This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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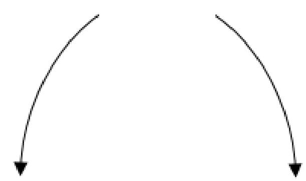
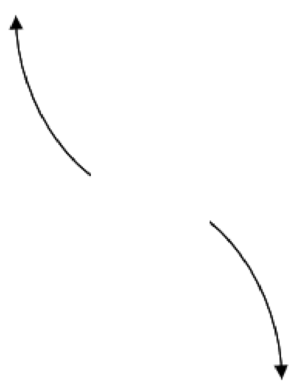
1. Describe the end behavior of the polynomial below.

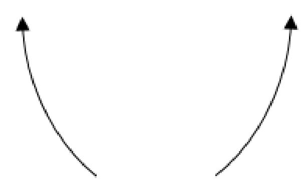
$$f(x) = -8(x+9)^4(x-9)^5(x-2)^5(x+2)^6$$

The solution is the graph below, which is option B.

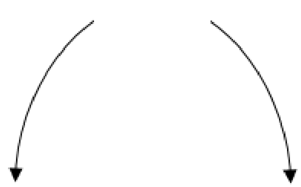




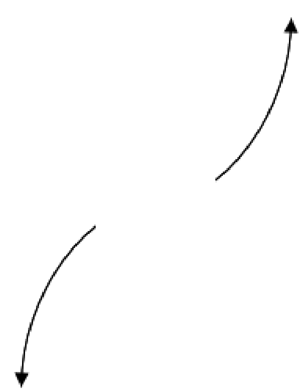
A.



C.



B.



D.

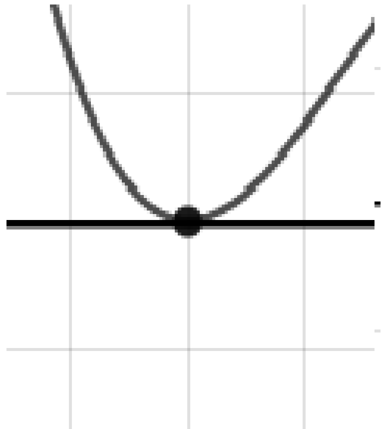
E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

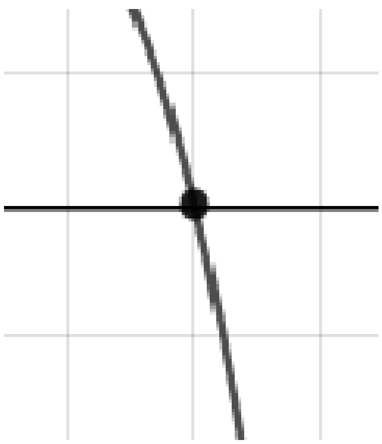
2. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

$$f(x) = -5(x+8)^9(x-8)^5(x-2)^6(x+2)^5$$

The solution is the graph below, which is option C.

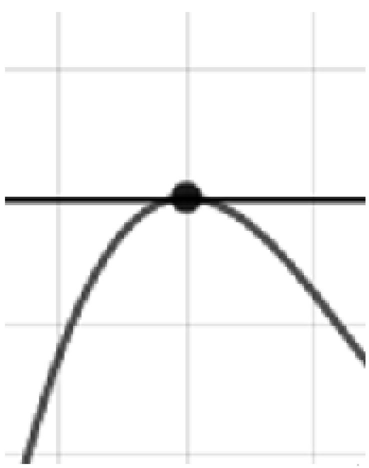


A.

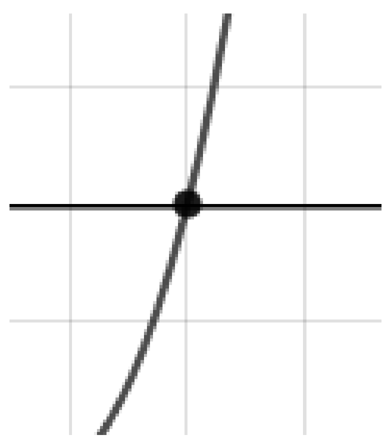


C.

B.



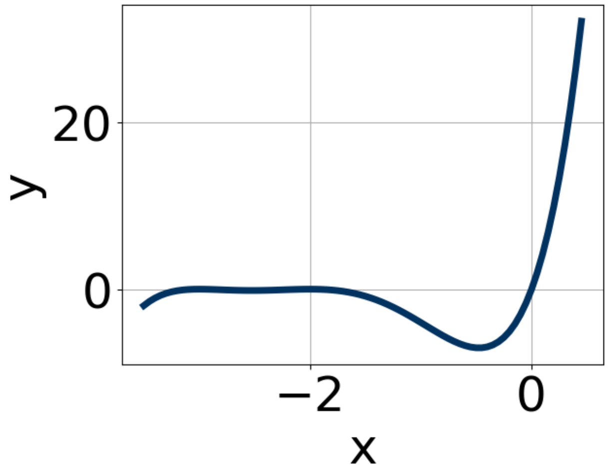
D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

3. Which of the following equations *could* be of the graph presented below?



The solution is $12x^7(x+3)^{10}(x+2)^4$, which is option A.

A. $12x^7(x+3)^{10}(x+2)^4$

   * This is the correct option.

B. $14x^8(x+3)^6(x+2)^5$

   The factor $(x+2)$ should have an even power and the factor x should have an odd power.

C. $-5x^6(x+3)^4(x+2)^6$

   The factor x should have an odd power and the leading coefficient should be the opposite sign.

D. $15x^5(x+3)^6(x+2)^7$

   The factor $(x+2)$ should have an even power.

E. $-17x^5(x+3)^6(x+2)^6$

   This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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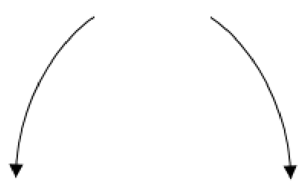
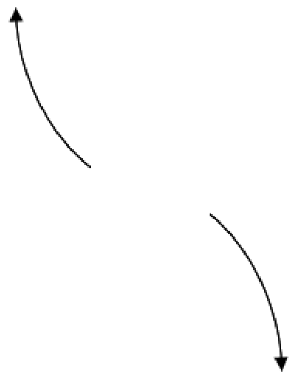
4. Describe the end behavior of the polynomial below.

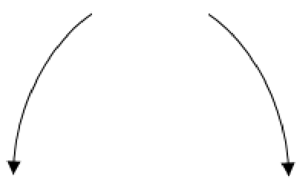
$$f(x) = -3(x-5)^4(x+5)^5(x+9)^3(x-9)^4$$

The solution is the graph below, which is option B.

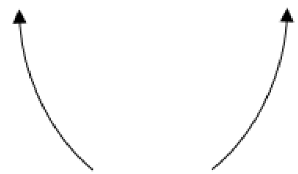




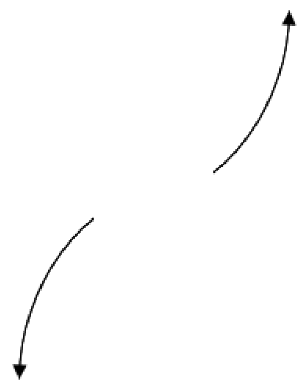
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 2i \text{ and } -2$$

The solution is $x^3 + 12x^2 + 49x + 58$, which is option B.

A. $b \in [0, 3], c \in [-4, 3],$ and $d \in [-7, -1]$

$x^3 + x^2 - 4$, which corresponds to multiplying out $(x - 2)(x + 2)$.

B. $b \in [11, 17], c \in [48, 50],$ and $d \in [57, 61]$

* $x^3 + 12x^2 + 49x + 58$, which is the correct option.

C. $b \in [0, 3], c \in [4, 10],$ and $d \in [3, 13]$

$x^3 + x^2 + 7x + 10$, which corresponds to multiplying out $(x + 5)(x + 2)$.

D. $b \in [-13, -10], c \in [48, 50],$ and $d \in [-61, -53]$

$x^3 - 12x^2 + 49x - 58$, which corresponds to multiplying out $(x - (-5 + 2i))(x - (-5 - 2i))(x - 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 2i))(x - (-5 - 2i))(x - (-2))$.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{3}, \frac{2}{5}, \text{ and } \frac{1}{2}$$

The solution is $30x^3 - 37x^2 + 15x - 2$, which is option E.

A. $a \in [29, 32], b \in [36, 40], c \in [10, 16], \text{ and } d \in [0.7, 3.2]$

$30x^3 + 37x^2 + 15x + 2$, which corresponds to multiplying out $(3x + 1)(5x + 2)(2x + 1)$.

B. $a \in [29, 32], b \in [1, 12], c \in [-11, -6], \text{ and } d \in [-2.2, -0.1]$

$30x^3 + 7x^2 - 7x - 2$, which corresponds to multiplying out $(3x + 3)(5x + 5)(2x - 2)$.

C. $a \in [29, 32], b \in [-40, -33], c \in [10, 16], \text{ and } d \in [0.7, 3.2]$

$30x^3 - 37x^2 + 15x + 2$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [29, 32], b \in [-21, -15], c \in [-3, 0], \text{ and } d \in [0.7, 3.2]$

$30x^3 - 17x^2 - 3x + 2$, which corresponds to multiplying out $(3x + 3)(5x - 5)(2x - 2)$.

E. $a \in [29, 32], b \in [-40, -33], c \in [10, 16], \text{ and } d \in [-2.2, -0.1]$

* $30x^3 - 37x^2 + 15x - 2$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 1)(5x - 2)(2x - 1)$

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{2}{5}, \frac{-7}{5}, \text{ and } 3$$

The solution is $25x^3 - 50x^2 - 89x + 42$, which is option A.

A. $a \in [22, 30], b \in [-53, -44], c \in [-89, -85], \text{ and } d \in [32, 48]$

* $25x^3 - 50x^2 - 89x + 42$, which is the correct option.

B. $a \in [22, 30], b \in [-53, -44], c \in [-89, -85], \text{ and } d \in [-44, -39]$

$25x^3 - 50x^2 - 89x - 42$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [22, 30], b \in [-30, -29], c \in [-128, -119], \text{ and } d \in [-44, -39]$

$25x^3 - 30x^2 - 121x - 42$, which corresponds to multiplying out $(5x + 5)(5x - 5)(x - 1)$.

D. $a \in [22, 30], b \in [50, 51], c \in [-89, -85], \text{ and } d \in [-44, -39]$

$25x^3 + 50x^2 - 89x - 42$, which corresponds to multiplying out $(5x + 2)(5x - 7)(x + 3)$.

E. $a \in [22, 30], b \in [-108, -96], c \in [60, 64], \text{ and } d \in [32, 48]$

$25x^3 - 100x^2 + 61x + 42$, which corresponds to multiplying out $(5x + 5)(5x + 5)(x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 2)(5x + 7)(x - 3)$

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 + 2i \text{ and } -3$$

The solution is $x^3 + 9x^2 + 31x + 39$, which is option C.

A. $b \in [-2, 6], c \in [2, 11]$, and $d \in [7, 14]$

$x^3 + x^2 + 6x + 9$, which corresponds to multiplying out $(x + 3)(x + 3)$.

B. $b \in [-2, 6], c \in [1, 2]$, and $d \in [-12, 2]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x - 2)(x + 3)$.

C. $b \in [5, 21], c \in [23, 38]$, and $d \in [39, 45]$

* $x^3 + 9x^2 + 31x + 39$, which is the correct option.

D. $b \in [-10, -8], c \in [23, 38]$, and $d \in [-40, -33]$

$x^3 - 9x^2 + 31x - 39$, which corresponds to multiplying out $(x - (-3 + 2i))(x - (-3 - 2i))(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

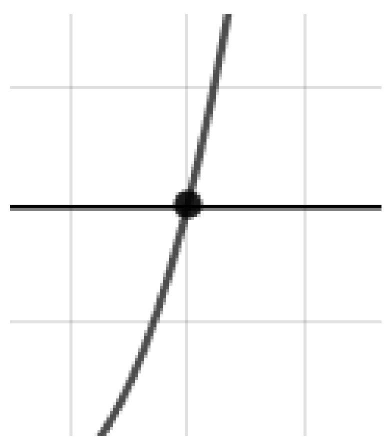
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 2i))(x - (-3 - 2i))(x - (-3))$.

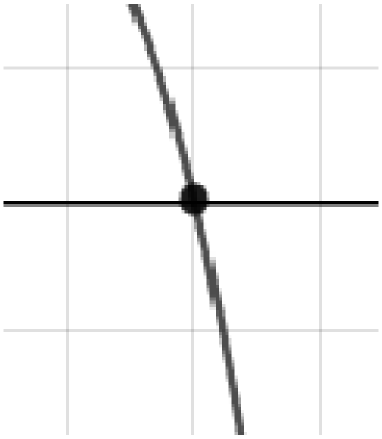
---

9. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

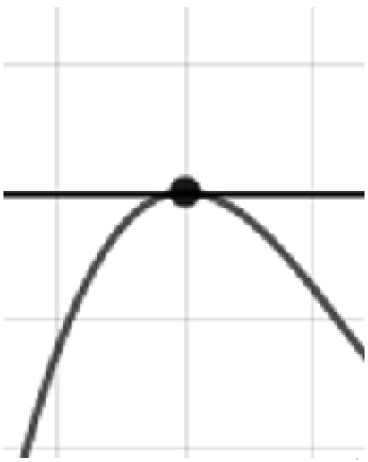
$$f(x) = 2(x + 3)^{13}(x - 3)^9(x + 8)^{12}(x - 8)^7$$

The solution is the graph below, which is option D.

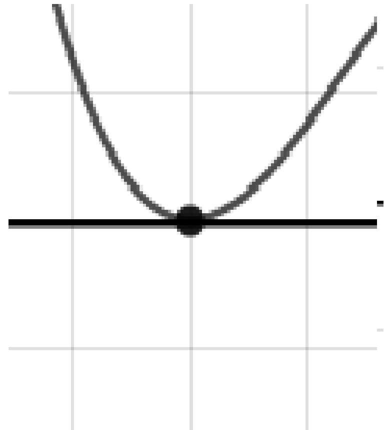




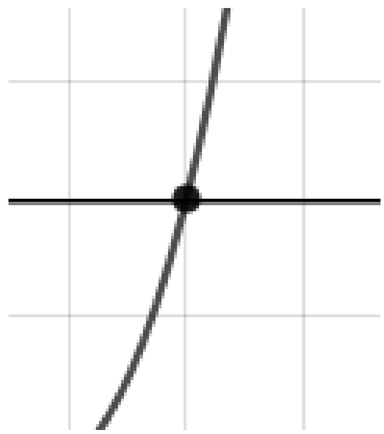
A.



B.



C.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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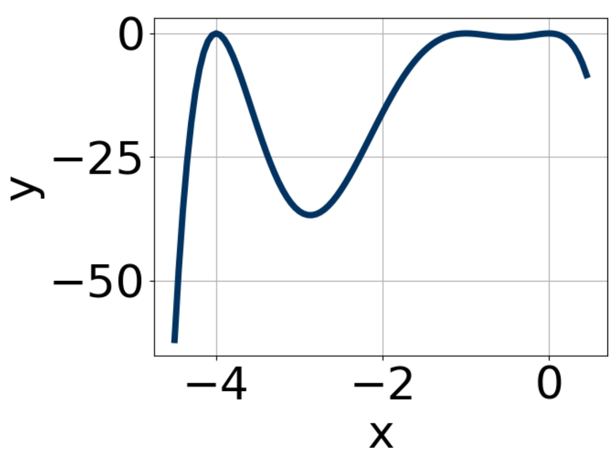
10. Which of the following equations *could* be of the graph presented below?



The solution is $-19x^4(x+4)^6(x+1)^4$, which is option B.

A. $19x^{10}(x+4)^4(x+1)^5$

The factor $(x+1)$ should have an even power and the leading coefficient should be the opposite sign.

B. $-19x^4(x+4)^6(x+1)^4$

* This is the correct option.

C. $-18x^8(x+4)^{10}(x+1)^{11}$

The factor $(x+1)$ should have an even power.

D. $-11x^7(x+4)^6(x+1)^5$

The factors x and $(x+1)$ should both have even powers.

E. $4x^4(x+4)^8(x+1)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).