

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

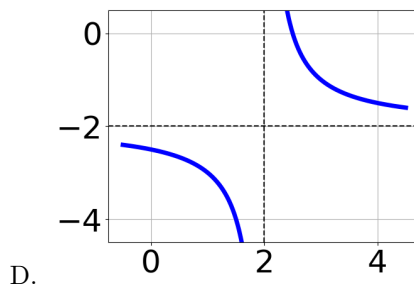
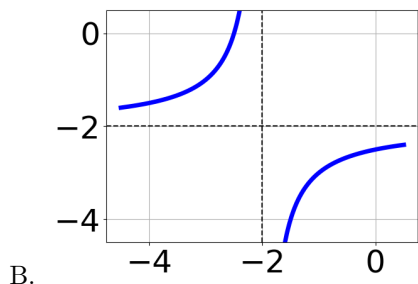
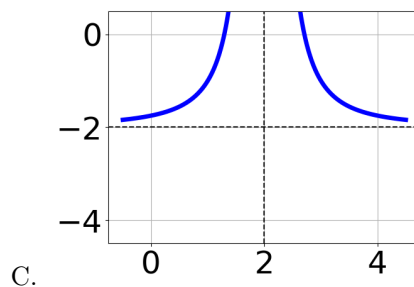
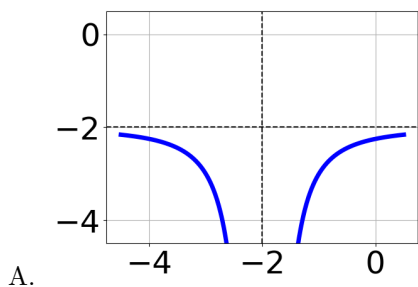
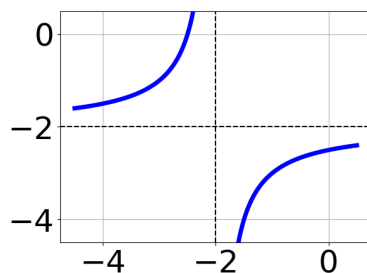
If you have a suggestion to make the keys better, please fill out the short survey [here](#).

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

1. Choose the graph of the equation below.

$$f(x) = \frac{-1}{x+2} - 2$$

The solution is the graph below, which is option B.



- E. None of the above.

**General Comment:** Remember that the general form of a basic rational equation is  $f(x) = \frac{a}{(x-h)^n} + k$ , where  $a$  is the leading coefficient (and in this case, we assume is either 1 or  $-1$ ),  $n$  is the degree (in this case, either 1 or 2), and  $(h, k)$  is the intersection of the asymptotes.

2. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{63}{54x + 27} + 1 = \frac{63}{54x + 27}$$

The solution is all solutions are invalid or lead to complex values in the equation., which is option D.

A.  $x \in [-0.5, 0.5]$

$x = -0.500$ , which corresponds to not checking if this value leads to dividing by 0 in the original equation and thus is not a valid solution.

B.  $x_1 \in [-1.5, 0.2]$  and  $x_2 \in [0.3, 1.4]$

$x = -0.500$  and  $x = 0.500$ , which corresponds to getting the correct solution and believing there should be a second solution to the equation.

C.  $x_1 \in [-1.5, 0.2]$  and  $x_2 \in [-1.4, 0.1]$

$x = -0.500$  and  $x = -0.500$ , which corresponds to getting the correct solution and believing there should be a second solution to the equation.

D. All solutions lead to invalid or complex values in the equation.

\* $x = -0.500$  leads to dividing by 0 in the original equation and thus is not a valid solution, which is the correct option.

E.  $x \in [0.3, 0.8]$

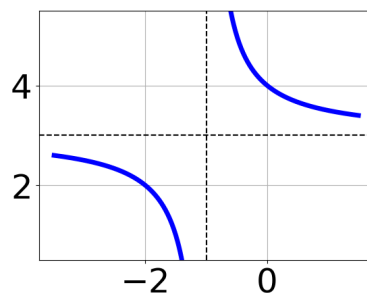
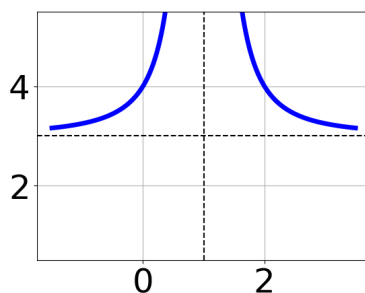
$x = 0.500$ , which corresponds to not distributing the factor  $54x + 27$  correctly when trying to eliminate the fraction.

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

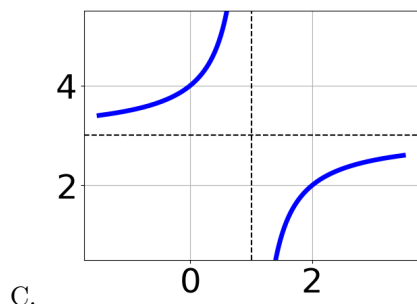
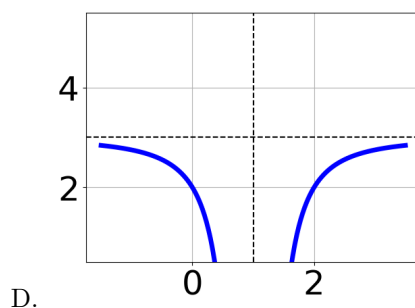
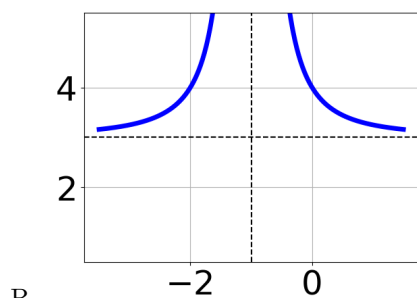
3. Choose the graph of the equation below.

$$f(x) = \frac{1}{(x-1)^2} + 3$$

The solution is the graph below, which is option E.



A.



E. None of the above.

**General Comment:** Remember that the general form of a basic rational equation is  $f(x) = \frac{a}{(x-h)^n} + k$ , where  $a$  is the leading coefficient (and in this case, we assume is either 1 or  $-1$ ),  $n$  is the degree (in this case, either 1 or 2), and  $(h, k)$  is the intersection of the asymptotes.

4. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{-6}{-6x-4} + -8 = \frac{8}{48x+32}$$

The solution is  $x = -0.562$ , which is option A.

A.  $x \in [-2.56, 1.44]$

\*  $x = -0.562$ , which is the correct option.

B.  $x_1 \in [-1.3, 0.4]$  and  $x_2 \in [0.77, 1.77]$

$x = -0.562$  and  $x = 0.771$ , which corresponds to getting the correct solution and believing there should be a second solution to the equation.

C. All solutions lead to invalid or complex values in the equation.

This corresponds to thinking  $x = -0.562$  leads to dividing by zero in the original equation, which it does not.

D.  $x_1 \in [-1.3, 0.4]$  and  $x_2 \in [-1.38, 0.62]$

$x = -0.562$  and  $x = -0.375$ , which corresponds to getting the correct solution and believing there should be a second solution to the equation.

E.  $x \in [0.2, 1]$

$x = 0.771$ , which corresponds to not distributing the factor  $-6x - 4$  correctly when trying to eliminate the fraction.

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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5. Determine the domain of the function below.

$$f(x) = \frac{6}{15x^2 - 8x - 16}$$

The solution is All Real numbers except  $x = -0.800$  and  $x = 1.333$ ., which is option D.

- A. All Real numbers except  $x = a$  and  $x = b$ , where  $a \in [-20, -18]$  and  $b \in [10, 15]$

All Real numbers except  $x = -20.000$  and  $x = 12.000$ , which corresponds to not factoring the denominator correctly.

- B. All Real numbers except  $x = a$ , where  $a \in [-0.8, 1.2]$

All Real numbers except  $x = -0.800$ , which corresponds to removing only 1 value from the denominator.

- C. All Real numbers except  $x = a$ , where  $a \in [-20, -18]$

All Real numbers except  $x = -20.000$ , which corresponds to removing a distractor value from the denominator.

- D. All Real numbers except  $x = a$  and  $x = b$ , where  $a \in [-0.8, 1.2]$  and  $b \in [0.33, 5.33]$

All Real numbers except  $x = -0.800$  and  $x = 1.333$ , which is the correct option.

- E. All Real numbers.

This corresponds to thinking the denominator has complex roots or that rational functions have a domain of all Real numbers.

**General Comment:** Recall that dividing by zero is not a real number. Therefore the domain is all real numbers **except** those that make the denominator 0.

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6. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{-2x}{6x - 7} + \frac{-3x^2}{24x^2 + 8x - 42} = \frac{-6}{4x + 6}$$

The solution is All solutions are invalid or lead to complex values in the equation., which is option A.

- A. All solutions lead to invalid or complex values in the equation.

\* The equation leads to solving  $-5x^2 + 24x - 42 = 0$ , which leads to complex solutions. This is the correct option.

- B.  $x_1 \in [3.5, 4.4]$  and  $x_2 \in [-1.22, 3.78]$

$x = 4.025$  and  $x = 0.775$ , which corresponds to making the discriminant from the Quadratic Formula positive to avoid complex solutions.

- C.  $x_1 \in [0.1, 1.4]$  and  $x_2 \in [-6.5, 0.5]$

$x = 1.167$  and  $x = -1.500$ , which corresponds to solving  $6x - 7 = 0$  and  $4x + 6 = 0$  and treating them as solutions to the equation.

- D.  $x \in [-3.7, 0.3]$

$x = -1.500$ , which corresponds to solving  $4x + 6 = 0$  and treating it as a solution to the equation.

E.  $x \in [0.1, 1.4]$

$x = 1.167$ , which corresponds to solving  $6x - 7 = 0$  and treating it as a solution to the equation.

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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7. Determine the domain of the function below.

$$f(x) = \frac{5}{18x^2 + 15x - 25}$$

The solution is All Real numbers except  $x = -1.667$  and  $x = 0.833$ , which is option E.

A. All Real numbers except  $x = a$  and  $x = b$ , where  $a \in [-17, -13]$  and  $b \in [29, 32]$

All Real numbers except  $x = -15.000$  and  $x = 30.000$ , which corresponds to not factoring the denominator correctly.

B. All Real numbers.

This corresponds to thinking the denominator has complex roots or that rational functions have a domain of all Real numbers.

C. All Real numbers except  $x = a$ , where  $a \in [-3.67, -0.67]$

All Real numbers except  $x = -1.667$ , which corresponds to removing only 1 value from the denominator.

D. All Real numbers except  $x = a$ , where  $a \in [-17, -13]$

All Real numbers except  $x = -15.000$ , which corresponds to removing a distractor value from the denominator.

E. All Real numbers except  $x = a$  and  $x = b$ , where  $a \in [-3.67, -0.67]$  and  $b \in [0.83, 3.83]$

All Real numbers except  $x = -1.667$  and  $x = 0.833$ , which is the correct option.

**General Comment:** Recall that dividing by zero is not a real number. Therefore the domain is all real numbers **except** those that make the denominator 0.

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8. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{-3x}{-2x + 6} + \frac{-2x^2}{12x^2 - 48x + 36} = \frac{5}{-6x + 6}$$

The solution is There are two solutions:  $x = -1.142$  and  $x = 1.642$ , which is option E.

A.  $x \in [1.34, 1.87]$

B.  $x_1 \in [-1.15, -0.96]$  and  $x_2 \in [3, 6]$

C.  $x \in [0.46, 1.62]$

D. All solutions lead to invalid or complex values in the equation.

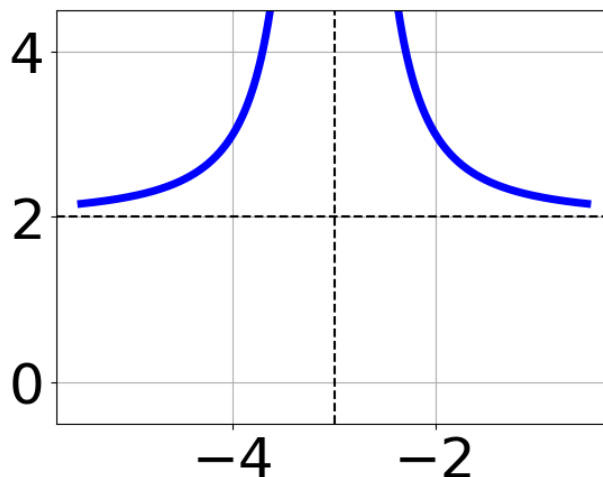
E.  $x_1 \in [-1.15, -0.96]$  and  $x_2 \in [-2.36, 2.64]$

\*  $x = -1.142$  and  $x = 1.642$ , which is the correct option.

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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9. Choose the equation of the function graphed below.



The solution is None of the above as it should be  $f(x) = \frac{1}{(x+3)^2} + 2$ , which is option E.

A.  $f(x) = \frac{-1}{(x+3)^2} + 2$

Corresponds to using the general form  $f(x) = \frac{a}{(x-h)^2} + k$  and the opposite leading coefficient.

B.  $f(x) = \frac{-1}{x+3} + 2$

Corresponds to thinking the graph was a shifted version of  $\frac{1}{x}$ , using the general form  $f(x) = \frac{a}{(x-h)^2} + k$ , and the opposite leading coefficient.

C.  $f(x) = \frac{1}{(x-3)^2} + 2$

The  $x$ -value of the equation does not match the graph.

D.  $f(x) = \frac{1}{x-3} + 2$

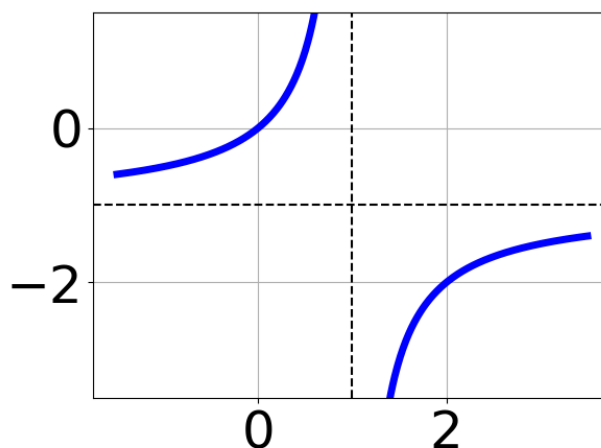
Corresponds to thinking the graph was a shifted version of  $\frac{1}{x}$ .

E. None of the above

None of the equation options were the correct equation.

**General Comment:** Remember that the general form of a basic rational equation is  $f(x) = \frac{a}{(x-h)^n} + k$ , where  $a$  is the leading coefficient (and in this case, we assume is either 1 or  $-1$ ),  $n$  is the degree (in this case, either 1 or 2), and  $(h, k)$  is the intersection of the asymptotes.

10. Choose the equation of the function graphed below.



The solution is  $f(x) = \frac{-1}{x-1} - 1$ , which is option C.

A.  $f(x) = \frac{1}{x+1} - 1$

Corresponds to using the general form  $f(x) = \frac{a}{x+h} + k$  and the opposite leading coefficient.

B.  $f(x) = \frac{-1}{(x-1)^2} - 1$

Corresponds to thinking the graph was a shifted version of  $\frac{1}{x^2}$ .

C.  $f(x) = \frac{-1}{x-1} - 1$

This is the correct option.

D.  $f(x) = \frac{1}{(x+1)^2} - 1$

Corresponds to thinking the graph was a shifted version of  $\frac{1}{x^2}$ , using the general form  $f(x) = \frac{a}{x+h} + k$ , and the opposite leading coefficient.

E. None of the above

This corresponds to believing the vertex of the graph was not correct.

**General Comment:** Remember that the general form of a basic rational equation is  $f(x) = \frac{a}{(x-h)^n} + k$ , where  $a$  is the leading coefficient (and in this case, we assume is either 1 or  $-1$ ),  $n$  is the degree (in this case, either 1 or 2), and  $(h, k)$  is the intersection of the asymptotes.

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