This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{16x^3 - 49x + 32}{x + 2}$$

The solution is $16x^2 - 32x + 15 + \dfrac{2}{x+2}$, which is option E.

A. $a \in [16, 18], b \in [31, 38], c \in [12, 17],$ and $r \in [59, 67]$.

You divided by the opposite of the factor.

B. $a \in [-34, -25], b \in [-69, -63], c \in [-182, -175],$ and $r \in [-324, -318]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [-34, -25], b \in [57, 67], c \in [-182, -175],$ and $r \in [385, 392]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [16, 18], b \in [-49, -47], c \in [91, 99],$ and $r \in [-253, -246]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [16, 18], b \in [-40, -28], c \in [12, 17],$ and $r \in [-1, 5]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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2. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 - 35x^2 + 66x - 40$$

The solution is $[1.33, 2, 2.5]$, which is option C.

A. $z_1 \in [-2.69, -2.1], \; z_2 \in [-3, -1.8],$ and $z_3 \in [-1.45, -1.14]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5.19, -4.42], \; z_2 \in [-3, -1.8],$ and $z_3 \in [-0.8, -0.42]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [1.1, 1.67], \; z_2 \in [1, 2.5],$ and $z_3 \in [2.32, 2.71]$

* This is the solution!

D. $z_1 \in [-2.18, -1.47]$, $z_2 \in [-1.1, -0.6]$, and $z_3 \in [-0.62, -0.23]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [0.05, 0.53]$, $z_2 \in [0.4, 1.5]$, and $z_3 \in [1.95, 2.11]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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3. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 - 45x^2 - 82x - 24$$

The solution is $[-0.8, -0.4, 3]$, which is option C.

A. $z_1 \in [-3.11, -2.79]$, $z_2 \in [0.24, 0.6]$, and $z_3 \in [0.38, 0.88]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-3.11, -2.79]$, $z_2 \in [1.07, 1.31]$, and $z_3 \in [2.14, 2.54]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-1.16, -0.39]$, $z_2 \in [-0.67, -0.28]$, and $z_3 \in [2.54, 3.27]$

* This is the solution!

D. $z_1 \in [-2.65, -2.14]$, $z_2 \in [-1.42, -1.09]$, and $z_3 \in [2.54, 3.27]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-3.11, -2.79]$, $z_2 \in [0.09, 0.19]$, and $z_3 \in [1.46, 2.4]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 35x^2 + 42}{x - 3}$$

The solution is $10x^2 - 5x - 15 + \frac{-3}{x-3}$, which is option C.

A. $a \in [28, 31], b \in [54, 58], c \in [160, 169]$, and $r \in [535, 539]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [28, 31], b \in [-126, -122], c \in [369, 376]$, and $r \in [-1084, -1081]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [5, 15], b \in [-6, -2], c \in [-20, -6]$, and $r \in [-5, 1]$.

* This is the solution!

D. $a \in [5, 15], b \in [-17, -7], c \in [-34, -25]$, and $r \in [-20, -12]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [5, 15], b \in [-65, -61], c \in [193, 197]$, and $r \in [-545, -541]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 - 34x^2 - 10x + 7}{x - 3}$$

The solution is $12x^2 + 2x - 4 + \frac{-5}{x-3}$, which is option C.

A. $a \in [31, 39]$, $b \in [71, 77]$, $c \in [211, 218]$, and $r \in [643, 650]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [10, 17]$, $b \in [-11, -8]$, $c \in [-30, -25]$, and $r \in [-58, -51]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [10, 17]$, $b \in [-2, 3]$, $c \in [-5, -2]$, and $r \in [-6, 0]$.

* This is the solution!

D. $a \in [10, 17]$, $b \in [-75, -64]$, $c \in [194, 203]$, and $r \in [-596, -584]$.

You divided by the opposite of the factor.

E. $a \in [31, 39]$, $b \in [-148, -138]$, $c \in [415, 418]$, and $r \in [-1243, -1237]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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6. Factor the polynomial below completely, knowing that $x + 3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 + 26x^3 - 37x^2 - 159x - 90$$

The solution is $[-3, -2, -0.75, 2.5]$, which is option C.

A. $z_1 \in [-1.23, -0.19]$, $z_2 \in [0.77, 1.43], z_3 \in [1.38, 2.29]$, and $z_4 \in [2.6, 4.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5.3, -4.32]$, $z_2 \in [0.15, 0.47], z_3 \in [1.38, 2.29]$, and $z_4 \in [2.6, 4.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-3.63, -2.76]$, $z_2 \in [-2.18, -1.85], z_3 \in [-0.83, -0.16]$, and $z_4 \in [0.6, 2.9]$

* This is the solution!

D. $z_1 \in [-3.63, -2.76]$, $z_2 \in [-2.18, -1.85], z_3 \in [-1.36, -0.79]$, and $z_4 \in [-0.4, 1.4]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-2.68, -2.27]$, $z_2 \in [0.61, 0.85], z_3 \in [1.38, 2.29]$, and $z_4 \in [2.6, 4.3]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. Factor the polynomial below completely, knowing that $x + 2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 - 29x^3 - 33x^2 + 116x - 60$$

The solution is $[-2, 0.75, 1.667, 2]$, which is option B.

A. $z_1 \in [-2.5, -1.9]$, $z_2 \in [-1.75, -1.65], z_3 \in [-0.91, -0.74]$, and $z_4 \in [1, 5]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-2.5, -1.9]$, $z_2 \in [0.67, 0.79], z_3 \in [1.58, 1.81]$, and $z_4 \in [1, 5]$

* This is the solution!

C. $z_1 \in [-2.5, -1.9]$, $z_2 \in [0.58, 0.66], z_3 \in [1.23, 1.34]$, and $z_4 \in [1, 5]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-2.5, -1.9]$, $z_2 \in [-1.42, -1.31], z_3 \in [-0.72, -0.45]$, and $z_4 \in [1, 5]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-3.2, -2.7]$, $z_2 \in [-2.01, -1.99], z_3 \in [-0.58, -0.21]$, and $z_4 \in [1, 5]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 45x^2 - 15x + 45}{x - 2}$$

The solution is $20x^2 - 5x - 25 + \dfrac{-5}{x - 2}$, which is option A.

A. $a \in [18, 23]$, $b \in [-8, -2]$, $c \in [-30, -22]$, and $r \in [-5, -2]$.

* This is the solution!

B. $a \in [40, 42]$, $b \in [-130, -123]$, $c \in [233, 239]$, and $r \in [-425, -423]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [18, 23]$, $b \in [-87, -83]$, $c \in [152, 156]$, and $r \in [-269, -264]$.

You divided by the opposite of the factor.

D. $a \in [18, 23], \; b \in [-27, -22], \; c \in [-40, -39], \;$ and $r \in [5, 10]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [40, 42], \; b \in [31, 36], \; c \in [52, 57], \;$ and $r \in [155, 161]$.

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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9. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 6x^3 + 2x^2 + 2x + 2$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$, which is option C.

A. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

B. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$

* This is the solution **since we asked for the possible Rational roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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10. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 3x^4 + 2x^3 + 6x^2 + 7x + 7$$

The solution is $\pm 1, \pm 7$, which is option C.

A. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 3}$

This would have been the solution **if asked for the possible Rational roots**!

C. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

D. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

   Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.