This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Using the scenario below, model the population of bacteria α in terms of the number of minutes, t that pass. Then, choose the correct approximate *(rounded to the nearest minute)* replication rate of bacteria-α.

    A newly discovered bacteria, α, is being examined in a lab. The lab started with a petri dish of 2 bacteria-α. After 3 hours, the petri dish has 358 bacteria-α. Based on similar bacteria, the lab believes bacteria-α doubles after some undetermined number of minutes.

    The solution is About 24 minutes, which is option D.

    A. About 42 minutes

       This does not solve for the constant correctly.

    B. About 254 minutes

       This does not solve for the constant correctly AND converted incorrectly.

    C. About 144 minutes

       This solves for the constant correctly but converted incorrectly.

    D. About 24 minutes

       * This is the correct option.

    E. None of the above

       Please contact the coordinator to discuss why you believe none of the answers above are correct.

    General Comment: Your model should be $P(t) = P_0(b)^{kt}$, where $P(t)$ is the population at some time t, P_0 is the initial population, and k is the replication rate. Be sure you convert the hours into minutes!

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2. Determine the appropriate model for the graph of points below.
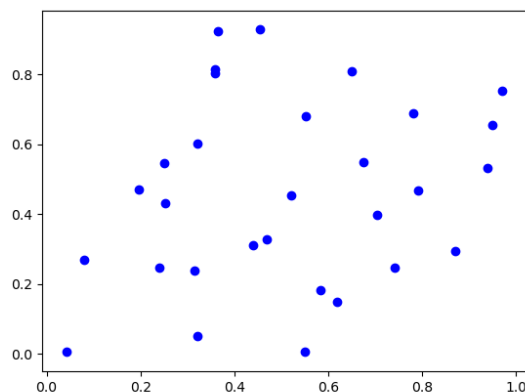
The solution is None of the above, which is option E.

A. Logarithmic model

For this to be the correct option, we want a rapid change early, then an extremely slow change later.

B. Exponential model

For this to be the correct option, we want an extremely slow change early, then a rapid change later.

C. Linear model

For this to be the correct option, we need to see a mostly straight line of points.

D. Non-linear Power model

For this to be the correct option, we need to see a polynomial or rational shape.

E. None of the above

For this to be the correct option, we want to see no pattern in the points.

General Comment: This question is testing if you can associate the models with their graphical representation. If you are having trouble, go back to the corresponding Core module to learn about the specific function you are having trouble recognizing.

___

3. Using the scenario below, model the population of bacteria α in terms of the number of minutes, t that pass. Then, choose the correct approximate *(rounded to the nearest minute)* replication rate of bacteria-α.

A newly discovered bacteria, α, is being examined in a lab. The lab started with a petri dish of 3 bacteria-α. After 3 hours, the petri dish has 2064 bacteria-α. Based on similar bacteria, the lab believes bacteria-α quadruples after some undetermined number of minutes.

The solution is About 38 minutes, which is option B.

A. About 351 minutes

This does not solve for the constant correctly AND converted incorrectly.

B. About 38 minutes

* This is the correct option.

C. About 229 minutes

This solves for the constant correctly but converted incorrectly.

D. About 58 minutes

This does not solve for the constant correctly.

E. None of the above

Please contact the coordinator to discuss why you believe none of the answers above are correct.

General Comment: Your model should be $P(t) = P_0(b)^{kt}$, where $P(t)$ is the population at some time t, P_0 is the initial population, and k is the replication rate. Be sure you convert the hours into minutes!

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4. A town has an initial population of 100000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

Year	1	2	3	4	5	6	7	8	9
Pop	100000	100020	100032	100041	100048	100053	100058	100062	100065

The solution is Logarithmic, which is option A.

A. Logarithmic

This suggests the slowest of growths that we know.

B. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

C. Exponential

This suggests the fastest of growths that we know.

D. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

General Comment: We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

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5. Determine the appropriate model for the graph of points below.
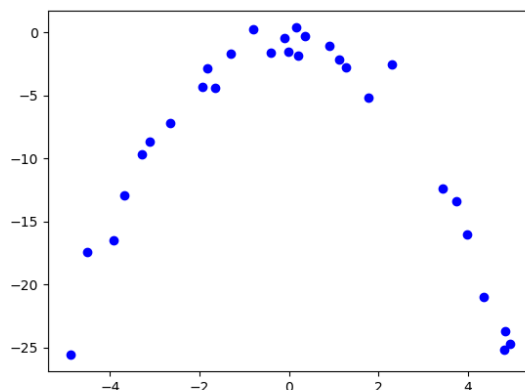
The solution is Non-linear Power model, which is option B.

A. Logarithmic model

   For this to be the correct option, we want a rapid change early, then an extremely slow change later.

B. Non-linear Power model

   For this to be the correct option, we need to see a polynomial or rational shape.

C. Linear model

   For this to be the correct option, we need to see a mostly straight line of points.

D. Exponential model

   For this to be the correct option, we want an extremely slow change early, then a rapid change later.

E. None of the above

   For this to be the correct option, we want to see no pattern in the points.

General Comment: This question is testing if you can associate the models with their graphical representation. If you are having trouble, go back to the corresponding Core module to learn about the specific function you are having trouble recognizing.

6. Using the scenario below, model the situation using an exponential function and a base of $\frac{1}{2}$. Then, solve for the half-life of the element, rounding to the nearest day.

   The half-life of an element is the amount of time it takes for the element to decay to half of its initial starting amount. There is initially 727 grams of element X and after 10 years there is 103 grams remaining.

   The solution is About 1095 days, which is option B.

   A. About 365 days

      This models half-life as a linear function.

   B. About 1095 days

      * This is the correct option.

C. About 1825 days

   This uses the correct model but a base of e rather than $\frac{1}{2}$.

D. About 4380 days

   This uses the correct model but solves for the exponential constant incorrectly.

E. None of the above

   Please contact the coordinator if you believe all the options above are incorrect.

General Comment: The model should be $A(t) = A_0(\frac{1}{2})^{kt}$, where $A(t)$ is the amount after t years, A_0 is the initial amount, and k is decay constant. To find the half-life, you need to solve for k by using the amount after x years, then solve for the time t when $A = \frac{A_0}{2}$. Your answer would be in years, so convert to days.

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7. A town has an initial population of 60000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

Year	1	2	3	4	5	6	7	8	9
Pop	60120	60360	61080	63240	69720	89160	147480	322440	847320

The solution is Exponential, which is option D.

A. Non-Linear Power

   This suggests a growth faster than constant but slower than exponential.

B. Logarithmic

   This suggests the slowest of growths that we know.

C. Linear

   This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

D. Exponential

   This suggests the fastest of growths that we know.

E. None of the above

   Please contact the coordinator to discuss why you believe none of the options model the population.

General Comment: We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

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8. The temperature of an object, T, in a different surrounding temperature T_s will behave according to the formula $T(t) = Ae^{kt} + T_s$, where t is minutes, A is a constant, and k is a constant. Use this formula and the situation below to construct a model that describes the uranium's temperature, T, based on the amount of time t (in minutes) that have passed. Choose the correct constant k from the options below.

   Uranium is taken out of the reactor with a temperature of $150°$ C and is placed into a $19°$ C bath to cool. After 10 minutes, the uranium has cooled to $93°$ C.

The solution is None of the above, which is option E.

   A. $k = -0.07066$

      This uses A as the initial temperature and solves for k incorrectly.

   B. $k = -0.07326$

      This uses A correctly and solves for k incorrectly.

   C. $k = -0.07066$

      This uses A as the initial temperature and solves for k correctly.

   D. $k = -0.07161$

      This uses A as the initial temperature and solves for k incorrectly.

   E. None of the above

      * This is the correct answer as $k = -0.05711$.

General Comment: The initial temperature is when $t = 0$. Unlike power models, that means A is not the initial temperature!

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9. Using the scenario below, model the situation using an exponential function and a base of $\frac{1}{2}$. Then, solve for the half-life of the element, rounding to the nearest day.

   The half-life of an element is the amount of time it takes for the element to decay to half of its initial starting amount. There is initially 664 grams of element X and after 7 years there is 94 grams remaining.

The solution is About 730 days, which is option B.

   A. About 0 days

      This models half-life as a linear function.

   B. About 730 days

      * This is the correct option.

   C. About 1095 days

      This uses the correct model but a base of e rather than $\frac{1}{2}$.

   D. About 2920 days

      This uses the correct model but solves for the exponential constant incorrectly.

   E. None of the above

      Please contact the coordinator if you believe all the options above are incorrect.

General Comment: The model should be $A(t) = A_0(\frac{1}{2})^{kt}$, where $A(t)$ is the amount after t years, A_0 is the initial amount, and k is decay constant. To find the half-life, you need to solve for k by using the amount after x years, then solve for the time t when $A = \frac{A_0}{2}$. Your answer would be in years, so convert to days.

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10. The temperature of an object, T, in a different surrounding temperature T_s will behave according to the formula $T(t) = Ae^{kt} + T_s$, where t is minutes, A is a constant, and k is a constant. Use this formula and the situation below to construct a model that describes the uranium's temperature, T, based on

the amount of time t (in minutes) that have passed. Choose the correct constant k from the options below.

Uranium is taken out of the reactor with a temperature of $140°$ C and is placed into a $17°$ C bath to cool. After 22 minutes, the uranium has cooled to $84°$ C.

The solution is None of the above, which is option E.

A. $k = -0.03217$

This uses A as the initial temperature and solves for k incorrectly.

B. $k = -0.03350$

This uses A as the initial temperature and solves for k correctly.

C. $k = -0.03288$

This uses A correctly and solves for k incorrectly.

D. $k = -0.03350$

This uses A as the initial temperature and solves for k incorrectly.

E. None of the above

* This is the correct answer as $k = -0.02761$.

General Comment: The initial temperature is when $t = 0$. Unlike power models, that means A is not the initial temperature!