This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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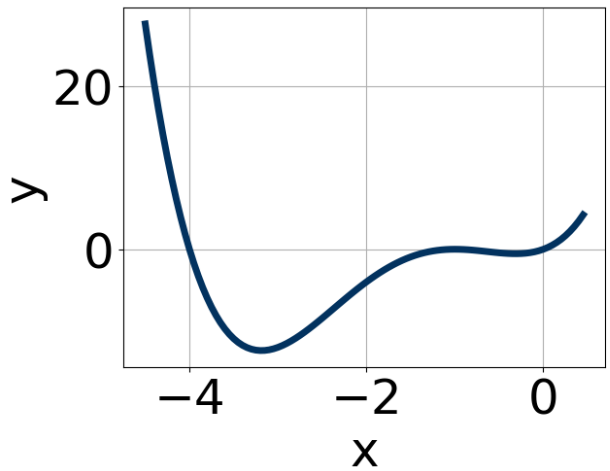
1. Which of the following equations *could* be of the graph presented below?



The solution is $19x^5(x+1)^{10}(x+4)^9$, which is option A.

A. $19x^5(x+1)^{10}(x+4)^9$

* This is the correct option.

B. $-12x^{11}(x+1)^4(x+4)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $13x^5(x+1)^6(x+4)^4$

The factor $(x+4)$ should have an odd power.

D. $5x^7(x+1)^{11}(x+4)^4$

The factor -1 should have an even power and the factor -4 should have an odd power.

E. $-2x^8(x+1)^{10}(x+4)^{11}$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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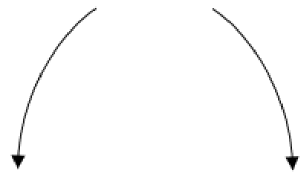
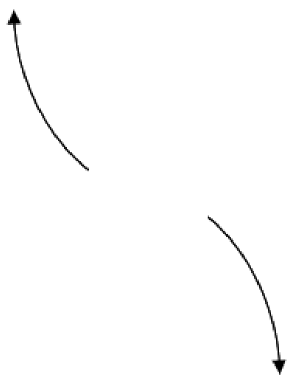
2. Describe the end behavior of the polynomial below.

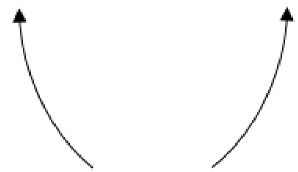
$$f(x) = -9(x+4)^5(x-4)^6(x+3)^2(x-3)^3$$

The solution is the graph below, which is option B.

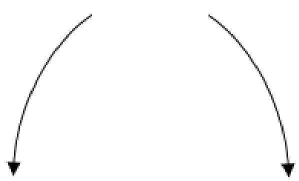




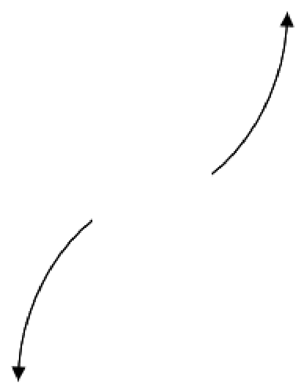
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-4}{3}, -3, \text{ and } \frac{-6}{5}$$

The solution is $15x^3 + 83x^2 + 138x + 72$, which is option C.

A. $a \in [9, 22], b \in [42, 48], c \in [-30, -28], \text{ and } d \in [-76, -66]$

$15x^3 + 43x^2 - 30x - 72$, which corresponds to multiplying out $(3x - 4)(x + 3)(5x + 6)$.

B. $a \in [9, 22], b \in [81, 91], c \in [136, 139], \text{ and } d \in [-76, -66]$

$15x^3 + 83x^2 + 138x - 72$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [9, 22], b \in [81, 91], c \in [136, 139], \text{ and } d \in [71, 75]$

* $15x^3 + 83x^2 + 138x + 72$, which is the correct option.

D. $a \in [9, 22], b \in [-85, -80], c \in [136, 139], \text{ and } d \in [-76, -66]$

$15x^3 - 83x^2 + 138x - 72$, which corresponds to multiplying out $(3x - 4)(x - 3)(5x - 6)$.

E. $a \in [9, 22], b \in [-51, -46], c \in [-18, -13], \text{ and } d \in [71, 75]$

$15x^3 - 47x^2 - 18x + 72$, which corresponds to multiplying out $(3x - 4)(x - 3)(5x + 6)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 4)(x + 3)(5x + 6)$

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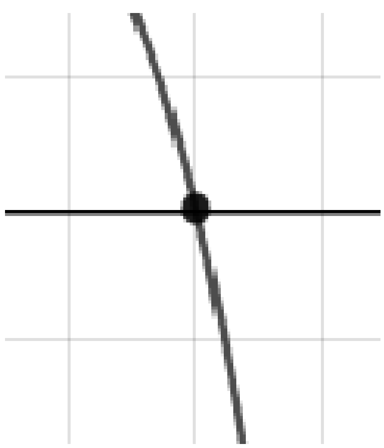
4. Describe the zero behavior of the zero $x = -2$ of the polynomial below.

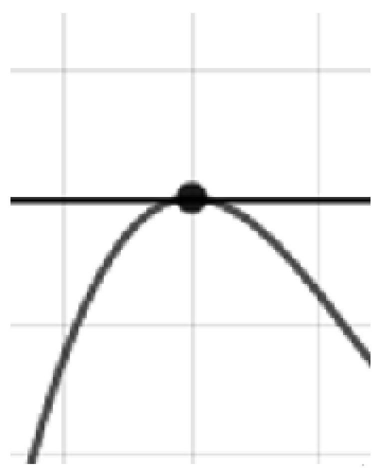
$$f(x) = 5(x - 2)^9(x + 2)^{14}(x + 6)^2(x - 6)^3$$

The solution is the graph below, which is option C.

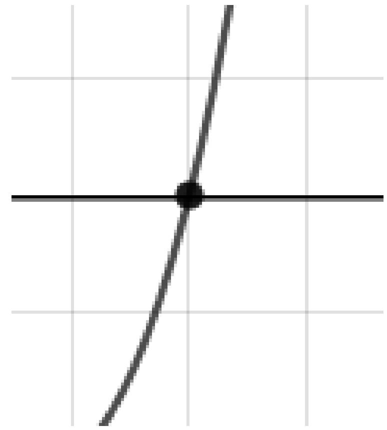
A.



B.



C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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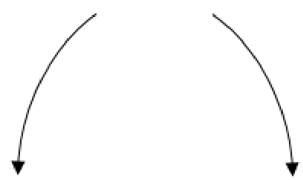
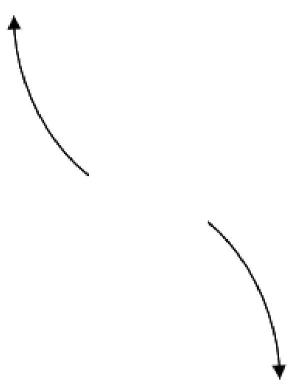
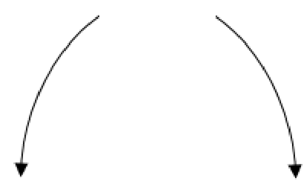
5. Describe the end behavior of the polynomial below.

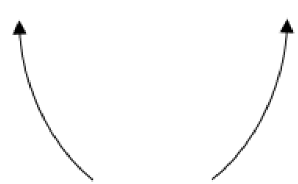
$$f(x) = -6(x-6)^5(x+6)^6(x-8)^2(x+8)^3$$

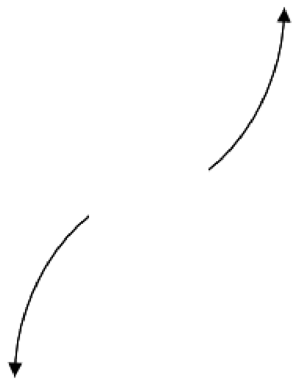
The solution is the graph below, which is option B.



A. 

B. 

C. 



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 2i \text{ and } -1$$

The solution is $x^3 + 7x^2 + 19x + 13$, which is option C.

A. $b \in [-0.9, 1.1], c \in [3.8, 5.6], \text{ and } d \in [2.51, 3.71]$

$x^3 + x^2 + 4x + 3$, which corresponds to multiplying out $(x + 3)(x + 1)$.

B. $b \in [-0.9, 1.1], c \in [2.8, 3.5], \text{ and } d \in [1.06, 2.89]$

$x^3 + x^2 + 3x + 2$, which corresponds to multiplying out $(x + 2)(x + 1)$.

C. $b \in [5.9, 10.8], c \in [17.6, 19.4], \text{ and } d \in [12.15, 13.95]$

* $x^3 + 7x^2 + 19x + 13$, which is the correct option.

D. $b \in [-8, -3.3], c \in [17.6, 19.4], \text{ and } d \in [-13.5, -12.06]$

$x^3 - 7x^2 + 19x - 13$, which corresponds to multiplying out $(x - (-3 - 2i))(x - (-3 + 2i))(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 2i))(x - (-3 + 2i))(x - (-1))$.

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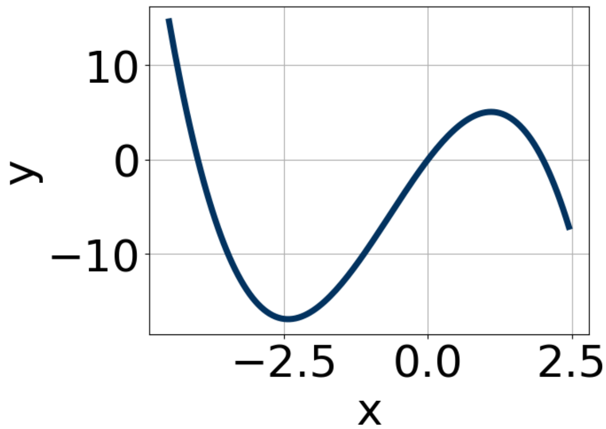
7. Which of the following equations *could* be of the graph presented below?



The solution is $-17x^9(x+4)^7(x-2)^{11}$, which is option E.

A. $-12x^4(x+4)^{10}(x-2)^{11}$

The factors 0 and -4 have have been odd power.

B. $15x^8(x+4)^{11}(x-2)^{11}$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

C. $-15x^6(x+4)^{11}(x-2)^7$

The factor 0 should have been an odd power.

D. $17x^{11}(x+4)^7(x-2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-17x^9(x+4)^7(x-2)^{11}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 2i \text{ and } -3$$

The solution is $x^3 - 7x^2 - x + 87$, which is option B.

A. $b \in [5, 11], c \in [-1.05, -0.7],$ and $d \in [-96, -81]$

$x^3 + 7x^2 - x - 87$, which corresponds to multiplying out $(x-(5+2i))(x-(5-2i))(x-3)$.

B. $b \in [-13, -2], c \in [-1.05, -0.7],$ and $d \in [87, 93]$

* $x^3 - 7x^2 - x + 87$, which is the correct option.

C. $b \in [-2, 5], c \in [-2.32, -1.02],$ and $d \in [-17, -12]$

$x^3 + x^2 - 2x - 15$, which corresponds to multiplying out $(x-5)(x+3)$.

D. $b \in [-2, 5], c \in [0.87, 1.64],$ and $d \in [-6, -1]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x-2)(x+3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 2i))(x - (5 - 2i))(x - (-3))$.

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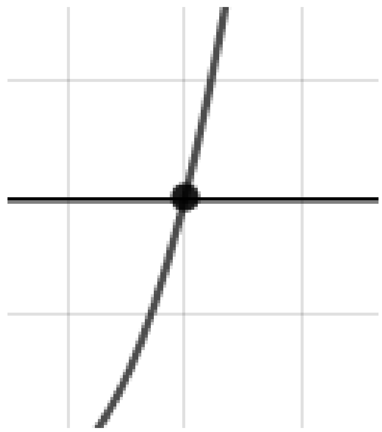
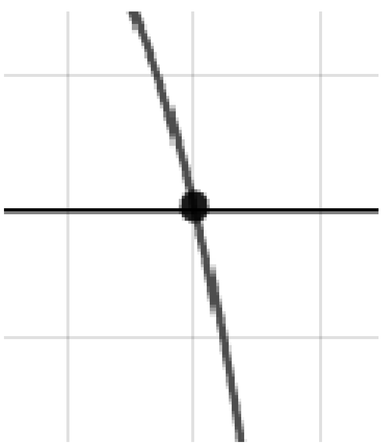
9. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = -8(x + 3)^8(x - 3)^{13}(x - 7)^3(x + 7)^7$$

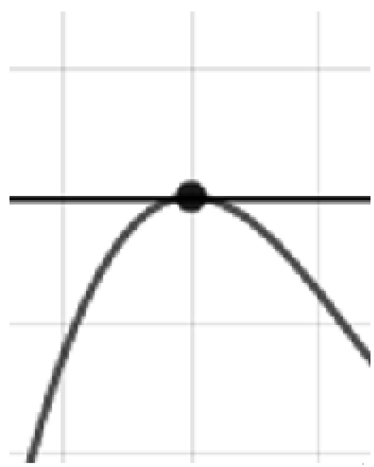
The solution is the graph below, which is option D.



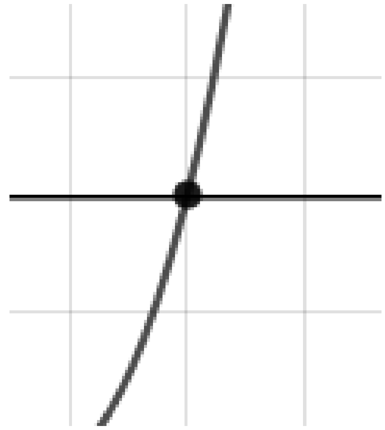
A.



B.



C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{6}{5}, \frac{2}{3}, \text{ and } \frac{-1}{5}$$

The solution is $75x^3 - 125x^2 + 32x + 12$, which is option D.

A. $a \in [70, 76], b \in [-125, -119], c \in [32, 39], \text{ and } d \in [-13, 1]$

$75x^3 - 125x^2 + 32x - 12$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [70, 76], b \in [125, 126], c \in [32, 39], \text{ and } d \in [-13, 1]$

$75x^3 + 125x^2 + 32x - 12$, which corresponds to multiplying out $(5x + 6)(3x + 2)(5x - 1)$.

C. $a \in [70, 76], b \in [53, 60], c \in [-59, -48], \text{ and } d \in [-13, 1]$

$75x^3 + 55x^2 - 52x - 12$, which corresponds to multiplying out $(5x + 6)(3x - 2)(5x + 1)$.

D. $a \in [70, 76], b \in [-125, -119], c \in [32, 39], \text{ and } d \in [11, 16]$

* $75x^3 - 125x^2 + 32x + 12$, which is the correct option.

E. $a \in [70, 76], b \in [154, 164], c \in [88, 90], \text{ and } d \in [11, 16]$

$75x^3 + 155x^2 + 88x + 12$, which corresponds to multiplying out $(5x + 6)(3x + 2)(5x + 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 6)(3x - 2)(5x + 1)$
