This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{4}, \frac{-7}{5}, \text{ and } \frac{5}{2}$$

The solution is $40x^3 - 114x^2 - 63x + 245$, which is option C.

A. $a \in [39, 49], b \in [114, 119], c \in [-65, -62],$ and $d \in [-247, -238]$

$40x^3 + 114x^2 - 63x - 245$, which corresponds to multiplying out $(4x + 7)(5x - 7)(2x + 5)$.

B. $a \in [39, 49], b \in [-87, -83], c \in [-137, -129],$ and $d \in [241, 248]$

$40x^3 - 86x^2 - 133x + 245$, which corresponds to multiplying out $(4x + 7)(5x - 7)(2x - 5)$.

C. $a \in [39, 49], b \in [-115, -112], c \in [-65, -62],$ and $d \in [241, 248]$

* $40x^3 - 114x^2 - 63x + 245$, which is the correct option.

D. $a \in [39, 49], b \in [-115, -112], c \in [-65, -62],$ and $d \in [-247, -238]$

$40x^3 - 114x^2 - 63x - 245$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [39, 49], b \in [25, 29], c \in [-220, -214],$ and $d \in [-247, -238]$

$40x^3 + 26x^2 - 217x - 245$, which corresponds to multiplying out $(4x + 7)(5x + 7)(2x - 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 7)(5x + 7)(2x - 5)$

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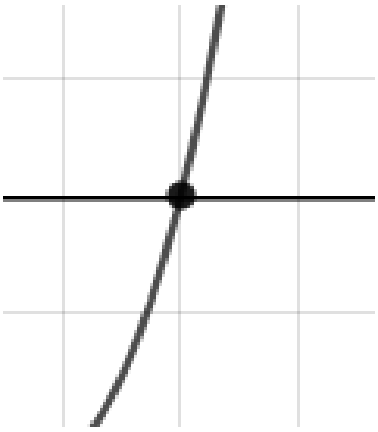
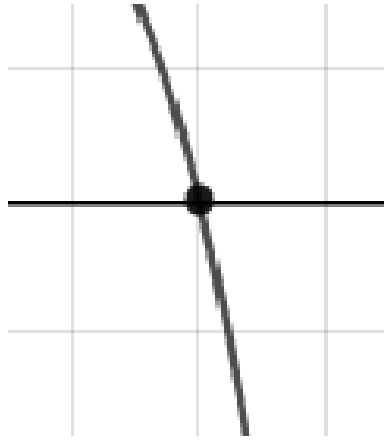
2. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

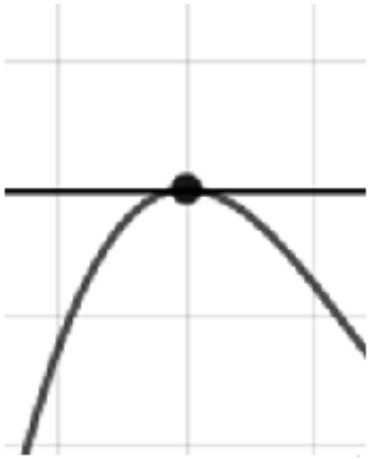
$$f(x) = 3(x + 7)^{11}(x - 7)^9(x - 8)^8(x + 8)^5$$

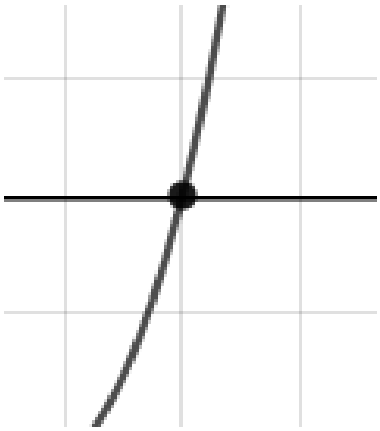
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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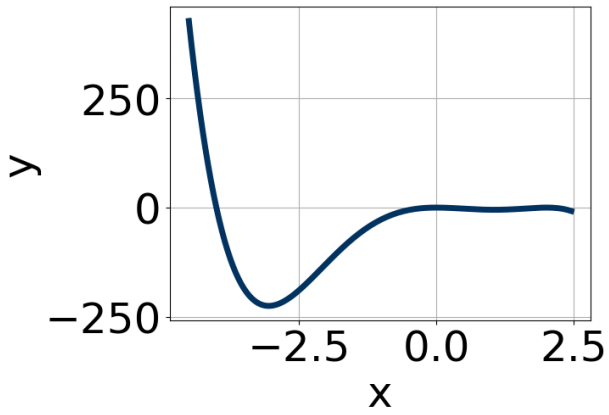
3. Which of the following equations *could* be of the graph presented below?



The solution is $-20x^4(x-2)^8(x+4)^{11}$, which is option A.

A. $-20x^4(x-2)^8(x+4)^{11}$

* This is the correct option.

B. $-4x^8(x-2)^{11}(x+4)^7$

The factor $(x-2)$ should have an even power.

C. $6x^4(x-2)^6(x+4)^8$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-7x^8(x-2)^5(x+4)^8$

The factor $(x-2)$ should have an even power and the factor $(x+4)$ should have an odd power.

E. $14x^6(x-2)^{10}(x+4)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 - 3i \text{ and } -2$$

The solution is $x^3 + 12x^2 + 54x + 68$, which is option D.

A. $b \in [-1, 10], c \in [6.99, 8.99],$ and $d \in [7, 12]$

$x^3 + x^2 + 7x + 10$, which corresponds to multiplying out $(x+5)(x+2)$.

B. $b \in [-16, -10], c \in [53.47, 55.02],$ and $d \in [-72, -63]$

$x^3 - 12x^2 + 54x - 68$, which corresponds to multiplying out $(x-(-5-3i))(x-(-5+3i))(x-2)$.

C. $b \in [-1, 10], c \in [4.6, 5.04],$ and $d \in [1, 7]$

$x^3 + x^2 + 5x + 6$, which corresponds to multiplying out $(x+3)(x+2)$.

D. $b \in [11, 13], c \in [53.47, 55.02],$ and $d \in [68, 69]$

* $x^3 + 12x^2 + 54x + 68$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-5-3i))(x-(-5+3i))(x-(-2))$.

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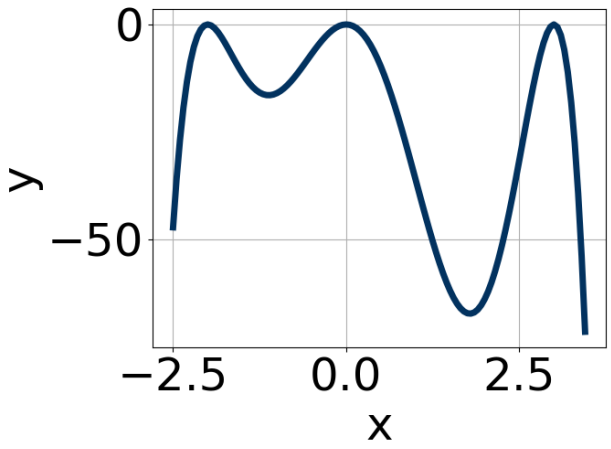
5. Which of the following equations *could* be of the graph presented below?



The solution is $-17x^4(x-3)^{10}(x+2)^4$, which is option C.

A. $-13x^{10}(x-3)^4(x+2)^{11}$

The factor $(x+2)$ should have an even power.

B. $11x^4(x-3)^4(x+2)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-17x^4(x-3)^{10}(x+2)^4$

* This is the correct option.

D. $-8x^8(x-3)^7(x+2)^7$

The factors $(x-3)$ and $(x+2)$ should both have even powers.

E. $12x^{10}(x-3)^{10}(x+2)^{11}$

The factor $(x+2)$ should have an even power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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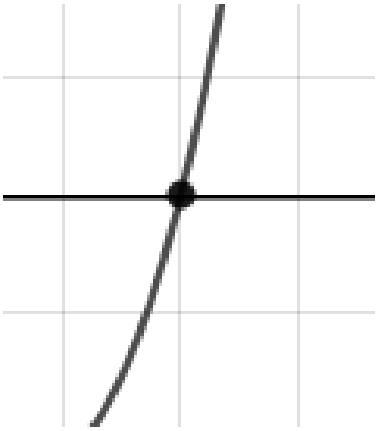
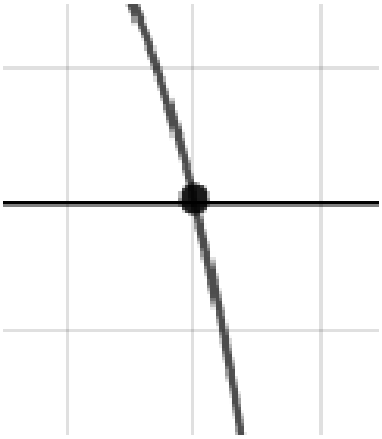
6. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

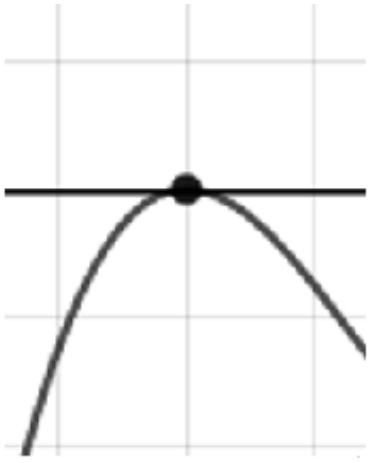
$$f(x) = 5(x-3)^5(x+3)^{10}(x+9)^6(x-9)^{10}$$

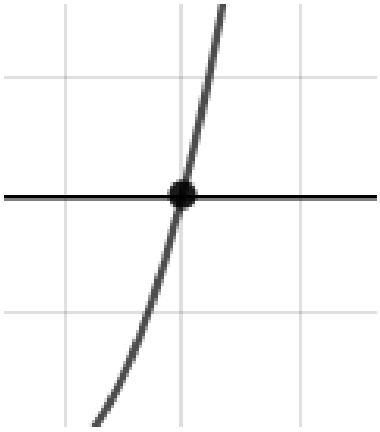
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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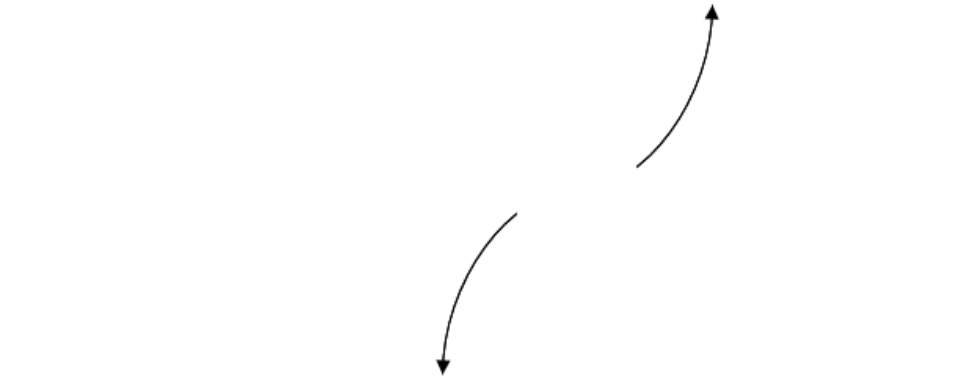
7. Describe the end behavior of the polynomial below.

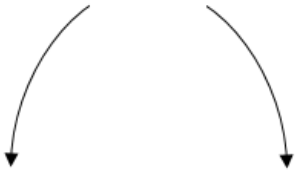
$$f(x) = 2(x+7)^3(x-7)^8(x-2)^2(x+2)^4$$

The solution is the graph below, which is option D.

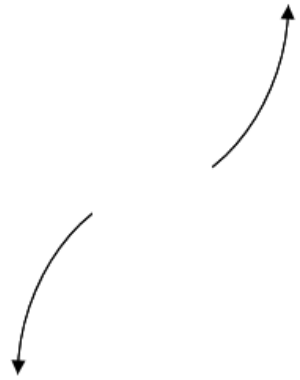


A.

C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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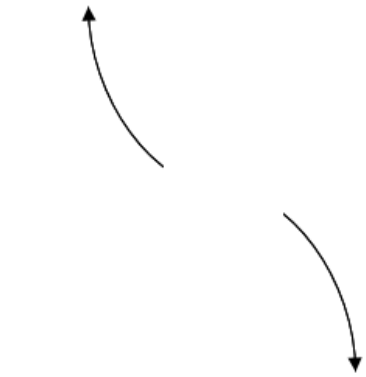
8. Describe the end behavior of the polynomial below.

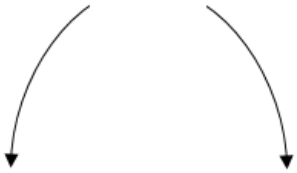
$$f(x) = -8(x-9)^4(x+9)^5(x+2)^4(x-2)^5$$

The solution is the graph below, which is option B.

A.

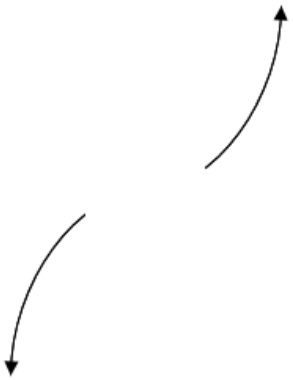


B.



C.

D.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{4}, \frac{-1}{5}, \text{ and } \frac{4}{5}$$

The solution is $100x^3 - 35x^2 - 31x - 4$, which is option D.

A. $a \in [97, 105], b \in [-39, -33], c \in [-38, -27],$ and $d \in [2, 6]$

$100x^3 - 35x^2 - 31x + 4$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [97, 105], b \in [-125, -121], c \in [36, 47],$ and $d \in [-7, -2]$

$100x^3 - 125x^2 + 41x - 4$, which corresponds to multiplying out $(4x - 1)(5x - 1)(5x - 4)$.

C. $a \in [97, 105], b \in [-87, -79], c \in [-1, 8],$ and $d \in [2, 6]$

$100x^3 - 85x^2 - x + 4$, which corresponds to multiplying out $(4x - 1)(5x + 1)(5x - 4)$.

D. $a \in [97, 105], b \in [-39, -33], c \in [-38, -27],$ and $d \in [-7, -2]$

* $100x^3 - 35x^2 - 31x - 4$, which is the correct option.

E. $a \in [97, 105], b \in [32, 40], c \in [-38, -27],$ and $d \in [2, 6]$

$100x^3 + 35x^2 - 31x + 4$, which corresponds to multiplying out $(4x - 1)(5x - 1)(5x + 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 1)(5x + 1)(5x - 4)$

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 3i \text{ and } 4$$

The solution is $x^3 - 10x^2 + 42x - 72$, which is option D.

A. $b \in [-1, 7], c \in [-6, 0],$ and $d \in [-14, -11]$

$x^3 + x^2 - x - 12$, which corresponds to multiplying out $(x + 3)(x - 4)$.

B. $b \in [-1, 7], c \in [-8, -2],$ and $d \in [10, 13]$

$x^3 + x^2 - 7x + 12$, which corresponds to multiplying out $(x - 3)(x - 4)$.

C. $b \in [5, 20], c \in [34, 44],$ and $d \in [72, 78]$

$x^3 + 10x^2 + 42x + 72$, which corresponds to multiplying out $(x - (3 - 3i))(x - (3 + 3i))(x + 4)$.

D. $b \in [-10, -5], c \in [34, 44],$ and $d \in [-77, -69]$

* $x^3 - 10x^2 + 42x - 72$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 - 3i))(x - (3 + 3i))(x - (4))$.
