This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{3}, \frac{-5}{2}, \text{ and } \frac{-2}{3}$$

The solution is $18x^3 + 15x^2 - 103x - 70$, which is option B.

A. $a \in [16, 21], b \in [-17, -12], c \in [-104, -99], \text{ and } d \in [68, 73]$

$18x^3 - 15x^2 - 103x + 70$, which corresponds to multiplying out $(3x + 7)(2x - 5)(3x - 2)$.

B. $a \in [16, 21], b \in [11, 20], c \in [-104, -99], \text{ and } d \in [-70, -68]$

* $18x^3 + 15x^2 - 103x - 70$, which is the correct option.

C. $a \in [16, 21], b \in [8, 14], c \in [-111, -104], \text{ and } d \in [-70, -68]$

$18x^3 + 9x^2 - 107x - 70$, which corresponds to multiplying out $(3x + 7)(2x - 5)(3x + 2)$.

D. $a \in [16, 21], b \in [11, 20], c \in [-104, -99], \text{ and } d \in [68, 73]$

$18x^3 + 15x^2 - 103x + 70$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [16, 21], b \in [96, 104], c \in [162, 167], \text{ and } d \in [68, 73]$

$18x^3 + 99x^2 + 163x + 70$, which corresponds to multiplying out $(3x + 7)(2x + 5)(3x + 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 7)(2x + 5)(3x + 2)$

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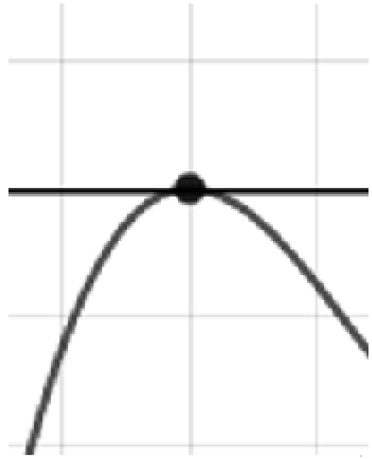
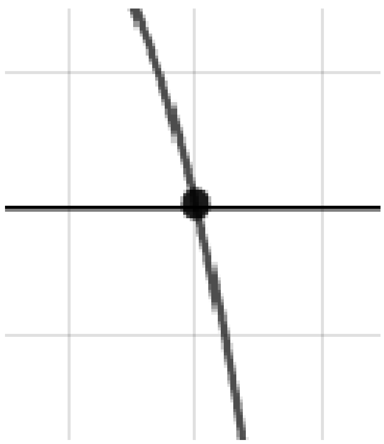
2. Describe the zero behavior of the zero $x = -2$ of the polynomial below.

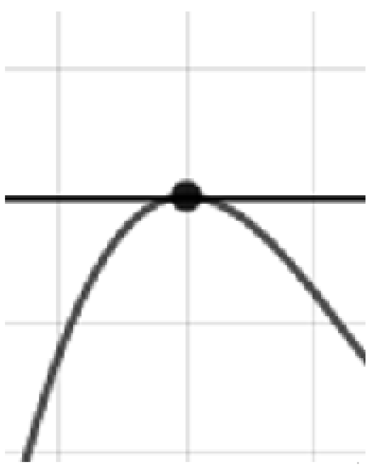
$$f(x) = 2(x + 2)^8(x - 2)^{11}(x + 9)^3(x - 9)^6$$

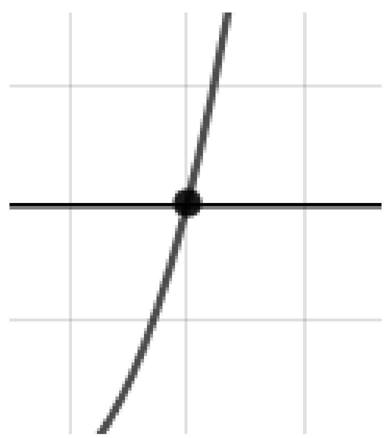
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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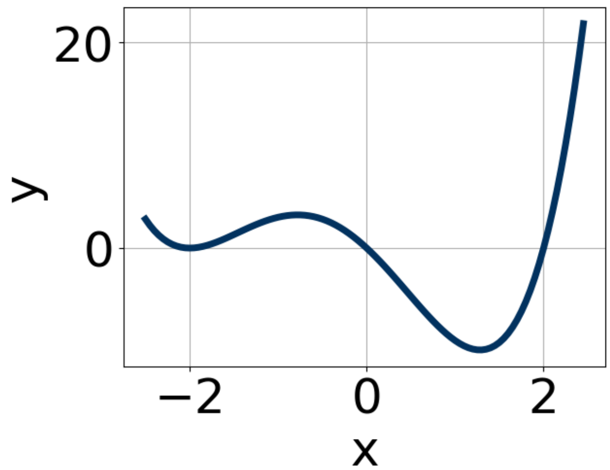
3. Which of the following equations *could* be of the graph presented below?



The solution is $8x^5(x+2)^{10}(x-2)^7$, which is option D.

A. $19x^{10}(x+2)^{10}(x-2)^7$

The factor x should have an odd power.

B. $-4x^7(x+2)^8(x-2)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-6x^9(x+2)^6(x-2)^4$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $8x^5(x+2)^{10}(x-2)^7$

* This is the correct option.

E. $5x^4(x+2)^9(x-2)^7$

The factor -2 should have an even power and the factor 0 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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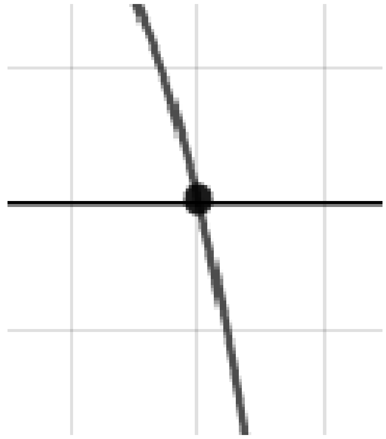
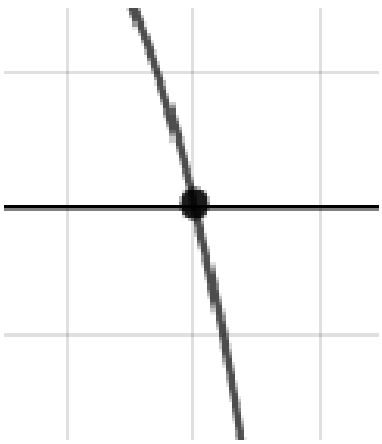
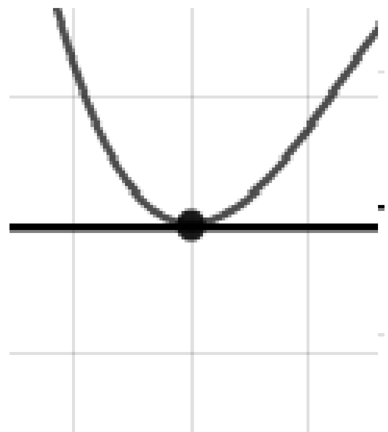
4. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

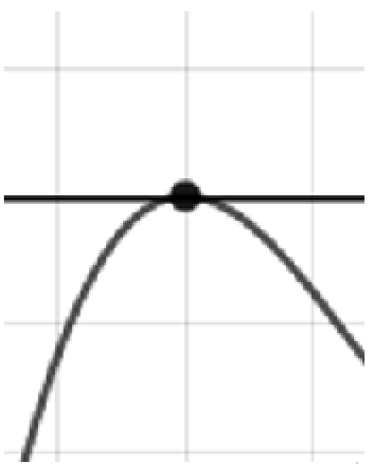
$$f(x) = -2(x+9)^{10}(x-9)^8(x+7)^8(x-7)^5$$

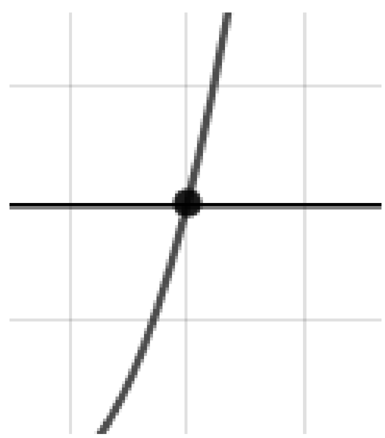
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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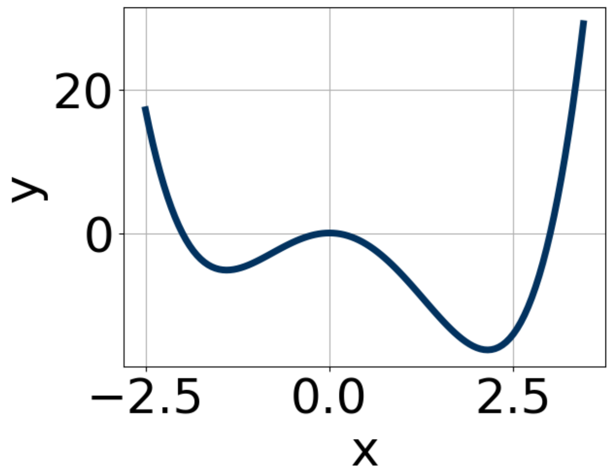
5. Which of the following equations *could* be of the graph presented below?



The solution is $12x^4(x-3)^{11}(x+2)^{11}$, which is option C.

A. $-10x^8(x-3)^5(x+2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-5x^8(x-3)^9(x+2)^4$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $12x^4(x-3)^{11}(x+2)^{11}$

* This is the correct option.

D. $11x^4(x-3)^{10}(x+2)^7$

The factor $(x-3)$ should have an odd power.

E. $19x^7(x-3)^8(x+2)^9$

The factor 0 should have an even power and the factor 3 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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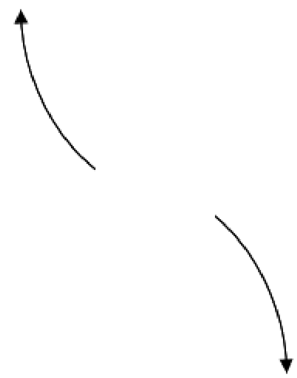
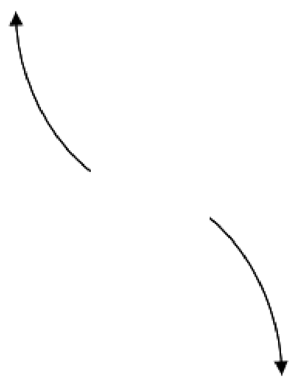
6. Describe the end behavior of the polynomial below.

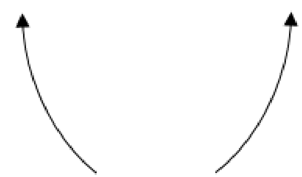
$$f(x) = -5(x-9)^3(x+9)^6(x-7)^3(x+7)^5$$

The solution is the graph below, which is option A.

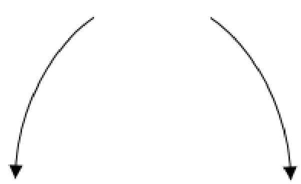




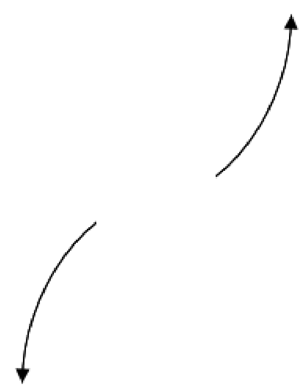
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 2i \text{ and } -3$$

The solution is $x^3 + 11x^2 + 44x + 60$, which is option C.

A. $b \in [-17, -4], c \in [38, 48]$, and $d \in [-61, -58]$

$x^3 - 11x^2 + 44x - 60$, which corresponds to multiplying out $(x - (-4 + 2i))(x - (-4 - 2i))(x - 3)$.

B. $b \in [-4, 7], c \in [6, 11]$, and $d \in [7, 18]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 4)(x + 3)$.

C. $b \in [6, 12], c \in [38, 48]$, and $d \in [59, 70]$

* $x^3 + 11x^2 + 44x + 60$, which is the correct option.

D. $b \in [-4, 7], c \in [-3, 2]$, and $d \in [-8, -3]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x - 2)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 2i))(x - (-4 - 2i))(x - (-3))$.

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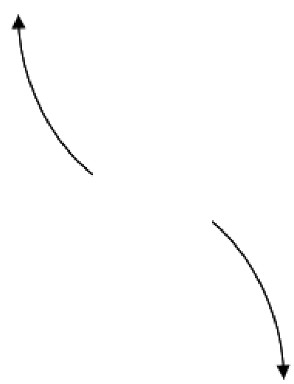
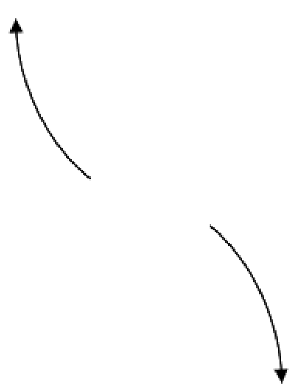
8. Describe the end behavior of the polynomial below.

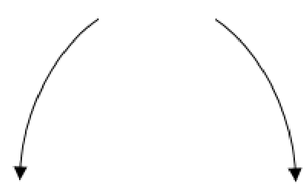
$$f(x) = -2(x - 2)^2(x + 2)^7(x + 8)^3(x - 8)^5$$

The solution is the graph below, which is option A.

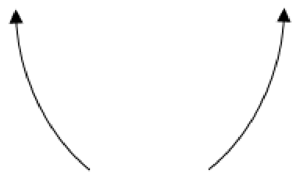


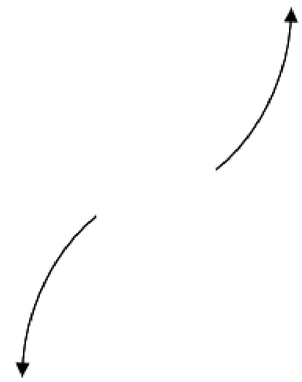


A.



B.

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 4i \text{ and } -2$$

The solution is $x^3 + 6x^2 + 28x + 40$, which is option C.

A. $b \in [-11, -3], c \in [27.08, 28.99], \text{ and } d \in [-40.7, -36.4]$

$x^3 - 6x^2 + 28x - 40$, which corresponds to multiplying out $(x - (-2 - 4i))(x - (-2 + 4i))(x - 2)$.

B. $b \in [-3, 5], c \in [4.37, 7.77], \text{ and } d \in [5.9, 8.6]$

$x^3 + x^2 + 6x + 8$, which corresponds to multiplying out $(x + 4)(x + 2)$.

C. $b \in [2, 15], c \in [27.08, 28.99], \text{ and } d \in [36.9, 41.9]$

* $x^3 + 6x^2 + 28x + 40$, which is the correct option.

D. $b \in [-3, 5], c \in [2.08, 5.16], \text{ and } d \in [2.9, 4.7]$

$x^3 + x^2 + 4x + 4$, which corresponds to multiplying out $(x + 2)(x + 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 4i))(x - (-2 + 4i))(x - (-2))$.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$5, \frac{-7}{4}, \text{ and } \frac{-4}{5}$$

The solution is $20x^3 - 49x^2 - 227x - 140$, which is option E.

A. $a \in [19, 30], b \in [-50, -45], c \in [-227, -224], \text{ and } d \in [140, 146]$

$20x^3 - 49x^2 - 227x + 140$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [19, 30], b \in [46, 52], c \in [-227, -224], \text{ and } d \in [140, 146]$

$20x^3 + 49x^2 - 227x + 140$, which corresponds to multiplying out $(x + 5)(4x - 7)(5x - 4)$.

C. $a \in [19, 30], b \in [142, 154], c \in [277, 289], \text{ and } d \in [140, 146]$

$20x^3 + 151x^2 + 283x + 140$, which corresponds to multiplying out $(x + 5)(4x + 7)(5x + 4)$.

D. $a \in [19, 30], b \in [77, 83], c \in [-125, -118], \text{ and } d \in [-140, -135]$

$20x^3 + 81x^2 - 123x - 140$, which corresponds to multiplying out $(x + 5)(4x - 7)(5x + 4)$.

E. $a \in [19, 30], b \in [-50, -45], c \in [-227, -224], \text{ and } d \in [-140, -135]$

* $20x^3 - 49x^2 - 227x - 140$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-5)(4x+7)(5x+4)$