This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

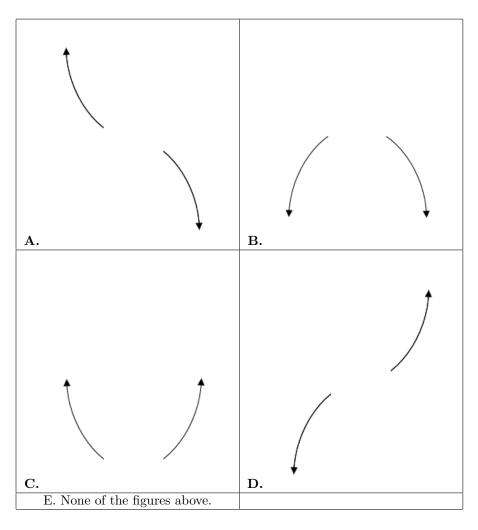
Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

26. Describe the end behavior of the polynomial below.

$$f(x) = -7(x-8)^{2}(x+8)^{5}(x-7)^{4}(x+7)^{5}$$

The solution is





**General Comments:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

27. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$4-2i$$
 and  $4$ 

The solution is  $x^3 - 12x^2 + 52x - 80$ 

A. 
$$b \in [-2, 2], c \in [-12, -6], \text{ and } d \in [15, 19]$$
  
 $x^3 + x^2 - 8x + 16, \text{ which corresponds to multiplying out } (x - 4)(x - 4).$ 

B. 
$$b \in [5, 17], c \in [44, 60]$$
, and  $d \in [79, 89]$   
 $x^3 + 12x^2 + 52x + 80$ , which corresponds to multiplying out  $(x - (4 - 2i))(x - (4 + 2i))(x + 4)$ .

C. 
$$b \in [-2, 2], c \in [-7, 2], \text{ and } d \in [-12, -5]$$
  
 $x^3 + x^2 - 2x - 8, \text{ which corresponds to multiplying out } (x + 2)(x - 4).$ 

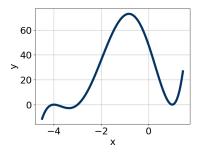
D. 
$$b \in [-19, -9], c \in [44, 60]$$
, and  $d \in [-83, -74]$   
\*  $x^3 - 12x^2 + 52x - 80$ , which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comments: Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (4 - 2i))(x - (4 + 2i))(x - (4)).

28. Which of the following equations *could* be of the graph presented below?



The solution is  $2(x-1)^6(x+4)^8(x+3)^5$ 

A. 
$$3(x-1)^4(x+4)^7(x+3)^{11}$$

The factor (x + 4) should have an even power.

B. 
$$2(x-1)^6(x+4)^8(x+3)^5$$

\* This is the correct option.

C. 
$$16(x-1)^6(x+4)^5(x+3)^{10}$$

The factor (x + 4) should have an even power and the factor (x + 3) should have an odd power.

D. 
$$-17(x-1)^8(x+4)^8(x+3)^{10}$$

The factor (x+3) should have an odd power and the leading coefficient should be the opposite sign.

E. 
$$-13(x-1)^8(x+4)^4(x+3)^{11}$$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

29. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$-2, -3, \text{ and } \frac{2}{5}$$

The solution is  $5x^3 + 23x^2 + 20x - 12$ 

A.  $a \in [3, 6], b \in [22.5, 25.8], c \in [18, 22],$  and  $d \in [8, 19]$  $5x^3 + 23x^2 + 20x + 12$ , which corresponds to multiplying everything correctly except the constant term.

B. 
$$a \in [3, 6], b \in [-28.1, -24.8], c \in [37, 45], \text{ and } d \in [-13, -8]$$
  
 $5x^3 - 27x^2 + 40x - 12$ , which corresponds to multiplying out  $(x + 1)(x + 1)(5x - 5)$ .

C. 
$$a \in [3, 6], b \in [-26.2, -20.1], c \in [18, 22], \text{ and } d \in [8, 19]$$
  
 $5x^3 - 23x^2 + 20x + 12$ , which corresponds to multiplying out  $(x - 2)(x - 3)(5x + 2)$ .

D. 
$$a \in [3, 6], b \in [22.5, 25.8], c \in [18, 22], \text{ and } d \in [-13, -8]$$
  
\*  $5x^3 + 23x^2 + 20x - 12$ , which is the correct option.

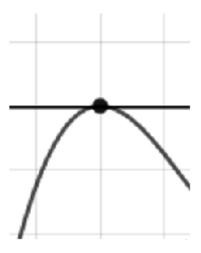
E. 
$$a \in [3, 6], b \in [0.9, 6.3], c \in [-39, -23], \text{ and } d \in [8, 19]$$
  
 $5x^3 + 3x^2 - 32x + 12$ , which corresponds to multiplying out  $(x + 1)(x - 1)(5x - 5)$ .

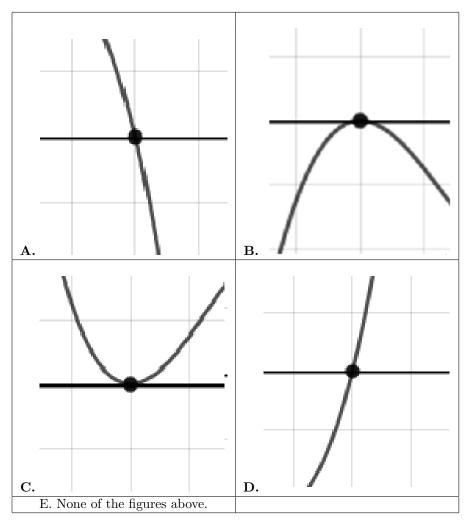
General Comments: To construct the lowest-degree polynomial, you want to multiply out (x+2)(x+3)(5x-2)

30. Describe the zero behavior of the zero x=2 of the polynomial below.

$$f(x) = -5(x-6)^{10}(x+6)^9(x-2)^8(x+2)^7$$

The solution is





**General Comments:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

 $\operatorname{Summer} \operatorname{C} 2020$