This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-4}{5}, \frac{-1}{2}, \text{ and } \frac{2}{5}$$

The solution is $50x^3 + 45x^2 - 6x - 8$, which is option D.

A. $a \in [47, 51], b \in [44, 46], c \in [-10, -4],$ and $d \in [5, 10]$

$50x^3 + 45x^2 - 6x + 8$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [47, 51], b \in [-86, -79], c \in [41, 48],$ and $d \in [-11, -2]$

$50x^3 - 85x^2 + 46x - 8$, which corresponds to multiplying out $(5x - 4)(2x - 1)(5x - 2)$.

C. $a \in [47, 51], b \in [-50, -44], c \in [-10, -4],$ and $d \in [5, 10]$

$50x^3 - 45x^2 - 6x + 8$, which corresponds to multiplying out $(5x - 4)(2x - 1)(5x + 2)$.

D. $a \in [47, 51], b \in [44, 46], c \in [-10, -4],$ and $d \in [-11, -2]$

* $50x^3 + 45x^2 - 6x - 8$, which is the correct option.

E. $a \in [47, 51], b \in [-35, -28], c \in [-16, -10],$ and $d \in [5, 10]$

$50x^3 - 35x^2 - 14x + 8$, which corresponds to multiplying out $(5x - 4)(2x + 1)(5x - 2)$.

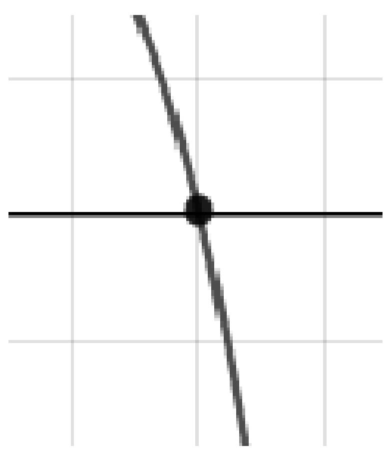
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 4)(2x + 1)(5x - 2)$

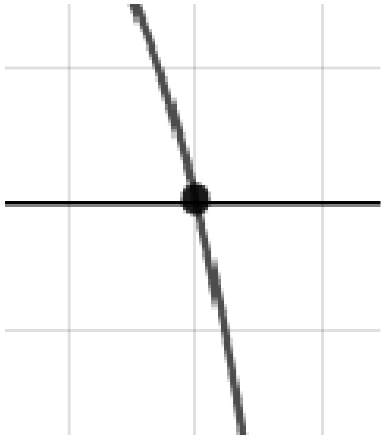
---

2. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

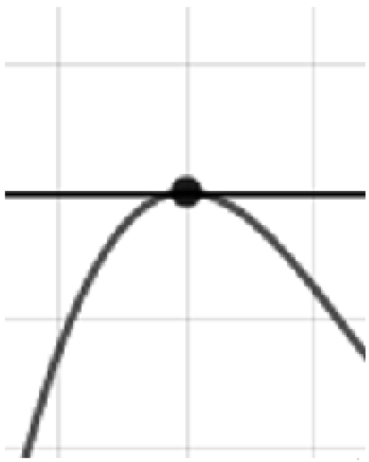
$$f(x) = -5(x + 4)^6(x - 4)^7(x + 5)^3(x - 5)^6$$

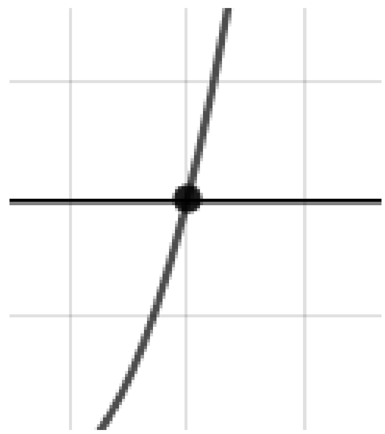
The solution is the graph below, which is option A.




A.


C.


B.

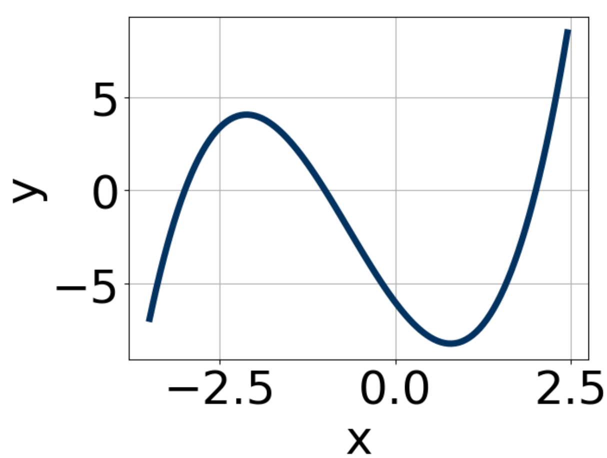

D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

3. Which of the following equations *could* be of the graph presented below?



The solution is $15(x-2)^7(x+1)^5(x+3)^7$, which is option C.

A. $17(x-2)^8(x+1)^9(x+3)^{11}$

The factor 2 should have been an odd power.

B. $-7(x-2)^4(x+1)^7(x+3)^{11}$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $15(x-2)^7(x+1)^5(x+3)^7$

* This is the correct option.

D. $-2(x-2)^{11}(x+1)^9(x+3)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $7(x-2)^{10}(x+1)^8(x+3)^{11}$

The factors 2 and -1 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 2i \text{ and } -1$$

The solution is $x^3 + 9x^2 + 28x + 20$, which is option B.

A. $b \in [0, 4], c \in [4.37, 5.42], \text{ and } d \in [2.7, 4.8]$

$x^3 + x^2 + 5x + 4$, which corresponds to multiplying out $(x+4)(x+1)$.

B. $b \in [6, 11], c \in [27.08, 28.41], \text{ and } d \in [14.4, 20.2]$

* $x^3 + 9x^2 + 28x + 20$, which is the correct option.

C. $b \in [-9, -7], c \in [27.08, 28.41], \text{ and } d \in [-20.3, -18.9]$

$x^3 - 9x^2 + 28x - 20$, which corresponds to multiplying out $(x-(-4-2i))(x-(-4+2i))(x-1)$.

D. $b \in [0, 4], c \in [0.79, 4.62], \text{ and } d \in [-0.4, 3.2]$

$x^3 + x^2 + 3x + 2$, which corresponds to multiplying out $(x+2)(x+1)$.

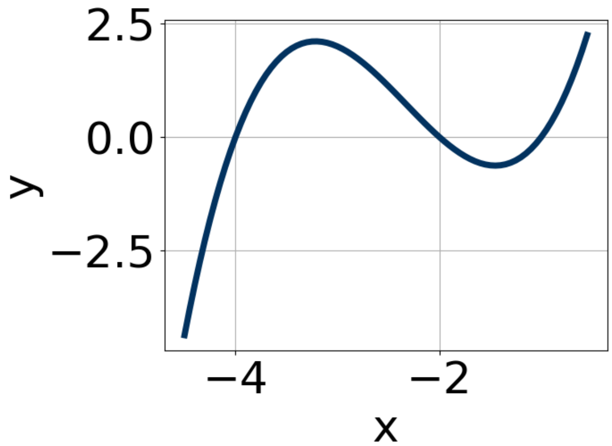
E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-4-2i))(x-(-4+2i))(x-(-1))$.

---

5. Which of the following equations *could* be of the graph presented below?



The solution is $9(x+1)^9(x+2)^7(x+4)^{11}$, which is option C.

A. $-7(x+1)^5(x+2)^9(x+4)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $10(x+1)^{10}(x+2)^5(x+4)^7$

The factor -1 should have been an odd power.

C. $9(x+1)^9(x+2)^7(x+4)^{11}$

* This is the correct option.

D. $-2(x+1)^{10}(x+2)^5(x+4)^5$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $20(x+1)^{10}(x+2)^8(x+4)^9$

The factors -1 and -2 have have been odd power.

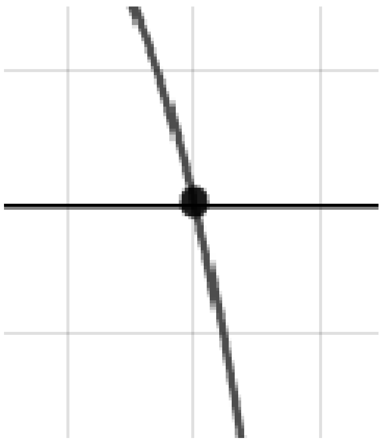
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

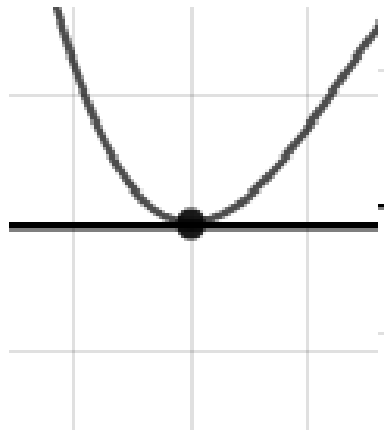
---

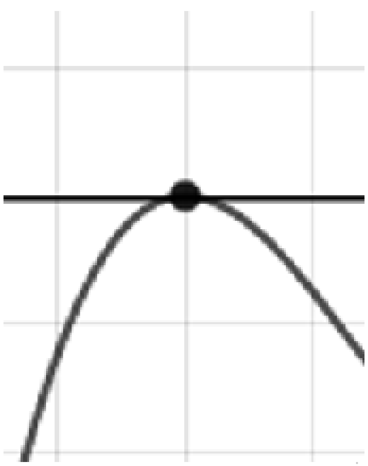
6. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

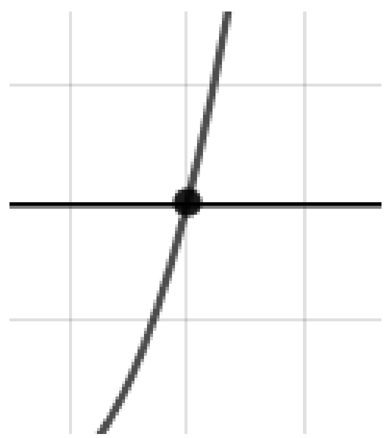
$$f(x) = 6(x+5)^4(x-5)^2(x+3)^{13}(x-3)^8$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

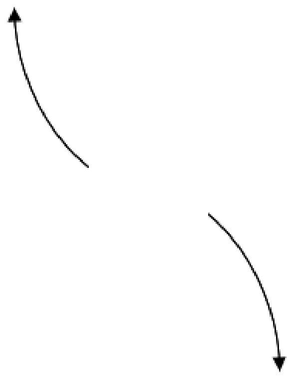
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

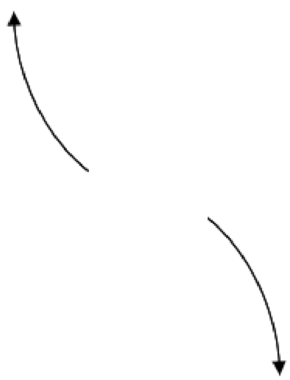
---

7. Describe the end behavior of the polynomial below.

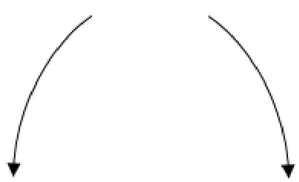
$$f(x) = -9(x-5)^3(x+5)^8(x-6)^4(x+6)^6$$

The solution is the graph below, which is option A.

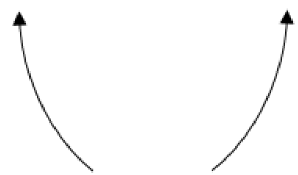




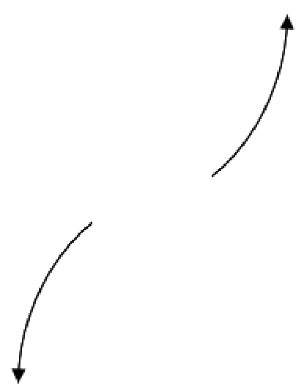
A.



B.



C.



D.

E. None of the above.

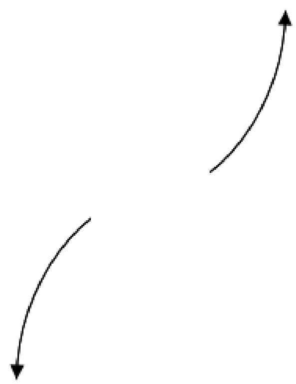
General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

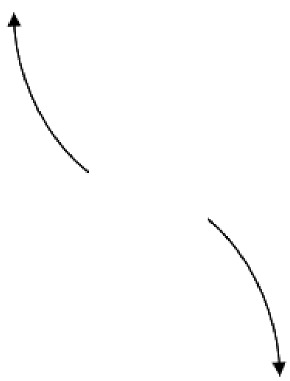
---

8. Describe the end behavior of the polynomial below.

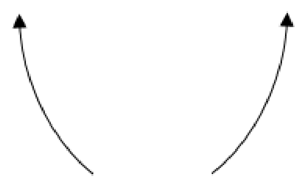
$$f(x) = 2(x+2)^2(x-2)^7(x+8)^4(x-8)^4$$

The solution is the graph below, which is option D.

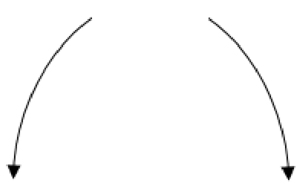




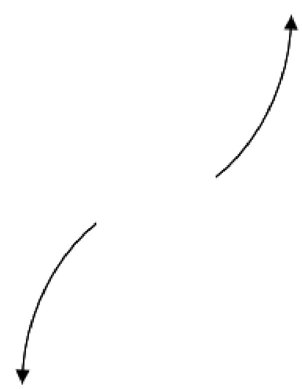
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{4}{3}, \frac{7}{5}, \text{ and } \frac{-1}{3}$$

The solution is $45x^3 - 108x^2 + 43x + 28$, which is option D.

A. $a \in [44, 48], b \in [-108, -105], c \in [40, 50], \text{ and } d \in [-28, -27]$

$45x^3 - 108x^2 + 43x - 28$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [44, 48], b \in [9, 14], c \in [-86, -82]$, and $d \in [-28, -27]$

$45x^3 + 12x^2 - 85x - 28$, which corresponds to multiplying out $(3x + 4)(5x - 7)(3x + 1)$.

C. $a \in [44, 48], b \in [127, 141], c \in [121, 128]$, and $d \in [25, 34]$

$45x^3 + 138x^2 + 125x + 28$, which corresponds to multiplying out $(3x + 4)(5x + 7)(3x + 1)$.

D. $a \in [44, 48], b \in [-108, -105], c \in [40, 50]$, and $d \in [25, 34]$

* $45x^3 - 108x^2 + 43x + 28$, which is the correct option.

E. $a \in [44, 48], b \in [107, 110], c \in [40, 50]$, and $d \in [-28, -27]$

$45x^3 + 108x^2 + 43x - 28$, which corresponds to multiplying out $(3x + 4)(5x + 7)(3x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 4)(5x - 7)(3x + 1)$

---

10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 - 3i \text{ and } 1$$

The solution is $x^3 + 9x^2 + 24x - 34$, which is option A.

A. $b \in [4, 14], c \in [23.5, 25.2]$, and $d \in [-34.6, -33]$

* $x^3 + 9x^2 + 24x - 34$, which is the correct option.

B. $b \in [-5, 6], c \in [3.8, 6.7]$, and $d \in [-6.8, -4]$

$x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

C. $b \in [-10, -2], c \in [23.5, 25.2]$, and $d \in [33.9, 36.6]$

$x^3 - 9x^2 + 24x + 34$, which corresponds to multiplying out $(x - (-5 - 3i))(x - (-5 + 3i))(x + 1)$.

D. $b \in [-5, 6], c \in [-1.7, 3.3]$, and $d \in [-3.6, -0.7]$

$x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 - 3i))(x - (-5 + 3i))(x - (1))$.
