This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Determine whether the function below is 1-1.

$$f(x) = (5x - 20)^3$$

The solution is yes, which is option C.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. Yes, the function is 1-1.

* This is the solution.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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2. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^-1(8)$ belongs to.

$$f(x) = e^{x-4} - 4$$

The solution is $f^{-1}(8) = 6.485$, which is option B.

A. $f^{-1}(8) \in [-1.52, -0.52]$

This solution corresponds to distractor 1.

B. $f^{-1}(8) \in [6.48, 8.48]$

This is the solution.

C. $f^{-1}(8) \in [-4.61, -1.61]$

This solution corresponds to distractor 4.

D. $f^{-1}(8) \in [-1.52, -0.52]$

This solution corresponds to distractor 3.

E. $f^{-1}(8) \in [-4.61, -1.61]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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3. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 7x^2 + 7x + 9 \text{ and } g(x) = 2x + 8$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers except $x = a,$ where $a \in [-9.17, -0.17]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [0.4, 7.4]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-0.33, 9.67]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-4.6, 7.4]$ and $b \in [-4.75, 0.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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4. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 11$ and choose the interval that $f^-1(11)$ belongs to.

$$f(x) = \sqrt[3]{3x - 2}$$

The solution is 444.3333333333333, which is option C.

A. $f^{-1}(11) \in [-443.95, -442.28]$

This solution corresponds to distractor 3.

B. $f^{-1}(11) \in [441.04, 444.2]$

Distractor 1: This corresponds to

C. $f^{-1}(11) \in [444.01, 444.69]$

* This is the correct solution.

D. $f^{-1}(11) \in [-445.64, -444.03]$

This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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5. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^-1(10)$ belongs to.

$$f(x) = \ln(x - 5) + 3$$

The solution is $f^{-1}(10) = 1101.633$, which is option E.

A. $f^{-1}(10) \in [3269019.37, 3269022.37]$

This solution corresponds to distractor 2.

B. $f^{-1}(10) \in [1084.63, 1094.63]$

This solution corresponds to distractor 3.

C. $f^{-1}(10) \in [442415.39, 442424.39]$

This solution corresponds to distractor 1.

D. $f^{-1}(10) \in [147.41, 155.41]$

This solution corresponds to distractor 4.

E. $f^{-1}(10) \in [1098.63, 1103.63]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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6. Determine whether the function below is 1-1.

$$f(x) = (4x - 29)^3$$

The solution is yes, which is option A.

A. Yes, the function is 1-1.

* This is the solution.

B. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -10$ and choose the interval that $f^-1(-10)$ belongs to.

$$f(x) = 3x^2 - 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-10) \in [4.25, 4.48]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(-10) \in [2.74, 3.91]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(-10) \in [2.01, 2.17]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(-10) \in [0.95, 2.12]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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8. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 8x + 9 \text{ and } g(x) = 2x + 9$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [3.25, 11.25]$

B. The domain is all Real numbers except $x = a,$ where $a \in [-8.33, -3.33]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [1.83, 3.83]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-6.6, -0.6]$ and $b \in [4.2, 13.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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9. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 2x^3 - 1x^2 + 2x - 3 \text{ and } g(x) = x^3 + x^2 + 3x - 4$$

The solution is 0.0, which is option A.

A. $(f \circ g)(1) \in [-0.8, 2]$

* This is the correct solution

B. $(f \circ g)(1) \in [-7.6, -5.6]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-4.3, -3.3]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(1) \in [-11.7, -9.2]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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10. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 2x^3 + 2x^2 - 2x \text{ and } g(x) = -2x^3 - 1x^2 + x + 1$$

The solution is 2.0, which is option B.

A. $(f \circ g)(1) \in [-28, -25]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [2, 8]$

* This is the correct solution

C. $(f \circ g)(1) \in [-10, 1]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-19, -16]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!
