This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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66. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 - 21x^2 + 10}{x - 2}$$

The solution is $9x^2 - 3x - 6 + \dfrac{-2}{x-2}$

A. $a \in [3, 12], b \in [-40, -38], c \in [73, 83],$ and $r \in [-151, -144]$.

You divided by the opposite of the factor.

B. $a \in [3, 12], b \in [-16, -10], c \in [-13, -10],$ and $r \in [-5, 1]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [15, 20], b \in [14, 20], c \in [29, 38],$ and $r \in [62, 75]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [15, 20], b \in [-59, -54], c \in [113, 120],$ and $r \in [-220, -210]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [3, 12], b \in [-8, -2], c \in [-11, -1],$ and $r \in [-5, 1]$.

* This is the solution!

General Comments: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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67. Factor the polynomial below completely, knowing that $x - 2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^4 + 31x^3 + 5x^2 - 122x - 120$$

The solution is $[-4, -1.6666666666666667, -1.5, 2]$

A. $z_1 \in [-3, 2],\ z_2 \in [0.56, 0.81], z_3 \in [0.47, 1],$ and $z_4 \in [3.2, 4.6]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-8, -3],\ z_2 \in [-0.8, -0.41], z_3 \in [-0.66, -0.15],$ and $z_4 \in [1.8, 3.3]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-3, 2],\ z_2 \in [0.66, 0.85], z_3 \in [2.76, 4.17],$ and $z_4 \in [3.2, 4.6]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-3, 2],\ z_2 \in [1.5, 1.54], z_3 \in [0.96, 2.43],$ and $z_4 \in [3.2, 4.6]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-8, -3],\ z_2 \in [-1.78, -1.63], z_3 \in [-1.97, -1.17],$ and $z_4 \in [1.8, 3.3]$

* This is the solution!

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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68. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 - 38x^2 + 76x - 50}{x - 3}$$

The solution is $6x^2 - 20x + 16 + \dfrac{-2}{x-3}$

A. $a \in [15, 20],\ b \in [9, 20],\ c \in [123, 131],$ and $r \in [319, 324]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [4, 8],\ b \in [-29, -21],\ c \in [20, 26],$ and $r \in [-3, 3]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [15, 20],\ b \in [-99, -91],\ c \in [347, 355],$ and $r \in [-1112, -1102]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

D. $a \in [4, 8],\ b \in [-24, -19],\ c \in [14, 22],$ and $r \in [-3, 3]$.

* This is the solution!

E. $a \in [4, 8],\ b \in [-59, -54],\ c \in [243, 247],$ and $r \in [-783, -777]$.

You divided by the opposite of the factor.

General Comments: Be sure to synthetically divide by the zero of the denominator!

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69. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 9x^3 + 9x^2 - 28x - 20$$

The solution is $[-2, -0.6666666666666666, 1.6666666666666667]$

A. $z_1 \in [-1.92, -1.5],\ z_2 \in [0.54, 0.69],$ and $z_3 \in [1.89, 2.3]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5.17, -4.86],\ z_2 \in [0.01, 0.57],$ and $z_3 \in [1.89, 2.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-2.19, -1.97],\ z_2 \in [-1.64, -1.25],$ and $z_3 \in [0.37, 0.66]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-0.88, -0.21],\ z_2 \in [1.24, 1.7],$ and $z_3 \in [1.89, 2.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-2.19, -1.97],\ z_2 \in [-1.1, -0.25],$ and $z_3 \in [1.49, 1.7]$

* This is the solution!

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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70. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 2x^2 + 5x + 4$$

The solution is $\pm 1, \pm 2, \pm 4$

A. $\pm 1, \pm 2$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. $\pm 1, \pm 2, \pm 4$

* This is the solution **since we asked for the possible Integer roots**!

C. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comments: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.
