This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 4x^3 + x^2 + 5x + 3 \text{ and } g(x) = \frac{4}{6x + 19}$$

The solution is The domain is all Real numbers except $x = -3.17$, which is option A.

A. The domain is all Real numbers except $x = a,$ where $a \in [-3.17, 2.83]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-10.2, -0.2]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-13.5, -6.5]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-6.6, -5.6]$ and $b \in [-7.33, -3.33]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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2. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = 2x^3 - 3x^2 - 4x - 1 \text{ and } g(x) = 3x^3 - 1x^2 - 3x + 3$$

The solution is -5.0, which is option A.

A. $(f \circ g)(-1) \in [-9, -3]$

* This is the correct solution

B. $(f \circ g)(-1) \in [0, 4]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [-21, -12]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [-29, -26]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 4x^3 - 1x^2 - 2x \text{ and } g(x) = x^3 + x^2 - x + 2$$

The solution is 93.0, which is option A.

A. $(f \circ g)(1) \in [88, 97]$

   * This is the correct solution

B. $(f \circ g)(1) \in [-5, -4]$

   Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [2, 6]$

   Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(1) \in [100, 103]$

   Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^-1(8)$ belongs to.

$$f(x) = e^{x-4} - 4$$

The solution is $f^{-1}(8) = 6.485$, which is option C.

A. $f^{-1}(8) \in [-4.61, -1.61]$

   This solution corresponds to distractor 4.

B. $f^{-1}(8) \in [-1.52, -0.52]$

   This solution corresponds to distractor 1.

C. $f^{-1}(8) \in [1.48, 11.48]$

   This is the solution.

D. $f^{-1}(8) \in [-4.61, -1.61]$

   This solution corresponds to distractor 2.

E. $f^{-1}(8) \in [-1.52, -0.52]$

   This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -13$ and choose the interval that $f^-1(-13)$ belongs to.

$$f(x) = 4x^2 - 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-13) \in [2.52, 3.29]$

   Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B. $f^{-1}(-13) \in [3.81, 4.71]$

   Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(-13) \in [1.81, 2.5]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(-13) \in [1.15, 1.95]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^-1(9)$ belongs to.

$$f(x) = e^{x+5} - 2$$

The solution is $f^{-1}(9) = -2.602$, which is option E.

A. $f^{-1}(9) \in [-0.77, -0.59]$

This solution corresponds to distractor 3.

B. $f^{-1}(9) \in [7.29, 7.8]$

This solution corresponds to distractor 1.

C. $f^{-1}(9) \in [0.07, 0.74]$

This solution corresponds to distractor 4.

D. $f^{-1}(9) \in [-0.08, 0.22]$

This solution corresponds to distractor 2.

E. $f^{-1}(9) \in [-2.69, -2.51]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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7. Determine whether the function below is 1-1.

$$f(x) = 9x^2 - 60x + 100$$

The solution is no, which is option E.

A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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8. Determine whether the function below is 1-1.

$$f(x) = (3x - 18)^3$$

The solution is yes, which is option B.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. Yes, the function is 1-1.

* This is the solution.

C. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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9. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{2}{3x - 13} \text{ and } g(x) = \frac{2}{3x - 19}$$

The solution is The domain is all Real numbers except $x = 4.33$ and $x = 6.33$, which is option D.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-4.17, -2.17]$

B. The domain is all Real numbers except $x = a,$ where $a \in [-8.8, -1.8]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [3.33, 10.33]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [2.33, 8.33]$ and $b \in [4.33, 10.33]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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10. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -12$ and choose the interval that $f^-1(-12)$ belongs to.

$$f(x) = 4x^2 - 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-12) \in [1.56, 2.12]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(-12) \in [4.48, 4.58]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(-12) \in [7.34, 7.65]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(-12) \in [1.37, 1.68]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!
