This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval that $f^{-1}(11)$ belongs to.

$$f(x) = 2x^2 + 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(11) \in [1.65, 1.89]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(11) \in [4.83, 5.13]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(11) \in [2.82, 3.03]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(11) \in [2.59, 2.81]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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2. Determine whether the function below is 1-1.

$$f(x) = 36x^2 + 420x + 1225$$

The solution is no, which is option D.

A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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3. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 3x^3 - 3x^2 - 2x + 3 \text{ and } g(x) = -x^3 - 1x^2 - 3x$$

The solution is 51.0, which is option D.

A. $(f \circ g)(-1) \in [-11, -1]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [2, 4]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [42, 45]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [48, 58]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{1}{6x + 29} \text{ and } g(x) = \frac{4}{5x - 27}$$

The solution is The domain is all Real numbers except $x = -4.833333333333333$ and $x = 5.4$, which is option D.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-5.6, 3.4]$

B. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-8.67, -1.67]$

C. The domain is all Real numbers except $x = a$, where $a \in [-8.25, 0.75]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-6.83, -3.83]$ and $b \in [3.4, 8.4]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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5. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = e^{x+3} + 2$$

The solution is $f^{-1}(7) = -1.391$, which is option B.

A. $f^{-1}(7) \in [4.38, 4.67]$

This solution corresponds to distractor 1.

B. $f^{-1}(7) \in [-1.58, -1.34]$

This is the solution.

C. $f^{-1}(7) \in [4, 4.2]$

This solution corresponds to distractor 2.

D. $f^{-1}(7) \in [4.21, 4.54]$

This solution corresponds to distractor 4.

E. $f^{-1}(7) \in [3.37, 3.41]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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6. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{1}{4x + 25} \text{ and } g(x) = \frac{3}{3x - 20}$$

The solution is The domain is all Real numbers except $x = -6.25$ and $x = 6.666666666666667$, which is option D.

A. The domain is all Real numbers except $x = a$, where $a \in [-5.17, -2.17]$

B. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-15.2, -4.2]$

C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [0.33, 3.33]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-9.25, -2.25]$ and $b \in [3.67, 16.67]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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7. Determine whether the function below is 1-1.

$$f(x) = (5x - 31)^3$$

The solution is yes, which is option C.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. Yes, the function is 1-1.

* This is the solution.

D. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval the $f^{-1}(11)$ belongs to.

$$f(x) = \sqrt[3]{4x+2}$$

The solution is 332.25, which is option A.

A. $f^{-1}(11) \in [332.1, 332.6]$

* This is the correct solution.

B. $f^{-1}(11) \in [332.7, 334.5]$

Distractor 1: This corresponds to

C. $f^{-1}(11) \in [-333, -332]$

This solution corresponds to distractor 2.

D. $f^{-1}(11) \in [-335.1, -332.4]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = \ln(x+4) - 2$$

The solution is $f^{-1}(9) = 59870.142$, which is option D.

A. $f^{-1}(9) \in [1089.63, 1096.63]$

This solution corresponds to distractor 1.

B. $f^{-1}(9) \in [59877.14, 59887.14]$

This solution corresponds to distractor 3.

C. $f^{-1}(9) \in [442404.39, 442424.39]$

This solution corresponds to distractor 4.

D. $f^{-1}(9) \in [59870.14, 59872.14]$

This is the solution.

E. $f^{-1}(9) \in [144.41, 150.41]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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10. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 2x^3 + 3x^2 - x \text{ and } g(x) = 4x^3 + 3x^2 + x + 4$$

The solution is 26.0, which is option A.

A. $(f \circ g)(-1) \in [25, 29]$

* This is the correct solution

B. $(f \circ g)(-1) \in [42, 43]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [46, 51]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [32, 38]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!