This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -14$ and choose the interval that $f^-1(-14)$ belongs to.

$$f(x) = 3x^2 + 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-14) \in [2.77, 3.81]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B. $f^{-1}(-14) \in [2.33, 3.07]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C. $f^{-1}(-14) \in [5.4, 6.07]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(-14) \in [0.99, 1.76]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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2. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^-1(10)$ belongs to.

$$f(x) = e^{x-5} - 2$$

The solution is $f^{-1}(10) = 7.485$, which is option B.

A. $f^{-1}(10) \in [-2.64, -2.2]$

This solution corresponds to distractor 1.

B. $f^{-1}(10) \in [7.04, 7.51]$

This is the solution.

C. $f^{-1}(10) \in [0.21, 1.68]$

This solution corresponds to distractor 3.

D. $f^{-1}(10) \in [-0.58, -0.36]$

This solution corresponds to distractor 4.

E. $f^{-1}(10) \in [-0.38, 0.5]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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3. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^-1(8)$ belongs to.

$$f(x) = e^{x-5} - 5$$

The solution is $f^{-1}(8) = 7.565$, which is option A.

A. $f^{-1}(8) \in [7.2, 8]$

This is the solution.

B. $f^{-1}(8) \in [-3.8, -1.4]$

This solution corresponds to distractor 1.

C. $f^{-1}(8) \in [-6.4, -2.6]$

This solution corresponds to distractor 2.

D. $f^{-1}(8) \in [-3.8, -1.4]$

This solution corresponds to distractor 3.

E. $f^{-1}(8) \in [-6.4, -2.6]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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4. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{2}{5x - 28} \text{ and } g(x) = 4x^2 + 6x + 2$$

The solution is The domain is all Real numbers except $x = 5.6$, which is option B.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-0.4, 7.6]$

B. The domain is all Real numbers except $x = a$, where $a \in [-0.4, 6.6]$

C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [0.5, 9.5]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [4.17, 12.17]$ and $b \in [3.25, 8.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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5. Determine whether the function below is 1-1.

$$f(x) = 18x^2 + 312x + 1014$$

The solution is no, which is option D.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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6. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = x^3 + 4x^2 - 3x - 3 \text{ and } g(x) = 3x^3 - 1x^2 - x - 1$$

The solution is -3.0, which is option C.

A. $(f \circ g)(1) \in [4.2, 9.3]$

Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-7.2, -3.6]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(1) \in [-3.9, 0.9]$

* This is the correct solution

D. $(f \circ g)(1) \in [1.7, 4.4]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 12$ and choose the interval that $f^-1(12)$ belongs to.

$$f(x) = 5x^2 + 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(12) \in [1.53, 2.27]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(12) \in [2.76, 3.9]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(12) \in [0.8, 1.48]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

D. $f^{-1}(12) \in [4.01, 5.49]$

Distractor 4: This corresponds to both distractors 2 and 3.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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8. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -x^3 + x^2 - x \text{ and } g(x) = -x^3 + 4x^2 + 4x$$

The solution is -1.0, which is option B.

A. $(f \circ g)(-1) \in [13, 17]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [-2, 0]$

* This is the correct solution

C. $(f \circ g)(-1) \in [-12, -5]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [21, 23]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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9. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-6x - 19} \text{ and } g(x) = 4x^2 + 6x + 4$$

The solution is The domain is all Real numbers less than or equal to$x = -3.17.$, which is option A.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-3.17, -1.17]$

B. The domain is all Real numbers except $x = a,$ where $a \in [4.17, 11.17]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [0.6, 9.6]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [6.25, 11.25]$ and $b \in [3.2, 10.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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10. Determine whether the function below is 1-1.

$$f(x) = 9x^2 + 120x + 400$$

The solution is no, which is option E.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.
