**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 3i \text{ and } 3$$

The solution is $x^3 - 13x^2 + 64x - 102$, which is option B.

A. $b \in [10, 18], c \in [62.11, 64.76],$ and $d \in [101, 106.3]$

$x^3 + 13x^2 + 64x + 102$, which corresponds to multiplying out $(x - (5 + 3i))(x - (5 - 3i))(x + 3)$.

B. $b \in [-21, -5], c \in [62.11, 64.76],$ and $d \in [-105.1, -100.1]$

* $x^3 - 13x^2 + 64x - 102$, which is the correct option.

C. $b \in [1, 3], c \in [-8.79, -7.83],$ and $d \in [11.8, 17]$

$x^3 + x^2 - 8x + 15$, which corresponds to multiplying out $(x - 5)(x - 3)$.

D. $b \in [1, 3], c \in [-7.39, -4.76],$ and $d \in [5.2, 9.5]$

$x^3 + x^2 - 6x + 9$, which corresponds to multiplying out $(x - 3)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

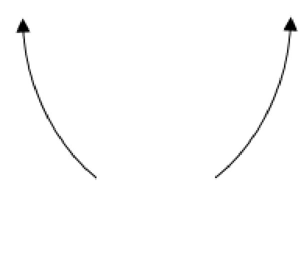
**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 3i))(x - (5 - 3i))(x - (3))$.

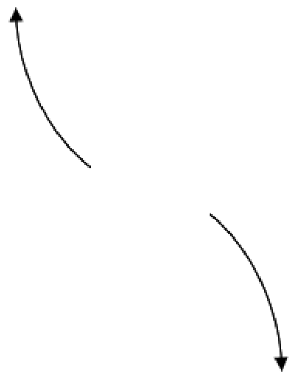
---

2. Describe the end behavior of the polynomial below.

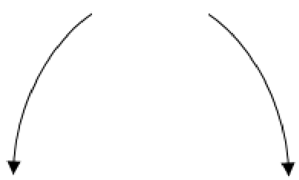
$$f(x) = 8(x - 6)^4(x + 6)^9(x + 9)^4(x - 9)^5$$

The solution is the graph below, which is option C.

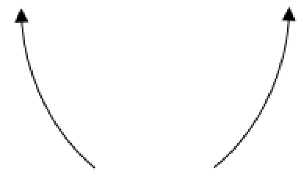




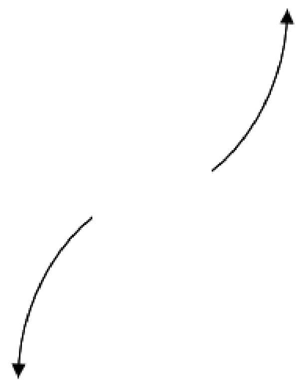
A.



B.



C.



D.

E. None of the above.

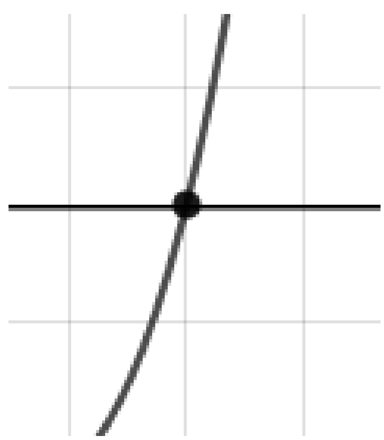
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

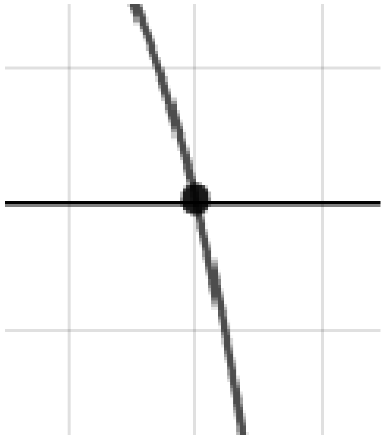
---

3. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

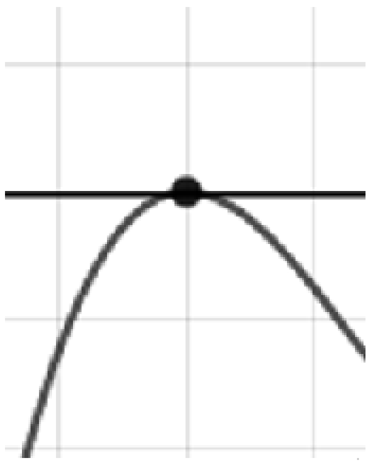
$$f(x) = 9(x+7)^{8}(x-7)^{11}(x-3)^{7}(x+3)^{10}$$

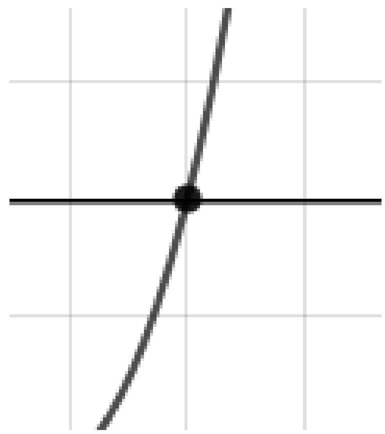
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-5}{3}, -3, \text{ and } \frac{5}{4}$$

The solution is $12x^3 + 41x^2 - 10x - 75$, which is option D.

A. $a \in [12, 15], b \in [-43, -40], c \in [-16, -2], \text{ and } d \in [71, 81]$

$12x^3 - 41x^2 - 10x + 75$, which corresponds to multiplying out $(3x - 5)(x - 3)(4x + 5)$.

B. $a \in [12, 15], b \in [37, 49], c \in [-16, -2], \text{ and } d \in [71, 81]$

$12x^3 + 41x^2 - 10x + 75$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [12, 15], b \in [-2, 3], c \in [-84, -79], \text{ and } d \in [71, 81]$

$12x^3 + x^2 - 80x + 75$, which corresponds to multiplying out $(3x - 5)(x + 3)(4x - 5)$.

D. $a \in [12, 15], b \in [37, 49], c \in [-16, -2], \text{ and } d \in [-78, -73]$

* $12x^3 + 41x^2 - 10x - 75$, which is the correct option.

E. $a \in [12, 15], b \in [-85, -66], c \in [126, 135]$, and $d \in [-78, -73]$

$12x^3 - 71x^2 + 130x - 75$, which corresponds to multiplying out $(3x - 5)(x - 3)(4x - 5)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(3x + 5)(x + 3)(4x - 5)$

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-6}{5}, \frac{-4}{3}, \text{ and } -3$$

The solution is $15x^3 + 83x^2 + 138x + 72$, which is option E.

A. $a \in [13, 21], b \in [76, 90], c \in [133, 140]$, and $d \in [-75, -70]$

$15x^3 + 83x^2 + 138x - 72$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [13, 21], b \in [42, 53], c \in [-19, -16]$, and $d \in [-75, -70]$

$15x^3 + 47x^2 - 18x - 72$, which corresponds to multiplying out $(5x - 6)(3x + 4)(x + 3)$.

C. $a \in [13, 21], b \in [3, 8], c \in [-91, -85]$, and $d \in [66, 76]$

$15x^3 + 7x^2 - 90x + 72$, which corresponds to multiplying out $(5x - 6)(3x - 4)(x + 3)$.

D. $a \in [13, 21], b \in [-84, -78], c \in [133, 140]$, and $d \in [-75, -70]$

$15x^3 - 83x^2 + 138x - 72$, which corresponds to multiplying out $(5x - 6)(3x - 4)(x - 3)$.

E. $a \in [13, 21], b \in [76, 90], c \in [133, 140]$, and $d \in [66, 76]$

* $15x^3 + 83x^2 + 138x + 72$, which is the correct option.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(5x + 6)(3x + 4)(x + 3)$

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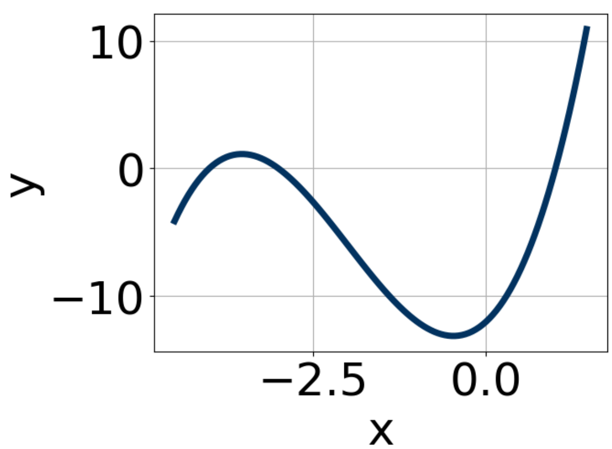
6. Which of the following equations *could* be of the graph presented below?




The solution is $20(x + 4)^9(x - 1)^{11}(x + 3)^7$, which is option E.

A. $-11(x + 4)^7(x - 1)^7(x + 3)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-3(x+4)^8(x-1)^{11}(x+3)^{11}$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $18(x+4)^8(x-1)^7(x+3)^5$

The factor $-4$ should have been an odd power.

D. $6(x+4)^4(x-1)^{10}(x+3)^5$

The factors $-4$ and $1$ have have been odd power.

E. $20(x+4)^9(x-1)^{11}(x+3)^7$

* This is the correct option.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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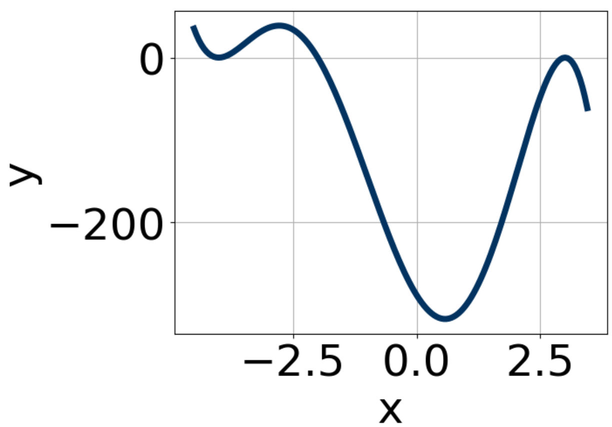
7. Which of the following equations *could* be of the graph presented below?




The solution is $-11(x+4)^6(x-3)^{10}(x+2)^5$, which is option B.

A. $-5(x+4)^{10}(x-3)^{11}(x+2)^7$

The factor $(x-3)$ should have an even power.

B. $-11(x+4)^6(x-3)^{10}(x+2)^5$

* This is the correct option.

C. $9(x+4)^6(x-3)^{10}(x+2)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-19(x+4)^4(x-3)^9(x+2)^6$

The factor $(x-3)$ should have an even power and the factor $(x+2)$ should have an odd power.

E. $19(x+4)^4(x-3)^8(x+2)^4$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

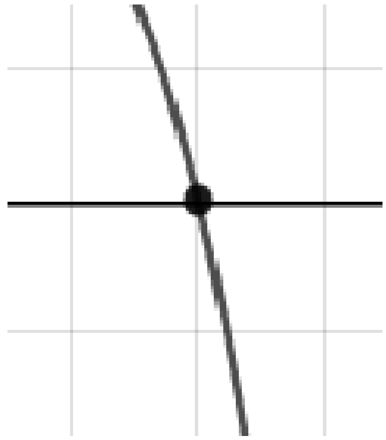
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

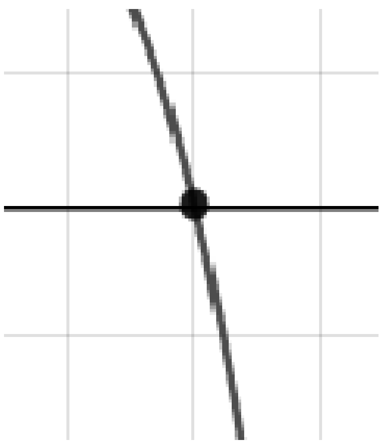
---

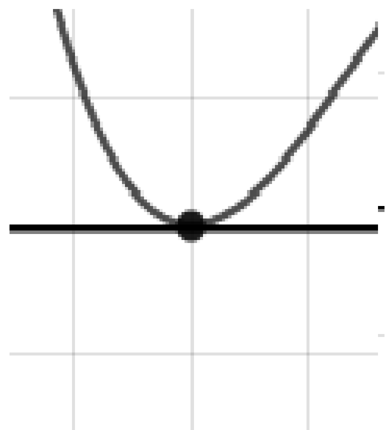
8. Describe the zero behavior of the zero $x = -7$ of the polynomial below.

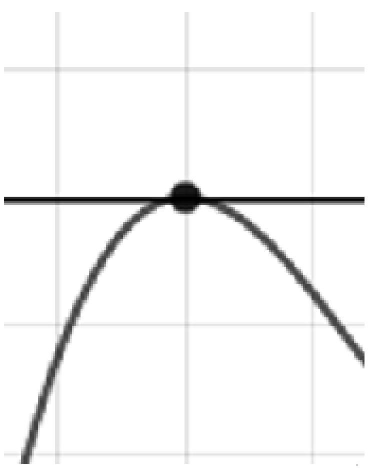
$$f(x) = -8(x-7)^8(x+7)^{13}(x-9)^4(x+9)^7$$

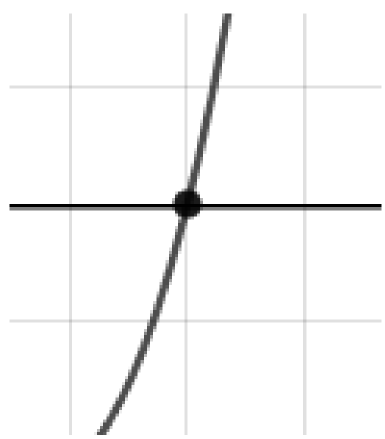
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 3i \text{ and } 1$$

The solution is $x^3 + 9x^2 + 24x - 34$, which is option A.

A. $b \in [4, 10], c \in [22, 26]$, and $d \in [-38, -33]$

   * $x^3 + 9x^2 + 24x - 34$, which is the correct option.

B. $b \in [-16, -7], c \in [22, 26]$, and $d \in [27, 36]$

   $x^3 - 9x^2 + 24x + 34$, which corresponds to multiplying out $(x - (-5 + 3i))(x - (-5 - 3i))(x + 1)$.

C. $b \in [-4, 5], c \in [1, 6]$, and $d \in [-12, -3]$

   $x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

D. $b \in [-4, 5], c \in [-4, 0]$, and $d \in [2, 4]$

   $x^3 + x^2 - 4x + 3$, which corresponds to multiplying out $(x - 3)(x - 1)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

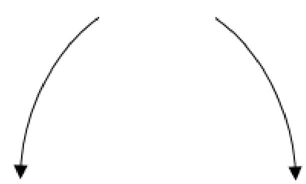
**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 3i))(x - (-5 - 3i))(x - (1))$.

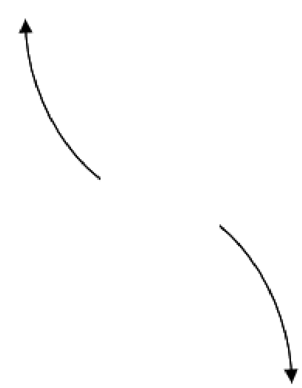
---

10. Describe the end behavior of the polynomial below.

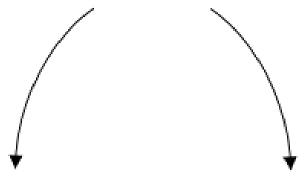
$$f(x) = -4(x + 4)^3(x - 4)^6(x - 5)^2(x + 5)^3$$

The solution is the graph below, which is option B.

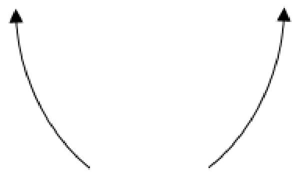




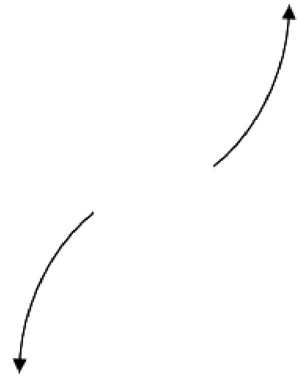
A.



B.



C.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.