This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^4 + 2x^3 + 2x^2 + 2x + 6$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$, which is option C.

A. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

* This is the solution **since we asked for the possible Rational roots**!

D. $\pm 1, \pm 2, \pm 3, \pm 6$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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2. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^4 + 6x^3 + 3x^2 + 2x + 4$$

The solution is $\pm 1, \pm 2, \pm 4$, which is option A.

A. $\pm 1, \pm 2, \pm 4$

* This is the solution **since we asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

D. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Factor the polynomial below completely, knowing that $x + 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 14x^3 - 167x^2 + 455x - 300$$

The solution is $[-5, 1.25, 1.5, 4]$, which is option B.

A. $z_1 \in [-4.25, -3.95]$, $z_2 \in [-0.88, -0.53]$, $z_3 \in [-0.67, -0.65]$, and $z_4 \in [4.9, 5.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5.08, -4.64]$, $z_2 \in [0.79, 1.43]$, $z_3 \in [1.47, 1.53]$, and $z_4 \in [2.5, 4.4]$

* This is the solution!

C. $z_1 \in [-5.08, -4.64]$, $z_2 \in [-0.31, 0.74]$, $z_3 \in [0.76, 0.84]$, and $z_4 \in [2.5, 4.4]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-4.25, -3.95]$, $z_2 \in [-3.36, -2.87]$, $z_3 \in [-0.63, -0.53]$, and $z_4 \in [4.9, 5.1]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-4.25, -3.95]$, $z_2 \in [-2.41, -0.84]$, $z_3 \in [-1.26, -1.22]$, and $z_4 \in [4.9, 5.1]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 - 12x + 6}{x + 2}$$

The solution is $4x^2 - 8x + 4 + \frac{-2}{x+2}$, which is option D.

A. $a \in [3, 8], b \in [-13, -10], c \in [15, 25]$, and $r \in [-67, -61]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [3, 8], b \in [8, 10], c \in [-1, 8]$, and $r \in [8, 15]$.

You divided by the opposite of the factor.

C. $a \in [-10, -4], b \in [10, 17], c \in [-48, -42]$, and $r \in [94, 97]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [3, 8], b \in [-9, 0], c \in [-1, 8]$, and $r \in [-5, 4]$.

* This is the solution!

E. $a \in [-10, -4], b \in [-20, -15], c \in [-48, -42],$ and $r \in [-85, -81]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 63x^2 + 23}{x - 3}$$

The solution is $20x^2 - 3x - 9 + \frac{-4}{x-3}$, which is option D.

A. $a \in [57, 65], b \in [113, 120], c \in [350, 355],$ and $r \in [1074, 1078]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [17, 22], b \in [-130, -118], c \in [369, 371],$ and $r \in [-1085, -1082]$.

You divided by the opposite of the factor.

C. $a \in [57, 65], b \in [-245, -241], c \in [729, 731],$ and $r \in [-2169, -2161]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [17, 22], b \in [-5, 0], c \in [-13, -7],$ and $r \in [-6, 4]$.

* This is the solution!

E. $a \in [17, 22], b \in [-29, -22], c \in [-47, -42],$ and $r \in [-70, -68]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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6. Factor the polynomial below completely, knowing that $x - 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 - 113x^3 + 338x^2 - 395x + 150$$

The solution is $[0.75, 1.667, 2, 5]$, which is option C.

A. $z_1 \in [-5.35, -4.91], \; z_2 \in [-6.33, -4.95], z_3 \in [-2.42, -1.69],$ and $z_4 \in [-0.34, -0.14]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-5.35, -4.91], \; z_2 \in [-2.13, -1.6], z_3 \in [-1.8, -1.62],$ and $z_4 \in [-0.84, -0.74]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [0.72, 1.07], \; z_2 \in [1.35, 1.95], z_3 \in [1.61, 2.61],$ and $z_4 \in [4.79, 5.08]$

* This is the solution!

D. $z_1 \in [0.59, 0.69], \; z_2 \in [1.11, 1.62], z_3 \in [1.61, 2.61],$ and $z_4 \in [4.79, 5.08]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-5.35, -4.91]$, $z_2 \in [-2.13, -1.6], z_3 \in [-1.44, -1.27]$, and $z_4 \in [-0.68, -0.44]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 - 100x^2 - 4x + 16$$

The solution is $[-0.4, 0.4, 4]$, which is option D.

A. $z_1 \in [-4.6, -3.3]$, $z_2 \in [-2.65, -2.36]$, and $z_3 \in [2.09, 3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-2.9, -2.4]$, $z_2 \in [1.93, 2.94]$, and $z_3 \in [3.93, 4.24]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-4.6, -3.3]$, $z_2 \in [-2.25, -1.76]$, and $z_3 \in [-0.2, 0.15]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-1.6, 0.4]$, $z_2 \in [0.19, 0.79]$, and $z_3 \in [3.93, 4.24]$

* This is the solution!

E. $z_1 \in [-4.6, -3.3]$, $z_2 \in [-0.56, -0.04]$, and $z_3 \in [0.11, 1.11]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 + 31x^2 - 38x - 40$$

The solution is $[-2, -0.8, 1.25]$, which is option E.

A. $z_1 \in [-0.92, -0.65]$, $z_2 \in [1.12, 1.47]$, and $z_3 \in [1.78, 2.59]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5.08, -4.9]$, $z_2 \in [-0.09, 0.27]$, and $z_3 \in [1.78, 2.59]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-1.46, -1.02]$, $z_2 \in [0.78, 1.08]$, and $z_3 \in [1.78, 2.59]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-2.2, -1.8]$, $z_2 \in [-1.59, -1.18]$, and $z_3 \in [0.41, 0.86]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-2.2, -1.8]$, $z_2 \in [-0.86, -0.52]$, and $z_3 \in [1.1, 1.49]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 - 2x^2 - 20x + 19}{x + 2}$$

The solution is $6x^2 - 14x + 8 + \frac{3}{x+2}$, which is option E.

A. $a \in [-15, -8]$, $b \in [-28, -25]$, $c \in [-72, -68]$, and $r \in [-132, -124]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [-15, -8]$, $b \in [22, 24]$, $c \in [-66, -63]$, and $r \in [144, 149]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [1, 11]$, $b \in [-21, -19]$, $c \in [34, 46]$, and $r \in [-103, -97]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [1, 11]$, $b \in [8, 17]$, $c \in [-3, 4]$, and $r \in [15, 20]$.

You divided by the opposite of the factor.

E. $a \in [1, 11]$, $b \in [-14, -9]$, $c \in [7, 9]$, and $r \in [2, 4]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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10. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 - 20x^2 - 2x + 19}{x - 3}$$

The solution is $6x^2 - 2x - 8 + \frac{-5}{x-3}$, which is option D.

A. $a \in [5, 9]$, $b \in [-40, -34]$, $c \in [111, 115]$, and $r \in [-322, -313]$.

You divided by the opposite of the factor.

B. $a \in [15, 21]$, $b \in [-76, -72]$, $c \in [218, 224]$, and $r \in [-646, -637]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [5, 9]$, $b \in [-14, -3]$, $c \in [-23, -16]$, and $r \in [-18, -15]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [5, 9]$, $b \in [-6, 6]$, $c \in [-15, -7]$, and $r \in [-6, -3]$.

* This is the solution!

E. $a \in [15, 21]$, $b \in [29, 35]$, $c \in [95, 104]$, and $r \in [318, 321]$.

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!