This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 2i \text{ and } -4$$

The solution is $x^3 - 4x^2 - 12x + 80$, which is option B.

A. $b \in [2.8, 5.1], c \in [-12, -11]$, and $d \in [-87, -78]$

$x^3 + 4x^2 - 12x - 80$, which corresponds to multiplying out $(x - (4 - 2i))(x - (4 + 2i))(x - 4)$.

B. $b \in [-7.8, -3.5], c \in [-12, -11]$, and $d \in [78, 84]$

* $x^3 - 4x^2 - 12x + 80$, which is the correct option.

C. $b \in [-0.6, 1.1], c \in [3, 7]$, and $d \in [3, 9]$

$x^3 + x^2 + 6x + 8$, which corresponds to multiplying out $(x + 2)(x + 4)$.

D. $b \in [-0.6, 1.1], c \in [0, 5]$, and $d \in [-20, -13]$

$x^3 + x^2 - 16$, which corresponds to multiplying out $(x - 4)(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 2i))(x - (4 + 2i))(x - (-4))$.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-6, \frac{1}{3}, \text{ and } \frac{-3}{2}$$

The solution is $6x^3 + 43x^2 + 39x - 18$, which is option B.

A. $a \in [6, 12], b \in [-25.3, -24.5], c \in [-64, -61]$, and $d \in [-24, -15]$

$6x^3 - 25x^2 - 63x - 18$, which corresponds to multiplying out $(x - 6)(3x + 1)(2x + 3)$.

B. $a \in [6, 12], b \in [40.1, 45.7], c \in [33, 40]$, and $d \in [-24, -15]$

* $6x^3 + 43x^2 + 39x - 18$, which is the correct option.

C. $a \in [6, 12], b \in [-30.7, -26], c \in [-53, -38]$, and $d \in [11, 26]$

$6x^3 - 29x^2 - 45x + 18$, which corresponds to multiplying out $(x - 6)(3x - 1)(2x + 3)$.

D. $a \in [6, 12], b \in [-44.1, -41], c \in [33, 40],$ and $d \in [11, 26]$

$6x^3 - 43x^2 + 39x + 18$, which corresponds to multiplying out $(x - 6)(3x + 1)(2x - 3)$.

E. $a \in [6, 12], b \in [40.1, 45.7], c \in [33, 40],$ and $d \in [11, 26]$

$6x^3 + 43x^2 + 39x + 18$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+6)(3x-1)(2x+3)$

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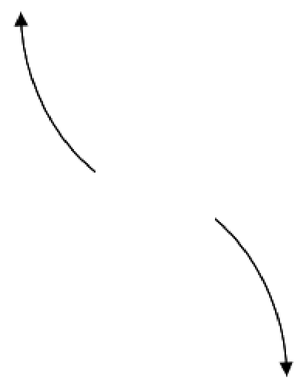
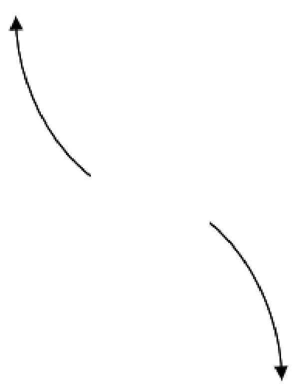
3. Describe the end behavior of the polynomial below.

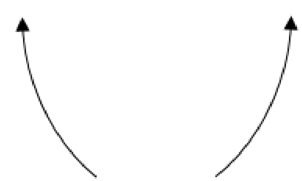
$$f(x) = -6(x + 7)^5(x - 7)^{10}(x - 8)^3(x + 8)^3$$

The solution is the graph below, which is option A.





A.



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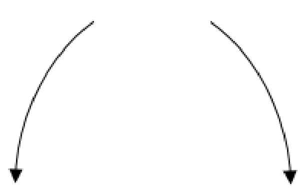

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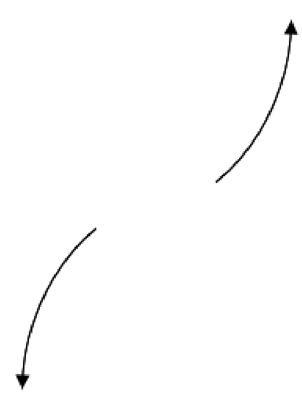

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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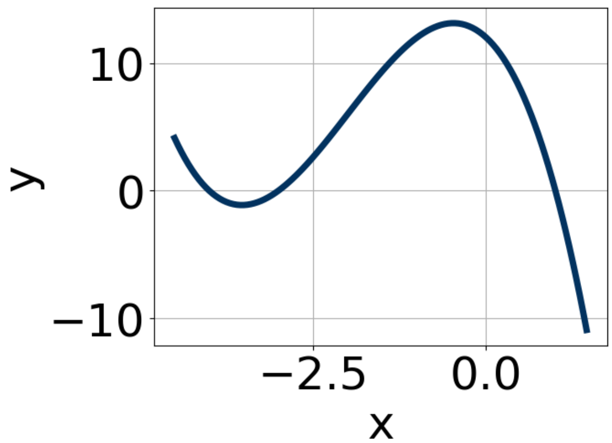
4. Which of the following equations *could* be of the graph presented below?



The solution is $-13(x+3)^9(x-1)^7(x+4)^5$, which is option E.

A. $-10(x+3)^{10}(x-1)^{11}(x+4)^9$

The factor -3 should have been an odd power.

B. $9(x+3)^{10}(x-1)^5(x+4)^5$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $16(x+3)^7(x-1)^7(x+4)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-17(x+3)^{10}(x-1)^6(x+4)^{11}$

The factors -3 and 1 have have been odd power.

E. $-13(x+3)^9(x-1)^7(x+4)^5$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, \frac{7}{4}, \text{ and } \frac{-2}{3}$$

The solution is $48x^3 - 64x^2 - 43x + 14$, which is option C.

A. $a \in [45, 50], b \in [123, 130], c \in [83, 89], \text{ and } d \in [12, 19]$

$48x^3 + 128x^2 + 85x + 14$, which corresponds to multiplying out $(4x+1)(4x+7)(3x+2)$.

B. $a \in [45, 50], b \in [-41, -33], c \in [-70, -66],$ and $d \in [-20, -13]$

$48x^3 - 40x^2 - 69x - 14$, which corresponds to multiplying out $(4x + 1)(4x - 7)(3x + 2)$.

C. $a \in [45, 50], b \in [-66, -60], c \in [-43, -33],$ and $d \in [12, 19]$

* $48x^3 - 64x^2 - 43x + 14$, which is the correct option.

D. $a \in [45, 50], b \in [-66, -60], c \in [-43, -33],$ and $d \in [-20, -13]$

$48x^3 - 64x^2 - 43x - 14$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [45, 50], b \in [64, 70], c \in [-43, -33],$ and $d \in [-20, -13]$

$48x^3 + 64x^2 - 43x - 14$, which corresponds to multiplying out $(4x + 1)(4x + 7)(3x - 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(4x - 7)(3x + 2)$

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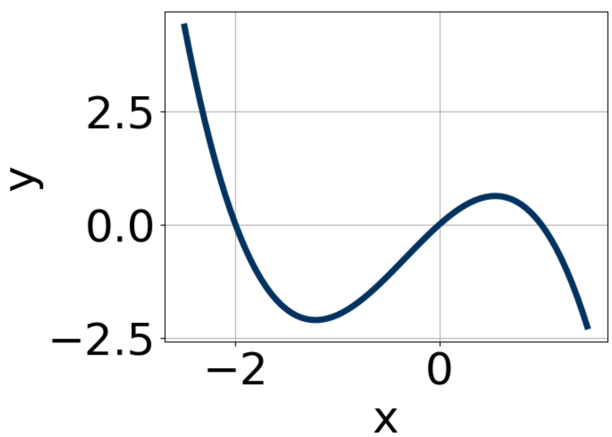
6. Which of the following equations *could* be of the graph presented below?



The solution is $-12x^5(x + 2)^5(x - 1)^9$, which is option C.

A. $11x^9(x + 2)^6(x - 1)^5$

The factor $(x + 2)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $11x^{11}(x + 2)^5(x - 1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-12x^5(x + 2)^5(x - 1)^9$

* This is the correct option.

D. $-6x^7(x + 2)^{10}(x - 1)^7$

The factor -2 should have been an odd power.

E. $-9x^6(x + 2)^4(x - 1)^7$

The factors -2 and 0 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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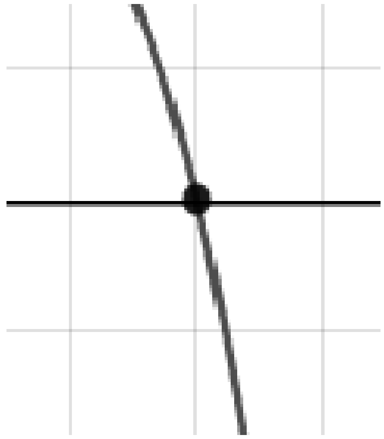
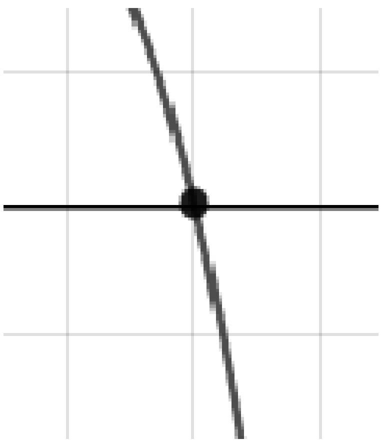
7. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

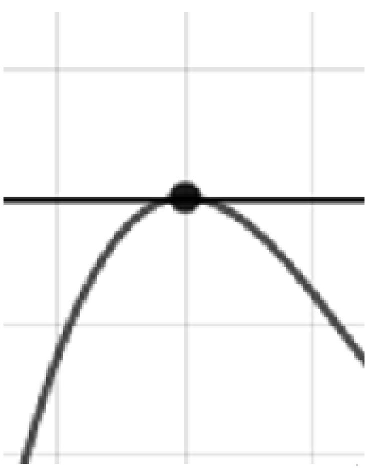
$$f(x) = 3(x+2)^5(x-2)^2(x+8)^7(x-8)^2$$

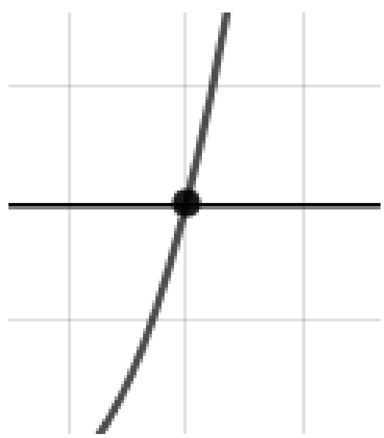
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

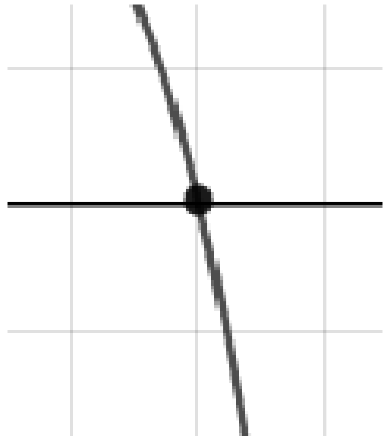
E. None of the above.

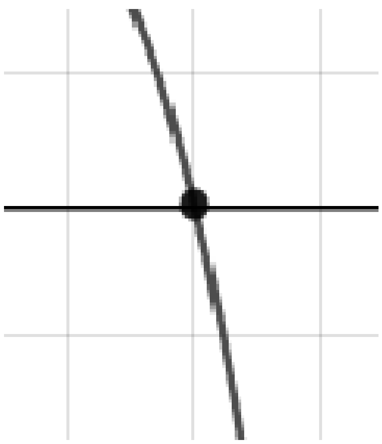
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

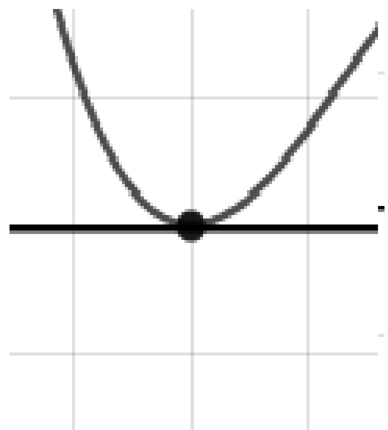
8. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

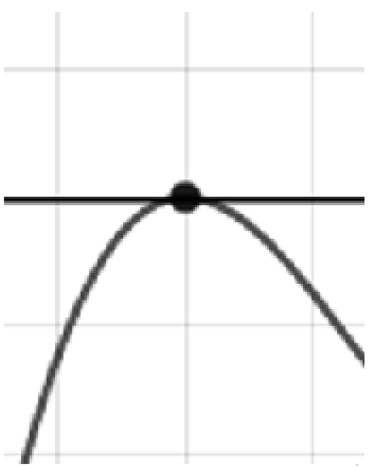
$$f(x) = 7(x-5)^2(x+5)^5(x+9)^8(x-9)^{11}$$

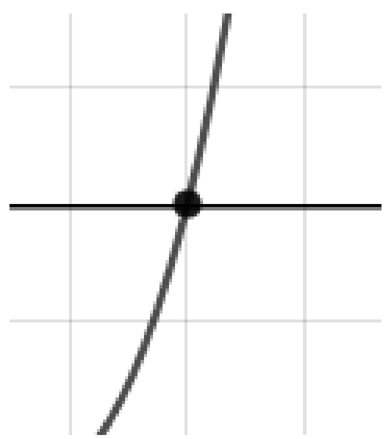
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 3i \text{ and } 3$$

The solution is $x^3 + 5x^2 + x - 75$, which is option B.

A. $b \in [-0.5, 2], c \in [-15, -4], \text{ and } d \in [7, 12]$

$x^3 + x^2 - 6x + 9$, which corresponds to multiplying out $(x - 3)(x - 3)$.

B. $b \in [3.6, 8.1], c \in [0, 5], \text{ and } d \in [-77, -74]$

* $x^3 + 5x^2 + x - 75$, which is the correct option.

C. $b \in [-5.2, 0.5], c \in [0, 5], \text{ and } d \in [70, 77]$

$x^3 - 5x^2 + x + 75$, which corresponds to multiplying out $(x - (-4 + 3i))(x - (-4 - 3i))(x + 3)$.

D. $b \in [-0.5, 2], c \in [0, 5], \text{ and } d \in [-14, -5]$

$x^3 + x^2 + x - 12$, which corresponds to multiplying out $(x + 4)(x - 3)$.

E. None of the above.

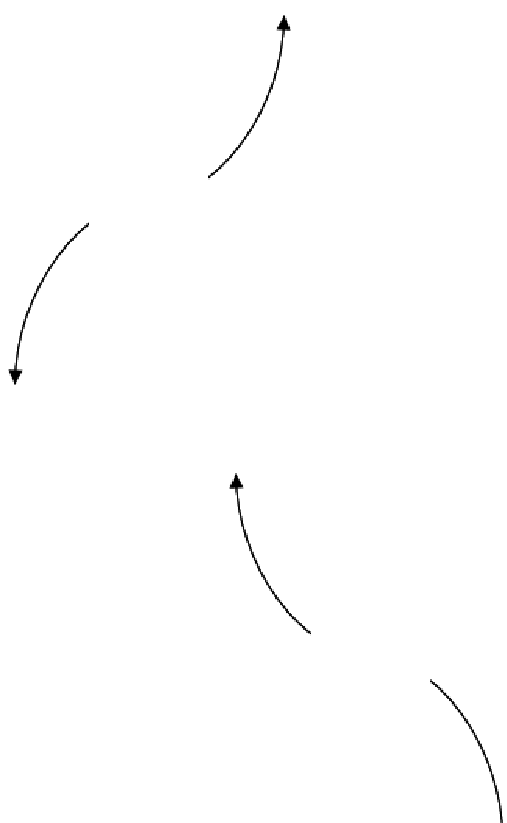
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

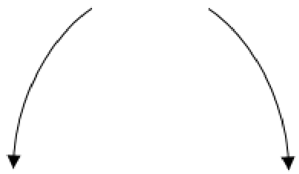
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 3i))(x - (-4 - 3i))(x - (3))$.

10. Describe the end behavior of the polynomial below.

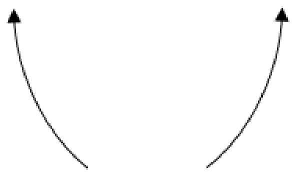
$$f(x) = 5(x - 6)^2(x + 6)^3(x + 3)^5(x - 3)^7$$

The solution is the graph below, which is option D.

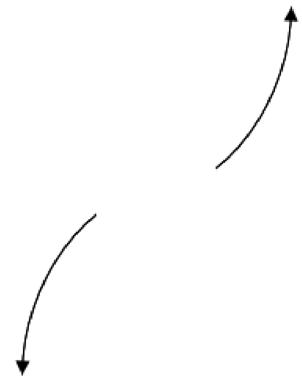




B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.