This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-6, \frac{7}{2}, \text{ and } -7$$

The solution is $2x^3 + 19x^2 - 7x - 294$, which is option B.

A. $a \in [1, 3], b \in [4, 18], c \in [-85, -74], \text{ and } d \in [-296, -290]$

$2x^3 + 9x^2 - 77x - 294$, which corresponds to multiplying out $(x - 6)(2x + 7)(x + 7)$.

B. $a \in [1, 3], b \in [19, 24], c \in [-10, -5], \text{ and } d \in [-296, -290]$

* $2x^3 + 19x^2 - 7x - 294$, which is the correct option.

C. $a \in [1, 3], b \in [-21, -18], c \in [-10, -5], \text{ and } d \in [290, 297]$

$2x^3 - 19x^2 - 7x + 294$, which corresponds to multiplying out $(x - 6)(2x + 7)(x - 7)$.

D. $a \in [1, 3], b \in [19, 24], c \in [-10, -5], \text{ and } d \in [290, 297]$

$2x^3 + 19x^2 - 7x + 294$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [1, 3], b \in [-9, -4], c \in [-96, -85], \text{ and } d \in [290, 297]$

$2x^3 - 5x^2 - 91x + 294$, which corresponds to multiplying out $(x - 6)(2x - 7)(x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+6)(2x-7)(x+7)$

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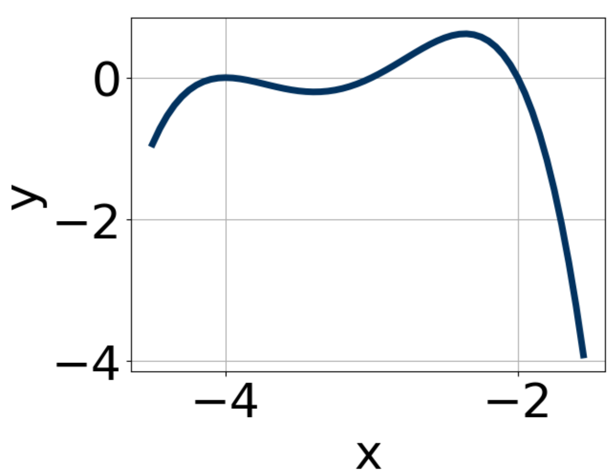
2. Which of the following equations *could* be of the graph presented below?



The solution is $-3(x+4)^4(x+2)^5(x+3)^5$, which is option A.

A. $-3(x+4)^4(x+2)^5(x+3)^5$

   * This is the correct option.

B. $-7(x+4)^9(x+2)^{10}(x+3)^5$

   The factor -4 should have an even power and the factor -2 should have an odd power.

C. $10(x+4)^6(x+2)^5(x+3)^6$

   The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-4(x+4)^4(x+2)^4(x+3)^{11}$

   The factor $(x+2)$ should have an odd power.

E. $18(x+4)^4(x+2)^7(x+3)^5$

   This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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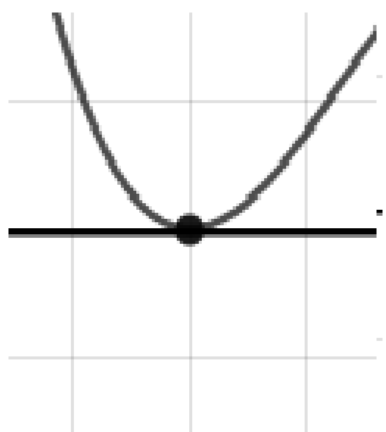
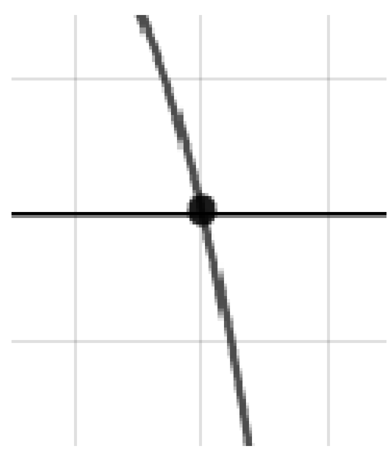
3. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

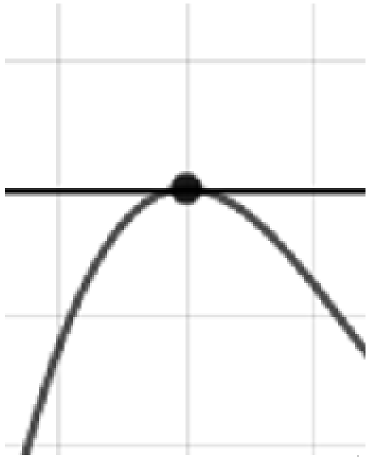
$$f(x) = 7(x-8)^5(x+8)^2(x+4)^{10}(x-4)^5$$

The solution is the graph below, which is option C.

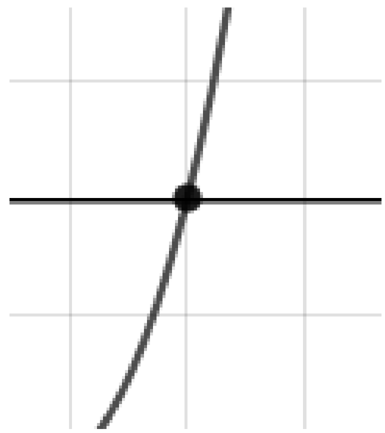




A.

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 4i \text{ and } 4$$

The solution is $x^3 - 12x^2 + 64x - 128$, which is option A.

A. $b \in [-15, -11], c \in [61, 66]$, and $d \in [-132, -126]$

   * $x^3 - 12x^2 + 64x - 128$, which is the correct option.

B. $b \in [4, 17], c \in [61, 66]$, and $d \in [124, 131]$

   $x^3 + 12x^2 + 64x + 128$, which corresponds to multiplying out $(x - (4 - 4i))(x - (4 + 4i))(x + 4)$.

C. $b \in [-3, 6], c \in [-4, 2]$, and $d \in [-19, -15]$

   $x^3 + x^2 - 16$, which corresponds to multiplying out $(x + 4)(x - 4)$.

D. $b \in [-3, 6], c \in [-8, -5]$, and $d \in [16, 20]$

   $x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(4-4i))(x-(4+4i))(x-(4))$.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$-2, \frac{-3}{5}, \text{ and } \frac{1}{3}$$

The solution is $15x^3+34x^2+5x-6$, which is option D.

A. $a \in [9, 26], b \in [-32, -20], c \in [-11, -8], \text{ and } d \in [2, 11]$

$15x^3-26x^2-11x+6$, which corresponds to multiplying out $(x-2)(5x+3)(3x-1)$.

B. $a \in [9, 26], b \in [31, 43], c \in [0, 11], \text{ and } d \in [2, 11]$

$15x^3+34x^2+5x+6$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [9, 26], b \in [-48, -38], c \in [29, 33], \text{ and } d \in [-6, -3]$

$15x^3-44x^2+31x-6$, which corresponds to multiplying out $(x-2)(5x-3)(3x-1)$.

D. $a \in [9, 26], b \in [31, 43], c \in [0, 11], \text{ and } d \in [-6, -3]$

* $15x^3+34x^2+5x-6$, which is the correct option.

E. $a \in [9, 26], b \in [-34, -33], c \in [0, 11], \text{ and } d \in [2, 11]$

$15x^3-34x^2+5x+6$, which corresponds to multiplying out $(x-2)(5x-3)(3x+1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+2)(5x+3)(3x-1)$

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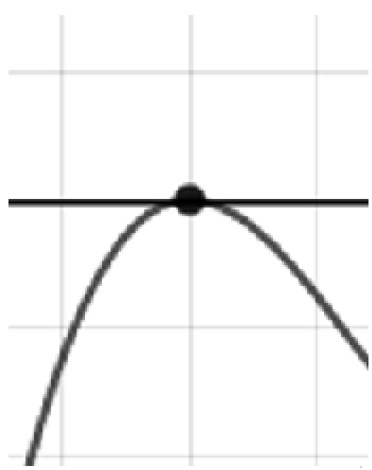
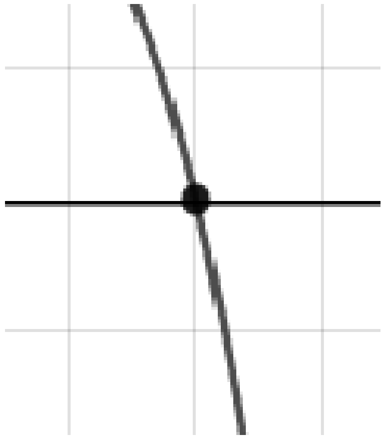
6. Describe the zero behavior of the zero $x=5$ of the polynomial below.

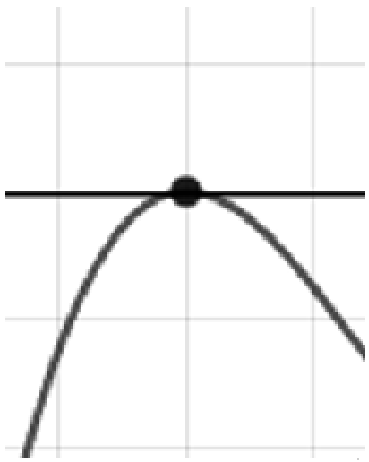
$$f(x) = 4(x-5)^2(x+5)^5(x+8)^6(x-8)^9$$

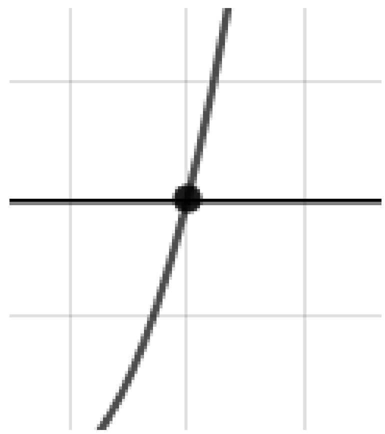
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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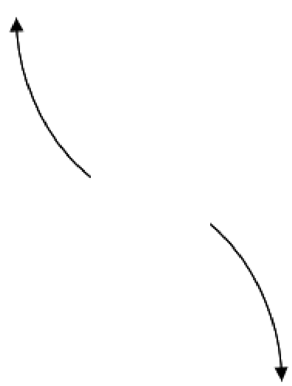
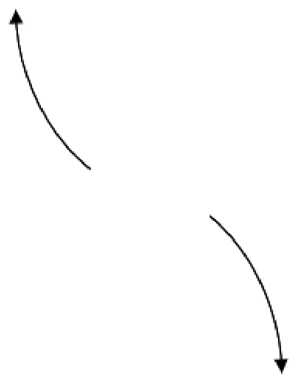
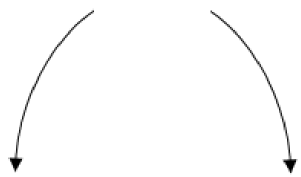
7. Describe the end behavior of the polynomial below.

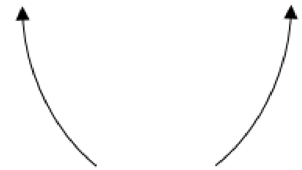
$$f(x) = -7(x+8)^4(x-8)^9(x+6)^5(x-6)^5$$

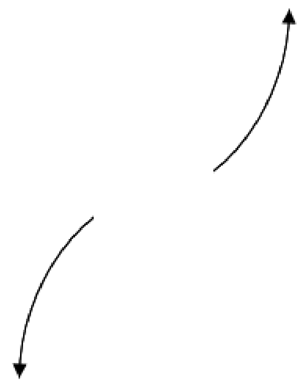
The solution is the graph below, which is option A.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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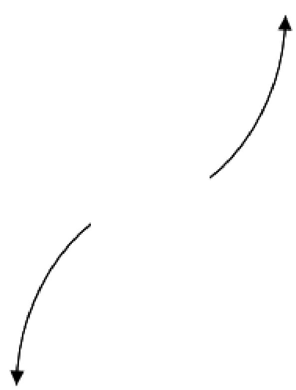
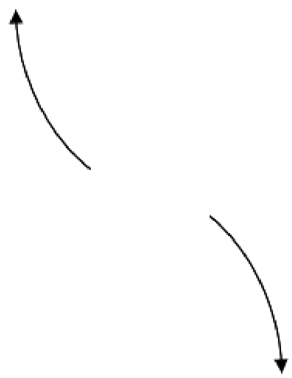
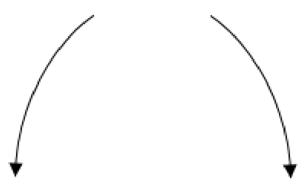
8. Describe the end behavior of the polynomial below.

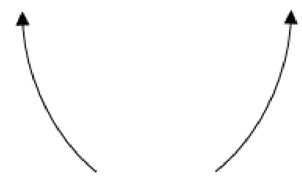
$$f(x) = 6(x+9)^4(x-9)^9(x+5)^5(x-5)^7$$

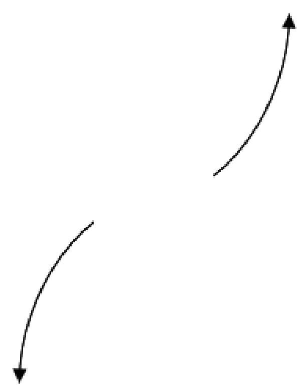
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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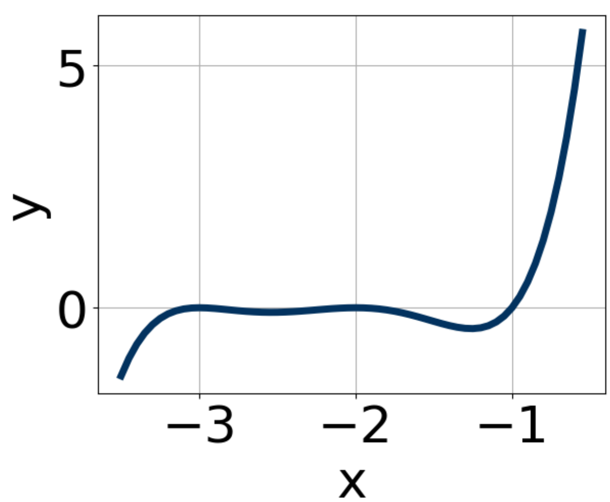
9. Which of the following equations *could* be of the graph presented below?



The solution is $20(x+3)^6(x+2)^6(x+1)^5$, which is option A.

A. $20(x+3)^6(x+2)^6(x+1)^5$

   * This is the correct option.

B. $11(x+3)^8(x+2)^5(x+1)^7$

The factor $(x+2)$ should have an even power.

C. $-11(x+3)^6(x+2)^6(x+1)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $18(x+3)^8(x+2)^7(x+1)^{10}$

The factor $(x+2)$ should have an even power and the factor $(x+1)$ should have an odd power.

E. $-14(x+3)^8(x+2)^4(x+1)^{10}$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 5i \text{ and } 3$$

The solution is $x^3 - 13x^2 + 80x - 150$, which is option A.

A. $b \in [-16, -9], c \in [79, 86], \text{ and } d \in [-157, -147]$

* $x^3 - 13x^2 + 80x - 150$, which is the correct option.

B. $b \in [-8, 10], c \in [2, 7], \text{ and } d \in [-19, -11]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x+5)(x-3)$.

C. $b \in [10, 14], c \in [79, 86], \text{ and } d \in [149, 151]$

$x^3 + 13x^2 + 80x + 150$, which corresponds to multiplying out $(x-(5-5i))(x-(5+5i))(x+3)$.

D. $b \in [-8, 10], c \in [-9, -2], \text{ and } d \in [12, 16]$

$x^3 + x^2 - 8x + 15$, which corresponds to multiplying out $(x-5)(x-3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x-(5-5i))(x-(5+5i))(x-(3))$.
