**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 14$ and choose the interval the $f^{-1}(14)$ belongs to.

$$f(x) = \sqrt[3]{5x - 3}$$

The solution is 549.4, which is option D.

A. $f^{-1}(14) \in [-550.33, -549.18]$

This solution corresponds to distractor 2.

B. $f^{-1}(14) \in [548, 548.78]$

Distractor 1: This corresponds to

C. $f^{-1}(14) \in [-548.46, -546.6]$

This solution corresponds to distractor 3.

D. $f^{-1}(14) \in [548.34, 551.9]$

* This is the correct solution.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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2. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 5x^3 + 8x^2 + 4x + 3 \text{ and } g(x) = \sqrt{4x + 17}$$

The solution is The domain is all Real numbers greater than or equal to$x = -4.25$., which is option A.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-6.25, 4.75]$

B. The domain is all Real numbers except $x = a,$ where $a \in [0.4, 6.4]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [4.8, 5.8]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [2.2, 5.2]$ and $b \in [-7.67, -2.67]$

E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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3. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-4x - 12} \text{ and } g(x) = 9x + 5$$

The solution is The domain is all Real numbers less than or equal to$x = -3.0$., which is option B.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-7.5, -2.5]$

B. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-4, 6]$

C. The domain is all Real numbers except $x = a$, where $a \in [1.8, 5.8]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-4.67, -1.67]$ and $b \in [2.25, 10.25]$

E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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4. Choose the interval below that $f$ composed with $g$ at $x = -2$ is in.

$$f(x) = 3x^3 + 3x^2 - 2x + 4 \text{ and } g(x) = x^3 + 4x^2 + 3x$$

The solution is 36.0, which is option A.

A. $(f \circ g)(-2) \in [31, 37]$

* This is the correct solution

B. $(f \circ g)(-2) \in [-13, -10]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-2) \in [19, 29]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-2) \in [-24, -20]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** $f$ composed with $g$ at $x$ means $f(g(x))$. The order matters!

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5. Choose the interval below that $f$ composed with $g$ at $x = 2$ is in.

$$f(x) = -x^3 + 3x^2 - 3x - 1 \text{ and } g(x) = -x^3 + 3x^2 - x$$

The solution is $-3.0$, which is option A.

A. $(f \circ g)(2) \in [-7, -1]$

* This is the correct solution

B. $(f \circ g)(2) \in [66, 69]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(2) \in [56, 59]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(2) \in [-10, -6]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** $f$ composed with $g$ at $x$ means $f(g(x))$. The order matters!

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6. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -12$ and choose the interval that $f^{-1}(-12)$ belongs to.

$$f(x) = 3x^2 + 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-12) \in [2.13, 2.95]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(-12) \in [1.24, 1.71]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

C. $f^{-1}(-12) \in [5.19, 6.21]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(-12) \in [3.3, 3.8]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

E. The function is not invertible for all Real numbers.

* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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7. Determine whether the function below is 1-1.

$$f(x) = -15x^2 + 212x - 672$$

The solution is no, which is option B.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is a $y$-value that goes to 2 different $x$-values.

* This is the solution.

C. No, because there is an $x$-value that goes to 2 different $y$-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a $y$-value that goes to 2 different $x$-values.

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8. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = e^{x+4} + 3$$

The solution is $f^{-1}(9) = -2.208$, which is option D.

A. $f^{-1}(9) \in [5.72, 5.84]$

This solution corresponds to distractor 1.

B. $f^{-1}(9) \in [5.52, 5.71]$

This solution corresponds to distractor 4.

C. $f^{-1}(9) \in [4.43, 4.65]$

This solution corresponds to distractor 3.

D. $f^{-1}(9) \in [-2.43, -2.1]$

This is the solution.

E. $f^{-1}(9) \in [5.36, 5.53]$

This solution corresponds to distractor 2.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the $x$ and $y$, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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9. Determine whether the function below is 1-1.

$$f(x) = \sqrt{5x - 21}$$

The solution is yes, which is option D.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is a $y$-value that goes to 2 different $x$-values.

Corresponds to the Horizontal Line test, which this function passes.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. Yes, the function is 1-1.

* This is the solution.

E. No, because there is an $x$-value that goes to 2 different $y$-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a $y$-value that goes to 2 different $x$-values.

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10. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = e^{x-4} + 3$$

The solution is $f^{-1}(9) = 5.792$, which is option B.

A. $f^{-1}(9) \in [5.33, 5.56]$

This solution corresponds to distractor 2.

B. $f^{-1}(9) \in [5.6, 5.89]$

This is the solution.

C. $f^{-1}(9) \in [4.59, 4.75]$

This solution corresponds to distractor 4.

D. $f^{-1}(9) \in [-2.44, -2.16]$

This solution corresponds to distractor 1.

E. $f^{-1}(9) \in [5.5, 5.62]$

This solution corresponds to distractor 3.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the $x$ and $y$, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.