This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{4}{5}, \frac{3}{2}, \text{ and } \frac{7}{4}$$

The solution is $40x^3 - 162x^2 + 209x - 84$, which is option B.

A. $a \in [38, 45], b \in [-98, -95], c \in [-1, 4], \text{ and } d \in [76, 87]$

$40x^3 - 98x^2 + x + 84$, which corresponds to multiplying out $(5x + 4)(2x - 3)(4x - 7)$.

B. $a \in [38, 45], b \in [-167, -157], c \in [204, 222], \text{ and } d \in [-85, -78]$

* $40x^3 - 162x^2 + 209x - 84$, which is the correct option.

C. $a \in [38, 45], b \in [22, 24], c \in [-118, -107], \text{ and } d \in [-85, -78]$

$40x^3 + 22x^2 - 113x - 84$, which corresponds to multiplying out $(5x + 4)(2x + 3)(4x - 7)$.

D. $a \in [38, 45], b \in [-167, -157], c \in [204, 222], \text{ and } d \in [76, 87]$

$40x^3 - 162x^2 + 209x + 84$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [38, 45], b \in [158, 163], c \in [204, 222], \text{ and } d \in [76, 87]$

$40x^3 + 162x^2 + 209x + 84$, which corresponds to multiplying out $(5x + 4)(2x + 3)(4x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 4)(2x - 3)(4x - 7)$

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 2i \text{ and } 2$$

The solution is $x^3 + 2x^2 - 16$, which is option A.

A. $b \in [1.07, 2.48], c \in [-1.6, 0.7], \text{ and } d \in [-21, -13]$

* $x^3 + 2x^2 - 16$, which is the correct option.

B. $b \in [-3.13, -1.69], c \in [-1.6, 0.7], \text{ and } d \in [13, 22]$

$x^3 - 2x^2 + 16$, which corresponds to multiplying out $(x - (-2 + 2i))(x - (-2 - 2i))(x + 2)$.

C. $b \in [0.96, 1.95], c \in [-5.5, -3.4], \text{ and } d \in [4, 9]$

$x^3 + x^2 - 4x + 4$, which corresponds to multiplying out $(x - 2)(x - 2)$.

D. $b \in [0.96, 1.95], c \in [-1.6, 0.7], \text{ and } d \in [-9, -3]$

$x^3 + x^2 - 4$, which corresponds to multiplying out $(x + 2)(x - 2)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 2i))(x - (-2 - 2i))(x - (2))$.

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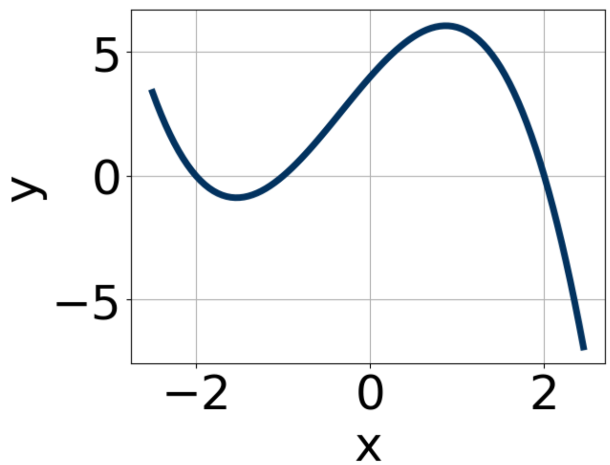
3. Which of the following equations *could* be of the graph presented below?



The solution is $-14(x+1)^{11}(x-2)^{9}(x+2)^{7}$, which is option B.

A. $15(x+1)^{9}(x-2)^{5}(x+2)^{7}$

   This corresponds to the leading coefficient being the opposite value than it should be.

B. $-14(x+1)^{11}(x-2)^{9}(x+2)^{7}$

   * This is the correct option.

C. $19(x+1)^{6}(x-2)^{5}(x+2)^{9}$

   The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-13(x+1)^{10}(x-2)^{10}(x+2)^{11}$

   The factors -1 and 2 have have been odd power.

E. $-8(x+1)^{10}(x-2)^{7}(x+2)^{9}$

   The factor -1 should have been an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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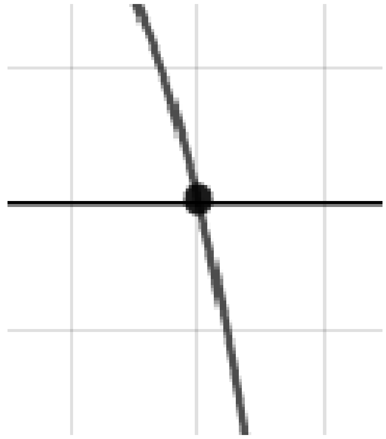
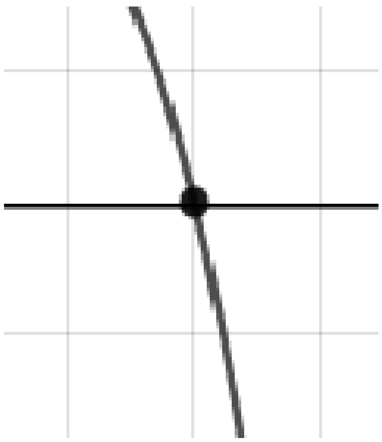
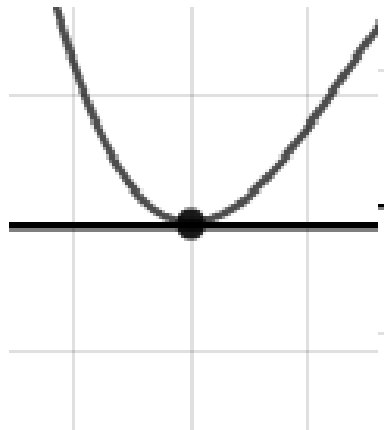
4. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

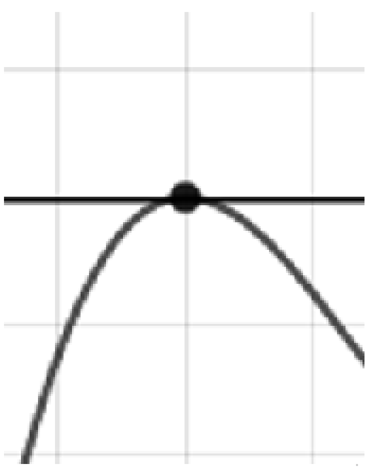
$$f(x) = -2(x+9)^{2}(x-9)^{7}(x-4)^{5}(x+4)^{9}$$

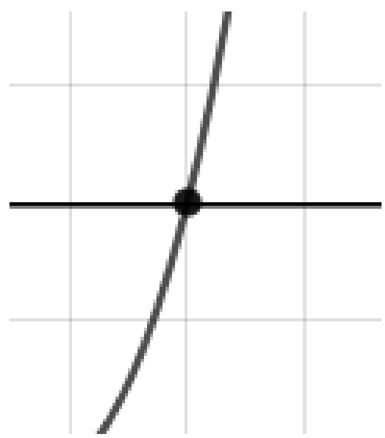
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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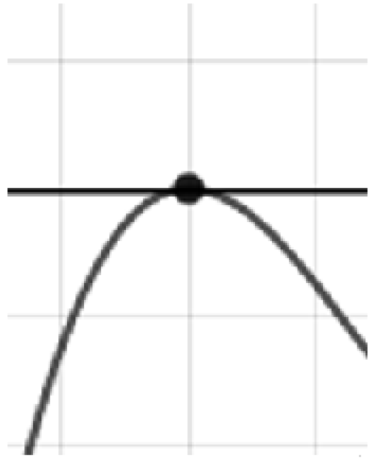
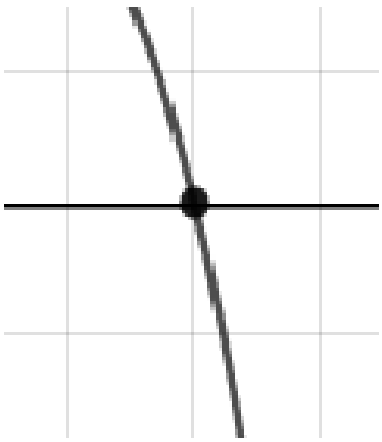
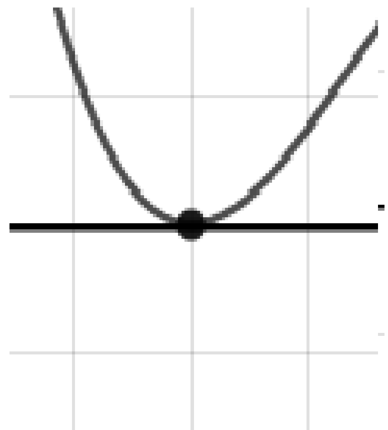
5. Describe the zero behavior of the zero $x = 6$ of the polynomial below.

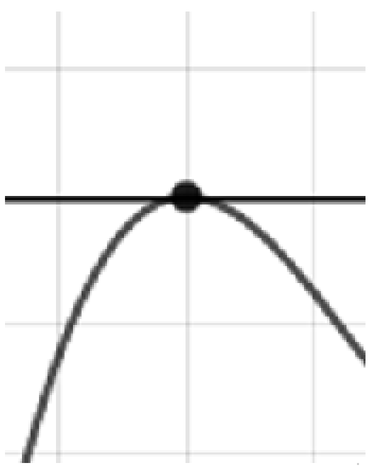
$$f(x) = -7(x+2)^4(x-2)^2(x+6)^5(x-6)^4$$

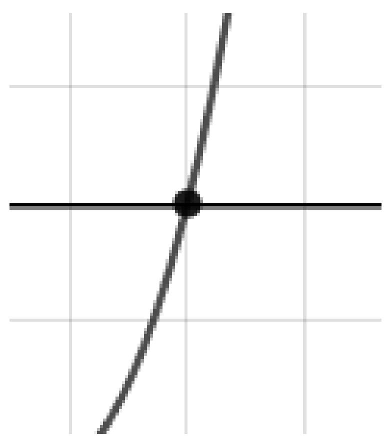
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 - 2i \text{ and } 2$$

The solution is $x^3 + 8x^2 + 9x - 58$, which is option D.

A. $b \in [-12, -4], c \in [4.8, 9.5]$, and $d \in [58, 60]$

$x^3 - 8x^2 + 9x + 58$, which corresponds to multiplying out $(x - (-5 - 2i))(x - (-5 + 2i))(x + 2)$.

B. $b \in [1, 4], c \in [-7.7, 1.2]$, and $d \in [-5, 3]$

$x^3 + x^2 - 4$, which corresponds to multiplying out $(x + 2)(x - 2)$.

C. $b \in [1, 4], c \in [2.7, 3.6]$, and $d \in [-11, -8]$

$x^3 + x^2 + 3x - 10$, which corresponds to multiplying out $(x + 5)(x - 2)$.

D. $b \in [3, 14], c \in [4.8, 9.5]$, and $d \in [-63, -54]$

* $x^3 + 8x^2 + 9x - 58$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 - 2i))(x - (-5 + 2i))(x - (2))$.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$4, \frac{7}{3}, \text{ and } \frac{1}{5}$$

The solution is $15x^3 - 98x^2 + 159x - 28$, which is option D.

A. $a \in [11, 20], b \in [13, 26], c \in [-145, -141]$, and $d \in [21, 36]$

$15x^3 + 22x^2 - 145x + 28$, which corresponds to multiplying out $(x + 4)(3x - 7)(5x - 1)$.

B. $a \in [11, 20], b \in [98, 105], c \in [158, 162]$, and $d \in [21, 36]$

$15x^3 + 98x^2 + 159x + 28$, which corresponds to multiplying out $(x + 4)(3x + 7)(5x + 1)$.

C. $a \in [11, 20], b \in [-101, -96], c \in [158, 162]$, and $d \in [21, 36]$

$15x^3 - 98x^2 + 159x + 28$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [11, 20], b \in [-101, -96], c \in [158, 162]$, and $d \in [-31, -27]$

* $15x^3 - 98x^2 + 159x - 28$, which is the correct option.

E. $a \in [11, 20], b \in [89, 93], c \in [115, 123]$, and $d \in [-31, -27]$

$15x^3 + 92x^2 + 121x - 28$, which corresponds to multiplying out $(x + 4)(3x + 7)(5x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x - 4)(3x - 7)(5x - 1)$

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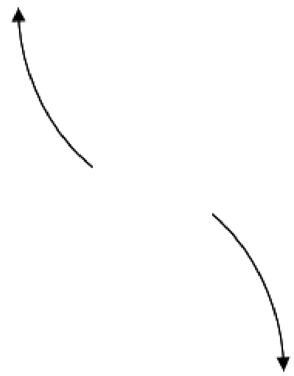
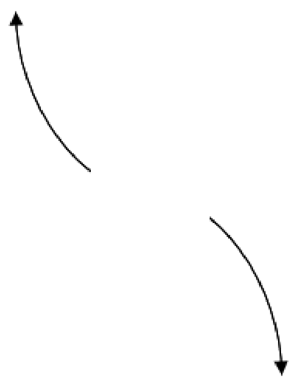
8. Describe the end behavior of the polynomial below.

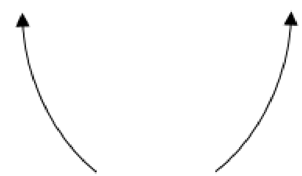
$$f(x) = -3(x + 2)^5(x - 2)^6(x + 6)^4(x - 6)^6$$

The solution is the graph below, which is option A.

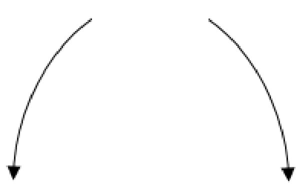




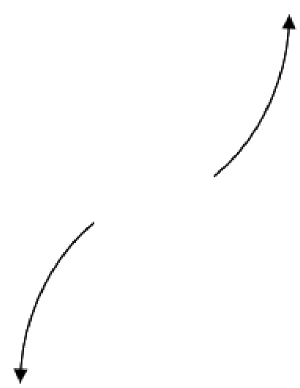
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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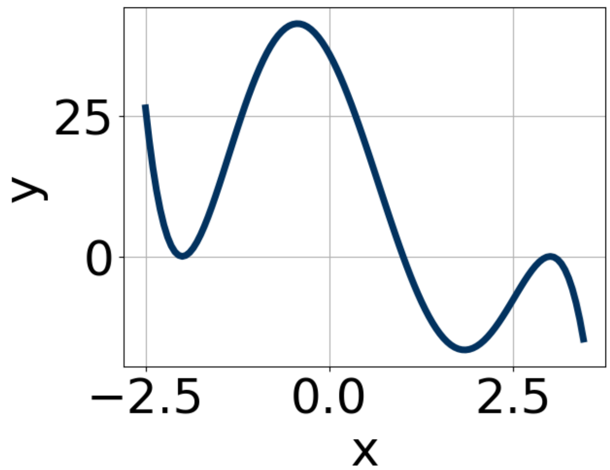
9. Which of the following equations *could* be of the graph presented below?



The solution is $-17(x-3)^8(x+2)^4(x-1)^{11}$, which is option D.

A. $-5(x-3)^4(x+2)^5(x-1)^4$

The factor $(x+2)$ should have an even power and the factor $(x-1)$ should have an odd power.

B. $-8(x-3)^{10}(x+2)^5(x-1)^9$

The factor $(x+2)$ should have an even power.

C. $4(x-3)^{10}(x+2)^8(x-1)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-17(x-3)^8(x+2)^4(x-1)^{11}$

* This is the correct option.

E. $20(x-3)^{10}(x+2)^6(x-1)^6$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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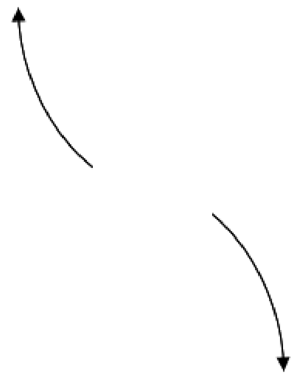
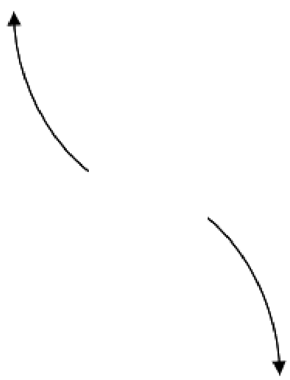
10. Describe the end behavior of the polynomial below.

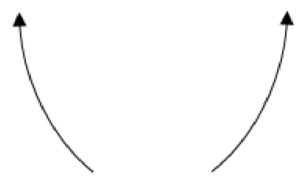
$$f(x) = -8(x+2)^2(x-2)^7(x-8)^2(x+8)^2$$

The solution is the graph below, which is option A.

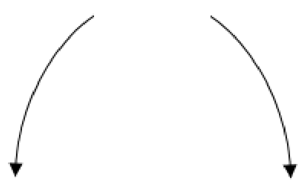




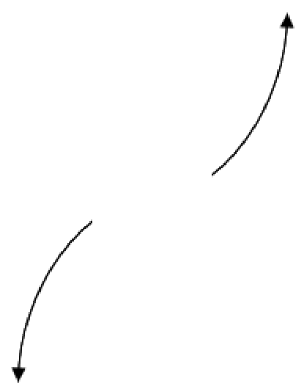
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.