This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 + 75x^2 - 16x - 48$$

The solution is $[-3, -0.8, 0.8]$, which is option C.

A. $z_1 \in [-3.16, -2.71], \; z_2 \in [-1.32, -1.21], \text{ and } z_3 \in [1.09, 1.63]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-1.28, -1.18], \; z_2 \in [1.09, 1.35], \text{ and } z_3 \in [2.81, 3.32]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-3.16, -2.71], \; z_2 \in [-0.86, -0.49], \text{ and } z_3 \in [0.52, 0.91]$

* This is the solution!

D. $z_1 \in [-4.08, -3.92], \; z_2 \in [0.08, 0.21], \text{ and } z_3 \in [2.81, 3.32]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-0.9, -0.7], \; z_2 \in [0.52, 0.96], \text{ and } z_3 \in [2.81, 3.32]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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2. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 106x^2 + 112x - 30}{x - 4}$$

The solution is $20x^2 - 26x + 8 + \frac{2}{x-4}$, which is option D.

A. $a \in [79, 82], \; b \in [-426, -424], \; c \in [1811, 1818], \text{ and } r \in [-7295, -7290].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [79, 82], \; b \in [212, 216], \; c \in [965, 973], \text{ and } r \in [3836, 3844].$

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [17, 26], \; b \in [-47, -44], \; c \in [-27, -22], \text{ and } r \in [-109, -104].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [17, 26]$, $b \in [-28, -23]$, $c \in [4, 11]$, and $r \in [-1, 5]$.

* This is the solution!

E. $a \in [17, 26]$, $b \in [-192, -184]$, $c \in [855, 861]$, and $r \in [-3457, -3450]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 + 28x^2 - 68}{x + 4}$$

The solution is $6x^2 + 4x - 16 + \dfrac{-4}{x+4}$, which is option B.

A. $a \in [-27, -23], b \in [123, 125], c \in [-498, -495]$, and $r \in [1913, 1919]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [3, 9], b \in [4, 9], c \in [-19, -11]$, and $r \in [-5, -3]$.

* This is the solution!

C. $a \in [-27, -23], b \in [-68, -63], c \in [-277, -267]$, and $r \in [-1157, -1153]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [3, 9], b \in [51, 53], c \in [208, 211]$, and $r \in [762, 767]$.

You divided by the opposite of the factor.

E. $a \in [3, 9], b \in [-6, 1], c \in [4, 15]$, and $r \in [-125, -117]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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4. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 21x^2 - 135x - 50$$

The solution is $[-2.5, -0.4, 5]$, which is option B.

A. $z_1 \in [-4.5, -1.5]$, $z_2 \in [-0.52, -0.38]$, and $z_3 \in [5, 7]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-4.5, -1.5]$, $z_2 \in [-0.52, -0.38]$, and $z_3 \in [5, 7]$

* This is the solution!

C. $z_1 \in [-6, -4]$, $z_2 \in [0.36, 0.46]$, and $z_3 \in [1.5, 4.5]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-6, -4]$, $z_2 \in [0.01, 0.37]$, and $z_3 \in [5, 7]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-6, -4]$, $z_2 \in [0.36, 0.46]$, and $z_3 \in [1.5, 4.5]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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5. Factor the polynomial below completely, knowing that $x - 3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 9x^4 + 9x^3 - 163x^2 + 115x + 150$$

The solution is $[-5, -0.667, 1.667, 3]$, which is option D.

A. $z_1 \in [-5.2, -4.7]$, $z_2 \in [-1.62, -1.48]$, $z_3 \in [0.52, 0.63]$, and $z_4 \in [2.4, 3.2]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-5.2, -4.7]$, $z_2 \in [-3.05, -2.99]$, $z_3 \in [0.12, 0.28]$, and $z_4 \in [4, 5.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-3.7, -2]$, $z_2 \in [-0.65, -0.6]$, $z_3 \in [1.47, 1.5]$, and $z_4 \in [4, 5.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-5.2, -4.7]$, $z_2 \in [-0.74, -0.64]$, $z_3 \in [1.64, 1.74]$, and $z_4 \in [2.4, 3.2]$

* This is the solution!

E. $z_1 \in [-3.7, -2]$, $z_2 \in [-1.71, -1.63]$, $z_3 \in [0.66, 0.71]$, and $z_4 \in [4, 5.3]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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6. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 + 26x^2 - 68x - 53}{x + 4}$$

The solution is $10x^2 - 14x - 12 + \frac{-5}{x+4}$, which is option D.

A. $a \in [-42, -33]$, $b \in [-134, -130]$, $c \in [-607, -603]$, and $r \in [-2473, -2463]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [-42, -33]$, $b \in [185, 188]$, $c \in [-818, -809]$, and $r \in [3195, 3197]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [9, 11]$, $b \in [-30, -21]$, $c \in [52, 57]$, and $r \in [-321, -310]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [9, 11]$, $b \in [-16, -13]$, $c \in [-14, -10]$, and $r \in [-8, -1]$.

* This is the solution!

E. $a \in [9, 11], \; b \in [66, 70], \; c \in [196, 202], \text{ and } r \in [722, 739]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 + 28x^2 - 62}{x + 4}$$

The solution is $6x^2 + 4x - 16 + \dfrac{2}{x+4}$, which is option D.

A. $a \in [4, 10], b \in [51, 53], c \in [207, 212], \text{ and } r \in [762, 773]$.

You divided by the opposite of the factor.

B. $a \in [4, 10], b \in [-6, 1], c \in [8, 13], \text{ and } r \in [-115, -105]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-25, -21], b \in [120, 125], c \in [-503, -493], \text{ and } r \in [1917, 1926]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [4, 10], b \in [3, 8], c \in [-17, -14], \text{ and } r \in [2, 3]$.

* This is the solution!

E. $a \in [-25, -21], b \in [-72, -65], c \in [-277, -268], \text{ and } r \in [-1150, -1148]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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8. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^2 + 2x + 3$$

The solution is $\pm 1, \pm 3$, which is option A.

A. $\pm 1, \pm 3$

* This is the solution **since we asked for the possible Integer roots**!

B. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 3}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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9. Factor the polynomial below completely, knowing that $x-3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 90x^3 + 343x^2 - 510x + 225$$

The solution is $[0.75, 2.5, 3, 5]$, which is option D.

A. $z_1 \in [-5.86, -4.88]$, $z_2 \in [-3.65, -2.93]$, $z_3 \in [-3.38, -2.77]$, and $z_4 \in [-0.71, -0.43]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-5.86, -4.88]$, $z_2 \in [-3.65, -2.93]$, $z_3 \in [-2.14, -0.63]$, and $z_4 \in [-0.49, -0.24]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.86, -4.88]$, $z_2 \in [-3.65, -2.93]$, $z_3 \in [-2.84, -2.19]$, and $z_4 \in [-0.83, -0.74]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [0.6, 0.85]$, $z_2 \in [2.02, 3.75]$, $z_3 \in [2.3, 3.15]$, and $z_4 \in [4.99, 5.07]$

* This is the solution!

E. $z_1 \in [0.14, 0.74]$, $z_2 \in [1.06, 1.46]$, $z_3 \in [2.3, 3.15]$, and $z_4 \in [4.99, 5.07]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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10. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^3 + 5x^2 + 2x + 5$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 7}$, which is option D.

A. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 7}$

* This is the solution **since we asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.
