This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 5i \text{ and } 3$$

The solution is $x^3 + x^2 + 17x - 87$, which is option A.

A. $b \in [-0.9, 3.8], c \in [16, 20.1], \text{ and } d \in [-91, -82]$

* $x^3 + x^2 + 17x - 87$, which is the correct option.

B. $b \in [-1.5, 0.8], c \in [16, 20.1], \text{ and } d \in [84, 91]$

$x^3 - 1x^2 + 17x + 87$, which corresponds to multiplying out $(x - (-2 - 5i))(x - (-2 + 5i))(x + 3)$.

C. $b \in [-0.9, 3.8], c \in [-3.2, -0.5], \text{ and } d \in [-6, -1]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x + 2)(x - 3)$.

D. $b \in [-0.9, 3.8], c \in [0.2, 7.4], \text{ and } d \in [-15, -11]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x + 5)(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 5i))(x - (-2 + 5i))(x - (3))$.

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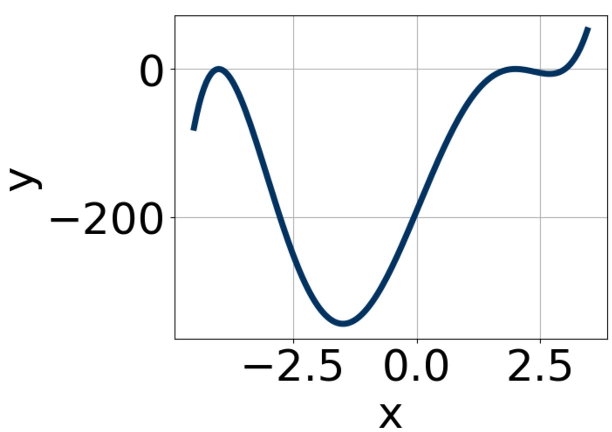
2. Which of the following equations *could* be of the graph presented below?



The solution is $8(x + 4)^6(x - 2)^8(x - 3)^9$, which is option C.

A. $13(x+4)^6(x-2)^5(x-3)^9$

The factor $(x-2)$ should have an even power.

B. $-18(x+4)^4(x-2)^{10}(x-3)^6$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $8(x+4)^6(x-2)^8(x-3)^9$

* This is the correct option.

D. $3(x+4)^8(x-2)^9(x-3)^4$

The factor $(x-2)$ should have an even power and the factor $(x-3)$ should have an odd power.

E. $-8(x+4)^4(x-2)^8(x-3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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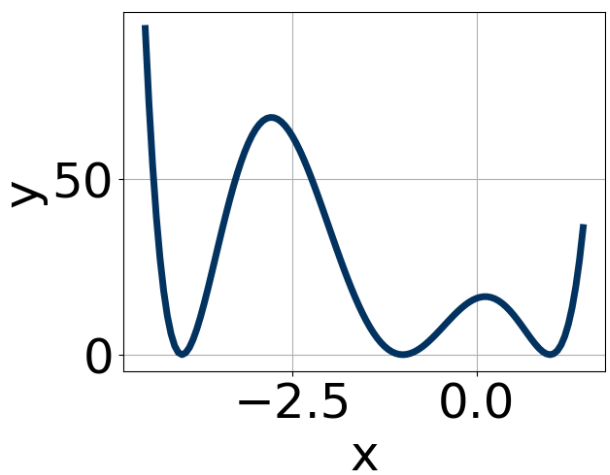
3. Which of the following equations *could* be of the graph presented below?



The solution is $18(x+4)^6(x+1)^4(x-1)^8$, which is option C.

A. $-18(x+4)^{10}(x+1)^4(x-1)^8$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $20(x+4)^8(x+1)^5(x-1)^7$

The factors $(x+1)$ and $(x-1)$ should both have even powers.

C. $18(x+4)^6(x+1)^4(x-1)^8$

* This is the correct option.

D. $-4(x+4)^{10}(x+1)^{10}(x-1)^7$

The factor $(x-1)$ should have an even power and the leading coefficient should be the opposite sign.

E. $6(x+4)^8(x+1)^4(x-1)^7$

The factor $(x-1)$ should have an even power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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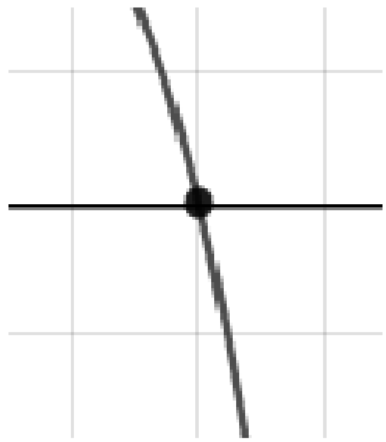
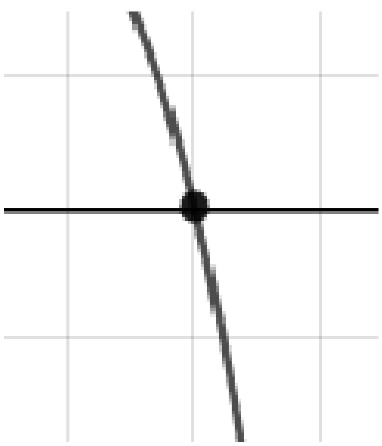
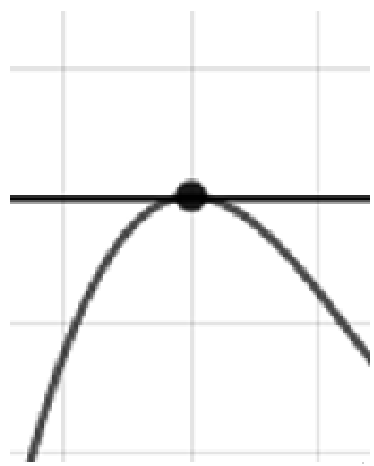
4. Describe the zero behavior of the zero $x = 6$ of the polynomial below.

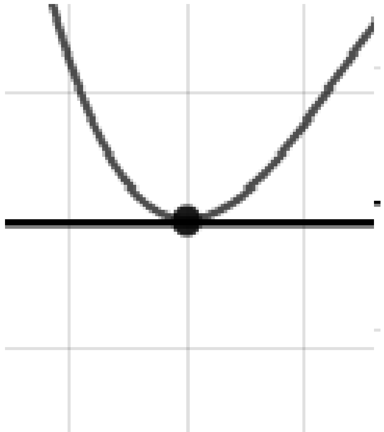
$$f(x) = -3(x+4)^8(x-4)^5(x+6)^6(x-6)^5$$

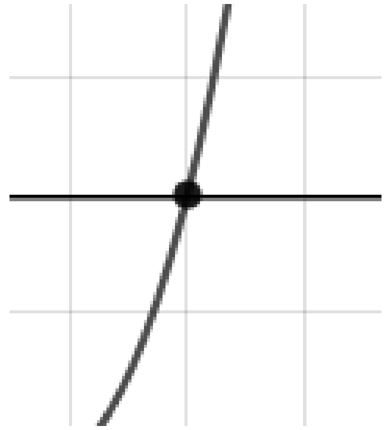
The solution is the graph below, which is option A.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 5i \text{ and } 1$$

The solution is $x^3 + 3x^2 + 25x - 29$, which is option A.

A. $b \in [2.3, 3.6], c \in [24, 32], \text{ and } d \in [-30, -20]$

* $x^3 + 3x^2 + 25x - 29$, which is the correct option.

B. $b \in [-1.2, 1.7], c \in [-10, -3], \text{ and } d \in [0, 12]$

$x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x - 5)(x - 1)$.

C. $b \in [-1.2, 1.7], c \in [-1, 13], \text{ and } d \in [-5, 0]$

$x^3 + x^2 + x - 2$, which corresponds to multiplying out $(x + 2)(x - 1)$.

D. $b \in [-5.5, -1.7], c \in [24, 32], \text{ and } d \in [23, 32]$

$x^3 - 3x^2 + 25x + 29$, which corresponds to multiplying out $(x - (-2 + 5i))(x - (-2 - 5i))(x + 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 5i))(x - (-2 - 5i))(x - (1))$.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$2, \frac{1}{5}, \text{ and } \frac{-1}{4}$$

The solution is $20x^3 - 39x^2 - 3x + 2$, which is option C.

A. $a \in [17, 29], b \in [-39.3, -36.6], c \in [-5.1, -0.4],$ and $d \in [-4, 0]$

$20x^3 - 39x^2 - 3x - 2$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [17, 29], b \in [47.6, 51.5], c \in [18.7, 20.3],$ and $d \in [2, 8]$

$20x^3 + 49x^2 + 19x + 2$, which corresponds to multiplying out $(x + 2)(5x + 1)(4x + 1)$.

C. $a \in [17, 29], b \in [-39.3, -36.6], c \in [-5.1, -0.4],$ and $d \in [2, 8]$

* $20x^3 - 39x^2 - 3x + 2$, which is the correct option.

D. $a \in [17, 29], b \in [33.4, 40.5], c \in [-5.1, -0.4],$ and $d \in [-4, 0]$

$20x^3 + 39x^2 - 3x - 2$, which corresponds to multiplying out $(x + 2)(5x + 1)(4x - 1)$.

E. $a \in [17, 29], b \in [39.7, 41.9], c \in [-2, 2.2],$ and $d \in [-4, 0]$

$20x^3 + 41x^2 + x - 2$, which corresponds to multiplying out $(x + 2)(5x - 1)(4x + 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x-2)(5x-1)(4x+1)$

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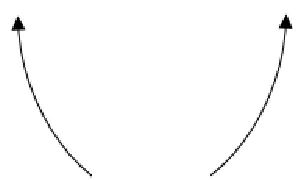
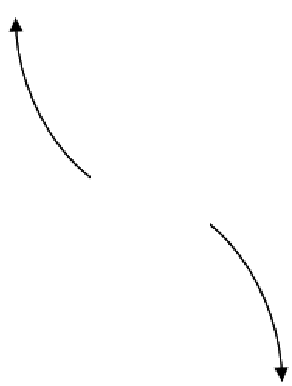
7. Describe the end behavior of the polynomial below.

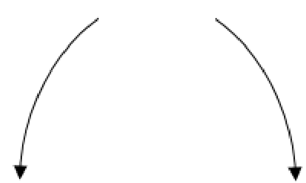
$$f(x) = 5(x - 5)^4(x + 5)^5(x - 6)^5(x + 6)^6$$

The solution is the graph below, which is option C.

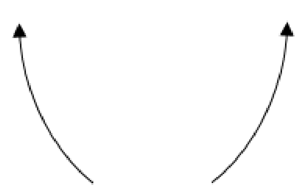




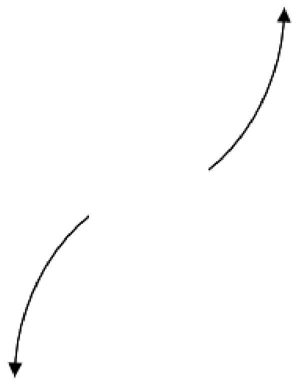
A.



B.



C.

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-5, \frac{-2}{3}, \text{ and } \frac{-7}{5}$$

The solution is $15x^3 + 106x^2 + 169x + 70$, which is option E.

A. $a \in [12, 16], b \in [103, 110], c \in [161, 178],$ and $d \in [-74, -62]$

$15x^3 + 106x^2 + 169x - 70$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [12, 16], b \in [-72, -57], c \in [-75, -65],$ and $d \in [68, 74]$

$15x^3 - 64x^2 - 69x + 70$, which corresponds to multiplying out $(x - 5)(3x - 2)(5x + 7)$.

C. $a \in [12, 16], b \in [-45, -43], c \in [-142, -136],$ and $d \in [-74, -62]$

$15x^3 - 44x^2 - 141x - 70$, which corresponds to multiplying out $(x - 5)(3x + 2)(5x + 7)$.

D. $a \in [12, 16], b \in [-113, -105], c \in [161, 178],$ and $d \in [-74, -62]$

$15x^3 - 106x^2 + 169x - 70$, which corresponds to multiplying out $(x - 5)(3x - 2)(5x - 7)$.

E. $a \in [12, 16], b \in [103, 110], c \in [161, 178],$ and $d \in [68, 74]$

* $15x^3 + 106x^2 + 169x + 70$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+5)(3x+2)(5x+7)$

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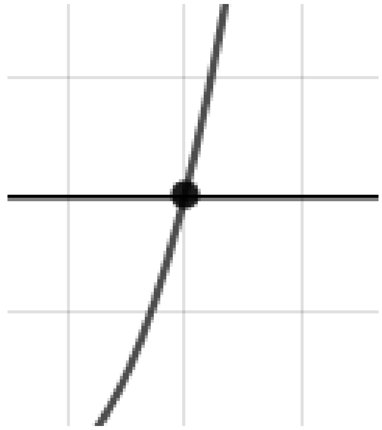
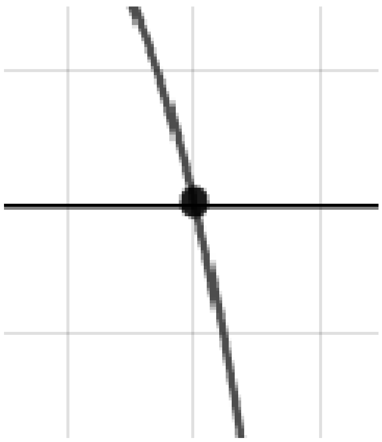
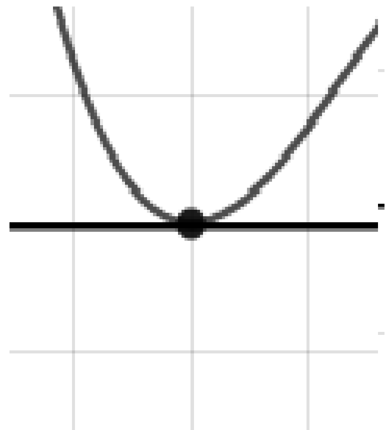
9. Describe the zero behavior of the zero $x = -9$ of the polynomial below.

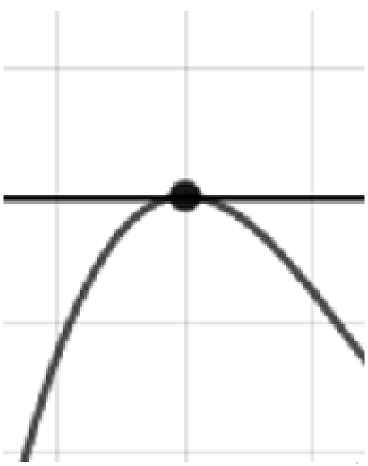
$$f(x) = -9(x - 9)^4(x + 9)^5(x + 3)^9(x - 3)^{10}$$

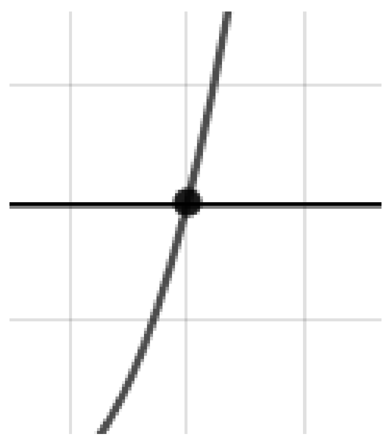
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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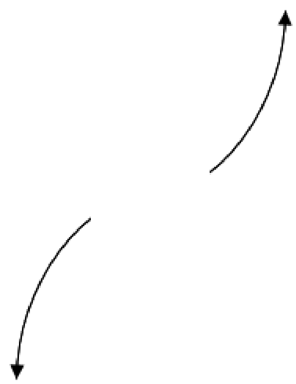
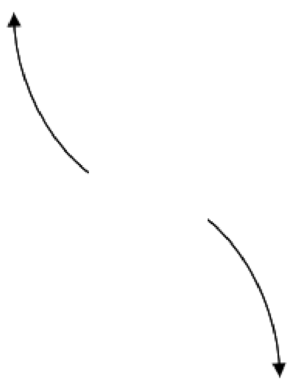
10. Describe the end behavior of the polynomial below.

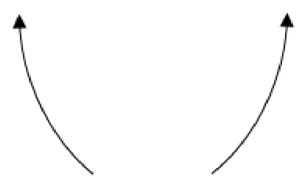
$$f(x) = 5(x-5)^5(x+5)^{10}(x-8)^5(x+8)^7$$

The solution is the graph below, which is option D.

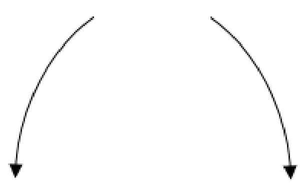




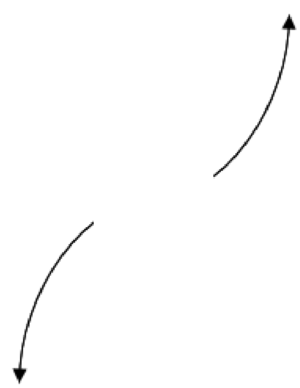
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.