This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 3i \text{ and } -3$$

The solution is $x^3 - 7x^2 + 4x + 102$, which is option B.

A. $b \in [7, 11], c \in [2.8, 4.9], \text{ and } d \in [-111, -99]$

$x^3 + 7x^2 + 4x - 102$, which corresponds to multiplying out $(x - (5 + 3i))(x - (5 - 3i))(x - 3)$.

B. $b \in [-13, -3], c \in [2.8, 4.9], \text{ and } d \in [92, 105]$

* $x^3 - 7x^2 + 4x + 102$, which is the correct option.

C. $b \in [-5, 2], c \in [-5.1, -1], \text{ and } d \in [-19, -11]$

$x^3 + x^2 - 2x - 15$, which corresponds to multiplying out $(x - 5)(x + 3)$.

D. $b \in [-5, 2], c \in [-1.5, 3.7], \text{ and } d \in [-13, -7]$

$x^3 + x^2 - 9$, which corresponds to multiplying out $(x - 3)(x + 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 3i))(x - (5 - 3i))(x - (-3))$.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-5}{3}, \frac{7}{2}, \text{ and } \frac{3}{5}$$

The solution is $30x^3 - 73x^2 - 142x + 105$, which is option D.

A. $a \in [30, 34], b \in [-76, -69], c \in [-148, -138], \text{ and } d \in [-107, -100]$

$30x^3 - 73x^2 - 142x - 105$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [30, 34], b \in [68, 75], c \in [-148, -138], \text{ and } d \in [-107, -100]$

$30x^3 + 73x^2 - 142x - 105$, which corresponds to multiplying out $(3x - 5)(2x + 7)(5x + 3)$.

C. $a \in [30, 34], b \in [-174, -166], c \in [267, 273], \text{ and } d \in [-107, -100]$

$30x^3 - 173x^2 + 268x - 105$, which corresponds to multiplying out $(3x - 5)(2x - 7)(5x - 3)$.

D. $a \in [30, 34], b \in [-76, -69], c \in [-148, -138]$, and $d \in [104, 107]$

* $30x^3 - 73x^2 - 142x + 105$, which is the correct option.

E. $a \in [30, 34], b \in [36, 39], c \in [-208, -198]$, and $d \in [104, 107]$

$30x^3 + 37x^2 - 208x + 105$, which corresponds to multiplying out $(3x - 5)(2x + 7)(5x - 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 5)(2x - 7)(5x - 3)$

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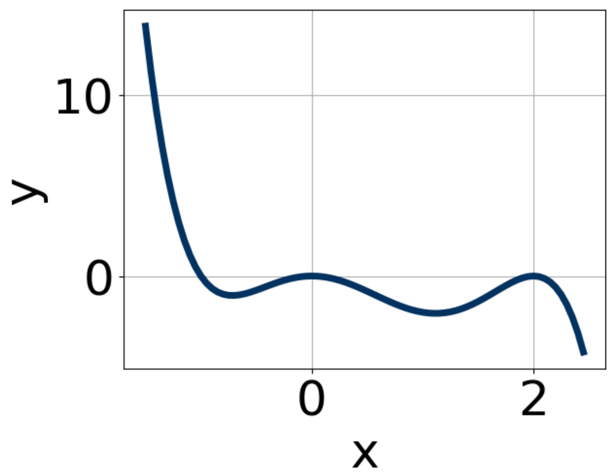
3. Which of the following equations *could* be of the graph presented below?



The solution is $-14x^6(x - 2)^{10}(x + 1)^9$, which is option A.

A. $-14x^6(x - 2)^{10}(x + 1)^9$

* This is the correct option.

B. $6x^6(x - 2)^8(x + 1)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $20x^{10}(x - 2)^6(x + 1)^8$

The factor $(x + 1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-4x^{10}(x - 2)^5(x + 1)^9$

The factor $(x - 2)$ should have an even power.

E. $-8x^{10}(x - 2)^7(x + 1)^8$

The factor $(x - 2)$ should have an even power and the factor $(x + 1)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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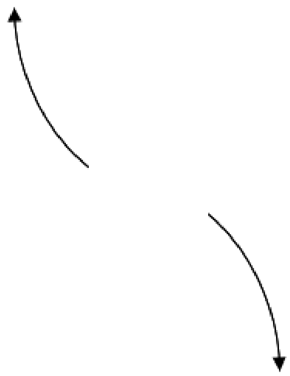
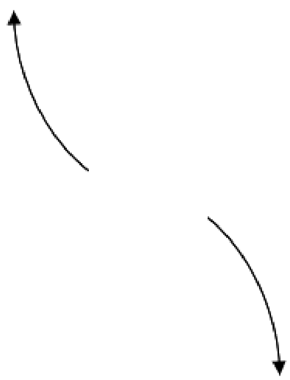
4. Describe the end behavior of the polynomial below.

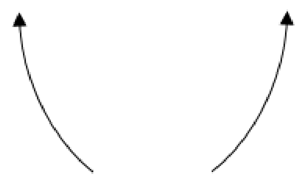
$$f(x) = -9(x + 5)^3(x - 5)^6(x + 2)^2(x - 2)^4$$

The solution is the graph below, which is option A.

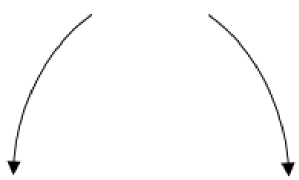




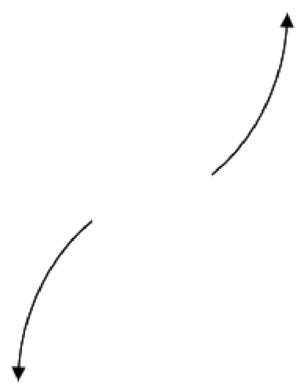
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$1, 5, \text{ and } \frac{-3}{5}$$

The solution is $5x^3 - 27x^2 + 7x + 15$, which is option D.

A. $a \in [4, 15], b \in [32, 36], c \in [41, 45], \text{ and } d \in [10, 17]$

$5x^3 + 33x^2 + 43x + 15$, which corresponds to multiplying out $(x + 1)(x + 5)(5x + 3)$.

B. $a \in [4, 15], b \in [19, 30], c \in [1, 17],$ and $d \in [-17, -11]$

$5x^3 + 27x^2 + 7x - 15$, which corresponds to multiplying out $(x + 1)(x + 5)(5x - 3)$.

C. $a \in [4, 15], b \in [-24, -14], c \in [-50, -34],$ and $d \in [-17, -11]$

$5x^3 - 17x^2 - 37x - 15$, which corresponds to multiplying out $(x + 1)(x - 5)(5x + 3)$.

D. $a \in [4, 15], b \in [-33, -26], c \in [1, 17],$ and $d \in [10, 17]$

* $5x^3 - 27x^2 + 7x + 15$, which is the correct option.

E. $a \in [4, 15], b \in [-33, -26], c \in [1, 17],$ and $d \in [-17, -11]$

$5x^3 - 27x^2 + 7x - 15$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-1)(x-5)(5x+3)$

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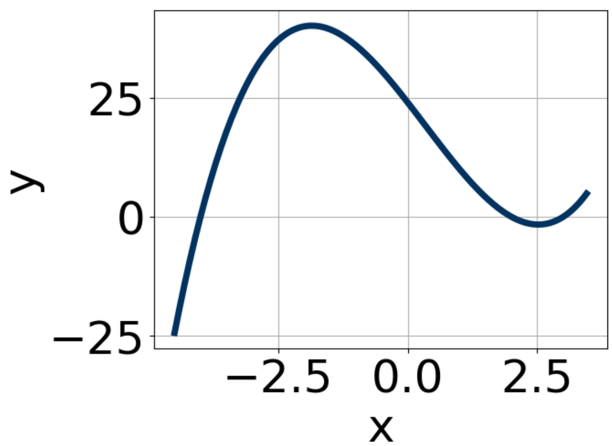
6. Which of the following equations *could* be of the graph presented below?



The solution is $14(x - 2)^{11}(x - 3)^5(x + 4)^5$, which is option A.

A. $14(x - 2)^{11}(x - 3)^5(x + 4)^5$

* This is the correct option.

B. $-6(x - 2)^5(x - 3)^9(x + 4)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $4(x - 2)^6(x - 3)^5(x + 4)^{11}$

The factor 2 should have been an odd power.

D. $7(x - 2)^{10}(x - 3)^{10}(x + 4)^7$

The factors 2 and 3 have have been odd power.

E. $-15(x - 2)^4(x - 3)^9(x + 4)^5$

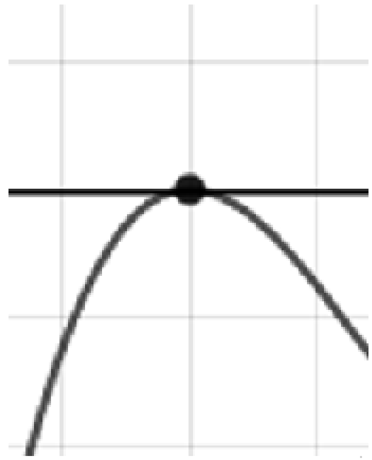
The factor $(x - 2)$ should have an odd power and the leading coefficient should be the opposite sign.

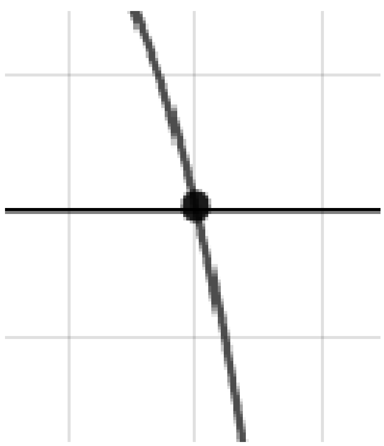
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

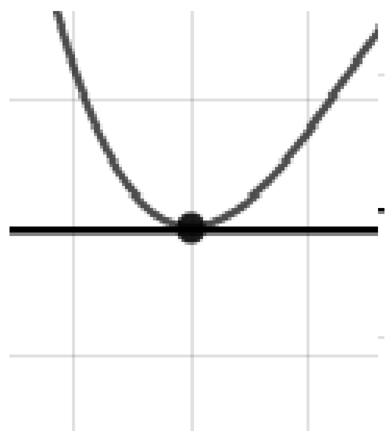
7. Describe the zero behavior of the zero $x = -9$ of the polynomial below.

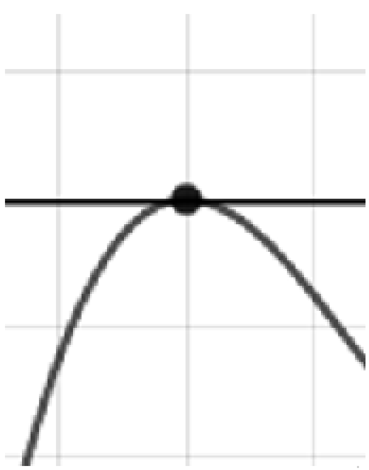
$$f(x) = 5(x+4)^{12}(x-4)^{8}(x-9)^{9}(x+9)^{8}$$

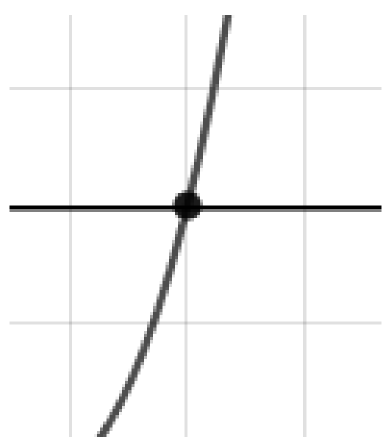
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 5i \text{ and } 4$$

The solution is $x^3 - 12x^2 + 73x - 164$, which is option D.

A. $b \in [1, 11], c \in [-11, 0]$, and $d \in [10, 17]$

   $x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

B. $b \in [3, 13], c \in [71, 75]$, and $d \in [163, 167]$

   $x^3 + 12x^2 + 73x + 164$, which corresponds to multiplying out $(x - (4 - 5i))(x - (4 + 5i))(x + 4)$.

C. $b \in [1, 11], c \in [-2, 6]$, and $d \in [-27, -17]$

   $x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

D. $b \in [-16, -8], c \in [71, 75]$, and $d \in [-165, -163]$

   * $x^3 - 12x^2 + 73x - 164$, which is the correct option.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 5i))(x - (4 + 5i))(x - (4))$.

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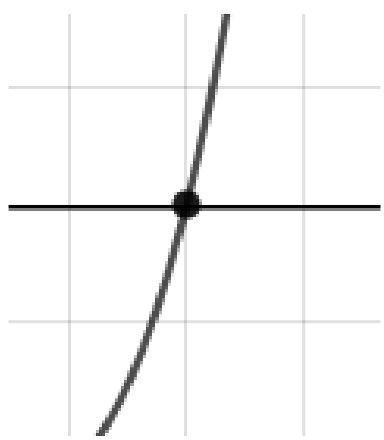
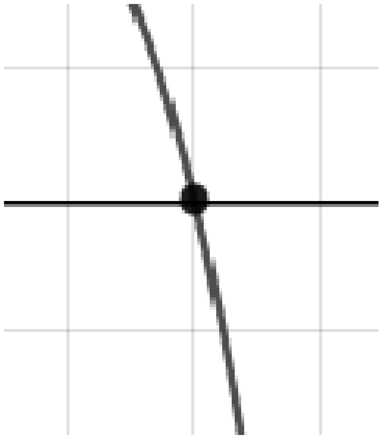
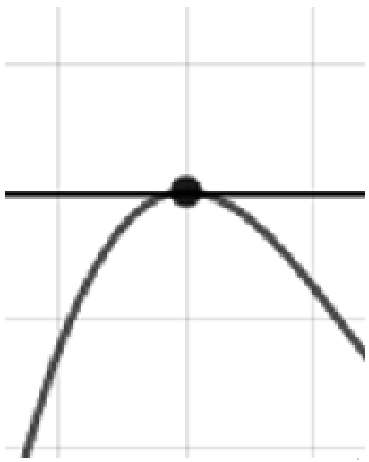
9. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

$$f(x) = 4(x + 9)^8(x - 9)^9(x + 8)^9(x - 8)^{10}$$

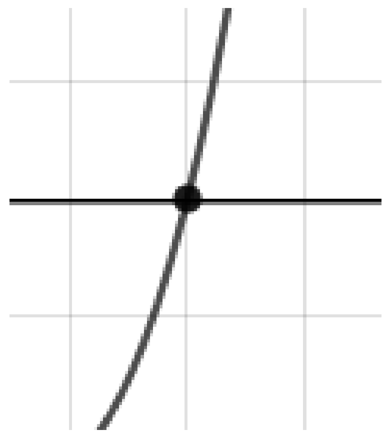
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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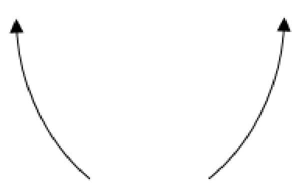
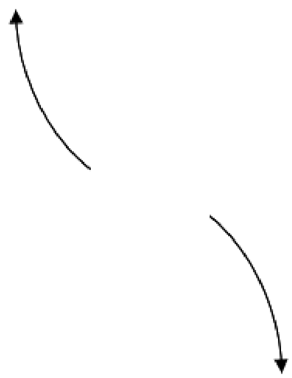
10. Describe the end behavior of the polynomial below.

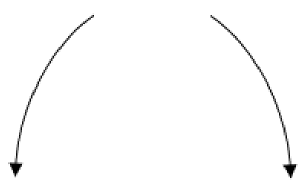
$$f(x) = 2(x-8)^4(x+8)^5(x-6)^2(x+6)^3$$

The solution is the graph below, which is option C.

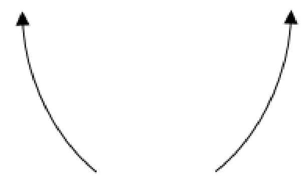




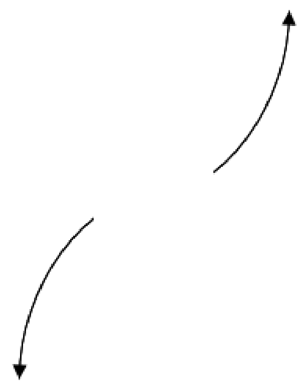
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.