This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 23x^2 - 88x + 80$$

The solution is $[-2.5, 0.8, 4]$, which is option E.

A. $z_1 \in [-5, -3]$, $z_2 \in [-1.35, -1.06]$, and $z_3 \in [-0.2, 1.4]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-0.4, 1.6]$, $z_2 \in [1.11, 1.3]$, and $z_3 \in [3.2, 4.7]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-5, -3]$, $z_2 \in [-0.93, -0.79]$, and $z_3 \in [2, 3]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-5, -3]$, $z_2 \in [-0.43, -0.09]$, and $z_3 \in [4.1, 5.5]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-2.5, -1.5]$, $z_2 \in [0.6, 0.82]$, and $z_3 \in [3.2, 4.7]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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2. Factor the polynomial below completely, knowing that $x + 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^4 + 13x^3 - 144x^2 - 325x - 150$$

The solution is $[-5, -1.5, -0.6666666666666666, 5]$, which is option B.

A. $z_1 \in [-5, -4]$, $z_2 \in [0.56, 0.73]$, $z_3 \in [-0.3, 2.1]$, and $z_4 \in [5, 7]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5, -4]$, $z_2 \in [-1.55, -1.48]$, $z_3 \in [-2.7, -0.6]$, and $z_4 \in [5, 7]$

* This is the solution!

C. $z_1 \in [-5, -4]$, $z_2 \in [0.56, 0.73]$, $z_3 \in [-0.3, 2.1]$, and $z_4 \in [5, 7]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-5, -4]$, $z_2 \in [0.21, 0.5]$, $z_3 \in [2.8, 3.9]$, and $z_4 \in [5, 7]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-5, -4], \; z_2 \in [-1.55, -1.48], z_3 \in [-2.7, -0.6], \text{ and } z_4 \in [5, 7]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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3. Factor the polynomial below completely, knowing that $x+5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 + 154x^3 + 461x^2 + 290x - 200$$

The solution is $[-5, -4, -1.6666666666666667, 0.4]$, which is option D.

A. $z_1 \in [-3.21, -1.62], \; z_2 \in [-0.5, 1], z_3 \in [3, 4.3], \text{ and } z_4 \in [4, 6]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-0.56, -0.26], \; z_2 \in [0.9, 3], z_3 \in [3, 4.3], \text{ and } z_4 \in [4, 6]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-5.03, -4.39], \; z_2 \in [-4.5, -3.5], z_3 \in [-0.7, 0.1], \text{ and } z_4 \in [2.5, 4.5]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-5.03, -4.39], \; z_2 \in [-4.5, -3.5], z_3 \in [-2.5, -1.1], \text{ and } z_4 \in [0.4, 1.4]$

* This is the solution!

E. $z_1 \in [-0.27, 0.65], \; z_2 \in [3.8, 4.1], z_3 \in [4.9, 5.5], \text{ and } z_4 \in [4, 6]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 + 35x^2 - 15}{x + 2}$$

The solution is $15x^2 + 5x - 10 + \frac{5}{x+2}$, which is option E.

A. $a \in [12, 18], b \in [-13, -6], c \in [23, 32], \text{ and } r \in [-106, -103].$

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [-30, -27], b \in [-26, -21], c \in [-55, -49], \text{ and } r \in [-115, -112].$

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [12, 18], b \in [63, 67], c \in [124, 131], \text{ and } r \in [241, 248].$

You divided by the opposite of the factor.

D. $a \in [-30, -27], b \in [92, 96], c \in [-191, -185], \text{ and } r \in [362, 368].$

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [12, 18], b \in [4, 10], c \in [-13, -5]$, and $r \in [4, 7]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 + 44x^2 + 4x - 57}{x + 3}$$

The solution is $12x^2 + 8x - 20 + \frac{3}{x+3}$, which is option E.

A. $a \in [-41, -27]$, $b \in [146, 153]$, $c \in [-456, -449]$, and $r \in [1295, 1303]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [10, 15]$, $b \in [-4, 5]$, $c \in [20, 21]$, and $r \in [-147, -136]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [10, 15]$, $b \in [79, 85]$, $c \in [239, 252]$, and $r \in [673, 676]$.

You divided by the opposite of the factor.

D. $a \in [-41, -27]$, $b \in [-65, -61]$, $c \in [-189, -186]$, and $r \in [-621, -617]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [10, 15]$, $b \in [8, 9]$, $c \in [-23, -15]$, and $r \in [-1, 6]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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6. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 + 47x^2 + 16x - 48$$

The solution is $[-4, -1.5, 0.8]$, which is option D.

A. $z_1 \in [-0.5, -0.16]$, $z_2 \in [2.92, 3.62]$, and $z_3 \in [3.5, 4.58]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-4.21, -3.82]$, $z_2 \in [-0.83, -0.6]$, and $z_3 \in [1.18, 1.38]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-1.65, -0.91]$, $z_2 \in [0.19, 1.25]$, and $z_3 \in [3.5, 4.58]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-4.21, -3.82]$, $z_2 \in [-1.76, -1.17]$, and $z_3 \in [0.53, 0.83]$

* This is the solution!

E. $z_1 \in [-0.82, -0.77]$, $z_2 \in [1.31, 1.88]$, and $z_3 \in [3.5, 4.58]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 6x^2 + 7x + 4$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$, which is option C.

A. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

* This is the solution **since we asked for the possible Rational roots**!

D. $\pm 1, \pm 2, \pm 4$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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8. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 + 42x^2 - 53}{x + 5}$$

The solution is $8x^2 + 2x - 10 + \dfrac{-3}{x+5}$, which is option D.

A. $a \in [-42, -37], b \in [-165, -155], c \in [-792, -788]$, and $r \in [-4006, -3999]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

B. $a \in [8, 14], b \in [-7, -2], c \in [33, 37]$, and $r \in [-270, -265]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-42, -37], b \in [242, 244], c \in [-1217, -1206]$, and $r \in [5996, 5999]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [8, 14], b \in [0, 6], c \in [-11, -4]$, and $r \in [-4, 2]$.

* This is the solution!

E. $a \in [8, 14], b \in [78, 85], c \in [404, 411]$, and $r \in [1995, 1999]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 - 4x^2 - 32x + 45}{x + 3}$$

The solution is $4x^2 - 16x + 16 + \frac{-3}{x+3}$, which is option B.

A. $a \in [2, 7], \ b \in [6, 14], \ c \in [-9, -5], \text{ and } r \in [13, 24].$

You divided by the opposite of the factor.

B. $a \in [2, 7], \ b \in [-17, -8], \ c \in [13, 18], \text{ and } r \in [-6, -1].$

* This is the solution!

C. $a \in [2, 7], \ b \in [-22, -19], \ c \in [43, 50], \text{ and } r \in [-151, -144].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [-16, -6], \ b \in [32, 35], \ c \in [-128, -127], \text{ and } r \in [428, 432].$

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [-16, -6], \ b \in [-45, -36], \ c \in [-153, -146], \text{ and } r \in [-417, -404].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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10. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 6x^2 + 5x + 5$$

The solution is All combinations of: $\frac{\pm 1, \pm 5}{\pm 1, \pm 2, \pm 3, \pm 6}$, which is option A.

A. All combinations of: $\frac{\pm 1, \pm 5}{\pm 1, \pm 2, \pm 3, \pm 6}$

* This is the solution **since we asked for the possible Rational roots**!

B. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots**!

D. All combinations of: $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.