This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 2i \text{ and } 1$$

The solution is $x^3 - 11x^2 + 39x - 29$, which is option A.

A. $b \in [-18, -7], c \in [35.4, 40.7], \text{ and } d \in [-30.8, -28.8]$

* $x^3 - 11x^2 + 39x - 29$, which is the correct option.

B. $b \in [-6, 7], c \in [-10.4, -5.5], \text{ and } d \in [2.6, 6.3]$

$x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x - 5)(x - 1)$.

C. $b \in [-6, 7], c \in [-4.6, -2.6], \text{ and } d \in [-4.2, 2.7]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x - 2)(x - 1)$.

D. $b \in [10, 12], c \in [35.4, 40.7], \text{ and } d \in [24.9, 29.1]$

$x^3 + 11x^2 + 39x + 29$, which corresponds to multiplying out $(x - (5 + 2i))(x - (5 - 2i))(x + 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 2i))(x - (5 - 2i))(x - (1))$.

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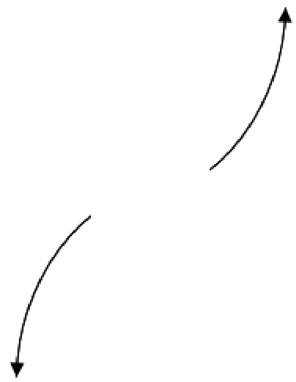
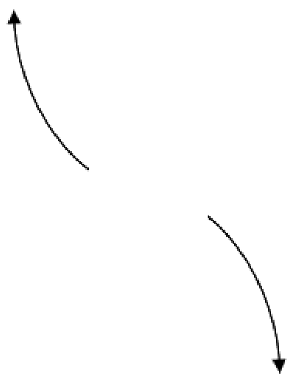
2. Describe the end behavior of the polynomial below.

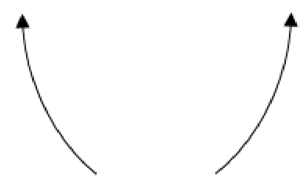
$$f(x) = 7(x - 4)^3(x + 4)^4(x - 8)^2(x + 8)^2$$

The solution is the graph below, which is option D.

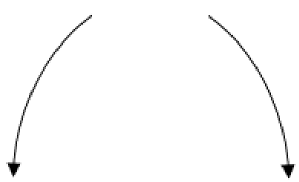




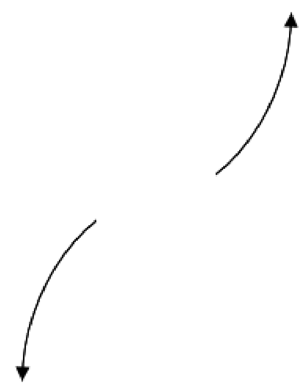
A.



C.



B.



D.

E. None of the above.

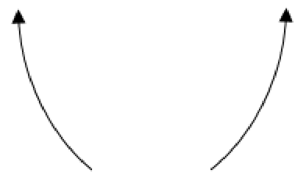
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

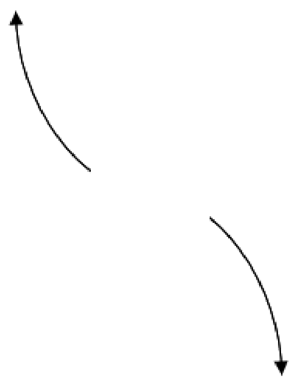
---

3. Describe the end behavior of the polynomial below.

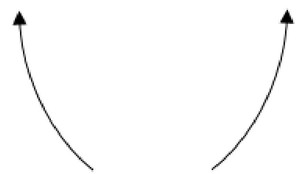
$$f(x) = 4(x+3)^2(x-3)^7(x+8)^5(x-8)^6$$

The solution is the graph below, which is option C.

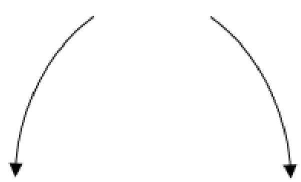




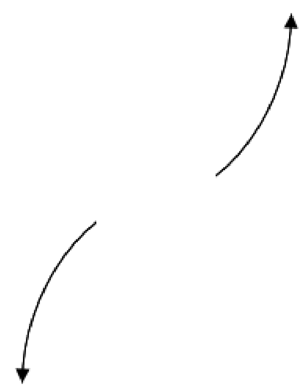
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{4}, -7, \text{ and } \frac{-1}{3}$$

The solution is $12x^3 + 97x^2 + 94x + 21$, which is option A.

A. $a \in [12, 14], b \in [94, 98], c \in [90, 105],$ and $d \in [20, 26]$

   * $12x^3 + 97x^2 + 94x + 21$, which is the correct option.

B. $a \in [12, 14], b \in [-99, -94], c \in [90, 105],$ and $d \in [-26, -20]$

   $12x^3 - 97x^2 + 94x - 21$, which corresponds to multiplying out $(4x - 3)(x - 7)(3x - 1)$.

C. $a \in [12, 14], b \in [-93, -88], c \in [31, 33],$ and $d \in [20, 26]$

   $12x^3 - 89x^2 + 32x + 21$, which corresponds to multiplying out $(4x - 3)(x - 7)(3x + 1)$.

D. $a \in [12, 14], b \in [78, 86], c \in [-43, -37],$ and $d \in [-26, -20]$

$12x^3 + 79x^2 - 38x - 21$, which corresponds to multiplying out $(4x - 3)(x + 7)(3x + 1)$.

E. $a \in [12, 14], b \in [94, 98], c \in [90, 105],$ and $d \in [-26, -20]$

$12x^3 + 97x^2 + 94x - 21$, which corresponds to multiplying everything correctly except the constant term.

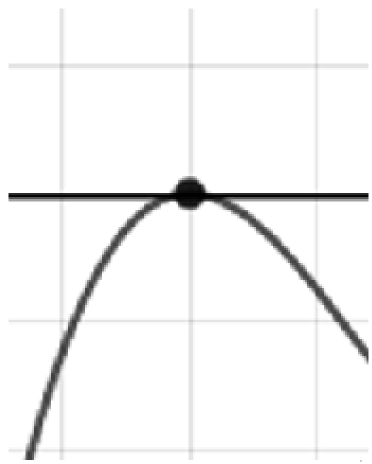
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 3)(x + 7)(3x + 1)$

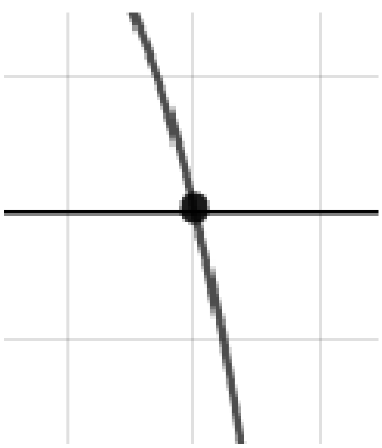
---

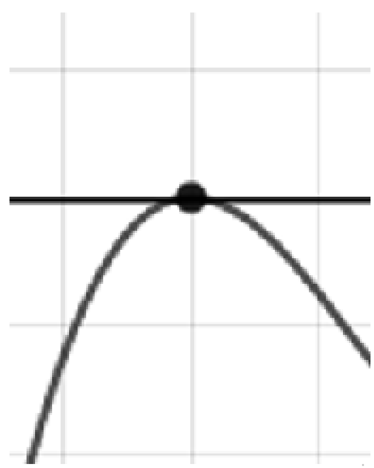
5. Describe the zero behavior of the zero $x = -9$ of the polynomial below.

$$f(x) = 2(x-4)^{10}(x+4)^{6}(x+9)^{10}(x-9)^{7}$$

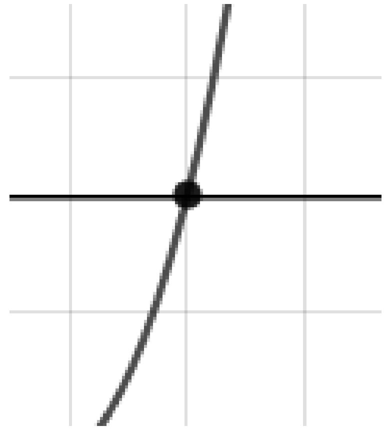
The solution is the graph below, which is option B.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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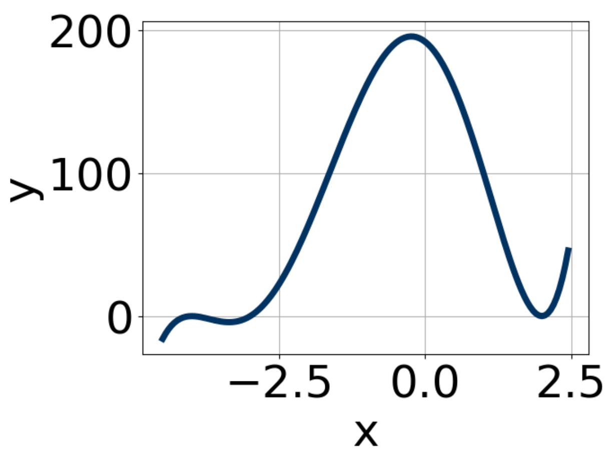
6. Which of the following equations *could* be of the graph presented below?



The solution is $6(x+4)^4(x-2)^8(x+3)^7$, which is option E.

A. $15(x+4)^6(x-2)^7(x+3)^5$

The factor $(x-2)$ should have an even power.

B. $-6(x+4)^{10}(x-2)^6(x+3)^8$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $17(x+4)^{10}(x-2)^7(x+3)^{10}$

The factor $(x-2)$ should have an even power and the factor $(x+3)$ should have an odd power.

D. $-19(x+4)^8(x-2)^8(x+3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $6(x+4)^4(x-2)^8(x+3)^7$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 + 3i \text{ and } 1$$

The solution is $x^3 - 5x^2 + 17x - 13$, which is option B.

A. $b \in [-1.8, 1.9], c \in [-4.15, -3.21]$, and $d \in [2.99, 3.53]$

$x^3 + x^2 - 4x + 3$, which corresponds to multiplying out $(x - 3)(x - 1)$.

B. $b \in [-9.1, -3.5], c \in [16.78, 18.65]$, and $d \in [-14.03, -11.89]$

* $x^3 - 5x^2 + 17x - 13$, which is the correct option.

C. $b \in [4.7, 5.3], c \in [16.78, 18.65]$, and $d \in [11.64, 13.41]$

$x^3 + 5x^2 + 17x + 13$, which corresponds to multiplying out $(x - (2 + 3i))(x - (2 - 3i))(x + 1)$.

D. $b \in [-1.8, 1.9], c \in [-3.38, -1.37]$, and $d \in [1.75, 2.85]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x - 2)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 + 3i))(x - (2 - 3i))(x - (1))$.

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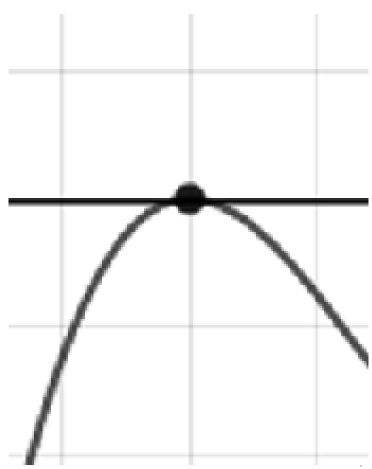
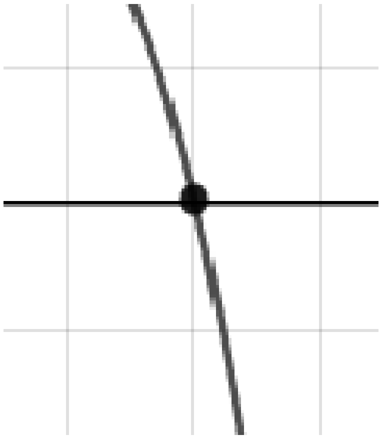
8. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

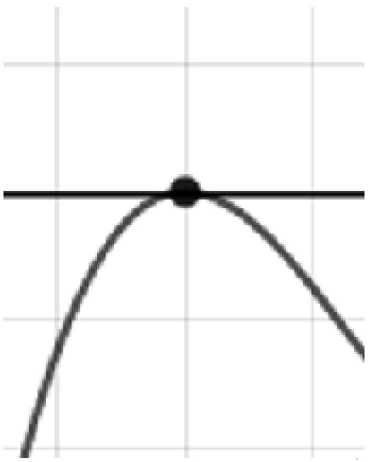
$$f(x) = 6(x - 4)^7(x + 4)^{12}(x + 3)^4(x - 3)^6$$

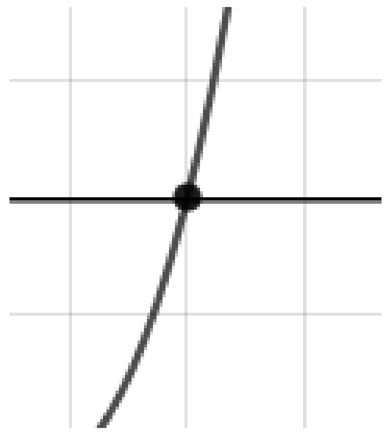
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{2}, \frac{5}{4}, \text{ and } \frac{-1}{5}$$

The solution is $40x^3 - 62x^2 + 11x + 5$, which is option A.

A. $a \in [37, 43], b \in [-63, -59], c \in [10, 12], \text{ and } d \in [4, 9]$

* $40x^3 - 62x^2 + 11x + 5$, which is the correct option.

B. $a \in [37, 43], b \in [-24, -15], c \in [-38, -26], \text{ and } d \in [-12, -4]$

$40x^3 - 22x^2 - 31x - 5$, which corresponds to multiplying out $(2x + 1)(4x - 5)(5x + 1)$.

C. $a \in [37, 43], b \in [-63, -59], c \in [10, 12], \text{ and } d \in [-12, -4]$

$40x^3 - 62x^2 + 11x - 5$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [37, 43], b \in [74, 85], c \in [38, 41], \text{ and } d \in [4, 9]$

$40x^3 + 78x^2 + 39x + 5$, which corresponds to multiplying out $(2x + 1)(4x + 5)(5x + 1)$.

E. $a \in [37, 43], b \in [55, 66], c \in [10, 12]$, and $d \in [-12, -4]$

$40x^3 + 62x^2 + 11x - 5$, which corresponds to multiplying out $(2x + 1)(4x + 5)(5x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 1)(4x - 5)(5x + 1)$

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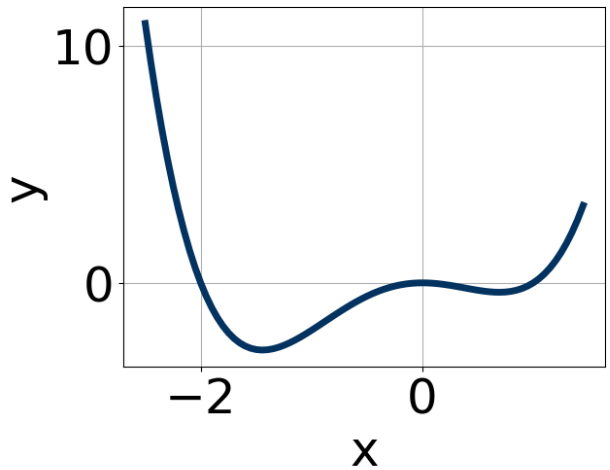
10. Which of the following equations *could* be of the graph presented below?



The solution is $4x^4(x - 1)^{11}(x + 2)^{11}$, which is option D.

A. $-2x^8(x - 1)^5(x + 2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $13x^9(x - 1)^4(x + 2)^9$

The factor 0 should have an even power and the factor 1 should have an odd power.

C. $-14x^8(x - 1)^7(x + 2)^{10}$

The factor $(x + 2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $4x^4(x - 1)^{11}(x + 2)^{11}$

* This is the correct option.

E. $7x^8(x - 1)^6(x + 2)^7$

The factor $(x - 1)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).
