This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 5x^2 + 3x + 6$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 5}$, which is option D.

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 2, \pm 3, \pm 6$

This would have been the solution **if asked for the possible Integer roots**!

C. $\pm 1, \pm 5$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 5}$

* This is the solution **since we asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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2. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 4x^3 + 5x^2 + 7x + 5$$

The solution is $\pm 1, \pm 5$, which is option B.

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 2, \pm 4}$

This would have been the solution **if asked for the possible Rational roots**!

B. $\pm 1, \pm 5$

* This is the solution **since we asked for the possible Integer roots**!

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Factor the polynomial below completely, knowing that $x-2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 - 71x^3 + 12x^2 + 116x + 48$$

The solution is $[-0.667, -0.6, 2, 4]$, which is option D.

A. $z_1 \in [-5.2, -2.7], \; z_2 \in [-2.28, -1.89], z_3 \in [0.55, 0.73], \text{ and } z_4 \in [-0.06, 1]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5.2, -2.7], \; z_2 \in [-2.28, -1.89], z_3 \in [1.36, 1.63], \text{ and } z_4 \in [1.29, 2.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.2, -2.7], \; z_2 \in [-2.28, -1.89], z_3 \in [0.08, 0.3], \text{ and } z_4 \in [2.98, 3.64]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-0.8, -0.3], \; z_2 \in [-0.67, -0.27], z_3 \in [1.8, 2.48], \text{ and } z_4 \in [3.99, 4.54]$

* This is the solution!

E. $z_1 \in [-2, -1], \; z_2 \in [-1.72, -1.24], z_3 \in [1.8, 2.48], \text{ and } z_4 \in [3.99, 4.54]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 - 24x^2 + 27}{x - 2}$$

The solution is $8x^2 - 8x - 16 + \frac{-5}{x-2}$, which is option C.

A. $a \in [14, 18], b \in [8, 9], c \in [16, 17], \text{ and } r \in [58, 60]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [14, 18], b \in [-56, -55], c \in [109, 118], \text{ and } r \in [-197, -196]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [5, 10], b \in [-11, -2], c \in [-16, -11], \text{ and } r \in [-5, -4]$.

* This is the solution!

D. $a \in [5, 10], b \in [-17, -12], c \in [-16, -11], \text{ and } r \in [7, 17]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [5, 10], b \in [-43, -39], c \in [74, 87],$ and $r \in [-135, -130].$

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{16x^3 + 84x^2 - 97}{x + 5}$$

The solution is $16x^2 + 4x - 20 + \frac{3}{x+5}$, which is option B.

A. $a \in [16, 19], b \in [164, 167], c \in [820, 821],$ and $r \in [4001, 4004].$

You divided by the opposite of the factor.

B. $a \in [16, 19], b \in [1, 6], c \in [-20, -18],$ and $r \in [-1, 4].$

* This is the solution!

C. $a \in [-82, -76], b \in [-320, -311], c \in [-1583, -1577],$ and $r \in [-7999, -7993].$

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [-82, -76], b \in [482, 491], c \in [-2420, -2411],$ and $r \in [12002, 12004].$

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [16, 19], b \in [-12, -9], c \in [69, 77],$ and $r \in [-535, -527].$

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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6. Factor the polynomial below completely, knowing that $x - 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 30x^3 - 87x^2 + 155x + 150$$

The solution is $[-2.5, -0.75, 2, 5]$, which is option E.

A. $z_1 \in [-6.2, -4.4], \ z_2 \in [-2.15, -1.97], z_3 \in [0.56, 0.64],$ and $z_4 \in [2.55, 3.08]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-1.8, -1.1], \ z_2 \in [-0.43, -0.15], z_3 \in [1.84, 2.03],$ and $z_4 \in [4.4, 5.57]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-6.2, -4.4], \ z_2 \in [-2.15, -1.97], z_3 \in [0.34, 0.55],$ and $z_4 \in [0.91, 1.86]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-6.2, -4.4], \ z_2 \in [-2.15, -1.97], z_3 \in [0.67, 0.99],$ and $z_4 \in [2.14, 2.86]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-4, -2.2], \;\; z_2 \in [-0.79, -0.71], z_3 \in [1.84, 2.03], \;$ and $\; z_4 \in [4.4, 5.57]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 - 83x^2 - 95x + 50$$

The solution is $[-1.25, 0.4, 5]$, which is option E.

A. $z_1 \in [-1.16, -0.26], \;\; z_2 \in [2.38, 3.36], \;$ and $\; z_3 \in [4.32, 5.39]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-5.24, -4.84], \;\; z_2 \in [-2.8, -1.73], \;$ and $\; z_3 \in [0.34, 0.82]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.24, -4.84], \;\; z_2 \in [-0.35, -0.02], \;$ and $\; z_3 \in [4.32, 5.39]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-5.24, -4.84], \;\; z_2 \in [-1.03, -0.32], \;$ and $\; z_3 \in [1.09, 1.48]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-1.45, -1.22], \;\; z_2 \in [-0.07, 0.58], \;$ and $\; z_3 \in [4.32, 5.39]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 - 77x^2 + 89x - 30$$

The solution is $[0.6, 1.25, 2]$, which is option B.

A. $z_1 \in [0.74, 0.84], \;\; z_2 \in [1.47, 1.68], \;$ and $\; z_3 \in [1.89, 2.27]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [0.49, 0.73], \;\; z_2 \in [1.14, 1.3], \;$ and $\; z_3 \in [1.89, 2.27]$

* This is the solution!

C. $z_1 \in [-2.22, -1.99], \;\; z_2 \in [-1.85, -1.62], \;$ and $\; z_3 \in [-0.9, -0.71]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-2.22, -1.99], \;\; z_2 \in [-1.28, -1.13], \;$ and $\; z_3 \in [-0.63, -0.44]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-3.19, -2.76], \;\; z_2 \in [-2.02, -1.87], \;$ and $\; z_3 \in [-0.35, -0.18]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 - 46x^2 + 88x - 43}{x - 5}$$

The solution is $6x^2 - 16x + 8 + \frac{-3}{x-5}$, which is option E.

A. $a \in [1, 13]$, $b \in [-26, -19]$, $c \in [-4, 3]$, and $r \in [-43, -40]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [1, 13]$, $b \in [-77, -70]$, $c \in [465, 475]$, and $r \in [-2386, -2380]$.

You divided by the opposite of the factor.

C. $a \in [26, 32]$, $b \in [102, 112]$, $c \in [605, 610]$, and $r \in [2996, 3001]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [26, 32]$, $b \in [-198, -189]$, $c \in [1064, 1072]$, and $r \in [-5386, -5379]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [1, 13]$, $b \in [-19, -13]$, $c \in [8, 14]$, and $r \in [-7, 2]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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10. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 - 64x^2 + 100x - 52}{x - 3}$$

The solution is $12x^2 - 28x + 16 + \frac{-4}{x-3}$, which is option A.

A. $a \in [8, 17]$, $b \in [-28, -25]$, $c \in [14, 17]$, and $r \in [-4, 0]$.

* This is the solution!

B. $a \in [8, 17]$, $b \in [-100, -98]$, $c \in [400, 402]$, and $r \in [-1254, -1246]$.

You divided by the opposite of the factor.

C. $a \in [33, 45]$, $b \in [39, 48]$, $c \in [226, 233]$, and $r \in [642, 646]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [33, 45]$, $b \in [-175, -166]$, $c \in [616, 624]$, and $r \in [-1903, -1893]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [8, 17]$, $b \in [-44, -38]$, $c \in [19, 21]$, and $r \in [-19, -11]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!