This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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66. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 - 16x^2 - 108x - 75}{x - 4}$$

The solution is $12x^2 + 32x + 20 + \dfrac{5}{x-4}$

A. $a \in [47, 51],\ b \in [-209, -207],\ c \in [717, 728],\ \text{and}\ r \in [-2976, -2965].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [8, 17],\ b \in [17, 21],\ c \in [-52, -46],\ \text{and}\ r \in [-220, -216].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [8, 17],\ b \in [31, 40],\ c \in [16, 23],\ \text{and}\ r \in [2, 10].$

* This is the solution!

D. $a \in [8, 17],\ b \in [-67, -62],\ c \in [145, 149],\ \text{and}\ r \in [-670, -662].$

You divided by the opposite of the factor.

E. $a \in [47, 51],\ b \in [173, 177],\ c \in [587, 598],\ \text{and}\ r \in [2305, 2313].$

You multiplied by the synthetic number rather than bringing the first factor down.

General Comments: Be sure to synthetically divide by the zero of the denominator!

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67. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 + 13x^2 - 40x - 75$$

The solution is $[-3, -1.6666666666666667, 2.5]$

A. $z_1 \in [-1.05, -0.83],\ z_2 \in [2.61, 3.51],\ \text{and}\ z_3 \in [4.84, 5.5]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-2.55, -2.4],\ z_2 \in [1, 2.02],\ \text{and}\ z_3 \in [2.54, 3.31]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-0.59, -0.3],\ z_2 \in [0.53, 0.84],\ \text{and}\ z_3 \in [2.54, 3.31]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-3.02, -2.94],\ z_2 \in [-0.92, -0.12],\ \text{and}\ z_3 \in [0.06, 0.76]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-3.02, -2.94]$, $z_2 \in [-2.29, -1.21]$, and $z_3 \in [2.31, 2.87]$

* This is the solution!

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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68. Factor the polynomial below completely, knowing that $x - 2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 9x^4 - 63x^3 + 74x^2 + 112x - 160$$

The solution is $[-1.3333333333333333, 1.3333333333333333, 2, 5]$

A. $z_1 \in [-6.6, -2.9]$, $z_2 \in [-3.01, -1.54]$, $z_3 \in [-0.7, -0.27]$, and $z_4 \in [3.8, 4.1]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-6.6, -2.9]$, $z_2 \in [-3.01, -1.54]$, $z_3 \in [-1.52, -0.93]$, and $z_4 \in [0.9, 1.7]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-2, -1.1]$, $z_2 \in [1.09, 1.58]$, $z_3 \in [1.9, 2.44]$, and $z_4 \in [4.1, 5.1]$

* This is the solution!

D. $z_1 \in [-6.6, -2.9]$, $z_2 \in [-3.01, -1.54]$, $z_3 \in [-1.03, -0.53]$, and $z_4 \in [0.3, 1.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-0.8, 0.5]$, $z_2 \in [0.42, 0.86]$, $z_3 \in [1.9, 2.44]$, and $z_4 \in [4.1, 5.1]$

Distractor 2: Corresponds to inversing rational roots.

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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69. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 60x + 42}{x + 2}$$

The solution is $20x^2 - 40x + 20 + \dfrac{2}{x + 2}$

A. $a \in [18, 24], b \in [31, 44], c \in [16, 24]$, and $r \in [80, 87]$.

You divided by the opposite of the factor.

B. $a \in [18, 24], b \in [-65, -55], c \in [118, 121]$, and $r \in [-324, -317]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-41, -37], b \in [79, 84], c \in [-222, -211]$, and $r \in [477, 488]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [18, 24], b \in [-47, -37], c \in [16, 24]$, and $r \in [-2, 3]$.

* This is the solution!

E. $a \in [-41, -37], b \in [-83, -76], c \in [-222, -211]$, and $r \in [-399, -397]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comments: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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70. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^2 + 3x + 3$$

The solution is $\pm 1, \pm 3$

A. $\pm 1, \pm 3$

   * This is the solution **since we asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 3, \pm 6}$

   This would have been the solution **if asked for the possible Rational roots**!

C. $\pm 1, \pm 2, \pm 3, \pm 6$

   Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 3}$

   Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Integer roots.

   Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comments: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.
