This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Determine whether the function below is 1-1.

$$f(x) = (6x + 33)^3$$

The solution is yes, which is option E.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

E. Yes, the function is 1-1.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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2. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -2x^3 + 2x^2 + 3x \text{ and } g(x) = 2x^3 + 3x^2 - 2x - 1$$

The solution is -2.0, which is option A.

A. $(f \circ g)(-1) \in [-3.2, 1.6]$

* This is the correct solution

B. $(f \circ g)(-1) \in [1.9, 2.6]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [2.4, 6]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [6.2, 9]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 4x + 5 \text{ and } g(x) = \sqrt{6x + 32}$$

The solution is The domain is all Real numbers greater than or equal to$x = -5.333333333333333.$, which is option C.

A. The domain is all Real numbers except $x = a,$ where $a \in [-8.67, -0.67]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [4.75, 7.75]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-8.33, -3.33]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-0.83, 7.17]$ and $b \in [1.67, 12.67]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = e^{x-3} + 5$$

The solution is $f^{-1}(7) = 3.693$, which is option D.

A. $f^{-1}(7) \in [-2.42, -2.13]$

This solution corresponds to distractor 1.

B. $f^{-1}(7) \in [7.47, 7.57]$

This solution corresponds to distractor 2.

C. $f^{-1}(7) \in [7.08, 7.35]$

This solution corresponds to distractor 3.

D. $f^{-1}(7) \in [3.44, 4]$

This is the solution.

E. $f^{-1}(7) \in [6.34, 6.52]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 15$ and choose the interval that $f^{-1}(15)$ belongs to.

$$f(x) = 5x^2 - 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(15) \in [1.15, 1.57]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(15) \in [2.41, 3.18]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(15) \in [3.77, 4.14]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(15) \in [1.86, 2.05]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{5x - 31} \text{ and } g(x) = 9x + 2$$

The solution is The domain is all Real numbers greater than or equal to$x = 6.2$., which is option C.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [1.33, 6.33]$

B. The domain is all Real numbers except $x = a$, where $a \in [2.25, 5.25]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [4.2, 10.2]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [2.33, 4.33]$ and $b \in [-9.33, -5.33]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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7. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 4x^3 - 3x^2 + 2x - 3 \text{ and } g(x) = 4x^3 - 1x^2 - 3x - 1$$

The solution is -12.0, which is option C.

A. $(f \circ g)(1) \in [-1, 0]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(1) \in [0, 9]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-12, -6]$

* This is the correct solution

D. $(f \circ g)(1) \in [-8, -3]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^{-1}(-15)$ belongs to.

$$f(x) = 2x^2 - 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [7.06, 7.48]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(-15) \in [4.07, 5.74]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(-15) \in [1.92, 2.72]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

D. $f^{-1}(-15) \in [2.94, 3.02]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below. Then, evaluate the inverse at $x = 6$ and choose the interval that $f^{-1}(6)$ belongs to.

$$f(x) = e^{x-2} - 3$$

The solution is $f^{-1}(6) = 4.197$, which is option E.

A. $f^{-1}(6) \in [-1.64, -1.28]$

This solution corresponds to distractor 4.

B. $f^{-1}(6) \in [0.14, 0.28]$

This solution corresponds to distractor 1.

C. $f^{-1}(6) \in [-2.08, -1.81]$

This solution corresponds to distractor 2.

D. $f^{-1}(6) \in [-1.37, -0.78]$

This solution corresponds to distractor 3.

E. $f^{-1}(6) \in [3.76, 4.25]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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10. Determine whether the function below is 1-1.

$$f(x) = \sqrt{3x + 20}$$

The solution is yes, which is option E.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. Yes, the function is 1-1.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.
