**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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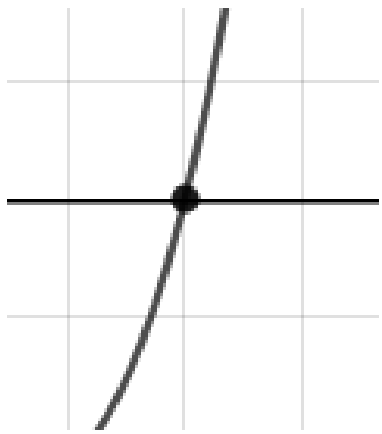
1. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

$$f(x) = 7(x+2)^{12}(x-2)^{8}(x+7)^{12}(x-7)^{9}$$

The solution is the graph below.



**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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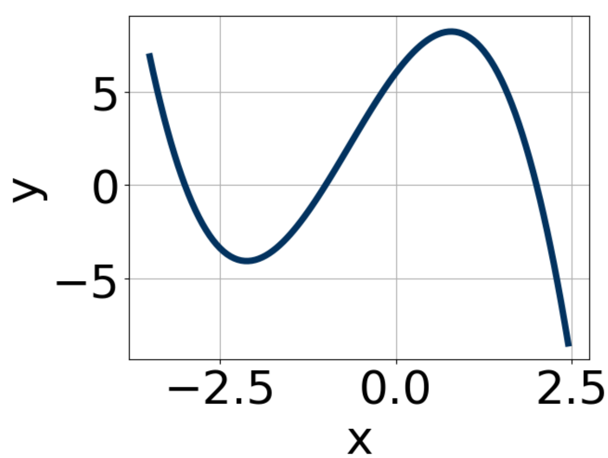
2. Write an equation that *could* represent the graph below.




The solution is $-19(x+1)^5(x-2)^9(x+3)^5$.

**Plausible alternative answers include:**The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign. The factor $-1$ should have been an odd power.

The factors $-1$ and 2 have have been odd power. * This is the correct option. This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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3. Construct the lowest-degree polynomial given the zeros below.

$$-4, \frac{-2}{5}, \text{ and } \frac{-5}{3}$$

The solution is $15x^3 + 91x^2 + 134x + 40$.

**Plausible alternative answers include:*** $15x^3 + 91x^2 + 134x + 40$, which is the correct option. $15x^3 - 41x^2 - 86x + 40$, which corresponds to multiplying out $(x-4)(5x-2)(3x+5)$. $15x^3 - 91x^2 + 134x - 40$, which corresponds to multiplying out $(x-4)(5x-2)(3x-5)$. $15x^3 + 91x^2 + 134x - 40$, which corresponds to multiplying everything correctly except the constant term. $15x^3 - 29x^2 - 114x - 40$, which corresponds to multiplying out $(x-4)(5x+2)(3x+5)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(x+4)(5x+2)(3x+5)$

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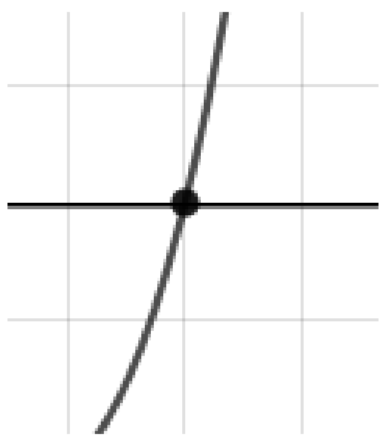
4. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

$$f(x) = -3(x-2)^7(x+2)^6(x+3)^7(x-3)^4$$

The solution is the graph below.



**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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5. Construct the lowest-degree polynomial given the zeros below.

$$\frac{4}{3}, \frac{-7}{2}, \text{ and } \frac{6}{5}$$

The solution is $30x^3 + 29x^2 - 218x + 168$.

**Plausible alternative answers include:**$30x^3 + 109x^2 - 34x - 168$, which corresponds to multiplying out $(3x+4)(2x+7)(5x-6)$. $30x^3 + 29x^2 - 218x - 168$, which corresponds to multiplying everything correctly except the constant term. $30x^3 - 101x^2 - 62x + 168$, which corresponds to multiplying out $(3x+4)(2x-7)(5x-6)$. * $30x^3 + 29x^2 - 218x + 168$, which is the correct option. $30x^3 - 29x^2 - 218x - 168$, which corresponds to multiplying out $(3x+4)(2x-7)(5x+6)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(3x - 4)(2x + 7)(5x - 6)$

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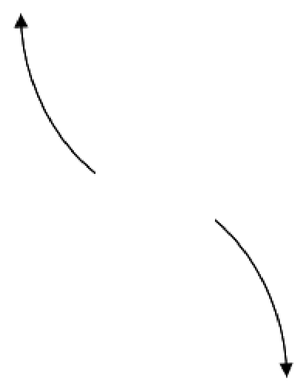
6. Describe the end behavior of the polynomial below.

$$f(x) = -3(x - 2)^3(x + 2)^4(x + 8)^5(x - 8)^5$$

The solution is the graph below.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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7. Construct the lowest-degree polynomial given the zeros below.

$$5 - 3i \text{ and } -1$$

The solution is $x^3 - 9x^2 + 24x + 34$.

> **Plausible alternative answers include:** $x^3 + x^2 + 4x + 3$, which corresponds to multiplying out $(x+3)(x+1)$. $x^3+9x^2+24x-34$, which corresponds to multiplying out $(x-(5-3i))(x-(5+3i))(x-1)$. $x^3+x^2-4x-5$, which corresponds to multiplying out $(x-5)(x+1)$. * $x^3-9x^2+24x+34$, which is the correct option. This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 - 3i))(x - (5 + 3i))(x - (-1))$.

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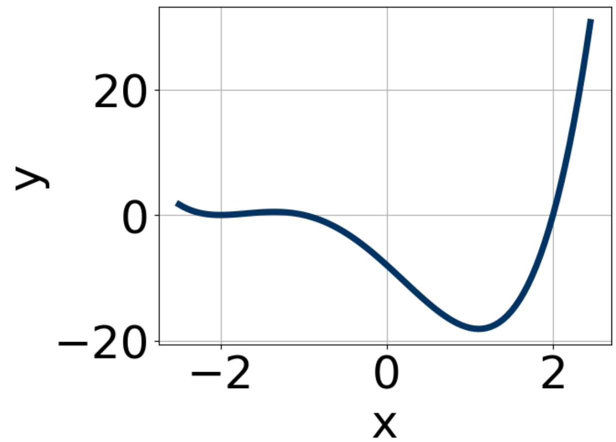
8. Write an equation that *could* represent the graph below.




The solution is $20(x+2)^6(x+1)^{11}(x-2)^7$.

> **Plausible alternative answers include:**The factor $-2$ should have an even power and the factor $-1$ should have an odd power. * This is the correct option. This corresponds to the leading coefficient being the opposite value than it should be. The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign. The factor $(x+1)$ should have an odd power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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9. Construct the lowest-degree polynomial given the zeros below.

$$-5+3i \text{ and } -2$$

The solution is $x^3+12x^2+54x+68$.

> **Plausible alternative answers include:**$x^3+x^2-x-6$, which corresponds to multiplying out $(x-3)(x+2)$. * $x^3+12x^2+54x+68$, which is the correct option. $x^3+x^2+7x+10$, which corresponds to multiplying out $(x+5)(x+2)$. $x^3-12x^2+54x-68$, which corresponds to multiplying out $(x-(-5+3i))(x-(-5-3i))(x-2)$. This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-5+3i))(x-(-5-3i))(x-(-2))$.

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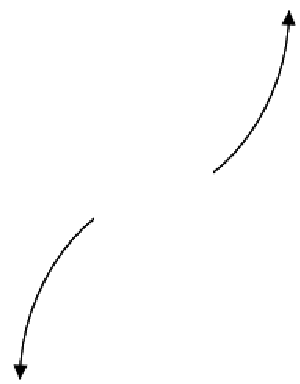
10. Describe the end behavior of the polynomial below.

$$f(x)=5(x+8)^5(x-8)^{10}(x+3)^5(x-3)^5$$

The solution is the graph below.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.