This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 5i \text{ and } 1$$

The solution is $x^3 + 7x^2 + 33x - 41$, which is option D.

A. $b \in [-3, 2], c \in [3.3, 7.3]$, and $d \in [-5.77, -4.52]$

$x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

B. $b \in [-19, -6], c \in [32, 33.7]$, and $d \in [39.4, 41.41]$

$x^3 - 7x^2 + 33x + 41$, which corresponds to multiplying out $(x - (-4 - 5i))(x - (-4 + 5i))(x + 1)$.

C. $b \in [-3, 2], c \in [-1, 3.5]$, and $d \in [-4.47, -3.55]$

$x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

D. $b \in [2, 9], c \in [32, 33.7]$, and $d \in [-41.76, -39.64]$

* $x^3 + 7x^2 + 33x - 41$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 - 5i))(x - (-4 + 5i))(x - (1))$.

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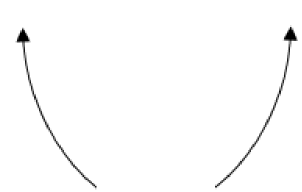
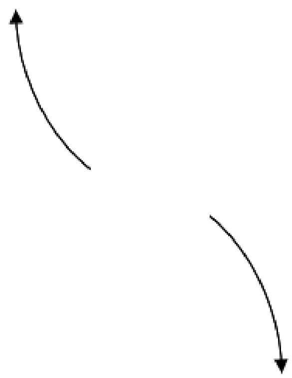
2. Describe the end behavior of the polynomial below.

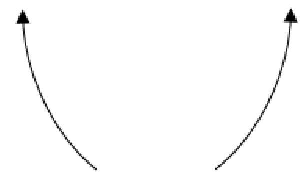
$$f(x) = 2(x + 9)^2(x - 9)^5(x + 6)^3(x - 6)^4$$

The solution is the graph below, which is option C.

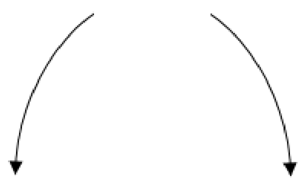




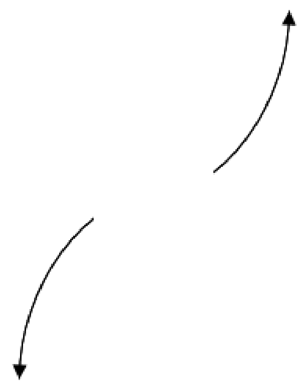
A.



C.



B.



D.

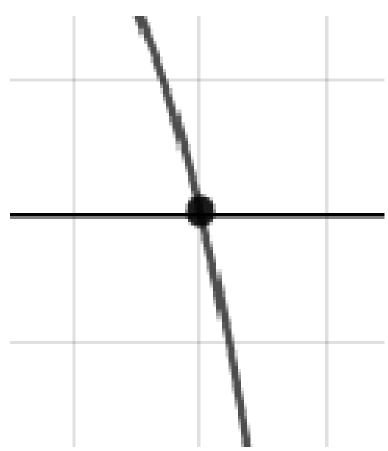
E. None of the above.

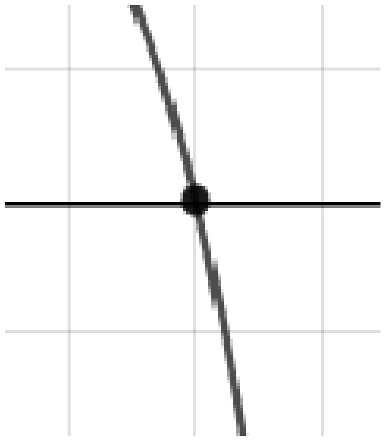
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

3. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

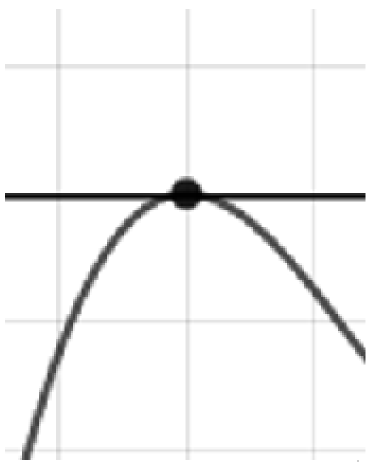
$$f(x) = -6(x+7)^{10}(x-7)^{8}(x+5)^{12}(x-5)^{9}$$

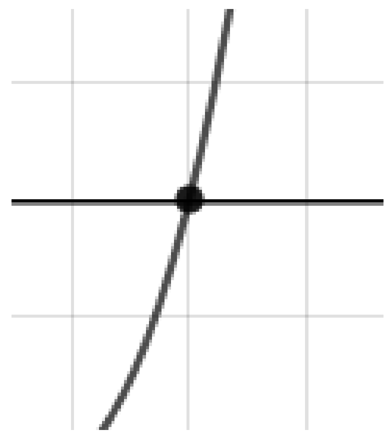
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{5}, -4, \text{ and } \frac{-6}{5}$$

The solution is $25x^3 + 165x^2 + 302x + 168$, which is option A.

A. $a \in [24, 27], b \in [159, 167], c \in [296, 309], \text{ and } d \in [166, 170]$

   * $25x^3 + 165x^2 + 302x + 168$, which is the correct option.

B. $a \in [24, 27], b \in [159, 167], c \in [296, 309], \text{ and } d \in [-168, -161]$

   $25x^3 + 165x^2 + 302x - 168$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [24, 27], b \in [-109, -104], c \in [-28, -20], \text{ and } d \in [166, 170]$

   $25x^3 - 105x^2 - 22x + 168$, which corresponds to multiplying out $(5x - 7)(x - 4)(5x + 6)$.

D. $a \in [24, 27], b \in [90, 102], c \in [-63, -56], \text{ and } d \in [-168, -161]$

   $25x^3 + 95x^2 - 62x - 168$, which corresponds to multiplying out $(5x - 7)(x + 4)(5x + 6)$.

E. $a \in [24, 27], b \in [-166, -163], c \in [296, 309],$ and $d \in [-168, -161]$

$25x^3 - 165x^2 + 302x - 168$, which corresponds to multiplying out $(5x - 7)(x - 4)(5x - 6)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 7)(x + 4)(5x + 6)$

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{2}, \frac{-7}{5}, \text{ and } -5$$

The solution is $10x^3 + 39x^2 - 90x - 175$, which is option D.

A. $a \in [9, 12], b \in [38, 45], c \in [-91, -85],$ and $d \in [171, 181]$

$10x^3 + 39x^2 - 90x + 175$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [9, 12], b \in [-45, -32], c \in [-91, -85],$ and $d \in [171, 181]$

$10x^3 - 39x^2 - 90x + 175$, which corresponds to multiplying out $(2x + 5)(5x - 7)(x - 5)$.

C. $a \in [9, 12], b \in [55, 66], c \in [17, 22],$ and $d \in [-178, -168]$

$10x^3 + 61x^2 + 20x - 175$, which corresponds to multiplying out $(2x + 5)(5x - 7)(x + 5)$.

D. $a \in [9, 12], b \in [38, 45], c \in [-91, -85],$ and $d \in [-178, -168]$

* $10x^3 + 39x^2 - 90x - 175$, which is the correct option.

E. $a \in [9, 12], b \in [83, 95], c \in [227, 235],$ and $d \in [171, 181]$

$10x^3 + 89x^2 + 230x + 175$, which corresponds to multiplying out $(2x + 5)(5x + 7)(x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 5)(5x + 7)(x + 5)$

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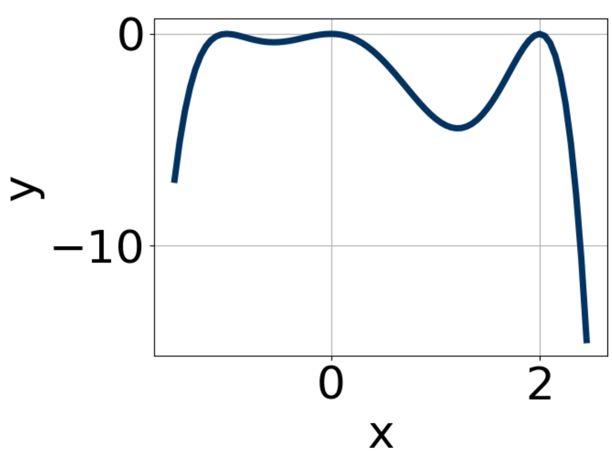
6. Which of the following equations *could* be of the graph presented below?



The solution is $-2x^4(x - 2)^8(x + 1)^4$, which is option A.

A. $-2x^4(x - 2)^8(x + 1)^4$

* This is the correct option.

B. $17x^6(x-2)^4(x+1)^9$

The factor $(x+1)$ should have an even power and the leading coefficient should be the opposite sign.

C. $-12x^9(x-2)^{10}(x+1)^7$

The factors x and $(x+1)$ should both have even powers.

D. $-17x^4(x-2)^6(x+1)^9$

The factor $(x+1)$ should have an even power.

E. $11x^6(x-2)^4(x+1)^8$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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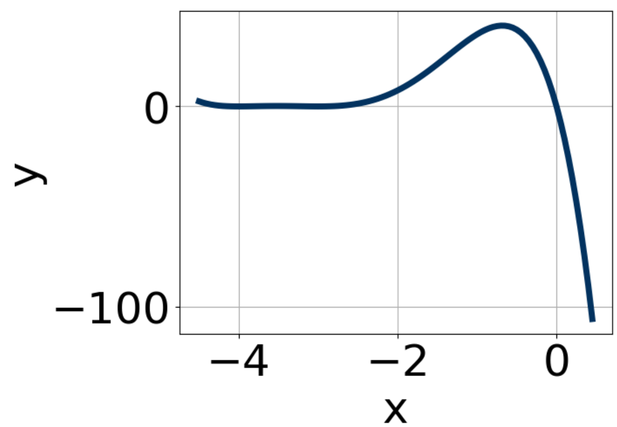
7. Which of the following equations *could* be of the graph presented below?



The solution is $-16x^{11}(x+3)^4(x+4)^4$, which is option A.

A. $-16x^{11}(x+3)^4(x+4)^4$

* This is the correct option.

B. $-10x^{10}(x+3)^{10}(x+4)^{11}$

The factor $(x+4)$ should have an even power and the factor x should have an odd power.

C. $10x^9(x+3)^8(x+4)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $18x^6(x+3)^6(x+4)^{10}$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

E. $-15x^{11}(x+3)^8(x+4)^9$

The factor $(x+4)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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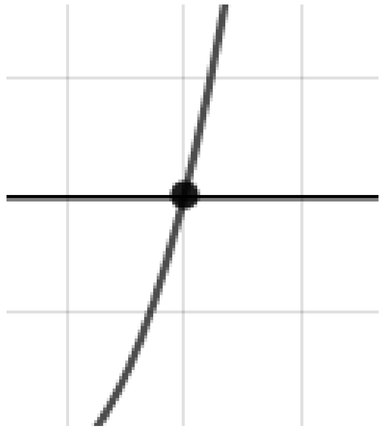
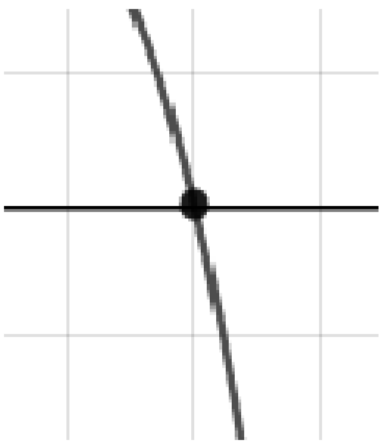
8. Describe the zero behavior of the zero $x = -2$ of the polynomial below.

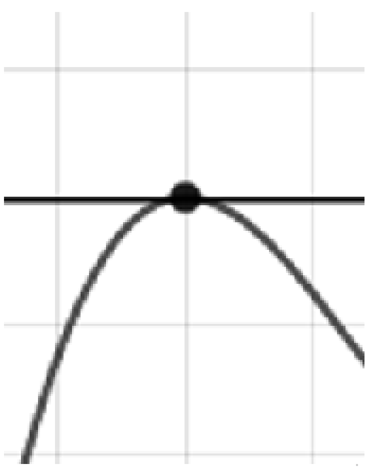
$$f(x) = 9(x+2)^9(x-2)^{14}(x+7)^4(x-7)^8$$

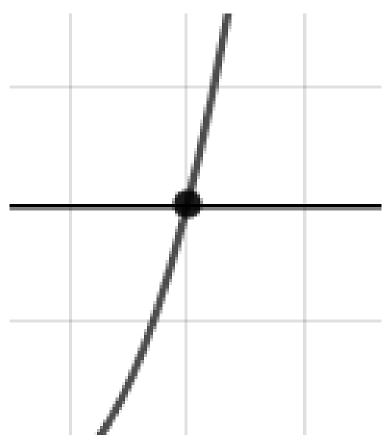
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 2i \text{ and } 3$$

The solution is $x^3 - 11x^2 + 44x - 60$, which is option B.

A. $b \in [-3, 2], c \in [-8.19, -6.43]$, and $d \in [12, 18]$

$x^3 + x^2 - 7x + 12$, which corresponds to multiplying out $(x - 4)(x - 3)$.

B. $b \in [-18, -8], c \in [42.96, 45.21]$, and $d \in [-60, -55]$

* $x^3 - 11x^2 + 44x - 60$, which is the correct option.

C. $b \in [10, 12], c \in [42.96, 45.21]$, and $d \in [53, 64]$

$x^3 + 11x^2 + 44x + 60$, which corresponds to multiplying out $(x - (4 + 2i))(x - (4 - 2i))(x + 3)$.

D. $b \in [-3, 2], c \in [-5.57, -3.76]$, and $d \in [-2, 10]$

$x^3 + x^2 - 5x + 6$, which corresponds to multiplying out $(x - 2)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 + 2i))(x - (4 - 2i))(x - (3))$.

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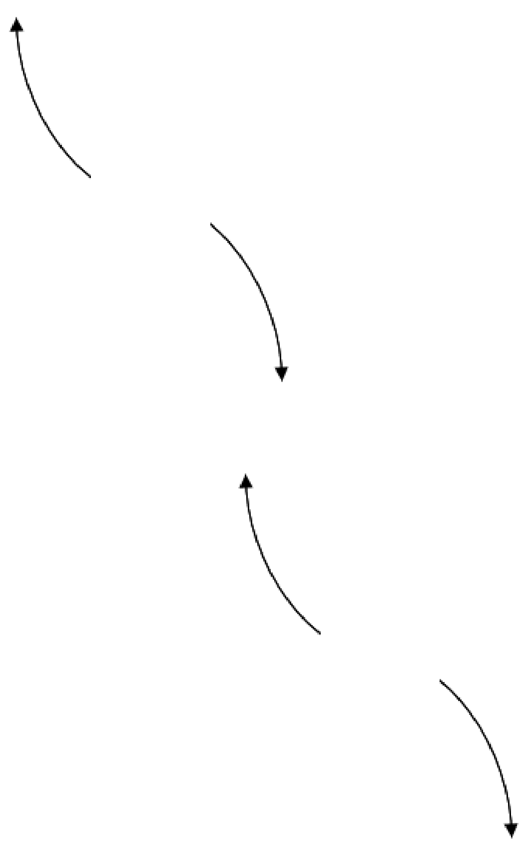
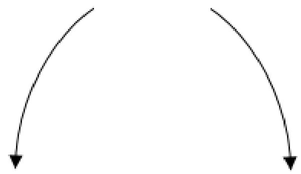
10. Describe the end behavior of the polynomial below.

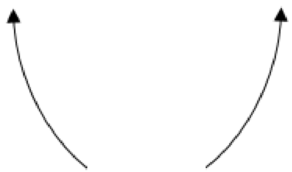
$$f(x) = -4(x + 4)^5(x - 4)^6(x - 5)^2(x + 5)^2$$

The solution is the graph below, which is option A.

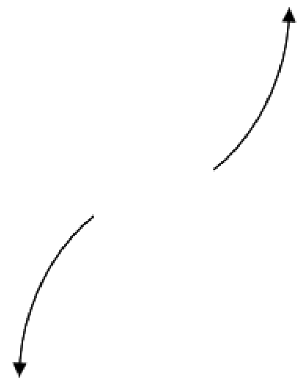




B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.