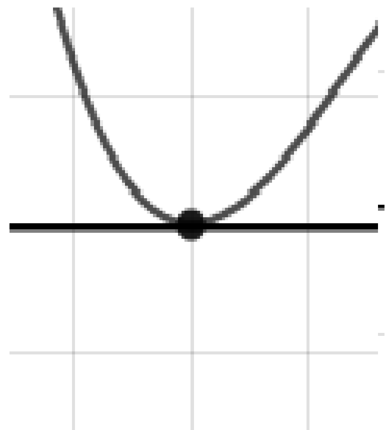
6. Describe the zero behavior of the zero $x=7$ of the polynomial below.

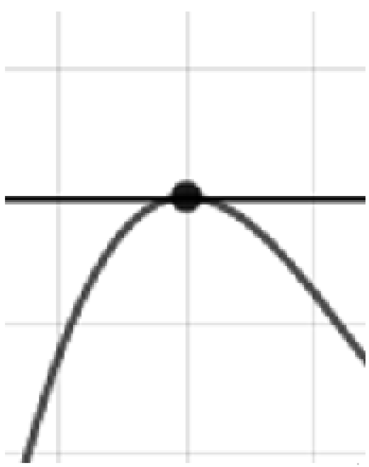
$$f(x) = 2(x-7)^9(x+7)^{14}(x-8)^8(x+8)^{11}$$

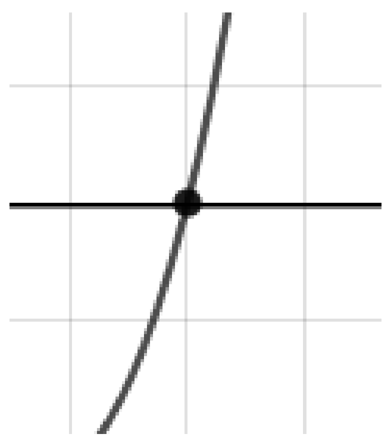
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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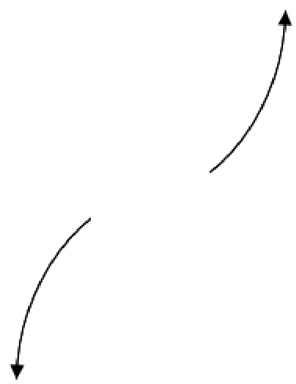
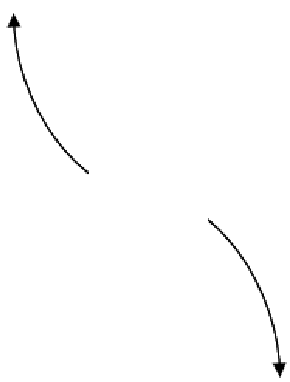
7. Describe the end behavior of the polynomial below.

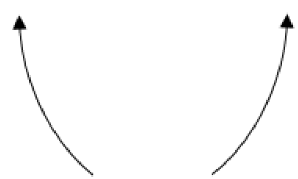
$$f(x) = 8(x-5)^3(x+5)^4(x-9)^4(x+9)^4$$

The solution is the graph below, which is option D.

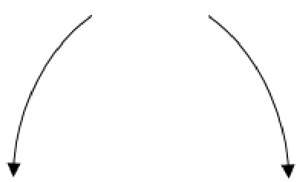




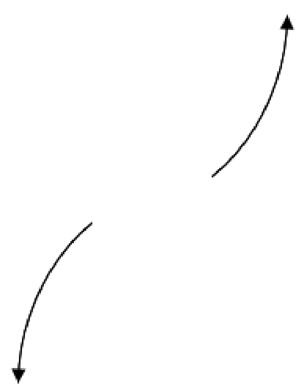
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 2i \text{ and } 3$$

The solution is $x^3 - 13x^2 + 59x - 87$, which is option B.

A. $b \in [-5, 5], c \in [-10, -7], \text{ and } d \in [10, 18]$

$x^3 + x^2 - 8x + 15$, which corresponds to multiplying out $(x - 5)(x - 3)$.

B. $b \in [-19, -8], c \in [57, 68], \text{ and } d \in [-88, -85]$

* $x^3 - 13x^2 + 59x - 87$, which is the correct option.

C. $b \in [-5, 5], c \in [-1, 0], \text{ and } d \in [-14, 2]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x + 2)(x - 3)$.

D. $b \in [13, 15], c \in [57, 68], \text{ and } d \in [82, 95]$

$x^3 + 13x^2 + 59x + 87$, which corresponds to multiplying out $(x - (5 - 2i))(x - (5 + 2i))(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 - 2i))(x - (5 + 2i))(x - (3))$.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{5}, \frac{4}{5}, \text{ and } \frac{-3}{2}$$

The solution is $50x^3 + 55x^2 - 46x - 24$, which is option A.

A. $a \in [48, 53], b \in [51, 63], c \in [-47, -44], \text{ and } d \in [-25, -20]$

* $50x^3 + 55x^2 - 46x - 24$, which is the correct option.

B. $a \in [48, 53], b \in [15, 17], c \in [-76, -71], \text{ and } d \in [20, 33]$

$50x^3 + 15x^2 - 74x + 24$, which corresponds to multiplying out $(5x + 5)(5x - 5)(2x - 2)$.

C. $a \in [48, 53], b \in [94, 96], c \in [10, 19], \text{ and } d \in [-25, -20]$

$50x^3 + 95x^2 + 14x - 24$, which corresponds to multiplying out $(5x + 5)(5x + 5)(2x - 2)$.

D. $a \in [48, 53], b \in [-61, -54], c \in [-47, -44], \text{ and } d \in [20, 33]$

$50x^3 - 55x^2 - 46x + 24$, which corresponds to multiplying out $(5x - 2)(5x + 4)(2x - 3)$.

E. $a \in [48, 53], b \in [51, 63], c \in [-47, -44], \text{ and } d \in [20, 33]$

$50x^3 + 55x^2 - 46x + 24$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 2)(5x - 4)(2x + 3)$

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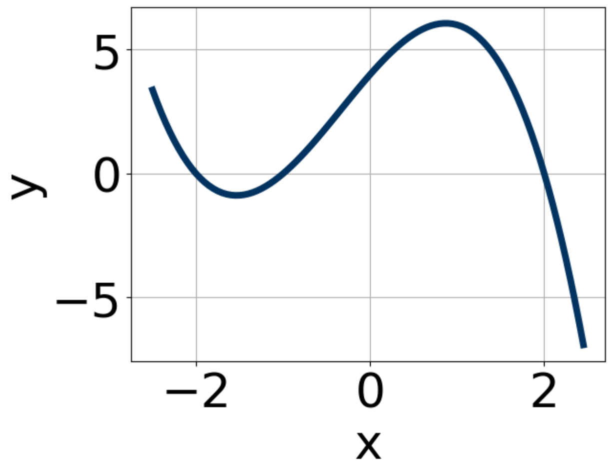
10. Which of the following equations *could* be of the graph presented below?



The solution is $-14(x-2)^9(x+2)^7(x+1)^{11}$, which is option A.

A. $-14(x-2)^9(x+2)^7(x+1)^{11}$

   * This is the correct option.

B. $-15(x-2)^{10}(x+2)^8(x+1)^7$

   The factors 2 and -2 have have been odd power.

C. $-9(x-2)^6(x+2)^7(x+1)^9$

   The factor 2 should have been an odd power.

D. $18(x-2)^{10}(x+2)^9(x+1)^9$

   The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $19(x-2)^5(x+2)^5(x+1)^{11}$

   This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).