This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = e^{x-4} + 2$$

The solution is $f^{-1}(8) = 5.792$, which is option C.

A. $f^{-1}(8) \in [4.33, 4.59]$

This solution corresponds to distractor 3.

B. $f^{-1}(8) \in [4.17, 4.34]$

This solution corresponds to distractor 2.

C. $f^{-1}(8) \in [5.36, 6]$

This is the solution.

D. $f^{-1}(8) \in [-2.43, -2.18]$

This solution corresponds to distractor 1.

E. $f^{-1}(8) \in [3.28, 3.69]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

---

2. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 6x^2 + 6x + 5 \text{ and } g(x) = 5x + 1$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [0.2, 9.2]$

B. The domain is all Real numbers except $x = a$, where $a \in [1.75, 7.75]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [3.33, 9.33]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [2.2, 9.2]$ and $b \in [4.25, 6.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

---

3. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = x^3 + x^2 + 3x \text{ and } g(x) = 4x^3 + 3x^2 - 3x + 2$$

The solution is 92.0, which is option D.

A. $(f \circ g)(-1) \in [-71, -67]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(-1) \in [100, 107]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [-82, -78]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [91, 95]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

---

4. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = 3x^2 - 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(10) \in [1.92, 3.29]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(10) \in [4.08, 5.39]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(10) \in [0.22, 2.01]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(10) \in [5.04, 6.9]$

Distractor 4: This corresponds to both distractors 2 and 3.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

---

5. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{4}{6x + 19} \text{ and } g(x) = \frac{4}{5x - 26}$$

The solution is The domain is all Real numbers except $x = -3.1666666666666665$ and $x = 5.2$, which is option D.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [1.17, 5.17]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [5.25, 10.25]$

C. The domain is all Real numbers except $x = a,$ where $a \in [-7.6, -0.6]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-5.17, 2.83]$ and $b \in [1.2, 12.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

---

6. Determine whether the function below is 1-1.

$$f(x) = \sqrt{-3x - 8}$$

The solution is yes, which is option C.

A. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. Yes, the function is 1-1.

* This is the solution.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

---

7. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -2x^3 - 4x^2 + 3x \text{ and } g(x) = -x^3 - 3x^2 + x + 1$$

The solution is -6.0, which is option C.

A. $(f \circ g)(-1) \in [34, 44]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [3, 5]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [-6, -3]$

* This is the correct solution

D. $(f \circ g)(-1) \in [42, 50]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

---

8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^{-1}(-15)$ belongs to.

$$f(x) = 4x^2 - 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [0.4, 2.05]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(-15) \in [3.75, 4.87]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(-15) \in [2.15, 2.66]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(-15) \in [6.34, 7.06]$

Distractor 4: This corresponds to both distractors 2 and 3.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

---

9. Determine whether the function below is 1-1.

$$f(x) = \sqrt{3x + 13}$$

The solution is yes, which is option E.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. Yes, the function is 1-1.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

---

10. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = e^{x+4} - 5$$

The solution is $f^{-1}(10) = -1.292$, which is option E.

A. $f^{-1}(10) \in [-2.43, -2.36]$

This solution corresponds to distractor 4.

B. $f^{-1}(10) \in [-3.33, -3.13]$

This solution corresponds to distractor 3.

C. $f^{-1}(10) \in [-3.54, -3.21]$

This solution corresponds to distractor 2.

D. $f^{-1}(10) \in [6.68, 6.81]$

This solution corresponds to distractor 1.

E. $f^{-1}(10) \in [-1.34, -1.16]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.
