This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-5}{3}, \frac{3}{5}, \text{ and } \frac{-4}{3}$$

The solution is $45x^3 + 108x^2 + 19x - 60$, which is option A.

A. $a \in [42, 46], b \in [107, 114], c \in [12, 24], \text{ and } d \in [-67, -56]$

* $45x^3 + 108x^2 + 19x - 60$, which is the correct option.

B. $a \in [42, 46], b \in [-42, -40], c \in [-92, -85], \text{ and } d \in [59, 61]$

$45x^3 - 42x^2 - 91x + 60$, which corresponds to multiplying out $(3x - 5)(5x - 3)(3x + 4)$.

C. $a \in [42, 46], b \in [9, 14], c \in [-111, -107], \text{ and } d \in [-67, -56]$

$45x^3 + 12x^2 - 109x - 60$, which corresponds to multiplying out $(3x - 5)(5x + 3)(3x + 4)$.

D. $a \in [42, 46], b \in [-114, -106], c \in [12, 24], \text{ and } d \in [59, 61]$

$45x^3 - 108x^2 + 19x + 60$, which corresponds to multiplying out $(3x - 5)(5x + 3)(3x - 4)$.

E. $a \in [42, 46], b \in [107, 114], c \in [12, 24], \text{ and } d \in [59, 61]$

$45x^3 + 108x^2 + 19x + 60$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 5)(5x - 3)(3x + 4)$

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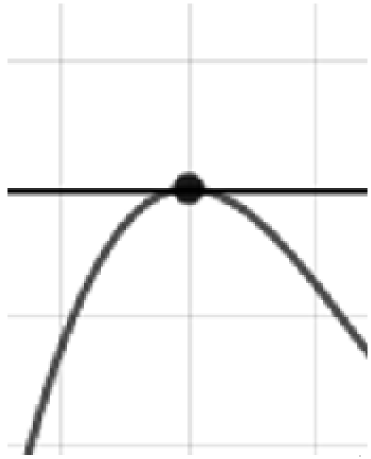
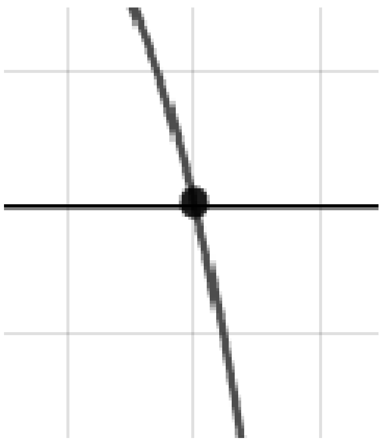
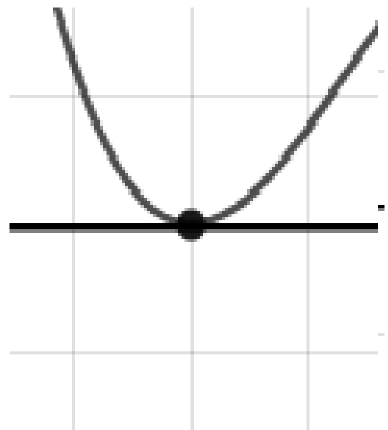
2. Describe the zero behavior of the zero $x = 6$ of the polynomial below.

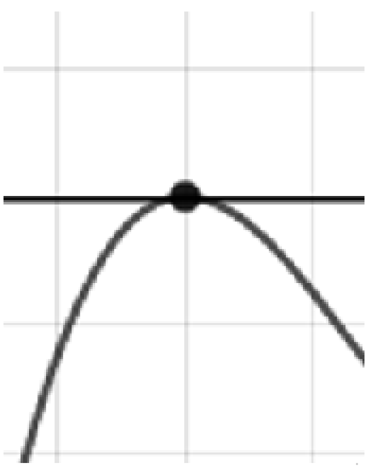
$$f(x) = -3(x + 5)^{10}(x - 5)^{7}(x - 6)^{12}(x + 6)^{9}$$

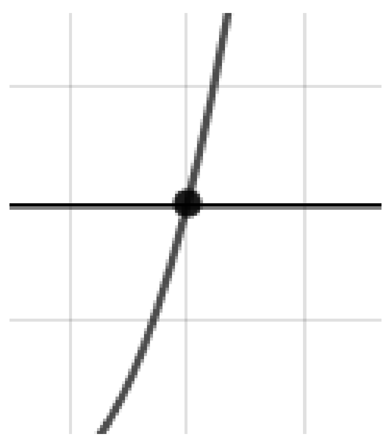
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 5i \text{ and } -2$$

The solution is $x^3 - 6x^2 + 25x + 82$, which is option D.

A. $b \in [1, 2], c \in [6, 8],$ and $d \in [9, 16]$

$x^3 + x^2 + 7x + 10$, which corresponds to multiplying out $(x + 5)(x + 2)$.

B. $b \in [6, 8], c \in [22, 31],$ and $d \in [-87, -77]$

$x^3 + 6x^2 + 25x - 82$, which corresponds to multiplying out $(x - (4 - 5i))(x - (4 + 5i))(x - 2)$.

C. $b \in [1, 2], c \in [-8, 5],$ and $d \in [-12, -6]$

$x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x - 4)(x + 2)$.

D. $b \in [-6, -2], c \in [22, 31],$ and $d \in [75, 87]$

* $x^3 - 6x^2 + 25x + 82$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 5i))(x - (4 + 5i))(x - (-2))$.

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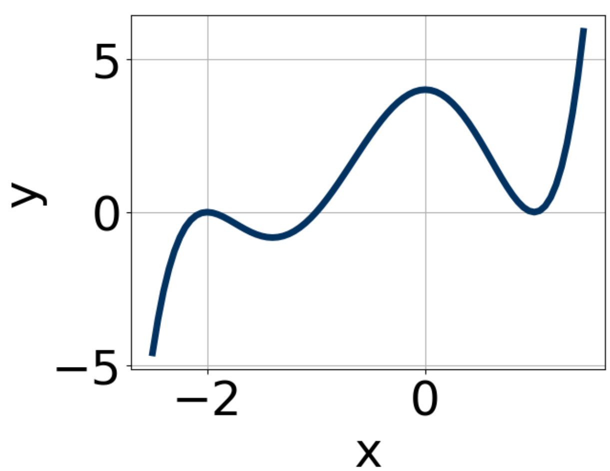
4. Which of the following equations *could* be of the graph presented below?



The solution is $10(x + 2)^6(x - 1)^6(x + 1)^5$, which is option B.

A. $-8(x + 2)^{10}(x - 1)^6(x + 1)^8$

The factor $(x + 1)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $10(x + 2)^6(x - 1)^6(x + 1)^5$

* This is the correct option.

C. $-15(x + 2)^6(x - 1)^4(x + 1)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $5(x + 2)^{10}(x - 1)^{11}(x + 1)^7$

The factor $(x - 1)$ should have an even power.

E. $15(x+2)^{10}(x-1)^9(x+1)^6$

The factor $(x-1)$ should have an even power and the factor $(x+1)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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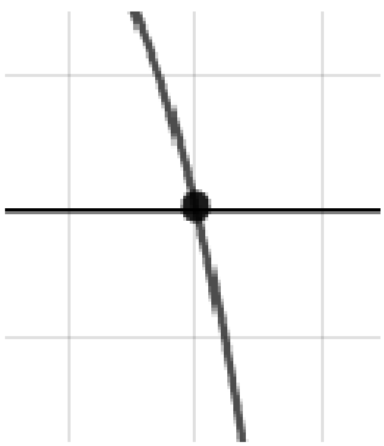
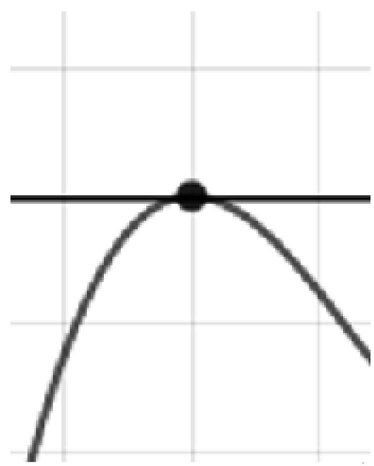
5. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

$$f(x) = -9(x-6)^9(x+6)^{14}(x+3)^4(x-3)^6$$

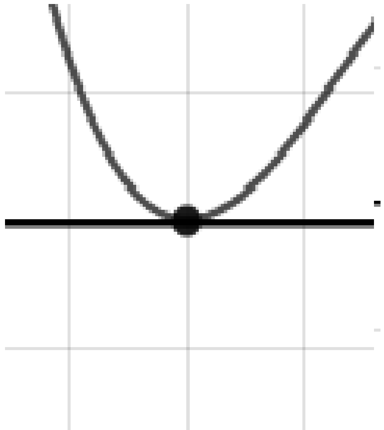
The solution is the graph below, which is option C.

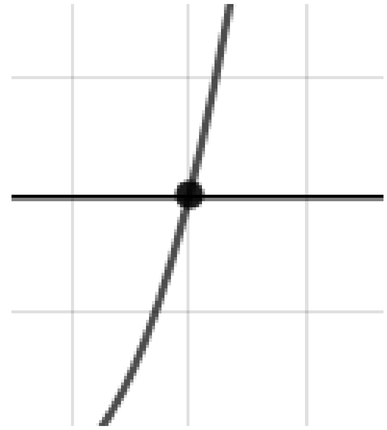


A.



B.

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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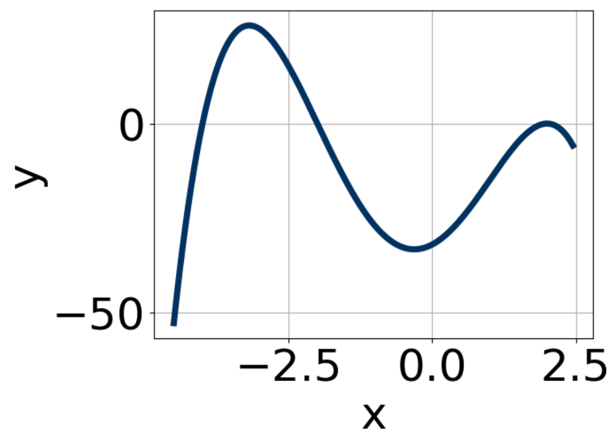
6. Which of the following equations *could* be of the graph presented below?



The solution is $-11(x-2)^6(x+2)^{11}(x+4)^7$, which is option E.

A. $13(x-2)^8(x+2)^{11}(x+4)^{10}$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $4(x-2)^6(x+2)^5(x+4)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-12(x-2)^{10}(x+2)^8(x+4)^7$

The factor $(x+2)$ should have an odd power.

D. $-14(x-2)^7(x+2)^4(x+4)^9$

The factor 2 should have an even power and the factor -2 should have an odd power.

E. $-11(x-2)^6(x+2)^{11}(x+4)^7$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } -3$$

The solution is $x^3 + 13x^2 + 71x + 123$, which is option D.

A. $b \in [-7, 6], c \in [1, 11]$, and $d \in [8, 23]$

$x^3 + x^2 + 8x + 15$, which corresponds to multiplying out $(x + 5)(x + 3)$.

B. $b \in [-7, 6], c \in [-6, 2]$, and $d \in [-15, -11]$

$x^3 + x^2 - x - 12$, which corresponds to multiplying out $(x - 4)(x + 3)$.

C. $b \in [-22, -12], c \in [69, 77]$, and $d \in [-125, -114]$

$x^3 - 13x^2 + 71x - 123$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x - 3)$.

D. $b \in [10, 21], c \in [69, 77]$, and $d \in [115, 125]$

* $x^3 + 13x^2 + 71x + 123$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (-3))$.

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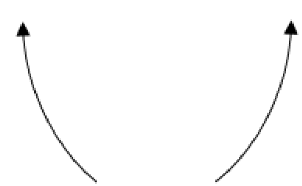
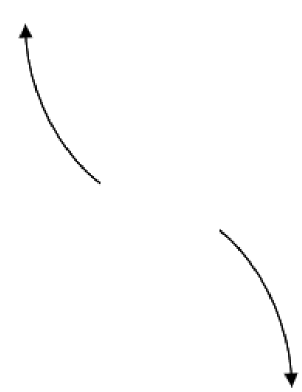
8. Describe the end behavior of the polynomial below.

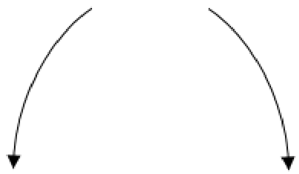
$$f(x) = 2(x + 9)^3(x - 9)^8(x + 5)^3(x - 5)^4$$

The solution is the graph below, which is option C.

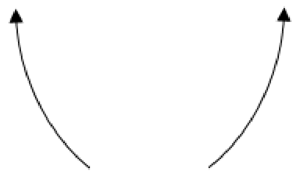




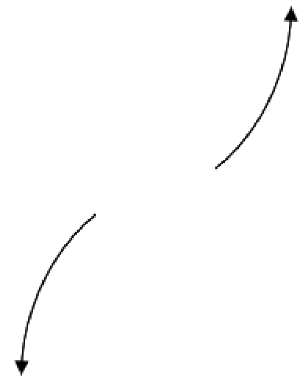
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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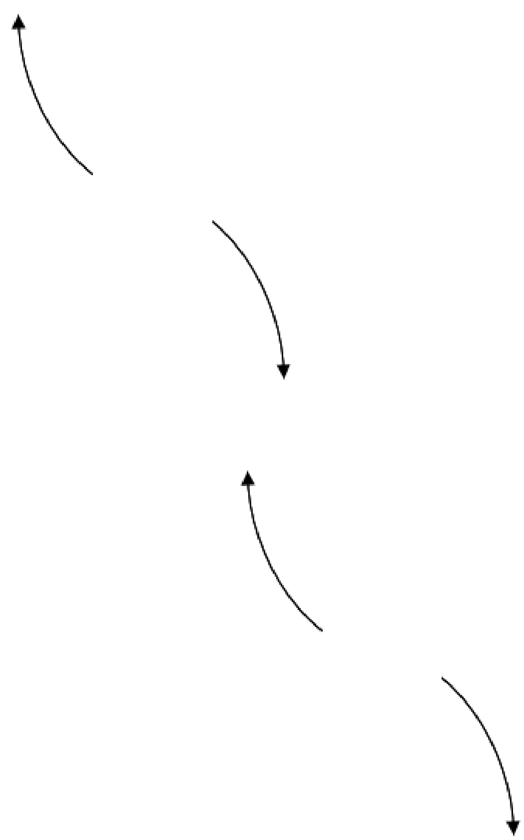
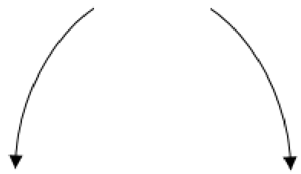
9. Describe the end behavior of the polynomial below.

$$f(x) = -7(x-4)^5(x+4)^6(x-5)^4(x+5)^6$$

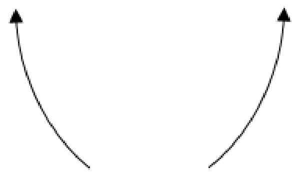
The solution is the graph below, which is option A.



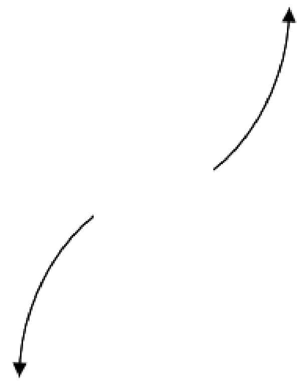
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{3}, 1, \text{ and } \frac{-2}{5}$$

The solution is $15x^3 - 4x^2 - 9x - 2$, which is option C.

A. $a \in [10, 17], b \in [3, 11], c \in [-9.39, -8.23], \text{ and } d \in [-0.3, 4.6]$

$15x^3 + 4x^2 - 9x + 2$, which corresponds to multiplying out $(3x - 1)(x + 1)(5x - 2)$.

B. $a \in [10, 17], b \in [10, 23], c \in [-1.88, -0.96], \text{ and } d \in [-2.8, -0.2]$

$15x^3 + 16x^2 - x - 2$, which corresponds to multiplying out $(3x - 1)(x + 1)(5x + 2)$.

C. $a \in [10, 17], b \in [-7, -3], c \in [-9.39, -8.23], \text{ and } d \in [-2.8, -0.2]$

* $15x^3 - 4x^2 - 9x - 2$, which is the correct option.

D. $a \in [10, 17], b \in [-7, -3], c \in [-9.39, -8.23], \text{ and } d \in [-0.3, 4.6]$

$15x^3 - 4x^2 - 9x + 2$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [10, 17], b \in [-18, -11], c \in [-4.09, -2.6], \text{ and } d \in [-0.3, 4.6]$

$15x^3 - 14x^2 - 3x + 2$, which corresponds to multiplying out $(3x - 1)(x - 1)(5x + 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 1)(x - 1)(5x + 2)$
