This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{2}, \frac{-1}{2}, \text{ and } -7$$

The solution is $4x^3 + 20x^2 - 61x - 35$, which is option B.

A. $a \in [2, 10], b \in [36.1, 42.5], c \in [86, 95], \text{ and } d \in [30, 40]$

$4x^3 + 40x^2 + 89x + 35$, which corresponds to multiplying out $(2x + 5)(2x + 1)(x + 7)$.

B. $a \in [2, 10], b \in [18.8, 21.8], c \in [-65, -58], \text{ and } d \in [-35, -32]$

* $4x^3 + 20x^2 - 61x - 35$, which is the correct option.

C. $a \in [2, 10], b \in [35.9, 37.2], c \in [42, 60], \text{ and } d \in [-35, -32]$

$4x^3 + 36x^2 + 51x - 35$, which corresponds to multiplying out $(2x + 5)(2x - 1)(x + 7)$.

D. $a \in [2, 10], b \in [-22.7, -19.9], c \in [-65, -58], \text{ and } d \in [30, 40]$

$4x^3 - 20x^2 - 61x + 35$, which corresponds to multiplying out $(2x + 5)(2x - 1)(x - 7)$.

E. $a \in [2, 10], b \in [18.8, 21.8], c \in [-65, -58], \text{ and } d \in [30, 40]$

$4x^3 + 20x^2 - 61x + 35$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 5)(2x + 1)(x + 7)$

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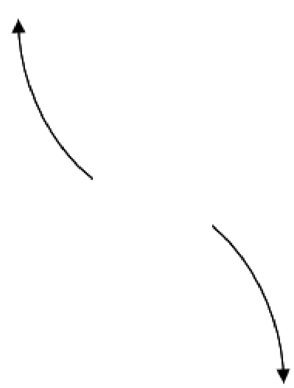
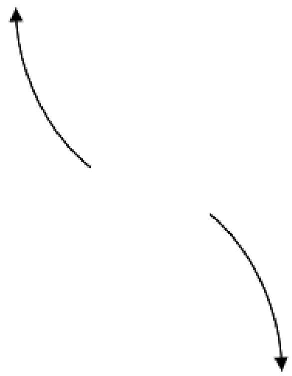
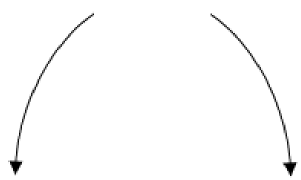
2. Describe the end behavior of the polynomial below.

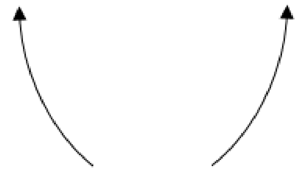
$$f(x) = -6(x - 6)^3(x + 6)^6(x + 2)^3(x - 2)^3$$

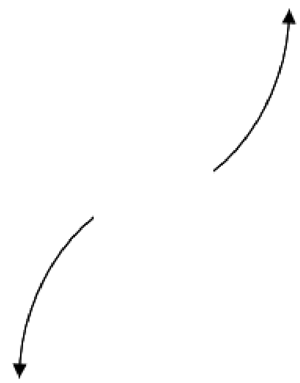
The solution is the graph below, which is option A.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 3i \text{ and } 2$$

The solution is $x^3 - 10x^2 + 41x - 50$, which is option A.

A. $b \in [-15, -8], c \in [35, 44], \text{ and } d \in [-50, -44]$

   * $x^3 - 10x^2 + 41x - 50$, which is the correct option.

B. $b \in [-5, 4], c \in [1, 7], \text{ and } d \in [-8, 2]$

   $x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x + 3)(x - 2)$.

C. $b \in [10, 15], c \in [35, 44], \text{ and } d \in [50, 56]$

   $x^3 + 10x^2 + 41x + 50$, which corresponds to multiplying out $(x - (4 - 3i))(x - (4 + 3i))(x + 2)$.

D. $b \in [-5, 4], c \in [-9, 0], \text{ and } d \in [6, 11]$

   $x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 4)(x - 2)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(4-3i))(x-(4+3i))(x-(2))$.

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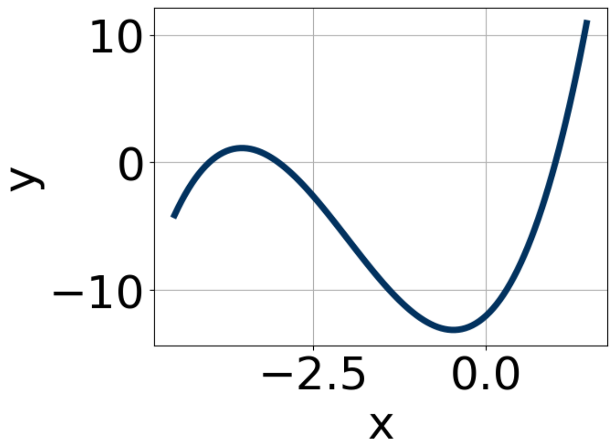
4. Which of the following equations *could* be of the graph presented below?



The solution is $3(x-1)^7(x+3)^9(x+4)^9$, which is option D.

A. $15(x-1)^4(x+3)^4(x+4)^9$

The factors 1 and -3 have have been odd power.

B. $-12(x-1)^{10}(x+3)^{11}(x+4)^{11}$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-11(x-1)^{11}(x+3)^7(x+4)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $3(x-1)^7(x+3)^9(x+4)^9$

* This is the correct option.

E. $20(x-1)^8(x+3)^5(x+4)^9$

The factor 1 should have been an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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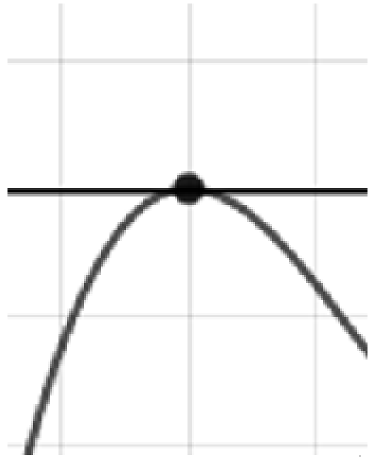
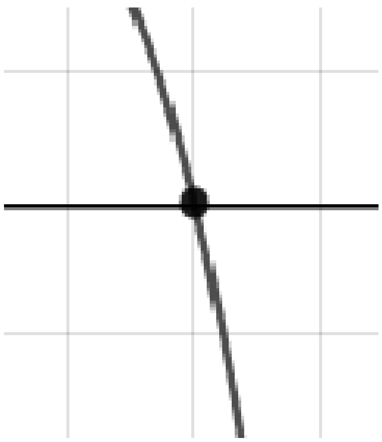
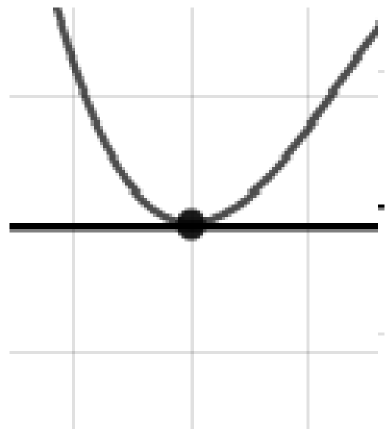
5. Describe the zero behavior of the zero $x=4$ of the polynomial below.

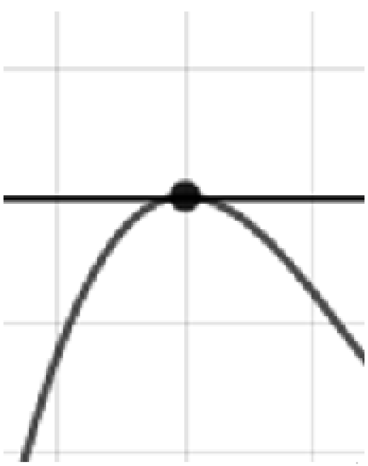
$$f(x) = -6(x-4)^2(x+4)^3(x-8)^2(x+8)^5$$

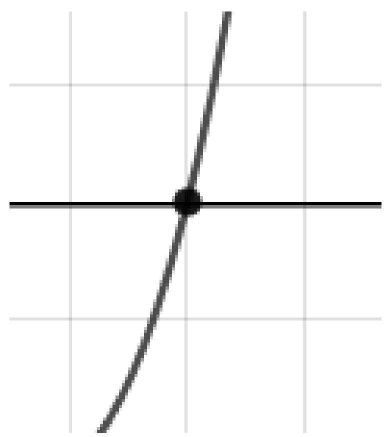
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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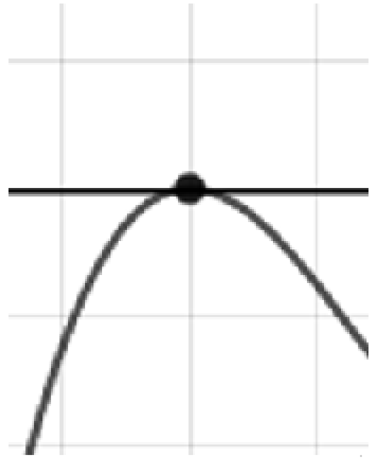
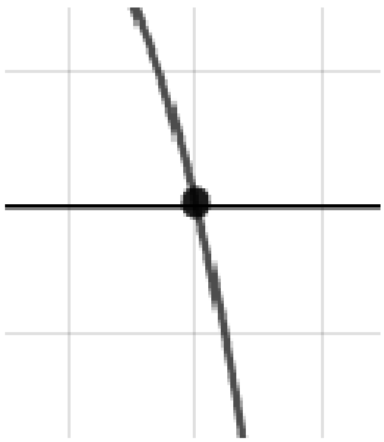
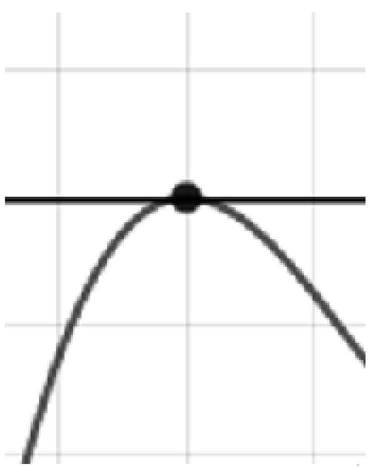
6. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

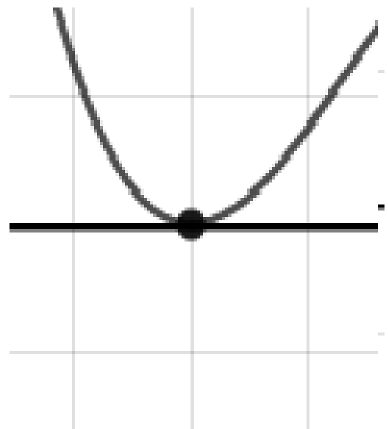
$$f(x) = -5(x+5)^3(x-5)^4(x+7)^2(x-7)^4$$

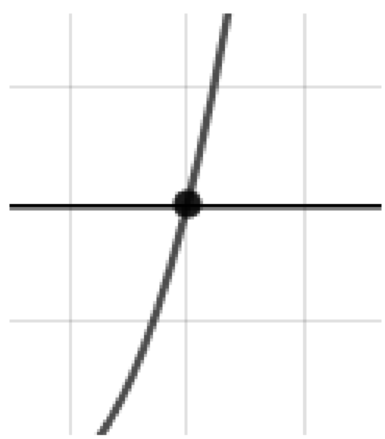
The solution is the graph below, which is option B.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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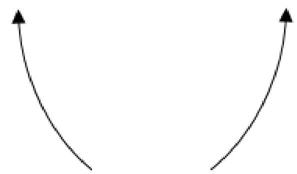
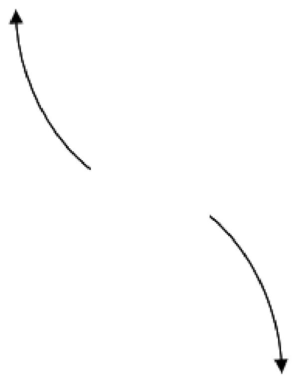
7. Describe the end behavior of the polynomial below.

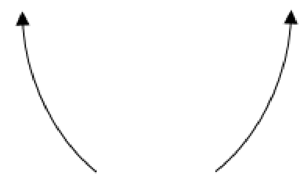
$$f(x) = 6(x-6)^4(x+6)^7(x-5)^3(x+5)^4$$

The solution is the graph below, which is option C.

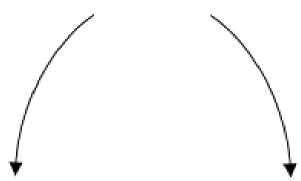




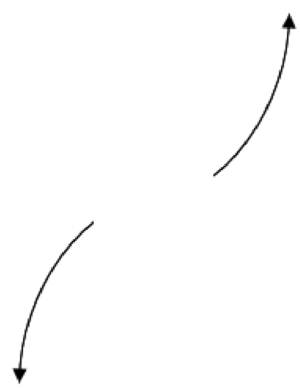
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 4i \text{ and } 1$$

The solution is $x^3 + 5x^2 + 19x - 25$, which is option D.

A. $b \in [-2.7, 4.7], c \in [0.81, 2.83]$, and $d \in [-3.5, -2.56]$

   $x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

B. $b \in [-7.9, -3.5], c \in [17.28, 19.46]$, and $d \in [24.68, 25.62]$

   $x^3 - 5x^2 + 19x + 25$, which corresponds to multiplying out $(x - (-3 - 4i))(x - (-3 + 4i))(x + 1)$.

C. $b \in [-2.7, 4.7], c \in [2.24, 5.06]$, and $d \in [-4.56, -3.05]$

   $x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

D. $b \in [3.6, 7.4], c \in [17.28, 19.46]$, and $d \in [-25.02, -24.7]$

* $x^3 + 5x^2 + 19x - 25$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 4i))(x - (-3 + 4i))(x - (1))$.

---

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{3}, \frac{1}{3}, \text{ and } \frac{5}{4}$$

The solution is $36x^3 - 33x^2 - 23x + 10$, which is option B.

A. $a \in [35, 37], b \in [-38, -31], c \in [-31, -22]$, and $d \in [-13, -7]$

$36x^3 - 33x^2 - 23x - 10$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [35, 37], b \in [-38, -31], c \in [-31, -22]$, and $d \in [8, 17]$

* $36x^3 - 33x^2 - 23x + 10$, which is the correct option.

C. $a \in [35, 37], b \in [-60, -55], c \in [4, 16]$, and $d \in [8, 17]$

$36x^3 - 57x^2 + 7x + 10$, which corresponds to multiplying out $(3x - 2)(3x + 1)(4x - 5)$.

D. $a \in [35, 37], b \in [-86, -76], c \in [53, 57]$, and $d \in [-13, -7]$

$36x^3 - 81x^2 + 53x - 10$, which corresponds to multiplying out $(3x - 2)(3x - 1)(4x - 5)$.

E. $a \in [35, 37], b \in [32, 39], c \in [-31, -22]$, and $d \in [-13, -7]$

$36x^3 + 33x^2 - 23x - 10$, which corresponds to multiplying out $(3x - 2)(3x + 1)(4x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 2)(3x - 1)(4x - 5)$

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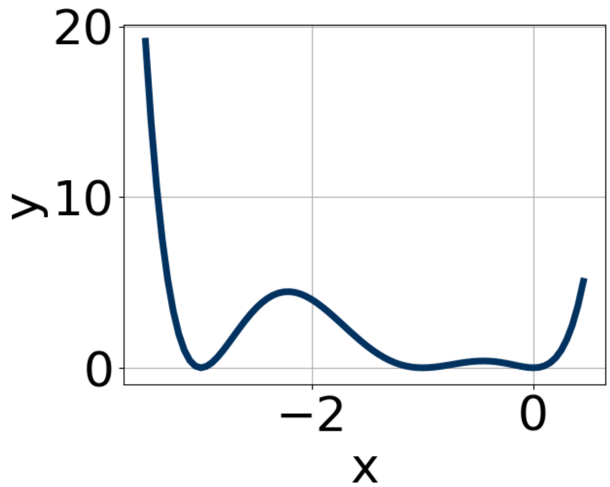
10. Which of the following equations *could* be of the graph presented below?



The solution is $4x^{10}(x+3)^{10}(x+1)^6$, which is option A.

A. $4x^{10}(x+3)^{10}(x+1)^6$

* This is the correct option.

B. $10x^8(x+3)^8(x+1)^{11}$

The factor $(x+1)$ should have an even power.

C. $6x^5(x+3)^4(x+1)^9$

The factors x and $(x+1)$ should both have even powers.

D. $-15x^{10}(x+3)^4(x+1)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-6x^6(x+3)^8(x+1)^5$

The factor $(x+1)$ should have an even power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).