This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 + 39x^2 - 30}{x + 3}$$

The solution is $12x^2 + 3x - 9 + \frac{-3}{x+3}$, which is option C.

A. $a \in [-38, -33], b \in [147, 149], c \in [-441, -436], \text{ and } r \in [1291, 1296]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [11, 13], b \in [75, 77], c \in [220, 232], \text{ and } r \in [644, 646]$.

You divided by the opposite of the factor.

C. $a \in [11, 13], b \in [-3, 5], c \in [-12, -2], \text{ and } r \in [-7, 2]$.

* This is the solution!

D. $a \in [-38, -33], b \in [-70, -65], c \in [-207, -199], \text{ and } r \in [-654, -650]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [11, 13], b \in [-14, -8], c \in [29, 38], \text{ and } r \in [-181, -171]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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2. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 - 44x^2 - 79x + 60$$

The solution is $[-1.67, 0.6, 4]$, which is option D.

A. $z_1 \in [-5, -2], \; z_2 \in [-0.9, -0.2], \text{ and } z_3 \in [0.92, 1.85]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5, -2], \; z_2 \in [-2.2, -0.9], \text{ and } z_3 \in [0.42, 0.84]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-0.6, 0.4], \; z_2 \in [1.1, 2.8], \text{ and } z_3 \in [3.7, 4.42]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-1.67, -0.67]$, $z_2 \in [-0.3, 0.8]$, and $z_3 \in [3.7, 4.42]$

* This is the solution!

E. $z_1 \in [-5, -2]$, $z_2 \in [-3.3, -2.1]$, and $z_3 \in [-0.02, 0.34]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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3. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 + 3x^2 - 79x - 60$$

The solution is $[-2.5, -0.8, 3]$, which is option C.

A. $z_1 \in [-3.1, -2.7]$, $z_2 \in [0, 0.71]$, and $z_3 \in [4.9, 5.12]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-3.1, -2.7]$, $z_2 \in [0, 0.71]$, and $z_3 \in [1.12, 1.66]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-2.8, -1.6]$, $z_2 \in [-0.89, -0.5]$, and $z_3 \in [2.53, 3.23]$

* This is the solution!

D. $z_1 \in [-3.1, -2.7]$, $z_2 \in [0.74, 1.16]$, and $z_3 \in [2.35, 2.78]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-1.5, -0.9]$, $z_2 \in [-0.58, -0.22]$, and $z_3 \in [2.53, 3.23]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 - 75x - 129}{x - 5}$$

The solution is $4x^2 + 20x + 25 + \frac{-4}{x-5}$, which is option E.

A. $a \in [2, 7], b \in [12, 18], c \in [-14, -6]$, and $r \in [-181, -171]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [2, 7], b \in [-26, -14], c \in [21, 26]$, and $r \in [-257, -246]$.

You divided by the opposite of the factor.

C. $a \in [17, 22], b \in [-103, -96], c \in [424, 427]$, and $r \in [-2255, -2252]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [17, 22], b \in [96, 105], c \in [424, 427]$, and $r \in [1992, 1999]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [2, 7], b \in [17, 23], c \in [21, 26]$, and $r \in [-7, -3]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 55x^2 - 30x - 43}{x + 3}$$

The solution is $20x^2 - 5x - 15 + \frac{2}{x+3}$, which is option E.

A. $a \in [-62, -56]$, $b \in [-130, -124]$, $c \in [-406, -400]$, and $r \in [-1263, -1256]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [19, 26]$, $b \in [111, 121]$, $c \in [310, 319]$, and $r \in [898, 908]$.

You divided by the opposite of the factor.

C. $a \in [-62, -56]$, $b \in [231, 237]$, $c \in [-735, -733]$, and $r \in [2160, 2164]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [19, 26]$, $b \in [-26, -22]$, $c \in [66, 73]$, and $r \in [-327, -319]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [19, 26]$, $b \in [-6, -1]$, $c \in [-18, -14]$, and $r \in [1, 8]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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6. Factor the polynomial below completely, knowing that $x - 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 - 113x^3 + 434x^2 - 655x + 300$$

The solution is $[0.8, 2.5, 3, 5]$, which is option C.

A. $z_1 \in [-1.5, 0.7]$, $z_2 \in [0.88, 2.15], z_3 \in [2.83, 3.07]$, and $z_4 \in [4.76, 5.22]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-6.1, -4.5]$, $z_2 \in [-3.06, -1.3], z_3 \in [-1.58, -0.95]$, and $z_4 \in [-0.43, -0.37]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [0.5, 0.9]$, $z_2 \in [2.3, 2.76], z_3 \in [2.83, 3.07]$, and $z_4 \in [4.76, 5.22]$

* This is the solution!

D. $z_1 \in [-6.1, -4.5]$, $z_2 \in [-3.06, -1.3], z_3 \in [-2.56, -2.44]$, and $z_4 \in [-0.91, -0.66]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-6.1, -4.5]$, $z_2 \in [-4.78, -3.6]$, $z_3 \in [-3.23, -2.63]$, and $z_4 \in [-0.61, -0.48]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. Factor the polynomial below completely, knowing that $x-2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 - 83x^3 + 197x^2 - 188x + 60$$

The solution is $[0.667, 1.25, 2, 3]$, which is option E.

A. $z_1 \in [-3.21, -2.92]$, $z_2 \in [-2.11, -1.9]$, $z_3 \in [-1.87, -1.4]$, and $z_4 \in [-0.97, -0.76]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-3.21, -2.92]$, $z_2 \in [-2.11, -1.9]$, $z_3 \in [-2.06, -1.61]$, and $z_4 \in [-0.48, -0.26]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [0.79, 1.04]$, $z_2 \in [1.45, 1.69]$, $z_3 \in [1.79, 2.39]$, and $z_4 \in [2.98, 3.13]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.21, -2.92]$, $z_2 \in [-2.11, -1.9]$, $z_3 \in [-1.42, -1.16]$, and $z_4 \in [-0.73, -0.63]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [0.42, 0.78]$, $z_2 \in [0.65, 1.49]$, $z_3 \in [1.79, 2.39]$, and $z_4 \in [2.98, 3.13]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 46x^2 + 40x + 22}{x - 3}$$

The solution is $10x^2 - 16x - 8 + \frac{-2}{x-3}$, which is option B.

A. $a \in [29, 35]$, $b \in [41, 48]$, $c \in [169, 174]$, and $r \in [534, 540]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [10, 11]$, $b \in [-18, -9]$, $c \in [-8, -7]$, and $r \in [-5, 2]$.

* This is the solution!

C. $a \in [10, 11]$, $b \in [-76, -75]$, $c \in [265, 271]$, and $r \in [-787, -778]$.

You divided by the opposite of the factor.

D. $a \in [29, 35]$, $b \in [-137, -132]$, $c \in [448, 452]$, and $r \in [-1326, -1315]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [10, 11]$, $b \in [-30, -22]$, $c \in [-12, -9]$, and $r \in [-5, 2]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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9. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^2 + 7x + 7$$

The solution is $\pm 1, \pm 7$, which is option B.

A. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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10. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^2 + 6x + 4$$

The solution is $\pm 1, \pm 2, \pm 4$, which is option A.

A. $\pm 1, \pm 2, \pm 4$

* This is the solution **since we asked for the possible Integer roots**!

B. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.