This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{3}, \frac{-1}{4}, \text{ and } \frac{-5}{2}$$

The solution is $24x^3 + 26x^2 - 95x - 25$, which is option D.

A. $a \in [23, 25], b \in [-28, -20], c \in [-98, -94]$, and $d \in [21, 27]$

$24x^3 - 26x^2 - 95x + 25$, which corresponds to multiplying out $(3x + 5)(4x - 1)(2x - 5)$.

B. $a \in [23, 25], b \in [106, 107], c \in [123, 131]$, and $d \in [21, 27]$

$24x^3 + 106x^2 + 125x + 25$, which corresponds to multiplying out $(3x + 5)(4x + 1)(2x + 5)$.

C. $a \in [23, 25], b \in [26, 29], c \in [-98, -94]$, and $d \in [21, 27]$

$24x^3 + 26x^2 - 95x + 25$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [23, 25], b \in [26, 29], c \in [-98, -94]$, and $d \in [-26, -21]$

* $24x^3 + 26x^2 - 95x - 25$, which is the correct option.

E. $a \in [23, 25], b \in [90, 99], c \in [74, 78]$, and $d \in [-26, -21]$

$24x^3 + 94x^2 + 75x - 25$, which corresponds to multiplying out $(3x + 5)(4x - 1)(2x + 5)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 5)(4x + 1)(2x + 5)$

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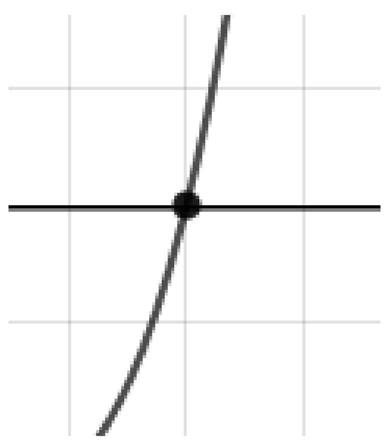
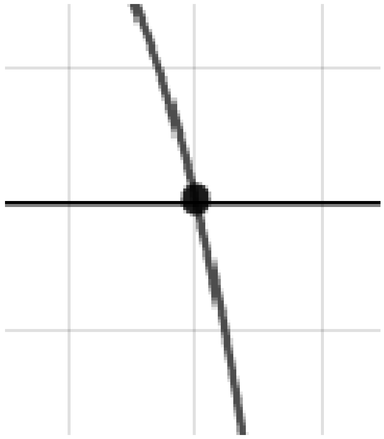
2. Describe the zero behavior of the zero $x = -3$ of the polynomial below.

$$f(x) = -2(x - 3)^4(x + 3)^7(x + 2)^7(x - 2)^{10}$$

The solution is the graph below, which is option D.



A. 

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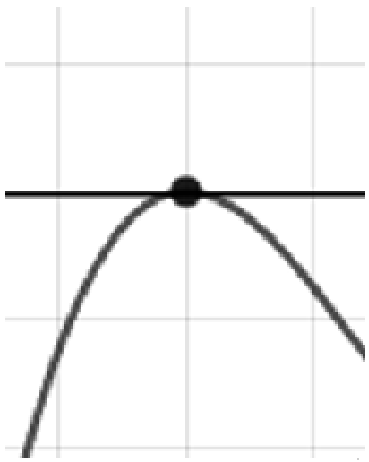
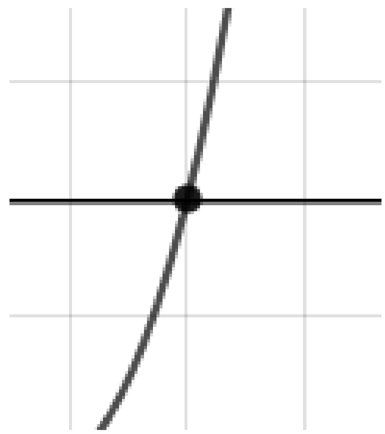
B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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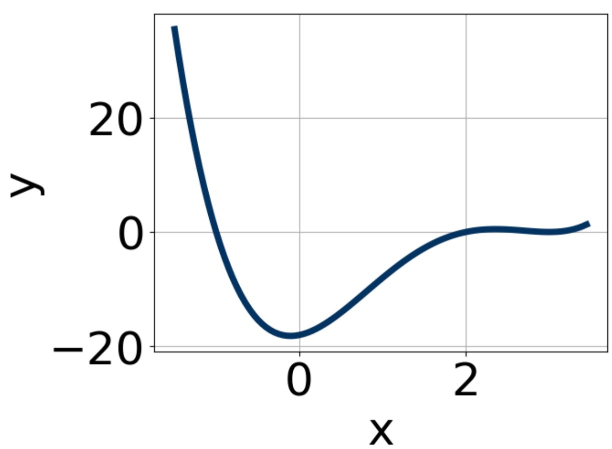
3. Which of the following equations *could* be of the graph presented below?



The solution is $5(x-3)^{10}(x-2)^{11}(x+1)^{11}$, which is option B.

A. $18(x-3)^4(x-2)^{10}(x+1)^{11}$

The factor $(x-2)$ should have an odd power.

B. $5(x-3)^{10}(x-2)^{11}(x+1)^{11}$

* This is the correct option.

C. $-14(x-3)^{10}(x-2)^{11}(x+1)^{4}$

The factor $(x+1)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $18(x-3)^{7}(x-2)^{4}(x+1)^{7}$

The factor 3 should have an even power and the factor 2 should have an odd power.

E. $-10(x-3)^{4}(x-2)^{9}(x+1)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 - 4i \text{ and } -3$$

The solution is $x^3 + 9x^2 + 43x + 75$, which is option C.

A. $b \in [-3, 3], c \in [6.2, 9.3], \text{ and } d \in [12, 14]$

$x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x+4)(x+3)$.

B. $b \in [-12, -8], c \in [42, 47.2], \text{ and } d \in [-75, -74]$

$x^3 - 9x^2 + 43x - 75$, which corresponds to multiplying out $(x-(-3-4i))(x-(-3+4i))(x-3)$.

C. $b \in [9, 13], c \in [42, 47.2], \text{ and } d \in [72, 82]$

* $x^3 + 9x^2 + 43x + 75$, which is the correct option.

D. $b \in [-3, 3], c \in [2.5, 6.7], \text{ and } d \in [5, 11]$

$x^3 + x^2 + 6x + 9$, which corresponds to multiplying out $(x+3)(x+3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x-(-3-4i))(x-(-3+4i))(x-(-3))$.

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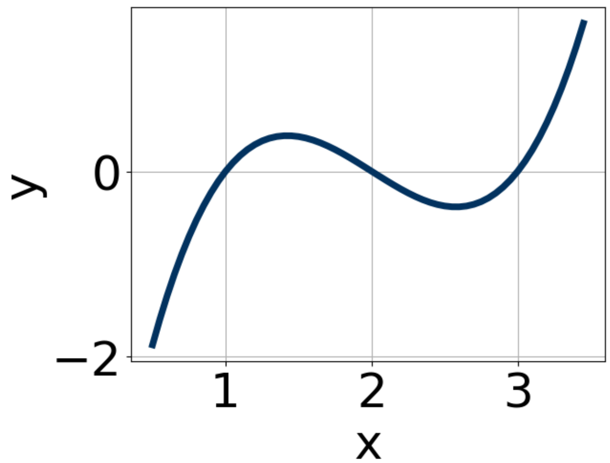
5. Which of the following equations *could* be of the graph presented below?



The solution is $4(x-1)^7(x-2)^5(x-3)^9$, which is option D.

A. $-20(x-1)^8(x-2)^9(x-3)^5$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $20(x-1)^8(x-2)^5(x-3)^7$

The factor 1 should have been an odd power.

C. $-5(x-1)^{11}(x-2)^{11}(x-3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $4(x-1)^7(x-2)^5(x-3)^9$

* This is the correct option.

E. $7(x-1)^8(x-2)^6(x-3)^7$

The factors 1 and 2 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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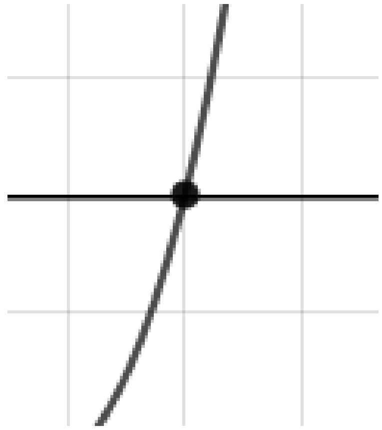
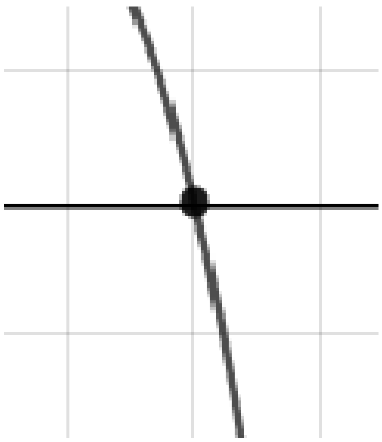
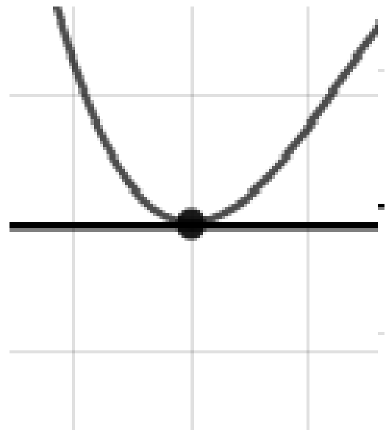
6. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

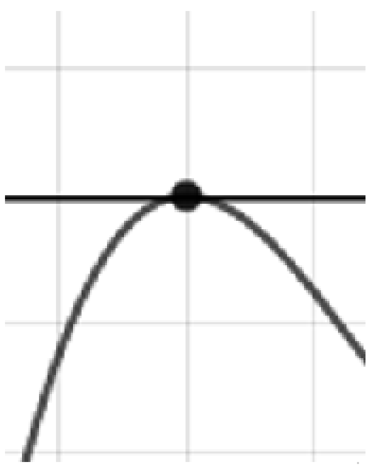
$$f(x) = 4(x-2)^6(x+2)^3(x-5)^7(x+5)^2$$

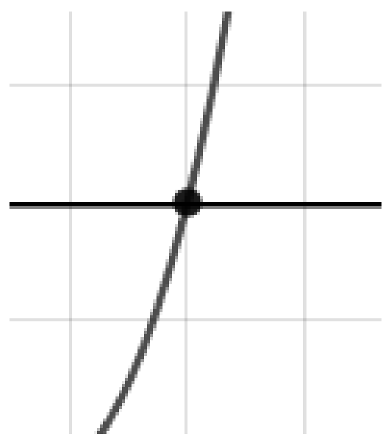
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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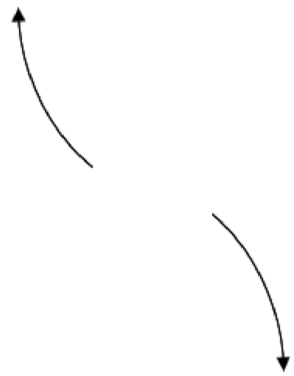
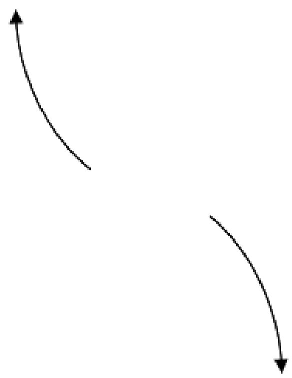
7. Describe the end behavior of the polynomial below.

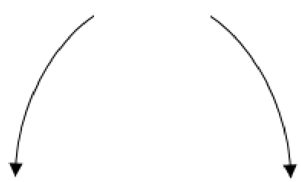
$$f(x) = -2(x+2)^3(x-2)^8(x-9)^4(x+9)^6$$

The solution is the graph below, which is option A.





A.



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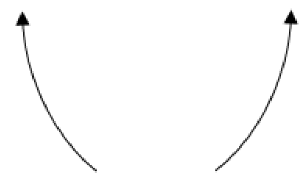
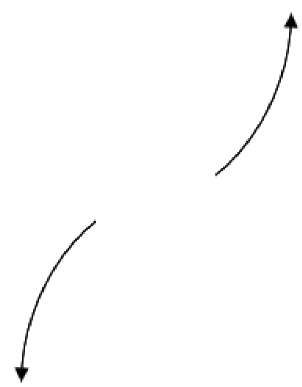


C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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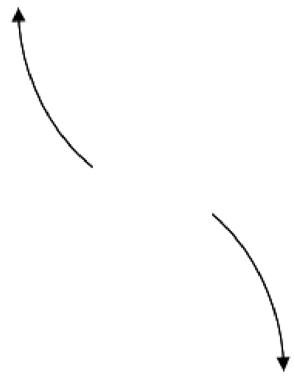
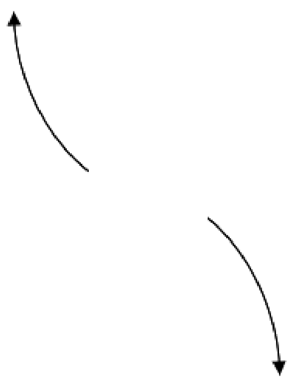
8. Describe the end behavior of the polynomial below.

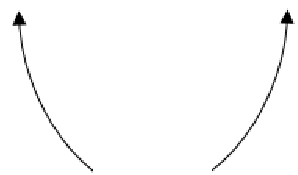
$$f(x) = -4(x-5)^5(x+5)^8(x-4)^5(x+4)^5$$

The solution is the graph below, which is option A.

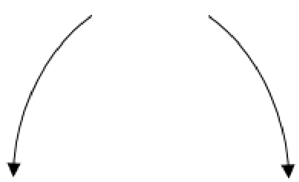




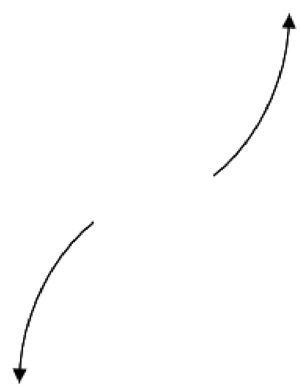
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{4}{3}, \frac{1}{4}, \text{ and } 1$$

The solution is $12x^3 - 31x^2 + 23x - 4$, which is option D.

A. $a \in [11, 21], b \in [-0.6, 2.1], c \in [-19.5, -16.4],$ and $d \in [2, 6]$

$12x^3 + x^2 - 17x + 4$, which corresponds to multiplying out $(3x + 4)(4x - 1)(x - 1)$.

B. $a \in [11, 21], b \in [1.5, 8.2], c \in [-16, -14.6], \text{ and } d \in [-6, 0]$

$12x^3 + 7x^2 - 15x - 4$, which corresponds to multiplying out $(3x + 4)(4x + 1)(x - 1)$.

C. $a \in [11, 21], b \in [30.9, 32.5], c \in [20.5, 27.3], \text{ and } d \in [2, 6]$

$12x^3 + 31x^2 + 23x + 4$, which corresponds to multiplying out $(3x + 4)(4x + 1)(x + 1)$.

D. $a \in [11, 21], b \in [-32.9, -28.5], c \in [20.5, 27.3], \text{ and } d \in [-6, 0]$

* $12x^3 - 31x^2 + 23x - 4$, which is the correct option.

E. $a \in [11, 21], b \in [-32.9, -28.5], c \in [20.5, 27.3], \text{ and } d \in [2, 6]$

$12x^3 - 31x^2 + 23x + 4$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 4)(4x - 1)(x - 1)$

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 3i \text{ and } -1$$

The solution is $x^3 + 5x^2 + 17x + 13$, which is option D.

A. $b \in [0.1, 3.9], c \in [-7, 0], \text{ and } d \in [-5.5, -1.1]$

$x^3 + x^2 - 2x - 3$, which corresponds to multiplying out $(x - 3)(x + 1)$.

B. $b \in [-10.8, -3], c \in [12, 26], \text{ and } d \in [-14.5, -9.4]$

$x^3 - 5x^2 + 17x - 13$, which corresponds to multiplying out $(x - (-2 + 3i))(x - (-2 - 3i))(x - 1)$.

C. $b \in [0.1, 3.9], c \in [2, 8], \text{ and } d \in [0.4, 3.8]$

$x^3 + x^2 + 3x + 2$, which corresponds to multiplying out $(x + 2)(x + 1)$.

D. $b \in [2.5, 6.7], c \in [12, 26], \text{ and } d \in [12.3, 16.1]$

* $x^3 + 5x^2 + 17x + 13$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 3i))(x - (-2 - 3i))(x - (-1))$.
