This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

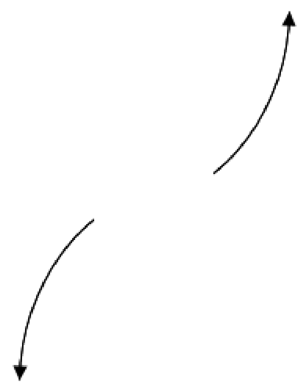
If you have a suggestion to make the keys better, please fill out the short survey here.

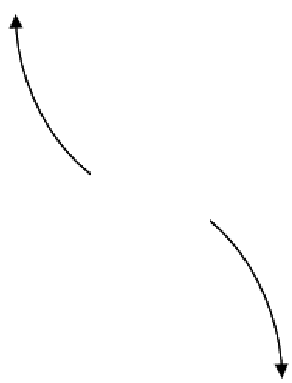
Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Describe the end behavior of the polynomial below.

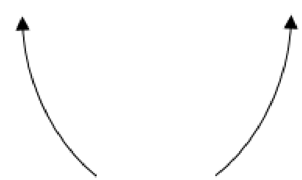
$$f(x) = 9(x+6)^5(x-6)^{10}(x+3)^3(x-3)^5$$

The solution is the graph below, which is option D.

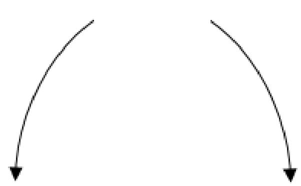




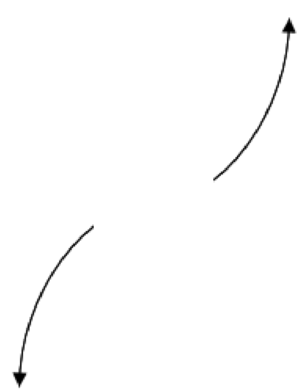
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$7, 6, \text{ and } \frac{-7}{4}$$

The solution is $4x^3 - 45x^2 + 77x + 294$, which is option C.

A. $a \in [-1, 5], b \in [-51, -40], c \in [74, 78], \text{ and } d \in [-295, -287]$

$4x^3 - 45x^2 + 77x - 294$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [-1, 5], b \in [43, 49], c \in [74, 78], \text{ and } d \in [-295, -287]$

$4x^3 + 45x^2 + 77x - 294$, which corresponds to multiplying out $(x + 7)(x + 6)(4x - 7)$.

C. $a \in [-1, 5], b \in [-51, -40], c \in [74, 78], \text{ and } d \in [287, 297]$

* $4x^3 - 45x^2 + 77x + 294$, which is the correct option.

D. $a \in [-1, 5], b \in [58, 62], c \in [256, 261], \text{ and } d \in [287, 297]$

$4x^3 + 59x^2 + 259x + 294$, which corresponds to multiplying out $(x + 1)(x + 1)(4x - 4)$.

E. $a \in [-1, 5], b \in [11, 13], c \in [-164, -157], \text{ and } d \in [-295, -287]$

$4x^3 + 11x^2 - 161x - 294$, which corresponds to multiplying out $(x + 1)(x - 1)(4x - 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-7)(x-6)(4x+7)$

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 2i \text{ and } -3$$

The solution is $x^3 + 11x^2 + 44x + 60$, which is option B.

A. $b \in [-1, 7], c \in [-1, 4], \text{ and } d \in [-13, -3]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x - 2)(x + 3)$.

B. $b \in [10, 18], c \in [41, 45], \text{ and } d \in [55, 73]$

* $x^3 + 11x^2 + 44x + 60$, which is the correct option.

C. $b \in [-11, -4], c \in [41, 45], \text{ and } d \in [-60, -58]$

$x^3 - 11x^2 + 44x - 60$, which corresponds to multiplying out $(x - (-4 + 2i))(x - (-4 - 2i))(x - 3)$.

D. $b \in [-1, 7], c \in [7, 13], \text{ and } d \in [8, 17]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 4)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-4+2i))(x-(-4-2i))(x-(-3))$.

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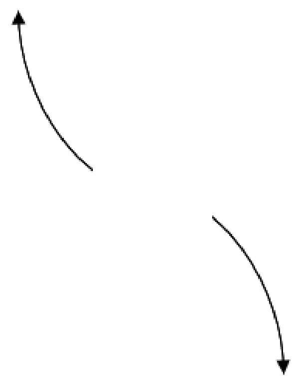
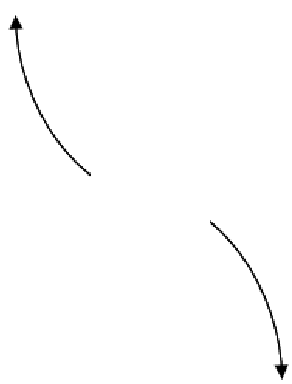
4. Describe the end behavior of the polynomial below.

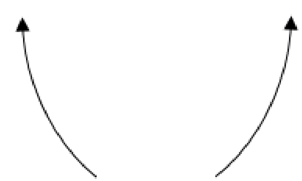
$$f(x) = -8(x-4)^5(x+4)^8(x+3)^4(x-3)^6$$

The solution is the graph below, which is option A.

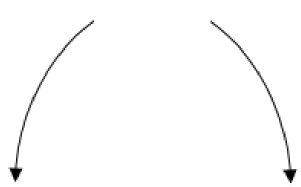




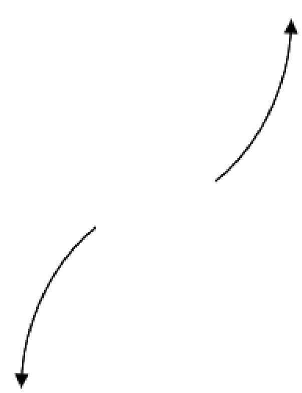
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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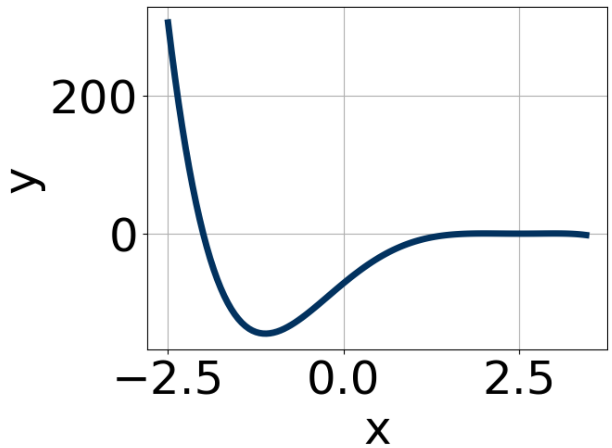
5. Which of the following equations *could* be of the graph presented below?



The solution is $-8(x-2)^{10}(x-3)^4(x+2)^9$, which is option C.

A. $13(x-2)^{10}(x-3)^4(x+2)^4$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $2(x-2)^8(x-3)^{10}(x+2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-8(x-2)^{10}(x-3)^4(x+2)^9$

* This is the correct option.

D. $-3(x-2)^4(x-3)^5(x+2)^9$

The factor $(x-3)$ should have an even power.

E. $-16(x-2)^6(x-3)^7(x+2)^{10}$

The factor $(x-3)$ should have an even power and the factor $(x+2)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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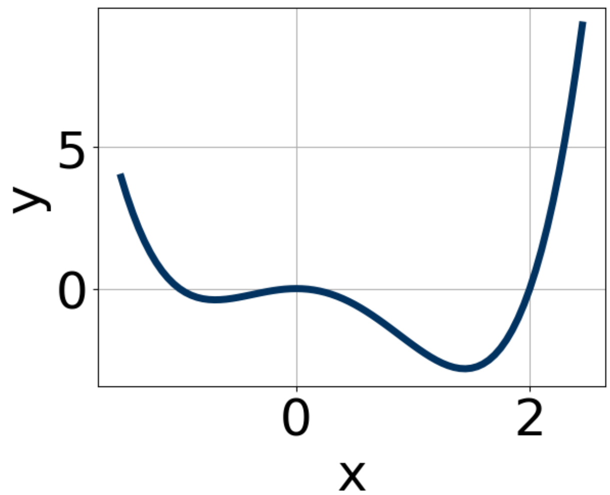
6. Which of the following equations *could* be of the graph presented below?



The solution is $13x^8(x-2)^5(x+1)^7$, which is option E.

A. $13x^{10}(x-2)^4(x+1)^{11}$

The factor $(x-2)$ should have an odd power.

B. $-8x^6(x-2)^5(x+1)^4$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-16x^8(x-2)^5(x+1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $3x^{11}(x-2)^6(x+1)^9$

The factor 0 should have an even power and the factor 2 should have an odd power.

E. $13x^8(x-2)^5(x+1)^7$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-4}{5}, \frac{-2}{3}, \text{ and } 3$$

The solution is $15x^3 - 23x^2 - 58x - 24$, which is option A.

A. $a \in [10, 22], b \in [-24, -20], c \in [-64, -57], \text{ and } d \in [-24, -18]$

* $15x^3 - 23x^2 - 58x - 24$, which is the correct option.

B. $a \in [10, 22], b \in [-51, -42], c \in [-8, 5], \text{ and } d \in [22, 29]$

$15x^3 - 47x^2 - 2x + 24$, which corresponds to multiplying out $(5x + 5)(3x - 3)(x - 1)$.

C. $a \in [10, 22], b \in [-70, -60], c \in [74, 80], \text{ and } d \in [-24, -18]$

$15x^3 - 67x^2 + 74x - 24$, which corresponds to multiplying out $(5x + 5)(3x + 3)(x - 1)$.

D. $a \in [10, 22], b \in [-24, -20], c \in [-64, -57],$ and $d \in [22, 29]$

$15x^3 - 23x^2 - 58x + 24$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [10, 22], b \in [18, 27], c \in [-64, -57],$ and $d \in [22, 29]$

$15x^3 + 23x^2 - 58x + 24$, which corresponds to multiplying out $(5x - 4)(3x - 2)(x + 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 4)(3x + 2)(x - 3)$

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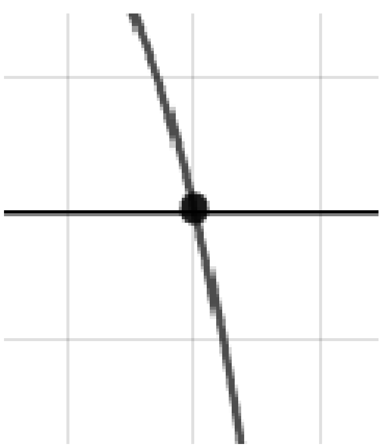
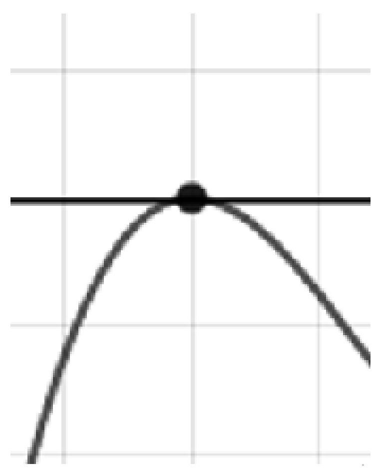
8. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

$$f(x) = 7(x - 9)^6(x + 9)^7(x - 6)^7(x + 6)^{11}$$

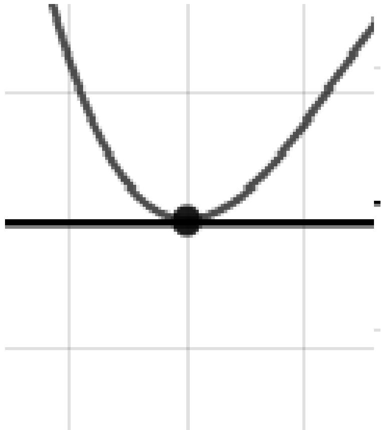
The solution is the graph below, which is option C.

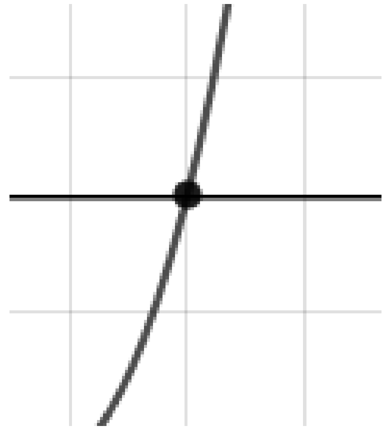


A.



B.

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 3i \text{ and } -2$$

The solution is $x^3 + 10x^2 + 41x + 50$, which is option C.

A. $b \in [0, 7], c \in [3.09, 5.81]$, and $d \in [5.4, 7.3]$

   $x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 3)(x + 2)$.

B. $b \in [-10, -3], c \in [40.53, 42.29]$, and $d \in [-52, -49.7]$

   $x^3 - 10x^2 + 41x - 50$, which corresponds to multiplying out $(x - (-4 - 3i))(x - (-4 + 3i))(x - 2)$.

C. $b \in [9, 16], c \in [40.53, 42.29]$, and $d \in [49, 50.4]$

   * $x^3 + 10x^2 + 41x + 50$, which is the correct option.

D. $b \in [0, 7], c \in [5.46, 7.06]$, and $d \in [7, 11.7]$

   $x^3 + x^2 + 6x + 8$, which corresponds to multiplying out $(x + 4)(x + 2)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 - 3i))(x - (-4 + 3i))(x - (-2))$.

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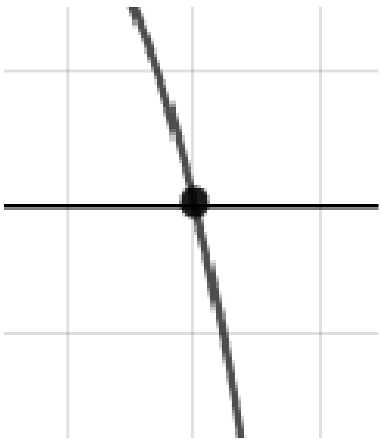
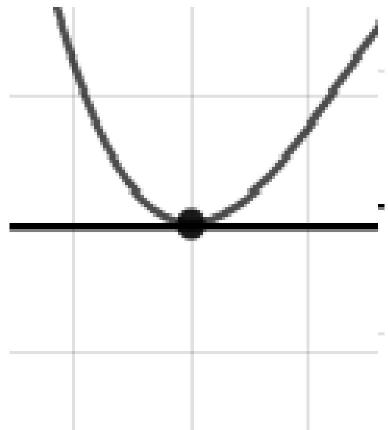
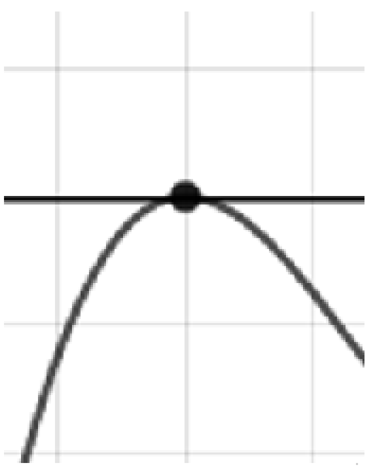
10. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

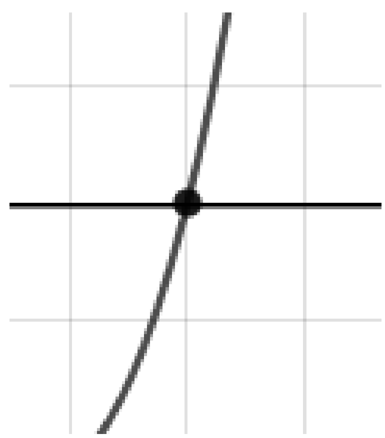
$$f(x) = 2(x + 5)^3(x - 5)^8(x + 7)^9(x - 7)^{10}$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.