This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-3, \frac{-3}{4}, \text{ and } \frac{7}{3}$$

The solution is $12x^3 + 17x^2 - 78x - 63$, which is option D.

A. $a \in [12, 13], b \in [14, 21], c \in [-78, -75], \text{ and } d \in [61, 66]$

$12x^3 + 17x^2 - 78x + 63$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [12, 13], b \in [-57, -54], c \in [22, 39], \text{ and } d \in [61, 66]$

$12x^3 - 55x^2 + 36x + 63$, which corresponds to multiplying out $(x - 3)(4x + 3)(3x - 7)$.

C. $a \in [12, 13], b \in [-23, -13], c \in [-78, -75], \text{ and } d \in [61, 66]$

$12x^3 - 17x^2 - 78x + 63$, which corresponds to multiplying out $(x - 3)(4x - 3)(3x + 7)$.

D. $a \in [12, 13], b \in [14, 21], c \in [-78, -75], \text{ and } d \in [-63, -57]$

* $12x^3 + 17x^2 - 78x - 63$, which is the correct option.

E. $a \in [12, 13], b \in [-78, -63], c \in [131, 136], \text{ and } d \in [-63, -57]$

$12x^3 - 73x^2 + 132x - 63$, which corresponds to multiplying out $(x - 3)(4x - 3)(3x - 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+3)(4x+3)(3x-7)$

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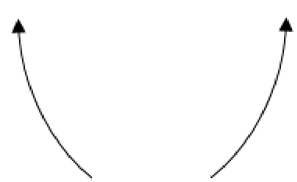
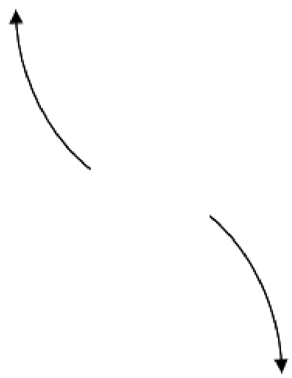
2. Describe the end behavior of the polynomial below.

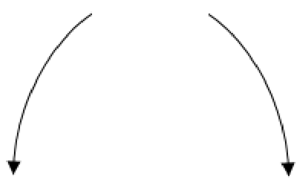
$$f(x) = 3(x - 9)^5(x + 9)^6(x - 3)^3(x + 3)^4$$

The solution is the graph below, which is option C.

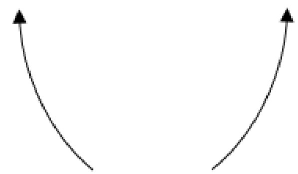




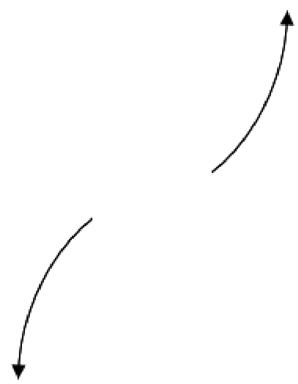
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 4i \text{ and } 2$$

The solution is $x^3 + 2x^2 + 12x - 40$, which is option D.

A. $b \in [0.5, 1.3], c \in [-0.75, 0.06],$ and $d \in [-6.3, -3.5]$

$x^3 + x^2 - 4$, which corresponds to multiplying out $(x + 2)(x - 2)$.

B. $b \in [0.5, 1.3], c \in [1.85, 3.06],$ and $d \in [-10.4, -7.1]$

$x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x + 4)(x - 2)$.

C. $b \in [-2.2, 0.7], c \in [11.7, 12.65],$ and $d \in [39.5, 42.6]$

$x^3 - 2x^2 + 12x + 40$, which corresponds to multiplying out $(x - (-2 - 4i))(x - (-2 + 4i))(x + 2)$.

D. $b \in [1.5, 2.9], c \in [11.7, 12.65],$ and $d \in [-41.8, -38.7]$

* $x^3 + 2x^2 + 12x - 40$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-2-4i))(x-(-2+4i))(x-(2))$.

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$\frac{7}{3}, \frac{-1}{4}, \text{ and } \frac{6}{5}$$

The solution is $60x^3-197x^2+115x+42$, which is option D.

A. $a \in [57,65], b \in [73,89], c \in [-153,-150],$ and $d \in [-43,-39]$

$60x^3+83x^2-151x-42$, which corresponds to multiplying out $(3x+7)(4x+1)(5x-6)$.

B. $a \in [57,65], b \in [-199,-195], c \in [108,120],$ and $d \in [-43,-39]$

$60x^3-197x^2+115x-42$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [57,65], b \in [196,200], c \in [108,120],$ and $d \in [-43,-39]$

$60x^3+197x^2+115x-42$, which corresponds to multiplying out $(3x+7)(4x-1)(5x+6)$.

D. $a \in [57,65], b \in [-199,-195], c \in [108,120],$ and $d \in [33,43]$

* $60x^3-197x^2+115x+42$, which is the correct option.

E. $a \in [57,65], b \in [47,60], c \in [-185,-182],$ and $d \in [33,43]$

$60x^3+53x^2-185x+42$, which corresponds to multiplying out $(3x+7)(4x-1)(5x-6)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x-7)(4x+1)(5x-6)$

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form x^3+bx^2+cx+d.

$$5-4i \text{ and } -1$$

The solution is $x^3-9x^2+31x+41$, which is option B.

A. $b \in [-7,7], c \in [-10,4],$ and $d \in [-13,-4]$

x^3+x^2-4x-5, which corresponds to multiplying out $(x-5)(x+1)$.

B. $b \in [-9,-4], c \in [29,36],$ and $d \in [37,44]$

* $x^3-9x^2+31x+41$, which is the correct option.

C. $b \in [4,12], c \in [29,36],$ and $d \in [-43,-39]$

$x^3+9x^2+31x-41$, which corresponds to multiplying out $(x-(5-4i))(x-(5+4i))(x-1)$.

D. $b \in [-7,7], c \in [3,13],$ and $d \in [0,8]$

x^3+x^2+5x+4, which corresponds to multiplying out $(x+4)(x+1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 - 4i))(x - (5 + 4i))(x - (-1))$.

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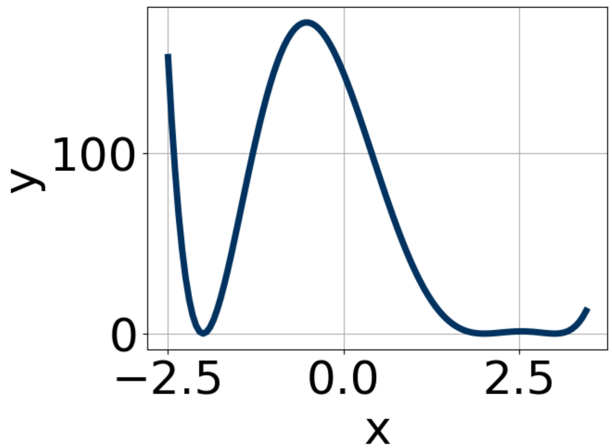
6. Which of the following equations *could* be of the graph presented below?



The solution is $16(x - 2)^4(x + 2)^4(x - 3)^8$, which is option D.

A. $5(x - 2)^6(x + 2)^{11}(x - 3)^7$

The factors $(x + 2)$ and $(x - 3)$ should both have even powers.

B. $17(x - 2)^6(x + 2)^{10}(x - 3)^7$

The factor $(x - 3)$ should have an even power.

C. $-5(x - 2)^4(x + 2)^{10}(x - 3)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $16(x - 2)^4(x + 2)^4(x - 3)^8$

* This is the correct option.

E. $-18(x - 2)^8(x + 2)^4(x - 3)^7$

The factor $(x - 3)$ should have an even power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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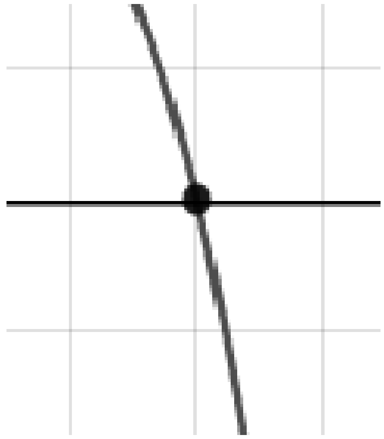
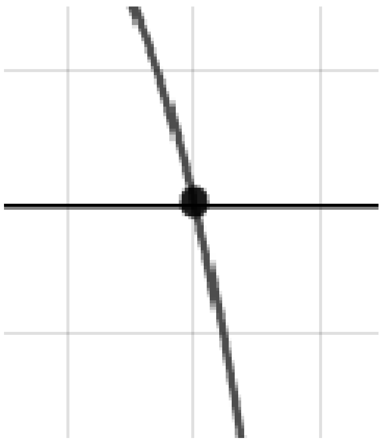
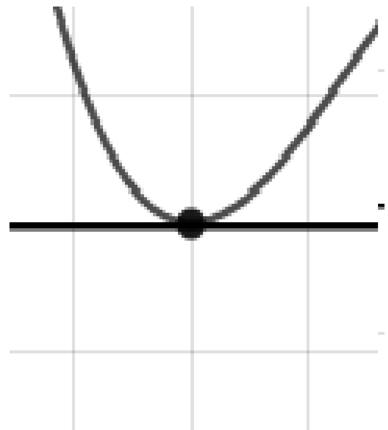
7. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

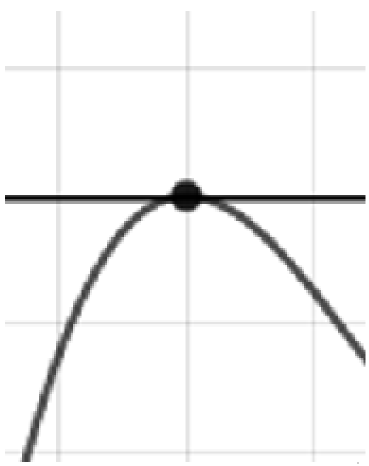
$$f(x) = -2(x - 3)^2(x + 3)^3(x - 4)^4(x + 4)^7$$

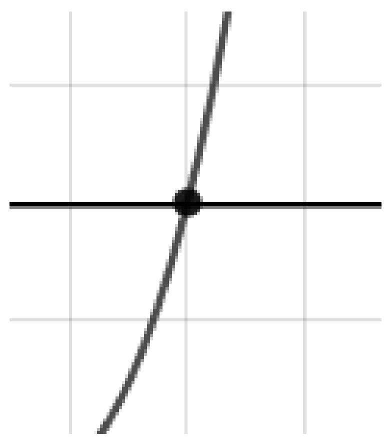
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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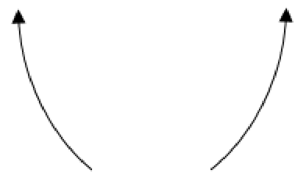
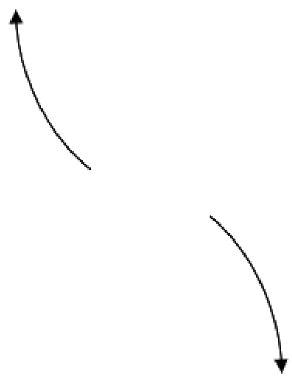
8. Describe the end behavior of the polynomial below.

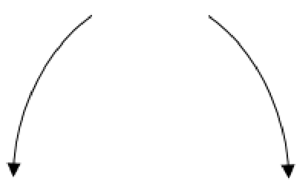
$$f(x) = 9(x-7)^4(x+7)^5(x+6)^3(x-6)^4$$

The solution is the graph below, which is option C.

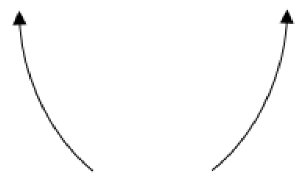




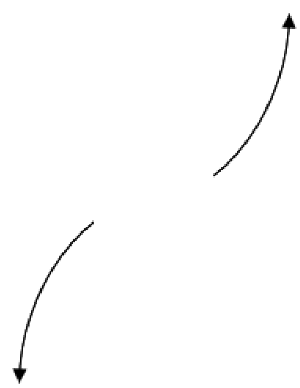
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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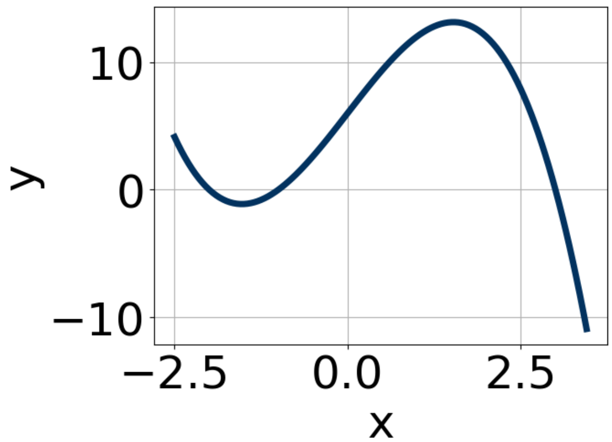
9. Which of the following equations *could* be of the graph presented below?



The solution is $-12(x+2)^{11}(x+1)^5(x-3)^9$, which is option D.

A. $-3(x+2)^{10}(x+1)^{10}(x-3)^9$

   The factors -2 and -1 have have been odd power.

B. $20(x+2)^7(x+1)^{11}(x-3)^7$

   This corresponds to the leading coefficient being the opposite value than it should be.

C. $-2(x+2)^8(x+1)^9(x-3)^7$

   The factor -2 should have been an odd power.

D. $-12(x+2)^{11}(x+1)^5(x-3)^9$

   * This is the correct option.

E. $10(x+2)^6(x+1)^7(x-3)^{11}$

   The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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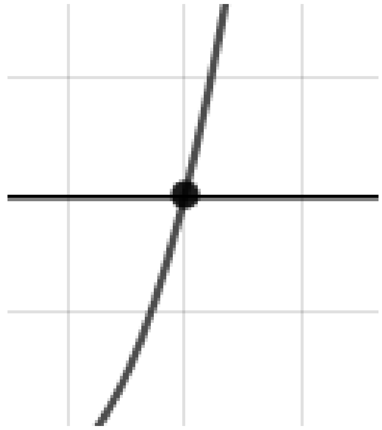
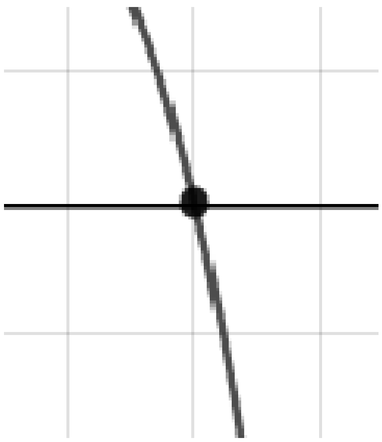
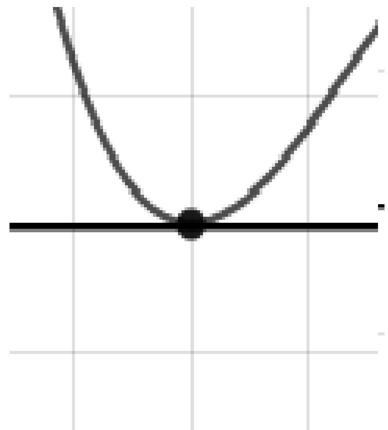
10. Describe the zero behavior of the zero $x=-3$ of the polynomial below.

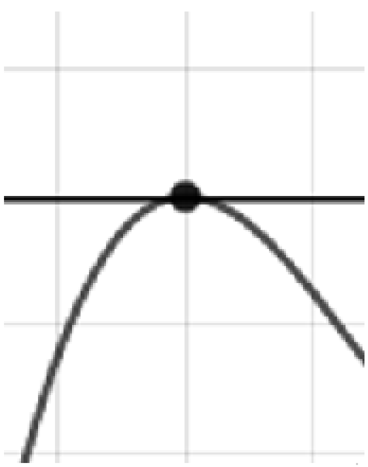
$$f(x) = 3(x-3)^4(x+3)^5(x-9)^8(x+9)^{10}$$

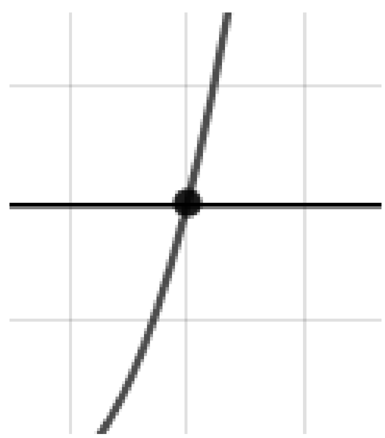
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.