This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Find the inverse of the function below. Then, evaluate the inverse at x = 8 and choose the interval that $f^{-}1(8)$ belongs to.

$$f(x) = \ln(x+5) + 2$$

The solution is $f^{-1}(8) = 398.429$, which is option E.

A. $f^{-1}(8) \in [22020.47, 22024.47]$

This solution corresponds to distractor 1.

B. $f^{-1}(8) \in [21.09, 27.09]$

This solution corresponds to distractor 2.

C. $f^{-1}(8) \in [442414.39, 442420.39]$

This solution corresponds to distractor 4.

D. $f^{-1}(8) \in [405.43, 413.43]$

This solution corresponds to distractor 3.

E. $f^{-1}(8) \in [388.43, 399.43]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

2. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{6x - 28}$$
 and $g(x) = x + 6$

The solution is The domain is all Real numbers greater than or equal to x = 4.67., which is option B.

- A. The domain is all Real numbers except x = a, where $a \in [1.17, 5.17]$
- B. The domain is all Real numbers greater than or equal to x = a, where $a \in [0.67, 5.67]$
- C. The domain is all Real numbers less than or equal to x = a, where $a \in [-5.5, -0.5]$
- D. The domain is all Real numbers except x = a and x = b, where $a \in [-1.67, 4.33]$ and $b \in [-4.2, -3.2]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

3. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -15 and choose the interval that $f^{-1}(-15)$ belongs to.

$$f(x) = \sqrt[3]{3x+4}$$

8497-6012 Summer C 2021

A. $f^{-1}(-15) \in [1125.5, 1129.3]$

This solution corresponds to distractor 2.

B. $f^{-1}(-15) \in [1122.7, 1126]$

This solution corresponds to distractor 3.

C. $f^{-1}(-15) \in [-1125.4, -1122.4]$

Distractor 1: This corresponds to

D. $f^{-1}(-15) \in [-1129, -1126.3]$

* This is the correct solution.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

4. Determine whether the function below is 1-1.

$$f(x) = 9x^2 - 39x - 230$$

The solution is no, which is option B.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

- B. No, because there is a y-value that goes to 2 different x-values.
 - * This is the solution.
- C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

5. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{3}{3x - 16}$$
 and $g(x) = \frac{2}{3x + 16}$

The solution is The domain is all Real numbers except x = 5.33 and x = -5.33, which is option D.

- A. The domain is all Real numbers less than or equal to x = a, where $a \in [-6.6, 5.4]$
- B. The domain is all Real numbers except x = a, where $a \in [-8.25, -4.25]$
- C. The domain is all Real numbers greater than or equal to x = a, where $a \in [5, 13]$
- D. The domain is all Real numbers except x = a and x = b, where $a \in [0.33, 6.33]$ and $b \in [-11.33, -1.33]$

8497-6012 Summer C 2021

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

6. Find the inverse of the function below. Then, evaluate the inverse at x = 4 and choose the interval that $f^{-}1(4)$ belongs to.

$$f(x) = e^{x+2} + 2$$

The solution is $f^{-1}(4) = -1.307$, which is option B.

A.
$$f^{-1}(4) \in [-0.7, 2.7]$$

This solution corresponds to distractor 1.

B.
$$f^{-1}(4) \in [-3.5, -0.7]$$

This is the solution.

C.
$$f^{-1}(4) \in [-0.7, 2.7]$$

This solution corresponds to distractor 3.

D.
$$f^{-1}(4) \in [2.8, 5.5]$$

This solution corresponds to distractor 4.

E.
$$f^{-1}(4) \in [2.8, 5.5]$$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

7. Choose the interval below that f composed with g at x = -1 is in.

$$f(x) = -4x^3 - 2x^2 + 4x - 1$$
 and $g(x) = -2x^3 - 2x^2 - x$

The solution is -3.0, which is option D.

A.
$$(f \circ g)(-1) \in [42, 47]$$

Distractor 3: Corresponds to being slightly off from the solution.

B.
$$(f \circ g)(-1) \in [38, 40]$$

Distractor 1: Corresponds to reversing the composition.

C.
$$(f \circ g)(-1) \in [4, 6]$$

Distractor 2: Corresponds to being slightly off from the solution.

D.
$$(f \circ g)(-1) \in [-12, 0]$$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means f(g(x)). The order matters!

8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = 13 and choose the interval that $f^{-1}(13)$ belongs to.

$$f(x) = \sqrt[3]{5x+3}$$

The solution is 438.8, which is option A.

- A. $f^{-1}(13) \in [438.67, 438.81]$
 - * This is the correct solution.
- B. $f^{-1}(13) \in [439.45, 441.34]$

Distractor 1: This corresponds to

C. $f^{-1}(13) \in [-439.21, -438.65]$

This solution corresponds to distractor 2.

D. $f^{-1}(13) \in [-440.29, -439.8]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

9. Choose the interval below that f composed with q at x = -1 is in.

$$f(x) = -2x^3 + 3x^2 + 4x$$
 and $g(x) = 3x^3 - 1x^2 - 2x$

The solution is 20.0, which is option B.

A. $(f \circ g)(-1) \in [24, 37]$

Distractor 2: Corresponds to being slightly off from the solution.

- B. $(f \circ g)(-1) \in [20, 21]$
 - * This is the correct solution
- C. $(f \circ g)(-1) \in [1, 14]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-5, 3]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means f(g(x)). The order matters!

10. Determine whether the function below is 1-1.

$$f(x) = 15x^2 - 56x - 396$$

The solution is no, which is option A.

- A. No, because there is a y-value that goes to 2 different x-values.
 - * This is the solution.
- B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

- D. No, because the domain of the function is not $(-\infty, \infty)$.
 - Corresponds to believing 1-1 means the domain is all Real numbers.
- E. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

Summer C 2021