This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 + 75x^2 - 16x - 48$$

The solution is $[-3, -0.8, 0.8]$, which is option C.

A. $z_1 \in [-3.16, -2.71], \; z_2 \in [-1.32, -1.21], \text{ and } z_3 \in [1.09, 1.63]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-1.28, -1.18], \; z_2 \in [1.09, 1.35], \text{ and } z_3 \in [2.81, 3.32]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-3.16, -2.71], \; z_2 \in [-0.86, -0.49], \text{ and } z_3 \in [0.52, 0.91]$

* This is the solution!

D. $z_1 \in [-4.08, -3.92], \; z_2 \in [0.08, 0.21], \text{ and } z_3 \in [2.81, 3.32]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-0.9, -0.7], \; z_2 \in [0.52, 0.96], \text{ and } z_3 \in [2.81, 3.32]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

2. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 106x^2 + 112x - 30}{x - 4}$$

The solution is $20x^2 - 26x + 8 + \frac{2}{x-4}$, which is option D.

A. $a \in [79, 82], \; b \in [-426, -424], \; c \in [1811, 1818], \text{ and } r \in [-7295, -7290]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [79, 82], \; b \in [212, 216], \; c \in [965, 973], \text{ and } r \in [3836, 3844]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [17, 26], \; b \in [-47, -44], \; c \in [-27, -22], \text{ and } r \in [-109, -104]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [17, 26], \ b \in [-28, -23], \ c \in [4, 11], \text{ and } r \in [-1, 5]$.

* This is the solution!

E. $a \in [17, 26], \ b \in [-192, -184], \ c \in [855, 861], \text{ and } r \in [-3457, -3450]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 + 28x^2 - 68}{x + 4}$$

The solution is $6x^2 + 4x - 16 + \frac{-4}{x+4}$, which is option B.

A. $a \in [-27, -23], b \in [123, 125], c \in [-498, -495], \text{ and } r \in [1913, 1919]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [3, 9], b \in [4, 9], c \in [-19, -11], \text{ and } r \in [-5, -3]$.

* This is the solution!

C. $a \in [-27, -23], b \in [-68, -63], c \in [-277, -267], \text{ and } r \in [-1157, -1153]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [3, 9], b \in [51, 53], c \in [208, 211], \text{ and } r \in [762, 767]$.

You divided by the opposite of the factor.

E. $a \in [3, 9], b \in [-6, 1], c \in [4, 15], \text{ and } r \in [-125, -117]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

4. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 21x^2 - 135x - 50$$

The solution is $[-2.5, -0.4, 5]$, which is option B.

A. $z_1 \in [-4.5, -1.5], \ z_2 \in [-0.52, -0.38], \text{ and } z_3 \in [5, 7]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-4.5, -1.5], \ z_2 \in [-0.52, -0.38], \text{ and } z_3 \in [5, 7]$

* This is the solution!

C. $z_1 \in [-6, -4], \ z_2 \in [0.36, 0.46], \text{ and } z_3 \in [1.5, 4.5]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-6, -4], \ z_2 \in [0.01, 0.37], \text{ and } z_3 \in [5, 7]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-6, -4]$, $z_2 \in [0.36, 0.46]$, and $z_3 \in [1.5, 4.5]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

5. Factor the polynomial below completely, knowing that $x-3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 9x^4 + 9x^3 - 163x^2 + 115x + 150$$

The solution is $[-5, -0.667, 1.667, 3]$, which is option D.

A. $z_1 \in [-5.2, -4.7]$, $z_2 \in [-1.62, -1.48]$, $z_3 \in [0.52, 0.63]$, and $z_4 \in [2.4, 3.2]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-5.2, -4.7]$, $z_2 \in [-3.05, -2.99]$, $z_3 \in [0.12, 0.28]$, and $z_4 \in [4, 5.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-3.7, -2]$, $z_2 \in [-0.65, -0.6]$, $z_3 \in [1.47, 1.5]$, and $z_4 \in [4, 5.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-5.2, -4.7]$, $z_2 \in [-0.74, -0.64]$, $z_3 \in [1.64, 1.74]$, and $z_4 \in [2.4, 3.2]$

* This is the solution!

E. $z_1 \in [-3.7, -2]$, $z_2 \in [-1.71, -1.63]$, $z_3 \in [0.66, 0.71]$, and $z_4 \in [4, 5.3]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

6. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 + 26x^2 - 68x - 53}{x + 4}$$

The solution is $10x^2 - 14x - 12 + \frac{-5}{x+4}$, which is option D.

A. $a \in [-42, -33]$, $b \in [-134, -130]$, $c \in [-607, -603]$, and $r \in [-2473, -2463]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [-42, -33]$, $b \in [185, 188]$, $c \in [-818, -809]$, and $r \in [3195, 3197]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [9, 11]$, $b \in [-30, -21]$, $c \in [52, 57]$, and $r \in [-321, -310]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [9, 11]$, $b \in [-16, -13]$, $c \in [-14, -10]$, and $r \in [-8, -1]$.

* This is the solution!

E. $a \in [9, 11], \; b \in [66, 70], \; c \in [196, 202], \text{ and } r \in [722, 739]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 + 28x^2 - 62}{x + 4}$$

The solution is $6x^2 + 4x - 16 + \frac{2}{x+4}$, which is option D.

A. $a \in [4, 10], b \in [51, 53], c \in [207, 212], \text{ and } r \in [762, 773]$.

You divided by the opposite of the factor.

B. $a \in [4, 10], b \in [-6, 1], c \in [8, 13], \text{ and } r \in [-115, -105]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-25, -21], b \in [120, 125], c \in [-503, -493], \text{ and } r \in [1917, 1926]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [4, 10], b \in [3, 8], c \in [-17, -14], \text{ and } r \in [2, 3]$.

* This is the solution!

E. $a \in [-25, -21], b \in [-72, -65], c \in [-277, -268], \text{ and } r \in [-1150, -1148]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

8. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^2 + 2x + 3$$

The solution is $\pm 1, \pm 3$, which is option A.

A. $\pm 1, \pm 3$

* This is the solution **since we asked for the possible Integer roots**!

B. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\frac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

D. All combinations of: $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 3}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

9. Factor the polynomial below completely, knowing that $x-3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 90x^3 + 343x^2 - 510x + 225$$

The solution is $[0.75, 2.5, 3, 5]$, which is option D.

A. $z_1 \in [-5.86, -4.88]$, $z_2 \in [-3.65, -2.93]$, $z_3 \in [-3.38, -2.77]$, and $z_4 \in [-0.71, -0.43]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-5.86, -4.88]$, $z_2 \in [-3.65, -2.93]$, $z_3 \in [-2.14, -0.63]$, and $z_4 \in [-0.49, -0.24]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.86, -4.88]$, $z_2 \in [-3.65, -2.93]$, $z_3 \in [-2.84, -2.19]$, and $z_4 \in [-0.83, -0.74]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [0.6, 0.85]$, $z_2 \in [2.02, 3.75]$, $z_3 \in [2.3, 3.15]$, and $z_4 \in [4.99, 5.07]$

* This is the solution!

E. $z_1 \in [0.14, 0.74]$, $z_2 \in [1.06, 1.46]$, $z_3 \in [2.3, 3.15]$, and $z_4 \in [4.99, 5.07]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

10. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^3 + 5x^2 + 2x + 5$$

The solution is All combinations of: $\frac{\pm 1, \pm 5}{\pm 1, \pm 7}$, which is option D.

A. All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots**!

D. All combinations of: $\frac{\pm 1, \pm 5}{\pm 1, \pm 7}$

* This is the solution **since we asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

11. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 41x^2 + 27x + 18$$

The solution is $[-0.4, 1.5, 3]$, which is option E.

A. $z_1 \in [-3.1, -2.9]$, $z_2 \in [-1.7, -0.7]$, and $z_3 \in [0.23, 0.73]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-2.9, -1.5]$, $z_2 \in [0.2, 0.8]$, and $z_3 \in [2.77, 3.08]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-3.1, -2.9]$, $z_2 \in [-1.1, 0]$, and $z_3 \in [2.35, 2.69]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-3.1, -2.9]$, $z_2 \in [-3.2, -2.7]$, and $z_3 \in [0, 0.29]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-1.9, 0]$, $z_2 \in [0.8, 2]$, and $z_3 \in [2.77, 3.08]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

12. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 + 11x^2 - 106x + 44}{x + 4}$$

The solution is $10x^2 - 29x + 10 + \frac{4}{x+4}$, which is option E.

A. $a \in [9, 11]$, $b \in [-46, -31]$, $c \in [84, 95]$, and $r \in [-401, -396]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [-44, -39]$, $b \in [-152, -148]$, $c \in [-703, -700]$, and $r \in [-2765, -2760]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [9, 11]$, $b \in [46, 56]$, $c \in [98, 100]$, and $r \in [425, 441]$.

You divided by the opposite of the factor.

D. $a \in [-44, -39]$, $b \in [168, 175]$, $c \in [-796, -787]$, and $r \in [3196, 3209]$.

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [9, 11]$, $b \in [-29, -25]$, $c \in [8, 15]$, and $r \in [3, 7]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

13. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 - 28x - 19}{x - 2}$$

The solution is $9x^2 + 18x + 8 + \frac{-3}{x-2}$, which is option E.

A. $a \in [14, 26], b \in [-36, -34], c \in [40, 45],$ and $r \in [-108, -105]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

B. $a \in [6, 12], b \in [-22, -17], c \in [0, 12],$ and $r \in [-35, -34]$.

You divided by the opposite of the factor.

C. $a \in [6, 12], b \in [6, 15], c \in [-23, -18],$ and $r \in [-42, -37]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [14, 26], b \in [36, 38], c \in [40, 45],$ and $r \in [65, 74]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [6, 12], b \in [16, 20], c \in [0, 12],$ and $r \in [-8, 1]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

14. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^3 + 35x^2 - 9x - 18$$

The solution is $[-3, -0.67, 0.75]$, which is option D.

A. $z_1 \in [-1.5, -1.26],\ z_2 \in [0.92, 1.85],$ and $z_3 \in [2.5, 3.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-0.8, -0.64],\ z_2 \in [0.48, 0.77],$ and $z_3 \in [2.5, 3.1]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-3.17, -2.34],\ z_2 \in [-1.51, -1.46],$ and $z_3 \in [1.1, 2.4]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.17, -2.34],\ z_2 \in [-0.74, -0.58],$ and $z_3 \in [0, 1.1]$

* This is the solution!

E. $z_1 \in [-0.34, -0.12],\ z_2 \in [1.58, 2.09],$ and $z_3 \in [2.5, 3.1]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

15. Factor the polynomial below completely, knowing that $x-4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 - 14x^3 - 248x^2 + 224x + 128$$

The solution is $[-4, -0.4, 1.333, 4]$, which is option C.

A. $z_1 \in [-5, -2], \; z_2 \in [-4.5, -3.89], z_3 \in [-0.08, 0.26], \text{ and } z_4 \in [2, 8]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-5, -2], \; z_2 \in [-3.04, -2.45], z_3 \in [0.73, 0.85], \text{ and } z_4 \in [2, 8]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-5, -2], \; z_2 \in [-0.52, -0.21], z_3 \in [1.24, 1.46], \text{ and } z_4 \in [2, 8]$

* This is the solution!

D. $z_1 \in [-5, -2], \; z_2 \in [-1.72, -1.22], z_3 \in [0.18, 0.71], \text{ and } z_4 \in [2, 8]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-5, -2], \; z_2 \in [-0.94, -0.69], z_3 \in [2.44, 2.56], \text{ and } z_4 \in [2, 8]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

16. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 + 27x^2 - 25x - 77}{x + 3}$$

The solution is $9x^2 - 25 + \dfrac{-2}{x+3}$, which is option A.

A. $a \in [9, 10], \; b \in [-2, 2], \; c \in [-27, -24], \text{ and } r \in [-9, 0]$.

* This is the solution!

B. $a \in [-32, -24], \; b \in [107, 111], \; c \in [-350, -348], \text{ and } r \in [968, 976]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [9, 10], \; b \in [53, 60], \; c \in [133, 143], \text{ and } r \in [334, 340]$.

You divided by the opposite of the factor.

D. $a \in [9, 10], \; b \in [-10, -7], \; c \in [11, 16], \text{ and } r \in [-121, -118]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [-32, -24], \; b \in [-55, -51], \; c \in [-188, -186], \text{ and } r \in [-642, -633]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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17. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 + 30x^2 - 44}{x + 2}$$

The solution is $10x^2 + 10x - 20 + \frac{-4}{x+2}$, which is option D.

A. $a \in [-21, -16], b \in [69, 73], c \in [-140, -137],$ and $r \in [232, 243]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [-21, -16], b \in [-12, -7], c \in [-21, -15],$ and $r \in [-88, -81]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [6, 11], b \in [46, 51], c \in [97, 101],$ and $r \in [153, 159]$.

You divided by the opposite of the factor.

D. $a \in [6, 11], b \in [9, 17], c \in [-21, -15],$ and $r \in [-4, -3]$.

* This is the solution!

E. $a \in [6, 11], b \in [-5, 6], c \in [-2, 3],$ and $r \in [-50, -42]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

18. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 3x^3 + 2x^2 + 4x + 7$$

The solution is All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 3}$, which is option C.

A. All combinations of: $\frac{\pm 1, \pm 3}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 3}$

* This is the solution **since we asked for the possible Rational roots**!

D. $\pm 1, \pm 7$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

19. Factor the polynomial below completely, knowing that $x+3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 + 51x^3 - 28x^2 - 333x - 180$$

The solution is $[-4, -3, -0.6, 2.5]$, which is option B.

A. $z_1 \in [-0.74, -0.5]$, $z_2 \in [2.94, 3.06]$, $z_3 \in [1.8, 3.3]$, and $z_4 \in [4, 5]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-4.18, -3.74]$, $z_2 \in [-3.13, -2.89]$, $z_3 \in [-0.8, -0.4]$, and $z_4 \in [2.5, 3.5]$

* This is the solution!

C. $z_1 \in [-0.47, -0.25]$, $z_2 \in [1.43, 2.14]$, $z_3 \in [1.8, 3.3]$, and $z_4 \in [4, 5]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-2.55, -2.47]$, $z_2 \in [-0.59, 1.11]$, $z_3 \in [1.8, 3.3]$, and $z_4 \in [4, 5]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-4.18, -3.74]$, $z_2 \in [-3.13, -2.89]$, $z_3 \in [-2.8, -1.2]$, and $z_4 \in [0.4, 1.4]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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20. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^3 + 4x^2 + 6x + 6$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$, which is option D.

A. $\pm 1, \pm 2, \pm 3, \pm 6$

This would have been the solution **if asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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21. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^3 - 6x^2 - 45x - 27$$

The solution is $[-1.5, -0.75, 3]$, which is option C.

A. $z_1 \in [-3.3, -1.7]$, $z_2 \in [0.68, 0.83]$, and $z_3 \in [1.44, 1.51]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-3.3, -1.7]$, $z_2 \in [0.64, 0.69]$, and $z_3 \in [1.17, 1.48]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-2.4, -1.4]$, $z_2 \in [-0.77, -0.75]$, and $z_3 \in [2.78, 3.13]$

* This is the solution!

D. $z_1 \in [-3.3, -1.7]$, $z_2 \in [0.3, 0.41]$, and $z_3 \in [2.78, 3.13]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-1.4, -1.1]$, $z_2 \in [-0.7, -0.63]$, and $z_3 \in [2.78, 3.13]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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22. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 - 8x^2 - 40x - 29}{x - 3}$$

The solution is $8x^2 + 16x + 8 + \frac{-5}{x-3}$, which is option A.

A. $a \in [6, 12]$, $b \in [14, 19]$, $c \in [6, 9]$, and $r \in [-5, 2]$.

* This is the solution!

B. $a \in [6, 12]$, $b \in [3, 10]$, $c \in [-27, -22]$, and $r \in [-80, -72]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [6, 12]$, $b \in [-32, -31]$, $c \in [54, 57]$, and $r \in [-197, -193]$.

You divided by the opposite of the factor.

D. $a \in [24, 32]$, $b \in [63, 69]$, $c \in [152, 155]$, and $r \in [427, 428]$.

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [24, 32]$, $b \in [-83, -75]$, $c \in [199, 207]$, and $r \in [-634, -628]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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23. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 + 26x^2 - 28}{x + 4}$$

The solution is $6x^2 + 2x - 8 + \frac{4}{x+4}$, which is option C.

A. $a \in [1, 9], b \in [48, 55], c \in [200, 202]$, and $r \in [771, 774]$.

You divided by the opposite of the factor.

B. $a \in [1, 9], b \in [-4, 0], c \in [19, 28]$, and $r \in [-130, -124]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [1, 9], b \in [1, 5], c \in [-12, -4]$, and $r \in [-1, 10]$.

* This is the solution!

D. $a \in [-24, -22], b \in [-73, -66], c \in [-283, -279]$, and $r \in [-1154, -1140]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [-24, -22], b \in [121, 123], c \in [-491, -483]$, and $r \in [1921, 1929]$.

You multipled by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

24. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 16x^3 - 40x^2 + x + 30$$

The solution is $[-0.75, 1.25, 2]$, which is option D.

A. $z_1 \in [-2.25, -1.94]$, $z_2 \in [-1.04, -0.34]$, and $z_3 \in [0.79, 1.94]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-1.34, -0.77]$, $z_2 \in [0.55, 0.88]$, and $z_3 \in [1.9, 2.19]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-2.25, -1.94]$, $z_2 \in [-1.46, -1.14]$, and $z_3 \in [0.51, 0.89]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-1.28, -0.5]$, $z_2 \in [1.07, 1.34]$, and $z_3 \in [1.9, 2.19]$

* This is the solution!

E. $z_1 \in [-5.28, -4.63]$, $z_2 \in [-2.05, -1.9]$, and $z_3 \in [-0.1, 0.58]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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25. Factor the polynomial below completely, knowing that $x-4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 - 59x^3 - 50x^2 + 208x - 96$$

The solution is $[-2, 0.6, 1.333, 4]$, which is option E.

A. $z_1 \in [-4, -3]$, $z_2 \in [-1.68, -1.66]$, $z_3 \in [-0.81, -0.63]$, and $z_4 \in [1.7, 2.7]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-2, 1]$, $z_2 \in [0.62, 0.9]$, $z_3 \in [1.57, 1.69]$, and $z_4 \in [3.9, 5.2]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-4, -3]$, $z_2 \in [-1.44, -1.27]$, $z_3 \in [-0.67, -0.6]$, and $z_4 \in [1.7, 2.7]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-4, -3]$, $z_2 \in [-3.02, -2.95]$, $z_3 \in [-0.44, -0.14]$, and $z_4 \in [1.7, 2.7]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-2, 1]$, $z_2 \in [0.55, 0.65]$, $z_3 \in [1.28, 1.46]$, and $z_4 \in [3.9, 5.2]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

26. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 - 4x^2 - 40x + 37}{x+2}$$

The solution is $12x^2 - 28x + 16 + \frac{5}{x+2}$, which is option C.

A. $a \in [-32, -22]$, $b \in [-55, -47]$, $c \in [-151, -142]$, and $r \in [-251, -245]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [-32, -22]$, $b \in [36, 49]$, $c \in [-129, -124]$, and $r \in [292, 296]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [12, 13]$, $b \in [-32, -24]$, $c \in [11, 19]$, and $r \in [4, 9]$.

* This is the solution!

D. $a \in [12, 13]$, $b \in [-43, -38]$, $c \in [79, 82]$, and $r \in [-205, -196]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [12, 13]$, $b \in [18, 26]$, $c \in [0, 1]$, and $r \in [29, 47]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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27. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{16x^3 - 48x - 28}{x - 2}$$

The solution is $16x^2 + 32x + 16 + \frac{4}{x-2}$, which is option C.

A. $a \in [32, 33], b \in [-64, -59], c \in [80, 83]$, and $r \in [-195, -187]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

B. $a \in [12, 23], b \in [-35, -27], c \in [9, 21]$, and $r \in [-60, -53]$.

You divided by the opposite of the factor.

C. $a \in [12, 23], b \in [26, 33], c \in [9, 21]$, and $r \in [3, 5]$.

* This is the solution!

D. $a \in [32, 33], b \in [61, 67], c \in [80, 83]$, and $r \in [130, 141]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [12, 23], b \in [15, 17], c \in [-39, -28]$, and $r \in [-60, -53]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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28. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 2x^2 + 3x + 7$$

The solution is $\pm 1, \pm 7$, which is option C.

A. All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 2}$

This would have been the solution **if asked for the possible Rational roots**!

C. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

D. $\pm 1, \pm 2$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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29. Factor the polynomial below completely, knowing that $x-4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^4 - 80x^3 - 132x^2 + 224x - 64$$

The solution is $[-2, 0.4, 0.8, 4]$, which is option A.

A. $z_1 \in [-2, 2]$, $z_2 \in [-0.17, 0.41]$, $z_3 \in [0.76, 0.9]$, and $z_4 \in [3, 6]$

* This is the solution!

B. $z_1 \in [-5, -3]$, $z_2 \in [-1.71, -0.25]$, $z_3 \in [-0.63, -0.31]$, and $z_4 \in [1, 3]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-5, -3]$, $z_2 \in [-2.13, -1.06]$, $z_3 \in [-0.19, 0.06]$, and $z_4 \in [1, 3]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-2, 2]$, $z_2 \in [1.21, 2.56]$, $z_3 \in [2.4, 2.52]$, and $z_4 \in [3, 6]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-5, -3]$, $z_2 \in [-2.8, -2.06]$, $z_3 \in [-1.34, -1.19]$, and $z_4 \in [1, 3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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30. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^3 + 4x^2 + 4x + 7$$

The solution is $\pm 1, \pm 7$, which is option C.

A. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.
