This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^-1(7)$ belongs to.

$$f(x) = \ln(x + 2) - 5$$

The solution is $f^{-1}(7) = 162752.791$, which is option A.

A. $f^{-1}(7) \in [162750.79, 162754.79]$

This is the solution.

B. $f^{-1}(7) \in [-0.61, 7.39]$

This solution corresponds to distractor 1.

C. $f^{-1}(7) \in [143.41, 144.41]$

This solution corresponds to distractor 2.

D. $f^{-1}(7) \in [162753.79, 162764.79]$

This solution corresponds to distractor 3.

E. $f^{-1}(7) \in [8098.08, 8102.08]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

2. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{2}{4x + 21} \text{ and } g(x) = \frac{2}{6x - 23}$$

The solution is The domain is all Real numbers except $x = -5.25$ and $x = 3.83$, which is option D.

A. The domain is all Real numbers except $x = a$, where $a \in [0.4, 11.4]$

B. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [1, 9]$

C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-6.67, -2.67]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-7.25, -4.25]$ and $b \in [0.83, 7.83]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

3. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval that $f^{-1}(11)$ belongs to.

$$f(x) = \sqrt[3]{2x + 3}$$

The solution is 664.0, which is option A.

A. $f^{-1}(11) \in [663.3, 664.8]$

   * This is the correct solution.

B. $f^{-1}(11) \in [664.9, 667.9]$

   Distractor 1: This corresponds to

C. $f^{-1}(11) \in [-664.5, -661.8]$

   This solution corresponds to distractor 2.

D. $f^{-1}(11) \in [-669.6, -664.4]$

   This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

   This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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4. Determine whether the function below is 1-1.

$$f(x) = -9x^2 + 15x + 234$$

The solution is no, which is option E.

A. No, because the domain of the function is not $(-\infty, \infty)$.

   Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

   Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because the range of the function is not $(-\infty, \infty)$.

   Corresponds to believing 1-1 means the range is all Real numbers.

D. Yes, the function is 1-1.

   Corresponds to believing the function passes the Horizontal Line test.

E. No, because there is a y-value that goes to 2 different x-values.

   * This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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5. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = x + 6 \text{ and } g(x) = \frac{1}{4x - 13}$$

The solution is The domain is all Real numbers except $x = 3.25$, which is option A.

A. The domain is all Real numbers except $x = a$, where $a \in [2.25, 6.25]$

    B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-6.4, -2.4]$

    C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-6.75, -2.75]$

    D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-12.33, 2.67]$ and $b \in [-8.67, -3.67]$

    E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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6. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^-1(7)$ belongs to.

$$f(x) = e^{x-4} + 5$$

The solution is $f^{-1}(7) = 4.693,$ which is option B.

    A. $f^{-1}(7) \in [6.02, 6.21]$

        This solution corresponds to distractor 4.

    B. $f^{-1}(7) \in [4.66, 4.73]$

        This is the solution.

    C. $f^{-1}(7) \in [7.35, 7.45]$

        This solution corresponds to distractor 3.

    D. $f^{-1}(7) \in [-3.34, -3.28]$

        This solution corresponds to distractor 1.

    E. $f^{-1}(7) \in [7.41, 7.5]$

        This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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7. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 - 2x^2 + 2x \text{ and } g(x) = -2x^3 - 3x^2 + 3x + 1$$

The solution is $-2.0,$ which is option D.

    A. $(f \circ g)(1) \in [-1.78, -0.74]$

        Distractor 1: Corresponds to reversing the composition.

    B. $(f \circ g)(1) \in [2.64, 3.93]$

        Distractor 2: Corresponds to being slightly off from the solution.

    C. $(f \circ g)(1) \in [-6.26, -5.68]$

        Distractor 3: Corresponds to being slightly off from the solution.

    D. $(f \circ g)(1) \in [-2.2, -1.72]$

        * This is the correct solution

    E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 12$ and choose the interval that $f^{-1}(12)$ belongs to.

$$f(x) = \sqrt[3]{3x + 4}$$

The solution is 574.6666666666666, which is option A.

A. $f^{-1}(12) \in [574, 576.8]$

   * This is the correct solution.

B. $f^{-1}(12) \in [577.3, 578.8]$

   Distractor 1: This corresponds to

C. $f^{-1}(12) \in [-580.7, -574.7]$

   This solution corresponds to distractor 3.

D. $f^{-1}(12) \in [-575.6, -573.9]$

   This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

   This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -3x^3 - 2x^2 + 3x + 4 \text{ and } g(x) = x^3 - 2x^2 + 3x$$

The solution is -22.0, which is option C.

A. $(f \circ g)(1) \in [-30, -24]$

   Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [6, 11]$

   Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(1) \in [-26, -20]$

   * This is the correct solution

D. $(f \circ g)(1) \in [-6, 1]$

   Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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10. Determine whether the function below is 1-1.

$$f(x) = (4x + 13)^3$$

The solution is yes, which is option C.

A. No, because the domain of the function is not $(-\infty, \infty)$.

   Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. Yes, the function is 1-1.

* This is the solution.

D. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.