This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{2}, 4, \text{ and } \frac{7}{2}$$

The solution is $4x^3 - 40x^2 + 131x - 140$, which is option E.

A. $a \in [-1, 6], b \in [36, 45], c \in [129, 132], \text{ and } d \in [133, 144]$

$4x^3 + 40x^2 + 131x + 140$, which corresponds to multiplying out $(2x + 5)(x + 4)(2x + 7)$.

B. $a \in [-1, 6], b \in [7, 14], c \in [-52, -45], \text{ and } d \in [-143, -137]$

$4x^3 + 12x^2 - 51x - 140$, which corresponds to multiplying out $(2x + 5)(x + 4)(2x - 7)$.

C. $a \in [-1, 6], b \in [-48, -32], c \in [129, 132], \text{ and } d \in [133, 144]$

$4x^3 - 40x^2 + 131x + 140$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [-1, 6], b \in [-26, -19], c \in [-27, -17], \text{ and } d \in [133, 144]$

$4x^3 - 20x^2 - 19x + 140$, which corresponds to multiplying out $(2x + 5)(x - 4)(2x - 7)$.

E. $a \in [-1, 6], b \in [-48, -32], c \in [129, 132], \text{ and } d \in [-143, -137]$

* $4x^3 - 40x^2 + 131x - 140$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 5)(x - 4)(2x - 7)$

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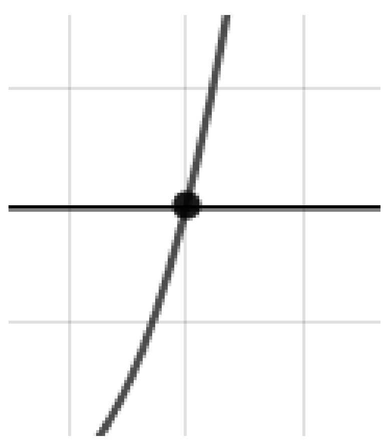
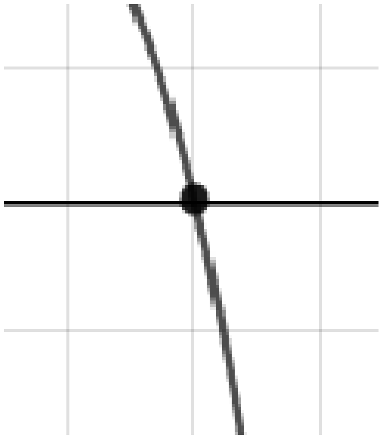
2. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

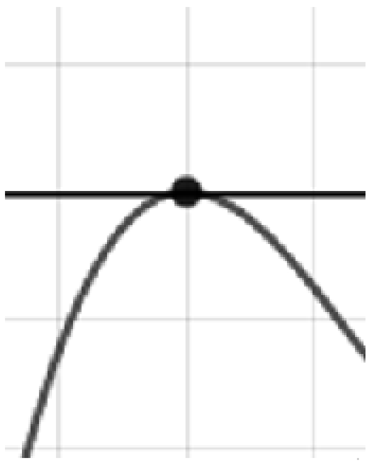
$$f(x) = 7(x + 8)^8(x - 8)^7(x + 9)^{12}(x - 9)^7$$

The solution is the graph below, which is option D.




A.


C.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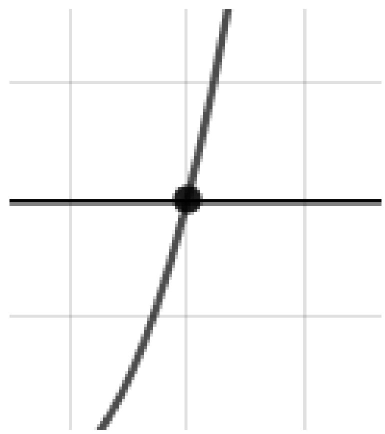
D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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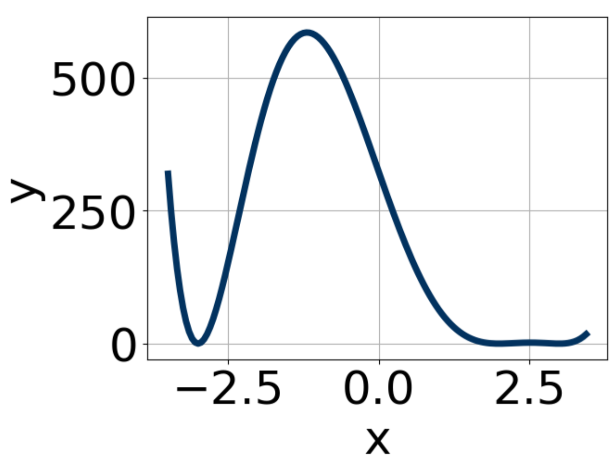
3. Which of the following equations *could* be of the graph presented below?



The solution is $6(x-3)^6(x+3)^8(x-2)^8$, which is option D.

A. $-4(x-3)^6(x+3)^8(x-2)^7$

The factor $(x-2)$ should have an even power and the leading coefficient should be the opposite sign.

B. $-11(x-3)^4(x+3)^{10}(x-2)^8$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $6(x-3)^4(x+3)^9(x-2)^9$

The factors $(x+3)$ and $(x-2)$ should both have even powers.

D. $6(x-3)^6(x+3)^8(x-2)^8$

* This is the correct option.

E. $19(x-3)^8(x+3)^{10}(x-2)^7$

The factor $(x-2)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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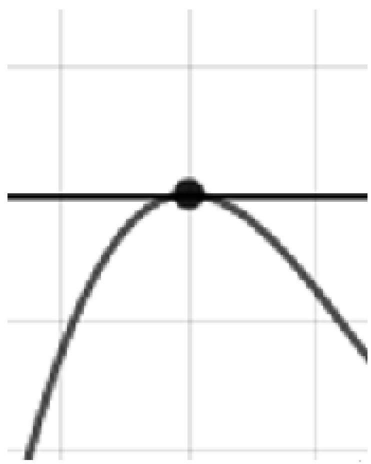
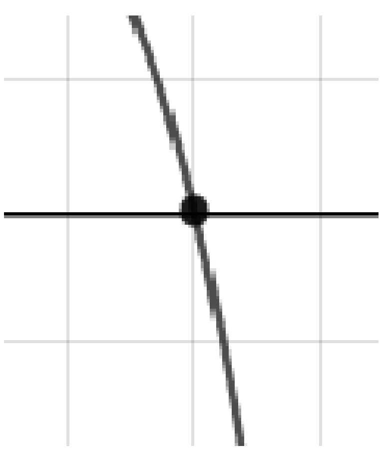
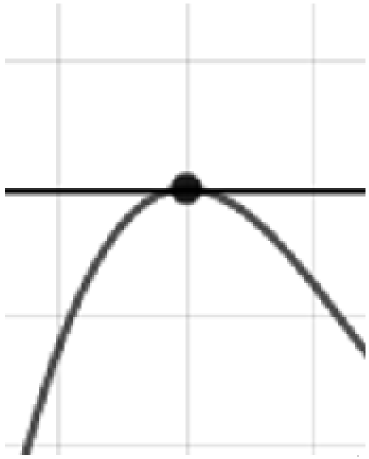
4. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

$$f(x) = -4(x+9)^9(x-9)^{10}(x-2)^7(x+2)^{10}$$

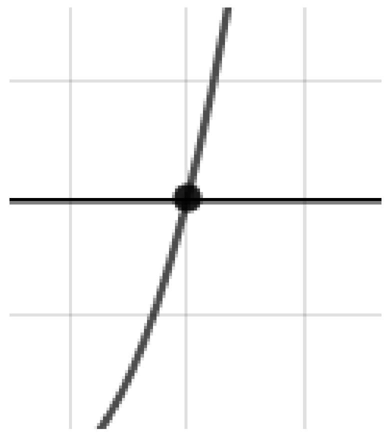
The solution is the graph below, which is option B.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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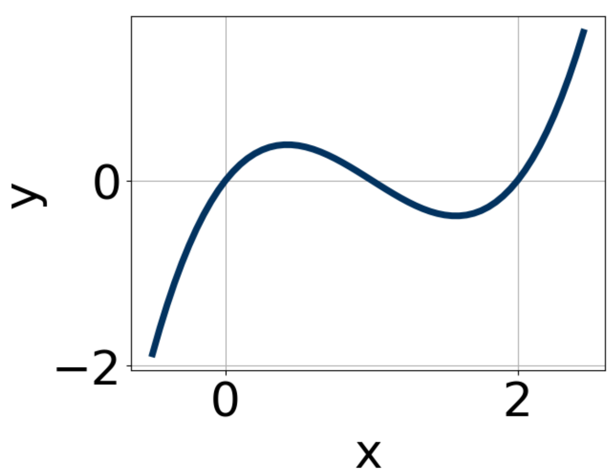
5. Which of the following equations *could* be of the graph presented below?



The solution is $12x^{11}(x-2)^{11}(x-1)^{11}$, which is option E.

A. $-12x^9(x-2)^5(x-1)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $18x^4(x-2)^6(x-1)^{11}$

The factors 0 and 2 have have been odd power.

C. $10x^{10}(x-2)^5(x-1)^{11}$

The factor 0 should have been an odd power.

D. $-10x^6(x-2)^{11}(x-1)^9$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

E. $12x^{11}(x-2)^{11}(x-1)^{11}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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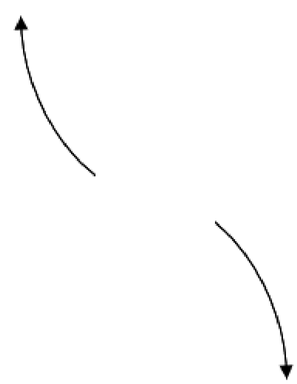
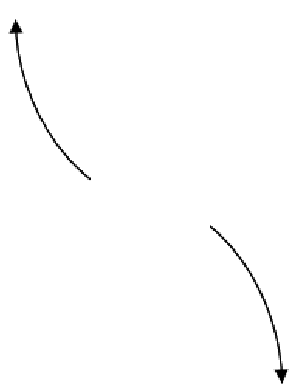
6. Describe the end behavior of the polynomial below.

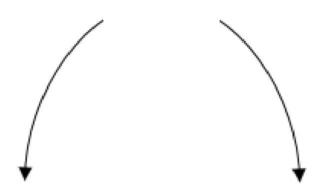
$$f(x) = -9(x+9)^2(x-9)^3(x+7)^2(x-7)^2$$

The solution is the graph below, which is option A.

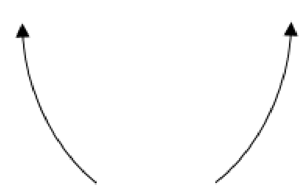




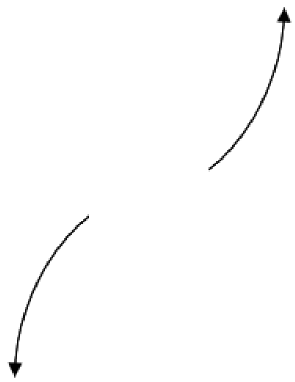
A.



B.



C.

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 5i \text{ and } 1$$

The solution is $x^3 + 5x^2 + 28x - 34$, which is option C.

A. $b \in [-6.1, -3.5], c \in [26.18, 29.01], \text{ and } d \in [32.07, 34.51]$

$x^3 - 5x^2 + 28x + 34$, which corresponds to multiplying out $(x - (-3 - 5i))(x - (-3 + 5i))(x + 1)$.

B. $b \in [0.1, 4.8], c \in [0.84, 2.43], \text{ and } d \in [-3.52, -2.55]$

$x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

C. $b \in [3.2, 10.8], c \in [26.18, 29.01], \text{ and } d \in [-35.08, -33.01]$

* $x^3 + 5x^2 + 28x - 34$, which is the correct option.

D. $b \in [0.1, 4.8], c \in [3.67, 5.55], \text{ and } d \in [-5.68, -3.99]$

$x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 5i))(x - (-3 + 5i))(x - (1))$.

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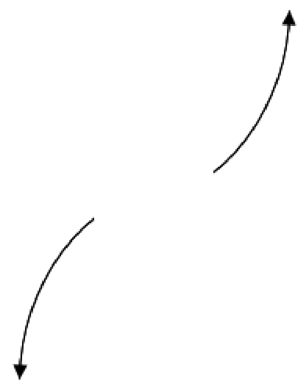
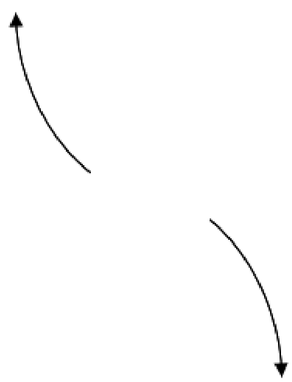
8. Describe the end behavior of the polynomial below.

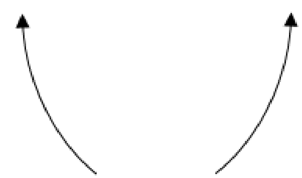
$$f(x) = 3(x - 6)^4(x + 6)^7(x - 5)^2(x + 5)^2$$

The solution is the graph below, which is option D.

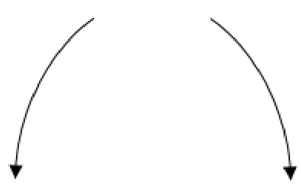




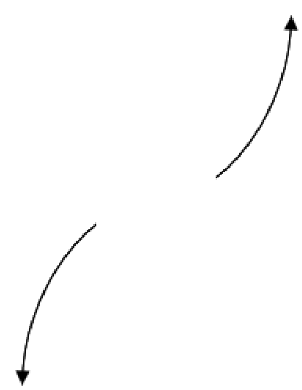
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 4i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 40x - 32$, which is option C.

A. $b \in [5, 10], c \in [31, 46],$ and $d \in [24, 34]$

$x^3 + 9x^2 + 40x + 32$, which corresponds to multiplying out $(x - (4 - 4i))(x - (4 + 4i))(x + 1)$.

B. $b \in [1, 2], c \in [-10, -1]$, and $d \in [3, 12]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

C. $b \in [-14, -8], c \in [31, 46]$, and $d \in [-34, -30]$

* $x^3 - 9x^2 + 40x - 32$, which is the correct option.

D. $b \in [1, 2], c \in [3, 4]$, and $d \in [-5, 0]$

$x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 4i))(x - (4 + 4i))(x - (1))$.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, -5, \text{ and } \frac{-7}{5}$$

The solution is $20x^3 + 123x^2 + 108x - 35$, which is option D.

A. $a \in [17, 24], b \in [-128, -121], c \in [108, 114]$, and $d \in [32, 38]$

$20x^3 - 123x^2 + 108x + 35$, which corresponds to multiplying out $(4x + 1)(x - 5)(5x - 7)$.

B. $a \in [17, 24], b \in [132, 140], c \in [166, 173]$, and $d \in [32, 38]$

$20x^3 + 133x^2 + 172x + 35$, which corresponds to multiplying out $(4x + 1)(x + 5)(5x + 7)$.

C. $a \in [17, 24], b \in [118, 125], c \in [108, 114]$, and $d \in [32, 38]$

$20x^3 + 123x^2 + 108x + 35$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [17, 24], b \in [118, 125], c \in [108, 114]$, and $d \in [-42, -31]$

* $20x^3 + 123x^2 + 108x - 35$, which is the correct option.

E. $a \in [17, 24], b \in [-70, -65], c \in [-163, -155]$, and $d \in [-42, -31]$

$20x^3 - 67x^2 - 158x - 35$, which corresponds to multiplying out $(4x + 1)(x - 5)(5x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(x + 5)(5x + 7)$
