This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 + 5x^2 - 80x - 79}{x - 3}$$

The solution is $10x^2 + 35x + 25 + \frac{-4}{x-3}$, which is option B.

A. $a \in [8, 13], \ b \in [-27, -24], \ c \in [-7, -3], \text{ and } r \in [-67, -59].$

You divided by the opposite of the factor.

B. $a \in [8, 13], \ b \in [33, 37], \ c \in [18, 32], \text{ and } r \in [-7, -3].$

* This is the solution!

C. $a \in [26, 34], \ b \in [-87, -79], \ c \in [174, 178], \text{ and } r \in [-608, -597].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

D. $a \in [26, 34], \ b \in [91, 99], \ c \in [196, 207], \text{ and } r \in [534, 543].$

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [8, 13], \ b \in [24, 27], \ c \in [-31, -27], \text{ and } r \in [-143, -134].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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2. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 3x^4 + 2x^3 + 3x^2 + 7x + 5$$

The solution is $\pm 1, \pm 5$, which is option C.

A. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. All combinations of: $\frac{\pm 1, \pm 3}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 5$

* This is the solution **since we asked for the possible Integer roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 3}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{16x^3 - 52x^2 + 33}{x - 3}$$

The solution is $16x^2 - 4x - 12 + \dfrac{-3}{x-3}$, which is option D.

A. $a \in [12, 25], b \in [-20, -18], c \in [-41, -36], \text{ and } r \in [-52, -45]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [47, 51], b \in [-199, -188], c \in [586, 591], \text{ and } r \in [-1736, -1727]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [47, 51], b \in [85, 98], c \in [275, 280], \text{ and } r \in [852, 865]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [12, 25], b \in [-9, -2], c \in [-14, -9], \text{ and } r \in [-6, -1]$.

* This is the solution!

E. $a \in [12, 25], b \in [-102, -93], c \in [296, 308], \text{ and } r \in [-868, -866]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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4. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 3x^2 + 6x + 2$$

The solution is $\pm 1, \pm 2$, which is option D.

A. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 3}$

This would have been the solution **if asked for the possible Rational roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. $\pm 1, \pm 2$

* This is the solution **since we asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 + 4x^2 - 28x - 27}{x - 2}$$

The solution is $8x^2 + 20x + 12 + \frac{-3}{x-2}$, which is option D.

A. $a \in [3, 15]$, $b \in [-15, -10]$, $c \in [-4, 1]$, and $r \in [-21, -18]$.

You divided by the opposite of the factor.

B. $a \in [3, 15]$, $b \in [8, 13]$, $c \in [-16, -15]$, and $r \in [-51, -38]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [13, 20]$, $b \in [36, 39]$, $c \in [38, 48]$, and $r \in [60, 67]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [3, 15]$, $b \in [17, 21]$, $c \in [9, 18]$, and $r \in [-6, -2]$.

* This is the solution!

E. $a \in [13, 20]$, $b \in [-31, -24]$, $c \in [25, 36]$, and $r \in [-86, -81]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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6. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 - 26x^2 + 13}{x - 3}$$

The solution is $8x^2 - 2x - 6 + \frac{-5}{x-3}$, which is option E.

A. $a \in [22, 30], b \in [45, 50], c \in [134, 140]$, and $r \in [425, 428]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [22, 30], b \in [-100, -94], c \in [293, 299]$, and $r \in [-870, -867]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [2, 11], b \in [-12, -5], c \in [-25, -15]$, and $r \in [-32, -26]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [2, 11], b \in [-52, -45], c \in [147, 152]$, and $r \in [-442, -429]$.

You divided by the opposite of the factor.

E. $a \in [2, 11], b \in [-7, 6], c \in [-6, -4]$, and $r \in [-8, 4]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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7. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 + 49x^2 + 68x + 20$$

The solution is $[-2.5, -2, -0.4]$, which is option C.

A. $z_1 \in [-2.62, -2.31]$, $z_2 \in [-3, -1]$, and $z_3 \in [-0.4, 1.6]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [0.36, 0.41]$, $z_2 \in [2, 3]$, and $z_3 \in [2.5, 4.5]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-2.62, -2.31]$, $z_2 \in [-3, -1]$, and $z_3 \in [-0.4, 1.6]$

* This is the solution!

D. $z_1 \in [0.36, 0.41]$, $z_2 \in [2, 3]$, and $z_3 \in [2.5, 4.5]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [0.18, 0.35]$, $z_2 \in [2, 3]$, and $z_3 \in [5, 8]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 - 50x^2 - 9x + 18$$

The solution is $[-0.6, 0.6, 2]$, which is option C.

A. $z_1 \in [-1.9, -1.2]$, $z_2 \in [1.33, 2.15]$, and $z_3 \in [1.9, 2.26]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-2.3, -1.8]$, $z_2 \in [-0.74, -0.57]$, and $z_3 \in [0.47, 0.92]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-1.1, 0.4]$, $z_2 \in [0.5, 0.94]$, and $z_3 \in [1.9, 2.26]$

* This is the solution!

D. $z_1 \in [-2.3, -1.8]$, $z_2 \in [-0.25, 0.02]$, and $z_3 \in [2.6, 3.82]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-2.3, -1.8]$, $z_2 \in [-1.74, -1.03]$, and $z_3 \in [1.55, 1.94]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Factor the polynomial below completely, knowing that $x - 2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 + 18x^3 - 75x^2 - 46x + 120$$

The solution is $[-4, -1.5, 1.25, 2]$, which is option B.

A. $z_1 \in [-3, 2]$, $z_2 \in [-1.26, -1.24]$, $z_3 \in [1.34, 1.54]$, and $z_4 \in [3.3, 5.1]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-4, -3]$, $z_2 \in [-1.5, -1.48]$, $z_3 \in [1.16, 1.4]$, and $z_4 \in [1.1, 3.9]$

* This is the solution!

C. $z_1 \in [-3, 2]$, $z_2 \in [-0.95, -0.78]$, $z_3 \in [0.62, 0.76]$, and $z_4 \in [3.3, 5.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-3, 2]$, $z_2 \in [-0.63, -0.57]$, $z_3 \in [2.75, 3.25]$, and $z_4 \in [3.3, 5.1]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-4, -3]$, $z_2 \in [-0.74, -0.65]$, $z_3 \in [0.68, 0.81]$, and $z_4 \in [1.1, 3.9]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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10. Factor the polynomial below completely, knowing that $x - 2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 - 71x^3 + 174x^2 - 171x + 54$$

The solution is $[0.6, 1.5, 2, 3]$, which is option B.

A. $z_1 \in [0.65, 0.69]$, $z_2 \in [1.66, 1.97]$, $z_3 \in [1.59, 2.31]$, and $z_4 \in [2.92, 3.18]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [0.53, 0.63]$, $z_2 \in [1.16, 1.61]$, $z_3 \in [1.59, 2.31]$, and $z_4 \in [2.92, 3.18]$

* This is the solution!

C. $z_1 \in [-3.11, -3]$, $z_2 \in [-2, -1.87]$, $z_3 \in [-1.86, -1.65]$, and $z_4 \in [-0.71, -0.66]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-3.11, -3]$, $z_2 \in [-2, -1.87]$, $z_3 \in [-1.54, -1.11]$, and $z_4 \in [-0.6, -0.49]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-3.11, -3]$, $z_2 \in [-3.1, -2.85]$, $z_3 \in [-2.21, -1.79]$, and $z_4 \in [-0.32, -0.21]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.