This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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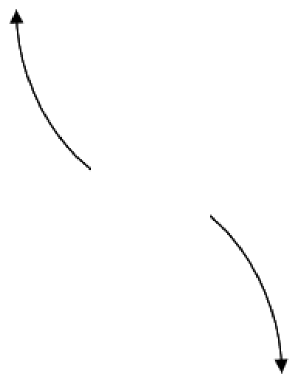
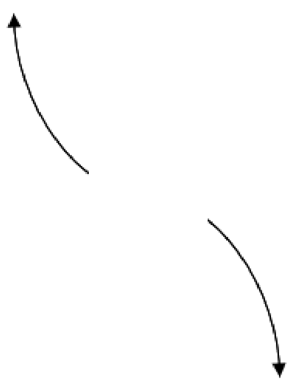
1. Describe the end behavior of the polynomial below.

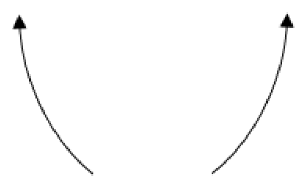
$$f(x) = -4(x-3)^4(x+3)^5(x-6)^2(x+6)^2$$

The solution is the graph below, which is option A.

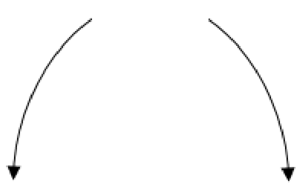




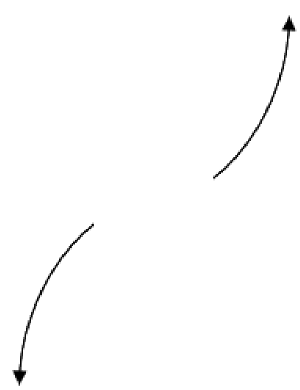
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-5, \frac{-3}{5}, \text{ and } \frac{6}{5}$$

The solution is $25x^3 + 110x^2 - 93x - 90$, which is option D.

A. $a \in [25, 34], b \in [106, 125], c \in [-95, -91], \text{ and } d \in [87, 95]$

$25x^3 + 110x^2 - 93x + 90$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [25, 34], b \in [-141, -137], c \in [52, 58], \text{ and } d \in [87, 95]$

$25x^3 - 140x^2 + 57x + 90$, which corresponds to multiplying out $(x + 1)(5x - 5)(5x - 5)$.

C. $a \in [25, 34], b \in [-112, -103], c \in [-95, -91], \text{ and } d \in [87, 95]$

$25x^3 - 110x^2 - 93x + 90$, which corresponds to multiplying out $(x - 5)(5x - 3)(5x + 6)$.

D. $a \in [25, 34], b \in [106, 125], c \in [-95, -91], \text{ and } d \in [-91, -88]$

* $25x^3 + 110x^2 - 93x - 90$, which is the correct option.

E. $a \in [25, 34], b \in [-174, -169], c \in [239, 248], \text{ and } d \in [-91, -88]$

$25x^3 - 170x^2 + 243x - 90$, which corresponds to multiplying out $(x + 1)(5x + 5)(5x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+5)(5x+3)(5x-6)$

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 3i \text{ and } -4$$

The solution is $x^3 + 8x^2 + 29x + 52$, which is option B.

A. $b \in [-9, -7], c \in [28.54, 29.25], \text{ and } d \in [-53, -48]$

$x^3 - 8x^2 + 29x - 52$, which corresponds to multiplying out $(x - (-2 - 3i))(x - (-2 + 3i))(x - 4)$.

B. $b \in [4, 13], c \in [28.54, 29.25], \text{ and } d \in [47, 55]$

* $x^3 + 8x^2 + 29x + 52$, which is the correct option.

C. $b \in [-7, 3], c \in [5.19, 6.76], \text{ and } d \in [3, 9]$

$x^3 + x^2 + 6x + 8$, which corresponds to multiplying out $(x + 2)(x + 4)$.

D. $b \in [-7, 3], c \in [6.86, 8.71], \text{ and } d \in [10, 20]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 3)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-2-3i))(x-(-2+3i))(x-(-4))$.

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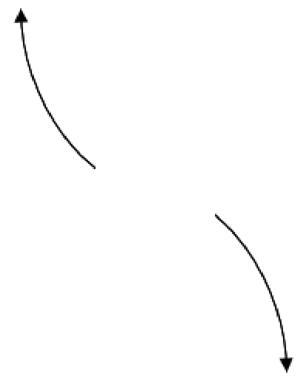
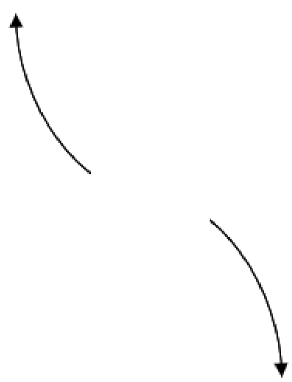
4. Describe the end behavior of the polynomial below.

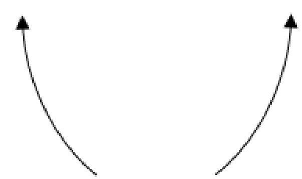
$$f(x) = -5(x-3)^5(x+3)^{10}(x-2)^5(x+2)^5$$

The solution is the graph below, which is option A.

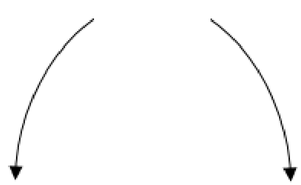




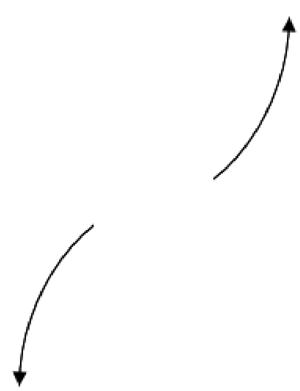
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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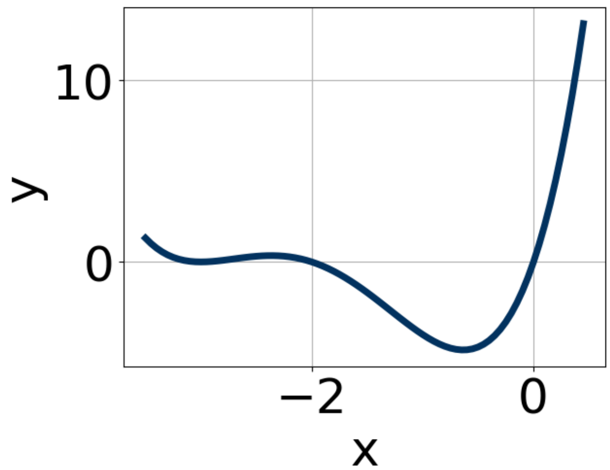
5. Which of the following equations *could* be of the graph presented below?



The solution is $4x^9(x+3)^6(x+2)^9$, which is option C.

A. $-15x^7(x+3)^8(x+2)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-20x^4(x+3)^8(x+2)^9$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

C. $4x^9(x+3)^6(x+2)^9$

* This is the correct option.

D. $18x^7(x+3)^6(x+2)^4$

The factor $(x+2)$ should have an odd power.

E. $13x^7(x+3)^5(x+2)^{10}$

The factor -3 should have an even power and the factor -2 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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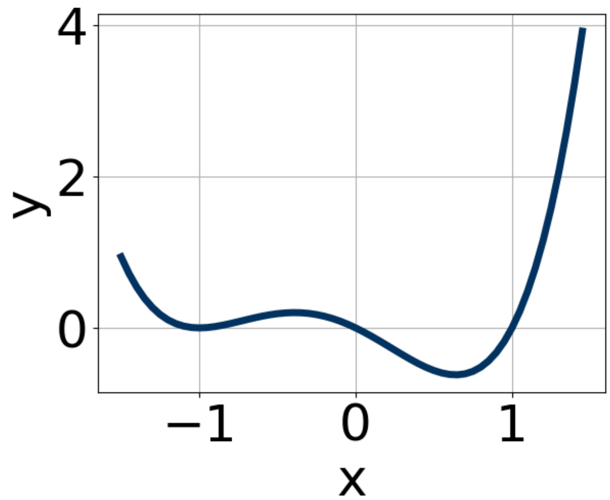
6. Which of the following equations *could* be of the graph presented below?



The solution is $16x^5(x+1)^4(x-1)^9$, which is option A.

A. $16x^5(x+1)^4(x-1)^9$

  * This is the correct option.

B. $3x^7(x+1)^6(x-1)^8$

  The factor $(x-1)$ should have an odd power.

C. $-6x^6(x+1)^{10}(x-1)^9$

  The factor x should have an odd power and the leading coefficient should be the opposite sign.

D. $6x^5(x+1)^{11}(x-1)^{10}$

  The factor -1 should have an even power and the factor 1 should have an odd power.

E. $-4x^{11}(x+1)^6(x-1)^{11}$

  This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-6, \frac{-1}{2}, \text{ and } \frac{-4}{3}$$

The solution is $6x^3 + 47x^2 + 70x + 24$, which is option D.

A. $a \in [0, 14], b \in [-27, -22], c \in [-65, -57], \text{ and } d \in [-24, -21]$

  $6x^3 - 25x^2 - 62x - 24$, which corresponds to multiplying out $(x+1)(2x-2)(3x-3)$.

B. $a \in [0, 14], b \in [-51, -41], c \in [70, 76], \text{ and } d \in [-24, -21]$

  $6x^3 - 47x^2 + 70x - 24$, which corresponds to multiplying out $(x-6)(2x-1)(3x-4)$.

C. $a \in [0, 14], b \in [39, 51], c \in [70, 76], \text{ and } d \in [-24, -21]$

  $6x^3 + 47x^2 + 70x - 24$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [0, 14], b \in [39, 51], c \in [70, 76],$ and $d \in [19, 25]$

* $6x^3 + 47x^2 + 70x + 24$, which is the correct option.

E. $a \in [0, 14], b \in [-36, -27], c \in [-37, -31],$ and $d \in [19, 25]$

$6x^3 - 31x^2 - 34x + 24$, which corresponds to multiplying out $(x + 1)(2x + 2)(3x - 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+6)(2x+1)(3x+4)$

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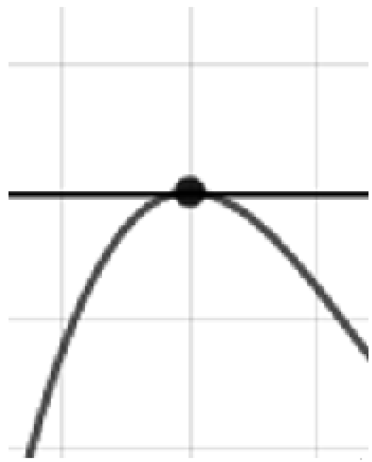
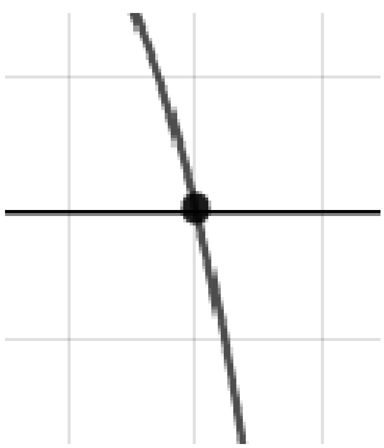
8. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = -2(x + 3)^7(x - 3)^8(x - 2)^9(x + 2)^{11}$$

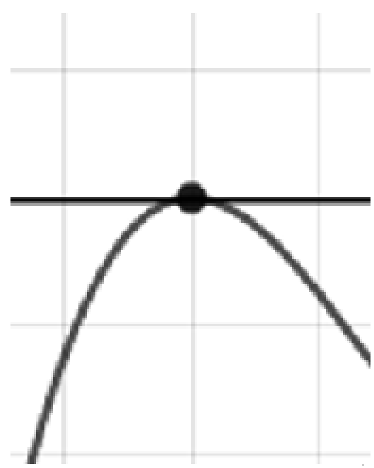
The solution is the graph below, which is option B.



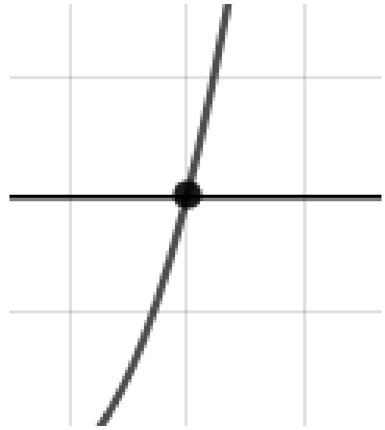
A.



B.



C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 5i \text{ and } 1$$

The solution is $x^3 + 7x^2 + 33x - 41$, which is option B.

A. $b \in [-1, 6], c \in [-10, -1],$ and $d \in [2, 7]$

$x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x - 5)(x - 1)$.

B. $b \in [4, 11], c \in [32, 39],$ and $d \in [-43, -38]$

* $x^3 + 7x^2 + 33x - 41$, which is the correct option.

C. $b \in [-13, -3], c \in [32, 39],$ and $d \in [35, 44]$

$x^3 - 7x^2 + 33x + 41$, which corresponds to multiplying out $(x - (-4 + 5i))(x - (-4 - 5i))(x + 1)$.

D. $b \in [-1, 6], c \in [2, 4],$ and $d \in [-5, 4]$

$x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 5i))(x - (-4 - 5i))(x - (1))$.

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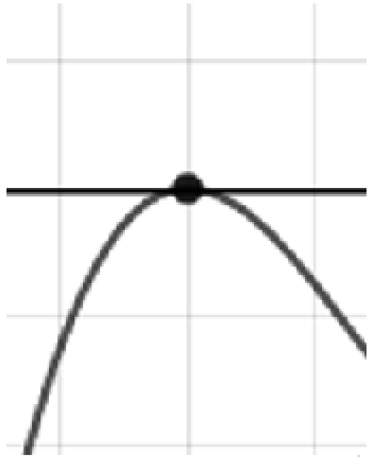
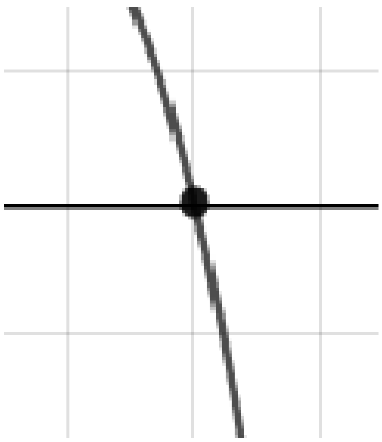
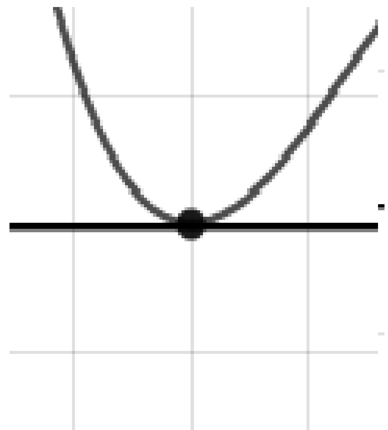
10. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

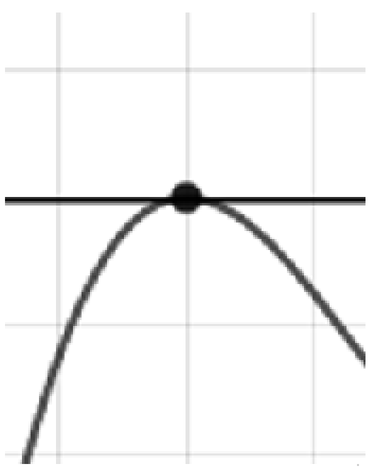
$$f(x) = -7(x + 4)^6(x - 4)^9(x - 7)^5(x + 7)^7$$

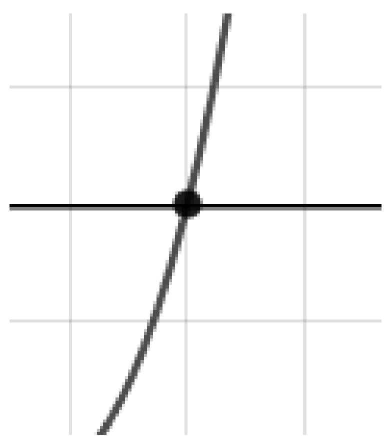
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.