**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 12$ and choose the interval that $f^{-1}(12)$ belongs to.

$$f(x) = 4x^2 + 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(12) \in [3.29, 4.04]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(12) \in [1.45, 1.52]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C. $f^{-1}(12) \in [2.03, 3.45]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(12) \in [1.76, 2.33]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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2. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{2}{4x + 29} \text{ and } g(x) = \frac{1}{3x + 10}$$

The solution is The domain is all Real numbers except $x = -7.25$ and $x = -3.3333333333333335$, which is option D.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-5, -2]$

B. The domain is all Real numbers except $x = a$, where $a \in [-3.25, -2.25]$

C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [3, 5]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-7.25, -3.25]$ and $b \in [-7.33, -1.33]$

E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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3. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 5x^3 + 7x^2 + 1 \text{ and } g(x) = 8x + 7$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-4.67, -1.67]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [1, 5]$

C. The domain is all Real numbers except $x = a,$ where $a \in [1.4, 5.4]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [5.2, 8.2]$ and $b \in [-7.2, -3.2]$

E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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4. Choose the interval below that $f$ composed with $g$ at $x = 1$ is in.

$$f(x) = -x^3 + 4x^2 - 4x \text{ and } g(x) = -3x^3 + 4x^2 - 2x$$

The solution is 9.0, which is option C.

A. $(f \circ g)(1) \in [12, 18]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-2, 4]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [7, 11]$

* This is the correct solution

D. $(f \circ g)(1) \in [7, 11]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

**General Comment:** $f$ composed with $g$ at $x$ means $f(g(x))$. The order matters!

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5. Choose the interval below that $f$ composed with $g$ at $x = -2$ is in.

$$f(x) = -2x^3 - 1x^2 + 3x - 4 \text{ and } g(x) = x^3 - 1x^2 - 4x$$

The solution is 96.0, which is option A.

A. $(f \circ g)(-2) \in [95, 104]$

* This is the correct solution

B. $(f \circ g)(-2) \in [-8, -1]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-2) \in [85, 91]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-2) \in [4, 12]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** $f$ composed with $g$ at $x$ means $f(g(x))$. The order matters!

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6. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval that $f^{-1}(11)$ belongs to.

$$f(x) = 5x^2 + 2$$

The solution is  The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(11) \in [4.54, 5.73]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(11) \in [2.68, 4.39]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(11) \in [0.87, 1.6]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

D. $f^{-1}(11) \in [1.39, 1.65]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E.  The function is not invertible for all Real numbers.

* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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7. Determine whether the function below is 1-1.

$$f(x) = 16x^2 + 112x + 196$$

The solution is no, which is option B.

A. No, because there is an $x$-value that goes to 2 different $y$-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because there is a $y$-value that goes to 2 different $x$-values.

* This is the solution.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a $y$-value that goes to 2 different $x$-values.

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8. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = e^{x+4} + 3$$

The solution is $f^{-1}(8) = -2.391$, which is option B.

A. $f^{-1}(8) \in [5.38, 5.44]$

This solution corresponds to distractor 2.

B. $f^{-1}(8) \in [-2.4, -2.36]$

This is the solution.

C. $f^{-1}(8) \in [5.48, 5.53]$

This solution corresponds to distractor 4.

D. $f^{-1}(8) \in [5.6, 5.63]$

This solution corresponds to distractor 1.

E. $f^{-1}(8) \in [4.25, 4.39]$

This solution corresponds to distractor 3.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the $x$ and $y$, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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9. Determine whether the function below is 1-1.

$$f(x) = (4x + 14)^3$$

The solution is yes, which is option E.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an $x$-value that goes to 2 different $y$-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because there is a $y$-value that goes to 2 different $x$-values.

Corresponds to the Horizontal Line test, which this function passes.

E. Yes, the function is 1-1.

* This is the solution.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a $y$-value that goes to 2 different $x$-values.

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10. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = e^{x-5} + 2$$

The solution is $f^{-1}(8) = 6.792$, which is option A.

A. $f^{-1}(8) \in [6.77, 6.95]$

This is the solution.

B. $f^{-1}(8) \in [4.14, 4.45]$

This solution corresponds to distractor 2.

C. $f^{-1}(8) \in [2.84, 3.45]$

This solution corresponds to distractor 4.

D. $f^{-1}(8) \in [4.34, 4.67]$

This solution corresponds to distractor 3.

E. $f^{-1}(8) \in [-3.23, -3.06]$

This solution corresponds to distractor 1.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the $x$ and $y$, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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