This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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66. Factor the polynomial below completely, knowing that $x + 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 - 7x^3 - 172x^2 + 112x + 192$$

The solution is $[-4, -0.8, 1.5, 4]$

A. $z_1 \in [-5, -3]$, $z_2 \in [-1.44, -1.13]$, $z_3 \in [0.36, 0.71]$, and $z_4 \in [2, 6]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-5, -3]$, $z_2 \in [-0.37, -0.25]$, $z_3 \in [3.94, 4.03]$, and $z_4 \in [2, 6]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-5, -3]$, $z_2 \in [-1.65, -1.31]$, $z_3 \in [0.67, 0.93]$, and $z_4 \in [2, 6]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-5, -3]$, $z_2 \in [-0.99, -0.67]$, $z_3 \in [1.33, 1.68]$, and $z_4 \in [2, 6]$

* This is the solution!

E. $z_1 \in [-5, -3]$, $z_2 \in [-0.75, -0.48]$, $z_3 \in [1.19, 1.42]$, and $z_4 \in [2, 6]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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67. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 4x^3 - 24x^2 + 5x + 75$$

The solution is $[-1.5, 2.5, 5]$

A. $z_1 \in [-5.6, -3.7]$, $z_2 \in [-2.9, -2.3]$, and $z_3 \in [0.7, 2]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5.6, -3.7]$, $z_2 \in [-0.8, -0.1]$, and $z_3 \in [0.6, 0.9]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-1.2, 0.2]$, $z_2 \in [-0.1, 1.3]$, and $z_3 \in [4.3, 6.5]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.1, -0.7]$, $z_2 \in [1.8, 4.1]$, and $z_3 \in [4.3, 6.5]$

* This is the solution!

E. $z_1 \in [-5.6, -3.7],\ z_2 \in [-1.8, -0.5],$ and $z_3 \in [2, 4.1]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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68. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 - 56x^2 + 12x + 84}{x - 4}$$

The solution is $12x^2 - 8x - 20 + \frac{4}{x-4}$

A. $a \in [11, 13],\ b \in [-13, -4],\ c \in [-27, -16],$ and $r \in [-4, 8]$.

* This is the solution!

B. $a \in [41, 54],\ b \in [132, 138],\ c \in [553, 566],$ and $r \in [2307, 2309]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [11, 13],\ b \in [-24, -16],\ c \in [-53, -46],$ and $r \in [-62, -57]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [41, 54],\ b \in [-254, -243],\ c \in [999, 1006],$ and $r \in [-3936, -3929]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [11, 13],\ b \in [-112, -92],\ c \in [426, 437],$ and $r \in [-1631, -1620]$.

You divided by the opposite of the factor.

General Comments: Be sure to synthetically divide by the zero of the denominator!

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69. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 + 65x^2 - 84}{x + 4}$$

The solution is $15x^2 + 5x - 20 + \frac{-4}{x+4}$

A. $a \in [13, 20], b \in [122, 127], c \in [492, 504],$ and $r \in [1910, 1918]$.

You divided by the opposite of the factor.

B. $a \in [13, 20], b \in [4, 10], c \in [-21, -18],$ and $r \in [-5, 3]$.

* This is the solution!

C. $a \in [-63, -59], b \in [301, 309], c \in [-1222, -1219],$ and $r \in [4792, 4797]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [-63, -59], b \in [-181, -169], c \in [-703, -698],$ and $r \in [-2886, -2879]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [13, 20], b \in [-14, -5], c \in [48, 54]$, and $r \in [-338, -329]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comments: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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70. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^3 + 3x^2 + 2x + 2$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

A. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

B. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comments: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.
