

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

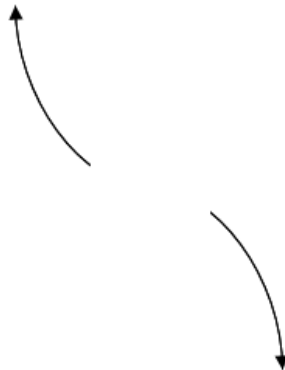
If you have a suggestion to make the keys better, please fill out the short survey [here](#).

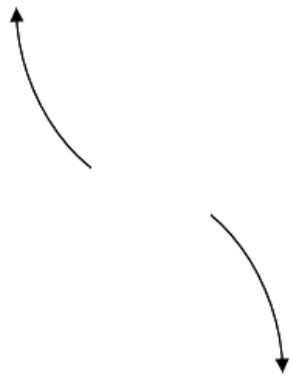
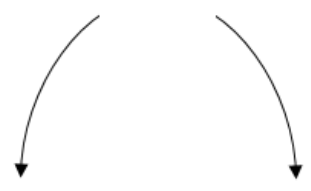
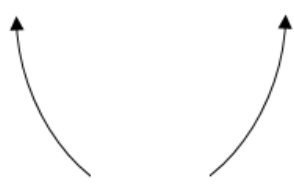
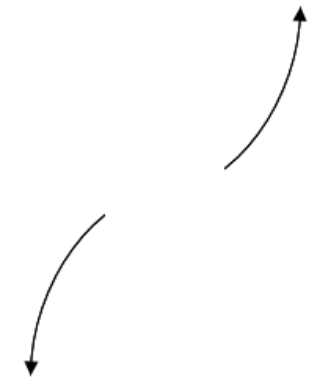
Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Describe the end behavior of the polynomial below.

$$f(x) = -8(x + 8)^2(x - 8)^5(x - 4)^4(x + 4)^6$$

The solution is



 <p>A.</p>	 <p>B.</p>
 <p>C.</p>	 <p>D.</p>
<p>E. None of the figures above.</p>	

- A. The function is above the x -axis, then passes through.
- B. The function is below the x -axis, then touches.
- C. The function is above the x -axis, then touches.
- D. The function is below the x -axis, then passes through.

General Comment: General Comments: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{3}{5}, \frac{2}{5}, \text{ and } \frac{-3}{2}$$

The solution is $50x^3 + 25x^2 - 63x + 18$

- A. $a \in [45, 52], b \in [118, 126], c \in [83, 94],$ and $d \in [15, 27]$
 $50x^3 + 125x^2 + 87x + 18$, which corresponds to multiplying out $(5x + 5)(5x + 5)(2x - 2)$.
- B. $a \in [45, 52], b \in [22, 27], c \in [-66, -59],$ and $d \in [-25, -16]$
 $50x^3 + 25x^2 - 63x - 18$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [45, 52], b \in [84, 90], c \in [-4, 4]$, and $d \in [-25, -16]$

$50x^3 + 85x^2 + 3x - 18$, which corresponds to multiplying out $(5x + 5)(5x - 5)(2x - 2)$.

D. $a \in [45, 52], b \in [22, 27], c \in [-66, -59]$, and $d \in [15, 27]$

* $50x^3 + 25x^2 - 63x + 18$, which is the correct option.

E. $a \in [45, 52], b \in [-27, -22], c \in [-66, -59]$, and $d \in [-25, -16]$

$50x^3 - 25x^2 - 63x - 18$, which corresponds to multiplying out $(5x + 3)(5x + 2)(2x - 3)$.

General Comment: General Comments: To construct the lowest-degree polynomial, you want to multiply out $(5x - 3)(5x - 2)(2x + 3)$

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 + 3i \text{ and } x - 1$$

The solution is $x^3 - 3x^2 + 9x + 13$

A. $b \in [-1.9, 1.3], c \in [-2.08, -1.93]$, and $d \in [-3.61, -2.43]$

$x^3 + x^2 - 2x - 3$, which corresponds to multiplying out $(x - 3)(x + 1)$.

B. $b \in [2.3, 4.1], c \in [7.06, 9.19]$, and $d \in [-13.04, -11.95]$

$x^3 + 3x^2 + 9x - 13$, which corresponds to multiplying out $(x - (2 + 3i))(x - (2 - 3i))(x - 1)$.

C. $b \in [-3.5, -0.9], c \in [7.06, 9.19]$, and $d \in [12.15, 13.07]$

* $x^3 - 3x^2 + 9x + 13$, which is the correct option.

D. $b \in [-1.9, 1.3], c \in [-1.85, -0.7]$, and $d \in [-2.24, -0.56]$

$x^3 + x^2 - x - 2$, which corresponds to multiplying out $(x - 2)(x + 1)$.

E. None of the above.

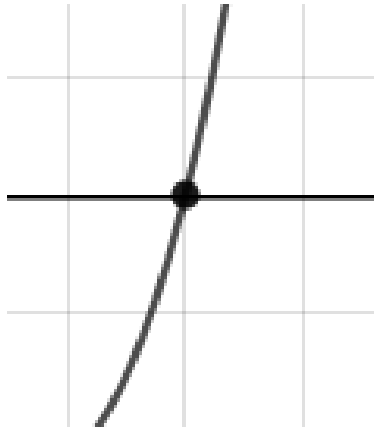
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 + 3i))(x - (2 - 3i))(x - (x - 1))$.

4. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = 7(x - 3)^9(x + 3)^{12}(x + 2)^6(x - 2)^{10}$$

The solution is

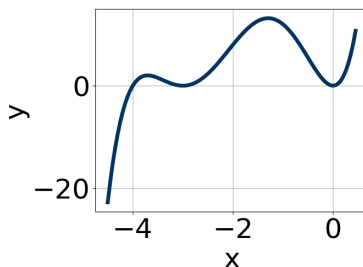


<p>A.</p> <p>Graph A shows a cubic function passing through the origin (0,0), marked with a black dot. The curve has a negative slope at the origin, indicating it is decreasing. The curve is concave up for x < 0 and concave down for x > 0.</p>	<p>B.</p> <p>Graph B shows a cubic function passing through the origin (0,0), marked with a black dot. The curve has a horizontal tangent at the origin, indicating it is neither increasing nor decreasing at that point. The curve is concave down for x < 0 and concave up for x > 0.</p>
<p>C.</p> <p>Graph C shows a cubic function passing through the origin (0,0), marked with a black dot. The curve has a horizontal tangent at the origin, indicating it is neither increasing nor decreasing at that point. The curve is concave up for x < 0 and concave down for x > 0.</p>	<p>D.</p> <p>Graph D shows a cubic function passing through the origin (0,0), marked with a black dot. The curve has a positive slope at the origin, indicating it is increasing. The curve is concave down for x < 0 and concave up for x > 0.</p>
<p>E. None of the figures above.</p>	

- A.
- B.
- C.
- D.

General Comment: General Comments: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

0. Which of the following equations *could* be of the graph presented below?



The solution is $6x^{10}(x+3)^8(x+4)^{11}$

A. $7x^8(x+3)^5(x+4)^6$

The factor $(x+3)$ should have an even power and the factor $(x+4)$ should have an odd power.

B. $6x^{10}(x+3)^8(x+4)^{11}$

* This is the correct option.

C. $-12x^6(x+3)^6(x+4)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-16x^4(x+3)^{10}(x+4)^6$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $13x^{10}(x+3)^5(x+4)^7$

The factor $(x+3)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).
