This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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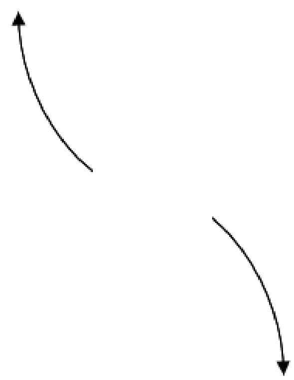
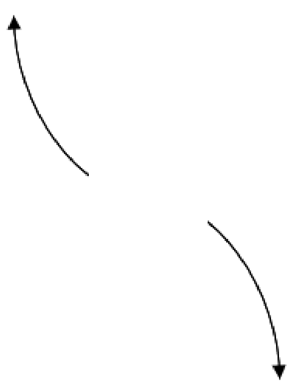
1. Describe the end behavior of the polynomial below.

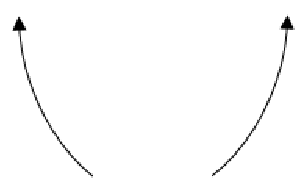
$$f(x) = -9(x-4)^3(x+4)^8(x+9)^3(x-9)^3$$

The solution is the graph below, which is option A.

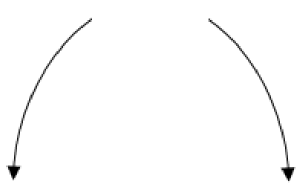




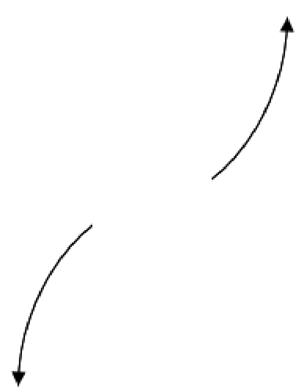
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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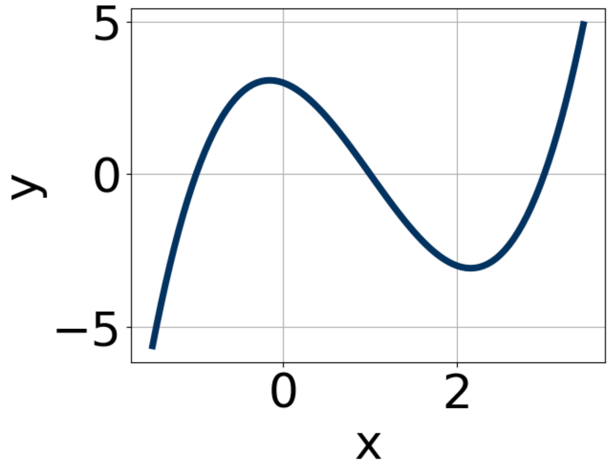
2. Which of the following equations *could* be of the graph presented below?



The solution is $6(x+1)^{11}(x-3)^{11}(x-1)^{9}$, which is option E.

A. $-14(x+1)^{5}(x-3)^{11}(x-1)^{9}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $20(x+1)^{6}(x-3)^{10}(x-1)^{5}$

The factors -1 and 3 have have been odd power.

C. $12(x+1)^{4}(x-3)^{5}(x-1)^{9}$

The factor -1 should have been an odd power.

D. $-11(x+1)^{10}(x-3)^{5}(x-1)^{7}$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $6(x+1)^{11}(x-3)^{11}(x-1)^{9}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$\frac{3}{5}, -5, \text{ and } -7$$

The solution is $5x^3+57x^2+139x-105$, which is option E.

A. $a \in [3, 17], b \in [-61, -55], c \in [131, 141], \text{ and } d \in [104, 111]$

$5x^3-57x^2+139x+105$, which corresponds to multiplying out $(5x+3)(x-5)(x-7)$.

B. $a \in [3, 17], b \in [62, 68], c \in [208, 220]$, and $d \in [104, 111]$

$5x^3 + 63x^2 + 211x + 105$, which corresponds to multiplying out $(5x + 3)(x + 5)(x + 7)$.

C. $a \in [3, 17], b \in [56, 59], c \in [131, 141]$, and $d \in [104, 111]$

$5x^3+57x^2+139x+105$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [3, 17], b \in [4, 14], c \in [-173, -167]$, and $d \in [-106, -101]$

$5x^3 + 13x^2 - 169x - 105$, which corresponds to multiplying out $(5x + 3)(x - 5)(x + 7)$.

E. $a \in [3, 17], b \in [56, 59], c \in [131, 141]$, and $d \in [-106, -101]$

* $5x^3 + 57x^2 + 139x - 105$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 3)(x + 5)(x + 7)$

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 3i \text{ and } -4$$

The solution is $x^3 - 4x^2 - 7x + 100$, which is option D.

A. $b \in [0.1, 1.1], c \in [-0.68, 0.29]$, and $d \in [-17.9, -15.3]$

$x^3 + x^2 - 16$, which corresponds to multiplying out $(x - 4)(x + 4)$.

B. $b \in [3, 6.4], c \in [-7.36, -6.73]$, and $d \in [-101.4, -98.9]$

$x^3 + 4x^2 - 7x - 100$, which corresponds to multiplying out $(x - (4 + 3i))(x - (4 - 3i))(x - 4)$.

C. $b \in [0.1, 1.1], c \in [0.54, 2.17]$, and $d \in [-13.2, -10.4]$

$x^3 + x^2 + x - 12$, which corresponds to multiplying out $(x - 3)(x + 4)$.

D. $b \in [-5, -2.6], c \in [-7.36, -6.73]$, and $d \in [98.6, 102.8]$

* $x^3 - 4x^2 - 7x + 100$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 + 3i))(x - (4 - 3i))(x - (-4))$.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-4}{3}, -2, \text{ and } \frac{3}{2}$$

The solution is $6x^3 + 11x^2 - 14x - 24$, which is option C.

A. $a \in [6, 11], b \in [-7, -3], c \in [-23, -21]$, and $d \in [23, 25]$

$6x^3 - 5x^2 - 22x + 24$, which corresponds to multiplying out $(3x - 4)(x + 2)(2x - 3)$.

B. $a \in [6, 11], b \in [-15, -9], c \in [-14, -11],$ and $d \in [23, 25]$

$6x^3 - 11x^2 - 14x + 24$, which corresponds to multiplying out $(3x - 4)(x - 2)(2x + 3)$.

C. $a \in [6, 11], b \in [8, 16], c \in [-14, -11],$ and $d \in [-31, -20]$

* $6x^3 + 11x^2 - 14x - 24$, which is the correct option.

D. $a \in [6, 11], b \in [-35, -27], c \in [46, 47],$ and $d \in [-31, -20]$

$6x^3 - 29x^2 + 46x - 24$, which corresponds to multiplying out $(3x - 4)(x - 2)(2x - 3)$.

E. $a \in [6, 11], b \in [8, 16], c \in [-14, -11],$ and $d \in [23, 25]$

$6x^3 + 11x^2 - 14x + 24$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 4)(x + 2)(2x - 3)$

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } -3$$

The solution is $x^3 + 13x^2 + 71x + 123$, which is option A.

A. $b \in [12, 15], c \in [60, 72],$ and $d \in [122, 124]$

* $x^3 + 13x^2 + 71x + 123$, which is the correct option.

B. $b \in [-6, 3], c \in [-9, 0],$ and $d \in [-15, -4]$

$x^3 + x^2 - x - 12$, which corresponds to multiplying out $(x - 4)(x + 3)$.

C. $b \in [-18, -8], c \in [60, 72],$ and $d \in [-124, -121]$

$x^3 - 13x^2 + 71x - 123$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x - 3)$.

D. $b \in [-6, 3], c \in [6, 14],$ and $d \in [6, 19]$

$x^3 + x^2 + 8x + 15$, which corresponds to multiplying out $(x + 5)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (-3))$.

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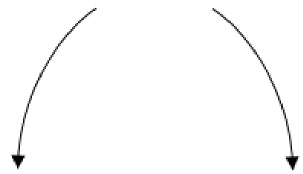
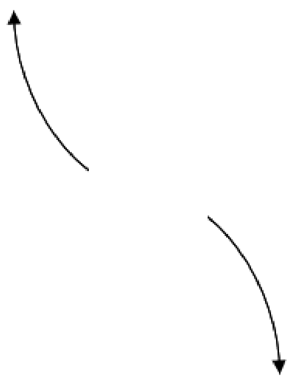
7. Describe the end behavior of the polynomial below.

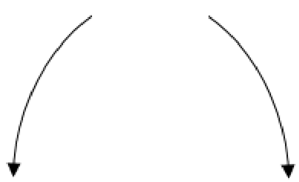
$$f(x) = -8(x - 2)^4(x + 2)^7(x + 8)^2(x - 8)^3$$

The solution is the graph below, which is option B.

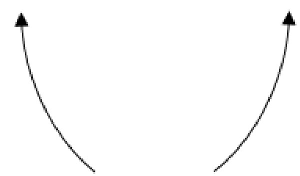




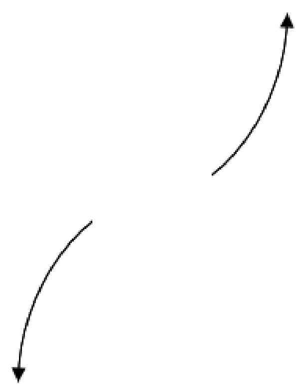
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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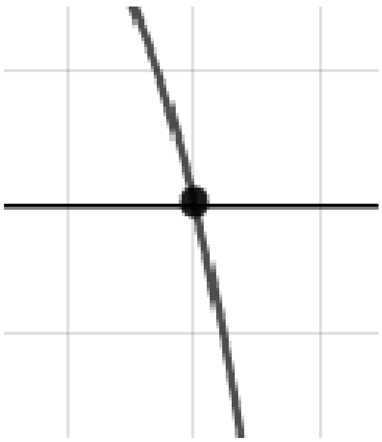
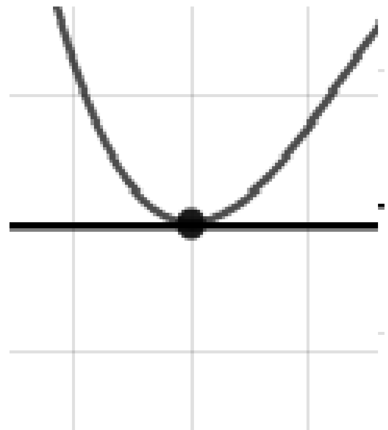
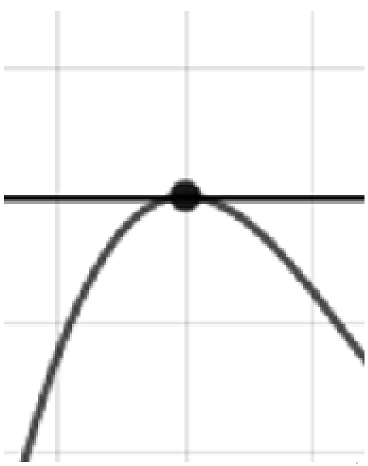
8. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

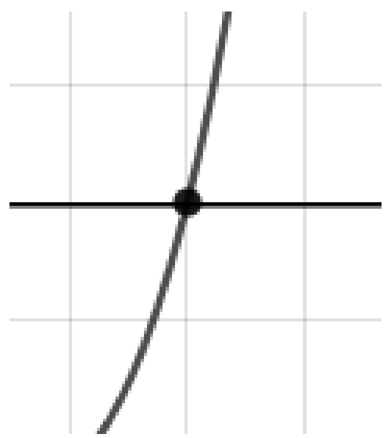
$$f(x) = 6(x-2)^4(x+2)^3(x+8)^6(x-8)^3$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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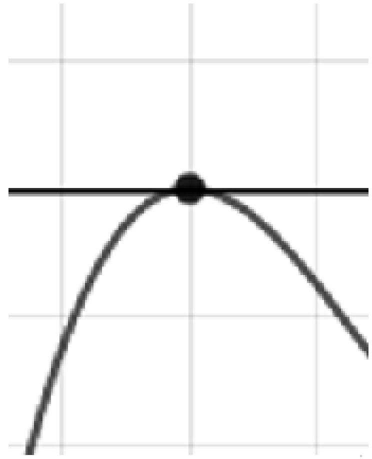
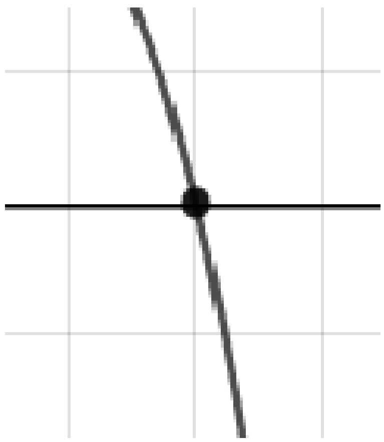
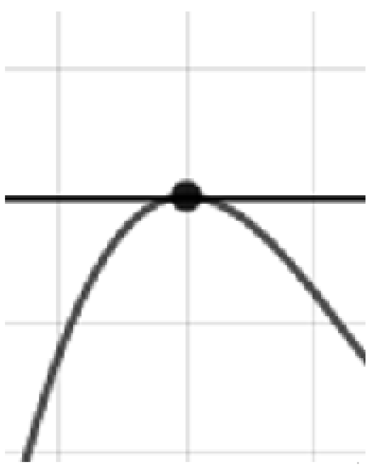
9. Describe the zero behavior of the zero $x = -9$ of the polynomial below.

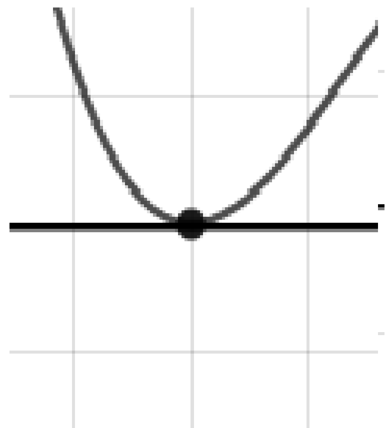
$$f(x) = 3(x+9)^8(x-9)^9(x-4)^3(x+4)^7$$

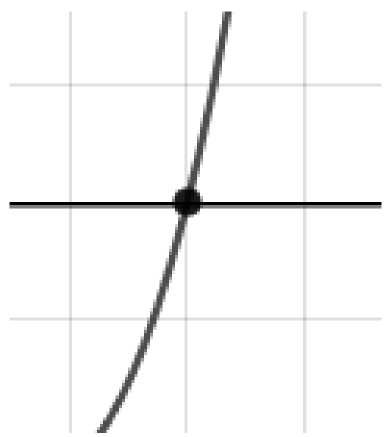
The solution is the graph below, which is option B.



A. 

B. 

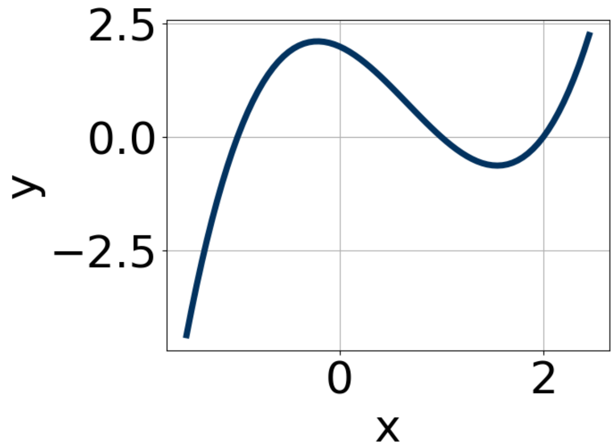
C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

10. Which of the following equations *could* be of the graph presented below?



The solution is $13(x-1)^{11}(x+1)^{11}(x-2)^5$, which is option A.

A. $13(x-1)^{11}(x+1)^{11}(x-2)^5$

   * This is the correct option.

B. $14(x-1)^4(x+1)^9(x-2)^9$

   The factor 1 should have been an odd power.

C. $-10(x-1)^5(x+1)^9(x-2)^{11}$

   This corresponds to the leading coefficient being the opposite value than it should be.

D. $-19(x-1)^{10}(x+1)^9(x-2)^9$

   The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $9(x-1)^4(x+1)^8(x-2)^{11}$

   The factors 1 and -1 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).