This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 2x^3 + 4x^2 + 3x + 1 \text{ and } g(x) = 2x^3 - 1x^2 - 4x - 3$$

The solution is -5.0, which is option C.

A. $(f \circ g)(-1) \in [-13, -7]$

Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [-3, 1]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [-6, -4]$

* This is the correct solution

D. $(f \circ g)(-1) \in [-1, 4]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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2. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 3x^3 - 1x^2 + 3x - 4 \text{ and } g(x) = -2x^3 - 3x^2 + 4x$$

The solution is -11.0, which is option A.

A. $(f \circ g)(1) \in [-12, -9]$

* This is the correct solution

B. $(f \circ g)(1) \in [-23, -15]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [3, 8]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-2, 3]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 4x^4 + 7x^3 + 2x^2 + 7 \text{ and } g(x) = 3x^4 + 9x^3 + x^2 + 4x + 5$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers except $x = a,$ where $a \in [-7.8, 1.2]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-2.2, 4.8]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-9.67, 4.33]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-8.4, -1.4]$ and $b \in [3.33, 9.33]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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4. Determine whether the function below is 1-1.

$$f(x) = (3x + 14)^3$$

The solution is yes, which is option B.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. Yes, the function is 1-1.

* This is the solution.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = 4x^2 + 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(10) \in [4.08, 4.2]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(10) \in [1.76, 1.96]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

C. $f^{-1}(10) \in [2, 2.2]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(10) \in [1.1, 1.13]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval the $f^{-1}(-15)$ belongs to.

$$f(x) = \sqrt[3]{4x + 5}$$

The solution is -845.0, which is option B.

A. $f^{-1}(-15) \in [-843.34, -841.99]$

Distractor 1: This corresponds to

B. $f^{-1}(-15) \in [-845.3, -844.73]$

* This is the correct solution.

C. $f^{-1}(-15) \in [843.94, 846.01]$

This solution corresponds to distractor 2.

D. $f^{-1}(-15) \in [842.15, 843.44]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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7. Determine whether the function below is 1-1.

$$f(x) = (6x - 19)^3$$

The solution is yes, which is option D.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. Yes, the function is 1-1.

* This is the solution.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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8. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = \ln(x - 2) - 4$$

The solution is $f^{-1}(7) = 59876.142$, which is option B.

A. $f^{-1}(7) \in [8097.3, 8103.4]$

This solution corresponds to distractor 2.

B. $f^{-1}(7) \in [59874.5, 59878.1]$

This is the solution.

C. $f^{-1}(7) \in [21.5, 26.5]$

This solution corresponds to distractor 1.

D. $f^{-1}(7) \in [59871.2, 59872.5]$

This solution corresponds to distractor 3.

E. $f^{-1}(7) \in [141.8, 145.4]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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9. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{6x - 42} \text{ and } g(x) = 5x + 4$$

The solution is The domain is all Real numbers greater than or equal to$x = 7.0.$, which is option C.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [0.8, 5.8]$

B. The domain is all Real numbers except $x = a,$ where $a \in [3.33, 13.33]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [1, 9]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [6.25, 9.25]$ and $b \in [6.67, 9.67]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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10. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = e^{x-3} - 3$$

The solution is $f^{-1}(7) = 5.303$, which is option D.

A. $f^{-1}(7) \in [-1.75, -1.29]$

This solution corresponds to distractor 2.

B. $f^{-1}(7) \in [-1.04, -0.1]$

This solution corresponds to distractor 1.

C. $f^{-1}(7) \in [-1.75, -1.29]$

This solution corresponds to distractor 4.

D. $f^{-1}(7) \in [5.24, 6.53]$

This is the solution.

E. $f^{-1}(7) \in [-1.04, -0.1]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.
