This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{4}, \frac{-1}{4}, \text{ and } \frac{-5}{2}$$

The solution is $32x^3 + 48x^2 - 90x - 25$, which is option B.

A. $a \in [31, 39], b \in [125, 129], c \in [125, 134], \text{ and } d \in [25, 29]$

$32x^3 + 128x^2 + 130x + 25$, which corresponds to multiplying out $(4x + 5)(4x + 1)(2x + 5)$.

B. $a \in [31, 39], b \in [44, 51], c \in [-91, -83], \text{ and } d \in [-30, -22]$

* $32x^3 + 48x^2 - 90x - 25$, which is the correct option.

C. $a \in [31, 39], b \in [-57, -45], c \in [-91, -83], \text{ and } d \in [25, 29]$

$32x^3 - 48x^2 - 90x + 25$, which corresponds to multiplying out $(4x + 5)(4x - 1)(2x - 5)$.

D. $a \in [31, 39], b \in [111, 113], c \in [70, 76], \text{ and } d \in [-30, -22]$

$32x^3 + 112x^2 + 70x - 25$, which corresponds to multiplying out $(4x + 5)(4x - 1)(2x + 5)$.

E. $a \in [31, 39], b \in [44, 51], c \in [-91, -83], \text{ and } d \in [25, 29]$

$32x^3 + 48x^2 - 90x + 25$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 5)(4x + 1)(2x + 5)$

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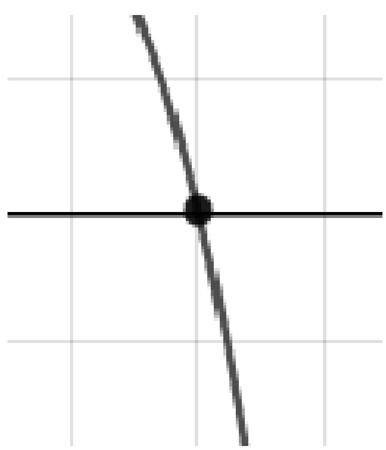
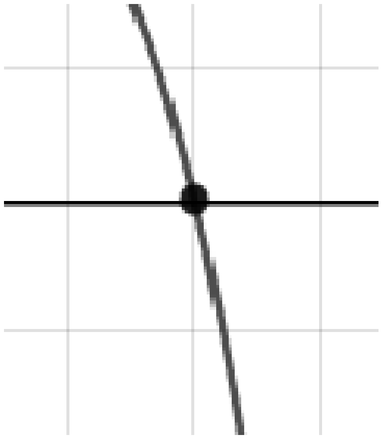
2. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

$$f(x) = 8(x + 3)^3(x - 3)^4(x + 8)^2(x - 8)^3$$

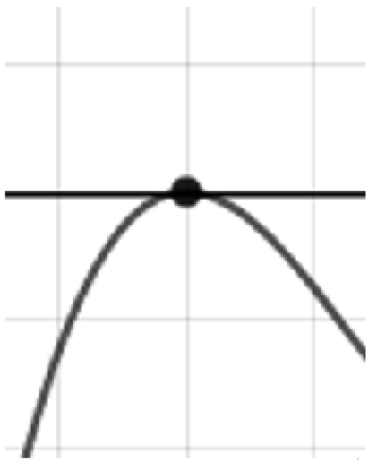
The solution is the graph below, which is option A.





A.

C.



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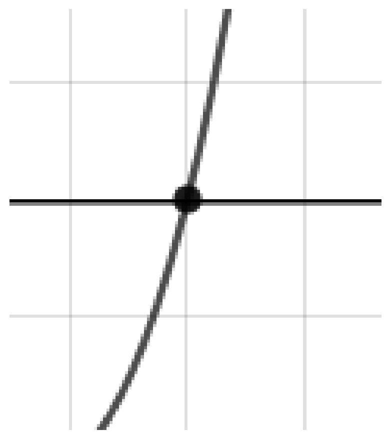

D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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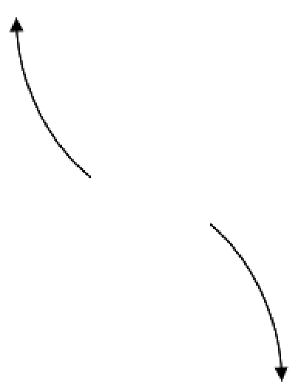
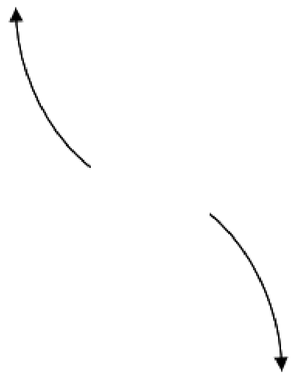
3. Describe the end behavior of the polynomial below.

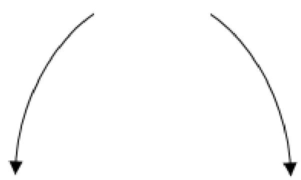
$$f(x) = -4(x-9)^3(x+9)^8(x+3)^5(x-3)^7$$

The solution is the graph below, which is option A.





A.



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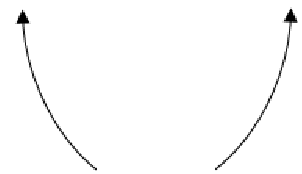
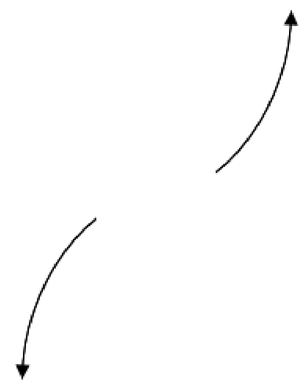


C.



D.

E. None of the above.

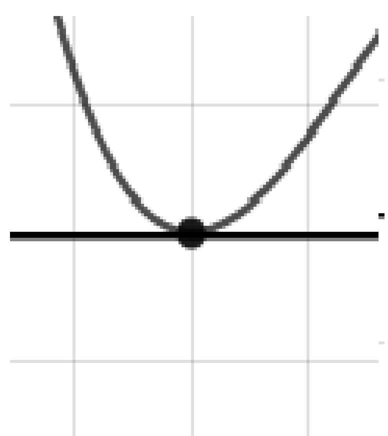
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

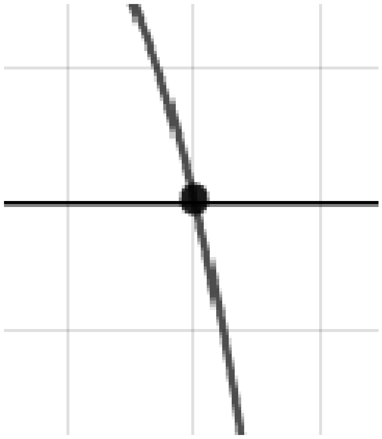
---

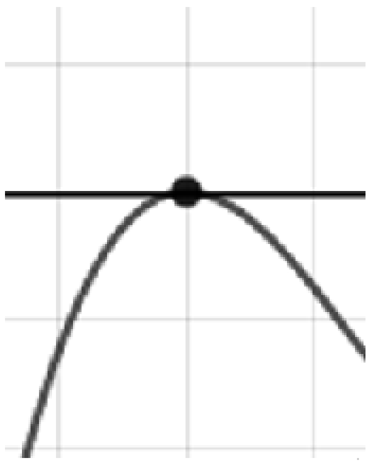
4. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

$$f(x) = 7(x+2)^8(x-2)^7(x-3)^{11}(x+3)^8$$

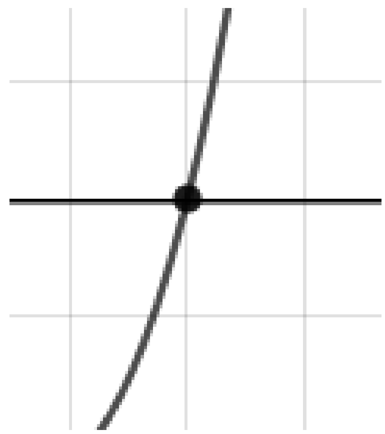
The solution is the graph below, which is option C.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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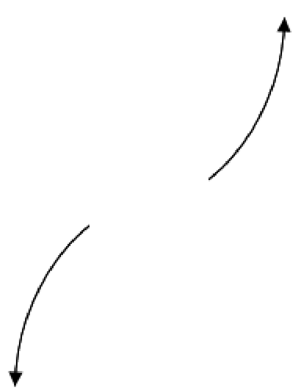
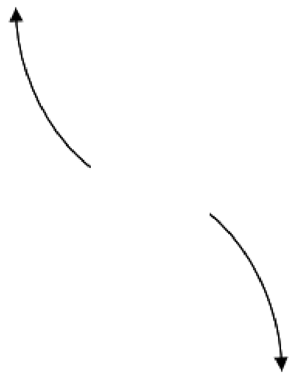
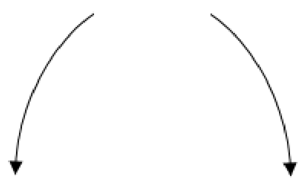
5. Describe the end behavior of the polynomial below.

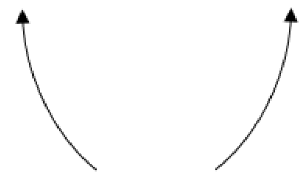
$$f(x) = 4(x+6)^2(x-6)^7(x-8)^5(x+8)^7$$

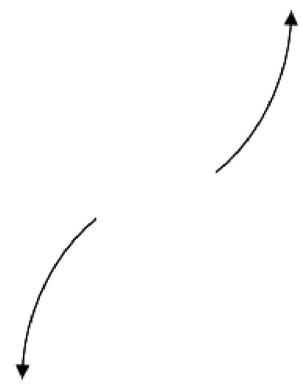
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 + 2i \text{ and } 4$$

The solution is $x^3 + 2x^2 - 11x - 52$, which is option A.

A. $b \in [1.3, 4.5], c \in [-12.6, -8.5], \text{ and } d \in [-54, -46]$

   * $x^3 + 2x^2 - 11x - 52$, which is the correct option.

B. $b \in [-0.4, 1.2], c \in [-6.2, -5.7], \text{ and } d \in [-1, 10]$

   $x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 2)(x - 4)$.

C. $b \in [-0.4, 1.2], c \in [-2.3, -0.7], \text{ and } d \in [-14, -8]$

   $x^3 + x^2 - x - 12$, which corresponds to multiplying out $(x + 3)(x - 4)$.

D. $b \in [-2.6, -1.3], c \in [-12.6, -8.5], \text{ and } d \in [48, 58]$

   $x^3 - 2x^2 - 11x + 52$, which corresponds to multiplying out $(x - (-3 + 2i))(x - (-3 - 2i))(x + 4)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 2i))(x - (-3 - 2i))(x - (4))$.

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 2i \text{ and } 1$$

The solution is $x^3 + 7x^2 + 12x - 20$, which is option D.

A. $b \in [-1, 5], c \in [3, 4], \text{ and } d \in [-6, -3]$

$x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

B. $b \in [-12, -5], c \in [6, 24], \text{ and } d \in [20, 27]$

$x^3 - 7x^2 + 12x + 20$, which corresponds to multiplying out $(x - (-4 + 2i))(x - (-4 - 2i))(x + 1)$.

C. $b \in [-1, 5], c \in [-3, 1], \text{ and } d \in [1, 6]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x - 2)(x - 1)$.

D. $b \in [5, 11], c \in [6, 24], \text{ and } d \in [-23, -18]$

* $x^3 + 7x^2 + 12x - 20$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 2i))(x - (-4 - 2i))(x - (1))$.

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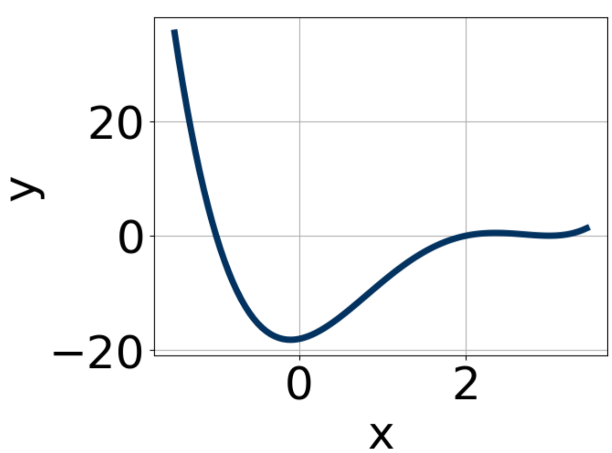
8. Which of the following equations *could* be of the graph presented below?



The solution is $8(x - 3)^4(x - 2)^5(x + 1)^5$, which is option E.

A. $-14(x - 3)^6(x - 2)^{11}(x + 1)^{10}$

The factor $(x + 1)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $7(x-3)^9(x-2)^6(x+1)^{11}$

The factor 3 should have an even power and the factor 2 should have an odd power.

C. $-7(x-3)^8(x-2)^7(x+1)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $9(x-3)^6(x-2)^4(x+1)^{11}$

The factor $(x-2)$ should have an odd power.

E. $8(x-3)^4(x-2)^5(x+1)^5$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-1, \frac{-1}{2}, \text{ and } \frac{4}{3}$$

The solution is $6x^3 + x^2 - 9x - 4$, which is option A.

A. $a \in [4, 11], b \in [-0.8, 1.7], c \in [-20, -8], \text{ and } d \in [-6, -2]$

* $6x^3 + x^2 - 9x - 4$, which is the correct option.

B. $a \in [4, 11], b \in [-0.8, 1.7], c \in [-20, -8], \text{ and } d \in [3, 5]$

$6x^3+x^2-9x+4$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [4, 11], b \in [-3, -0.3], c \in [-20, -8], \text{ and } d \in [3, 5]$

$6x^3 - 1x^2 - 9x + 4$, which corresponds to multiplying out $(x-1)(2x-1)(3x+4)$.

D. $a \in [4, 11], b \in [-18.5, -15.8], c \in [12, 20], \text{ and } d \in [-6, -2]$

$6x^3 - 17x^2 + 15x - 4$, which corresponds to multiplying out $(x-1)(2x-1)(3x-4)$.

E. $a \in [4, 11], b \in [-12.4, -10.2], c \in [-2, 7], \text{ and } d \in [3, 5]$

$6x^3 - 11x^2 + x + 4$, which corresponds to multiplying out $(x-1)(2x+1)(3x-4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+1)(2x+1)(3x-4)$

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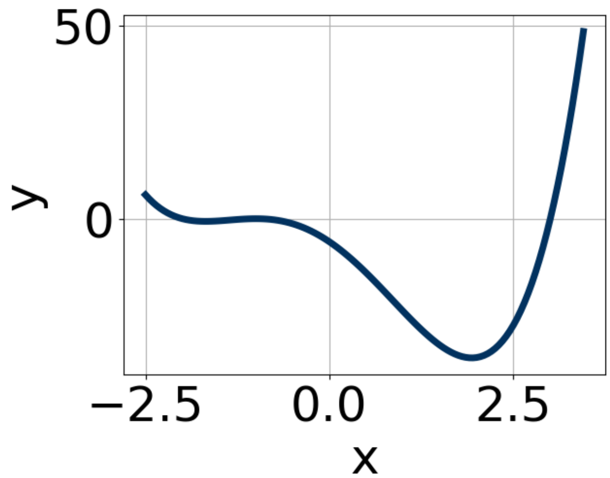
10. Which of the following equations *could* be of the graph presented below?



The solution is $20(x+1)^4(x+2)^9(x-3)^{11}$, which is option B.

A. $-20(x+1)^8(x+2)^9(x-3)^{11}$

   This corresponds to the leading coefficient being the opposite value than it should be.

B. $20(x+1)^4(x+2)^9(x-3)^{11}$

   * This is the correct option.

C. $11(x+1)^9(x+2)^{10}(x-3)^9$

   The factor -1 should have an even power and the factor -2 should have an odd power.

D. $-3(x+1)^{10}(x+2)^5(x-3)^4$

   The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $4(x+1)^4(x+2)^{10}(x-3)^5$

   The factor $(x+2)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).