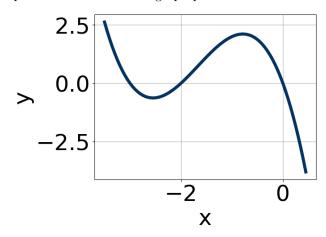
This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Which of the following equations *could* be of the graph presented below?



The solution is  $-19x^{11}(x+2)^9(x+3)^9$ , which is option A.

A. 
$$-19x^{11}(x+2)^9(x+3)^9$$

\* This is the correct option.

B. 
$$8x^7(x+2)^{11}(x+3)^9$$

This corresponds to the leading coefficient being the opposite value than it should be.

C. 
$$-2x^9(x+2)^{10}(x+3)^8$$

The factors -2 and -3 have have been odd power.

D. 
$$-19x^5(x+2)^8(x+3)^9$$

The factor -2 should have been an odd power.

E. 
$$15x^9(x+2)^6(x+3)^7$$

The factor (x + 2) should have an odd power and the leading coefficient should be the opposite sign.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-3}{2}$$
, -7, and  $\frac{7}{2}$ 

The solution is  $4x^3 + 20x^2 - 77x - 147$ , which is option A.

## Answer Key for Makeup Progress Quiz 2 Version ALL

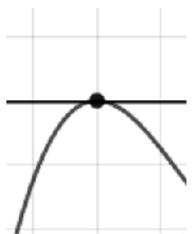
- A.  $a \in [1,7], b \in [20,23], c \in [-77,-71], \text{ and } d \in [-149,-143]$ \*  $4x^3 + 20x^2 - 77x - 147$ , which is the correct option.
- B.  $a \in [1,7], b \in [-20,-13], c \in [-77,-71], \text{ and } d \in [147,150]$  $4x^3 - 20x^2 - 77x + 147, \text{ which corresponds to multiplying out } (2x - 3)(x - 7)(2x + 7).$
- C.  $a \in [1,7], b \in [6,15], c \in [-120,-118], \text{ and } d \in [147,150]$  $4x^3 + 8x^2 - 119x + 147, \text{ which corresponds to multiplying out } (2x - 3)(x + 7)(2x - 7).$
- D.  $a \in [1,7], b \in [20,23], c \in [-77,-71]$ , and  $d \in [147,150]$  $4x^3 + 20x^2 - 77x + 147$ , which corresponds to multiplying everything correctly except the constant term.
- E.  $a \in [1,7], b \in [-52, -47], c \in [158, 164], \text{ and } d \in [-149, -143]$  $4x^3 - 48x^2 + 161x - 147, \text{ which corresponds to multiplying out } (2x - 3)(x - 7)(2x - 7).$

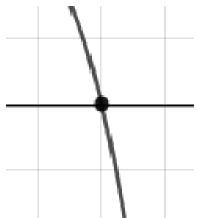
**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (2x + 3)(x + 7)(2x - 7)

3. Describe the zero behavior of the zero x = -7 of the polynomial below.

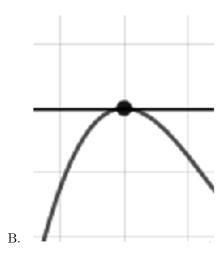
$$f(x) = -2(x-4)^8(x+4)^5(x+7)^{10}(x-7)^9$$

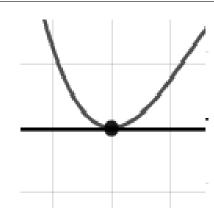
The solution is the graph below, which is option B.





A.





D.

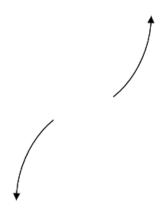
E. None of the above.

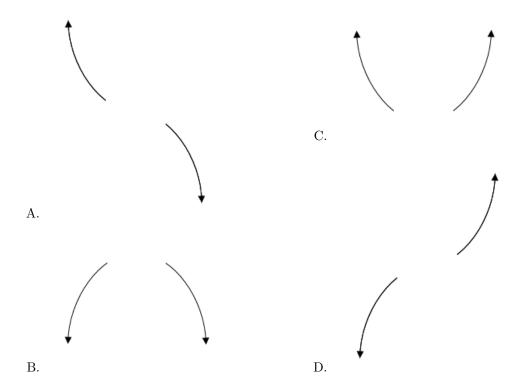
**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

4. Describe the end behavior of the polynomial below.

$$f(x) = 7(x-7)^{2}(x+7)^{3}(x-8)^{5}(x+8)^{5}$$

The solution is the graph below, which is option D.



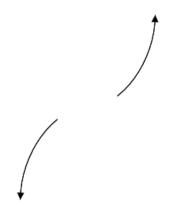


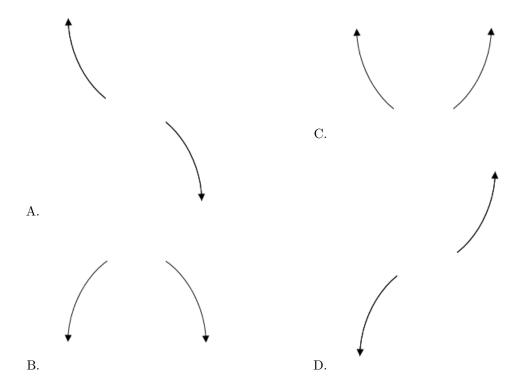
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

5. Describe the end behavior of the polynomial below.

$$f(x) = 2(x-2)^4(x+2)^7(x-4)^2(x+4)^2$$

The solution is the graph below, which is option D.





**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-4 + 4i$$
 and 4

The solution is  $x^3 + 4x^2 - 128$ , which is option D.

A. 
$$b \in [0.9,3.2], c \in [-3,3]$$
, and  $d \in [-18,-14]$  
$$x^3 + x^2 - 16$$
, which corresponds to multiplying out  $(x+4)(x-4)$ .

B. 
$$b \in [0.9, 3.2], c \in [-10, -7]$$
, and  $d \in [13, 21]$   
 $x^3 + x^2 - 8x + 16$ , which corresponds to multiplying out  $(x - 4)(x - 4)$ .

C. 
$$b \in [-7.8, -3.9], c \in [-3, 3], \text{ and } d \in [121, 134]$$
  
 $x^3 - 4x^2 + 128$ , which corresponds to multiplying out  $(x - (-4 + 4i))(x - (-4 - 4i))(x + 4)$ .

D. 
$$b \in [3.1, 5.5], c \in [-3, 3], \text{ and } d \in [-130, -123]$$
  
\*  $x^3 + 4x^2 - 128$ , which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-4 + 4i))(x - (-4 - 4i))(x - (4)).

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$-3, \frac{-5}{2}$$
, and  $\frac{-1}{5}$ 

The solution is  $10x^3 + 57x^2 + 86x + 15$ , which is option A.

- A.  $a \in [10, 15], b \in [51, 61], c \in [80, 87], \text{ and } d \in [11, 19]$ 
  - \*  $10x^3 + 57x^2 + 86x + 15$ , which is the correct option.
- B.  $a \in [10, 15], b \in [-53, -46], c \in [53, 71], \text{ and } d \in [11, 19]$

 $10x^3 - 53x^2 + 64x + 15$ , which corresponds to multiplying out (x-3)(2x-5)(5x+1).

C.  $a \in [10, 15], b \in [-57, -54], c \in [80, 87], \text{ and } d \in [-17, -7]$ 

 $10x^3 - 57x^2 + 86x - 15$ , which corresponds to multiplying out (x-3)(2x-5)(5x-1).

D.  $a \in [10, 15], b \in [-3, 6], c \in [-81, -75], \text{ and } d \in [-17, -7]$ 

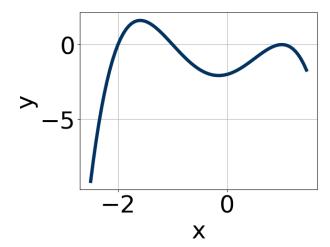
 $10x^3 - 3x^2 - 76x - 15$ , which corresponds to multiplying out (x-3)(2x+5)(5x+1).

E.  $a \in [10, 15], b \in [51, 61], c \in [80, 87], \text{ and } d \in [-17, -7]$ 

 $10x^3 + 57x^2 + 86x - 15$ , which corresponds to multiplying everything correctly except the constant term.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (x+3)(2x+5)(5x+1)

8. Which of the following equations *could* be of the graph presented below?



The solution is  $-15(x-1)^8(x+1)^{11}(x+2)^7$ , which is option A.

A. 
$$-15(x-1)^8(x+1)^{11}(x+2)^7$$

\* This is the correct option.

B. 
$$-11(x-1)^8(x+1)^6(x+2)^9$$

The factor (x + 1) should have an odd power.

C. 
$$16(x-1)^8(x+1)^9(x+2)^{11}$$

This corresponds to the leading coefficient being the opposite value than it should be.

D. 
$$-14(x-1)^7(x+1)^8(x+2)^9$$

The factor 1 should have an even power and the factor -1 should have an odd power.

E. 
$$11(x-1)^4(x+1)^{11}(x+2)^4$$

The factor (x + 2) should have an odd power and the leading coefficient should be the opposite sign.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-3 - 4i$$
 and  $-2$ 

The solution is  $x^3 + 8x^2 + 37x + 50$ , which is option C.

A. 
$$b \in [-1, 5], c \in [1.8, 5.3], \text{ and } d \in [5.8, 6.4]$$

 $x^3 + x^2 + 5x + 6$ , which corresponds to multiplying out (x+3)(x+2).

B. 
$$b \in [-1, 5], c \in [5.2, 8.8], \text{ and } d \in [7.7, 10.7]$$

 $x^3 + x^2 + 6x + 8$ , which corresponds to multiplying out (x + 4)(x + 2).

C. 
$$b \in [4, 9], c \in [35.8, 38.9], \text{ and } d \in [46.8, 52.1]$$

\*  $x^3 + 8x^2 + 37x + 50$ , which is the correct option.

D. 
$$b \in [-9, -3], c \in [35.8, 38.9], \text{ and } d \in [-50.4, -49]$$

$$x^3 - 8x^2 + 37x - 50$$
, which corresponds to multiplying out  $(x - (-3 - 4i))(x - (-3 + 4i))(x - 2)$ .

E. None of the above.

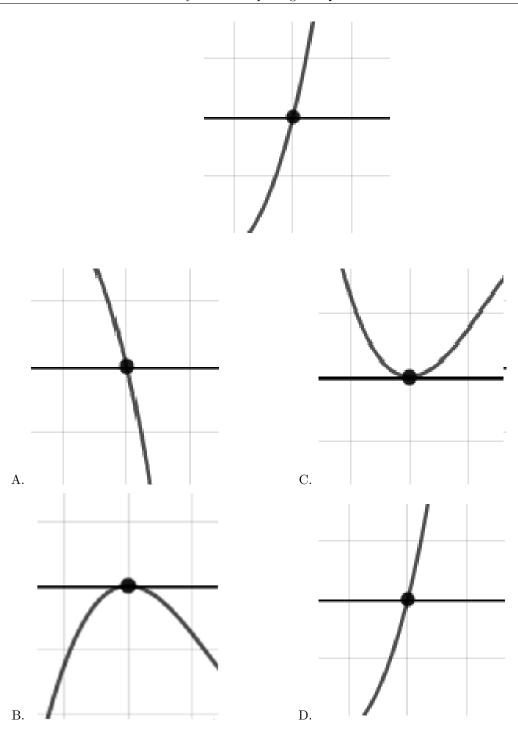
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-3 - 4i))(x - (-3 + 4i))(x - (-2)).

10. Describe the zero behavior of the zero x = -8 of the polynomial below.

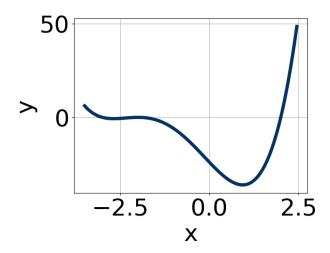
$$f(x) = -2(x-8)^8(x+8)^{11}(x+9)^9(x-9)^{13}$$

The solution is the graph below, which is option D.



**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

11. Which of the following equations *could* be of the graph presented below?



The solution is  $13(x+2)^4(x-2)^{11}(x+3)^{11}$ , which is option C.

A. 
$$19(x+2)^7(x-2)^4(x+3)^9$$

The factor -2 should have an even power and the factor 2 should have an odd power.

B. 
$$20(x+2)^{10}(x-2)^8(x+3)^7$$

The factor (x-2) should have an odd power.

C. 
$$13(x+2)^4(x-2)^{11}(x+3)^{11}$$

\* This is the correct option.

D. 
$$-18(x+2)^4(x-2)^7(x+3)^4$$

The factor (x + 3) should have an odd power and the leading coefficient should be the opposite sign.

E. 
$$-4(x+2)^{10}(x-2)^{11}(x+3)^5$$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

12. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{1}{4}, \frac{7}{5}$$
, and 2

The solution is  $20x^3 - 73x^2 + 73x - 14$ , which is option C.

A.  $a \in [17, 26], b \in [68, 75], c \in [69, 74], \text{ and } d \in [10, 16]$ 

 $20x^3 + 73x^2 + 73x + 14$ , which corresponds to multiplying out (4x + 1)(5x + 7)(x + 2).

B.  $a \in [17, 26], b \in [-7, -6], c \in [-59, -56], \text{ and } d \in [-18, -13]$ 

 $20x^3 - 7x^2 - 59x - 14$ , which corresponds to multiplying out (4x + 1)(5x + 7)(x - 2).

C.  $a \in [17, 26], b \in [-73, -66], c \in [69, 74], \text{ and } d \in [-18, -13]$ 

\*  $20x^3 - 73x^2 + 73x - 14$ , which is the correct option.

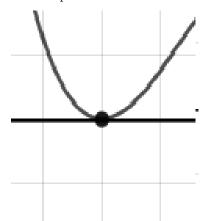
- D.  $a \in [17, 26], b \in [-73, -66], c \in [69, 74],$  and  $d \in [10, 16]$   $20x^3 73x^2 + 73x + 14,$  which corresponds to multiplying everything correctly except the constant
- E.  $a \in [17, 26], b \in [-66, -58], c \in [34, 45], \text{ and } d \in [10, 16]$  $20x^3 - 63x^2 + 39x + 14$ , which corresponds to multiplying out (4x + 1)(5x - 7)(x - 2).

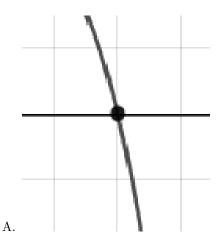
**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (4x - 1)(5x - 7)(x - 2)

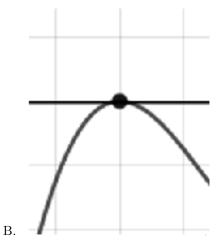
13. Describe the zero behavior of the zero x = 2 of the polynomial below.

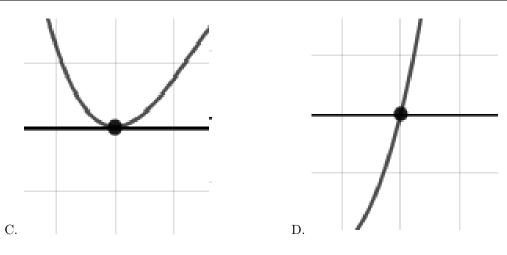
$$f(x) = -9(x-7)^{7}(x+7)^{4}(x-2)^{12}(x+2)^{9}$$

The solution is the graph below, which is option C.







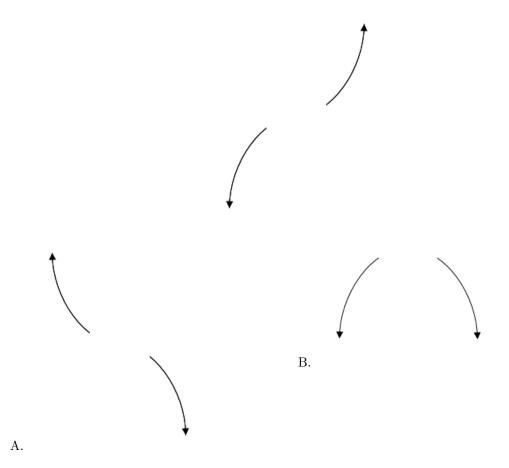


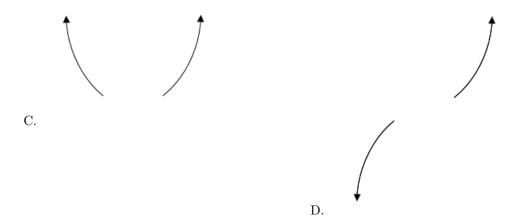
**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

14. Describe the end behavior of the polynomial below.

$$f(x) = 8(x-9)^{2}(x+9)^{5}(x-7)^{4}(x+7)^{6}$$

The solution is the graph below, which is option D.



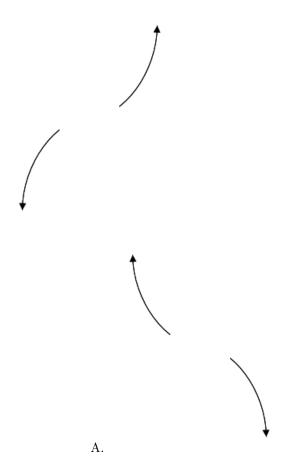


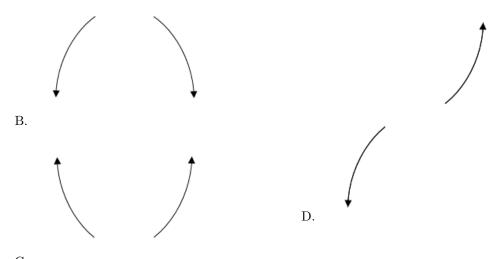
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

15. Describe the end behavior of the polynomial below.

$$f(x) = 9(x+5)^3(x-5)^6(x-3)^5(x+3)^7$$

The solution is the graph below, which is option D.





С.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

16. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-2+5i$$
 and 3

The solution is  $x^3 + x^2 + 17x - 87$ , which is option A.

A.  $b \in [-0.1, 2.4], c \in [15, 24], \text{ and } d \in [-94, -80]$ \*  $x^3 + x^2 + 17x - 87$ , which is the correct option.

B.  $b \in [-2.5, -0.9], c \in [15, 24], \text{ and } d \in [75, 89]$  $x^3 - 1x^2 + 17x + 87, \text{ which corresponds to multiplying out } (x - (-2 + 5i))(x - (-2 - 5i))(x + 3).$ 

C.  $b \in [-0.1, 2.4], c \in [-8, -3], \text{ and } d \in [10, 24]$  $x^3 + x^2 - 8x + 15, \text{ which corresponds to multiplying out } (x - 5)(x - 3).$ 

D.  $b \in [-0.1, 2.4], c \in [-2, 5], \text{ and } d \in [-10, -2]$  $x^3 + x^2 - x - 6, \text{ which corresponds to multiplying out } (x + 2)(x - 3).$ 

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-2 + 5i))(x - (-2 - 5i))(x - (3)).

17. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-3}{2}, \frac{-7}{3}, \text{ and } -4$$

The solution is  $6x^3 + 47x^2 + 113x + 84$ , which is option D.

- A.  $a \in [1, 13], b \in [42, 51], c \in [111, 117], \text{ and } d \in [-87, -83]$ 
  - $6x^3 + 47x^2 + 113x 84$ , which corresponds to multiplying everything correctly except the constant term.
- B.  $a \in [1, 13], b \in [-4, 2], c \in [-76, -68], \text{ and } d \in [84, 88]$

$$6x^3 + x^2 - 71x + 84$$
, which corresponds to multiplying out  $(2x - 3)(3x - 7)(x + 4)$ .

C.  $a \in [1, 13], b \in [-50, -41], c \in [111, 117], \text{ and } d \in [-87, -83]$ 

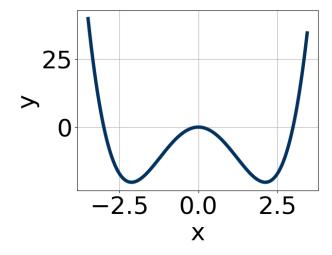
$$6x^3 - 47x^2 + 113x - 84$$
, which corresponds to multiplying out  $(2x - 3)(3x - 7)(x - 4)$ .

- D.  $a \in [1, 13], b \in [42, 51], c \in [111, 117], \text{ and } d \in [84, 88]$ 
  - \*  $6x^3 + 47x^2 + 113x + 84$ , which is the correct option.
- E.  $a \in [1, 13], b \in [28, 30], c \in [-3, 5], \text{ and } d \in [-87, -83]$

$$6x^3 + 29x^2 - x - 84$$
, which corresponds to multiplying out  $(2x - 3)(3x + 7)(x + 4)$ .

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (2x + 3)(3x + 7)(x + 4)

## 18. Which of the following equations *could* be of the graph presented below?



The solution is  $18x^8(x+3)^9(x-3)^7$ , which is option A.

- A.  $18x^8(x+3)^9(x-3)^7$ 
  - \* This is the correct option.
- B.  $-19x^4(x+3)^5(x-3)^4$

The factor (x-3) should have an odd power and the leading coefficient should be the opposite sign.

C.  $19x^4(x+3)^8(x-3)^{11}$ 

The factor (x+3) should have an odd power.

D.  $8x^5(x+3)^8(x-3)^7$ 

The factor 0 should have an even power and the factor -3 should have an odd power.

E. 
$$-3x^4(x+3)^{11}(x-3)^7$$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

19. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$5 + 2i$$
 and 4

The solution is  $x^3 - 14x^2 + 69x - 116$ , which is option B.

A. 
$$b \in [-6, 2], c \in [-9.4, -7.5], \text{ and } d \in [18, 22]$$
  
 $x^3 + x^2 - 9x + 20$ , which corresponds to multiplying out  $(x - 5)(x - 4)$ .

B. 
$$b \in [-21, -8], c \in [67.5, 70.9], \text{ and } d \in [-126, -113]$$
  
\*  $x^3 - 14x^2 + 69x - 116$ , which is the correct option.

C. 
$$b \in [-6, 2], c \in [-8, -3.8]$$
, and  $d \in [5, 12]$   
 $x^3 + x^2 - 6x + 8$ , which corresponds to multiplying out  $(x - 2)(x - 4)$ .

D. 
$$b \in [12, 21], c \in [67.5, 70.9]$$
, and  $d \in [115, 119]$   
 $x^3 + 14x^2 + 69x + 116$ , which corresponds to multiplying out  $(x - (5 + 2i))(x - (5 - 2i))(x + 4)$ .

E. None of the above.

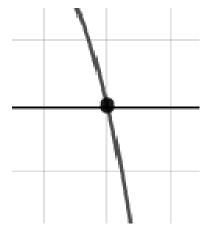
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

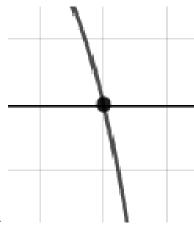
**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (5 + 2i))(x - (5 - 2i))(x - (4)).

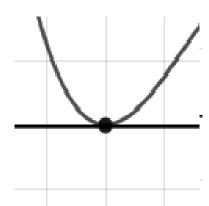
20. Describe the zero behavior of the zero x=2 of the polynomial below.

$$f(x) = 8(x+2)^{2}(x-2)^{7}(x-4)^{9}(x+4)^{11}$$

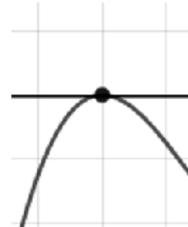
The solution is the graph below, which is option A.



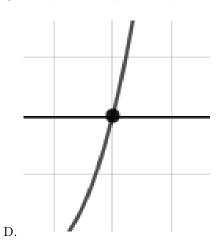




A.



С.

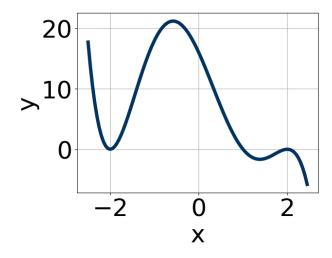


В.

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

21. Which of the following equations *could* be of the graph presented below?



The solution is  $-6(x-2)^{10}(x+2)^4(x-1)^7$ , which is option D.

A. 
$$-13(x-2)^{10}(x+2)^5(x-1)^{10}$$

The factor (x+2) should have an even power and the factor (x-1) should have an odd power.

B. 
$$-14(x-2)^{10}(x+2)^9(x-1)^{11}$$

The factor (x+2) should have an even power.

C. 
$$18(x-2)^{10}(x+2)^4(x-1)^{10}$$

The factor (x-1) should have an odd power and the leading coefficient should be the opposite sign.

D. 
$$-6(x-2)^{10}(x+2)^4(x-1)^7$$

\* This is the correct option.

E. 
$$16(x-2)^8(x+2)^8(x-1)^5$$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

22. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-3}{5}, \frac{7}{4}$$
, and  $\frac{1}{3}$ 

The solution is  $60x^3 - 89x^2 - 40x + 21$ , which is option B.

A.  $a \in [53, 63], b \in [-91, -78], c \in [-46, -37], \text{ and } d \in [-21, -18]$ 

 $60x^3 - 89x^2 - 40x - 21$ , which corresponds to multiplying everything correctly except the constant term.

- B.  $a \in [53, 63], b \in [-91, -78], c \in [-46, -37], \text{ and } d \in [20, 23]$ 
  - \*  $60x^3 89x^2 40x + 21$ , which is the correct option.
- C.  $a \in [53, 63], b \in [83, 95], c \in [-46, -37], \text{ and } d \in [-21, -18]$

 $60x^3 + 89x^2 - 40x - 21$ , which corresponds to multiplying out (5x - 3)(4x + 7)(3x + 1).

D.  $a \in [53, 63], b \in [49, 51], c \in [-88, -79], \text{ and } d \in [20, 23]$ 

 $60x^3 + 49x^2 - 86x + 21$ , which corresponds to multiplying out (5x - 3)(4x + 7)(3x - 1).

E.  $a \in [53, 63], b \in [-165, -159], c \in [107, 112], \text{ and } d \in [-21, -18]$ 

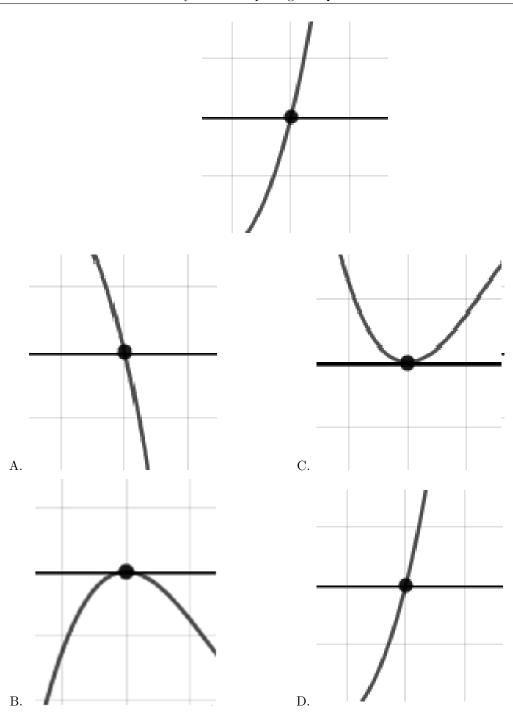
 $60x^3 - 161x^2 + 110x - 21$ , which corresponds to multiplying out (5x - 3)(4x - 7)(3x - 1).

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (5x + 3)(4x - 7)(3x - 1)

23. Describe the zero behavior of the zero x = 8 of the polynomial below.

$$f(x) = 3(x+8)^8(x-8)^{11}(x-7)^9(x+7)^{13}$$

The solution is the graph below, which is option D.

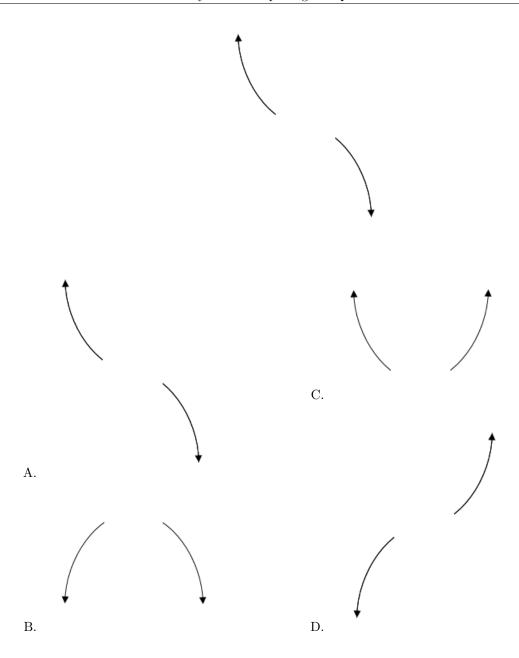


**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

24. Describe the end behavior of the polynomial below.

$$f(x) = -4(x+6)^4(x-6)^5(x+2)^5(x-2)^5$$

The solution is the graph below, which is option A.



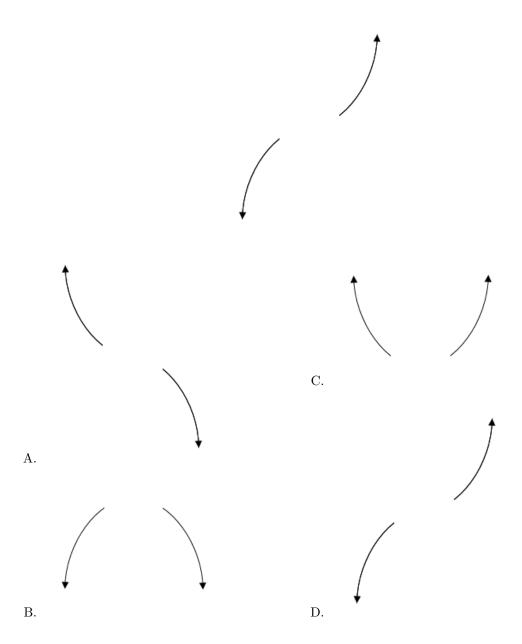
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

25. Describe the end behavior of the polynomial below.

$$f(x) = 4(x+2)^5(x-2)^8(x+9)^3(x-9)^3$$

The solution is the graph below, which is option D.

2790-1423



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

26. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-5 + 5i$$
 and  $-1$ 

The solution is  $x^3 + 11x^2 + 60x + 50$ , which is option B.

A. 
$$b \in [-16, -10], c \in [59, 67], \text{ and } d \in [-58, -48]$$

$$x^3 - 11x^2 + 60x - 50$$
, which corresponds to multiplying out  $(x - (-5 + 5i))(x - (-5 - 5i))(x - 1)$ .

- B.  $b \in [4, 19], c \in [59, 67], \text{ and } d \in [46, 58]$ 
  - \*  $x^3 + 11x^2 + 60x + 50$ , which is the correct option.
- C.  $b \in [-8, 6], c \in [-1, 13], \text{ and } d \in [3, 6]$

 $x^3 + x^2 + 6x + 5$ , which corresponds to multiplying out (x + 5)(x + 1).

D.  $b \in [-8, 6], c \in [-6, 3], \text{ and } d \in [-7, 3]$ 

 $x^3 + x^2 - 4x - 5$ , which corresponds to multiplying out (x - 5)(x + 1).

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (-5 + 5i))(x - (-5 - 5i))(x - (-1)).

27. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-2}{5}, \frac{-3}{2}, \text{ and } \frac{1}{5}$$

The solution is  $50x^3 + 85x^2 + 11x - 6$ , which is option C.

- A.  $a \in [48, 62], b \in [44, 50], c \in [-44, -38], \text{ and } d \in [1, 10]$ 
  - $50x^3 + 45x^2 41x + 6$ , which corresponds to multiplying out (5x 2)(2x + 3)(5x 1).
- B.  $a \in [48, 62], b \in [-106, -98], c \in [42, 50], \text{ and } d \in [-8, 2]$

 $50x^3 - 105x^2 + 49x - 6$ , which corresponds to multiplying out (5x - 2)(2x - 3)(5x - 1).

- C.  $a \in [48, 62], b \in [79, 88], c \in [7, 18], \text{ and } d \in [-8, 2]$ 
  - \*  $50x^3 + 85x^2 + 11x 6$ , which is the correct option.
- D.  $a \in [48, 62], b \in [79, 88], c \in [7, 18], \text{ and } d \in [1, 10]$

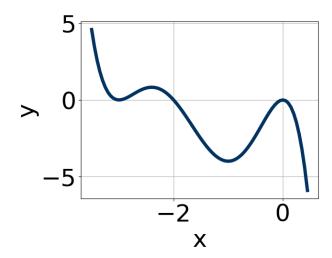
 $50x^3 + 85x^2 + 11x + 6$ , which corresponds to multiplying everything correctly except the constant term.

E.  $a \in [48, 62], b \in [-85, -84], c \in [7, 18], \text{ and } d \in [1, 10]$ 

 $50x^3 - 85x^2 + 11x + 6$ , which corresponds to multiplying out (5x - 2)(2x - 3)(5x + 1).

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out (5x + 2)(2x + 3)(5x - 1)

28. Which of the following equations *could* be of the graph presented below?



The solution is  $-13x^8(x+3)^{10}(x+2)^5$ , which is option D.

A. 
$$14x^{10}(x+3)^{10}(x+2)^5$$

This corresponds to the leading coefficient being the opposite value than it should be.

B. 
$$18x^{10}(x+3)^4(x+2)^4$$

The factor (x + 2) should have an odd power and the leading coefficient should be the opposite sign.

C. 
$$-11x^9(x+3)^6(x+2)^5$$

The factor x should have an even power.

D. 
$$-13x^8(x+3)^{10}(x+2)^5$$

\* This is the correct option.

E. 
$$-8x^{11}(x+3)^{10}(x+2)^{10}$$

The factor x should have an even power and the factor (x+2) should have an odd power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

29. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$4-3i$$
 and 3

The solution is  $x^3 - 11x^2 + 49x - 75$ , which is option B.

A. 
$$b \in [9, 12], c \in [40, 52], \text{ and } d \in [74, 86]$$

$$x^3 + 11x^2 + 49x + 75$$
, which corresponds to multiplying out  $(x - (4-3i))(x - (4+3i))(x + 3)$ .

Summer C 2021

B. 
$$b \in [-14, -5], c \in [40, 52], \text{ and } d \in [-77, -72]$$

\* 
$$x^3 - 11x^2 + 49x - 75$$
, which is the correct option.

C. 
$$b \in [0, 3], c \in [0, 5], \text{ and } d \in [-9, -8]$$

$$x^3 + x^2 - 9$$
, which corresponds to multiplying out  $(x+3)(x-3)$ .

2790-1423

- D.  $b \in [0, 3], c \in [-10, -6]$ , and  $d \in [7, 16]$  $x^3 + x^2 - 7x + 12$ , which corresponds to multiplying out (x - 4)(x - 3).
- E. None of the above.

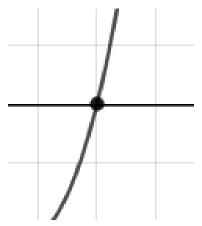
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

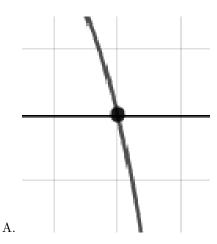
**General Comment:** Remember that the conjugate of a + bi is a - bi. Since these zeros always come in pairs, we need to multiply out (x - (4 - 3i))(x - (4 + 3i))(x - (3)).

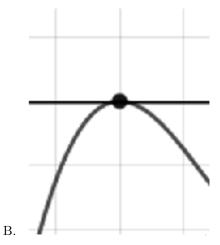
30. Describe the zero behavior of the zero x = 3 of the polynomial below.

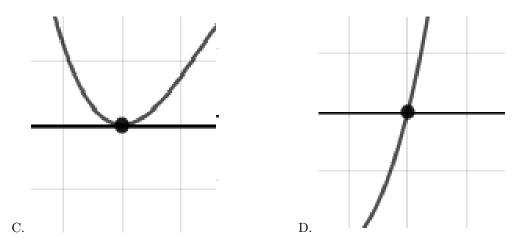
$$f(x) = 3(x-3)^5(x+3)^{10}(x+8)^5(x-8)^6$$

The solution is the graph below, which is option D.









**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.