This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 2i \text{ and } -3$$

The solution is $x^3 + 11x^2 + 44x + 60$, which is option A.

A. $b \in [11, 13], c \in [42, 45],$ and $d \in [52, 62]$

* $x^3 + 11x^2 + 44x + 60$, which is the correct option.

B. $b \in [-1, 6], c \in [0, 3],$ and $d \in [-13, -2]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x - 2)(x + 3)$.

C. $b \in [-11, -7], c \in [42, 45],$ and $d \in [-60, -51]$

$x^3 - 11x^2 + 44x - 60$, which corresponds to multiplying out $(x - (-4 + 2i))(x - (-4 - 2i))(x - 3)$.

D. $b \in [-1, 6], c \in [5, 16],$ and $d \in [9, 15]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 4)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 2i))(x - (-4 - 2i))(x - (-3))$.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{5}, \frac{-1}{2}, \text{ and } \frac{-5}{2}$$

The solution is $20x^3 + 56x^2 + 13x - 5$, which is option D.

A. $a \in [17, 24], b \in [44, 46], c \in [-22, -11],$ and $d \in [-5, 2]$

$20x^3 + 44x^2 - 17x - 5$, which corresponds to multiplying out $(5x + 1)(2x - 1)(2x + 5)$.

B. $a \in [17, 24], b \in [48, 59], c \in [5, 16],$ and $d \in [-4, 7]$

$20x^3 + 56x^2 + 13x + 5$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [17, 24], b \in [63, 71], c \in [35, 38],$ and $d \in [-4, 7]$

$20x^3 + 64x^2 + 37x + 5$, which corresponds to multiplying out $(5x + 1)(2x + 1)(2x + 5)$.

D. $a \in [17, 24], b \in [48, 59], c \in [5, 16],$ and $d \in [-5, 2]$

* $20x^3 + 56x^2 + 13x - 5$, which is the correct option.

E. $a \in [17, 24], b \in [-60, -53], c \in [5, 16],$ and $d \in [-4, 7]$

$20x^3 - 56x^2 + 13x + 5$, which corresponds to multiplying out $(5x + 1)(2x - 1)(2x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 1)(2x + 1)(2x + 5)$

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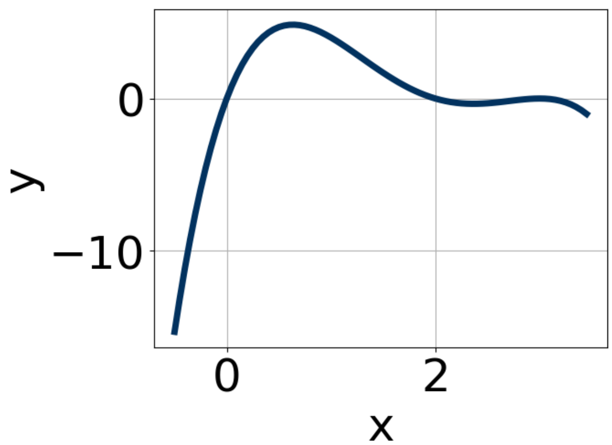
3. Which of the following equations *could* be of the graph presented below?



The solution is $-20x^7(x - 3)^4(x - 2)^{11}$, which is option D.

A. $-4x^7(x - 3)^4(x - 2)^6$

The factor $(x - 2)$ should have an odd power.

B. $14x^{11}(x - 3)^8(x - 2)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-7x^5(x - 3)^5(x - 2)^{10}$

The factor 3 should have an even power and the factor 2 should have an odd power.

D. $-20x^7(x - 3)^4(x - 2)^{11}$

* This is the correct option.

E. $16x^8(x - 3)^6(x - 2)^9$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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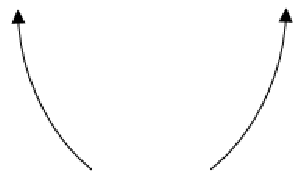
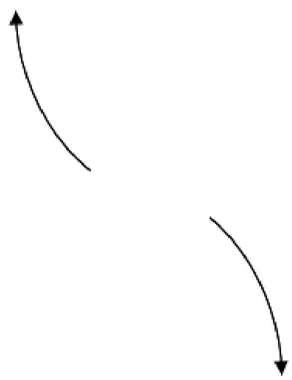
4. Describe the end behavior of the polynomial below.

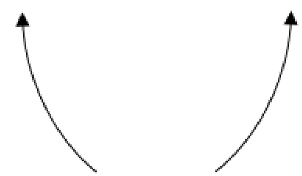
$$f(x) = 6(x + 5)^5(x - 5)^8(x + 9)^4(x - 9)^5$$

The solution is the graph below, which is option C.

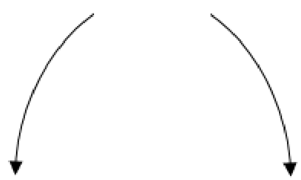




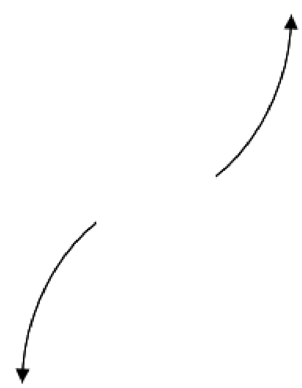
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{5}, -3, \text{ and } 6$$

The solution is $5x^3 - 8x^2 - 111x - 126$, which is option D.

A. $a \in [3, 6], b \in [-55, -49], c \in [153, 155],$ and $d \in [-130, -115]$

   $5x^3 - 52x^2 + 153x - 126$, which corresponds to multiplying out $(5x - 7)(x - 3)(x - 6)$.

B. $a \in [3, 6], b \in [4, 14], c \in [-114, -110],$ and $d \in [124, 127]$

   $5x^3 + 8x^2 - 111x + 126$, which corresponds to multiplying out $(5x - 7)(x - 3)(x + 6)$.

C. $a \in [3, 6], b \in [-11, -4], c \in [-114, -110],$ and $d \in [124, 127]$

$5x^3 - 8x^2 - 111x + 126$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [3, 6], b \in [-11, -4], c \in [-114, -110],$ and $d \in [-130, -115]$

* $5x^3 - 8x^2 - 111x - 126$, which is the correct option.

E. $a \in [3, 6], b \in [-25, -18], c \in [-72, -66],$ and $d \in [124, 127]$

$5x^3 - 22x^2 - 69x + 126$, which corresponds to multiplying out $(5x - 7)(x + 3)(x - 6)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 7)(x + 3)(x - 6)$

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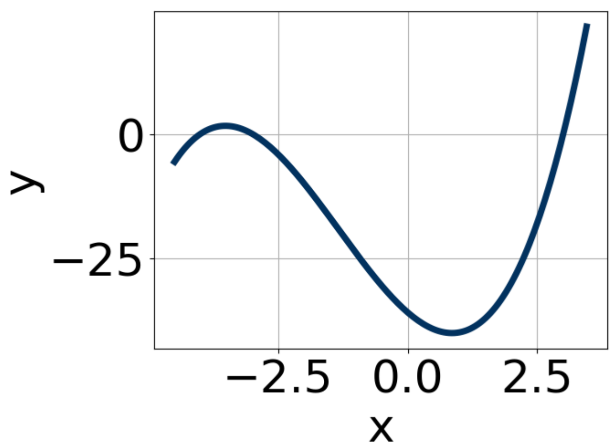
6. Which of the following equations *could* be of the graph presented below?



The solution is $2(x - 3)^7(x + 3)^9(x + 4)^7$, which is option D.

A. $15(x - 3)^4(x + 3)^{10}(x + 4)^{11}$

The factors 3 and -3 have have been odd power.

B. $-19(x - 3)^6(x + 3)^9(x + 4)^9$

The factor $(x - 3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $15(x - 3)^6(x + 3)^9(x + 4)^7$

The factor 3 should have been an odd power.

D. $2(x - 3)^7(x + 3)^9(x + 4)^7$

* This is the correct option.

E. $-10(x - 3)^7(x + 3)^{11}(x + 4)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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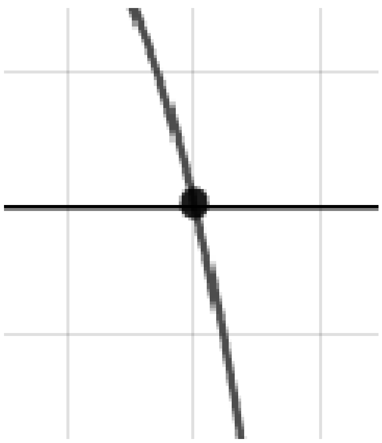
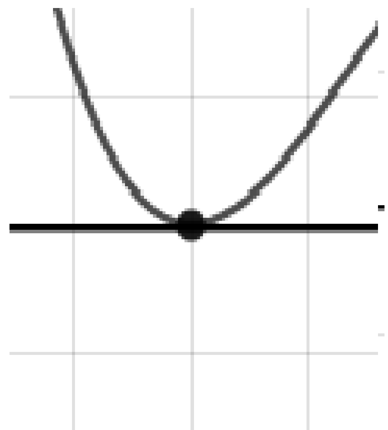
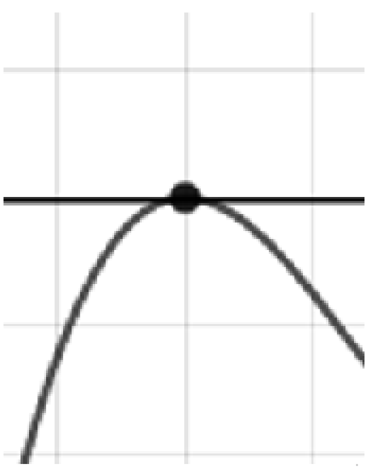
7. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

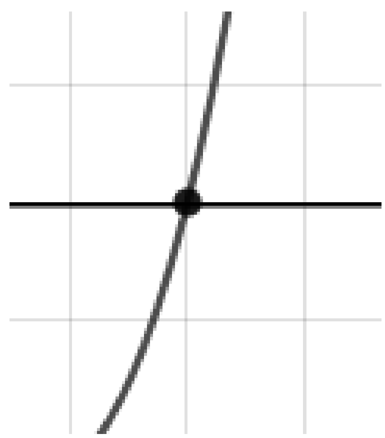
$$f(x) = 4(x + 5)^5(x - 5)^8(x - 2)^4(x + 2)^8$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain

the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 3i \text{ and } -2$$

The solution is $x^3 - 8x^2 + 14x + 68$, which is option C.

A. $b \in [0, 5], c \in [-4, 2],$ and $d \in [-16, -1]$

$x^3 + x^2 - 3x - 10$, which corresponds to multiplying out $(x - 5)(x + 2)$.

B. $b \in [6, 12], c \in [8, 22],$ and $d \in [-76, -63]$

$x^3 + 8x^2 + 14x - 68$, which corresponds to multiplying out $(x - (5 - 3i))(x - (5 + 3i))(x - 2)$.

C. $b \in [-8, -4], c \in [8, 22],$ and $d \in [63, 75]$

* $x^3 - 8x^2 + 14x + 68$, which is the correct option.

D. $b \in [0, 5], c \in [4, 13],$ and $d \in [-1, 10]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 3)(x + 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 - 3i))(x - (5 + 3i))(x - (-2))$.

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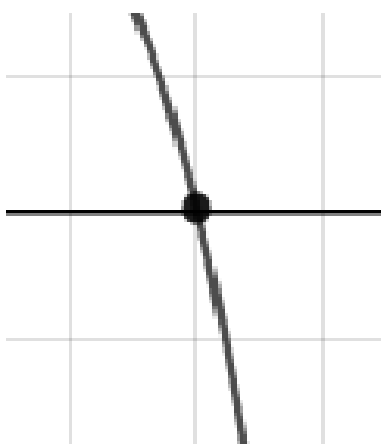
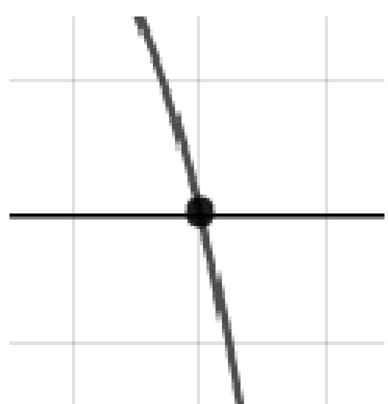
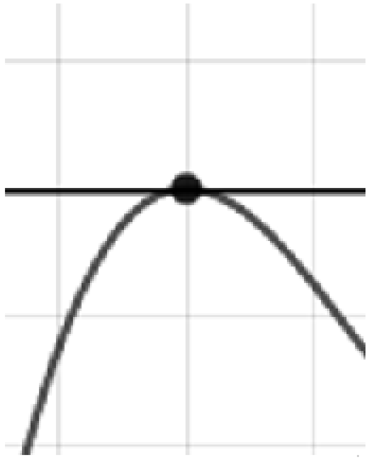
9. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

$$f(x) = -5(x + 9)^2(x - 9)^3(x - 8)^2(x + 8)^5$$

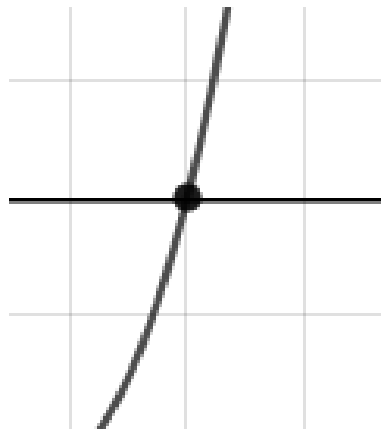
The solution is the graph below, which is option A.





B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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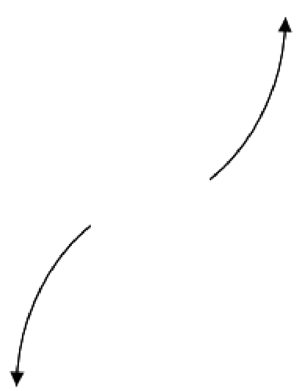
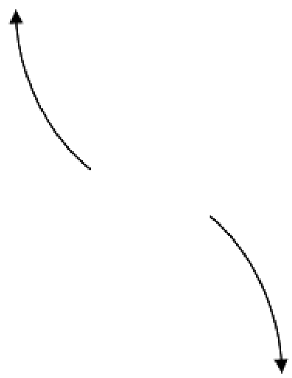
10. Describe the end behavior of the polynomial below.

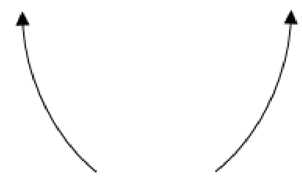
$$f(x) = 8(x+2)^5(x-2)^8(x+3)^2(x-3)^2$$

The solution is the graph below, which is option D.

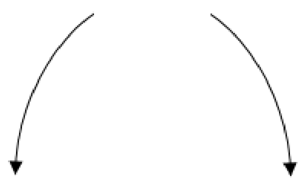




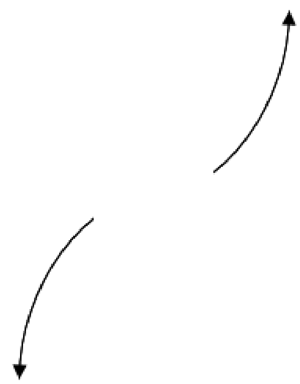
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.