This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{25x^3 - 85x^2 + 82x - 20}{x - 2}$$

The solution is $25x^2 - 35x + 12 + \dfrac{4}{x-2}$, which is option C.

A. $a \in [18, 29], \; b \in [-141, -129], \; c \in [348, 356], \text{ and } r \in [-724.68, -722.52]$.

You divided by the opposite of the factor.

B. $a \in [18, 29], \; b \in [-60, -57], \; c \in [16, 28], \text{ and } r \in [1.08, 3.15]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [18, 29], \; b \in [-42, -33], \; c \in [4, 14], \text{ and } r \in [3.6, 5.08]$.

* This is the solution!

D. $a \in [48, 58], \; b \in [-191, -182], \; c \in [452, 458], \text{ and } r \in [-925.19, -923.81]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [48, 58], \; b \in [13, 20], \; c \in [109, 116], \text{ and } r \in [203.09, 204.16]$.

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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2. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 2x^4 + 7x^3 + 2x^2 + 3x + 7$$

The solution is $\pm 1, \pm 7$, which is option A.

A. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

B. $\pm 1, \pm 2$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2}$

This would have been the solution **if asked for the possible Rational roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 - 28x - 14}{x - 2}$$

The solution is $9x^2 + 18x + 8 + \frac{2}{x-2}$, which is option C.

A. $a \in [14, 22], b \in [30, 40], c \in [42, 45]$, and $r \in [72, 77]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [7, 13], b \in [8, 14], c \in [-19, -16]$, and $r \in [-36, -31]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [7, 13], b \in [10, 22], c \in [4, 15]$, and $r \in [0, 10]$.

* This is the solution!

D. $a \in [7, 13], b \in [-18, -11], c \in [4, 15]$, and $r \in [-32, -25]$.

You divided by the opposite of the factor.

E. $a \in [14, 22], b \in [-38, -30], c \in [42, 45]$, and $r \in [-105, -97]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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4. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^4 + 3x^3 + 5x^2 + 5x + 2$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 7}$, which is option B.

A. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 7}$

* This is the solution **since we asked for the possible Rational roots**!

C. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 + 31x^2 - 45x - 32}{x + 4}$$

The solution is $10x^2 - 9x - 9 + \dfrac{4}{x+4}$, which is option A.

A. $a \in [5, 14], \; b \in [-16, -7], \; c \in [-13, -1], \text{ and } r \in [-3, 7].$

* This is the solution!

B. $a \in [-41, -36], \; b \in [-131, -128], \; c \in [-563, -559], \text{ and } r \in [-2285, -2267].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [5, 14], \; b \in [-26, -15], \; c \in [45, 57], \text{ and } r \in [-289, -275].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [5, 14], \; b \in [70, 72], \; c \in [232, 244], \text{ and } r \in [920, 926].$

You divided by the opposite of the factor.

E. $a \in [-41, -36], \; b \in [187, 196], \; c \in [-810, -807], \text{ and } r \in [3203, 3209].$

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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6. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 - 26x^2 + 15}{x - 3}$$

The solution is $8x^2 - 2x - 6 + \dfrac{-3}{x-3}$, which is option C.

A. $a \in [5, 10], b \in [-52, -44], c \in [150, 153], \text{ and } r \in [-437, -434].$

You divided by the opposite of the factor.

B. $a \in [5, 10], b \in [-15, -5], c \in [-26, -16], \text{ and } r \in [-25, -22].$

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [5, 10], b \in [-7, -1], c \in [-8, -4]$, and $r \in [-9, -1]$.

* This is the solution!

D. $a \in [19, 27], b \in [-104, -92], c \in [293, 295]$, and $r \in [-868, -865]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [19, 27], b \in [43, 49], c \in [138, 143]$, and $r \in [423, 432]$.

You multipled by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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7. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 + 29x^2 - 81x + 36$$

The solution is $[-3, 0.75, 0.8]$, which is option B.

A. $z_1 \in [-4.26, -3.91]$, $z_2 \in [-0.21, 0.28]$, and $z_3 \in [2.72, 3.15]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-3.55, -2.84]$, $z_2 \in [0.39, 0.8]$, and $z_3 \in [0.52, 1.13]$

* This is the solution!

C. $z_1 \in [-1.41, -1.2]$, $z_2 \in [-1.38, -1.15]$, and $z_3 \in [2.72, 3.15]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-0.99, -0.41]$, $z_2 \in [-0.99, -0.26]$, and $z_3 \in [2.72, 3.15]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-3.55, -2.84]$, $z_2 \in [0.85, 1.65]$, and $z_3 \in [1.17, 1.6]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 + 31x^2 - 38x - 40$$

The solution is $[-2, -0.8, 1.25]$, which is option A.

A. $z_1 \in [-2.05, -1.76]$, $z_2 \in [-0.86, -0.28]$, and $z_3 \in [1.08, 1.34]$

* This is the solution!

B. $z_1 \in [-2.05, -1.76]$, $z_2 \in [-1.7, -1.15]$, and $z_3 \in [0.44, 1.08]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-0.51, -0.23]$, $z_2 \in [1.94, 2.57]$, and $z_3 \in [3.41, 4.51]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-0.84, -0.78]$, $z_2 \in [0.9, 1.41]$, and $z_3 \in [1.62, 2.38]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-1.42, -1.14]$, $z_2 \in [0.26, 1]$, and $z_3 \in [1.62, 2.38]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Factor the polynomial below completely, knowing that $x+2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 26x^3 - 69x^2 + 130x + 200$$

The solution is $[-2, -1.25, 2.5, 4]$, which is option D.

A. $z_1 \in [-3.6, -0.1]$, $z_2 \in [-0.83, -0.64]$, $z_3 \in [0.22, 0.54]$, and $z_4 \in [3.28, 4.12]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-4.7, -2.9]$, $z_2 \in [-2.58, -2.37]$, $z_3 \in [1.06, 1.66]$, and $z_4 \in [1.27, 2.9]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-4.7, -2.9]$, $z_2 \in [-0.4, -0.15]$, $z_3 \in [0.66, 1.04]$, and $z_4 \in [1.27, 2.9]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-3.6, -0.1]$, $z_2 \in [-1.33, -1.01]$, $z_3 \in [2.41, 2.53]$, and $z_4 \in [3.28, 4.12]$

* This is the solution!

E. $z_1 \in [-4.7, -2.9]$, $z_2 \in [-0.73, -0.57]$, $z_3 \in [1.86, 2.33]$, and $z_4 \in [4.81, 5.51]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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10. Factor the polynomial below completely, knowing that $x+3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 - 39x^3 - 127x^2 + 315x + 225$$

The solution is $[-3, -0.6, 2.5, 5]$, which is option D.

A. $z_1 \in [-6.3, -3.9]$, $z_2 \in [-2.7, -1.91]$, $z_3 \in [0.49, 0.71]$, and $z_4 \in [2.6, 3.9]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-3.3, 0.2]$, $z_2 \in [-2.23, -1.58]$, $z_3 \in [0.34, 0.41]$, and $z_4 \in [3.8, 5.4]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-6.3, -3.9]$, $z_2 \in [-5.42, -5]$, $z_3 \in [0.11, 0.38]$, and $z_4 \in [2.6, 3.9]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-3.3, 0.2]$, $z_2 \in [-0.66, -0.58]$, $z_3 \in [2.41, 2.52]$, and $z_4 \in [3.8, 5.4]$

* This is the solution!

E. $z_1 \in [-6.3, -3.9]$, $z_2 \in [-0.55, 0.6]$, $z_3 \in [1.58, 1.88]$, and $z_4 \in [2.6, 3.9]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.