This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^3 - 6x^2 - 45x - 27$$

The solution is $[-1.5, -0.75, 3]$, which is option C.

A. $z_1 \in [-3.3, -1.7], \; z_2 \in [0.68, 0.83], \text{ and } z_3 \in [1.44, 1.51]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-3.3, -1.7], \; z_2 \in [0.64, 0.69], \text{ and } z_3 \in [1.17, 1.48]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-2.4, -1.4], \; z_2 \in [-0.77, -0.75], \text{ and } z_3 \in [2.78, 3.13]$

* This is the solution!

D. $z_1 \in [-3.3, -1.7], \; z_2 \in [0.3, 0.41], \text{ and } z_3 \in [2.78, 3.13]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-1.4, -1.1], \; z_2 \in [-0.7, -0.63], \text{ and } z_3 \in [2.78, 3.13]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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2. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 - 8x^2 - 40x - 29}{x - 3}$$

The solution is $8x^2 + 16x + 8 + \frac{-5}{x-3}$, which is option A.

A. $a \in [6, 12], \; b \in [14, 19], \; c \in [6, 9], \text{ and } r \in [-5, 2].$

* This is the solution!

B. $a \in [6, 12], \; b \in [3, 10], \; c \in [-27, -22], \text{ and } r \in [-80, -72].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [6, 12], \; b \in [-32, -31], \; c \in [54, 57], \text{ and } r \in [-197, -193].$

You divided by the opposite of the factor.

D. $a \in [24, 32], \; b \in [63, 69], \; c \in [152, 155], \text{ and } r \in [427, 428].$

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [24, 32]$, $b \in [-83, -75]$, $c \in [199, 207]$, and $r \in [-634, -628]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 + 26x^2 - 28}{x + 4}$$

The solution is $6x^2 + 2x - 8 + \dfrac{4}{x+4}$, which is option C.

A. $a \in [1, 9], b \in [48, 55], c \in [200, 202]$, and $r \in [771, 774]$.

You divided by the opposite of the factor.

B. $a \in [1, 9], b \in [-4, 0], c \in [19, 28]$, and $r \in [-130, -124]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [1, 9], b \in [1, 5], c \in [-12, -4]$, and $r \in [-1, 10]$.

* This is the solution!

D. $a \in [-24, -22], b \in [-73, -66], c \in [-283, -279]$, and $r \in [-1154, -1140]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [-24, -22], b \in [121, 123], c \in [-491, -483]$, and $r \in [1921, 1929]$.

You multipled by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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4. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 16x^3 - 40x^2 + x + 30$$

The solution is $[-0.75, 1.25, 2]$, which is option D.

A. $z_1 \in [-2.25, -1.94]$, $z_2 \in [-1.04, -0.34]$, and $z_3 \in [0.79, 1.94]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-1.34, -0.77]$, $z_2 \in [0.55, 0.88]$, and $z_3 \in [1.9, 2.19]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-2.25, -1.94]$, $z_2 \in [-1.46, -1.14]$, and $z_3 \in [0.51, 0.89]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-1.28, -0.5]$, $z_2 \in [1.07, 1.34]$, and $z_3 \in [1.9, 2.19]$

* This is the solution!

E. $z_1 \in [-5.28, -4.63]$, $z_2 \in [-2.05, -1.9]$, and $z_3 \in [-0.1, 0.58]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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5. Factor the polynomial below completely, knowing that $x-4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 - 59x^3 - 50x^2 + 208x - 96$$

The solution is $[-2, 0.6, 1.333, 4]$, which is option E.

A. $z_1 \in [-4, -3]$, $z_2 \in [-1.68, -1.66], z_3 \in [-0.81, -0.63]$, and $z_4 \in [1.7, 2.7]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-2, 1]$, $z_2 \in [0.62, 0.9], z_3 \in [1.57, 1.69]$, and $z_4 \in [3.9, 5.2]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-4, -3]$, $z_2 \in [-1.44, -1.27], z_3 \in [-0.67, -0.6]$, and $z_4 \in [1.7, 2.7]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-4, -3]$, $z_2 \in [-3.02, -2.95], z_3 \in [-0.44, -0.14]$, and $z_4 \in [1.7, 2.7]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-2, 1]$, $z_2 \in [0.55, 0.65], z_3 \in [1.28, 1.46]$, and $z_4 \in [3.9, 5.2]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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6. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 - 4x^2 - 40x + 37}{x + 2}$$

The solution is $12x^2 - 28x + 16 + \dfrac{5}{x+2}$, which is option C.

A. $a \in [-32, -22]$, $b \in [-55, -47]$, $c \in [-151, -142]$, and $r \in [-251, -245]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [-32, -22]$, $b \in [36, 49]$, $c \in [-129, -124]$, and $r \in [292, 296]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [12, 13]$, $b \in [-32, -24]$, $c \in [11, 19]$, and $r \in [4, 9]$.

* This is the solution!

D. $a \in [12, 13]$, $b \in [-43, -38]$, $c \in [79, 82]$, and $r \in [-205, -196]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [12, 13]$, $b \in [18, 26]$, $c \in [0, 1]$, and $r \in [29, 47]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{16x^3 - 48x - 28}{x - 2}$$

The solution is $16x^2 + 32x + 16 + \frac{4}{x-2}$, which is option C.

A. $a \in [32, 33], b \in [-64, -59], c \in [80, 83]$, and $r \in [-195, -187]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

B. $a \in [12, 23], b \in [-35, -27], c \in [9, 21]$, and $r \in [-60, -53]$.

You divided by the opposite of the factor.

C. $a \in [12, 23], b \in [26, 33], c \in [9, 21]$, and $r \in [3, 5]$.

* This is the solution!

D. $a \in [32, 33], b \in [61, 67], c \in [80, 83]$, and $r \in [130, 141]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [12, 23], b \in [15, 17], c \in [-39, -28]$, and $r \in [-60, -53]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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8. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 2x^2 + 3x + 7$$

The solution is $\pm 1, \pm 7$, which is option C.

A. All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 2}$

This would have been the solution **if asked for the possible Rational roots**!

C. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

D. $\pm 1, \pm 2$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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9. Factor the polynomial below completely, knowing that $x-4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^4 - 80x^3 - 132x^2 + 224x - 64$$

The solution is $[-2, 0.4, 0.8, 4]$, which is option A.

A. $z_1 \in [-2, 2], \; z_2 \in [-0.17, 0.41], z_3 \in [0.76, 0.9], \text{ and } z_4 \in [3, 6]$

* This is the solution!

B. $z_1 \in [-5, -3], \; z_2 \in [-1.71, -0.25], z_3 \in [-0.63, -0.31], \text{ and } z_4 \in [1, 3]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-5, -3], \; z_2 \in [-2.13, -1.06], z_3 \in [-0.19, 0.06], \text{ and } z_4 \in [1, 3]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-2, 2], \; z_2 \in [1.21, 2.56], z_3 \in [2.4, 2.52], \text{ and } z_4 \in [3, 6]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-5, -3], \; z_2 \in [-2.8, -2.06], z_3 \in [-1.34, -1.19], \text{ and } z_4 \in [1, 3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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10. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^3 + 4x^2 + 4x + 7$$

The solution is $\pm 1, \pm 7$, which is option C.

A. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.
