This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Determine whether the function below is 1-1.

$$f(x) = 9x^2 + 15x - 456$$

The solution is no, which is option A.

A. No, because there is a y-value that goes to 2 different x-values.

   * This is the solution.

B. No, because there is an x-value that goes to 2 different y-values.

   Corresponds to the Vertical Line test, which checks if an expression is a function.

C. Yes, the function is 1-1.

   Corresponds to believing the function passes the Horizontal Line test.

D. No, because the domain of the function is not $(-\infty, \infty)$.

   Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because the range of the function is not $(-\infty, \infty)$.

   Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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2. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = x^3 - 4x^2 + x \text{ and } g(x) = x^3 + 4x^2 - x$$

The solution is 4.0, which is option A.

A. $(f \circ g)(-1) \in [2, 8]$

   * This is the correct solution

B. $(f \circ g)(-1) \in [-67, -61]$

   Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [-3, 1]$

   Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-83, -73]$

   Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = x^4 + 5x^3 + 5x^2 + 2 \text{ and } g(x) = \frac{5}{4x + 15}$$

The solution is The domain is all Real numbers except $x = -3.75$, which is option A.

A. The domain is all Real numbers except $x = a$, where $a \in [-3.75, 0.25]$

B. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-9.25, -2.25]$

C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-2, 0]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-8.33, 1.67]$ and $b \in [6.2, 7.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = e^{x-4} - 3$$

The solution is $f^{-1}(9) = 6.485$, which is option E.

A. $f^{-1}(9) \in [-1.36, -1.19]$

This solution corresponds to distractor 2.

B. $f^{-1}(9) \in [-0.61, -0.43]$

This solution corresponds to distractor 3.

C. $f^{-1}(9) \in [-1.43, -1.22]$

This solution corresponds to distractor 4.

D. $f^{-1}(9) \in [-1.76, -1.5]$

This solution corresponds to distractor 1.

E. $f^{-1}(9) \in [6.36, 6.51]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -11$ and choose the interval the $f^{-1}(-11)$ belongs to.

$$f(x) = \sqrt[3]{2x - 3}$$

The solution is -664.0, which is option C.

A. $f^{-1}(-11) \in [663.1, 665.4]$

This solution corresponds to distractor 2.

B. $f^{-1}(-11) \in [-668.4, -665.4]$

Distractor 1: This corresponds to

C. $f^{-1}(-11) \in [-665.8, -660.6]$

* This is the correct solution.

D. $f^{-1}(-11) \in [666.8, 669.9]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{5}{3x-16} \text{ and } g(x) = \frac{3}{4x+21}$$

The solution is The domain is all Real numbers except $x = 5.333333333333333$ and $x = -5.25$, which is option D.

A. The domain is all Real numbers except $x = a$, where $a \in [4.33, 12.33]$

B. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-5.67, 2.33]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [3.75, 7.75]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [4.33, 7.33]$ and $b \in [-9.25, -3.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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7. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -2x^3 + 2x^2 + 4x + 3 \text{ and } g(x) = 2x^3 + x^2 - 4x + 1$$

The solution is -77.0, which is option A.

A. $(f \circ g)(-1) \in [-79, -74]$

* This is the correct solution

B. $(f \circ g)(-1) \in [-68, -67]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [51, 55]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [39, 48]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 10$ and choose the interval the $f^{-1}(10)$ belongs to.

$$f(x) = \sqrt[3]{3x+2}$$

The solution is 332.6666666666667, which is option B.

A. $f^{-1}(10) \in [-334.41, -333.58]$

This solution corresponds to distractor 3.

B. $f^{-1}(10) \in [332.26, 333.46]$

* This is the correct solution.

C. $f^{-1}(10) \in [333.98, 334.06]$

Distractor 1: This corresponds to

D. $f^{-1}(10) \in [-333.03, -332.01]$

This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = e^{x-2} + 5$$

The solution is $f^{-1}(9) = 3.386$, which is option A.

A. $f^{-1}(9) \in [3.15, 3.4]$

This is the solution.

B. $f^{-1}(9) \in [7.34, 7.43]$

This solution corresponds to distractor 3.

C. $f^{-1}(9) \in [7.59, 7.66]$

This solution corresponds to distractor 2.

D. $f^{-1}(9) \in [6.77, 7.03]$

This solution corresponds to distractor 4.

E. $f^{-1}(9) \in [-0.78, -0.44]$

This solution corresponds to distractor 1.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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10. Determine whether the function below is 1-1.

$$f(x) = 36x^2 - 348x + 841$$

The solution is no, which is option A.

A. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.
