This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^4 + 2x^3 + 2x^2 + 2x + 6$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$, which is option C.

A. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

* This is the solution **since we asked for the possible Rational roots**!

D. $\pm 1, \pm 2, \pm 3, \pm 6$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

2. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^4 + 6x^3 + 3x^2 + 2x + 4$$

The solution is $\pm 1, \pm 2, \pm 4$, which is option A.

A. $\pm 1, \pm 2, \pm 4$

* This is the solution **since we asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

This would have been the solution **if asked for the possible Rational roots**!

D. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

3. Factor the polynomial below completely, knowing that $x+5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 14x^3 - 167x^2 + 455x - 300$$

The solution is $[-5, 1.25, 1.5, 4]$, which is option B.

A. $z_1 \in [-4.25, -3.95]$, $z_2 \in [-0.88, -0.53]$, $z_3 \in [-0.67, -0.65]$, and $z_4 \in [4.9, 5.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5.08, -4.64]$, $z_2 \in [0.79, 1.43]$, $z_3 \in [1.47, 1.53]$, and $z_4 \in [2.5, 4.4]$

* This is the solution!

C. $z_1 \in [-5.08, -4.64]$, $z_2 \in [-0.31, 0.74]$, $z_3 \in [0.76, 0.84]$, and $z_4 \in [2.5, 4.4]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-4.25, -3.95]$, $z_2 \in [-3.36, -2.87]$, $z_3 \in [-0.63, -0.53]$, and $z_4 \in [4.9, 5.1]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-4.25, -3.95]$, $z_2 \in [-2.41, -0.84]$, $z_3 \in [-1.26, -1.22]$, and $z_4 \in [4.9, 5.1]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 - 12x + 6}{x + 2}$$

The solution is $4x^2 - 8x + 4 + \frac{-2}{x+2}$, which is option D.

A. $a \in [3, 8], b \in [-13, -10], c \in [15, 25]$, and $r \in [-67, -61]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [3, 8], b \in [8, 10], c \in [-1, 8]$, and $r \in [8, 15]$.

You divided by the opposite of the factor.

C. $a \in [-10, -4], b \in [10, 17], c \in [-48, -42]$, and $r \in [94, 97]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [3, 8], b \in [-9, 0], c \in [-1, 8]$, and $r \in [-5, 4]$.

* This is the solution!

E. $a \in [-10, -4], b \in [-20, -15], c \in [-48, -42]$, and $r \in [-85, -81]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 63x^2 + 23}{x - 3}$$

The solution is $20x^2 - 3x - 9 + \frac{-4}{x-3}$, which is option D.

A. $a \in [57, 65], b \in [113, 120], c \in [350, 355]$, and $r \in [1074, 1078]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [17, 22], b \in [-130, -118], c \in [369, 371]$, and $r \in [-1085, -1082]$.

You divided by the opposite of the factor.

C. $a \in [57, 65], b \in [-245, -241], c \in [729, 731]$, and $r \in [-2169, -2161]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [17, 22], b \in [-5, 0], c \in [-13, -7]$, and $r \in [-6, 4]$.

* This is the solution!

E. $a \in [17, 22], b \in [-29, -22], c \in [-47, -42]$, and $r \in [-70, -68]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

6. Factor the polynomial below completely, knowing that $x - 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 - 113x^3 + 338x^2 - 395x + 150$$

The solution is $[0.75, 1.667, 2, 5]$, which is option C.

A. $z_1 \in [-5.35, -4.91]$, $z_2 \in [-6.33, -4.95], z_3 \in [-2.42, -1.69]$, and $z_4 \in [-0.34, -0.14]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-5.35, -4.91]$, $z_2 \in [-2.13, -1.6], z_3 \in [-1.8, -1.62]$, and $z_4 \in [-0.84, -0.74]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [0.72, 1.07]$, $z_2 \in [1.35, 1.95], z_3 \in [1.61, 2.61]$, and $z_4 \in [4.79, 5.08]$

* This is the solution!

D. $z_1 \in [0.59, 0.69]$, $z_2 \in [1.11, 1.62], z_3 \in [1.61, 2.61]$, and $z_4 \in [4.79, 5.08]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-5.35, -4.91]$, $z_2 \in [-2.13, -1.6], z_3 \in [-1.44, -1.27]$, and $z_4 \in [-0.68, -0.44]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

7. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 - 100x^2 - 4x + 16$$

The solution is $[-0.4, 0.4, 4]$, which is option D.

A. $z_1 \in [-4.6, -3.3]$, $z_2 \in [-2.65, -2.36]$, and $z_3 \in [2.09, 3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-2.9, -2.4]$, $z_2 \in [1.93, 2.94]$, and $z_3 \in [3.93, 4.24]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-4.6, -3.3]$, $z_2 \in [-2.25, -1.76]$, and $z_3 \in [-0.2, 0.15]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-1.6, 0.4]$, $z_2 \in [0.19, 0.79]$, and $z_3 \in [3.93, 4.24]$

* This is the solution!

E. $z_1 \in [-4.6, -3.3]$, $z_2 \in [-0.56, -0.04]$, and $z_3 \in [0.11, 1.11]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 + 31x^2 - 38x - 40$$

The solution is $[-2, -0.8, 1.25]$, which is option E.

A. $z_1 \in [-0.92, -0.65]$, $z_2 \in [1.12, 1.47]$, and $z_3 \in [1.78, 2.59]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-5.08, -4.9]$, $z_2 \in [-0.09, 0.27]$, and $z_3 \in [1.78, 2.59]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-1.46, -1.02]$, $z_2 \in [0.78, 1.08]$, and $z_3 \in [1.78, 2.59]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-2.2, -1.8]$, $z_2 \in [-1.59, -1.18]$, and $z_3 \in [0.41, 0.86]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-2.2, -1.8]$, $z_2 \in [-0.86, -0.52]$, and $z_3 \in [1.1, 1.49]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 - 2x^2 - 20x + 19}{x + 2}$$

The solution is $6x^2 - 14x + 8 + \frac{3}{x+2}$, which is option E.

A. $a \in [-15, -8]$, $b \in [-28, -25]$, $c \in [-72, -68]$, and $r \in [-132, -124]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [-15, -8]$, $b \in [22, 24]$, $c \in [-66, -63]$, and $r \in [144, 149]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [1, 11]$, $b \in [-21, -19]$, $c \in [34, 46]$, and $r \in [-103, -97]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [1, 11]$, $b \in [8, 17]$, $c \in [-3, 4]$, and $r \in [15, 20]$.

You divided by the opposite of the factor.

E. $a \in [1, 11]$, $b \in [-14, -9]$, $c \in [7, 9]$, and $r \in [2, 4]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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10. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 - 20x^2 - 2x + 19}{x - 3}$$

The solution is $6x^2 - 2x - 8 + \frac{-5}{x-3}$, which is option D.

A. $a \in [5, 9]$, $b \in [-40, -34]$, $c \in [111, 115]$, and $r \in [-322, -313]$.

You divided by the opposite of the factor.

B. $a \in [15, 21]$, $b \in [-76, -72]$, $c \in [218, 224]$, and $r \in [-646, -637]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [5, 9]$, $b \in [-14, -3]$, $c \in [-23, -16]$, and $r \in [-18, -15]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [5, 9]$, $b \in [-6, 6]$, $c \in [-15, -7]$, and $r \in [-6, -3]$.

* This is the solution!

E. $a \in [15, 21]$, $b \in [29, 35]$, $c \in [95, 104]$, and $r \in [318, 321]$.

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

11. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^3 + 7x^2 + 3x + 5$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 7}$, which is option A.

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 7}$

* This is the solution **since we asked for the possible Rational roots**!

B. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

12. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^4 + 3x^3 + 3x^2 + 3x + 6$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$, which is option A.

A. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. $\pm 1, \pm 2, \pm 3, \pm 6$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

13. Factor the polynomial below completely, knowing that $x+5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 9x^4 + 27x^3 - 127x^2 - 155x + 150$$

The solution is $[-5, -1.667, 0.667, 3]$, which is option A.

A. $z_1 \in [-5.2, -4.9], \; z_2 \in [-1.81, -1.57], z_3 \in [0.65, 0.78], \text{ and } z_4 \in [1.9, 4.5]$

* This is the solution!

B. $z_1 \in [-5.2, -4.9], \; z_2 \in [-0.62, -0.5], z_3 \in [1.46, 1.56], \text{ and } z_4 \in [1.9, 4.5]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-4.5, -1.9], \; z_2 \in [-0.8, -0.62], z_3 \in [1.62, 1.74], \text{ and } z_4 \in [3.6, 5.7]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-4.5, -1.9], \; z_2 \in [-0.25, -0.17], z_3 \in [4.88, 5.04], \text{ and } z_4 \in [3.6, 5.7]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-4.5, -1.9], \; z_2 \in [-1.51, -1.43], z_3 \in [0.59, 0.62], \text{ and } z_4 \in [3.6, 5.7]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

14. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 62x^2 - 16}{x + 3}$$

The solution is $20x^2 + 2x - 6 + \dfrac{2}{x+3}$, which is option C.

A. $a \in [-60, -57], b \in [242, 243], c \in [-730, -721], \text{ and } r \in [2161, 2168].$

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [-60, -57], b \in [-119, -110], c \in [-354, -349], \text{ and } r \in [-1080, -1074].$

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [18, 23], b \in [-2, 7], c \in [-13, -1], \text{ and } r \in [-2, 3].$

* This is the solution!

D. $a \in [18, 23], b \in [-18, -15], c \in [70, 76], \text{ and } r \in [-311, -303].$

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [18, 23], b \in [120, 128], c \in [364, 373], \text{ and } r \in [1076, 1088].$

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

15. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 30x^2 + 43}{x - 2}$$

The solution is $10x^2 - 10x - 20 + \frac{3}{x-2}$, which is option D.

A. $a \in [16, 23], b \in [9, 11], c \in [12, 29]$, and $r \in [81, 88]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [16, 23], b \in [-70, -67], c \in [139, 141]$, and $r \in [-239, -231]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [6, 13], b \in [-20, -15], c \in [-21, -18]$, and $r \in [21, 24]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [6, 13], b \in [-19, -7], c \in [-21, -18]$, and $r \in [-1, 4]$.

* This is the solution!

E. $a \in [6, 13], b \in [-50, -48], c \in [95, 104]$, and $r \in [-158, -154]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

16. Factor the polynomial below completely, knowing that $x + 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 10x^3 - 101x^2 + 238x - 120$$

The solution is $[-4, 0.75, 2, 2.5]$, which is option B.

A. $z_1 \in [-2.15, -1.66]$, $z_2 \in [-1.58, -1.33]$, $z_3 \in [-0.52, -0.16]$, and $z_4 \in [3.33, 4.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-4.34, -3.6]$, $z_2 \in [0.46, 0.95]$, $z_3 \in [1.97, 2.28]$, and $z_4 \in [2.01, 2.65]$

* This is the solution!

C. $z_1 \in [-2.68, -2.27]$, $z_2 \in [-2.16, -1.71]$, $z_3 \in [-0.92, -0.7]$, and $z_4 \in [3.33, 4.3]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-4.34, -3.6]$, $z_2 \in [0.37, 0.42]$, $z_3 \in [1.07, 1.43]$, and $z_4 \in [0.8, 2.23]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-3.35, -2.63]$, $z_2 \in [-2.16, -1.71]$, $z_3 \in [-0.64, -0.62]$, and $z_4 \in [3.33, 4.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

17. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 - 43x^2 - 3x + 18$$

The solution is $[-0.6, 0.75, 2]$, which is option D.

A. $z_1 \in [-1.74, -1.55]$, $z_2 \in [1.33, 1.4]$, and $z_3 \in [1.74, 2.19]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-2.07, -1.88]$, $z_2 \in [-0.9, -0.68]$, and $z_3 \in [0.53, 0.61]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-2.07, -1.88]$, $z_2 \in [-0.47, 0.06]$, and $z_3 \in [2.75, 3.01]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-0.8, -0.48]$, $z_2 \in [0.43, 0.86]$, and $z_3 \in [1.74, 2.19]$

* This is the solution!

E. $z_1 \in [-2.07, -1.88]$, $z_2 \in [-1.41, -1.21]$, and $z_3 \in [1.07, 1.95]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

18. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 + 50x^2 - 9x - 18$$

The solution is $[-2, -0.6, 0.6]$, which is option E.

A. $z_1 \in [-3.1, -2.6]$, $z_2 \in [0.06, 0.51]$, and $z_3 \in [1.92, 2.63]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-1.9, -0.7]$, $z_2 \in [1.54, 1.86]$, and $z_3 \in [1.92, 2.63]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-1.5, 0.1]$, $z_2 \in [0.29, 1.22]$, and $z_3 \in [1.92, 2.63]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-2.1, -1.8]$, $z_2 \in [-1.91, -1.44]$, and $z_3 \in [1.3, 1.89]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-2.1, -1.8]$, $z_2 \in [-1.34, -0.43]$, and $z_3 \in [0.54, 0.97]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

19. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 29x^2 - 50x + 26}{x - 4}$$

The solution is $10x^2 + 11x - 6 + \frac{2}{x-4}$, which is option E.

A. $a \in [38, 42]$, $b \in [130, 134]$, $c \in [471, 482]$, and $r \in [1920, 1925]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [5, 15]$, $b \in [-73, -61]$, $c \in [220, 231]$, and $r \in [-883, -870]$.

You divided by the opposite of the factor.

C. $a \in [38, 42]$, $b \in [-190, -187]$, $c \in [704, 707]$, and $r \in [-2800, -2793]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

D. $a \in [5, 15]$, $b \in [0, 4]$, $c \in [-47, -45]$, and $r \in [-117, -112]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [5, 15]$, $b \in [9, 14]$, $c \in [-13, -3]$, and $r \in [0, 7]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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20. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 85x^2 + 200x - 129}{x - 5}$$

The solution is $10x^2 - 35x + 25 + \frac{-4}{x-5}$, which is option C.

A. $a \in [49, 53]$, $b \in [-335, -331]$, $c \in [1872, 1879]$, and $r \in [-9510, -9502]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [3, 11]$, $b \in [-136, -134]$, $c \in [875, 882]$, and $r \in [-4504, -4494]$.

You divided by the opposite of the factor.

C. $a \in [3, 11]$, $b \in [-41, -33]$, $c \in [25, 28]$, and $r \in [-4, 1]$.

* This is the solution!

D. $a \in [49, 53]$, $b \in [164, 171]$, $c \in [1020, 1033]$, and $r \in [4988, 5000]$.

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [3, 11]$, $b \in [-45, -44]$, $c \in [18, 23]$, and $r \in [-51, -48]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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21. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 5x^2 + 3x + 6$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 5}$, which is option D.

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 2, \pm 3, \pm 6$

This would have been the solution **if asked for the possible Integer roots**!

C. $\pm 1, \pm 5$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 5}$

* This is the solution **since we asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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22. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 4x^3 + 5x^2 + 7x + 5$$

The solution is $\pm 1, \pm 5$, which is option B.

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 2, \pm 4}$

This would have been the solution **if asked for the possible Rational roots**!

B. $\pm 1, \pm 5$

* This is the solution **since we asked for the possible Integer roots**!

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

23. Factor the polynomial below completely, knowing that $x-2$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 - 71x^3 + 12x^2 + 116x + 48$$

The solution is $[-0.667, -0.6, 2, 4]$, which is option D.

A. $z_1 \in [-5.2, -2.7], \ z_2 \in [-2.28, -1.89], z_3 \in [0.55, 0.73], \text{ and } z_4 \in [-0.06, 1]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-5.2, -2.7], \ z_2 \in [-2.28, -1.89], z_3 \in [1.36, 1.63], \text{ and } z_4 \in [1.29, 2.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.2, -2.7], \ z_2 \in [-2.28, -1.89], z_3 \in [0.08, 0.3], \text{ and } z_4 \in [2.98, 3.64]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-0.8, -0.3], \ z_2 \in [-0.67, -0.27], z_3 \in [1.8, 2.48], \text{ and } z_4 \in [3.99, 4.54]$

* This is the solution!

E. $z_1 \in [-2, -1], \ z_2 \in [-1.72, -1.24], z_3 \in [1.8, 2.48], \text{ and } z_4 \in [3.99, 4.54]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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24. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 - 24x^2 + 27}{x - 2}$$

The solution is $8x^2 - 8x - 16 + \frac{-5}{x-2}$, which is option C.

A. $a \in [14, 18], b \in [8, 9], c \in [16, 17], \text{ and } r \in [58, 60]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [14, 18], b \in [-56, -55], c \in [109, 118], \text{ and } r \in [-197, -196]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [5, 10], b \in [-11, -2], c \in [-16, -11], \text{ and } r \in [-5, -4]$.

* This is the solution!

D. $a \in [5, 10], b \in [-17, -12], c \in [-16, -11], \text{ and } r \in [7, 17]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [5, 10], b \in [-43, -39], c \in [74, 87], \text{ and } r \in [-135, -130]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

25. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{16x^3 + 84x^2 - 97}{x + 5}$$

The solution is $16x^2 + 4x - 20 + \frac{3}{x+5}$, which is option B.

A. $a \in [16, 19], b \in [164, 167], c \in [820, 821], \text{ and } r \in [4001, 4004]$.

You divided by the opposite of the factor.

B. $a \in [16, 19], b \in [1, 6], c \in [-20, -18], \text{ and } r \in [-1, 4]$.

* This is the solution!

C. $a \in [-82, -76], b \in [-320, -311], c \in [-1583, -1577], \text{ and } r \in [-7999, -7993]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [-82, -76], b \in [482, 491], c \in [-2420, -2411], \text{ and } r \in [12002, 12004]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [16, 19], b \in [-12, -9], c \in [69, 77], \text{ and } r \in [-535, -527]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

26. Factor the polynomial below completely, knowing that $x - 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 30x^3 - 87x^2 + 155x + 150$$

The solution is $[-2.5, -0.75, 2, 5]$, which is option E.

A. $z_1 \in [-6.2, -4.4], \; z_2 \in [-2.15, -1.97], z_3 \in [0.56, 0.64], \text{ and } z_4 \in [2.55, 3.08]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-1.8, -1.1], \; z_2 \in [-0.43, -0.15], z_3 \in [1.84, 2.03], \text{ and } z_4 \in [4.4, 5.57]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-6.2, -4.4], \; z_2 \in [-2.15, -1.97], z_3 \in [0.34, 0.55], \text{ and } z_4 \in [0.91, 1.86]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-6.2, -4.4], \; z_2 \in [-2.15, -1.97], z_3 \in [0.67, 0.99], \text{ and } z_4 \in [2.14, 2.86]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-4, -2.2], \; z_2 \in [-0.79, -0.71], z_3 \in [1.84, 2.03], \text{ and } z_4 \in [4.4, 5.57]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

27. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 - 83x^2 - 95x + 50$$

The solution is $[-1.25, 0.4, 5]$, which is option E.

A. $z_1 \in [-1.16, -0.26]$, $z_2 \in [2.38, 3.36]$, and $z_3 \in [4.32, 5.39]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-5.24, -4.84]$, $z_2 \in [-2.8, -1.73]$, and $z_3 \in [0.34, 0.82]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.24, -4.84]$, $z_2 \in [-0.35, -0.02]$, and $z_3 \in [4.32, 5.39]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-5.24, -4.84]$, $z_2 \in [-1.03, -0.32]$, and $z_3 \in [1.09, 1.48]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-1.45, -1.22]$, $z_2 \in [-0.07, 0.58]$, and $z_3 \in [4.32, 5.39]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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28. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 - 77x^2 + 89x - 30$$

The solution is $[0.6, 1.25, 2]$, which is option B.

A. $z_1 \in [0.74, 0.84]$, $z_2 \in [1.47, 1.68]$, and $z_3 \in [1.89, 2.27]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [0.49, 0.73]$, $z_2 \in [1.14, 1.3]$, and $z_3 \in [1.89, 2.27]$

* This is the solution!

C. $z_1 \in [-2.22, -1.99]$, $z_2 \in [-1.85, -1.62]$, and $z_3 \in [-0.9, -0.71]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-2.22, -1.99]$, $z_2 \in [-1.28, -1.13]$, and $z_3 \in [-0.63, -0.44]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-3.19, -2.76]$, $z_2 \in [-2.02, -1.87]$, and $z_3 \in [-0.35, -0.18]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

29. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6x^3 - 46x^2 + 88x - 43}{x - 5}$$

The solution is $6x^2 - 16x + 8 + \dfrac{-3}{x-5}$, which is option E.

A. $a \in [1, 13]$, $b \in [-26, -19]$, $c \in [-4, 3]$, and $r \in [-43, -40]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [1, 13]$, $b \in [-77, -70]$, $c \in [465, 475]$, and $r \in [-2386, -2380]$.

You divided by the opposite of the factor.

C. $a \in [26, 32]$, $b \in [102, 112]$, $c \in [605, 610]$, and $r \in [2996, 3001]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [26, 32]$, $b \in [-198, -189]$, $c \in [1064, 1072]$, and $r \in [-5386, -5379]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [1, 13]$, $b \in [-19, -13]$, $c \in [8, 14]$, and $r \in [-7, 2]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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30. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 - 64x^2 + 100x - 52}{x - 3}$$

The solution is $12x^2 - 28x + 16 + \dfrac{-4}{x-3}$, which is option A.

A. $a \in [8, 17]$, $b \in [-28, -25]$, $c \in [14, 17]$, and $r \in [-4, 0]$.

* This is the solution!

B. $a \in [8, 17]$, $b \in [-100, -98]$, $c \in [400, 402]$, and $r \in [-1254, -1246]$.

You divided by the opposite of the factor.

C. $a \in [33, 45]$, $b \in [39, 48]$, $c \in [226, 233]$, and $r \in [642, 646]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [33, 45]$, $b \in [-175, -166]$, $c \in [616, 624]$, and $r \in [-1903, -1893]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [8, 17]$, $b \in [-44, -38]$, $c \in [19, 21]$, and $r \in [-19, -11]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!
