This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{3}, 7, \text{ and } \frac{5}{3}$$

The solution is $9x^3 - 81x^2 + 131x - 35$, which is option A.

A. $a \in [2, 13], b \in [-83, -77], c \in [129, 133], \text{ and } d \in [-38, -33]$

* $9x^3 - 81x^2 + 131x - 35$, which is the correct option.

B. $a \in [2, 13], b \in [-83, -77], c \in [129, 133], \text{ and } d \in [35, 38]$

$9x^3 - 81x^2 + 131x + 35$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [2, 13], b \in [79, 93], c \in [129, 133], \text{ and } d \in [35, 38]$

$9x^3 + 81x^2 + 131x + 35$, which corresponds to multiplying out $(3x + 1)(x + 7)(3x + 5)$.

D. $a \in [2, 13], b \in [-77, -74], c \in [79, 80], \text{ and } d \in [35, 38]$

$9x^3 - 75x^2 + 79x + 35$, which corresponds to multiplying out $(3x + 1)(x - 7)(3x - 5)$.

E. $a \in [2, 13], b \in [51, 52], c \in [-90, -84], \text{ and } d \in [-38, -33]$

$9x^3 + 51x^2 - 89x - 35$, which corresponds to multiplying out $(3x + 1)(x + 7)(3x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 1)(x - 7)(3x - 5)$

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 5i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 49x - 41$, which is option D.

A. $b \in [1, 3], c \in [-11, -4], \text{ and } d \in [2, 9]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

B. $b \in [5, 20], c \in [49, 51], \text{ and } d \in [38, 46]$

$x^3 + 9x^2 + 49x + 41$, which corresponds to multiplying out $(x - (4 - 5i))(x - (4 + 5i))(x + 1)$.

C. $b \in [1, 3], c \in [0, 9], \text{ and } d \in [-11, 2]$

$x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

D. $b \in [-17, -5], c \in [49, 51], \text{ and } d \in [-44, -38]$

* $x^3 - 9x^2 + 49x - 41$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 5i))(x - (4 + 5i))(x - (1))$.

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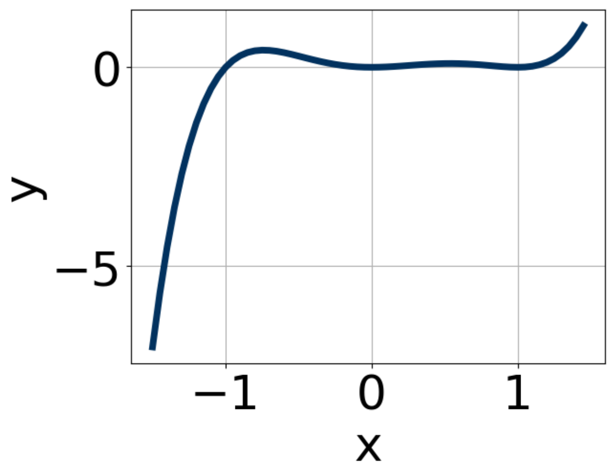
3. Which of the following equations *could* be of the graph presented below?



The solution is $8x^4(x-1)^4(x+1)^5$, which is option E.

A. $9x^9(x-1)^8(x+1)^{11}$

The factor x should have an even power.

B. $-9x^8(x-1)^8(x+1)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-20x^8(x-1)^4(x+1)^4$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $8x^9(x-1)^8(x+1)^6$

The factor x should have an even power and the factor $(x+1)$ should have an odd power.

E. $8x^4(x-1)^4(x+1)^5$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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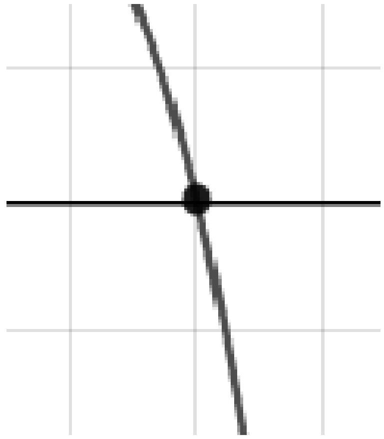
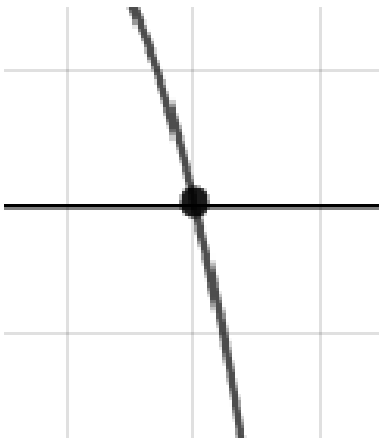
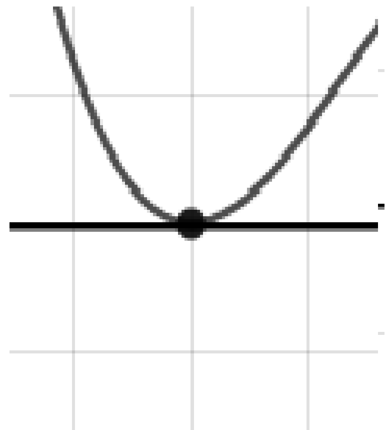
4. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

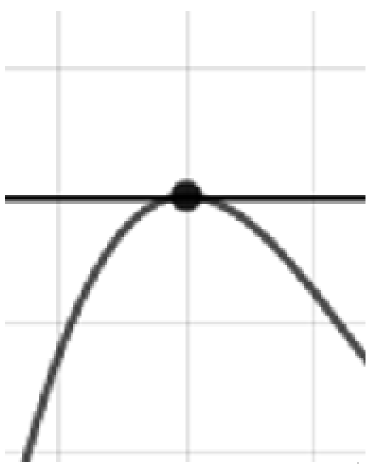
$$f(x) = 8(x+4)^9(x-4)^{12}(x-7)^5(x+7)^9$$

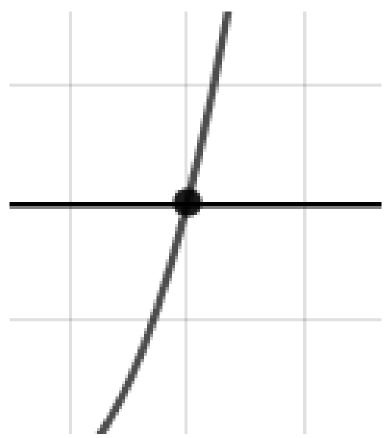
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

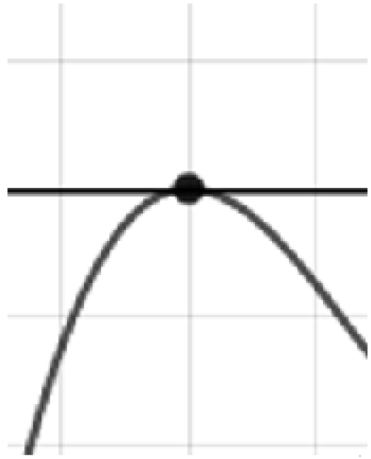
E. None of the above.

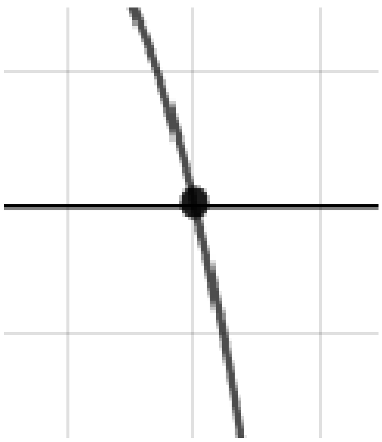
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

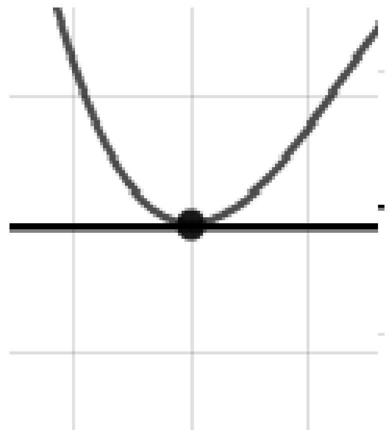
5. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

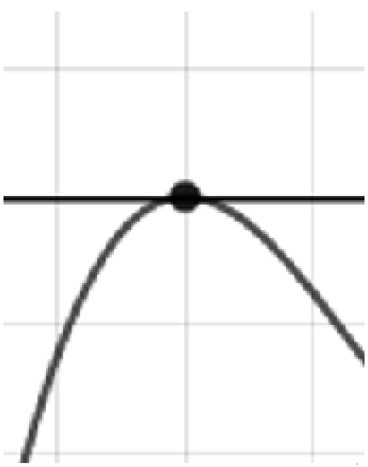
$$f(x) = 3(x+6)^6(x-6)^2(x-8)^9(x+8)^6$$

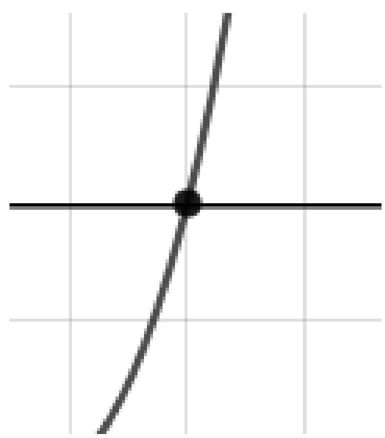
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 5i \text{ and } 2$$

The solution is $x^3 + 8x^2 + 30x - 100$, which is option C.

A. $b \in [-5, 6], c \in [-9, 1],$ and $d \in [7, 22]$

   $x^3 + x^2 - 7x + 10$, which corresponds to multiplying out $(x - 5)(x - 2)$.

B. $b \in [-12, -2], c \in [29, 38],$ and $d \in [92, 102]$

   $x^3 - 8x^2 + 30x + 100$, which corresponds to multiplying out $(x - (-5 + 5i))(x - (-5 - 5i))(x + 2)$.

C. $b \in [6, 14], c \in [29, 38],$ and $d \in [-106, -99]$

   * $x^3 + 8x^2 + 30x - 100$, which is the correct option.

D. $b \in [-5, 6], c \in [-1, 4],$ and $d \in [-21, -9]$

   $x^3 + x^2 + 3x - 10$, which corresponds to multiplying out $(x + 5)(x - 2)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 5i))(x - (-5 - 5i))(x - (2))$.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, 7, \text{ and } \frac{-7}{5}$$

The solution is $20x^3 - 117x^2 - 168x + 49$, which is option A.

A. $a \in [20, 22], b \in [-122, -116], c \in [-170, -163],$ and $d \in [46, 52]$

   * $20x^3 - 117x^2 - 168x + 49$, which is the correct option.

B. $a \in [20, 22], b \in [165, 181], c \in [238, 243],$ and $d \in [46, 52]$

   $20x^3 + 173x^2 + 238x + 49$, which corresponds to multiplying out $(4x + 1)(x + 7)(5x + 7)$.

C. $a \in [20, 22], b \in [-109, -104], c \in [-227, -217],$ and $d \in [-53, -44]$

   $20x^3 - 107x^2 - 224x - 49$, which corresponds to multiplying out $(4x + 1)(x - 7)(5x + 7)$.

D. $a \in [20, 22], b \in [114, 125], c \in [-170, -163],$ and $d \in [-53, -44]$

   $20x^3 + 117x^2 - 168x - 49$, which corresponds to multiplying out $(4x + 1)(x + 7)(5x - 7)$.

E. $a \in [20, 22], b \in [-122, -116], c \in [-170, -163],$ and $d \in [-53, -44]$

   $20x^3 - 117x^2 - 168x - 49$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(x - 7)(5x + 7)$

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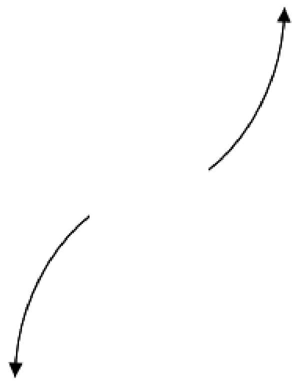
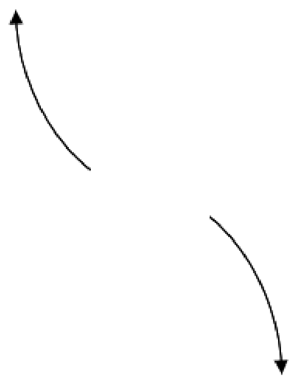
8. Describe the end behavior of the polynomial below.

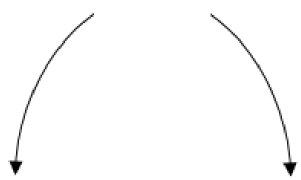
$$f(x) = 7(x + 4)^3(x - 4)^8(x - 5)^4(x + 5)^4$$

The solution is the graph below, which is option D.

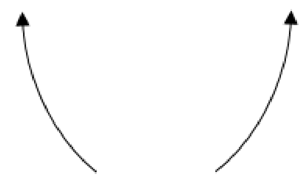




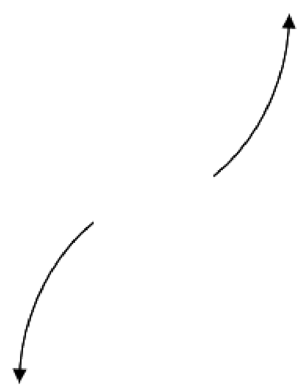
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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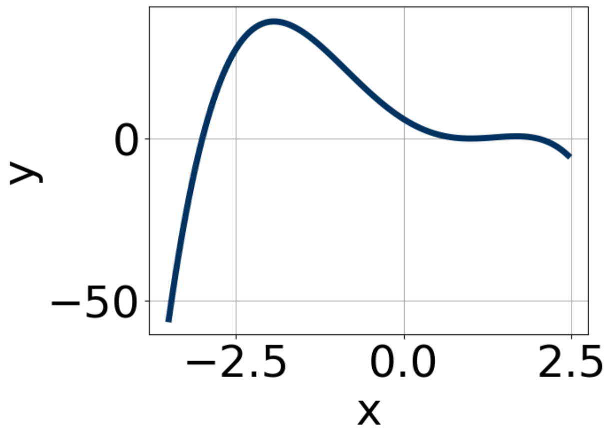
9. Which of the following equations *could* be of the graph presented below?



The solution is $-4(x-1)^4(x+3)^5(x-2)^9$, which is option E.

A. $-5(x-1)^{10}(x+3)^6(x-2)^9$

The factor $(x+3)$ should have an odd power.

B. $3(x-1)^{10}(x+3)^9(x-2)^6$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-3(x-1)^{11}(x+3)^6(x-2)^9$

The factor 1 should have an even power and the factor -3 should have an odd power.

D. $18(x-1)^6(x+3)^9(x-2)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-4(x-1)^4(x+3)^5(x-2)^9$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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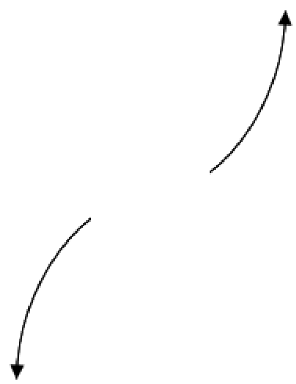
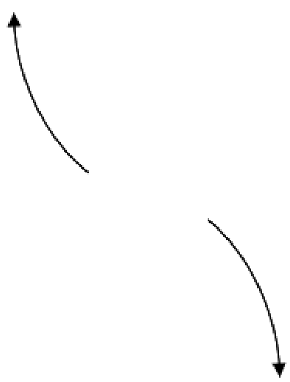
10. Describe the end behavior of the polynomial below.

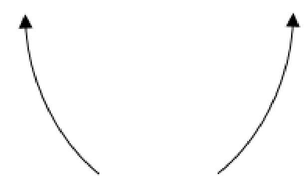
$$f(x) = 9(x-6)^2(x+6)^3(x+8)^5(x-8)^7$$

The solution is the graph below, which is option D.

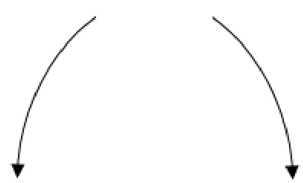




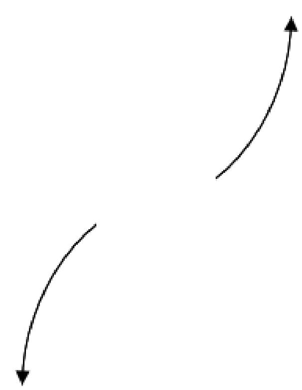
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.