This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. The temperature of an object, T, in a different surrounding temperature  $T_s$  will behave according to the formula  $T(t) = Ae^{kt} + T_s$ , where t is minutes, A is a constant, and k is a constant. Use this formula and the situation below to construct a model that describes the uranium's temperature, T, based on the amount of time t (in minutes) that have passed. Choose the correct constant k from the options below.

Uranium is taken out of the reactor with a temperature of 200° C and is placed into a 20° C bath to cool. After 30 minutes, the uranium has cooled to 139° C.

The solution is k = -0.01379, which is option D.

A. k = -0.01731

This uses A as the initial temperature and solves for k correctly.

B. k = -0.02572

This uses A correctly but solves for k incorrectly.

C. k = -0.02529

This uses A as the initial temperature and solves for k incorrectly.

- D. k = -0.01379
  - \* This is the correct option.
- E. None of the above

If you chose this, please contact the coordinator to discuss why you believe none of the other answers are correct.

**General Comment:** The initial temperature is when t = 0. Unlike power models, that means A is not the initial temperature!

2. A town has an initial population of 100000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

	1									The solution is Logarithmic,
Pop	100000	99965	99945	99930	99919	99910	99902	99896	99890	-The solution is Logarithmic,
which is	option D									

A. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

B. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

C. Exponential

This suggests the fastest of growths that we know.

D. Logarithmic

This suggests the slowest of growths that we know.

E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

**General Comment:** We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

3. The temperature of an object, T, in a different surrounding temperature  $T_s$  will behave according to the formula  $T(t) = Ae^{kt} + T_s$ , where t is minutes, A is a constant, and k is a constant. Use this formula and the situation below to construct a model that describes the uranium's temperature, T, based on the amount of time t (in minutes) that have passed. Choose the correct constant k from the options below.

Uranium is taken out of the reactor with a temperature of 150° C and is placed into a 20° C bath to cool. After 18 minutes, the uranium has cooled to 91° C.

The solution is k = -0.03360, which is option C.

A. k = -0.04036

This uses A correctly but solves for k incorrectly.

B. k = -0.04155

This uses A as the initial temperature and solves for k correctly.

C. k = -0.03360

\* This is the correct option.

D. k = -0.03940

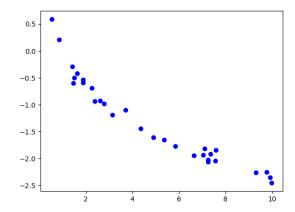
This uses A as the initial temperature and solves for k incorrectly.

E. None of the above

If you chose this, please contact the coordinator to discuss why you believe none of the other answers are correct.

**General Comment:** The initial temperature is when t = 0. Unlike power models, that means A is not the initial temperature!

4. Determine the appropriate model for the graph of points below.



The solution is Logarithmic model, which is option B.

# A. Exponential model

For this to be the correct option, we want an extremely slow change early, then a rapid change later.

### B. Logarithmic model

For this to be the correct option, we want a rapid change early, then an extremely slow change later.

# C. Linear model

For this to be the correct option, we need to see a mostly straight line of points.

#### D. Non-linear Power model

For this to be the correct option, we need to see a polynomial or rational shape.

# E. None of the above

For this to be the correct option, we want to see no pattern in the points.

**General Comment:** This question is testing if you can associate the models with their graphical representation. If you are having trouble, go back to the corresponding Core module to learn about the specific function you are having trouble recognizing.

5. Using the scenario below, model the population of bacteria  $\alpha$  in terms of the number of minutes, t that pass. Then, choose the correct approximate (rounded to the nearest minute) replication rate of bacteria- $\alpha$ .

A newly discovered bacteria, α, is being examined in a lab. The lab started with a petri dish of 4 bacteria-α. After 1 hours, the petri dish has 14 bacteria-α. Based on similar bacteria, the lab believes bacteria-α doubles after some undetermined number of minutes.

The solution is About 32 minutes, which is option B.

## A. About 47 minutes

This does not solve for the constant correctly.

# B. About 32 minutes

<sup>\*</sup> This is the correct option.

### C. About 282 minutes

This does not solve for the constant correctly AND converted incorrectly.

### D. About 197 minutes

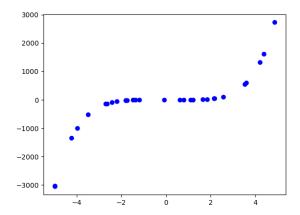
This solves for the constant correctly but converted incorrectly.

# E. None of the above

Please contact the coordinator to discuss why you believe none of the answers above are correct.

**General Comment:** Your model should be  $P(t) = P_0(b)^{kt}$ , where P(t) is the population at some time t,  $P_0$  is the initial population, and k is the replication rate. Be sure you convert the hours into minutes!

# 6. Determine the appropriate model for the graph of points below.



The solution is Non-linear Power model, which is option A.

### A. Non-linear Power model

For this to be the correct option, we need to see a polynomial or rational shape.

### B. Linear model

For this to be the correct option, we need to see a mostly straight line of points.

### C. Exponential model

For this to be the correct option, we want an extremely slow change early, then a rapid change later.

# D. Logarithmic model

For this to be the correct option, we want a rapid change early, then an extremely slow change later.

### E. None of the above

For this to be the correct option, we want to see no pattern in the points.

**General Comment:** This question is testing if you can associate the models with their graphical representation. If you are having trouble, go back to the corresponding Core module to learn about the specific function you are having trouble recognizing.

7. Using the scenario below, model the situation using an exponential function and a base of  $\frac{1}{2}$ . Then, solve for the half-life of the element, rounding to the nearest day.

The half-life of an element is the amount of time it takes for the element to decay to half of its initial starting amount. There is initially 606 grams of element X and after 4 years there is 75 grams remaining.

The solution is About 365 days, which is option A.

- A. About 365 days
  - \* This is the correct option.
- B. About 365 days

This uses the correct model but a base of e rather than  $\frac{1}{2}$ .

C. About 1 day

This models half-life as a linear function.

D. About 1825 days

This uses the correct model but solves for the exponential constant incorrectly.

E. None of the above

Please contact the coordinator if you believe all the options above are incorrect.

**General Comment:** The model should be  $A(t) = A_0(\frac{1}{2})^{kt}$ , where A(t) is the amount after t years,  $A_0$  is the initial amount, and k is decay constant. To find the half-life, you need to solve for k by using the amount after x years, then solve for the time t when  $A = \frac{A_0}{2}$ . Your answer would be in years, so convert to days.

8. Using the scenario below, model the situation using an exponential function and a base of  $\frac{1}{2}$ . Then, solve for the half-life of the element, rounding to the nearest day.

The half-life of an element is the amount of time it takes for the element to decay to half of its initial starting amount. There is initially 908 grams of element X and after 7 years there is 113 grams remaining.

The solution is About 730 days, which is option D.

A. About 1 day

This models half-life as a linear function.

B. About 1095 days

This uses the correct model but a base of e rather than  $\frac{1}{2}$ .

C. About 3285 days

This uses the correct model but solves for the exponential constant incorrectly.

D. About 730 days

\* This is the correct option.

E. None of the above

Please contact the coordinator if you believe all the options above are incorrect.

**General Comment:** The model should be  $A(t) = A_0(\frac{1}{2})^{kt}$ , where A(t) is the amount after t years,  $A_0$  is the initial amount, and k is decay constant. To find the half-life, you need to solve for k by using the amount after x years, then solve for the time t when  $A = \frac{A_0}{2}$ . Your answer would be in years, so convert to days.

9. Using the scenario below, model the population of bacteria  $\alpha$  in terms of the number of minutes, t that pass. Then, choose the correct approximate (rounded to the nearest minute) replication rate of bacteria- $\alpha$ .

A newly discovered bacteria, α, is being examined in a lab. The lab started with a petri dish of 4 bacteria-α. After 3 hours, the petri dish has 147 bacteria-α. Based on similar bacteria, the lab believes bacteria-α doubles after some undetermined number of minutes.

The solution is About 34 minutes, which is option C.

A. About 74 minutes

This does not solve for the constant correctly.

B. About 207 minutes

This solves for the constant correctly but converted incorrectly.

- C. About 34 minutes
  - \* This is the correct option.
- D. About 449 minutes

This does not solve for the constant correctly AND converted incorrectly.

E. None of the above

Please contact the coordinator to discuss why you believe none of the answers above are correct.

**General Comment:** Your model should be  $P(t) = P_0(b)^{kt}$ , where P(t) is the population at some time t,  $P_0$  is the initial population, and k is the replication rate. Be sure you convert the hours into minutes!

10. A town has an initial population of 80000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

$\mathbf{Y}\mathbf{e}\mathbf{a}\mathbf{r}$	1	2	3	4	5	6	7	8	9	-The solu-		
Pop	80080	80320	81280	85120	100480	161920	407680	1390720	5322880	-1 He solu-		
tion is Exponential, which is option D.												

A. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

B. Logarithmic

This suggests the slowest of growths that we know.

C. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

D. Exponential

This suggests the fastest of growths that we know.

E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

**General Comment:** We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

11. The temperature of an object, T, in a different surrounding temperature  $T_s$  will behave according to the formula  $T(t) = Ae^{kt} + T_s$ , where t is minutes, A is a constant, and k is a constant. Use this formula and the situation below to construct a model that describes the uranium's temperature, T, based on the amount of time t (in minutes) that have passed. Choose the correct constant k from the options below.

Uranium is taken out of the reactor with a temperature of 160° C and is placed into a 13° C bath to cool. After 14 minutes, the uranium has cooled to 102° C.

The solution is k = -0.03584, which is option C.

A. k = -0.04190

This uses A as the initial temperature and solves for k correctly.

B. k = -0.05277

This uses A as the initial temperature and solves for k incorrectly.

C. k = -0.03584

\* This is the correct option.

D. k = -0.05352

This uses A correctly but solves for k incorrectly.

E. None of the above

If you chose this, please contact the coordinator to discuss why you believe none of the other answers are correct.

**General Comment:** The initial temperature is when t = 0. Unlike power models, that means A is not the initial temperature!

12. A town has an initial population of 50000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

Year	1	2	3	4	5	6	7	8	9	-The solution is Linear,		
Pop	50030	50060	50090	50120	50150	50180	50210	50240	50270	-The solution is Linear		
which is option C.												

A. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

B. Exponential

This suggests the fastest of growths that we know.

C. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

D. Logarithmic

This suggests the slowest of growths that we know.

E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

**General Comment:** We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

13. The temperature of an object, T, in a different surrounding temperature  $T_s$  will behave according to the formula  $T(t) = Ae^{kt} + T_s$ , where t is minutes, A is a constant, and k is a constant. Use this formula and the situation below to construct a model that describes the uranium's temperature, T, based on the amount of time t (in minutes) that have passed. Choose the correct constant k from the options below.

Uranium is taken out of the reactor with a temperature of 150° C and is placed into a 15° C bath to cool. After 26 minutes, the uranium has cooled to 87° C.

The solution is k = -0.02418, which is option A.

- A. k = -0.02418
  - \* This is the correct option.
- B. k = -0.02823

This uses A as the initial temperature and solves for k correctly.

C. k = -0.02737

This uses A as the initial temperature and solves for k incorrectly.

D. k = -0.02785

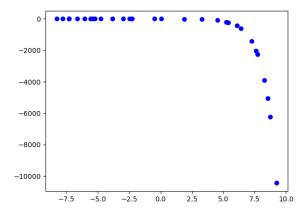
This uses A correctly but solves for k incorrectly.

E. None of the above

If you chose this, please contact the coordinator to discuss why you believe none of the other answers are correct.

**General Comment:** The initial temperature is when t = 0. Unlike power models, that means A is not the initial temperature!

14. Determine the appropriate model for the graph of points below.



The solution is Exponential model, which is option A.

# A. Exponential model

For this to be the correct option, we want an extremely slow change early, then a rapid change later.

### B. Logarithmic model

For this to be the correct option, we want a rapid change early, then an extremely slow change later.

# C. Linear model

For this to be the correct option, we need to see a mostly straight line of points.

#### D. Non-linear Power model

For this to be the correct option, we need to see a polynomial or rational shape.

# E. None of the above

For this to be the correct option, we want to see no pattern in the points.

**General Comment:** This question is testing if you can associate the models with their graphical representation. If you are having trouble, go back to the corresponding Core module to learn about the specific function you are having trouble recognizing.

15. Using the scenario below, model the population of bacteria  $\alpha$  in terms of the number of minutes, t that pass. Then, choose the correct approximate (rounded to the nearest minute) replication rate of bacteria- $\alpha$ .

A newly discovered bacteria, α, is being examined in a lab. The lab started with a petri dish of 4 bacteria-α. After 3 hours, the petri dish has 664 bacteria-α. Based on similar bacteria, the lab believes bacteria-α triples after some undetermined number of minutes.

The solution is None of the above, which is option E.

## A. About 345 minutes

This uses the wrong base, does not solve for the constant correctly, AND converted incorrectly.

# B. About 146 minutes

This uses the wrong base and solves for the constant correctly but converted incorrectly.

#### C. About 57 minutes

This uses the wrong base and does not solve for the constant correctly.

### D. About 24 minutes

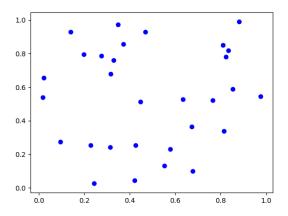
This uses the wrong base.

# E. None of the above

\* This is the correct option as all other options used the wrong base in their model.

**General Comment:** Your model should be  $P(t) = P_0(b)^{kt}$ , where P(t) is the population at some time t,  $P_0$  is the initial population, and k is the replication rate. Be sure you convert the hours into minutes!

# 16. Determine the appropriate model for the graph of points below.



The solution is None of the above, which is option E.

### A. Linear model

For this to be the correct option, we need to see a mostly straight line of points.

### B. Logarithmic model

For this to be the correct option, we want a rapid change early, then an extremely slow change later.

# C. Exponential model

For this to be the correct option, we want an extremely slow change early, then a rapid change later.

# D. Non-linear Power model

For this to be the correct option, we need to see a polynomial or rational shape.

### E. None of the above

For this to be the correct option, we want to see no pattern in the points.

**General Comment:** This question is testing if you can associate the models with their graphical representation. If you are having trouble, go back to the corresponding Core module to learn about the specific function you are having trouble recognizing.

17. Using the scenario below, model the situation using an exponential function and a base of  $\frac{1}{2}$ . Then, solve for the half-life of the element, rounding to the nearest day.

The half-life of an element is the amount of time it takes for the element to decay to half of its initial starting amount. There is initially 759 grams of element X and after 6 years there is 75 grams remaining.

The solution is About 365 days, which is option A.

- A. About 365 days
  - \* This is the correct option.
- B. About 730 days

This uses the correct model but a base of e rather than  $\frac{1}{2}$ .

C. About 1 day

This models half-life as a linear function.

D. About 2920 days

This uses the correct model but solves for the exponential constant incorrectly.

E. None of the above

Please contact the coordinator if you believe all the options above are incorrect.

**General Comment:** The model should be  $A(t) = A_0(\frac{1}{2})^{kt}$ , where A(t) is the amount after t years,  $A_0$  is the initial amount, and k is decay constant. To find the half-life, you need to solve for k by using the amount after x years, then solve for the time t when  $A = \frac{A_0}{2}$ . Your answer would be in years, so convert to days.

18. Using the scenario below, model the situation using an exponential function and a base of  $\frac{1}{2}$ . Then, solve for the half-life of the element, rounding to the nearest day.

The half-life of an element is the amount of time it takes for the element to decay to half of its initial starting amount. There is initially 848 grams of element X and after 3 years there is 212 grams remaining.

The solution is About 365 days, which is option C.

A. About 1095 days

This uses the correct model but solves for the exponential constant incorrectly.

B. About 1 day

This models half-life as a linear function.

- C. About 365 days
  - \* This is the correct option.
- D. About 730 days

This uses the correct model but a base of e rather than  $\frac{1}{2}$ .

E. None of the above

Please contact the coordinator if you believe all the options above are incorrect.

**General Comment:** The model should be  $A(t) = A_0(\frac{1}{2})^{kt}$ , where A(t) is the amount after t years,  $A_0$  is the initial amount, and k is decay constant. To find the half-life, you need to solve for k by using the amount after x years, then solve for the time t when  $A = \frac{A_0}{2}$ . Your answer would be in years, so convert to days.

19. Using the scenario below, model the population of bacteria  $\alpha$  in terms of the number of minutes, t that pass. Then, choose the correct approximate (rounded to the nearest minute) replication rate of bacteria- $\alpha$ .

A newly discovered bacteria, α, is being examined in a lab. The lab started with a petri dish of 2 bacteria-α. After 3 hours, the petri dish has 384 bacteria-α. Based on similar bacteria, the lab believes bacteria-α triples after some undetermined number of minutes.

The solution is None of the above, which is option E.

A. About 23 minutes

This uses the wrong base.

B. About 251 minutes

This uses the wrong base, does not solve for the constant correctly, AND converted incorrectly.

C. About 142 minutes

This uses the wrong base and solves for the constant correctly but converted incorrectly.

D. About 41 minutes

This uses the wrong base and does not solve for the constant correctly.

- E. None of the above
  - \* This is the correct option as all other options used the wrong base in their model.

**General Comment:** Your model should be  $P(t) = P_0(b)^{kt}$ , where P(t) is the population at some time t,  $P_0$  is the initial population, and k is the replication rate. Be sure you convert the hours into minutes!

20. A town has an initial population of 50000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

$\mathbf{Y}\mathbf{e}\mathbf{a}\mathbf{r}$	1	2	3	4	5	6	7	8	9	-The solution is Exponential,
Pop	49900	49800	49600	49200	48400	46800	43600	37200	24400	-The solution is Exponential,
which is	option (	Ċ.								

A. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

B. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

C. Exponential

This suggests the fastest of growths that we know.

D. Logarithmic

This suggests the slowest of growths that we know.

#### E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

**General Comment:** We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

21. The temperature of an object, T, in a different surrounding temperature  $T_s$  will behave according to the formula  $T(t) = Ae^{kt} + T_s$ , where t is minutes, A is a constant, and k is a constant. Use this formula and the situation below to construct a model that describes the uranium's temperature, T, based on the amount of time t (in minutes) that have passed. Choose the correct constant k from the options below.

Uranium is taken out of the reactor with a temperature of 120° C and is placed into a 12° C bath to cool. After 22 minutes, the uranium has cooled to 61° C.

The solution is k = -0.03592, which is option A.

- A. k = -0.03592
  - \* This is the correct option.
- B. k = -0.04071

This uses A as the initial temperature and solves for k correctly.

C. k = -0.03113

This uses A correctly but solves for k incorrectly.

D. k = -0.03057

This uses A as the initial temperature and solves for k incorrectly.

E. None of the above

If you chose this, please contact the coordinator to discuss why you believe none of the other answers are correct.

**General Comment:** The initial temperature is when t = 0. Unlike power models, that means A is not the initial temperature!

22. A town has an initial population of 60000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

Yea	ar	1	2	3	4	5	6	7	8	9	-The solution is Non-Linear Power,
Po	$\mathbf{p}$	59964	59924	59884	59844	59804	59764	59724	59684	59644	-The solution is Non-Emeal Tower,
which is option D.											

A. Exponential

This suggests the fastest of growths that we know.

B. Logarithmic

This suggests the slowest of growths that we know.

C. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

D. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

**General Comment:** We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?

23. The temperature of an object, T, in a different surrounding temperature  $T_s$  will behave according to the formula  $T(t) = Ae^{kt} + T_s$ , where t is minutes, A is a constant, and k is a constant. Use this formula and the situation below to construct a model that describes the uranium's temperature, T, based on the amount of time t (in minutes) that have passed. Choose the correct constant k from the options below.

Uranium is taken out of the reactor with a temperature of 180° C and is placed into a 11° C bath to cool. After 14 minutes, the uranium has cooled to 110° C.

The solution is None of the above, which is option E.

A. k = -0.09660

This uses A as the bath temperature and solves for k incorrectly.

B. k = -0.04270

This uses A as the initial temperature and solves for k correctly.

C. k = -0.05354

This uses A correctly and solves for k incorrectly.

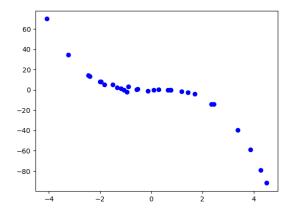
D. k = -0.05300

This uses A as the initial temperature and solves for k incorrectly.

- E. None of the above
  - \* This is the correct answer as k = -0.03820.

**General Comment:** The initial temperature is when t = 0. Unlike power models, that means A is not the initial temperature!

24. Determine the appropriate model for the graph of points below.



The solution is Non-linear Power model, which is option D.

#### A. Linear model

For this to be the correct option, we need to see a mostly straight line of points.

# B. Exponential model

For this to be the correct option, we want an extremely slow change early, then a rapid change later.

# C. Logarithmic model

For this to be the correct option, we want a rapid change early, then an extremely slow change later.

#### D. Non-linear Power model

For this to be the correct option, we need to see a polynomial or rational shape.

# E. None of the above

For this to be the correct option, we want to see no pattern in the points.

**General Comment:** This question is testing if you can associate the models with their graphical representation. If you are having trouble, go back to the corresponding Core module to learn about the specific function you are having trouble recognizing.

25. Using the scenario below, model the population of bacteria  $\alpha$  in terms of the number of minutes, t that pass. Then, choose the correct approximate (rounded to the nearest minute) replication rate of bacteria- $\alpha$ .

A newly discovered bacteria, α, is being examined in a lab. The lab started with a petri dish of 2 bacteria-α. After 2 hours, the petri dish has 704 bacteria-α. Based on similar bacteria, the lab believes bacteria-α triples after some undetermined number of minutes.

The solution is None of the above, which is option E.

## A. About 25 minutes

This uses the wrong base and does not solve for the constant correctly.

# B. About 14 minutes

This uses the wrong base.

### C. About 152 minutes

This uses the wrong base, does not solve for the constant correctly, AND converted incorrectly.

### D. About 85 minutes

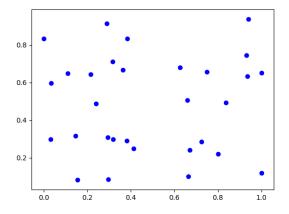
This uses the wrong base and solves for the constant correctly but converted incorrectly.

# E. None of the above

\* This is the correct option as all other options used the wrong base in their model.

**General Comment:** Your model should be  $P(t) = P_0(b)^{kt}$ , where P(t) is the population at some time t,  $P_0$  is the initial population, and k is the replication rate. Be sure you convert the hours into minutes!

# 26. Determine the appropriate model for the graph of points below.



The solution is None of the above, which is option E.

### A. Linear model

For this to be the correct option, we need to see a mostly straight line of points.

### B. Non-linear Power model

For this to be the correct option, we need to see a polynomial or rational shape.

### C. Logarithmic model

For this to be the correct option, we want a rapid change early, then an extremely slow change later.

# D. Exponential model

For this to be the correct option, we want an extremely slow change early, then a rapid change later.

### E. None of the above

For this to be the correct option, we want to see no pattern in the points.

**General Comment:** This question is testing if you can associate the models with their graphical representation. If you are having trouble, go back to the corresponding Core module to learn about the specific function you are having trouble recognizing.

27. Using the scenario below, model the situation using an exponential function and a base of  $\frac{1}{2}$ . Then, solve for the half-life of the element, rounding to the nearest day.

The half-life of an element is the amount of time it takes for the element to decay to half of its initial starting amount. There is initially 551 grams of element X and after 3 years there is 91 grams remaining.

The solution is About 365 days, which is option B.

A. About 1095 days

This uses the correct model but solves for the exponential constant incorrectly.

- B. About 365 days
  - \* This is the correct option.
- C. About 365 days

This uses the correct model but a base of e rather than  $\frac{1}{2}$ .

D. About 1 day

This models half-life as a linear function.

E. None of the above

Please contact the coordinator if you believe all the options above are incorrect.

**General Comment:** The model should be  $A(t) = A_0(\frac{1}{2})^{kt}$ , where A(t) is the amount after t years,  $A_0$  is the initial amount, and k is decay constant. To find the half-life, you need to solve for k by using the amount after x years, then solve for the time t when  $A = \frac{A_0}{2}$ . Your answer would be in years, so convert to days.

28. Using the scenario below, model the situation using an exponential function and a base of  $\frac{1}{2}$ . Then, solve for the half-life of the element, rounding to the nearest day.

The half-life of an element is the amount of time it takes for the element to decay to half of its initial starting amount. There is initially 740 grams of element X and after 9 years there is 82 grams remaining.

The solution is About 730 days, which is option D.

A. About 1 day

This models half-life as a linear function.

B. About 4380 days

This uses the correct model but solves for the exponential constant incorrectly.

C. About 1460 days

This uses the correct model but a base of e rather than  $\frac{1}{2}$ .

- D. About 730 days
  - \* This is the correct option.
- E. None of the above

Please contact the coordinator if you believe all the options above are incorrect.

**General Comment:** The model should be  $A(t) = A_0(\frac{1}{2})^{kt}$ , where A(t) is the amount after t years,  $A_0$  is the initial amount, and k is decay constant. To find the half-life, you need to solve for k by using the amount after x years, then solve for the time t when  $A = \frac{A_0}{2}$ . Your answer would be in years, so convert to days.

29. Using the scenario below, model the population of bacteria  $\alpha$  in terms of the number of minutes, t that pass. Then, choose the correct approximate (rounded to the nearest minute) replication rate of bacteria- $\alpha$ .

A newly discovered bacteria, α, is being examined in a lab. The lab started with a petri dish of 3 bacteria-α. After 3 hours, the petri dish has 877 bacteria-α. Based on similar bacteria, the lab believes bacteria-α triples after some undetermined number of minutes.

The solution is None of the above, which is option E.

A. About 285 minutes

This uses the wrong base, does not solve for the constant correctly, AND converted incorrectly.

B. About 131 minutes

This uses the wrong base and solves for the constant correctly but converted incorrectly.

C. About 21 minutes

This uses the wrong base.

D. About 47 minutes

This uses the wrong base and does not solve for the constant correctly.

E. None of the above

\* This is the correct option as all other options used the wrong base in their model.

**General Comment:** Your model should be  $P(t) = P_0(b)^{kt}$ , where P(t) is the population at some time t,  $P_0$  is the initial population, and k is the replication rate. Be sure you convert the hours into minutes!

30. A town has an initial population of 90000. The town's population for the next 10 years is provided below. Which type of function would be most appropriate to model the town's population?

$\mathbf{Y}\mathbf{e}\mathbf{a}\mathbf{r}$	1	2	3	4	5	6	7	8	9	-The solution is Exponential,
Pop	89940	89880	89760	89520	89040	88080	86160	82320	74640	—The solution is Exponential,
which is	option 1	Ä.								

A. Exponential

This suggests the fastest of growths that we know.

B. Linear

This suggests a constant growth. You would be able to add or subtract the same amount year-to-year if this is the correct answer.

C. Non-Linear Power

This suggests a growth faster than constant but slower than exponential.

D. Logarithmic

This suggests the slowest of growths that we know.

### E. None of the above

Please contact the coordinator to discuss why you believe none of the options model the population.

**General Comment:** We are trying to compare the growth rate of the population. Growth rates can be characterized from slowest to fastest as: logarithmic, indirect, linear, direct, exponential. The best way to approach this is to first compare it to linear (is it linear, faster than linear, or slower than linear)? If faster, is it as fast as exponential? If slower, is it as slow as logarithmic?