This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{5}, 3, \text{ and } \frac{7}{4}$$

The solution is $20x^3 - 87x^2 + 67x + 42$, which is option B.

A. $a \in [19, 21], b \in [7, 23], c \in [-120, -111], \text{ and } d \in [34, 44]$

$20x^3 + 17x^2 - 115x + 42$, which corresponds to multiplying out $(5x - 2)(x + 3)(4x - 7)$.

B. $a \in [19, 21], b \in [-89, -86], c \in [61, 71], \text{ and } d \in [34, 44]$

* $20x^3 - 87x^2 + 67x + 42$, which is the correct option.

C. $a \in [19, 21], b \in [87, 90], c \in [61, 71], \text{ and } d \in [-46, -36]$

$20x^3 + 87x^2 + 67x - 42$, which corresponds to multiplying out $(5x - 2)(x + 3)(4x + 7)$.

D. $a \in [19, 21], b \in [-89, -86], c \in [61, 71], \text{ and } d \in [-46, -36]$

$20x^3 - 87x^2 + 67x - 42$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [19, 21], b \in [-104, -101], c \in [136, 145], \text{ and } d \in [-46, -36]$

$20x^3 - 103x^2 + 143x - 42$, which corresponds to multiplying out $(5x - 2)(x - 3)(4x - 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 2)(x - 3)(4x - 7)$

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 4i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 40x - 32$, which is option D.

A. $b \in [9, 11], c \in [39, 42], \text{ and } d \in [29, 35]$

$x^3 + 9x^2 + 40x + 32$, which corresponds to multiplying out $(x - (4 - 4i))(x - (4 + 4i))(x + 1)$.

B. $b \in [1, 6], c \in [-8, -1], \text{ and } d \in [0, 5]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

C. $b \in [1, 6], c \in [3, 11], \text{ and } d \in [-4, 3]$

$x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

D. $b \in [-12, -6], c \in [39, 42], \text{ and } d \in [-35, -30]$

* $x^3 - 9x^2 + 40x - 32$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(4-4i))(x-(4+4i))(x-(1))$.

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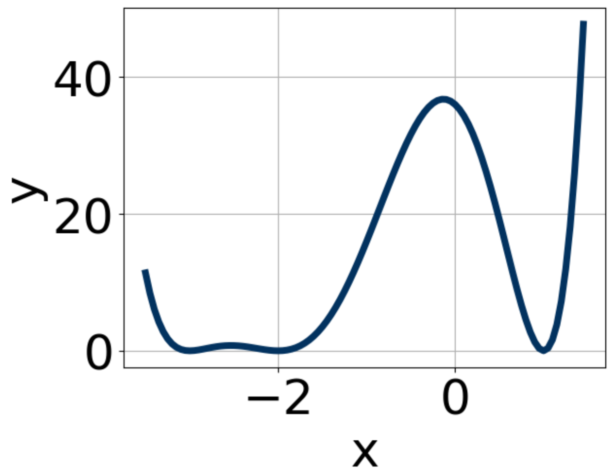
3. Which of the following equations *could* be of the graph presented below?



The solution is $6(x+2)^8(x-1)^6(x+3)^{10}$, which is option A.

A. $6(x+2)^8(x-1)^6(x+3)^{10}$

* This is the correct option.

B. $2(x+2)^{10}(x-1)^4(x+3)^5$

The factor $(x+3)$ should have an even power.

C. $16(x+2)^8(x-1)^9(x+3)^7$

The factors $(x-1)$ and $(x+3)$ should both have even powers.

D. $-12(x+2)^6(x-1)^6(x+3)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-12(x+2)^{10}(x-1)^{10}(x+3)^{11}$

The factor $(x+3)$ should have an even power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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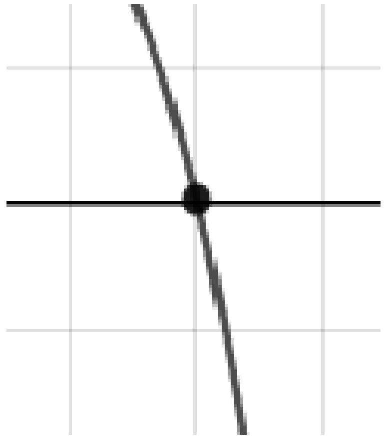
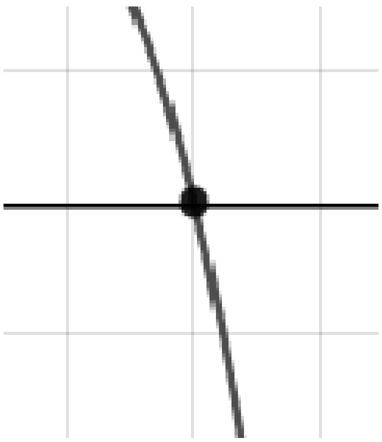
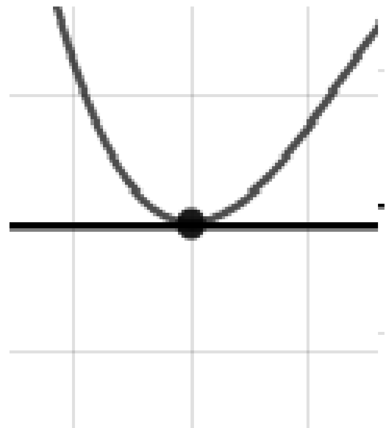
4. Describe the zero behavior of the zero $x=-6$ of the polynomial below.

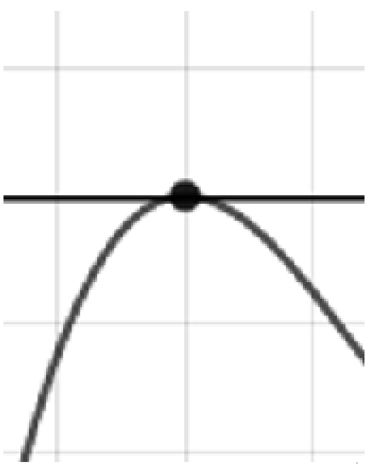
$$f(x)=5(x-8)^9(x+8)^6(x-6)^{14}(x+6)^9$$

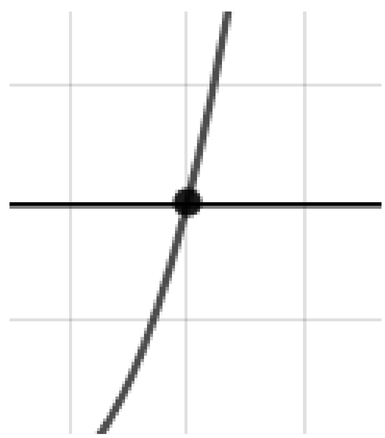
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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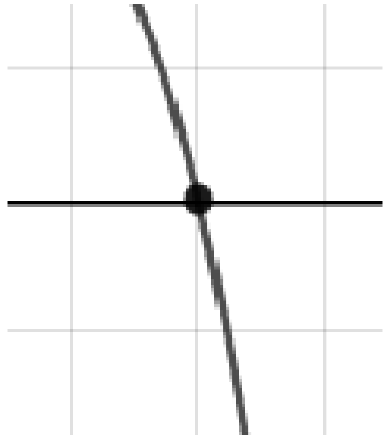
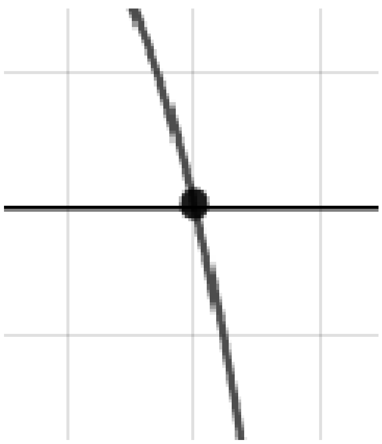
5. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

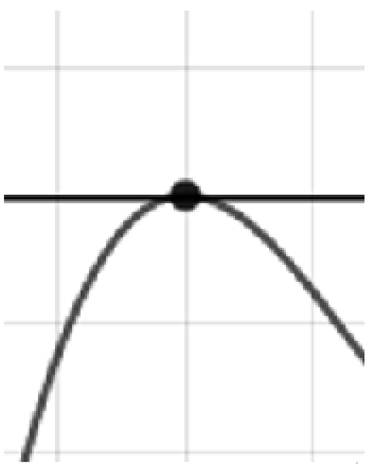
$$f(x) = 3(x-2)^3(x+2)^8(x+5)^6(x-5)^7$$

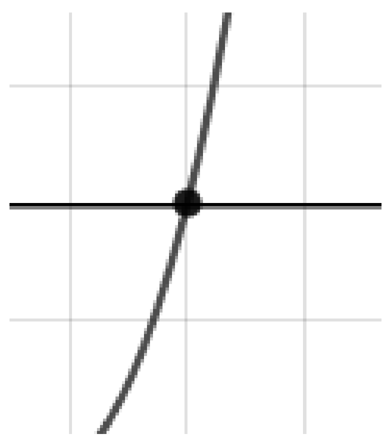
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 3i \text{ and } 1$$

The solution is $x^3 - 11x^2 + 44x - 34$, which is option D.

A. $b \in [7, 14], c \in [43.1, 45.1]$, and $d \in [32.85, 34.42]$

$x^3 + 11x^2 + 44x + 34$, which corresponds to multiplying out $(x - (5 + 3i))(x - (5 - 3i))(x + 1)$.

B. $b \in [-4, 2], c \in [-6.2, -5.8]$, and $d \in [4.36, 5.61]$

$x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x - 5)(x - 1)$.

C. $b \in [-4, 2], c \in [-4.9, -1.2]$, and $d \in [1.56, 3.56]$

$x^3 + x^2 - 4x + 3$, which corresponds to multiplying out $(x - 3)(x - 1)$.

D. $b \in [-13, -2], c \in [43.1, 45.1]$, and $d \in [-35.09, -33.44]$

* $x^3 - 11x^2 + 44x - 34$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 3i))(x - (5 - 3i))(x - (1))$.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{2}{3}, \frac{-3}{2}, \text{ and } \frac{-5}{3}$$

The solution is $18x^3 + 45x^2 + 7x - 30$, which is option A.

A. $a \in [18, 20], b \in [44, 49], c \in [4, 9]$, and $d \in [-33, -23]$

* $18x^3 + 45x^2 + 7x - 30$, which is the correct option.

B. $a \in [18, 20], b \in [10, 22], c \in [-45, -37]$, and $d \in [-33, -23]$

$18x^3 + 15x^2 - 43x - 30$, which corresponds to multiplying out $(3x + 2)(2x - 3)(3x + 5)$.

C. $a \in [18, 20], b \in [44, 49], c \in [4, 9]$, and $d \in [22, 35]$

$18x^3 + 45x^2 + 7x + 30$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [18, 20], b \in [-52, -44], c \in [4, 9]$, and $d \in [22, 35]$

$18x^3 - 45x^2 + 7x + 30$, which corresponds to multiplying out $(3x + 2)(2x - 3)(3x - 5)$.

E. $a \in [18, 20], b \in [69, 80], c \in [79, 85]$, and $d \in [22, 35]$

$18x^3 + 69x^2 + 83x + 30$, which corresponds to multiplying out $(3x + 2)(2x + 3)(3x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 2)(2x + 3)(3x + 5)$

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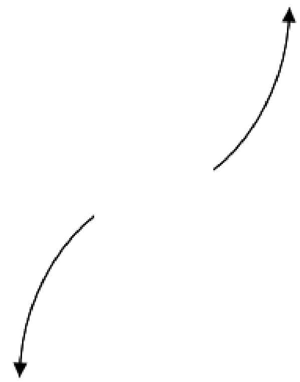
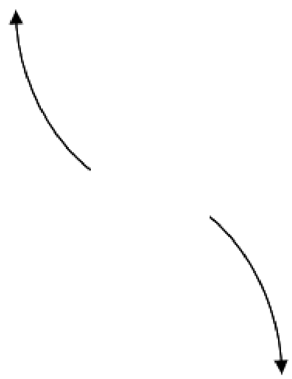
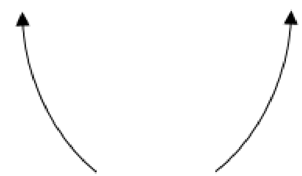
8. Describe the end behavior of the polynomial below.

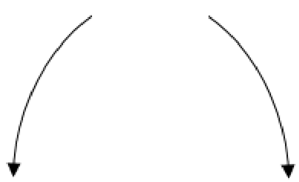
$$f(x) = 7(x - 2)^4(x + 2)^7(x + 7)^4(x - 7)^4$$

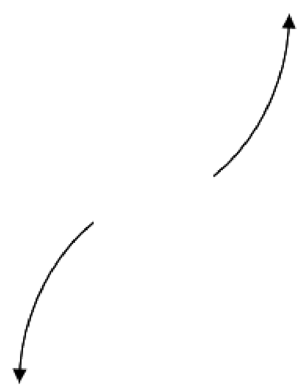
The solution is the graph below, which is option D.



A. 

C. 

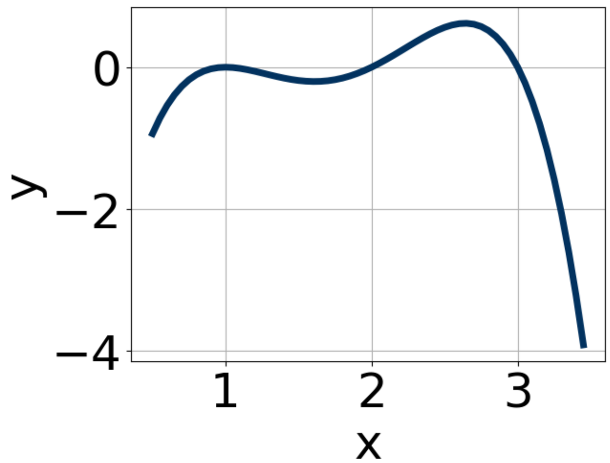
B. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

9. Which of the following equations *could* be of the graph presented below?



The solution is $-2(x-1)^4(x-2)^5(x-3)^{11}$, which is option B.

A. $10(x-1)^6(x-2)^{11}(x-3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-2(x-1)^4(x-2)^5(x-3)^{11}$

* This is the correct option.

C. $-10(x-1)^8(x-2)^6(x-3)^9$

The factor $(x-2)$ should have an odd power.

D. $-18(x-1)^7(x-2)^{10}(x-3)^7$

The factor 1 should have an even power and the factor 2 should have an odd power.

E. $19(x-1)^{10}(x-2)^7(x-3)^8$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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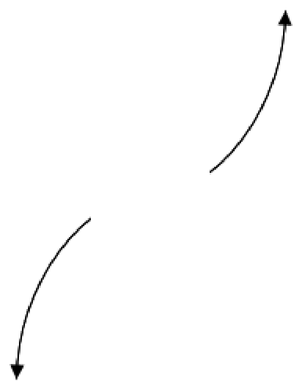
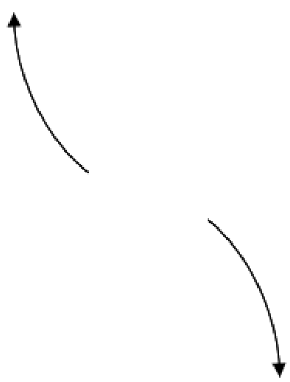
10. Describe the end behavior of the polynomial below.

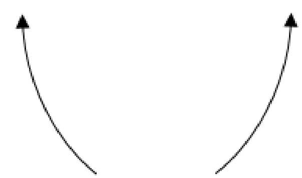
$$f(x) = 6(x-3)^5(x+3)^8(x+6)^2(x-6)^2$$

The solution is the graph below, which is option D.

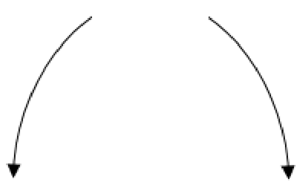




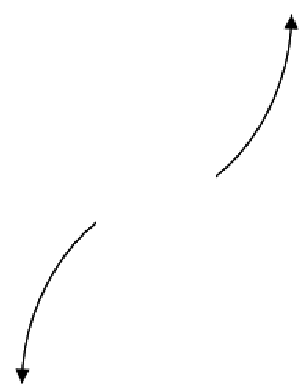
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.