This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } 4$$

The solution is $x^3 + 6x^2 + x - 164$, which is option A.

A. $b \in [5, 14], c \in [-1, 5],$ and $d \in [-165, -162]$

* $x^3 + 6x^2 + x - 164$, which is the correct option.

B. $b \in [-2, 4], c \in [-1, 5],$ and $d \in [-22, -18]$

$x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

C. $b \in [-2, 4], c \in [-10, -7],$ and $d \in [11, 20]$

$x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

D. $b \in [-12, -3], c \in [-1, 5],$ and $d \in [164, 169]$

$x^3 - 6x^2 + x + 164$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x + 4)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (4))$.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{4}, 5, \text{ and } \frac{7}{3}$$

The solution is $12x^3 - 109x^2 + 294x - 245$, which is option D.

A. $a \in [12, 14], b \in [-112, -107], c \in [293, 297],$ and $d \in [245, 252]$

$12x^3 - 109x^2 + 294x + 245$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [12, 14], b \in [-76, -65], c \in [-15, -8],$ and $d \in [245, 252]$

$12x^3 - 67x^2 - 14x + 245$, which corresponds to multiplying out $(4x + 7)(x - 5)(3x - 7)$.

C. $a \in [12, 14], b \in [108, 115], c \in [293, 297],$ and $d \in [245, 252]$

$12x^3 + 109x^2 + 294x + 245$, which corresponds to multiplying out $(4x + 7)(x + 5)(3x + 7)$.

D. $a \in [12, 14], b \in [-112, -107], c \in [293, 297]$, and $d \in [-246, -240]$

* $12x^3 - 109x^2 + 294x - 245$, which is the correct option.

E. $a \in [12, 14], b \in [49, 58], c \in [-88, -83]$, and $d \in [-246, -240]$

$12x^3 + 53x^2 - 84x - 245$, which corresponds to multiplying out $(4x + 7)(x + 5)(3x - 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 7)(x - 5)(3x - 7)$

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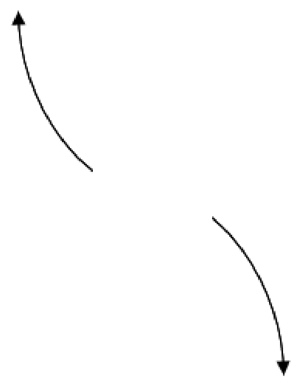
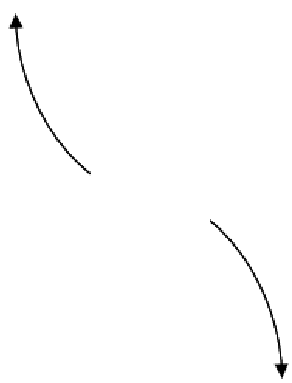
3. Describe the end behavior of the polynomial below.

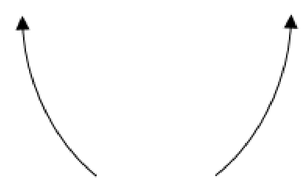
$$f(x) = -5(x + 3)^4(x - 3)^5(x + 2)^3(x - 2)^5$$

The solution is the graph below, which is option A.





A.



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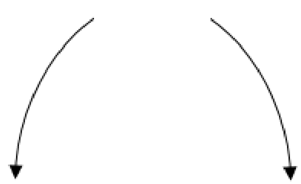

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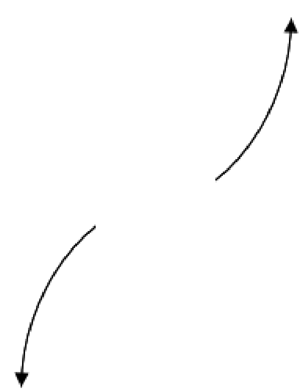

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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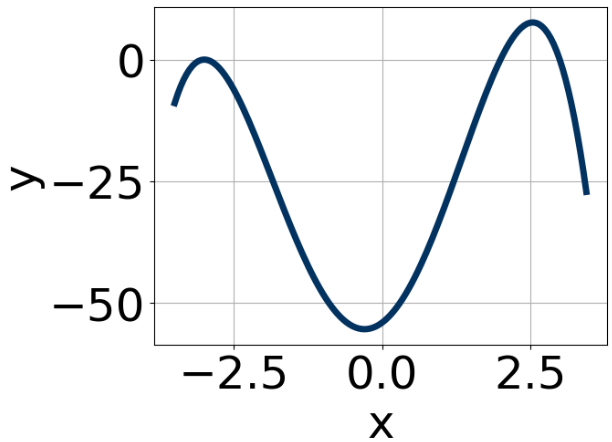
4. Which of the following equations *could* be of the graph presented below?



The solution is $-5(x+3)^6(x-3)^5(x-2)^9$, which is option B.

A. $18(x+3)^{10}(x-3)^9(x-2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-5(x+3)^6(x-3)^5(x-2)^9$

* This is the correct option.

C. $4(x+3)^4(x-3)^9(x-2)^4$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-16(x+3)^5(x-3)^8(x-2)^7$

The factor -3 should have an even power and the factor 3 should have an odd power.

E. $-13(x+3)^{10}(x-3)^8(x-2)^{11}$

The factor $(x-3)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{2}, \frac{5}{4}, \text{ and } \frac{7}{5}$$

The solution is $40x^3 - 86x^2 + 17x + 35$, which is option A.

A. $a \in [35, 45], b \in [-86, -81], c \in [16, 18], \text{ and } d \in [32, 42]$

* $40x^3 - 86x^2 + 17x + 35$, which is the correct option.

B. $a \in [35, 45], b \in [-128, -124], c \in [121, 126], \text{ and } d \in [-38, -33]$

$40x^3 - 126x^2 + 123x - 35$, which corresponds to multiplying out $(2x-1)(4x-5)(5x-7)$.

C. $a \in [35, 45], b \in [-33, -17], c \in [-75, -64]$, and $d \in [32, 42]$

$40x^3 - 26x^2 - 67x + 35$, which corresponds to multiplying out $(2x - 1)(4x + 5)(5x - 7)$.

D. $a \in [35, 45], b \in [-86, -81], c \in [16, 18]$, and $d \in [-38, -33]$

$40x^3 - 86x^2 + 17x - 35$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [35, 45], b \in [81, 94], c \in [16, 18]$, and $d \in [-38, -33]$

$40x^3 + 86x^2 + 17x - 35$, which corresponds to multiplying out $(2x - 1)(4x + 5)(5x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 1)(4x - 5)(5x - 7)$

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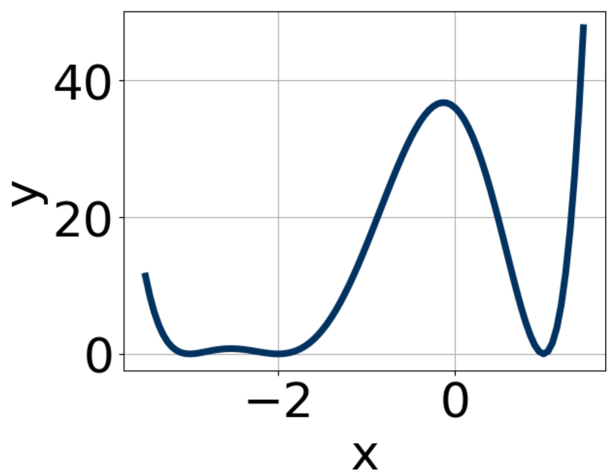
6. Which of the following equations *could* be of the graph presented below?



The solution is $8(x + 2)^8(x + 3)^6(x - 1)^6$, which is option A.

A. $8(x + 2)^8(x + 3)^6(x - 1)^6$

* This is the correct option.

B. $-12(x + 2)^4(x + 3)^8(x - 1)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $10(x + 2)^6(x + 3)^6(x - 1)^5$

The factor $(x - 1)$ should have an even power.

D. $-13(x + 2)^{10}(x + 3)^8(x - 1)^9$

The factor $(x - 1)$ should have an even power and the leading coefficient should be the opposite sign.

E. $19(x + 2)^{10}(x + 3)^{11}(x - 1)^9$

The factors $(x + 3)$ and $(x - 1)$ should both have even powers.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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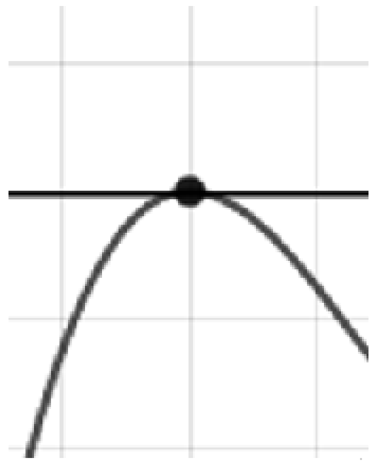
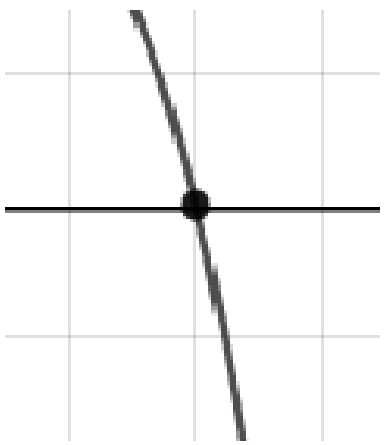
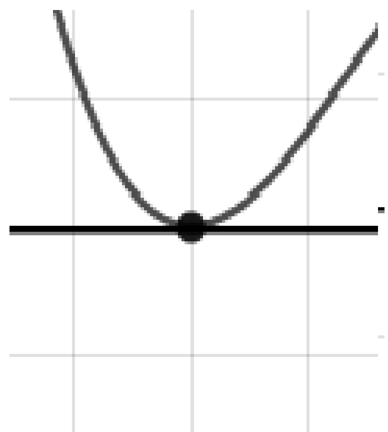
7. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

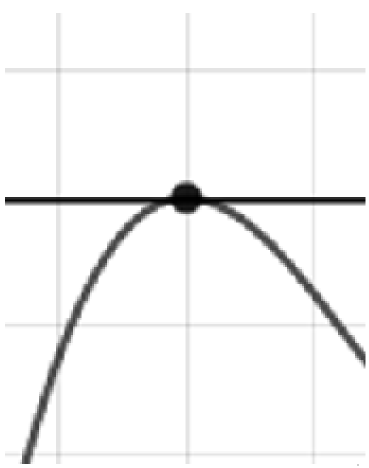
$$f(x) = -2(x-2)^6(x+2)^3(x+3)^{11}(x-3)^6$$

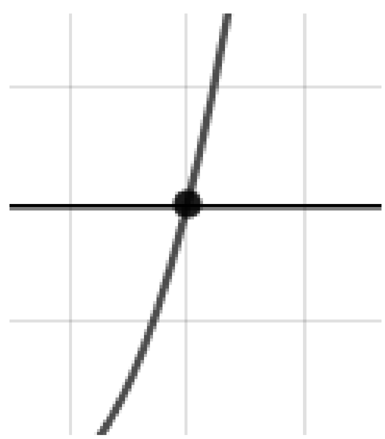
The solution is the graph below, which is option B.



A.


C.


B.


D.


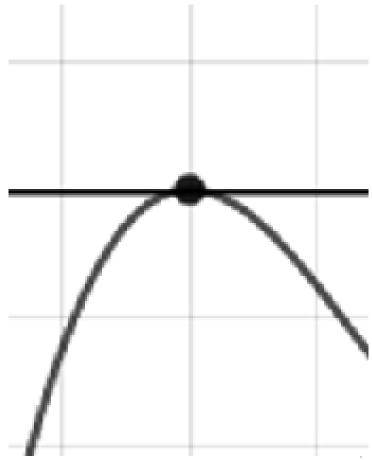
E. None of the above.

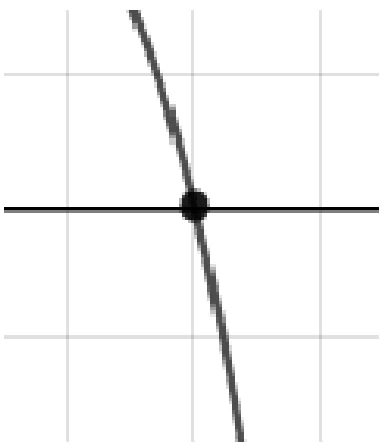
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

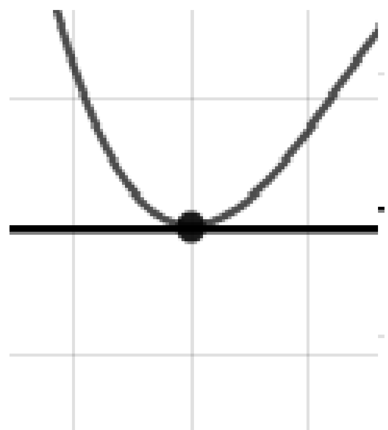
8. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

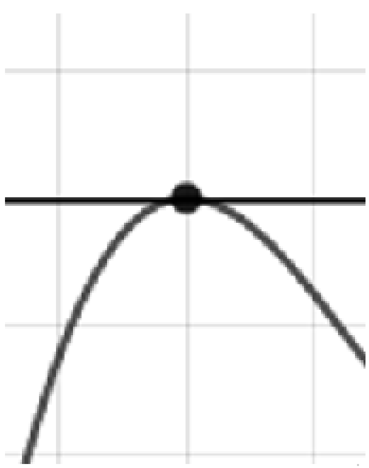
$$f(x) = -2(x-2)^2(x+2)^7(x+9)^4(x-9)^8$$

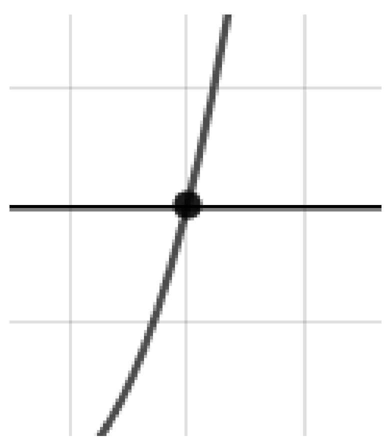
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 2i \text{ and } 1$$

The solution is $x^3 + 3x^2 + 4x - 8$, which is option C.

A. $b \in [-3.8, -1.5], c \in [2.1, 5.2],$ and $d \in [6.4, 9.3]$

$x^3 - 3x^2 + 4x + 8$, which corresponds to multiplying out $(x - (-2 + 2i))(x - (-2 - 2i))(x + 1)$.

B. $b \in [0.7, 2.7], c \in [-3.6, -0.8],$ and $d \in [0.3, 4.3]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x - 2)(x - 1)$.

C. $b \in [2, 4.5], c \in [2.1, 5.2],$ and $d \in [-8.4, -7.4]$

* $x^3 + 3x^2 + 4x - 8$, which is the correct option.

D. $b \in [0.7, 2.7], c \in [-0.3, 2.5],$ and $d \in [-4.2, -1.1]$

$x^3 + x^2 + x - 2$, which corresponds to multiplying out $(x + 2)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 2i))(x - (-2 - 2i))(x - (1))$.

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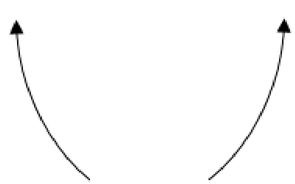
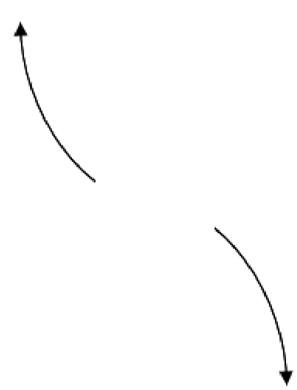
10. Describe the end behavior of the polynomial below.

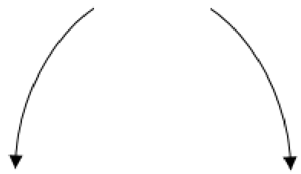
$$f(x) = 6(x + 3)^3(x - 3)^6(x - 2)^5(x + 2)^6$$

The solution is the graph below, which is option C.

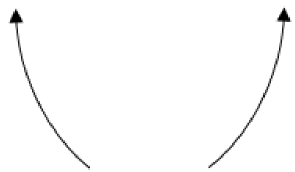




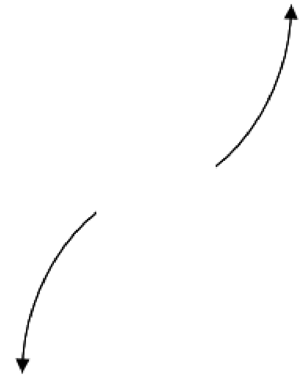
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.