This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-4}{3}, \frac{4}{5}, \text{ and } \frac{6}{5}$$

The solution is $75x^3 - 50x^2 - 128x + 96$, which is option A.

A. $a \in [70, 77], b \in [-53, -41], c \in [-130, -119], \text{ and } d \in [86, 98]$

* $75x^3 - 50x^2 - 128x + 96$, which is the correct option.

B. $a \in [70, 77], b \in [50, 55], c \in [-130, -119], \text{ and } d \in [-100, -88]$

$75x^3 + 50x^2 - 128x - 96$, which corresponds to multiplying out $(3x - 4)(5x + 4)(5x + 6)$.

C. $a \in [70, 77], b \in [-53, -41], c \in [-130, -119], \text{ and } d \in [-100, -88]$

$75x^3 - 50x^2 - 128x - 96$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [70, 77], b \in [-251, -249], c \in [271, 273], \text{ and } d \in [-100, -88]$

$75x^3 - 250x^2 + 272x - 96$, which corresponds to multiplying out $(3x - 4)(5x - 4)(5x - 6)$.

E. $a \in [70, 77], b \in [-131, -126], c \in [-33, -27], \text{ and } d \in [86, 98]$

$75x^3 - 130x^2 - 32x + 96$, which corresponds to multiplying out $(3x - 4)(5x + 4)(5x - 6)$.

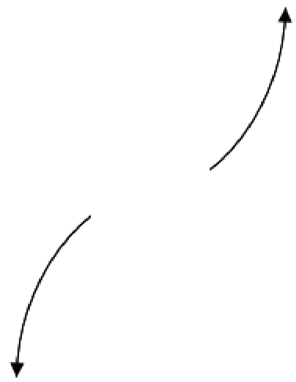
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 4)(5x - 4)(5x - 6)$

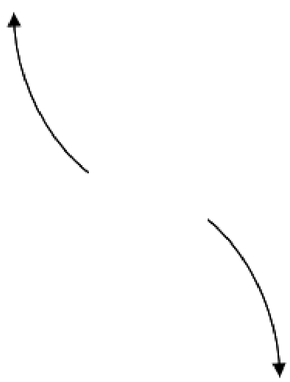
---

2. Describe the end behavior of the polynomial below.

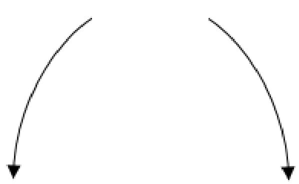
$$f(x) = 5(x - 7)^4(x + 7)^5(x - 4)^3(x + 4)^3$$

The solution is the graph below, which is option D.

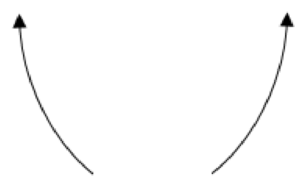




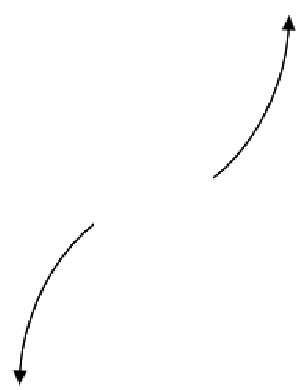
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 4i \text{ and } 1$$

The solution is $x^3 + 3x^2 + 16x - 20$, which is option D.

A. $b \in [0.7, 1.4], c \in [-6, -3],$ and $d \in [2, 11]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

B. $b \in [0.7, 1.4], c \in [-1, 5],$ and $d \in [-9, 0]$

$x^3 + x^2 + x - 2$, which corresponds to multiplying out $(x + 2)(x - 1)$.

C. $b \in [-6.9, -1.6], c \in [14, 23],$ and $d \in [13, 26]$

$x^3 - 3x^2 + 16x + 20$, which corresponds to multiplying out $(x - (-2 + 4i))(x - (-2 - 4i))(x + 1)$.

D. $b \in [1.6, 6.2], c \in [14, 23],$ and $d \in [-25, -14]$

* $x^3 + 3x^2 + 16x - 20$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

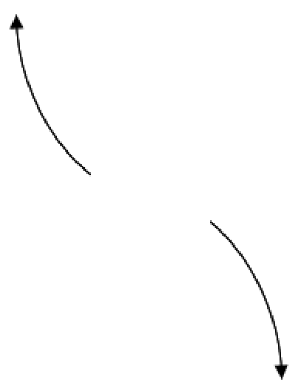
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 4i))(x - (-2 - 4i))(x - (1))$.

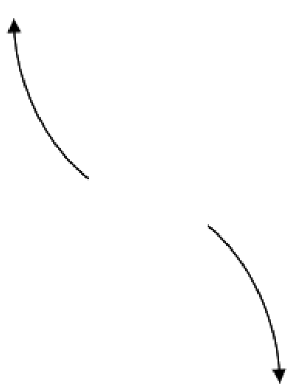
---

4. Describe the end behavior of the polynomial below.

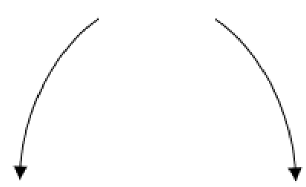
$$f(x) = -4(x - 4)^5(x + 4)^{10}(x + 6)^3(x - 6)^5$$

The solution is the graph below, which is option A.

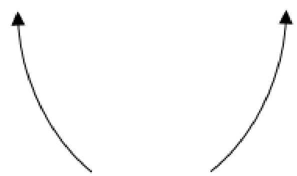




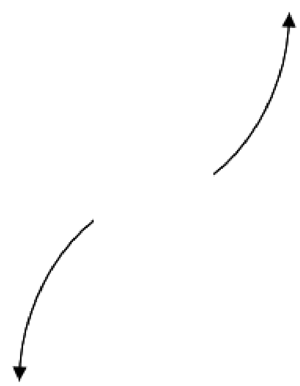
A.



B.



C.



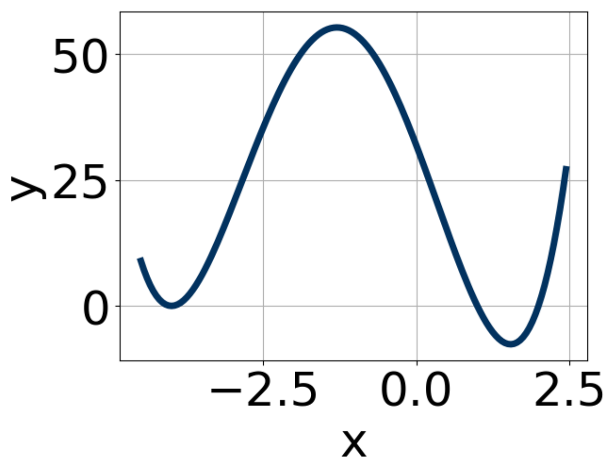
D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

5. Which of the following equations *could* be of the graph presented below?



The solution is $10(x+4)^6(x-1)^{11}(x-2)^5$, which is option A.

A. $10(x+4)^6(x-1)^{11}(x-2)^5$

   * This is the correct option.

B. $10(x+4)^8(x-1)^{10}(x-2)^5$

   The factor $(x-1)$ should have an odd power.

C. $-12(x+4)^4(x-1)^5(x-2)^8$

   The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-6(x+4)^{10}(x-1)^{11}(x-2)^{11}$

   This corresponds to the leading coefficient being the opposite value than it should be.

E. $17(x+4)^7(x-1)^{10}(x-2)^5$

The factor -4 should have an even power and the factor 1 should have an odd power.

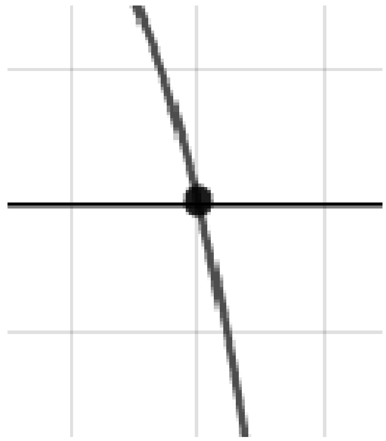
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

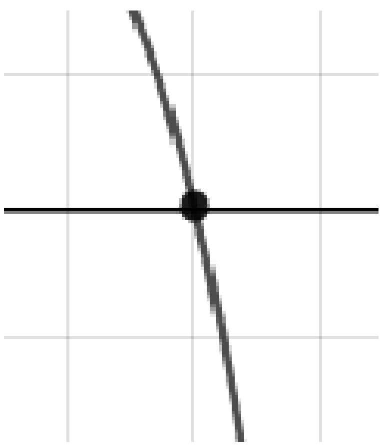
---

6. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

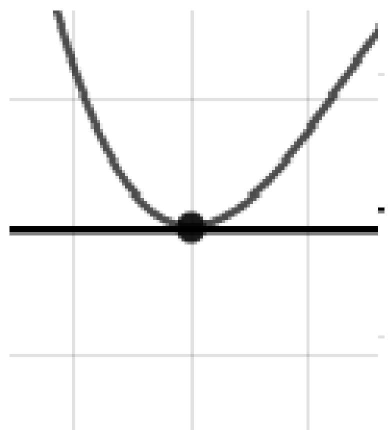
$$f(x) = -9(x-5)^2(x+5)^7(x+7)^8(x-7)^{10}$$

The solution is the graph below, which is option A.

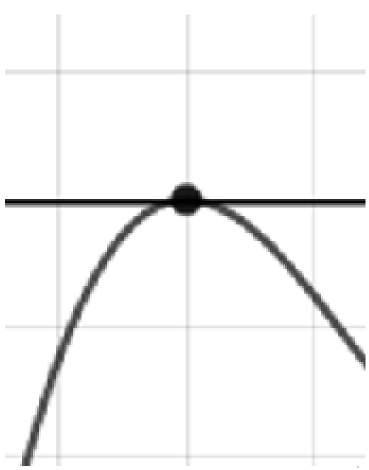




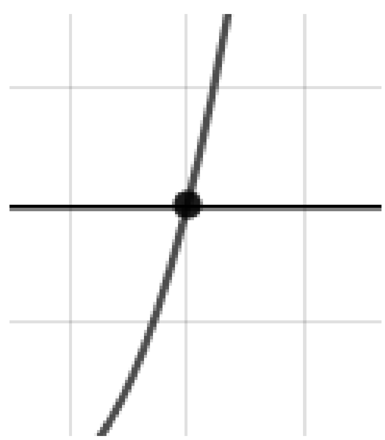
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 3i \text{ and } 2$$

The solution is $x^3 + 2x^2 + 5x - 26$, which is option B.

A. $b \in [-2.71, -1.5], c \in [3.3, 9.2], \text{ and } d \in [22.8, 26.7]$

$x^3 - 2x^2 + 5x + 26$, which corresponds to multiplying out $(x - (-2 - 3i))(x - (-2 + 3i))(x + 2)$.

B. $b \in [1.42, 2.12], c \in [3.3, 9.2], \text{ and } d \in [-26.6, -24.8]$

* $x^3 + 2x^2 + 5x - 26$, which is the correct option.

C. $b \in [0.14, 1.15], c \in [0.2, 1.1], \text{ and } d \in [-7.3, -4.4]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x + 3)(x - 2)$.

D. $b \in [0.14, 1.15], c \in [-3.6, 0.6], \text{ and } d \in [-4.4, -1.3]$

$x^3 + x^2 - 4$, which corresponds to multiplying out $(x + 2)(x - 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 3i))(x - (-2 + 3i))(x - (2))$.

---

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{4}, 7, \text{ and } \frac{7}{5}$$

The solution is $20x^3 - 163x^2 + 154x + 49$, which is option A.

A. $a \in [18, 22], b \in [-170, -159], c \in [152, 163], \text{ and } d \in [42, 51]$

* $20x^3 - 163x^2 + 154x + 49$, which is the correct option.

B. $a \in [18, 22], b \in [106, 113], c \in [-227, -221], \text{ and } d \in [42, 51]$

$20x^3 + 107x^2 - 224x + 49$, which corresponds to multiplying out $(4x - 1)(x + 7)(5x - 7)$.

C. $a \in [18, 22], b \in [-178, -171], c \in [231, 239], \text{ and } d \in [-53, -47]$

$20x^3 - 173x^2 + 238x - 49$, which corresponds to multiplying out $(4x - 1)(x - 7)(5x - 7)$.

D. $a \in [18, 22], b \in [159, 164], c \in [152, 163], \text{ and } d \in [-53, -47]$

$20x^3 + 163x^2 + 154x - 49$, which corresponds to multiplying out $(4x - 1)(x + 7)(5x + 7)$.

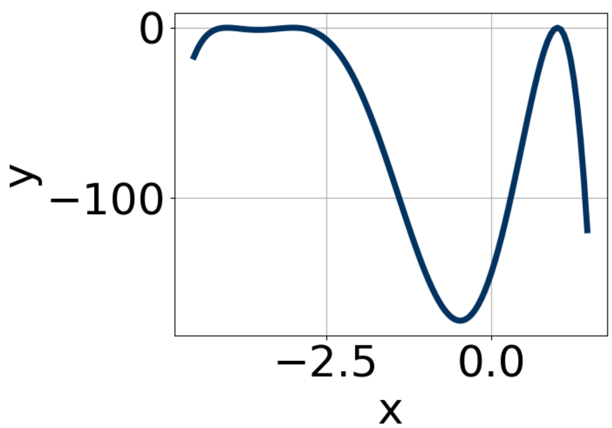
E. $a \in [18, 22], b \in [-170, -159], c \in [152, 163], \text{ and } d \in [-53, -47]$

$20x^3 - 163x^2 + 154x - 49$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x+1)(x-7)(5x-7)$

---

9. Which of the following equations *could* be of the graph presented below?



The solution is $-3(x+4)^6(x+3)^6(x-1)^8$, which is option D.

A. $-20(x+4)^{10}(x+3)^5(x-1)^7$

The factors $(x+3)$ and $(x-1)$ should both have even powers.

B. $9(x+4)^8(x+3)^8(x-1)^9$

The factor $(x-1)$ should have an even power and the leading coefficient should be the opposite sign.

C. $10(x+4)^4(x+3)^{10}(x-1)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-3(x+4)^6(x+3)^6(x-1)^8$

* This is the correct option.

E. $-16(x+4)^6(x+3)^4(x-1)^5$

The factor $(x-1)$ should have an even power.

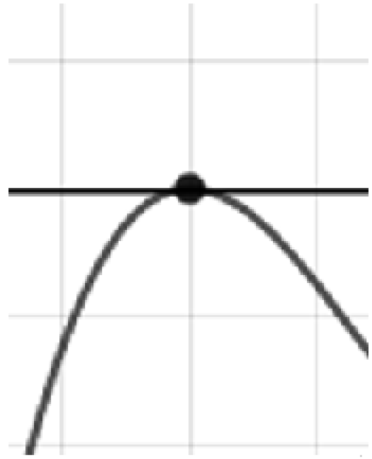
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

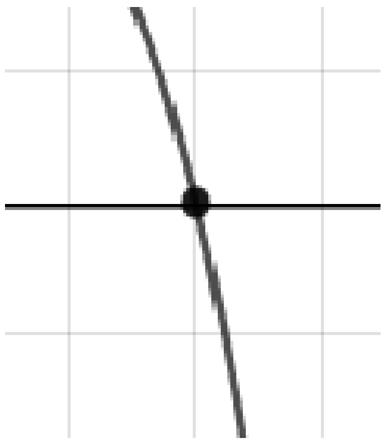
---

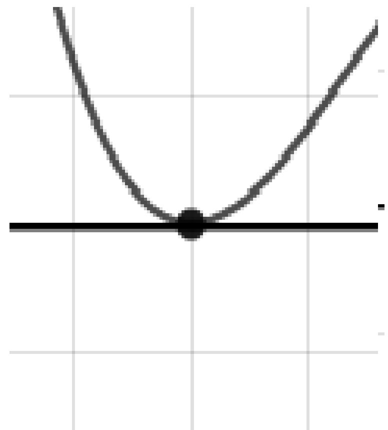
10. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

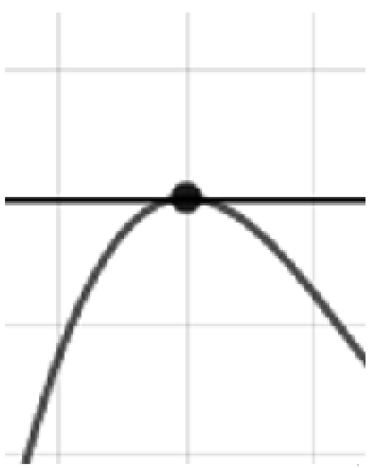
$$f(x) = -7(x+7)^5(x-7)^{10}(x-4)^4(x+4)^7$$

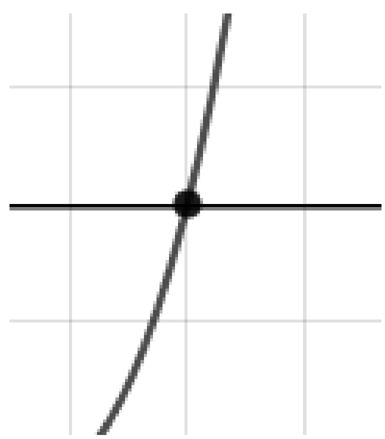
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.