This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } 1$$

The solution is $x^3 + 9x^2 + 31x - 41$, which is option D.

A. $b \in [0, 8], c \in [-9, -1],$ and $d \in [-1, 6]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

B. $b \in [0, 8], c \in [0, 10],$ and $d \in [-6, 2]$

$x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

C. $b \in [-14, -8], c \in [30, 39],$ and $d \in [40, 48]$

$x^3 - 9x^2 + 31x + 41$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x + 1)$.

D. $b \in [7, 15], c \in [30, 39],$ and $d \in [-46, -29]$

* $x^3 + 9x^2 + 31x - 41$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (1))$.

---

2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, \frac{2}{3}, \text{ and } -7$$

The solution is $12x^3 + 73x^2 - 75x + 14$, which is option D.

A. $a \in [3, 16], b \in [70, 76], c \in [-84, -73],$ and $d \in [-18, -11]$

$12x^3 + 73x^2 - 75x - 14$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [3, 16], b \in [92, 96], c \in [72, 87],$ and $d \in [8, 15]$

$12x^3 + 95x^2 + 79x + 14$, which corresponds to multiplying out $(4x + 1)(3x + 2)(x + 7)$.

C. $a \in [3, 16], b \in [78, 85], c \in [-44, -33],$ and $d \in [-18, -11]$

$12x^3 + 79x^2 - 37x - 14$, which corresponds to multiplying out $(4x + 1)(3x - 2)(x + 7)$.

D. $a \in [3, 16], b \in [70, 76], c \in [-84, -73],$ and $d \in [8, 15]$

* $12x^3 + 73x^2 - 75x + 14$, which is the correct option.

E. $a \in [3, 16], b \in [-76, -67], c \in [-84, -73],$ and $d \in [-18, -11]$

$12x^3 - 73x^2 - 75x - 14$, which corresponds to multiplying out $(4x + 1)(3x + 2)(x - 7)$.

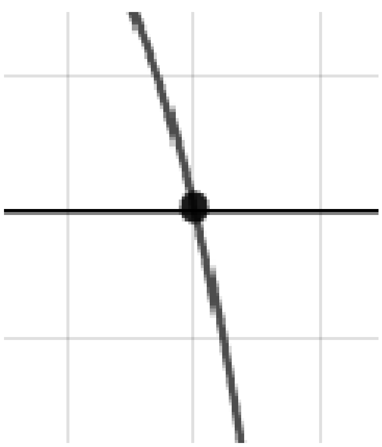
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(3x - 2)(x + 7)$

---

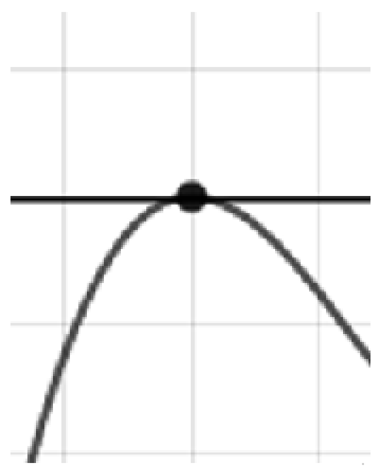
3. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = -9(x-6)^5(x+6)^4(x+3)^{11}(x-3)^8$$

The solution is the graph below, which is option C.

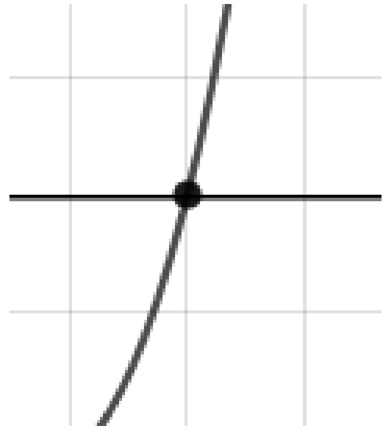


A.



B.

C. 

D. 

E. None of the above.

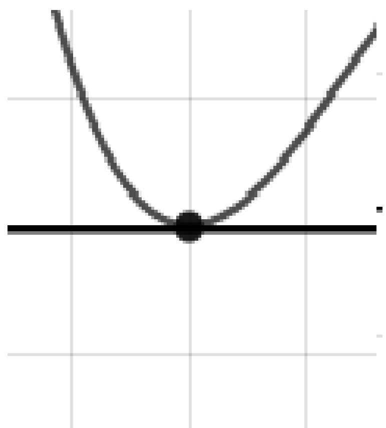
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

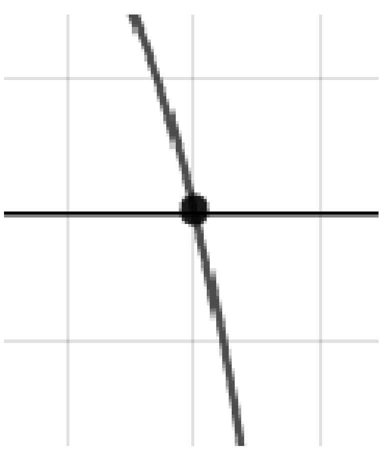
---

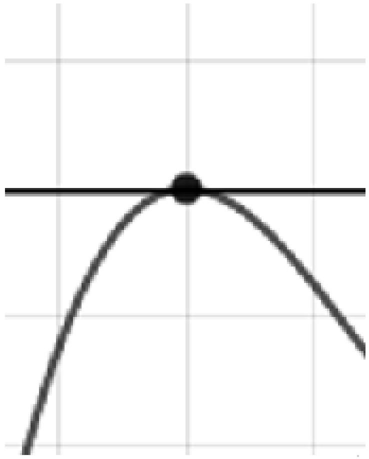
4. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

$$f(x) = -8(x-6)^7(x+6)^3(x+8)^8(x-8)^7$$

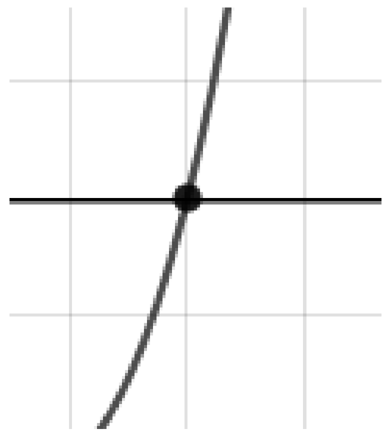
The solution is the graph below, which is option C.



A. 

B. 

C. 

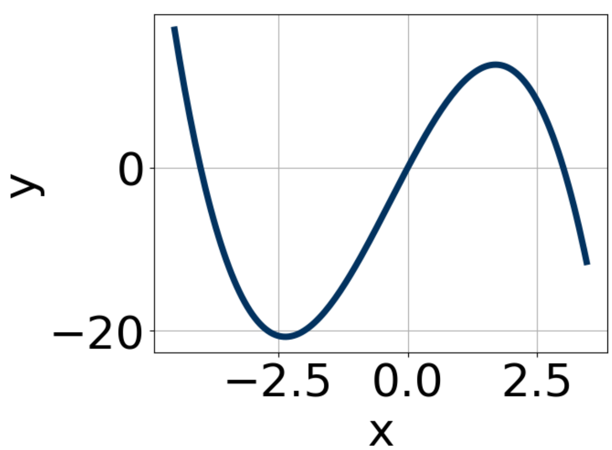
D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

5. Which of the following equations *could* be of the graph presented below?



The solution is $-8x^9(x+4)^9(x-3)^7$, which is option C.

A. $-17x^9(x+4)^6(x-3)^{11}$

The factor -4 should have been an odd power.

B. $-19x^{10}(x+4)^6(x-3)^7$

The factors -4 and 0 have have been odd power.

C. $-8x^9(x+4)^9(x-3)^7$

* This is the correct option.

D. $13x^{11}(x+4)^7(x-3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $16x^7(x+4)^{10}(x-3)^9$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

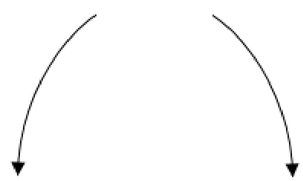
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

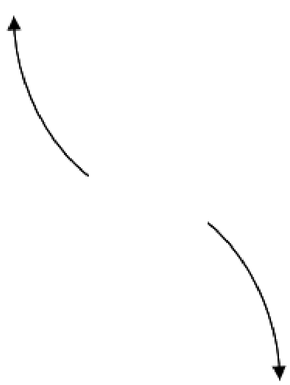
---

6. Describe the end behavior of the polynomial below.

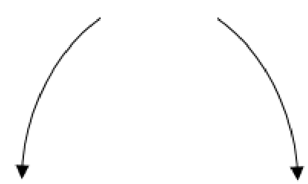
$$f(x) = -3(x+5)^5(x-5)^8(x+9)^2(x-9)^3$$

The solution is the graph below, which is option B.

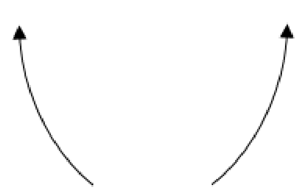




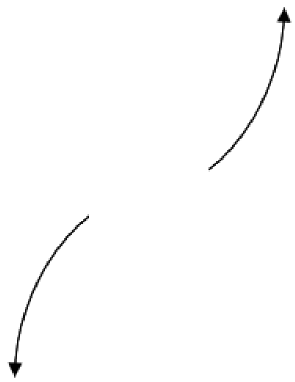
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 3i \text{ and } -1$$

The solution is $x^3 - 7x^2 + 17x + 25$, which is option B.

A. $b \in [6, 11], c \in [16, 18], \text{ and } d \in [-26, -24]$

   $x^3 + 7x^2 + 17x - 25$, which corresponds to multiplying out $(x - (4 - 3i))(x - (4 + 3i))(x - 1)$.

B. $b \in [-9, -4], c \in [16, 18], \text{ and } d \in [20, 28]$

   * $x^3 - 7x^2 + 17x + 25$, which is the correct option.

C. $b \in [-2, 6], c \in [-4, -2], \text{ and } d \in [-4, -1]$

   $x^3 + x^2 - 3x - 4$, which corresponds to multiplying out $(x - 4)(x + 1)$.

D. $b \in [-2, 6], c \in [3, 5], \text{ and } d \in [-1, 8]$

   $x^3 + x^2 + 4x + 3$, which corresponds to multiplying out $(x + 3)(x + 1)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 3i))(x - (4 + 3i))(x - (-1))$.

---

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{3}, \frac{-1}{4}, \text{ and } \frac{7}{4}$$

The solution is $48x^3 - 40x^2 - 69x - 14$, which is option B.

A. $a \in [44, 54], b \in [-110, -98], c \in [21, 29], \text{ and } d \in [9, 22]$

$48x^3 - 104x^2 + 27x + 14$, which corresponds to multiplying out $(3x - 2)(4x + 1)(4x - 7)$.

B. $a \in [44, 54], b \in [-41, -35], c \in [-74, -64], \text{ and } d \in [-15, -11]$

* $48x^3 - 40x^2 - 69x - 14$, which is the correct option.

C. $a \in [44, 54], b \in [-41, -35], c \in [-74, -64], \text{ and } d \in [9, 22]$

$48x^3 - 40x^2 - 69x + 14$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [44, 54], b \in [-134, -124], c \in [85, 86], \text{ and } d \in [-15, -11]$

$48x^3 - 128x^2 + 85x - 14$, which corresponds to multiplying out $(3x - 2)(4x - 1)(4x - 7)$.

E. $a \in [44, 54], b \in [40, 42], c \in [-74, -64], \text{ and } d \in [9, 22]$

$48x^3 + 40x^2 - 69x + 14$, which corresponds to multiplying out $(3x - 2)(4x - 1)(4x + 7)$.

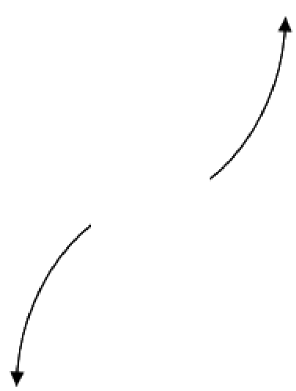
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 2)(4x + 1)(4x - 7)$

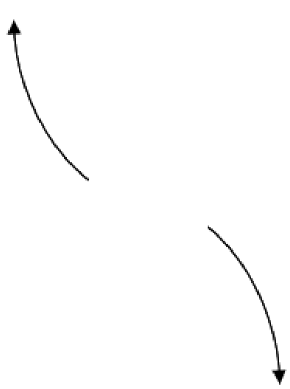
---

9. Describe the end behavior of the polynomial below.

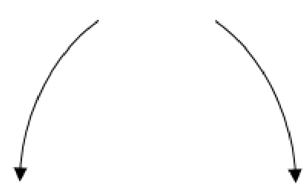
$$f(x) = 3(x - 3)^2(x + 3)^7(x + 9)^5(x - 9)^7$$

The solution is the graph below, which is option D.

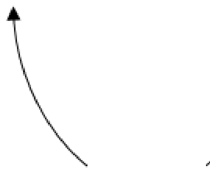


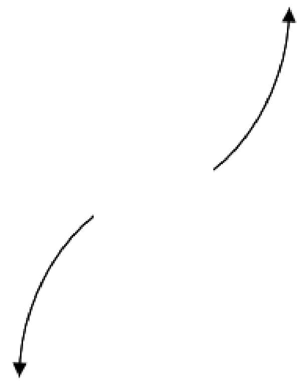


A.



B.

C. 

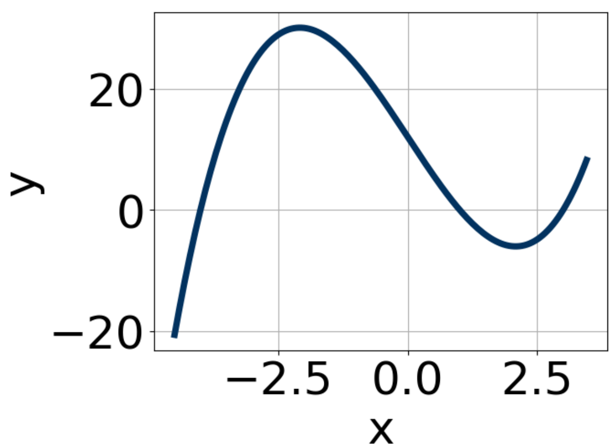
D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

10. Which of the following equations *could* be of the graph presented below?



The solution is $4(x-1)^{11}(x-3)^{11}(x+4)^{9}$, which is option E.

A. $17(x-1)^{10}(x-3)^{6}(x+4)^{11}$

The factors 1 and 3 have have been odd power.

B. $-20(x-1)^{10}(x-3)^{11}(x+4)^{11}$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $4(x-1)^{8}(x-3)^{11}(x+4)^{5}$

The factor 1 should have been an odd power.

D. $-20(x-1)^{11}(x-3)^{5}(x+4)^{5}$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $4(x-1)^{11}(x-3)^{11}(x+4)^{9}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).