**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

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1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^-1(-15)$ belongs to.

$$f(x) = 5x^2 + 4$$

The solution is  The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [2.55, 3.24]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B. $f^{-1}(-15) \in [0.22, 1.57]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

C. $f^{-1}(-15) \in [5.71, 6.76]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(-15) \in [1.92, 2.48]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E.  The function is not invertible for all Real numbers.

* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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2. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^-1(9)$ belongs to.

$$f(x) = e^{x-5} - 2$$

The solution is $f^{-1}(9) = 7.398$, which is option B.

A. $f^{-1}(9) \in [-0.52, 0.09]$

This solution corresponds to distractor 2.

B. $f^{-1}(9) \in [6.77, 7.99]$

This is the solution.

C. $f^{-1}(9) \in [-0.67, -0.3]$

This solution corresponds to distractor 4.

D. $f^{-1}(9) \in [-2.85, -2.48]$

This solution corresponds to distractor 1.

E. $f^{-1}(9) \in [0.5, 1.03]$

This solution corresponds to distractor 3.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the $x$ and $y$, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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3. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^-1(8)$ belongs to.

$$f(x) = e^{x-2} - 3$$

The solution is $f^{-1}(8) = 4.398$, which is option A.

A. $f^{-1}(8) \in [3.79, 5.33]$

This is the solution.

B. $f^{-1}(8) \in [0.3, 0.72]$

This solution corresponds to distractor 1.

C. $f^{-1}(8) \in [-1.81, -1.38]$

This solution corresponds to distractor 2.

D. $f^{-1}(8) \in [-0.85, -0.59]$

This solution corresponds to distractor 3.

E. $f^{-1}(8) \in [-1.27, -1.04]$

This solution corresponds to distractor 4.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the $x$ and $y$, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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4. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{4}{6x + 29} \text{ and } g(x) = \frac{4}{4x - 17}$$

The solution is  The domain is all Real numbers except $x = -4.83$ and $x = 4.25$, which is option D.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-8.33, 4.67]$

B. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-4.25, -2.25]$

C. The domain is all Real numbers except $x = a$, where $a \in [-9.17, -1.17]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-8.83, -2.83]$ and $b \in [4.25, 8.25]$

E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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5. Determine whether the function below is 1-1.

$$f(x) = -36x^2 - 342x - 756$$

The solution is no, which is option B.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is a $y$-value that goes to 2 different $x$-values.

* This is the solution.

C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because there is an $x$-value that goes to 2 different $y$-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a $y$-value that goes to 2 different $x$-values.

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6. Choose the interval below that $f$ composed with $g$ at $x = -1$ is in.

$$f(x) = -x^3 + 2x^2 + 2x - 2 \text{ and } g(x) = -4x^3 - 2x^2 + x$$

The solution is 1.0, which is option A.

A. $(f \circ g)(-1) \in [-3, 5]$

* This is the correct solution

B. $(f \circ g)(-1) \in [-3, 5]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [-6, -1]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-8, -5]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** $f$ composed with $g$ at $x$ means $f(g(x))$. The order matters!

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7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 14$ and choose the interval that $f^-1(14)$ belongs to.

$$f(x) = 5x^2 + 3$$

The solution is  The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(14) \in [5.96, 6.85]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(14) \in [3.46, 3.64]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(14) \in [1.66, 2.16]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(14) \in [1.43, 1.51]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

**General Comment:** Be sure you check that the function is 1-1 before trying to find the inverse!

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8. Choose the interval below that $f$ composed with $g$ at $x = -1$ is in.

$$f(x) = 4x^3 + x^2 - x - 1 \text{ and } g(x) = -x^3 - 1x^2 + x$$

The solution is $-3.0$, which is option B.

A. $(f \circ g)(-1) \in [21, 24]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [-9, 1]$

* This is the correct solution

C. $(f \circ g)(-1) \in [11, 16]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [3, 7]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** $f$ composed with $g$ at $x$ means $f(g(x))$. The order matters!

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9. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-6x - 23} \text{ and } g(x) = 7x^2 + 4x + 8$$

The solution is  The domain is all Real numbers less than or equal to$x = -3.83.$, which is option C.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-10, -5]$

B. The domain is all Real numbers except $x = a,$ where $a \in [-6.67, -3.67]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-8.83, 4.17]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-0.75, 9.25]$ and $b \in [4.25, 6.25]$

E. The domain is all Real numbers.

**General Comment:** The new domain is the intersection of the previous domains.

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10. Determine whether the function below is 1-1.

$$f(x) = -12x^2 - 57x - 63$$

The solution is no, which is option C.

A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

B. No, because there is an $x$-value that goes to 2 different $y$-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because there is a $y$-value that goes to 2 different $x$-values.

* This is the solution.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a $y$-value that goes to 2 different $x$-values.

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