This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Determine whether the function below is 1-1.

$$f(x) = -24x^2 - 12x + 336$$

The solution is no, which is option C.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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2. Determine whether the function below is 1-1.

$$f(x) = 36x^2 + 480x + 1600$$

The solution is no, which is option E.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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3. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -10$ and choose the interval that $f^-1(-10)$ belongs to.

$$f(x) = \sqrt[3]{4x + 5}$$

The solution is -251.25, which is option B.

A. $f^{-1}(-10) \in [249.3, 253.6]$

This solution corresponds to distractor 2.

B. $f^{-1}(-10) \in [-253.5, -249.2]$

* This is the correct solution.

C. $f^{-1}(-10) \in [246.5, 250.6]$

This solution corresponds to distractor 3.

D. $f^{-1}(-10) \in [-250.2, -248.6]$

Distractor 1: This corresponds to

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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4. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 3x^2 + x + 5 \text{ and } g(x) = 8x^3 + 5x^2 + 5x$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers except $x = a,$ where $a \in [-10.25, 1.75]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [5.33, 12.33]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-13.67, -2.67]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [5.83, 7.83]$ and $b \in [4.67, 6.67]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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5. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 2x^3 - 4x^2 + 4x \text{ and } g(x) = -2x^3 + 4x^2 + x + 1$$

The solution is 80.0, which is option D.

A. $(f \circ g)(1) \in [-8, 2]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [88, 95]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [1, 5]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(1) \in [77, 87]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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6. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^-1(7)$ belongs to.

$$f(x) = e^{x-5} + 3$$

The solution is $f^{-1}(7) = 6.386$, which is option E.

A. $f^{-1}(7) \in [2.62, 3.88]$

This solution corresponds to distractor 4.

B. $f^{-1}(7) \in [-4.27, -3.07]$

This solution corresponds to distractor 1.

C. $f^{-1}(7) \in [5.41, 5.89]$

This solution corresponds to distractor 3.

D. $f^{-1}(7) \in [4.86, 5.34]$

This solution corresponds to distractor 2.

E. $f^{-1}(7) \in [6.08, 7.06]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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7. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -2x^3 + x^2 - x \text{ and } g(x) = -2x^3 - 1x^2 - x + 4$$

The solution is 0.0, which is option C.

A. $(f \circ g)(1) \in [23.1, 25.2]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [8.9, 9.9]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-1.3, 3.9]$

* This is the correct solution

D. $(f \circ g)(1) \in [17.6, 18.8]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 5x^2 + 8x + 9 \text{ and } g(x) = 2x^3 + 4x^2 + x + 8$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-7.75, 2.25]$

B. The domain is all Real numbers except $x = a$, where $a \in [1.67, 10.67]$

C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [3.5, 8.5]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [3.2, 10.2]$ and $b \in [-8.67, -4.67]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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9. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -10$ and choose the interval that $f^-1(-10)$ belongs to.

$$f(x) = 3x^2 - 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-10) \in [1.29, 1.31]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(-10) \in [2.28, 2.31]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(-10) \in [3.27, 3.35]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(-10) \in [2.18, 2.29]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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10. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^-1(9)$ belongs to.

$$f(x) = e^{x-5} + 3$$

The solution is $f^{-1}(9) = 6.792$, which is option E.

A. $f^{-1}(9) \in [5.57, 5.67]$

This solution corresponds to distractor 3.

B. $f^{-1}(9) \in [4.16, 4.4]$

This solution corresponds to distractor 4.

C. $f^{-1}(9) \in [5.3, 5.53]$

This solution corresponds to distractor 2.

D. $f^{-1}(9) \in [-3.25, -2.83]$

This solution corresponds to distractor 1.

E. $f^{-1}(9) \in [6.79, 7.24]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.