This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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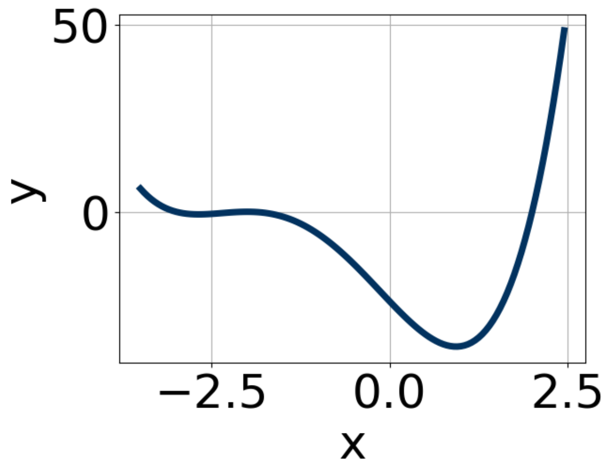
1. Which of the following equations *could* be of the graph presented below?



The solution is $13(x+2)^4(x-2)^{11}(x+3)^{11}$, which is option C.

A. $19(x+2)^7(x-2)^4(x+3)^9$

The factor -2 should have an even power and the factor 2 should have an odd power.

B. $20(x+2)^{10}(x-2)^8(x+3)^7$

The factor $(x-2)$ should have an odd power.

C. $13(x+2)^4(x-2)^{11}(x+3)^{11}$

* This is the correct option.

D. $-18(x+2)^4(x-2)^7(x+3)^4$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-4(x+2)^{10}(x-2)^{11}(x+3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, \frac{7}{5}, \text{ and } 2$$

The solution is $20x^3 - 73x^2 + 73x - 14$, which is option C.

A. $a \in [17, 26], b \in [68, 75], c \in [69, 74],$ and $d \in [10, 16]$

$20x^3 + 73x^2 + 73x + 14$, which corresponds to multiplying out $(4x + 1)(5x + 7)(x + 2)$.

B. $a \in [17, 26], b \in [-7, -6], c \in [-59, -56],$ and $d \in [-18, -13]$

$20x^3 - 7x^2 - 59x - 14$, which corresponds to multiplying out $(4x + 1)(5x + 7)(x - 2)$.

C. $a \in [17, 26], b \in [-73, -66], c \in [69, 74],$ and $d \in [-18, -13]$

* $20x^3 - 73x^2 + 73x - 14$, which is the correct option.

D. $a \in [17, 26], b \in [-73, -66], c \in [69, 74],$ and $d \in [10, 16]$

$20x^3 - 73x^2 + 73x + 14$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [17, 26], b \in [-66, -58], c \in [34, 45],$ and $d \in [10, 16]$

$20x^3 - 63x^2 + 39x + 14$, which corresponds to multiplying out $(4x + 1)(5x - 7)(x - 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(5x - 7)(x - 2)$

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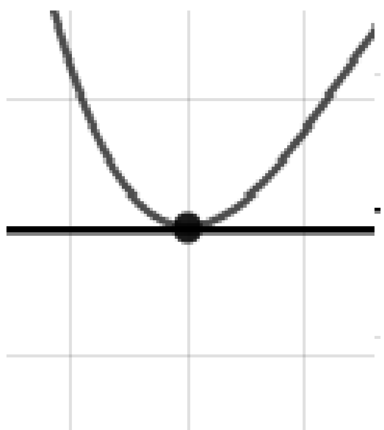
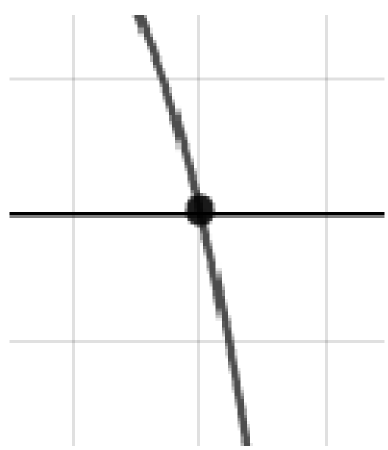
3. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

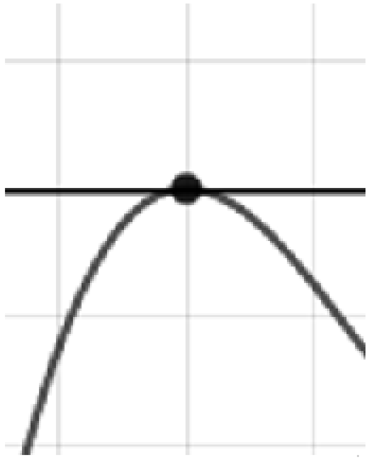
$$f(x) = -9(x - 7)^7(x + 7)^4(x - 2)^{12}(x + 2)^9$$

The solution is the graph below, which is option C.



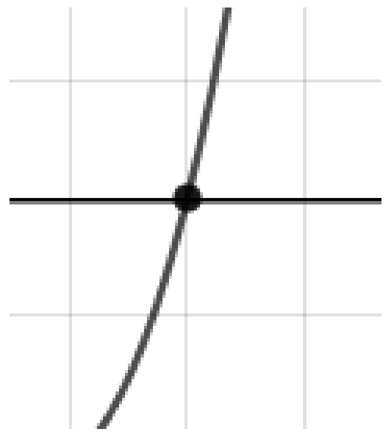


A.



B.

C.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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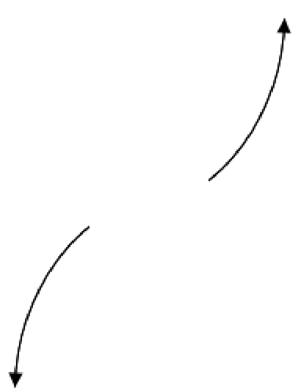
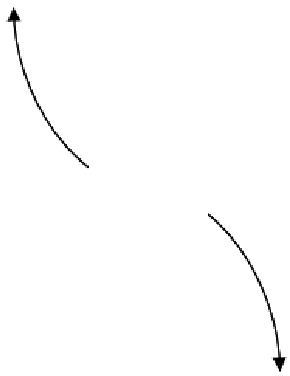
4. Describe the end behavior of the polynomial below.

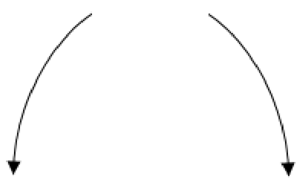
$$f(x) = 8(x-9)^2(x+9)^5(x-7)^4(x+7)^6$$

The solution is the graph below, which is option D.

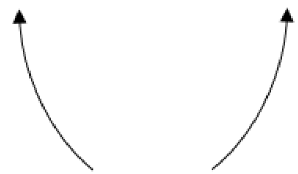




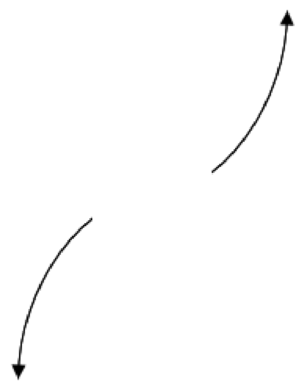
A.



B.



C.



D.

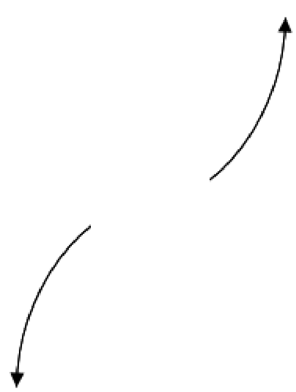
E. None of the above.

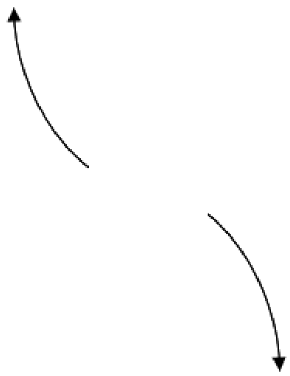
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

5. Describe the end behavior of the polynomial below.

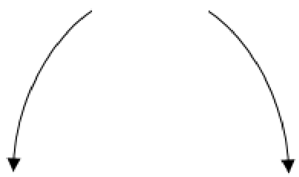
$$f(x) = 9(x+5)^3(x-5)^6(x-3)^5(x+3)^7$$

The solution is the graph below, which is option D.

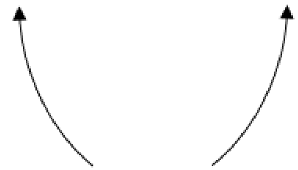




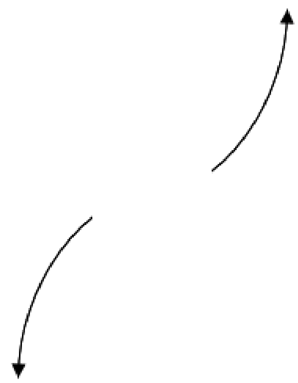
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 5i \text{ and } 3$$

The solution is $x^3 + x^2 + 17x - 87$, which is option A.

A. $b \in [-0.1, 2.4], c \in [15, 24],$ and $d \in [-94, -80]$

   * $x^3 + x^2 + 17x - 87$, which is the correct option.

B. $b \in [-2.5, -0.9], c \in [15, 24],$ and $d \in [75, 89]$

   $x^3 - 1x^2 + 17x + 87$, which corresponds to multiplying out $(x - (-2 + 5i))(x - (-2 - 5i))(x + 3)$.

C. $b \in [-0.1, 2.4], c \in [-8, -3],$ and $d \in [10, 24]$

   $x^3 + x^2 - 8x + 15$, which corresponds to multiplying out $(x - 5)(x - 3)$.

D. $b \in [-0.1, 2.4], c \in [-2, 5],$ and $d \in [-10, -2]$

   $x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x + 2)(x - 3)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 5i))(x - (-2 - 5i))(x - (3))$.

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{2}, \frac{-7}{3}, \text{ and } -4$$

The solution is $6x^3 + 47x^2 + 113x + 84$, which is option D.

A. $a \in [1, 13], b \in [42, 51], c \in [111, 117], \text{ and } d \in [-87, -83]$

$6x^3 + 47x^2 + 113x - 84$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [1, 13], b \in [-4, 2], c \in [-76, -68], \text{ and } d \in [84, 88]$

$6x^3 + x^2 - 71x + 84$, which corresponds to multiplying out $(2x - 3)(3x - 7)(x + 4)$.

C. $a \in [1, 13], b \in [-50, -41], c \in [111, 117], \text{ and } d \in [-87, -83]$

$6x^3 - 47x^2 + 113x - 84$, which corresponds to multiplying out $(2x - 3)(3x - 7)(x - 4)$.

D. $a \in [1, 13], b \in [42, 51], c \in [111, 117], \text{ and } d \in [84, 88]$

* $6x^3 + 47x^2 + 113x + 84$, which is the correct option.

E. $a \in [1, 13], b \in [28, 30], c \in [-3, 5], \text{ and } d \in [-87, -83]$

$6x^3 + 29x^2 - x - 84$, which corresponds to multiplying out $(2x - 3)(3x + 7)(x + 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 3)(3x + 7)(x + 4)$

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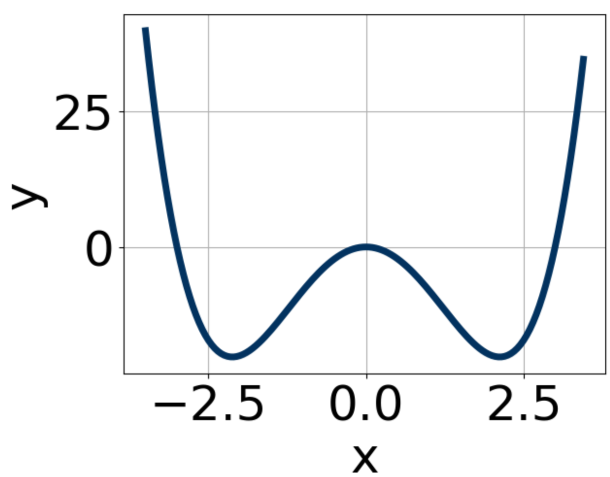
8. Which of the following equations *could* be of the graph presented below?



The solution is $18x^8(x + 3)^9(x - 3)^7$, which is option A.

A. $18x^8(x + 3)^9(x - 3)^7$

* This is the correct option.

B. $-19x^4(x+3)^5(x-3)^4$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $19x^4(x+3)^8(x-3)^{11}$

The factor $(x+3)$ should have an odd power.

D. $8x^5(x+3)^8(x-3)^7$

The factor 0 should have an even power and the factor -3 should have an odd power.

E. $-3x^4(x+3)^{11}(x-3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 2i \text{ and } 4$$

The solution is $x^3 - 14x^2 + 69x - 116$, which is option B.

A. $b \in [-6, 2], c \in [-9.4, -7.5],$ and $d \in [18, 22]$

$x^3 + x^2 - 9x + 20$, which corresponds to multiplying out $(x-5)(x-4)$.

B. $b \in [-21, -8], c \in [67.5, 70.9],$ and $d \in [-126, -113]$

* $x^3 - 14x^2 + 69x - 116$, which is the correct option.

C. $b \in [-6, 2], c \in [-8, -3.8],$ and $d \in [5, 12]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x-2)(x-4)$.

D. $b \in [12, 21], c \in [67.5, 70.9],$ and $d \in [115, 119]$

$x^3 + 14x^2 + 69x + 116$, which corresponds to multiplying out $(x-(5+2i))(x-(5-2i))(x+4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x-(5+2i))(x-(5-2i))(x-(4))$.

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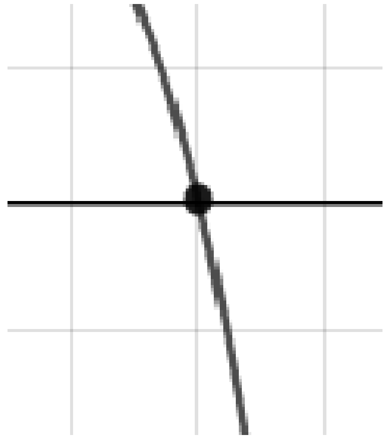
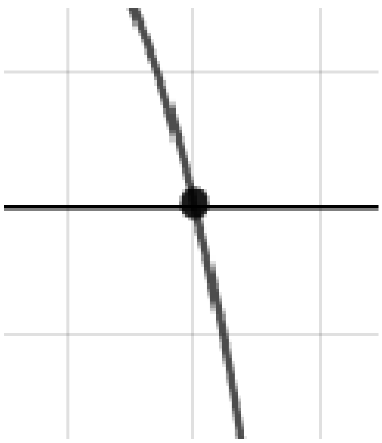
10. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

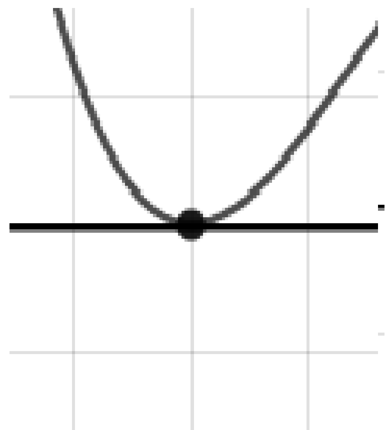
$$f(x) = 8(x+2)^2(x-2)^7(x-4)^9(x+4)^{11}$$

The solution is the graph below, which is option A.

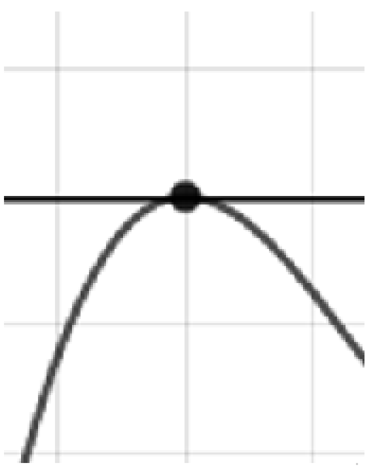




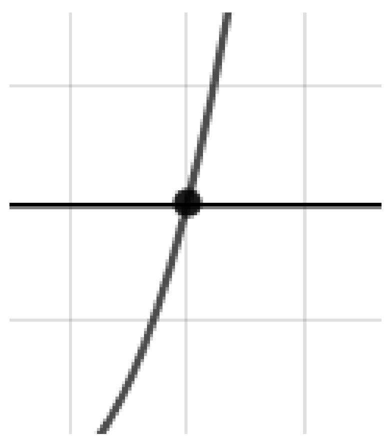
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.