This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^-1(7)$ belongs to.

$$f(x) = e^{x+3} + 3$$

The solution is $f^{-1}(7) = -1.614$, which is option E.

A. $f^{-1}(7) \in [4.3, 4.7]$

This solution corresponds to distractor 3.

B. $f^{-1}(7) \in [4.4, 5.9]$

This solution corresponds to distractor 4.

C. $f^{-1}(7) \in [4.4, 5.9]$

This solution corresponds to distractor 2.

D. $f^{-1}(7) \in [4.3, 4.7]$

This solution corresponds to distractor 1.

E. $f^{-1}(7) \in [-2.8, -1.3]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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2. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 3x^2 + 5x + 8 \text{ and } g(x) = \frac{2}{4x + 27}$$

The solution is The domain is all Real numbers except $x = -6.75$, which is option B.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-9.5, -2.5]$

B. The domain is all Real numbers except $x = a,$ where $a \in [-13.75, -4.75]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-2, 5]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [3.33, 9.33]$ and $b \in [-3.2, -2.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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3. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -10$ and choose the interval that $f^-1(-10)$ belongs to.

$$f(x) = \sqrt[3]{2x + 5}$$

The solution is -502.5, which is option D.

A. $f^{-1}(-10) \in [-499.5, -496.5]$

   Distractor 1: This corresponds to

B. $f^{-1}(-10) \in [501.1, 503.1]$

   This solution corresponds to distractor 2.

C. $f^{-1}(-10) \in [495.1, 500.2]$

   This solution corresponds to distractor 3.

D. $f^{-1}(-10) \in [-503.9, -500.4]$

   * This is the correct solution.

E. The function is not invertible for all Real numbers.

   This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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4. Determine whether the function below is 1-1.

$$f(x) = (5x - 35)^3$$

The solution is yes, which is option A.

A. Yes, the function is 1-1.

   * This is the solution.

B. No, because the range of the function is not $(-\infty, \infty)$.

   Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is a y-value that goes to 2 different x-values.

   Corresponds to the Horizontal Line test, which this function passes.

D. No, because there is an x-value that goes to 2 different y-values.

   Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because the domain of the function is not $(-\infty, \infty)$.

   Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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5. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{3}{6x + 37} \text{ and } g(x) = x + 7$$

The solution is The domain is all Real numbers except $x = -6.17$, which is option B.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-6.5, -2.5]$

B. The domain is all Real numbers except $x = a$, where $a \in [-6.17, -2.17]$

C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [0, 3]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-3.8, 1.2]$ and $b \in [4.33, 8.33]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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6. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^-1(10)$ belongs to.

$$f(x) = \ln(x+4) - 5$$

The solution is $f^{-1}(10) = 3269013.372$, which is option C.

A. $f^{-1}(10) \in [142.41, 149.41]$

This solution corresponds to distractor 1.

B. $f^{-1}(10) \in [396.43, 399.43]$

This solution corresponds to distractor 2.

C. $f^{-1}(10) \in [3269012.37, 3269019.37]$

This is the solution.

D. $f^{-1}(10) \in [1202597.28, 1202600.28]$

This solution corresponds to distractor 4.

E. $f^{-1}(10) \in [3269019.37, 3269022.37]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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7. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -2x^3 + x^2 + 2x \text{ and } g(x) = -x^3 - 2x^2 - 3x - 4$$

The solution is 16.0, which is option A.

A. $(f \circ g)(-1) \in [13, 17]$

* This is the correct solution

B. $(f \circ g)(-1) \in [-16, -6]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [-8, -3]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [5, 12]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^{-}1(-15)$ belongs to.

$$f(x) = 3x^2 - 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [1.82, 1.95]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(-15) \in [6.9, 7.22]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(-15) \in [3.76, 4.2]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(-15) \in [2.22, 2.55]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 2x^3 - 2x^2 + 2x + 3 \text{ and } g(x) = x^3 + 2x^2 + 3x$$

The solution is -25.0, which is option C.

A. $(f \circ g)(-1) \in [-18.39, -17.32]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(-1) \in [-13.09, -12.75]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [-25.46, -22.42]$

* This is the correct solution

D. $(f \circ g)(-1) \in [-21.79, -18.43]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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10. Determine whether the function below is 1-1.

$$f(x) = (3x + 21)^3$$

The solution is yes, which is option D.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

   Corresponds to the Horizontal Line test, which this function passes.

C. No, because there is an x-value that goes to 2 different y-values.

   Corresponds to the Vertical Line test, which checks if an expression is a function.

D. Yes, the function is 1-1.

   * This is the solution.

E. No, because the domain of the function is not $(-\infty, \infty)$.

   Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.