This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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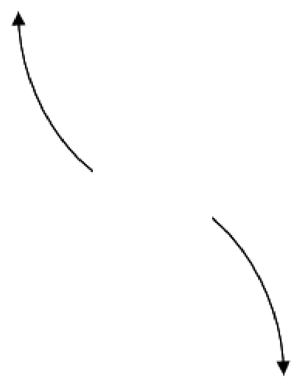
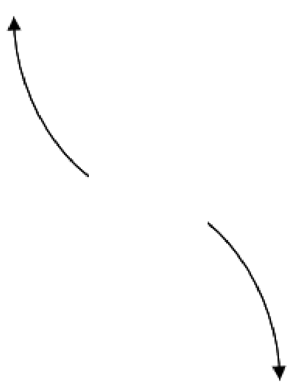
1. Describe the end behavior of the polynomial below.

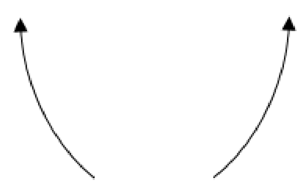
$$f(x) = -7(x+8)^3(x-8)^6(x+2)^3(x-2)^3$$

The solution is the graph below, which is option A.

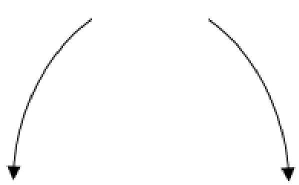




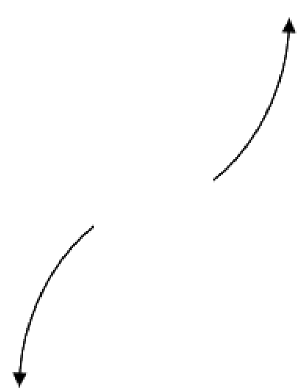
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-4}{5}, \frac{-1}{5}, \text{ and } \frac{-4}{3}$$

The solution is $75x^3 + 175x^2 + 112x + 16$, which is option B.

A. $a \in [74, 76], b \in [53, 57], c \in [-76, -63],$ and $d \in [-16, -13]$

$75x^3 + 55x^2 - 72x - 16$, which corresponds to multiplying out $(5x + 5)(5x - 5)(3x - 3)$.

B. $a \in [74, 76], b \in [175, 179], c \in [112, 115],$ and $d \in [16, 17]$

* $75x^3 + 175x^2 + 112x + 16$, which is the correct option.

C. $a \in [74, 76], b \in [175, 179], c \in [112, 115],$ and $d \in [-16, -13]$

$75x^3 + 175x^2 + 112x - 16$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [74, 76], b \in [-175, -173], c \in [112, 115],$ and $d \in [-16, -13]$

$75x^3 - 175x^2 + 112x - 16$, which corresponds to multiplying out $(5x - 4)(5x - 1)(3x - 4)$.

E. $a \in [74, 76], b \in [22, 27], c \in [-91, -82],$ and $d \in [16, 17]$

$75x^3 + 25x^2 - 88x + 16$, which corresponds to multiplying out $(5x + 5)(5x + 5)(3x - 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 4)(5x + 1)(3x + 4)$

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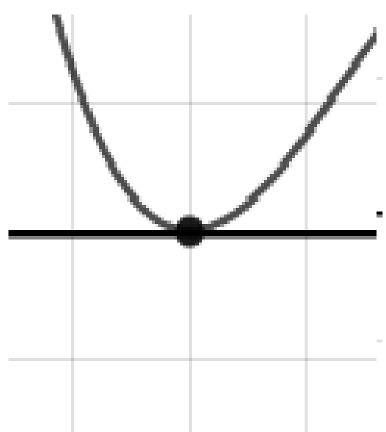
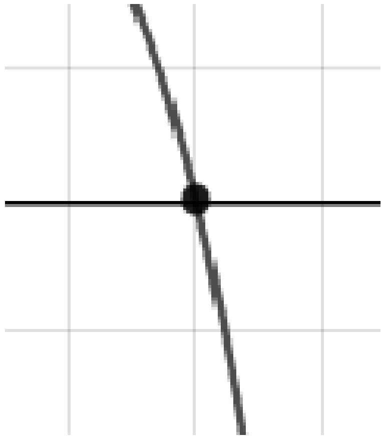
3. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

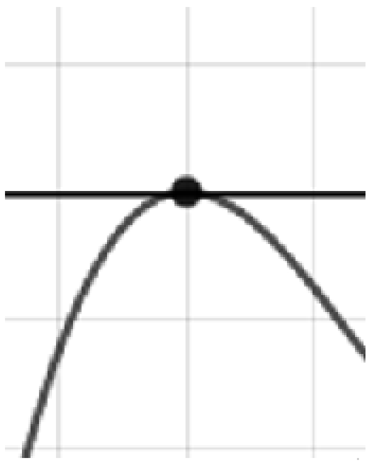
$$f(x) = 2(x + 5)^9(x - 5)^{14}(x + 4)^3(x - 4)^4$$

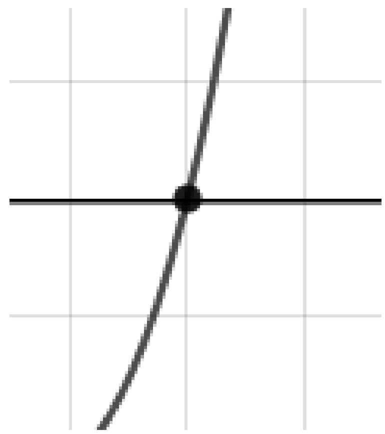
The solution is the graph below, which is option C.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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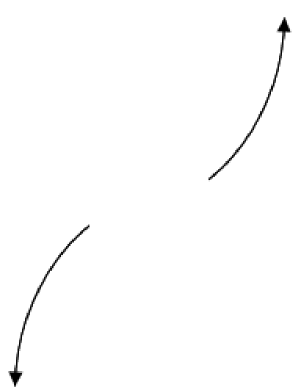
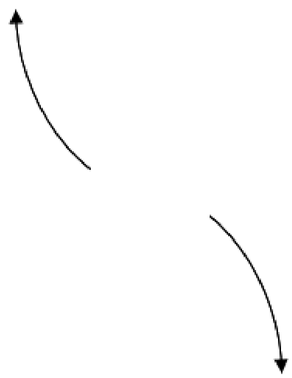
4. Describe the end behavior of the polynomial below.

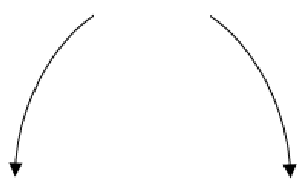
$$f(x) = 7(x+8)^5(x-8)^8(x-5)^4(x+5)^4$$

The solution is the graph below, which is option D.

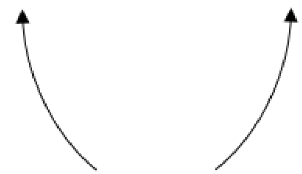




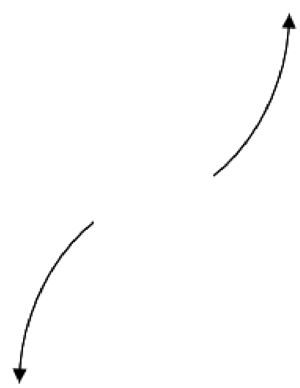
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 + 4i \text{ and } -3$$

The solution is $x^3 - 3x^2 + 7x + 75$, which is option A.

A. $b \in [-6.9, -0.4], c \in [5.58, 7.49], \text{ and } d \in [71.2, 77.6]$

   * $x^3 - 3x^2 + 7x + 75$, which is the correct option.

B. $b \in [-0.2, 1.3], c \in [-0.66, 2.32], \text{ and } d \in [-10.7, -7.6]$

   $x^3 + x^2 - 9$, which corresponds to multiplying out $(x - 3)(x + 3)$.

C. $b \in [1.9, 4.8], c \in [5.58, 7.49], \text{ and } d \in [-77.3, -73.5]$

   $x^3 + 3x^2 + 7x - 75$, which corresponds to multiplying out $(x - (3 + 4i))(x - (3 - 4i))(x - 3)$.

D. $b \in [-0.2, 1.3], c \in [-2.47, -0.78], \text{ and } d \in [-13.7, -9.8]$

   $x^3 + x^2 - x - 12$, which corresponds to multiplying out $(x - 4)(x + 3)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 + 4i))(x - (3 - 4i))(x - (-3))$.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-7, \frac{1}{3}, \text{ and } \frac{5}{2}$$

The solution is $6x^3 + 25x^2 - 114x + 35$, which is option D.

A. $a \in [4, 11], b \in [-55, -52], c \in [80, 87], \text{ and } d \in [29, 36]$

$6x^3 - 55x^2 + 86x + 35$, which corresponds to multiplying out $(x + 1)(3x + 3)(2x - 2)$.

B. $a \in [4, 11], b \in [-66, -57], c \in [118, 125], \text{ and } d \in [-41, -31]$

$6x^3 - 59x^2 + 124x - 35$, which corresponds to multiplying out $(x + 1)(3x - 3)(2x - 2)$.

C. $a \in [4, 11], b \in [-25, -20], c \in [-122, -107], \text{ and } d \in [-41, -31]$

$6x^3 - 25x^2 - 114x - 35$, which corresponds to multiplying out $(x - 7)(3x + 1)(2x + 5)$.

D. $a \in [4, 11], b \in [19, 27], c \in [-122, -107], \text{ and } d \in [29, 36]$

* $6x^3 + 25x^2 - 114x + 35$, which is the correct option.

E. $a \in [4, 11], b \in [19, 27], c \in [-122, -107], \text{ and } d \in [-41, -31]$

$6x^3 + 25x^2 - 114x - 35$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+7)(3x-1)(2x-5)$

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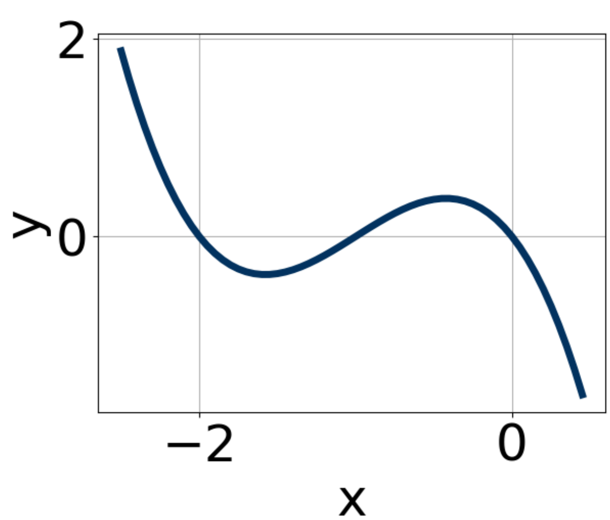
7. Which of the following equations *could* be of the graph presented below?



The solution is $-20x^7(x + 1)^5(x + 2)^9$, which is option D.

A. $7x^9(x + 1)^7(x + 2)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-7x^9(x+1)^{10}(x+2)^{11}$

The factor -1 should have been an odd power.

C. $3x^{11}(x+1)^8(x+2)^7$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-20x^7(x+1)^5(x+2)^9$

* This is the correct option.

E. $-3x^5(x+1)^{10}(x+2)^8$

The factors -1 and -2 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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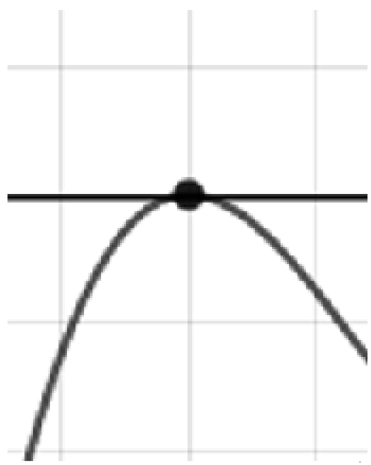
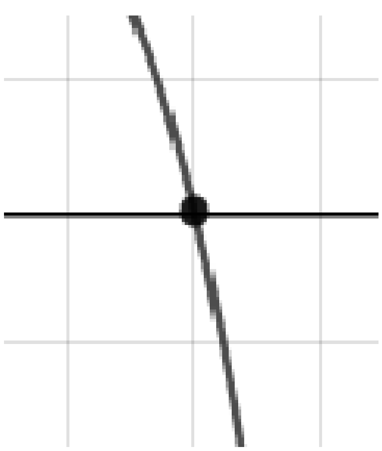
8. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = 8(x-6)^{11}(x+6)^8(x+3)^7(x-3)^2$$

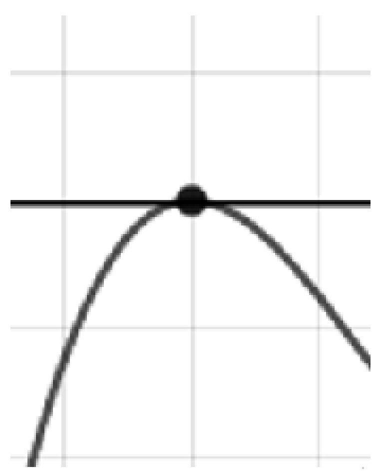
The solution is the graph below, which is option B.



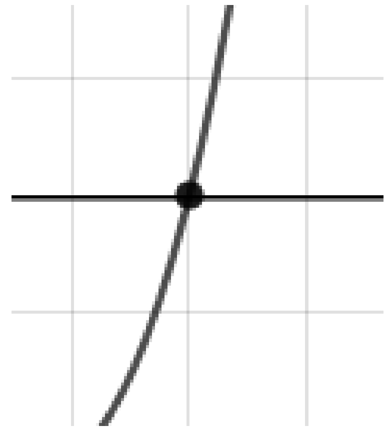
A.



B.



C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 4i \text{ and } 4$$

The solution is $x^3 + 4x^2 - 128$, which is option D.

A. $b \in [-2.3, 2.2], c \in [-3, 10],$ and $d \in [-22, -15]$

$x^3 + x^2 - 16$, which corresponds to multiplying out $(x + 4)(x - 4)$.

B. $b \in [-2.3, 2.2], c \in [-14, -6],$ and $d \in [11, 19]$

$x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

C. $b \in [-5.4, -1], c \in [-3, 10],$ and $d \in [123, 134]$

$x^3 - 4x^2 + 128$, which corresponds to multiplying out $(x - (-4 + 4i))(x - (-4 - 4i))(x + 4)$.

D. $b \in [2.1, 5.3], c \in [-3, 10],$ and $d \in [-138, -126]$

* $x^3 + 4x^2 - 128$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 4i))(x - (-4 - 4i))(x - (4))$.

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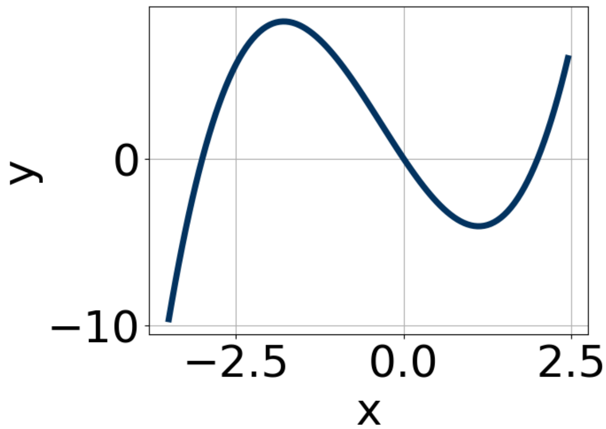
10. Which of the following equations *could* be of the graph presented below?



The solution is $3x^5(x+3)^9(x-2)^5$, which is option E.

A. $9x^8(x+3)^6(x-2)^{11}$

   The factors -3 and 0 have have been odd power.

B. $10x^9(x+3)^6(x-2)^{11}$

   The factor -3 should have been an odd power.

C. $-14x^{11}(x+3)^{10}(x-2)^7$

   The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-10x^9(x+3)^5(x-2)^7$

   This corresponds to the leading coefficient being the opposite value than it should be.

E. $3x^5(x+3)^9(x-2)^5$

   * This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).