This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$f(x) = \ln(x + 2) + 4$$

The solution is $f^{-1}(9) = 146.413$, which is option A.

A. $f^{-1}(9) \in [135.41, 149.41]$

This is the solution.

B. $f^{-1}(9) \in [59871.14, 59879.14]$

This solution corresponds to distractor 4.

C. $f^{-1}(9) \in [442410.39, 442412.39]$

This solution corresponds to distractor 1.

D. $f^{-1}(9) \in [148.41, 154.41]$

This solution corresponds to distractor 3.

E. $f^{-1}(9) \in [1096.63, 1103.63]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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2. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 4x^4 + 4x^3 + 3x^2 + x + 5 \text{ and } g(x) = \frac{5}{4x + 25}$$

The solution is The domain is all Real numbers except $x = -6.25$, which is option A.

A. The domain is all Real numbers except $x = a$, where $a \in [-8.25, -3.25]$

B. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-9.4, -0.4]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-8.6, -4.6]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-7.25, -0.25]$ and $b \in [-8.4, -5.4]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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3. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 - 2x^2 + 4x - 1 \text{ and } g(x) = -4x^3 - 1x^2 + 3x$$

The solution is -1.0, which is option B.

A. $(f \circ g)(1) \in [3.41, 4.26]$

Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-1.1, -0.45]$

* This is the correct solution

C. $(f \circ g)(1) \in [-0.14, 0.31]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(1) \in [7.8, 8.24]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^{-1}(-15)$ belongs to.

$$f(x) = 2x^2 - 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [4.29, 4.73]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B. $f^{-1}(-15) \in [1.98, 2.74]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C. $f^{-1}(-15) \in [4.96, 5.57]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(-15) \in [2.51, 3.02]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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5. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 2x^3 + 8x^2 + x + 3 \text{ and } g(x) = x^2 + 1$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-3.67, -1.67]$

B. The domain is all Real numbers except $x = a$, where $a \in [-6.67, -0.67]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-10, -3]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-6.2, -5.2]$ and $b \in [-8.8, -4.8]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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6. Determine whether the function below is 1-1.

$$f(x) = (6x + 40)^3$$

The solution is yes, which is option C.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. Yes, the function is 1-1.

* This is the solution.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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7. Choose the interval below that f composed with g at $x = -2$ is in.

$$f(x) = -3x^3 - 4x^2 + 3x \text{ and } g(x) = -2x^3 - 2x^2 + 3x$$

The solution is -34.0, which is option A.

A. $(f \circ g)(-2) \in [-39, -31]$

* This is the correct solution

B. $(f \circ g)(-2) \in [-29, -24]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-2) \in [-44, -39]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-2) \in [-20, -16]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 12$ and choose the interval the $f^{-1}(12)$ belongs to.

$$f(x) = \sqrt[3]{4x + 5}$$

The solution is 430.75, which is option C.

A. $f^{-1}(12) \in [-432.1, -430.2]$

This solution corresponds to distractor 2.

B. $f^{-1}(12) \in [433, 433.6]$

Distractor 1: This corresponds to

C. $f^{-1}(12) \in [429.5, 432.6]$

* This is the correct solution.

D. $f^{-1}(12) \in [-435.8, -432.9]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Determine whether the function below is 1-1.

$$f(x) = (5x + 30)^3$$

The solution is yes, which is option D.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. Yes, the function is 1-1.

* This is the solution.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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10. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = \ln(x - 3) + 2$$

The solution is $f^{-1}(7) = 151.413$, which is option E.

A. $f^{-1}(7) \in [145.2, 146.4]$

This solution corresponds to distractor 3.

B. $f^{-1}(7) \in [22028.1, 22028.9]$

This solution corresponds to distractor 2.

C. $f^{-1}(7) \in [52.3, 58.4]$

This solution corresponds to distractor 4.

D. $f^{-1}(7) \in [8104.6, 8107.1]$

This solution corresponds to distractor 1.

E. $f^{-1}(7) \in [148.5, 152.3]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.
