This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Factor the polynomial below completely, knowing that $x-2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^4 - 143x^3 + 212x^2 + 33x - 90$$

The solution is $[-0.6, 0.75, 2, 5]$, which is option C.

A. $z_1 \in [-2, -1.6]$, $z_2 \in [0.98, 1.85]$, $z_3 \in [1.93, 2.08]$, and $z_4 \in [4.45, 5.68]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-5.3, -3.4]$, $z_2 \in [-2.67, -1.65]$, $z_3 \in [-1.14, -0.62]$, and $z_4 \in [-0.5, 1.45]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-1.5, 0.4]$, $z_2 \in [0.49, 1.03]$, $z_3 \in [1.93, 2.08]$, and $z_4 \in [4.45, 5.68]$

* This is the solution!

D. $z_1 \in [-5.3, -3.4]$, $z_2 \in [-2.67, -1.65]$, $z_3 \in [-0.68, 0.13]$, and $z_4 \in [2.83, 3.55]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-5.3, -3.4]$, $z_2 \in [-2.67, -1.65]$, $z_3 \in [-1.51, -1.1]$, and $z_4 \in [0.65, 2.43]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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2. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 64x^2 + 74x - 25}{x - 5}$$

The solution is $10x^2 - 14x + 4 + \frac{-5}{x-5}$, which is option A.

A. $a \in [9, 14]$, $b \in [-15, -6]$, $c \in [2, 5]$, and $r \in [-9, -1]$.

* This is the solution!

B. $a \in [9, 14]$, $b \in [-119, -108]$, $c \in [642, 645]$, and $r \in [-3249, -3242]$.

You divided by the opposite of the factor.

C. $a \in [43, 53]$, $b \in [-320, -306]$, $c \in [1641, 1645]$, and $r \in [-8246, -8238]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

D. $a \in [43, 53]$, $b \in [180, 194]$, $c \in [1000, 1007]$, and $r \in [4994, 4996]$.

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [9, 14]$, $b \in [-32, -17]$, $c \in [-24, -21]$, and $r \in [-119, -110]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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3. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 16x^3 + 40x^2 + x - 30$$

The solution is $[-2, -1.25, 0.75]$, which is option E.

A. $z_1 \in [-0.65, -0.05]$, $z_2 \in [1.8, 2.3]$, and $z_3 \in [4.98, 5.07]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-2.18, -1.53]$, $z_2 \in [-0.83, -0.62]$, and $z_3 \in [1.25, 1.67]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-1.67, -1.19]$, $z_2 \in [0.74, 1.09]$, and $z_3 \in [1.9, 2.6]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-0.91, -0.62]$, $z_2 \in [1.04, 1.44]$, and $z_3 \in [1.9, 2.6]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-2.18, -1.53]$, $z_2 \in [-1.34, -1.21]$, and $z_3 \in [0.49, 0.83]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 7x^3 + 3x^2 + 4x + 4$$

The solution is $\pm 1, \pm 2, \pm 4$, which is option A.

A. $\pm 1, \pm 2, \pm 4$

* This is the solution **since we asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 7}$

This would have been the solution **if asked for the possible Rational roots**!

C. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 4}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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5. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 + 71x^2 + 32x - 48$$

The solution is $[-4, -1.33, 0.6]$, which is option E.

A. $z_1 \in [-0.31, 0]$, $z_2 \in [3.4, 5.2]$, and $z_3 \in [2.8, 5.1]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-1.74, -1.25]$, $z_2 \in [0, 1.2]$, and $z_3 \in [2.8, 5.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-4.13, -3.86]$, $z_2 \in [-1.2, -0.2]$, and $z_3 \in [1.4, 2.1]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-0.81, -0.46]$, $z_2 \in [0.9, 2.3]$, and $z_3 \in [2.8, 5.1]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-4.13, -3.86]$, $z_2 \in [-2.6, -1.1]$, and $z_3 \in [0.3, 1.1]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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6. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 105x^2 - 122}{x + 5}$$

The solution is $20x^2 + 5x - 25 + \frac{3}{x+5}$, which is option C.

A. $a \in [-103, -96], b \in [598, 607], c \in [-3033, -3021]$, and $r \in [15001, 15007]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [17, 23], b \in [200, 209], c \in [1024, 1033]$, and $r \in [4999, 5011]$.

You divided by the opposite of the factor.

C. $a \in [17, 23], b \in [-2, 9], c \in [-25, -22]$, and $r \in [-2, 10]$.

* This is the solution!

D. $a \in [-103, -96], b \in [-400, -393], c \in [-1977, -1970]$, and $r \in [-10001, -9989]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [17, 23], b \in [-15, -14], c \in [88, 95]$, and $r \in [-665, -656]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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7. Factor the polynomial below completely, knowing that $x-4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 + 14x^3 - 163x^2 - 129x + 180$$

The solution is $[-5, -1.5, 0.75, 4]$, which is option C.

A. $z_1 \in [-4.9, -3.6], \ z_2 \in [-3.06, -2.98], z_3 \in [0.27, 0.47], \text{ and } z_4 \in [4.4, 6.6]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-4.9, -3.6], \ z_2 \in [-1.49, -1.28], z_3 \in [0.66, 0.7], \text{ and } z_4 \in [4.4, 6.6]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.6, -4.8], \ z_2 \in [-1.51, -1.49], z_3 \in [0.7, 0.79], \text{ and } z_4 \in [2.9, 4.2]$

* This is the solution!

D. $z_1 \in [-5.6, -4.8], \ z_2 \in [-0.68, -0.65], z_3 \in [1.25, 1.37], \text{ and } z_4 \in [2.9, 4.2]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-4.9, -3.6], \ z_2 \in [-0.78, -0.72], z_3 \in [1.49, 1.51], \text{ and } z_4 \in [4.4, 6.6]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 3x^4 + 4x^3 + 6x^2 + 3x + 5$$

The solution is $\pm 1, \pm 5$, which is option A.

A. $\pm 1, \pm 5$

* This is the solution **since we asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 3}$

This would have been the solution **if asked for the possible Rational roots**!

C. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 + 70x^2 + 105x + 53}{x + 2}$$

The solution is $15x^2 + 40x + 25 + \frac{3}{x+2}$, which is option C.

A. $a \in [-30, -29]$, $b \in [128, 137]$, $c \in [-163, -151]$, and $r \in [358, 366]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [14, 17]$, $b \in [97, 101]$, $c \in [298, 307]$, and $r \in [663, 668]$.

You divided by the opposite of the factor.

C. $a \in [14, 17]$, $b \in [39, 45]$, $c \in [23, 27]$, and $r \in [3, 4]$.

* This is the solution!

D. $a \in [14, 17]$, $b \in [20, 29]$, $c \in [30, 33]$, and $r \in [-37, -31]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [-30, -29]$, $b \in [9, 11]$, $c \in [123, 128]$, and $r \in [296, 309]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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10. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 + 26x^2 - 29}{x + 4}$$

The solution is $6x^2 + 2x - 8 + \frac{3}{x+4}$, which is option A.

A. $a \in [3, 10], b \in [2, 4], c \in [-11, -5]$, and $r \in [-6, 5]$.

* This is the solution!

B. $a \in [-27, -20], b \in [117, 124], c \in [-488, -487]$, and $r \in [1917, 1927]$.

You multipled by the synthetic number rather than bringing the first factor down.

C. $a \in [3, 10], b \in [-9, 1], c \in [18, 21]$, and $r \in [-129, -128]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [-27, -20], b \in [-73, -64], c \in [-280, -274]$, and $r \in [-1151, -1146]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [3, 10], b \in [47, 52], c \in [194, 202]$, and $r \in [769, 776]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.
