This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{4}, \frac{7}{4}, \text{ and } \frac{1}{5}$$

The solution is $80x^3 - 96x^2 - 89x + 21$, which is option C.

A. $a \in [71, 88], b \in [-99, -95], c \in [-89, -81], \text{ and } d \in [-25, -15]$

$80x^3 - 96x^2 - 89x - 21$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [71, 88], b \in [-222, -215], c \in [143, 147], \text{ and } d \in [-25, -15]$

$80x^3 - 216x^2 + 145x - 21$, which corresponds to multiplying out $(4x - 3)(4x - 7)(5x - 1)$.

C. $a \in [71, 88], b \in [-99, -95], c \in [-89, -81], \text{ and } d \in [16, 31]$

* $80x^3 - 96x^2 - 89x + 21$, which is the correct option.

D. $a \in [71, 88], b \in [57, 67], c \in [-121, -119], \text{ and } d \in [16, 31]$

$80x^3 + 64x^2 - 121x + 21$, which corresponds to multiplying out $(4x - 3)(4x + 7)(5x - 1)$.

E. $a \in [71, 88], b \in [94, 102], c \in [-89, -81], \text{ and } d \in [-25, -15]$

$80x^3 + 96x^2 - 89x - 21$, which corresponds to multiplying out $(4x - 3)(4x + 7)(5x + 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 3)(4x - 7)(5x - 1)$

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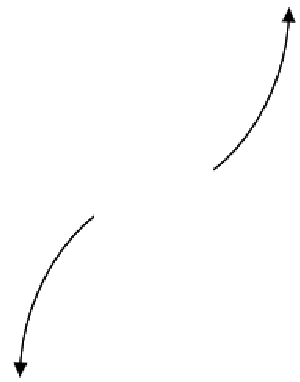
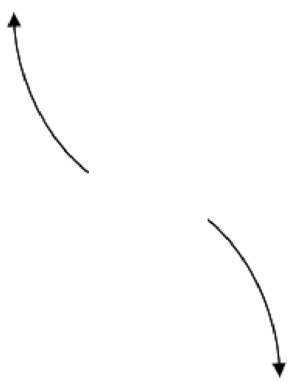
2. Describe the end behavior of the polynomial below.

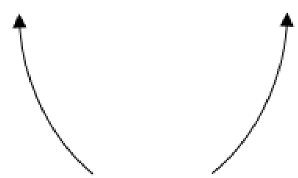
$$f(x) = 7(x - 7)^4(x + 7)^7(x - 3)^2(x + 3)^2$$

The solution is the graph below, which is option D.

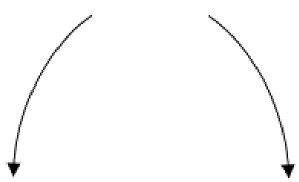




A.



C.



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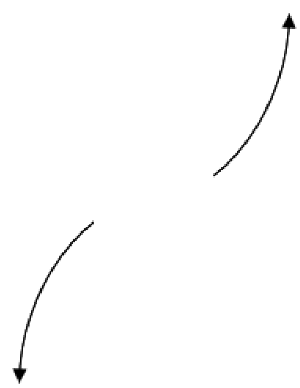

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 4i \text{ and } -4$$

The solution is $x^3 - 6x^2 + x + 164$, which is option C.

A. $b \in [3, 20], c \in [-0.68, 1.9],$ and $d \in [-168, -161]$

$x^3 + 6x^2 + x - 164$, which corresponds to multiplying out $(x - (5 - 4i))(x - (5 + 4i))(x - 4)$.

B. $b \in [-1, 4], c \in [5.04, 8.21],$ and $d \in [16, 23]$

$x^3 + x^2 + 8x + 16$, which corresponds to multiplying out $(x + 4)(x + 4)$.

C. $b \in [-6, -4], c \in [-0.68, 1.9],$ and $d \in [164, 166]$

* $x^3 - 6x^2 + x + 164$, which is the correct option.

D. $b \in [-1, 4], c \in [-1.45, 0.42],$ and $d \in [-20, -17]$

$x^3 + x^2 - x - 20$, which corresponds to multiplying out $(x - 5)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 - 4i))(x - (5 + 4i))(x - (-4))$.

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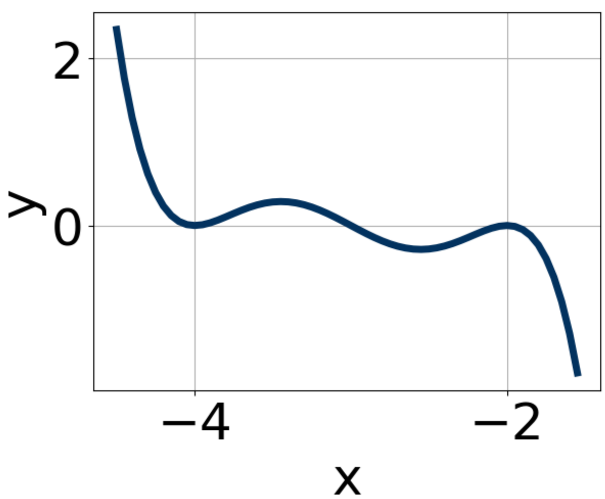
4. Which of the following equations *could* be of the graph presented below?



The solution is $-6(x + 2)^{10}(x + 4)^6(x + 3)^5$, which is option D.

A. $-13(x + 2)^{10}(x + 4)^7(x + 3)^6$

The factor $(x + 4)$ should have an even power and the factor $(x + 3)$ should have an odd power.

B. $18(x + 2)^{10}(x + 4)^6(x + 3)^{10}$

The factor $(x + 3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $16(x + 2)^4(x + 4)^4(x + 3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-6(x + 2)^{10}(x + 4)^6(x + 3)^5$

* This is the correct option.

E. $-15(x + 2)^6(x + 4)^7(x + 3)^{11}$

The factor $(x + 4)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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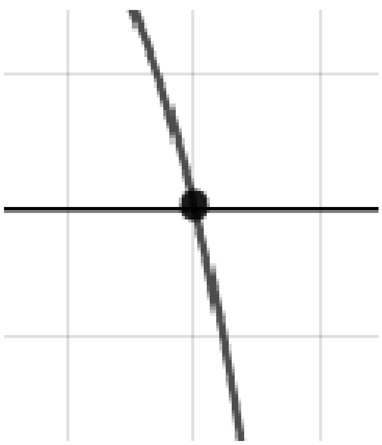
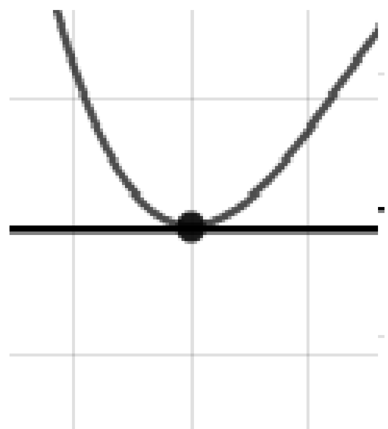
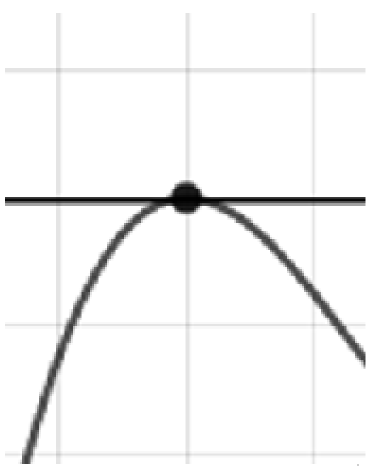
5. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

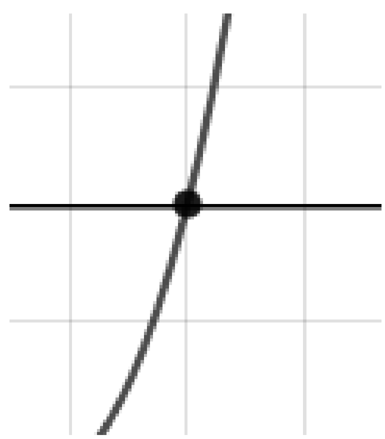
$$f(x) = 6(x+7)^3(x-7)^2(x-8)^5(x+8)^4$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

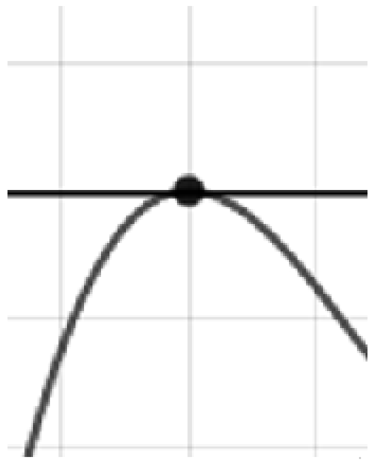
E. None of the above.

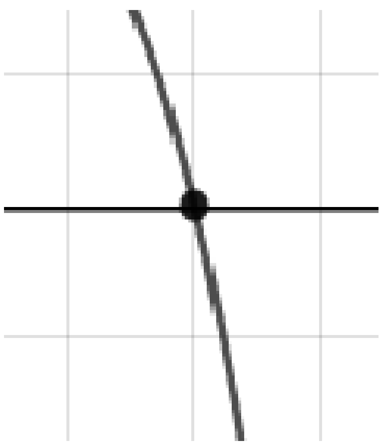
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

6. Describe the zero behavior of the zero $x = -9$ of the polynomial below.

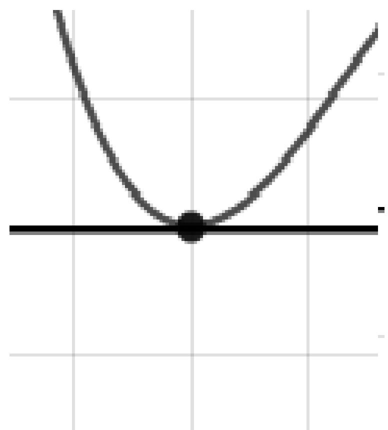
$$f(x) = -2(x+9)^6(x-9)^9(x-8)^2(x+8)^5$$

The solution is the graph below, which is option B.

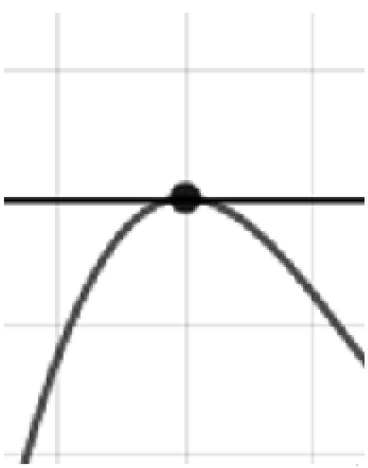




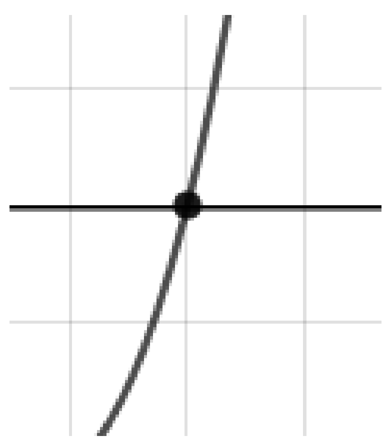
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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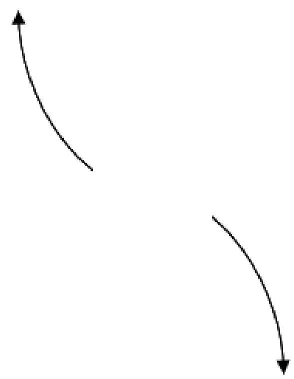
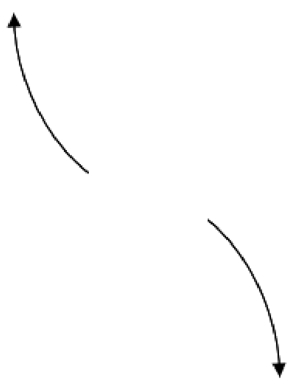
7. Describe the end behavior of the polynomial below.

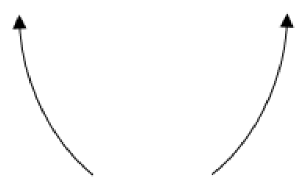
$$f(x) = -4(x-2)^3(x+2)^4(x-9)^5(x+9)^5$$

The solution is the graph below, which is option A.

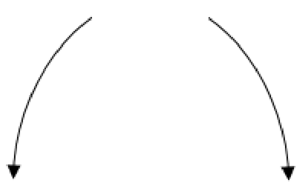




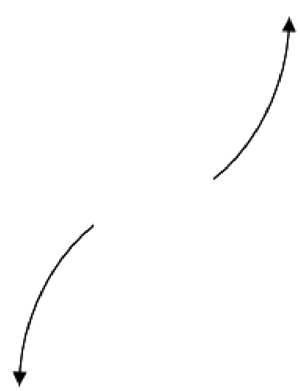
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 + 2i \text{ and } 1$$

The solution is $x^3 + 5x^2 + 7x - 13$, which is option B.

A. $b \in [-6.2, -2.7], c \in [7, 14]$, and $d \in [10, 14]$

$x^3 - 5x^2 + 7x + 13$, which corresponds to multiplying out $(x - (-3 + 2i))(x - (-3 - 2i))(x + 1)$.

B. $b \in [1.3, 5.6], c \in [7, 14]$, and $d \in [-16, -7]$

* $x^3 + 5x^2 + 7x - 13$, which is the correct option.

C. $b \in [-0.3, 3.3], c \in [-2, 3]$, and $d \in [-5, 0]$

$x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

D. $b \in [-0.3, 3.3], c \in [-6, -1]$, and $d \in [0, 3]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x - 2)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 2i))(x - (-3 - 2i))(x - (1))$.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-5}{4}, \frac{-3}{4}, \text{ and } 5$$

The solution is $16x^3 - 48x^2 - 145x - 75$, which is option C.

A. $a \in [15, 19], b \in [42, 52], c \in [-151, -140]$, and $d \in [68, 80]$

$16x^3 + 48x^2 - 145x + 75$, which corresponds to multiplying out $(4x - 5)(4x - 3)(x + 5)$.

B. $a \in [15, 19], b \in [-48, -41], c \in [-151, -140]$, and $d \in [68, 80]$

$16x^3 - 48x^2 - 145x + 75$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [15, 19], b \in [-48, -41], c \in [-151, -140]$, and $d \in [-76, -69]$

* $16x^3 - 48x^2 - 145x - 75$, which is the correct option.

D. $a \in [15, 19], b \in [-94, -79], c \in [23, 34]$, and $d \in [68, 80]$

$16x^3 - 88x^2 + 25x + 75$, which corresponds to multiplying out $(4x - 5)(4x + 3)(x - 5)$.

E. $a \in [15, 19], b \in [-119, -111], c \in [173, 179]$, and $d \in [-76, -69]$

$16x^3 - 112x^2 + 175x - 75$, which corresponds to multiplying out $(4x - 5)(4x - 3)(x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 5)(4x + 3)(x - 5)$

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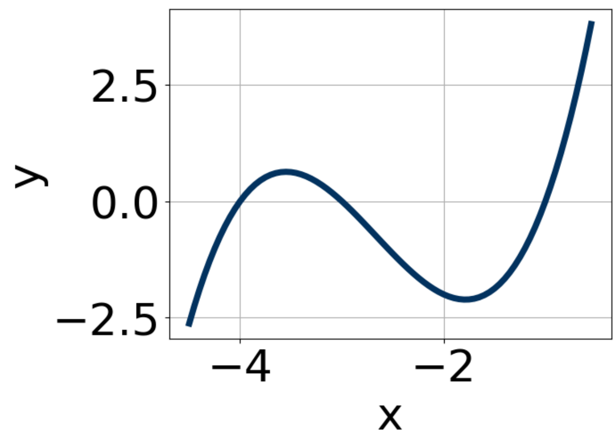
10. Which of the following equations *could* be of the graph presented below?



The solution is $10(x+1)^{11}(x+3)^{7}(x+4)^{11}$, which is option C.

A. $5(x+1)^{8}(x+3)^{4}(x+4)^{5}$

   The factors -1 and -3 have have been odd power.

B. $-17(x+1)^{4}(x+3)^{5}(x+4)^{5}$

   The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $10(x+1)^{11}(x+3)^{7}(x+4)^{11}$

   * This is the correct option.

D. $5(x+1)^{4}(x+3)^{11}(x+4)^{11}$

   The factor -1 should have been an odd power.

E. $-9(x+1)^{7}(x+3)^{11}(x+4)^{9}$

   This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).