This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{2}{3}, \frac{1}{2}, \text{ and } \frac{-7}{2}$$

The solution is $12x^3 + 28x^2 - 45x + 14$, which is option A.

A. $a \in [11, 24], b \in [26, 35], c \in [-50, -44], \text{ and } d \in [14, 21]$

* $12x^3 + 28x^2 - 45x + 14$, which is the correct option.

B. $a \in [11, 24], b \in [-29, -26], c \in [-50, -44], \text{ and } d \in [-16, -12]$

$12x^3 - 28x^2 - 45x - 14$, which corresponds to multiplying out $(3x + 2)(2x + 1)(2x - 7)$.

C. $a \in [11, 24], b \in [39, 45], c \in [3, 4], \text{ and } d \in [-16, -12]$

$12x^3 + 44x^2 + 3x - 14$, which corresponds to multiplying out $(3x + 2)(2x - 1)(2x + 7)$.

D. $a \in [11, 24], b \in [26, 35], c \in [-50, -44], \text{ and } d \in [-16, -12]$

$12x^3 + 28x^2 - 45x - 14$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [11, 24], b \in [50, 60], c \in [53, 61], \text{ and } d \in [14, 21]$

$12x^3 + 56x^2 + 53x + 14$, which corresponds to multiplying out $(3x + 2)(2x + 1)(2x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 2)(2x - 1)(2x + 7)$

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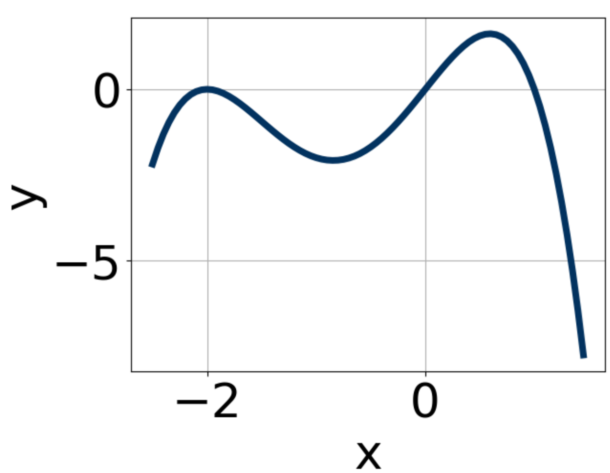
2. Which of the following equations *could* be of the graph presented below?



The solution is $-3x^7(x+2)^8(x-1)^{11}$, which is option D.

A. $9x^8(x+2)^4(x-1)^9$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

B. $-2x^5(x+2)^8(x-1)^4$

The factor $(x-1)$ should have an odd power.

C. $13x^{11}(x+2)^{10}(x-1)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-3x^7(x+2)^8(x-1)^{11}$

* This is the correct option.

E. $-9x^5(x+2)^9(x-1)^8$

The factor -2 should have an even power and the factor 1 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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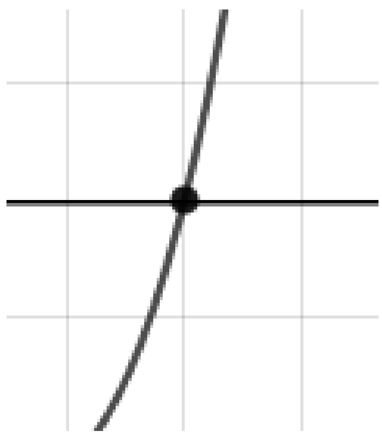
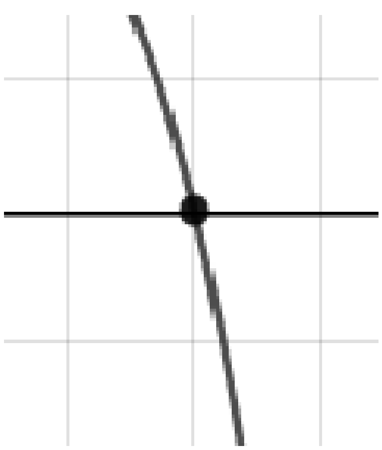
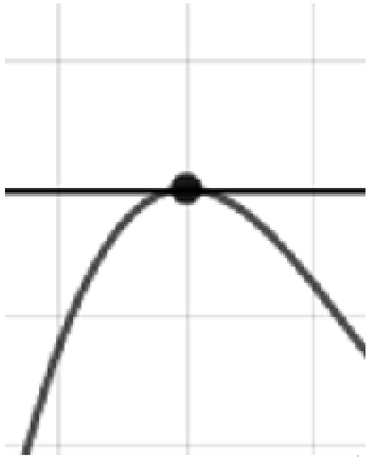
3. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

$$f(x) = 8(x+2)^8(x-2)^7(x+5)^{10}(x-5)^5$$

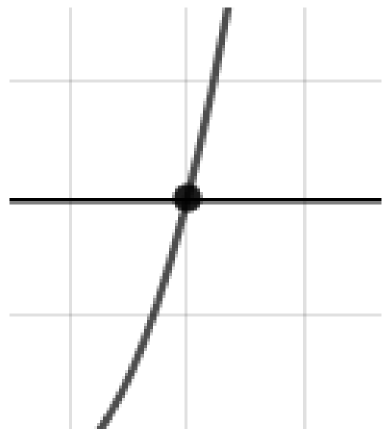
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 3i \text{ and } 2$$

The solution is $x^3 + 6x^2 + 9x - 50$, which is option B.

A. $b \in [-1, 4], c \in [-0.36, 1.69],$ and $d \in [-6.6, -3.6]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x + 3)(x - 2)$.

B. $b \in [5, 9], c \in [8.74, 10.07],$ and $d \in [-50.9, -49.8]$

* $x^3 + 6x^2 + 9x - 50$, which is the correct option.

C. $b \in [-1, 4], c \in [1.92, 2.59],$ and $d \in [-10.8, -6.9]$

$x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x + 4)(x - 2)$.

D. $b \in [-6, 0], c \in [8.74, 10.07],$ and $d \in [49.5, 51.7]$

$x^3 - 6x^2 + 9x + 50$, which corresponds to multiplying out $(x - (-4 - 3i))(x - (-4 + 3i))(x + 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-4-3i))(x-(-4+3i))(x-(2))$.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$\frac{-7}{5}, \frac{1}{3}, \text{ and } \frac{1}{5}$$

The solution is $75x^3+65x^2-51x+7$, which is option B.

A. $a \in [75, 82], b \in [-95, -92], c \in [-21, -16], \text{ and } d \in [1, 10]$

$75x^3-95x^2-19x+7$, which corresponds to multiplying out $(5x-7)(3x+1)(5x-1)$.

B. $a \in [75, 82], b \in [61, 66], c \in [-52, -46], \text{ and } d \in [1, 10]$

* $75x^3+65x^2-51x+7$, which is the correct option.

C. $a \in [75, 82], b \in [61, 66], c \in [-52, -46], \text{ and } d \in [-14, -6]$

$75x^3+65x^2-51x-7$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [75, 82], b \in [-68, -60], c \in [-52, -46], \text{ and } d \in [-14, -6]$

$75x^3-65x^2-51x-7$, which corresponds to multiplying out $(5x-7)(3x+1)(5x+1)$.

E. $a \in [75, 82], b \in [-148, -139], c \in [57, 65], \text{ and } d \in [-14, -6]$

$75x^3-145x^2+61x-7$, which corresponds to multiplying out $(5x-7)(3x-1)(5x-1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x+7)(3x-1)(5x-1)$

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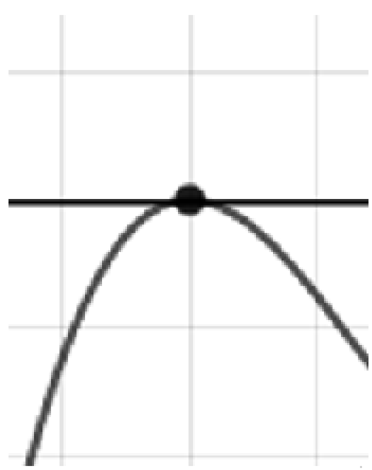
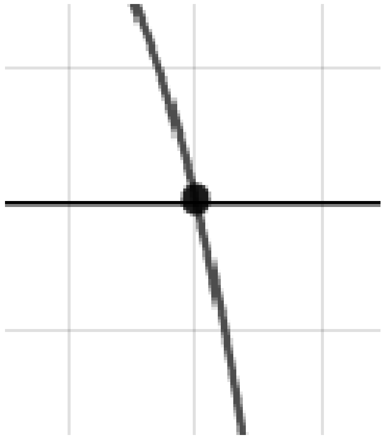
6. Describe the zero behavior of the zero $x=3$ of the polynomial below.

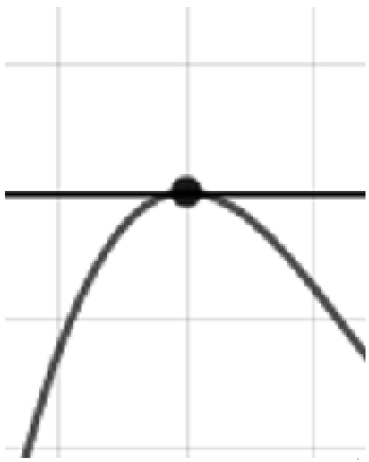
$$f(x) = -2(x+3)^5(x-3)^{10}(x-6)^4(x+6)^5$$

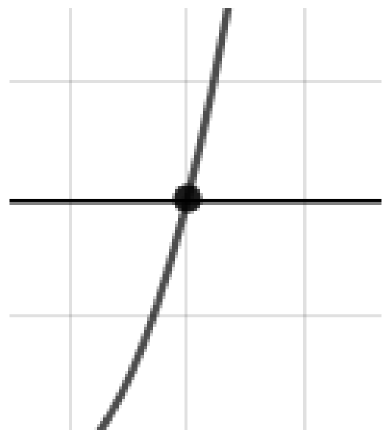
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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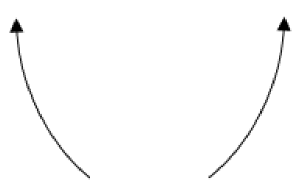
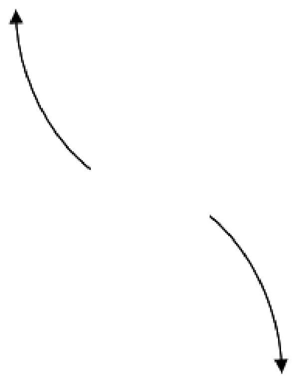
7. Describe the end behavior of the polynomial below.

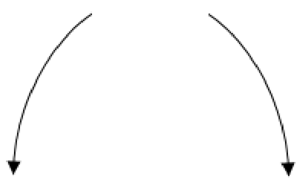
$$f(x) = 9(x-7)^3(x+7)^8(x+8)^3(x-8)^4$$

The solution is the graph below, which is option C.

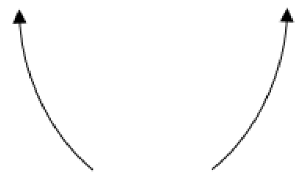




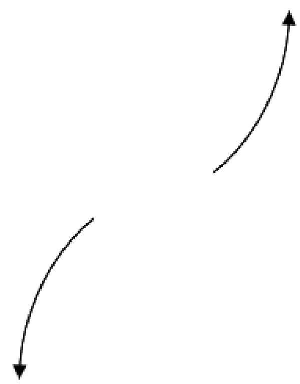
A.



B.



C.



D.

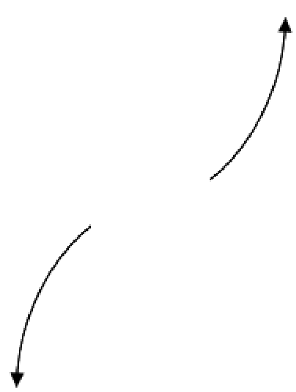
E. None of the above.

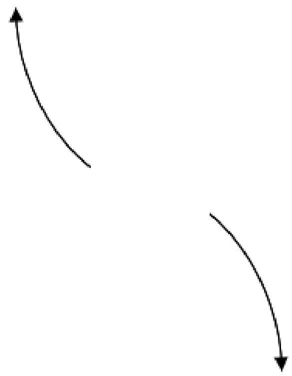
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

8. Describe the end behavior of the polynomial below.

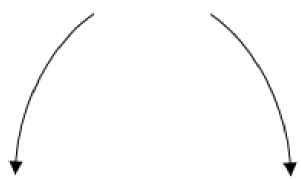
$$f(x) = 4(x-3)^3(x+3)^4(x-5)^5(x+5)^7$$

The solution is the graph below, which is option D.

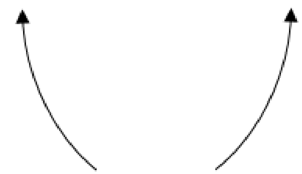




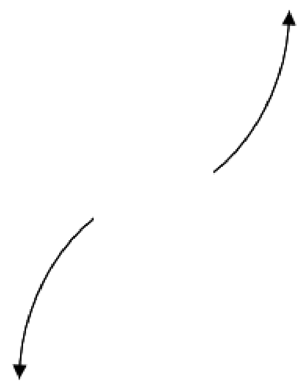
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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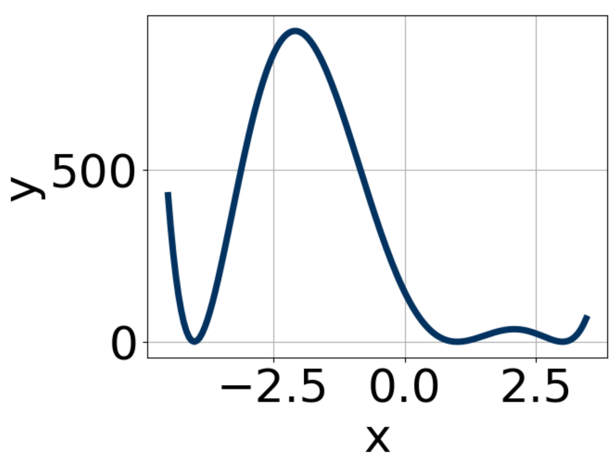
9. Which of the following equations *could* be of the graph presented below?



The solution is $5(x+4)^6(x-1)^4(x-3)^4$, which is option C.

A. $-20(x+4)^{10}(x-1)^4(x-3)^{10}$

   This corresponds to the leading coefficient being the opposite value than it should be.

B. $11(x+4)^4(x-1)^5(x-3)^{11}$

   The factors $(x-1)$ and $(x-3)$ should both have even powers.

C. $5(x+4)^6(x-1)^4(x-3)^4$

* This is the correct option.

D. $14(x+4)^6(x-1)^{10}(x-3)^7$

The factor $(x-3)$ should have an even power.

E. $-8(x+4)^{10}(x-1)^{10}(x-3)^7$

The factor $(x-3)$ should have an even power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form x^3+bx^2+cx+d.

$$-4+2i \text{ and } 4$$

The solution is $x^3+4x^2-12x-80$, which is option A.

A. $b\in[2.8,4.2], c\in[-12,-8], \text{ and } d\in[-83,-75]$

* $x^3+4x^2-12x-80$, which is the correct option.

B. $b\in[0.1,2.1], c\in[0,3], \text{ and } d\in[-19,-12]$

x^3+x^2-16, which corresponds to multiplying out $(x+4)(x-4)$.

C. $b\in[-6.4,-3.6], c\in[-12,-8], \text{ and } d\in[79,82]$

$x^3-4x^2-12x+80$, which corresponds to multiplying out $(x-(-4+2i))(x-(-4-2i))(x+4)$.

D. $b\in[0.1,2.1], c\in[-10,-2], \text{ and } d\in[7,12]$

x^3+x^2-6x+8, which corresponds to multiplying out $(x-2)(x-4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-4+2i))(x-(-4-2i))(x-(4))$.
