This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

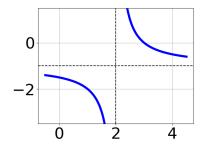
If you have a suggestion to make the keys better, please fill out the short survey here.

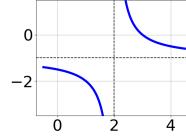
Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

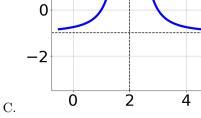
1. Choose the graph of the equation below.

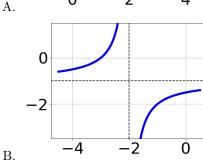
$$f(x) = \frac{1}{x-2} - 1$$

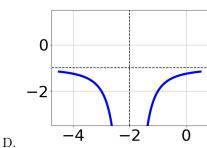
The solution is the graph below, which is option A.







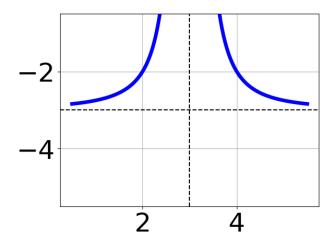




E. None of the above.

**General Comment:** Remember that the general form of a basic rational equation is  $f(x) = \frac{a}{(x-h)^n} + k$ , where a is the leading coefficient (and in this case, we assume is either 1 or -1), n is the degree (in this case, either 1 or 2), and (h,k) is the intersection of the asymptotes.

2. Choose the equation of the function graphed below.



The solution is  $f(x) = \frac{1}{(x-3)^2} - 3$ , which is option D.

A. 
$$f(x) = \frac{1}{x-3} - 3$$

Corresponds to thinking the graph was a shifted version of  $\frac{1}{x}$ .

B. 
$$f(x) = \frac{-1}{x+3} - 3$$

Corresponds to thinking the graph was a shifted version of  $\frac{1}{x}$ , using the general form  $f(x) = \frac{a}{(x+h)^2} + k$ , and the opposite leading coefficient.

C. 
$$f(x) = \frac{-1}{(x+3)^2} - 3$$

Corresponds to using the general form  $f(x) = \frac{a}{(x+h)^2} + k$  and the opposite leading coefficient.

D. 
$$f(x) = \frac{1}{(x-3)^2} - 3$$

This is the correct option.

## E. None of the above

This corresponds to believing the vertex of the graph was not correct.

**General Comment:** Remember that the general form of a basic rational equation is  $f(x) = \frac{a}{(x-h)^n} + k$ , where a is the leading coefficient (and in this case, we assume is either 1 or -1), n is the degree (in this case, either 1 or 2), and (h,k) is the intersection of the asymptotes.

3. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{-4x}{-7x+4} + \frac{-6x^2}{-28x^2 + 37x - 12} = \frac{3}{4x-3}$$

The solution is There are two solutions: x = 0.619 and x = 0.881, which is option E.

A. 
$$x \in [0.71, 0.88]$$

B. 
$$x_1 \in [0.58, 0.67]$$
 and  $x_2 \in [0.56, 0.72]$ 

C. All solutions lead to invalid or complex values in the equation.

D. 
$$x \in [0.8, 0.93]$$

E. 
$$x_1 \in [0.58, 0.67]$$
 and  $x_2 \in [0.73, 1.12]$ 

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

4. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{7}{7x-3} + -6 = \frac{5}{-21x+9}$$

The solution is x = 0.635, which is option D.

A. 
$$x \in [-0.26, -0.19]$$

x = -0.222, which corresponds to not distributing the factor 7x - 3 correctly when trying to eliminate the fraction.

B. 
$$x_1 \in [0.39, 0.58]$$
 and  $x_2 \in [0.63, 2.63]$ 

x = 0.476 and x = 0.635, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

C. All solutions lead to invalid or complex values in the equation.

This corresponds to thinking x = 0.635 leads to dividing by zero in the original equation, which it does not.

D. 
$$x \in [0.63, 1.63]$$

\* x = 0.635, which is the correct option.

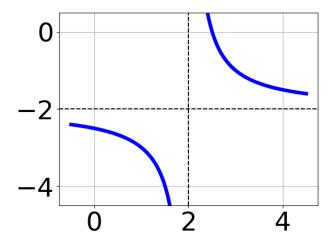
E. 
$$x_1 \in [-0.26, -0.19]$$
 and  $x_2 \in [0.63, 2.63]$ 

x = -0.222 and x = 0.635, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

General Comment: Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

5. Choose the equation of the function graphed below.

<sup>\*</sup> x = 0.619 and x = 0.881, which is the correct option.



The solution is  $f(x) = \frac{1}{x-2} - 2$ , which is option C.

A. 
$$f(x) = \frac{1}{(x-2)^2} - 2$$

Corresponds to thinking the graph was a shifted version of  $\frac{1}{x^2}$ .

B. 
$$f(x) = \frac{-1}{x+2} - 2$$

Corresponds to using the general form  $f(x) = \frac{a}{x+h} + k$  and the opposite leading coefficient.

C. 
$$f(x) = \frac{1}{x-2} - 2$$

This is the correct option.

D. 
$$f(x) = \frac{-1}{(x+2)^2} - 2$$

Corresponds to thinking the graph was a shifted version of  $\frac{1}{x^2}$ , using the general form  $f(x) = \frac{a}{x+h} + k$ , and the opposite leading coefficient.

## E. None of the above

This corresponds to believing the vertex of the graph was not correct.

**General Comment:** Remember that the general form of a basic rational equation is  $f(x) = \frac{a}{(x-h)^n} + k$ , where a is the leading coefficient (and in this case, we assume is either 1 or -1), n is the degree (in this case, either 1 or 2), and (h, k) is the intersection of the asymptotes.

6. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{24}{48x+18}+1=\frac{24}{48x+18}$$

The solution is all solutions are invalid or lead to complex values in the equation., which is option B.

A. 
$$x \in [-0.1, 1]$$

x = 0.375, which corresponds to not distributing the factor 48x + 18 correctly when trying to eliminate the fraction.

B. All solutions lead to invalid or complex values in the equation.

\*x = -0.375 leads to dividing by 0 in the original equation and thus is not a valid solution, which is the correct option.

C.  $x \in [-1.38, 1.62]$ 

x = -0.375, which corresponds to not checking if this value leads to dividing by 0 in the original equation and thus is not a valid solution.

D.  $x_1 \in [-0.5, -0.1]$  and  $x_2 \in [-0.1, 1.3]$ 

x = -0.375 and x = 0.375, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

E.  $x_1 \in [-0.5, -0.1]$  and  $x_2 \in [-0.5, 0.2]$ 

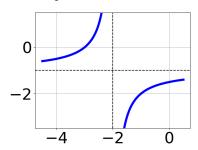
x = -0.375 and x = -0.375, which corresponds to getting the correct solution and believing there should be a second solution to the equation.

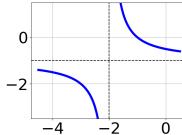
**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!

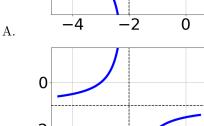
7. Choose the graph of the equation below.

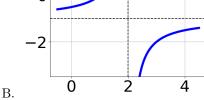
$$f(x) = \frac{-1}{x+2} - 1$$

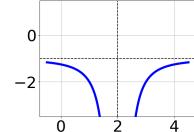
The solution is the graph below, which is option E.

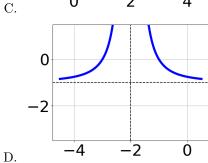












Summer C 2021

E. None of the above.

**General Comment:** Remember that the general form of a basic rational equation is  $f(x) = \frac{a}{(x-h)^n} + k$ , where a is the leading coefficient (and in this case, we assume is either 1 or -1), n is the degree (in this case, either 1 or 2), and (h, k) is the intersection of the asymptotes.

8. Determine the domain of the function below.

$$f(x) = \frac{6}{20x^2 - 8x - 12}$$

The solution is All Real numbers except x = -0.600 and x = 1.000, which is option B.

A. All Real numbers except x = a and x = b, where  $a \in [-17, -11]$  and  $b \in [16, 17]$ 

All Real numbers except x = -15.000 and x = 16.000, which corresponds to not factoring the denominator correctly.

B. All Real numbers except x = a and x = b, where  $a \in [-1.6, 0.4]$  and  $b \in [0, 6]$ 

All Real numbers except x = -0.600 and x = 1.000, which is the correct option.

C. All Real numbers except x = a, where  $a \in [-17, -11]$ 

All Real numbers except x = -15.000, which corresponds to removing a distractor value from the denominator.

D. All Real numbers.

This corresponds to thinking the denominator has complex roots or that rational functions have a domain of all Real numbers.

E. All Real numbers except x = a, where  $a \in [-1.6, 0.4]$ 

All Real numbers except x = -0.600, which corresponds to removing only 1 value from the denominator.

**General Comment:** Recall that dividing by zero is not a real number. Therefore the domain is all real numbers **except** those that make the denominator 0.

9. Determine the domain of the function below.

$$f(x) = \frac{3}{25x^2 - 10x - 24}$$

The solution is All Real numbers except x = -0.800 and x = 1.200., which is option C.

A. All Real numbers except x = a, where  $a \in [-1.4, 0.4]$ 

All Real numbers except x = -0.800, which corresponds to removing only 1 value from the denominator.

B. All Real numbers except x = a, where  $a \in [-21, -19.1]$ 

All Real numbers except x = -20.000, which corresponds to removing a distractor value from the denominator.

C. All Real numbers except x = a and x = b, where  $a \in [-1.4, 0.4]$  and  $b \in [0.1, 2.7]$ 

All Real numbers except x = -0.800 and x = 1.200, which is the correct option.

D. All Real numbers except x = a and x = b, where  $a \in [-21, -19.1]$  and  $b \in [28.7, 30.2]$ 

All Real numbers except x = -20.000 and x = 30.000, which corresponds to not factoring the denominator correctly.

E. All Real numbers.

This corresponds to thinking the denominator has complex roots or that rational functions have a domain of all Real numbers.

**General Comment:** Recall that dividing by zero is not a real number. Therefore the domain is all real numbers **except** those that make the denominator 0.

10. Solve the rational equation below. Then, choose the interval(s) that the solution(s) belongs to.

$$\frac{3x}{-6x-4} + \frac{-7x^2}{-24x^2 + 2x + 12} = \frac{4}{4x-3}$$

The solution is All solutions are invalid or lead to complex values in the equation., which is option D.

A.  $x_1 \in [-1.55, -0.69]$  and  $x_2 \in [-0.21, 0.48]$ 

x = -1.223 and x = 0.434, which corresponds to making the discriminant from the Quadratic Formula positive to avoid complex solutions.

B.  $x \in [0.45, 0.96]$ 

x = 0.750, which corresponds to solving 4x - 3 = 0 and treating it as a solution to the equation.

C.  $x \in [-0.84, -0.5]$ 

x = -0.667, which corresponds to solving -6x-4=0 and treating it as a solution to the equation.

- D. All solutions lead to invalid or complex values in the equation.
  - \* The equation leads to solving  $19x^2 + 15x + 16 = 0$ , which leads to complex solutions. This is the correct option.
- E.  $x_1 \in [-0.84, -0.5]$  and  $x_2 \in [0.56, 0.89]$

x = -0.667 and x = 0.750, which corresponds to solving -6x - 4 = 0 and 4x - 3 = 0 and treating them as solutions to the equation.

**General Comment:** Distractors are different based on the number of solutions. Remember that after solving, we need to make sure our solution does not make the original equation divide by zero!