This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Choose the graph of the equation below.

$$f(x) = \frac{1}{(x-2)^2} - 3$$

The solution is the graph below, which is option D.
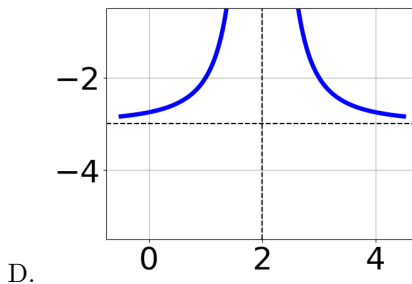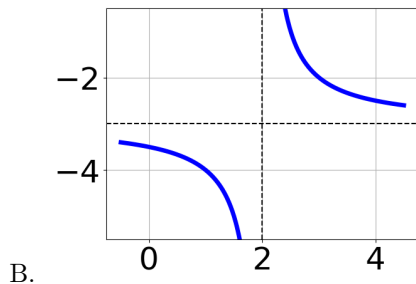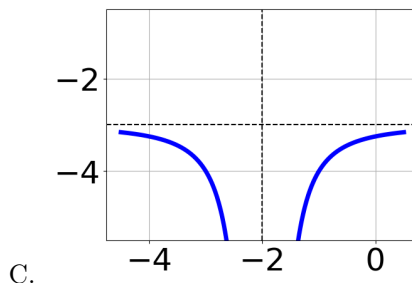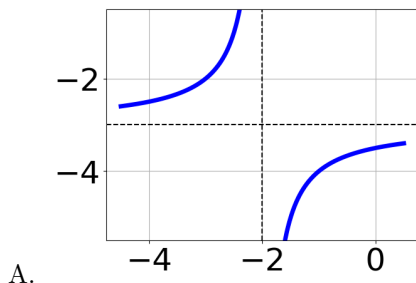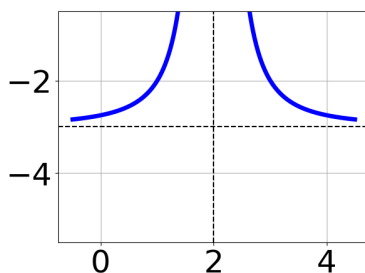
A.

C.

B.

D.

E. None of the above.

General Comment: Remember that the general form of a basic rational equation is $f(x) = \frac{a}{(x-h)^n} + k$, where a is the leading coefficient (and in this case, we assume is either 1 or -1), n is the degree (in this case, either 1 or 2), and (h, k) is the intersection of the asymptotes.

2. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{8}{5x-5} + 5 = \frac{-2}{40x-40}$$

The solution is $x = 0.670$, which is option C.

A. $x \in [-1.35, -1.3]$

   $x = -1.330$, which corresponds to not distributing the factor $5x - 5$ correctly when trying to eliminate the fraction.

B. $x_1 \in [0.58, 0.63]$ and $x_2 \in [-0.33, 2.67]$

   $x = 0.600$ and $x = 0.670$, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

C. $x \in [0.67, 2.67]$

   * $x = 0.670$, which is the correct option.

D. All solutions lead to invalid or complex values in the equation.

   This corresponds to thinking $x = 0.670$ leads to dividing by zero in the original equation, which it does not.

E. $x_1 \in [-1.35, -1.3]$ and $x_2 \in [-0.33, 2.67]$

   $x = -1.330$ and $x = 0.670$, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

General Comment: Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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3. Choose the graph of the equation below.

$$f(x) = \frac{-1}{(x-2)^2} - 1$$

The solution is the graph below, which is option A.

A.

B.
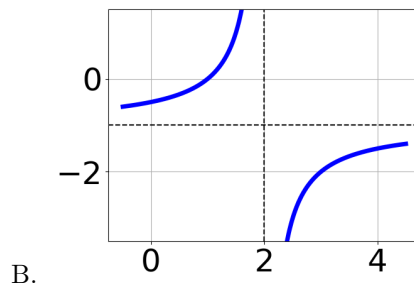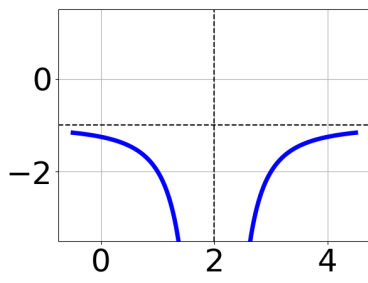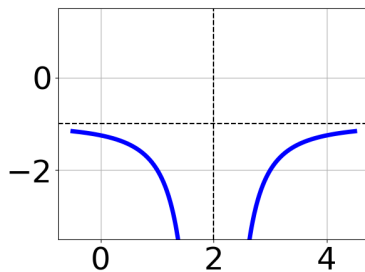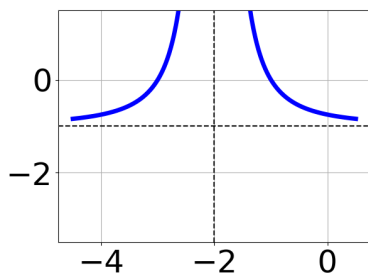
C.

D.

E. None of the above.

General Comment: Remember that the general form of a basic rational equation is $f(x) = \dfrac{a}{(x - h)^n} + k$, where a is the leading coefficient (and in this case, we assume is either 1 or -1), n is the degree (in this case, either 1 or 2), and (h, k) is the intersection of the asymptotes.

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4. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{50}{20x + 90} + 1 = \frac{50}{20x + 90}$$

The solution is all solutions are invalid or lead to complex values in the equation., which is option C.

A. $x \in [-5.5, -2.5]$

$x = -4.500$, which corresponds to not checking if this value leads to dividing by 0 in the original equation and thus is not a valid solution.

B. $x \in [3.5, 6.5]$

$x = 4.500$, which corresponds to not distributing the factor $20x + 90$ correctly when trying to eliminate the fraction.

C. All solutions lead to invalid or complex values in the equation.

*$x = -4.500$ leads to dividing by 0 in the original equation and thus is not a valid solution, which is the correct option.

D. $x_1 \in [-5.5, -3.5]$ and $x_2 \in [-5.5, -2.5]$

$x = -4.500$ and $x = -4.500$, which corresponds to getting the correct solution and believing there should be a second solution to the equation.
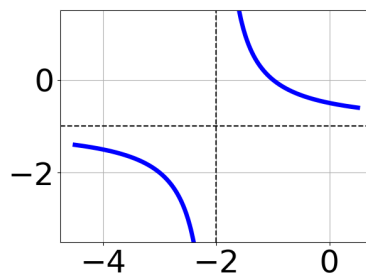
E. $x_1 \in [-5.5, -3.5]$ and $x_2 \in [3.5, 6.5]$

$x = -4.500$ and $x = 4.500$, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

General Comment: Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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5. Determine the domain of the function below.

$$f(x) = \frac{3}{20x^2 - 45x + 25}$$

The solution is All Real numbers except $x = 1.000$ and $x = 1.250$., which is option C.

A. All Real numbers except $x = a$, where $a \in [-0.9, 1.2]$

All Real numbers except $x = 1.000$, which corresponds to removing only 1 value from the denominator.

B. All Real numbers.

This corresponds to thinking the denominator has complex roots or that rational functions have a domain of all Real numbers.

C. All Real numbers except $x = a$ and $x = b$, where $a \in [-0.9, 1.2]$ and $b \in [1.1, 1.6]$

All Real numbers except $x = 1.000$ and $x = 1.250$, which is the correct option.

D. All Real numbers except $x = a$, where $a \in [19.7, 21.6]$

All Real numbers except $x = 20.000$, which corresponds to removing a distractor value from the denominator.

E. All Real numbers except $x = a$ and $x = b$, where $a \in [19.7, 21.6]$ and $b \in [23.9, 26.9]$

All Real numbers except $x = 20.000$ and $x = 25.000$, which corresponds to not factoring the denominator correctly.

General Comment: Recall that dividing by zero is not a real number. Therefore the domain is all real numbers **except** those that make the denominator 0.

6. Choose the equation of the function graphed below.
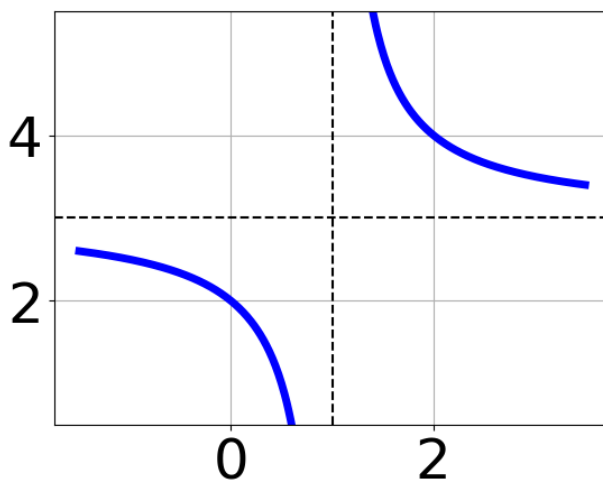
The solution is $f(x) = \dfrac{1}{x - 1} + 3$, which is option A.

A. $f(x) = \dfrac{1}{x - 1} + 3$

This is the correct option.

B. $f(x) = \dfrac{1}{(x - 1)^2} + 3$

Corresponds to thinking the graph was a shifted version of $\dfrac{1}{x^2}$.

C. $f(x) = \dfrac{-1}{x + 1} + 3$

Corresponds to using the general form $f(x) = \dfrac{a}{x+h} + k$ and the opposite leading coefficient.

D. $f(x) = \dfrac{-1}{(x+1)^2} + 3$

Corresponds to thinking the graph was a shifted version of $\dfrac{1}{x^2}$, using the general form $f(x) = \dfrac{a}{x+h} + k$, and the opposite leading coefficient.

E. None of the above

This corresponds to believing the vertex of the graph was not correct.

General Comment: Remember that the general form of a basic rational equation is $f(x) = \dfrac{a}{(x-h)^n} + k$, where a is the leading coefficient (and in this case, we assume is either 1 or -1), n is the degree (in this case, either 1 or 2), and (h, k) is the intersection of the asymptotes.

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7. Determine the domain of the function below.

$$f(x) = \frac{5}{30x^2 + 12x - 18}$$

The solution is All Real numbers except $x = -1.000$ and $x = 0.600.$, which is option E.

A. All Real numbers.

This corresponds to thinking the denominator has complex roots or that rational functions have a domain of all Real numbers.

B. All Real numbers except $x = a$, where $a \in [-38, -35]$

All Real numbers except $x = -36.000$, which corresponds to removing a distractor value from the denominator.

C. All Real numbers except $x = a$ and $x = b$, where $a \in [-38, -35]$ and $b \in [15, 17]$

All Real numbers except $x = -36.000$ and $x = 15.000$, which corresponds to not factoring the denominator correctly.

D. All Real numbers except $x = a$, where $a \in [-3, 0]$

All Real numbers except $x = -1.000$, which corresponds to removing only 1 value from the denominator.

E. All Real numbers except $x = a$ and $x = b$, where $a \in [-3, 0]$ and $b \in [0.6, 1.6]$

All Real numbers except $x = -1.000$ and $x = 0.600$, which is the correct option.

General Comment: Recall that dividing by zero is not a real number. Therefore the domain is all real numbers **except** those that make the denominator 0.

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8. Choose the equation of the function graphed below.
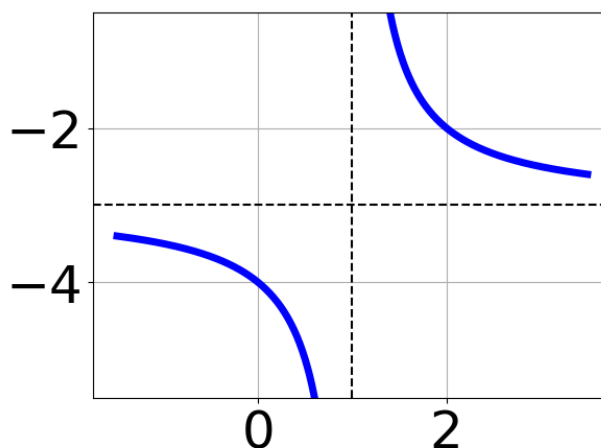
The solution is $f(x) = \dfrac{1}{x-1} - 3$, which is option C.

A. $f(x) = \dfrac{1}{(x-1)^2} - 3$

Corresponds to thinking the graph was a shifted version of $\dfrac{1}{x^2}$.

B. $f(x) = \dfrac{-1}{(x+1)^2} - 3$

Corresponds to thinking the graph was a shifted version of $\dfrac{1}{x^2}$, using the general form $f(x) = \dfrac{a}{x+h} + k$, and the opposite leading coefficient.

C. $f(x) = \dfrac{1}{x-1} - 3$

This is the correct option.

D. $f(x) = \dfrac{-1}{x+1} - 3$

Corresponds to using the general form $f(x) = \dfrac{a}{x+h} + k$ and the opposite leading coefficient.

E. None of the above

This corresponds to believing the vertex of the graph was not correct.

General Comment: Remember that the general form of a basic rational equation is $f(x) = \dfrac{a}{(x-h)^n} + k$, where a is the leading coefficient (and in this case, we assume is either 1 or -1), n is the degree (in this case, either 1 or 2), and (h, k) is the intersection of the asymptotes.

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9. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{-5x}{-6x+3} + \frac{-7x^2}{24x^2 - 36x + 12} = \frac{5}{-4x+4}$$

The solution is There are two solutions: $x = 0.756$ and $x = -1.526$, which is option A.

A. $x_1 \in [0.64, 0.78]$ and $x_2 \in [-2.37, 0.11]$

  * $x = 0.756$ and $x = -1.526$, which is the correct option.

B. $x \in [0.86, 1.04]$

C. $x_1 \in [0.64, 0.78]$ and $x_2 \in [-0.58, 1.58]$

D. All solutions lead to invalid or complex values in the equation.

E. $x \in [-1.77, -1.5]$

General Comment: Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

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10. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{5x}{3x - 6} + \frac{-2x^2}{-12x^2 + 12x + 24} = \frac{-4}{-4x - 4}$$

The solution is All solutions are invalid or lead to complex values in the equation., which is option E.

A. $x_1 \in [1.77, 3.6]$ and $x_2 \in [-1.17, -0.95]$

$x = 2.000$ and $x = -1.000$, which corresponds to solving $3x - 6 = 0$ and $-4x - 4 = 0$ and treating them as solutions to the equation.

B. $x \in [-2.56, 0.83]$

$x = -1.000$, which corresponds to solving $-4x - 4 = 0$ and treating it as a solution to the equation.

C. $x \in [1.77, 3.6]$

$x = 2.000$, which corresponds to solving $3x - 6 = 0$ and treating it as a solution to the equation.

D. $x_1 \in [-0.3, 0.93]$ and $x_2 \in [-2.17, -1.08]$

$x = 0.911$ and $x = -1.355$, which corresponds to making the discriminant from the Quadratic Formula positive to avoid complex solutions.

E. All solutions lead to invalid or complex values in the equation.

* The equation leads to solving $-18x^2 - 8x - 24 = 0$, which leads to complex solutions. This is the correct option.

General Comment: Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!