**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 5i \text{ and } 3$$

The solution is $x^3 + 7x^2 + 20x - 150$, which is option C.

A. $b \in [-11, -3], c \in [16, 22], \text{ and } d \in [145, 151]$

$x^3 - 7x^2 + 20x + 150$, which corresponds to multiplying out $(x - (-5 + 5i))(x - (-5 - 5i))(x + 3)$.

B. $b \in [1, 6], c \in [-1, 7], \text{ and } d \in [-18, -13]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x + 5)(x - 3)$.

C. $b \in [2, 12], c \in [16, 22], \text{ and } d \in [-156, -141]$

* $x^3 + 7x^2 + 20x - 150$, which is the correct option.

D. $b \in [1, 6], c \in [-9, 1], \text{ and } d \in [13, 20]$

$x^3 + x^2 - 8x + 15$, which corresponds to multiplying out $(x - 5)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 5i))(x - (-5 - 5i))(x - (3))$.

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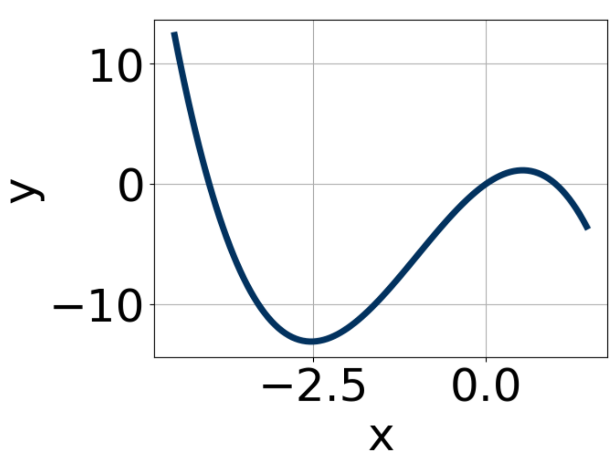
2. Which of the following equations *could* be of the graph presented below?



The solution is $-9x^5(x-1)^9(x+4)^9$, which is option A.

A. $-9x^5(x-1)^9(x+4)^9$

  * This is the correct option.

B. $17x^{11}(x-1)^9(x+4)^9$

  This corresponds to the leading coefficient being the opposite value than it should be.

C. $6x^5(x-1)^8(x+4)^9$

  The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-7x^7(x-1)^8(x+4)^4$

  The factors 1 and $-4$ have have been odd power.

E. $-17x^7(x-1)^{10}(x+4)^9$

  The factor 1 should have been an odd power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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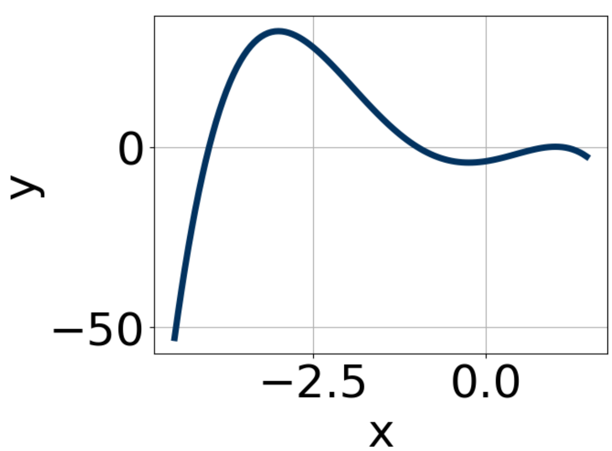
3. Which of the following equations *could* be of the graph presented below?




The solution is $-4(x-1)^{10}(x+4)^7(x+1)^9$, which is option E.

A. $-17(x-1)^4(x+4)^8(x+1)^9$

  The factor $(x+4)$ should have an odd power.

B. $20(x-1)^{10}(x+4)^7(x+1)^4$

  The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $2(x-1)^{10}(x+4)^9(x+1)^9$

  This corresponds to the leading coefficient being the opposite value than it should be.

D. $-6(x-1)^7(x+4)^8(x+1)^{11}$

  The factor 1 should have an even power and the factor $-4$ should have an odd power.

E. $-4(x-1)^{10}(x+4)^7(x+1)^9$

* This is the correct option.

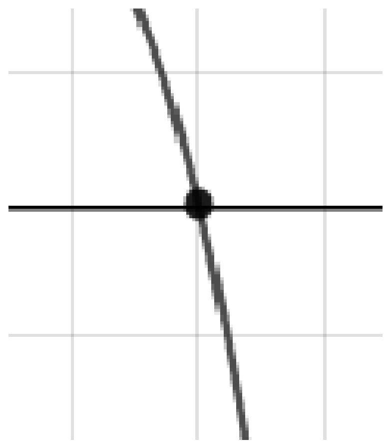
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

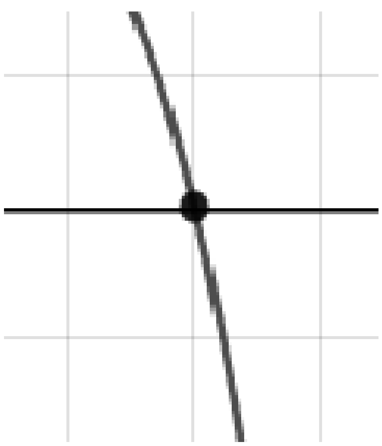
---

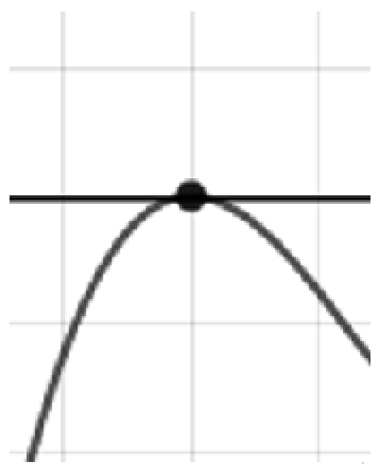
4. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

$$f(x) = -4(x-8)^6(x+8)^2(x+6)^9(x-6)^6$$

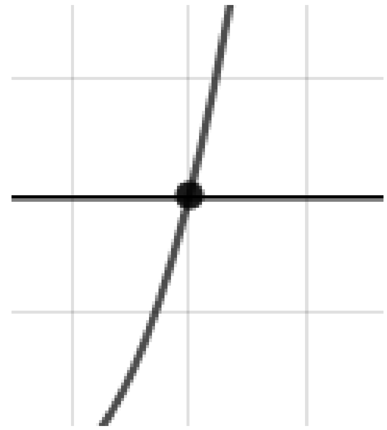
The solution is the graph below, which is option A.



A. 

B. 

C. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 2i \text{ and } -2$$

The solution is $x^3 + 10x^2 + 36x + 40$, which is option A.

A. $b \in [8, 13], c \in [34.2, 37], \text{ and } d \in [40, 42]$

   * $x^3 + 10x^2 + 36x + 40$, which is the correct option.

B. $b \in [-4, 6], c \in [4.5, 8.2], \text{ and } d \in [6, 9]$

   $x^3 + x^2 + 6x + 8$, which corresponds to multiplying out $(x + 4)(x + 2)$.

C. $b \in [-4, 6], c \in [3, 4.3], \text{ and } d \in [2, 5]$

   $x^3 + x^2 + 4x + 4$, which corresponds to multiplying out $(x + 2)(x + 2)$.

D. $b \in [-14, -8], c \in [34.2, 37], \text{ and } d \in [-40, -32]$

   $x^3 - 10x^2 + 36x - 40$, which corresponds to multiplying out $(x - (-4 - 2i))(x - (-4 + 2i))(x - 2)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 - 2i))(x - (-4 + 2i))(x - (-2))$.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{5}, \frac{5}{2}, \text{ and } \frac{-1}{4}$$

The solution is $40x^3 - 34x^2 - 151x - 35$, which is option D.

A. $a \in [35, 41], b \in [48, 58], c \in [-131, -125],$ and $d \in [-42, -32]$

$40x^3 + 54x^2 - 129x - 35$, which corresponds to multiplying out $(5x - 7)(2x + 5)(4x + 1)$.

B. $a \in [35, 41], b \in [-36, -27], c \in [-158, -150],$ and $d \in [33, 37]$

$40x^3 - 34x^2 - 151x + 35$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [35, 41], b \in [-148, -145], c \in [95, 106],$ and $d \in [33, 37]$

$40x^3 - 146x^2 + 101x + 35$, which corresponds to multiplying out $(5x - 7)(2x - 5)(4x + 1)$.

D. $a \in [35, 41], b \in [-36, -27], c \in [-158, -150],$ and $d \in [-42, -32]$

* $40x^3 - 34x^2 - 151x - 35$, which is the correct option.

E. $a \in [35, 41], b \in [32, 40], c \in [-158, -150],$ and $d \in [33, 37]$

$40x^3 + 34x^2 - 151x + 35$, which corresponds to multiplying out $(5x - 7)(2x + 5)(4x - 1)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(5x + 7)(2x - 5)(4x + 1)$

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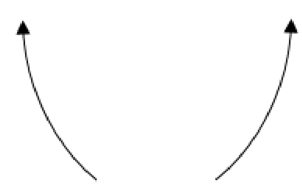
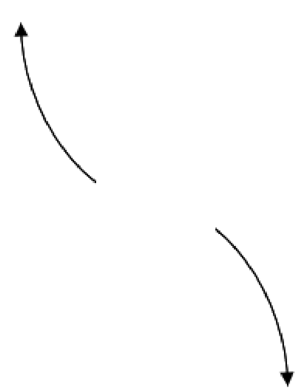
7. Describe the end behavior of the polynomial below.

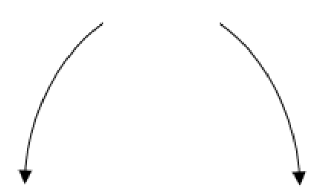
$$f(x) = 8(x - 4)^4(x + 4)^9(x + 8)^4(x - 8)^5$$

The solution is the graph below, which is option C.

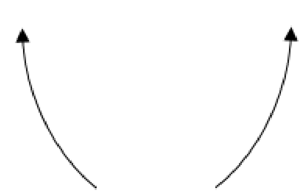




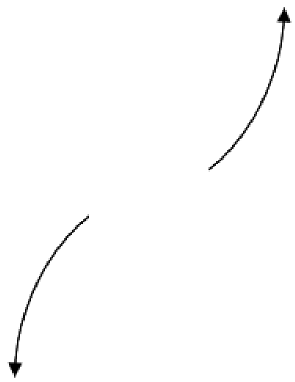
A.



B.



C.

D. 

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{2}, \frac{7}{4}, \text{ and } 4$$

The solution is $8x^3 - 50x^2 + 79x - 28$, which is option B.

A. $a \in [7, 12], b \in [-23, -12], c \in [-71, -59], \text{ and } d \in [-32, -21]$

$8x^3 - 14x^2 - 65x - 28$, which corresponds to multiplying out $(2x + 1)(4x + 7)(x - 4)$.

B. $a \in [7, 12], b \in [-51, -44], c \in [77, 82], \text{ and } d \in [-32, -21]$

* $8x^3 - 50x^2 + 79x - 28$, which is the correct option.

C. $a \in [7, 12], b \in [49, 55], c \in [77, 82], \text{ and } d \in [28, 30]$

$8x^3 + 50x^2 + 79x + 28$, which corresponds to multiplying out $(2x + 1)(4x + 7)(x + 4)$.

D. $a \in [7, 12], b \in [-51, -44], c \in [77, 82], \text{ and } d \in [28, 30]$

$8x^3 - 50x^2 + 79x + 28$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [7, 12], b \in [-44, -34], c \in [31, 43], \text{ and } d \in [28, 30]$

$8x^3 - 42x^2 + 33x + 28$, which corresponds to multiplying out $(2x + 1)(4x - 7)(x - 4)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(2x - 1)(4x - 7)(x - 4)$

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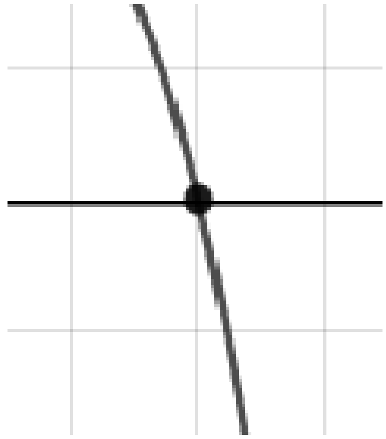
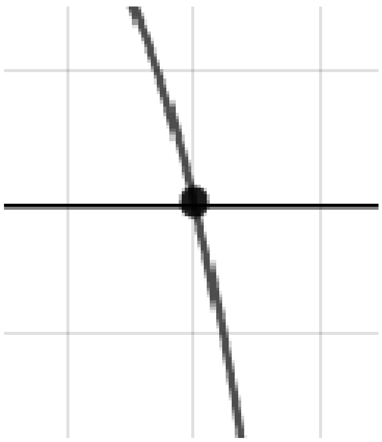
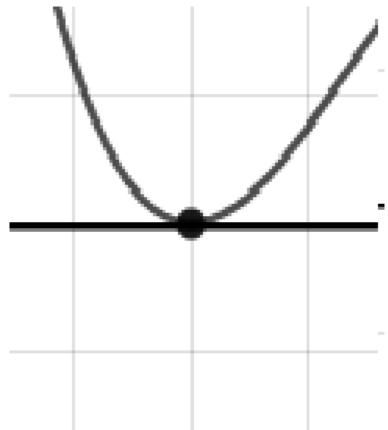
9. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

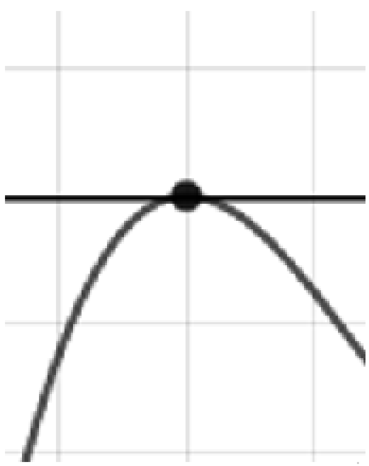
$$f(x) = -4(x - 2)^5(x + 2)^{10}(x - 3)^6(x + 3)^{10}$$

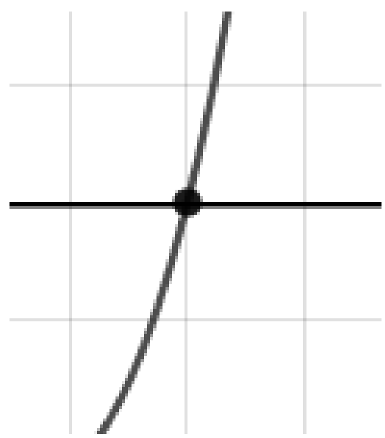
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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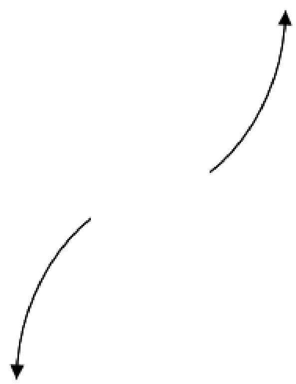
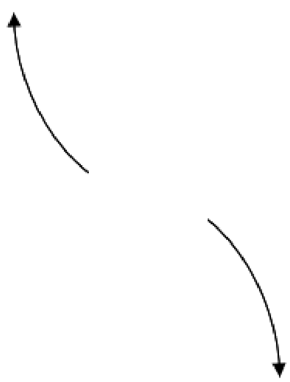
10. Describe the end behavior of the polynomial below.

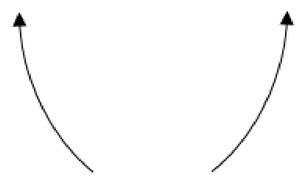
$$f(x) = 2(x+5)^3(x-5)^6(x+3)^3(x-3)^5$$

The solution is the graph below, which is option D.

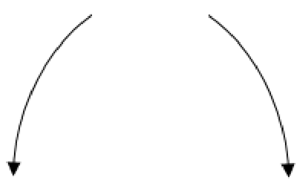




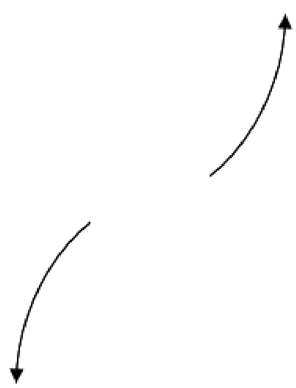
A.



C.



B.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.