This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{4}{3}, \frac{2}{3}, \text{ and } \frac{3}{5}$$

The solution is $45x^3 - 117x^2 + 94x - 24$, which is option A.

A. $a \in [43, 49], b \in [-121, -109], c \in [94, 102], \text{ and } d \in [-30, -23]$

* $45x^3 - 117x^2 + 94x - 24$, which is the correct option.

B. $a \in [43, 49], b \in [115, 126], c \in [94, 102], \text{ and } d \in [24, 25]$

$45x^3 + 117x^2 + 94x + 24$, which corresponds to multiplying out $(3x + 4)(3x + 2)(5x + 3)$.

C. $a \in [43, 49], b \in [2, 5], c \in [-61, -52], \text{ and } d \in [24, 25]$

$45x^3 + 3x^2 - 58x + 24$, which corresponds to multiplying out $(3x + 4)(3x - 2)(5x - 3)$.

D. $a \in [43, 49], b \in [63, 65], c \in [-21, -6], \text{ and } d \in [-30, -23]$

$45x^3 + 63x^2 - 14x - 24$, which corresponds to multiplying out $(3x + 4)(3x + 2)(5x - 3)$.

E. $a \in [43, 49], b \in [-121, -109], c \in [94, 102], \text{ and } d \in [24, 25]$

$45x^3 - 117x^2 + 94x + 24$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 4)(3x - 2)(5x - 3)$

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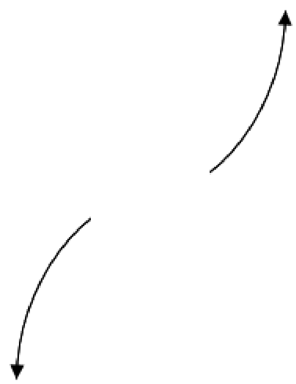
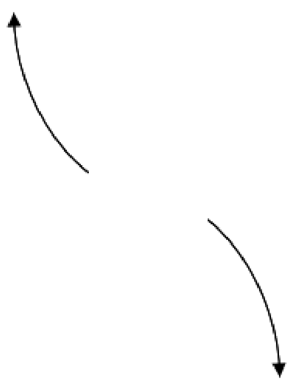
2. Describe the end behavior of the polynomial below.

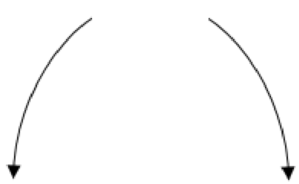
$$f(x) = 8(x - 9)^5(x + 9)^{10}(x - 3)^3(x + 3)^5$$

The solution is the graph below, which is option D.

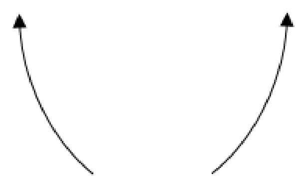




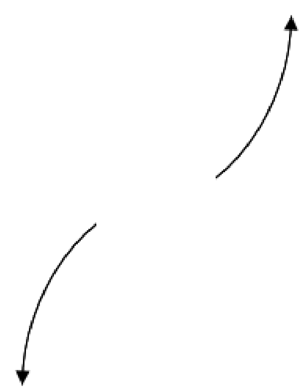
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 + 4i \text{ and } 1$$

The solution is $x^3 - 5x^2 + 24x - 20$, which is option C.

A. $b \in [4.8, 7.3], c \in [22.72, 24.73],$ and $d \in [18.8, 23.2]$

$x^3 + 5x^2 + 24x + 20$, which corresponds to multiplying out $(x - (2 + 4i))(x - (2 - 4i))(x + 1)$.

B. $b \in [-0.5, 1.6], c \in [-4.03, -2.68],$ and $d \in [0.5, 2.3]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x-2)(x-1)$.

C. $b \in [-8.6, -2.1], c \in [22.72, 24.73],$ and $d \in [-21.3, -19.8]$

* $x^3 - 5x^2 + 24x - 20$, which is the correct option.

D. $b \in [-0.5, 1.6], c \in [-5.22, -3.99],$ and $d \in [2.7, 7]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x-4)(x-1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 + 4i))(x - (2 - 4i))(x - (1))$.

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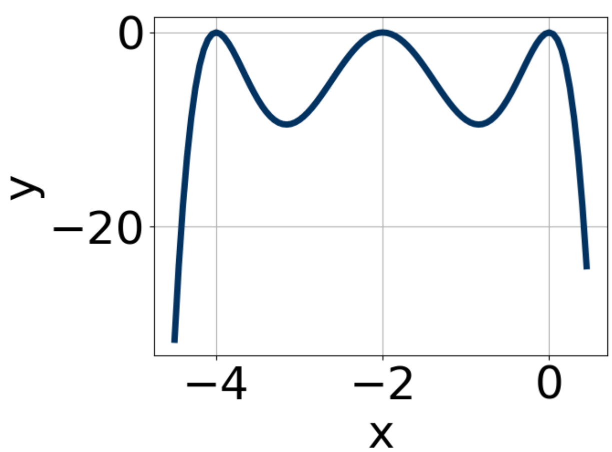
4. Which of the following equations *could* be of the graph presented below?



The solution is $-3x^4(x+2)^8(x+4)^8$, which is option D.

A. $18x^5(x+2)^8(x+4)^4$

The factor x should have an even power and the leading coefficient should be the opposite sign.

B. $9x^{10}(x+2)^8(x+4)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-6x^5(x+2)^8(x+4)^6$

The factor x should have an even power.

D. $-3x^4(x+2)^8(x+4)^8$

* This is the correct option.

E. $-14x^5(x+2)^6(x+4)^5$

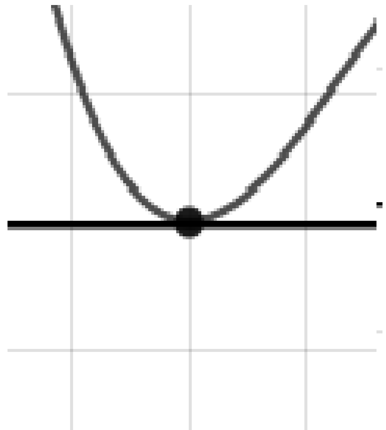
The factors $(x+4)$ and x should both have even powers.

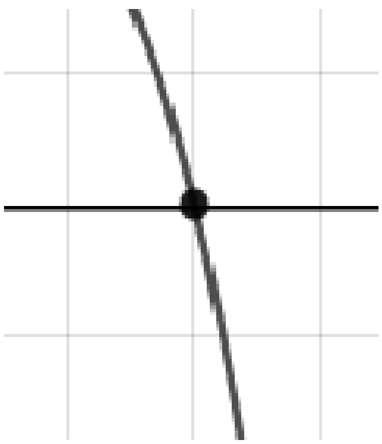
General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

5. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

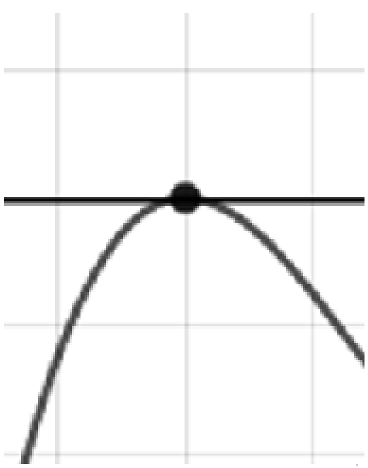
$$f(x) = 9(x-3)^{11}(x+3)^{8}(x-9)^{10}(x+9)^{9}$$

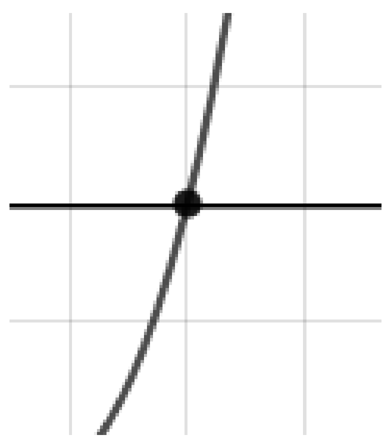
The solution is the graph below, which is option C.



A. 

C. 

B. 

D. 

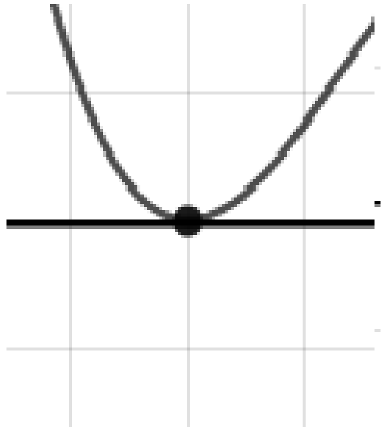
E. None of the above.

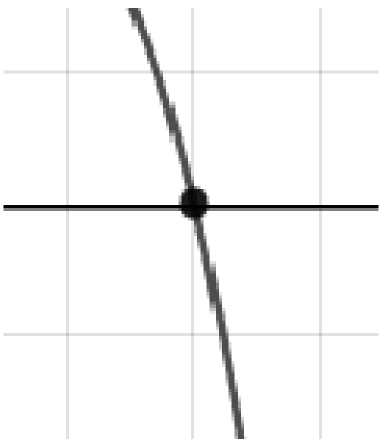
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

6. Describe the zero behavior of the zero $x = 4$ of the polynomial below.

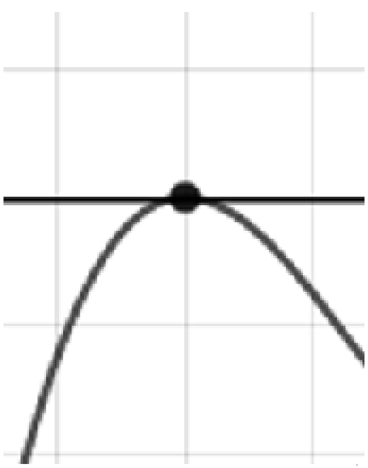
$$f(x) = 5(x+6)^5(x-6)^4(x+4)^9(x-4)^6$$

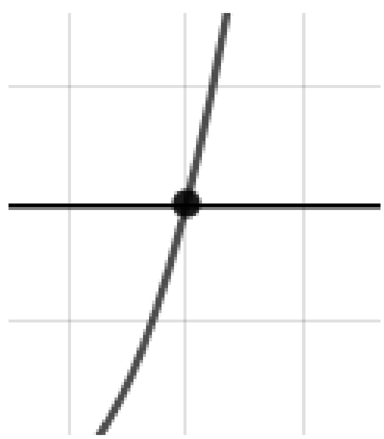
The solution is the graph below, which is option C.



A. 

C. 

B. 

D. 

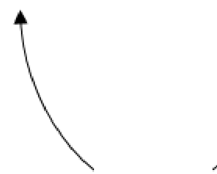
E. None of the above.

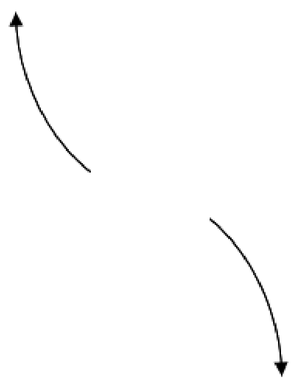
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

7. Describe the end behavior of the polynomial below.

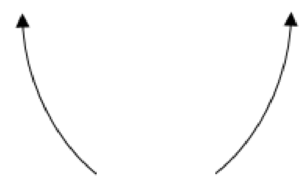
$$f(x) = 2(x+4)^3(x-4)^8(x-5)^5(x+5)^6$$

The solution is the graph below, which is option C.

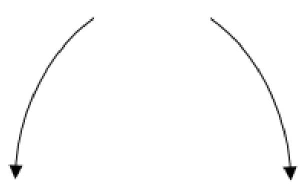




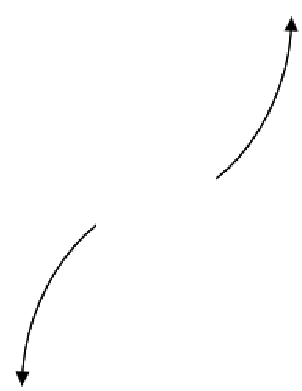
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 - 3i \text{ and } 1$$

The solution is $x^3 + 7x^2 + 17x - 25$, which is option C.

A. $b \in [-2.3, 1.9], c \in [1.81, 2.56], \text{ and } d \in [-3.42, -2.96]$

$x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x+3)(x-1)$.

B. $b \in [-8.3, -5.2], c \in [15.94, 17.54],$ and $d \in [24.44, 26.01]$

$x^3 - 7x^2 + 17x + 25$, which corresponds to multiplying out $(x - (-4 - 3i))(x - (-4 + 3i))(x + 1)$.

C. $b \in [4.9, 8.8], c \in [15.94, 17.54],$ and $d \in [-26.86, -23.36]$

* $x^3 + 7x^2 + 17x - 25$, which is the correct option.

D. $b \in [-2.3, 1.9], c \in [2.49, 3.29],$ and $d \in [-4.03, -3.82]$

$x^3 + x^2 + 3x - 4$, which corresponds to multiplying out $(x + 4)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 - 3i))(x - (-4 + 3i))(x - (1))$.

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{4}, \frac{7}{2}, \text{ and } \frac{-3}{5}$$

The solution is $40x^3 - 46x^2 - 287x - 147$, which is option A.

A. $a \in [33, 45], b \in [-46, -44], c \in [-295, -277],$ and $d \in [-147, -143]$

* $40x^3 - 46x^2 - 287x - 147$, which is the correct option.

B. $a \in [33, 45], b \in [90, 98], c \in [-205, -195],$ and $d \in [-147, -143]$

$40x^3 + 94x^2 - 203x - 147$, which corresponds to multiplying out $(4x - 7)(2x + 7)(5x + 3)$.

C. $a \in [33, 45], b \in [35, 50], c \in [-295, -277],$ and $d \in [146, 151]$

$40x^3 + 46x^2 - 287x + 147$, which corresponds to multiplying out $(4x - 7)(2x + 7)(5x - 3)$.

D. $a \in [33, 45], b \in [-186, -184], c \in [111, 127],$ and $d \in [146, 151]$

$40x^3 - 186x^2 + 119x + 147$, which corresponds to multiplying out $(4x - 7)(2x - 7)(5x + 3)$.

E. $a \in [33, 45], b \in [-46, -44], c \in [-295, -277],$ and $d \in [146, 151]$

$40x^3 - 46x^2 - 287x + 147$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 7)(2x - 7)(5x + 3)$

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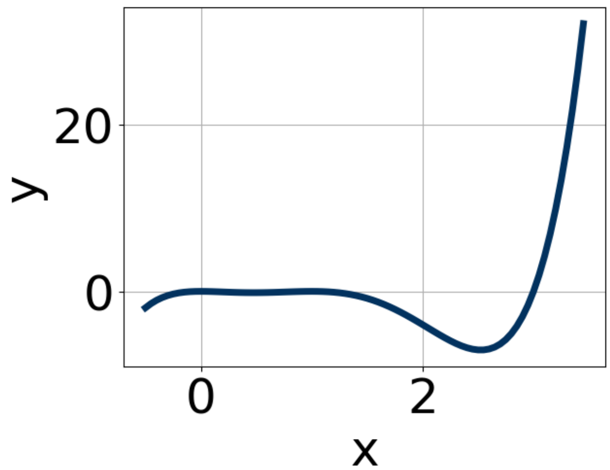
10. Which of the following equations *could* be of the graph presented below?



The solution is $9x^4(x-1)^{10}(x-3)^9$, which is option C.

A. $6x^9(x-1)^4(x-3)^5$

The factor x should have an even power.

B. $19x^{11}(x-1)^8(x-3)^6$

The factor x should have an even power and the factor $(x-3)$ should have an odd power.

C. $9x^4(x-1)^{10}(x-3)^9$

* This is the correct option.

D. $-8x^{10}(x-1)^8(x-3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-11x^4(x-1)^8(x-3)^8$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).