This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$3, 1, \text{ and } \frac{7}{3}$$

The solution is $3x^3 - 19x^2 + 37x - 21$, which is option A.

A. $a \in [1, 9], b \in [-19, -12], c \in [36, 42], \text{ and } d \in [-22, -19]$

* $3x^3 - 19x^2 + 37x - 21$, which is the correct option.

B. $a \in [1, 9], b \in [-12, 4], c \in [-23, -22], \text{ and } d \in [21, 28]$

$3x^3 - 1x^2 - 23x + 21$, which corresponds to multiplying out $(x + 3)(x - 1)(3x - 7)$.

C. $a \in [1, 9], b \in [5, 11], c \in [-20, -16], \text{ and } d \in [-22, -19]$

$3x^3 + 5x^2 - 19x - 21$, which corresponds to multiplying out $(x + 3)(x + 1)(3x - 7)$.

D. $a \in [1, 9], b \in [-19, -12], c \in [36, 42], \text{ and } d \in [21, 28]$

$3x^3 - 19x^2 + 37x + 21$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [1, 9], b \in [9, 25], c \in [36, 42], \text{ and } d \in [21, 28]$

$3x^3 + 19x^2 + 37x + 21$, which corresponds to multiplying out $(x + 3)(x + 1)(3x + 7)$.

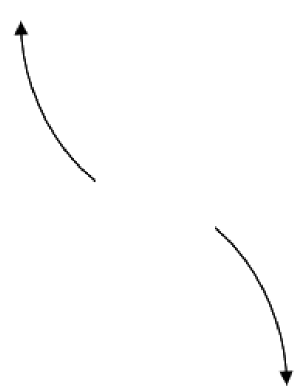
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-3)(x-1)(3x-7)$

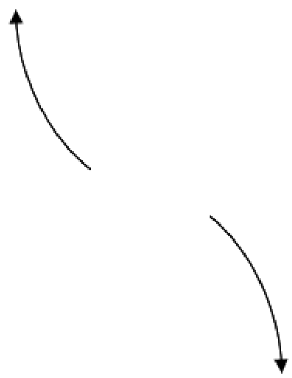
---

2. Describe the end behavior of the polynomial below.

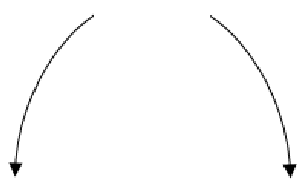
$$f(x) = -2(x + 3)^2(x - 3)^3(x + 4)^2(x - 4)^4$$

The solution is the graph below, which is option A.

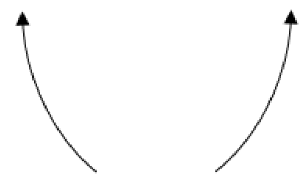




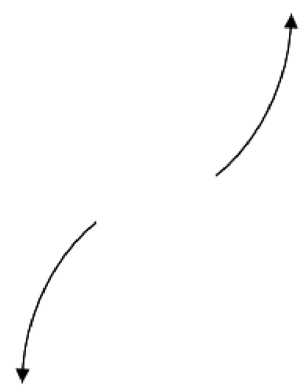
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 3i \text{ and } 2$$

The solution is $x^3 + 6x^2 + 9x - 50$, which is option D.

A. $b \in [-7.2, -5.5], c \in [6.8, 10.2],$ and $d \in [48.9, 50.9]$

$x^3 - 6x^2 + 9x + 50$, which corresponds to multiplying out $(x - (-4 - 3i))(x - (-4 + 3i))(x + 2)$.

B. $b \in [-0.8, 2.4], c \in [1.5, 2.3],$ and $d \in [-8.1, -6.2]$

$x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x + 4)(x - 2)$.

C. $b \in [-0.8, 2.4], c \in [-0.7, 1.3],$ and $d \in [-7.9, -5.2]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x + 3)(x - 2)$.

D. $b \in [2.7, 8.3], c \in [6.8, 10.2],$ and $d \in [-53.6, -47.8]$

* $x^3 + 6x^2 + 9x - 50$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 - 3i))(x - (-4 + 3i))(x - (2))$.

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{4}, \frac{-2}{3}, \text{ and } \frac{-4}{3}$$

The solution is $36x^3 + 9x^2 - 94x - 56$, which is option B.

A. $a \in [35, 45], b \in [129, 138], c \in [152, 162],$ and $d \in [53, 59]$

$36x^3 + 135x^2 + 158x + 56$, which corresponds to multiplying out $(4x + 7)(3x + 2)(3x + 4)$.

B. $a \in [35, 45], b \in [9, 14], c \in [-96, -83],$ and $d \in [-56, -51]$

* $36x^3 + 9x^2 - 94x - 56$, which is the correct option.

C. $a \in [35, 45], b \in [83, 91], c \in [10, 14],$ and $d \in [-56, -51]$

$36x^3 + 87x^2 + 10x - 56$, which corresponds to multiplying out $(4x + 7)(3x - 2)(3x + 4)$.

D. $a \in [35, 45], b \in [-10, 0], c \in [-96, -83],$ and $d \in [53, 59]$

$36x^3 - 9x^2 - 94x + 56$, which corresponds to multiplying out $(4x + 7)(3x - 2)(3x - 4)$.

E. $a \in [35, 45], b \in [9, 14], c \in [-96, -83],$ and $d \in [53, 59]$

$36x^3 + 9x^2 - 94x + 56$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 7)(3x + 2)(3x + 4)$

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 3i \text{ and } -2$$

The solution is $x^3 - 4x^2 + 6x + 36$, which is option A.

A. $b \in [-8, 0], c \in [5.4, 8.1],$ and $d \in [35, 43]$

* $x^3 - 4x^2 + 6x + 36$, which is the correct option.

B. $b \in [2, 8], c \in [5.4, 8.1],$ and $d \in [-39, -29]$

$x^3 + 4x^2 + 6x - 36$, which corresponds to multiplying out $(x - (3 - 3i))(x - (3 + 3i))(x - 2)$.

C. $b \in [1, 2], c \in [-2.7, 2.2],$ and $d \in [-8, -2]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x - 3)(x + 2)$.

D. $b \in [1, 2], c \in [4.6, 5.1],$ and $d \in [4, 8]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 3)(x + 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 - 3i))(x - (3 + 3i))(x - (-2))$.

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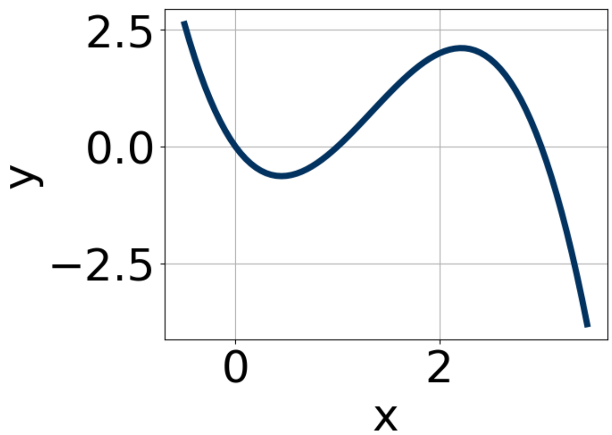
6. Which of the following equations *could* be of the graph presented below?



The solution is $-17x^5(x - 3)^{11}(x - 1)^{11}$, which is option B.

A. $-11x^4(x - 3)^6(x - 1)^7$

The factors 0 and 3 have have been odd power.

B. $-17x^5(x - 3)^{11}(x - 1)^{11}$

* This is the correct option.

C. $8x^{10}(x - 3)^9(x - 1)^5$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

D. $20x^5(x - 3)^{11}(x - 1)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-10x^8(x - 3)^5(x - 1)^5$

The factor 0 should have been an odd power.

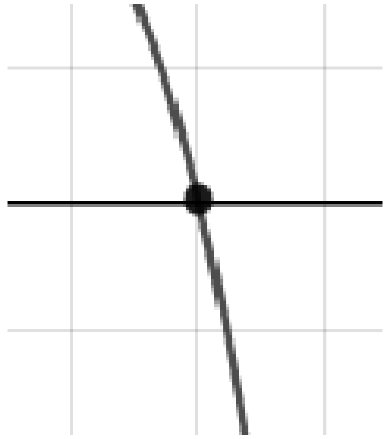
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

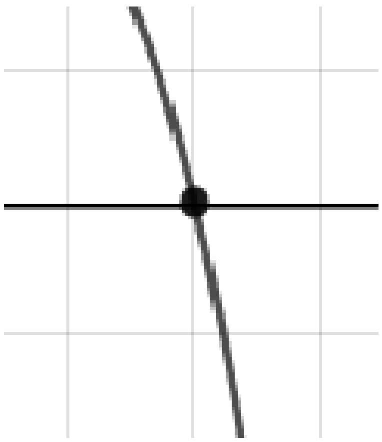
---

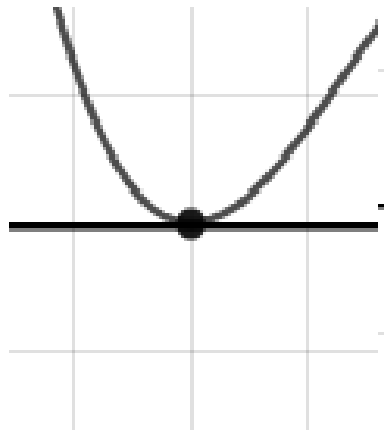
7. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

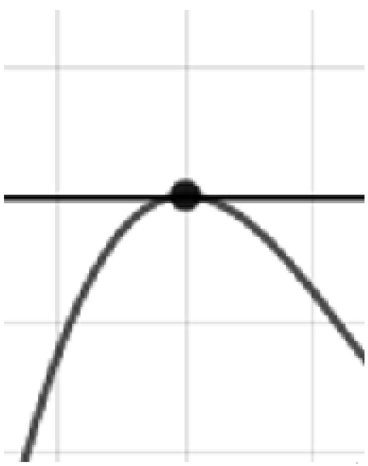
$$f(x) = -8(x + 3)^{12}(x - 3)^8(x + 8)^6(x - 8)^3$$

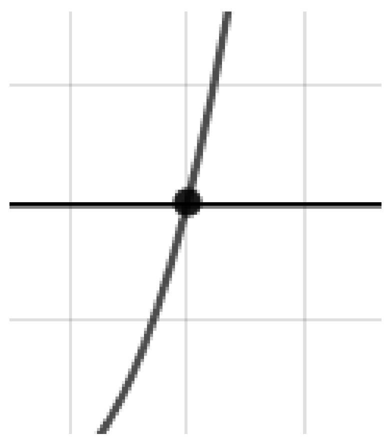
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

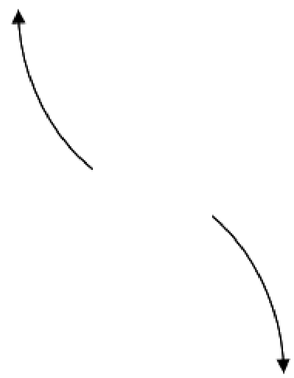
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

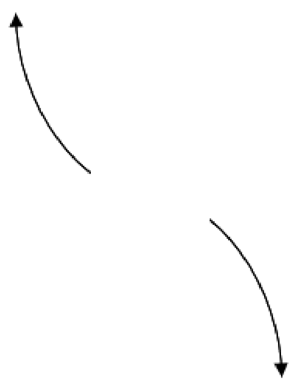
---

8. Describe the end behavior of the polynomial below.

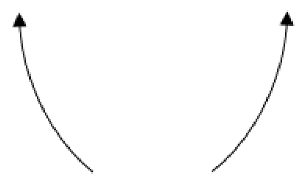
$$f(x) = -3(x+3)^2(x-3)^3(x+2)^5(x-2)^7$$

The solution is the graph below, which is option A.

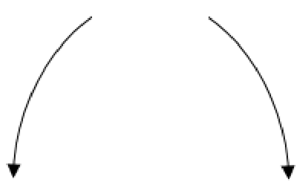




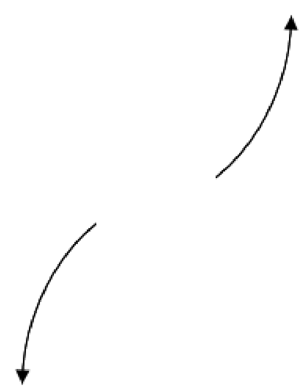
A.



C.



B.



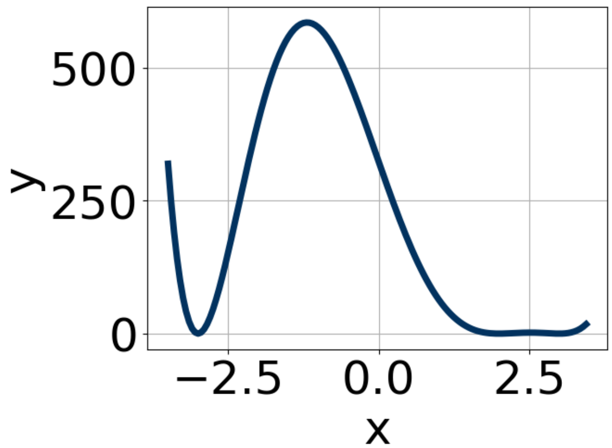
D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

9. Which of the following equations *could* be of the graph presented below?



The solution is $17(x+3)^{10}(x-3)^6(x-2)^8$, which is option A.

A. $17(x+3)^{10}(x-3)^6(x-2)^8$

* This is the correct option.

B. $-18(x+3)^{10}(x-3)^{10}(x-2)^{11}$

The factor $(x-2)$ should have an even power and the leading coefficient should be the opposite sign.

C. $-18(x+3)^{10}(x-3)^4(x-2)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $10(x+3)^4(x-3)^7(x-2)^5$

The factors $(x-3)$ and $(x-2)$ should both have even powers.

E. $4(x+3)^4(x-3)^4(x-2)^5$

The factor $(x-2)$ should have an even power.

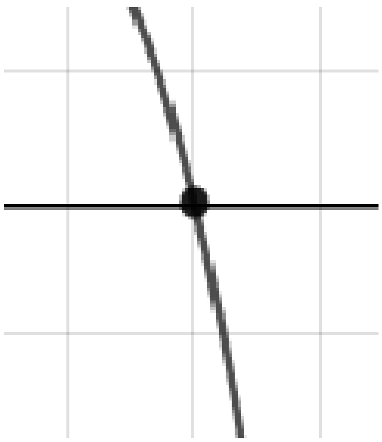
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

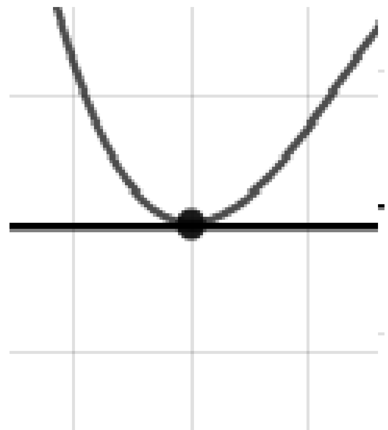
10. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

$$f(x) = 4(x+9)^4(x-9)^2(x+4)^{13}(x-4)^8$$

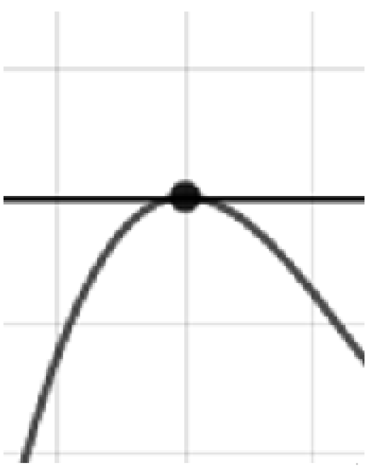
The solution is the graph below, which is option C.



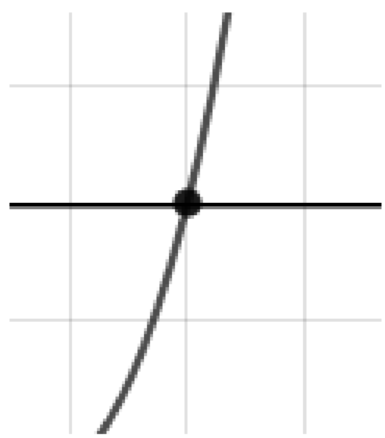
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.