This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -x^3 + 3x^2 + 4x \text{ and } g(x) = -4x^3 - 4x^2 + 4x + 3$$

The solution is 0.0, which is option B.

A. $(f \circ g)(-1) \in [-7, -4]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [-3, 1]$

* This is the correct solution

C. $(f \circ g)(-1) \in [3, 10]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [-12, -8]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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2. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 + 2x^2 - 2x \text{ and } g(x) = x^3 - 2x^2 + x$$

The solution is 0.0, which is option A.

A. $(f \circ g)(1) \in [-0.7, 1.9]$

* This is the correct solution

B. $(f \circ g)(1) \in [-16.8, -11.2]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-5.9, -3.1]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-19.7, -15.7]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Determine whether the function below is 1-1.

$$f(x) = 15x^2 - 189x + 594$$

The solution is no, which is option D.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^-1(8)$ belongs to.

$$f(x) = \ln(x - 2) - 5$$

The solution is $f^{-1}(8) = 442415.392$, which is option A.

A. $f^{-1}(8) \in [442414.39, 442417.39]$

This is the solution.

B. $f^{-1}(8) \in [22016.47, 22027.47]$

This solution corresponds to distractor 2.

C. $f^{-1}(8) \in [15.09, 25.09]$

This solution corresponds to distractor 1.

D. $f^{-1}(8) \in [396.43, 399.43]$

This solution corresponds to distractor 4.

E. $f^{-1}(8) \in [442405.39, 442412.39]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 15$ and choose the interval that $f^-1(15)$ belongs to.

$$f(x) = \sqrt[3]{4x - 3}$$

The solution is 844.5, which is option A.

A. $f^{-1}(15) \in [843.5, 844.8]$

* This is the correct solution.

B. $f^{-1}(15) \in [-847.1, -843.8]$

This solution corresponds to distractor 2.

C. $f^{-1}(15) \in [841.1, 843.1]$

Distractor 1: This corresponds to

D. $f^{-1}(15) \in [-843.1, -839.4]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^-1(9)$ belongs to.

$$f(x) = e^{x+4} - 3$$

The solution is $f^{-1}(9) = -1.515$, which is option B.

A. $f^{-1}(9) \in [-1.44, -1.23]$

This solution corresponds to distractor 3.

B. $f^{-1}(9) \in [-1.58, -1.46]$

This is the solution.

C. $f^{-1}(9) \in [-0.58, -0.38]$

This solution corresponds to distractor 4.

D. $f^{-1}(9) \in [-1.36, -1.19]$

This solution corresponds to distractor 2.

E. $f^{-1}(9) \in [6.47, 6.65]$

This solution corresponds to distractor 1.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -11$ and choose the interval that $f^-1(-11)$ belongs to.

$$f(x) = \sqrt[3]{2x + 4}$$

The solution is -667.5, which is option C.

A. $f^{-1}(-11) \in [-663.5, -660.5]$

Distractor 1: This corresponds to

B. $f^{-1}(-11) \in [662.5, 664.5]$

This solution corresponds to distractor 3.

C. $f^{-1}(-11) \in [-674.5, -665.5]$

* This is the correct solution.

D. $f^{-1}(-11) \in [664.5, 668.5]$

This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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8. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 8x^2 + 8 \text{ and } g(x) = \sqrt{3x + 15}$$

The solution is The domain is all Real numbers greater than or equal to$x = -5.0$., which is option A.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-6, -1]$

B. The domain is all Real numbers except $x = a$, where $a \in [0.83, 5.83]$

C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-0.6, 8.4]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-9.67, -1.67]$ and $b \in [-3.75, 1.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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9. Determine whether the function below is 1-1.

$$f(x) = (5x - 18)^3$$

The solution is yes, which is option A.

A. Yes, the function is 1-1.

* This is the solution.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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10. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 9x^3 + 8x^2 + 6x \text{ and } g(x) = \sqrt{-3x + 10}$$

The solution is The domain is all Real numbers less than or equal to$x = 3.33$., which is option C.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [4.5, 10.5]$

B. The domain is all Real numbers except $x = a,$ where $a \in [-8.25, 0.75]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [3.33, 4.33]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [3.75, 5.75]$ and $b \in [-6.2, -3.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.
