This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-5}{2}, \frac{4}{3}, \text{ and } -1$$

The solution is $6x^3 + 13x^2 - 13x - 20$, which is option D.

A. $a \in [1, 10], b \in [13, 15], c \in [-15, -4],$ and $d \in [10, 26]$

$6x^3 + 13x^2 - 13x + 20$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [1, 10], b \in [-2, 5], c \in [-31, -24],$ and $d \in [-22, -12]$

$6x^3 - 1x^2 - 27x - 20$, which corresponds to multiplying out $(2x - 5)(3x + 4)(x + 1)$.

C. $a \in [1, 10], b \in [-14, -11], c \in [-15, -4],$ and $d \in [10, 26]$

$6x^3 - 13x^2 - 13x + 20$, which corresponds to multiplying out $(2x - 5)(3x + 4)(x - 1)$.

D. $a \in [1, 10], b \in [13, 15], c \in [-15, -4],$ and $d \in [-22, -12]$

* $6x^3 + 13x^2 - 13x - 20$, which is the correct option.

E. $a \in [1, 10], b \in [-19, -15], c \in [-3, -2],$ and $d \in [10, 26]$

$6x^3 - 17x^2 - 3x + 20$, which corresponds to multiplying out $(2x - 5)(3x - 4)(x + 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 5)(3x - 4)(x + 1)$

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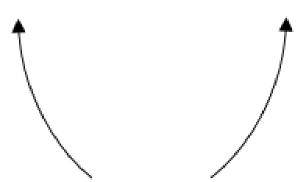
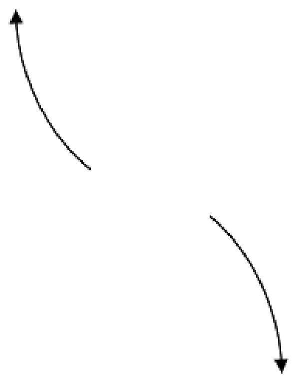
2. Describe the end behavior of the polynomial below.

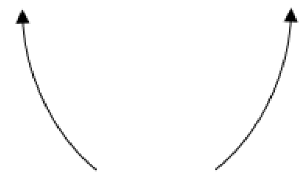
$$f(x) = 2(x - 3)^5(x + 3)^{10}(x + 7)^4(x - 7)^5$$

The solution is the graph below, which is option C.

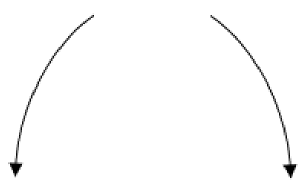




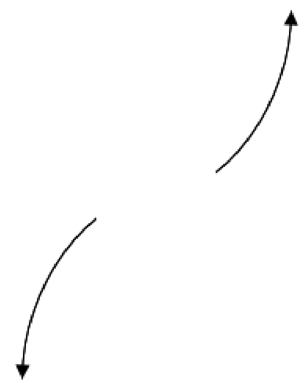
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 5i \text{ and } -4$$

The solution is $x^3 + 10x^2 + 58x + 136$, which is option C.

A. $b \in [-11, -8], c \in [57.4, 58.57],$ and $d \in [-141, -128]$

$x^3 - 10x^2 + 58x - 136$, which corresponds to multiplying out $(x - (-3 - 5i))(x - (-3 + 5i))(x - 4)$.

B. $b \in [1, 5], c \in [8.96, 9.07],$ and $d \in [16, 25]$

$x^3 + x^2 + 9x + 20$, which corresponds to multiplying out $(x + 5)(x + 4)$.

C. $b \in [9, 15], c \in [57.4, 58.57],$ and $d \in [136, 145]$

* $x^3 + 10x^2 + 58x + 136$, which is the correct option.

D. $b \in [1, 5], c \in [6.8, 8.11],$ and $d \in [12, 18]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 3)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-3-5i))(x-(-3+5i))(x-(-4))$.

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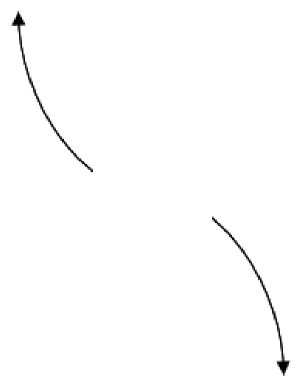
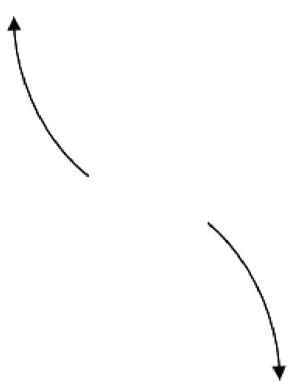
4. Describe the end behavior of the polynomial below.

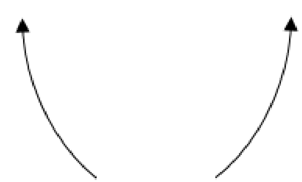
$$f(x) = -2(x-8)^4(x+8)^5(x+4)^2(x-4)^2$$

The solution is the graph below, which is option A.

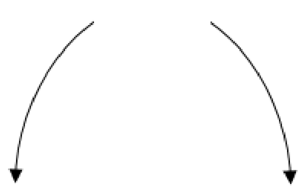




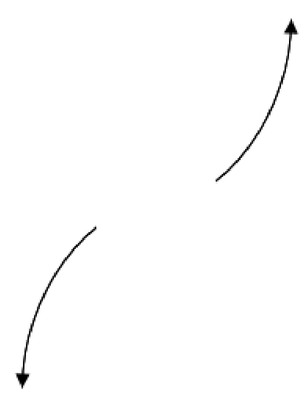
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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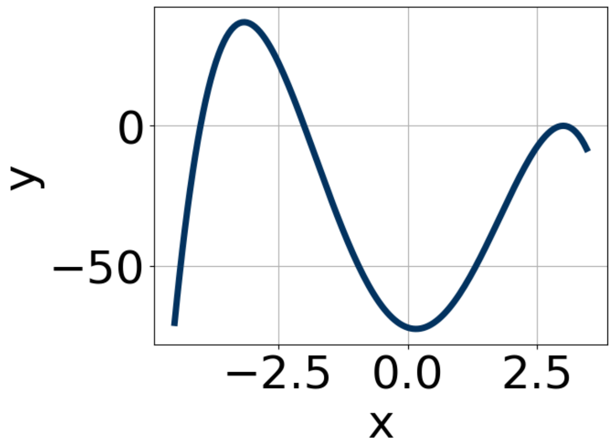
5. Which of the following equations *could* be of the graph presented below?



The solution is $-14(x-3)^8(x+4)^{11}(x+2)^5$, which is option E.

A. $-2(x-3)^6(x+4)^{10}(x+2)^5$

   The factor $(x+4)$ should have an odd power.

B. $6(x-3)^{10}(x+4)^{11}(x+2)^7$

   This corresponds to the leading coefficient being the opposite value than it should be.

C. $19(x-3)^6(x+4)^9(x+2)^{10}$

   The factor $(x+2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-19(x-3)^9(x+4)^6(x+2)^{11}$

   The factor 3 should have an even power and the factor -4 should have an odd power.

E. $-14(x-3)^8(x+4)^{11}(x+2)^5$

   * This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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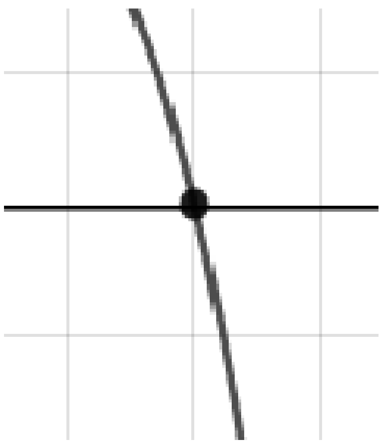
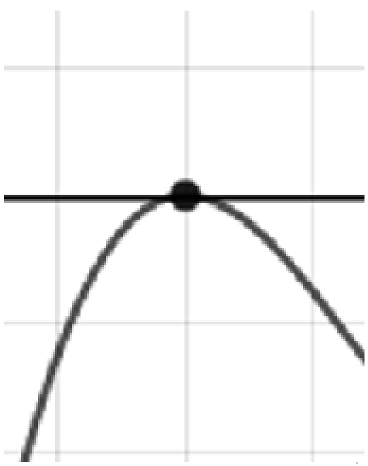
6. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

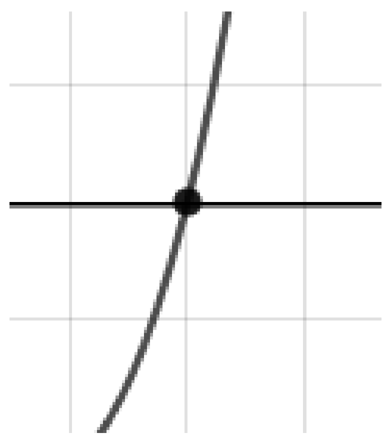
$$f(x) = 2(x+5)^4(x-5)^2(x+9)^{11}(x-9)^8$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 3i \text{ and } -2$$

The solution is $x^3 - 8x^2 + 14x + 68$, which is option D.

A. $b \in [-3, 4], c \in [-1, 3],$ and $d \in [-8, -3]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x - 3)(x + 2)$.

B. $b \in [5, 14], c \in [7, 19],$ and $d \in [-75, -65]$

$x^3 + 8x^2 + 14x - 68$, which corresponds to multiplying out $(x - (5 + 3i))(x - (5 - 3i))(x - 2)$.

C. $b \in [-3, 4], c \in [-7, -2],$ and $d \in [-10, -8]$

$x^3 + x^2 - 3x - 10$, which corresponds to multiplying out $(x - 5)(x + 2)$.

D. $b \in [-12, -7], c \in [7, 19],$ and $d \in [67, 75]$

* $x^3 - 8x^2 + 14x + 68$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 3i))(x - (5 - 3i))(x - (-2))$.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-2}{3}, \frac{7}{3}, \text{ and } 6$$

The solution is $9x^3 - 69x^2 + 76x + 84$, which is option A.

A. $a \in [3, 10], b \in [-71, -67], c \in [69, 84],$ and $d \in [82, 94]$

* $9x^3 - 69x^2 + 76x + 84$, which is the correct option.

B. $a \in [3, 10], b \in [-71, -67], c \in [69, 84],$ and $d \in [-86, -79]$

$9x^3 - 69x^2 + 76x - 84$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [3, 10], b \in [66, 74], c \in [69, 84],$ and $d \in [-86, -79]$

$9x^3 + 69x^2 + 76x - 84$, which corresponds to multiplying out $(3x - 2)(3x + 7)(x + 6)$.

D. $a \in [3, 10], b \in [-43, -34], c \in [-106, -100],$ and $d \in [82, 94]$

$9x^3 - 39x^2 - 104x + 84$, which corresponds to multiplying out $(3x - 2)(3x + 7)(x - 6)$.

E. $a \in [3, 10], b \in [-81, -79], c \in [175, 180],$ and $d \in [-86, -79]$

$9x^3 - 81x^2 + 176x - 84$, which corresponds to multiplying out $(3x - 2)(3x - 7)(x - 6)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 2)(3x - 7)(x - 6)$

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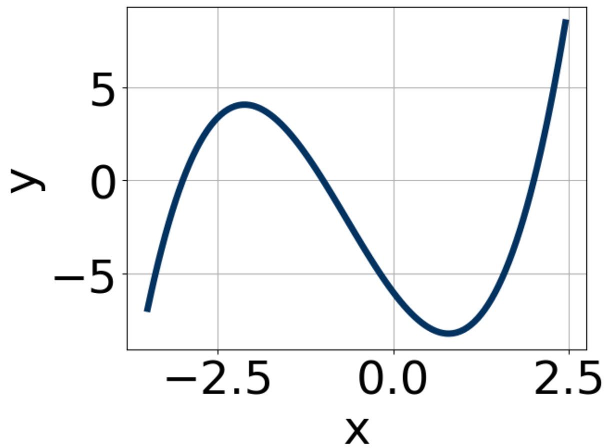
9. Which of the following equations *could* be of the graph presented below?



The solution is $20(x-2)^7(x+3)^5(x+1)^9$, which is option B.

A. $12(x-2)^6(x+3)^5(x+1)^7$

   The factor 2 should have been an odd power.

B. $20(x-2)^7(x+3)^5(x+1)^9$

   * This is the correct option.

C. $-17(x-2)^4(x+3)^7(x+1)^5$

   The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-14(x-2)^7(x+3)^{11}(x+1)^7$

   This corresponds to the leading coefficient being the opposite value than it should be.

E. $9(x-2)^4(x+3)^6(x+1)^9$

   The factors 2 and -3 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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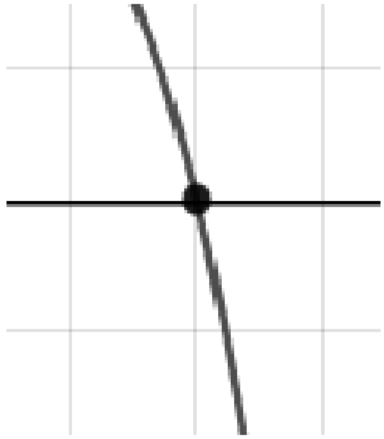
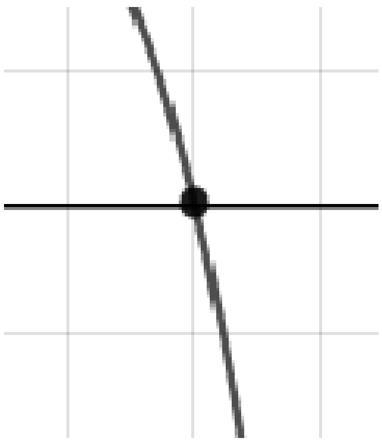
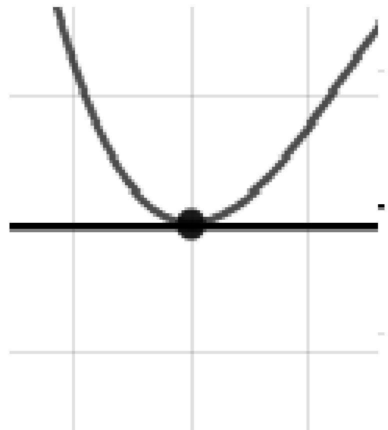
10. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

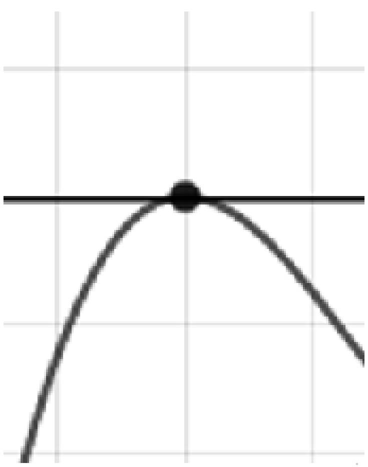
$$f(x) = 5(x-4)^9(x+4)^{10}(x-7)^9(x+7)^{10}$$

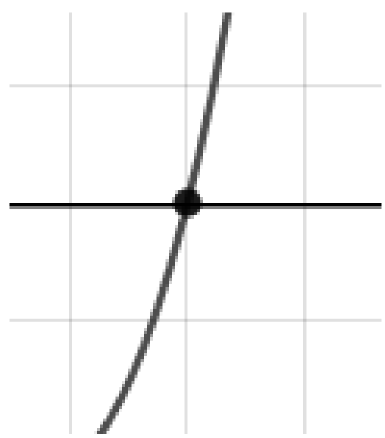
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.