This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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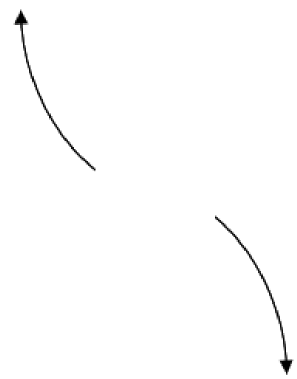
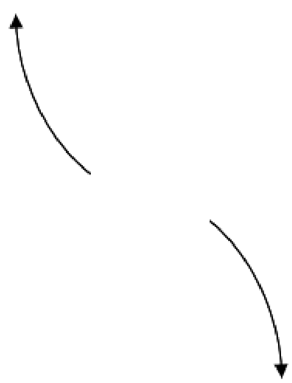
1. Describe the end behavior of the polynomial below.

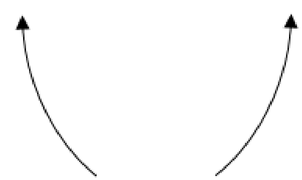
$$f(x) = -6(x+4)^5(x-4)^{10}(x+8)^2(x-8)^2$$

The solution is the graph below, which is option A.

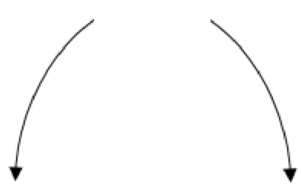




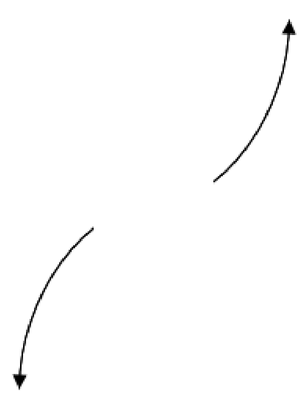
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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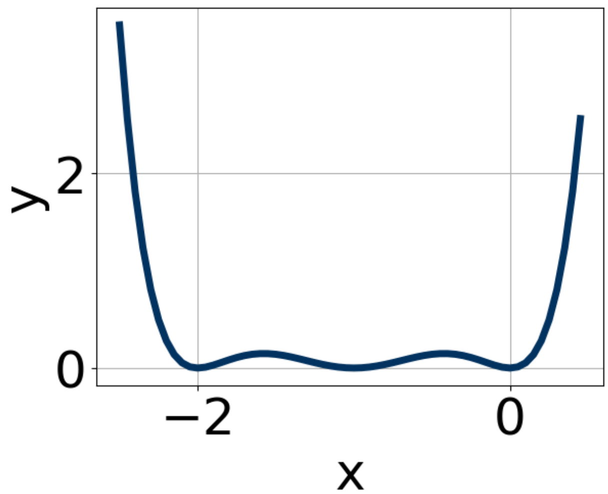
2. Which of the following equations *could* be of the graph presented below?



The solution is $18x^8(x+1)^8(x+2)^8$, which is option E.

A. $-3x^4(x+1)^8(x+2)^7$

The factor $(x+2)$ should have an even power and the leading coefficient should be the opposite sign.

B. $3x^{10}(x+1)^9(x+2)^9$

The factors $(x+1)$ and $(x+2)$ should both have even powers.

C. $-10x^6(x+1)^6(x+2)^8$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $14x^8(x+1)^{10}(x+2)^5$

The factor $(x+2)$ should have an even power.

E. $18x^8(x+1)^8(x+2)^8$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, \frac{-1}{5}, \text{ and } \frac{1}{2}$$

The solution is $40x^3 - 22x^2 - x + 1$, which is option A.

A. $a \in [38, 42], b \in [-22.1, -20.7], c \in [-1.03, 0.65],$ and $d \in [0.99, 2.17]$

* $40x^3 - 22x^2 - x + 1$, which is the correct option.

B. $a \in [38, 42], b \in [-6.3, -1.7], c \in [-8.44, -6.66],$ and $d \in [-2.34, -0.38]$

$40x^3 - 2x^2 - 7x - 1$, which corresponds to multiplying out $(4x + 1)(5x + 1)(2x - 1)$.

C. $a \in [38, 42], b \in [-18.8, -14.4], c \in [-4.18, -1.92],$ and $d \in [0.99, 2.17]$

$40x^3 - 18x^2 - 3x + 1$, which corresponds to multiplying out $(4x + 1)(5x - 1)(2x - 1)$.

D. $a \in [38, 42], b \in [19.4, 22.1], c \in [-1.03, 0.65],$ and $d \in [-2.34, -0.38]$

$40x^3 + 22x^2 - x - 1$, which corresponds to multiplying out $(4x + 1)(5x - 1)(2x + 1)$.

E. $a \in [38, 42], b \in [-22.1, -20.7], c \in [-1.03, 0.65],$ and $d \in [-2.34, -0.38]$

$40x^3 - 22x^2 - x - 1$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(5x + 1)(2x - 1)$

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, \frac{-7}{3}, \text{ and } \frac{3}{4}$$

The solution is $48x^3 + 64x^2 - 103x + 21$, which is option C.

A. $a \in [48, 52], b \in [61, 71], c \in [-104, -99],$ and $d \in [-22, -14]$

$48x^3 + 64x^2 - 103x - 21$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [48, 52], b \in [83, 93], c \in [-72, -63],$ and $d \in [-22, -14]$

$48x^3 + 88x^2 - 65x - 21$, which corresponds to multiplying out $(4x + 1)(3x + 7)(4x - 3)$.

C. $a \in [48, 52], b \in [61, 71], c \in [-104, -99],$ and $d \in [19, 28]$

* $48x^3 + 64x^2 - 103x + 21$, which is the correct option.

D. $a \in [48, 52], b \in [-65, -63], c \in [-104, -99],$ and $d \in [-22, -14]$

$48x^3 - 64x^2 - 103x - 21$, which corresponds to multiplying out $(4x + 1)(3x - 7)(4x + 3)$.

E. $a \in [48, 52], b \in [-144, -132], c \in [38, 50],$ and $d \in [19, 28]$

$48x^3 - 136x^2 + 47x + 21$, which corresponds to multiplying out $(4x + 1)(3x - 7)(4x - 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(3x + 7)(4x - 3)$

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 + 4i \text{ and } -1$$

The solution is $x^3 - 5x^2 + 19x + 25$, which is option A.

A. $b \in [-7, -4], c \in [18.23, 21.41]$, and $d \in [23.7, 26.32]$

  * $x^3 - 5x^2 + 19x + 25$, which is the correct option.

B. $b \in [1, 3], c \in [-2.11, -1.71]$, and $d \in [-3.94, -2.28]$

  $x^3 + x^2 - 2x - 3$, which corresponds to multiplying out $(x - 3)(x + 1)$.

C. $b \in [1, 3], c \in [-3.03, -2.44]$, and $d \in [-6.22, -3.95]$

  $x^3 + x^2 - 3x - 4$, which corresponds to multiplying out $(x - 4)(x + 1)$.

D. $b \in [3, 17], c \in [18.23, 21.41]$, and $d \in [-25.16, -24.62]$

  $x^3 + 5x^2 + 19x - 25$, which corresponds to multiplying out $(x - (3 + 4i))(x - (3 - 4i))(x - 1)$.

E. None of the above.

  This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 + 4i))(x - (3 - 4i))(x - (-1))$.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 2i \text{ and } -3$$

The solution is $x^3 + 9x^2 + 31x + 39$, which is option D.

A. $b \in [-13, -6], c \in [30.93, 31.32]$, and $d \in [-42.1, -37.4]$

  $x^3 - 9x^2 + 31x - 39$, which corresponds to multiplying out $(x - (-3 - 2i))(x - (-3 + 2i))(x - 3)$.

B. $b \in [-7, 3], c \in [5.16, 6.14]$, and $d \in [8.1, 9.3]$

  $x^3 + x^2 + 6x + 9$, which corresponds to multiplying out $(x + 3)(x + 3)$.

C. $b \in [-7, 3], c \in [4.11, 5.57]$, and $d \in [3.2, 8.6]$

  $x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 2)(x + 3)$.

D. $b \in [7, 11], c \in [30.93, 31.32]$, and $d \in [38.3, 41.1]$

  * $x^3 + 9x^2 + 31x + 39$, which is the correct option.

E. None of the above.

  This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 2i))(x - (-3 + 2i))(x - (-3))$.

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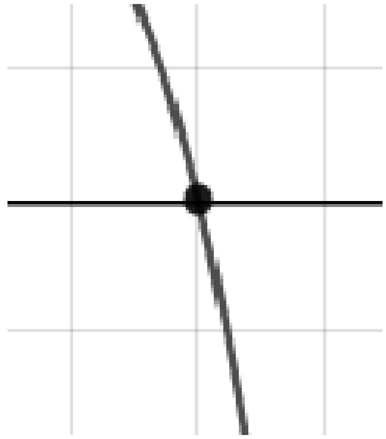
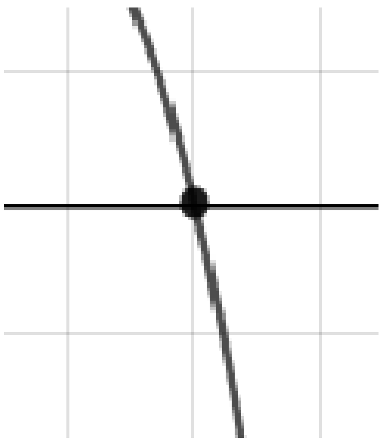
7. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

$$f(x) = -7(x - 8)^4(x + 8)^3(x - 9)^{11}(x + 9)^8$$

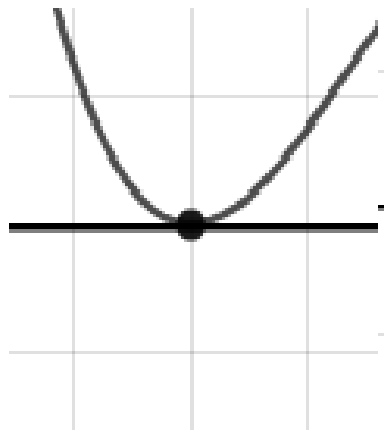
The solution is the graph below, which is option A.



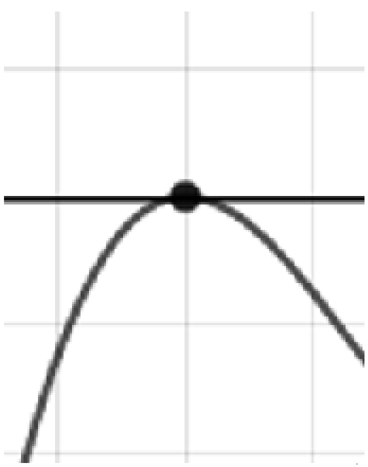
A.



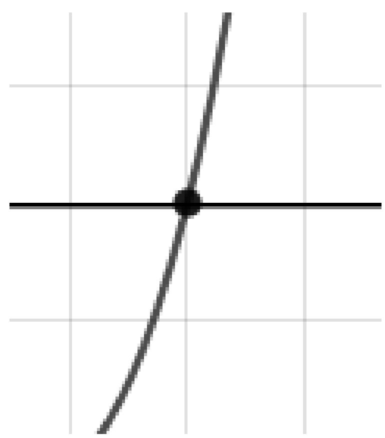
C.



B.



D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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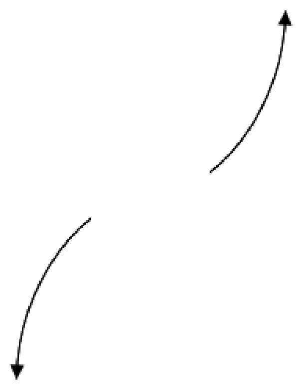
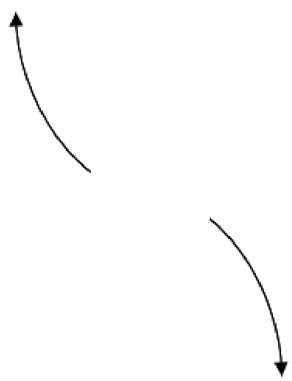
8. Describe the end behavior of the polynomial below.

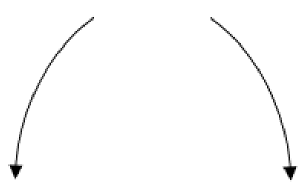
$$f(x) = 8(x-4)^5(x+4)^8(x+5)^2(x-5)^4$$

The solution is the graph below, which is option D.

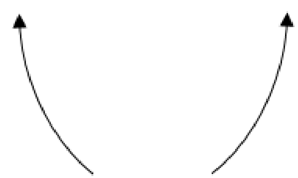




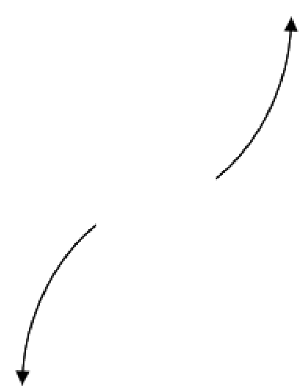
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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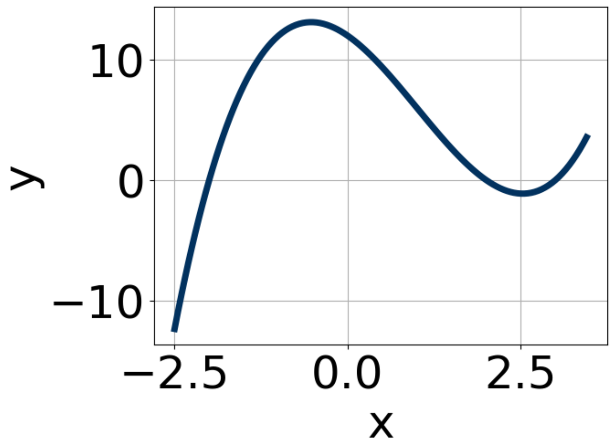
9. Which of the following equations *could* be of the graph presented below?



The solution is $3(x-2)^{11}(x-3)^5(x+2)^{11}$, which is option E.

A. $-16(x-2)^8(x-3)^5(x+2)^7$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-2(x-2)^7(x-3)^9(x+2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $2(x-2)^{10}(x-3)^8(x+2)^9$

The factors 2 and 3 have have been odd power.

D. $6(x-2)^4(x-3)^5(x+2)^9$

The factor 2 should have been an odd power.

E. $3(x-2)^{11}(x-3)^5(x+2)^{11}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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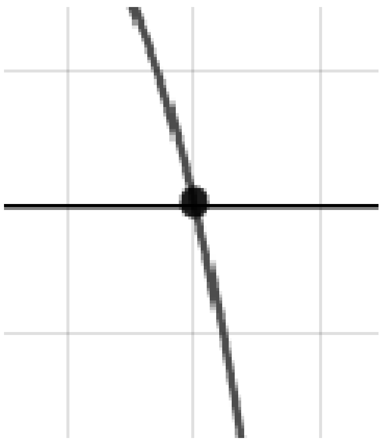
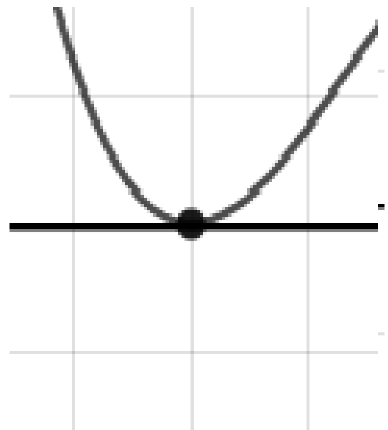
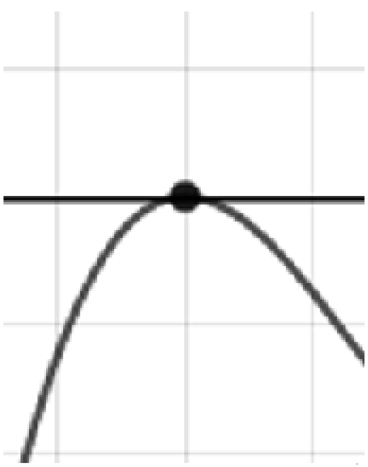
10. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

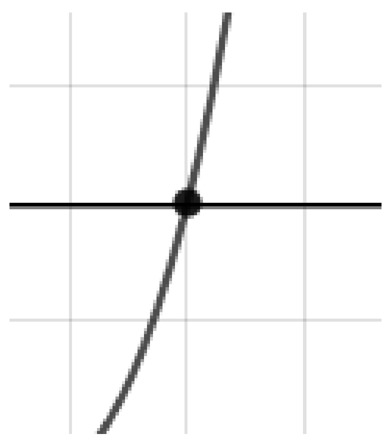
$$f(x) = -5(x+7)^9(x-7)^{14}(x-9)^5(x+9)^6$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.