This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

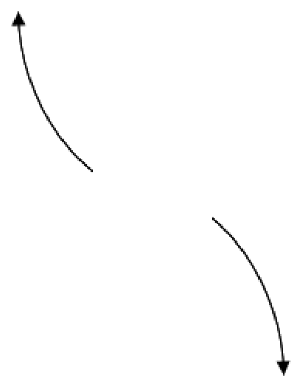
If you have a suggestion to make the keys better, please fill out the short survey here.

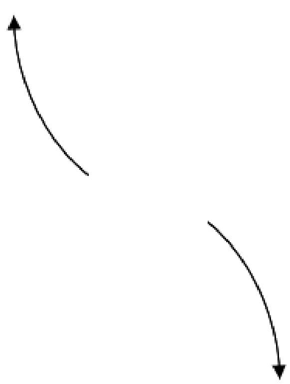
Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Describe the end behavior of the polynomial below.

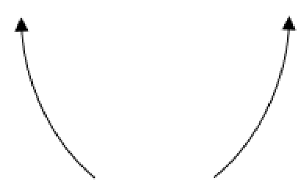
$$f(x) = -6(x-7)^3(x+7)^8(x+2)^3(x-2)^3$$

The solution is the graph below, which is option A.

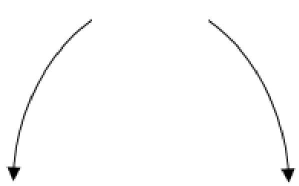




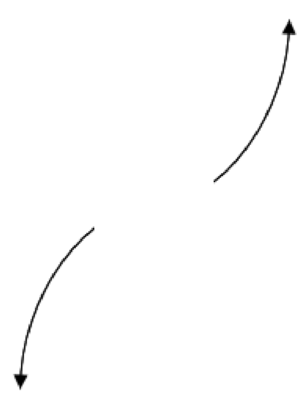
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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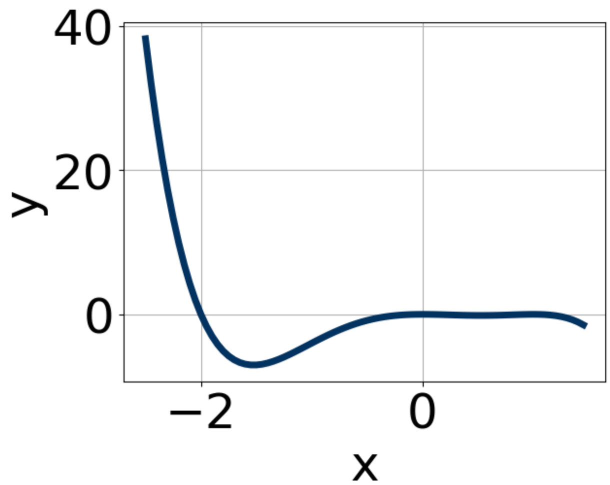
2. Which of the following equations *could* be of the graph presented below?



The solution is $-20x^4(x-1)^6(x+2)^7$, which is option E.

A. $8x^4(x-1)^4(x+2)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-15x^4(x-1)^{11}(x+2)^{10}$

The factor $(x-1)$ should have an even power and the factor $(x+2)$ should have an odd power.

C. $-3x^{10}(x-1)^9(x+2)^{11}$

The factor $(x-1)$ should have an even power.

D. $16x^8(x-1)^8(x+2)^4$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-20x^4(x-1)^6(x+2)^7$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{3}, \frac{4}{5}, \text{ and } \frac{-5}{4}$$

The solution is $60x^3 + 67x^2 - 42x - 40$, which is option C.

A. $a \in [51, 65], b \in [-78, -65], c \in [-43, -39], \text{ and } d \in [40, 46]$

$60x^3 - 67x^2 - 42x + 40$, which corresponds to multiplying out $(3x-2)(5x+4)(4x-5)$.

B. $a \in [51, 65], b \in [64, 74], c \in [-43, -39],$ and $d \in [40, 46]$

$60x^3+67x^2-42x+40$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [51, 65], b \in [64, 74], c \in [-43, -39],$ and $d \in [-41, -38]$

* $60x^3 + 67x^2 - 42x - 40$, which is the correct option.

D. $a \in [51, 65], b \in [82, 84], c \in [-26, -18],$ and $d \in [-41, -38]$

$60x^3 + 83x^2 - 22x - 40$, which corresponds to multiplying out $(3x - 2)(5x + 4)(4x + 5)$.

E. $a \in [51, 65], b \in [-13, -11], c \in [-83, -76],$ and $d \in [40, 46]$

$60x^3 - 13x^2 - 78x + 40$, which corresponds to multiplying out $(3x - 2)(5x - 4)(4x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 2)(5x - 4)(4x + 5)$

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$5, \frac{7}{4}, \text{ and } \frac{1}{4}$$

The solution is $16x^3 - 112x^2 + 167x - 35$, which is option A.

A. $a \in [12, 26], b \in [-113, -109], c \in [157, 171],$ and $d \in [-45, -34]$

* $16x^3 - 112x^2 + 167x - 35$, which is the correct option.

B. $a \in [12, 26], b \in [102, 105], c \in [113, 118],$ and $d \in [-45, -34]$

$16x^3 + 104x^2 + 113x - 35$, which corresponds to multiplying out $(x + 5)(4x + 7)(4x - 1)$.

C. $a \in [12, 26], b \in [48, 52], c \in [-153, -150],$ and $d \in [34, 42]$

$16x^3 + 48x^2 - 153x + 35$, which corresponds to multiplying out $(x + 5)(4x - 7)(4x - 1)$.

D. $a \in [12, 26], b \in [112, 116], c \in [157, 171],$ and $d \in [34, 42]$

$16x^3 + 112x^2 + 167x + 35$, which corresponds to multiplying out $(x + 5)(4x + 7)(4x + 1)$.

E. $a \in [12, 26], b \in [-113, -109], c \in [157, 171],$ and $d \in [34, 42]$

$16x^3 - 112x^2 + 167x + 35$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-5)(4x-7)(4x-1)$

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 5i \text{ and } -2$$

The solution is $x^3 - 6x^2 + 25x + 82$, which is option D.

A. $b \in [0.3, 3.1], c \in [5, 10],$ and $d \in [8, 12]$

$x^3 + x^2 + 7x + 10$, which corresponds to multiplying out $(x + 5)(x + 2)$.

B. $b \in [2.1, 8.9], c \in [15, 28]$, and $d \in [-89, -71]$

$x^3 + 6x^2 + 25x - 82$, which corresponds to multiplying out $(x - (4 - 5i))(x - (4 + 5i))(x - 2)$.

C. $b \in [0.3, 3.1], c \in [-3, -1]$, and $d \in [-8, -3]$

$x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x - 4)(x + 2)$.

D. $b \in [-7.3, -5.9], c \in [15, 28]$, and $d \in [82, 85]$

* $x^3 - 6x^2 + 25x + 82$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 5i))(x - (4 + 5i))(x - (-2))$.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 5i \text{ and } 3$$

The solution is $x^3 + 3x^2 + 16x - 102$, which is option D.

A. $b \in [-0.42, 1.63], c \in [-3.2, 1]$, and $d \in [-14, -6]$

$x^3 + x^2 - 9$, which corresponds to multiplying out $(x + 3)(x - 3)$.

B. $b \in [-4.7, -1.52], c \in [14.7, 18.4]$, and $d \in [102, 106]$

$x^3 - 3x^2 + 16x + 102$, which corresponds to multiplying out $(x - (-3 - 5i))(x - (-3 + 5i))(x + 3)$.

C. $b \in [-0.42, 1.63], c \in [1.8, 4.8]$, and $d \in [-17, -13]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x + 5)(x - 3)$.

D. $b \in [2.39, 3.09], c \in [14.7, 18.4]$, and $d \in [-107, -92]$

* $x^3 + 3x^2 + 16x - 102$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 5i))(x - (-3 + 5i))(x - (3))$.

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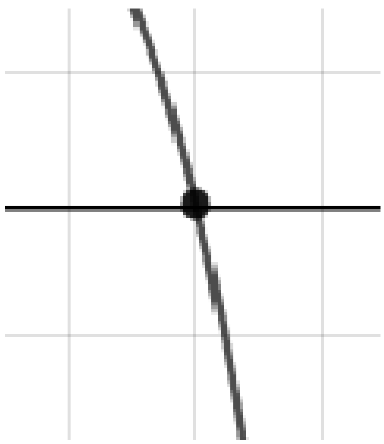
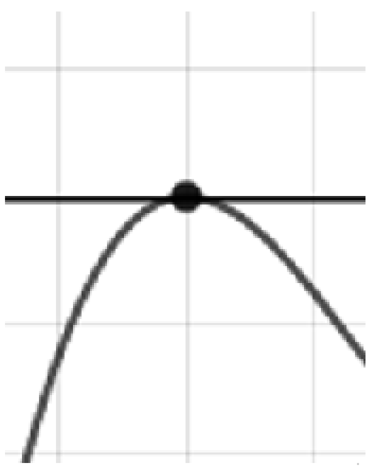
7. Describe the zero behavior of the zero $x = -2$ of the polynomial below.

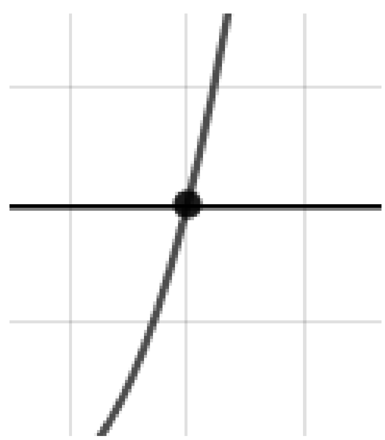
$$f(x) = -9(x - 5)^6(x + 5)^5(x - 2)^{11}(x + 2)^6$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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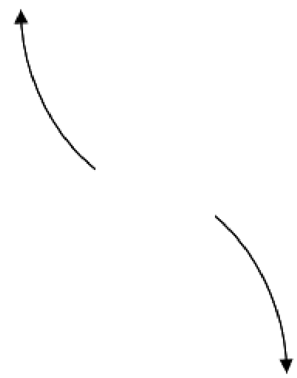
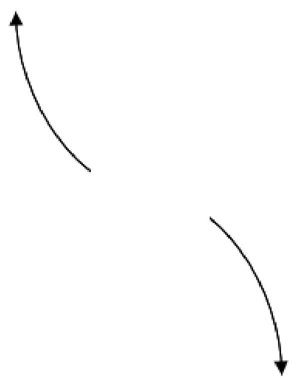
8. Describe the end behavior of the polynomial below.

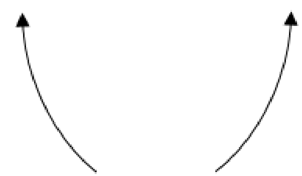
$$f(x) = -5(x+8)^2(x-8)^5(x+7)^3(x-7)^3$$

The solution is the graph below, which is option A.

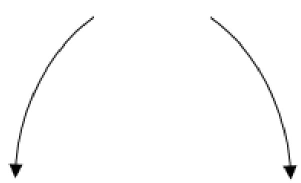




A.



C.



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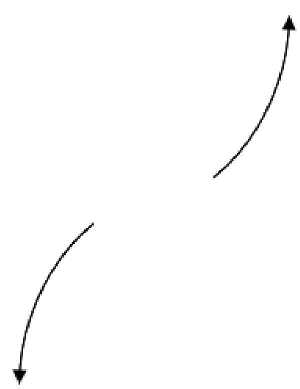

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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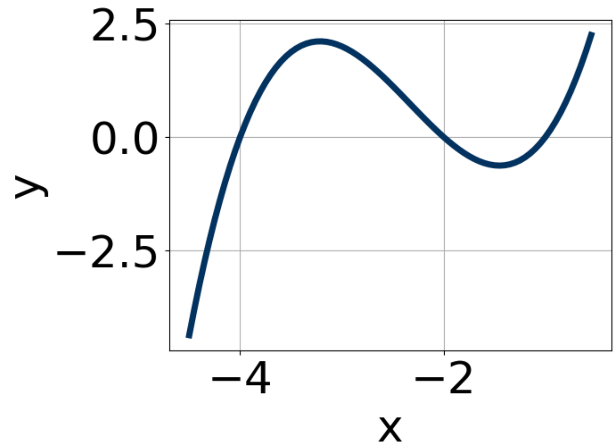
9. Which of the following equations *could* be of the graph presented below?



The solution is $20(x+4)^{11}(x+1)^7(x+2)^{11}$, which is option C.

A. $-4(x+4)^9(x+1)^9(x+2)^5$

   This corresponds to the leading coefficient being the opposite value than it should be.

B. $-2(x+4)^4(x+1)^5(x+2)^{11}$

   The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $20(x+4)^{11}(x+1)^7(x+2)^{11}$

   * This is the correct option.

D. $10(x+4)^4(x+1)^9(x+2)^{11}$

   The factor -4 should have been an odd power.

E. $12(x+4)^{10}(x+1)^8(x+2)^7$

   The factors -4 and -1 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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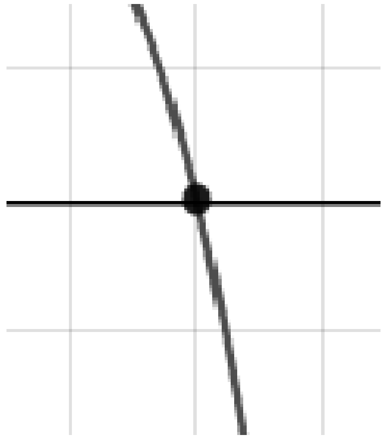
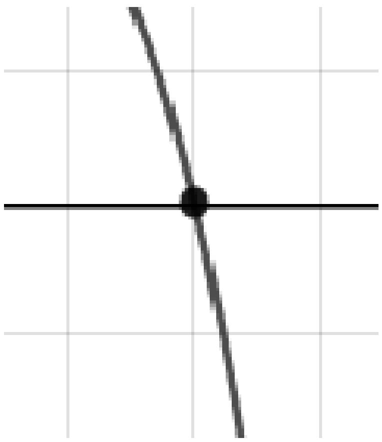
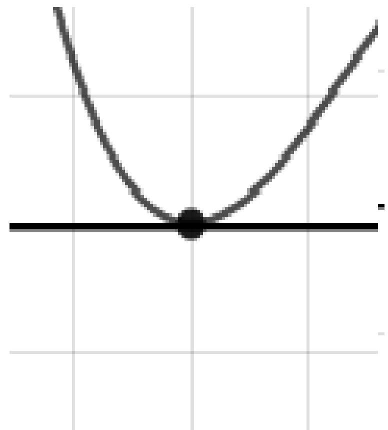
10. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

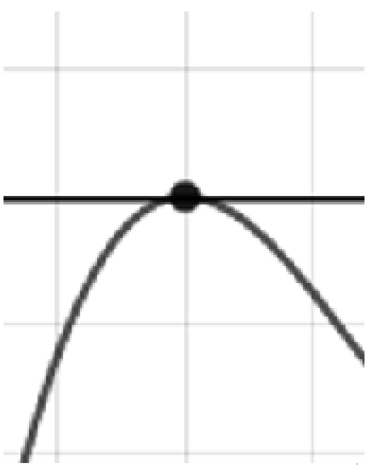
$$f(x) = -7(x+2)^2(x-2)^7(x+7)^3(x-7)^4$$

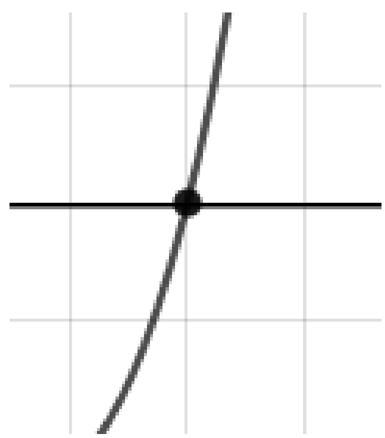
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.