This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Choose the interval below that f composed with g at x = -1 is in.

$$f(x) = 4x^3 - 2x^2 - 4x$$
 and  $g(x) = 3x^3 + 3x^2 - x + 2$ 

The solution is 78.0, which is option B.

A. 
$$(f \circ g)(-1) \in [-11, -5]$$

Distractor 1: Corresponds to reversing the composition.

B. 
$$(f \circ g)(-1) \in [75, 79]$$

\* This is the correct solution

C. 
$$(f \circ g)(-1) \in [81, 90]$$

Distractor 2: Corresponds to being slightly off from the solution.

D. 
$$(f \circ g)(-1) \in [-5, 0]$$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** f composed with g at x means f(g(x)). The order matters!

2. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -15 and choose the interval the  $f^{-1}(-15)$  belongs to.

$$f(x) = \sqrt[3]{3x+5}$$

The solution is -1126.666666666667, which is option A.

A. 
$$f^{-1}(-15) \in [-1128.5, -1124.9]$$

\* This is the correct solution.

B. 
$$f^{-1}(-15) \in [-1124.7, -1122.4]$$

Distractor 1: This corresponds to

C. 
$$f^{-1}(-15) \in [1122.2, 1125.6]$$

This solution corresponds to distractor 3.

D. 
$$f^{-1}(-15) \in [1124.6, 1127.9]$$

This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

3. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -10 and choose the interval that  $f^{-1}(-10)$  belongs to.

$$f(x) = 4x^2 - 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A.  $f^{-1}(-10) \in [1.58, 2.34]$ 

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift

B.  $f^{-1}(-10) \in [2.26, 2.49]$ 

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C.  $f^{-1}(-10) \in [4.29, 5.04]$ 

Distractor 4: This corresponds to both distractors 2 and 3.

D.  $f^{-1}(-10) \in [1.22, 1.62]$ 

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

\* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

4. Choose the interval below that f composed with g at x = -1 is in.

$$f(x) = 2x^3 + 4x^2 + 2x + 1$$
 and  $g(x) = 2x^3 + 2x^2 + 2x$ 

The solution is -3.0, which is option C.

A.  $(f \circ g)(-1) \in [9, 20]$ 

Distractor 3: Corresponds to being slightly off from the solution.

B.  $(f \circ g)(-1) \in [-15, -9]$ 

Distractor 2: Corresponds to being slightly off from the solution.

C.  $(f \circ g)(-1) \in [-4, 0]$ 

\* This is the correct solution

D.  $(f \circ g)(-1) \in [4, 9]$ 

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

**General Comment:** f composed with g at x means f(g(x)). The order matters!

5. Determine whether the function below is 1-1.

$$f(x) = -24x^2 - 270x - 729$$

The solution is no, which is option E.

A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

B. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because there is a y-value that goes to 2 different x-values.

\* This is the solution.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

6. Find the inverse of the function below. Then, evaluate the inverse at x = 5 and choose the interval that  $f^{-1}(5)$  belongs to.

$$f(x) = \ln(x+3) - 2$$

The solution is  $f^{-1}(5) = 1093.633$ , which is option A.

A.  $f^{-1}(5) \in [1090.63, 1097.63]$ 

This is the solution.

B.  $f^{-1}(5) \in [3.39, 6.39]$ 

This solution corresponds to distractor 2.

C.  $f^{-1}(5) \in [1097.63, 1106.63]$ 

This solution corresponds to distractor 3.

D.  $f^{-1}(5) \in [2977.96, 2983.96]$ 

This solution corresponds to distractor 4.

E.  $f^{-1}(5) \in [15.09, 18.09]$ 

This solution corresponds to distractor 1.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

7. Find the inverse of the function below. Then, evaluate the inverse at x = 9 and choose the interval that  $f^{-1}(9)$  belongs to.

$$f(x) = \ln(x-4) - 5$$

The solution is  $f^{-1}(9) = 1202608.284$ , which is option B.

A.  $f^{-1}(9) \in [1202596.28, 1202605.28]$ 

This solution corresponds to distractor 3.

B.  $f^{-1}(9) \in [1202604.28, 1202609.28]$ 

This is the solution.

C.  $f^{-1}(9) \in [141.41, 144.41]$ 

This solution corresponds to distractor 4.

D.  $f^{-1}(9) \in [442408.39, 442416.39]$ 

This solution corresponds to distractor 2.

E.  $f^{-1}(9) \in [55.6, 61.6]$ 

This solution corresponds to distractor 1.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

8. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 8x^2 + 5x + 5$$
 and  $g(x) = 3x + 6$ 

The solution is  $(-\infty, \infty)$ , which is option E.

- A. The domain is all Real numbers greater than or equal to x=a, where  $a \in [-8, -2]$
- B. The domain is all Real numbers less than or equal to x = a, where  $a \in [2.33, 3.33]$
- C. The domain is all Real numbers except x = a, where  $a \in [1.75, 6.75]$
- D. The domain is all Real numbers except x = a and x = b, where  $a \in [4.2, 10.2]$  and  $b \in [-7.17, -4.17]$
- E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

9. Determine whether the function below is 1-1.

$$f(x) = (5x - 23)^3$$

The solution is yes, which is option A.

- A. Yes, the function is 1-1.
  - \* This is the solution.
- B. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

10. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = x + 3$$
 and  $g(x) = 6x + 7$ 

The solution is  $(-\infty, \infty)$ , which is option E.

- A. The domain is all Real numbers greater than or equal to  $x=a, \text{ where } a \in [-10, -5]$
- B. The domain is all Real numbers except x = a, where  $a \in [-5.67, -2.67]$
- C. The domain is all Real numbers less than or equal to x = a, where  $a \in [-5.33, -0.33]$
- D. The domain is all Real numbers except x = a and x = b, where  $a \in [4.6, 11.6]$  and  $b \in [6.2, 11.2]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.