This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{3}, 1, \text{ and } \frac{-7}{2}$$

The solution is $6x^3 + x^2 - 56x + 49$, which is option C.

A. $a \in [0, 14], b \in [28, 31.1], c \in [12, 15],$ and $d \in [-57, -44]$

$6x^3 + 29x^2 + 14x - 49$, which corresponds to multiplying out $(3x + 7)(x - 1)(2x + 7)$.

B. $a \in [0, 14], b \in [0.9, 2], c \in [-60, -55],$ and $d \in [-57, -44]$

$6x^3 + x^2 - 56x - 49$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [0, 14], b \in [0.9, 2], c \in [-60, -55],$ and $d \in [48, 54]$

* $6x^3 + x^2 - 56x + 49$, which is the correct option.

D. $a \in [0, 14], b \in [40.6, 41.8], c \in [82, 89],$ and $d \in [48, 54]$

$6x^3 + 41x^2 + 84x + 49$, which corresponds to multiplying out $(3x + 7)(x + 1)(2x + 7)$.

E. $a \in [0, 14], b \in [-4.2, 0.2], c \in [-60, -55],$ and $d \in [-57, -44]$

$6x^3 - 1x^2 - 56x - 49$, which corresponds to multiplying out $(3x + 7)(x + 1)(2x - 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 7)(x - 1)(2x + 7)$

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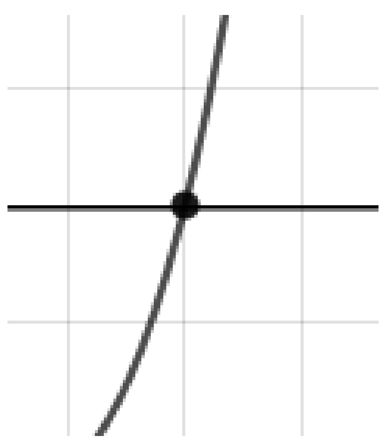
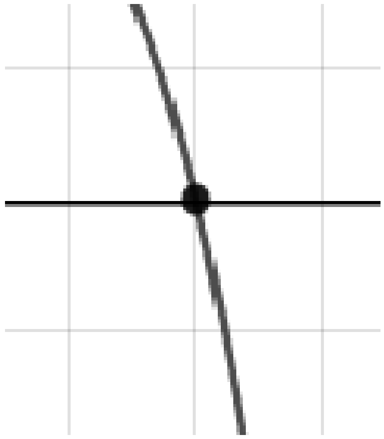
2. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

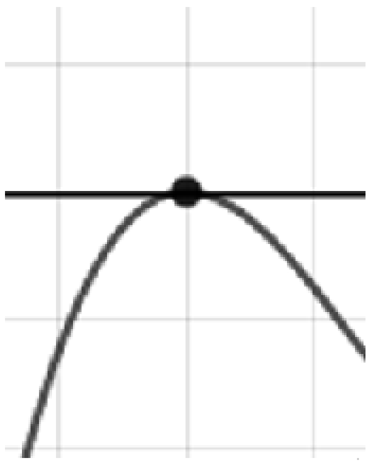
$$f(x) = -4(x + 4)^5(x - 4)^8(x - 9)^3(x + 9)^4$$

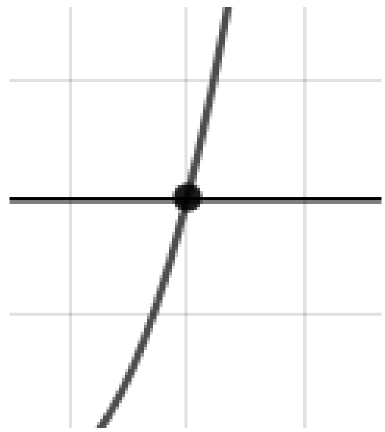
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 2i \text{ and } 1$$

The solution is $x^3 - 11x^2 + 39x - 29$, which is option A.

A. $b \in [-14, -9], c \in [37, 44], \text{ and } d \in [-34, -23]$

  * $x^3 - 11x^2 + 39x - 29$, which is the correct option.

B. $b \in [2, 13], c \in [37, 44], \text{ and } d \in [27, 33]$

  $x^3 + 11x^2 + 39x + 29$, which corresponds to multiplying out $(x - (5 - 2i))(x - (5 + 2i))(x + 1)$.

C. $b \in [-2, 6], c \in [-2, 5], \text{ and } d \in [-6, 1]$

  $x^3 + x^2 + x - 2$, which corresponds to multiplying out $(x + 2)(x - 1)$.

D. $b \in [-2, 6], c \in [-6, -5], \text{ and } d \in [0, 8]$

  $x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x - 5)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(5-2i))(x-(5+2i))(x-(1))$.

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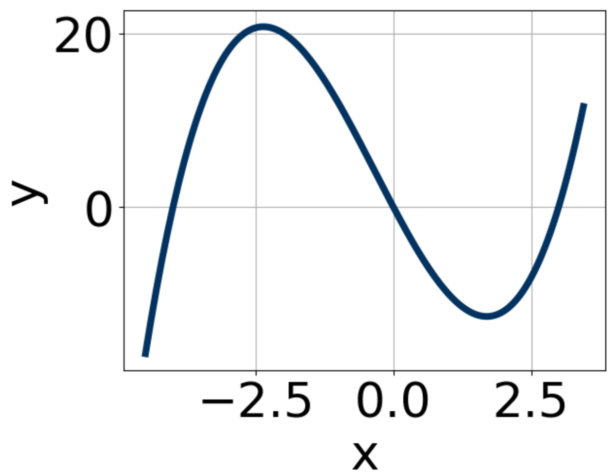
4. Which of the following equations *could* be of the graph presented below?



The solution is $7x^9(x+4)^{11}(x-3)^9$, which is option B.

A. $-11x^6(x+4)^{11}(x-3)^5$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

B. $7x^9(x+4)^{11}(x-3)^9$

* This is the correct option.

C. $-6x^7(x+4)^5(x-3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $6x^8(x+4)^4(x-3)^{11}$

The factors 0 and -4 have have been odd power.

E. $8x^8(x+4)^5(x-3)^7$

The factor 0 should have been an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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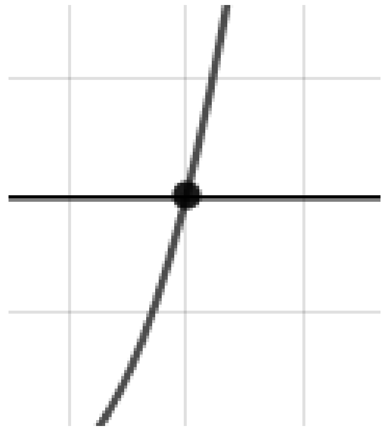
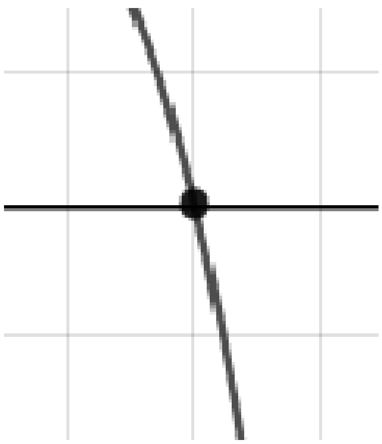
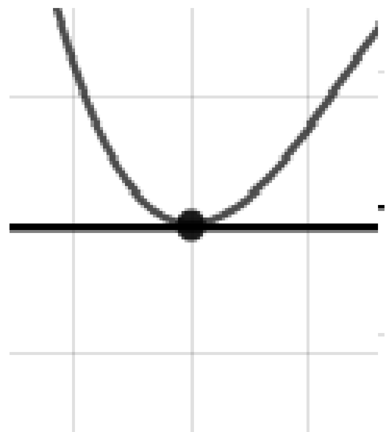
5. Describe the zero behavior of the zero $x=-8$ of the polynomial below.

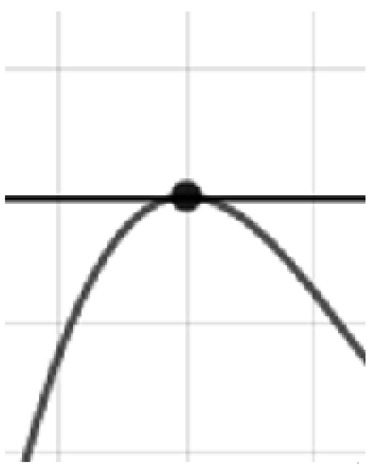
$$f(x)=4(x-7)^5(x+7)^3(x+8)^9(x-8)^8$$

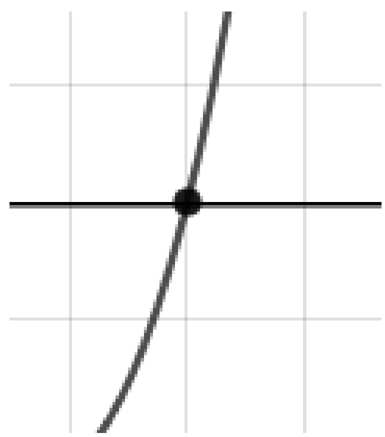
The solution is the graph below, which is option D.



A. 

C. 

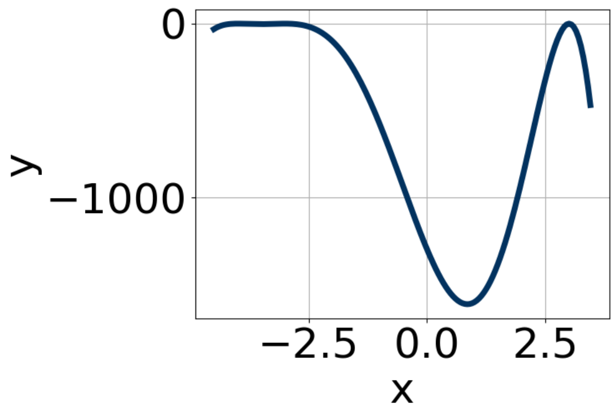
B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

6. Which of the following equations *could* be of the graph presented below?



The solution is $-4(x+4)^4(x-3)^{10}(x+3)^4$, which is option B.

A. $16(x+4)^{10}(x-3)^{10}(x+3)^6$

   This corresponds to the leading coefficient being the opposite value than it should be.

B. $-4(x+4)^4(x-3)^{10}(x+3)^4$

   * This is the correct option.

C. $14(x+4)^{10}(x-3)^4(x+3)^{11}$

   The factor $(x+3)$ should have an even power and the leading coefficient should be the opposite sign.

D. $-7(x+4)^4(x-3)^{11}(x+3)^7$

   The factors $(x-3)$ and $(x+3)$ should both have even powers.

E. $-10(x+4)^{10}(x-3)^6(x+3)^{11}$

   The factor $(x+3)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 2i \text{ and } 4$$

The solution is $x^3 - 14x^2 + 69x - 116$, which is option A.

A. $b \in [-17, -13], c \in [69, 79],$ and $d \in [-116, -115]$

   * $x^3 - 14x^2 + 69x - 116$, which is the correct option.

B. $b \in [-7, 5], c \in [-5, 6],$ and $d \in [-9, -2]$

   $x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x+2)(x-4)$.

C. $b \in [-7, 5], c \in [-13, -6],$ and $d \in [10, 27]$

   $x^3 + x^2 - 9x + 20$, which corresponds to multiplying out $(x-5)(x-4)$.

D. $b \in [14, 16], c \in [69, 79],$ and $d \in [114, 119]$

   $x^3 + 14x^2 + 69x + 116$, which corresponds to multiplying out $(x-(5-2i))(x-(5+2i))(x+4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(5-2i))(x-(5+2i))(x-(4))$.

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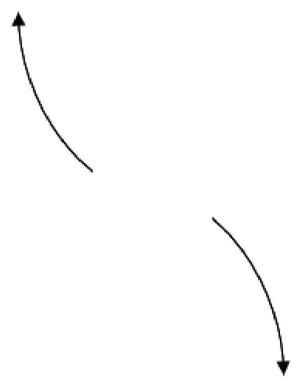
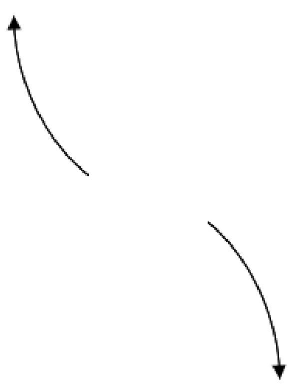
8. Describe the end behavior of the polynomial below.

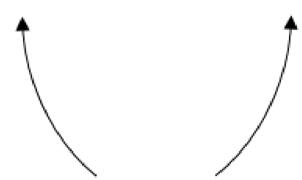
$$f(x) = -9(x+2)^4(x-2)^5(x-6)^5(x+6)^7$$

The solution is the graph below, which is option A.

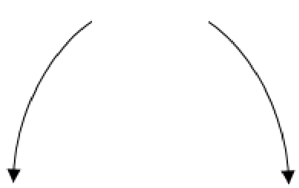




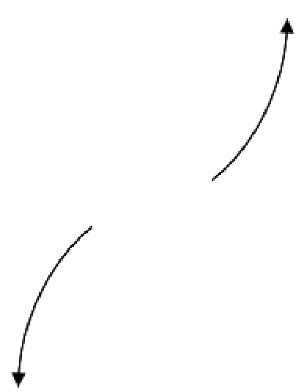
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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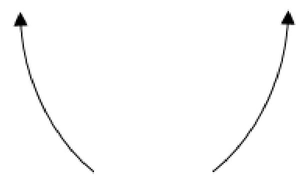
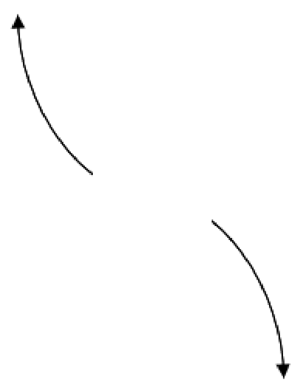
9. Describe the end behavior of the polynomial below.

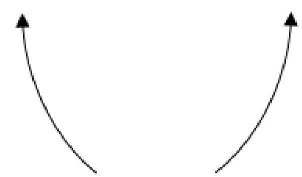
$$f(x) = 9(x+3)^2(x-3)^3(x-4)^4(x+4)^5$$

The solution is the graph below, which is option C.

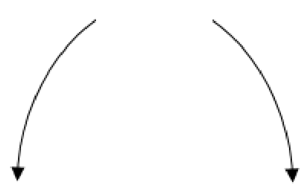




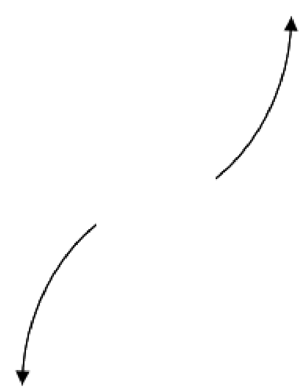
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{4}, \frac{-7}{5}, \text{ and } 4$$

The solution is $20x^3 - 17x^2 - 203x - 196$, which is option C.

A. $a \in [20, 23], b \in [-144, -139], c \in [301, 307], \text{ and } d \in [-196, -195]$

$20x^3 - 143x^2 + 301x - 196$, which corresponds to multiplying out $(4x - 7)(5x - 7)(x - 4)$.

B. $a \in [20, 23], b \in [9, 18], c \in [-207, -199], \text{ and } d \in [189, 200]$

$20x^3 + 17x^2 - 203x + 196$, which corresponds to multiplying out $(4x - 7)(5x - 7)(x + 4)$.

C. $a \in [20, 23], b \in [-19, -15], c \in [-207, -199], \text{ and } d \in [-196, -195]$

* $20x^3 - 17x^2 - 203x - 196$, which is the correct option.

D. $a \in [20, 23], b \in [-92, -78], c \in [-26, -20], \text{ and } d \in [189, 200]$

$20x^3 - 87x^2 - 21x + 196$, which corresponds to multiplying out $(4x - 7)(5x + 7)(x - 4)$.

E. $a \in [20, 23], b \in [-19, -15], c \in [-207, -199], \text{ and } d \in [189, 200]$

$20x^3 - 17x^2 - 203x + 196$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 7)(5x + 7)(x - 4)$