

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

If you have a suggestion to make the keys better, please fill out the short survey [here](#).

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 3x^3 + 8x + 5 \text{ and } g(x) = \sqrt{-5x - 18}$$

The solution is The domain is all Real numbers less than or equal to $x = -3.6$, which is option A.

- A. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-7.6, 1.4]$
- B. The domain is all Real numbers except $x = a$, where $a \in [-8.6, -3.6]$
- C. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [4, 9]$
- D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [3.75, 10.75]$ and $b \in [1.2, 8.2]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

2. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^{-1}(7)$ belongs to.

$$f(x) = \ln(x - 5) - 3$$

The solution is $f^{-1}(7) = 22031.466$, which is option A.

- A. $f^{-1}(7) \in [22026.47, 22032.47]$

This is the solution.

- B. $f^{-1}(7) \in [162748.79, 162754.79]$

This solution corresponds to distractor 2.

- C. $f^{-1}(7) \in [58.6, 62.6]$

This solution corresponds to distractor 1.

- D. $f^{-1}(7) \in [22015.47, 22023.47]$

This solution corresponds to distractor 3.

- E. $f^{-1}(7) \in [-2.61, 10.39]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y , use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

3. Determine whether the function below is 1-1.

$$f(x) = (4x + 22)^3$$

The solution is yes, which is option B.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. Yes, the function is 1-1.

* This is the solution.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. No, because there is an x -value that goes to 2 different y -values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because there is a y -value that goes to 2 different x -values.

Corresponds to the Horizontal Line test, which this function passes.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y -value that goes to 2 different x -values.

4. Find the inverse of the function below. Then, evaluate the inverse at $x = 4$ and choose the interval that $f^{-1}(4)$ belongs to.

$$f(x) = e^{x-2} - 2$$

The solution is $f^{-1}(4) = 3.792$, which is option E.

A. $f^{-1}(4) \in [-0.26, 0.56]$

This solution corresponds to distractor 3.

B. $f^{-1}(4) \in [-2.74, -0.56]$

This solution corresponds to distractor 2.

C. $f^{-1}(4) \in [-0.26, 0.56]$

This solution corresponds to distractor 1.

D. $f^{-1}(4) \in [-2.74, -0.56]$

This solution corresponds to distractor 4.

E. $f^{-1}(4) \in [3.68, 4.18]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y , use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

5. Determine whether the function below is 1-1.

$$f(x) = 9x^2 + 126x + 441$$

The solution is no, which is option A.

A. No, because there is a y -value that goes to 2 different x -values.

* This is the solution.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is an x -value that goes to 2 different y -values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y -value that goes to 2 different x -values.

6. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 3x^3 + 2x^2 - 4x \text{ and } g(x) = 3x^3 + 2x^2 - 4x + 1$$

The solution is 24.0, which is option B.

A. $(f \circ g)(1) \in [-13, -3]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [21, 25]$

* This is the correct solution

C. $(f \circ g)(1) \in [16, 21]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [1, 4]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

7. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = x^3 - 2x^2 - x + 2 \text{ and } g(x) = -x^3 + 4x^2 + x$$

The solution is 30.0, which is option B.

A. $(f \circ g)(-1) \in [35, 40]$

Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [29, 31]$

* This is the correct solution

C. $(f \circ g)(-1) \in [-4, 1]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [7, 18]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -14$ and choose the interval that $f^{-1}(-14)$ belongs to.

$$f(x) = \sqrt[3]{5x - 4}$$

The solution is -548.0 , which is option C.

A. $f^{-1}(-14) \in [547, 548.6]$

This solution corresponds to distractor 2.

B. $f^{-1}(-14) \in [-551.3, -548.6]$

Distractor 1: This corresponds to

C. $f^{-1}(-14) \in [-548.6, -546.7]$

* This is the correct solution.

D. $f^{-1}(-14) \in [549, 552.1]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

9. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^{-1}(-15)$ belongs to.

$$f(x) = 3x^2 - 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [2.27, 2.72]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(-15) \in [2.73, 3.19]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(-15) \in [5.61, 6.24]$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(-15) \in [1.77, 2.01]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

10. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{3}{3x - 14} \text{ and } g(x) = 2x^2 + 3x + 7$$

The solution is The domain is all Real numbers except $x = 4.67$, which is option A.

- A. The domain is all Real numbers except $x = a$, where $a \in [3.67, 13.67]$
- B. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-8.67, -1.67]$
- C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-3, 0]$
- D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [2.67, 6.67]$ and $b \in [-4.67, 1.33]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.
