This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{2}, \frac{3}{2}, \text{ and } \frac{7}{3}$$

The solution is $12x^3 - 76x^2 + 157x - 105$, which is option D.

A. $a \in [12, 15], b \in [13, 23], c \in [-67, -64], \text{ and } d \in [-114, -101]$

$12x^3 + 20x^2 - 67x - 105$, which corresponds to multiplying out $(2x + 5)(2x + 3)(3x - 7)$.

B. $a \in [12, 15], b \in [-77, -70], c \in [156, 158], \text{ and } d \in [105, 106]$

$12x^3 - 76x^2 + 157x + 105$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [12, 15], b \in [-24, -14], c \in [-83, -68], \text{ and } d \in [105, 106]$

$12x^3 - 16x^2 - 73x + 105$, which corresponds to multiplying out $(2x + 5)(2x - 3)(3x - 7)$.

D. $a \in [12, 15], b \in [-77, -70], c \in [156, 158], \text{ and } d \in [-114, -101]$

* $12x^3 - 76x^2 + 157x - 105$, which is the correct option.

E. $a \in [12, 15], b \in [70, 79], c \in [156, 158], \text{ and } d \in [105, 106]$

$12x^3 + 76x^2 + 157x + 105$, which corresponds to multiplying out $(2x + 5)(2x + 3)(3x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 5)(2x - 3)(3x - 7)$

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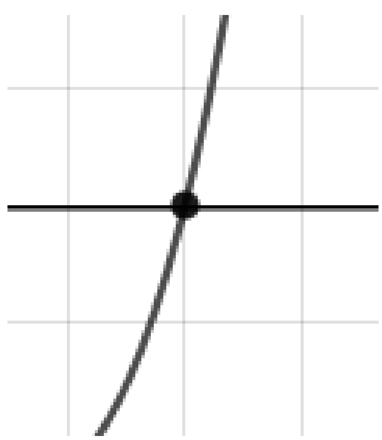
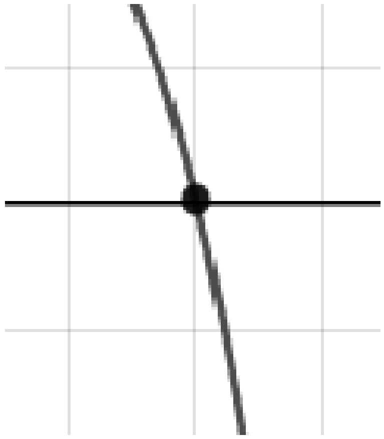
2. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

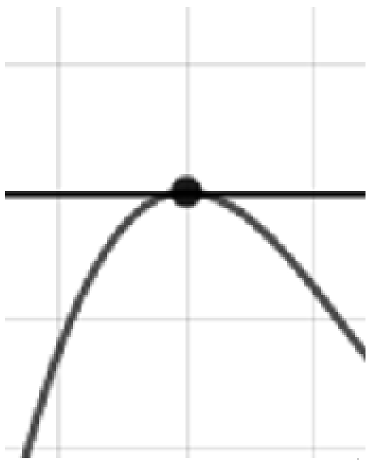
$$f(x) = 4(x - 2)^7(x + 2)^5(x - 4)^6(x + 4)^3$$

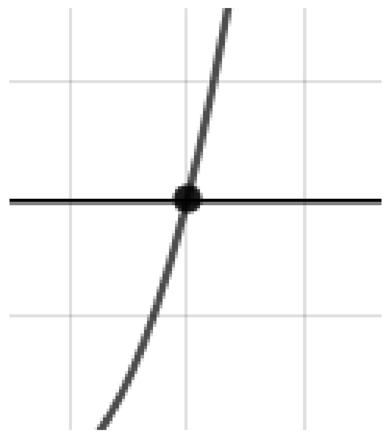
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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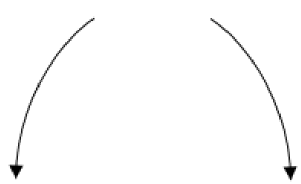
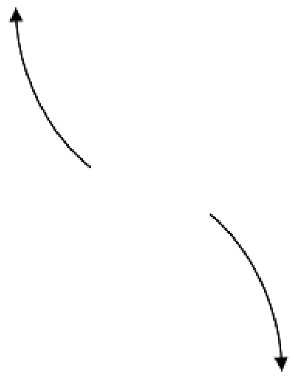
3. Describe the end behavior of the polynomial below.

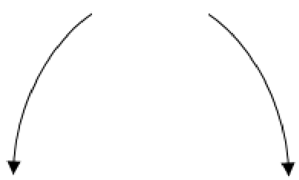
$$f(x) = -3(x-5)^2(x+5)^3(x-8)^5(x+8)^6$$

The solution is the graph below, which is option B.

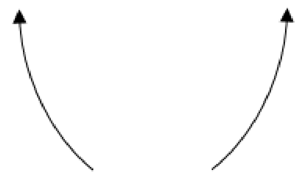




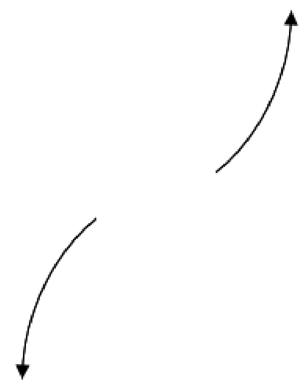
A.



B.



C.



D.

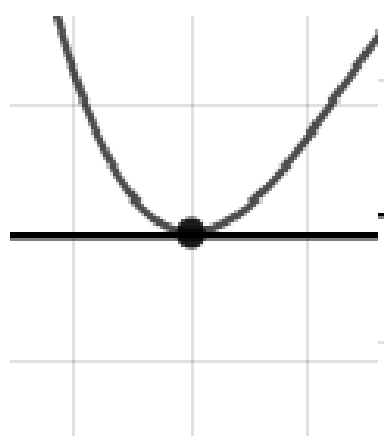
E. None of the above.

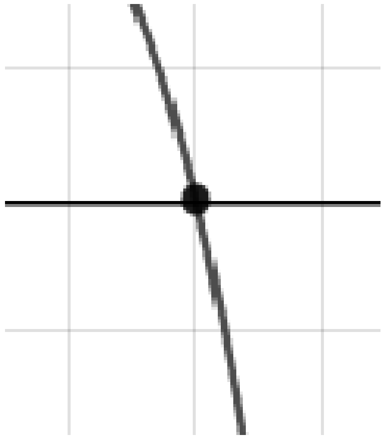
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

4. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

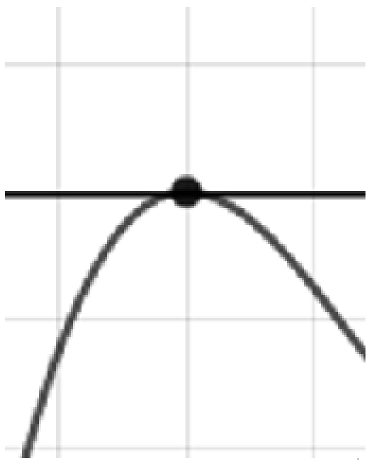
$$f(x) = 3(x-4)^8(x+4)^7(x-5)^{14}(x+5)^9$$

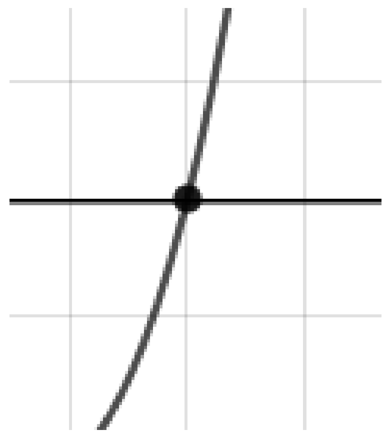
The solution is the graph below, which is option C.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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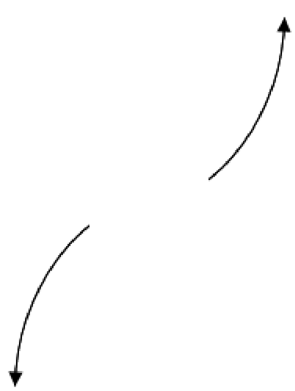
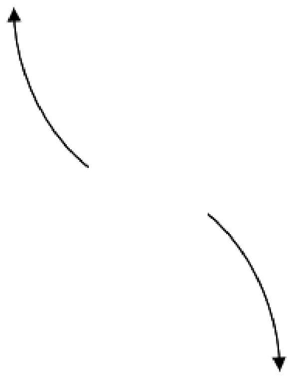
5. Describe the end behavior of the polynomial below.

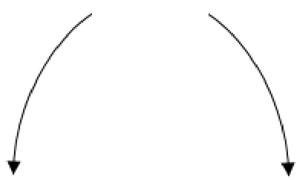
$$f(x) = 4(x+7)^2(x-7)^5(x-6)^2(x+6)^4$$

The solution is the graph below, which is option D.

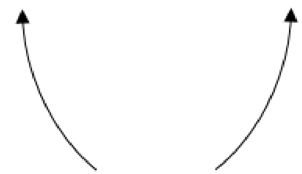




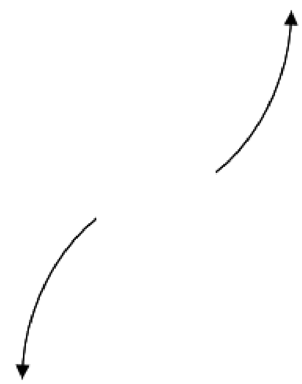
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 4i \text{ and } -3$$

The solution is $x^3 + 11x^2 + 56x + 96$, which is option C.

A. $b \in [1, 4], c \in [0, 11]$, and $d \in [12, 13]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 4)(x + 3)$.

B. $b \in [1, 4], c \in [-4, 2]$, and $d \in [-16, -4]$

$x^3 + x^2 - x - 12$, which corresponds to multiplying out $(x - 4)(x + 3)$.

C. $b \in [6, 13], c \in [54, 60]$, and $d \in [89, 97]$

* $x^3 + 11x^2 + 56x + 96$, which is the correct option.

D. $b \in [-13, -8], c \in [54, 60]$, and $d \in [-97, -90]$

$x^3 - 11x^2 + 56x - 96$, which corresponds to multiplying out $(x - (-4 + 4i))(x - (-4 - 4i))(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-4+4i))(x-(-4-4i))(x-(-3))$.

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form x^3+bx^2+cx+d.

$$4+2i \text{ and } -4$$

The solution is $x^3-4x^2-12x+80$, which is option B.

A. $b \in [0.3, 1.4], c \in [1.4, 6.2], \text{ and } d \in [-12, -6]$

x^3+x^2+2x-8, which corresponds to multiplying out $(x-2)(x+4)$.

B. $b \in [-7.4, -2], c \in [-12.6, -9.7], \text{ and } d \in [79, 85]$

* $x^3-4x^2-12x+80$, which is the correct option.

C. $b \in [3.9, 6.7], c \in [-12.6, -9.7], \text{ and } d \in [-83, -73]$

$x^3+4x^2-12x-80$, which corresponds to multiplying out $(x-(4+2i))(x-(4-2i))(x-4)$.

D. $b \in [0.3, 1.4], c \in [-0.1, 0.9], \text{ and } d \in [-23, -9]$

x^3+x^2-16, which corresponds to multiplying out $(x-4)(x+4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(4+2i))(x-(4-2i))(x-(-4))$.

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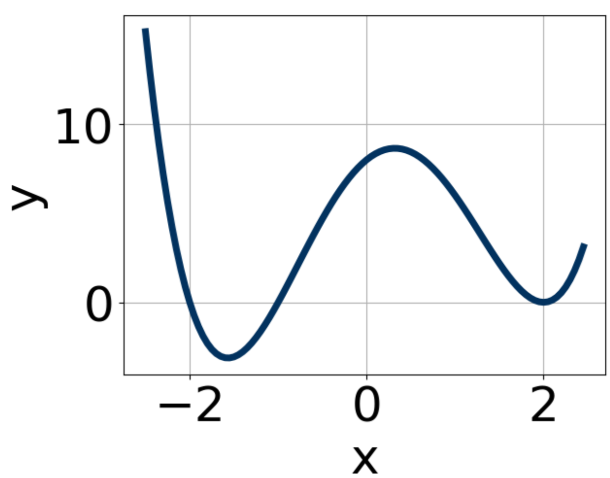
8. Which of the following equations *could* be of the graph presented below?



The solution is $6(x-2)^4(x+1)^{11}(x+2)^7$, which is option E.

A. $15(x-2)^7(x+1)^6(x+2)^9$

The factor 2 should have an even power and the factor -1 should have an odd power.

B. $-15(x-2)^6(x+1)^5(x+2)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $11(x-2)^4(x+1)^8(x+2)^{11}$

The factor $(x+1)$ should have an odd power.

D. $-18(x-2)^4(x+1)^{11}(x+2)^{10}$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $6(x-2)^4(x+1)^{11}(x+2)^7$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{4}{3}, -3, \text{ and } \frac{4}{5}$$

The solution is $15x^3 + 13x^2 - 80x + 48$, which is option E.

A. $a \in [15, 17], b \in [-14, -5], c \in [-84, -73], \text{ and } d \in [-50, -47]$

$15x^3 - 13x^2 - 80x - 48$, which corresponds to multiplying out $(3x+4)(x-3)(5x+4)$.

B. $a \in [15, 17], b \in [6, 16], c \in [-84, -73], \text{ and } d \in [-50, -47]$

$15x^3 + 13x^2 - 80x - 48$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [15, 17], b \in [52, 62], c \in [5, 13], \text{ and } d \in [-50, -47]$

$15x^3 + 53x^2 + 8x - 48$, which corresponds to multiplying out $(3x+4)(x+3)(5x-4)$.

D. $a \in [15, 17], b \in [-41, -33], c \in [-42, -35], \text{ and } d \in [43, 55]$

$15x^3 - 37x^2 - 40x + 48$, which corresponds to multiplying out $(3x+4)(x-3)(5x-4)$.

E. $a \in [15, 17], b \in [6, 16], c \in [-84, -73], \text{ and } d \in [43, 55]$

* $15x^3 + 13x^2 - 80x + 48$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x-4)(x+3)(5x-4)$

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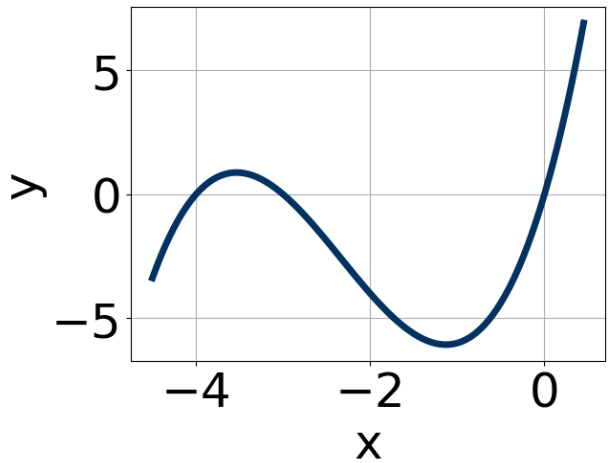
10. Which of the following equations *could* be of the graph presented below?



The solution is $7x^9(x+4)^{11}(x+3)^{11}$, which is option A.

A. $7x^9(x+4)^{11}(x+3)^{11}$

   * This is the correct option.

B. $11x^9(x+4)^6(x+3)^7$

   The factor -4 should have been an odd power.

C. $-17x^{11}(x+4)^8(x+3)^{11}$

   The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-15x^5(x+4)^9(x+3)^5$

   This corresponds to the leading coefficient being the opposite value than it should be.

E. $14x^{11}(x+4)^4(x+3)^4$

   The factors -4 and -3 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).