**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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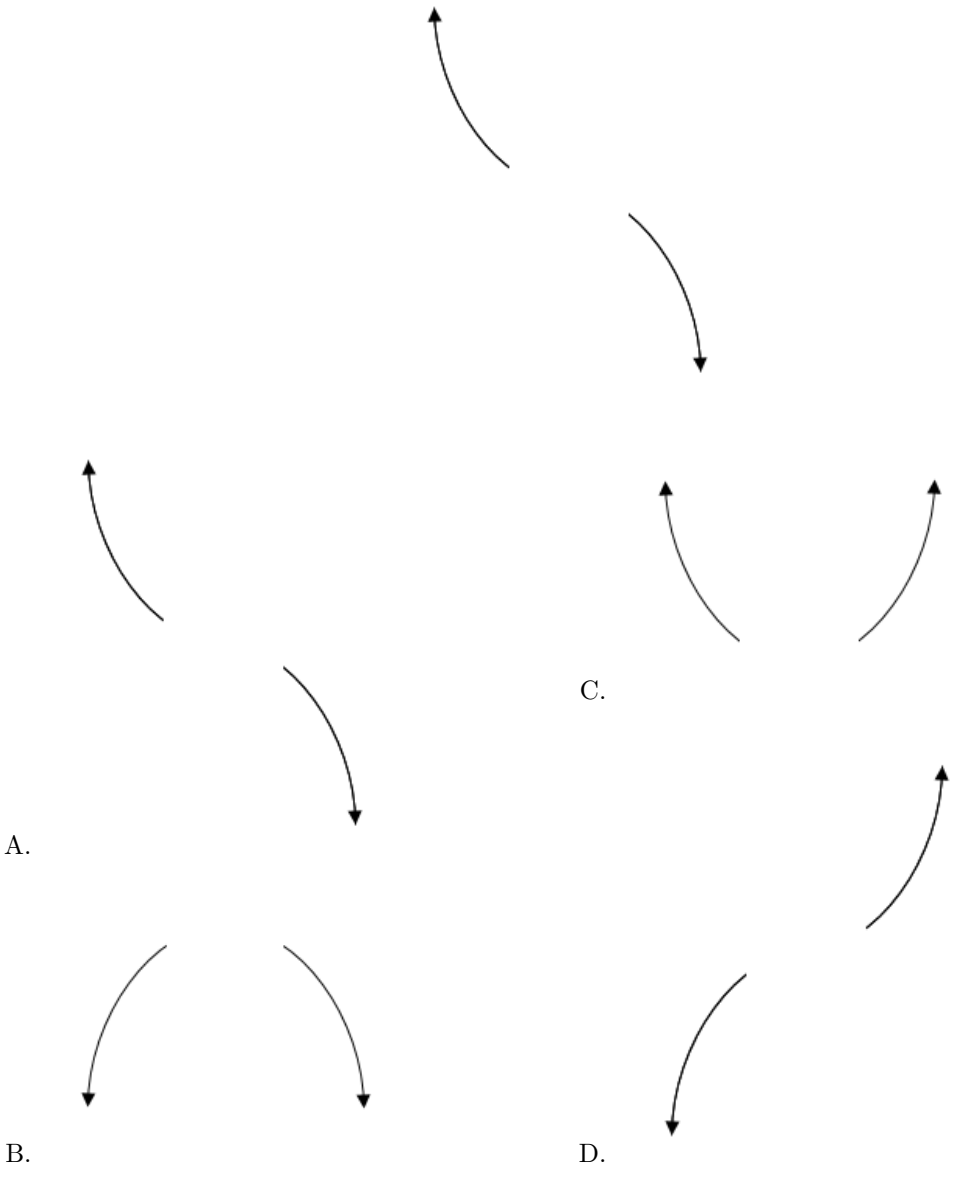
1. Describe the end behavior of the polynomial below.

$$f(x) = -3(x-8)^2(x+8)^5(x+9)^3(x-9)^3$$

The solution is the graph below, which is option A.



A.

C.

B.

D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{3}, \frac{-2}{5}, \text{ and } \frac{5}{3}$$

The solution is $45x^3 - 162x^2 + 103x + 70$, which is option D.

A. $a \in [40, 47], b \in [157, 164], c \in [98, 105],$ and $d \in [-78, -62]$

$45x^3 + 162x^2 + 103x - 70$, which corresponds to multiplying out $(3x + 7)(5x - 2)(3x + 5)$.

B. $a \in [40, 47], b \in [10, 18], c \in [-187, -180],$ and $d \in [67, 80]$

$45x^3 + 12x^2 - 187x + 70$, which corresponds to multiplying out $(3x + 3)(5x + 5)(3x - 3)$.

C. $a \in [40, 47], b \in [44, 51], c \in [-164, -157],$ and $d \in [-78, -62]$

$45x^3 + 48x^2 - 163x - 70$, which corresponds to multiplying out $(3x + 3)(5x - 5)(3x - 3)$.

D. $a \in [40, 47], b \in [-162, -159], c \in [98, 105],$ and $d \in [67, 80]$

* $45x^3 - 162x^2 + 103x + 70$, which is the correct option.

E. $a \in [40, 47], b \in [-162, -159], c \in [98, 105],$ and $d \in [-78, -62]$

$45x^3 - 162x^2 + 103x - 70$, which corresponds to multiplying everything correctly except the constant term.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(3x - 7)(5x + 2)(3x - 5)$

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 3i \text{ and } 1$$

The solution is $x^3 - 11x^2 + 44x - 34$, which is option B.

A. $b \in [-5, 2], c \in [-4, -1],$ and $d \in [2.3, 3.8]$

$x^3 + x^2 - 4x + 3$, which corresponds to multiplying out $(x - 3)(x - 1)$.

B. $b \in [-21, -10], c \in [43, 45],$ and $d \in [-36.5, -33.9]$

* $x^3 - 11x^2 + 44x - 34$, which is the correct option.

C. $b \in [9, 16], c \in [43, 45],$ and $d \in [33.9, 36.3]$

$x^3 + 11x^2 + 44x + 34$, which corresponds to multiplying out $(x - (5 + 3i))(x - (5 - 3i))(x + 1)$.

D. $b \in [-5, 2], c \in [-9, -5],$ and $d \in [4.4, 5.8]$

$x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x - 5)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 3i))(x - (5 - 3i))(x - (1))$.

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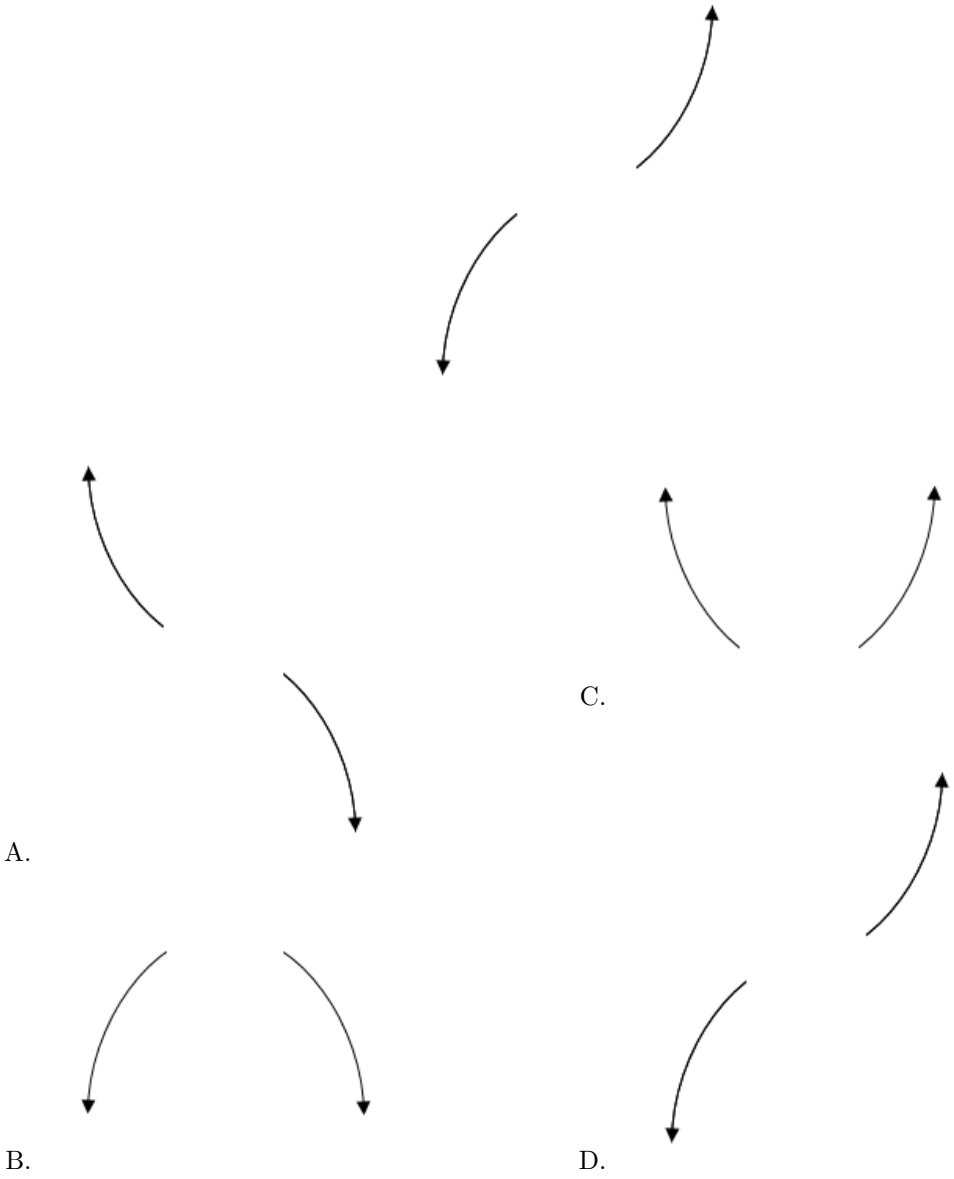
4. Describe the end behavior of the polynomial below.

$$f(x) = 2(x + 7)^3(x - 7)^4(x - 3)^4(x + 3)^6$$

The solution is the graph below, which is option D.



A.



C.



B.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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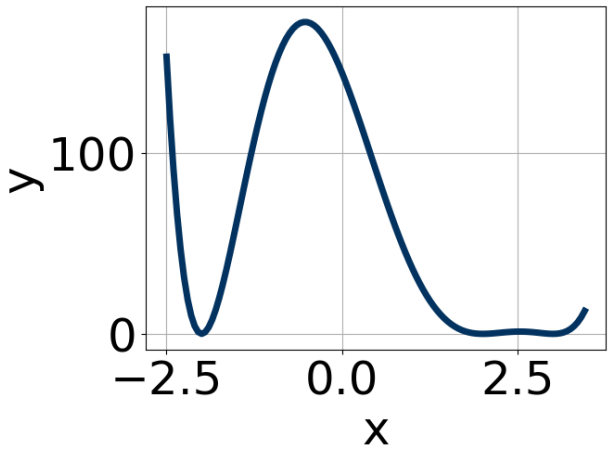
5. Which of the following equations *could* be of the graph presented below?




The solution is $20(x-2)^{10}(x+2)^{8}(x-3)^{6}$, which is option B.

A. $-14(x-2)^{6}(x+2)^{8}(x-3)^{6}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $20(x-2)^{10}(x+2)^{8}(x-3)^{6}$

* This is the correct option.

C. $-5(x-2)^{10}(x+2)^{10}(x-3)^{5}$

The factor $(x-3)$ should have an even power and the leading coefficient should be the opposite sign.

D. $4(x-2)^{8}(x+2)^{6}(x-3)^{5}$

The factor $(x-3)$ should have an even power.

E. $9(x-2)^{8}(x+2)^{11}(x-3)^{5}$

The factors $(x+2)$ and $(x-3)$ should both have even powers.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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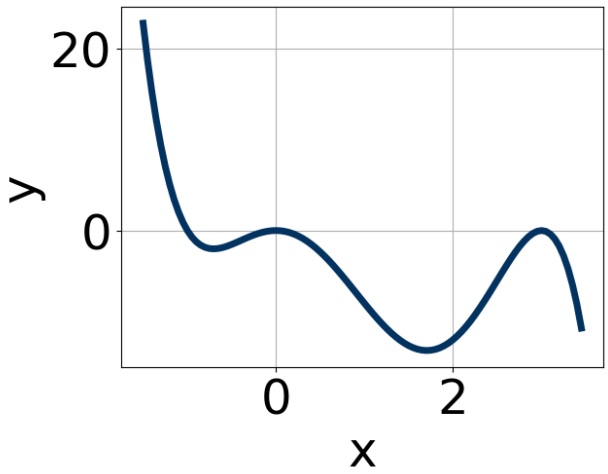
6. Which of the following equations *could* be of the graph presented below?




The solution is $-5x^8(x-3)^8(x+1)^9$, which is option C.

A. $6x^4(x-3)^{10}(x+1)^4$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $9x^{10}(x-3)^6(x+1)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-5x^8(x-3)^8(x+1)^9$

* This is the correct option.

D. $-19x^7(x-3)^6(x+1)^6$

The factor $x$ should have an even power and the factor $(x+1)$ should have an odd power.

E. $-10x^{11}(x-3)^4(x+1)^9$

The factor $x$ should have an even power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3+bx^2+cx+d$.

$$\frac{-7}{5}, 5, \text{ and } 2$$

The solution is $5x^3-28x^2+x+70$, which is option C.

A. $a \in [4,9], b \in [-45,-40], c \in [95,103],$ and $d \in [-73,-67]$

$5x^3-42x^2+99x-70$, which corresponds to multiplying out $(5x+5)(x-1)(x-1)$.

B. $a \in [4,9], b \in [22,34], c \in [-8,8],$ and $d \in [-73,-67]$

$5x^3+28x^2+x-70$, which corresponds to multiplying out $(5x-7)(x+5)(x+2)$.

C. $a \in [4,9], b \in [-34,-27], c \in [-8,8],$ and $d \in [70,73]$

* $5x^3-28x^2+x+70$, which is the correct option.

D. $a \in [4, 9], b \in [5, 9], c \in [-74, -67],$ and $d \in [70, 73]$

$5x^3 + 8x^2 - 71x + 70$, which corresponds to multiplying out $(5x + 5)(x + 1)(x - 1)$.

E. $a \in [4, 9], b \in [-34, -27], c \in [-8, 8],$ and $d \in [-73, -67]$

$5x^3 - 28x^2 + x - 70$, which corresponds to multiplying everything correctly except the constant term.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(5x + 7)(x - 5)(x - 2)$

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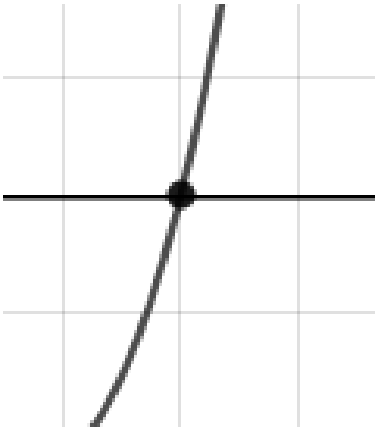
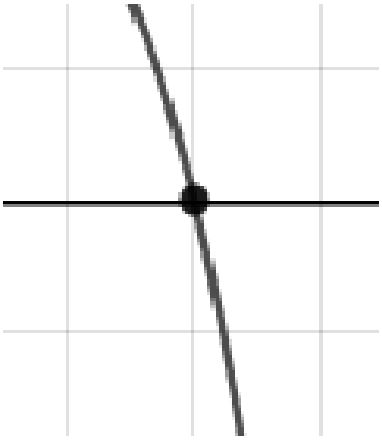
8. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

$$f(x) = 9(x + 9)^6(x - 9)^9(x - 3)^5(x + 3)^7$$

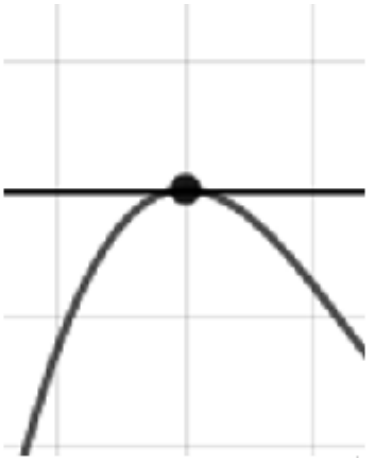
The solution is the graph below, which is option D.

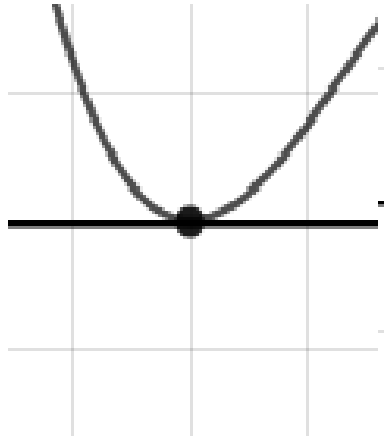


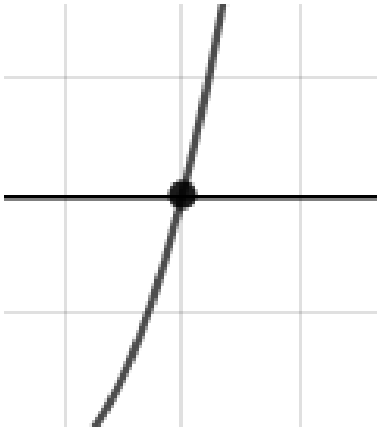
A.



B.



C. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 + 5i \text{ and } 3$$

The solution is $x^3 - 7x^2 + 41x - 87$, which is option B.

A. $b \in [1, 2], c \in [-9.1, -7.3],$ and $d \in [10, 22]$

$x^3 + x^2 - 8x + 15$, which corresponds to multiplying out $(x - 5)(x - 3)$.

B. $b \in [-7, -4], c \in [40.5, 41.2],$ and $d \in [-93, -85]$

* $x^3 - 7x^2 + 41x - 87$, which is the correct option.

C. $b \in [1, 2], c \in [-5.6, -1.6],$ and $d \in [3, 7]$

$x^3 + x^2 - 5x + 6$, which corresponds to multiplying out $(x - 2)(x - 3)$.

D. $b \in [6, 14], c \in [40.5, 41.2],$ and $d \in [87, 89]$

$x^3 + 7x^2 + 41x + 87$, which corresponds to multiplying out $(x - (2 + 5i))(x - (2 - 5i))(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 + 5i))(x - (2 - 5i))(x - (3))$.

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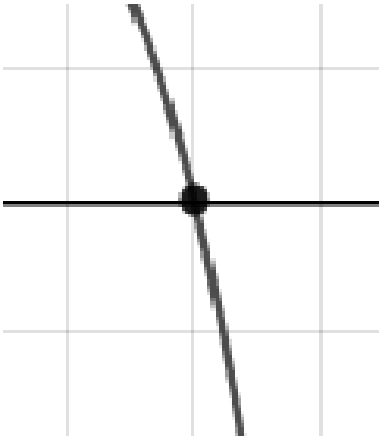
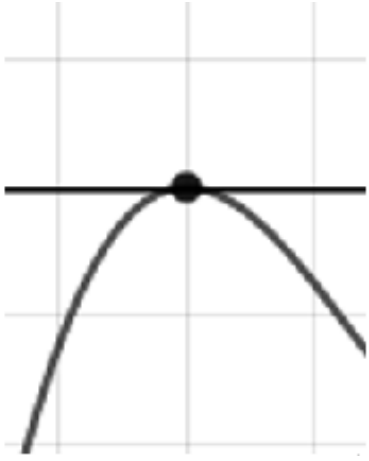
10. Describe the zero behavior of the zero $x = -7$ of the polynomial below.

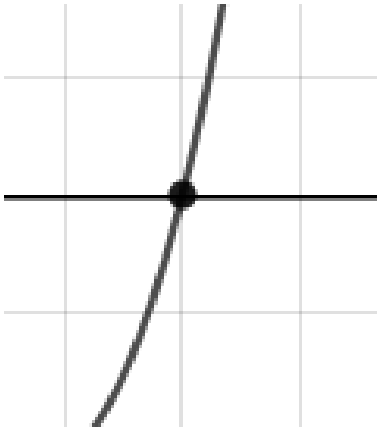
$$f(x) = 2(x - 9)^{13}(x + 9)^9(x - 7)^7(x + 7)^4$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.