This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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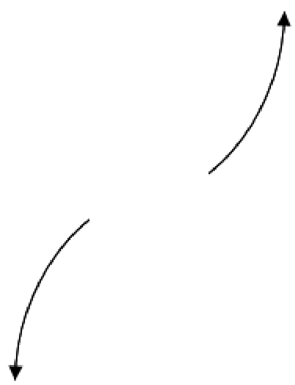
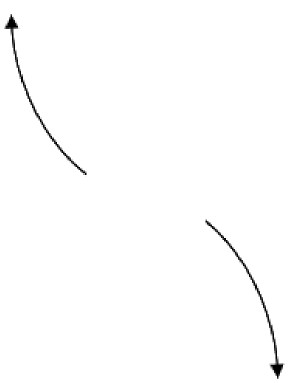
1. Describe the end behavior of the polynomial below.

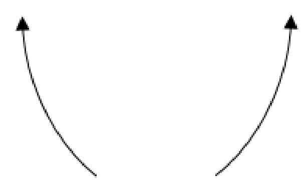
$$f(x) = 8(x-3)^4(x+3)^5(x+4)^5(x-4)^5$$

The solution is the graph below, which is option D.

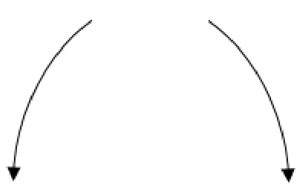




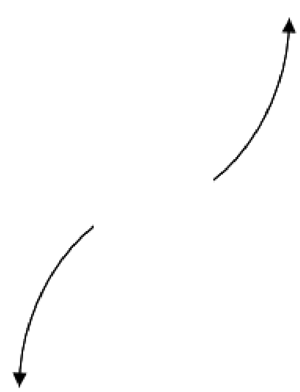
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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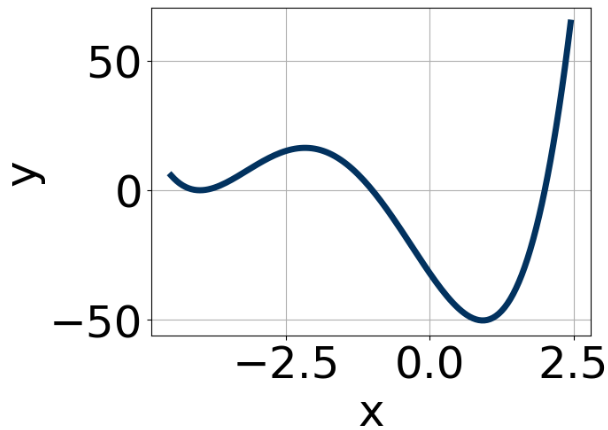
2. Which of the following equations *could* be of the graph presented below?



The solution is $19(x+4)^4(x-2)^7(x+1)^7$, which is option E.

A. $-4(x+4)^6(x-2)^9(x+1)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $16(x+4)^6(x-2)^4(x+1)^5$

The factor $(x-2)$ should have an odd power.

C. $-16(x+4)^{10}(x-2)^{11}(x+1)^{10}$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $13(x+4)^9(x-2)^8(x+1)^7$

The factor -4 should have an even power and the factor 2 should have an odd power.

E. $19(x+4)^4(x-2)^7(x+1)^7$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$\frac{-2}{3}, \frac{3}{5}, \text{ and } \frac{2}{5}$$

The solution is $75x^3-25x^2-32x+12$, which is option B.

A. $a \in [72, 77], b \in [-130, -118], c \in [63, 76], \text{ and } d \in [-16, -11]$

$75x^3-125x^2+68x-12$, which corresponds to multiplying out $(3x-2)(5x-3)(5x-2)$.

B. $a \in [72, 77], b \in [-30, -21], c \in [-35, -29]$, and $d \in [9, 15]$

* $75x^3 - 25x^2 - 32x + 12$, which is the correct option.

C. $a \in [72, 77], b \in [-38, -33], c \in [-30, -21]$, and $d \in [9, 15]$

$75x^3 - 35x^2 - 28x + 12$, which corresponds to multiplying out $(3x - 2)(5x + 3)(5x - 2)$.

D. $a \in [72, 77], b \in [24, 26], c \in [-35, -29]$, and $d \in [-16, -11]$

$75x^3 + 25x^2 - 32x - 12$, which corresponds to multiplying out $(3x - 2)(5x + 3)(5x + 2)$.

E. $a \in [72, 77], b \in [-30, -21], c \in [-35, -29]$, and $d \in [-16, -11]$

$75x^3 - 25x^2 - 32x - 12$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 2)(5x - 3)(5x - 2)$

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 5i \text{ and } 1$$

The solution is $x^3 + 7x^2 + 33x - 41$, which is option D.

A. $b \in [-11, -2], c \in [30.6, 36.6]$, and $d \in [35.9, 45.1]$

$x^3 - 7x^2 + 33x + 41$, which corresponds to multiplying out $(x - (-4 - 5i))(x - (-4 + 5i))(x + 1)$.

B. $b \in [0, 5], c \in [3.2, 6.3]$, and $d \in [-5.5, -4.6]$

$x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

C. $b \in [0, 5], c \in [2.1, 3.5]$, and $d \in [-4.1, -3.1]$

$x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

D. $b \in [6, 10], c \in [30.6, 36.6]$, and $d \in [-41.6, -37.1]$

* $x^3 + 7x^2 + 33x - 41$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 - 5i))(x - (-4 + 5i))(x - (1))$.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$7, -4, \text{ and } \frac{1}{3}$$

The solution is $3x^3 - 10x^2 - 81x + 28$, which is option A.

A. $a \in [-3, 7], b \in [-12, -3], c \in [-85, -77]$, and $d \in [25, 30]$

* $3x^3 - 10x^2 - 81x + 28$, which is the correct option.

B. $a \in [-3, 7], b \in [-12, -3], c \in [-85, -77],$ and $d \in [-33, -24]$

$3x^3 - 10x^2 - 81x - 28$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [-3, 7], b \in [9, 15], c \in [-85, -77],$ and $d \in [-33, -24]$

$3x^3 + 10x^2 - 81x - 28$, which corresponds to multiplying out $(x + 7)(x - 4)(3x + 1)$.

D. $a \in [-3, 7], b \in [28, 35], c \in [71, 78],$ and $d \in [-33, -24]$

$3x^3 + 32x^2 + 73x - 28$, which corresponds to multiplying out $(x + 7)(x + 4)(3x - 1)$.

E. $a \in [-3, 7], b \in [8, 9], c \in [-89, -86],$ and $d \in [25, 30]$

$3x^3 + 8x^2 - 87x + 28$, which corresponds to multiplying out $(x + 7)(x - 4)(3x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-7)(x+4)(3x-1)$

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 3i \text{ and } 1$$

The solution is $x^3 + 3x^2 + 9x - 13$, which is option C.

A. $b \in [-4.28, -2.11], c \in [7.59, 9.76],$ and $d \in [12.72, 14.46]$

$x^3 - 3x^2 + 9x + 13$, which corresponds to multiplying out $(x - (-2 - 3i))(x - (-2 + 3i))(x + 1)$.

B. $b \in [0.04, 2.08], c \in [-0.53, 1.89],$ and $d \in [-2.5, -1.82]$

$x^3 + x^2 + x - 2$, which corresponds to multiplying out $(x + 2)(x - 1)$.

C. $b \in [2.7, 3.9], c \in [7.59, 9.76],$ and $d \in [-13.96, -12.08]$

* $x^3 + 3x^2 + 9x - 13$, which is the correct option.

D. $b \in [0.04, 2.08], c \in [1.44, 2.84],$ and $d \in [-3.54, -2.67]$

$x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 3i))(x - (-2 + 3i))(x - (1))$.

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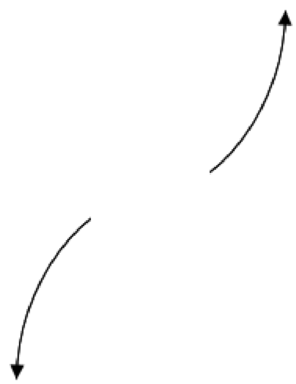
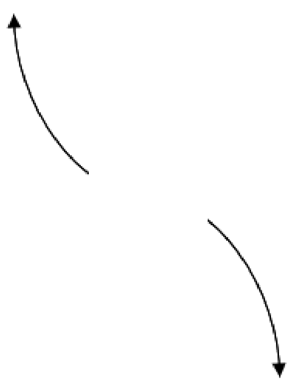
7. Describe the end behavior of the polynomial below.

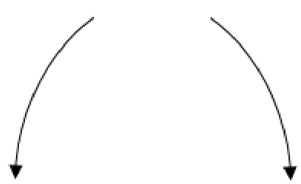
$$f(x) = 8(x + 4)^4(x - 4)^7(x - 3)^4(x + 3)^4$$

The solution is the graph below, which is option D.

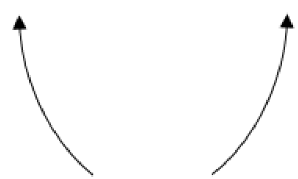




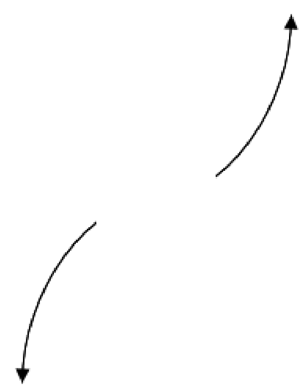
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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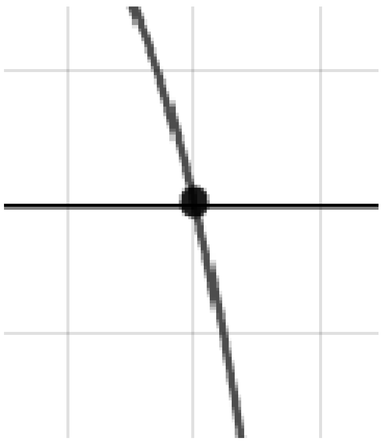
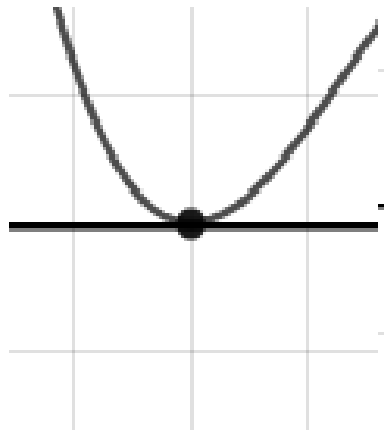
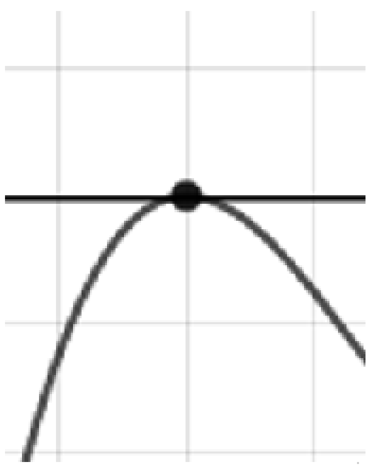
8. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

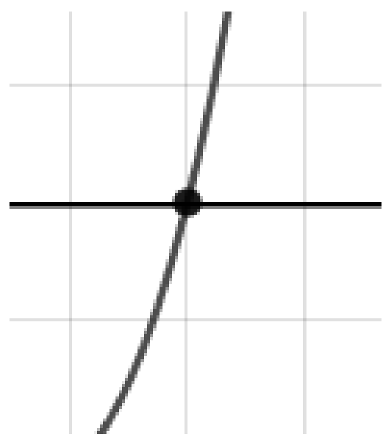
$$f(x) = 2(x-2)^3(x+2)^2(x-7)^{10}(x+7)^7$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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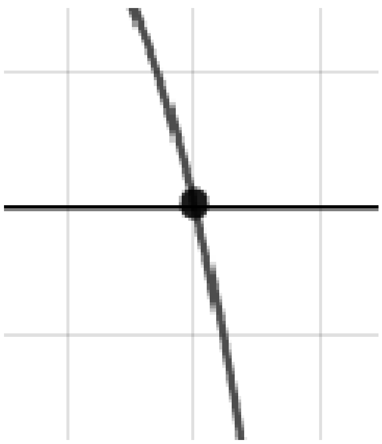
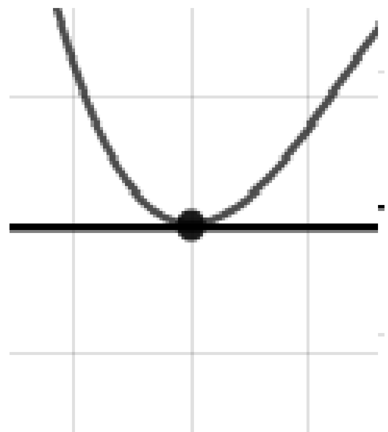
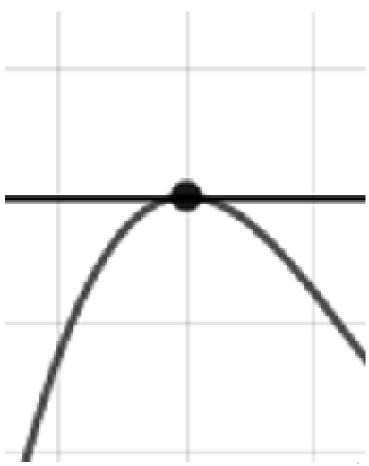
9. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

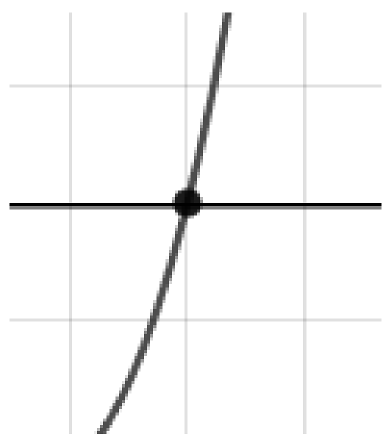
$$f(x) = 2(x+4)^{13}(x-4)^{9}(x+9)^{5}(x-9)^{4}$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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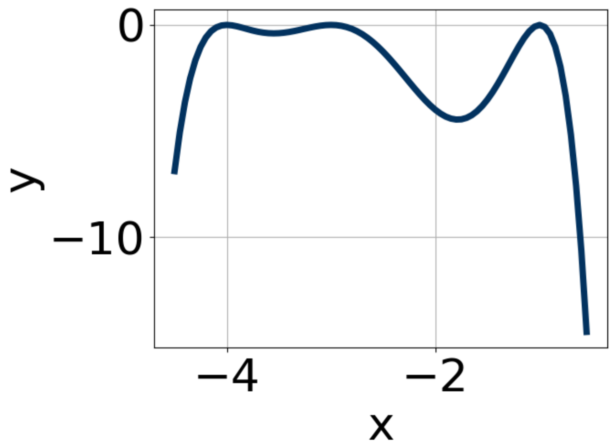
10. Which of the following equations *could* be of the graph presented below?



The solution is $-15(x+1)^4(x+4)^6(x+3)^6$, which is option C.

A. $-11(x+1)^6(x+4)^{11}(x+3)^7$

The factors $(x+4)$ and $(x+3)$ should both have even powers.

B. $18(x+1)^4(x+4)^{10}(x+3)^5$

The factor $(x+3)$ should have an even power and the leading coefficient should be the opposite sign.

C. $-15(x+1)^4(x+4)^6(x+3)^6$

* This is the correct option.

D. $19(x+1)^{10}(x+4)^4(x+3)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-15(x+1)^{10}(x+4)^6(x+3)^{11}$

The factor $(x+3)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).