

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

If you have a suggestion to make the keys better, please fill out the short survey [here](#).

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

1. Choose the interval below that  $f$  composed with  $g$  at  $x = -1$  is in.

$$f(x) = x^3 - 3x^2 - 2x + 1 \text{ and } g(x) = 4x^3 + 4x^2 - x$$

The solution is  $-3.0$ , which is option B.

A.  $(f \circ g)(-1) \in [9.3, 10.38]$

Distractor 3: Corresponds to being slightly off from the solution.

B.  $(f \circ g)(-1) \in [-3.32, -2.98]$

\* This is the correct solution

C.  $(f \circ g)(-1) \in [1.64, 2.57]$

Distractor 2: Corresponds to being slightly off from the solution.

D.  $(f \circ g)(-1) \in [-0.35, 1.66]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

**General Comment:**  $f$  composed with  $g$  at  $x$  means  $f(g(x))$ . The order matters!

2. Determine whether the function below is 1-1.

$$f(x) = 36x^2 + 480x + 1600$$

The solution is no, which is option A.

A. No, because there is a  $y$ -value that goes to 2 different  $x$ -values.

\* This is the solution.

B. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

C. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because there is an  $x$ -value that goes to 2 different  $y$ -values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a  $y$ -value that goes to 2 different  $x$ -values.

3. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-4x + 11} \text{ and } g(x) = 6x + 4$$

The solution is The domain is all Real numbers less than or equal to  $x = 2.75$ ., which is option A.

- A. The domain is all Real numbers less than or equal to  $x = a$ , where  $a \in [-0.25, 5.75]$
- B. The domain is all Real numbers except  $x = a$ , where  $a \in [-6.8, -0.8]$
- C. The domain is all Real numbers greater than or equal to  $x = a$ , where  $a \in [-12.4, -2.4]$
- D. The domain is all Real numbers except  $x = a$  and  $x = b$ , where  $a \in [-9.4, -1.4]$  and  $b \in [2.33, 14.33]$
- E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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4. Find the inverse of the function below. Then, evaluate the inverse at  $x = 10$  and choose the interval that  $f^{-1}(10)$  belongs to.

$$f(x) = e^{x-5} + 2$$

The solution is  $f^{-1}(10) = 7.079$ , which is option B.

- A.  $f^{-1}(10) \in [3.11, 3.77]$

This solution corresponds to distractor 4.

- B.  $f^{-1}(10) \in [6.93, 7.45]$

This is the solution.

- C.  $f^{-1}(10) \in [-2.94, -2.59]$

This solution corresponds to distractor 1.

- D.  $f^{-1}(10) \in [4.02, 4.57]$

This solution corresponds to distractor 2.

- E.  $f^{-1}(10) \in [4.57, 4.97]$

This solution corresponds to distractor 3.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the  $x$  and  $y$ , use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at  $x = -13$  and choose the interval that  $f^{-1}(-13)$  belongs to.

$$f(x) = \sqrt[3]{3x + 5}$$

The solution is  $-734.0$ , which is option C.

- A.  $f^{-1}(-13) \in [722.67, 732.67]$

This solution corresponds to distractor 3.

- B.  $f^{-1}(-13) \in [734, 740]$

This solution corresponds to distractor 2.

- C.  $f^{-1}(-13) \in [-735, -733]$

\* This is the correct solution.

D.  $f^{-1}(-13) \in [-732.67, -723.67]$

Distractor 1: This corresponds to

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Choose the interval below that  $f$  composed with  $g$  at  $x = 1$  is in.

$$f(x) = 2x^3 - 1x^2 + 4x - 4 \text{ and } g(x) = -2x^3 + x^2 + 2x + 1$$

The solution is 16.0, which is option A.

A.  $(f \circ g)(1) \in [15, 25]$

\* This is the correct solution

B.  $(f \circ g)(1) \in [-8, -5]$

Distractor 3: Corresponds to being slightly off from the solution.

C.  $(f \circ g)(1) \in [-1, 3]$

Distractor 1: Corresponds to reversing the composition.

D.  $(f \circ g)(1) \in [23, 36]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:**  $f$  composed with  $g$  at  $x$  means  $f(g(x))$ . The order matters!

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7. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 8x^4 + 8x^3 + 4x^2 + x \text{ and } g(x) = \frac{5}{5x + 22}$$

The solution is The domain is all Real numbers except  $x = -4.4$ , which is option A.

A. The domain is all Real numbers except  $x = a$ , where  $a \in [-6.4, 0.6]$

B. The domain is all Real numbers greater than or equal to  $x = a$ , where  $a \in [-8.33, -0.33]$

C. The domain is all Real numbers less than or equal to  $x = a$ , where  $a \in [-1.17, 6.83]$

D. The domain is all Real numbers except  $x = a$  and  $x = b$ , where  $a \in [3.67, 16.67]$  and  $b \in [-9.17, -5.17]$

E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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8. Find the inverse of the function below. Then, evaluate the inverse at  $x = 8$  and choose the interval that  $f^{-1}(8)$  belongs to.

$$f(x) = e^{x+2} + 5$$

The solution is  $f^{-1}(8) = -0.901$ , which is option C.

A.  $f^{-1}(8) \in [6.73, 6.89]$

This solution corresponds to distractor 3.

B.  $f^{-1}(8) \in [7.35, 7.89]$

This solution corresponds to distractor 2.

C.  $f^{-1}(8) \in [-1.17, -0.58]$

This is the solution.

D.  $f^{-1}(8) \in [2.92, 3.25]$

This solution corresponds to distractor 1.

E.  $f^{-1}(8) \in [6.92, 7.48]$

This solution corresponds to distractor 4.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the  $x$  and  $y$ , use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

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9. Determine whether the function below is 1-1.

$$f(x) = 20x^2 - 68x - 736$$

The solution is no, which is option C.

- A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

- B. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

- C. No, because there is a  $y$ -value that goes to 2 different  $x$ -values.

\* This is the solution.

- D. No, because there is an  $x$ -value that goes to 2 different  $y$ -values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

- E. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a  $y$ -value that goes to 2 different  $x$ -values.

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10. Find the inverse of the function below (if it exists). Then, evaluate the inverse at  $x = -10$  and choose the interval that  $f^{-1}(-10)$  belongs to.

$$f(x) = 4x^2 - 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A.  $f^{-1}(-10) \in [1.26, 2.12]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B.  $f^{-1}(-10) \in [2.98, 3.66]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C.  $f^{-1}(-10) \in [1.05, 1.21]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

D.  $f^{-1}(-10) \in [3.84, 4.42]$

Distractor 4: This corresponds to both distractors 2 and 3.

E. The function is not invertible for all Real numbers.

\* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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