This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 + 39x^2 - 30}{x + 3}$$

The solution is $12x^2 + 3x - 9 + \frac{-3}{x+3}$, which is option C.

A. $a \in [-38, -33], b \in [147, 149], c \in [-441, -436]$, and $r \in [1291, 1296]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [11, 13], b \in [75, 77], c \in [220, 232]$, and $r \in [644, 646]$.

You divided by the opposite of the factor.

C. $a \in [11, 13], b \in [-3, 5], c \in [-12, -2]$, and $r \in [-7, 2]$.

* This is the solution!

D. $a \in [-38, -33], b \in [-70, -65], c \in [-207, -199]$, and $r \in [-654, -650]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

E. $a \in [11, 13], b \in [-14, -8], c \in [29, 38]$, and $r \in [-181, -171]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

2. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 - 44x^2 - 79x + 60$$

The solution is $[-1.67, 0.6, 4]$, which is option D.

A. $z_1 \in [-5, -2]$, $z_2 \in [-0.9, -0.2]$, and $z_3 \in [0.92, 1.85]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5, -2]$, $z_2 \in [-2.2, -0.9]$, and $z_3 \in [0.42, 0.84]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-0.6, 0.4]$, $z_2 \in [1.1, 2.8]$, and $z_3 \in [3.7, 4.42]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-1.67, -0.67],\ z_2 \in [-0.3, 0.8],$ and $z_3 \in [3.7, 4.42]$

* This is the solution!

E. $z_1 \in [-5, -2],\ z_2 \in [-3.3, -2.1],$ and $z_3 \in [-0.02, 0.34]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

3. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 + 3x^2 - 79x - 60$$

The solution is $[-2.5, -0.8, 3]$, which is option C.

A. $z_1 \in [-3.1, -2.7],\ z_2 \in [0, 0.71],$ and $z_3 \in [4.9, 5.12]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-3.1, -2.7],\ z_2 \in [0, 0.71],$ and $z_3 \in [1.12, 1.66]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-2.8, -1.6],\ z_2 \in [-0.89, -0.5],$ and $z_3 \in [2.53, 3.23]$

* This is the solution!

D. $z_1 \in [-3.1, -2.7],\ z_2 \in [0.74, 1.16],$ and $z_3 \in [2.35, 2.78]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-1.5, -0.9],\ z_2 \in [-0.58, -0.22],$ and $z_3 \in [2.53, 3.23]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 - 75x - 129}{x - 5}$$

The solution is $4x^2 + 20x + 25 + \frac{-4}{x-5}$, which is option E.

A. $a \in [2, 7], b \in [12, 18], c \in [-14, -6],$ and $r \in [-181, -171]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [2, 7], b \in [-26, -14], c \in [21, 26],$ and $r \in [-257, -246]$.

You divided by the opposite of the factor.

C. $a \in [17, 22], b \in [-103, -96], c \in [424, 427],$ and $r \in [-2255, -2252]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [17, 22], b \in [96, 105], c \in [424, 427]$, and $r \in [1992, 1999]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [2, 7], b \in [17, 23], c \in [21, 26]$, and $r \in [-7, -3]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 55x^2 - 30x - 43}{x + 3}$$

The solution is $20x^2 - 5x - 15 + \dfrac{2}{x+3}$, which is option E.

A. $a \in [-62, -56],\ b \in [-130, -124],\ c \in [-406, -400]$, and $r \in [-1263, -1256]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [19, 26],\ b \in [111, 121],\ c \in [310, 319]$, and $r \in [898, 908]$.

You divided by the opposite of the factor.

C. $a \in [-62, -56],\ b \in [231, 237],\ c \in [-735, -733]$, and $r \in [2160, 2164]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [19, 26],\ b \in [-26, -22],\ c \in [66, 73]$, and $r \in [-327, -319]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [19, 26],\ b \in [-6, -1],\ c \in [-18, -14]$, and $r \in [1, 8]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

6. Factor the polynomial below completely, knowing that $x - 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 - 113x^3 + 434x^2 - 655x + 300$$

The solution is $[0.8, 2.5, 3, 5]$, which is option C.

A. $z_1 \in [-1.5, 0.7],\ z_2 \in [0.88, 2.15], z_3 \in [2.83, 3.07]$, and $z_4 \in [4.76, 5.22]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-6.1, -4.5],\ z_2 \in [-3.06, -1.3], z_3 \in [-1.58, -0.95]$, and $z_4 \in [-0.43, -0.37]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [0.5, 0.9],\ z_2 \in [2.3, 2.76], z_3 \in [2.83, 3.07]$, and $z_4 \in [4.76, 5.22]$

* This is the solution!

D. $z_1 \in [-6.1, -4.5],\ z_2 \in [-3.06, -1.3], z_3 \in [-2.56, -2.44]$, and $z_4 \in [-0.91, -0.66]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-6.1, -4.5]$, $z_2 \in [-4.78, -3.6]$, $z_3 \in [-3.23, -2.63]$, and $z_4 \in [-0.61, -0.48]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

7. Factor the polynomial below completely, knowing that $x - 2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 - 83x^3 + 197x^2 - 188x + 60$$

The solution is $[0.667, 1.25, 2, 3]$, which is option E.

A. $z_1 \in [-3.21, -2.92]$, $z_2 \in [-2.11, -1.9]$, $z_3 \in [-1.87, -1.4]$, and $z_4 \in [-0.97, -0.76]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-3.21, -2.92]$, $z_2 \in [-2.11, -1.9]$, $z_3 \in [-2.06, -1.61]$, and $z_4 \in [-0.48, -0.26]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [0.79, 1.04]$, $z_2 \in [1.45, 1.69]$, $z_3 \in [1.79, 2.39]$, and $z_4 \in [2.98, 3.13]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.21, -2.92]$, $z_2 \in [-2.11, -1.9]$, $z_3 \in [-1.42, -1.16]$, and $z_4 \in [-0.73, -0.63]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [0.42, 0.78]$, $z_2 \in [0.65, 1.49]$, $z_3 \in [1.79, 2.39]$, and $z_4 \in [2.98, 3.13]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

8. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 46x^2 + 40x + 22}{x - 3}$$

The solution is $10x^2 - 16x - 8 + \frac{-2}{x-3}$, which is option B.

A. $a \in [29, 35]$, $b \in [41, 48]$, $c \in [169, 174]$, and $r \in [534, 540]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [10, 11]$, $b \in [-18, -9]$, $c \in [-8, -7]$, and $r \in [-5, 2]$.

* This is the solution!

C. $a \in [10, 11]$, $b \in [-76, -75]$, $c \in [265, 271]$, and $r \in [-787, -778]$.

You divided by the opposite of the factor.

D. $a \in [29, 35]$, $b \in [-137, -132]$, $c \in [448, 452]$, and $r \in [-1326, -1315]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [10, 11], \; b \in [-30, -22], \; c \in [-12, -9], \text{ and } r \in [-5, 2].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

9. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^2 + 7x + 7$$

The solution is $\pm 1, \pm 7$, which is option B.

A. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

10. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^2 + 6x + 4$$

The solution is $\pm 1, \pm 2, \pm 4$, which is option A.

A. $\pm 1, \pm 2, \pm 4$

* This is the solution **since we asked for the possible Integer roots**!

B. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

11. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 105x^2 - 128}{x + 5}$$

The solution is $20x^2 + 5x - 25 + \frac{-3}{x+5}$, which is option B.

A. $a \in [19, 27], b \in [-15, -11], c \in [89, 92]$, and $r \in [-670, -662]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [19, 27], b \in [2, 11], c \in [-30, -24]$, and $r \in [-7, -2]$.

* This is the solution!

C. $a \in [-105, -94], b \in [-397, -394], c \in [-1976, -1973]$, and $r \in [-10008, -9998]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [-105, -94], b \in [602, 607], c \in [-3027, -3024]$, and $r \in [14989, 15000]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [19, 27], b \in [203, 206], c \in [1023, 1026]$, and $r \in [4997, 5002]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

12. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 39x^2 - 61x + 30$$

The solution is $[-1.5, 0.4, 5]$, which is option A.

A. $z_1 \in [-2.4, -0.9]$, $z_2 \in [0.36, 0.97]$, and $z_3 \in [4.87, 5.67]$

* This is the solution!

B. $z_1 \in [-5.1, -4.1]$, $z_2 \in [-0.78, -0.09]$, and $z_3 \in [0.97, 1.69]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-1.4, 0.1]$, $z_2 \in [2.08, 3.12]$, and $z_3 \in [4.87, 5.67]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-5.1, -4.1]$, $z_2 \in [-3.19, -2.32]$, and $z_3 \in [0.45, 0.75]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-5.1, -4.1]$, $z_2 \in [-2.36, -1.87]$, and $z_3 \in [0.14, 0.36]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

13. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 - 1x^2 - 52x + 20$$

The solution is $[-2, 0.4, 1.67]$, which is option E.

A. $z_1 \in [-1.85, -1.24]$, $z_2 \in [-0.43, -0.35]$, and $z_3 \in [1.81, 2.03]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-2.78, -2.35]$, $z_2 \in [-0.6, -0.46]$, and $z_3 \in [1.81, 2.03]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.02, -4.61]$, $z_2 \in [-0.17, -0.02]$, and $z_3 \in [1.81, 2.03]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-2.33, -1.98]$, $z_2 \in [0.59, 0.64]$, and $z_3 \in [2.15, 2.76]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-2.33, -1.98]$, $z_2 \in [0.38, 0.48]$, and $z_3 \in [1.36, 1.85]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

14. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 - 62x + 33}{x + 3}$$

The solution is $8x^2 - 24x + 10 + \frac{3}{x+3}$, which is option E.

A. $a \in [4, 9], b \in [-39, -31], c \in [62, 69]$, and $r \in [-232, -225]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [-27, -21], b \in [-72, -67], c \in [-280, -277]$, and $r \in [-804, -800]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [4, 9], b \in [20, 26], c \in [7, 15]$, and $r \in [58, 66]$.

You divided by the opposite of the factor.

D. $a \in [-27, -21], b \in [71, 77], c \in [-280, -277]$, and $r \in [867, 868]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [4, 9], b \in [-28, -21], c \in [7, 15]$, and $r \in [2, 5]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

15. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 - 22x^2 + 4x + 26}{x - 5}$$

The solution is $4x^2 - 2x - 6 + \frac{-4}{x-5}$, which is option A.

A. $a \in [2, 5],\ b \in [-2, 2],\ c \in [-6, -5],$ and $r \in [-7, -1]$.

* This is the solution!

B. $a \in [20, 23],\ b \in [75, 79],\ c \in [394, 399],$ and $r \in [1991, 1997]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [2, 5],\ b \in [-8, -5],\ c \in [-24, -18],$ and $r \in [-58, -52]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [2, 5],\ b \in [-43, -39],\ c \in [213, 221],$ and $r \in [-1044, -1043]$.

You divided by the opposite of the factor.

E. $a \in [20, 23],\ b \in [-125, -115],\ c \in [610, 618],$ and $r \in [-3050, -3036]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

16. Factor the polynomial below completely, knowing that $x + 3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 4x^4 + 4x^3 - 51x^2 - 36x + 135$$

The solution is $[-3, -2.5, 1.5, 3]$, which is option A.

A. $z_1 \in [-5, 1],\ z_2 \in [-2.54, -2.45], z_3 \in [1.24, 1.53],$ and $z_4 \in [3, 4]$

* This is the solution!

B. $z_1 \in [-5, 1],\ z_2 \in [-0.8, -0.68], z_3 \in [2.74, 3.16],$ and $z_4 \in [5, 7]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-5, 1],\ z_2 \in [-0.72, -0.56], z_3 \in [0.32, 0.59],$ and $z_4 \in [3, 4]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-5, 1],\ z_2 \in [-0.5, -0.35], z_3 \in [0.42, 0.9],$ and $z_4 \in [3, 4]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-5, 1],\ z_2 \in [-1.5, -1.46], z_3 \in [2.24, 2.69],$ and $z_4 \in [3, 4]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

17. Factor the polynomial below completely, knowing that $x+4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 + 101x^3 + 165x^2 - 248x - 240$$

The solution is $[-5, -4, -0.75, 1.333]$, which is option E.

A. $z_1 \in [-0.46, 0.02]$, $z_2 \in [2.74, 3.09]$, $z_3 \in [3.87, 4.03]$, and $z_4 \in [3.99, 5.65]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-5.22, -4.73]$, $z_2 \in [-4.54, -3.29]$, $z_3 \in [-2.25, -0.9]$, and $z_4 \in [-0.17, 1]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-1.56, -0.95]$, $z_2 \in [0.63, 0.84]$, $z_3 \in [3.87, 4.03]$, and $z_4 \in [3.99, 5.65]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-0.96, -0.61]$, $z_2 \in [1.26, 1.46]$, $z_3 \in [3.87, 4.03]$, and $z_4 \in [3.99, 5.65]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-5.22, -4.73]$, $z_2 \in [-4.54, -3.29]$, $z_3 \in [-1, -0.5]$, and $z_4 \in [0.79, 1.62]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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18. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{25x^3 - 85x^2 + 15x + 40}{x - 3}$$

The solution is $25x^2 - 10x - 15 + \frac{-5}{x-3}$, which is option E.

A. $a \in [73, 76]$, $b \in [-314, -306]$, $c \in [945, 951]$, and $r \in [-2795, -2791]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [25, 26]$, $b \in [-163, -157]$, $c \in [492, 496]$, and $r \in [-1445, -1441]$.

You divided by the opposite of the factor.

C. $a \in [73, 76]$, $b \in [136, 145]$, $c \in [432, 438]$, and $r \in [1340, 1346]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [25, 26]$, $b \in [-42, -31]$, $c \in [-60, -51]$, and $r \in [-71, -65]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [25, 26]$, $b \in [-19, -9]$, $c \in [-17, -12]$, and $r \in [-5, -1]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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19. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 3x^2 + 5x + 4$$

The solution is $\pm 1, \pm 2, \pm 4$, which is option A.

A. $\pm 1, \pm 2, \pm 4$

 * This is the solution **since we asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 4}$

 Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3}$

 This would have been the solution **if asked for the possible Rational roots**!

D. $\pm 1, \pm 3$

 Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

 Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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20. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 5x^2 + 5x + 2$$

The solution is $\pm 1, \pm 2$, which is option B.

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 2}$

 Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 2$

 * This is the solution **since we asked for the possible Integer roots**!

C. $\pm 1, \pm 5$

 Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 5}$

 This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

 Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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21. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{16x^3 - 49x + 32}{x + 2}$$

The solution is $16x^2 - 32x + 15 + \frac{2}{x+2}$, which is option E.

A. $a \in [16, 18], b \in [31, 38], c \in [12, 17],$ and $r \in [59, 67]$.

You divided by the opposite of the factor.

B. $a \in [-34, -25], b \in [-69, -63], c \in [-182, -175],$ and $r \in [-324, -318]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [-34, -25], b \in [57, 67], c \in [-182, -175],$ and $r \in [385, 392]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [16, 18], b \in [-49, -47], c \in [91, 99],$ and $r \in [-253, -246]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [16, 18], b \in [-40, -28], c \in [12, 17],$ and $r \in [-1, 5]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

22. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 - 35x^2 + 66x - 40$$

The solution is $[1.33, 2, 2.5]$, which is option C.

A. $z_1 \in [-2.69, -2.1], \; z_2 \in [-3, -1.8],$ and $z_3 \in [-1.45, -1.14]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5.19, -4.42], \; z_2 \in [-3, -1.8],$ and $z_3 \in [-0.8, -0.42]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [1.1, 1.67], \; z_2 \in [1, 2.5],$ and $z_3 \in [2.32, 2.71]$

* This is the solution!

D. $z_1 \in [-2.18, -1.47], \; z_2 \in [-1.1, -0.6],$ and $z_3 \in [-0.62, -0.23]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [0.05, 0.53], \; z_2 \in [0.4, 1.5],$ and $z_3 \in [1.95, 2.11]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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23. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 - 45x^2 - 82x - 24$$

The solution is $[-0.8, -0.4, 3]$, which is option C.

A. $z_1 \in [-3.11, -2.79]$, $z_2 \in [0.24, 0.6]$, and $z_3 \in [0.38, 0.88]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-3.11, -2.79]$, $z_2 \in [1.07, 1.31]$, and $z_3 \in [2.14, 2.54]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-1.16, -0.39]$, $z_2 \in [-0.67, -0.28]$, and $z_3 \in [2.54, 3.27]$

* This is the solution!

D. $z_1 \in [-2.65, -2.14]$, $z_2 \in [-1.42, -1.09]$, and $z_3 \in [2.54, 3.27]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-3.11, -2.79]$, $z_2 \in [0.09, 0.19]$, and $z_3 \in [1.46, 2.4]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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24. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 35x^2 + 42}{x - 3}$$

The solution is $10x^2 - 5x - 15 + \frac{-3}{x-3}$, which is option C.

A. $a \in [28, 31], b \in [54, 58], c \in [160, 169]$, and $r \in [535, 539]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [28, 31], b \in [-126, -122], c \in [369, 376]$, and $r \in [-1084, -1081]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [5, 15], b \in [-6, -2], c \in [-20, -6]$, and $r \in [-5, 1]$.

* This is the solution!

D. $a \in [5, 15], b \in [-17, -7], c \in [-34, -25]$, and $r \in [-20, -12]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [5, 15], b \in [-65, -61], c \in [193, 197]$, and $r \in [-545, -541]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

25. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 - 34x^2 - 10x + 7}{x - 3}$$

The solution is $12x^2 + 2x - 4 + \frac{-5}{x-3}$, which is option C.

A. $a \in [31, 39]$, $b \in [71, 77]$, $c \in [211, 218]$, and $r \in [643, 650]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [10, 17]$, $b \in [-11, -8]$, $c \in [-30, -25]$, and $r \in [-58, -51]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [10, 17]$, $b \in [-2, 3]$, $c \in [-5, -2]$, and $r \in [-6, 0]$.

* This is the solution!

D. $a \in [10, 17]$, $b \in [-75, -64]$, $c \in [194, 203]$, and $r \in [-596, -584]$.

You divided by the opposite of the factor.

E. $a \in [31, 39]$, $b \in [-148, -138]$, $c \in [415, 418]$, and $r \in [-1243, -1237]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

26. Factor the polynomial below completely, knowing that $x + 3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 + 26x^3 - 37x^2 - 159x - 90$$

The solution is $[-3, -2, -0.75, 2.5]$, which is option C.

A. $z_1 \in [-1.23, -0.19]$, $z_2 \in [0.77, 1.43]$, $z_3 \in [1.38, 2.29]$, and $z_4 \in [2.6, 4.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5.3, -4.32]$, $z_2 \in [0.15, 0.47]$, $z_3 \in [1.38, 2.29]$, and $z_4 \in [2.6, 4.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-3.63, -2.76]$, $z_2 \in [-2.18, -1.85]$, $z_3 \in [-0.83, -0.16]$, and $z_4 \in [0.6, 2.9]$

* This is the solution!

D. $z_1 \in [-3.63, -2.76]$, $z_2 \in [-2.18, -1.85]$, $z_3 \in [-1.36, -0.79]$, and $z_4 \in [-0.4, 1.4]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-2.68, -2.27]$, $z_2 \in [0.61, 0.85]$, $z_3 \in [1.38, 2.29]$, and $z_4 \in [2.6, 4.3]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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27. Factor the polynomial below completely, knowing that $x+2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 - 29x^3 - 33x^2 + 116x - 60$$

The solution is $[-2, 0.75, 1.667, 2]$, which is option B.

A. $z_1 \in [-2.5, -1.9]$, $z_2 \in [-1.75, -1.65]$, $z_3 \in [-0.91, -0.74]$, and $z_4 \in [1, 5]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-2.5, -1.9]$, $z_2 \in [0.67, 0.79]$, $z_3 \in [1.58, 1.81]$, and $z_4 \in [1, 5]$

* This is the solution!

C. $z_1 \in [-2.5, -1.9]$, $z_2 \in [0.58, 0.66]$, $z_3 \in [1.23, 1.34]$, and $z_4 \in [1, 5]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-2.5, -1.9]$, $z_2 \in [-1.42, -1.31]$, $z_3 \in [-0.72, -0.45]$, and $z_4 \in [1, 5]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-3.2, -2.7]$, $z_2 \in [-2.01, -1.99]$, $z_3 \in [-0.58, -0.21]$, and $z_4 \in [1, 5]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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28. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 45x^2 - 15x + 45}{x - 2}$$

The solution is $20x^2 - 5x - 25 + \dfrac{-5}{x-2}$, which is option A.

A. $a \in [18, 23]$, $b \in [-8, -2]$, $c \in [-30, -22]$, and $r \in [-5, -2]$.

* This is the solution!

B. $a \in [40, 42]$, $b \in [-130, -123]$, $c \in [233, 239]$, and $r \in [-425, -423]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [18, 23]$, $b \in [-87, -83]$, $c \in [152, 156]$, and $r \in [-269, -264]$.

You divided by the opposite of the factor.

D. $a \in [18, 23]$, $b \in [-27, -22]$, $c \in [-40, -39]$, and $r \in [5, 10]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [40, 42]$, $b \in [31, 36]$, $c \in [52, 57]$, and $r \in [155, 161]$.

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

29. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 6x^3 + 2x^2 + 2x + 2$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$, which is option C.

A. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

B. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$

* This is the solution **since we asked for the possible Rational roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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30. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 3x^4 + 2x^3 + 6x^2 + 7x + 7$$

The solution is $\pm 1, \pm 7$, which is option C.

A. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 3}$

This would have been the solution **if asked for the possible Rational roots**!

C. $\pm 1, \pm 7$

* This is the solution **since we asked for the possible Integer roots**!

D. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.
