This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{4}, -1, \text{ and } -3$$

The solution is $4x^3 + 23x^2 + 40x + 21$, which is option B.

A. $a \in [2, 5], b \in [21, 29], c \in [37, 41], \text{ and } d \in [-23, -18]$

$4x^3 + 23x^2 + 40x - 21$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [2, 5], b \in [21, 29], c \in [37, 41], \text{ and } d \in [20, 22]$

* $4x^3 + 23x^2 + 40x + 21$, which is the correct option.

C. $a \in [2, 5], b \in [-24, -16], c \in [37, 41], \text{ and } d \in [-23, -18]$

$4x^3 - 23x^2 + 40x - 21$, which corresponds to multiplying out $(4x - 7)(x - 1)(x - 3)$.

D. $a \in [2, 5], b \in [6, 12], c \in [-19, -11], \text{ and } d \in [-23, -18]$

$4x^3 + 9x^2 - 16x - 21$, which corresponds to multiplying out $(4x - 7)(x + 1)(x + 3)$.

E. $a \in [2, 5], b \in [0, 3], c \in [-33, -25], \text{ and } d \in [20, 22]$

$4x^3 + x^2 - 26x + 21$, which corresponds to multiplying out $(4x - 7)(x - 1)(x + 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 7)(x + 1)(x + 3)$

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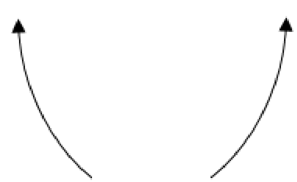
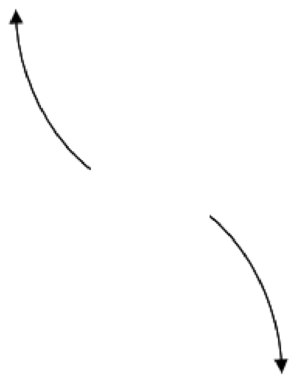
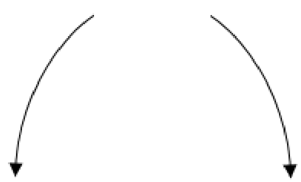
2. Describe the end behavior of the polynomial below.

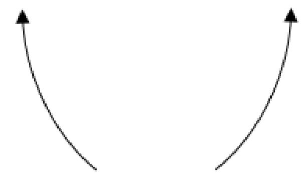
$$f(x) = 5(x + 4)^2(x - 4)^3(x + 8)^5(x - 8)^6$$

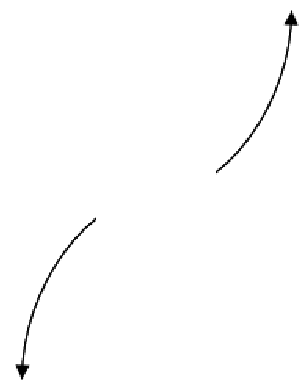
The solution is the graph below, which is option C.



A. 

B. 

C. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 5i \text{ and } 4$$

The solution is $x^3 - 12x^2 + 73x - 164$, which is option A.

A. $b \in [-13, -11], c \in [71, 74],$ and $d \in [-171, -156]$

   * $x^3 - 12x^2 + 73x - 164$, which is the correct option.

B. $b \in [9, 15], c \in [71, 74],$ and $d \in [156, 167]$

   $x^3 + 12x^2 + 73x + 164$, which corresponds to multiplying out $(x - (4 - 5i))(x - (4 + 5i))(x + 4)$.

C. $b \in [-6, 2], c \in [-11, -2],$ and $d \in [16, 20]$

   $x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

D. $b \in [-6, 2], c \in [-1, 11],$ and $d \in [-28, -19]$

   $x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 5i))(x - (4 + 5i))(x - (4))$.

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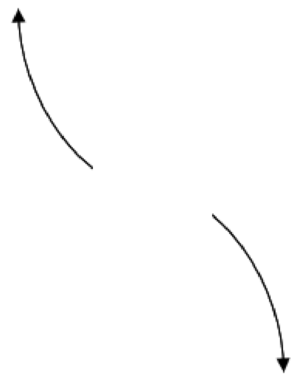
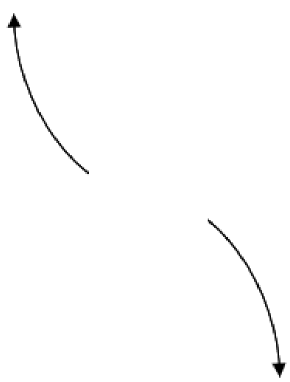
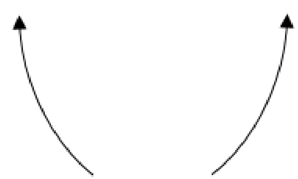
4. Describe the end behavior of the polynomial below.

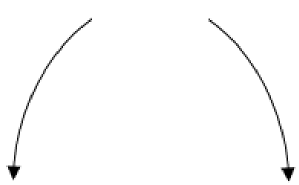
$$f(x) = -2(x + 7)^3(x - 7)^4(x - 8)^3(x + 8)^5$$

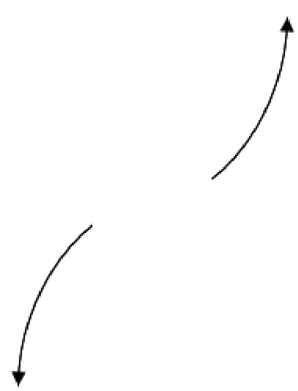
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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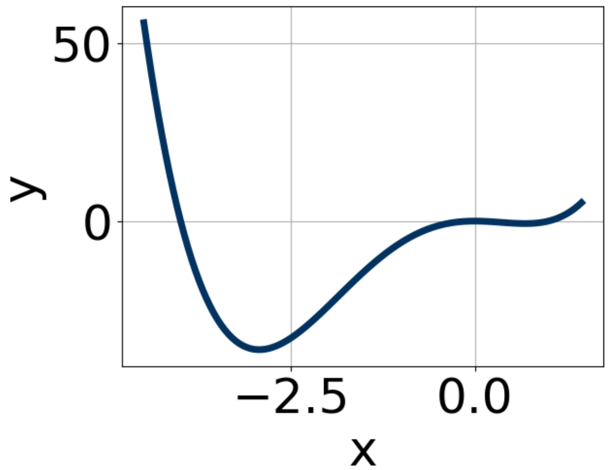
5. Which of the following equations *could* be of the graph presented below?



The solution is $16x^{10}(x-1)^5(x+4)^9$, which is option B.

A. $-7x^{10}(x-1)^9(x+4)^8$

The factor $(x+4)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $16x^{10}(x-1)^5(x+4)^9$

* This is the correct option.

C. $-12x^8(x-1)^9(x+4)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $13x^{10}(x-1)^8(x+4)^{11}$

The factor $(x-1)$ should have an odd power.

E. $4x^5(x-1)^6(x+4)^9$

The factor 0 should have an even power and the factor 1 should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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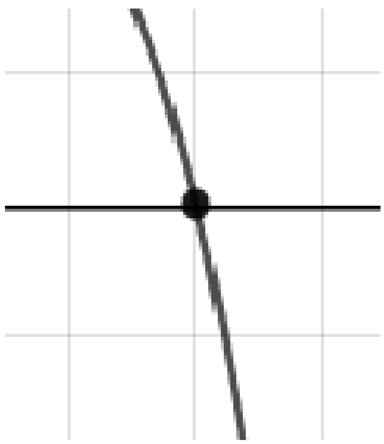
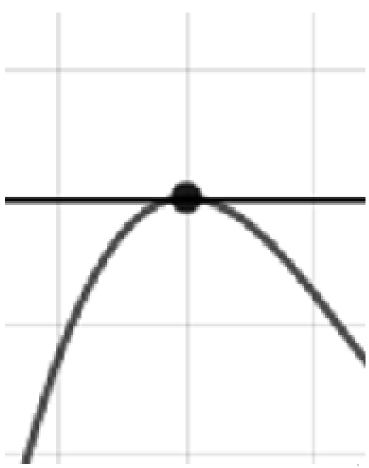
6. Describe the zero behavior of the zero $x=-7$ of the polynomial below.

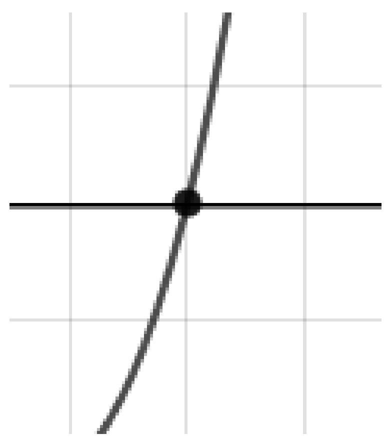
$$f(x) = -5(x-2)^6(x+2)^4(x+7)^6(x-7)^5$$

The solution is the graph below, which is option C.

A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 + 2i \text{ and } -4$$

The solution is $x^3 + 10x^2 + 37x + 52$, which is option A.

A. $b \in [10, 19], c \in [32, 39]$, and $d \in [51, 63]$

   * $x^3 + 10x^2 + 37x + 52$, which is the correct option.

B. $b \in [-1, 3], c \in [4, 8]$, and $d \in [11, 17]$

   $x^3 + x^2 + 7x + 12$, which corresponds to multiplying out $(x + 3)(x + 4)$.

C. $b \in [-1, 3], c \in [-4, 3]$, and $d \in [-12, -5]$

   $x^3 + x^2 + 2x - 8$, which corresponds to multiplying out $(x - 2)(x + 4)$.

D. $b \in [-11, -7], c \in [32, 39]$, and $d \in [-52, -50]$

   $x^3 - 10x^2 + 37x - 52$, which corresponds to multiplying out $(x - (-3 + 2i))(x - (-3 - 2i))(x - 4)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 2i))(x - (-3 - 2i))(x - (-4))$.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{5}, \frac{-5}{2}, \text{ and } \frac{1}{2}$$

The solution is $20x^3 + 12x^2 - 81x + 35$, which is option B.

A. $a \in [18, 21], b \in [65, 73], c \in [23, 32]$, and $d \in [-35, -33]$

   $20x^3 + 68x^2 + 31x - 35$, which corresponds to multiplying out $(5x + 7)(2x + 5)(2x - 1)$.

B. $a \in [18, 21], b \in [8, 17], c \in [-95, -77]$, and $d \in [33, 37]$

   * $20x^3 + 12x^2 - 81x + 35$, which is the correct option.

C. $a \in [18, 21], b \in [8, 17], c \in [-95, -77]$, and $d \in [-35, -33]$

   $20x^3 + 12x^2 - 81x - 35$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [18, 21], b \in [-33, -27], c \in [-68, -57]$, and $d \in [33, 37]$

   $20x^3 - 32x^2 - 59x + 35$, which corresponds to multiplying out $(5x + 7)(2x - 5)(2x - 1)$.

E. $a \in [18, 21], b \in [-18, -3], c \in [-95, -77]$, and $d \in [-35, -33]$

   $20x^3 - 12x^2 - 81x - 35$, which corresponds to multiplying out $(5x + 7)(2x - 5)(2x + 1)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x - 7)(2x + 5)(2x - 1)$

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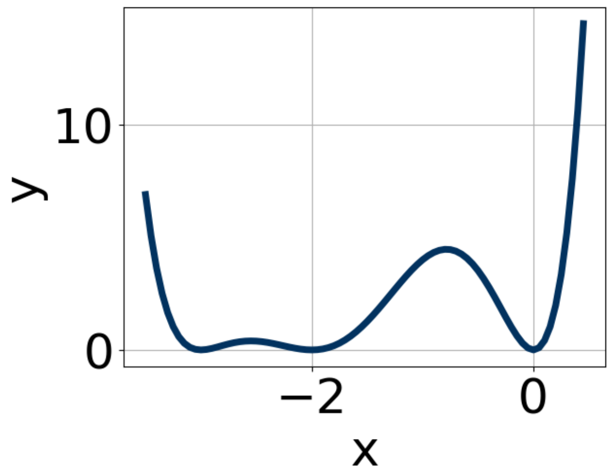
9. Which of the following equations *could* be of the graph presented below?



The solution is $8x^6(x+3)^{10}(x+2)^{10}$, which is option D.

A. $14x^{11}(x+3)^6(x+2)^7$

The factors x and $(x+2)$ should both have even powers.

B. $16x^{10}(x+3)^4(x+2)^{11}$

The factor $(x+2)$ should have an even power.

C. $-4x^8(x+3)^8(x+2)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $8x^6(x+3)^{10}(x+2)^{10}$

* This is the correct option.

E. $-5x^8(x+3)^6(x+2)^7$

The factor $(x+2)$ should have an even power and the leading coefficient should be the opposite sign.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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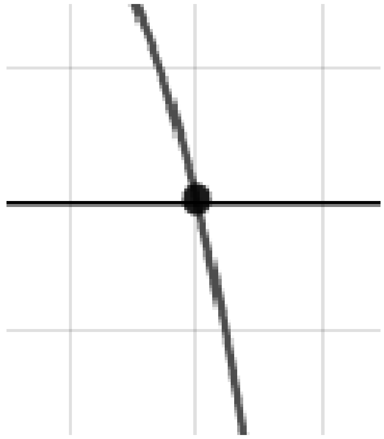
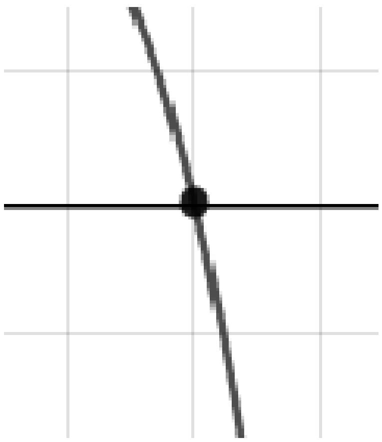
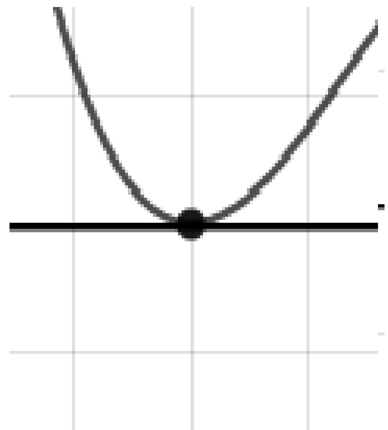
10. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

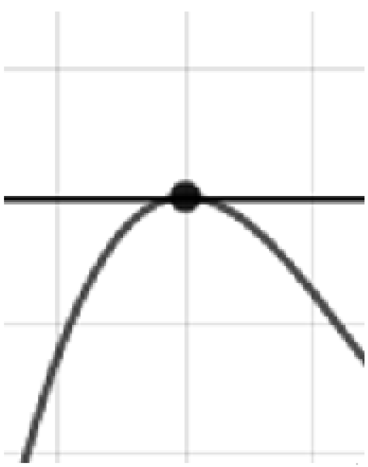
$$f(x) = -3(x+9)^6(x-9)^{11}(x-5)^8(x+5)^9$$

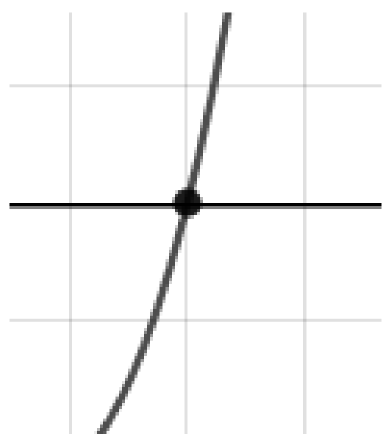
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.