This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 2i \text{ and } -3$$

The solution is $x^3 - 3x^2 - 5x + 39$, which is option D.

A. $b \in [1.1, 4], c \in [-5.1, -3.6], \text{ and } d \in [-40, -32]$

$x^3 + 3x^2 - 5x - 39$, which corresponds to multiplying out $(x - (3 - 2i))(x - (3 + 2i))(x - 3)$.

B. $b \in [-0.4, 2], c \in [-2.9, 0.1], \text{ and } d \in [-9, -4]$

$x^3 + x^2 - 9$, which corresponds to multiplying out $(x - 3)(x + 3)$.

C. $b \in [-0.4, 2], c \in [3.2, 8.7], \text{ and } d \in [4, 7]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 2)(x + 3)$.

D. $b \in [-6.7, -1], c \in [-5.1, -3.6], \text{ and } d \in [36, 41]$

* $x^3 - 3x^2 - 5x + 39$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 - 2i))(x - (3 + 2i))(x - (-3))$.

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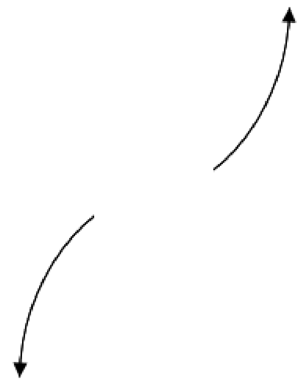
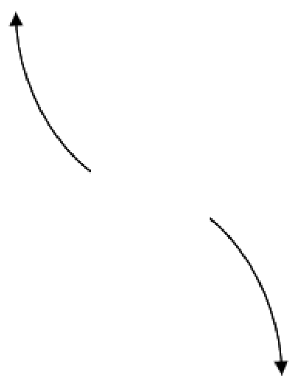
2. Describe the end behavior of the polynomial below.

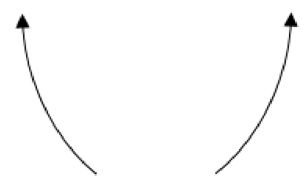
$$f(x) = 2(x - 9)^3(x + 9)^8(x - 7)^3(x + 7)^5$$

The solution is the graph below, which is option D.

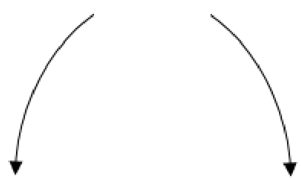




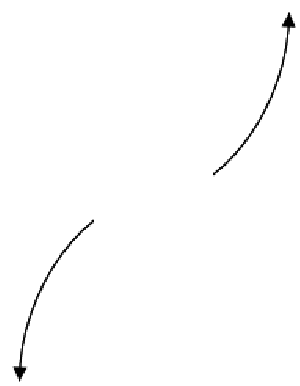
A.



C.



B.



D.

E. None of the above.

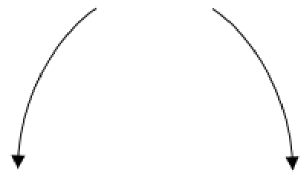
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

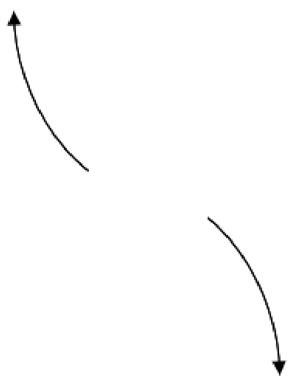
---

3. Describe the end behavior of the polynomial below.

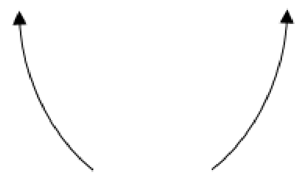
$$f(x) = -4(x-2)^5(x+2)^{10}(x-3)^5(x+3)^6$$

The solution is the graph below, which is option B.

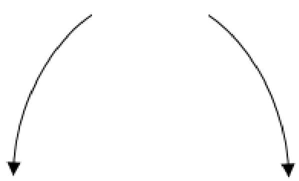




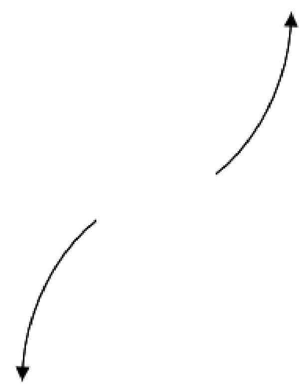
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-6, \frac{-3}{4}, \text{ and } \frac{7}{2}$$

The solution is $8x^3 + 26x^2 - 153x - 126$, which is option C.

A. $a \in [3, 10], b \in [-75, -66], c \in [108, 118], \text{ and } d \in [125, 128]$

$8x^3 - 70x^2 + 111x + 126$, which corresponds to multiplying out $(x - 6)(4x + 3)(2x - 7)$.

B. $a \in [3, 10], b \in [-26, -24], c \in [-154, -145], \text{ and } d \in [125, 128]$

$8x^3 - 26x^2 - 153x + 126$, which corresponds to multiplying out $(x - 6)(4x - 3)(2x + 7)$.

C. $a \in [3, 10], b \in [23, 33], c \in [-154, -145], \text{ and } d \in [-130, -119]$

* $8x^3 + 26x^2 - 153x - 126$, which is the correct option.

D. $a \in [3, 10], b \in [-89, -77], c \in [222, 233]$, and $d \in [-130, -119]$

$8x^3 - 82x^2 + 225x - 126$, which corresponds to multiplying out $(x - 6)(4x - 3)(2x - 7)$.

E. $a \in [3, 10], b \in [23, 33], c \in [-154, -145]$, and $d \in [125, 128]$

$8x^3 + 26x^2 - 153x + 126$, which corresponds to multiplying everything correctly except the constant term.

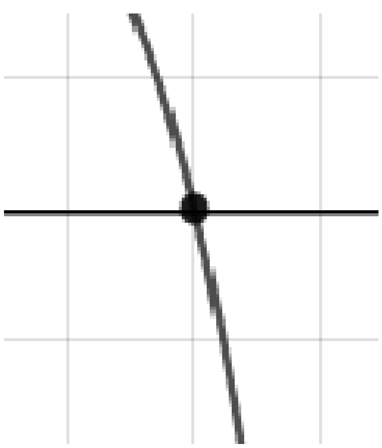
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+6)(4x+3)(2x - 7)$

---

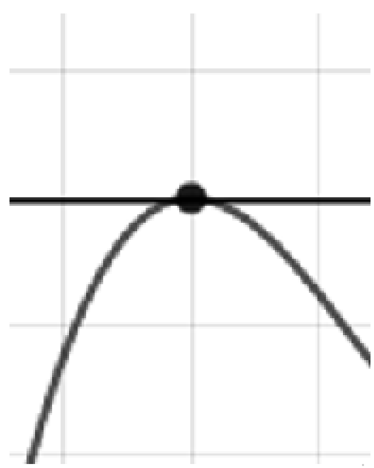
5. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = 6(x - 3)^4(x + 3)^9(x + 7)^4(x - 7)^8$$

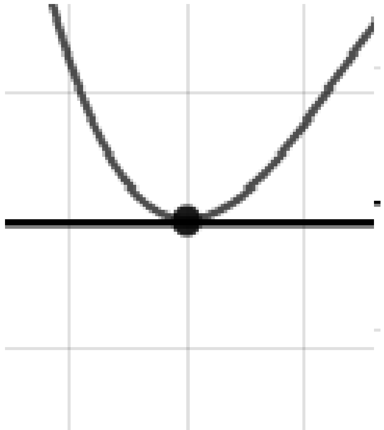
The solution is the graph below, which is option C.

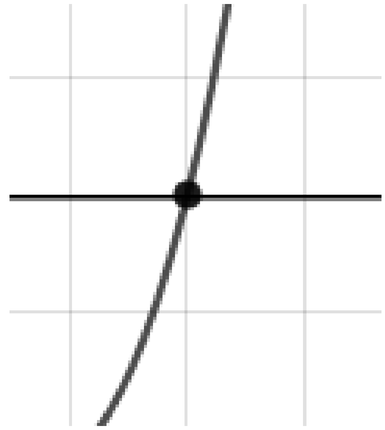


A.



B.

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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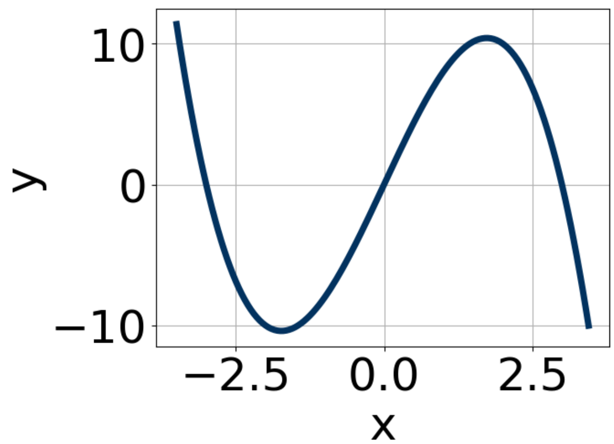
6. Which of the following equations *could* be of the graph presented below?



The solution is $-14x^{11}(x-3)^5(x+3)^9$, which is option C.

A. $5x^5(x-3)^{10}(x+3)^5$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-7x^9(x-3)^4(x+3)^9$

The factor 3 should have been an odd power.

C. $-14x^{11}(x-3)^5(x+3)^9$

* This is the correct option.

D. $-18x^9(x-3)^4(x+3)^8$

The factors 3 and -3 have have been odd power.

E. $17x^7(x-3)^5(x+3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 5i \text{ and } 3$$

The solution is $x^3 + x^2 + 17x - 87$, which is option A.

A. $b \in [0.2, 3.8], c \in [16.8, 19.7], \text{ and } d \in [-92, -81]$

* $x^3 + x^2 + 17x - 87$, which is the correct option.

B. $b \in [0.2, 3.8], c \in [-3.5, 0.3], \text{ and } d \in [-9, -3]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x + 2)(x - 3)$.

C. $b \in [-4.5, 0.5], c \in [16.8, 19.7], \text{ and } d \in [86, 92]$

$x^3 - 1x^2 + 17x + 87$, which corresponds to multiplying out $(x - (-2 - 5i))(x - (-2 + 5i))(x + 3)$.

D. $b \in [0.2, 3.8], c \in [1.8, 4.3], \text{ and } d \in [-18, -11]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x + 5)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

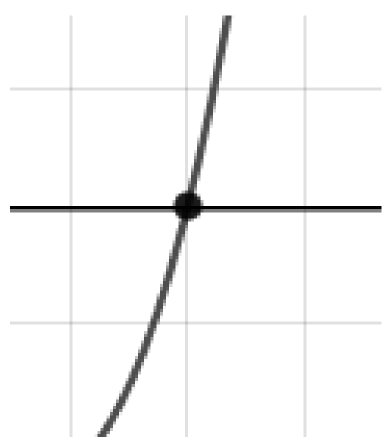
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 5i))(x - (-2 + 5i))(x - (3))$.

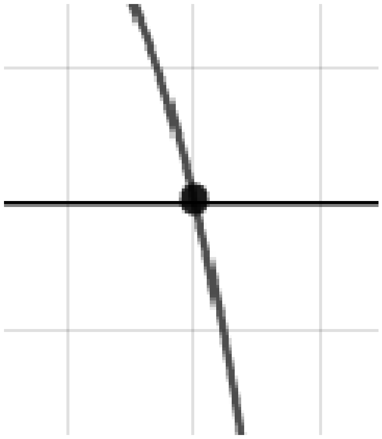
---

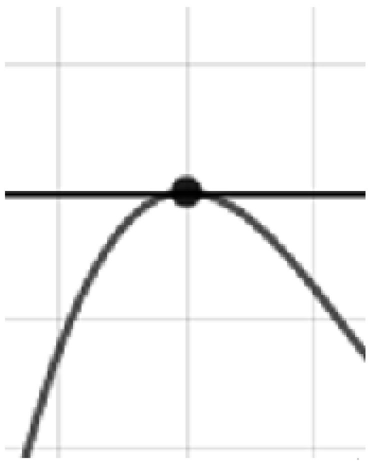
8. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

$$f(x) = -9(x - 6)^{11}(x + 6)^9(x - 5)^7(x + 5)^6$$

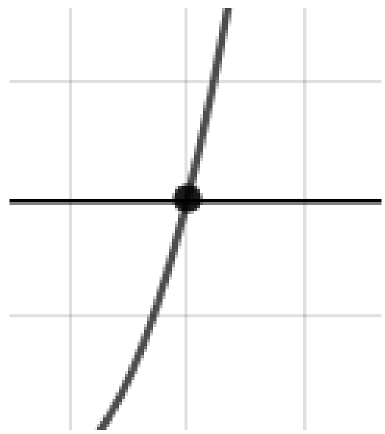
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{3}, 7, \text{ and } \frac{-7}{5}$$

The solution is $15x^3 - 109x^2 - 7x + 245$, which is option B.

A. $a \in [13, 24], b \in [143, 153], c \in [348, 358]$, and $d \in [239, 253]$

   $15x^3 + 151x^2 + 357x + 245$, which corresponds to multiplying out $(3x + 5)(x + 7)(5x + 7)$.

B. $a \in [13, 24], b \in [-110, -101], c \in [-10, -4]$, and $d \in [239, 253]$

   * $15x^3 - 109x^2 - 7x + 245$, which is the correct option.

C. $a \in [13, 24], b \in [-110, -101], c \in [-10, -4]$, and $d \in [-247, -238]$

   $15x^3 - 109x^2 - 7x - 245$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [13, 24], b \in [106, 114], c \in [-10, -4]$, and $d \in [-247, -238]$

   $15x^3 + 109x^2 - 7x - 245$, which corresponds to multiplying out $(3x + 5)(x + 7)(5x - 7)$.

E. $a \in [13, 24], b \in [-60, -56], c \in [-287, -277],$ and $d \in [-247, -238]$

$15x^3 - 59x^2 - 287x - 245$, which corresponds to multiplying out $(3x + 5)(x - 7)(5x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 5)(x - 7)(5x + 7)$

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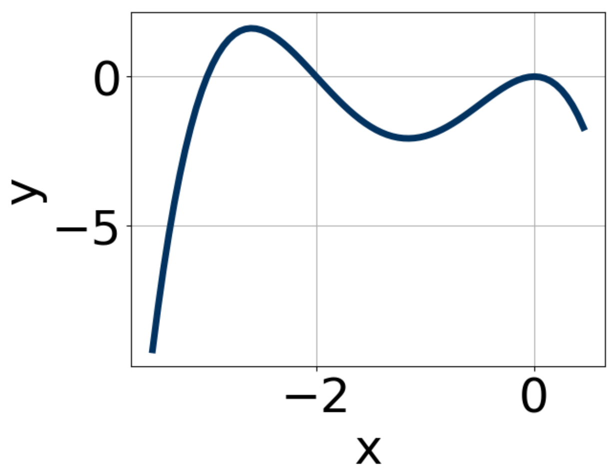
10. Which of the following equations *could* be of the graph presented below?



The solution is $-18x^6(x + 3)^{11}(x + 2)^9$, which is option E.

A. $9x^4(x + 3)^7(x + 2)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-20x^{10}(x + 3)^{10}(x + 2)^{11}$

The factor $(x + 3)$ should have an odd power.

C. $14x^4(x + 3)^9(x + 2)^8$

The factor $(x + 2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-13x^9(x + 3)^6(x + 2)^9$

The factor 0 should have an even power and the factor -3 should have an odd power.

E. $-18x^6(x + 3)^{11}(x + 2)^9$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).
