This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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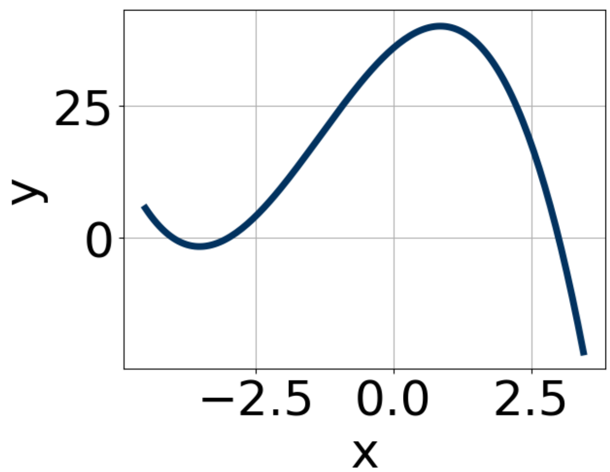
1. Which of the following equations *could* be of the graph presented below?



The solution is $-3(x-3)^7(x+4)^9(x+3)^{11}$, which is option B.

A. $-20(x-3)^6(x+4)^{11}(x+3)^5$

The factor 3 should have been an odd power.

B. $-3(x-3)^7(x+4)^9(x+3)^{11}$

* This is the correct option.

C. $6(x-3)^4(x+4)^{11}(x+3)^9$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-3(x-3)^4(x+4)^8(x+3)^9$

The factors 3 and -4 have have been odd power.

E. $18(x-3)^5(x+4)^5(x+3)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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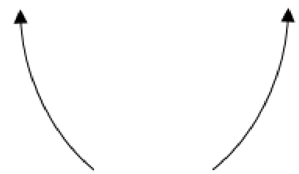
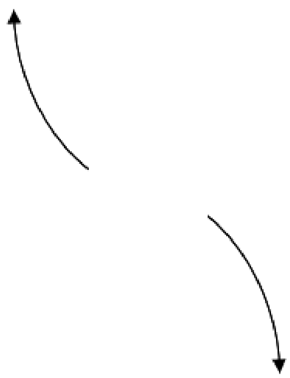
2. Describe the end behavior of the polynomial below.

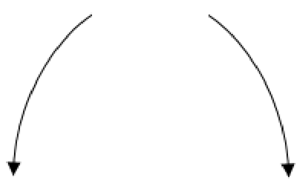
$$f(x) = 6(x+4)^5(x-4)^6(x-3)^4(x+3)^5$$

The solution is the graph below, which is option C.

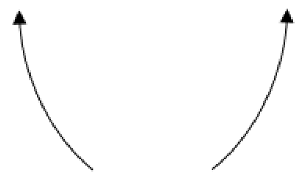




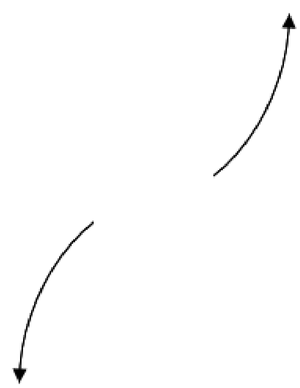
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 5i \text{ and } -3$$

The solution is $x^3 + 13x^2 + 80x + 150$, which is option B.

A. $b \in [-1, 9], c \in [-8, 1],$ and $d \in [-15, -11]$

$x^3 + x^2 - 2x - 15$, which corresponds to multiplying out $(x - 5)(x + 3)$.

B. $b \in [11, 16], c \in [80, 82],$ and $d \in [147, 158]$

* $x^3 + 13x^2 + 80x + 150$, which is the correct option.

C. $b \in [-1, 9], c \in [7, 11],$ and $d \in [11, 19]$

$x^3 + x^2 + 8x + 15$, which corresponds to multiplying out $(x + 5)(x + 3)$.

D. $b \in [-19, -12], c \in [80, 82],$ and $d \in [-158, -146]$

$x^3 - 13x^2 + 80x - 150$, which corresponds to multiplying out $(x - (-5 + 5i))(x - (-5 - 5i))(x - 3)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 5i))(x - (-5 - 5i))(x - (-3))$.

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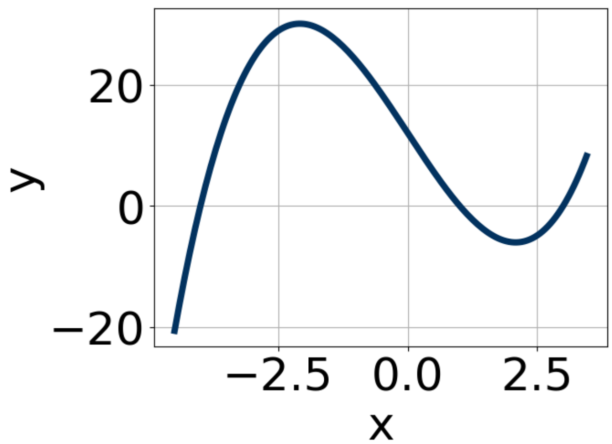
4. Which of the following equations *could* be of the graph presented below?



The solution is $9(x - 3)^7(x + 4)^{11}(x - 1)^{11}$, which is option C.

A. $3(x - 3)^4(x + 4)^8(x - 1)^5$

The factors 3 and -4 have have been odd power.

B. $-12(x - 3)^4(x + 4)^5(x - 1)^7$

The factor $(x - 3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $9(x - 3)^7(x + 4)^{11}(x - 1)^{11}$

* This is the correct option.

D. $-12(x - 3)^9(x + 4)^{11}(x - 1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $10(x - 3)^6(x + 4)^{11}(x - 1)^7$

The factor 3 should have been an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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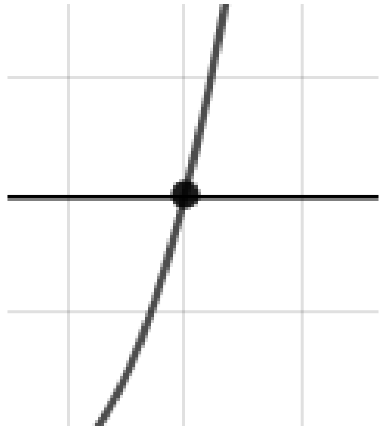
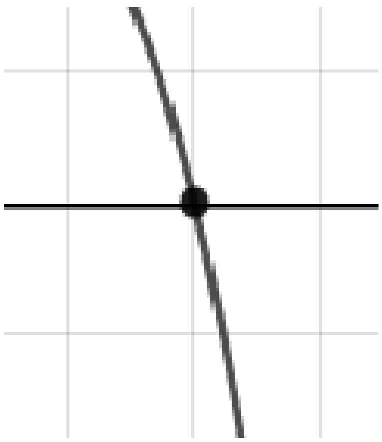
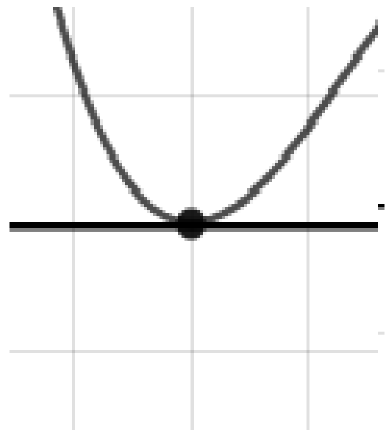
5. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

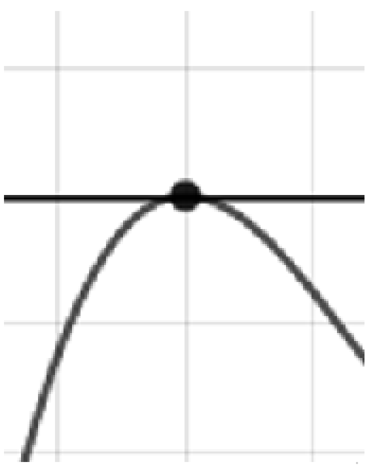
$$f(x) = 9(x + 2)^{11}(x - 2)^7(x + 8)^7(x - 8)^6$$

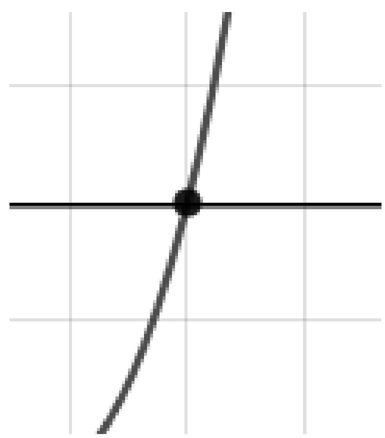
The solution is the graph below, which is option D.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{5}{2}, \frac{-1}{3}, \text{ and } \frac{-2}{3}$$

The solution is $18x^3 - 27x^2 - 41x - 10$, which is option B.

A. $a \in [17, 23], b \in [19, 28], c \in [-45, -36],$ and $d \in [8, 11]$

$18x^3 + 27x^2 - 41x + 10$, which corresponds to multiplying out $(2x + 5)(3x - 1)(3x - 2)$.

B. $a \in [17, 23], b \in [-27, -24], c \in [-45, -36],$ and $d \in [-17, -9]$

* $18x^3 - 27x^2 - 41x - 10$, which is the correct option.

C. $a \in [17, 23], b \in [50, 54], c \in [3, 12],$ and $d \in [-17, -9]$

$18x^3 + 51x^2 + 11x - 10$, which corresponds to multiplying out $(2x + 5)(3x - 1)(3x + 2)$.

D. $a \in [17, 23], b \in [58, 75], c \in [44, 53],$ and $d \in [8, 11]$

$18x^3 + 63x^2 + 49x + 10$, which corresponds to multiplying out $(2x + 5)(3x + 1)(3x + 2)$.

E. $a \in [17, 23], b \in [-27, -24], c \in [-45, -36],$ and $d \in [8, 11]$

$18x^3 - 27x^2 - 41x + 10$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 5)(3x + 1)(3x + 2)$

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 5i \text{ and } -2$$

The solution is $x^3 - 6x^2 + 25x + 82$, which is option D.

A. $b \in [1, 3], c \in [-4.5, -2.7],$ and $d \in [-10.1, -9.4]$

$x^3 + x^2 - 3x - 10$, which corresponds to multiplying out $(x - 5)(x + 2)$.

B. $b \in [1, 3], c \in [-2.53, -1.56],$ and $d \in [-8.9, -6.6]$

$x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x - 4)(x + 2)$.

C. $b \in [6, 11], c \in [22.96, 25.33],$ and $d \in [-83.9, -75.9]$

$x^3 + 6x^2 + 25x - 82$, which corresponds to multiplying out $(x - (4 + 5i))(x - (4 - 5i))(x - 2)$.

D. $b \in [-7, -3], c \in [22.96, 25.33],$ and $d \in [80.5, 82.2]$

* $x^3 - 6x^2 + 25x + 82$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 + 5i))(x - (4 - 5i))(x - (-2))$.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{5}, \frac{-1}{4}, \text{ and } \frac{2}{5}$$

The solution is $100x^3 - 155x^2 + 11x + 14$, which is option C.

A. $a \in [97, 103], b \in [75, 77], c \in [-81, -77],$ and $d \in [6, 19]$

$100x^3 + 75x^2 - 81x + 14$, which corresponds to multiplying out $(5x + 7)(4x - 1)(5x - 2)$.

B. $a \in [97, 103], b \in [-163, -151], c \in [5, 14],$ and $d \in [-14, -13]$

$100x^3 - 155x^2 + 11x - 14$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [97, 103], b \in [-163, -151], c \in [5, 14],$ and $d \in [6, 19]$

* $100x^3 - 155x^2 + 11x + 14$, which is the correct option.

D. $a \in [97, 103], b \in [147, 156], c \in [5, 14],$ and $d \in [-14, -13]$

$100x^3 + 155x^2 + 11x - 14$, which corresponds to multiplying out $(5x + 7)(4x - 1)(5x + 2)$.

E. $a \in [97, 103], b \in [119, 127], c \in [-37, -25],$ and $d \in [-14, -13]$

$100x^3 + 125x^2 - 31x - 14$, which corresponds to multiplying out $(5x + 7)(4x + 1)(5x - 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 7)(4x + 1)(5x - 2)$

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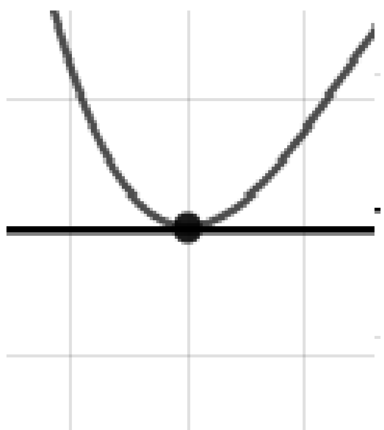
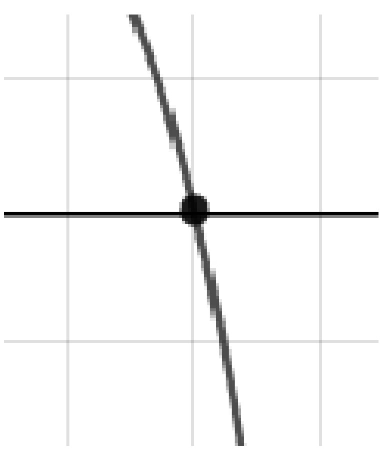
9. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

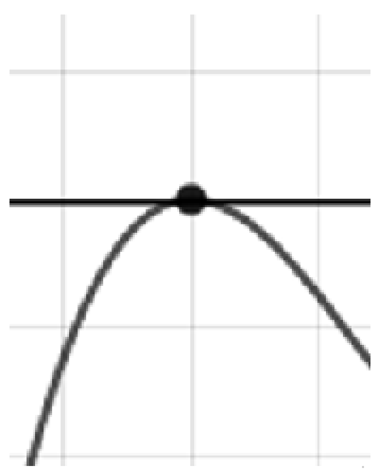
$$f(x) = 9(x - 6)^5(x + 6)^{10}(x - 9)^7(x + 9)^{11}$$

The solution is the graph below, which is option C.



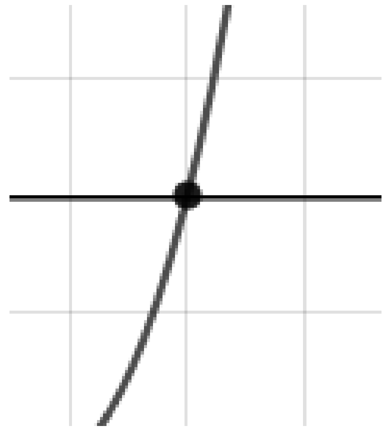


A.



B.

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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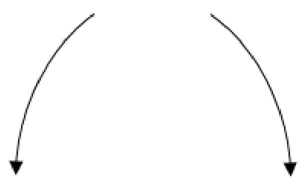
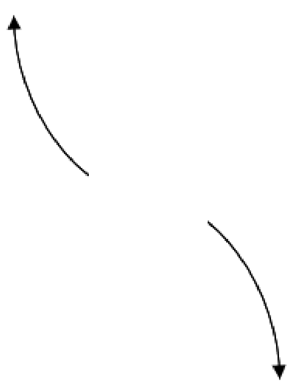
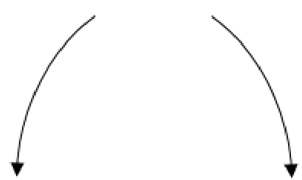
10. Describe the end behavior of the polynomial below.

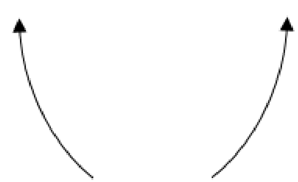
$$f(x) = -8(x-2)^4(x+2)^5(x+9)^5(x-9)^6$$

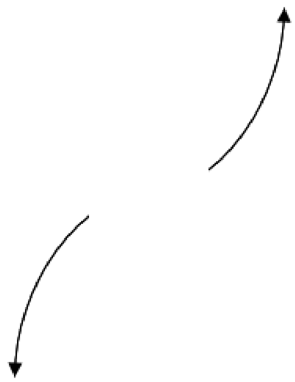
The solution is the graph below, which is option B.



A. 

B. 

C. 



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.