This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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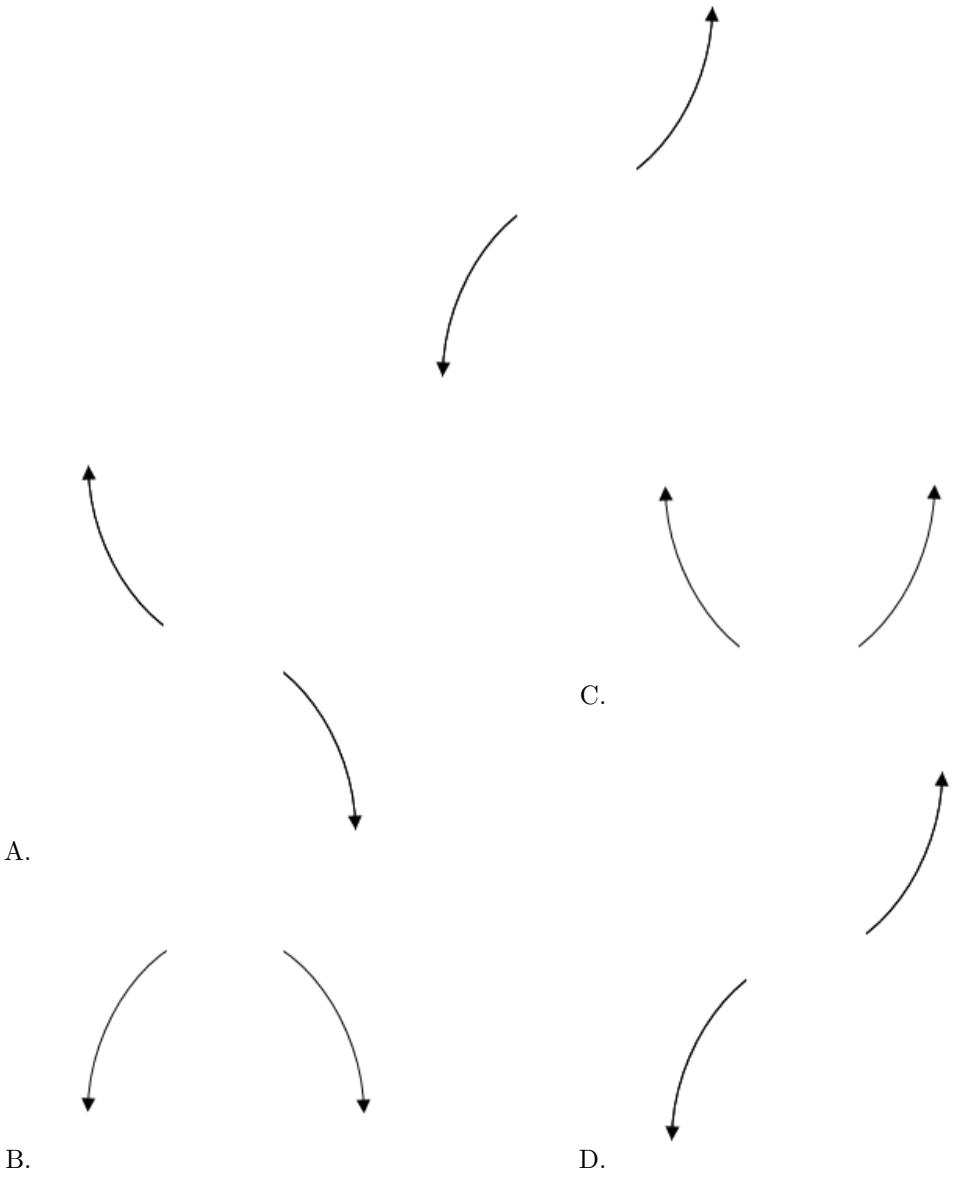
1. Describe the end behavior of the polynomial below.

$$f(x) = 9(x+2)^4(x-2)^9(x-9)^3(x+9)^5$$

The solution is the graph below, which is option D.



A.

C.

B.

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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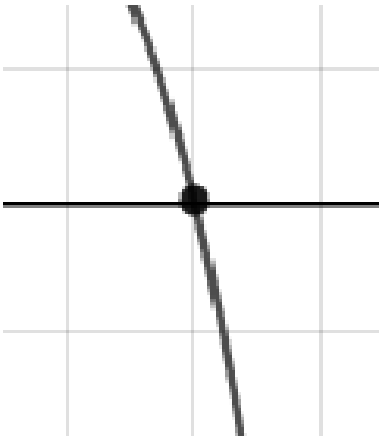
2. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

$$f(x) = 8(x+8)^7(x-8)^{12}(x-4)^4(x+4)^8$$

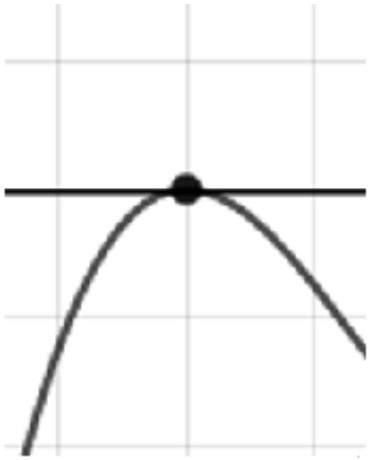
The solution is the graph below, which is option C.

A.

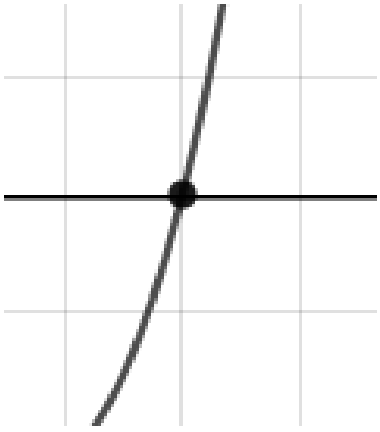


C.

B.



D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 2i \text{ and } 1$$

The solution is $x^3 + 7x^2 + 12x - 20$, which is option A.

A. $b \in [5, 17], c \in [12, 13],$ and $d \in [-24, -17]$

* $x^3 + 7x^2 + 12x - 20$, which is the correct option.

B. $b \in [-6, 2], c \in [-2, 6],$ and $d \in [-5, -2]$

$x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

C. $b \in [-13, -1], c \in [12, 13],$ and $d \in [19, 24]$

$x^3 - 7x^2 + 12x + 20$, which corresponds to multiplying out $(x - (-4 + 2i))(x - (-4 - 2i))(x + 1)$.

D. $b \in [-6, 2], c \in [-4, 0],$ and $d \in [-1, 7]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x - 2)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 2i))(x - (-4 - 2i))(x - (1))$.

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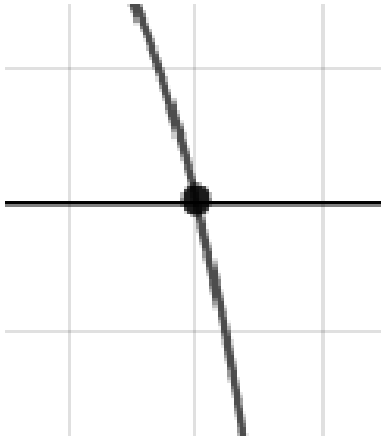
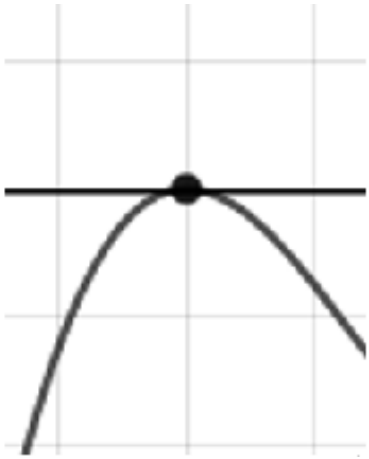
4. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

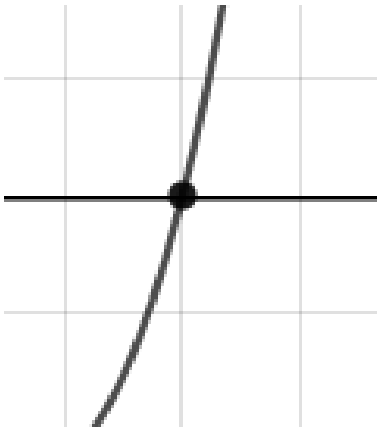
$$f(x) = 6(x + 4)^{12}(x - 4)^9(x - 5)^5(x + 5)^2$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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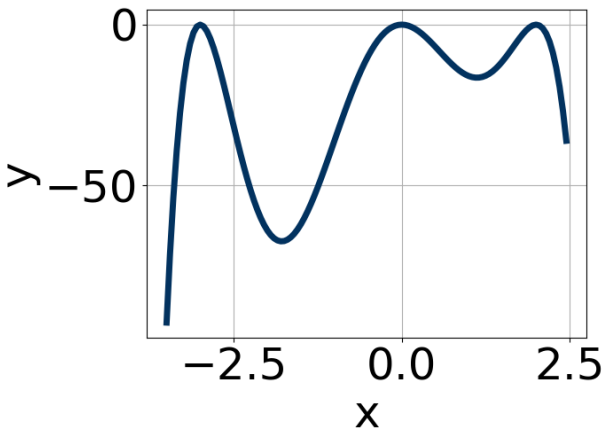
5. Which of the following equations *could* be of the graph presented below?



The solution is $-12x^6(x+3)^6(x-2)^{10}$, which is option E.

A. $16x^{10}(x+3)^{10}(x-2)^5$

The factor $(x-2)$ should have an even power and the leading coefficient should be the opposite sign.

B. $-8x^9(x+3)^8(x-2)^5$

The factors x and $(x-2)$ should both have even powers.

C. $-19x^4(x+3)^{10}(x-2)^{11}$

The factor $(x-2)$ should have an even power.

D. $5x^{10}(x+3)^{10}(x-2)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-12x^6(x+3)^6(x-2)^{10}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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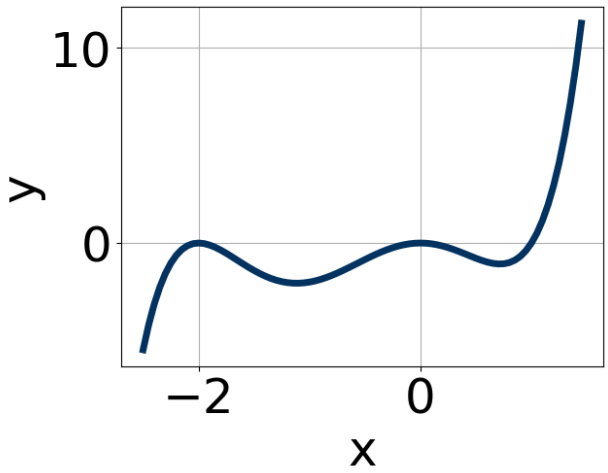
6. Which of the following equations *could* be of the graph presented below?



The solution is $2x^4(x+2)^6(x-1)^7$, which is option A.

A. $2x^4(x+2)^6(x-1)^7$

* This is the correct option.

B. $-20x^6(x+2)^8(x-1)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-20x^{10}(x+2)^6(x-1)^4$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $9x^5(x+2)^8(x-1)^9$

The factor x should have an even power.

E. $14x^5(x+2)^4(x-1)^8$

The factor x should have an even power and the factor $(x-1)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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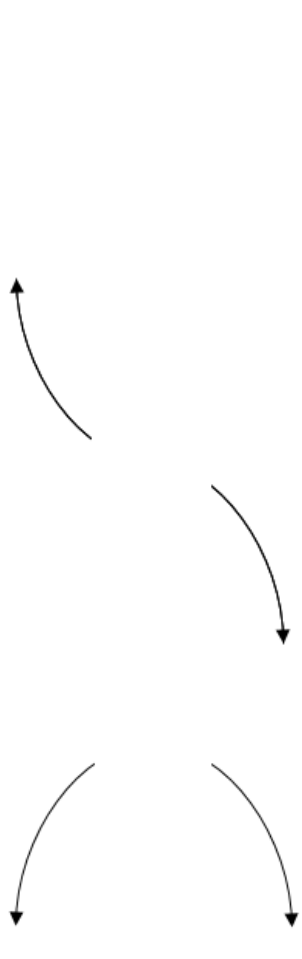
7. Describe the end behavior of the polynomial below.

$$f(x) = -8(x+8)^5(x-8)^6(x-2)^2(x+2)^3$$

The solution is the graph below, which is option B.

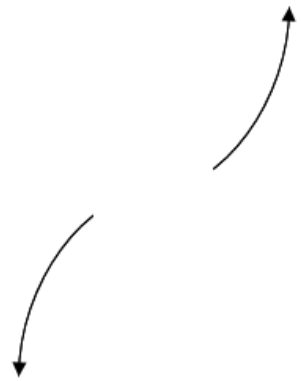


C.

A.

B.

D.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{2}{5}, \frac{1}{5}, \text{ and } 5$$

The solution is $25x^3 - 140x^2 + 77x - 10$, which is option A.

A. $a \in [20, 28], b \in [-140, -132], c \in [71, 78],$ and $d \in [-17, -9]$

* $25x^3 - 140x^2 + 77x - 10$, which is the correct option.

B. $a \in [20, 28], b \in [-111, -108], c \in [-73, -71],$ and $d \in [-17, -9]$

$25x^3 - 110x^2 - 73x - 10$, which corresponds to multiplying out $(5x + 2)(5x + 1)(x - 5)$.

C. $a \in [20, 28], b \in [-140, -132], c \in [71, 78],$ and $d \in [3, 13]$

$25x^3 - 140x^2 + 77x + 10$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [20, 28], b \in [137, 143], c \in [71, 78],$ and $d \in [3, 13]$

$25x^3 + 140x^2 + 77x + 10$, which corresponds to multiplying out $(5x + 2)(5x + 1)(x + 5)$.

E. $a \in [20, 28], b \in [-120, -116], c \in [-29, -19],$ and $d \in [3, 13]$

$25x^3 - 120x^2 - 27x + 10$, which corresponds to multiplying out $(5x + 2)(5x - 1)(x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 2)(5x - 1)(x - 5)$

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 5i \text{ and } -2$$

The solution is $x^3 + 12x^2 + 70x + 100$, which is option A.

A. $b \in [4, 22], c \in [68, 74],$ and $d \in [92, 107]$

* $x^3 + 12x^2 + 70x + 100$, which is the correct option.

B. $b \in [-1, 10], c \in [2, 12],$ and $d \in [9, 19]$

$x^3 + x^2 + 7x + 10$, which corresponds to multiplying out $(x + 5)(x + 2)$.

C. $b \in [-1, 10], c \in [-3, -1],$ and $d \in [-13, -9]$

$x^3 + x^2 - 3x - 10$, which corresponds to multiplying out $(x - 5)(x + 2)$.

D. $b \in [-13, -3], c \in [68, 74],$ and $d \in [-102, -96]$

$x^3 - 12x^2 + 70x - 100$, which corresponds to multiplying out $(x - (-5 + 5i))(x - (-5 - 5i))(x - 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 5i))(x - (-5 - 5i))(x - (-2))$.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{3}{4}, -4, \text{ and } \frac{-2}{3}$$

The solution is $12x^3 + 47x^2 - 10x - 24$, which is option E.

A. $a \in [10, 16], b \in [-35, -30], c \in [-67, -55],$ and $d \in [-26, -22]$

$12x^3 - 31x^2 - 62x - 24$, which corresponds to multiplying out $(4x + 3)(x - 4)(3x + 2)$.

B. $a \in [10, 16], b \in [41, 52], c \in [-10, -6],$ and $d \in [24, 25]$

$12x^3 + 47x^2 - 10x + 24$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [10, 16], b \in [-48, -40], c \in [-10, -6],$ and $d \in [24, 25]$

$12x^3 - 47x^2 - 10x + 24$, which corresponds to multiplying out $(4x + 3)(x - 4)(3x - 2)$.

D. $a \in [10, 16], b \in [64, 69], c \in [72, 82],$ and $d \in [24, 25]$

$12x^3 + 65x^2 + 74x + 24$, which corresponds to multiplying out $(4x + 3)(x + 4)(3x + 2)$.

E. $a \in [10, 16], b \in [41, 52], c \in [-10, -6],$ and $d \in [-26, -22]$

* $12x^3 + 47x^2 - 10x - 24$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 3)(x + 4)(3x + 2)$