1. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^4 + 2x^3 + 2x^2 + 2x + 6$$

- A.  $\pm 1, \pm 7$
- B. All combinations of:  $\frac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$
- C. All combinations of:  $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$
- D.  $\pm 1, \pm 2, \pm 3, \pm 6$
- E. There is no formula or theorem that tells us all possible Rational roots.

2. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^4 + 6x^3 + 3x^2 + 2x + 4$$

- A.  $\pm 1, \pm 2, \pm 4$
- B. All combinations of:  $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$
- C. All combinations of:  $\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$
- D.  $\pm 1, \pm 2, \pm 3, \pm 6$
- E. There is no formula or theorem that tells us all possible Integer roots.
- 3. Factor the polynomial below completely, knowing that x+5 is a factor. Then, choose the intervals the zeros of the polynomial belong to, where  $z_1 \leq z_2 \leq z_3 \leq z_4$ . To make the problem easier, all zeros are between -5 and 5.

$$f(x) = 8x^4 - 14x^3 - 167x^2 + 455x - 300$$

- A.  $z_1 \in [-4.25, -3.95], z_2 \in [-0.88, -0.53], z_3 \in [-0.67, -0.65], and z_4 \in [4.9, 5.1]$
- B.  $z_1 \in [-5.08, -4.64], z_2 \in [0.79, 1.43], z_3 \in [1.47, 1.53], \text{ and } z_4 \in [2.5, 4.4]$

- C.  $z_1 \in [-5.08, -4.64], z_2 \in [-0.31, 0.74], z_3 \in [0.76, 0.84], \text{ and } z_4 \in [2.5, 4.4]$
- D.  $z_1 \in [-4.25, -3.95], z_2 \in [-3.36, -2.87], z_3 \in [-0.63, -0.53], \text{ and } z_4 \in [4.9, 5.1]$
- E.  $z_1 \in [-4.25, -3.95], z_2 \in [-2.41, -0.84], z_3 \in [-1.26, -1.22], \text{ and } z_4 \in [4.9, 5.1]$
- 4. Perform the division below. Then, find the intervals that correspond to the quotient in the form  $ax^2 + bx + c$  and remainder r.

$$\frac{4x^3 - 12x + 6}{x + 2}$$

- A.  $a \in [3, 8], b \in [-13, -10], c \in [15, 25], \text{ and } r \in [-67, -61].$
- B.  $a \in [3, 8], b \in [8, 10], c \in [-1, 8], \text{ and } r \in [8, 15].$
- C.  $a \in [-10, -4], b \in [10, 17], c \in [-48, -42], \text{ and } r \in [94, 97].$
- D.  $a \in [3, 8], b \in [-9, 0], c \in [-1, 8], \text{ and } r \in [-5, 4].$
- E.  $a \in [-10, -4], b \in [-20, -15], c \in [-48, -42], \text{ and } r \in [-85, -81].$
- 5. Perform the division below. Then, find the intervals that correspond to the quotient in the form  $ax^2 + bx + c$  and remainder r.

$$\frac{20x^3 - 63x^2 + 23}{x - 3}$$

- A.  $a \in [57, 65], b \in [113, 120], c \in [350, 355], \text{ and } r \in [1074, 1078].$
- B.  $a \in [17, 22], b \in [-130, -118], c \in [369, 371], \text{ and } r \in [-1085, -1082].$
- C.  $a \in [57, 65], b \in [-245, -241], c \in [729, 731], \text{ and } r \in [-2169, -2161].$
- D.  $a \in [17, 22], b \in [-5, 0], c \in [-13, -7], \text{ and } r \in [-6, 4].$
- E.  $a \in [17, 22], b \in [-29, -22], c \in [-47, -42], \text{ and } r \in [-70, -68].$

6. Factor the polynomial below completely, knowing that x-5 is a factor. Then, choose the intervals the zeros of the polynomial belong to, where  $z_1 \leq z_2 \leq z_3 \leq z_4$ . To make the problem easier, all zeros are between -5 and 5.

$$f(x) = 12x^4 - 113x^3 + 338x^2 - 395x + 150$$

- A.  $z_1 \in [-5.35, -4.91], z_2 \in [-6.33, -4.95], z_3 \in [-2.42, -1.69], and z_4 \in [-0.34, -0.14]$
- B.  $z_1 \in [-5.35, -4.91], z_2 \in [-2.13, -1.6], z_3 \in [-1.8, -1.62], \text{ and } z_4 \in [-0.84, -0.74]$
- C.  $z_1 \in [0.72, 1.07], z_2 \in [1.35, 1.95], z_3 \in [1.61, 2.61], \text{ and } z_4 \in [4.79, 5.08]$
- D.  $z_1 \in [0.59, 0.69], z_2 \in [1.11, 1.62], z_3 \in [1.61, 2.61], \text{ and } z_4 \in [4.79, 5.08]$
- E.  $z_1 \in [-5.35, -4.91], z_2 \in [-2.13, -1.6], z_3 \in [-1.44, -1.27], \text{ and } z_4 \in [-0.68, -0.44]$
- 7. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where  $z_1 \leq z_2 \leq z_3$ . To make the problem easier, all zeros are between -5 and 5.

$$f(x) = 25x^3 - 100x^2 - 4x + 16$$

- A.  $z_1 \in [-4.6, -3.3], z_2 \in [-2.65, -2.36], \text{ and } z_3 \in [2.09, 3]$
- B.  $z_1 \in [-2.9, -2.4], z_2 \in [1.93, 2.94], \text{ and } z_3 \in [3.93, 4.24]$
- C.  $z_1 \in [-4.6, -3.3], z_2 \in [-2.25, -1.76], \text{ and } z_3 \in [-0.2, 0.15]$
- D.  $z_1 \in [-1.6, 0.4], z_2 \in [0.19, 0.79], \text{ and } z_3 \in [3.93, 4.24]$
- E.  $z_1 \in [-4.6, -3.3], z_2 \in [-0.56, -0.04], \text{ and } z_3 \in [0.11, 1.11]$
- 8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where  $z_1 \leq z_2 \leq z_3$ . To make the problem easier, all zeros are between -5 and 5.

$$f(x) = 20x^3 + 31x^2 - 38x - 40$$

A.  $z_1 \in [-0.92, -0.65], z_2 \in [1.12, 1.47], \text{ and } z_3 \in [1.78, 2.59]$ 

B. 
$$z_1 \in [-5.08, -4.9], z_2 \in [-0.09, 0.27], \text{ and } z_3 \in [1.78, 2.59]$$

C. 
$$z_1 \in [-1.46, -1.02], z_2 \in [0.78, 1.08], \text{ and } z_3 \in [1.78, 2.59]$$

D. 
$$z_1 \in [-2.2, -1.8], z_2 \in [-1.59, -1.18], \text{ and } z_3 \in [0.41, 0.86]$$

E. 
$$z_1 \in [-2.2, -1.8], z_2 \in [-0.86, -0.52], \text{ and } z_3 \in [1.1, 1.49]$$

9. Perform the division below. Then, find the intervals that correspond to the quotient in the form  $ax^2 + bx + c$  and remainder r.

$$\frac{6x^3 - 2x^2 - 20x + 19}{x + 2}$$

A. 
$$a \in [-15, -8], b \in [-28, -25], c \in [-72, -68], and  $r \in [-132, -124].$$$

B. 
$$a \in [-15, -8], b \in [22, 24], c \in [-66, -63], and r \in [144, 149].$$

C. 
$$a \in [1, 11], b \in [-21, -19], c \in [34, 46], and  $r \in [-103, -97].$$$

D. 
$$a \in [1, 11], b \in [8, 17], c \in [-3, 4], and r \in [15, 20].$$

E. 
$$a \in [1, 11], b \in [-14, -9], c \in [7, 9], and r \in [2, 4].$$

10. Perform the division below. Then, find the intervals that correspond to the quotient in the form  $ax^2 + bx + c$  and remainder r.

$$\frac{6x^3 - 20x^2 - 2x + 19}{x - 3}$$

A. 
$$a \in [5, 9], b \in [-40, -34], c \in [111, 115], and  $r \in [-322, -313].$$$

B. 
$$a \in [15, 21], b \in [-76, -72], c \in [218, 224], and  $r \in [-646, -637].$$$

C. 
$$a \in [5, 9], b \in [-14, -3], c \in [-23, -16], and  $r \in [-18, -15].$$$

D. 
$$a \in [5, 9], b \in [-6, 6], c \in [-15, -7], and r \in [-6, -3].$$

E. 
$$a \in [15, 21], b \in [29, 35], c \in [95, 104], and  $r \in [318, 321].$$$

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