

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

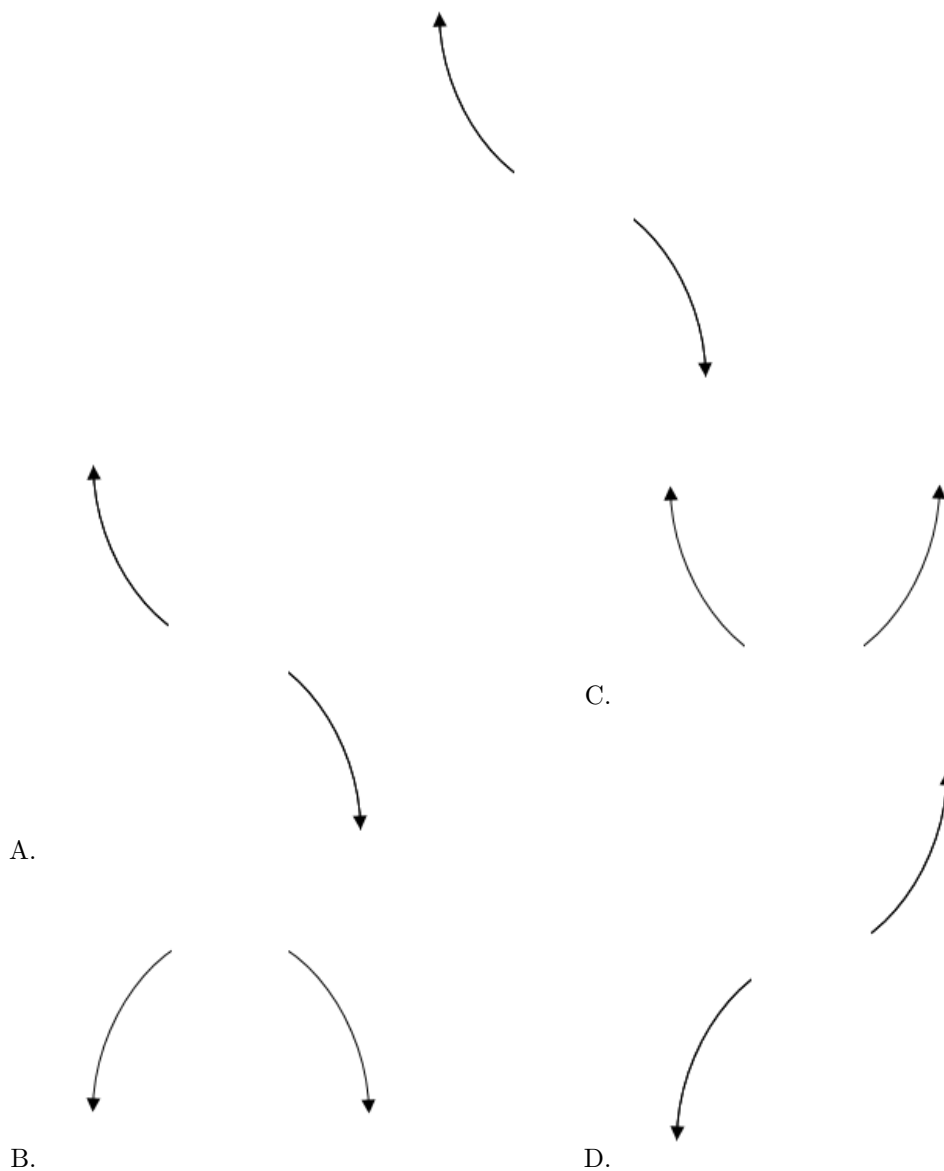
If you have a suggestion to make the keys better, please fill out the short survey [here](#).

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

- Describe the end behavior of the polynomial below.

$$f(x) = -3(x + 5)^4(x - 5)^7(x - 7)^4(x + 7)^6$$

The solution is the graph below, which is option A.



E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

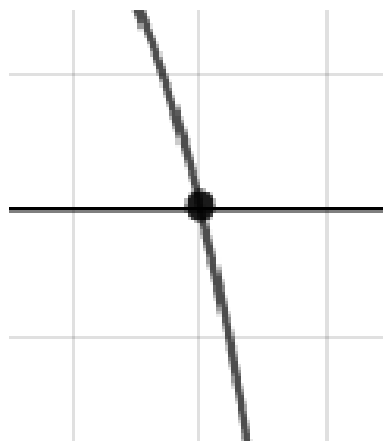
2. Describe the zero behavior of the zero  $x = 3$  of the polynomial below.

$$f(x) = 6(x - 3)^8(x + 3)^{13}(x - 4)^9(x + 4)^{12}$$

The solution is the graph below, which is option B.



A.



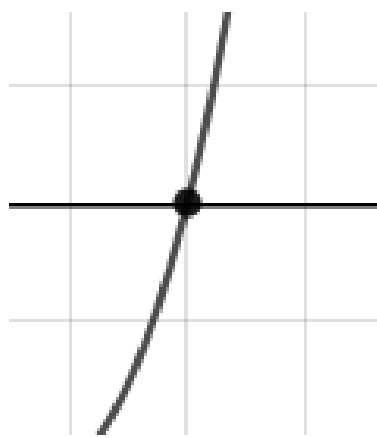
C.



B.



D.

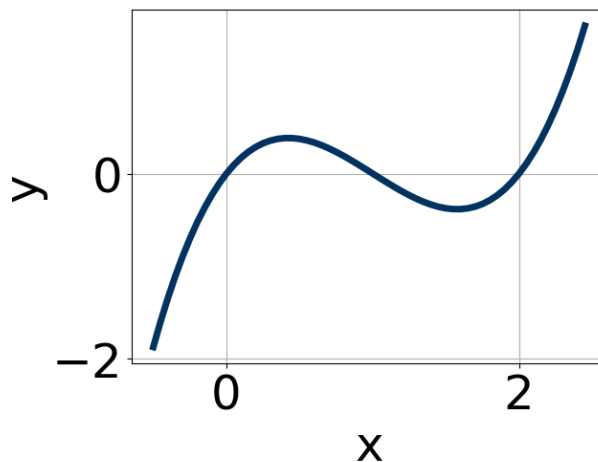


E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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3. Which of the following equations *could* be of the graph presented below?



The solution is  $18x^7(x-2)^7(x-1)^5$ , which is option C.

A.  $4x^7(x-2)^{10}(x-1)^5$

The factor 2 should have been an odd power.

B.  $-8x^9(x-2)^6(x-1)^5$

The factor  $(x-2)$  should have an odd power and the leading coefficient should be the opposite sign.

C.  $18x^7(x-2)^7(x-1)^5$

\* This is the correct option.

D.  $7x^{10}(x-2)^8(x-1)^{11}$

The factors 2 and 0 have have been odd power.

E.  $-12x^9(x-2)^5(x-1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$4 + 5i \text{ and } 2$$

The solution is  $x^3 - 10x^2 + 57x - 82$ , which is option D.

A.  $b \in [7, 16], c \in [56, 57.3], \text{ and } d \in [81, 86.3]$

$x^3 + 10x^2 + 57x + 82$ , which corresponds to multiplying out  $(x - (4 + 5i))(x - (4 - 5i))(x + 2)$ .

B.  $b \in [0, 2]$ ,  $c \in [-6.8, -1.8]$ , and  $d \in [3.8, 8.5]$

$x^3 + x^2 - 6x + 8$ , which corresponds to multiplying out  $(x - 4)(x - 2)$ .

C.  $b \in [0, 2]$ ,  $c \in [-8.8, -6.5]$ , and  $d \in [8.4, 13.1]$

$x^3 + x^2 - 7x + 10$ , which corresponds to multiplying out  $(x - 5)(x - 2)$ .

D.  $b \in [-13, -4]$ ,  $c \in [56, 57.3]$ , and  $d \in [-84.9, -79.5]$

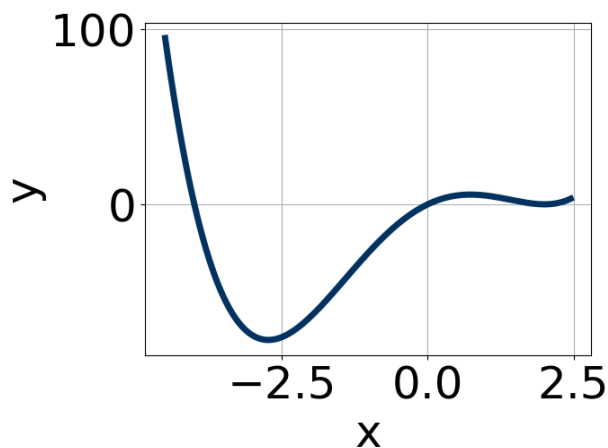
\*  $x^3 - 10x^2 + 57x - 82$ , which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (4 + 5i))(x - (4 - 5i))(x - (2))$ .

5. Which of the following equations *could* be of the graph presented below?



The solution is  $11x^9(x - 2)^8(x + 4)^7$ , which is option D.

A.  $-18x^9(x - 2)^{10}(x + 4)^8$

The factor  $(x + 4)$  should have an odd power and the leading coefficient should be the opposite sign.

B.  $19x^6(x - 2)^9(x + 4)^9$

The factor 2 should have an even power and the factor 0 should have an odd power.

C.  $-14x^9(x - 2)^6(x + 4)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D.  $11x^9(x - 2)^8(x + 4)^7$

\* This is the correct option.

E.  $19x^6(x - 2)^{10}(x + 4)^9$

The factor  $x$  should have an odd power.

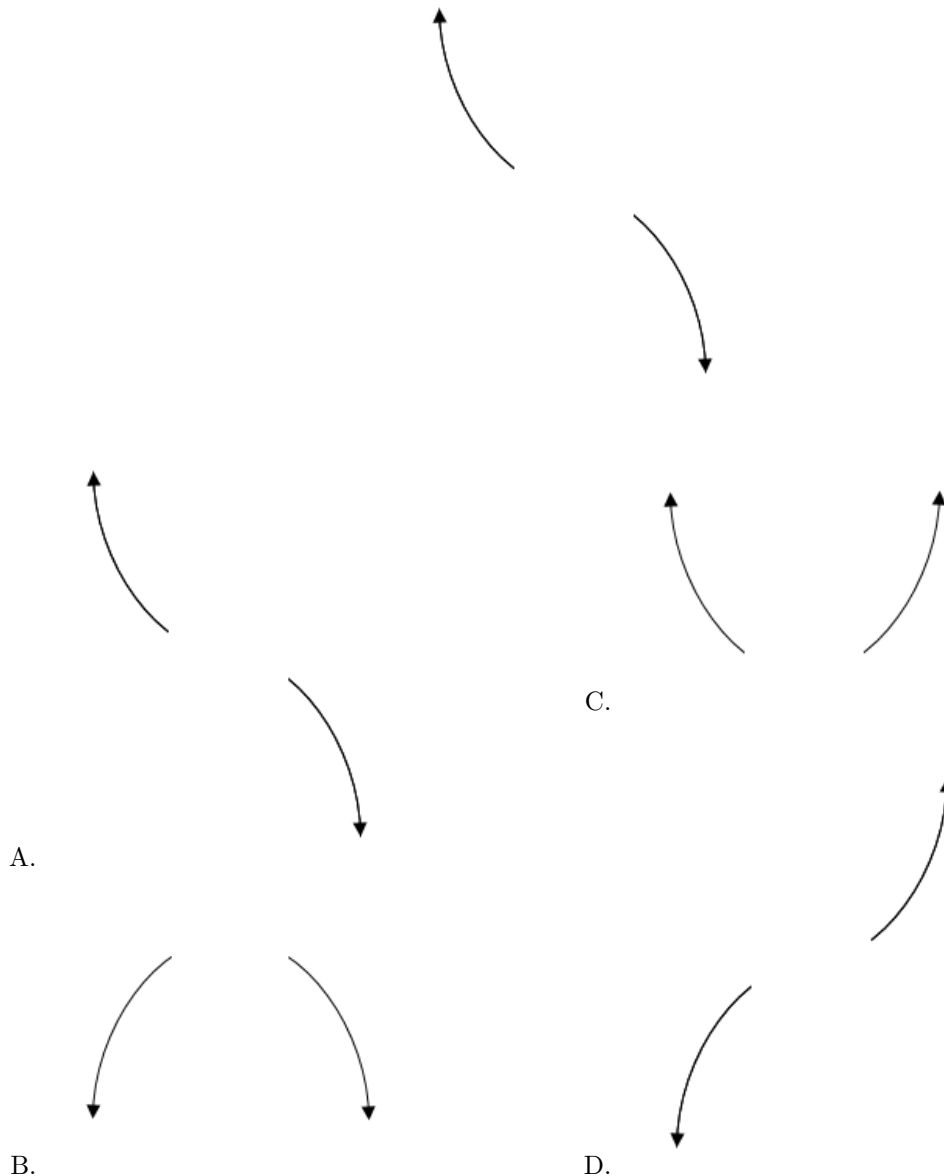
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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6. Describe the end behavior of the polynomial below.

$$f(x) = -3(x+2)^3(x-2)^8(x+6)^5(x-6)^5$$

The solution is the graph below, which is option A.



E. None of the above.

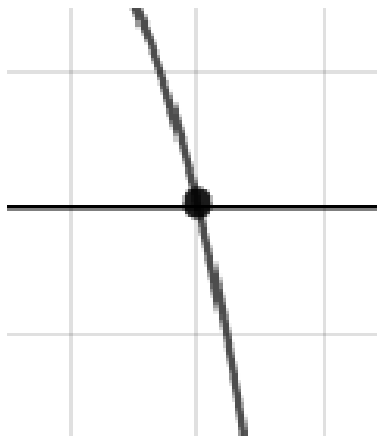
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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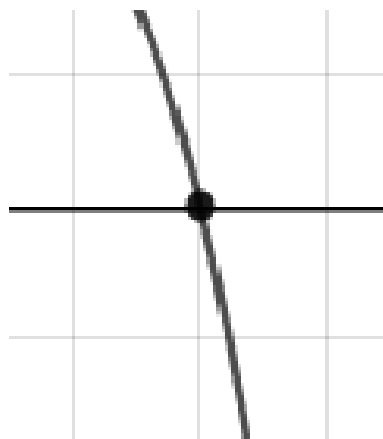
7. Describe the zero behavior of the zero  $x = 7$  of the polynomial below.

$$f(x) = 9(x + 8)^9(x - 8)^7(x - 7)^7(x + 7)^2$$

The solution is the graph below, which is option A.



A.



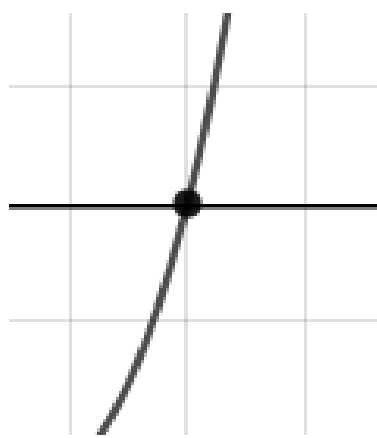
C.



B.



D.



E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-1}{5}, \frac{-1}{4}, \text{ and } 6$$

The solution is  $20x^3 - 111x^2 - 53x - 6$ , which is option E.

- A.  $a \in [16, 21], b \in [107, 116], c \in [-55, -47]$ , and  $d \in [-2, 13]$

$20x^3 + 111x^2 - 53x + 6$ , which corresponds to multiplying out  $(5x - 1)(4x - 1)(x + 6)$ .

- B.  $a \in [16, 21], b \in [-137, -123], c \in [51, 61]$ , and  $d \in [-9, -5]$

$20x^3 - 129x^2 + 55x - 6$ , which corresponds to multiplying out  $(5x - 1)(4x - 1)(x - 6)$ .

- C.  $a \in [16, 21], b \in [-113, -106], c \in [-55, -47]$ , and  $d \in [-2, 13]$

$20x^3 - 111x^2 - 53x + 6$ , which corresponds to multiplying everything correctly except the constant term.

- D.  $a \in [16, 21], b \in [-120, -116], c \in [-19, -4]$ , and  $d \in [-2, 13]$

$20x^3 - 119x^2 - 7x + 6$ , which corresponds to multiplying out  $(5x - 1)(4x + 1)(x - 6)$ .

- E.  $a \in [16, 21], b \in [-113, -106], c \in [-55, -47]$ , and  $d \in [-9, -5]$

\*  $20x^3 - 111x^2 - 53x - 6$ , which is the correct option.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(5x + 1)(4x + 1)(x - 6)$

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-5 + 4i \text{ and } -2$$

The solution is  $x^3 + 12x^2 + 61x + 82$ , which is option D.

- A.  $b \in [-14, -10], c \in [53, 67]$ , and  $d \in [-88, -77]$

$x^3 - 12x^2 + 61x - 82$ , which corresponds to multiplying out  $(x - (-5 + 4i))(x - (-5 - 4i))(x - 2)$ .

- B.  $b \in [1, 6], c \in [-5, -1]$ , and  $d \in [-8, 0]$

$x^3 + x^2 - 2x - 8$ , which corresponds to multiplying out  $(x - 4)(x + 2)$ .

- C.  $b \in [1, 6], c \in [7, 8]$ , and  $d \in [9, 17]$

$x^3 + x^2 + 7x + 10$ , which corresponds to multiplying out  $(x + 5)(x + 2)$ .

- D.  $b \in [9, 25], c \in [53, 67]$ , and  $d \in [82, 90]$

\*  $x^3 + 12x^2 + 61x + 82$ , which is the correct option.

- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (-5 + 4i))(x - (-5 - 4i))(x - (-2))$ .

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-5}{4}, \frac{-3}{4}, \text{ and } -5$$

The solution is  $16x^3 + 112x^2 + 175x + 75$ , which is option E.

- A.  $a \in [12, 18], b \in [44, 51], c \in [-145, -143],$  and  $d \in [73, 83]$

$16x^3 + 48x^2 - 145x + 75$ , which corresponds to multiplying out  $(4x - 5)(4x - 3)(x + 5)$ .

- B.  $a \in [12, 18], b \in [104, 114], c \in [171, 181],$  and  $d \in [-76, -74]$

$16x^3 + 112x^2 + 175x - 75$ , which corresponds to multiplying everything correctly except the constant term.

- C.  $a \in [12, 18], b \in [72, 73], c \in [-60, -50],$  and  $d \in [-76, -74]$

$16x^3 + 72x^2 - 55x - 75$ , which corresponds to multiplying out  $(4x - 5)(4x + 3)(x + 5)$ .

- D.  $a \in [12, 18], b \in [-114, -109], c \in [171, 181],$  and  $d \in [-76, -74]$

$16x^3 - 112x^2 + 175x - 75$ , which corresponds to multiplying out  $(4x - 5)(4x - 3)(x - 5)$ .

- E.  $a \in [12, 18], b \in [104, 114], c \in [171, 181],$  and  $d \in [73, 83]$

\*  $16x^3 + 112x^2 + 175x + 75$ , which is the correct option.

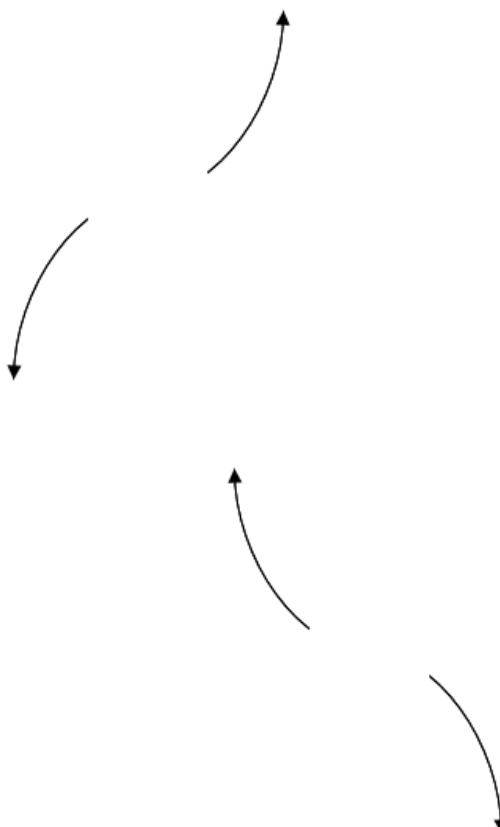
**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(4x + 5)(4x + 3)(x + 5)$

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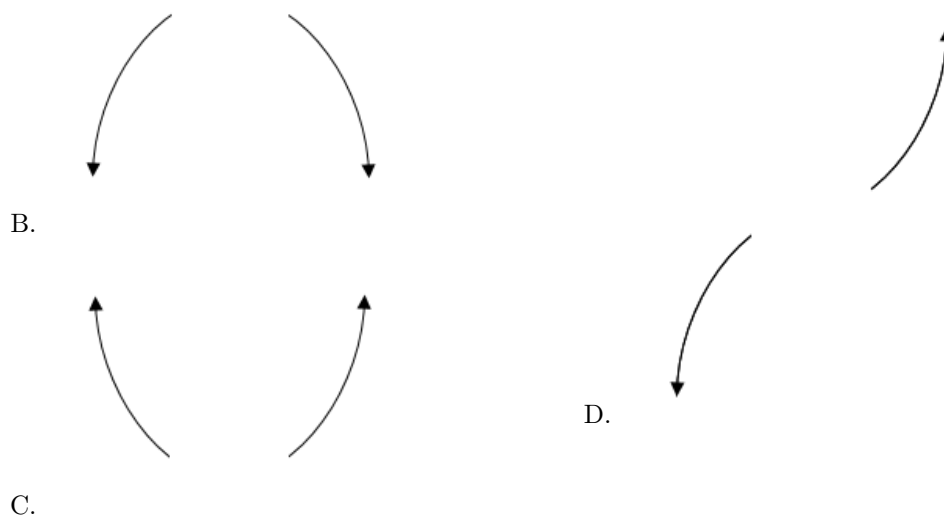
11. Describe the end behavior of the polynomial below.

$$f(x) = 4(x + 3)^4(x - 3)^9(x + 2)^4(x - 2)^6$$

The solution is the graph below, which is option D.







E. None of the above.

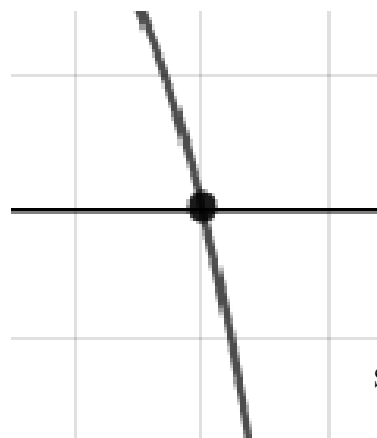
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

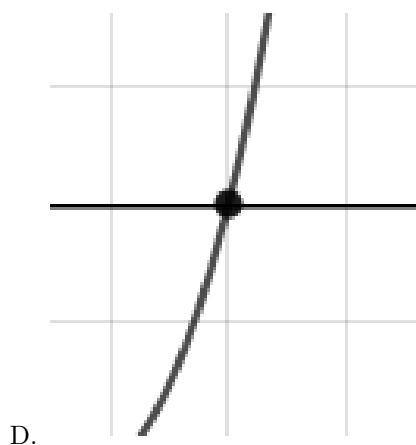
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12. Describe the zero behavior of the zero  $x = -8$  of the polynomial below.

$$f(x) = -3(x + 9)^6(x - 9)^5(x + 8)^{14}(x - 8)^9$$

The solution is the graph below, which is option B.

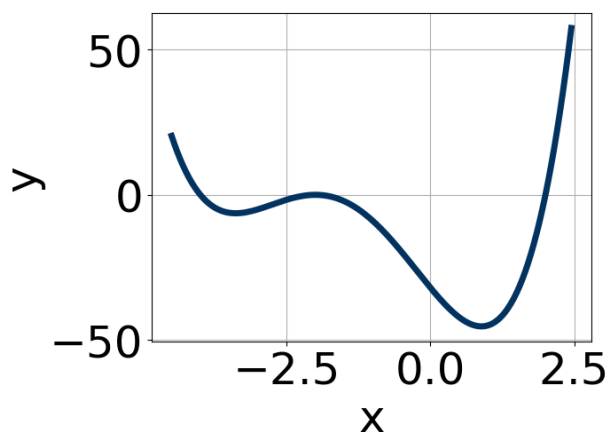




E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

13. Which of the following equations *could* be of the graph presented below?



The solution is  $17(x + 2)^{10}(x - 2)^7(x + 4)^5$ , which is option A.

A.  $17(x + 2)^{10}(x - 2)^7(x + 4)^5$

\* This is the correct option.

B.  $-9(x+2)^{10}(x-2)^9(x+4)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

C.  $-18(x+2)^{10}(x-2)^{11}(x+4)^{10}$

The factor  $(x+4)$  should have an odd power and the leading coefficient should be the opposite sign.

D.  $8(x+2)^7(x-2)^4(x+4)^5$

The factor  $-2$  should have an even power and the factor  $2$  should have an odd power.

E.  $20(x+2)^4(x-2)^4(x+4)^9$

The factor  $(x-2)$  should have an odd power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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14. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$3 + 2i \text{ and } 4$$

The solution is  $x^3 - 10x^2 + 37x - 52$ , which is option C.

A.  $b \in [1, 8], c \in [-7.52, -6.31]$ , and  $d \in [11, 13]$

$x^3 + x^2 - 7x + 12$ , which corresponds to multiplying out  $(x-3)(x-4)$ .

B.  $b \in [5, 14], c \in [36.76, 37.91]$ , and  $d \in [49, 57]$

$x^3 + 10x^2 + 37x + 52$ , which corresponds to multiplying out  $(x - (3 + 2i))(x - (3 - 2i))(x + 4)$ .

C.  $b \in [-15, -7], c \in [36.76, 37.91]$ , and  $d \in [-52, -48]$

\*  $x^3 - 10x^2 + 37x - 52$ , which is the correct option.

D.  $b \in [1, 8], c \in [-6.57, -5.83]$ , and  $d \in [6, 9]$

$x^3 + x^2 - 6x + 8$ , which corresponds to multiplying out  $(x-2)(x-4)$ .

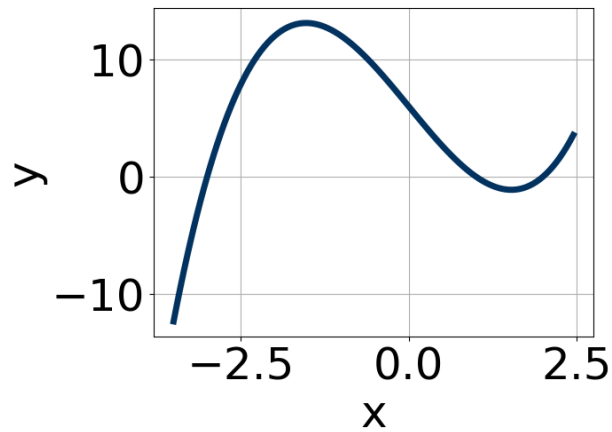
E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (3 + 2i))(x - (3 - 2i))(x - (4))$ .

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15. Which of the following equations *could* be of the graph presented below?



The solution is  $16(x - 2)^5(x + 3)^5(x - 1)^5$ , which is option B.

A.  $4(x - 2)^4(x + 3)^6(x - 1)^5$

The factors 2 and  $-3$  have have been odd power.

B.  $16(x - 2)^5(x + 3)^5(x - 1)^5$

\* This is the correct option.

C.  $-9(x - 2)^{10}(x + 3)^9(x - 1)^7$

The factor  $(x - 2)$  should have an odd power and the leading coefficient should be the opposite sign.

D.  $4(x - 2)^6(x + 3)^7(x - 1)^{11}$

The factor 2 should have been an odd power.

E.  $-10(x - 2)^{11}(x + 3)^5(x - 1)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

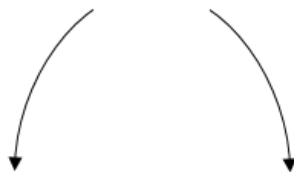
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

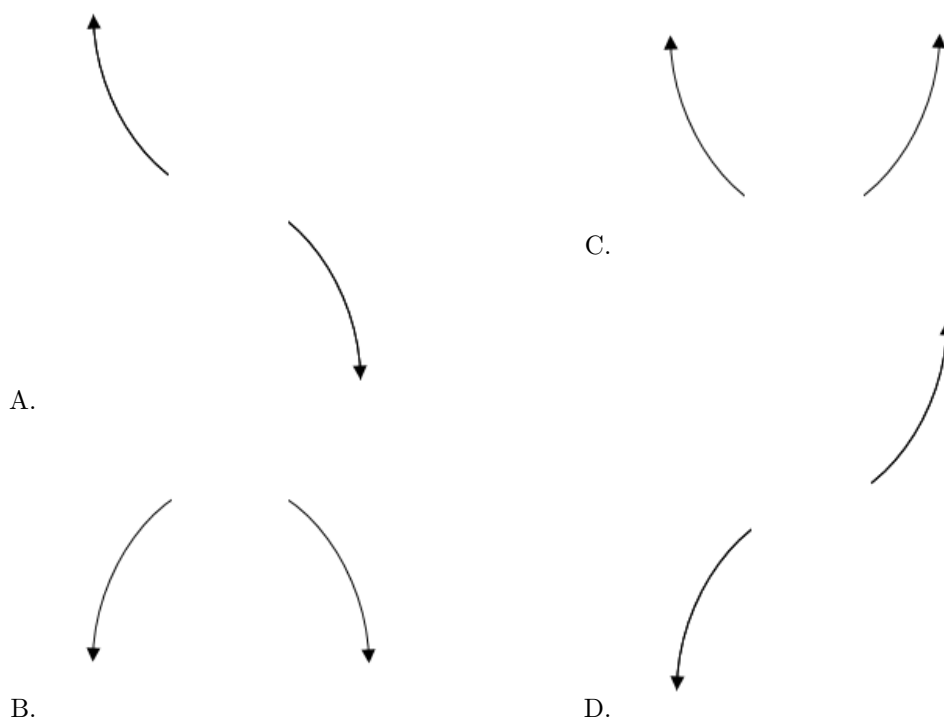
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16. Describe the end behavior of the polynomial below.

$$f(x) = -9(x + 8)^4(x - 8)^5(x - 6)^4(x + 6)^5$$

The solution is the graph below, which is option B.





E. None of the above.

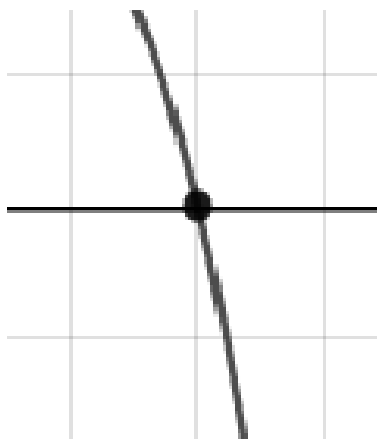
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

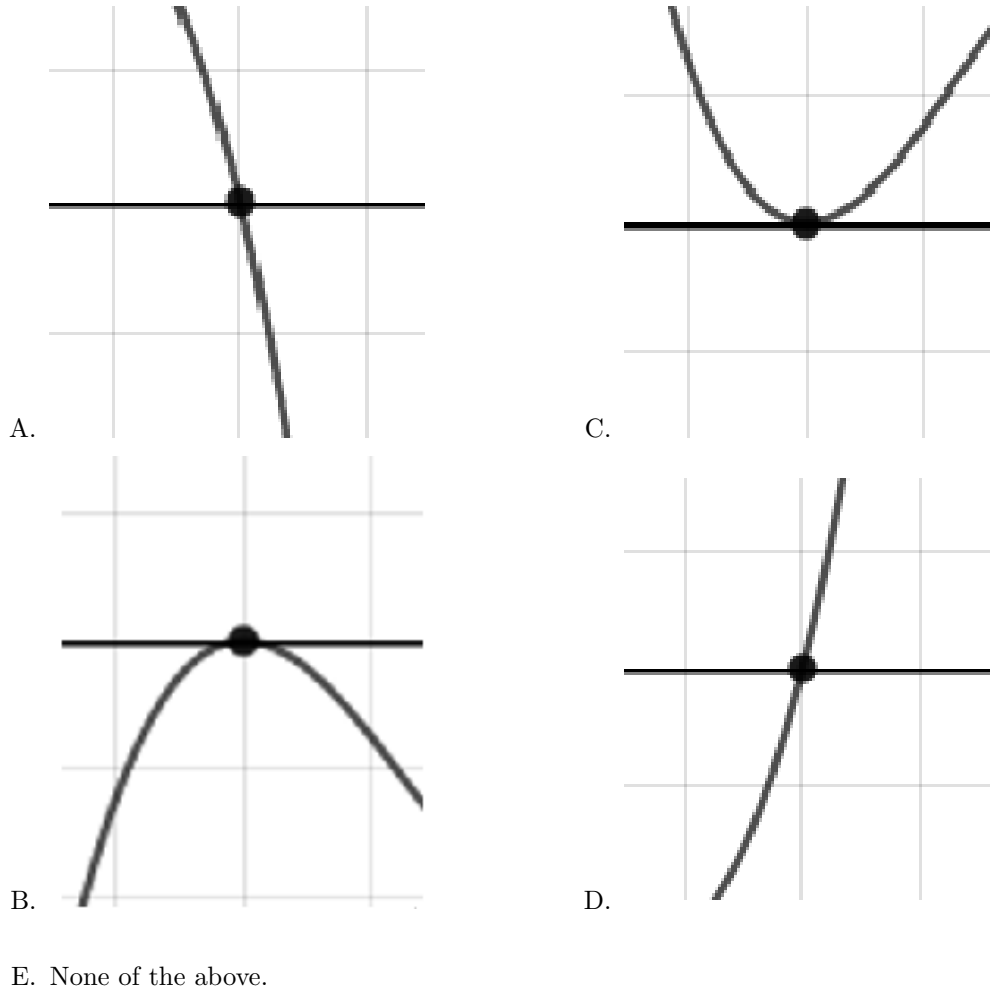
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17. Describe the zero behavior of the zero  $x = -5$  of the polynomial below.

$$f(x) = -9(x - 5)^4(x + 5)^7(x - 9)^4(x + 9)^8$$

The solution is the graph below, which is option A.





**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

18. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$1, \frac{-3}{4}, \text{ and } \frac{6}{5}$$

The solution is  $20x^3 - 29x^2 - 9x + 18$ , which is option A.

- A.  $a \in [20, 21], b \in [-35, -25], c \in [-9, 1]$ , and  $d \in [15, 24]$

\*  $20x^3 - 29x^2 - 9x + 18$ , which is the correct option.

- B.  $a \in [20, 21], b \in [-22, -14], c \in [-21, -16]$ , and  $d \in [15, 24]$

$20x^3 - 19x^2 - 21x + 18$ , which corresponds to multiplying out  $(x + 1)(4x - 3)(5x - 6)$ .

- C.  $a \in [20, 21], b \in [27, 36], c \in [-9, 1]$ , and  $d \in [-26, -17]$

$20x^3 + 29x^2 - 9x - 18$ , which corresponds to multiplying out  $(x + 1)(4x - 3)(5x + 6)$ .

- D.  $a \in [20, 21], b \in [-35, -25], c \in [-9, 1]$ , and  $d \in [-26, -17]$

$20x^3 - 29x^2 - 9x - 18$ , which corresponds to multiplying everything correctly except the constant term.

E.  $a \in [20, 21], b \in [10, 12], c \in [-33, -25]$ , and  $d \in [-26, -17]$

$20x^3 + 11x^2 - 27x - 18$ , which corresponds to multiplying out  $(x + 1)(4x + 3)(5x - 6)$ .

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(x - 1)(4x + 3)(5x - 6)$

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19. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$3 + 4i \text{ and } 4$$

The solution is  $x^3 - 10x^2 + 49x - 100$ , which is option D.

A.  $b \in [-5, 7], c \in [-7.9, -4.2]$ , and  $d \in [11, 13]$

$x^3 + x^2 - 7x + 12$ , which corresponds to multiplying out  $(x - 3)(x - 4)$ .

B.  $b \in [-5, 7], c \in [-8.4, -7.9]$ , and  $d \in [13, 18]$

$x^3 + x^2 - 8x + 16$ , which corresponds to multiplying out  $(x - 4)(x - 4)$ .

C.  $b \in [7, 19], c \in [48.6, 51.5]$ , and  $d \in [98, 101]$

$x^3 + 10x^2 + 49x + 100$ , which corresponds to multiplying out  $(x - (3 + 4i))(x - (3 - 4i))(x + 4)$ .

D.  $b \in [-10, -4], c \in [48.6, 51.5]$ , and  $d \in [-102, -94]$

\*  $x^3 - 10x^2 + 49x - 100$ , which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (3 + 4i))(x - (3 - 4i))(x - (4))$ .

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20. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{3}{4}, \frac{5}{2}, \text{ and } -4$$

The solution is  $8x^3 + 6x^2 - 89x + 60$ , which is option C.

A.  $a \in [5, 9], b \in [16, 29], c \in [-71, -68]$ , and  $d \in [-63, -58]$

$8x^3 + 18x^2 - 71x - 60$ , which corresponds to multiplying out  $(4x + 3)(2x - 5)(x + 4)$ .

B.  $a \in [5, 9], b \in [-13, -5], c \in [-95, -76]$ , and  $d \in [-63, -58]$

$8x^3 - 6x^2 - 89x - 60$ , which corresponds to multiplying out  $(4x + 3)(2x + 5)(x - 4)$ .

C.  $a \in [5, 9], b \in [2, 9], c \in [-95, -76]$ , and  $d \in [55, 66]$

\*  $8x^3 + 6x^2 - 89x + 60$ , which is the correct option.

D.  $a \in [5, 9], b \in [2, 9], c \in [-95, -76]$ , and  $d \in [-63, -58]$

$8x^3 + 6x^2 - 89x - 60$ , which corresponds to multiplying everything correctly except the constant term.

E.  $a \in [5, 9], b \in [57, 64], c \in [115, 125]$ , and  $d \in [55, 66]$

$8x^3 + 58x^2 + 119x + 60$ , which corresponds to multiplying out  $(4x + 3)(2x + 5)(x + 4)$ .

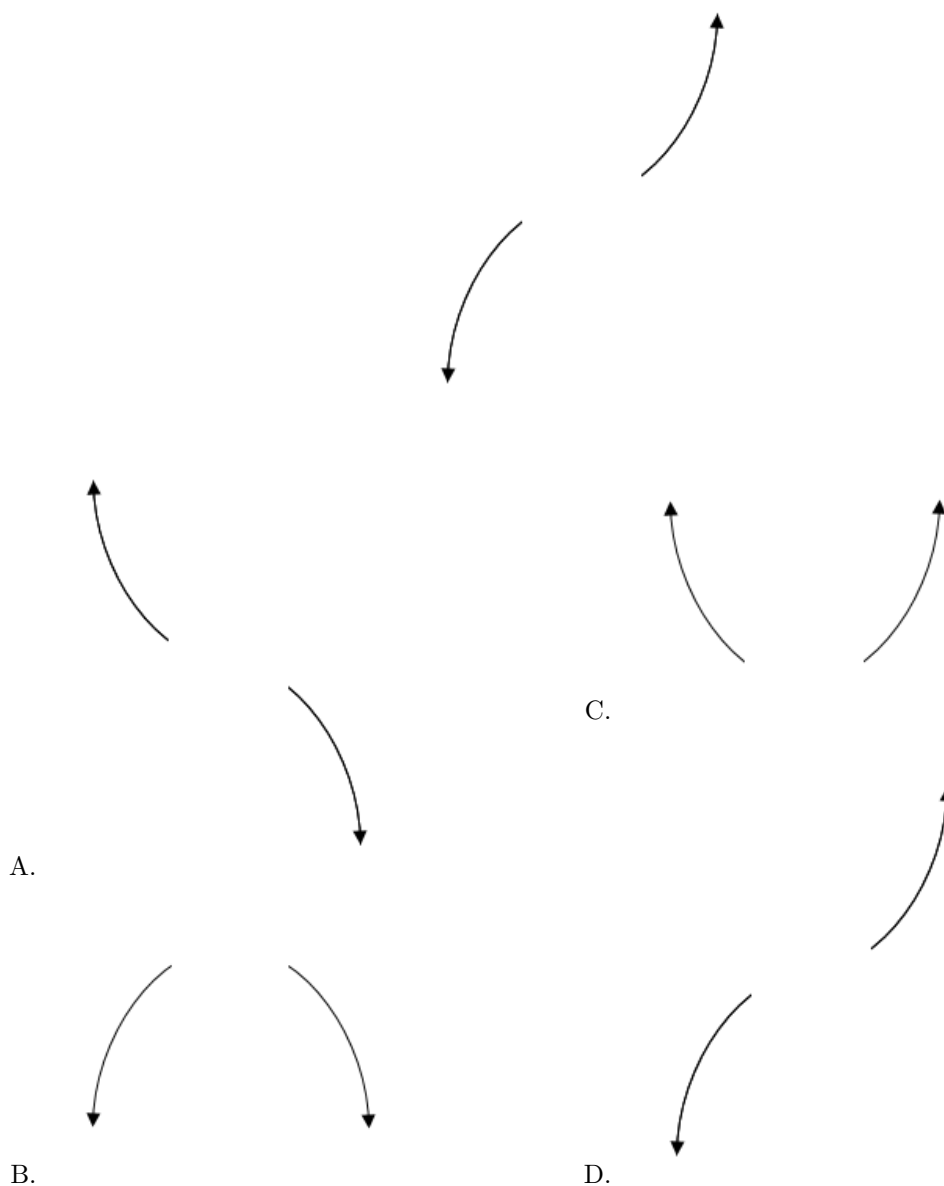
**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(4x - 3)(2x - 5)(x + 4)$

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21. Describe the end behavior of the polynomial below.

$$f(x) = 7(x + 5)^4(x - 5)^7(x - 9)^3(x + 9)^3$$

The solution is the graph below, which is option D.



E. None of the above.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

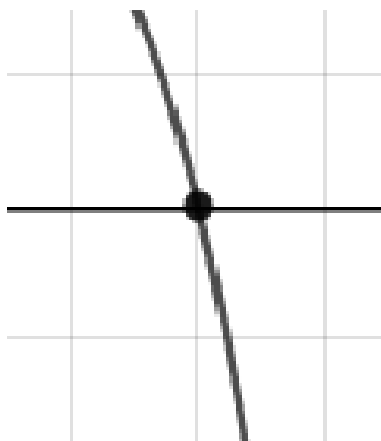
22. Describe the zero behavior of the zero  $x = -5$  of the polynomial below.

$$f(x) = 3(x - 3)^9(x + 3)^7(x + 5)^4(x - 5)^3$$

The solution is the graph below, which is option B.



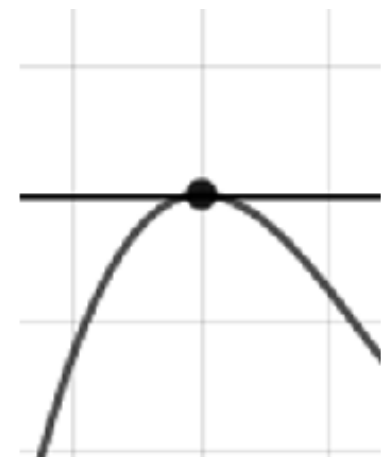
A.



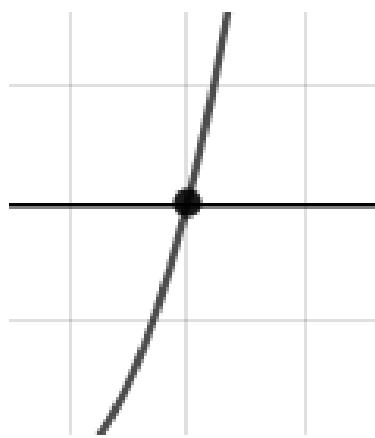
C.



B.



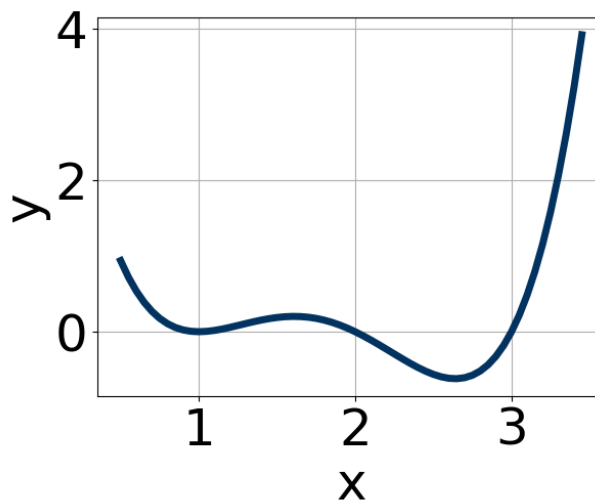
D.



E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

23. Which of the following equations *could* be of the graph presented below?



The solution is  $7(x-1)^6(x-2)^{11}(x-3)^7$ , which is option D.

A.  $9(x-1)^{10}(x-2)^8(x-3)^5$

The factor  $(x-2)$  should have an odd power.

B.  $19(x-1)^7(x-2)^{10}(x-3)^5$

The factor 1 should have an even power and the factor 2 should have an odd power.

C.  $-11(x-1)^4(x-2)^{11}(x-3)^4$

The factor  $(x-3)$  should have an odd power and the leading coefficient should be the opposite sign.

D.  $7(x-1)^6(x-2)^{11}(x-3)^7$

\* This is the correct option.

E.  $-6(x-1)^{10}(x-2)^{11}(x-3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

24. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$5 + 2i \text{ and } 4$$

The solution is  $x^3 - 14x^2 + 69x - 116$ , which is option D.

A.  $b \in [13, 15]$ ,  $c \in [64, 73]$ , and  $d \in [114, 122]$

$x^3 + 14x^2 + 69x + 116$ , which corresponds to multiplying out  $(x - (5 + 2i))(x - (5 - 2i))(x + 4)$ .

B.  $b \in [-4, 5]$ ,  $c \in [-19, -7]$ , and  $d \in [17, 23]$

$x^3 + x^2 - 9x + 20$ , which corresponds to multiplying out  $(x - 5)(x - 4)$ .

C.  $b \in [-4, 5]$ ,  $c \in [-8, -3]$ , and  $d \in [8, 14]$

$x^3 + x^2 - 6x + 8$ , which corresponds to multiplying out  $(x - 2)(x - 4)$ .

D.  $b \in [-16, -13]$ ,  $c \in [64, 73]$ , and  $d \in [-125, -112]$

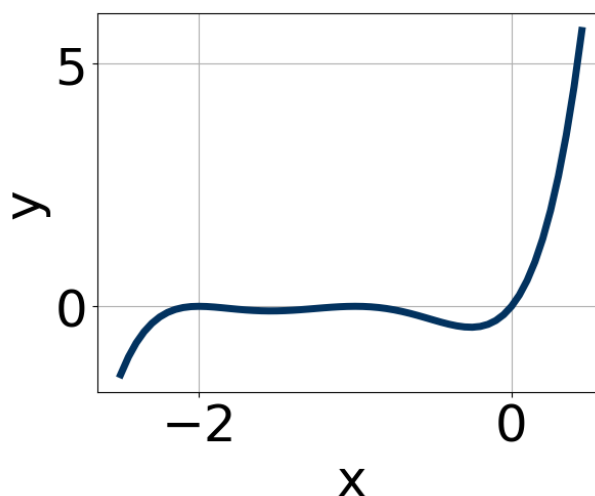
\*  $x^3 - 14x^2 + 69x - 116$ , which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (5 + 2i))(x - (5 - 2i))(x - (4))$ .

25. Which of the following equations *could* be of the graph presented below?



The solution is  $2x^9(x + 1)^8(x + 2)^{10}$ , which is option E.

A.  $19x^5(x + 1)^6(x + 2)^7$

The factor  $(x + 2)$  should have an even power.

B.  $-14x^7(x + 1)^{10}(x + 2)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

C.  $14x^6(x + 1)^{10}(x + 2)^9$

The factor  $(x + 2)$  should have an even power and the factor  $x$  should have an odd power.

D.  $-11x^4(x + 1)^{10}(x + 2)^4$

The factor  $x$  should have an odd power and the leading coefficient should be the opposite sign.

E.  $2x^9(x + 1)^8(x + 2)^{10}$

\* This is the correct option.

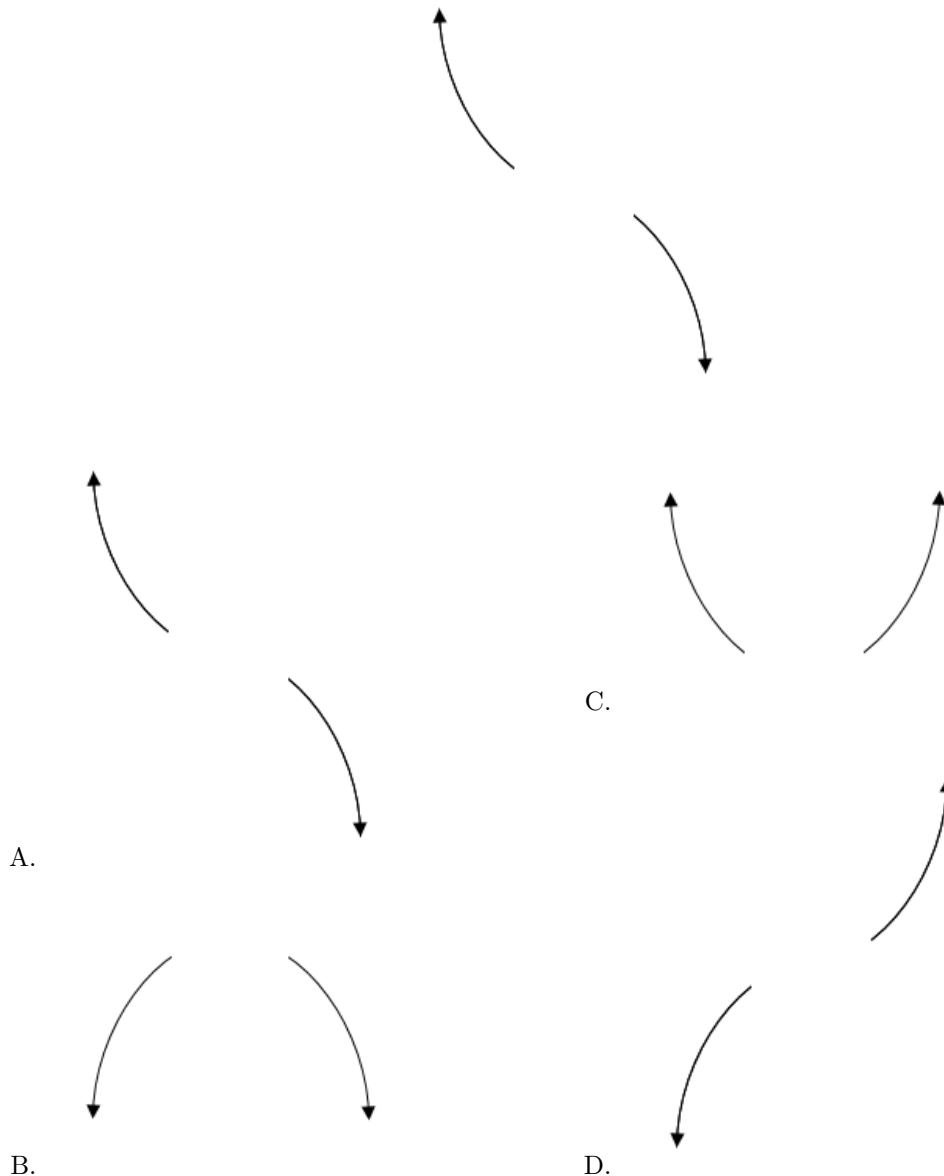
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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26. Describe the end behavior of the polynomial below.

$$f(x) = -4(x - 4)^3(x + 4)^6(x + 8)^5(x - 8)^5$$

The solution is the graph below, which is option A.



E. None of the above.

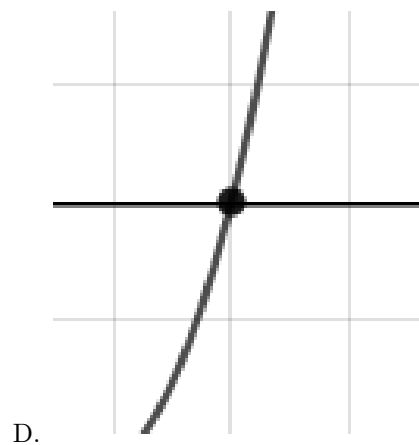
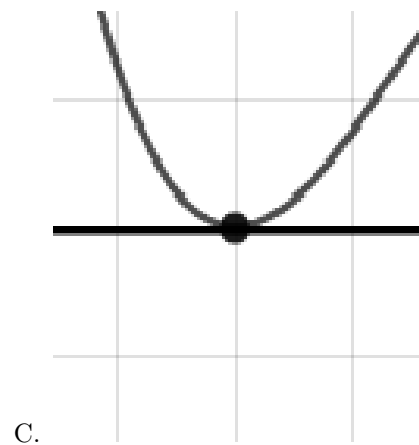
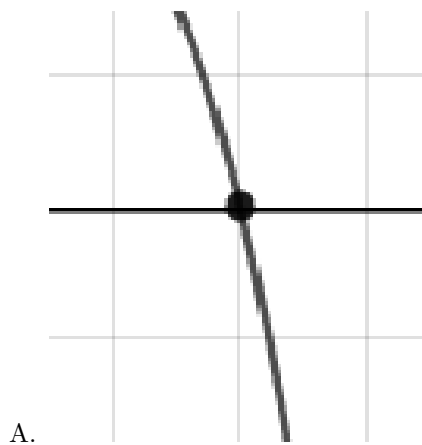
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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27. Describe the zero behavior of the zero  $x = 9$  of the polynomial below.

$$f(x) = -7(x - 9)^4(x + 9)^7(x + 2)^6(x - 2)^7$$

The solution is the graph below, which is option B.



E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

28. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{1}{4}, 4, \text{ and } \frac{3}{5}$$

The solution is  $20x^3 - 97x^2 + 71x - 12$ , which is option A.

- A.  $a \in [12, 21], b \in [-101, -93], c \in [61, 75]$ , and  $d \in [-15, -8]$

\*  $20x^3 - 97x^2 + 71x - 12$ , which is the correct option.

- B.  $a \in [12, 21], b \in [-101, -93], c \in [61, 75]$ , and  $d \in [8, 15]$

$20x^3 - 97x^2 + 71x + 12$ , which corresponds to multiplying everything correctly except the constant term.

- C.  $a \in [12, 21], b \in [-88, -82], c \in [23, 26]$ , and  $d \in [8, 15]$

$20x^3 - 87x^2 + 25x + 12$ , which corresponds to multiplying out  $(4x + 1)(x - 4)(5x - 3)$ .

- D.  $a \in [12, 21], b \in [95, 101], c \in [61, 75]$ , and  $d \in [8, 15]$

$20x^3 + 97x^2 + 71x + 12$ , which corresponds to multiplying out  $(4x + 1)(x + 4)(5x + 3)$ .

- E.  $a \in [12, 21], b \in [69, 77], c \in [-36, -28]$ , and  $d \in [-15, -8]$

$20x^3 + 73x^2 - 31x - 12$ , which corresponds to multiplying out  $(4x + 1)(x + 4)(5x - 3)$ .

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(4x - 1)(x - 4)(5x - 3)$

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29. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-3 + 2i \text{ and } 1$$

The solution is  $x^3 + 5x^2 + 7x - 13$ , which is option B.

- A.  $b \in [-8, -3], c \in [7, 8]$ , and  $d \in [11, 14]$

$x^3 - 5x^2 + 7x + 13$ , which corresponds to multiplying out  $(x - (-3 + 2i))(x - (-3 - 2i))(x + 1)$ .

- B.  $b \in [2, 8], c \in [7, 8]$ , and  $d \in [-16, -8]$

\*  $x^3 + 5x^2 + 7x - 13$ , which is the correct option.

- C.  $b \in [1, 2], c \in [0, 4]$ , and  $d \in [-5, -2]$

$x^3 + x^2 + 2x - 3$ , which corresponds to multiplying out  $(x + 3)(x - 1)$ .

- D.  $b \in [1, 2], c \in [-7, 1]$ , and  $d \in [0, 5]$

$x^3 + x^2 - 3x + 2$ , which corresponds to multiplying out  $(x - 2)(x - 1)$ .

- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (-3 + 2i))(x - (-3 - 2i))(x - (1))$ .

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30. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$4, \frac{-3}{5}, \text{ and } \frac{3}{4}$$

The solution is  $20x^3 - 83x^2 + 3x + 36$ , which is option C.

- A.  $a \in [19, 26], b \in [81, 88], c \in [3, 6],$  and  $d \in [-37, -33]$

$20x^3 + 83x^2 + 3x - 36$ , which corresponds to multiplying out  $(x + 4)(5x - 3)(4x + 3)$ .

- B.  $a \in [19, 26], b \in [76, 78], c \in [-22, -18],$  and  $d \in [-37, -33]$

$20x^3 + 77x^2 - 21x - 36$ , which corresponds to multiplying out  $(x + 4)(5x + 3)(4x - 3)$ .

- C.  $a \in [19, 26], b \in [-85, -80], c \in [3, 6],$  and  $d \in [29, 39]$

\*  $20x^3 - 83x^2 + 3x + 36$ , which is the correct option.

- D.  $a \in [19, 26], b \in [46, 63], c \in [-101, -94],$  and  $d \in [29, 39]$

$20x^3 + 53x^2 - 99x + 36$ , which corresponds to multiplying out  $(x + 4)(5x - 3)(4x - 3)$ .

- E.  $a \in [19, 26], b \in [-85, -80], c \in [3, 6],$  and  $d \in [-37, -33]$

$20x^3 - 83x^2 + 3x - 36$ , which corresponds to multiplying everything correctly except the constant term.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(x - 4)(5x + 3)(4x - 3)$

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