This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = 15 and choose the interval that  $f^{-1}(15)$  belongs to.

$$f(x) = \sqrt[3]{3x - 4}$$

The solution is 1126.33333333333, which is option B.

A.  $f^{-1}(15) \in [-1128.5, -1124.2]$ 

This solution corresponds to distractor 2.

- B.  $f^{-1}(15) \in [1124.3, 1128.9]$ 
  - \* This is the correct solution.
- C.  $f^{-1}(15) \in [-1124.2, -1120.8]$

This solution corresponds to distractor 3.

D.  $f^{-1}(15) \in [1121.9, 1125.8]$ 

Distractor 1: This corresponds to

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

2. Find the inverse of the function below. Then, evaluate the inverse at x = 7 and choose the interval that  $f^{-1}(7)$  belongs to.

$$f(x) = e^{x+5} - 3$$

The solution is  $f^{-1}(7) = -2.697$ , which is option C.

A. 
$$f^{-1}(7) \in [-2.55, -2.18]$$

This solution corresponds to distractor 3.

B. 
$$f^{-1}(7) \in [-1.84, -0.93]$$

This solution corresponds to distractor 2.

C. 
$$f^{-1}(7) \in [-3.14, -2.59]$$

This is the solution.

D. 
$$f^{-1}(7) \in [6.87, 7.36]$$

This solution corresponds to distractor 1.

E. 
$$f^{-1}(7) \in [-0.62, -0.27]$$

This solution corresponds to distractor 4.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

3. Find the inverse of the function below. Then, evaluate the inverse at x = 7 and choose the interval that  $f^{-1}(7)$  belongs to.

$$f(x) = e^{x-5} + 3$$

The solution is  $f^{-1}(7) = 6.386$ , which is option C.

A.  $f^{-1}(7) \in [5.43, 5.54]$ 

This solution corresponds to distractor 3.

B.  $f^{-1}(7) \in [5.12, 5.31]$ 

This solution corresponds to distractor 2.

C.  $f^{-1}(7) \in [6.27, 6.45]$ 

This is the solution.

D.  $f^{-1}(7) \in [3.54, 3.82]$ 

This solution corresponds to distractor 4.

E.  $f^{-1}(7) \in [-3.62, -3.52]$ 

This solution corresponds to distractor 1.

**General Comment:** Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion  $e^y = x \leftrightarrow y = \ln(x)$ .

4. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 6x^4 + 4x^2 + 7x + 3$$
 and  $g(x) = \sqrt{-6x - 27}$ 

The solution is The domain is all Real numbers less than or equal to x = -4.5, which is option C.

- A. The domain is all Real numbers except x = a, where  $a \in [6.25, 9.25]$
- B. The domain is all Real numbers greater than or equal to x = a, where  $a \in [3.5, 10.5]$
- C. The domain is all Real numbers less than or equal to x = a, where  $a \in [-12.5, -1.5]$
- D. The domain is all Real numbers except x = a and x = b, where  $a \in [1.4, 5.4]$  and  $b \in [1.25, 7.25]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

5. Determine whether the function below is 1-1.

$$f(x) = 18x^2 - 42x - 196$$

The solution is no, which is option A.

- A. No, because there is a y-value that goes to 2 different x-values.
  - \* This is the solution.
- B. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

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C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

6. Choose the interval below that f composed with g at x = -1 is in.

$$f(x) = 3x^3 + 2x^2 - 4x - 4$$
 and  $g(x) = 3x^3 + x^2 + 2x + 3$ 

The solution is -1.0, which is option B.

A.  $(f \circ g)(-1) \in [1.9, 4.3]$ 

Distractor 2: Corresponds to being slightly off from the solution.

B.  $(f \circ g)(-1) \in [-2.4, 0.1]$ 

\* This is the correct solution

C.  $(f \circ g)(-1) \in [-2.4, 0.1]$ 

Distractor 1: Corresponds to reversing the composition.

D.  $(f \circ g)(-1) \in [5.6, 7.5]$ 

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

**General Comment:** f composed with g at x means f(g(x)). The order matters!

7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = 12 and choose the interval that  $f^{-1}(12)$  belongs to.

$$f(x) = \sqrt[3]{5x+2}$$

The solution is 345.2, which is option D.

A. 
$$f^{-1}(12) \in [-345.36, -344.52]$$

This solution corresponds to distractor 2.

B.  $f^{-1}(12) \in [345.63, 346.11]$ 

Distractor 1: This corresponds to

C.  $f^{-1}(12) \in [-346.21, -345.48]$ 

This solution corresponds to distractor 3.

D.  $f^{-1}(12) \in [344.56, 345.27]$ 

\* This is the correct solution.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

8. Choose the interval below that f composed with g at x = -1 is in.

$$f(x) = x^3 - 1x^2 - 2x$$
 and  $q(x) = -3x^3 + 3x^2 - x - 2$ 

The solution is 90.0, which is option A.

- A.  $(f \circ g)(-1) \in [89, 92]$ 
  - \* This is the correct solution
- B.  $(f \circ g)(-1) \in [-13, -8]$

Distractor 3: Corresponds to being slightly off from the solution.

C.  $(f \circ g)(-1) \in [81, 89]$ 

Distractor 2: Corresponds to being slightly off from the solution.

D.  $(f \circ g)(-1) \in [-2, 0]$ 

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

**General Comment:** f composed with g at x means f(g(x)). The order matters!

9. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{3}{4x - 23}$$
 and  $g(x) = \frac{2}{4x - 29}$ 

The solution is The domain is all Real numbers except x = 5.75 and x = 7.25, which is option D.

- A. The domain is all Real numbers except x = a, where  $a \in [6.67, 12.67]$
- B. The domain is all Real numbers greater than or equal to x = a, where  $a \in [-12, 1]$
- C. The domain is all Real numbers less than or equal to x = a, where  $a \in [0.4, 5.4]$
- D. The domain is all Real numbers except x = a and x = b, where  $a \in [-2.25, 7.75]$  and  $b \in [6.25, 10.25]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

10. Determine whether the function below is 1-1.

$$f(x) = 18x^2 + 15x - 375$$

The solution is no, which is option C.

A. No, because the domain of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because there is a y-value that goes to 2 different x-values.

\* This is the solution.

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D. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

E. No, because the range of the function is not  $(-\infty, \infty)$ .

Corresponds to believing 1-1 means the range is all Real numbers.

**General Comment:** There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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