This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 + 2x^2 - 3x \text{ and } g(x) = 2x^3 + 2x^2 - 2x$$

The solution is -14.0, which is option B.

A. $(f \circ g)(1) \in [-36, -34]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-19, -13]$

* This is the correct solution

C. $(f \circ g)(1) \in [-10, 3]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-31, -29]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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2. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = x^3 - 4x^2 - 4x - 2 \text{ and } g(x) = -4x^3 - 1x^2 + 2x + 2$$

The solution is -23.0, which is option A.

A. $(f \circ g)(-1) \in [-24, -18]$

* This is the correct solution

B. $(f \circ g)(-1) \in [89, 99]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [97, 107]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-31, -26]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Determine whether the function below is 1-1.

$$f(x) = -16x^2 - 24x + 247$$

The solution is no, which is option B.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^-1(10)$ belongs to.

$$f(x) = e^{x+2} - 5$$

The solution is $f^{-1}(10) = 0.708$, which is option E.

A. $f^{-1}(10) \in [-3.55, -3.34]$

This solution corresponds to distractor 2.

B. $f^{-1}(10) \in [-3.16, -2.67]$

This solution corresponds to distractor 3.

C. $f^{-1}(10) \in [4.47, 4.86]$

This solution corresponds to distractor 1.

D. $f^{-1}(10) \in [-2.54, -2.41]$

This solution corresponds to distractor 4.

E. $f^{-1}(10) \in [0.55, 0.96]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 10$ and choose the interval that $f^-1(10)$ belongs to.

$$f(x) = 2x^2 - 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(10) \in [1.77, 2.8]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(10) \in [2.88, 4.02]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(10) \in [6.38, 7.96]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(10) \in [0.91, 2.03]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^-1(7)$ belongs to.

$$f(x) = \ln(x + 5) + 3$$

The solution is $f^{-1}(7) = 49.598$, which is option D.

A. $f^{-1}(7) \in [162755.79, 162763.79]$

This solution corresponds to distractor 4.

B. $f^{-1}(7) \in [58.6, 61.6]$

This solution corresponds to distractor 3.

C. $f^{-1}(7) \in [22020.47, 22024.47]$

This solution corresponds to distractor 1.

D. $f^{-1}(7) \in [47.6, 54.6]$

This is the solution.

E. $f^{-1}(7) \in [7.39, 11.39]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 14$ and choose the interval that $f^-1(14)$ belongs to.

$$f(x) = \sqrt[3]{3x - 4}$$

The solution is 916.0, which is option C.

A. $f^{-1}(14) \in [-916.4, -914.3]$

This solution corresponds to distractor 2.

B. $f^{-1}(14) \in [-913.6, -911.7]$

This solution corresponds to distractor 3.

C. $f^{-1}(14) \in [914.9, 919.4]$

* This is the correct solution.

D. $f^{-1}(14) \in [911.6, 915.6]$

Distractor 1: This corresponds to

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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8. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{1}{4x + 25} \text{ and } g(x) = \frac{4}{6x - 29}$$

The solution is The domain is all Real numbers except $x = -6.25$ and $x = 4.83$, which is option D.

A. The domain is all Real numbers except $x = a,$ where $a \in [5.67, 14.67]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [0.33, 12.33]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-8.5, -4.5]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-15.25, -2.25]$ and $b \in [2.83, 9.83]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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9. Determine whether the function below is 1-1.

$$f(x) = (4x - 18)^3$$

The solution is yes, which is option B.

A. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

B. Yes, the function is 1-1.

* This is the solution.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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10. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 6x + 4 \text{ and } g(x) = \frac{1}{4x - 21}$$

The solution is The domain is all Real numbers except $x = 5.25$, which is option C.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-1.5, 4.5]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-6.67, -0.67]$

C. The domain is all Real numbers except $x = a,$ where $a \in [4.25, 8.25]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [2.83, 7.83]$ and $b \in [-7.33, 1.67]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.
