This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 2i \text{ and } -3$$

The solution is $x^3 + 9x^2 + 31x + 39$, which is option C.

A. $b \in [-5, 3], c \in [5.4, 6.45]$, and $d \in [7.9, 9.4]$

$x^3 + x^2 + 6x + 9$, which corresponds to multiplying out $(x + 3)(x + 3)$.

B. $b \in [-5, 3], c \in [4.58, 5.53]$, and $d \in [1.9, 7.1]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x + 2)(x + 3)$.

C. $b \in [2, 13], c \in [30.15, 31.6]$, and $d \in [38.1, 39.8]$

* $x^3 + 9x^2 + 31x + 39$, which is the correct option.

D. $b \in [-17, -6], c \in [30.15, 31.6]$, and $d \in [-42, -38.7]$

$x^3 - 9x^2 + 31x - 39$, which corresponds to multiplying out $(x - (-3 - 2i))(x - (-3 + 2i))(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 - 2i))(x - (-3 + 2i))(x - (-3))$.

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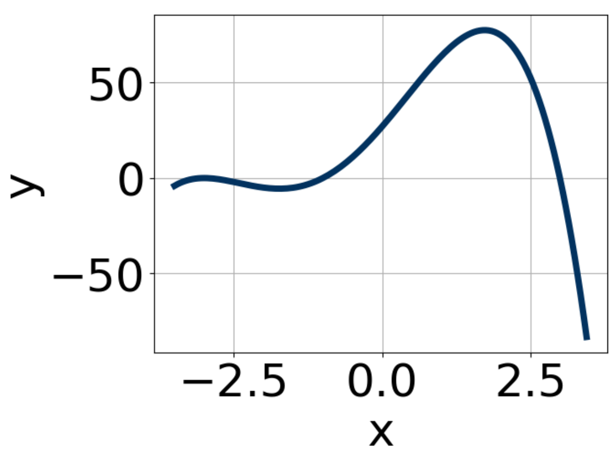
2. Which of the following equations *could* be of the graph presented below?



The solution is $-15(x+3)^{10}(x-3)^7(x+1)^{11}$, which is option A.

A. $-15(x+3)^{10}(x-3)^7(x+1)^{11}$

* This is the correct option.

B. $-9(x+3)^{11}(x-3)^8(x+1)^9$

The factor -3 should have an even power and the factor 3 should have an odd power.

C. $-7(x+3)^{10}(x-3)^6(x+1)^7$

The factor $(x-3)$ should have an odd power.

D. $5(x+3)^{10}(x-3)^5(x+1)^4$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $7(x+3)^6(x-3)^5(x+1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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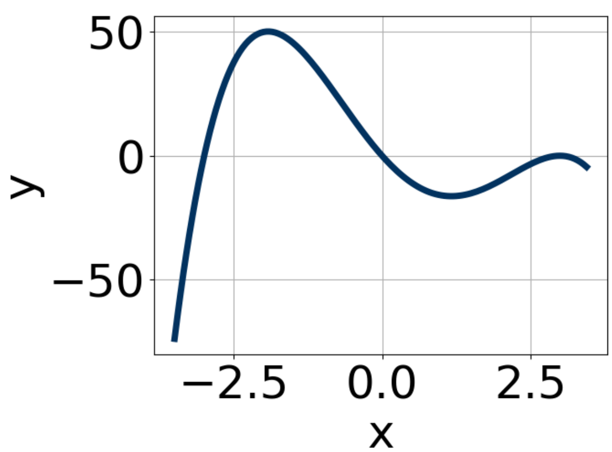
3. Which of the following equations *could* be of the graph presented below?



The solution is $-7x^7(x-3)^8(x+3)^5$, which is option D.

A. $10x^7(x-3)^4(x+3)^{10}$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $15x^{11}(x-3)^6(x+3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-20x^6(x-3)^9(x+3)^7$

The factor 3 should have an even power and the factor 0 should have an odd power.

D. $-7x^7(x-3)^8(x+3)^5$

* This is the correct option.

E. $-18x^4(x-3)^4(x+3)^5$

The factor x should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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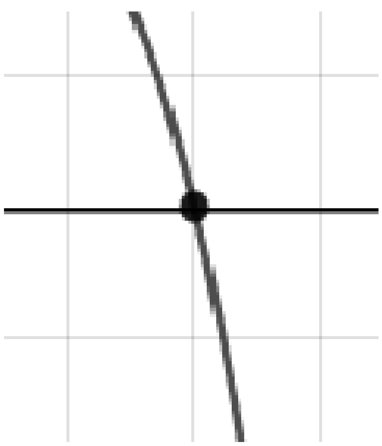
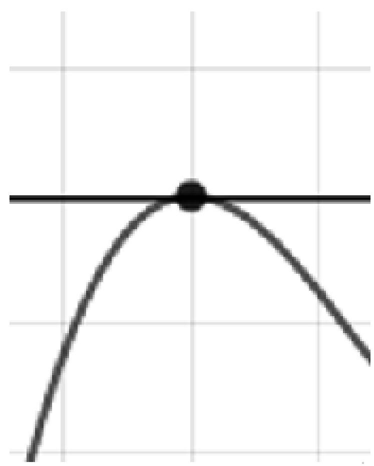
4. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

$$f(x) = 2(x+6)^8(x-6)^4(x-4)^{10}(x+4)^7$$

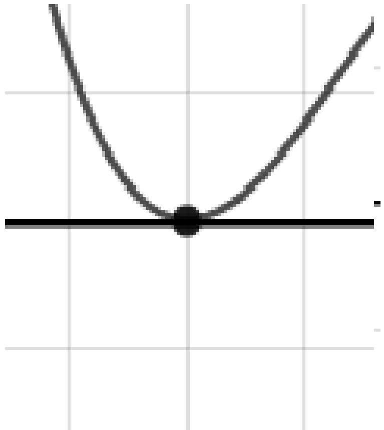
The solution is the graph below, which is option C.

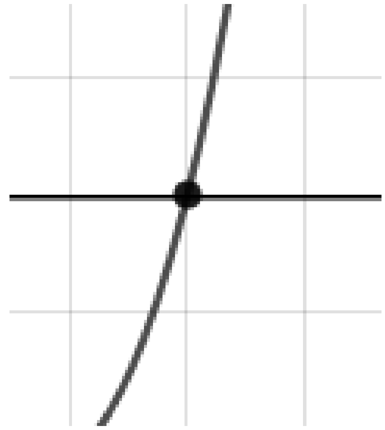


A.



B.

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 + 3i \text{ and } 3$$

The solution is $x^3 - 7x^2 + 25x - 39$, which is option D.

A. $b \in [4, 11], c \in [21.54, 25.63]$, and $d \in [33, 43]$

$x^3 + 7x^2 + 25x + 39$, which corresponds to multiplying out $(x - (2 + 3i))(x - (2 - 3i))(x + 3)$.

B. $b \in [-3, 5], c \in [-5.17, -2.87]$, and $d \in [0, 7]$

$x^3 + x^2 - 5x + 6$, which corresponds to multiplying out $(x - 2)(x - 3)$.

C. $b \in [-3, 5], c \in [-6.83, -5.89]$, and $d \in [9, 10]$

$x^3 + x^2 - 6x + 9$, which corresponds to multiplying out $(x - 3)(x - 3)$.

D. $b \in [-9, -4], c \in [21.54, 25.63]$, and $d \in [-46, -38]$

* $x^3 - 7x^2 + 25x - 39$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 + 3i))(x - (2 - 3i))(x - (3))$.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{3}{5}, \frac{-1}{3}, \text{ and } \frac{-1}{2}$$

The solution is $30x^3 + 7x^2 - 10x - 3$, which is option E.

A. $a \in [30, 39], b \in [-13, -1], c \in [-13, -6],$ and $d \in [-1, 7]$

$30x^3 - 7x^2 - 10x + 3$, which corresponds to multiplying out $(5x + 3)(3x - 1)(2x - 1)$.

B. $a \in [30, 39], b \in [40, 44], c \in [18, 24],$ and $d \in [-1, 7]$

$30x^3 + 43x^2 + 20x + 3$, which corresponds to multiplying out $(5x + 3)(3x + 1)(2x + 1)$.

C. $a \in [30, 39], b \in [22, 27], c \in [-2, 0],$ and $d \in [-3, -2]$

$30x^3 + 23x^2 - 2x - 3$, which corresponds to multiplying out $(5x + 3)(3x - 1)(2x + 1)$.

D. $a \in [30, 39], b \in [7, 13], c \in [-13, -6],$ and $d \in [-1, 7]$

$30x^3 + 7x^2 - 10x + 3$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [30, 39], b \in [7, 13], c \in [-13, -6],$ and $d \in [-3, -2]$

* $30x^3 + 7x^2 - 10x - 3$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 3)(3x + 1)(2x + 1)$

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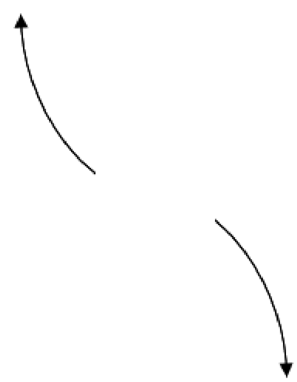
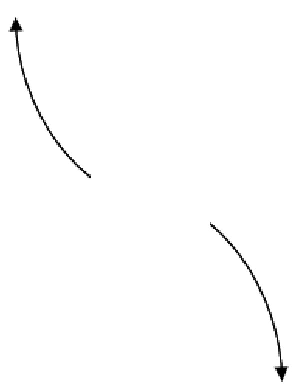
7. Describe the end behavior of the polynomial below.

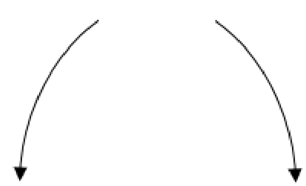
$$f(x) = -7(x - 9)^5(x + 9)^8(x + 4)^5(x - 4)^7$$

The solution is the graph below, which is option A.

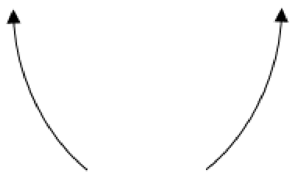




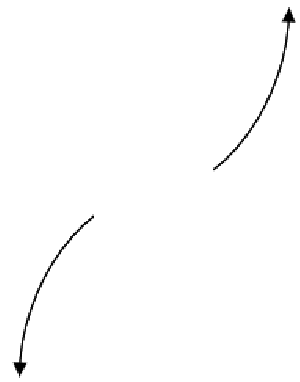
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-6}{5}, \frac{3}{5}, \text{ and } \frac{7}{2}$$

The solution is $50x^3 - 145x^2 - 141x + 126$, which is option E.

A. $a \in [48, 54], b \in [-154, -139], c \in [-141, -135], \text{ and } d \in [-128, -118]$

$50x^3 - 145x^2 - 141x - 126$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [48, 54], b \in [-206, -201], c \in [67, 73], \text{ and } d \in [125, 132]$

$50x^3 - 205x^2 + 69x + 126$, which corresponds to multiplying out $(5x - 6)(5x + 3)(2x - 7)$.

C. $a \in [48, 54], b \in [-267, -261], c \in [350, 357], \text{ and } d \in [-128, -118]$

$50x^3 - 265x^2 + 351x - 126$, which corresponds to multiplying out $(5x - 6)(5x - 3)(2x - 7)$.

D. $a \in [48, 54], b \in [142, 152], c \in [-141, -135], \text{ and } d \in [-128, -118]$

$50x^3 + 145x^2 - 141x - 126$, which corresponds to multiplying out $(5x - 6)(5x + 3)(2x + 7)$.

E. $a \in [48, 54], b \in [-154, -139], c \in [-141, -135], \text{ and } d \in [125, 132]$

* $50x^3 - 145x^2 - 141x + 126$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 6)(5x - 3)(2x - 7)$

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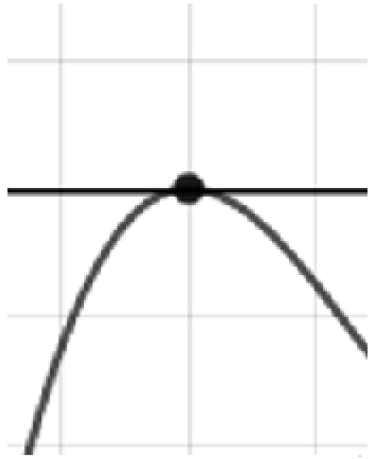
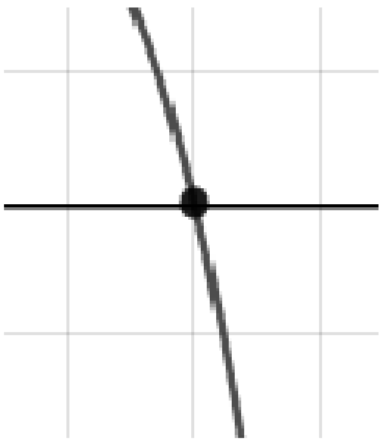
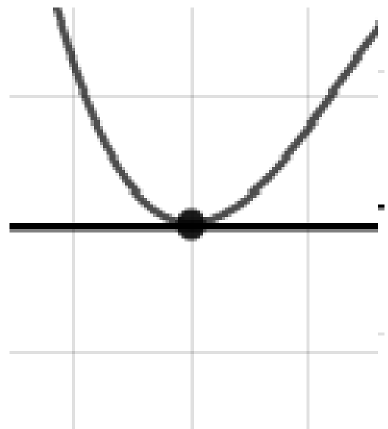
9. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

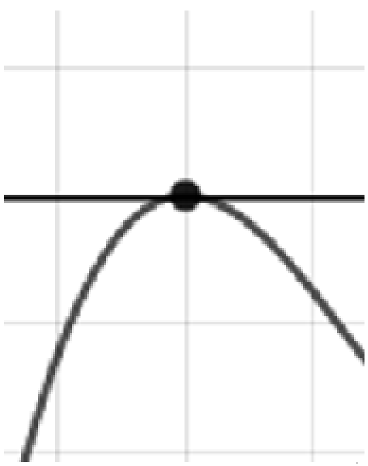
$$f(x) = -7(x - 3)^6(x + 3)^3(x - 5)^{10}(x + 5)^7$$

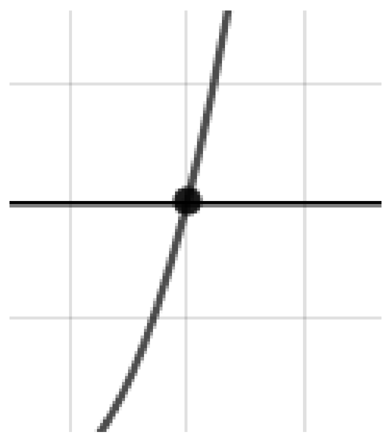
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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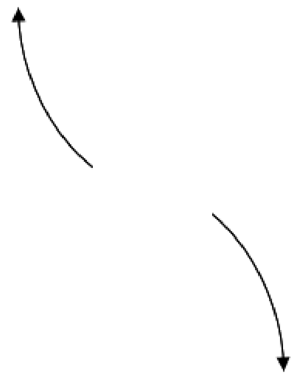
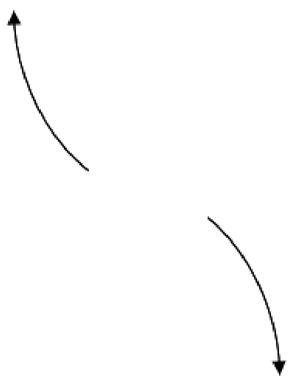
10. Describe the end behavior of the polynomial below.

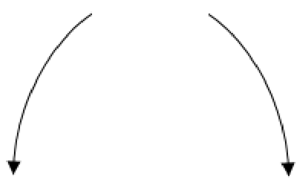
$$f(x) = -8(x+3)^4(x-3)^5(x+7)^3(x-7)^5$$

The solution is the graph below, which is option A.

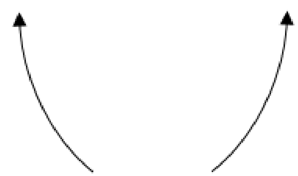




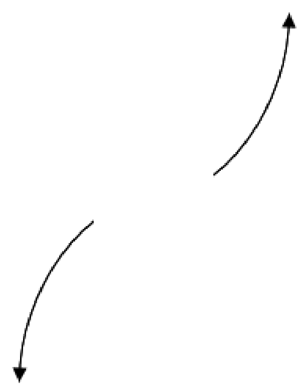
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.