This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 41x^2 + 27x + 18$$

The solution is $[-0.4, 1.5, 3]$, which is option E.

A. $z_1 \in [-3.1, -2.9], \ z_2 \in [-1.7, -0.7], \text{ and } z_3 \in [0.23, 0.73]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-2.9, -1.5], \ z_2 \in [0.2, 0.8], \text{ and } z_3 \in [2.77, 3.08]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-3.1, -2.9], \ z_2 \in [-1.1, 0], \text{ and } z_3 \in [2.35, 2.69]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-3.1, -2.9], \ z_2 \in [-3.2, -2.7], \text{ and } z_3 \in [0, 0.29]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-1.9, 0], \ z_2 \in [0.8, 2], \text{ and } z_3 \in [2.77, 3.08]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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2. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 + 11x^2 - 106x + 44}{x + 4}$$

The solution is $10x^2 - 29x + 10 + \dfrac{4}{x+4}$, which is option E.

A. $a \in [9, 11], \ b \in [-46, -31], \ c \in [84, 95], \text{ and } r \in [-401, -396].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [-44, -39], \ b \in [-152, -148], \ c \in [-703, -700], \text{ and } r \in [-2765, -2760].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [9, 11], \ b \in [46, 56], \ c \in [98, 100], \text{ and } r \in [425, 441].$

You divided by the opposite of the factor.

D. $a \in [-44, -39], \ b \in [168, 175], \ c \in [-796, -787], \text{ and } r \in [3196, 3209]$.

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [9, 11], \ b \in [-29, -25], \ c \in [8, 15], \text{ and } r \in [3, 7]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 - 28x - 19}{x - 2}$$

The solution is $9x^2 + 18x + 8 + \frac{-3}{x-2}$, which is option E.

A. $a \in [14, 26], b \in [-36, -34], c \in [40, 45], \text{ and } r \in [-108, -105]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

B. $a \in [6, 12], b \in [-22, -17], c \in [0, 12], \text{ and } r \in [-35, -34]$.

You divided by the opposite of the factor.

C. $a \in [6, 12], b \in [6, 15], c \in [-23, -18], \text{ and } r \in [-42, -37]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [14, 26], b \in [36, 38], c \in [40, 45], \text{ and } r \in [65, 74]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [6, 12], b \in [16, 20], c \in [0, 12], \text{ and } r \in [-8, 1]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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4. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^3 + 35x^2 - 9x - 18$$

The solution is $[-3, -0.67, 0.75]$, which is option D.

A. $z_1 \in [-1.5, -1.26], \ z_2 \in [0.92, 1.85], \text{ and } z_3 \in [2.5, 3.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-0.8, -0.64], \ z_2 \in [0.48, 0.77], \text{ and } z_3 \in [2.5, 3.1]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-3.17, -2.34], \ z_2 \in [-1.51, -1.46], \text{ and } z_3 \in [1.1, 2.4]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.17, -2.34], \ z_2 \in [-0.74, -0.58], \text{ and } z_3 \in [0, 1.1]$

* This is the solution!

E. $z_1 \in [-0.34, -0.12], \;\; z_2 \in [1.58, 2.09], \;$ and $\; z_3 \in [2.5, 3.1]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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5. Factor the polynomial below completely, knowing that $x-4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 - 14x^3 - 248x^2 + 224x + 128$$

The solution is $[-4, -0.4, 1.333, 4]$, which is option C.

A. $z_1 \in [-5, -2], \;\; z_2 \in [-4.5, -3.89], z_3 \in [-0.08, 0.26], \;$ and $\; z_4 \in [2, 8]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-5, -2], \;\; z_2 \in [-3.04, -2.45], z_3 \in [0.73, 0.85], \;$ and $\; z_4 \in [2, 8]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-5, -2], \;\; z_2 \in [-0.52, -0.21], z_3 \in [1.24, 1.46], \;$ and $\; z_4 \in [2, 8]$

* This is the solution!

D. $z_1 \in [-5, -2], \;\; z_2 \in [-1.72, -1.22], z_3 \in [0.18, 0.71], \;$ and $\; z_4 \in [2, 8]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-5, -2], \;\; z_2 \in [-0.94, -0.69], z_3 \in [2.44, 2.56], \;$ and $\; z_4 \in [2, 8]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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6. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 + 27x^2 - 25x - 77}{x+3}$$

The solution is $9x^2 - 25 + \dfrac{-2}{x+3}$, which is option A.

A. $a \in [9, 10], \;\; b \in [-2, 2], \;\; c \in [-27, -24], \;$ and $\; r \in [-9, 0]$.

* This is the solution!

B. $a \in [-32, -24], \;\; b \in [107, 111], \;\; c \in [-350, -348], \;$ and $\; r \in [968, 976]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [9, 10], \;\; b \in [53, 60], \;\; c \in [133, 143], \;$ and $\; r \in [334, 340]$.

You divided by the opposite of the factor.

D. $a \in [9, 10], \;\; b \in [-10, -7], \;\; c \in [11, 16], \;$ and $\; r \in [-121, -118]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [-32, -24], \; b \in [-55, -51], \; c \in [-188, -186], \text{ and } r \in [-642, -633]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 + 30x^2 - 44}{x + 2}$$

The solution is $10x^2 + 10x - 20 + \frac{-4}{x+2}$, which is option D.

A. $a \in [-21, -16], b \in [69, 73], c \in [-140, -137], \text{ and } r \in [232, 243]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [-21, -16], b \in [-12, -7], c \in [-21, -15], \text{ and } r \in [-88, -81]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [6, 11], b \in [46, 51], c \in [97, 101], \text{ and } r \in [153, 159]$.

You divided by the opposite of the factor.

D. $a \in [6, 11], b \in [9, 17], c \in [-21, -15], \text{ and } r \in [-4, -3]$.

* This is the solution!

E. $a \in [6, 11], b \in [-5, 6], c \in [-2, 3], \text{ and } r \in [-50, -42]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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8. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 3x^3 + 2x^2 + 4x + 7$$

The solution is All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 3}$, which is option C.

A. All combinations of: $\frac{\pm 1, \pm 3}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 3}$

* This is the solution **since we asked for the possible Rational roots**!

D. $\pm 1, \pm 7$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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9. Factor the polynomial below completely, knowing that $x+3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 + 51x^3 - 28x^2 - 333x - 180$$

The solution is $[-4, -3, -0.6, 2.5]$, which is option B.

A. $z_1 \in [-0.74, -0.5], \;\; z_2 \in [2.94, 3.06], z_3 \in [1.8, 3.3], \;$ and $\; z_4 \in [4, 5]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-4.18, -3.74], \;\; z_2 \in [-3.13, -2.89], z_3 \in [-0.8, -0.4], \;$ and $\; z_4 \in [2.5, 3.5]$

* This is the solution!

C. $z_1 \in [-0.47, -0.25], \;\; z_2 \in [1.43, 2.14], z_3 \in [1.8, 3.3], \;$ and $\; z_4 \in [4, 5]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-2.55, -2.47], \;\; z_2 \in [-0.59, 1.11], z_3 \in [1.8, 3.3], \;$ and $\; z_4 \in [4, 5]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-4.18, -3.74], \;\; z_2 \in [-3.13, -2.89], z_3 \in [-2.8, -1.2], \;$ and $\; z_4 \in [0.4, 1.4]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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10. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^3 + 4x^2 + 6x + 6$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$, which is option D.

A. $\pm 1, \pm 2, \pm 3, \pm 6$

This would have been the solution **if asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.