This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 87x^2 - 80x - 72}{x + 5}$$

The solution is $20x^2 - 13x - 15 + \frac{3}{x+5}$, which is option E.

A. $a \in [19, 26],\ b \in [185, 190],\ c \in [847, 860],$ and $r \in [4200, 4206]$.

You divided by the opposite of the factor.

B. $a \in [19, 26],\ b \in [-42, -29],\ c \in [112, 121],$ and $r \in [-785, -778]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-101, -95],\ b \in [-414, -407],\ c \in [-2147, -2143],$ and $r \in [-10799, -10794]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

D. $a \in [-101, -95],\ b \in [585, 592],\ c \in [-3015, -3012],$ and $r \in [14999, 15004]$.

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [19, 26],\ b \in [-16, -11],\ c \in [-16, -4],$ and $r \in [-2, 6]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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2. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^4 + 6x^3 + 7x^2 + 7x + 2$$

The solution is All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$, which is option B.

A. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

B. All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Factor the polynomial below completely, knowing that $x+3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^4 + 127x^3 + 46x^2 - 415x + 150$$

The solution is $[-5, -3, 0.4, 1.25]$, which is option D.

A. $z_1 \in [-3.5, -1.5], \; z_2 \in [-1.11, -0.49], z_3 \in [2.9, 3.27], \text{ and } z_4 \in [4.3, 6.2]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5, -3], \; z_2 \in [-3.21, -2.66], z_3 \in [0.66, 0.81], \text{ and } z_4 \in [2.3, 2.6]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-1.25, 2.75], \; z_2 \in [-0.65, -0.3], z_3 \in [2.9, 3.27], \text{ and } z_4 \in [4.3, 6.2]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-5, -3], \; z_2 \in [-3.21, -2.66], z_3 \in [-0.12, 0.4], \text{ and } z_4 \in [-0.3, 1.8]$

* This is the solution!

E. $z_1 \in [-5, -3], \; z_2 \in [-0.2, 0.37], z_3 \in [2.9, 3.27], \text{ and } z_4 \in [4.3, 6.2]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 76x^2 - 32x + 59}{x - 4}$$

The solution is $20x^2 + 4x - 16 + \dfrac{-5}{x-4}$, which is option E.

A. $a \in [18, 25], \; b \in [-158, -152], \; c \in [589, 597], \text{ and } r \in [-2311, -2305].$

You divided by the opposite of the factor.

B. $a \in [76, 85], \; b \in [-399, -395], \; c \in [1550, 1562], \text{ and } r \in [-6154, -6144].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [76, 85], \; b \in [242, 248], \; c \in [943, 945], \text{ and } r \in [3830, 3837].$

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [18, 25], \; b \in [-21, -11], \; c \in [-82, -77], \text{ and } r \in [-181, -175]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [18, 25], \; b \in [2, 6], \; c \in [-19, -12], \text{ and } r \in [-7, -1]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 - 35x^2 + 24}{x - 2}$$

The solution is $15x^2 - 5x - 10 + \frac{4}{x-2}$, which is option C.

A. $a \in [28, 31], b \in [23, 32], c \in [47, 54], \text{ and } r \in [116, 130]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [15, 19], b \in [-22, -14], c \in [-23, -16], \text{ and } r \in [0, 7]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [15, 19], b \in [-7, -1], c \in [-17, -8], \text{ and } r \in [0, 7]$.

* This is the solution!

D. $a \in [15, 19], b \in [-65, -59], c \in [129, 134], \text{ and } r \in [-241, -231]$.

You divided by the opposite of the factor.

E. $a \in [28, 31], b \in [-104, -92], c \in [189, 194], \text{ and } r \in [-362, -355]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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6. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 6x^3 + 2x^2 + 7x + 7$$

The solution is All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$, which is option A.

A. All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

* This is the solution **since we asked for the possible Rational roots**!

B. All combinations of: $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 7$

This would have been the solution **if asked for the possible Integer roots**!

D. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 - 42x + 38}{x + 3}$$

The solution is $6x^2 - 18x + 12 + \frac{2}{x+3}$, which is option A.

A. $a \in [1, 11], b \in [-18, -14], c \in [11, 20], \text{ and } r \in [2, 8]$.

* This is the solution!

B. $a \in [-21, -13], b \in [46, 58], c \in [-206, -203], \text{ and } r \in [644, 651]$.

You multipled by the synthetic number rather than bringing the first factor down.

C. $a \in [1, 11], b \in [15, 25], c \in [11, 20], \text{ and } r \in [70, 78]$.

You divided by the opposite of the factor.

D. $a \in [-21, -13], b \in [-59, -48], c \in [-206, -203], \text{ and } r \in [-576, -567]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [1, 11], b \in [-25, -23], c \in [54, 59], \text{ and } r \in [-181, -170]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 + 29x^2 - 20x - 75$$

The solution is $[-5, -1.5, 1.6666666666666667]$, which is option D.

A. $z_1 \in [-0.76, -0.43], \; z_2 \in [0.2, 1.1], \text{ and } z_3 \in [4, 8]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-1.72, -1.37], \; z_2 \in [1, 1.8], \text{ and } z_3 \in [4, 8]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-5.03, -4.73], \; z_2 \in [-1.4, 0.4], \text{ and } z_3 \in [0.6, 1.6]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-5.03, -4.73], \; z_2 \in [-3.4, -0.7], \text{ and } z_3 \in [0.67, 2.67]$

* This is the solution!

E. $z_1 \in [-1.13, -0.74], \; z_2 \in [2.5, 3.6], \text{ and } z_3 \in [4, 8]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 4x^3 - 49x - 60$$

The solution is $[-2.5, -1.5, 4]$, which is option A.

A. $z_1 \in [-2.5, -1.5], \; z_2 \in [-1.97, -0.97], \text{ and } z_3 \in [3.2, 4.3]$

* This is the solution!

B. $z_1 \in [-6, -3], \; z_2 \in [0.27, 0.52], \text{ and } z_3 \in [-0.1, 0.8]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-0.67, 0.33], \; z_2 \in [-0.59, -0.2], \text{ and } z_3 \in [3.2, 4.3]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-6, -3], \; z_2 \in [0.52, 0.99], \text{ and } z_3 \in [4.2, 6.5]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-6, -3], \; z_2 \in [1.43, 2.03], \text{ and } z_3 \in [2, 3.6]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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10. Factor the polynomial below completely, knowing that $x-3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^4 - 7x^3 - 118x^2 + 305x - 150$$

The solution is $[-5, 0.6666666666666666, 2.5, 3]$, which is option E.

A. $z_1 \in [-4.1, -2.1], \; z_2 \in [-1.61, -1.37], z_3 \in [-0.54, -0.06], \text{ and } z_4 \in [4.9, 6.7]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5.9, -3.7], \; z_2 \in [0.34, 0.44], z_3 \in [1.16, 1.7], \text{ and } z_4 \in [2.1, 3.9]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-4.1, -2.1], \; z_2 \in [-2.02, -1.74], z_3 \in [-0.91, -0.7], \text{ and } z_4 \in [4.9, 6.7]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-4.1, -2.1], \; z_2 \in [-2.63, -2.41], z_3 \in [-0.68, -0.44], \text{ and } z_4 \in [4.9, 6.7]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-5.9, -3.7]$, $z_2 \in [0.46, 0.72]$, $z_3 \in [2.32, 2.9]$, and $z_4 \in [2.1, 3.9]$

   * This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.