This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{2}{3}, -7, \text{ and } \frac{7}{5}$$

The solution is $15x^3 + 74x^2 - 203x + 98$, which is option A.

A. $a \in [14, 16], b \in [74, 75], c \in [-204, -195], \text{ and } d \in [97, 102]$

* $15x^3 + 74x^2 - 203x + 98$, which is the correct option.

B. $a \in [14, 16], b \in [74, 75], c \in [-204, -195], \text{ and } d \in [-98, -96]$

$15x^3 + 74x^2 - 203x - 98$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [14, 16], b \in [83, 101], c \in [-98, -83], \text{ and } d \in [-98, -96]$

$15x^3 + 94x^2 - 91x - 98$, which corresponds to multiplying out $(3x + 2)(x + 7)(5x - 7)$.

D. $a \in [14, 16], b \in [-81, -66], c \in [-204, -195], \text{ and } d \in [-98, -96]$

$15x^3 - 74x^2 - 203x - 98$, which corresponds to multiplying out $(3x + 2)(x - 7)(5x + 7)$.

E. $a \in [14, 16], b \in [-116, -113], c \in [62, 71], \text{ and } d \in [97, 102]$

$15x^3 - 116x^2 + 63x + 98$, which corresponds to multiplying out $(3x + 2)(x - 7)(5x - 7)$.

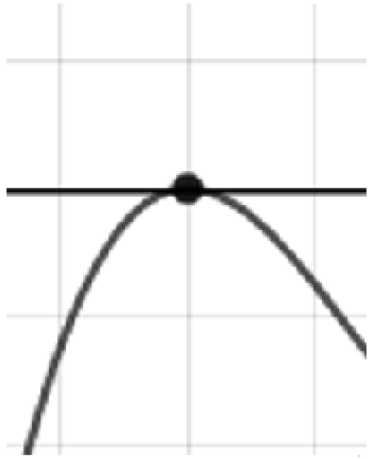
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 2)(x + 7)(5x - 7)$

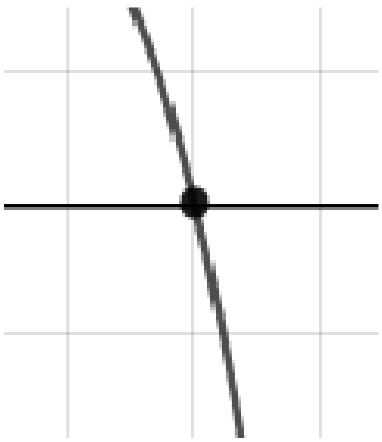
---

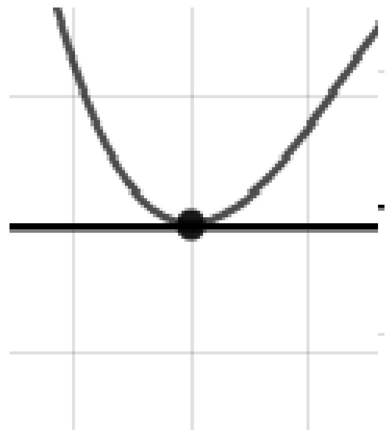
2. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

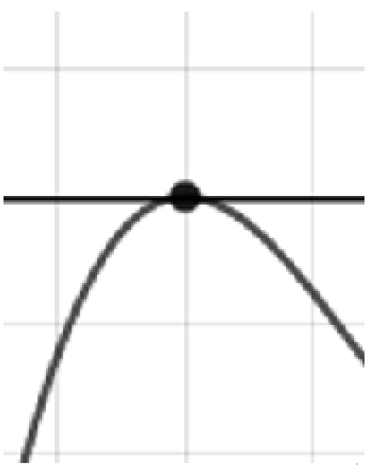
$$f(x) = -4(x + 8)^7(x - 8)^{10}(x - 4)^4(x + 4)^8$$

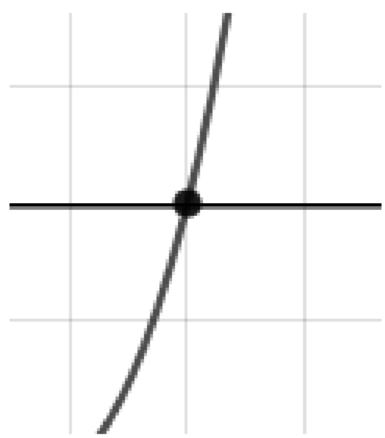
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 4i \text{ and } 4$$

The solution is $x^3 + 4x - 80$, which is option D.

A. $b \in [0.9, 2.6], c \in [-10, -4.4]$, and $d \in [15, 18]$

$x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x-4)(x-4)$.

B. $b \in [-3.1, 0.1], c \in [2.6, 4.7]$, and $d \in [79, 82]$

$x^3 + 4x + 80$, which corresponds to multiplying out $(x - (-2+4i))(x - (-2-4i))(x+4)$.

C. $b \in [0.9, 2.6], c \in [-6.7, 0.2]$, and $d \in [-12, -6]$

$x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x+2)(x-4)$.

D. $b \in [-3.1, 0.1], c \in [2.6, 4.7]$, and $d \in [-82, -75]$

* $x^3 + 4x - 80$, which is the correct option.

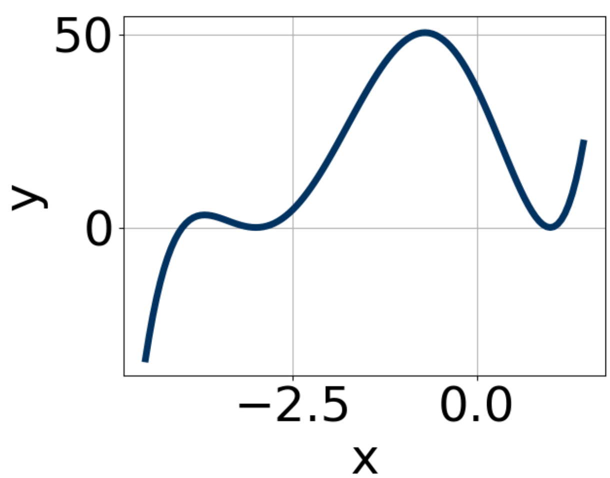
E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2+4i))(x - (-2-4i))(x - (4))$.

---

4. Which of the following equations *could* be of the graph presented below?



The solution is $15(x-1)^4(x+3)^8(x+4)^5$, which is option B.

A. $3(x-1)^8(x+3)^7(x+4)^9$

The factor $(x+3)$ should have an even power.

B. $15(x-1)^4(x+3)^8(x+4)^5$

* This is the correct option.

C. $13(x-1)^{10}(x+3)^7(x+4)^6$

The factor $(x+3)$ should have an even power and the factor $(x+4)$ should have an odd power.

D. $-5(x-1)^6(x+3)^4(x+4)^4$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-11(x-1)^{10}(x+3)^{10}(x+4)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

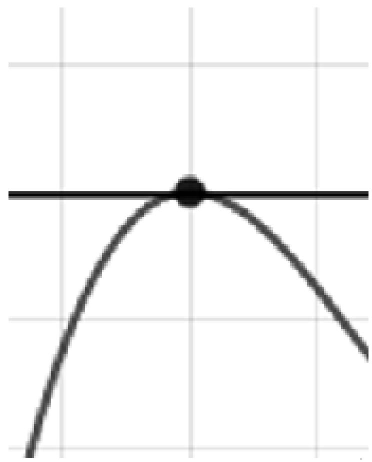
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

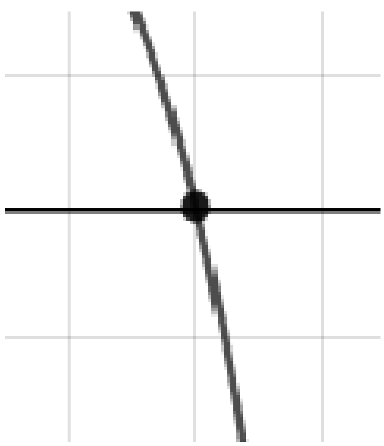
---

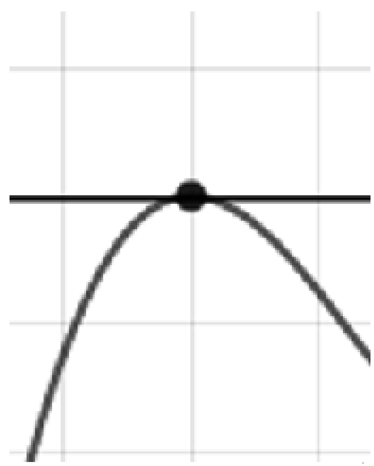
5. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

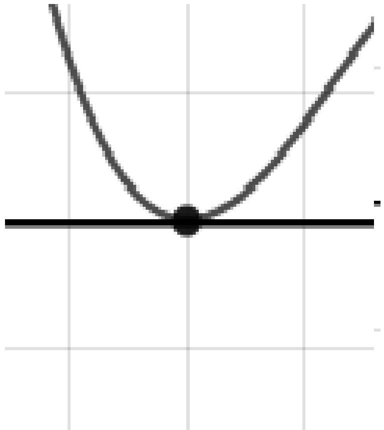
$$f(x) = 6(x+8)^4(x-8)^2(x-5)^5(x+5)^2$$

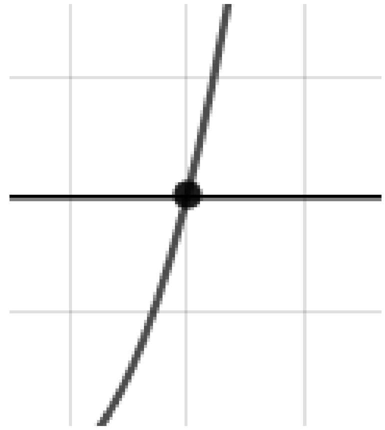
The solution is the graph below, which is option B.



A. 

B. 

C. 

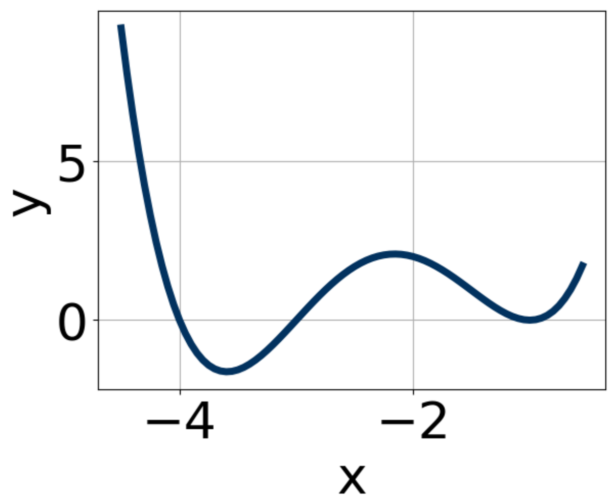
D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

6. Which of the following equations *could* be of the graph presented below?



The solution is $7(x+1)^8(x+3)^9(x+4)^{11}$, which is option C.

A. $-7(x+1)^6(x+3)^9(x+4)^7$

   This corresponds to the leading coefficient being the opposite value than it should be.

B. $3(x+1)^5(x+3)^4(x+4)^5$

   The factor -1 should have an even power and the factor -3 should have an odd power.

C. $7(x+1)^8(x+3)^9(x+4)^{11}$

   * This is the correct option.

D. $-7(x+1)^4(x+3)^9(x+4)^{10}$

   The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $17(x+1)^6(x+3)^8(x+4)^5$

The factor $(x+3)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 4i \text{ and } 2$$

The solution is $x^3 - 12x^2 + 61x - 82$, which is option A.

A. $b \in [-20, -7], c \in [60, 64.2], \text{ and } d \in [-82.1, -78.6]$

* $x^3 - 12x^2 + 61x - 82$, which is the correct option.

B. $b \in [-4, 6], c \in [-9.6, -6.6], \text{ and } d \in [8.9, 14]$

$x^3 + x^2 - 7x + 10$, which corresponds to multiplying out $(x-5)(x-2)$.

C. $b \in [12, 16], c \in [60, 64.2], \text{ and } d \in [79, 82.4]$

$x^3 + 12x^2 + 61x + 82$, which corresponds to multiplying out $(x-(5+4i))(x-(5-4i))(x+2)$.

D. $b \in [-4, 6], c \in [-6.7, -2.2], \text{ and } d \in [4.9, 9.8]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x-4)(x-2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

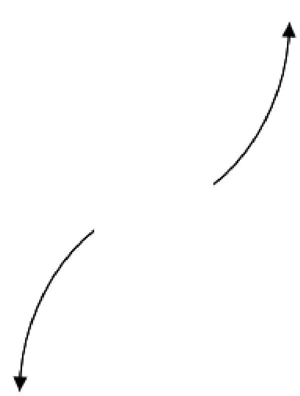
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x-(5+4i))(x-(5-4i))(x-(2))$.

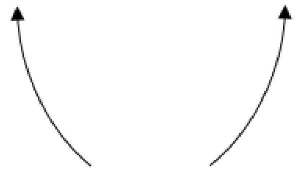
---

8. Describe the end behavior of the polynomial below.

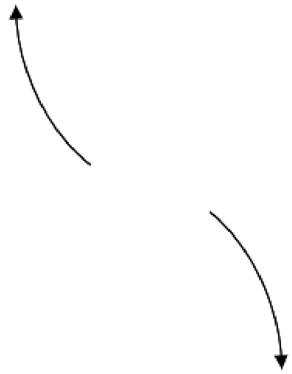
$$f(x) = 7(x+8)^4(x-8)^7(x+3)^3(x-3)^3$$

The solution is the graph below, which is option D.

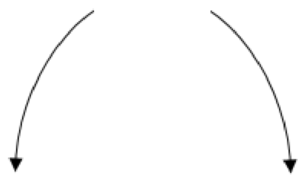




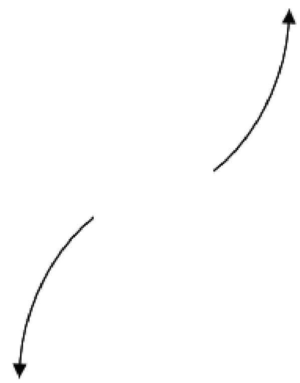
C.



A.



B.



D.

E. None of the above.

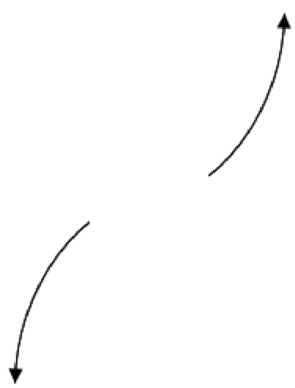
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

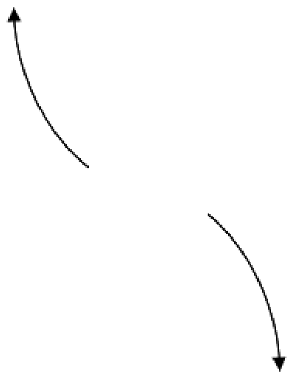
---

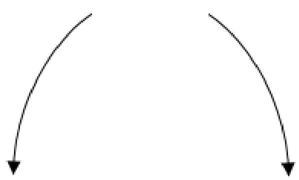
9. Describe the end behavior of the polynomial below.

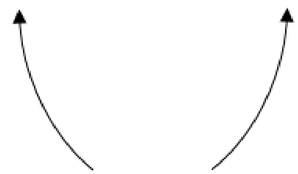
$$f(x) = 5(x+5)^3(x-5)^8(x-7)^3(x+7)^3$$

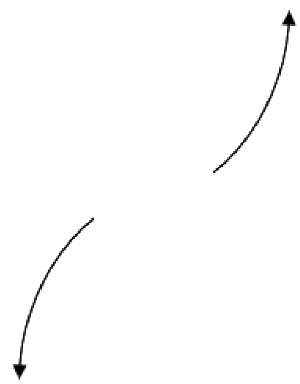
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{2}, \frac{-4}{3}, \text{ and } \frac{-1}{4}$$

The solution is $24x^3 + 74x^2 + 65x + 12$, which is option B.

A. $a \in [21, 26], b \in [2, 9], c \in [-61, -45], \text{ and } d \in [-12, -9]$

$24x^3 + 2x^2 - 49x - 12$, which corresponds to multiplying out $(2x - 3)(3x + 4)(4x + 1)$.

B. $a \in [21, 26], b \in [69, 75], c \in [60, 68], \text{ and } d \in [9, 16]$

* $24x^3 + 74x^2 + 65x + 12$, which is the correct option.

C. $a \in [21, 26], b \in [-68, -61], c \in [27, 37], \text{ and } d \in [9, 16]$

$24x^3 - 62x^2 + 31x + 12$, which corresponds to multiplying out $(2x - 3)(3x - 4)(4x + 1)$.

D. $a \in [21, 26], b \in [69, 75], c \in [60, 68], \text{ and } d \in [-12, -9]$

$24x^3 + 74x^2 + 65x - 12$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [21, 26], b \in [-77, -65], c \in [60, 68], \text{ and } d \in [-12, -9]$

$24x^3 - 74x^2 + 65x - 12$, which corresponds to multiplying out $(2x - 3)(3x - 4)(4x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 3)(3x + 4)(4x + 1)$