

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

If you have a suggestion to make the keys better, please fill out the short survey [here](#).

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$3 - 2i \text{ and } -3$$

The solution is $x^3 - 3x^2 - 5x + 39$, which is option D.

- A. $b \in [1.1, 4], c \in [-5.1, -3.6]$, and $d \in [-40, -32]$

$$x^3 + 3x^2 - 5x - 39, \text{ which corresponds to multiplying out } (x - (3 - 2i))(x - (3 + 2i))(x - 3).$$

- B. $b \in [-0.4, 2], c \in [-2.9, 0.1]$, and $d \in [-9, -4]$

$$x^3 + x^2 - 9, \text{ which corresponds to multiplying out } (x - 3)(x + 3).$$

- C. $b \in [-0.4, 2], c \in [3.2, 8.7]$, and $d \in [4, 7]$

$$x^3 + x^2 + 5x + 6, \text{ which corresponds to multiplying out } (x + 2)(x + 3).$$

- D. $b \in [-6.7, -1], c \in [-5.1, -3.6]$, and $d \in [36, 41]$

$$* x^3 - 3x^2 - 5x + 39, \text{ which is the correct option.}$$

- E. None of the above.

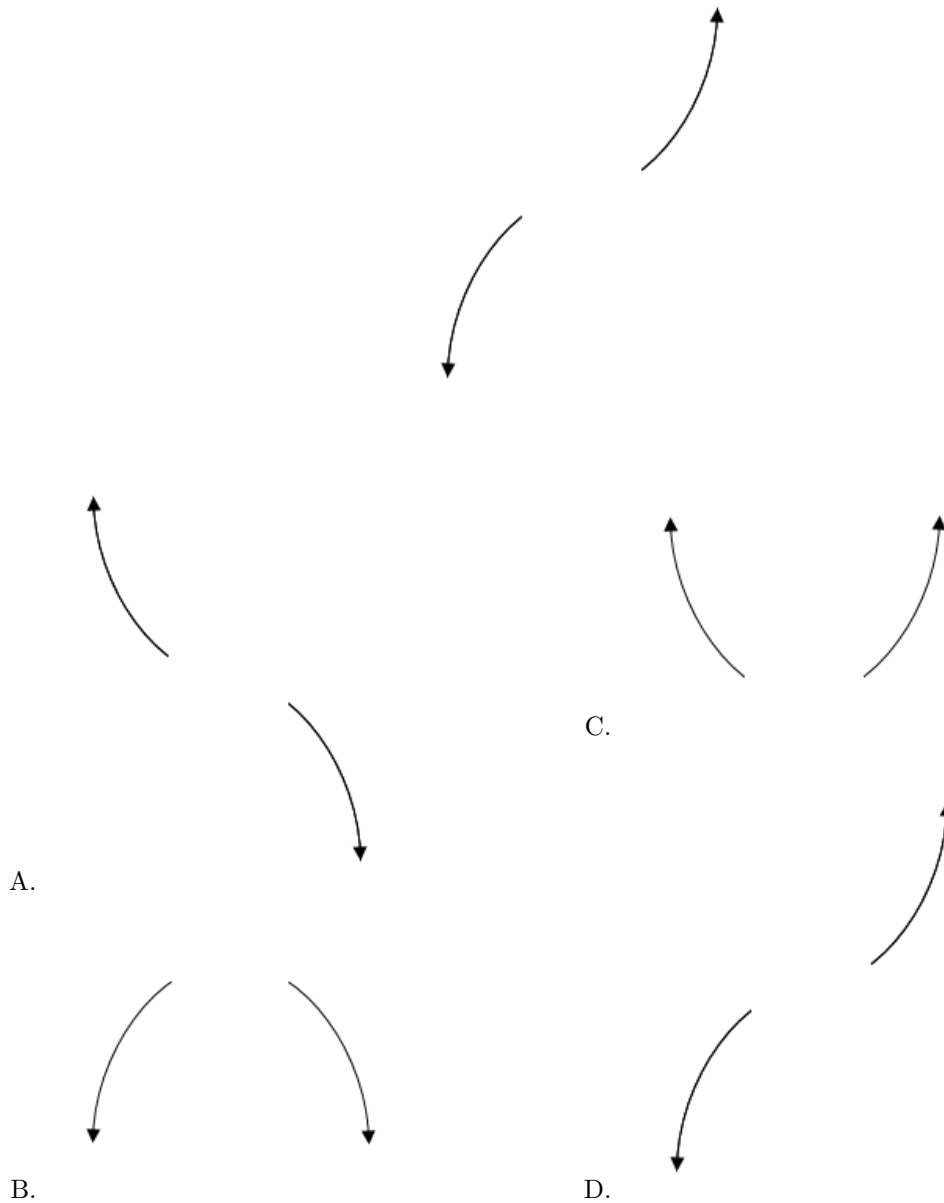
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (3 - 2i))(x - (3 + 2i))(x - (-3))$.

2. Describe the end behavior of the polynomial below.

$$f(x) = 2(x - 9)^3(x + 9)^8(x - 7)^3(x + 7)^5$$

The solution is the graph below, which is option D.



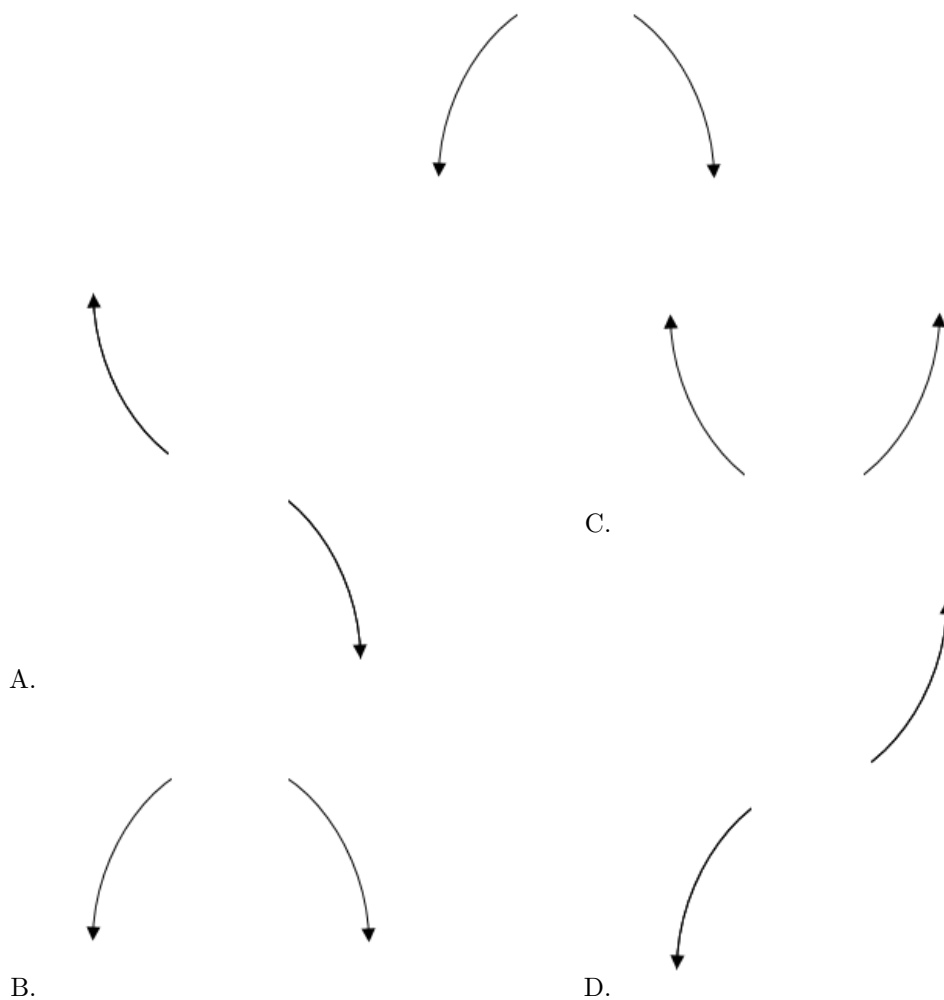
E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

3. Describe the end behavior of the polynomial below.

$$f(x) = -4(x - 2)^5(x + 2)^{10}(x - 3)^5(x + 3)^6$$

The solution is the graph below, which is option B.



General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-6, \frac{-3}{4}, \text{ and } \frac{7}{2}$$

The solution is $8x^3 + 26x^2 - 153x - 126$, which is option C.

- A. $a \in [3, 10], b \in [-75, -66], c \in [108, 118]$, and $d \in [125, 128]$

$8x^3 - 70x^2 + 111x + 126$, which corresponds to multiplying out $(x - 6)(4x + 3)(2x - 7)$.

- B. $a \in [3, 10], b \in [-26, -24], c \in [-154, -145]$, and $d \in [125, 128]$

$8x^3 - 26x^2 - 153x + 126$, which corresponds to multiplying out $(x - 6)(4x - 3)(2x + 7)$.

- C. $a \in [3, 10], b \in [23, 33], c \in [-154, -145]$, and $d \in [-130, -119]$

* $8x^3 + 26x^2 - 153x - 126$, which is the correct option.

D. $a \in [3, 10]$, $b \in [-89, -77]$, $c \in [222, 233]$, and $d \in [-130, -119]$

$8x^3 - 82x^2 + 225x - 126$, which corresponds to multiplying out $(x - 6)(4x - 3)(2x - 7)$.

E. $a \in [3, 10]$, $b \in [23, 33]$, $c \in [-154, -145]$, and $d \in [125, 128]$

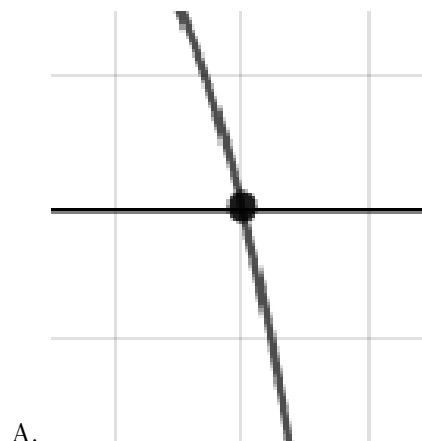
$8x^3 + 26x^2 - 153x + 126$, which corresponds to multiplying everything correctly except the constant term.

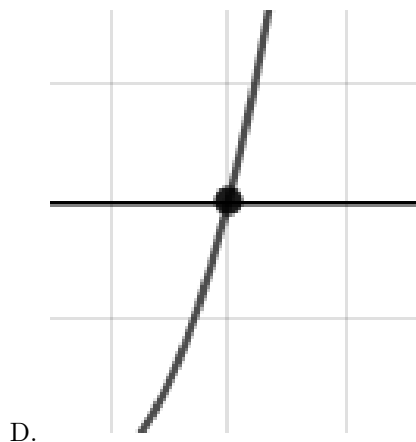
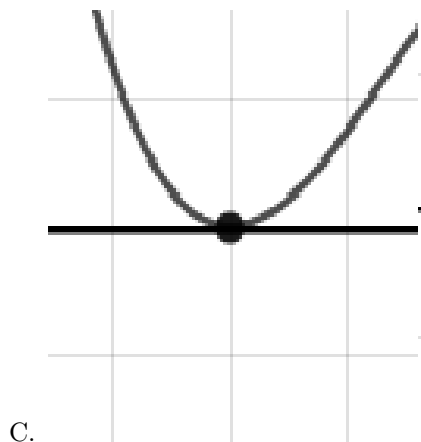
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+6)(4x+3)(2x-7)$

5. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = 6(x - 3)^4(x + 3)^9(x + 7)^4(x - 7)^8$$

The solution is the graph below, which is option C.

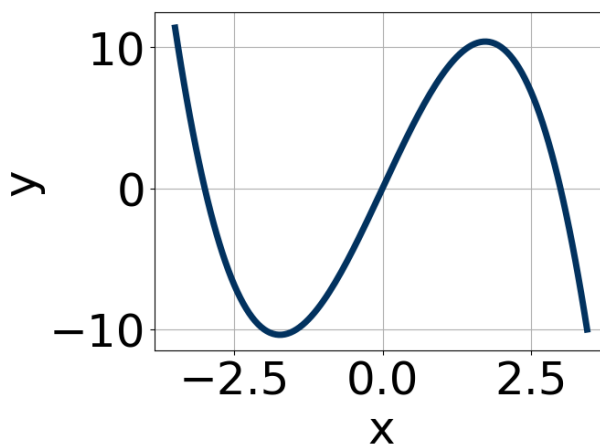




E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

6. Which of the following equations *could* be of the graph presented below?



The solution is $-14x^{11}(x-3)^5(x+3)^9$, which is option C.

A. $5x^5(x-3)^{10}(x+3)^5$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-7x^9(x-3)^4(x+3)^9$

The factor 3 should have been an odd power.

C. $-14x^{11}(x-3)^5(x+3)^9$

* This is the correct option.

D. $-18x^9(x-3)^4(x+3)^8$

The factors 3 and -3 have have been odd power.

E. $17x^7(x-3)^5(x+3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 - 5i \text{ and } 3$$

The solution is $x^3 + x^2 + 17x - 87$, which is option A.

- A. $b \in [0.2, 3.8]$, $c \in [16.8, 19.7]$, and $d \in [-92, -81]$

* $x^3 + x^2 + 17x - 87$, which is the correct option.

- B. $b \in [0.2, 3.8]$, $c \in [-3.5, 0.3]$, and $d \in [-9, -3]$

$x^3 + x^2 - x - 6$, which corresponds to multiplying out $(x + 2)(x - 3)$.

- C. $b \in [-4.5, 0.5]$, $c \in [16.8, 19.7]$, and $d \in [86, 92]$

$x^3 - 1x^2 + 17x + 87$, which corresponds to multiplying out $(x - (-2 - 5i))(x - (-2 + 5i))(x + 3)$.

- D. $b \in [0.2, 3.8]$, $c \in [1.8, 4.3]$, and $d \in [-18, -11]$

$x^3 + x^2 + 2x - 15$, which corresponds to multiplying out $(x + 5)(x - 3)$.

- E. None of the above.

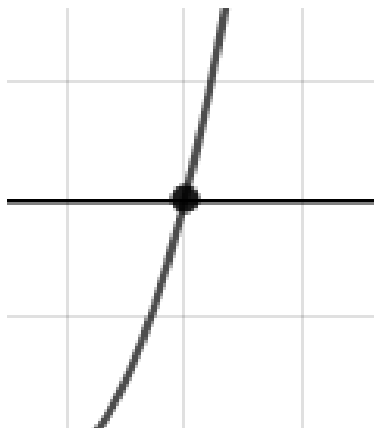
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

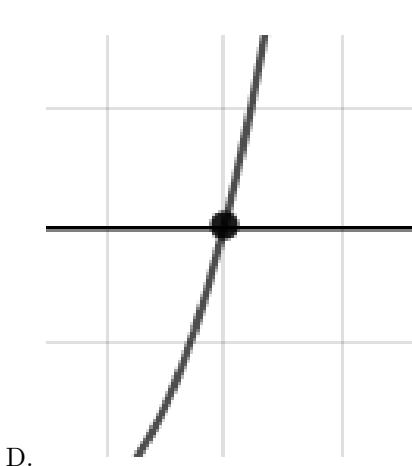
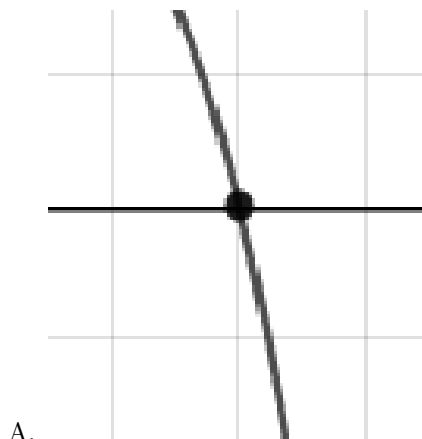
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 - 5i))(x - (-2 + 5i))(x - (3))$.

8. Describe the zero behavior of the zero $x = 5$ of the polynomial below.

$$f(x) = -9(x - 6)^{11}(x + 6)^9(x - 5)^7(x + 5)^6$$

The solution is the graph below, which is option D.





E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{3}, 7, \text{ and } \frac{-7}{5}$$

The solution is $15x^3 - 109x^2 - 7x + 245$, which is option B.

A. $a \in [13, 24], b \in [143, 153], c \in [348, 358], \text{ and } d \in [239, 253]$

$15x^3 + 151x^2 + 357x + 245$, which corresponds to multiplying out $(3x + 5)(x + 7)(5x + 7)$.

B. $a \in [13, 24], b \in [-110, -101], c \in [-10, -4], \text{ and } d \in [239, 253]$

* $15x^3 - 109x^2 - 7x + 245$, which is the correct option.

C. $a \in [13, 24], b \in [-110, -101], c \in [-10, -4], \text{ and } d \in [-247, -238]$

$15x^3 - 109x^2 - 7x - 245$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [13, 24], b \in [106, 114], c \in [-10, -4], \text{ and } d \in [-247, -238]$

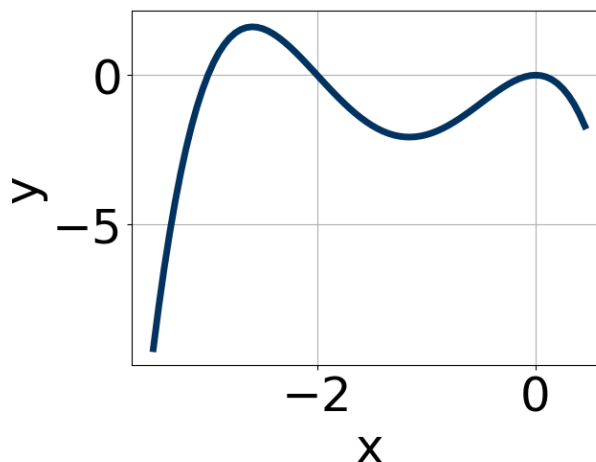
$15x^3 + 109x^2 - 7x - 245$, which corresponds to multiplying out $(3x + 5)(x + 7)(5x - 7)$.

E. $a \in [13, 24]$, $b \in [-60, -56]$, $c \in [-287, -277]$, and $d \in [-247, -238]$

$15x^3 - 59x^2 - 287x - 245$, which corresponds to multiplying out $(3x + 5)(x - 7)(5x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x - 5)(x - 7)(5x + 7)$

10. Which of the following equations *could* be of the graph presented below?



The solution is $-18x^6(x + 3)^{11}(x + 2)^9$, which is option E.

A. $9x^4(x + 3)^7(x + 2)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-20x^{10}(x + 3)^{10}(x + 2)^{11}$

The factor $(x + 3)$ should have an odd power.

C. $14x^4(x + 3)^9(x + 2)^8$

The factor $(x + 2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-13x^9(x + 3)^6(x + 2)^9$

The factor 0 should have an even power and the factor -3 should have an odd power.

E. $-18x^6(x + 3)^{11}(x + 2)^9$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

11. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 3i \text{ and } 2$$

The solution is $x^3 - 12x^2 + 54x - 68$, which is option C.

A. $b \in [-9, 6]$, $c \in [0, 6]$, and $d \in [-9, 1]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x + 3)(x - 2)$.

B. $b \in [10, 13]$, $c \in [52, 62]$, and $d \in [68, 76]$

$x^3 + 12x^2 + 54x + 68$, which corresponds to multiplying out $(x - (5 - 3i))(x - (5 + 3i))(x + 2)$.

C. $b \in [-14, -11]$, $c \in [52, 62]$, and $d \in [-76, -62]$

* $x^3 - 12x^2 + 54x - 68$, which is the correct option.

D. $b \in [-9, 6]$, $c \in [-13, -1]$, and $d \in [4, 16]$

$x^3 + x^2 - 7x + 10$, which corresponds to multiplying out $(x - 5)(x - 2)$.

E. None of the above.

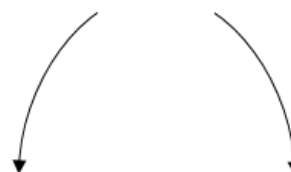
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 - 3i))(x - (5 + 3i))(x - (2))$.

12. Describe the end behavior of the polynomial below.

$$f(x) = 8(x + 3)^3(x - 3)^8(x - 2)^3(x + 2)^4$$

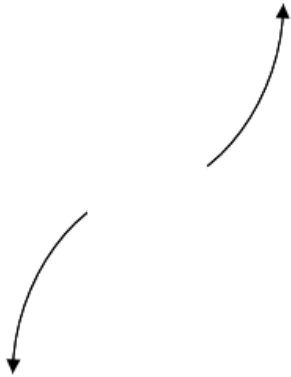
The solution is the graph below, which is option C.



B.

A.

C.



D.

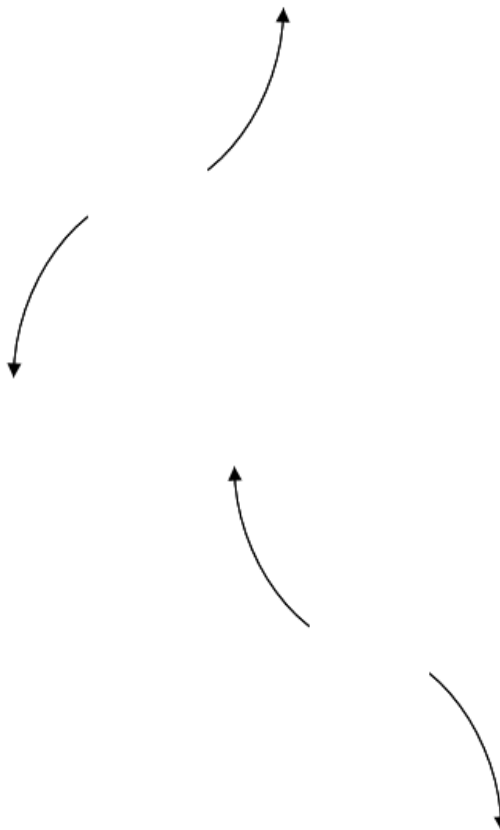
E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

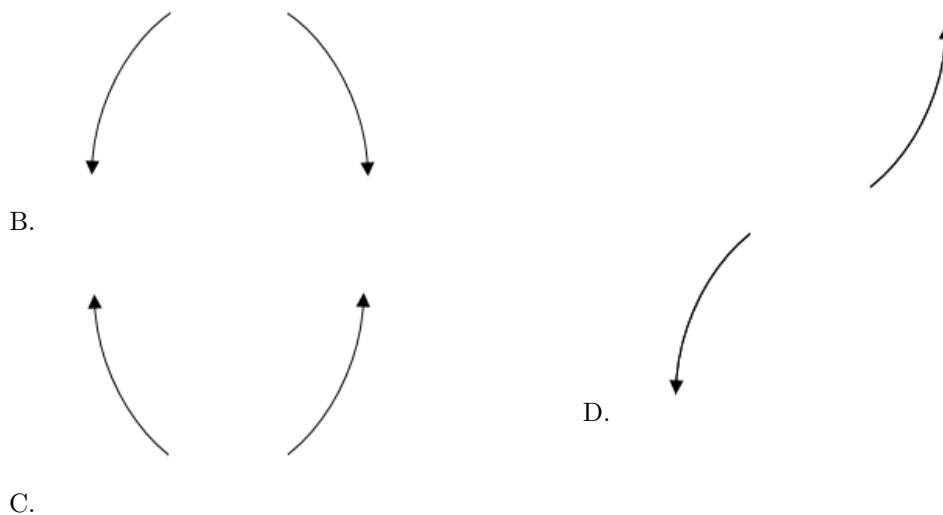
13. Describe the end behavior of the polynomial below.

$$f(x) = 7(x - 4)^4(x + 4)^5(x + 3)^3(x - 3)^5$$

The solution is the graph below, which is option D.



A.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

14. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$7, \frac{-1}{5}, \text{ and } \frac{2}{3}$$

The solution is $15x^3 - 112x^2 + 47x + 14$, which is option C.

- A. $a \in [15, 17], b \in [110.6, 112.3], c \in [44, 57], \text{ and } d \in [-17, -12]$

$15x^3 + 112x^2 + 47x - 14$, which corresponds to multiplying out $(x + 7)(5x - 1)(3x + 2)$.

- B. $a \in [15, 17], b \in [91.8, 95.9], c \in [-92, -82], \text{ and } d \in [14, 19]$

$15x^3 + 92x^2 - 89x + 14$, which corresponds to multiplying out $(x + 7)(5x - 1)(3x - 2)$.

- C. $a \in [15, 17], b \in [-112.5, -108.4], c \in [44, 57], \text{ and } d \in [14, 19]$

* $15x^3 - 112x^2 + 47x + 14$, which is the correct option.

- D. $a \in [15, 17], b \in [96, 100.8], c \in [-52, -44], \text{ and } d \in [-17, -12]$

$15x^3 + 98x^2 - 51x - 14$, which corresponds to multiplying out $(x + 7)(5x + 1)(3x - 2)$.

- E. $a \in [15, 17], b \in [-112.5, -108.4], c \in [44, 57], \text{ and } d \in [-17, -12]$

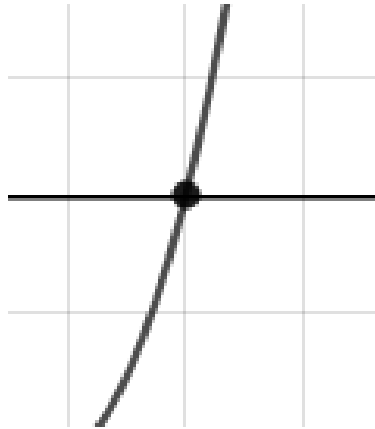
$15x^3 - 112x^2 + 47x - 14$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x - 7)(5x + 1)(3x - 2)$

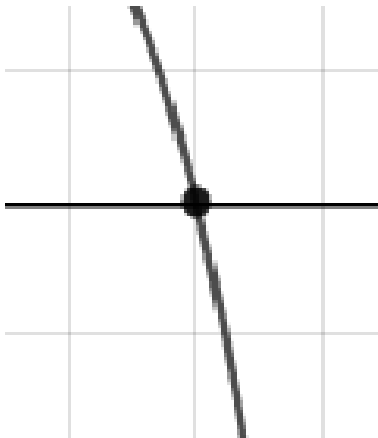
15. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

$$f(x) = -3(x + 2)^4(x - 2)^5(x - 7)^5(x + 7)^7$$

The solution is the graph below, which is option D.



A.



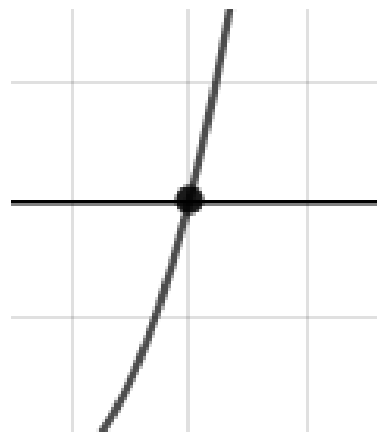
C.



B.



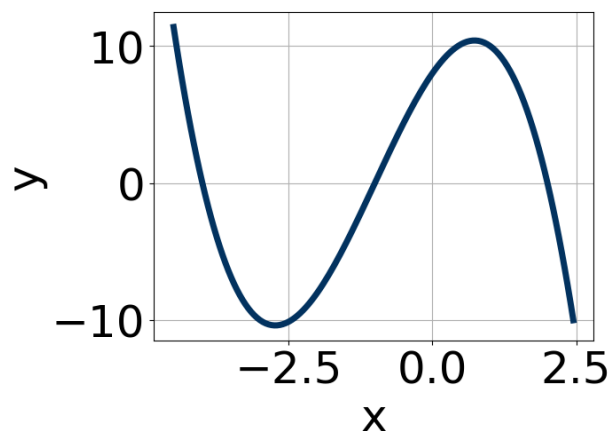
D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

16. Which of the following equations *could* be of the graph presented below?



The solution is $-10(x-2)^5(x+4)^7(x+1)^{11}$, which is option E.

A. $-8(x-2)^{10}(x+4)^6(x+1)^5$

The factors 2 and -4 have have been odd power.

B. $12(x-2)^8(x+4)^7(x+1)^7$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-4(x-2)^{10}(x+4)^{11}(x+1)^{11}$

The factor 2 should have been an odd power.

D. $11(x-2)^7(x+4)^9(x+1)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-10(x-2)^5(x+4)^7(x+1)^{11}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

17. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 5i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 49x - 41$, which is option A.

A. $b \in [-11, -5], c \in [48.79, 49.11]$, and $d \in [-41.09, -39.64]$

* $x^3 - 9x^2 + 49x - 41$, which is the correct option.

B. $b \in [1, 6], c \in [-5.16, -3.28]$, and $d \in [2.13, 4.62]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x-4)(x-1)$.

C. $b \in [1, 6], c \in [-6.36, -5.54]$, and $d \in [4.44, 5.18]$

$x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x-5)(x-1)$.

D. $b \in [3, 14], c \in [48.79, 49.11]$, and $d \in [39.48, 43]$

$x^3 + 9x^2 + 49x + 41$, which corresponds to multiplying out $(x - (4 + 5i))(x - (4 - 5i))(x + 1)$.

E. None of the above.

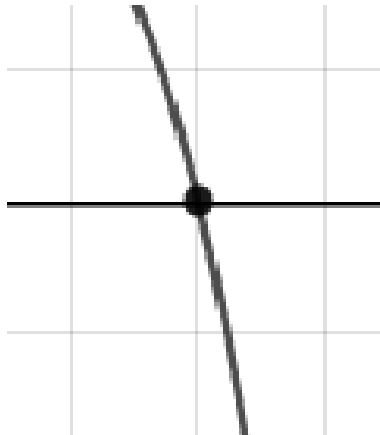
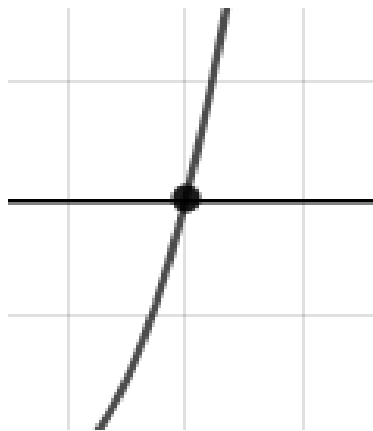
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 + 5i))(x - (4 - 5i))(x - (1))$.

18. Describe the zero behavior of the zero $x = -7$ of the polynomial below.

$$f(x) = -9(x - 4)^5(x + 4)^2(x + 7)^{11}(x - 7)^8$$

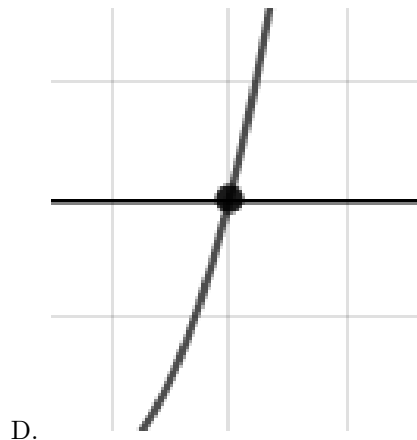
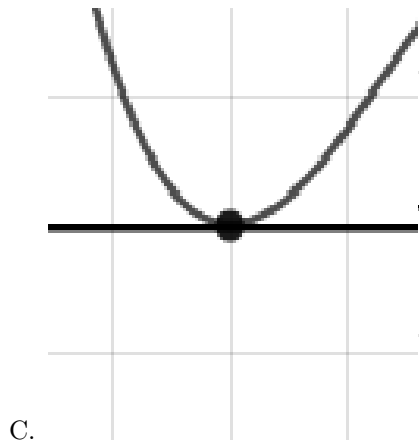
The solution is the graph below, which is option D.



A.



B.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

19. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-1, \frac{-4}{5}, \text{ and } \frac{3}{5}$$

The solution is $25x^3 + 30x^2 - 7x - 12$, which is option D.

- A. $a \in [19, 32], b \in [28, 33], c \in [-10, -3]$, and $d \in [12, 19]$

$25x^3 + 30x^2 - 7x + 12$, which corresponds to multiplying everything correctly except the constant term.

- B. $a \in [19, 32], b \in [-26, -18], c \in [-20, -16]$, and $d \in [12, 19]$

$25x^3 - 20x^2 - 17x + 12$, which corresponds to multiplying out $(x - 1)(5x + 4)(5x - 3)$.

- C. $a \in [19, 32], b \in [-64, -56], c \in [45, 49]$, and $d \in [-12, -9]$

$25x^3 - 60x^2 + 47x - 12$, which corresponds to multiplying out $(x - 1)(5x - 4)(5x - 3)$.

- D. $a \in [19, 32], b \in [28, 33], c \in [-10, -3]$, and $d \in [-12, -9]$

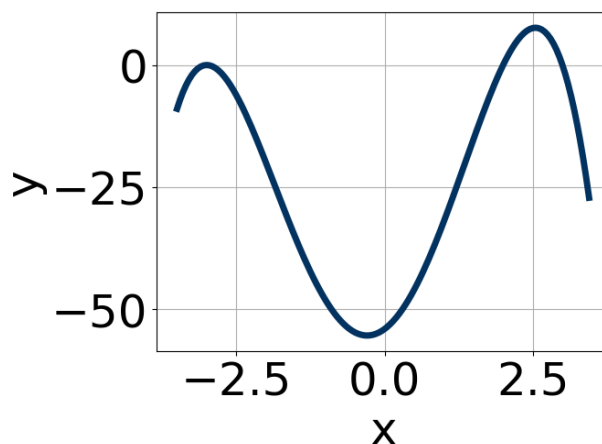
* $25x^3 + 30x^2 - 7x - 12$, which is the correct option.

- E. $a \in [19, 32], b \in [-32, -26], c \in [-10, -3]$, and $d \in [12, 19]$

$25x^3 - 30x^2 - 7x + 12$, which corresponds to multiplying out $(x - 1)(5x - 4)(5x + 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+1)(5x+4)(5x-3)$

20. Which of the following equations *could* be of the graph presented below?



The solution is $-5(x+3)^6(x-2)^5(x-3)^9$, which is option E.

A. $12(x+3)^8(x-2)^{11}(x-3)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-19(x+3)^6(x-2)^{10}(x-3)^{11}$

The factor $(x-2)$ should have an odd power.

C. $-20(x+3)^9(x-2)^6(x-3)^9$

The factor -3 should have an even power and the factor 2 should have an odd power.

D. $6(x+3)^4(x-2)^{11}(x-3)^4$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-5(x+3)^6(x-2)^5(x-3)^9$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

21. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 2i \text{ and } 1$$

The solution is $x^3 - 11x^2 + 39x - 29$, which is option A.

A. $b \in [-18, -7], c \in [35.4, 40.7], \text{ and } d \in [-30.8, -28.8]$

* $x^3 - 11x^2 + 39x - 29$, which is the correct option.

B. $b \in [-6, 7], c \in [-10.4, -5.5], \text{ and } d \in [2.6, 6.3]$

$x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x-5)(x-1)$.

C. $b \in [-6, 7], c \in [-4.6, -2.6], \text{ and } d \in [-4.2, 2.7]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x-2)(x-1)$.

D. $b \in [10, 12], c \in [35.4, 40.7], \text{ and } d \in [24.9, 29.1]$

$x^3 + 11x^2 + 39x + 29$, which corresponds to multiplying out $(x - (5 + 2i))(x - (5 - 2i))(x + 1)$.

E. None of the above.

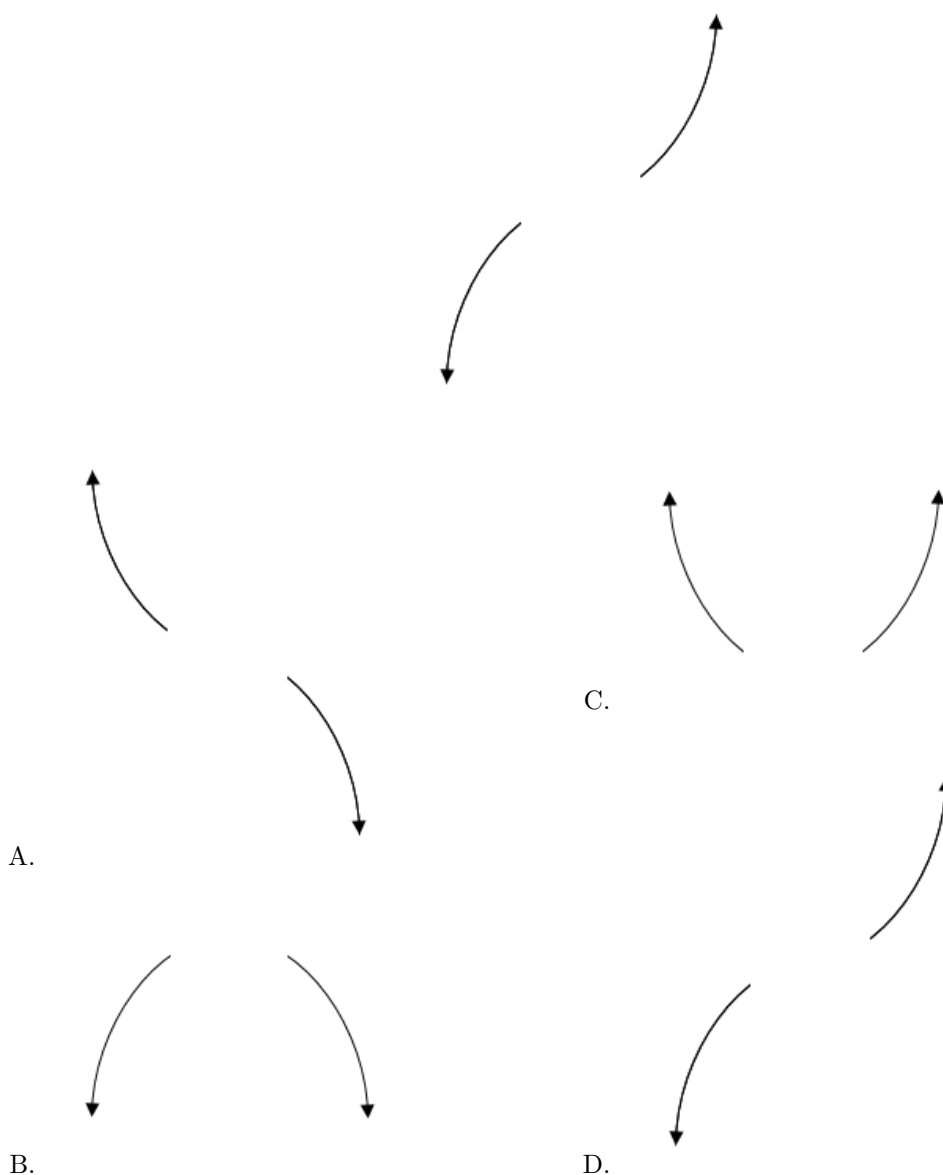
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 2i))(x - (5 - 2i))(x - (1))$.

22. Describe the end behavior of the polynomial below.

$$f(x) = 7(x - 4)^3(x + 4)^4(x - 8)^2(x + 8)^2$$

The solution is the graph below, which is option D.



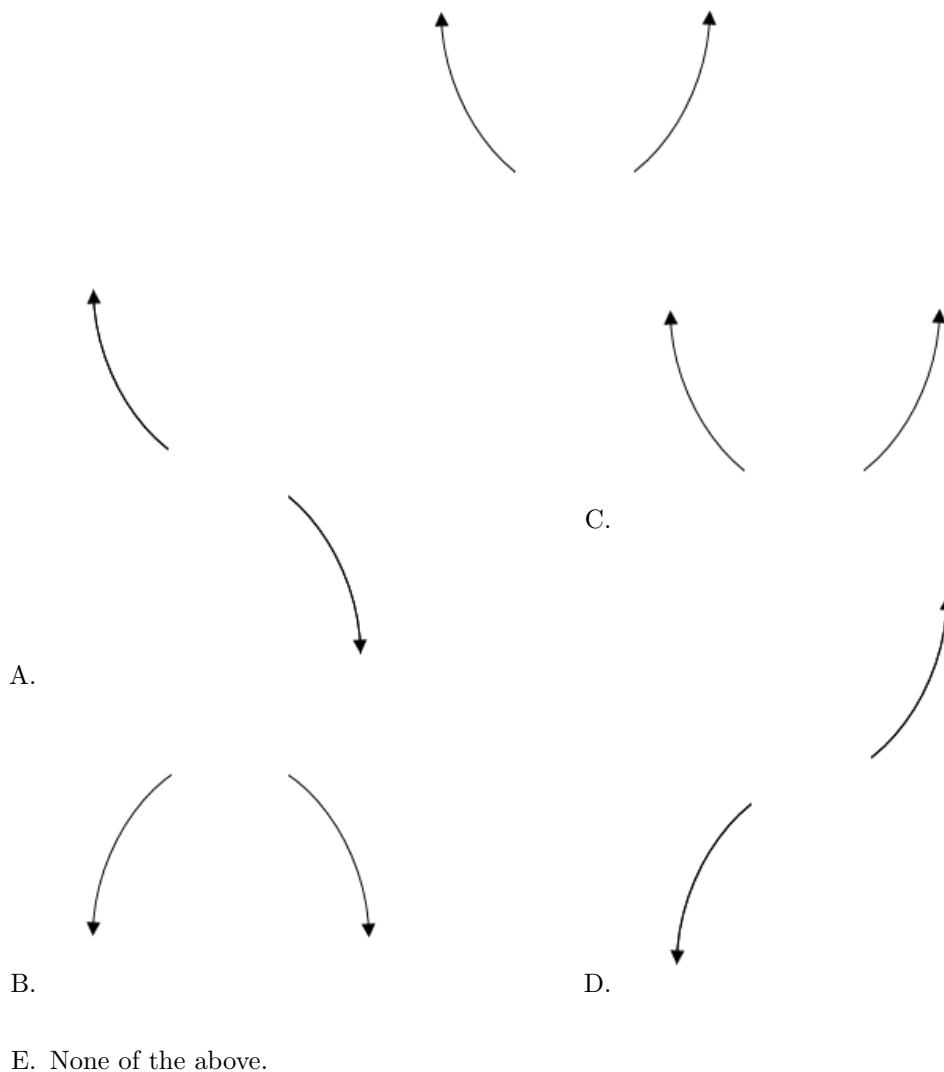
E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

23. Describe the end behavior of the polynomial below.

$$f(x) = 4(x + 3)^2(x - 3)^7(x + 8)^5(x - 8)^6$$

The solution is the graph below, which is option C.



General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

24. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{4}, -7, \text{ and } \frac{-1}{3}$$

The solution is $12x^3 + 97x^2 + 94x + 21$, which is option A.

A. $a \in [12, 14], b \in [94, 98], c \in [90, 105]$, and $d \in [20, 26]$

* $12x^3 + 97x^2 + 94x + 21$, which is the correct option.

B. $a \in [12, 14], b \in [-99, -94], c \in [90, 105]$, and $d \in [-26, -20]$

$12x^3 - 97x^2 + 94x - 21$, which corresponds to multiplying out $(4x - 3)(x - 7)(3x - 1)$.

C. $a \in [12, 14], b \in [-93, -88], c \in [31, 33]$, and $d \in [20, 26]$

$12x^3 - 89x^2 + 32x + 21$, which corresponds to multiplying out $(4x - 3)(x - 7)(3x + 1)$.

D. $a \in [12, 14], b \in [78, 86], c \in [-43, -37]$, and $d \in [-26, -20]$

$12x^3 + 79x^2 - 38x - 21$, which corresponds to multiplying out $(4x - 3)(x + 7)(3x + 1)$.

E. $a \in [12, 14], b \in [94, 98], c \in [90, 105]$, and $d \in [-26, -20]$

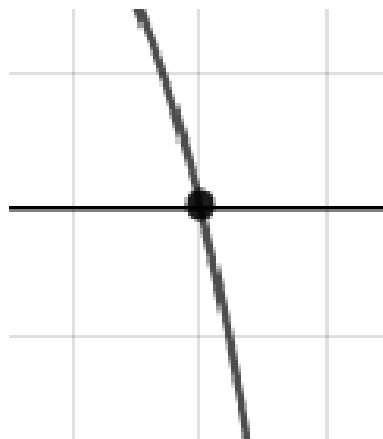
$12x^3 + 97x^2 + 94x - 21$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 3)(x + 7)(3x + 1)$

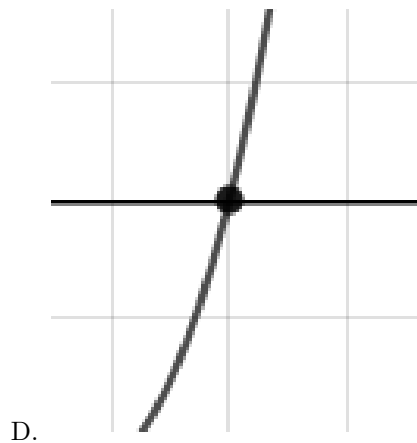
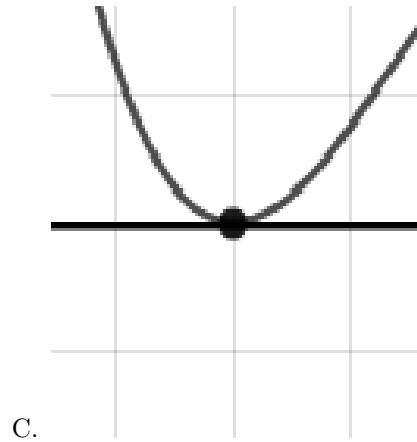
25. Describe the zero behavior of the zero $x = -9$ of the polynomial below.

$$f(x) = 2(x - 4)^{10}(x + 4)^6(x + 9)^{10}(x - 9)^7$$

The solution is the graph below, which is option B.



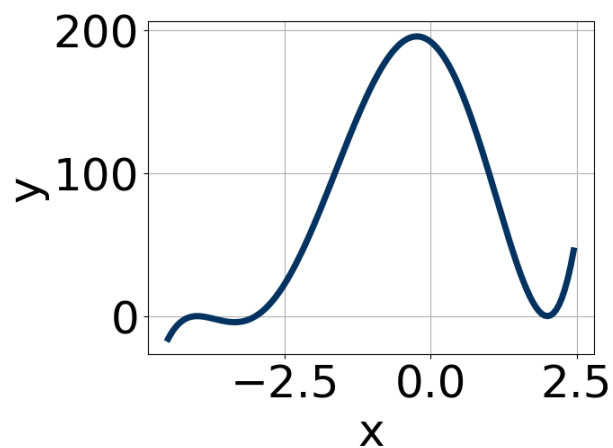
A.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

26. Which of the following equations *could* be of the graph presented below?



The solution is $6(x + 4)^4(x - 2)^8(x + 3)^7$, which is option E.

A. $15(x + 4)^6(x - 2)^7(x + 3)^5$

The factor $(x - 2)$ should have an even power.

B. $-6(x + 4)^{10}(x - 2)^6(x + 3)^8$

The factor $(x + 3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $17(x + 4)^{10}(x - 2)^7(x + 3)^{10}$

The factor $(x - 2)$ should have an even power and the factor $(x + 3)$ should have an odd power.

D. $-19(x + 4)^8(x - 2)^8(x + 3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $6(x + 4)^4(x - 2)^8(x + 3)^7$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

27. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$2 + 3i \text{ and } 1$$

The solution is $x^3 - 5x^2 + 17x - 13$, which is option B.

A. $b \in [-1.8, 1.9], c \in [-4.15, -3.21], \text{ and } d \in [2.99, 3.53]$

$x^3 + x^2 - 4x + 3$, which corresponds to multiplying out $(x - 3)(x - 1)$.

B. $b \in [-9.1, -3.5], c \in [16.78, 18.65], \text{ and } d \in [-14.03, -11.89]$

* $x^3 - 5x^2 + 17x - 13$, which is the correct option.

C. $b \in [4.7, 5.3], c \in [16.78, 18.65], \text{ and } d \in [11.64, 13.41]$

$x^3 + 5x^2 + 17x + 13$, which corresponds to multiplying out $(x - (2 + 3i))(x - (2 - 3i))(x + 1)$.

D. $b \in [-1.8, 1.9], c \in [-3.38, -1.37], \text{ and } d \in [1.75, 2.85]$

$x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x - 2)(x - 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (2 + 3i))(x - (2 - 3i))(x - (1))$.

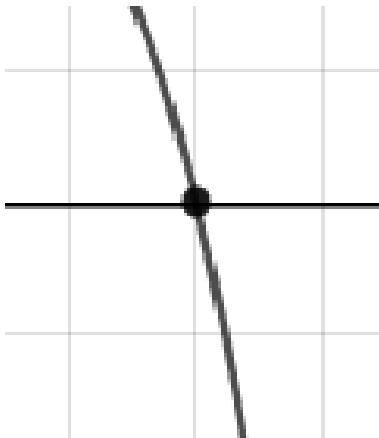
28. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

$$f(x) = 6(x - 4)^7(x + 4)^{12}(x + 3)^4(x - 3)^6$$

The solution is the graph below, which is option B.



A.



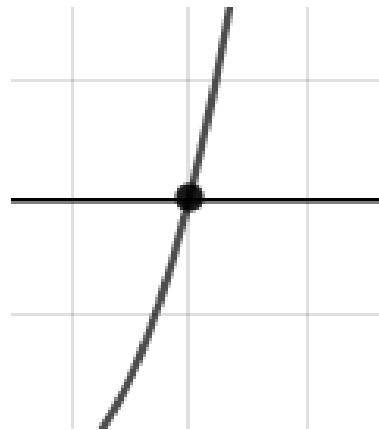
C.



B.



D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

-
29. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{2}, \frac{5}{4}, \text{ and } \frac{-1}{5}$$

The solution is $40x^3 - 62x^2 + 11x + 5$, which is option A.

A. $a \in [37, 43]$, $b \in [-63, -59]$, $c \in [10, 12]$, and $d \in [4, 9]$

* $40x^3 - 62x^2 + 11x + 5$, which is the correct option.

B. $a \in [37, 43]$, $b \in [-24, -15]$, $c \in [-38, -26]$, and $d \in [-12, -4]$

$40x^3 - 22x^2 - 31x - 5$, which corresponds to multiplying out $(2x + 1)(4x - 5)(5x + 1)$.

C. $a \in [37, 43]$, $b \in [-63, -59]$, $c \in [10, 12]$, and $d \in [-12, -4]$

$40x^3 - 62x^2 + 11x - 5$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [37, 43]$, $b \in [74, 85]$, $c \in [38, 41]$, and $d \in [4, 9]$

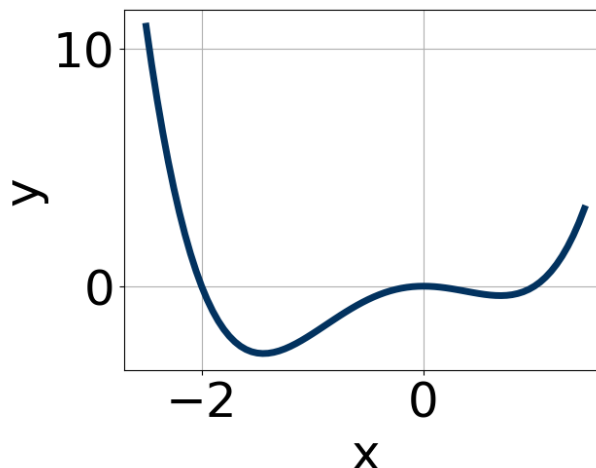
$40x^3 + 78x^2 + 39x + 5$, which corresponds to multiplying out $(2x + 1)(4x + 5)(5x + 1)$.

E. $a \in [37, 43]$, $b \in [55, 66]$, $c \in [10, 12]$, and $d \in [-12, -4]$

$40x^3 + 62x^2 + 11x - 5$, which corresponds to multiplying out $(2x + 1)(4x + 5)(5x - 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 1)(4x - 5)(5x + 1)$

30. Which of the following equations *could* be of the graph presented below?



The solution is $4x^4(x - 1)^{11}(x + 2)^{11}$, which is option D.

A. $-2x^8(x - 1)^5(x + 2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $13x^9(x - 1)^4(x + 2)^9$

The factor 0 should have an even power and the factor 1 should have an odd power.

C. $-14x^8(x - 1)^7(x + 2)^{10}$

The factor $(x + 2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $4x^4(x - 1)^{11}(x + 2)^{11}$

* This is the correct option.

E. $7x^8(x-1)^6(x+2)^7$

The factor $(x-1)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).
