This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 14$ and choose the interval that $f^{-1}(14)$ belongs to.

$$f(x) = 2x^2 + 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(14) \in [2.4, 3.69]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(14) \in [2.3, 2.6]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C. $f^{-1}(14) \in [3.69, 4.71]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(14) \in [5.04, 5.78]$

Distractor 4: This corresponds to both distractors 2 and 3.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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2. Determine whether the function below is 1-1.

$$f(x) = \sqrt{-6x + 33}$$

The solution is yes, which is option B.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. Yes, the function is 1-1.

* This is the solution.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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3. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 + 2x^2 + x - 3 \text{ and } g(x) = -x^3 + 2x^2 - 2x - 2$$

The solution is 66.0, which is option D.

A. $(f \circ g)(1) \in [9, 12]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [17, 25]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(1) \in [71, 76]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [63, 68]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{3x - 21} \text{ and } g(x) = 3x^3 + 8x^2 + 6x$$

The solution is The domain is all Real numbers greater than or equal to$x = 7.0$., which is option C.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-0.5, 9.5]$

B. The domain is all Real numbers except $x = a$, where $a \in [3.67, 14.67]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [2, 9]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-12.67, -2.67]$ and $b \in [1.17, 12.17]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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5. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = \ln(x - 4) - 5$$

The solution is $f^{-1}(8) = 442417.392$, which is option C.

A. $f^{-1}(8) \in [442407.39, 442415.39]$

This solution corresponds to distractor 3.

B. $f^{-1}(8) \in [162742.79, 162756.79]$

This solution corresponds to distractor 2.

C. $f^{-1}(8) \in [442417.39, 442419.39]$

This is the solution.

D. $f^{-1}(8) \in [20.09, 28.09]$

This solution corresponds to distractor 1.

E. $f^{-1}(8) \in [47.6, 50.6]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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6. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{4}{5x - 34} \text{ and } g(x) = \frac{1}{5x - 21}$$

The solution is The domain is all Real numbers except $x = 6.8$ and $x = 4.2$, which is option D.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-8, 2]$

B. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-5.2, -4.2]$

C. The domain is all Real numbers except $x = a$, where $a \in [-9.2, 2.8]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [5.8, 7.8]$ and $b \in [-1.8, 14.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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7. Determine whether the function below is 1-1.

$$f(x) = 16x^2 + 136x + 289$$

The solution is no, which is option B.

A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

B. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 13$ and choose the interval the $f^{-1}(13)$ belongs to.

$$f(x) = \sqrt[3]{3x - 4}$$

The solution is 733.6666666666666, which is option C.

A. $f^{-1}(13) \in [-734, -733.1]$

This solution corresponds to distractor 2.

B. $f^{-1}(13) \in [-732.5, -730]$

This solution corresponds to distractor 3.

C. $f^{-1}(13) \in [732.9, 735.1]$

* This is the correct solution.

D. $f^{-1}(13) \in [729.8, 732.7]$

Distractor 1: This corresponds to

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = \ln(x + 5) + 4$$

The solution is $f^{-1}(7) = 15.086$, which is option A.

A. $f^{-1}(7) \in [12.5, 18.2]$

This is the solution.

B. $f^{-1}(7) \in [22.8, 27]$

This solution corresponds to distractor 3.

C. $f^{-1}(7) \in [59867.2, 59870.4]$

This solution corresponds to distractor 1.

D. $f^{-1}(7) \in [162756.3, 162759.2]$

This solution corresponds to distractor 4.

E. $f^{-1}(7) \in [10, 15]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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10. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 4x^3 + 2x^2 + x \text{ and } g(x) = -2x^3 - 2x^2 - 4x - 4$$

The solution is 0.0, which is option B.

A. $(f \circ g)(-1) \in [32, 36]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [-2, 2]$

   * This is the correct solution

C. $(f \circ g)(-1) \in [44, 45]$

   Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [9, 16]$

   Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!
