This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 4i \text{ and } -4$$

The solution is $x^3 - 6x^2 + x + 164$, which is option B.

A. $b \in [-2.3, 4.4], c \in [-1.6, -0.93], \text{ and } d \in [-21.5, -18]$

$x^3 + x^2 - x - 20$, which corresponds to multiplying out $(x - 5)(x + 4)$.

B. $b \in [-8.4, -5.6], c \in [0.11, 1.85], \text{ and } d \in [163.6, 167.2]$

* $x^3 - 6x^2 + x + 164$, which is the correct option.

C. $b \in [3, 9.5], c \in [0.11, 1.85], \text{ and } d \in [-165.4, -158.5]$

$x^3 + 6x^2 + x - 164$, which corresponds to multiplying out $(x - (5 + 4i))(x - (5 - 4i))(x - 4)$.

D. $b \in [-2.3, 4.4], c \in [-0.84, 0.88], \text{ and } d \in [-17.4, -12]$

$x^3 + x^2 + 0x - 16$, which corresponds to multiplying out $(x - 4)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 4i))(x - (5 - 4i))(x - (-4))$.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{2}, \frac{-1}{4}, \text{ and } \frac{7}{5}$$

The solution is $40x^3 - 26x^2 - 37x - 7$, which is option E.

A. $a \in [35, 42], b \in [-88, -76], c \in [47, 56], \text{ and } d \in [-8, -2]$

$40x^3 - 86x^2 + 47x - 7$, which corresponds to multiplying out $(2x - 1)(4x - 1)(5x - 7)$.

B. $a \in [35, 42], b \in [-66, -63], c \in [4, 12], \text{ and } d \in [3, 13]$

$40x^3 - 66x^2 + 9x + 7$, which corresponds to multiplying out $(2x - 1)(4x + 1)(5x - 7)$.

C. $a \in [35, 42], b \in [20, 30], c \in [-38, -35], \text{ and } d \in [3, 13]$

$40x^3 + 26x^2 - 37x + 7$, which corresponds to multiplying out $(2x - 1)(4x - 1)(5x + 7)$.

D. $a \in [35, 42], b \in [-29, -24], c \in [-38, -35],$ and $d \in [3, 13]$

$40x^3 - 26x^2 - 37x + 7$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [35, 42], b \in [-29, -24], c \in [-38, -35],$ and $d \in [-8, -2]$

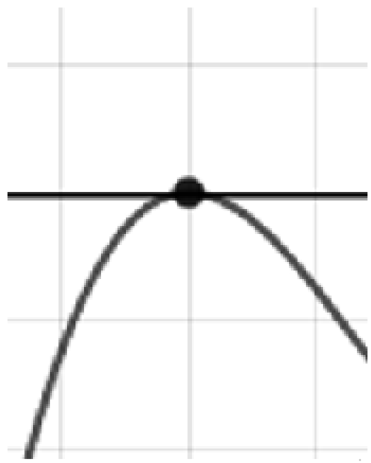
* $40x^3 - 26x^2 - 37x - 7$, which is the correct option.

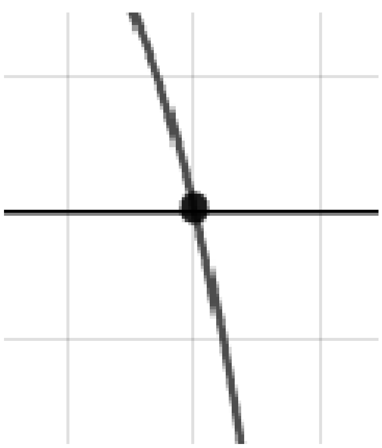
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 1)(4x + 1)(5x - 7)$

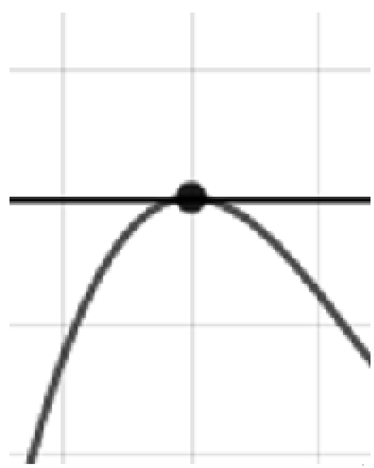
3. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

$$f(x) = -7(x - 2)^{10}(x + 2)^6(x + 8)^{11}(x - 8)^6$$

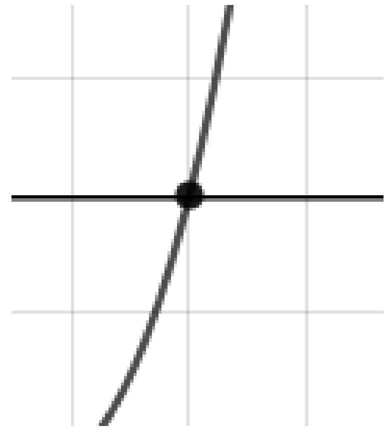
The solution is the graph below, which is option B.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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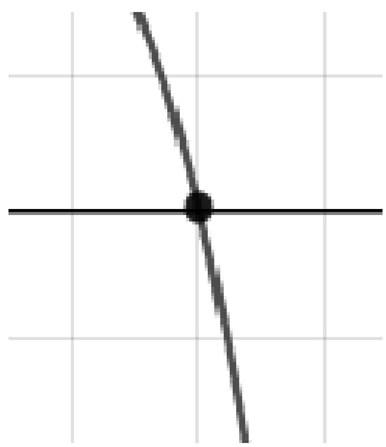
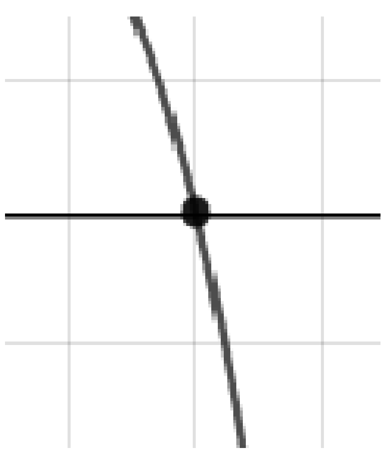
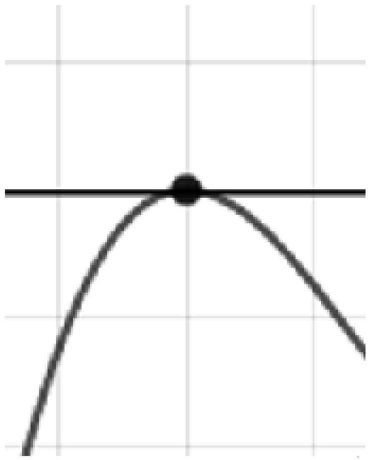
4. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

$$f(x) = -9(x-5)^{12}(x+5)^{9}(x+8)^{10}(x-8)^{7}$$

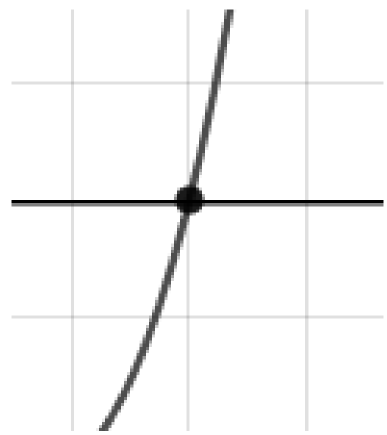
The solution is the graph below, which is option A.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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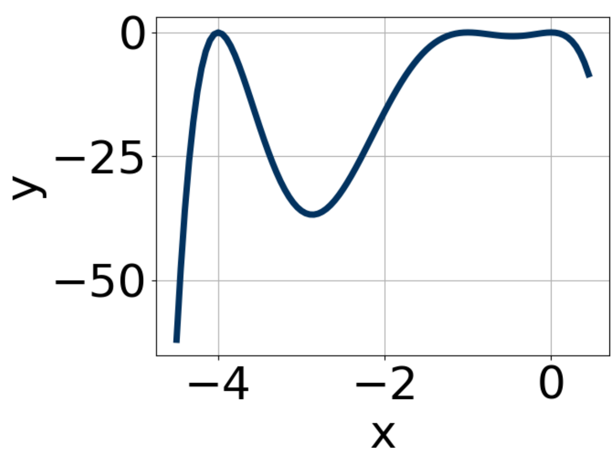
5. Which of the following equations *could* be of the graph presented below?



The solution is $-17x^4(x+4)^4(x+1)^6$, which is option C.

A. $-8x^7(x+4)^4(x+1)^{11}$

The factors x and $(x+1)$ should both have even powers.

B. $-14x^{10}(x+4)^8(x+1)^9$

The factor $(x+1)$ should have an even power.

C. $-17x^4(x+4)^4(x+1)^6$

* This is the correct option.

D. $14x^{10}(x+4)^6(x+1)^9$

The factor $(x+1)$ should have an even power and the leading coefficient should be the opposite sign.

E. $13x^8(x+4)^{10}(x+1)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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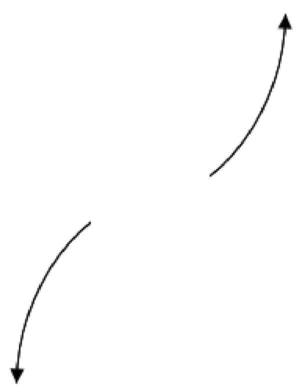
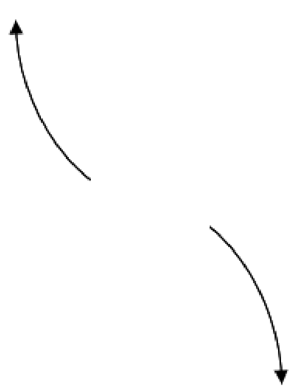
6. Describe the end behavior of the polynomial below.

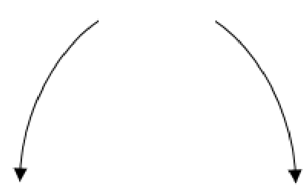
$$f(x) = 5(x+3)^3(x-3)^8(x+7)^3(x-7)^5$$

The solution is the graph below, which is option D.

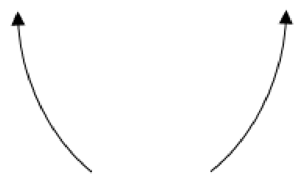


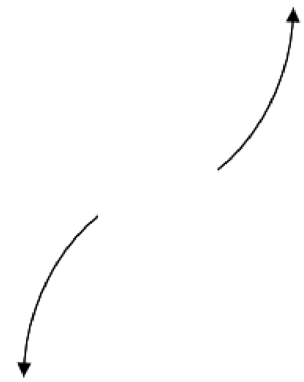


A.



B.

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 5i \text{ and } 4$$

The solution is $x^3 + 4x^2 + 9x - 164$, which is option C.

A. $b \in [-8, -3], c \in [8.79, 9.4],$ and $d \in [160, 168]$

$x^3 - 4x^2 + 9x + 164$, which corresponds to multiplying out $(x - (-4 - 5i))(x - (-4 + 5i))(x + 4)$.

B. $b \in [-1, 2], c \in [0.48, 1.62],$ and $d \in [-24, -18]$

$x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

C. $b \in [2, 5], c \in [8.79, 9.4],$ and $d \in [-165, -163]$

* $x^3 + 4x^2 + 9x - 164$, which is the correct option.

D. $b \in [-1, 2], c \in [-0.13, 0.06],$ and $d \in [-18, -14]$

$x^3 + x^2 + 0x - 16$, which corresponds to multiplying out $(x + 4)(x - 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 - 5i))(x - (-4 + 5i))(x - (4))$.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-7, \frac{-3}{2}, \text{ and } \frac{-5}{3}$$

The solution is $6x^3 + 61x^2 + 148x + 105$, which is option C.

A. $a \in [0, 7], b \in [-44, -36], c \in [-26, -17],$ and $d \in [101, 112]$

$6x^3 - 41x^2 - 22x + 105$, which corresponds to multiplying out $(x - 7)(2x - 3)(3x + 5)$.

B. $a \in [0, 7], b \in [59, 62], c \in [138, 153],$ and $d \in [-109, -103]$

$6x^3 + 61x^2 + 148x - 105$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [0, 7], b \in [59, 62], c \in [138, 153],$ and $d \in [101, 112]$

* $6x^3 + 61x^2 + 148x + 105$, which is the correct option.

D. $a \in [0, 7], b \in [-26, -18], c \in [-119, -111],$ and $d \in [-109, -103]$

$6x^3 - 23x^2 - 118x - 105$, which corresponds to multiplying out $(x - 7)(2x + 3)(3x + 5)$.

E. $a \in [0, 7], b \in [-65, -60], c \in [138, 153],$ and $d \in [-109, -103]$

$6x^3 - 61x^2 + 148x - 105$, which corresponds to multiplying out $(x - 7)(2x - 3)(3x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+7)(2x+3)(3x+5)$

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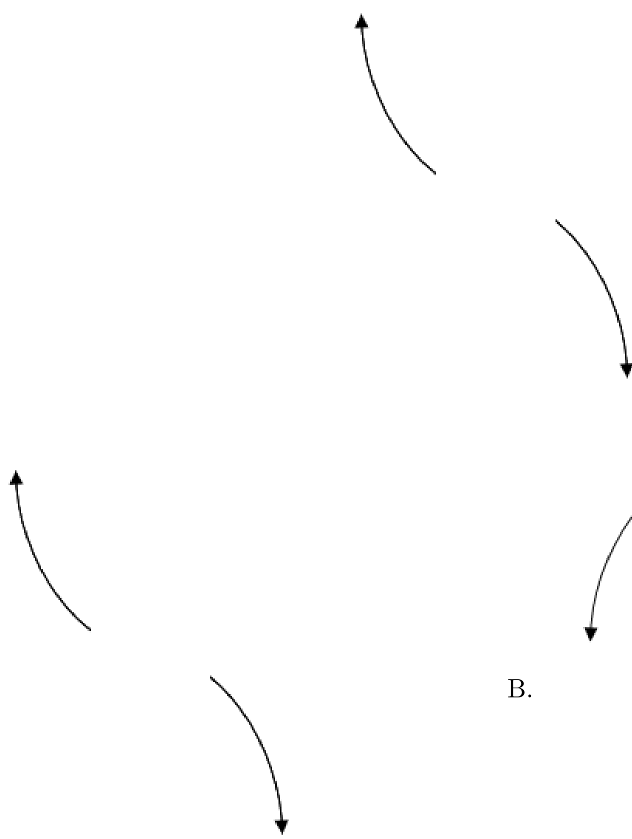
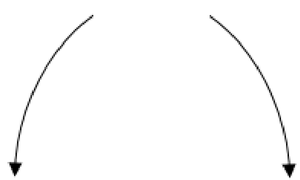
9. Describe the end behavior of the polynomial below.

$$f(x) = -4(x - 5)^3(x + 5)^4(x + 6)^3(x - 6)^3$$

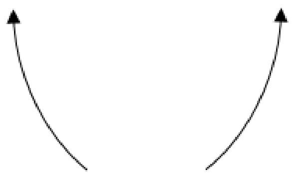
The solution is the graph below, which is option A.

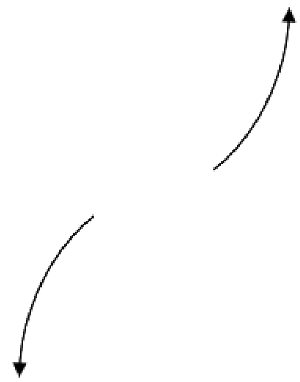




B.

A.

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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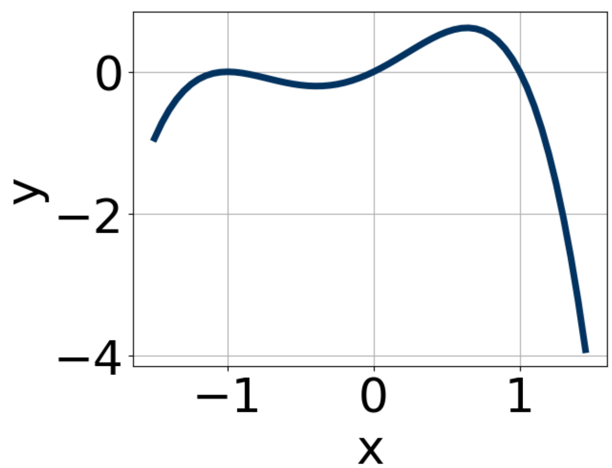
10. Which of the following equations *could* be of the graph presented below?



The solution is $-12x^9(x+1)^8(x-1)^5$, which is option C.

A. $10x^{11}(x+1)^4(x-1)^5$

   This corresponds to the leading coefficient being the opposite value than it should be.

B. $-8x^7(x+1)^8(x-1)^{10}$

   The factor $(x-1)$ should have an odd power.

C. $-12x^9(x+1)^8(x-1)^5$

   * This is the correct option.

D. $3x^{10}(x+1)^4(x-1)^{11}$

   The factor x should have an odd power and the leading coefficient should be the opposite sign.

E. $-10x^9(x+1)^9(x-1)^{10}$

   The factor -1 should have an even power and the factor 1 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).