This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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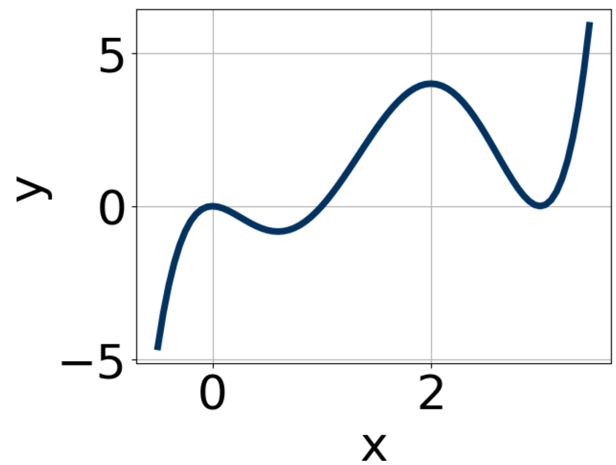
1. Which of the following equations *could* be of the graph presented below?



The solution is $3x^{10}(x-3)^{10}(x-1)^7$, which is option B.

A. $12x^5(x-3)^6(x-1)^8$

The factor x should have an even power and the factor $(x-1)$ should have an odd power.

B. $3x^{10}(x-3)^{10}(x-1)^7$

* This is the correct option.

C. $-16x^6(x-3)^6(x-1)^6$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $10x^7(x-3)^8(x-1)^9$

The factor x should have an even power.

E. $-4x^4(x-3)^6(x-1)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

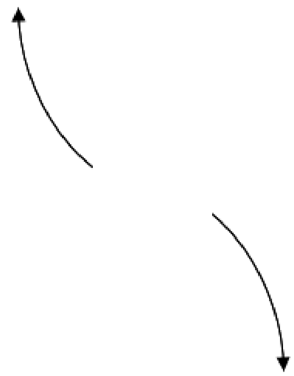
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

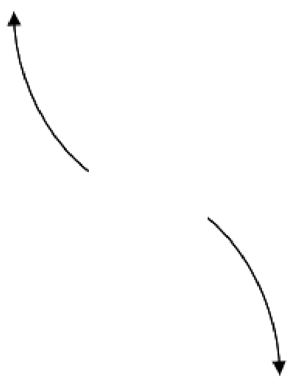
---

2. Describe the end behavior of the polynomial below.

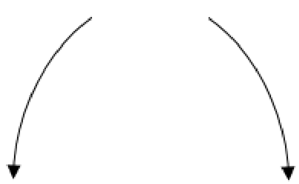
$$f(x) = -8(x-6)^4(x+6)^5(x-3)^3(x+3)^5$$

The solution is the graph below, which is option A.

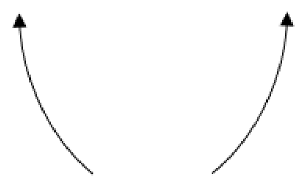




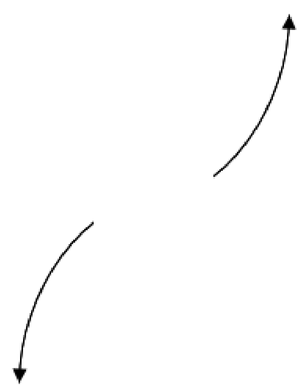
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{2}, 3, \text{ and } \frac{2}{5}$$

The solution is $10x^3 - 19x^2 - 39x + 18$, which is option C.

A. $a \in [9, 13], b \in [-20, -16], c \in [-42, -33], \text{ and } d \in [-19, -12]$

$10x^3 - 19x^2 - 39x - 18$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [9, 13], b \in [9, 17], c \in [-51, -46]$, and $d \in [13, 22]$

$10x^3 + 11x^2 - 51x + 18$, which corresponds to multiplying out $(2x - 3)(x + 3)(5x - 2)$.

C. $a \in [9, 13], b \in [-20, -16], c \in [-42, -33]$, and $d \in [13, 22]$

* $10x^3 - 19x^2 - 39x + 18$, which is the correct option.

D. $a \in [9, 13], b \in [-50, -48], c \in [63, 65]$, and $d \in [-19, -12]$

$10x^3 - 49x^2 + 63x - 18$, which corresponds to multiplying out $(2x - 3)(x - 3)(5x - 2)$.

E. $a \in [9, 13], b \in [19, 25], c \in [-42, -33]$, and $d \in [-19, -12]$

$10x^3 + 19x^2 - 39x - 18$, which corresponds to multiplying out $(2x - 3)(x + 3)(5x + 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 3)(x - 3)(5x - 2)$

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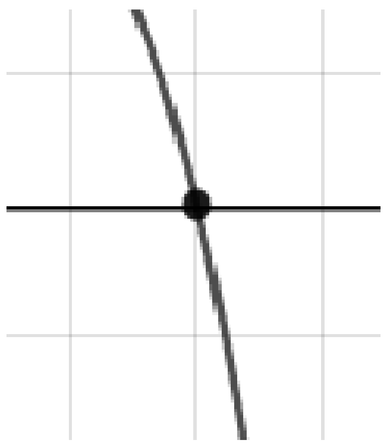
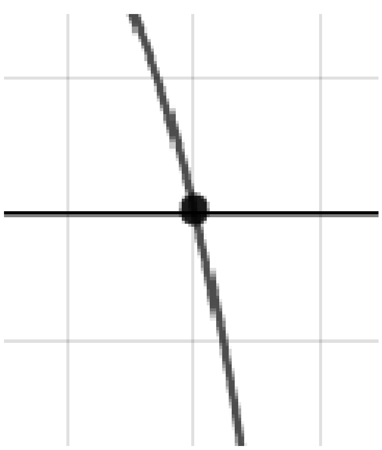
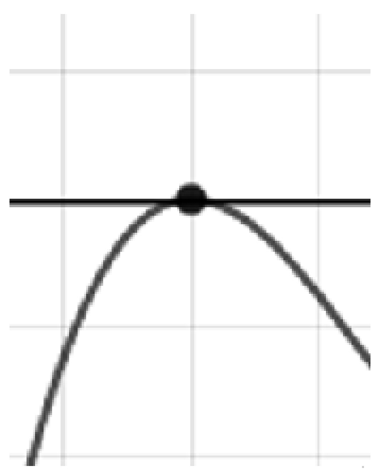
4. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

$$f(x) = -4(x + 8)^5(x - 8)^{10}(x + 2)^8(x - 2)^{12}$$

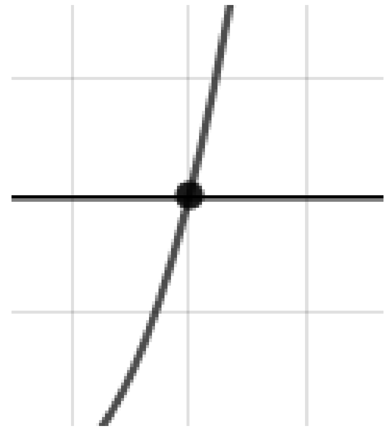
The solution is the graph below, which is option A.



A. 

B. 

C.



D.

E. None of the above.

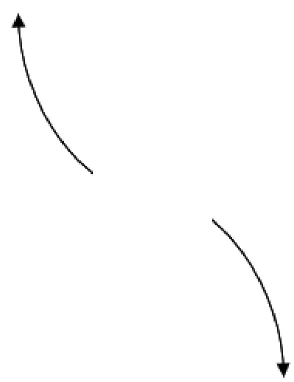
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

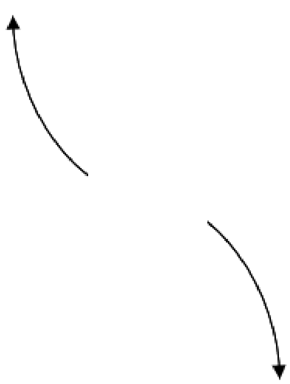
---

5. Describe the end behavior of the polynomial below.

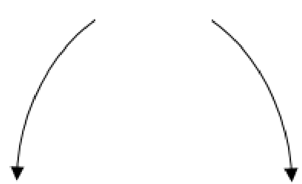
$$f(x) = -2(x-4)^5(x+4)^8(x+6)^5(x-6)^5$$

The solution is the graph below, which is option A.

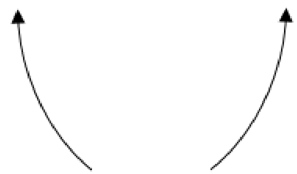




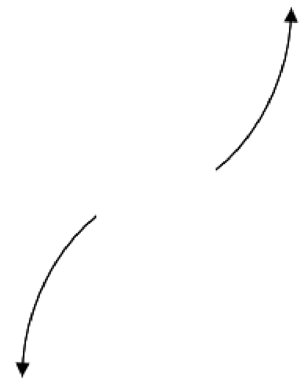
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 5i \text{ and } 2$$

The solution is $x^3 - 10x^2 + 57x - 82$, which is option B.

A. $b \in [-5, 5], c \in [-5, 4], \text{ and } d \in [-10, -7]$

$x^3 + x^2 + 3x - 10$, which corresponds to multiplying out $(x + 5)(x - 2)$.

B. $b \in [-11, -4], c \in [54, 62], \text{ and } d \in [-87, -81]$

* $x^3 - 10x^2 + 57x - 82$, which is the correct option.

C. $b \in [-5, 5], c \in [-10, 1], \text{ and } d \in [7, 9]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 4)(x - 2)$.

D. $b \in [9, 17], c \in [54, 62], \text{ and } d \in [79, 83]$

$x^3 + 10x^2 + 57x + 82$, which corresponds to multiplying out $(x - (4 - 5i))(x - (4 + 5i))(x + 2)$.

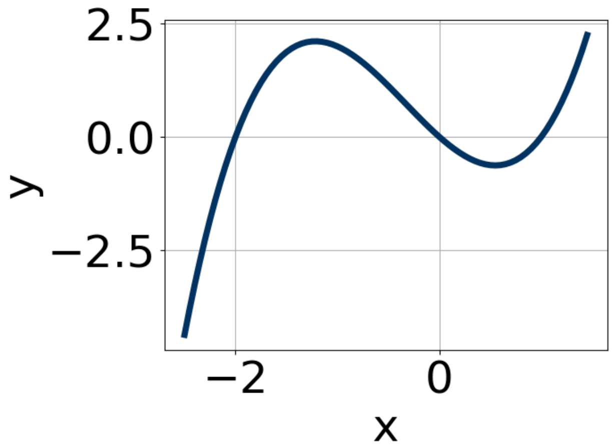
E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 5i))(x - (4 + 5i))(x - (2))$.

---

7. Which of the following equations *could* be of the graph presented below?



The solution is $13x^{11}(x-1)^{11}(x+2)^9$, which is option C.

A. $-13x^{11}(x-1)^7(x+2)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $13x^9(x-1)^8(x+2)^9$

The factor 1 should have been an odd power.

C. $13x^{11}(x-1)^{11}(x+2)^9$

* This is the correct option.

D. $-18x^7(x-1)^8(x+2)^7$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $18x^4(x-1)^{10}(x+2)^{11}$

The factors 1 and 0 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form x^3+bx^2+cx+d.

$$-3+5i \text{ and } 4$$

The solution is $x^3+2x^2+10x-136$, which is option C.

A. $b \in [-4.9, -0.1], c \in [9, 18],$ and $d \in [134, 142]$

$x^3-2x^2+10x+136$, which corresponds to multiplying out $(x-(-3+5i))(x-(-3-5i))(x+4)$.

B. $b \in [-0.2, 1.3], c \in [-1, 9],$ and $d \in [-14, -5]$

x^3+x^2-x-12, which corresponds to multiplying out $(x+3)(x-4)$.

C. $b \in [1.2, 8], c \in [9, 18],$ and $d \in [-137, -135]$

* $x^3+2x^2+10x-136$, which is the correct option.

D. $b \in [-0.2, 1.3], c \in [-14, -8],$ and $d \in [20, 25]$

$x^3+x^2-9x+20$, which corresponds to multiplying out $(x-5)(x-4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

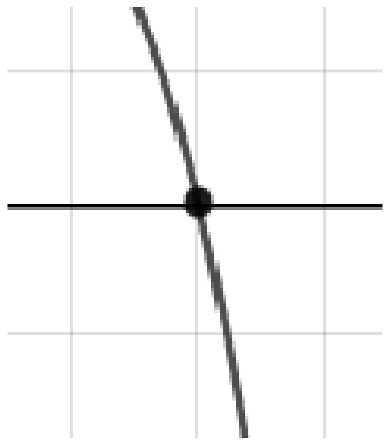
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 5i))(x - (-3 - 5i))(x - (4))$.

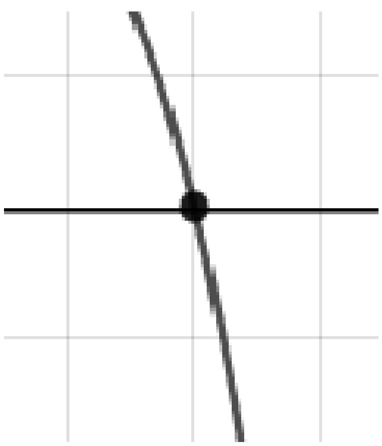
---

9. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

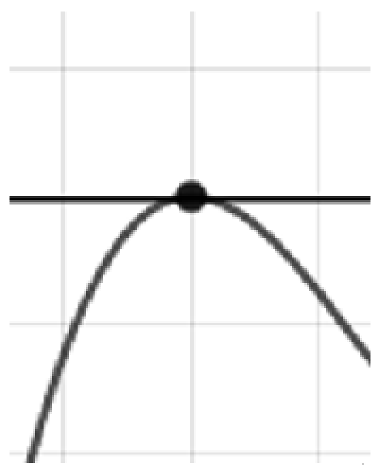
$$f(x) = -9(x - 3)^4(x + 3)^5(x - 8)^4(x + 8)^8$$

The solution is the graph below, which is option A.

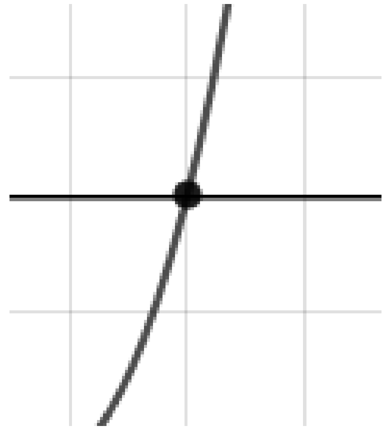




A.



B.

C.  D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{4}, \frac{-1}{3}, \text{ and } \frac{-7}{3}$$

The solution is $36x^3 + 51x^2 - 92x - 35$, which is option C.

A. $a \in [31, 37], b \in [51, 53], c \in [-96, -85], \text{ and } d \in [27, 44]$

$36x^3 + 51x^2 - 92x + 35$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [31, 37], b \in [-55, -44], c \in [-96, -85], \text{ and } d \in [27, 44]$

$36x^3 - 51x^2 - 92x + 35$, which corresponds to multiplying out $(4x + 5)(3x - 1)(3x - 7)$.

C. $a \in [31, 37], b \in [51, 53], c \in [-96, -85], \text{ and } d \in [-41, -27]$

* $36x^3 + 51x^2 - 92x - 35$, which is the correct option.

D. $a \in [31, 37], b \in [117, 121], c \in [62, 71], \text{ and } d \in [-41, -27]$

$36x^3 + 117x^2 + 62x - 35$, which corresponds to multiplying out $(4x + 5)(3x - 1)(3x + 7)$.

E. $a \in [31, 37], b \in [140, 143], c \in [142, 157], \text{ and } d \in [27, 44]$

$36x^3 + 141x^2 + 148x + 35$, which corresponds to multiplying out $(4x + 5)(3x + 1)(3x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 5)(3x + 1)(3x + 7)$
