This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = x^4 + 8x^3 + 7x + 7 \text{ and } g(x) = \frac{1}{5x + 36}$$

The solution is The domain is all Real numbers except $x = -7.2$, which is option C.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-7, -3]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-6.33, -0.33]$

C. The domain is all Real numbers except $x = a,$ where $a \in [-11.2, -2.2]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-6.33, -3.33]$ and $b \in [1.2, 8.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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2. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^-1(9)$ belongs to.

$$f(x) = \ln(x - 2) - 5$$

The solution is $f^{-1}(9) = 1202606.284$, which is option D.

A. $f^{-1}(9) \in [1090.63, 1098.63]$

This solution corresponds to distractor 4.

B. $f^{-1}(9) \in [59866.14, 59873.14]$

This solution corresponds to distractor 2.

C. $f^{-1}(9) \in [1202600.28, 1202606.28]$

This solution corresponds to distractor 3.

D. $f^{-1}(9) \in [1202603.28, 1202608.28]$

This is the solution.

E. $f^{-1}(9) \in [51.6, 59.6]$

This solution corresponds to distractor 1.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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3. Determine whether the function below is 1-1.

$$f(x) = -25x^2 + 30x + 391$$

The solution is no, which is option B.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^-1(8)$ belongs to.

$$f(x) = \ln(x + 5) - 2$$

The solution is $f^{-1}(8) = 22021.466$, which is option A.

A. $f^{-1}(8) \in [22019.47, 22022.47]$

This is the solution.

B. $f^{-1}(8) \in [442411.39, 442413.39]$

This solution corresponds to distractor 4.

C. $f^{-1}(8) \in [22029.47, 22035.47]$

This solution corresponds to distractor 3.

D. $f^{-1}(8) \in [13.09, 22.09]$

This solution corresponds to distractor 2.

E. $f^{-1}(8) \in [395.43, 399.43]$

This solution corresponds to distractor 1.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Determine whether the function below is 1-1.

$$f(x) = 9x^2 - 78x + 169$$

The solution is no, which is option C.

A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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6. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = x^3 + 2x^2 - 3x - 3 \text{ and } g(x) = -2x^3 - 2x^2 + 3x + 2$$

The solution is 1.0, which is option A.

A. $(f \circ g)(-1) \in [0.74, 1.77]$

* This is the correct solution

B. $(f \circ g)(-1) \in [0.74, 1.77]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [-6.62, -4.63]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-4.38, -3.54]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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7. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -3x^3 + 4x^2 + x - 3 \text{ and } g(x) = -2x^3 + 3x^2 + 3x - 2$$

The solution is -3.0, which is option A.

A. $(f \circ g)(-1) \in [-4, 1]$

* This is the correct solution

B. $(f \circ g)(-1) \in [-13, -6]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [1, 7]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-29, -19]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 15$ and choose the interval that $f^-1(15)$ belongs to.

$$f(x) = 4x^2 + 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(15) \in [1.41, 1.79]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(15) \in [2.37, 2.82]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(15) \in [5.29, 6]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(15) \in [2.06, 2.37]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval that $f^-1(11)$ belongs to.

$$f(x) = 2x^2 + 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(11) \in [2.52, 2.9]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(11) \in [1.19, 2.72]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C. $f^{-1}(11) \in [5.06, 7.17]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(11) \in [2.97, 4.59]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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10. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 5x^3 + 9x^2 + 3x + 3 \text{ and } g(x) = \frac{5}{5x - 34}$$

The solution is The domain is all Real numbers except $x = 6.8$, which is option C.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [7, 13]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-0.8, 5.2]$

C. The domain is all Real numbers except $x = a,$ where $a \in [5.8, 8.8]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-14.67, -1.67]$ and $b \in [-6.17, 4.83]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.
