This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 12$ and choose the interval that $f^{-1}(12)$ belongs to.

$$f(x) = 3x^2 - 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(12) \in [1.03, 1.59]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(12) \in [6.21, 6.82]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(12) \in [4.65, 6.05]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(12) \in [2.31, 2.5]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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2. Determine whether the function below is 1-1.

$$f(x) = \sqrt{-3x + 13}$$

The solution is yes, which is option C.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. Yes, the function is 1-1.

* This is the solution.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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3. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 4x^3 - 1x^2 - x \text{ and } g(x) = 3x^3 - 4x^2 + x$$

The solution is 0.0, which is option C.

A. $(f \circ g)(1) \in [5, 11]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(1) \in [15, 20]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-4, 2]$

* This is the correct solution

D. $(f \circ g)(1) \in [-7, -3]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{5}{3x + 20} \text{ and } g(x) = \frac{3}{3x + 20}$$

The solution is The domain is all Real numbers except $x = -6.666666666666667$ and $x = -6.666666666666667$, which is option D.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-8, -5]$

B. The domain is all Real numbers except $x = a$, where $a \in [-4.2, -1.2]$

C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-6, 0]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-7.67, -3.67]$ and $b \in [-8.67, -0.67]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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5. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = \ln(x + 5) - 4$$

The solution is $f^{-1}(10) = 1202599.284$, which is option D.

A. $f^{-1}(10) \in [392.43, 400.43]$

This solution corresponds to distractor 1.

B. $f^{-1}(10) \in [140.41, 148.41]$

This solution corresponds to distractor 2.

C. $f^{-1}(10) \in [1202609.28, 1202610.28]$

This solution corresponds to distractor 3.

D. $f^{-1}(10) \in [1202599.28, 1202603.28]$

This is the solution.

E. $f^{-1}(10) \in [3269011.37, 3269020.37]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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6. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{4}{5x + 33} \text{ and } g(x) = \frac{3}{5x + 28}$$

The solution is The domain is all Real numbers except $x = -6.6$ and $x = -5.6$, which is option D.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-5.67, -3.67]$

B. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [1.67, 7.67]$

C. The domain is all Real numbers except $x = a$, where $a \in [-5.25, 0.75]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-7.6, 1.4]$ and $b \in [-5.6, -1.6]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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7. Determine whether the function below is 1-1.

$$f(x) = \sqrt{-6x + 20}$$

The solution is yes, which is option C.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. Yes, the function is 1-1.

* This is the solution.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 12$ and choose the interval that $f^{-1}(12)$ belongs to.

$$f(x) = 3x^2 - 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(12) \in [1.27, 2.18]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(12) \in [4.75, 6.2]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(12) \in [3.01, 3.47]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(12) \in [1.97, 2.74]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = e^{x+2} - 5$$

The solution is $f^{-1}(10) = 0.708$, which is option B.

A. $f^{-1}(10) \in [-3.78, -3.1]$

This solution corresponds to distractor 2.

B. $f^{-1}(10) \in [0.67, 0.82]$

This is the solution.

C. $f^{-1}(10) \in [4.55, 5.02]$

This solution corresponds to distractor 1.

D. $f^{-1}(10) \in [-3.07, -2.86]$

This solution corresponds to distractor 3.

E. $f^{-1}(10) \in [-2.54, -2.23]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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10. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 + 3x^2 - x + 1 \text{ and } g(x) = -x^3 - 2x^2 + 4x - 2$$

The solution is 7.0, which is option B.

A. $(f \circ g)(1) \in [-2.75, -1.81]$

Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [6.96, 7.32]$

* This is the correct solution

C. $(f \circ g)(1) \in [-8.81, -7.52]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-1.87, 0.65]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!
