This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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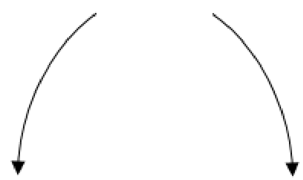
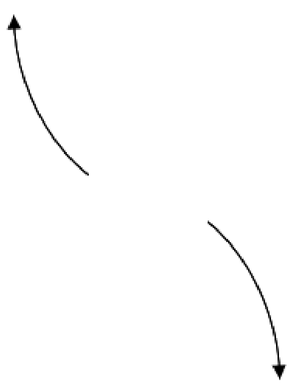
1. Describe the end behavior of the polynomial below.

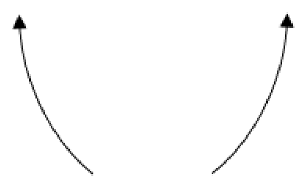
$$f(x) = -9(x+8)^3(x-8)^8(x+7)^5(x-7)^6$$

The solution is the graph below, which is option B.

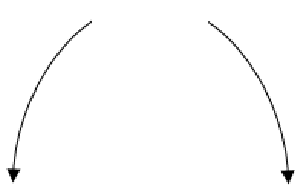




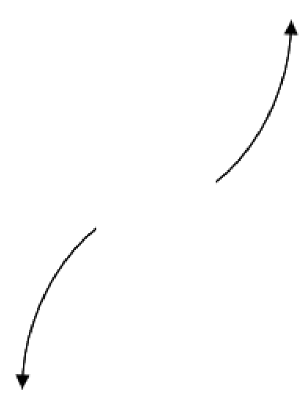
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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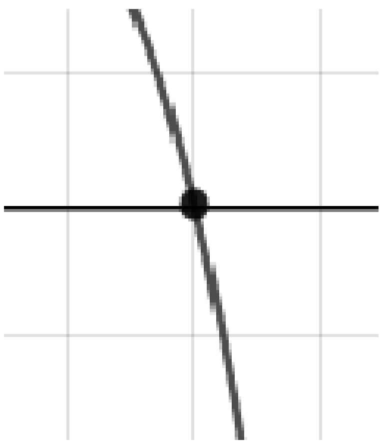
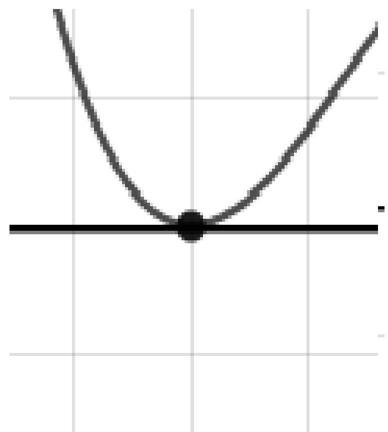
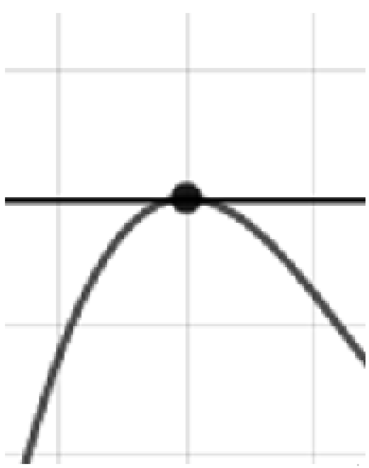
2. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

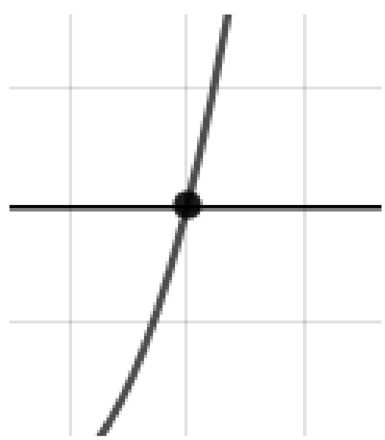
$$f(x) = 2(x+4)^7(x-4)^6(x-2)^{10}(x+2)^7$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 3i \text{ and } -1$$

The solution is $x^3 + 11x^2 + 44x + 34$, which is option B.

A. $b \in [-3, 5], c \in [5, 16]$, and $d \in [-2, 9]$

$x^3 + x^2 + 6x + 5$, which corresponds to multiplying out $(x + 5)(x + 1)$.

B. $b \in [10, 15], c \in [39, 51]$, and $d \in [33, 40]$

* $x^3 + 11x^2 + 44x + 34$, which is the correct option.

C. $b \in [-3, 5], c \in [-2, 1]$, and $d \in [-11, -2]$

$x^3 + x^2 - 2x - 3$, which corresponds to multiplying out $(x - 3)(x + 1)$.

D. $b \in [-16, -7], c \in [39, 51]$, and $d \in [-42, -25]$

$x^3 - 11x^2 + 44x - 34$, which corresponds to multiplying out $(x - (-5 + 3i))(x - (-5 - 3i))(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 3i))(x - (-5 - 3i))(x - (-1))$.

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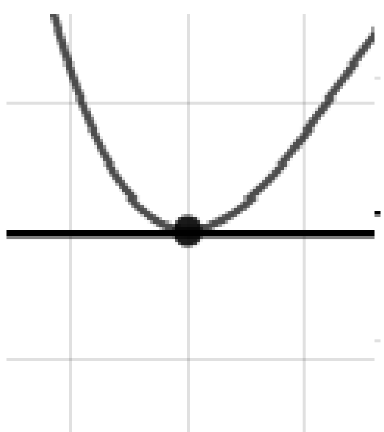
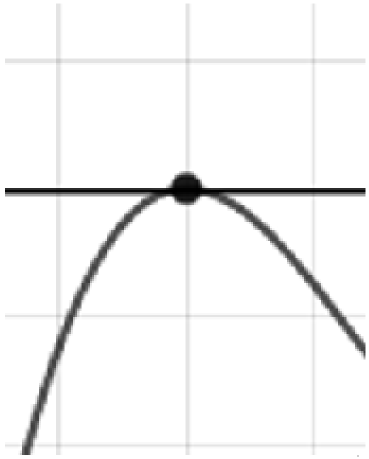
4. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

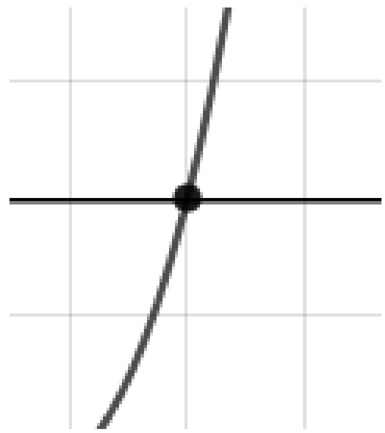
$$f(x) = 6(x + 2)^9(x - 2)^8(x + 8)^7(x - 8)^4$$

The solution is the graph below, which is option C.



B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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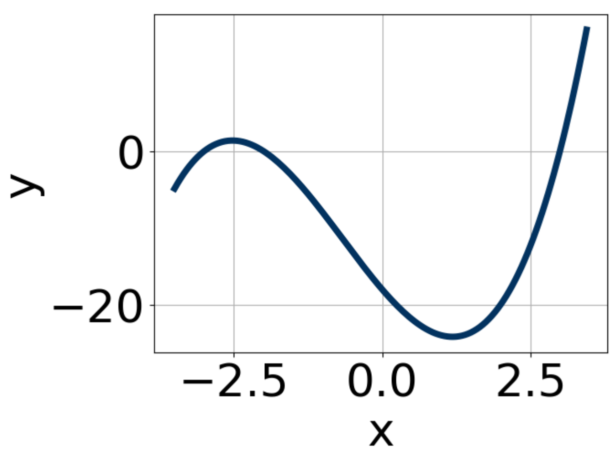
5. Which of the following equations *could* be of the graph presented below?



The solution is $13(x-3)^{11}(x+3)^{9}(x+2)^{5}$, which is option E.

A. $-20(x-3)^{11}(x+3)^{7}(x+2)^{7}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $15(x-3)^4(x+3)^{11}(x+2)^7$

The factor 3 should have been an odd power.

C. $-4(x-3)^4(x+3)^5(x+2)^9$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $19(x-3)^{10}(x+3)^8(x+2)^5$

The factors 3 and -3 have have been odd power.

E. $13(x-3)^{11}(x+3)^9(x+2)^5$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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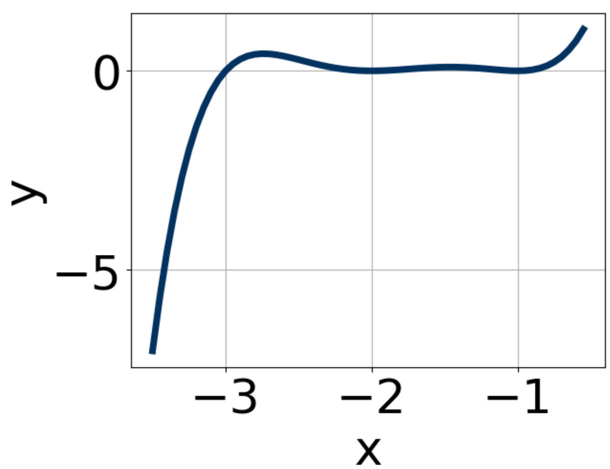
6. Which of the following equations *could* be of the graph presented below?



The solution is $16(x+1)^{10}(x+2)^8(x+3)^9$, which is option A.

A. $16(x+1)^{10}(x+2)^8(x+3)^9$

* This is the correct option.

B. $11(x+1)^4(x+2)^{11}(x+3)^7$

The factor $(x+2)$ should have an even power.

C. $-3(x+1)^{10}(x+2)^4(x+3)^4$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-16(x+1)^6(x+2)^6(x+3)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $12(x+1)^8(x+2)^{11}(x+3)^4$

The factor $(x+2)$ should have an even power and the factor $(x+3)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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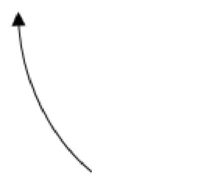
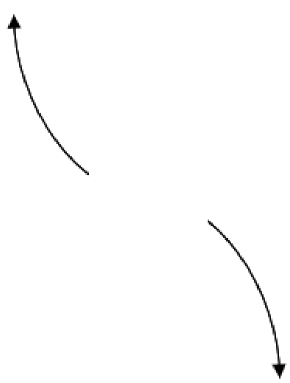
7. Describe the end behavior of the polynomial below.

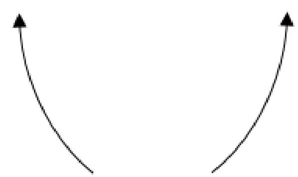
$$f(x) = 9(x+4)^2(x-4)^5(x+8)^3(x-8)^4$$

The solution is the graph below, which is option C.

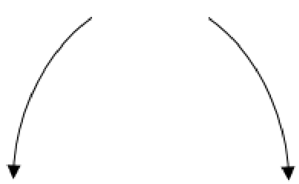




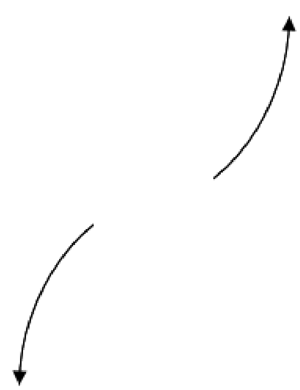
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{2}, -5, \text{ and } \frac{-3}{4}$$

The solution is $8x^3 + 42x^2 + 7x - 15$, which is option D.

A. $a \in [1, 9], b \in [50, 52], c \in [46, 55],$ and $d \in [12, 18]$

$8x^3 + 50x^2 + 53x + 15$, which corresponds to multiplying out $(2x + 1)(x + 5)(4x + 3)$.

B. $a \in [1, 9], b \in [35, 44], c \in [6, 12],$ and $d \in [12, 18]$

$8x^3 + 42x^2 + 7x + 15$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [1, 9], b \in [-32, -28], c \in [-48, -38],$ and $d \in [-16, -13]$

$8x^3 - 30x^2 - 47x - 15$, which corresponds to multiplying out $(2x + 1)(x - 5)(4x + 3)$.

D. $a \in [1, 9], b \in [35, 44], c \in [6, 12],$ and $d \in [-16, -13]$

* $8x^3 + 42x^2 + 7x - 15$, which is the correct option.

E. $a \in [1, 9], b \in [-50, -37], c \in [6, 12],$ and $d \in [12, 18]$

$8x^3 - 42x^2 + 7x + 15$, which corresponds to multiplying out $(2x + 1)(x - 5)(4x - 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 1)(x + 5)(4x + 3)$

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 3i \text{ and } 4$$

The solution is $x^3 - 10x^2 + 42x - 72$, which is option B.

A. $b \in [8, 14], c \in [39, 42.7],$ and $d \in [66, 81]$

$x^3 + 10x^2 + 42x + 72$, which corresponds to multiplying out $(x - (3 - 3i))(x - (3 + 3i))(x + 4)$.

B. $b \in [-11, -7], c \in [39, 42.7],$ and $d \in [-75, -67]$

* $x^3 - 10x^2 + 42x - 72$, which is the correct option.

C. $b \in [-8, 2], c \in [-7.7, -5.8],$ and $d \in [12, 17]$

$x^3 + x^2 - 7x + 12$, which corresponds to multiplying out $(x - 3)(x - 4)$.

D. $b \in [-8, 2], c \in [-2.9, 0.2],$ and $d \in [-19, -11]$

$x^3 + x^2 - x - 12$, which corresponds to multiplying out $(x + 3)(x - 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 - 3i))(x - (3 + 3i))(x - (4))$.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$5, \frac{6}{5}, \text{ and } 1$$

The solution is $5x^3 - 36x^2 + 61x - 30$, which is option A.

A. $a \in [-2, 6], b \in [-37, -32], c \in [56, 63],$ and $d \in [-34, -28]$

   * $5x^3 - 36x^2 + 61x - 30$, which is the correct option.

B. $a \in [-2, 6], b \in [-37, -32], c \in [56, 63],$ and $d \in [29, 33]$

   $5x^3 - 36x^2 + 61x + 30$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [-2, 6], b \in [8, 19], c \in [-51, -45],$ and $d \in [29, 33]$

   $5x^3 + 14x^2 - 49x + 30$, which corresponds to multiplying out $(x + 5)(5x - 6)(x - 1)$.

D. $a \in [-2, 6], b \in [35, 40], c \in [56, 63],$ and $d \in [29, 33]$

   $5x^3 + 36x^2 + 61x + 30$, which corresponds to multiplying out $(x + 5)(5x + 6)(x + 1)$.

E. $a \in [-2, 6], b \in [22, 27], c \in [-9, 0],$ and $d \in [-34, -28]$

   $5x^3 + 26x^2 - x - 30$, which corresponds to multiplying out $(x + 5)(5x + 6)(x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-5)(5x-6)(x-1)$