This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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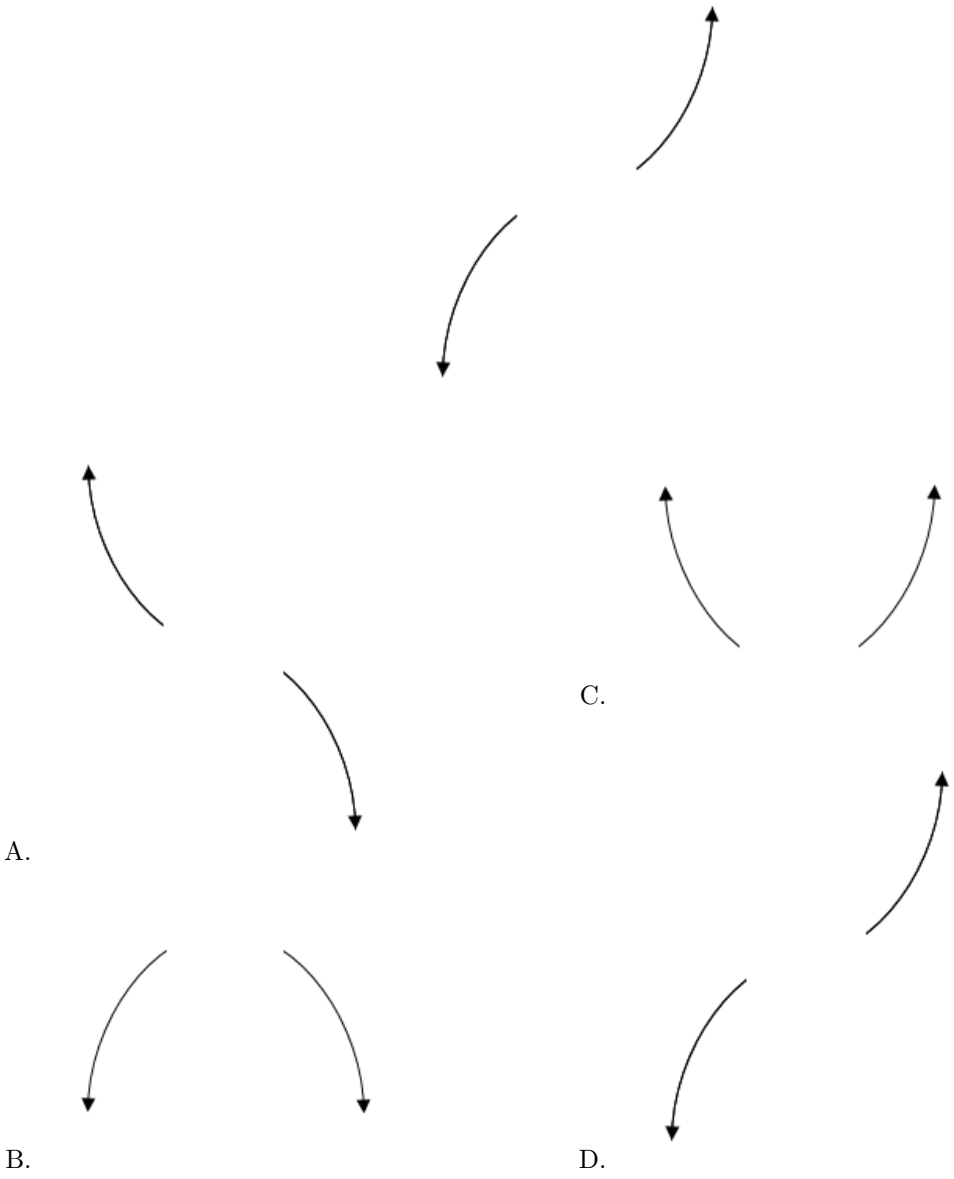
1. Describe the end behavior of the polynomial below.

$$f(x) = 6(x+4)^3(x-4)^4(x+9)^3(x-9)^3$$

The solution is the graph below, which is option D.



A.

B.

C.

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{2}, \frac{1}{5}, \text{ and } 6$$

The solution is $10x^3 - 47x^2 - 81x + 18$, which is option C.

A. $a \in [6, 11], b \in [47, 50], c \in [-83, -76],$ and $d \in [-21, -15]$

$10x^3 + 47x^2 - 81x - 18$, which corresponds to multiplying out $(2x - 3)(5x + 1)(x + 6)$.

B. $a \in [6, 11], b \in [-55, -41], c \in [-83, -76],$ and $d \in [-21, -15]$

$10x^3 - 47x^2 - 81x - 18$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [6, 11], b \in [-55, -41], c \in [-83, -76],$ and $d \in [16, 25]$

* $10x^3 - 47x^2 - 81x + 18$, which is the correct option.

D. $a \in [6, 11], b \in [-74, -70], c \in [75, 80],$ and $d \in [16, 25]$

$10x^3 - 73x^2 + 75x + 18$, which corresponds to multiplying out $(2x + 2)(5x + 5)(x - 1)$.

E. $a \in [6, 11], b \in [-80, -75], c \in [101, 114],$ and $d \in [-21, -15]$

$10x^3 - 77x^2 + 105x - 18$, which corresponds to multiplying out $(2x + 2)(5x - 5)(x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 3)(5x - 1)(x - 6)$

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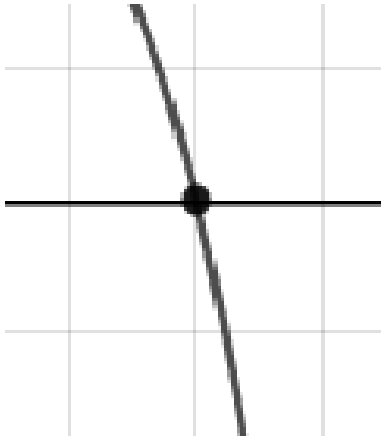
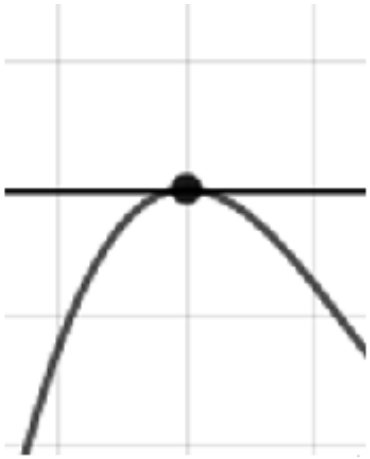
3. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

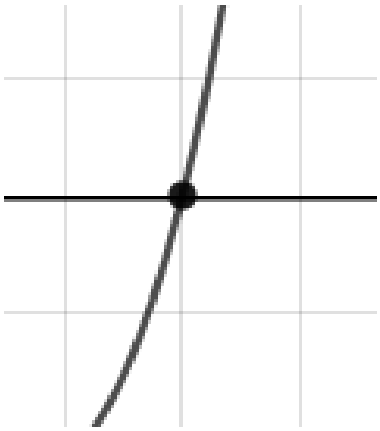
$$f(x) = 6(x - 4)^9(x + 4)^{12}(x + 8)^3(x - 8)^5$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 2i \text{ and } 3$$

The solution is $x^3 - 13x^2 + 59x - 87$, which is option A.

A. $b \in [-16, -12], c \in [59, 62],$ and $d \in [-89, -75]$

* $x^3 - 13x^2 + 59x - 87$, which is the correct option.

B. $b \in [-3, 6], c \in [-13, -5],$ and $d \in [14, 20]$

$x^3 + x^2 - 8x + 15$, which corresponds to multiplying out $(x - 5)(x - 3)$.

C. $b \in [-3, 6], c \in [-4, 0],$ and $d \in [-9, 1]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x + 2)(x - 3)$.

D. $b \in [10, 20], c \in [59, 62],$ and $d \in [84, 92]$

$x^3 + 13x^2 + 59x + 87$, which corresponds to multiplying out $(x - (5 - 2i))(x - (5 + 2i))(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(5-2i))(x-(5+2i))(x-(3))$.

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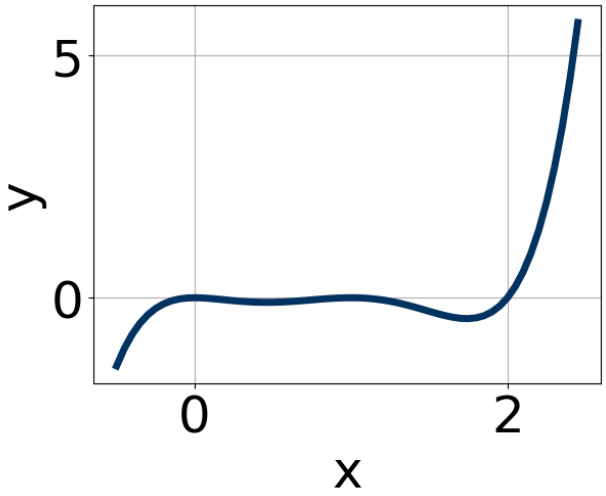
5. Which of the following equations *could* be of the graph presented below?



The solution is $10x^4(x-1)^8(x-2)^9$, which is option B.

A. $10x^4(x-1)^{11}(x-2)^5$

The factor $(x-1)$ should have an even power.

B. $10x^4(x-1)^8(x-2)^9$

* This is the correct option.

C. $-6x^{10}(x-1)^6(x-2)^6$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-16x^6(x-1)^{10}(x-2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $10x^{10}(x-1)^7(x-2)^4$

The factor $(x-1)$ should have an even power and the factor $(x-2)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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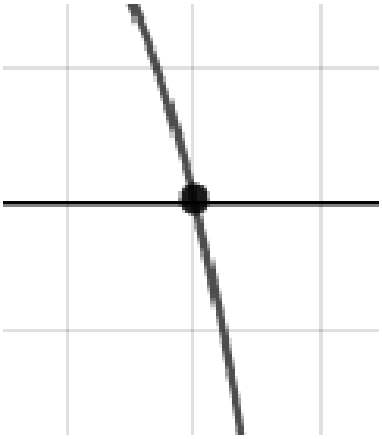
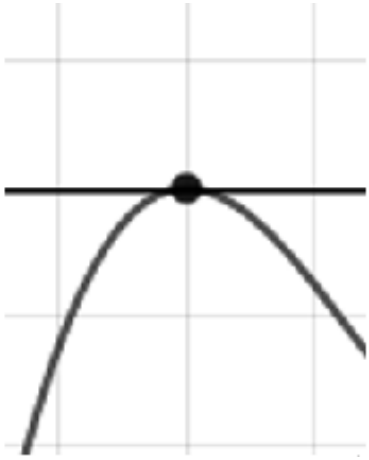
6. Describe the zero behavior of the zero $x=5$ of the polynomial below.

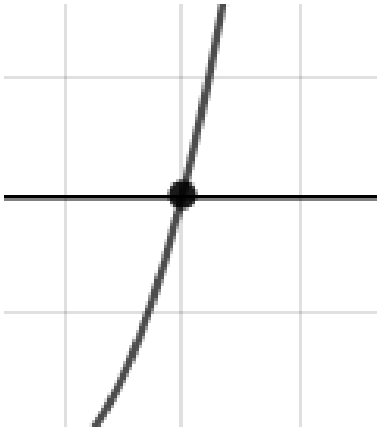
$$f(x)=6(x-5)^8(x+5)^9(x-9)^2(x+9)^6$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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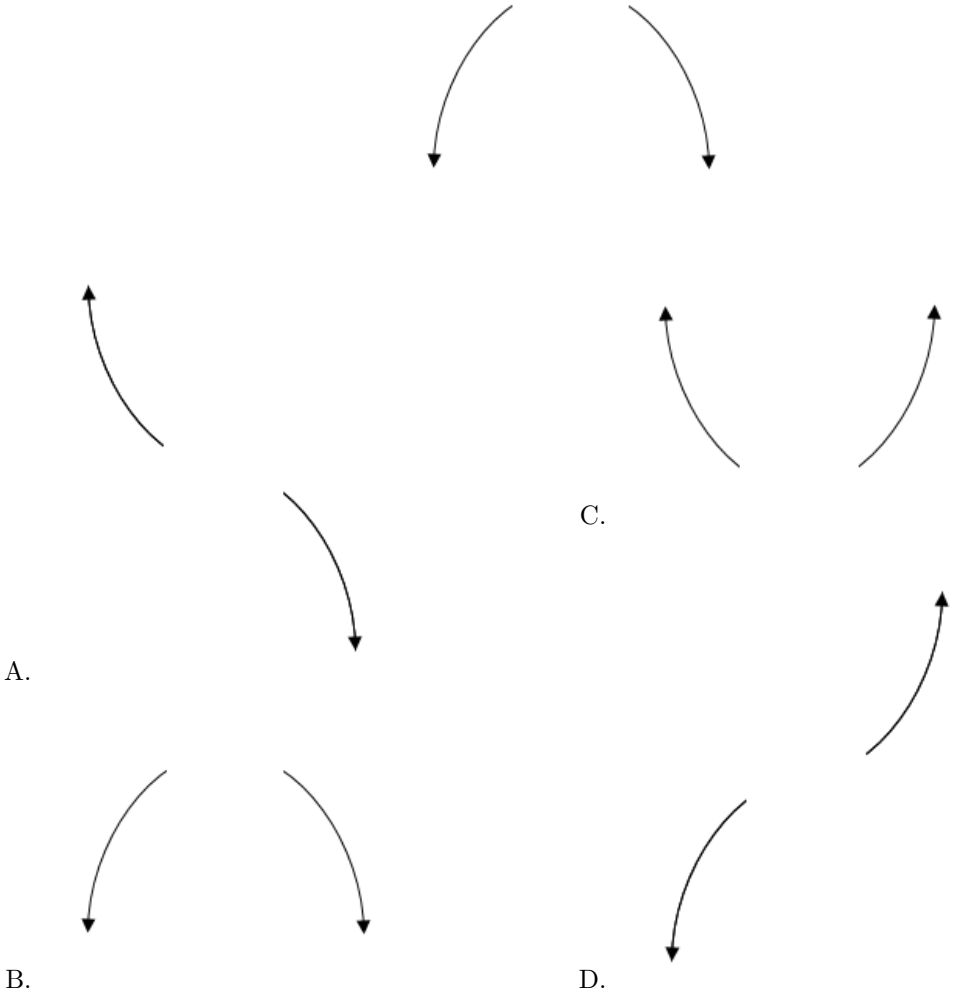
7. Describe the end behavior of the polynomial below.

$$f(x) = -9(x-9)^3(x+9)^4(x-8)^5(x+8)^6$$

The solution is the graph below, which is option B.



A.

B.

C.

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 4i \text{ and } -4$$

The solution is $x^3 + 8x^2 + 36x + 80$, which is option A.

A. $b \in [7, 12], c \in [33.6, 38.7],$ and $d \in [78, 88]$

* $x^3 + 8x^2 + 36x + 80$, which is the correct option.

B. $b \in [-8, -3], c \in [33.6, 38.7],$ and $d \in [-82, -75]$

$x^3 - 8x^2 + 36x - 80$, which corresponds to multiplying out $(x - (-2 - 4i))(x - (-2 + 4i))(x - 4)$.

C. $b \in [-2, 6], c \in [4.5, 7.9],$ and $d \in [3, 9]$

$x^3 + x^2 + 6x + 8$, which corresponds to multiplying out $(x + 2)(x + 4)$.

D. $b \in [-2, 6], c \in [6.8, 9.4],$ and $d \in [12, 24]$

$x^3 + x^2 + 8x + 16$, which corresponds to multiplying out $(x + 4)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 4i))(x - (-2 + 4i))(x - (-4))$.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{5}, \frac{-4}{5}, \text{ and } \frac{1}{2}$$

The solution is $50x^3 - 55x^2 - 41x + 28$, which is option D.

A. $a \in [50, 51], b \in [-63, -50], c \in [-43, -33],$ and $d \in [-32, -26]$

$50x^3 - 55x^2 - 41x - 28$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [50, 51], b \in [51, 56], c \in [-43, -33],$ and $d \in [-32, -26]$

$50x^3 + 55x^2 - 41x - 28$, which corresponds to multiplying out $(5x + 7)(5x - 4)(2x + 1)$.

C. $a \in [50, 51], b \in [80, 86], c \in [-2, 3],$ and $d \in [-32, -26]$

$50x^3 + 85x^2 + x - 28$, which corresponds to multiplying out $(5x + 5)(5x - 5)(2x - 2)$.

D. $a \in [50, 51], b \in [-63, -50], c \in [-43, -33],$ and $d \in [24, 36]$

* $50x^3 - 55x^2 - 41x + 28$, which is the correct option.

E. $a \in [50, 51], b \in [4, 8], c \in [-73, -67],$ and $d \in [24, 36]$

$50x^3 + 5x^2 - 71x + 28$, which corresponds to multiplying out $(5x + 5)(5x + 5)(2x - 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 7)(5x + 4)(2x - 1)$

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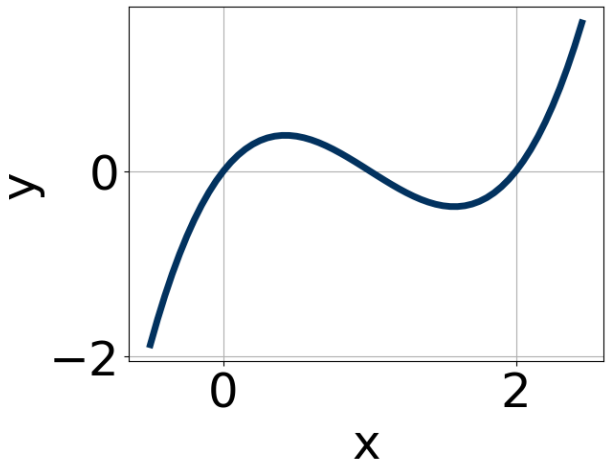
10. Which of the following equations *could* be of the graph presented below?



The solution is $15x^7(x-1)^{11}(x-2)^{11}$, which is option A.

A. $15x^7(x-1)^{11}(x-2)^{11}$

   * This is the correct option.

B. $-13x^7(x-1)^9(x-2)^7$

   This corresponds to the leading coefficient being the opposite value than it should be.

C. $19x^7(x-1)^8(x-2)^6$

   The factors 1 and 2 have have been odd power.

D. $-15x^5(x-1)^8(x-2)^7$

   The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $7x^{11}(x-1)^6(x-2)^9$

   The factor 1 should have been an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).
