This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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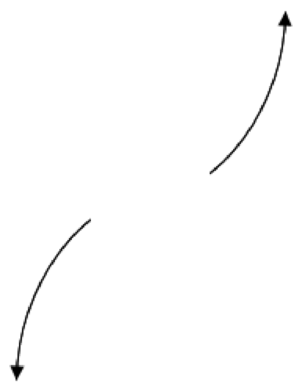
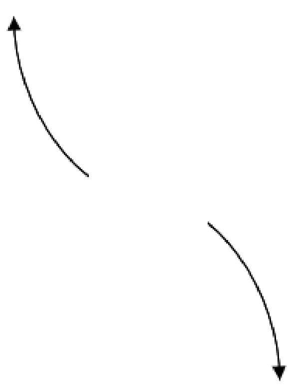
1. Describe the end behavior of the polynomial below.

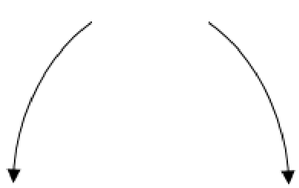
$$f(x) = 4(x+3)^4(x-3)^9(x+2)^4(x-2)^6$$

The solution is the graph below, which is option D.





A.



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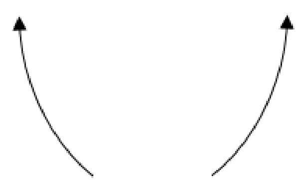
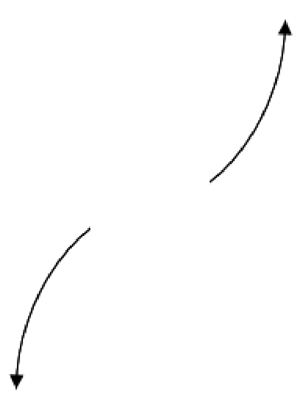


C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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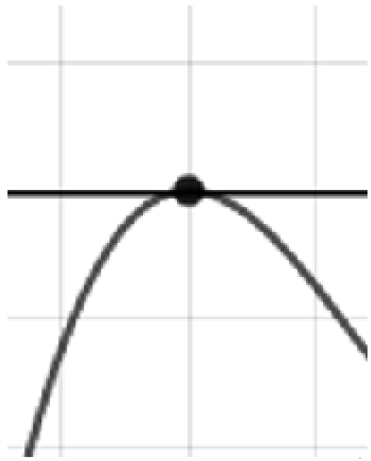
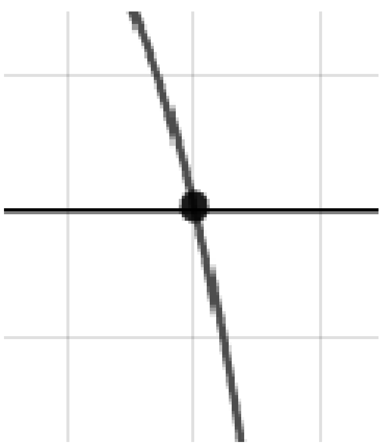
2. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

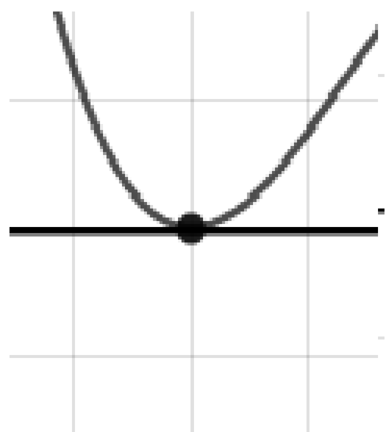
$$f(x) = -3(x+9)^6(x-9)^5(x+8)^{14}(x-8)^9$$

The solution is the graph below, which is option B.

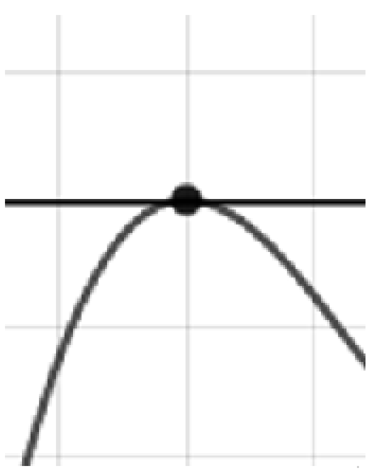




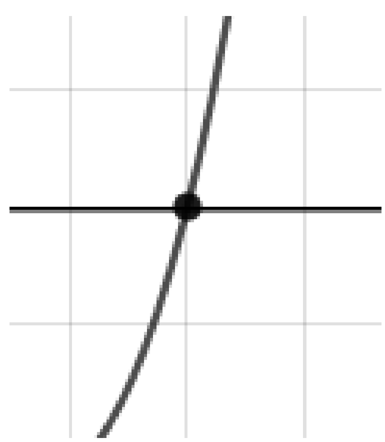
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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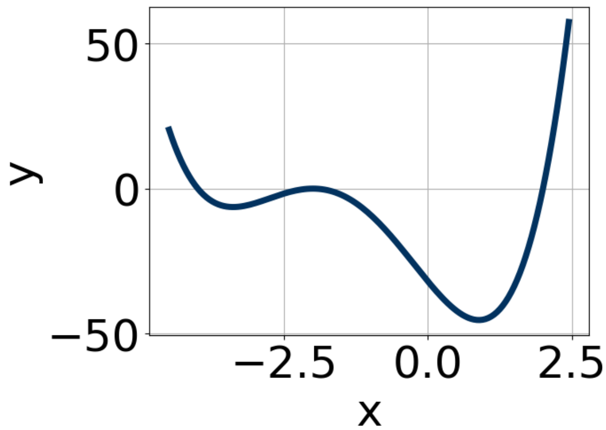
3. Which of the following equations *could* be of the graph presented below?



The solution is $17(x+2)^{10}(x-2)^7(x+4)^5$, which is option A.

A. $17(x+2)^{10}(x-2)^7(x+4)^5$

* This is the correct option.

B. $-9(x+2)^{10}(x-2)^9(x+4)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-18(x+2)^{10}(x-2)^{11}(x+4)^{10}$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $8(x+2)^7(x-2)^4(x+4)^5$

The factor -2 should have an even power and the factor 2 should have an odd power.

E. $20(x+2)^4(x-2)^4(x+4)^9$

The factor $(x-2)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form x^3+bx^2+cx+d.

$$3+2i \text{ and } 4$$

The solution is $x^3-10x^2+37x-52$, which is option C.

A. $b \in [1,8], c \in [-7.52,-6.31], \text{ and } d \in [11,13]$

$x^3+x^2-7x+12$, which corresponds to multiplying out $(x-3)(x-4)$.

B. $b \in [5,14], c \in [36.76,37.91], \text{ and } d \in [49,57]$

$x^3+10x^2+37x+52$, which corresponds to multiplying out $(x-(3+2i))(x-(3-2i))(x+4)$.

C. $b \in [-15, -7], c \in [36.76, 37.91]$, and $d \in [-52, -48]$

* $x^3 - 10x^2 + 37x - 52$, which is the correct option.

D. $b \in [1, 8], c \in [-6.57, -5.83]$, and $d \in [6, 9]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x-2)(x-4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 + 2i))(x - (3 - 2i))(x - (4))$.

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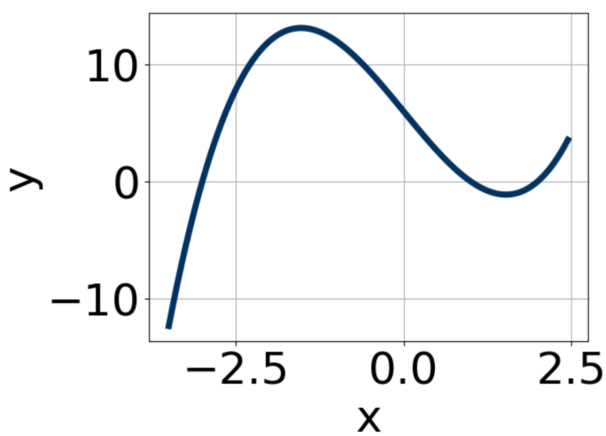
5. Which of the following equations *could* be of the graph presented below?



The solution is $16(x-2)^5(x+3)^5(x-1)^5$, which is option B.

A. $4(x-2)^4(x+3)^6(x-1)^5$

The factors 2 and -3 have have been odd power.

B. $16(x-2)^5(x+3)^5(x-1)^5$

* This is the correct option.

C. $-9(x-2)^{10}(x+3)^9(x-1)^7$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $4(x-2)^6(x+3)^7(x-1)^{11}$

The factor 2 should have been an odd power.

E. $-10(x-2)^{11}(x+3)^5(x-1)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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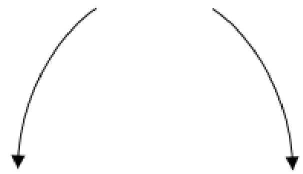
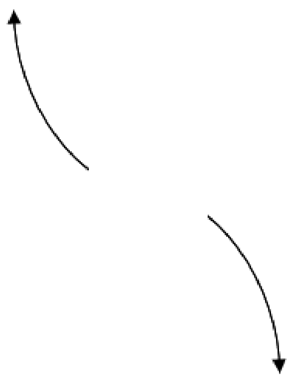
6. Describe the end behavior of the polynomial below.

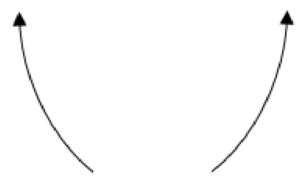
$$f(x) = -9(x+8)^4(x-8)^5(x-6)^4(x+6)^5$$

The solution is the graph below, which is option B.

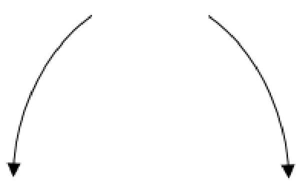




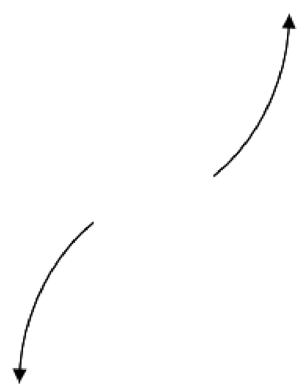
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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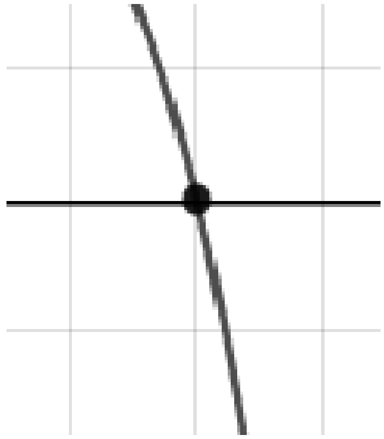
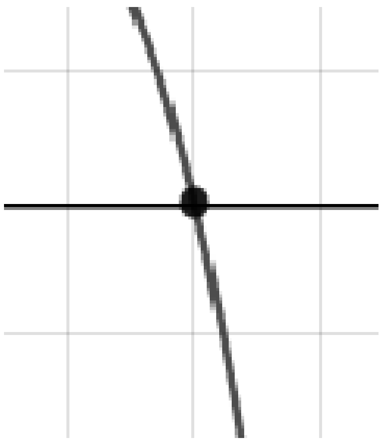
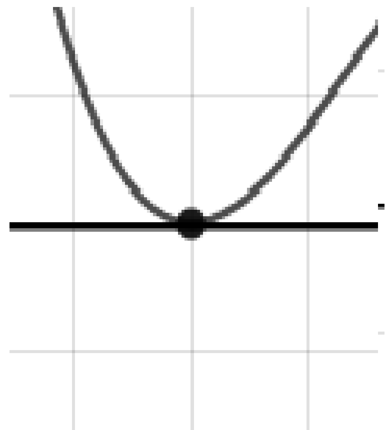
7. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

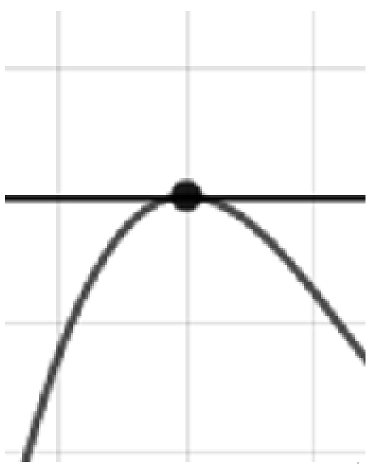
$$f(x) = -9(x-5)^4(x+5)^7(x-9)^4(x+9)^8$$

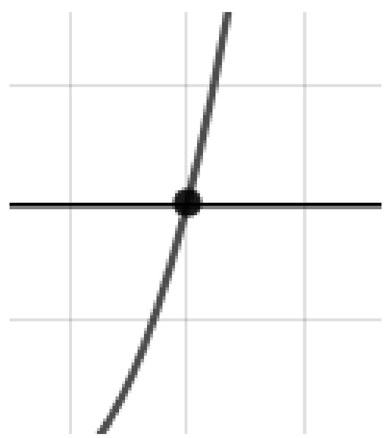
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$1, \frac{-3}{4}, \text{ and } \frac{6}{5}$$

The solution is $20x^3 - 29x^2 - 9x + 18$, which is option A.

A. $a \in [20, 21], b \in [-35, -25], c \in [-9, 1],$ and $d \in [15, 24]$

* $20x^3 - 29x^2 - 9x + 18$, which is the correct option.

B. $a \in [20, 21], b \in [-22, -14], c \in [-21, -16],$ and $d \in [15, 24]$

$20x^3 - 19x^2 - 21x + 18$, which corresponds to multiplying out $(x + 1)(4x - 3)(5x - 6)$.

C. $a \in [20, 21], b \in [27, 36], c \in [-9, 1],$ and $d \in [-26, -17]$

$20x^3 + 29x^2 - 9x - 18$, which corresponds to multiplying out $(x + 1)(4x - 3)(5x + 6)$.

D. $a \in [20, 21], b \in [-35, -25], c \in [-9, 1],$ and $d \in [-26, -17]$

$20x^3 - 29x^2 - 9x - 18$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [20, 21], b \in [10, 12], c \in [-33, -25],$ and $d \in [-26, -17]$

$20x^3 + 11x^2 - 27x - 18$, which corresponds to multiplying out $(x + 1)(4x + 3)(5x - 6)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-1)(4x+3)(5x-6)$

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 + 4i \text{ and } 4$$

The solution is $x^3 - 10x^2 + 49x - 100$, which is option D.

A. $b \in [-5, 7], c \in [-7.9, -4.2],$ and $d \in [11, 13]$

$x^3 + x^2 - 7x + 12$, which corresponds to multiplying out $(x - 3)(x - 4)$.

B. $b \in [-5, 7], c \in [-8.4, -7.9],$ and $d \in [13, 18]$

$x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

C. $b \in [7, 19], c \in [48.6, 51.5],$ and $d \in [98, 101]$

$x^3 + 10x^2 + 49x + 100$, which corresponds to multiplying out $(x - (3 + 4i))(x - (3 - 4i))(x + 4)$.

D. $b \in [-10, -4], c \in [48.6, 51.5],$ and $d \in [-102, -94]$

* $x^3 - 10x^2 + 49x - 100$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 + 4i))(x - (3 - 4i))(x - (4))$.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{3}{4}, \frac{5}{2}, \text{ and } -4$$

The solution is $8x^3 + 6x^2 - 89x + 60$, which is option C.

A. $a \in [5, 9], b \in [16, 29], c \in [-71, -68],$ and $d \in [-63, -58]$

$8x^3 + 18x^2 - 71x - 60$, which corresponds to multiplying out $(4x + 3)(2x - 5)(x + 4)$.

B. $a \in [5, 9], b \in [-13, -5], c \in [-95, -76],$ and $d \in [-63, -58]$

$8x^3 - 6x^2 - 89x - 60$, which corresponds to multiplying out $(4x + 3)(2x + 5)(x - 4)$.

C. $a \in [5, 9], b \in [2, 9], c \in [-95, -76],$ and $d \in [55, 66]$

* $8x^3 + 6x^2 - 89x + 60$, which is the correct option.

D. $a \in [5, 9], b \in [2, 9], c \in [-95, -76],$ and $d \in [-63, -58]$

$8x^3 + 6x^2 - 89x - 60$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [5, 9], b \in [57, 64], c \in [115, 125],$ and $d \in [55, 66]$

$8x^3 + 58x^2 + 119x + 60$, which corresponds to multiplying out $(4x + 3)(2x + 5)(x + 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 3)(2x - 5)(x + 4)$