**This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.**

**If you have a suggestion to make the keys better, please fill out the short survey here.**

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

---

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 - 3i \text{ and } 2$$

The solution is $x^3 - 12x^2 + 54x - 68$, which is option C.

A. $b \in [-9, 6], c \in [0, 6]$, and $d \in [-9, 1]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x + 3)(x - 2)$.

B. $b \in [10, 13], c \in [52, 62]$, and $d \in [68, 76]$

$x^3 + 12x^2 + 54x + 68$, which corresponds to multiplying out $(x - (5 - 3i))(x - (5 + 3i))(x + 2)$.

C. $b \in [-14, -11], c \in [52, 62]$, and $d \in [-76, -62]$

* $x^3 - 12x^2 + 54x - 68$, which is the correct option.

D. $b \in [-9, 6], c \in [-13, -1]$, and $d \in [4, 16]$

$x^3 + x^2 - 7x + 10$, which corresponds to multiplying out $(x - 5)(x - 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

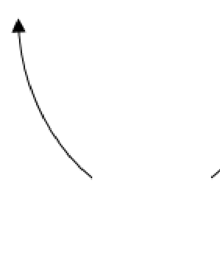
**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 - 3i))(x - (5 + 3i))(x - (2))$.

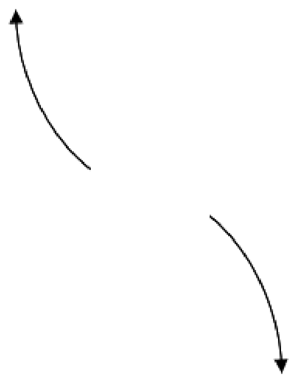
---

2. Describe the end behavior of the polynomial below.

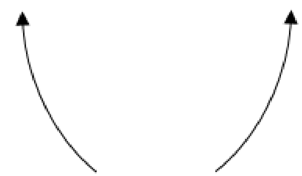
$$f(x) = 8(x + 3)^3(x - 3)^8(x - 2)^3(x + 2)^4$$

The solution is the graph below, which is option C.

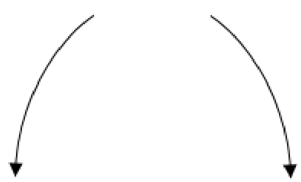




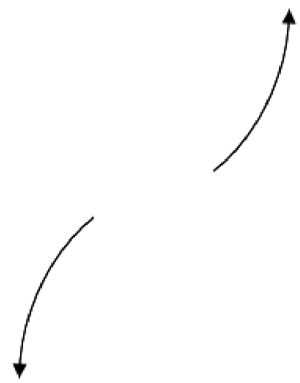
A.



C.



B.



D.

E. None of the above.

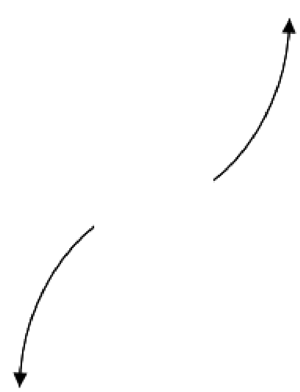
**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

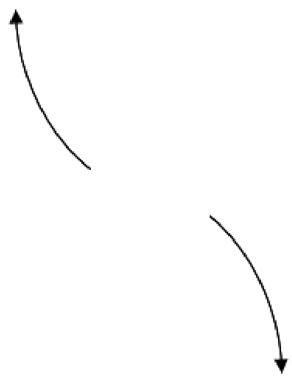
---

3. Describe the end behavior of the polynomial below.

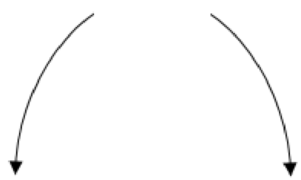
$$f(x) = 7(x-4)^4(x+4)^5(x+3)^3(x-3)^5$$

The solution is the graph below, which is option D.

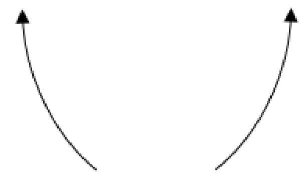




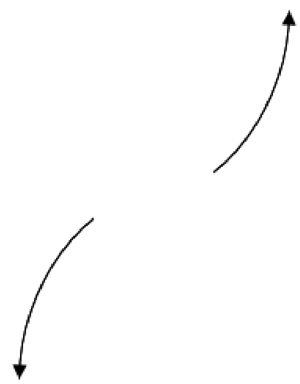
A.



B.



C.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$7, \frac{-1}{5}, \text{ and } \frac{2}{3}$$

The solution is $15x^3 - 112x^2 + 47x + 14$, which is option C.

A. $a \in [15, 17], b \in [110.6, 112.3], c \in [44, 57], \text{ and } d \in [-17, -12]$

   $15x^3 + 112x^2 + 47x - 14$, which corresponds to multiplying out $(x + 7)(5x - 1)(3x + 2)$.

B. $a \in [15, 17], b \in [91.8, 95.9], c \in [-92, -82], \text{ and } d \in [14, 19]$

   $15x^3 + 92x^2 - 89x + 14$, which corresponds to multiplying out $(x + 7)(5x - 1)(3x - 2)$.

C. $a \in [15, 17], b \in [-112.5, -108.4], c \in [44, 57], \text{ and } d \in [14, 19]$

   * $15x^3 - 112x^2 + 47x + 14$, which is the correct option.

D. $a \in [15, 17], b \in [96, 100.8], c \in [-52, -44], \text{ and } d \in [-17, -12]$

   $15x^3 + 98x^2 - 51x - 14$, which corresponds to multiplying out $(x + 7)(5x + 1)(3x - 2)$.

E. $a \in [15, 17], b \in [-112.5, -108.4], c \in [44, 57], \text{ and } d \in [-17, -12]$

   $15x^3 - 112x^2 + 47x - 14$, which corresponds to multiplying everything correctly except the constant term.

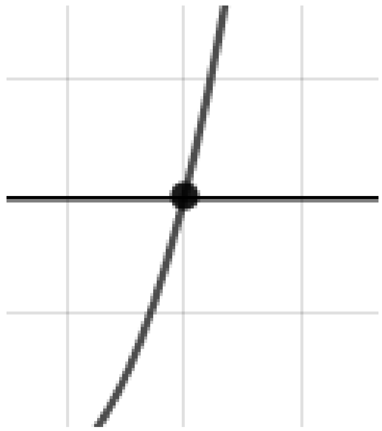
**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(x-7)(5x+1)(3x-2)$

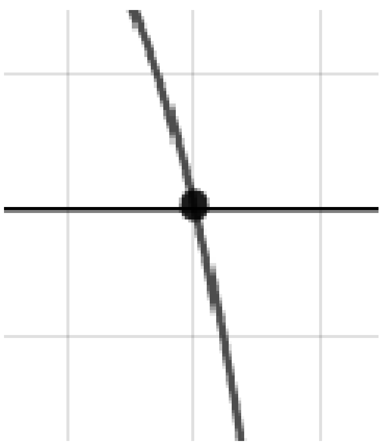
---

5. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

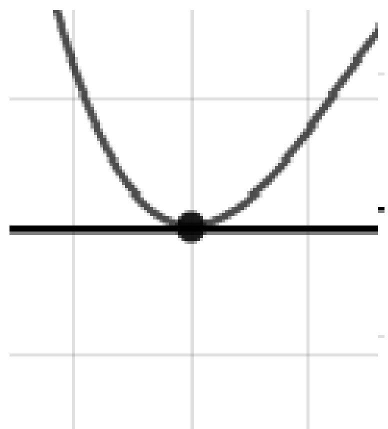
$$f(x) = -3(x+2)^4(x-2)^5(x-7)^5(x+7)^7$$

The solution is the graph below, which is option D.

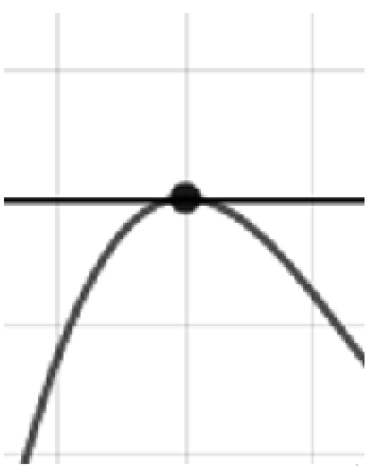




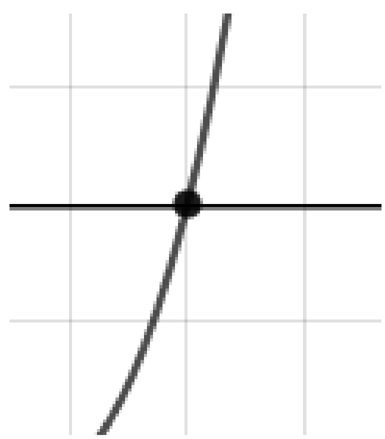
A.



C.



B.



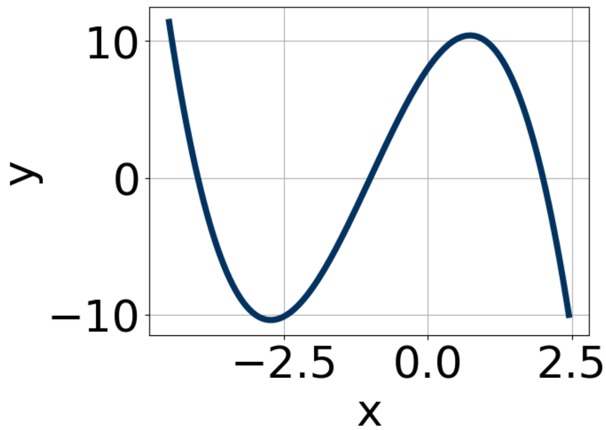
D.

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

6. Which of the following equations *could* be of the graph presented below?




The solution is $-10(x-2)^5(x+4)^7(x+1)^{11}$, which is option E.

A. $-8(x-2)^{10}(x+4)^6(x+1)^5$

The factors 2 and $-4$ have have been odd power.

B. $12(x-2)^8(x+4)^7(x+1)^7$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $-4(x-2)^{10}(x+4)^{11}(x+1)^{11}$

The factor 2 should have been an odd power.

D. $11(x-2)^7(x+4)^9(x+1)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-10(x-2)^5(x+4)^7(x+1)^{11}$

* This is the correct option.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 5i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 49x - 41$, which is option A.

A. $b \in [-11, -5], c \in [48.79, 49.11], \text{ and } d \in [-41.09, -39.64]$

* $x^3 - 9x^2 + 49x - 41$, which is the correct option.

B. $b \in [1, 6], c \in [-5.16, -3.28], \text{ and } d \in [2.13, 4.62]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x-4)(x-1)$.

C. $b \in [1, 6], c \in [-6.36, -5.54],$ and $d \in [4.44, 5.18]$

$x^3 + x^2 - 6x + 5$, which corresponds to multiplying out $(x - 5)(x - 1)$.

D. $b \in [3, 14], c \in [48.79, 49.11],$ and $d \in [39.48, 43]$

$x^3 + 9x^2 + 49x + 41$, which corresponds to multiplying out $(x - (4 + 5i))(x - (4 - 5i))(x + 1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

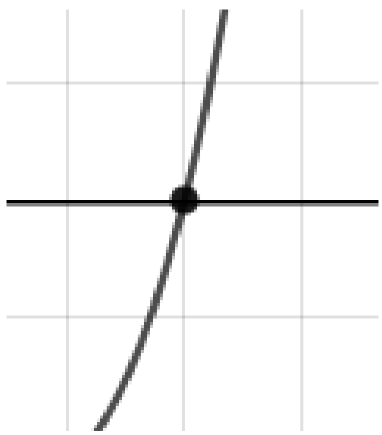
**General Comment:** Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 + 5i))(x - (4 - 5i))(x - (1))$.

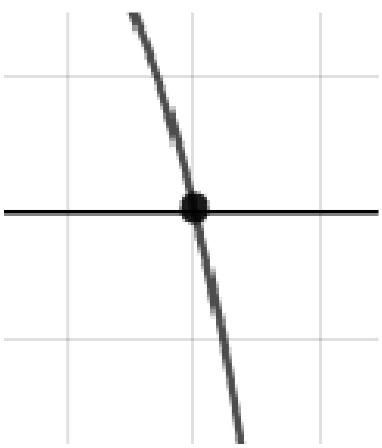
---

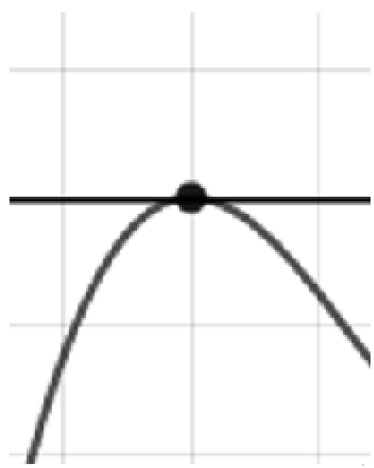
8. Describe the zero behavior of the zero $x = -7$ of the polynomial below.

$$f(x) = -9(x - 4)^5(x + 4)^2(x + 7)^{11}(x - 7)^8$$

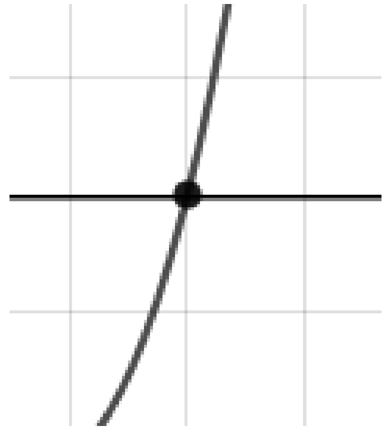
The solution is the graph below, which is option D.



A. 

B. 

C. 

D. 

E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-1, \frac{-4}{5}, \text{ and } \frac{3}{5}$$

The solution is $25x^3 + 30x^2 - 7x - 12$, which is option D.

A. $a \in [19, 32], b \in [28, 33], c \in [-10, -3], \text{ and } d \in [12, 19]$

$25x^3 + 30x^2 - 7x + 12$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [19, 32], b \in [-26, -18], c \in [-20, -16], \text{ and } d \in [12, 19]$

$25x^3 - 20x^2 - 17x + 12$, which corresponds to multiplying out $(x - 1)(5x + 4)(5x - 3)$.

C. $a \in [19, 32], b \in [-64, -56], c \in [45, 49], \text{ and } d \in [-12, -9]$

$25x^3 - 60x^2 + 47x - 12$, which corresponds to multiplying out $(x - 1)(5x - 4)(5x - 3)$.

D. $a \in [19, 32], b \in [28, 33], c \in [-10, -3], \text{ and } d \in [-12, -9]$

* $25x^3 + 30x^2 - 7x - 12$, which is the correct option.

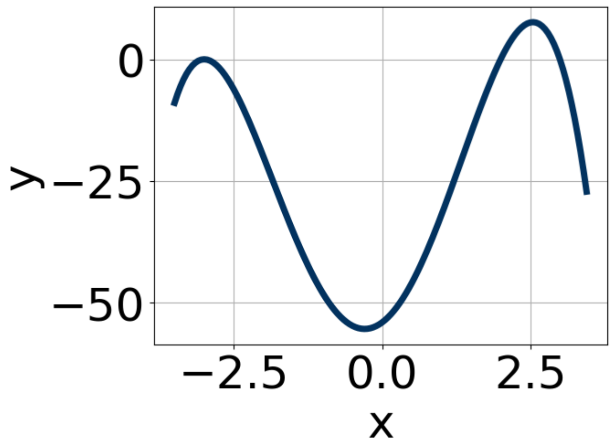
E. $a \in [19, 32], b \in [-32, -26], c \in [-10, -3], \text{ and } d \in [12, 19]$

$25x^3 - 30x^2 - 7x + 12$, which corresponds to multiplying out $(x - 1)(5x - 4)(5x + 3)$.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out $(x+1)(5x+4)(5x-3)$

---

10. Which of the following equations *could* be of the graph presented below?




The solution is $-5(x+3)^6(x-2)^5(x-3)^9$, which is option E.

A. $12(x+3)^8(x-2)^{11}(x-3)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-19(x+3)^6(x-2)^{10}(x-3)^{11}$

The factor $(x-2)$ should have an odd power.

C. $-20(x+3)^9(x-2)^6(x-3)^9$

The factor $-3$ should have an even power and the factor $2$ should have an odd power.

D. $6(x+3)^4(x-2)^{11}(x-3)^4$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $-5(x+3)^6(x-2)^5(x-3)^9$

* This is the correct option.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).