This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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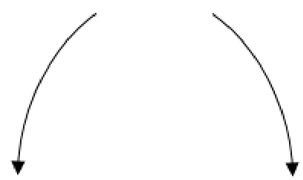
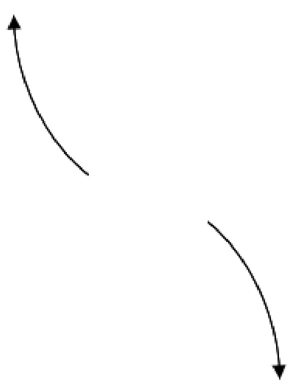
1. Describe the end behavior of the polynomial below.

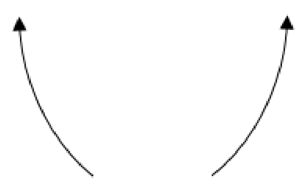
$$f(x) = -9(x+4)^3(x-4)^6(x+5)^2(x-5)^3$$

The solution is the graph below, which is option B.

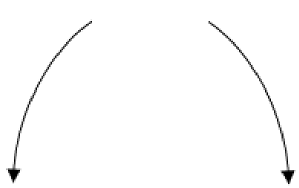




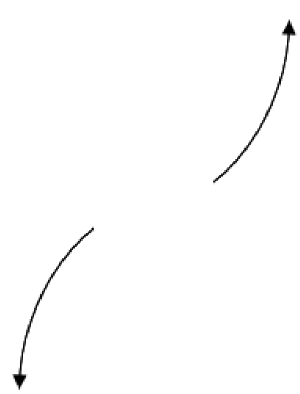
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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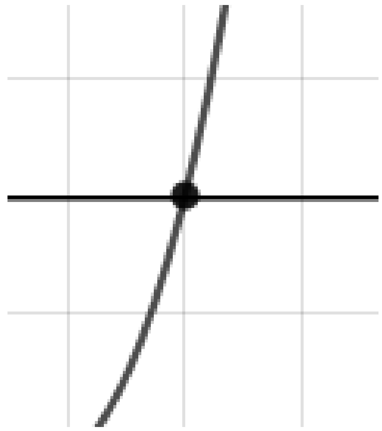
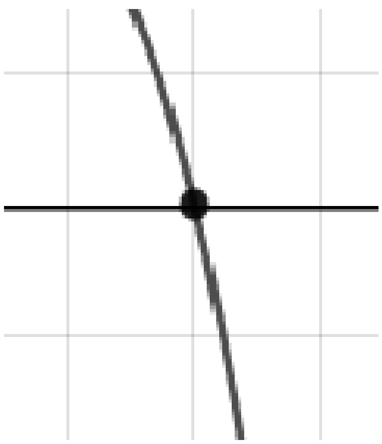
2. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

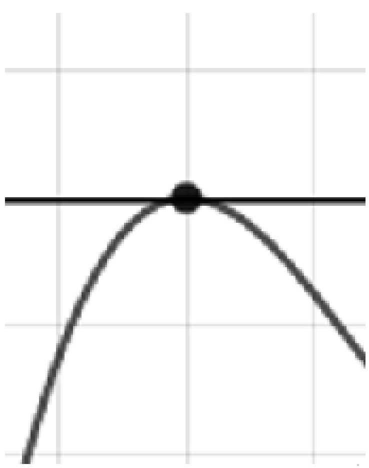
$$f(x) = 9(x+4)^6(x-4)^4(x-2)^9(x+2)^6$$

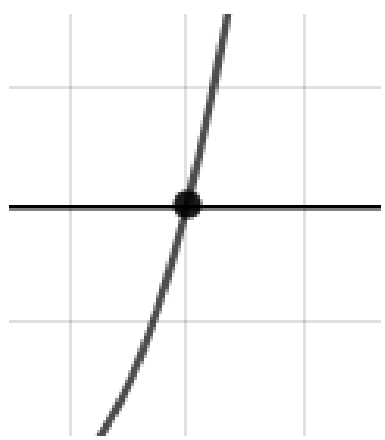
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 - 4i \text{ and } 2$$

The solution is $x^3 + 8x^2 + 21x - 82$, which is option A.

A. $b \in [7, 11], c \in [16.6, 24.7]$, and $d \in [-82.5, -81]$

   * $x^3 + 8x^2 + 21x - 82$, which is the correct option.

B. $b \in [0, 5], c \in [1.6, 2.3]$, and $d \in [-9.2, -7.7]$

   $x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x + 4)(x - 2)$.

C. $b \in [0, 5], c \in [2.8, 4.4]$, and $d \in [-11.9, -8.9]$

   $x^3 + x^2 + 3x - 10$, which corresponds to multiplying out $(x + 5)(x - 2)$.

D. $b \in [-10, -4], c \in [16.6, 24.7]$, and $d \in [78.5, 82.4]$

   $x^3 - 8x^2 + 21x + 82$, which corresponds to multiplying out $(x - (-5 - 4i))(x - (-5 + 4i))(x + 2)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 - 4i))(x - (-5 + 4i))(x - (2))$.

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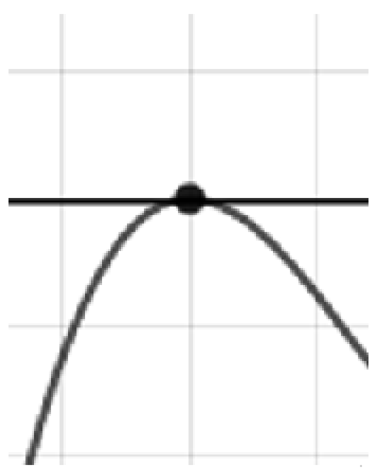
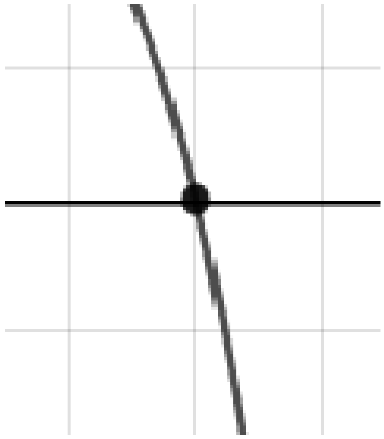
4. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

$$f(x) = -7(x + 6)^8(x - 6)^9(x - 3)^6(x + 3)^9$$

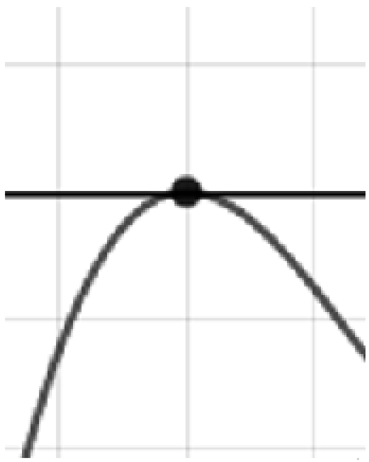
The solution is the graph below, which is option B.



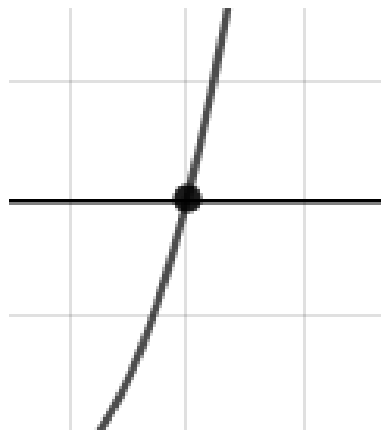


A.

C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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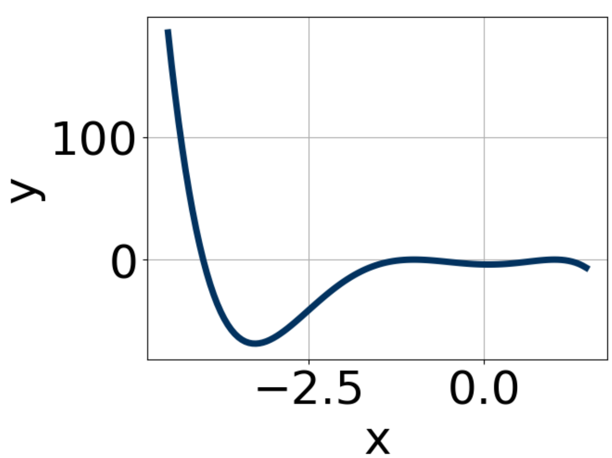
5. Which of the following equations *could* be of the graph presented below?



The solution is $-7(x+1)^6(x-1)^6(x+4)^9$, which is option D.

A. $6(x+1)^{10}(x-1)^{6}(x+4)^{6}$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-6(x+1)^{4}(x-1)^{5}(x+4)^{8}$

The factor $(x-1)$ should have an even power and the factor $(x+4)$ should have an odd power.

C. $-9(x+1)^{10}(x-1)^{9}(x+4)^{11}$

The factor $(x-1)$ should have an even power.

D. $-7(x+1)^{6}(x-1)^{6}(x+4)^{9}$

* This is the correct option.

E. $15(x+1)^{8}(x-1)^{4}(x+4)^{5}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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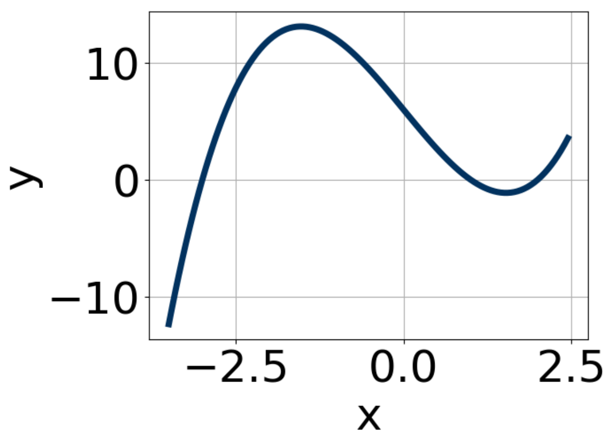
6. Which of the following equations *could* be of the graph presented below?



The solution is $6(x-2)^{9}(x+3)^{9}(x-1)^{7}$, which is option E.

A. $-3(x-2)^{9}(x+3)^{11}(x-1)^{9}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $17(x-2)^{10}(x+3)^{7}(x-1)^{7}$

The factor 2 should have been an odd power.

C. $-18(x-2)^{8}(x+3)^{9}(x-1)^{5}$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $20(x-2)^{4}(x+3)^{10}(x-1)^{9}$

The factors 2 and -3 have have been odd power.

E. $6(x-2)^{9}(x+3)^{9}(x-1)^{7}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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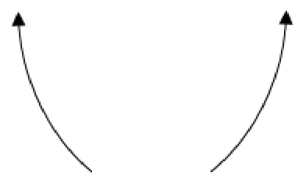
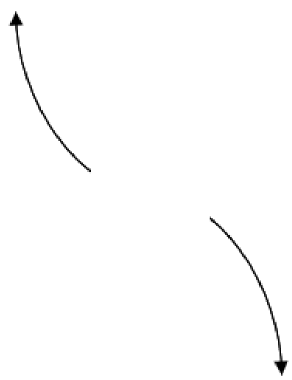
7. Describe the end behavior of the polynomial below.

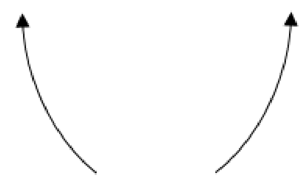
$$f(x) = 3(x-4)^4(x+4)^5(x+3)^5(x-3)^6$$

The solution is the graph below, which is option C.

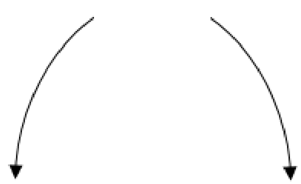




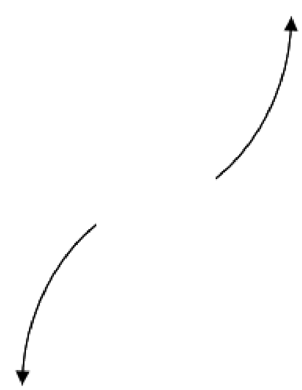
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{6}{5}, \frac{4}{5}, \text{ and } \frac{-5}{2}$$

The solution is $50x^3 + 25x^2 - 202x + 120$, which is option B.

A. $a \in [50, 56], b \in [225, 228], c \in [298, 302],$ and $d \in [113, 124]$

$50x^3 + 225x^2 + 298x + 120$, which corresponds to multiplying out $(5x + 6)(5x + 4)(2x + 5)$.

B. $a \in [50, 56], b \in [15, 26], c \in [-203, -196],$ and $d \in [113, 124]$

* $50x^3 + 25x^2 - 202x + 120$, which is the correct option.

C. $a \in [50, 56], b \in [140, 148], c \in [0, 13],$ and $d \in [-125, -119]$

$50x^3 + 145x^2 + 2x - 120$, which corresponds to multiplying out $(5x + 6)(5x - 4)(2x + 5)$.

D. $a \in [50, 56], b \in [15, 26], c \in [-203, -196],$ and $d \in [-125, -119]$

$50x^3 + 25x^2 - 202x - 120$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [50, 56], b \in [-26, -18], c \in [-203, -196],$ and $d \in [-125, -119]$

$50x^3 - 25x^2 - 202x - 120$, which corresponds to multiplying out $(5x + 6)(5x + 4)(2x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 6)(5x - 4)(2x + 5)$

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 5i \text{ and } -1$$

The solution is $x^3 + 11x^2 + 60x + 50$, which is option B.

A. $b \in [-12, -5], c \in [60, 64],$ and $d \in [-51, -40]$

$x^3 - 11x^2 + 60x - 50$, which corresponds to multiplying out $(x - (-5 + 5i))(x - (-5 - 5i))(x - 1)$.

B. $b \in [7, 18], c \in [60, 64],$ and $d \in [48, 53]$

* $x^3 + 11x^2 + 60x + 50$, which is the correct option.

C. $b \in [-3, 9], c \in [0, 11],$ and $d \in [2, 15]$

$x^3 + x^2 + 6x + 5$, which corresponds to multiplying out $(x + 5)(x + 1)$.

D. $b \in [-3, 9], c \in [-6, -2],$ and $d \in [-5, -2]$

$x^3 + x^2 - 4x - 5$, which corresponds to multiplying out $(x - 5)(x + 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 5i))(x - (-5 - 5i))(x - (-1))$.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-6}{5}, \frac{-1}{2}, \text{ and } \frac{4}{5}$$

The solution is $50x^3 + 45x^2 - 38x - 24$, which is option E.

A. $a \in [40, 53], b \in [43, 50], c \in [-38, -36],$ and $d \in [20, 26]$

   $50x^3 + 45x^2 - 38x + 24$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [40, 53], b \in [-133, -118], c \in [93, 99],$ and $d \in [-28, -21]$

   $50x^3 - 125x^2 + 98x - 24$, which corresponds to multiplying out $(5x - 6)(2x - 1)(5x - 4)$.

C. $a \in [40, 53], b \in [-78, -71], c \in [-6, 0],$ and $d \in [20, 26]$

   $50x^3 - 75x^2 - 2x + 24$, which corresponds to multiplying out $(5x - 6)(2x + 1)(5x - 4)$.

D. $a \in [40, 53], b \in [-54, -41], c \in [-38, -36],$ and $d \in [20, 26]$

   $50x^3 - 45x^2 - 38x + 24$, which corresponds to multiplying out $(5x - 6)(2x - 1)(5x + 4)$.

E. $a \in [40, 53], b \in [43, 50], c \in [-38, -36],$ and $d \in [-28, -21]$

   * $50x^3 + 45x^2 - 38x - 24$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 6)(2x + 1)(5x - 4)$