This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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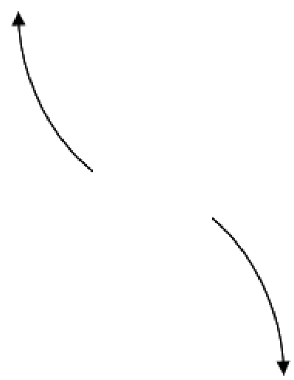
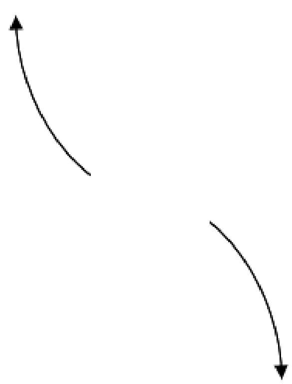
1. Describe the end behavior of the polynomial below.

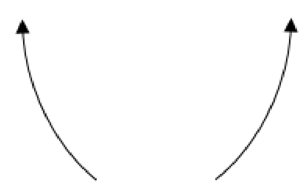
$$f(x) = -6(x+5)^2(x-5)^5(x+6)^5(x-6)^5$$

The solution is the graph below, which is option A.

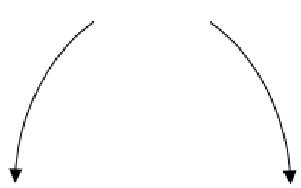




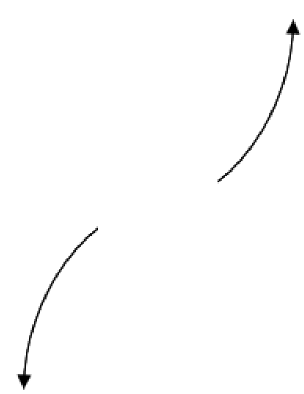
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 5i \text{ and } -1$$

The solution is $x^3 - 5x^2 + 28x + 34$, which is option C.

A. $b \in [0, 3.1], c \in [-11, 2]$, and $d \in [-8, 1]$

$x^3 + x^2 - 2x - 3$, which corresponds to multiplying out $(x - 3)(x + 1)$.

B. $b \in [1.1, 5.6], c \in [20, 29]$, and $d \in [-35, -31]$

$x^3 + 5x^2 + 28x - 34$, which corresponds to multiplying out $(x - (3 - 5i))(x - (3 + 5i))(x - 1)$.

C. $b \in [-5.2, -3], c \in [20, 29]$, and $d \in [31, 38]$

* $x^3 - 5x^2 + 28x + 34$, which is the correct option.

D. $b \in [0, 3.1], c \in [1, 9]$, and $d \in [0, 8]$

$x^3 + x^2 + 6x + 5$, which corresponds to multiplying out $(x + 5)(x + 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 - 5i))(x - (3 + 5i))(x - (-1))$.

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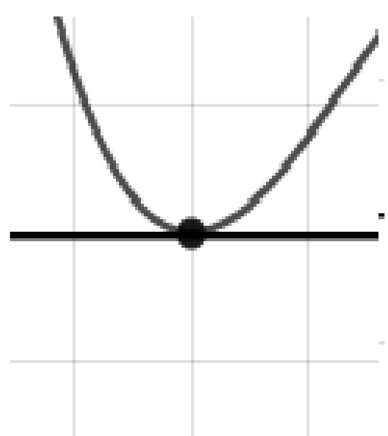
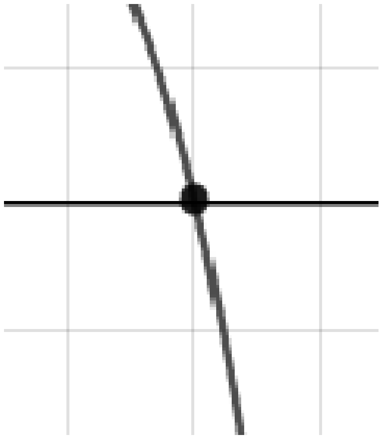
3. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

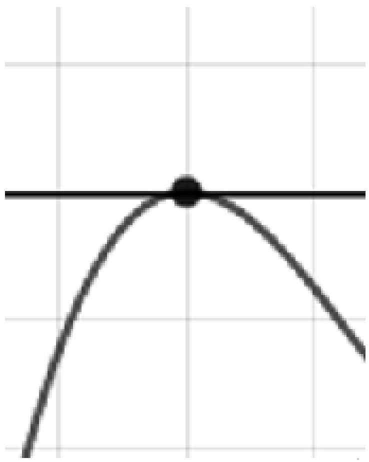
$$f(x) = 8(x + 9)^5(x - 9)^{10}(x - 8)^8(x + 8)^{12}$$

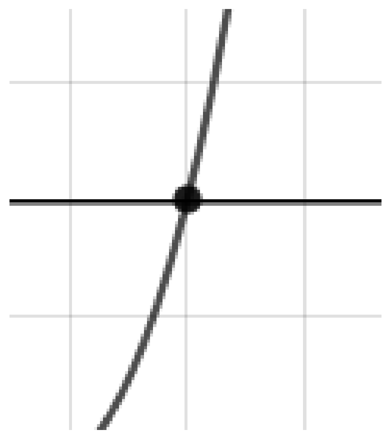
The solution is the graph below, which is option C.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$1, \frac{-3}{4}, \text{ and } \frac{-5}{3}$$

The solution is $12x^3 + 17x^2 - 14x - 15$, which is option D.

A. $a \in [12, 19], b \in [14.9, 17.6], c \in [-25, -7],$ and $d \in [10, 23]$

   $12x^3 + 17x^2 - 14x + 15$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [12, 19], b \in [22.9, 25], c \in [-9, -3],$ and $d \in [-15, -12]$

   $12x^3 + 23x^2 - 4x - 15$, which corresponds to multiplying out $(x + 1)(4x - 3)(3x + 5)$.

C. $a \in [12, 19], b \in [-19.1, -15.4], c \in [-25, -7],$ and $d \in [10, 23]$

   $12x^3 - 17x^2 - 14x + 15$, which corresponds to multiplying out $(x + 1)(4x - 3)(3x - 5)$.

D. $a \in [12, 19], b \in [14.9, 17.6], c \in [-25, -7],$ and $d \in [-15, -12]$

* $12x^3 + 17x^2 - 14x - 15$, which is the correct option.

E. $a \in [12, 19], b \in [40.2, 44], c \in [38, 47],$ and $d \in [10, 23]$

$12x^3 + 41x^2 + 44x + 15$, which corresponds to multiplying out $(x + 1)(4x + 3)(3x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-1)(4x+3)(3x + 5)$

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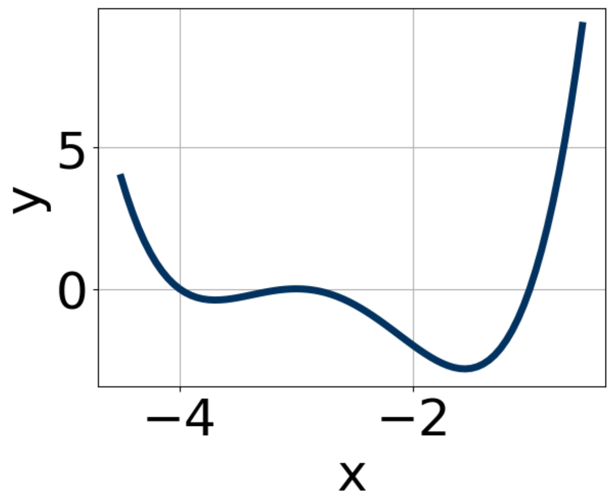
5. Which of the following equations *could* be of the graph presented below?



The solution is $2(x + 3)^8(x + 1)^7(x + 4)^5$, which is option C.

A. $16(x + 3)^5(x + 1)^6(x + 4)^{11}$

The factor -3 should have an even power and the factor -1 should have an odd power.

B. $19(x + 3)^6(x + 1)^{10}(x + 4)^7$

The factor $(x + 1)$ should have an odd power.

C. $2(x + 3)^8(x + 1)^7(x + 4)^5$

* This is the correct option.

D. $-3(x + 3)^8(x + 1)^9(x + 4)^{10}$

The factor $(x + 4)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-20(x + 3)^4(x + 1)^{11}(x + 4)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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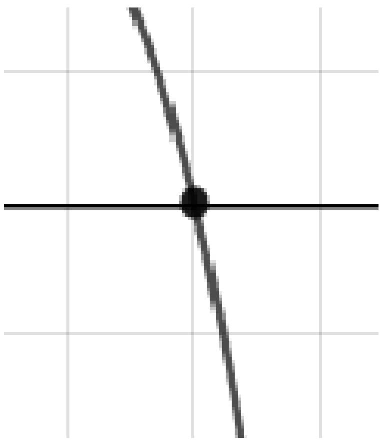
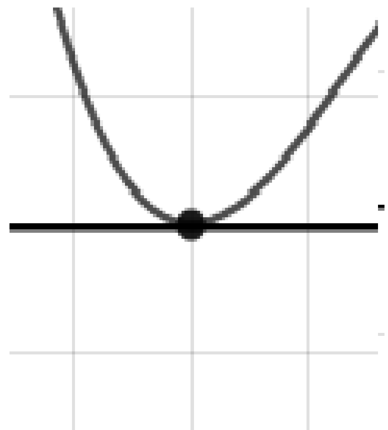
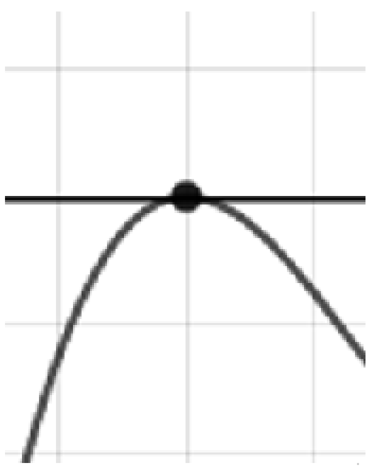
6. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

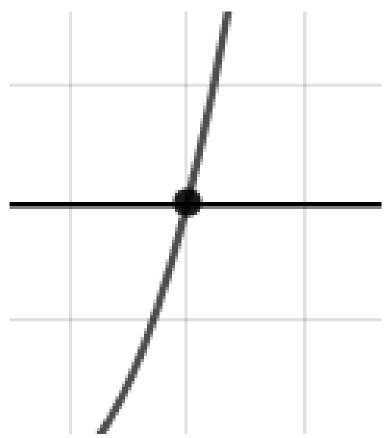
$$f(x) = 5(x - 6)^5(x + 6)^8(x - 9)^9(x + 9)^{11}$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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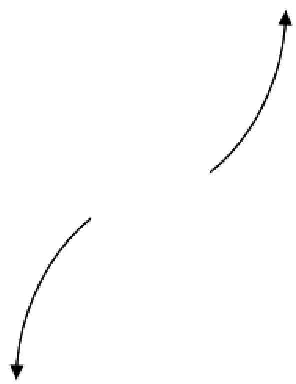
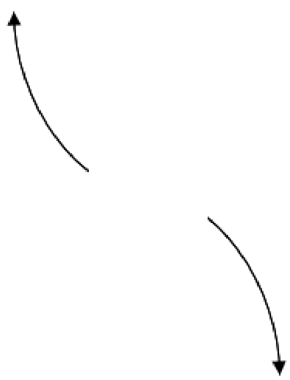
7. Describe the end behavior of the polynomial below.

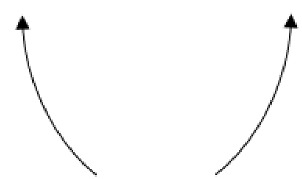
$$f(x) = 2(x+2)^2(x-2)^3(x+3)^3(x-3)^5$$

The solution is the graph below, which is option D.

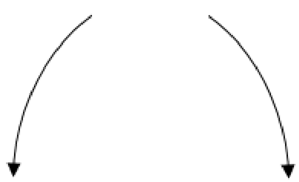




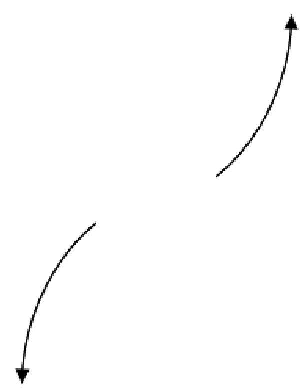
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 3i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 33x - 25$, which is option B.

A. $b \in [-2, 6], c \in [-1, 13],$ and $d \in [-5, -1]$

$x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

B. $b \in [-13, -8], c \in [31, 36],$ and $d \in [-26, -23]$

* $x^3 - 9x^2 + 33x - 25$, which is the correct option.

C. $b \in [8, 11], c \in [31, 36],$ and $d \in [25, 31]$

$x^3 + 9x^2 + 33x + 25$, which corresponds to multiplying out $(x - (4 - 3i))(x - (4 + 3i))(x + 1)$.

D. $b \in [-2, 6], c \in [-10, -1],$ and $d \in [0, 5]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 3i))(x - (4 + 3i))(x - (1))$.

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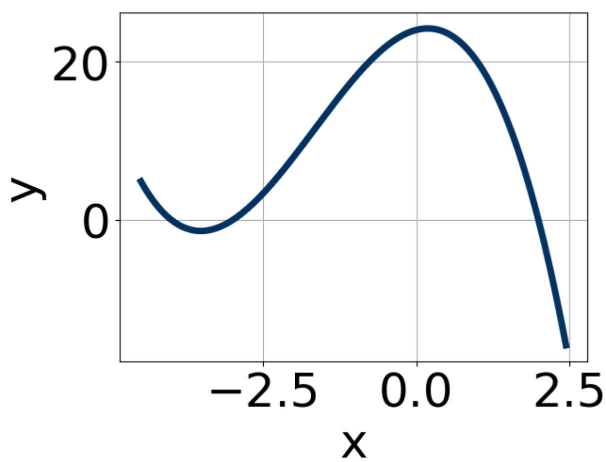
9. Which of the following equations *could* be of the graph presented below?



The solution is $-11(x + 3)^{11}(x + 4)^9(x - 2)^5$, which is option C.

A. $-3(x + 3)^8(x + 4)^6(x - 2)^{11}$

The factors -3 and -4 have have been odd power.

B. $16(x + 3)^{10}(x + 4)^{11}(x - 2)^7$

The factor $(x + 3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-11(x + 3)^{11}(x + 4)^9(x - 2)^5$

* This is the correct option.

D. $2(x + 3)^{11}(x + 4)^{11}(x - 2)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-9(x + 3)^4(x + 4)^7(x - 2)^{11}$

The factor -3 should have been an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$3, 4, \text{ and } \frac{-1}{2}$$

The solution is $2x^3 - 13x^2 + 17x + 12$, which is option D.

A. $a \in [-4, 5], b \in [14, 19], c \in [26, 35], \text{ and } d \in [7, 17]$

$2x^3 + 15x^2 + 31x + 12$, which corresponds to multiplying out $(x + 3)(x + 4)(2x + 1)$.

B. $a \in [-4, 5], b \in [-21, -10], c \in [11, 26], \text{ and } d \in [-12, -10]$

$2x^3 - 13x^2 + 17x - 12$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [-4, 5], b \in [-4, 10], c \in [-27, -20], \text{ and } d \in [-12, -10]$

$2x^3 - 1x^2 - 25x - 12$, which corresponds to multiplying out $(x + 3)(x - 4)(2x + 1)$.

D. $a \in [-4, 5], b \in [-21, -10], c \in [11, 26], \text{ and } d \in [7, 17]$

* $2x^3 - 13x^2 + 17x + 12$, which is the correct option.

E. $a \in [-4, 5], b \in [9, 14], c \in [11, 26], \text{ and } d \in [-12, -10]$

$2x^3 + 13x^2 + 17x - 12$, which corresponds to multiplying out $(x + 3)(x + 4)(2x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x - 3)(x - 4)(2x + 1)$
