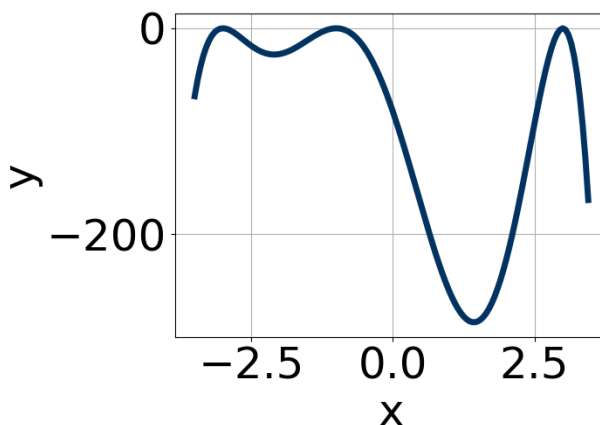


This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

If you have a suggestion to make the keys better, please fill out the short survey [here](#).

*Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.*

1. Which of the following equations *could* be of the graph presented below?



The solution is  $-18(x+3)^6(x+1)^4(x-3)^8$ , which is option A.

A.  $-18(x+3)^6(x+1)^4(x-3)^8$

\* This is the correct option.

B.  $12(x+3)^8(x+1)^4(x-3)^7$

The factor  $(x-3)$  should have an even power and the leading coefficient should be the opposite sign.

C.  $-7(x+3)^6(x+1)^{10}(x-3)^7$

The factor  $(x-3)$  should have an even power.

D.  $8(x+3)^6(x+1)^6(x-3)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

E.  $-12(x+3)^{10}(x+1)^{11}(x-3)^5$

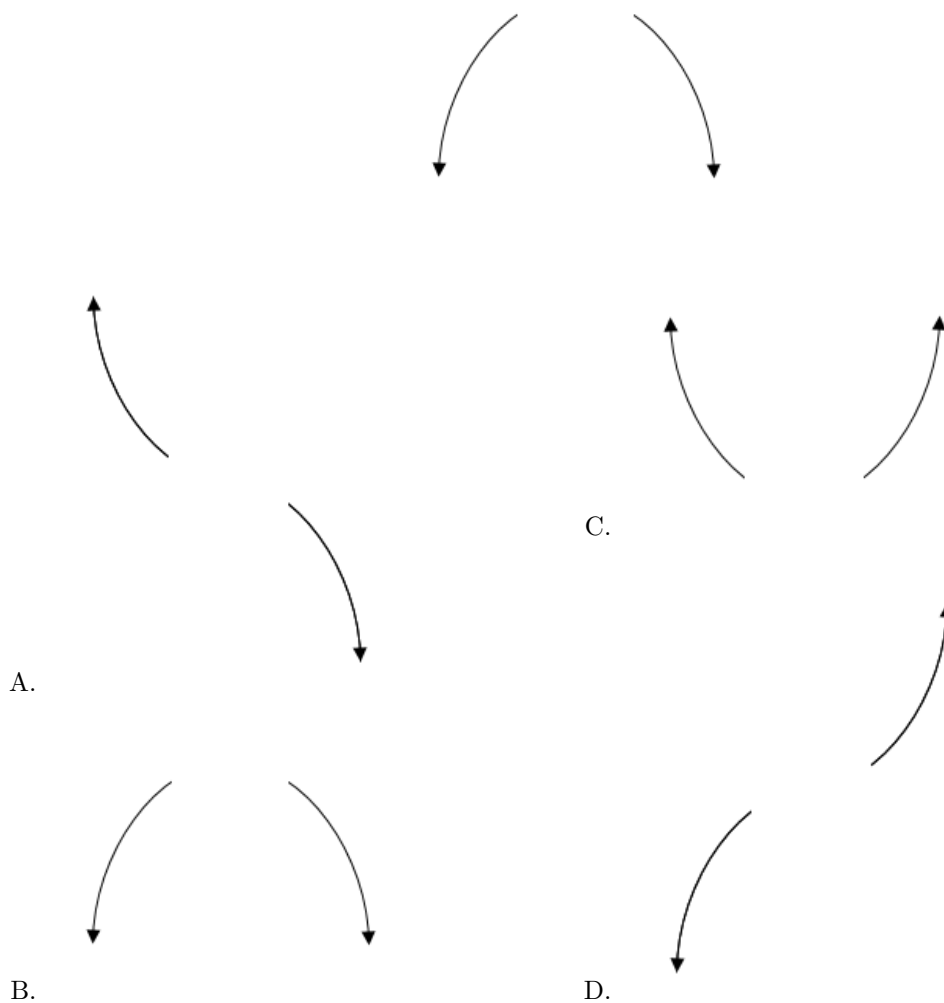
The factors  $(x+1)$  and  $(x-3)$  should both have even powers.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

2. Describe the end behavior of the polynomial below.

$$f(x) = -9(x+8)^2(x-8)^5(x-6)^2(x+6)^3$$

The solution is the graph below, which is option B.



**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$4 + 3i \text{ and } 1$$

The solution is  $x^3 - 9x^2 + 33x - 25$ , which is option C.

- A.  $b \in [1, 2]$ ,  $c \in [-6, -4.35]$ , and  $d \in [3.48, 4.06]$

$x^3 + x^2 - 5x + 4$ , which corresponds to multiplying out  $(x - 4)(x - 1)$ .

- B.  $b \in [1, 2]$ ,  $c \in [-4.24, -2.63]$ , and  $d \in [2.96, 3.37]$

$x^3 + x^2 - 4x + 3$ , which corresponds to multiplying out  $(x - 3)(x - 1)$ .

- C.  $b \in [-17, -7]$ ,  $c \in [31.51, 33.82]$ , and  $d \in [-25.2, -24.86]$

\*  $x^3 - 9x^2 + 33x - 25$ , which is the correct option.

D.  $b \in [8, 10]$ ,  $c \in [31.51, 33.82]$ , and  $d \in [24.44, 25.5]$

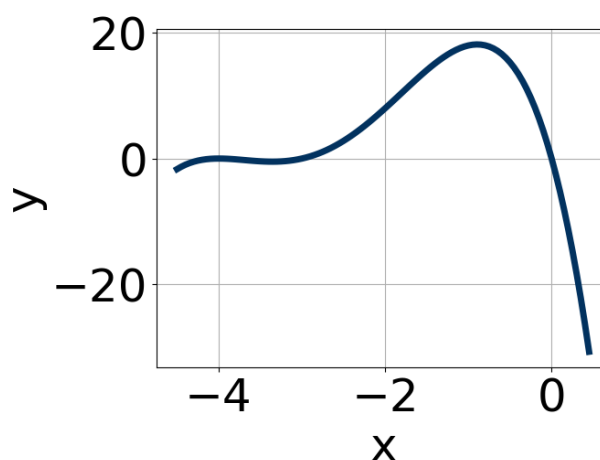
$x^3 + 9x^2 + 33x + 25$ , which corresponds to multiplying out  $(x - (4 + 3i))(x - (4 - 3i))(x + 1)$ .

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (4 + 3i))(x - (4 - 3i))(x - (1))$ .

4. Which of the following equations *could* be of the graph presented below?



The solution is  $-5x^5(x + 4)^8(x + 3)^7$ , which is option A.

A.  $-5x^5(x + 4)^8(x + 3)^7$

\* This is the correct option.

B.  $-19x^6(x + 4)^6(x + 3)^7$

The factor  $x$  should have an odd power.

C.  $-14x^{10}(x + 4)^9(x + 3)^5$

The factor  $-4$  should have an even power and the factor  $0$  should have an odd power.

D.  $10x^{11}(x + 4)^8(x + 3)^{10}$

The factor  $(x + 3)$  should have an odd power and the leading coefficient should be the opposite sign.

E.  $19x^{11}(x + 4)^4(x + 3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

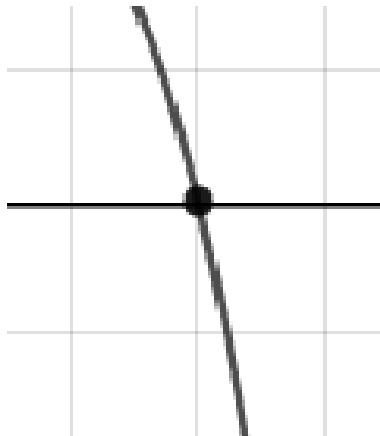
5. Describe the zero behavior of the zero  $x = 8$  of the polynomial below.

$$f(x) = -9(x - 6)^9(x + 6)^6(x - 8)^{12}(x + 8)^9$$

The solution is the graph below, which is option B.



A.



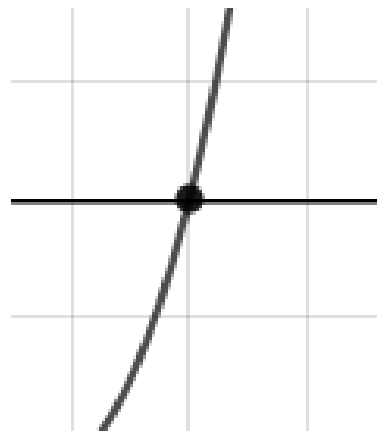
C.



B.



D.



E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

---

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain

the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{3}{2}, 5, \text{ and } \frac{-4}{3}$$

The solution is  $6x^3 - 31x^2 - 7x + 60$ , which is option B.

A.  $a \in [0, 10], b \in [45, 52], c \in [97, 103]$ , and  $d \in [55, 64]$

$6x^3 + 47x^2 + 97x + 60$ , which corresponds to multiplying out  $(2x + 3)(x + 5)(3x + 4)$ .

B.  $a \in [0, 10], b \in [-34, -27], c \in [-9, -4]$ , and  $d \in [55, 64]$

\*  $6x^3 - 31x^2 - 7x + 60$ , which is the correct option.

C.  $a \in [0, 10], b \in [-34, -27], c \in [-9, -4]$ , and  $d \in [-66, -56]$

$6x^3 - 31x^2 - 7x - 60$ , which corresponds to multiplying everything correctly except the constant term.

D.  $a \in [0, 10], b \in [26, 37], c \in [-9, -4]$ , and  $d \in [-66, -56]$

$6x^3 + 31x^2 - 7x - 60$ , which corresponds to multiplying out  $(2x + 3)(x + 5)(3x - 4)$ .

E.  $a \in [0, 10], b \in [-13, -7], c \in [-76, -68]$ , and  $d \in [-66, -56]$

$6x^3 - 13x^2 - 73x - 60$ , which corresponds to multiplying out  $(2x + 3)(x - 5)(3x + 4)$ .

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(2x - 3)(x - 5)(3x + 4)$

7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-3 + 2i \text{ and } 2$$

The solution is  $x^3 + 4x^2 + x - 26$ , which is option C.

A.  $b \in [-5.5, -1.2], c \in [-3, 3]$ , and  $d \in [24, 27]$

$x^3 - 4x^2 + x + 26$ , which corresponds to multiplying out  $(x - (-3 + 2i))(x - (-3 - 2i))(x + 2)$ .

B.  $b \in [0.7, 1.5], c \in [-6, -3]$ , and  $d \in [0, 9]$

$x^3 + x^2 - 4x + 4$ , which corresponds to multiplying out  $(x - 2)(x - 2)$ .

C.  $b \in [3.8, 5.3], c \in [-3, 3]$ , and  $d \in [-32, -24]$

\*  $x^3 + 4x^2 + x - 26$ , which is the correct option.

D.  $b \in [0.7, 1.5], c \in [-3, 3]$ , and  $d \in [-10, -3]$

$x^3 + x^2 + x - 6$ , which corresponds to multiplying out  $(x + 3)(x - 2)$ .

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (-3 + 2i))(x - (-3 - 2i))(x - (2))$ .

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-3}{5}, \frac{-7}{2}, \text{ and } \frac{-3}{2}$$

The solution is  $20x^3 + 112x^2 + 165x + 63$ , which is option C.

- A.  $a \in [15, 23]$ ,  $b \in [110, 119]$ ,  $c \in [165, 169]$ , and  $d \in [-64, -58]$

$20x^3 + 112x^2 + 165x - 63$ , which corresponds to multiplying everything correctly except the constant term.

- B.  $a \in [15, 23]$ ,  $b \in [-117, -109]$ ,  $c \in [165, 169]$ , and  $d \in [-64, -58]$

$20x^3 - 112x^2 + 165x - 63$ , which corresponds to multiplying out  $(5x - 3)(2x - 7)(2x - 3)$ .

- C.  $a \in [15, 23]$ ,  $b \in [110, 119]$ ,  $c \in [165, 169]$ , and  $d \in [54, 68]$

\*  $20x^3 + 112x^2 + 165x + 63$ , which is the correct option.

- D.  $a \in [15, 23]$ ,  $b \in [-53, -49]$ ,  $c \in [-85, -80]$ , and  $d \in [54, 68]$

$20x^3 - 52x^2 - 81x + 63$ , which corresponds to multiplying out  $(5x - 3)(2x - 7)(2x + 3)$ .

- E.  $a \in [15, 23]$ ,  $b \in [88, 93]$ ,  $c \in [36, 51]$ , and  $d \in [-64, -58]$

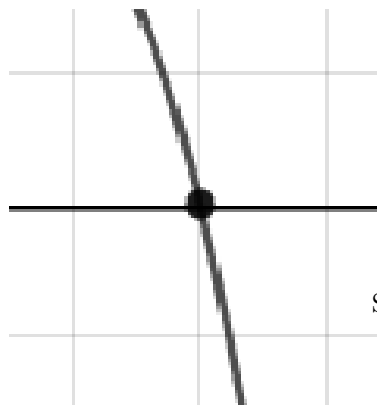
$20x^3 + 88x^2 + 45x - 63$ , which corresponds to multiplying out  $(5x - 3)(2x + 7)(2x + 3)$ .

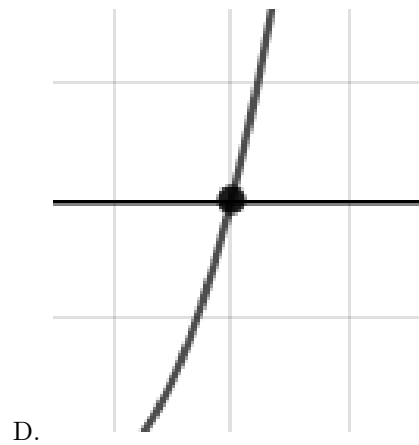
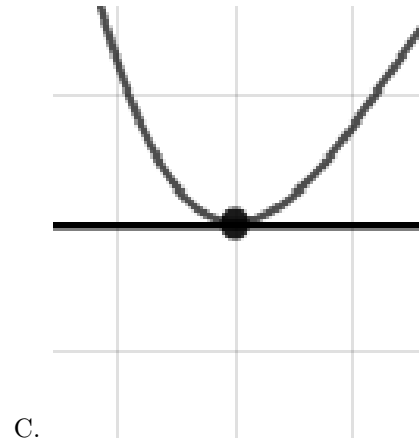
**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(5x + 3)(2x + 7)(2x + 3)$

9. Describe the zero behavior of the zero  $x = 7$  of the polynomial below.

$$f(x) = 2(x + 7)^7(x - 7)^{10}(x - 3)^4(x + 3)^8$$

The solution is the graph below, which is option C.





E. None of the above.

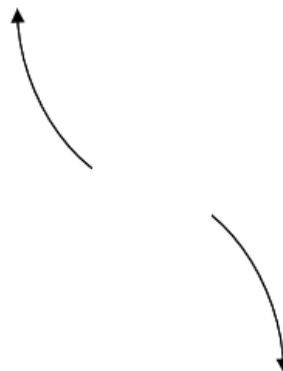
**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

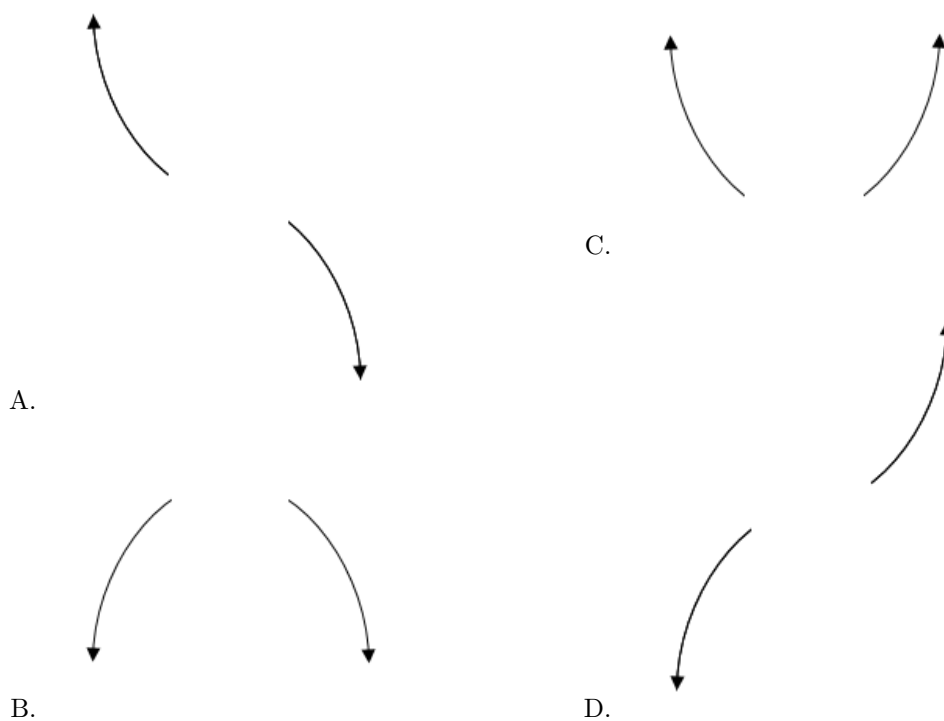
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10. Describe the end behavior of the polynomial below.

$$f(x) = -3(x + 4)^3(x - 4)^6(x - 5)^5(x + 5)^7$$

The solution is the graph below, which is option A.

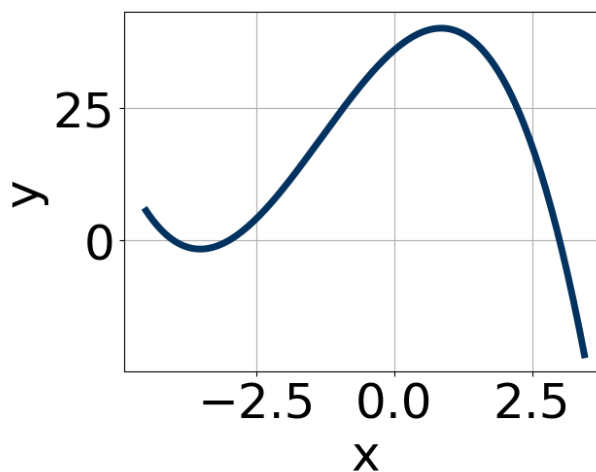




E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

11. Which of the following equations *could* be of the graph presented below?



The solution is  $-3(x - 3)^7(x + 4)^9(x + 3)^{11}$ , which is option B.

A.  $-20(x - 3)^6(x + 4)^{11}(x + 3)^5$

The factor 3 should have been an odd power.

B.  $-3(x - 3)^7(x + 4)^9(x + 3)^{11}$

\* This is the correct option.



C.  $6(x-3)^4(x+4)^{11}(x+3)^9$

The factor  $(x-3)$  should have an odd power and the leading coefficient should be the opposite sign.

D.  $-3(x-3)^4(x+4)^8(x+3)^9$

The factors 3 and  $-4$  have been odd power.

E.  $18(x-3)^5(x+4)^5(x+3)^{11}$

This corresponds to the leading coefficient being the opposite value than it should be.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

---

12. Describe the end behavior of the polynomial below.

$$f(x) = 6(x+4)^5(x-4)^6(x-3)^4(x+3)^5$$

The solution is the graph below, which is option C.



C.



A.



B.



D.

E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

13. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-5 + 5i \text{ and } -3$$

The solution is  $x^3 + 13x^2 + 80x + 150$ , which is option B.

A.  $b \in [-1, 9], c \in [-8, 1], \text{ and } d \in [-15, -11]$

$x^3 + x^2 - 2x - 15$ , which corresponds to multiplying out  $(x - 5)(x + 3)$ .

B.  $b \in [11, 16], c \in [80, 82], \text{ and } d \in [147, 158]$

\*  $x^3 + 13x^2 + 80x + 150$ , which is the correct option.

C.  $b \in [-1, 9], c \in [7, 11], \text{ and } d \in [11, 19]$

$x^3 + x^2 + 8x + 15$ , which corresponds to multiplying out  $(x + 5)(x + 3)$ .

D.  $b \in [-19, -12], c \in [80, 82], \text{ and } d \in [-158, -146]$

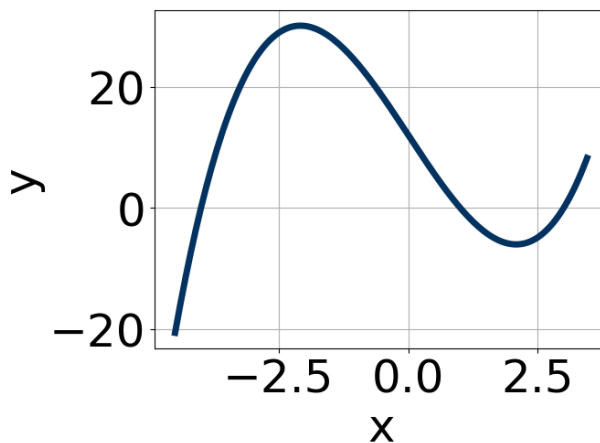
$x^3 - 13x^2 + 80x - 150$ , which corresponds to multiplying out  $(x - (-5 + 5i))(x - (-5 - 5i))(x - 3)$ .

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (-5 + 5i))(x - (-5 - 5i))(x - (-3))$ .

14. Which of the following equations *could* be of the graph presented below?



The solution is  $9(x - 3)^7(x + 4)^{11}(x - 1)^{11}$ , which is option C.

A.  $3(x - 3)^4(x + 4)^8(x - 1)^5$

The factors 3 and  $-4$  have have been odd power.

B.  $-12(x-3)^4(x+4)^5(x-1)^7$

The factor  $(x-3)$  should have an odd power and the leading coefficient should be the opposite sign.

C.  $9(x-3)^7(x+4)^{11}(x-1)^{11}$

\* This is the correct option.

D.  $-12(x-3)^9(x+4)^{11}(x-1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

E.  $10(x-3)^6(x+4)^{11}(x-1)^7$

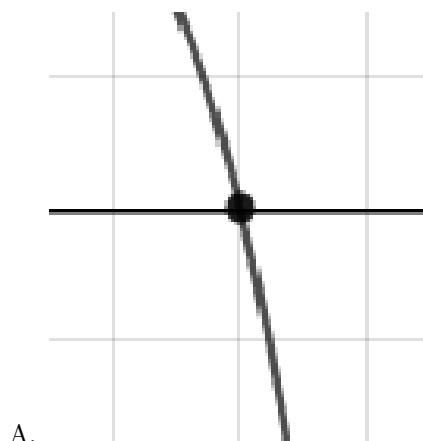
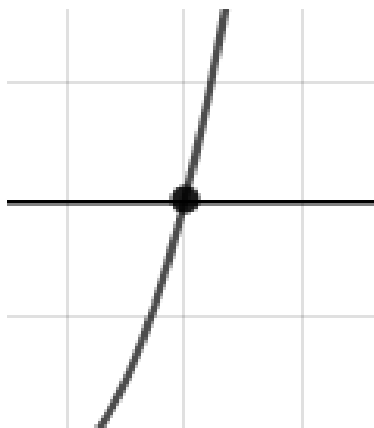
The factor 3 should have been an odd power.

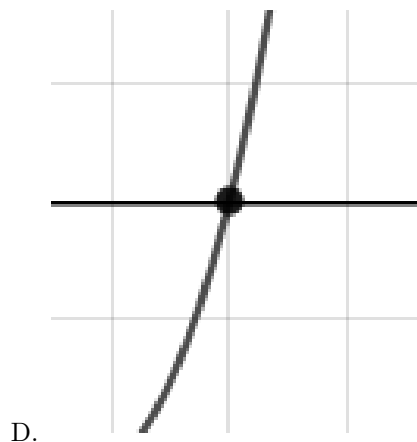
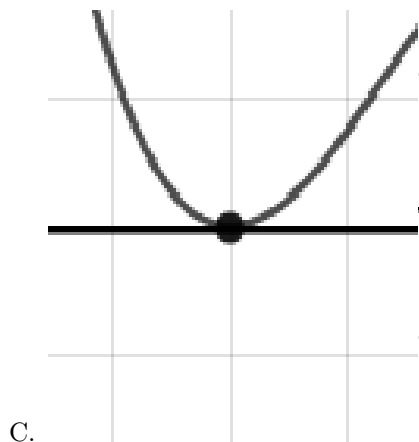
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

15. Describe the zero behavior of the zero  $x = -8$  of the polynomial below.

$$f(x) = 9(x+2)^{11}(x-2)^7(x+8)^7(x-8)^6$$

The solution is the graph below, which is option D.





E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

16. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{5}{2}, \frac{-1}{3}, \text{ and } \frac{-2}{3}$$

The solution is  $18x^3 - 27x^2 - 41x - 10$ , which is option B.

A.  $a \in [17, 23], b \in [19, 28], c \in [-45, -36]$ , and  $d \in [8, 11]$

$18x^3 + 27x^2 - 41x + 10$ , which corresponds to multiplying out  $(2x + 5)(3x - 1)(3x - 2)$ .

B.  $a \in [17, 23], b \in [-27, -24], c \in [-45, -36]$ , and  $d \in [-17, -9]$

\*  $18x^3 - 27x^2 - 41x - 10$ , which is the correct option.

C.  $a \in [17, 23], b \in [50, 54], c \in [3, 12]$ , and  $d \in [-17, -9]$

$18x^3 + 51x^2 + 11x - 10$ , which corresponds to multiplying out  $(2x + 5)(3x - 1)(3x + 2)$ .

D.  $a \in [17, 23], b \in [58, 75], c \in [44, 53]$ , and  $d \in [8, 11]$

$18x^3 + 63x^2 + 49x + 10$ , which corresponds to multiplying out  $(2x + 5)(3x + 1)(3x + 2)$ .

E.  $a \in [17, 23], b \in [-27, -24], c \in [-45, -36]$ , and  $d \in [8, 11]$

$18x^3 - 27x^2 - 41x + 10$ , which corresponds to multiplying everything correctly except the constant term.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(2x - 5)(3x + 1)(3x + 2)$

17. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$4 + 5i \text{ and } -2$$

The solution is  $x^3 - 6x^2 + 25x + 82$ , which is option D.

- A.  $b \in [1, 3]$ ,  $c \in [-4.5, -2.7]$ , and  $d \in [-10.1, -9.4]$

$x^3 + x^2 - 3x - 10$ , which corresponds to multiplying out  $(x - 5)(x + 2)$ .

- B.  $b \in [1, 3]$ ,  $c \in [-2.53, -1.56]$ , and  $d \in [-8.9, -6.6]$

$x^3 + x^2 - 2x - 8$ , which corresponds to multiplying out  $(x - 4)(x + 2)$ .

- C.  $b \in [6, 11]$ ,  $c \in [22.96, 25.33]$ , and  $d \in [-83.9, -75.9]$

$x^3 + 6x^2 + 25x - 82$ , which corresponds to multiplying out  $(x - (4 + 5i))(x - (4 - 5i))(x - 2)$ .

- D.  $b \in [-7, -3]$ ,  $c \in [22.96, 25.33]$ , and  $d \in [80.5, 82.2]$

\*  $x^3 - 6x^2 + 25x + 82$ , which is the correct option.

- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (4 + 5i))(x - (4 - 5i))(x - (-2))$ .

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18. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{7}{5}, \frac{-1}{4}, \text{ and } \frac{2}{5}$$

The solution is  $100x^3 - 155x^2 + 11x + 14$ , which is option C.

- A.  $a \in [97, 103]$ ,  $b \in [75, 77]$ ,  $c \in [-81, -77]$ , and  $d \in [6, 19]$

$100x^3 + 75x^2 - 81x + 14$ , which corresponds to multiplying out  $(5x + 7)(4x - 1)(5x - 2)$ .

- B.  $a \in [97, 103]$ ,  $b \in [-163, -151]$ ,  $c \in [5, 14]$ , and  $d \in [-14, -13]$

$100x^3 - 155x^2 + 11x - 14$ , which corresponds to multiplying everything correctly except the constant term.

- C.  $a \in [97, 103]$ ,  $b \in [-163, -151]$ ,  $c \in [5, 14]$ , and  $d \in [6, 19]$

\*  $100x^3 - 155x^2 + 11x + 14$ , which is the correct option.

- D.  $a \in [97, 103]$ ,  $b \in [147, 156]$ ,  $c \in [5, 14]$ , and  $d \in [-14, -13]$

$100x^3 + 155x^2 + 11x - 14$ , which corresponds to multiplying out  $(5x + 7)(4x - 1)(5x + 2)$ .

- E.  $a \in [97, 103]$ ,  $b \in [119, 127]$ ,  $c \in [-37, -25]$ , and  $d \in [-14, -13]$

$100x^3 + 125x^2 - 31x - 14$ , which corresponds to multiplying out  $(5x + 7)(4x + 1)(5x - 2)$ .

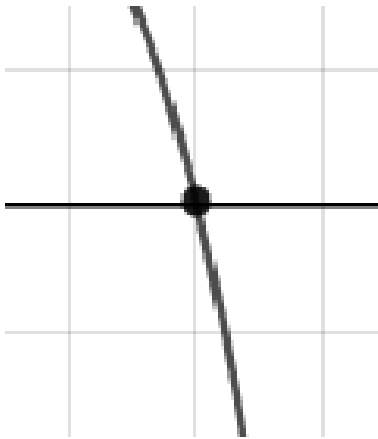
**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(5x - 7)(4x + 1)(5x - 2)$

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19. Describe the zero behavior of the zero  $x = -6$  of the polynomial below.

$$f(x) = 9(x - 6)^5(x + 6)^{10}(x - 9)^7(x + 9)^{11}$$

The solution is the graph below, which is option C.



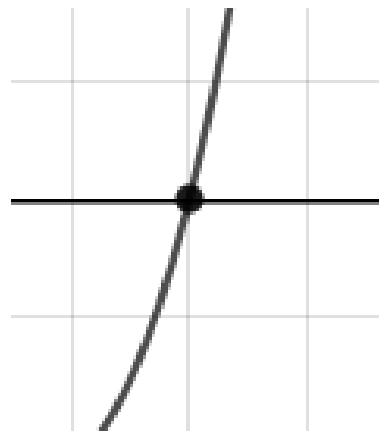
A.



C.



B.



D.

E. None of the above.

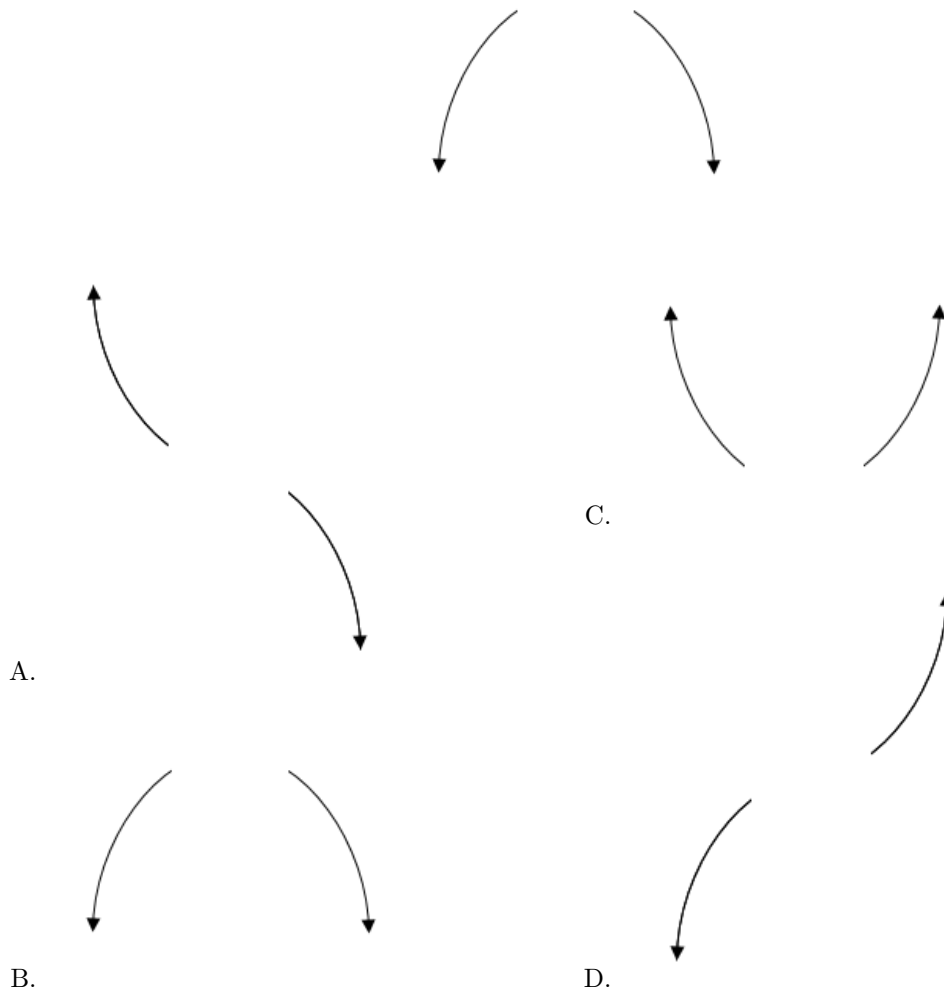
**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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20. Describe the end behavior of the polynomial below.

$$f(x) = -8(x - 2)^4(x + 2)^5(x + 9)^5(x - 9)^6$$

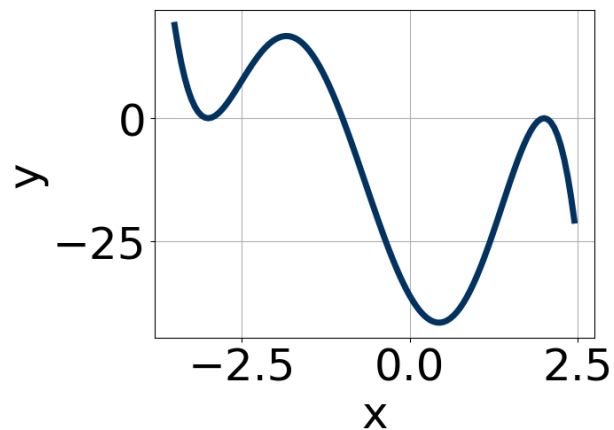
The solution is the graph below, which is option B.



E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

21. Which of the following equations *could* be of the graph presented below?



The solution is  $-18(x-2)^{10}(x+3)^6(x+1)^{11}$ , which is option B.

A.  $19(x-2)^{10}(x+3)^6(x+1)^8$

The factor  $(x+1)$  should have an odd power and the leading coefficient should be the opposite sign.

B.  $-18(x-2)^{10}(x+3)^6(x+1)^{11}$

\* This is the correct option.

C.  $-19(x-2)^8(x+3)^9(x+1)^{10}$

The factor  $(x+3)$  should have an even power and the factor  $(x+1)$  should have an odd power.

D.  $13(x-2)^{10}(x+3)^{10}(x+1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

E.  $-4(x-2)^{10}(x+3)^5(x+1)^5$

The factor  $(x+3)$  should have an even power.

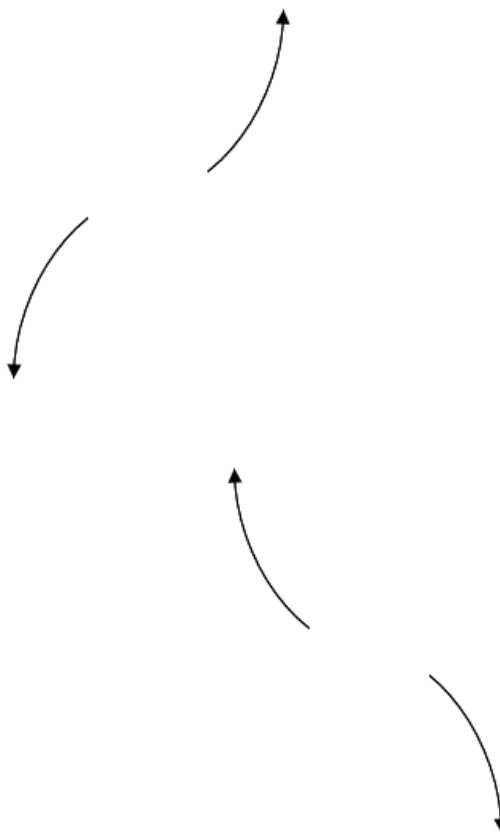
**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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22. Describe the end behavior of the polynomial below.

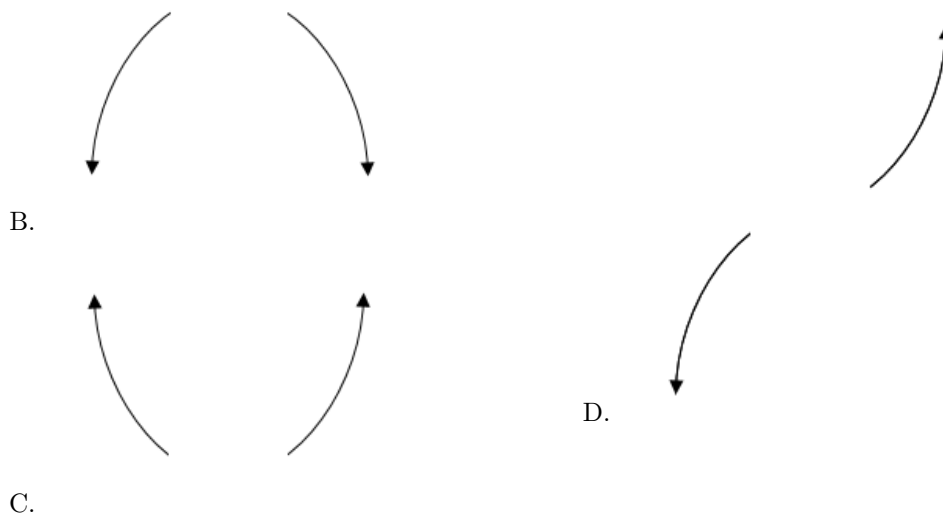
$$f(x) = 4(x+6)^4(x-6)^9(x+9)^3(x-9)^3$$

The solution is the graph below, which is option D.



A.





**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

---

23. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-3 - 5i \text{ and } -4$$

The solution is  $x^3 + 10x^2 + 58x + 136$ , which is option D.

- A.  $b \in [-1, 5]$ ,  $c \in [8, 9.7]$ , and  $d \in [16, 22]$

$x^3 + x^2 + 9x + 20$ , which corresponds to multiplying out  $(x + 5)(x + 4)$ .

- B.  $b \in [-1, 5]$ ,  $c \in [6.7, 8.6]$ , and  $d \in [10, 14]$

$x^3 + x^2 + 7x + 12$ , which corresponds to multiplying out  $(x + 3)(x + 4)$ .

- C.  $b \in [-15, -8]$ ,  $c \in [56.1, 58.3]$ , and  $d \in [-143, -134]$

$x^3 - 10x^2 + 58x - 136$ , which corresponds to multiplying out  $(x - (-3 - 5i))(x - (-3 + 5i))(x - 4)$ .

- D.  $b \in [6, 14]$ ,  $c \in [56.1, 58.3]$ , and  $d \in [130, 141]$

\*  $x^3 + 10x^2 + 58x + 136$ , which is the correct option.

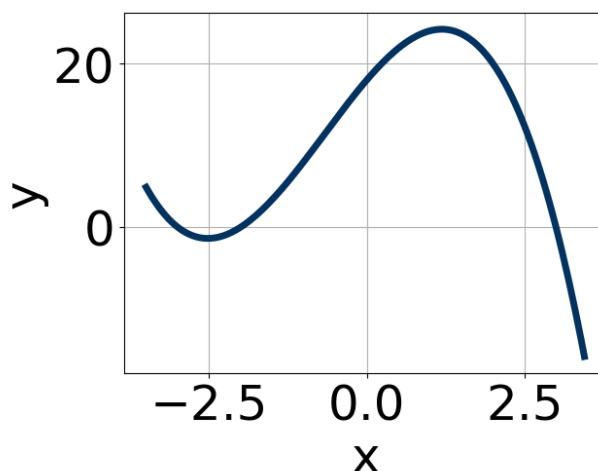
- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (-3 - 5i))(x - (-3 + 5i))(x - (-4))$ .

---

24. Which of the following equations *could* be of the graph presented below?



The solution is  $-2(x+2)^9(x+3)^{11}(x-3)^5$ , which is option D.

A.  $7(x+2)^6(x+3)^{11}(x-3)^7$

The factor  $(x+2)$  should have an odd power and the leading coefficient should be the opposite sign.

B.  $-10(x+2)^{10}(x+3)^9(x-3)^9$

The factor  $-2$  should have been an odd power.

C.  $11(x+2)^9(x+3)^9(x-3)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

D.  $-2(x+2)^9(x+3)^{11}(x-3)^5$

\* This is the correct option.

E.  $-5(x+2)^4(x+3)^8(x-3)^9$

The factors  $-2$  and  $-3$  have have been odd power.

**General Comment:** General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

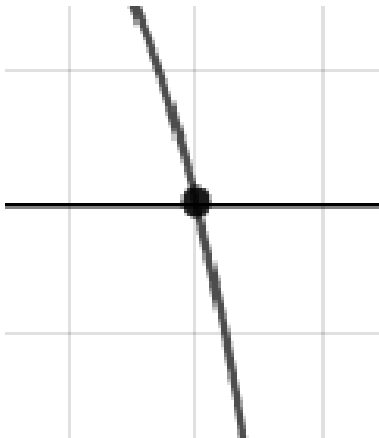
25. Describe the zero behavior of the zero  $x = -6$  of the polynomial below.

$$f(x) = -9(x-6)^9(x+6)^{10}(x+2)^9(x-2)^{12}$$

The solution is the graph below, which is option B.



A.



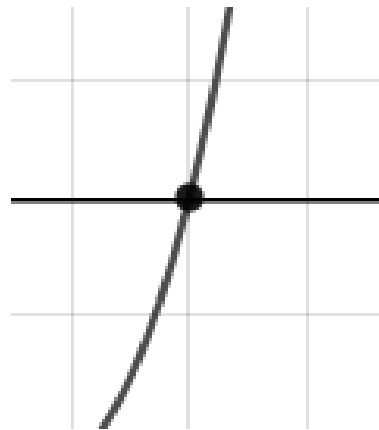
C.



B.



D.



E. None of the above.

**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

- 
26. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$-2, \frac{-7}{3}, \text{ and } \frac{3}{2}$$

The solution is  $6x^3 + 17x^2 - 11x - 42$ , which is option E.

A.  $a \in [5, 11], b \in [16, 20], c \in [-14, -9]$ , and  $d \in [40, 47]$

$6x^3 + 17x^2 - 11x + 42$ , which corresponds to multiplying everything correctly except the constant term.

B.  $a \in [5, 11], b \in [-12, -2], c \in [-33, -27]$ , and  $d \in [40, 47]$

$6x^3 - 7x^2 - 31x + 42$ , which corresponds to multiplying out  $(x - 2)(3x + 7)(2x - 3)$ .

C.  $a \in [5, 11], b \in [-43, -32], c \in [63, 68]$ , and  $d \in [-47, -37]$

$6x^3 - 35x^2 + 67x - 42$ , which corresponds to multiplying out  $(x - 2)(3x - 7)(2x - 3)$ .

D.  $a \in [5, 11], b \in [-21, -14], c \in [-14, -9]$ , and  $d \in [40, 47]$

$6x^3 - 17x^2 - 11x + 42$ , which corresponds to multiplying out  $(x - 2)(3x - 7)(2x + 3)$ .

E.  $a \in [5, 11], b \in [16, 20], c \in [-14, -9]$ , and  $d \in [-47, -37]$

\*  $6x^3 + 17x^2 - 11x - 42$ , which is the correct option.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(x+2)(3x+7)(2x-3)$

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27. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $x^3 + bx^2 + cx + d$ .

$$-4 - 3i \text{ and } -3$$

The solution is  $x^3 + 11x^2 + 49x + 75$ , which is option A.

A.  $b \in [11, 19], c \in [48.36, 49.78]$ , and  $d \in [72.6, 77.1]$

\*  $x^3 + 11x^2 + 49x + 75$ , which is the correct option.

B.  $b \in [0, 7], c \in [4.02, 6.83]$ , and  $d \in [7.9, 9.5]$

$x^3 + x^2 + 6x + 9$ , which corresponds to multiplying out  $(x + 3)(x + 3)$ .

C.  $b \in [0, 7], c \in [6.29, 9.06]$ , and  $d \in [11.8, 14.7]$

$x^3 + x^2 + 7x + 12$ , which corresponds to multiplying out  $(x + 4)(x + 3)$ .

D.  $b \in [-16, -10], c \in [48.36, 49.78]$ , and  $d \in [-76, -71.8]$

$x^3 - 11x^2 + 49x - 75$ , which corresponds to multiplying out  $(x - (-4 - 3i))(x - (-4 + 3i))(x - 3)$ .

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

**General Comment:** Remember that the conjugate of  $a + bi$  is  $a - bi$ . Since these zeros always come in pairs, we need to multiply out  $(x - (-4 - 3i))(x - (-4 + 3i))(x - (-3))$ .

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28. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form  $ax^3 + bx^2 + cx + d$ .

$$\frac{-7}{5}, \frac{-1}{4}, \text{ and } \frac{3}{5}$$

The solution is  $100x^3 + 105x^2 - 64x - 21$ , which is option E.

- A.  $a \in [100, 104]$ ,  $b \in [-230, -224]$ ,  $c \in [129, 136]$ , and  $d \in [-21, -15]$

$100x^3 - 225x^2 + 134x - 21$ , which corresponds to multiplying out  $(5x - 7)(4x - 1)(5x - 3)$ .

- B.  $a \in [100, 104]$ ,  $b \in [-177, -171]$ ,  $c \in [30, 38]$ , and  $d \in [20, 32]$

$100x^3 - 175x^2 + 34x + 21$ , which corresponds to multiplying out  $(5x - 7)(4x + 1)(5x - 3)$ .

- C.  $a \in [100, 104]$ ,  $b \in [-112, -104]$ ,  $c \in [-65, -60]$ , and  $d \in [20, 32]$

$100x^3 - 105x^2 - 64x + 21$ , which corresponds to multiplying out  $(5x - 7)(4x - 1)(5x + 3)$ .

- D.  $a \in [100, 104]$ ,  $b \in [103, 115]$ ,  $c \in [-65, -60]$ , and  $d \in [20, 32]$

$100x^3 + 105x^2 - 64x + 21$ , which corresponds to multiplying everything correctly except the constant term.

- E.  $a \in [100, 104]$ ,  $b \in [103, 115]$ ,  $c \in [-65, -60]$ , and  $d \in [-21, -15]$

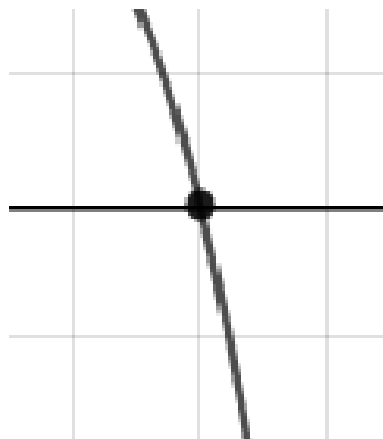
\*  $100x^3 + 105x^2 - 64x - 21$ , which is the correct option.

**General Comment:** To construct the lowest-degree polynomial, you want to multiply out  $(5x + 7)(4x + 1)(5x - 3)$

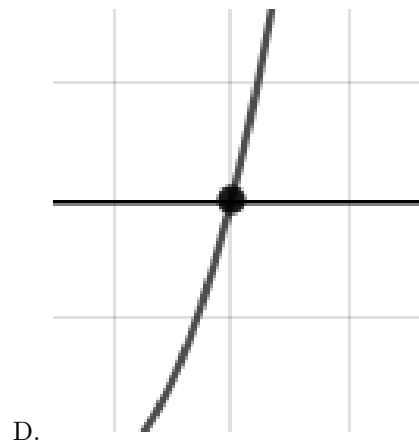
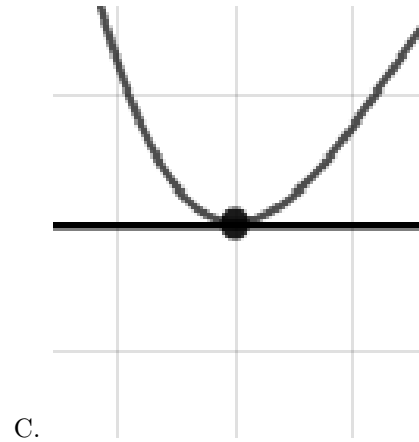
29. Describe the zero behavior of the zero  $x = 7$  of the polynomial below.

$$f(x) = -7(x - 3)^{11}(x + 3)^9(x - 7)^{14}(x + 7)^9$$

The solution is the graph below, which is option B.



A.



E. None of the above.

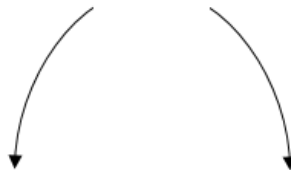
**General Comment:** You will need to sketch the entire graph, then zoom in on the zero the question asks about.

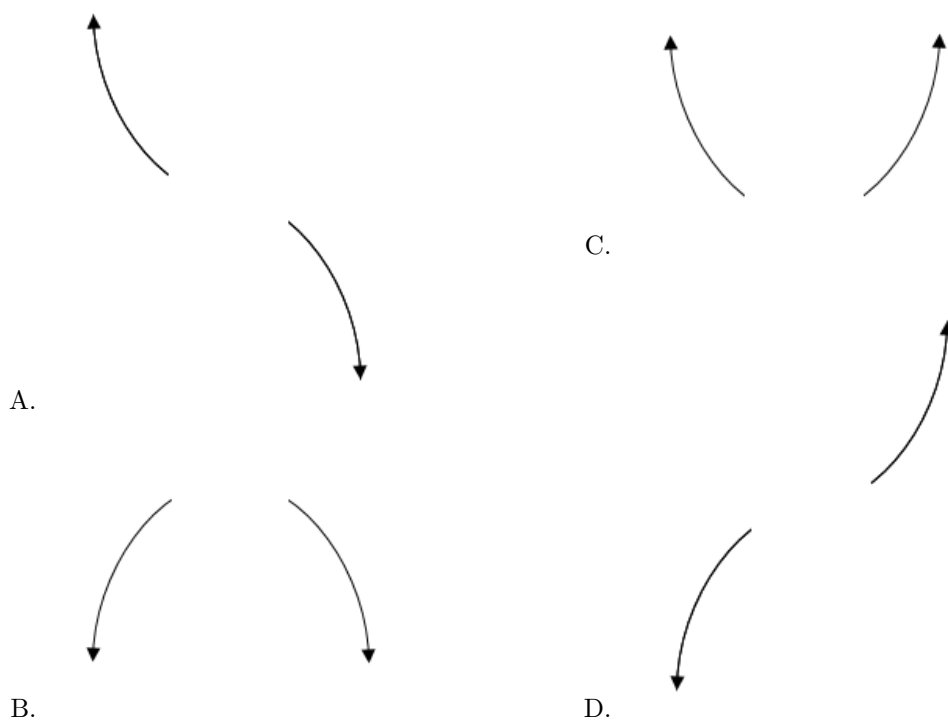
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30. Describe the end behavior of the polynomial below.

$$f(x) = -3(x + 9)^5(x - 9)^8(x - 3)^2(x + 3)^3$$

The solution is the graph below, which is option B.





E. None of the above.

**General Comment:** Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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