This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-3 - 2i$$
 and  $-3$ 

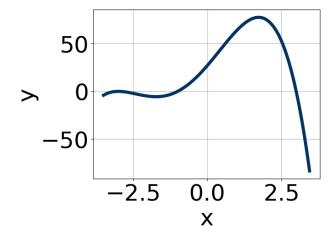
The solution is  $x^3 + 9x^2 + 31x + 39$ , which is option C.

- A.  $b \in [-5, 3], c \in [5.4, 6.45], \text{ and } d \in [7.9, 9.4]$  $x^3 + x^2 + 6x + 9$ , which corresponds to multiplying out (x + 3)(x + 3).
- B.  $b \in [-5, 3], c \in [4.58, 5.53]$ , and  $d \in [1.9, 7.1]$  $x^3 + x^2 + 5x + 6$ , which corresponds to multiplying out (x + 2)(x + 3).
- C.  $b \in [2, 13], c \in [30.15, 31.6]$ , and  $d \in [38.1, 39.8]$ \*  $x^3 + 9x^2 + 31x + 39$ , which is the correct option.
- D.  $b \in [-17, -6], c \in [30.15, 31.6]$ , and  $d \in [-42, -38.7]$  $x^3 - 9x^2 + 31x - 39$ , which corresponds to multiplying out (x - (-3 - 2i))(x - (-3 + 2i))(x - 3).
- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-3 - 2i))(x - (-3 + 2i))(x - (-3)).

2. Which of the following equations *could* be of the graph presented below?



The solution is  $-15(x+3)^{10}(x-3)^{7}(x+1)^{11}$ , which is option A.

A. 
$$-15(x+3)^{10}(x-3)^7(x+1)^{11}$$

\* This is the correct option.

B. 
$$-9(x+3)^{11}(x-3)^8(x+1)^9$$

The factor -3 should have an even power and the factor 3 should have an odd power.

C. 
$$-7(x+3)^{10}(x-3)^6(x+1)^7$$

The factor (x-3) should have an odd power.

D. 
$$5(x+3)^{10}(x-3)^5(x+1)^4$$

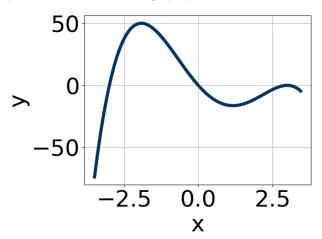
The factor (x + 1) should have an odd power and the leading coefficient should be the opposite sign.

E. 
$$7(x+3)^6(x-3)^5(x+1)^5$$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

## 3. Which of the following equations *could* be of the graph presented below?



The solution is  $-7x^7(x-3)^8(x+3)^5$ , which is option D.

A. 
$$10x^7(x-3)^4(x+3)^{10}$$

The factor (x + 3) should have an odd power and the leading coefficient should be the opposite sign.

B. 
$$15x^{11}(x-3)^6(x+3)^5$$

This corresponds to the leading coefficient being the opposite value than it should be.

C. 
$$-20x^6(x-3)^9(x+3)^7$$

The factor 3 should have an even power and the factor 0 should have an odd power.

D. 
$$-7x^7(x-3)^8(x+3)^5$$

\* This is the correct option.

E. 
$$-18x^4(x-3)^4(x+3)^5$$

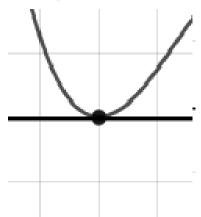
The factor x should have an odd power.

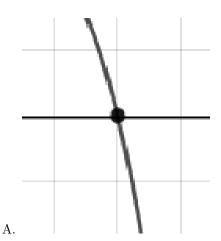
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

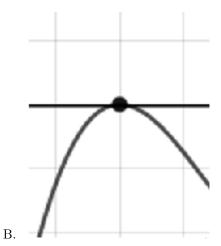
4. Describe the zero behavior of the zero x = 4 of the polynomial below.

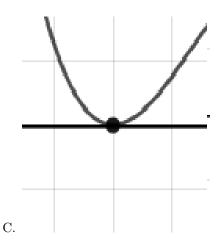
$$f(x) = 2(x+6)^8(x-6)^4(x-4)^{10}(x+4)^7$$

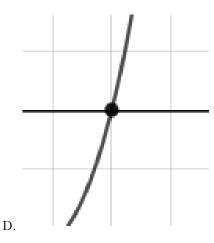
The solution is the graph below, which is option C.











**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$2+3i$$
 and  $3$ 

The solution is  $x^3 - 7x^2 + 25x - 39$ , which is option D.

A.  $b \in [4, 11], c \in [21.54, 25.63]$ , and  $d \in [33, 43]$ 

 $x^3 + 7x^2 + 25x + 39$ , which corresponds to multiplying out (x - (2+3i))(x - (2-3i))(x+3).

B.  $b \in [-3, 5], c \in [-5.17, -2.87], \text{ and } d \in [0, 7]$ 

 $x^3 + x^2 - 5x + 6$ , which corresponds to multiplying out (x - 2)(x - 3).

C.  $b \in [-3, 5], c \in [-6.83, -5.89], \text{ and } d \in [9, 10]$ 

 $x^3 + x^2 - 6x + 9$ , which corresponds to multiplying out (x - 3)(x - 3).

D.  $b \in [-9, -4], c \in [21.54, 25.63]$ , and  $d \in [-46, -38]$ 

\*  $x^3 - 7x^2 + 25x - 39$ , which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (2 + 3i))(x - (2 - 3i))(x - (3)).

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{3}{5}, \frac{-1}{3}$$
, and  $\frac{-1}{2}$ 

The solution is  $30x^3 + 7x^2 - 10x - 3$ , which is option E.

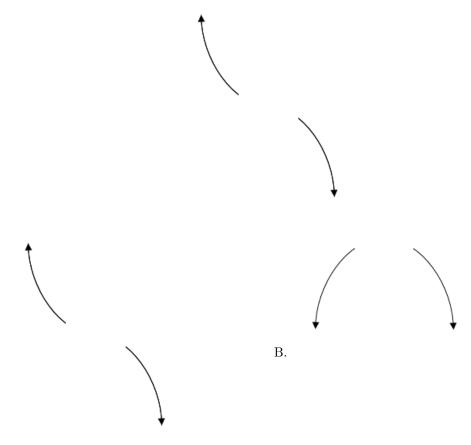
- A.  $a \in [30, 39], b \in [-13, -1], c \in [-13, -6], \text{ and } d \in [-1, 7]$  $30x^3 - 7x^2 - 10x + 3$ , which corresponds to multiplying out (5x + 3)(3x - 1)(2x - 1).
- B.  $a \in [30, 39], b \in [40, 44], c \in [18, 24], \text{ and } d \in [-1, 7]$  $30x^3 + 43x^2 + 20x + 3$ , which corresponds to multiplying out (5x + 3)(3x + 1)(2x + 1).
- C.  $a \in [30, 39], b \in [22, 27], c \in [-2, 0], \text{ and } d \in [-3, -2]$  $30x^3 + 23x^2 - 2x - 3, \text{ which corresponds to multiplying out } (5x + 3)(3x - 1)(2x + 1).$
- D.  $a \in [30, 39], b \in [7, 13], c \in [-13, -6]$ , and  $d \in [-1, 7]$  $30x^3 + 7x^2 - 10x + 3$ , which corresponds to multiplying everything correctly except the constant term.
- E.  $a \in [30, 39], b \in [7, 13], c \in [-13, -6], \text{ and } d \in [-3, -2]$ \*  $30x^3 + 7x^2 - 10x - 3$ , which is the correct option.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (5x - 3)(3x + 1)(2x + 1)

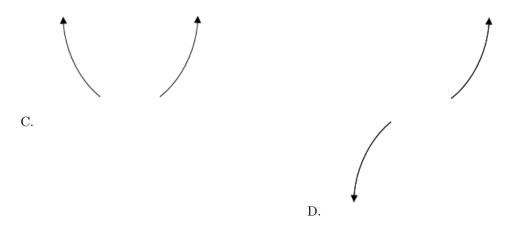
7. Describe the end behavior of the polynomial below.

$$f(x) = -7(x-9)^5(x+9)^8(x+4)^5(x-4)^7$$

The solution is the graph below, which is option A.



A.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-6}{5}, \frac{3}{5}$$
, and  $\frac{7}{2}$ 

The solution is  $50x^3 - 145x^2 - 141x + 126$ , which is option E.

A.  $a \in [48, 54], b \in [-154, -139], c \in [-141, -135]$ , and  $d \in [-128, -118]$  $50x^3 - 145x^2 - 141x - 126$ , which corresponds to multiplying everything correctly except the constant term.

B.  $a \in [48, 54], b \in [-206, -201], c \in [67, 73], \text{ and } d \in [125, 132]$  $50x^3 - 205x^2 + 69x + 126, \text{ which corresponds to multiplying out } (5x - 6)(5x + 3)(2x - 7).$ 

C.  $a \in [48, 54], b \in [-267, -261], c \in [350, 357], \text{ and } d \in [-128, -118]$  $50x^3 - 265x^2 + 351x - 126, \text{ which corresponds to multiplying out } (5x - 6)(5x - 3)(2x - 7).$ 

D.  $a \in [48, 54], b \in [142, 152], c \in [-141, -135], \text{ and } d \in [-128, -118]$  $50x^3 + 145x^2 - 141x - 126$ , which corresponds to multiplying out (5x - 6)(5x + 3)(2x + 7).

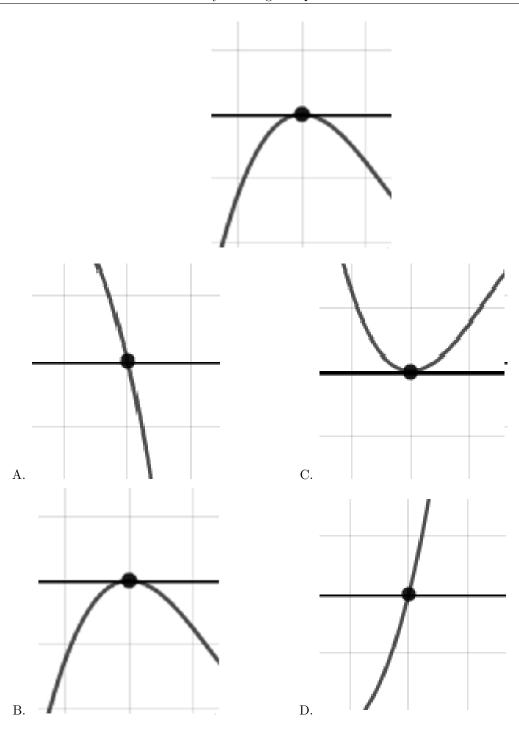
E.  $a \in [48, 54], b \in [-154, -139], c \in [-141, -135], \text{ and } d \in [125, 132]$ \*  $50x^3 - 145x^2 - 141x + 126$ , which is the correct option.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (5x + 6)(5x - 3)(2x - 7)

9. Describe the zero behavior of the zero x = 5 of the polynomial below.

$$f(x) = -7(x-3)^{6}(x+3)^{3}(x-5)^{10}(x+5)^{7}$$

The solution is the graph below, which is option B.

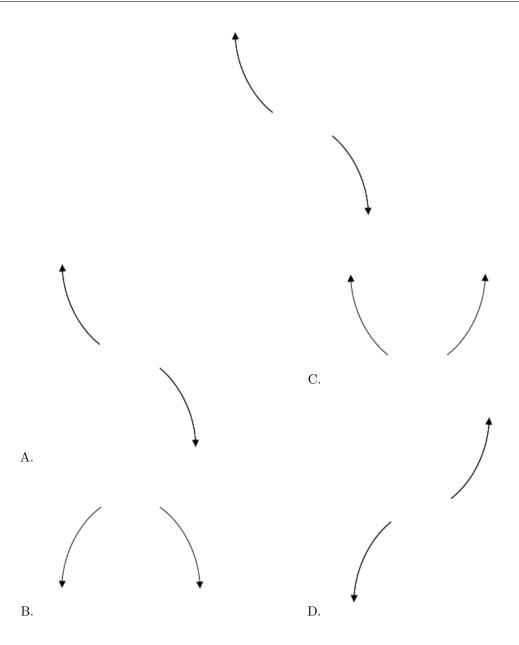


**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

10. Describe the end behavior of the polynomial below.

$$f(x) = -8(x+3)^4(x-3)^5(x+7)^3(x-7)^5$$

The solution is the graph below, which is option A.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

11. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-5 + 5i$$
 and 3

The solution is  $x^3 + 7x^2 + 20x - 150$ , which is option C.

A. 
$$b \in [-11, -3], c \in [16, 22], \text{ and } d \in [145, 151]$$

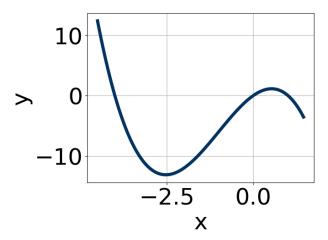
$$x^3 - 7x^2 + 20x + 150$$
, which corresponds to multiplying out  $(x - (-5 + 5i))(x - (-5 - 5i))(x + 3)$ .

- B.  $b \in [1, 6], c \in [-1, 7]$ , and  $d \in [-18, -13]$  $x^3 + x^2 + 2x - 15$ , which corresponds to multiplying out (x + 5)(x - 3).
- C.  $b \in [2, 12], c \in [16, 22]$ , and  $d \in [-156, -141]$ \*  $x^3 + 7x^2 + 20x - 150$ , which is the correct option.
- D.  $b \in [1,6], c \in [-9,1]$ , and  $d \in [13,20]$   $x^3+x^2-8x+15$ , which corresponds to multiplying out (x-5)(x-3).
- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-5 + 5i))(x - (-5 - 5i))(x - (3)).

## 12. Which of the following equations *could* be of the graph presented below?



The solution is  $-9x^5(x-1)^9(x+4)^9$ , which is option A.

A. 
$$-9x^5(x-1)^9(x+4)^9$$

\* This is the correct option.

B. 
$$17x^{11}(x-1)^9(x+4)^9$$

This corresponds to the leading coefficient being the opposite value than it should be.

C. 
$$6x^5(x-1)^8(x+4)^9$$

The factor (x-1) should have an odd power and the leading coefficient should be the opposite sign.

D. 
$$-7x^7(x-1)^8(x+4)^4$$

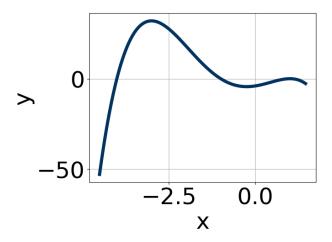
The factors 1 and -4 have have been odd power.

E. 
$$-17x^7(x-1)^{10}(x+4)^9$$

The factor 1 should have been an odd power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

## 13. Which of the following equations *could* be of the graph presented below?



The solution is  $-4(x-1)^{10}(x+4)^7(x+1)^9$ , which is option E.

A. 
$$-17(x-1)^4(x+4)^8(x+1)^9$$

The factor (x + 4) should have an odd power.

B. 
$$20(x-1)^{10}(x+4)^7(x+1)^4$$

The factor (x + 1) should have an odd power and the leading coefficient should be the opposite sign.

C. 
$$2(x-1)^{10}(x+4)^9(x+1)^9$$

This corresponds to the leading coefficient being the opposite value than it should be.

D. 
$$-6(x-1)^7(x+4)^8(x+1)^{11}$$

The factor 1 should have an even power and the factor -4 should have an odd power.

E. 
$$-4(x-1)^{10}(x+4)^7(x+1)^9$$

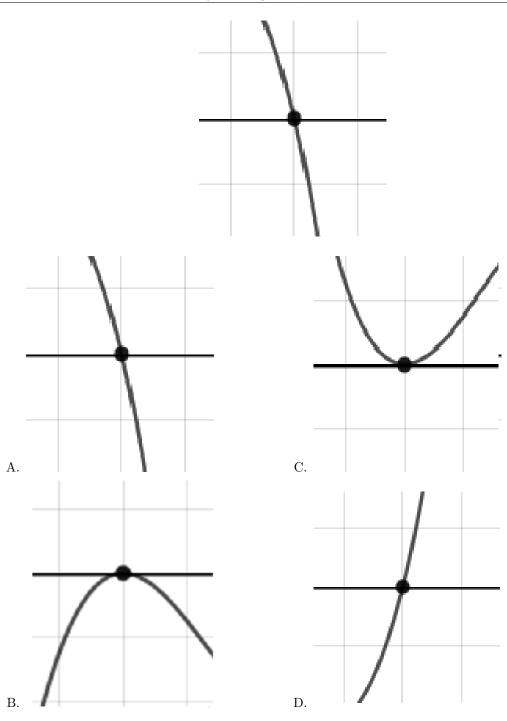
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

14. Describe the zero behavior of the zero x = -6 of the polynomial below.

$$f(x) = -4(x-8)^{6}(x+8)^{2}(x+6)^{9}(x-6)^{6}$$

The solution is the graph below, which is option A.

<sup>\*</sup> This is the correct option.



**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

15. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

-4-2i and -2

The solution is  $x^3 + 10x^2 + 36x + 40$ , which is option A.

- A.  $b \in [8, 13], c \in [34.2, 37], \text{ and } d \in [40, 42]$ 
  - \*  $x^3 + 10x^2 + 36x + 40$ , which is the correct option.
- B.  $b \in [-4, 6], c \in [4.5, 8.2], \text{ and } d \in [6, 9]$  $x^3 + x^2 + 6x + 8, \text{ which corresponds to multiplying out } (x + 4)(x + 2).$
- C.  $b \in [-4, 6], c \in [3, 4.3], \text{ and } d \in [2, 5]$  $x^3 + x^2 + 4x + 4, \text{ which corresponds to multiplying out } (x + 2)(x + 2).$
- D.  $b \in [-14, -8], c \in [34.2, 37]$ , and  $d \in [-40, -32]$  $x^3 - 10x^2 + 36x - 40$ , which corresponds to multiplying out (x - (-4 - 2i))(x - (-4 + 2i))(x - 2).
- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-4 - 2i))(x - (-4 + 2i))(x - (-2)).

16. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-7}{5}, \frac{5}{2}, \text{ and } \frac{-1}{4}$$

The solution is  $40x^3 - 34x^2 - 151x - 35$ , which is option D.

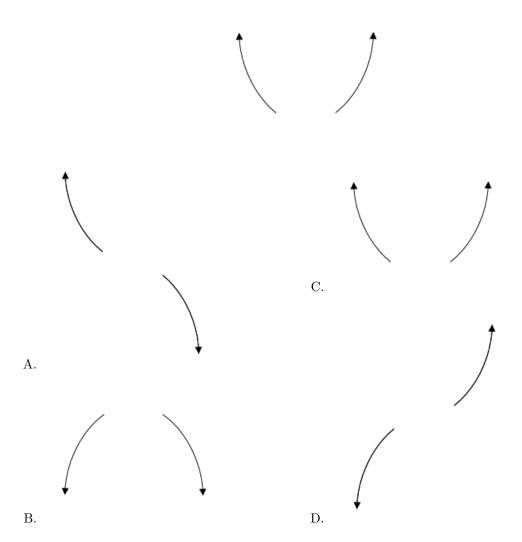
- A.  $a \in [35, 41], b \in [48, 58], c \in [-131, -125], \text{ and } d \in [-42, -32]$  $40x^3 + 54x^2 - 129x - 35, \text{ which corresponds to multiplying out } (5x - 7)(2x + 5)(4x + 1).$
- B.  $a \in [35, 41], b \in [-36, -27], c \in [-158, -150]$ , and  $d \in [33, 37]$  $40x^3 - 34x^2 - 151x + 35$ , which corresponds to multiplying everything correctly except the constant term.
- C.  $a \in [35, 41], b \in [-148, -145], c \in [95, 106], \text{ and } d \in [33, 37]$  $40x^3 - 146x^2 + 101x + 35, \text{ which corresponds to multiplying out } (5x - 7)(2x - 5)(4x + 1).$
- D.  $a \in [35, 41], b \in [-36, -27], c \in [-158, -150], \text{ and } d \in [-42, -32]$ \*  $40x^3 - 34x^2 - 151x - 35$ , which is the correct option.
- E.  $a \in [35, 41], b \in [32, 40], c \in [-158, -150], \text{ and } d \in [33, 37]$  $40x^3 + 34x^2 - 151x + 35$ , which corresponds to multiplying out (5x - 7)(2x + 5)(4x - 1).

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (5x + 7)(2x - 5)(4x + 1)

17. Describe the end behavior of the polynomial below.

$$f(x) = 8(x-4)^4(x+4)^9(x+8)^4(x-8)^5$$

The solution is the graph below, which is option C.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

18. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{1}{2}, \frac{7}{4}$$
, and 4

The solution is  $8x^3 - 50x^2 + 79x - 28$ , which is option B.

A.  $a \in [7, 12], b \in [-23, -12], c \in [-71, -59], \text{ and } d \in [-32, -21]$ 

 $8x^3 - 14x^2 - 65x - 28$ , which corresponds to multiplying out (2x+1)(4x+7)(x-4).

B.  $a \in [7, 12], b \in [-51, -44], c \in [77, 82], \text{ and } d \in [-32, -21]$ 

\*  $8x^3 - 50x^2 + 79x - 28$ , which is the correct option.

C.  $a \in [7,12], b \in [49,55], c \in [77,82], \text{ and } d \in [28,30]$ 

 $8x^3 + 50x^2 + 79x + 28$ , which corresponds to multiplying out (2x+1)(4x+7)(x+4).

D.  $a \in [7, 12], b \in [-51, -44], c \in [77, 82], \text{ and } d \in [28, 30]$ 

 $8x^3 - 50x^2 + 79x + 28$ , which corresponds to multiplying everything correctly except the constant term.

E.  $a \in [7,12], b \in [-44,-34], c \in [31,43], \text{ and } d \in [28,30]$ 

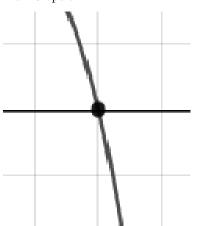
$$8x^3 - 42x^2 + 33x + 28$$
, which corresponds to multiplying out  $(2x + 1)(4x - 7)(x - 4)$ .

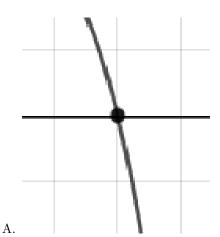
**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (2x - 1)(4x - 7)(x - 4)

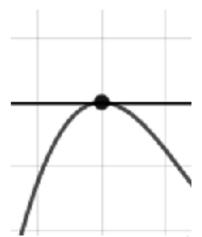
19. Describe the zero behavior of the zero x=2 of the polynomial below.

$$f(x) = -4(x-2)^5(x+2)^{10}(x-3)^6(x+3)^{10}$$

The solution is the graph below, which is option A.

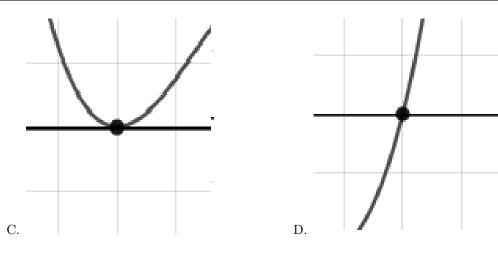






В.

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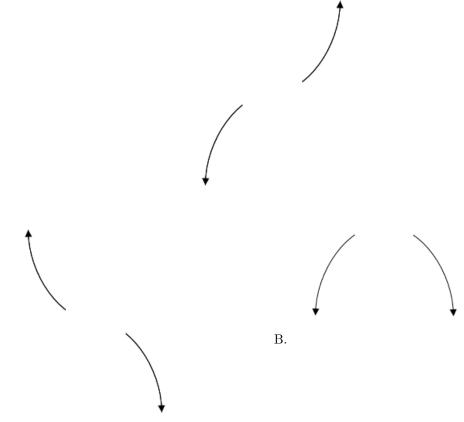


**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

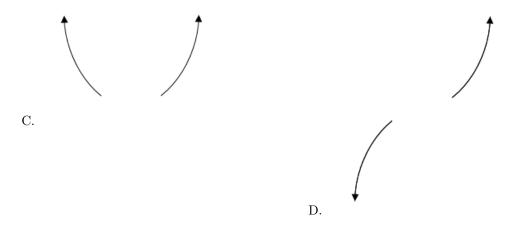
20. Describe the end behavior of the polynomial below.

$$f(x) = 2(x+5)^3(x-5)^6(x+3)^3(x-3)^5$$

The solution is the graph below, which is option D.



A.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

21. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-2 - 5i$$
 and 3

The solution is  $x^3 + x^2 + 17x - 87$ , which is option A.

A.  $b \in [-0.9, 3.8], c \in [16, 20.1], \text{ and } d \in [-91, -82]$ \*  $x^3 + x^2 + 17x - 87$ , which is the correct option.

B.  $b \in [-1.5, 0.8], c \in [16, 20.1], \text{ and } d \in [84, 91]$  $x^3 - 1x^2 + 17x + 87, \text{ which corresponds to multiplying out } (x - (-2 - 5i))(x - (-2 + 5i))(x + 3).$ 

C.  $b \in [-0.9, 3.8], c \in [-3.2, -0.5], \text{ and } d \in [-6, -1]$  $x^3 + x^2 - x - 6$ , which corresponds to multiplying out (x + 2)(x - 3).

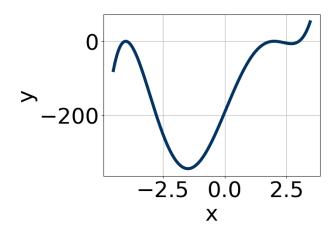
D.  $b \in [-0.9, 3.8], c \in [0.2, 7.4], \text{ and } d \in [-15, -11]$  $x^3 + x^2 + 2x - 15, \text{ which corresponds to multiplying out } (x + 5)(x - 3).$ 

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-2 - 5i))(x - (-2 + 5i))(x - (3)).

22. Which of the following equations *could* be of the graph presented below?



The solution is  $8(x+4)^6(x-2)^8(x-3)^9$ , which is option C.

A. 
$$13(x+4)^6(x-2)^5(x-3)^9$$

The factor (x-2) should have an even power.

B. 
$$-18(x+4)^4(x-2)^{10}(x-3)^6$$

The factor (x-3) should have an odd power and the leading coefficient should be the opposite sign.

C. 
$$8(x+4)^6(x-2)^8(x-3)^9$$

\* This is the correct option.

D. 
$$3(x+4)^8(x-2)^9(x-3)^4$$

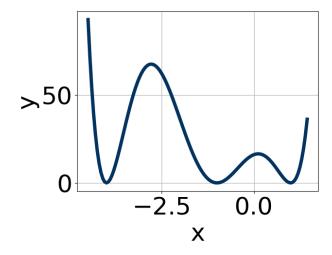
The factor (x-2) should have an even power and the factor (x-3) should have an odd power.

E. 
$$-8(x+4)^4(x-2)^8(x-3)^5$$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

23. Which of the following equations *could* be of the graph presented below?



The solution is  $18(x+4)^6(x+1)^4(x-1)^8$ , which is option C.

A. 
$$-18(x+4)^{10}(x+1)^4(x-1)^8$$

This corresponds to the leading coefficient being the opposite value than it should be.

B. 
$$20(x+4)^8(x+1)^5(x-1)^7$$

The factors (x+1) and (x-1) should both have even powers.

C. 
$$18(x+4)^6(x+1)^4(x-1)^8$$

\* This is the correct option.

D. 
$$-4(x+4)^{10}(x+1)^{10}(x-1)^7$$

The factor (x-1) should have an even power and the leading coefficient should be the opposite sign.

E. 
$$6(x+4)^8(x+1)^4(x-1)^7$$

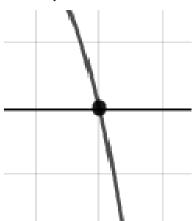
The factor (x-1) should have an even power.

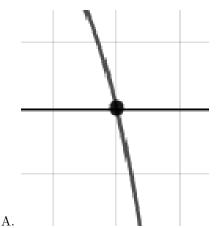
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

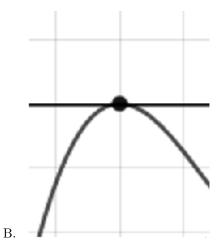
24. Describe the zero behavior of the zero x = 6 of the polynomial below.

$$f(x) = -3(x+4)^8(x-4)^5(x+6)^6(x-6)^5$$

The solution is the graph below, which is option A.

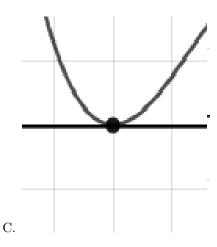


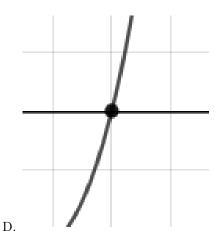




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**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

25. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-2 + 5i$$
 and 1

The solution is  $x^3 + 3x^2 + 25x - 29$ , which is option A.

- A.  $b \in [2.3, 3.6], c \in [24, 32], \text{ and } d \in [-30, -20]$ \*  $x^3 + 3x^2 + 25x - 29$ , which is the correct option.
- B.  $b \in [-1.2, 1.7], c \in [-10, -3], \text{ and } d \in [0, 12]$  $x^3 + x^2 - 6x + 5, \text{ which corresponds to multiplying out } (x - 5)(x - 1).$
- C.  $b \in [-1.2, 1.7], c \in [-1, 13]$ , and  $d \in [-5, 0]$  $x^3 + x^2 + x - 2$ , which corresponds to multiplying out (x + 2)(x - 1).
- D.  $b \in [-5.5, -1.7], c \in [24, 32], \text{ and } d \in [23, 32]$  $x^3 - 3x^2 + 25x + 29, \text{ which corresponds to multiplying out } (x - (-2 + 5i))(x - (-2 - 5i))(x + 1).$
- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-2 + 5i))(x - (-2 - 5i))(x - (1)).

26. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$2, \frac{1}{5}$$
, and  $\frac{-1}{4}$ 

The solution is  $20x^3 - 39x^2 - 3x + 2$ , which is option C.

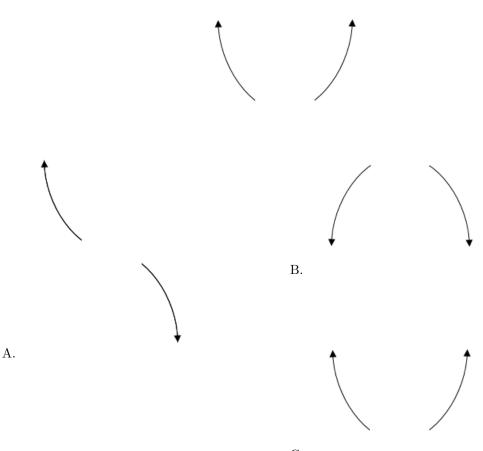
- A.  $a \in [17, 29], b \in [-39.3, -36.6], c \in [-5.1, -0.4],$  and  $d \in [-4, 0]$  $20x^3 - 39x^2 - 3x - 2$ , which corresponds to multiplying everything correctly except the constant term.
- B.  $a \in [17, 29], b \in [47.6, 51.5], c \in [18.7, 20.3], \text{ and } d \in [2, 8]$  $20x^3 + 49x^2 + 19x + 2$ , which corresponds to multiplying out (x + 2)(5x + 1)(4x + 1).
- C.  $a \in [17, 29], b \in [-39.3, -36.6], c \in [-5.1, -0.4], \text{ and } d \in [2, 8]$ \*  $20x^3 - 39x^2 - 3x + 2$ , which is the correct option.
- D.  $a \in [17, 29], b \in [33.4, 40.5], c \in [-5.1, -0.4], \text{ and } d \in [-4, 0]$  $20x^3 + 39x^2 - 3x - 2$ , which corresponds to multiplying out (x + 2)(5x + 1)(4x - 1).
- E.  $a \in [17, 29], b \in [39.7, 41.9], c \in [-2, 2.2], \text{ and } d \in [-4, 0]$  $20x^3 + 41x^2 + x - 2$ , which corresponds to multiplying out (x + 2)(5x - 1)(4x + 1).

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (x-2)(5x-1)(4x+1)

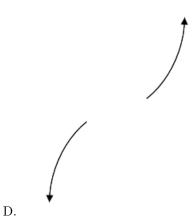
27. Describe the end behavior of the polynomial below.

$$f(x) = 5(x-5)^4(x+5)^5(x-6)^5(x+6)^6$$

The solution is the graph below, which is option C.



С.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

28. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$-5, \frac{-2}{3}, \text{ and } \frac{-7}{5}$$

The solution is  $15x^3 + 106x^2 + 169x + 70$ , which is option E.

A.  $a \in [12, 16], b \in [103, 110], c \in [161, 178]$ , and  $d \in [-74, -62]$  $15x^3 + 106x^2 + 169x - 70$ , which corresponds to multiplying everything correctly except the constant term.

B.  $a \in [12, 16], b \in [-72, -57], c \in [-75, -65], \text{ and } d \in [68, 74]$  $15x^3 - 64x^2 - 69x + 70, \text{ which corresponds to multiplying out } (x - 5)(3x - 2)(5x + 7).$ 

C.  $a \in [12, 16], b \in [-45, -43], c \in [-142, -136], \text{ and } d \in [-74, -62]$  $15x^3 - 44x^2 - 141x - 70$ , which corresponds to multiplying out (x - 5)(3x + 2)(5x + 7).

D.  $a \in [12, 16], b \in [-113, -105], c \in [161, 178], \text{ and } d \in [-74, -62]$  $15x^3 - 106x^2 + 169x - 70$ , which corresponds to multiplying out (x - 5)(3x - 2)(5x - 7).

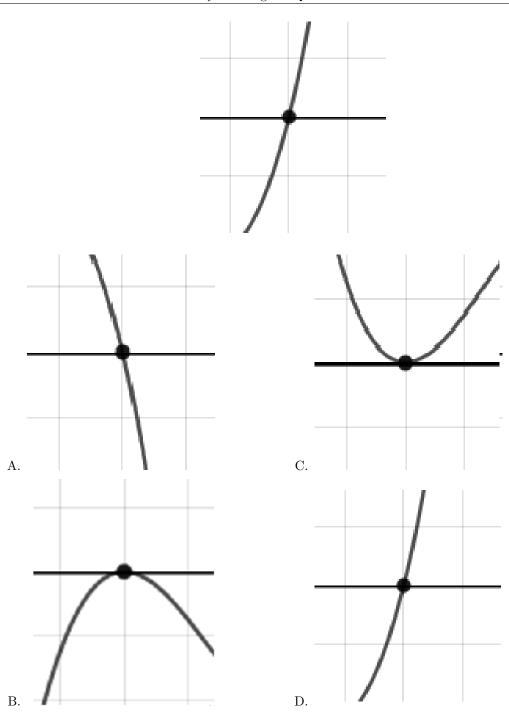
E.  $a \in [12, 16], b \in [103, 110], c \in [161, 178], \text{ and } d \in [68, 74]$ \*  $15x^3 + 106x^2 + 169x + 70$ , which is the correct option.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (x+5)(3x+2)(5x+7)

29. Describe the zero behavior of the zero x = -9 of the polynomial below.

$$f(x) = -9(x-9)^4(x+9)^5(x+3)^9(x-3)^{10}$$

The solution is the graph below, which is option D.



**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

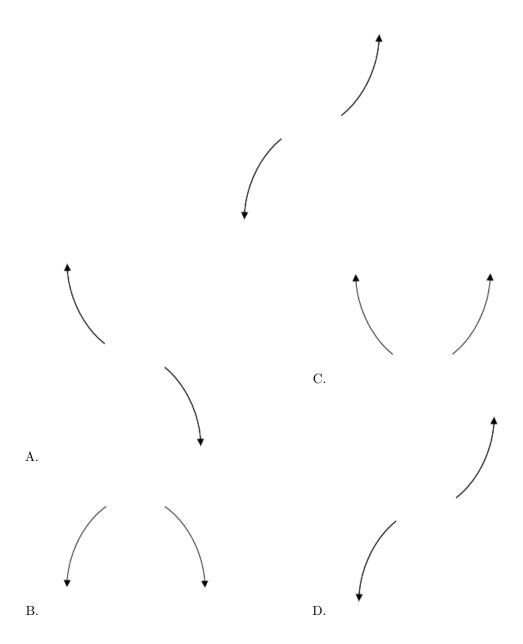
30. Describe the end behavior of the polynomial below.

$$f(x) = 5(x-5)^5(x+5)^{10}(x-8)^5(x+8)^7$$

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The solution is the graph below, which is option D.

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**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.