This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Determine whether the function below is 1-1.

$$f(x) = (6x - 42)^3$$

The solution is yes, which is option E.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. Yes, the function is 1-1.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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2. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -x^3 - 3x^2 + 2x + 2 \text{ and } g(x) = x^3 - 4x^2 - 4x + 1$$

The solution is 2.0, which is option A.

A. $(f \circ g)(-1) \in [0.5, 3.6]$

* This is the correct solution

B. $(f \circ g)(-1) \in [-23.9, -17.1]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [-4.7, -2.3]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-15.4, -13.6]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-3x+14} \text{ and } g(x) = 5x^4 + 6x^2 + 6x + 7$$

The solution is The domain is all Real numbers less than or equal to$x = 4.666666666666667.$, which is option B.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-6.8, -1.8]$

B. The domain is all Real numbers less than or equal to $x = a$, where $a \in [1.67, 13.67]$

C. The domain is all Real numbers except $x = a$, where $a \in [0.75, 8.75]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-12.67, -1.67]$ and $b \in [-9.25, -3.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = e^{x+5} + 3$$

The solution is $f^{-1}(8) = -3.391$, which is option E.

A. $f^{-1}(8) \in [6.59, 6.73]$

This solution corresponds to distractor 1.

B. $f^{-1}(8) \in [5.22, 5.5]$

This solution corresponds to distractor 2.

C. $f^{-1}(8) \in [5.46, 5.64]$

This solution corresponds to distractor 4.

D. $f^{-1}(8) \in [3.88, 4.13]$

This solution corresponds to distractor 3.

E. $f^{-1}(8) \in [-3.47, -3.25]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^{-1}(-15)$ belongs to.

$$f(x) = 2x^2 + 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [2.52, 3.2]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(-15) \in [5.44, 6.22]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(-15) \in [3.96, 4.25]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(-15) \in [2.07, 3.05]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{5}{4x+15} \text{ and } g(x) = \frac{2}{3x-16}$$

The solution is The domain is all Real numbers except $x = -3.75$ and $x = 5.333333333333333$, which is option D.

A. The domain is all Real numbers except $x = a$, where $a \in [-0.8, 7.2]$

B. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-5.33, 4.67]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-6.2, -4.2]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-6.75, 5.25]$ and $b \in [1.33, 8.33]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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7. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 4x^3 + x^2 - 3x + 2 \text{ and } g(x) = -2x^3 + 3x^2 - x$$

The solution is 2.0, which is option D.

A. $(f \circ g)(1) \in [-79, -78]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [8, 16]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-86, -83]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(1) \in [-1, 6]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -10$ and choose the interval the $f^{-1}(-10)$ belongs to.

$$f(x) = \sqrt[3]{5x-3}$$

The solution is -199.4, which is option B.

A. $f^{-1}(-10) \in [199.05, 200.26]$

   This solution corresponds to distractor 2.

B. $f^{-1}(-10) \in [-199.96, -198.03]$

   * This is the correct solution.

C. $f^{-1}(-10) \in [-201.02, -199.92]$

   Distractor 1: This corresponds to

D. $f^{-1}(-10) \in [200.39, 201.01]$

   This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

   This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = \ln(x+2) + 4$$

The solution is $f^{-1}(7) = 18.086$, which is option C.

A. $f^{-1}(7) \in [150.6, 155.8]$

   This solution corresponds to distractor 2.

B. $f^{-1}(7) \in [59870.8, 59876.5]$

   This solution corresponds to distractor 1.

C. $f^{-1}(7) \in [17.6, 20]$

   This is the solution.

D. $f^{-1}(7) \in [8106.5, 8108]$

   This solution corresponds to distractor 4.

E. $f^{-1}(7) \in [20.6, 23.4]$

   This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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10. Determine whether the function below is 1-1.

$$f(x) = \sqrt{3x+17}$$

The solution is yes, which is option E.

A. No, because there is an x-value that goes to 2 different y-values.

   Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. Yes, the function is 1-1.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.