This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{2}, 2, \text{ and } \frac{1}{2}$$

The solution is $4x^3 - 24x^2 + 39x - 14$, which is option C.

A. $a \in [3, 5], b \in [0, 7], c \in [-32, -29], \text{ and } d \in [13, 21]$

$4x^3 + 4x^2 - 31x + 14$, which corresponds to multiplying out $(2x + 7)(x - 2)(2x - 1)$.

B. $a \in [3, 5], b \in [-31, -17], c \in [36, 41], \text{ and } d \in [13, 21]$

$4x^3 - 24x^2 + 39x + 14$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [3, 5], b \in [-31, -17], c \in [36, 41], \text{ and } d \in [-19, -5]$

* $4x^3 - 24x^2 + 39x - 14$, which is the correct option.

D. $a \in [3, 5], b \in [16, 21], c \in [16, 19], \text{ and } d \in [-19, -5]$

$4x^3 + 20x^2 + 17x - 14$, which corresponds to multiplying out $(2x + 7)(x + 2)(2x - 1)$.

E. $a \in [3, 5], b \in [24, 25], c \in [36, 41], \text{ and } d \in [13, 21]$

$4x^3 + 24x^2 + 39x + 14$, which corresponds to multiplying out $(2x + 7)(x + 2)(2x + 1)$.

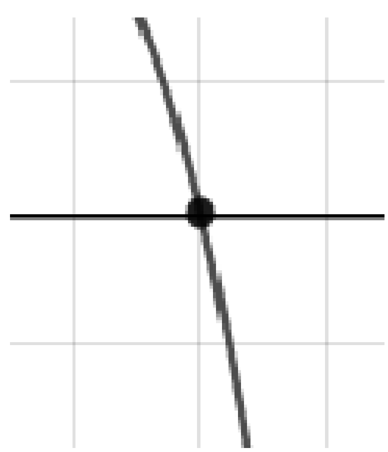
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 7)(x - 2)(2x - 1)$

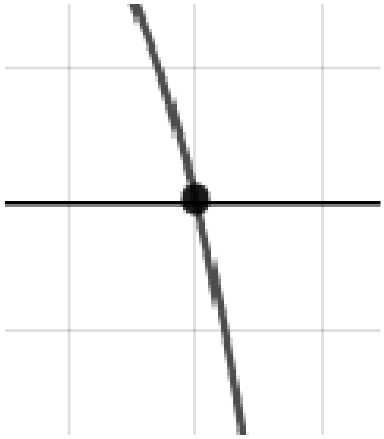
---

2. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

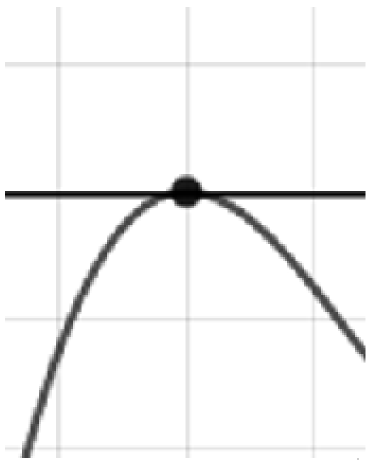
$$f(x) = -8(x + 2)^7(x - 2)^4(x + 9)^8(x - 9)^7$$

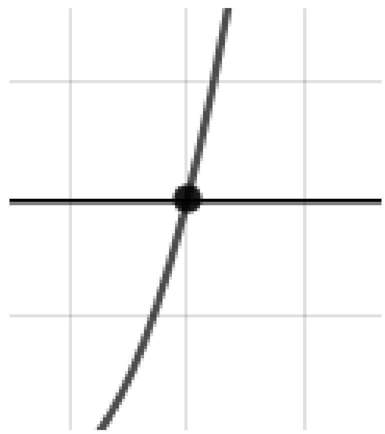
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

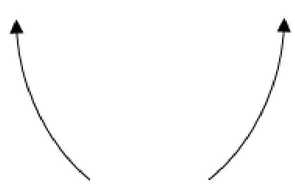
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

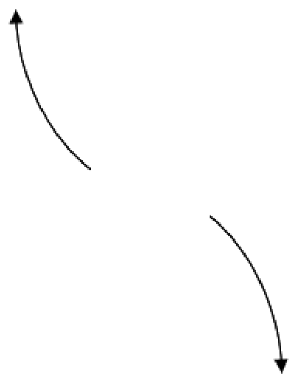
---

3. Describe the end behavior of the polynomial below.

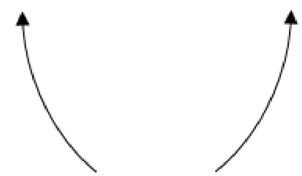
$$f(x) = 8(x+4)^3(x-4)^6(x+9)^4(x-9)^5$$

The solution is the graph below, which is option C.

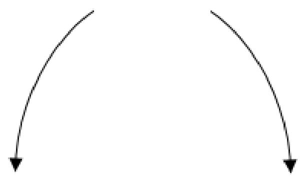




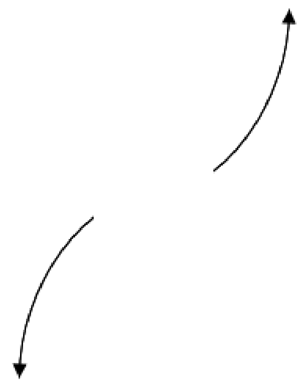
A.



C.



B.



D.

E. None of the above.

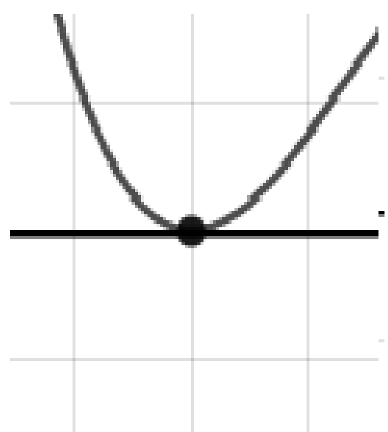
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

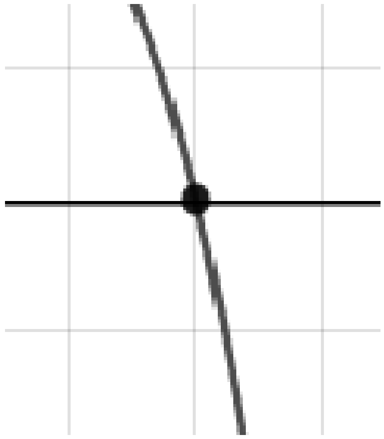
---

4. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

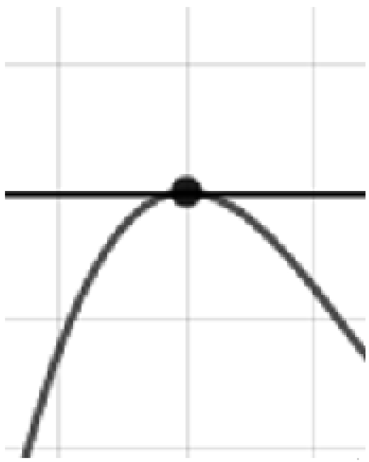
$$f(x) = 4(x+2)^5(x-2)^4(x+4)^5(x-4)^4$$

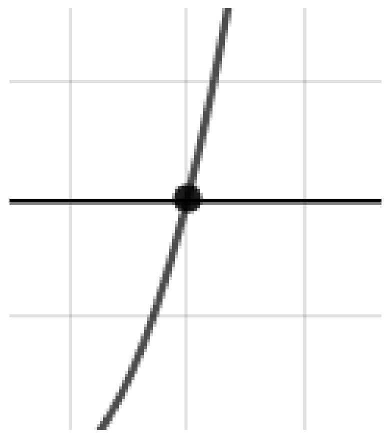
The solution is the graph below, which is option C.



A. 

C. 

B. 

D. 

E. None of the above.

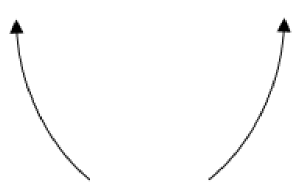
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

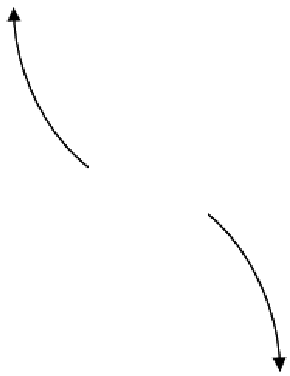
---

5. Describe the end behavior of the polynomial below.

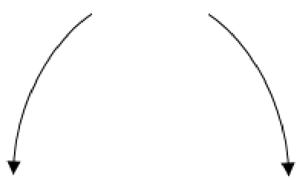
$$f(x) = 2(x+3)^4(x-3)^9(x+9)^4(x-9)^5$$

The solution is the graph below, which is option C.

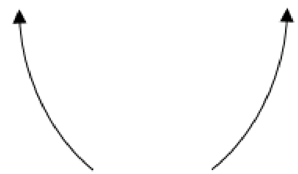




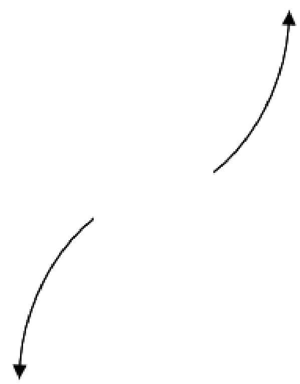
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 5i \text{ and } -4$$

The solution is $x^3 + 12x^2 + 73x + 164$, which is option A.

A. $b \in [6, 13], c \in [72.58, 73.76],$ and $d \in [162.4, 169]$

   * $x^3 + 12x^2 + 73x + 164$, which is the correct option.

B. $b \in [-3, 10], c \in [8.81, 9.58],$ and $d \in [19.5, 21.2]$

   $x^3 + x^2 + 9x + 20$, which corresponds to multiplying out $(x + 5)(x + 4)$.

C. $b \in [-3, 10], c \in [7.93, 8.59],$ and $d \in [15.7, 18.2]$

   $x^3 + x^2 + 8x + 16$, which corresponds to multiplying out $(x + 4)(x + 4)$.

D. $b \in [-15, -4], c \in [72.58, 73.76],$ and $d \in [-165.5, -160.5]$

   $x^3 - 12x^2 + 73x - 164$, which corresponds to multiplying out $(x - (-4 - 5i))(x - (-4 + 5i))(x - 4)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 - 5i))(x - (-4 + 5i))(x - (-4))$.

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 + 5i \text{ and } -4$$

The solution is $x^3 + 10x^2 + 58x + 136$, which is option B.

A. $b \in [-12, -7], c \in [50, 59]$, and $d \in [-140, -131]$

$x^3 - 10x^2 + 58x - 136$, which corresponds to multiplying out $(x - (-3 + 5i))(x - (-3 - 5i))(x - 4)$.

B. $b \in [5, 18], c \in [50, 59]$, and $d \in [136, 142]$

* $x^3 + 10x^2 + 58x + 136$, which is the correct option.

C. $b \in [-3, 9], c \in [-5, 5]$, and $d \in [-26, -19]$

$x^3 + x^2 - x - 20$, which corresponds to multiplying out $(x - 5)(x + 4)$.

D. $b \in [-3, 9], c \in [2, 14]$, and $d \in [7, 17]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 3)(x + 4)$.

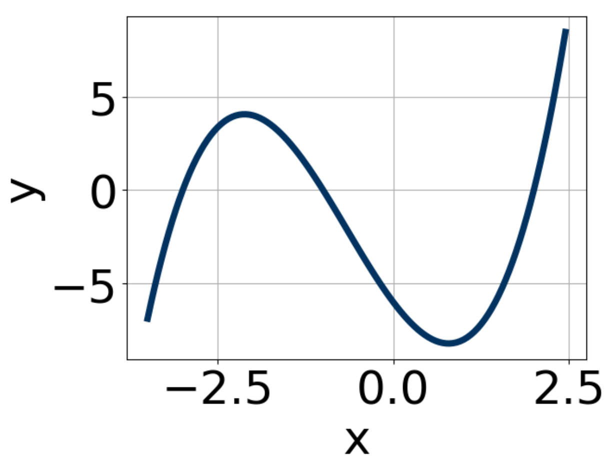
E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 5i))(x - (-3 - 5i))(x - (-4))$.

---

8. Which of the following equations *could* be of the graph presented below?



The solution is $13(x + 3)^7(x + 1)^5(x - 2)^7$, which is option E.

A. $4(x + 3)^4(x + 1)^4(x - 2)^{11}$

The factors -3 and -1 have have been odd power.

B. $18(x + 3)^4(x + 1)^9(x - 2)^7$

The factor -3 should have been an odd power.

C. $-15(x+3)^8(x+1)^9(x-2)^7$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-19(x+3)^7(x+1)^5(x-2)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $13(x+3)^7(x+1)^5(x-2)^7$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{5}, \frac{-5}{3}, \text{ and } \frac{3}{4}$$

The solution is $60x^3 + 139x^2 + 2x - 105$, which is option A.

A. $a \in [60, 61], b \in [135, 149], c \in [-4, 9], \text{ and } d \in [-115, -100]$

* $60x^3 + 139x^2 + 2x - 105$, which is the correct option.

B. $a \in [60, 61], b \in [-229, -223], c \in [275, 281], \text{ and } d \in [-115, -100]$

$60x^3 - 229x^2 + 278x - 105$, which corresponds to multiplying out $(5x-7)(3x-5)(4x-3)$.

C. $a \in [60, 61], b \in [135, 149], c \in [-4, 9], \text{ and } d \in [104, 108]$

$60x^3+139x^2+2x+105$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [60, 61], b \in [-32, -25], c \in [-153, -151], \text{ and } d \in [104, 108]$

$60x^3 - 29x^2 - 152x + 105$, which corresponds to multiplying out $(5x-7)(3x+5)(4x-3)$.

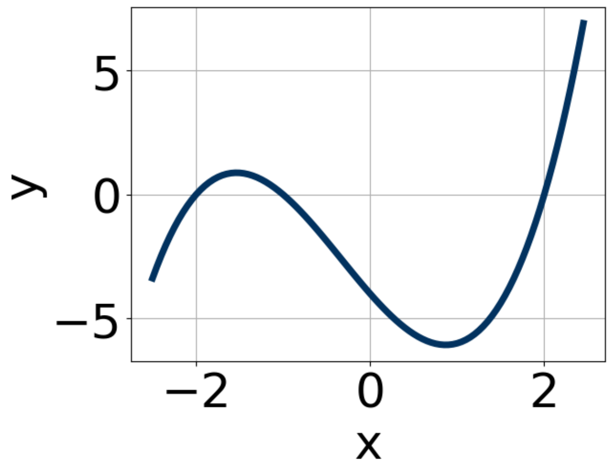
E. $a \in [60, 61], b \in [-142, -136], c \in [-4, 9], \text{ and } d \in [104, 108]$

$60x^3 - 139x^2 + 2x + 105$, which corresponds to multiplying out $(5x-7)(3x-5)(4x+3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x+7)(3x+5)(4x-3)$

---

10. Which of the following equations *could* be of the graph presented below?



The solution is $8(x+2)^7(x-2)^9(x+1)^7$, which is option B.

A. $-5(x+2)^4(x-2)^5(x+1)^5$

   The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $8(x+2)^7(x-2)^9(x+1)^7$

   * This is the correct option.

C. $17(x+2)^4(x-2)^{10}(x+1)^{11}$

   The factors -2 and 2 have have been odd power.

D. $18(x+2)^8(x-2)^9(x+1)^9$

   The factor -2 should have been an odd power.

E. $-16(x+2)^7(x-2)^{11}(x+1)^7$

   This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).