This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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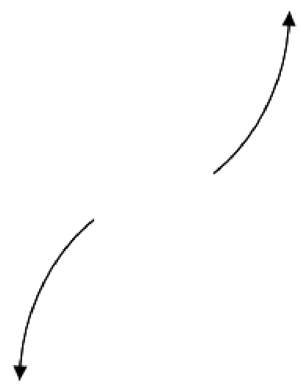
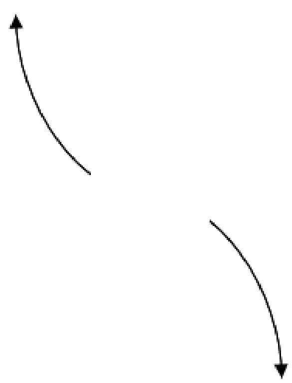
1. Describe the end behavior of the polynomial below.

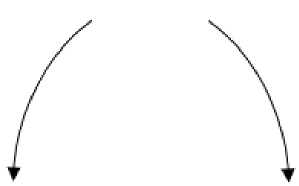
$$f(x) = 7(x+5)^4(x-5)^7(x-9)^3(x+9)^3$$

The solution is the graph below, which is option D.

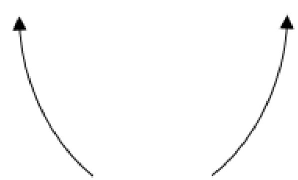




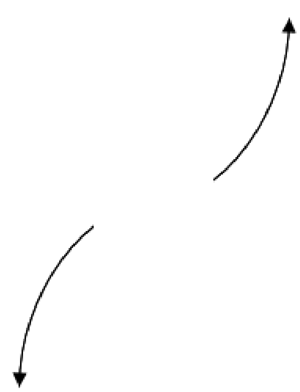
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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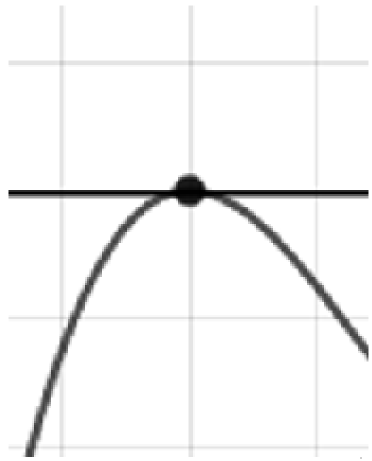
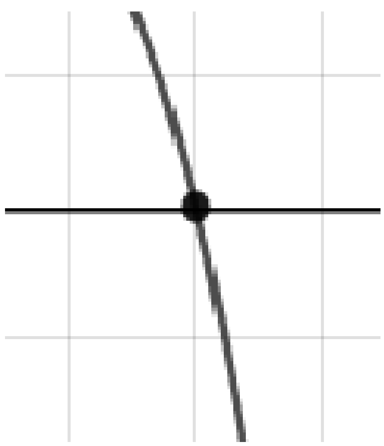
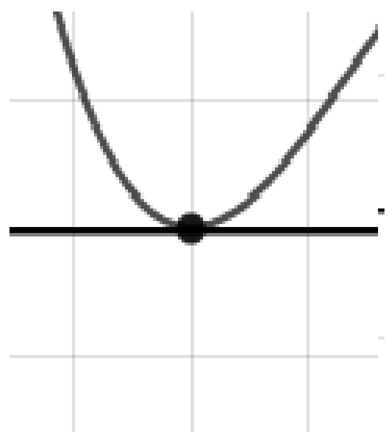
2. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

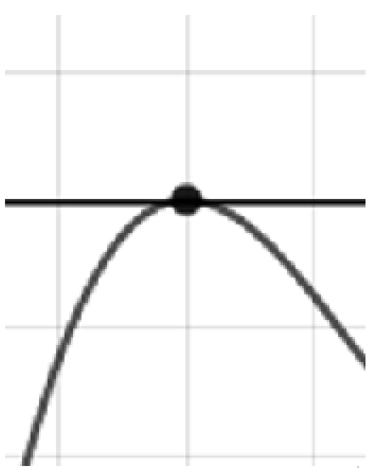
$$f(x) = 3(x-3)^9(x+3)^7(x+5)^4(x-5)^3$$

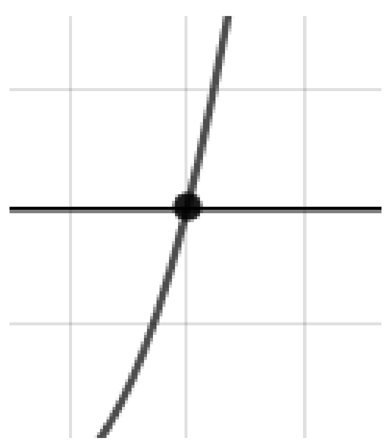
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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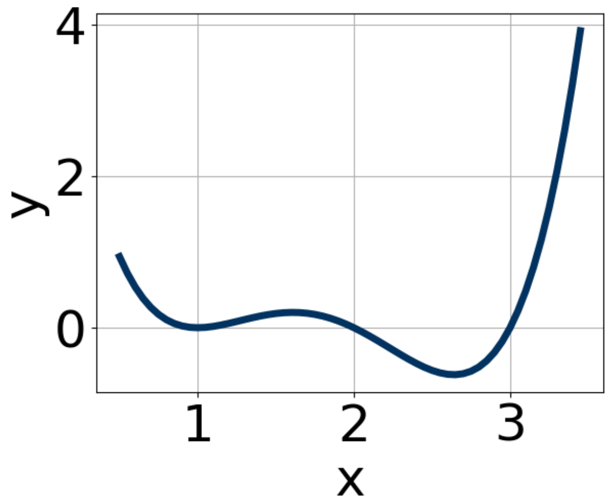
3. Which of the following equations *could* be of the graph presented below?



The solution is $7(x-1)^6(x-2)^{11}(x-3)^7$, which is option D.

A. $9(x-1)^{10}(x-2)^8(x-3)^5$

The factor $(x-2)$ should have an odd power.

B. $19(x-1)^7(x-2)^{10}(x-3)^5$

The factor 1 should have an even power and the factor 2 should have an odd power.

C. $-11(x-1)^4(x-2)^{11}(x-3)^4$

The factor $(x-3)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $7(x-1)^6(x-2)^{11}(x-3)^7$

* This is the correct option.

E. $-6(x-1)^{10}(x-2)^{11}(x-3)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$5 + 2i \text{ and } 4$$

The solution is $x^3 - 14x^2 + 69x - 116$, which is option D.

A. $b \in [13, 15], c \in [64, 73], \text{ and } d \in [114, 122]$

$x^3 + 14x^2 + 69x + 116$, which corresponds to multiplying out $(x-(5+2i))(x-(5-2i))(x+4)$.

B. $b \in [-4, 5], c \in [-19, -7],$ and $d \in [17, 23]$

$x^3 + x^2 - 9x + 20$, which corresponds to multiplying out $(x - 5)(x - 4)$.

C. $b \in [-4, 5], c \in [-8, -3],$ and $d \in [8, 14]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 2)(x - 4)$.

D. $b \in [-16, -13], c \in [64, 73],$ and $d \in [-125, -112]$

* $x^3 - 14x^2 + 69x - 116$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (5 + 2i))(x - (5 - 2i))(x - (4))$.

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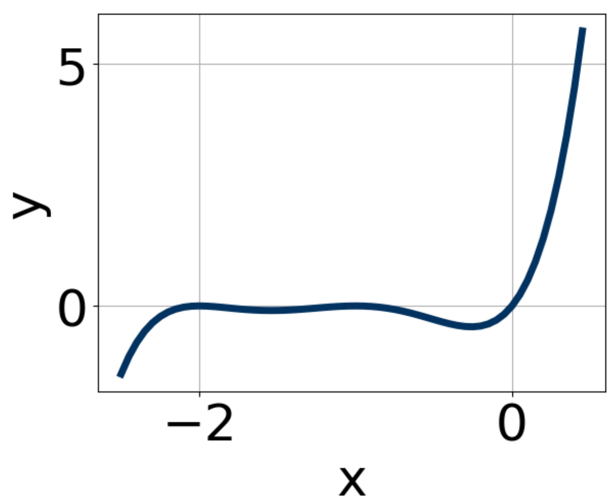
5. Which of the following equations *could* be of the graph presented below?



The solution is $2x^9(x + 1)^8(x + 2)^{10}$, which is option E.

A. $19x^5(x + 1)^6(x + 2)^7$

The factor $(x + 2)$ should have an even power.

B. $-14x^7(x + 1)^{10}(x + 2)^4$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $14x^6(x + 1)^{10}(x + 2)^9$

The factor $(x + 2)$ should have an even power and the factor x should have an odd power.

D. $-11x^4(x + 1)^{10}(x + 2)^4$

The factor x should have an odd power and the leading coefficient should be the opposite sign.

E. $2x^9(x + 1)^8(x + 2)^{10}$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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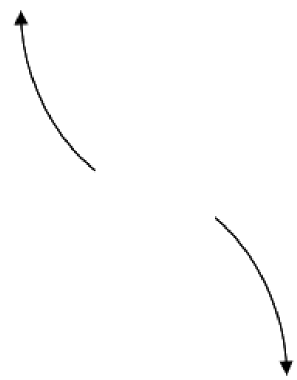
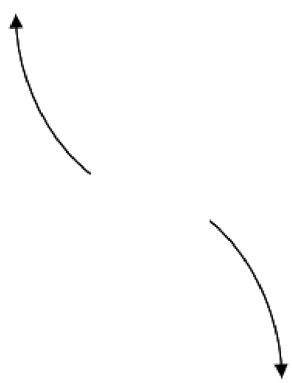
6. Describe the end behavior of the polynomial below.

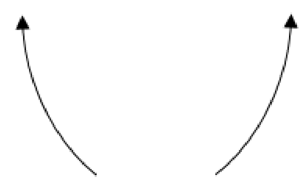
$$f(x) = -4(x-4)^3(x+4)^6(x+8)^5(x-8)^5$$

The solution is the graph below, which is option A.

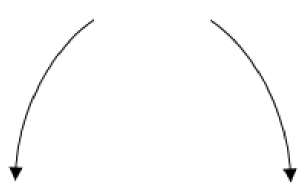




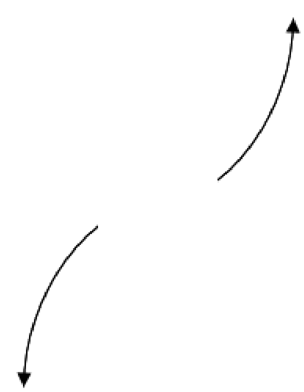
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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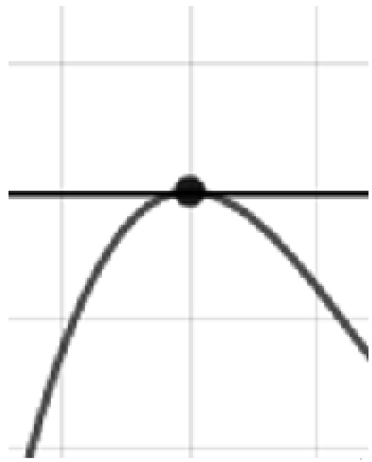
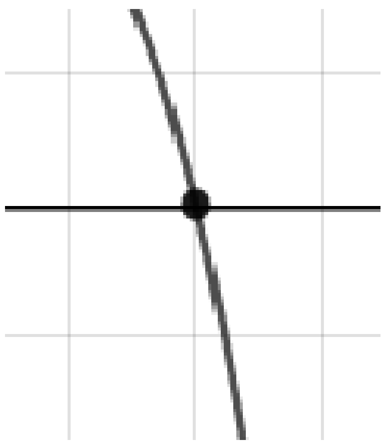
7. Describe the zero behavior of the zero $x = 9$ of the polynomial below.

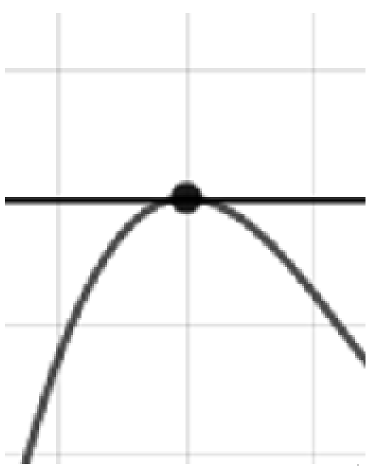
$$f(x) = -7(x-9)^4(x+9)^7(x+2)^6(x-2)^7$$

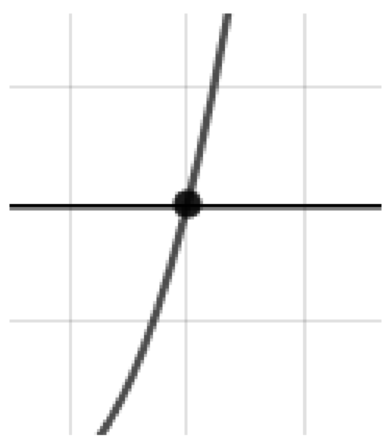
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, 4, \text{ and } \frac{3}{5}$$

The solution is $20x^3 - 97x^2 + 71x - 12$, which is option A.

A. $a \in [12, 21], b \in [-101, -93], c \in [61, 75], \text{ and } d \in [-15, -8]$

   * $20x^3 - 97x^2 + 71x - 12$, which is the correct option.

B. $a \in [12, 21], b \in [-101, -93], c \in [61, 75], \text{ and } d \in [8, 15]$

   $20x^3 - 97x^2 + 71x + 12$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [12, 21], b \in [-88, -82], c \in [23, 26], \text{ and } d \in [8, 15]$

   $20x^3 - 87x^2 + 25x + 12$, which corresponds to multiplying out $(4x + 1)(x - 4)(5x - 3)$.

D. $a \in [12, 21], b \in [95, 101], c \in [61, 75], \text{ and } d \in [8, 15]$

   $20x^3 + 97x^2 + 71x + 12$, which corresponds to multiplying out $(4x + 1)(x + 4)(5x + 3)$.

E. $a \in [12, 21], b \in [69, 77], c \in [-36, -28], \text{ and } d \in [-15, -8]$

   $20x^3 + 73x^2 - 31x - 12$, which corresponds to multiplying out $(4x + 1)(x + 4)(5x - 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(x - 4)(5x - 3)$

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 + 2i \text{ and } 1$$

The solution is $x^3 + 5x^2 + 7x - 13$, which is option B.

A. $b \in [-8, -3], c \in [7, 8], \text{ and } d \in [11, 14]$

   $x^3 - 5x^2 + 7x + 13$, which corresponds to multiplying out $(x - (-3 + 2i))(x - (-3 - 2i))(x + 1)$.

B. $b \in [2, 8], c \in [7, 8], \text{ and } d \in [-16, -8]$

   * $x^3 + 5x^2 + 7x - 13$, which is the correct option.

C. $b \in [1, 2], c \in [0, 4], \text{ and } d \in [-5, -2]$

   $x^3 + x^2 + 2x - 3$, which corresponds to multiplying out $(x + 3)(x - 1)$.

D. $b \in [1, 2], c \in [-7, 1], \text{ and } d \in [0, 5]$

   $x^3 + x^2 - 3x + 2$, which corresponds to multiplying out $(x - 2)(x - 1)$.

E. None of the above.

   This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 2i))(x - (-3 - 2i))(x - (1))$.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$4, \frac{-3}{5}, \text{ and } \frac{3}{4}$$

The solution is $20x^3 - 83x^2 + 3x + 36$, which is option C.

A. $a \in [19, 26], b \in [81, 88], c \in [3, 6], \text{ and } d \in [-37, -33]$

$20x^3 + 83x^2 + 3x - 36$, which corresponds to multiplying out $(x + 4)(5x - 3)(4x + 3)$.

B. $a \in [19, 26], b \in [76, 78], c \in [-22, -18], \text{ and } d \in [-37, -33]$

$20x^3 + 77x^2 - 21x - 36$, which corresponds to multiplying out $(x + 4)(5x + 3)(4x - 3)$.

C. $a \in [19, 26], b \in [-85, -80], c \in [3, 6], \text{ and } d \in [29, 39]$

* $20x^3 - 83x^2 + 3x + 36$, which is the correct option.

D. $a \in [19, 26], b \in [46, 63], c \in [-101, -94], \text{ and } d \in [29, 39]$

$20x^3 + 53x^2 - 99x + 36$, which corresponds to multiplying out $(x + 4)(5x - 3)(4x - 3)$.

E. $a \in [19, 26], b \in [-85, -80], c \in [3, 6], \text{ and } d \in [-37, -33]$

$20x^3 - 83x^2 + 3x - 36$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x - 4)(5x + 3)(4x - 3)$