

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

If you have a suggestion to make the keys better, please fill out the short survey [here](#).

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at  $x = 11$  and choose the interval that  $f^{-1}(11)$  belongs to.

$$f(x) = 3x^2 - 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

- A.  $f^{-1}(11) \in [1.93, 2.58]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

- B.  $f^{-1}(11) \in [4.79, 5.22]$

Distractor 4: This corresponds to both distractors 2 and 3.

- C.  $f^{-1}(11) \in [1.54, 2]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

- D.  $f^{-1}(11) \in [3.84, 4.62]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

- E. The function is not invertible for all Real numbers.

\* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

2. Determine whether the function below is 1-1.

$$f(x) = 15x^2 - 8x - 384$$

The solution is no, which is option C.

- A. No, because there is an  $x$ -value that goes to 2 different  $y$ -values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

- B. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

- C. No, because there is a  $y$ -value that goes to 2 different  $x$ -values.

\* This is the solution.

- D. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

- E. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a  $y$ -value that goes to 2 different  $x$ -values.

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3. Determine whether the function below is 1-1.

$$f(x) = -12x^2 - 44x + 240$$

The solution is no, which is option E.

- A. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

- B. No, because there is an  $x$ -value that goes to 2 different  $y$ -values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

- C. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

- D. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

- E. No, because there is a  $y$ -value that goes to 2 different  $x$ -values.

\* This is the solution.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a  $y$ -value that goes to 2 different  $x$ -values.

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4. Find the inverse of the function below. Then, evaluate the inverse at  $x = 6$  and choose the interval that  $f^{-1}(6)$  belongs to.

$$f(x) = e^{x-3} + 2$$

The solution is  $f^{-1}(6) = 4.386$ , which is option A.

- A.  $f^{-1}(6) \in [4.3, 4.46]$

This is the solution.

- B.  $f^{-1}(6) \in [3.85, 4.09]$

This solution corresponds to distractor 2.

- C.  $f^{-1}(6) \in [3.08, 3.25]$

This solution corresponds to distractor 4.

- D.  $f^{-1}(6) \in [-1.74, -1.49]$

This solution corresponds to distractor 1.

- E.  $f^{-1}(6) \in [4.15, 4.29]$

This solution corresponds to distractor 3.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the  $x$  and  $y$ , use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

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5. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 3x + 3 \text{ and } g(x) = 2x^3 + x^2 + 7x + 5$$

The solution is  $(-\infty, \infty)$ , which is option E.

- A. The domain is all Real numbers except  $x = a$ , where  $a \in [2.25, 10.25]$
- B. The domain is all Real numbers less than or equal to  $x = a$ , where  $a \in [-1.75, 6.25]$
- C. The domain is all Real numbers greater than or equal to  $x = a$ , where  $a \in [4.2, 5.2]$
- D. The domain is all Real numbers except  $x = a$  and  $x = b$ , where  $a \in [-7.67, -0.67]$  and  $b \in [-7.75, -0.75]$
- E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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6. Find the inverse of the function below (if it exists). Then, evaluate the inverse at  $x = 10$  and choose the interval that  $f^{-1}(10)$  belongs to.

$$f(x) = 2x^2 - 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

- A.  $f^{-1}(10) \in [0.99, 2.48]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

- B.  $f^{-1}(10) \in [1.92, 3.54]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

- C.  $f^{-1}(10) \in [4.5, 5.73]$

Distractor 4: This corresponds to both distractors 2 and 3.

- D.  $f^{-1}(10) \in [3.37, 3.66]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

- E. The function is not invertible for all Real numbers.

\* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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7. Choose the interval below that  $f$  composed with  $g$  at  $x = -1$  is in.

$$f(x) = -3x^3 - 3x^2 - 3x \text{ and } g(x) = 2x^3 - 1x^2 - 3x$$

The solution is 0.0, which is option D.

- A.  $(f \circ g)(-1) \in [-8, -5]$

Distractor 2: Corresponds to being slightly off from the solution.

- B.  $(f \circ g)(-1) \in [38, 47]$

Distractor 3: Corresponds to being slightly off from the solution.

C.  $(f \circ g)(-1) \in [36, 41]$

Distractor 1: Corresponds to reversing the composition.

D.  $(f \circ g)(-1) \in [-3, 1]$

\* This is the correct solution

E. It is not possible to compose the two functions.

**General Comment:**  $f$  composed with  $g$  at  $x$  means  $f(g(x))$ . The order matters!

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8. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-4x + 18} \text{ and } g(x) = 4x^3 + 7x^2 + 5x + 3$$

The solution is The domain is all Real numbers less than or equal to  $x = 4.5$ , which is option B.

A. The domain is all Real numbers greater than or equal to  $x = a$ , where  $a \in [1.33, 7.33]$

B. The domain is all Real numbers less than or equal to  $x = a$ , where  $a \in [-1.5, 5.5]$

C. The domain is all Real numbers except  $x = a$ , where  $a \in [-10.33, 1.67]$

D. The domain is all Real numbers except  $x = a$  and  $x = b$ , where  $a \in [1.4, 6.4]$  and  $b \in [-8.25, -5.25]$

E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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9. Choose the interval below that  $f$  composed with  $g$  at  $x = 1$  is in.

$$f(x) = 2x^3 + x^2 + 4x - 4 \text{ and } g(x) = -x^3 - 4x^2 + 4x$$

The solution is  $-9.0$ , which is option B.

A.  $(f \circ g)(1) \in [-46, -40]$

Distractor 3: Corresponds to being slightly off from the solution.

B.  $(f \circ g)(1) \in [-11, -8]$

\* This is the correct solution

C.  $(f \circ g)(1) \in [-23, -14]$

Distractor 2: Corresponds to being slightly off from the solution.

D.  $(f \circ g)(1) \in [-53, -49]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

**General Comment:**  $f$  composed with  $g$  at  $x$  means  $f(g(x))$ . The order matters!

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10. Find the inverse of the function below. Then, evaluate the inverse at  $x = 9$  and choose the interval that  $f^{-1}(9)$  belongs to.

$$f(x) = \ln(x + 2) - 4$$

The solution is  $f^{-1}(9) = 442411.392$ , which is option A.

A.  $f^{-1}(9) \in [442403.39, 442415.39]$

This is the solution.

B.  $f^{-1}(9) \in [1088.63, 1094.63]$

This solution corresponds to distractor 2.

C.  $f^{-1}(9) \in [442412.39, 442419.39]$

This solution corresponds to distractor 3.

D.  $f^{-1}(9) \in [59868.14, 59871.14]$

This solution corresponds to distractor 4.

E.  $f^{-1}(9) \in [140.41, 147.41]$

This solution corresponds to distractor 1.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the  $x$  and  $y$ , use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

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