This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 + 21x^2 - 7}{x + 2}$$

The solution is $9x^2 + 3x - 6 + \dfrac{5}{x+2}$, which is option D.

A. $a \in [8, 17], b \in [36, 48], c \in [77, 84],$ and $r \in [149, 151]$.

You divided by the opposite of the factor.

B. $a \in [8, 17], b \in [-8, -2], c \in [13, 21],$ and $r \in [-65, -60]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-18, -14], b \in [54, 60], c \in [-114, -113],$ and $r \in [219, 224]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [8, 17], b \in [3, 8], c \in [-11, -3],$ and $r \in [4, 14]$.

* This is the solution!

E. $a \in [-18, -14], b \in [-17, -10], c \in [-32, -25],$ and $r \in [-69, -66]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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2. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^4 + 6x^3 + 3x^2 + 7x + 2$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$, which is option B.

A. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 48x^2 - 116x - 43}{x - 4}$$

The solution is $20x^2 + 32x + 12 + \dfrac{5}{x-4}$, which is option C.

A. $a \in [19, 24], \; b \in [12, 13], \; c \in [-85, -79], \text{ and } r \in [-284, -280]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [19, 24], \; b \in [-131, -125], \; c \in [391, 398], \text{ and } r \in [-1632, -1622]$.

You divided by the opposite of the factor.

C. $a \in [19, 24], \; b \in [31, 38], \; c \in [8, 19], \text{ and } r \in [2, 9]$.

* This is the solution!

D. $a \in [80, 86], \; b \in [-370, -364], \; c \in [1356, 1360], \text{ and } r \in [-5472, -5466]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [80, 86], \; b \in [269, 275], \; c \in [969, 975], \text{ and } r \in [3842, 3846]$.

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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4. Factor the polynomial below completely, knowing that $x + 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^4 + 210x^3 + 507x^2 + 434x + 120$$

The solution is $[-5, -2, -0.8, -0.6]$, which is option D.

A. $z_1 \in [1.24, 1.46], \; z_2 \in [1.6, 1.94], z_3 \in [1.3, 2.2], \text{ and } z_4 \in [4.71, 5.09]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [0.1, 0.33], \; z_2 \in [1.89, 2.8], z_3 \in [2.8, 4.5], \text{ and } z_4 \in [4.71, 5.09]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-5.19, -4.79], \; z_2 \in [-2.35, -1.49], z_3 \in [-2, -1], \text{ and } z_4 \in [-1.56, -1.17]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-5.19, -4.79], \; z_2 \in [-2.35, -1.49], z_3 \in [-1.1, 1.6], \text{ and } z_4 \in [-0.93, 0.31]$

* This is the solution!

E. $z_1 \in [0.5, 0.72],\ z_2 \in [-0.1, 0.82], z_3 \in [1.3, 2.2],$ and $z_4 \in [4.71, 5.09]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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5. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 + 5x^2 - 22x - 24$$

The solution is $[-1.5, -1.33, 2]$, which is option A.

A. $z_1 \in [-1.67, -1.39],\ z_2 \in [-1.42, -1.18],$ and $z_3 \in [1.7, 2.6]$

* This is the solution!

B. $z_1 \in [-2.13, -1.96],\ z_2 \in [0.46, 0.55],$ and $z_3 \in [3.7, 4.4]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-2.13, -1.96],\ z_2 \in [0.62, 0.81],$ and $z_3 \in [-0.5, 1.2]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-2.13, -1.96],\ z_2 \in [1.22, 1.4],$ and $z_3 \in [1, 1.9]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-1.04, -0.67],\ z_2 \in [-0.83, -0.6],$ and $z_3 \in [1.7, 2.6]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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6. Factor the polynomial below completely, knowing that $x - 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 - 53x^3 - 23x^2 + 202x - 120$$

The solution is $[-2, 0.75, 1.667, 4]$, which is option A.

A. $z_1 \in [-3.4, -1.4],\ z_2 \in [0.68, 0.95], z_3 \in [1.54, 1.71],$ and $z_4 \in [4, 6]$

* This is the solution!

B. $z_1 \in [-3.4, -1.4],\ z_2 \in [0.52, 0.7], z_3 \in [1.2, 1.38],$ and $z_4 \in [4, 6]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-5.6, -4.6],\ z_2 \in [-4.05, -3.87], z_3 \in [-0.43, -0.2],$ and $z_4 \in [0, 3]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-4.7, -3.1],\ z_2 \in [-1.44, -1.16], z_3 \in [-0.71, -0.32],$ and $z_4 \in [0, 3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-4.7, -3.1],\ z_2 \in [-1.75, -1.65], z_3 \in [-0.85, -0.62],$ and $z_4 \in [0, 3]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 + 28x^2 - 33}{x + 3}$$

The solution is $8x^2 + 4x - 12 + \frac{3}{x+3}$, which is option A.

A. $a \in [5, 12], b \in [4, 6], c \in [-13, -3]$, and $r \in [0, 8]$.

   * This is the solution!

B. $a \in [5, 12], b \in [52, 57], c \in [156, 158]$, and $r \in [435, 437]$.

   You divided by the opposite of the factor.

C. $a \in [5, 12], b \in [-6, 1], c \in [13, 19]$, and $r \in [-104, -92]$.

   You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [-24, -23], b \in [97, 102], c \in [-300, -290]$, and $r \in [864, 875]$.

   You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [-24, -23], b \in [-48, -40], c \in [-135, -124]$, and $r \in [-432, -427]$.

   You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 41x^2 - 54x + 45$$

The solution is $[-1.5, 0.6, 5]$, which is option E.

A. $z_1 \in [-6, -4.8]$, $z_2 \in [-0.8, -0.3]$, and $z_3 \in [1, 1.6]$

   Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-6, -4.8]$, $z_2 \in [-3.3, -2.7]$, and $z_3 \in [-0.7, 0.6]$

   Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-6, -4.8]$, $z_2 \in [-2.9, -1.5]$, and $z_3 \in [0.6, 0.9]$

   Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-1, -0.1]$, $z_2 \in [0.8, 2.1]$, and $z_3 \in [4.4, 5.9]$

   Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-1.9, -1.1]$, $z_2 \in [-0.1, 1.2]$, and $z_3 \in [4.4, 5.9]$

   * This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 + 45x^2 - 21x - 39}{x + 4}$$

The solution is $12x^2 - 3x - 9 + \frac{-3}{x+4}$, which is option E.

A. $a \in [8, 13], \; b \in [90, 102], \; c \in [342, 359], \text{ and } r \in [1363, 1367].$

You divided by the opposite of the factor.

B. $a \in [-49, -44], \; b \in [-152, -144], \; c \in [-611, -607], \text{ and } r \in [-2475, -2471].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [-49, -44], \; b \in [232, 242], \; c \in [-970, -961], \text{ and } r \in [3834, 3839].$

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [8, 13], \; b \in [-16, -10], \; c \in [53, 55], \text{ and } r \in [-310, -305].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [8, 13], \; b \in [-3, 4], \; c \in [-19, -8], \text{ and } r \in [-5, 4].$

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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10. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 3x^2 + 6x + 5$$

The solution is All combinations of: $\frac{\pm 1, \pm 5}{\pm 1, \pm 3}$, which is option B.

A. All combinations of: $\frac{\pm 1, \pm 3}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\frac{\pm 1, \pm 5}{\pm 1, \pm 3}$

* This is the solution **since we asked for the possible Rational roots**!

C. $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots**!

D. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.