This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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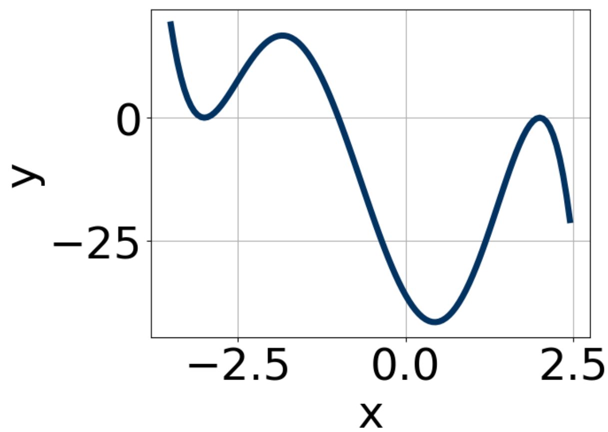
1. Which of the following equations *could* be of the graph presented below?



The solution is $-18(x-2)^{10}(x+3)^{6}(x+1)^{11}$, which is option B.

A. $19(x-2)^{10}(x+3)^{6}(x+1)^{8}$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-18(x-2)^{10}(x+3)^{6}(x+1)^{11}$

* This is the correct option.

C. $-19(x-2)^{8}(x+3)^{9}(x+1)^{10}$

The factor $(x+3)$ should have an even power and the factor $(x+1)$ should have an odd power.

D. $13(x-2)^{10}(x+3)^{10}(x+1)^{5}$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-4(x-2)^{10}(x+3)^{5}(x+1)^{5}$

The factor $(x+3)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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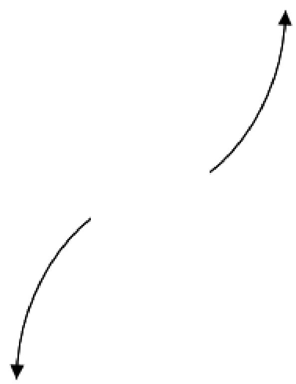
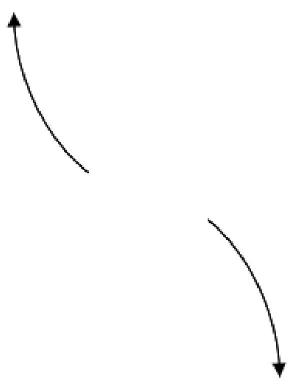
2. Describe the end behavior of the polynomial below.

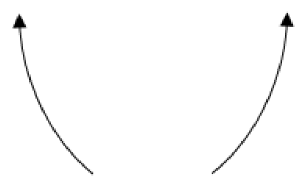
$$f(x) = 4(x+6)^{4}(x-6)^{9}(x+9)^{3}(x-9)^{3}$$

The solution is the graph below, which is option D.

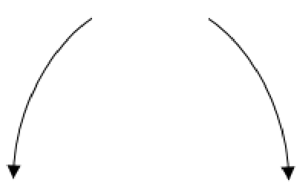




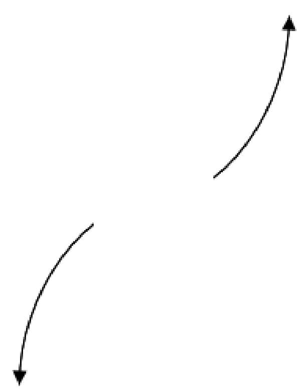
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 5i \text{ and } -4$$

The solution is $x^3 + 10x^2 + 58x + 136$, which is option D.

A. $b \in [-1, 5], c \in [8, 9.7],$ and $d \in [16, 22]$

$x^3 + x^2 + 9x + 20$, which corresponds to multiplying out $(x + 5)(x + 4)$.

B. $b \in [-1, 5], c \in [6.7, 8.6],$ and $d \in [10, 14]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 3)(x + 4)$.

C. $b \in [-15, -8], c \in [56.1, 58.3],$ and $d \in [-143, -134]$

$x^3 - 10x^2 + 58x - 136$, which corresponds to multiplying out $(x-(-3-5i))(x-(-3+5i))(x-4)$.

D. $b \in [6, 14], c \in [56.1, 58.3],$ and $d \in [130, 141]$

* $x^3 + 10x^2 + 58x + 136$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 5i))(x - (-3 + 5i))(x - (-4))$.

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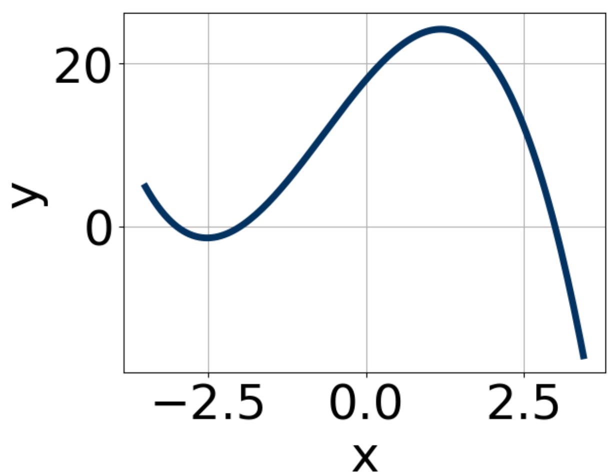
4. Which of the following equations *could* be of the graph presented below?



The solution is $-2(x + 2)^9(x + 3)^{11}(x - 3)^5$, which is option D.

A. $7(x + 2)^6(x + 3)^{11}(x - 3)^7$

The factor $(x + 2)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-10(x + 2)^{10}(x + 3)^9(x - 3)^9$

The factor -2 should have been an odd power.

C. $11(x + 2)^9(x + 3)^9(x - 3)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-2(x + 2)^9(x + 3)^{11}(x - 3)^5$

* This is the correct option.

E. $-5(x + 2)^4(x + 3)^8(x - 3)^9$

The factors -2 and -3 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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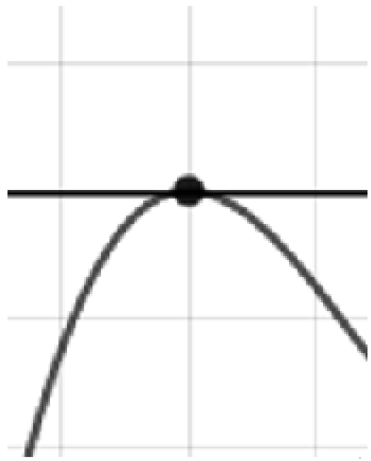
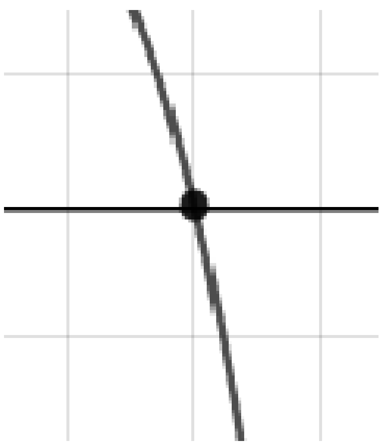
5. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

$$f(x) = -9(x-6)^9(x+6)^{10}(x+2)^9(x-2)^{12}$$

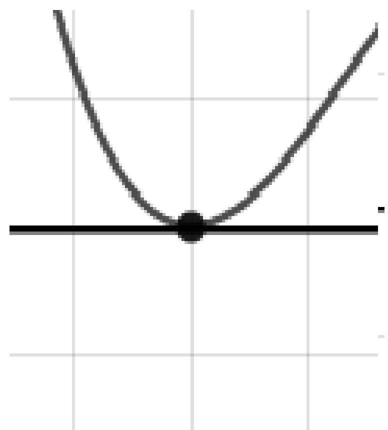
The solution is the graph below, which is option B.



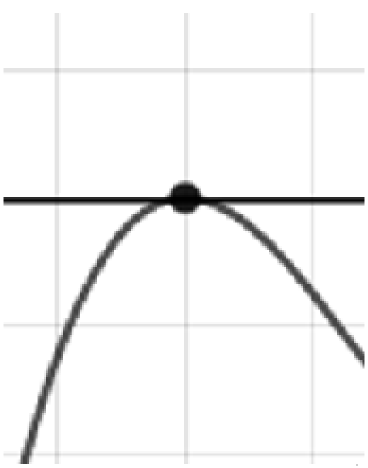
A.



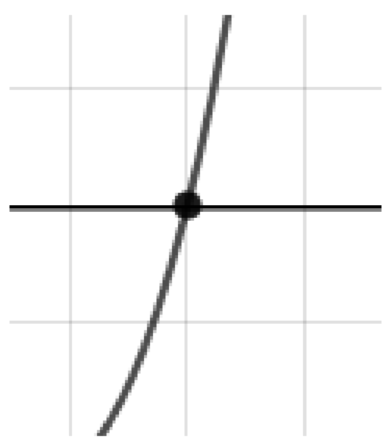
C.



B.



D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-2, \frac{-7}{3}, \text{ and } \frac{3}{2}$$

The solution is $6x^3 + 17x^2 - 11x - 42$, which is option E.

A. $a \in [5, 11], b \in [16, 20], c \in [-14, -9], \text{ and } d \in [40, 47]$

$6x^3 + 17x^2 - 11x + 42$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [5, 11], b \in [-12, -2], c \in [-33, -27], \text{ and } d \in [40, 47]$

$6x^3 - 7x^2 - 31x + 42$, which corresponds to multiplying out $(x - 2)(3x + 7)(2x - 3)$.

C. $a \in [5, 11], b \in [-43, -32], c \in [63, 68], \text{ and } d \in [-47, -37]$

$6x^3 - 35x^2 + 67x - 42$, which corresponds to multiplying out $(x - 2)(3x - 7)(2x - 3)$.

D. $a \in [5, 11], b \in [-21, -14], c \in [-14, -9], \text{ and } d \in [40, 47]$

$6x^3 - 17x^2 - 11x + 42$, which corresponds to multiplying out $(x - 2)(3x - 7)(2x + 3)$.

E. $a \in [5, 11], b \in [16, 20], c \in [-14, -9], \text{ and } d \in [-47, -37]$

* $6x^3 + 17x^2 - 11x - 42$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+2)(3x+7)(2x-3)$

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 3i \text{ and } -3$$

The solution is $x^3 + 11x^2 + 49x + 75$, which is option A.

A. $b \in [11, 19], c \in [48.36, 49.78], \text{ and } d \in [72.6, 77.1]$

* $x^3 + 11x^2 + 49x + 75$, which is the correct option.

B. $b \in [0, 7], c \in [4.02, 6.83], \text{ and } d \in [7.9, 9.5]$

$x^3 + x^2 + 6x + 9$, which corresponds to multiplying out $(x + 3)(x + 3)$.

C. $b \in [0, 7], c \in [6.29, 9.06], \text{ and } d \in [11.8, 14.7]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 4)(x + 3)$.

D. $b \in [-16, -10], c \in [48.36, 49.78], \text{ and } d \in [-76, -71.8]$

$x^3 - 11x^2 + 49x - 75$, which corresponds to multiplying out $(x - (-4 - 3i))(x - (-4 + 3i))(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-4-3i))(x-(-4+3i))(x-(-3))$.

---

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$\frac{-7}{5}, \frac{-1}{4}, \text{ and } \frac{3}{5}$$

The solution is $100x^3+105x^2-64x-21$, which is option E.

A. $a \in [100, 104], b \in [-230, -224], c \in [129, 136], \text{ and } d \in [-21, -15]$

$100x^3-225x^2+134x-21$, which corresponds to multiplying out $(5x-7)(4x-1)(5x-3)$.

B. $a \in [100, 104], b \in [-177, -171], c \in [30, 38], \text{ and } d \in [20, 32]$

$100x^3-175x^2+34x+21$, which corresponds to multiplying out $(5x-7)(4x+1)(5x-3)$.

C. $a \in [100, 104], b \in [-112, -104], c \in [-65, -60], \text{ and } d \in [20, 32]$

$100x^3-105x^2-64x+21$, which corresponds to multiplying out $(5x-7)(4x-1)(5x+3)$.

D. $a \in [100, 104], b \in [103, 115], c \in [-65, -60], \text{ and } d \in [20, 32]$

$100x^3+105x^2-64x+21$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [100, 104], b \in [103, 115], c \in [-65, -60], \text{ and } d \in [-21, -15]$

* $100x^3+105x^2-64x-21$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x+7)(4x+1)(5x-3)$

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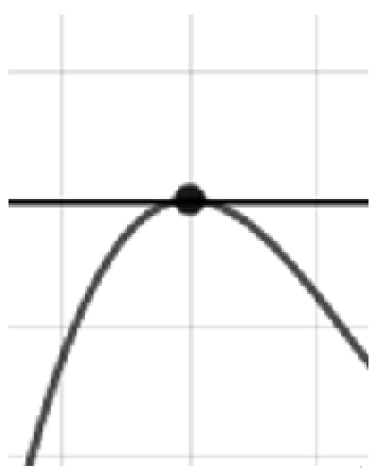
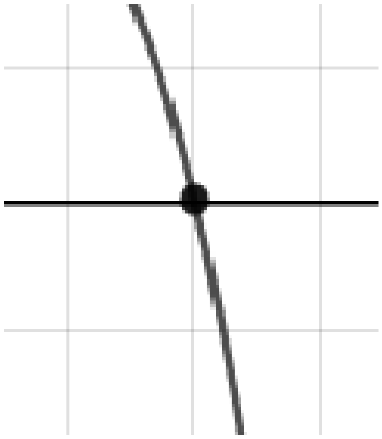
9. Describe the zero behavior of the zero $x=7$ of the polynomial below.

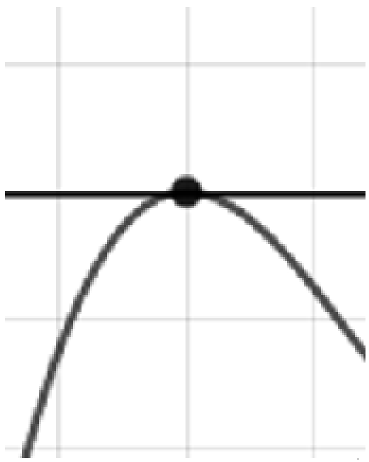
$$f(x) = -7(x-3)^{11}(x+3)^9(x-7)^{14}(x+7)^9$$

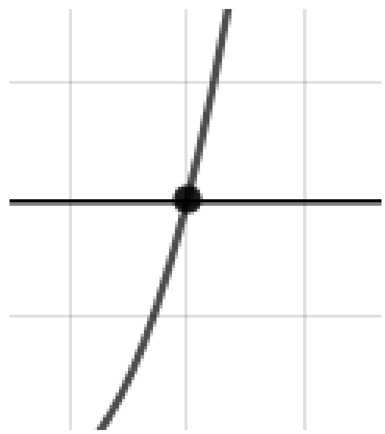
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

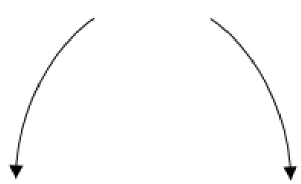
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

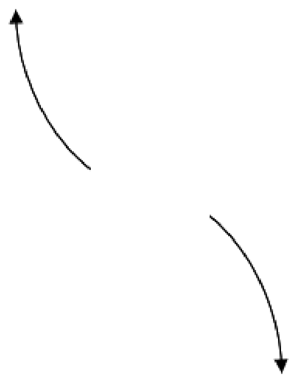
---

10. Describe the end behavior of the polynomial below.

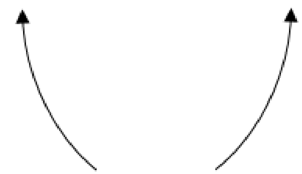
$$f(x) = -3(x+9)^5(x-9)^8(x-3)^2(x+3)^3$$

The solution is the graph below, which is option B.

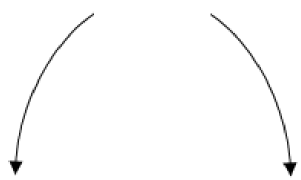




A.



C.



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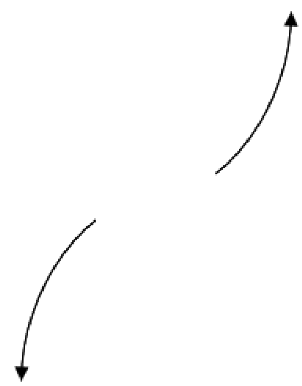

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.