This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } 1$$

The solution is $x^3 + 9x^2 + 31x - 41$, which is option A.

A. $b \in [7, 12], c \in [31, 38], \text{ and } d \in [-48, -38]$

* $x^3 + 9x^2 + 31x - 41$, which is the correct option.

B. $b \in [0, 5], c \in [-2, 10], \text{ and } d \in [-10, -2]$

$x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

C. $b \in [-10, -7], c \in [31, 38], \text{ and } d \in [35, 42]$

$x^3 - 9x^2 + 31x + 41$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x + 1)$.

D. $b \in [0, 5], c \in [-7, 0], \text{ and } d \in [2, 5]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (1))$.

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2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{3}, \frac{-3}{2}, \text{ and } -1$$

The solution is $6x^3 + 29x^2 + 44x + 21$, which is option E.

A. $a \in [0, 8], b \in [-18, -13], c \in [-7, -1], \text{ and } d \in [20, 27]$

$6x^3 - 17x^2 - 2x + 21$, which corresponds to multiplying out $(3x - 7)(2x - 3)(x + 1)$.

B. $a \in [0, 8], b \in [-31, -26], c \in [43, 48], \text{ and } d \in [-21, -18]$

$6x^3 - 29x^2 + 44x - 21$, which corresponds to multiplying out $(3x - 7)(2x - 3)(x - 1)$.

C. $a \in [0, 8], b \in [-2, 12], c \in [-29, -22], \text{ and } d \in [-21, -18]$

$6x^3 + x^2 - 26x - 21$, which corresponds to multiplying out $(3x - 7)(2x + 3)(x + 1)$.

D. $a \in [0, 8], b \in [26, 34], c \in [43, 48],$ and $d \in [-21, -18]$

$6x^3 + 29x^2 + 44x - 21$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [0, 8], b \in [26, 34], c \in [43, 48],$ and $d \in [20, 27]$

* $6x^3 + 29x^2 + 44x + 21$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 7)(2x + 3)(x + 1)$

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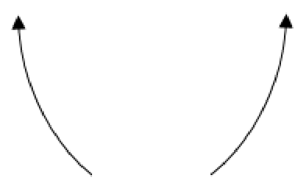
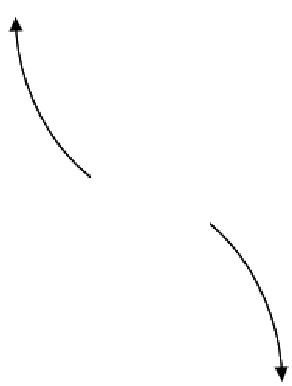
3. Describe the end behavior of the polynomial below.

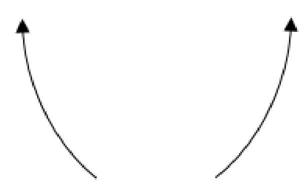
$$f(x) = 8(x + 3)^5(x - 3)^{10}(x + 9)^2(x - 9)^3$$

The solution is the graph below, which is option C.

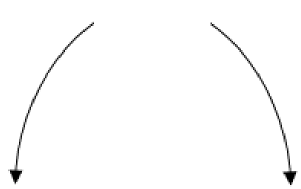




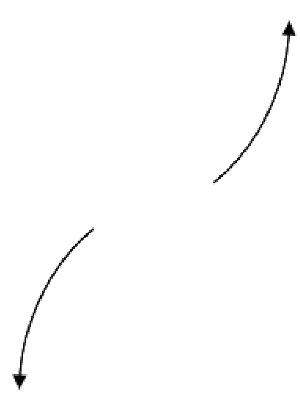
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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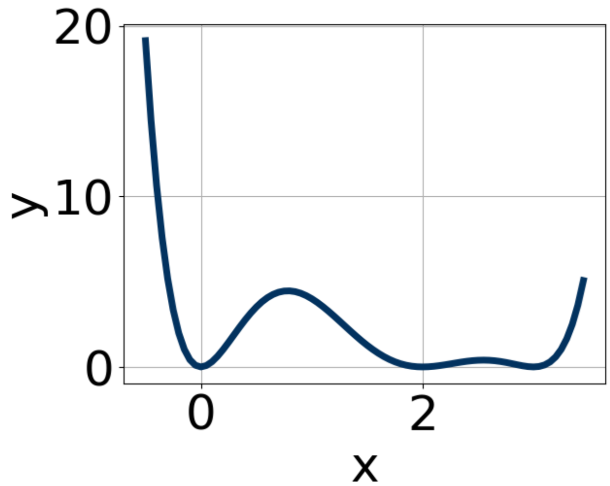
4. Which of the following equations *could* be of the graph presented below?



The solution is $20x^8(x-3)^4(x-2)^6$, which is option C.

A. $-6x^{10}(x-3)^8(x-2)^{11}$

The factor $(x-2)$ should have an even power and the leading coefficient should be the opposite sign.

B. $-12x^8(x-3)^{10}(x-2)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $20x^8(x-3)^4(x-2)^6$

* This is the correct option.

D. $19x^{10}(x-3)^{10}(x-2)^5$

The factor $(x-2)$ should have an even power.

E. $17x^7(x-3)^8(x-2)^7$

The factors x and $(x-2)$ should both have even powers.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form ax^3+bx^2+cx+d.

$$\frac{-3}{4}, 4, \text{ and } \frac{4}{3}$$

The solution is $12x^3-55x^2+16x+48$, which is option C.

A. $a \in [6,19], b \in [54,56], c \in [13,20], \text{ and } d \in [-52,-44]$

$12x^3+55x^2+16x-48$, which corresponds to multiplying out $(4x-3)(x+4)(3x+4)$.

B. $a \in [6,19], b \in [-58,-53], c \in [13,20], \text{ and } d \in [-52,-44]$

$12x^3-55x^2+16x-48$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [6, 19], b \in [-58, -53], c \in [13, 20]$, and $d \in [47, 52]$

* $12x^3 - 55x^2 + 16x + 48$, which is the correct option.

D. $a \in [6, 19], b \in [22, 29], c \in [-90, -84]$, and $d \in [47, 52]$

$12x^3 + 23x^2 - 88x + 48$, which corresponds to multiplying out $(4x - 3)(x + 4)(3x - 4)$.

E. $a \in [6, 19], b \in [-77, -65], c \in [111, 121]$, and $d \in [-52, -44]$

$12x^3 - 73x^2 + 112x - 48$, which corresponds to multiplying out $(4x - 3)(x - 4)(3x - 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 3)(x - 4)(3x - 4)$

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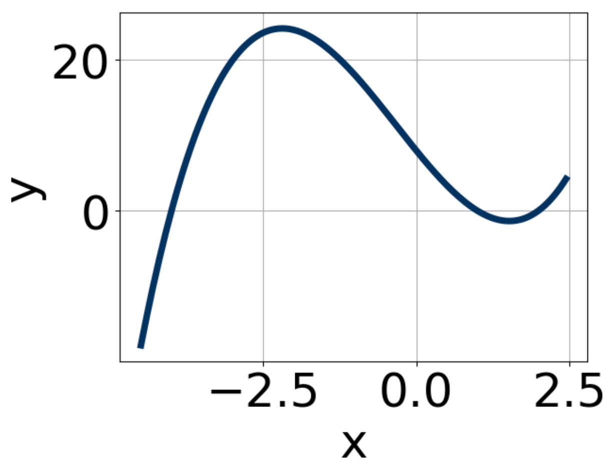
6. Which of the following equations *could* be of the graph presented below?



The solution is $11(x - 1)^7(x - 2)^9(x + 4)^7$, which is option C.

A. $-7(x - 1)^8(x - 2)^7(x + 4)^{11}$

The factor $(x - 1)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-17(x - 1)^5(x - 2)^9(x + 4)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $11(x - 1)^7(x - 2)^9(x + 4)^7$

* This is the correct option.

D. $20(x - 1)^{10}(x - 2)^8(x + 4)^{11}$

The factors 1 and 2 have have been odd power.

E. $18(x - 1)^6(x - 2)^{11}(x + 4)^5$

The factor 1 should have been an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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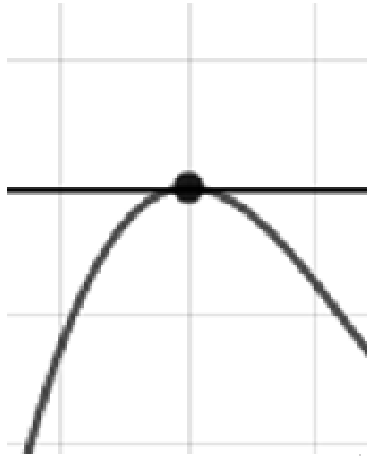
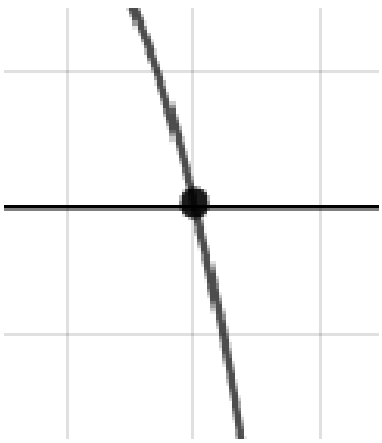
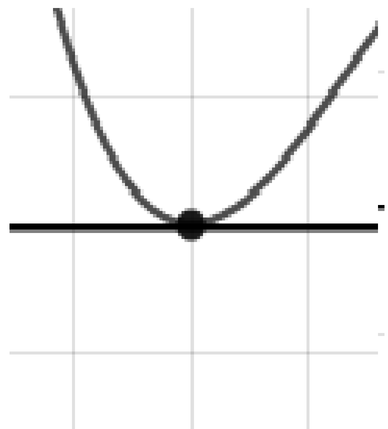
7. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

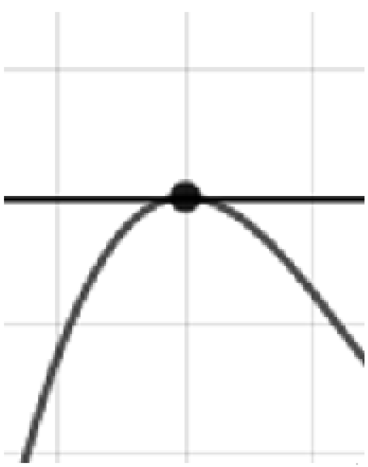
$$f(x) = 4(x + 4)^8(x - 4)^{13}(x - 8)^2(x + 8)^6$$

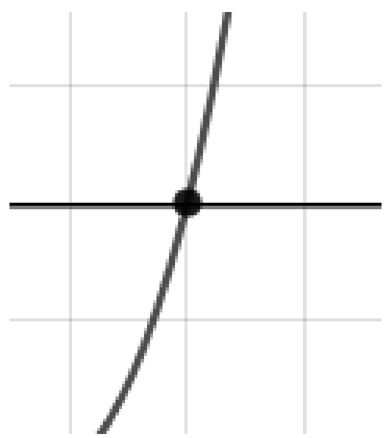
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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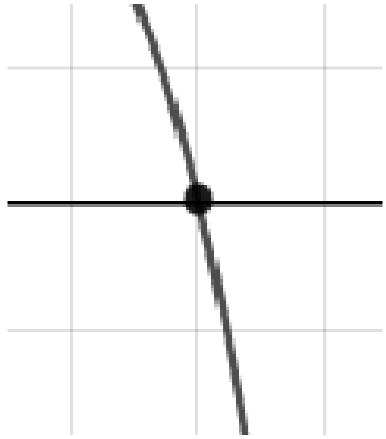
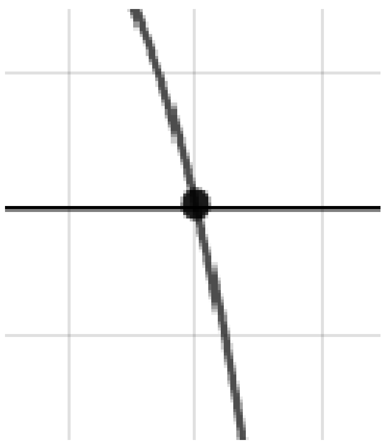
8. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

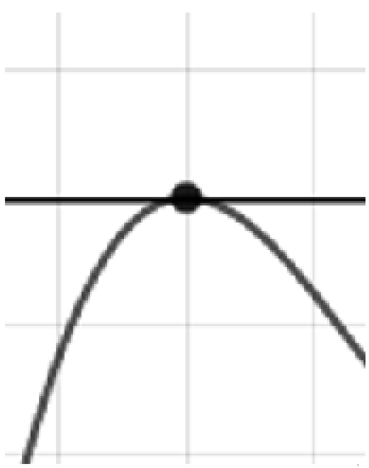
$$f(x) = -4(x-8)^{12}(x+8)^{8}(x+6)^{11}(x-6)^{8}$$

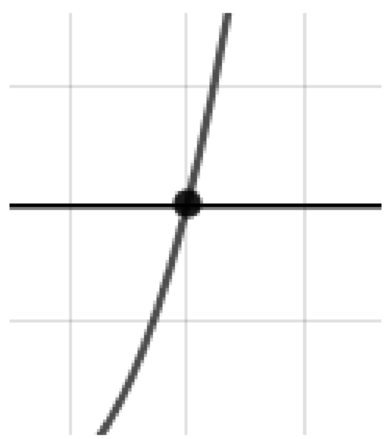
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain

the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } 4$$

The solution is $x^3 + 6x^2 + x - 164$, which is option D.

A. $b \in [-7.2, -3.1], c \in [-6, 9], \text{ and } d \in [157, 173]$

   $x^3 - 6x^2 + x + 164$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x + 4)$.

B. $b \in [-1.8, 4.3], c \in [-6, 9], \text{ and } d \in [-26, -16]$

   $x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

C. $b \in [-1.8, 4.3], c \in [-10, -5], \text{ and } d \in [16, 23]$

   $x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

D. $b \in [5.4, 7.5], c \in [-6, 9], \text{ and } d \in [-165, -163]$

   * $x^3 + 6x^2 + x - 164$, which is the correct option.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (4))$.

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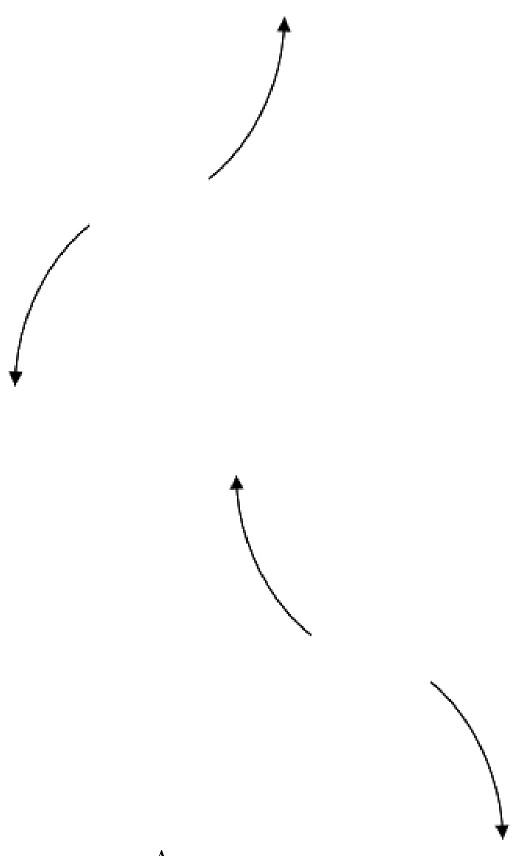
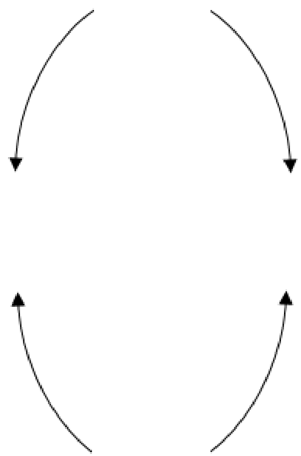
10. Describe the end behavior of the polynomial below.

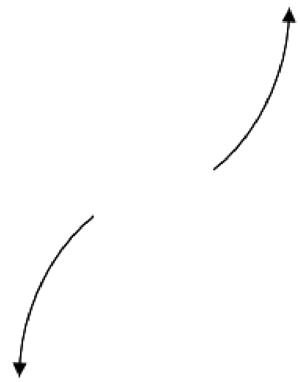
$$f(x) = 4(x - 9)^3(x + 9)^8(x - 8)^4(x + 8)^4$$

The solution is the graph below, which is option D.



B. 

C.

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.