This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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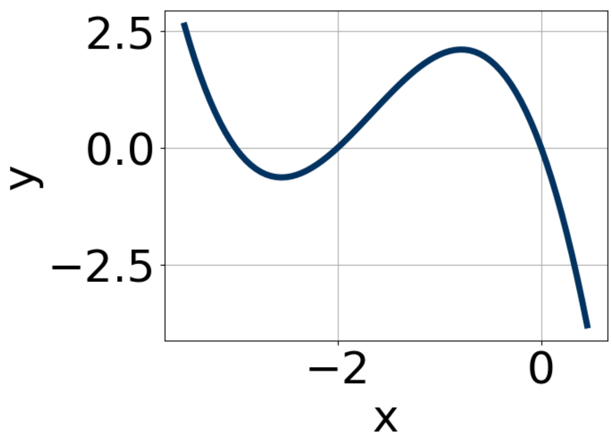
1. Which of the following equations *could* be of the graph presented below?



The solution is $-19x^{11}(x+2)^9(x+3)^9$, which is option A.

A. $-19x^{11}(x+2)^9(x+3)^9$

* This is the correct option.

B. $8x^7(x+2)^{11}(x+3)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-2x^9(x+2)^{10}(x+3)^8$

The factors -2 and -3 have have been odd power.

D. $-19x^5(x+2)^8(x+3)^9$

The factor -2 should have been an odd power.

E. $15x^9(x+2)^6(x+3)^7$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$\frac{-3}{2}, -7, \text{ and } \frac{7}{2}$$

The solution is $4x^3+20x^2-77x-147$, which is option A.

A. $a \in [1,7], b \in [20,23], c \in [-77,-71],$ and $d \in [-149,-143]$

* $4x^3 + 20x^2 - 77x - 147$, which is the correct option.

B. $a \in [1,7], b \in [-20,-13], c \in [-77,-71],$ and $d \in [147,150]$

$4x^3 - 20x^2 - 77x + 147$, which corresponds to multiplying out $(2x-3)(x-7)(2x+7)$.

C. $a \in [1,7], b \in [6,15], c \in [-120,-118],$ and $d \in [147,150]$

$4x^3 + 8x^2 - 119x + 147$, which corresponds to multiplying out $(2x-3)(x+7)(2x-7)$.

D. $a \in [1,7], b \in [20,23], c \in [-77,-71],$ and $d \in [147,150]$

$4x^3 + 20x^2 - 77x + 147$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [1,7], b \in [-52,-47], c \in [158,164],$ and $d \in [-149,-143]$

$4x^3 - 48x^2 + 161x - 147$, which corresponds to multiplying out $(2x-3)(x-7)(2x-7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x+3)(x+7)(2x-7)$

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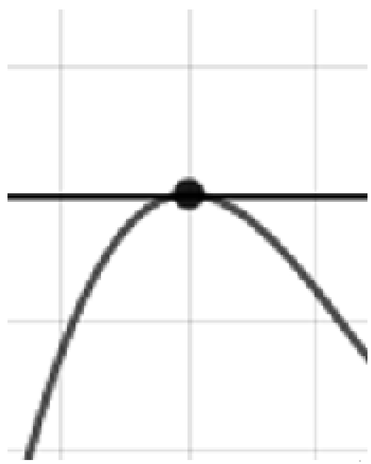
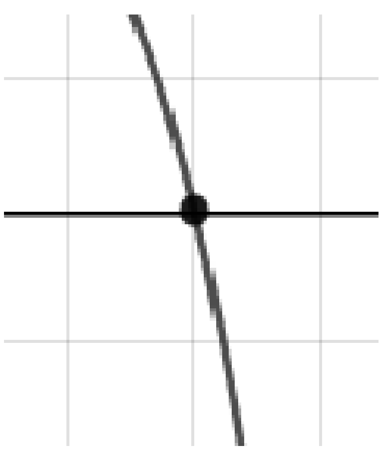
3. Describe the zero behavior of the zero $x = -7$ of the polynomial below.

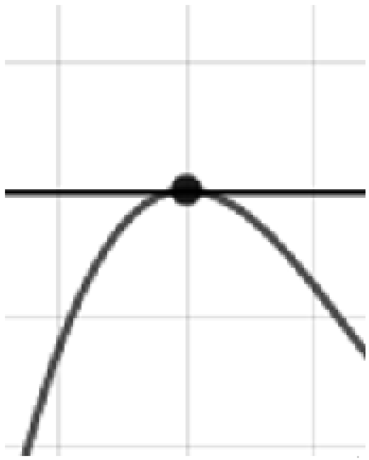
$$f(x) = -2(x-4)^8(x+4)^5(x+7)^{10}(x-7)^9$$

The solution is the graph below, which is option B.

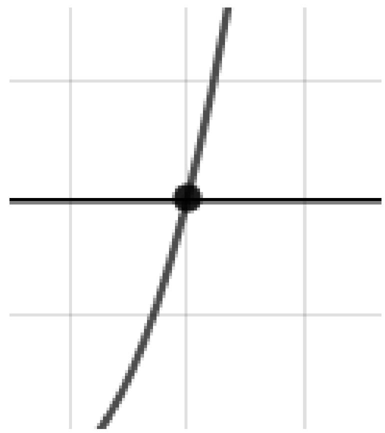




A.

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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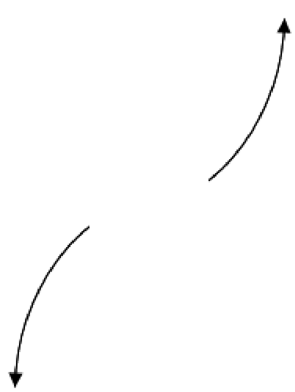
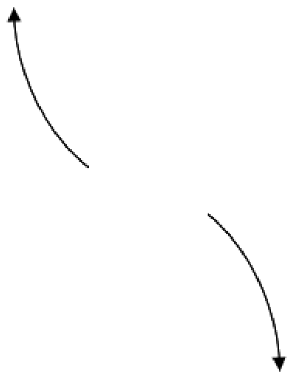
4. Describe the end behavior of the polynomial below.

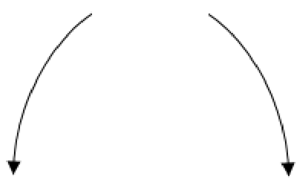
$$f(x) = 7(x-7)^2(x+7)^3(x-8)^5(x+8)^5$$

The solution is the graph below, which is option D.

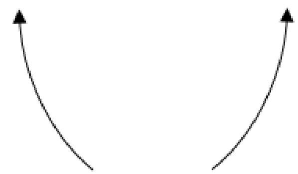




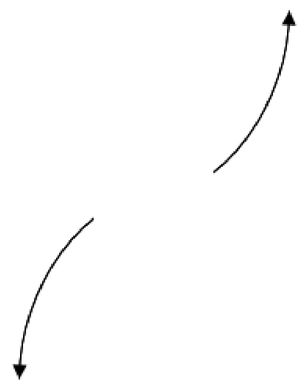
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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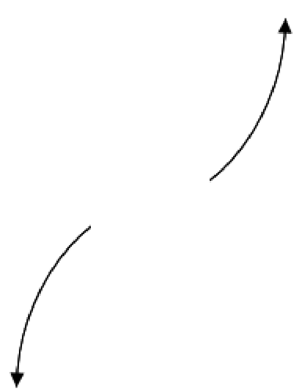
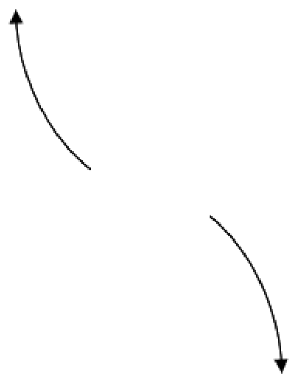
5. Describe the end behavior of the polynomial below.

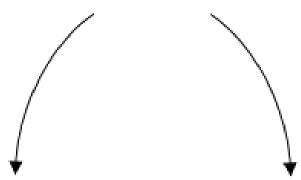
$$f(x) = 2(x-2)^4(x+2)^7(x-4)^2(x+4)^2$$

The solution is the graph below, which is option D.

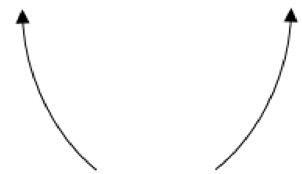




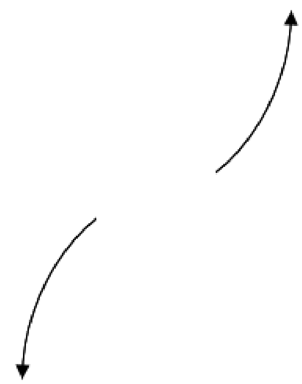
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 4i \text{ and } 4$$

The solution is $x^3 + 4x^2 - 128$, which is option D.

A. $b \in [0.9, 3.2], c \in [-3, 3], \text{ and } d \in [-18, -14]$

$x^3 + x^2 - 16$, which corresponds to multiplying out $(x + 4)(x - 4)$.

B. $b \in [0.9, 3.2], c \in [-10, -7], \text{ and } d \in [13, 21]$

$x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

C. $b \in [-7.8, -3.9], c \in [-3, 3], \text{ and } d \in [121, 134]$

$x^3 - 4x^2 + 128$, which corresponds to multiplying out $(x - (-4 + 4i))(x - (-4 - 4i))(x + 4)$.

D. $b \in [3.1, 5.5], c \in [-3, 3], \text{ and } d \in [-130, -123]$

* $x^3 + 4x^2 - 128$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 4i))(x - (-4 - 4i))(x - (4))$.

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-3, \frac{-5}{2}, \text{ and } \frac{-1}{5}$$

The solution is $10x^3 + 57x^2 + 86x + 15$, which is option A.

A. $a \in [10, 15], b \in [51, 61], c \in [80, 87], \text{ and } d \in [11, 19]$

* $10x^3 + 57x^2 + 86x + 15$, which is the correct option.

B. $a \in [10, 15], b \in [-53, -46], c \in [53, 71], \text{ and } d \in [11, 19]$

$10x^3 - 53x^2 + 64x + 15$, which corresponds to multiplying out $(x - 3)(2x - 5)(5x + 1)$.

C. $a \in [10, 15], b \in [-57, -54], c \in [80, 87], \text{ and } d \in [-17, -7]$

$10x^3 - 57x^2 + 86x - 15$, which corresponds to multiplying out $(x - 3)(2x - 5)(5x - 1)$.

D. $a \in [10, 15], b \in [-3, 6], c \in [-81, -75], \text{ and } d \in [-17, -7]$

$10x^3 - 3x^2 - 76x - 15$, which corresponds to multiplying out $(x - 3)(2x + 5)(5x + 1)$.

E. $a \in [10, 15], b \in [51, 61], c \in [80, 87], \text{ and } d \in [-17, -7]$

$10x^3 + 57x^2 + 86x - 15$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+3)(2x+5)(5x+1)$

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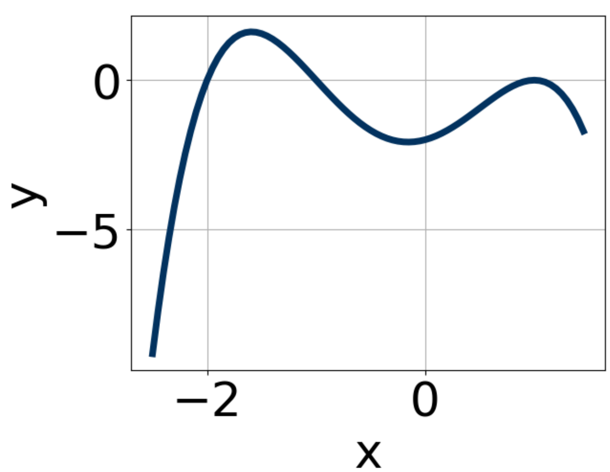
8. Which of the following equations *could* be of the graph presented below?



The solution is $-15(x - 1)^8(x + 1)^{11}(x + 2)^7$, which is option A.

A. $-15(x - 1)^8(x + 1)^{11}(x + 2)^7$

* This is the correct option.

B. $-11(x-1)^8(x+1)^6(x+2)^9$

The factor $(x+1)$ should have an odd power.

C. $16(x-1)^8(x+1)^9(x+2)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-14(x-1)^7(x+1)^8(x+2)^9$

The factor 1 should have an even power and the factor -1 should have an odd power.

E. $11(x-1)^4(x+1)^{11}(x+2)^4$

The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form x^3+bx^2+cx+d.

$$-3-4i \text{ and } -2$$

The solution is $x^3+8x^2+37x+50$, which is option C.

A. $b \in [-1,5], c \in [1.8,5.3], \text{ and } d \in [5.8,6.4]$

x^3+x^2+5x+6, which corresponds to multiplying out $(x+3)(x+2)$.

B. $b \in [-1,5], c \in [5.2,8.8], \text{ and } d \in [7.7,10.7]$

x^3+x^2+6x+8, which corresponds to multiplying out $(x+4)(x+2)$.

C. $b \in [4,9], c \in [35.8,38.9], \text{ and } d \in [46.8,52.1]$

* $x^3+8x^2+37x+50$, which is the correct option.

D. $b \in [-9,-3], c \in [35.8,38.9], \text{ and } d \in [-50.4,-49]$

$x^3-8x^2+37x-50$, which corresponds to multiplying out $(x-(-3-4i))(x-(-3+4i))(x-2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-3-4i))(x-(-3+4i))(x-(-2))$.

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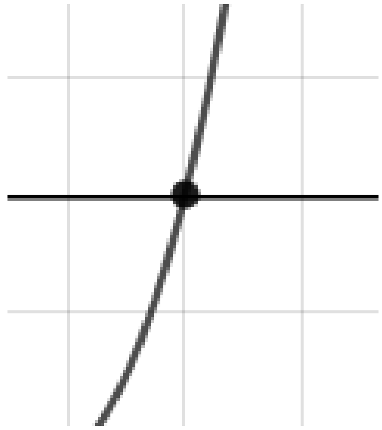
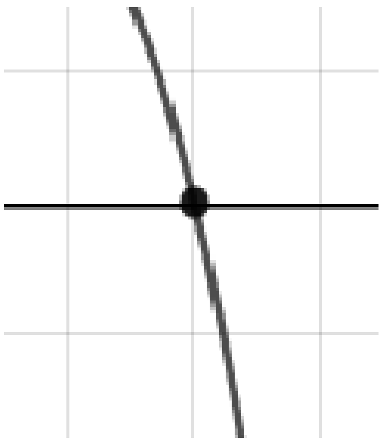
10. Describe the zero behavior of the zero $x=-8$ of the polynomial below.

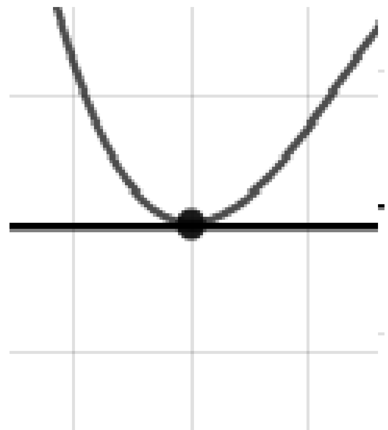
$$f(x) = -2(x-8)^8(x+8)^{11}(x+9)^9(x-9)^{13}$$

The solution is the graph below, which is option D.

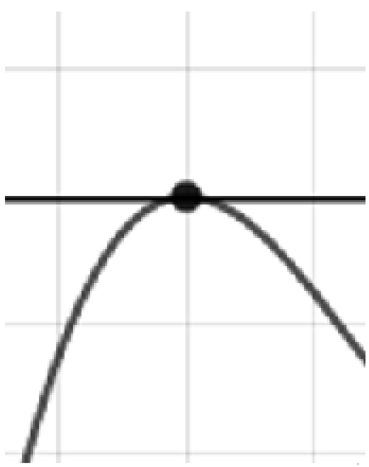




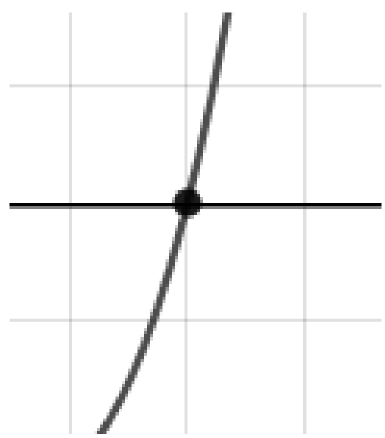
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.