This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Determine whether the function below is 1-1.

$$f(x) = 36x^2 - 252x + 441$$

The solution is no, which is option C.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

D. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

---

2. Determine whether the function below is 1-1.

$$f(x) = (3x + 19)^3$$

The solution is yes, which is option B.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. Yes, the function is 1-1.

* This is the solution.

C. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

---

3. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 13$ and choose the interval that $f^-1(13)$ belongs to.

$$f(x) = 3x^2 - 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(13) \in [2.17, 2.58]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(13) \in [4.98, 5.3]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(13) \in [1.51, 2.17]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(13) \in [3.23, 3.35]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

---

4. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = x^2 + 5x + 8 \text{ and } g(x) = \frac{3}{4x - 21}$$

The solution is The domain is all Real numbers except $x = 5.25$, which is option C.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-0.25, 6.75]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [4, 9]$

C. The domain is all Real numbers except $x = a,$ where $a \in [4.25, 9.25]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-5.4, -2.4]$ and $b \in [1.25, 9.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

---

5. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -3x^3 - 2x^2 - 2x - 2 \text{ and } g(x) = -3x^3 - 2x^2 + 4x$$

The solution is 67.0, which is option D.

A. $(f \circ g)(-1) \in [5, 14]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [58, 65]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [-3, 5]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(-1) \in [65, 70]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

---

6. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^-1(8)$ belongs to.

$$f(x) = \ln(x + 3) + 3$$

The solution is $f^{-1}(8) = 145.413$, which is option D.

A. $f^{-1}(8) \in [151.41, 152.41]$

This solution corresponds to distractor 3.

B. $f^{-1}(8) \in [59871.14, 59872.14]$

This solution corresponds to distractor 1.

C. $f^{-1}(8) \in [151.41, 152.41]$

This solution corresponds to distractor 2.

D. $f^{-1}(8) \in [141.41, 150.41]$

This is the solution.

E. $f^{-1}(8) \in [59877.14, 59879.14]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

---

7. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = x^3 - 4x^2 - 2x + 1 \text{ and } g(x) = -3x^3 - 4x^2 + 2x$$

The solution is -56.0, which is option A.

A. $(f \circ g)(-1) \in [-59, -51]$

* This is the correct solution

B. $(f \circ g)(-1) \in [4, 10]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(-1) \in [6, 16]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(-1) \in [-66, -64]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

---

8. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 4x^4 + 6x^3 + 7x^2 + 7x + 8 \text{ and } g(x) = x^2 + 7x + 1$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [2.25, 7.25]$

B. The domain is all Real numbers less than or equal to $x = a$, where $a \in [0.25, 8.25]$

C. The domain is all Real numbers except $x = a$, where $a \in [-5.75, -3.75]$

D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-10.33, -1.33]$ and $b \in [3.2, 6.2]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

---

9. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -11$ and choose the interval that $f^-1(-11)$ belongs to.

$$f(x) = \sqrt[3]{2x + 3}$$

The solution is -667.0, which is option B.

A. $f^{-1}(-11) \in [-665, -662.1]$

Distractor 1: This corresponds to

B. $f^{-1}(-11) \in [-669.5, -665.9]$

* This is the correct solution.

C. $f^{-1}(-11) \in [663.5, 665.9]$

This solution corresponds to distractor 3.

D. $f^{-1}(-11) \in [666.6, 667.4]$

This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

---

10. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^-1(9)$ belongs to.

$$f(x) = e^{x+3} - 5$$

The solution is $f^{-1}(9) = -0.361$, which is option D.

A. $f^{-1}(9) \in [-3.85, -3.55]$

This solution corresponds to distractor 2.

B. $f^{-1}(9) \in [-2.87, -2.39]$

This solution corresponds to distractor 4.

C. $f^{-1}(9) \in [5.29, 5.97]$

This solution corresponds to distractor 1.

D. $f^{-1}(9) \in [-0.68, -0.23]$

This is the solution.

E. $f^{-1}(9) \in [-3.3, -3.08]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.
