This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

---

1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 42x^2 + 37}{x - 4}$$

The solution is $10x^2 - 2x - 8 + \frac{5}{x-4}$, which is option B.

A. $a \in [7, 15], b \in [-89, -81], c \in [325, 333],$ and $r \in [-1276, -1272]$.

You divided by the opposite of the factor.

B. $a \in [7, 15], b \in [-3, 2], c \in [-8, -4],$ and $r \in [4, 9]$.

* This is the solution!

C. $a \in [7, 15], b \in [-15, -10], c \in [-38, -35],$ and $r \in [-74, -67]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [38, 46], b \in [117, 128], c \in [472, 476],$ and $r \in [1924, 1933]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [38, 46], b \in [-204, -197], c \in [808, 817],$ and $r \in [-3200, -3192]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

2. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 3x^4 + 6x^3 + 6x^2 + 3x + 6$$

The solution is $\pm 1, \pm 2, \pm 3, \pm 6$, which is option B.

A. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

B. $\pm 1, \pm 2, \pm 3, \pm 6$

* This is the solution **since we asked for the possible Integer roots**!

C. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 3}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 - 33x^2 - 96x - 50}{x - 4}$$

The solution is $15x^2 + 27x + 12 + \dfrac{-2}{x-4}$, which is option A.

A. $a \in [14, 16], \; b \in [24, 34], \; c \in [12, 16], \text{ and } r \in [-9, 0].$

* This is the solution!

B. $a \in [57, 64], \; b \in [207, 214], \; c \in [728, 736], \text{ and } r \in [2878, 2879].$

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [14, 16], \; b \in [-97, -88], \; c \in [275, 283], \text{ and } r \in [-1160, -1151].$

You divided by the opposite of the factor.

D. $a \in [14, 16], \; b \in [11, 15], \; c \in [-65, -59], \text{ and } r \in [-230, -226].$

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [57, 64], \; b \in [-277, -269], \; c \in [996, 1002], \text{ and } r \in [-4037, -4026].$

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

4. Factor the polynomial below completely, knowing that $x + 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^4 + 29x^3 - 233x^2 + 3x + 90$$

The solution is $[-5, -0.6, 0.667, 3]$, which is option C.

A. $z_1 \in [-4, 1], \; z_2 \in [-1.52, -1.5], z_3 \in [1.55, 1.68], \text{ and } z_4 \in [4, 8]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-7, -4], \; z_2 \in [-1.72, -1.66], z_3 \in [1.46, 1.55], \text{ and } z_4 \in [1, 4]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-7, -4], \; z_2 \in [-0.62, -0.57], z_3 \in [0.64, 0.75], \text{ and } z_4 \in [1, 4]$

* This is the solution!

D. $z_1 \in [-4, 1]$, $z_2 \in [-0.15, -0.04]$, $z_3 \in [2.95, 3.07]$, and $z_4 \in [4, 8]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-4, 1]$, $z_2 \in [-0.67, -0.65]$, $z_3 \in [0.55, 0.64]$, and $z_4 \in [4, 8]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

5. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 + 9x^2 - 28x - 12$$

The solution is $[-2, -0.4, 1.5]$, which is option C.

A. $z_1 \in [-3.33, -2.61]$, $z_2 \in [-0.02, 0.32]$, and $z_3 \in [1.78, 2.21]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-2.82, -2.12]$, $z_2 \in [-2.05, -1.75]$, and $z_3 \in [0.04, 0.93]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-2.38, -1.94]$, $z_2 \in [-0.71, -0.38]$, and $z_3 \in [1.45, 1.66]$

* This is the solution!

D. $z_1 \in [-0.73, -0.52]$, $z_2 \in [1.91, 2.19]$, and $z_3 \in [2.48, 2.61]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-1.57, -1.33]$, $z_2 \in [0.32, 0.59]$, and $z_3 \in [1.78, 2.21]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

6. Factor the polynomial below completely, knowing that $x - 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^4 - 99x^3 + 308x^2 - 333x + 90$$

The solution is $[0.4, 1.5, 3, 5]$, which is option E.

A. $z_1 \in [-5.14, -4.51]$, $z_2 \in [-3.1, -2.9]$, $z_3 \in [-2.91, -2.29]$, and $z_4 \in [-0.78, -0.64]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [0.57, 0.97]$, $z_2 \in [1.9, 3.4]$, $z_3 \in [2.25, 3.29]$, and $z_4 \in [4.93, 5.07]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-5.14, -4.51]$, $z_2 \in [-3.1, -2.9]$, $z_3 \in [-1.97, -1.46]$, and $z_4 \in [-0.44, -0.38]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-5.14, -4.51]$, $z_2 \in [-3.1, -2.9]$, $z_3 \in [-2.01, -1.57]$, and $z_4 \in [-0.3, -0.06]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [0.25, 0.54]$, $z_2 \in [0.4, 2.2]$, $z_3 \in [2.25, 3.29]$, and $z_4 \in [4.93, 5.07]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 + 39x^2 - 44}{x + 4}$$

The solution is $9x^2 + 3x - 12 + \frac{4}{x+4}$, which is option E.

A. $a \in [-39, -33], b \in [182, 186], c \in [-735, -729]$, and $r \in [2880, 2888]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [-39, -33], b \in [-106, -102], c \in [-420, -411]$, and $r \in [-1728, -1723]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [7, 16], b \in [-8, -5], c \in [28, 34]$, and $r \in [-196, -188]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [7, 16], b \in [71, 80], c \in [298, 307]$, and $r \in [1148, 1157]$.

You divided by the opposite of the factor.

E. $a \in [7, 16], b \in [0, 11], c \in [-14, -9]$, and $r \in [2, 10]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 - 19x^2 - 9x + 36$$

The solution is $[-1.33, 1.5, 3]$, which is option A.

A. $z_1 \in [-1.96, -1.17]$, $z_2 \in [1.18, 1.64]$, and $z_3 \in [2, 3.4]$

* This is the solution!

B. $z_1 \in [-3.26, -2.9]$, $z_2 \in [-0.79, -0.65]$, and $z_3 \in [0.2, 0.8]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-1.13, -0.74]$, $z_2 \in [0.49, 0.98]$, and $z_3 \in [2, 3.4]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.26, -2.9]$, $z_2 \in [-0.57, -0.38]$, and $z_3 \in [3.7, 5]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-3.26, -2.9],\ z_2 \in [-1.57, -1.18],$ and $z_3 \in [1.2, 1.4]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 113x^2 + 142x + 42}{x + 4}$$

The solution is $20x^2 + 33x + 10 + \dfrac{2}{x+4}$, which is option A.

A. $a \in [15, 21],\ b \in [29, 34],\ c \in [9, 12],$ and $r \in [2, 3]$.

* This is the solution!

B. $a \in [15, 21],\ b \in [191, 198],\ c \in [907, 915],$ and $r \in [3697, 3699]$.

You divided by the opposite of the factor.

C. $a \in [-84, -78],\ b \in [-207, -203],\ c \in [-694, -681],$ and $r \in [-2708, -2700]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

D. $a \in [15, 21],\ b \in [10, 14],\ c \in [73, 85],$ and $r \in [-347, -336]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [-84, -78],\ b \in [429, 434],\ c \in [-1591, -1588],$ and $r \in [6401, 6406]$.

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

10. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 7x^4 + 2x^3 + 4x^2 + 3x + 5$$

The solution is $\pm 1, \pm 5$, which is option A.

A. $\pm 1, \pm 5$

* This is the solution **since we asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 7}$

This would have been the solution **if asked for the possible Rational roots**!

C. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

11. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 - 65x^2 + 82}{x - 4}$$

The solution is $15x^2 - 5x - 20 + \frac{2}{x-4}$, which is option B.

A. $a \in [13, 16], b \in [-24, -15], c \in [-60, -55]$, and $r \in [-99, -97]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [13, 16], b \in [-11, -1], c \in [-25, -13]$, and $r \in [-5, 4]$.

* This is the solution!

C. $a \in [60, 61], b \in [175, 181], c \in [697, 708]$, and $r \in [2882, 2889]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [13, 16], b \in [-125, -123], c \in [495, 504]$, and $r \in [-1919, -1912]$.

You divided by the opposite of the factor.

E. $a \in [60, 61], b \in [-309, -304], c \in [1220, 1223]$, and $r \in [-4803, -4794]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

12. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 6x^2 + 5x + 2$$

The solution is All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$, which is option B.

A. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

B. All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$

* This is the solution **since we asked for the possible Rational roots**!

C. All combinations of: $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

13. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 38x^2 - 16x + 34}{x - 4}$$

The solution is $10x^2 + 2x - 8 + \frac{2}{x-4}$, which is option C.

A. $a \in [37, 41], \; b \in [119, 126], \; c \in [468, 475], \text{ and } r \in [1922, 1924]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [5, 14], \; b \in [-78, -74], \; c \in [296, 303], \text{ and } r \in [-1152, -1147]$.

You divided by the opposite of the factor.

C. $a \in [5, 14], \; b \in [-3, 4], \; c \in [-11, -3], \text{ and } r \in [-1, 3]$.

* This is the solution!

D. $a \in [37, 41], \; b \in [-201, -193], \; c \in [776, 778], \text{ and } r \in [-3074, -3063]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [5, 14], \; b \in [-10, -2], \; c \in [-42, -39], \text{ and } r \in [-86, -82]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

14. Factor the polynomial below completely, knowing that $x + 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^4 + 13x^3 - 253x^2 + 78x + 72$$

The solution is $[-4, -0.4, 0.75, 3]$, which is option E.

A. $z_1 \in [-3.3, -2.6], \; z_2 \in [-1.16, -0.5], z_3 \in [0.23, 0.44], \text{ and } z_4 \in [3.1, 4.6]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-3.3, -2.6], \; z_2 \in [-1.59, -1.31], z_3 \in [2.3, 2.69], \text{ and } z_4 \in [3.1, 4.6]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-4.7, -3.5], \; z_2 \in [-2.67, -2.31], z_3 \in [1.2, 1.91], \text{ and } z_4 \in [1.5, 3.2]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.3, -2.6], \; z_2 \in [-3.23, -2.61], z_3 \in [-0.05, 0.12], \text{ and } z_4 \in [3.1, 4.6]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-4.7, -3.5], \; z_2 \in [-0.5, 0.04], z_3 \in [0.72, 0.88], \text{ and } z_4 \in [1.5, 3.2]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

15. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 - 1x^2 - 39x - 36$$

The solution is $[-1.5, -1.33, 3]$, which is option E.

A. $z_1 \in [-0.79, -0.48]$, $z_2 \in [-0.68, -0.58]$, and $z_3 \in [2.6, 3.4]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-3.4, -2.82]$, $z_2 \in [1.28, 1.47]$, and $z_3 \in [1, 1.6]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-3.4, -2.82]$, $z_2 \in [0.56, 0.82]$, and $z_3 \in [-0.2, 1.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-3.4, -2.82]$, $z_2 \in [0.36, 0.66]$, and $z_3 \in [3.4, 5.4]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-2.03, -1.3]$, $z_2 \in [-1.4, -1.18]$, and $z_3 \in [2.6, 3.4]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

16. Factor the polynomial below completely, knowing that $x - 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 6x^3 - 189x^2 + 265x + 300$$

The solution is $[-5, -0.75, 2.5, 4]$, which is option A.

A. $z_1 \in [-5.9, -4.4]$, $z_2 \in [-0.82, -0.46]$, $z_3 \in [2.49, 2.51]$, and $z_4 \in [2.7, 4.9]$

* This is the solution!

B. $z_1 \in [-4.7, -2.8]$, $z_2 \in [-0.5, -0.38]$, $z_3 \in [1.33, 1.35]$, and $z_4 \in [4.7, 5.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.9, -4.4]$, $z_2 \in [-4.11, -3.8]$, $z_3 \in [0.35, 0.38]$, and $z_4 \in [4.7, 5.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-4.7, -2.8]$, $z_2 \in [-2.96, -2.39]$, $z_3 \in [0.74, 0.76]$, and $z_4 \in [4.7, 5.3]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-5.9, -4.4]$, $z_2 \in [-1.42, -1.05]$, $z_3 \in [0.39, 0.41]$, and $z_4 \in [2.7, 4.9]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

17. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 70x + 65}{x + 3}$$

The solution is $10x^2 - 30x + 20 + \dfrac{5}{x+3}$, which is option E.

A. $a \in [7, 12], b \in [30, 33], c \in [20, 26],$ and $r \in [124, 130]$.

You divided by the opposite of the factor.

B. $a \in [-38, -25], b \in [90, 93], c \in [-344, -335],$ and $r \in [1078, 1091]$.

You multipled by the synthetic number rather than bringing the first factor down.

C. $a \in [-38, -25], b \in [-91, -85], c \in [-344, -335],$ and $r \in [-958, -953]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [7, 12], b \in [-40, -39], c \in [89, 91],$ and $r \in [-298, -294]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [7, 12], b \in [-35, -29], c \in [20, 26],$ and $r \in [2, 13]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

18. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 - 33x^2 - 20x + 12$$

The solution is $[-0.75, 0.4, 2]$, which is option B.

A. $z_1 \in [-2.02, -1.65],\ z_2 \in [-2.77, -1.28],$ and $z_3 \in [0.09, 0.38]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-1.2, -0.31],\ z_2 \in [0.22, 0.44],$ and $z_3 \in [1.92, 2.22]$

* This is the solution!

C. $z_1 \in [-1.63, -1.11],\ z_2 \in [1.83, 2.91],$ and $z_3 \in [2.28, 2.58]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-2.55, -2.31],\ z_2 \in [-2.77, -1.28],$ and $z_3 \in [1.1, 1.38]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-2.02, -1.65],\ z_2 \in [-0.52, -0.21],$ and $z_3 \in [0.69, 0.98]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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19. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 + 67x^2 + 94x + 35}{x + 2}$$

The solution is $15x^2 + 37x + 20 + \frac{-5}{x+2}$, which is option A.

A. $a \in [13, 18]$, $b \in [37, 39]$, $c \in [16, 24]$, and $r \in [-11, -3]$.

   * This is the solution!

B. $a \in [-31, -28]$, $b \in [6, 13]$, $c \in [104, 113]$, and $r \in [251, 257]$.

   You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [-31, -28]$, $b \in [125, 129]$, $c \in [-161, -159]$, and $r \in [354, 357]$.

   You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [13, 18]$, $b \in [92, 101]$, $c \in [284, 289]$, and $r \in [606, 615]$.

   You divided by the opposite of the factor.

E. $a \in [13, 18]$, $b \in [20, 23]$, $c \in [24, 34]$, and $r \in [-50, -46]$.

   You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

20. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^4 + 4x^3 + 7x^2 + 4x + 7$$

The solution is $\pm 1, \pm 7$, which is option B.

A. All combinations of: $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

   Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 7$

   * This is the solution **since we asked for the possible Integer roots**!

C. $\pm 1, \pm 2, \pm 3, \pm 6$

   Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\frac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

   This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

   Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

21. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{9x^3 + 21x^2 - 7}{x + 2}$$

The solution is $9x^2 + 3x - 6 + \frac{5}{x+2}$, which is option D.

A. $a \in [8, 17], b \in [36, 48], c \in [77, 84]$, and $r \in [149, 151]$.

You divided by the opposite of the factor.

B. $a \in [8, 17], b \in [-8, -2], c \in [13, 21]$, and $r \in [-65, -60]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [-18, -14], b \in [54, 60], c \in [-114, -113]$, and $r \in [219, 224]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [8, 17], b \in [3, 8], c \in [-11, -3]$, and $r \in [4, 14]$.

* This is the solution!

E. $a \in [-18, -14], b \in [-17, -10], c \in [-32, -25]$, and $r \in [-69, -66]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

22. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^4 + 6x^3 + 3x^2 + 7x + 2$$

The solution is All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$, which is option B.

A. All combinations of: $\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

---

23. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 - 48x^2 - 116x - 43}{x - 4}$$

The solution is $20x^2 + 32x + 12 + \frac{5}{x-4}$, which is option C.

A. $a \in [19, 24],\ b \in [12, 13],\ c \in [-85, -79],$ and $r \in [-284, -280]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [19, 24],\ b \in [-131, -125],\ c \in [391, 398],$ and $r \in [-1632, -1622]$.

You divided by the opposite of the factor.

C. $a \in [19, 24],\ b \in [31, 38],\ c \in [8, 19],$ and $r \in [2, 9]$.

* This is the solution!

D. $a \in [80, 86],\ b \in [-370, -364],\ c \in [1356, 1360],$ and $r \in [-5472, -5466]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [80, 86],\ b \in [269, 275],\ c \in [969, 975],$ and $r \in [3842, 3846]$.

You multiplied by the synthetic number rather than bringing the first factor down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

---

24. Factor the polynomial below completely, knowing that $x + 5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^4 + 210x^3 + 507x^2 + 434x + 120$$

The solution is $[-5, -2, -0.8, -0.6]$, which is option D.

A. $z_1 \in [1.24, 1.46],\ z_2 \in [1.6, 1.94], z_3 \in [1.3, 2.2],$ and $z_4 \in [4.71, 5.09]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [0.1, 0.33],\ z_2 \in [1.89, 2.8], z_3 \in [2.8, 4.5],$ and $z_4 \in [4.71, 5.09]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-5.19, -4.79],\ z_2 \in [-2.35, -1.49], z_3 \in [-2, -1],$ and $z_4 \in [-1.56, -1.17]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-5.19, -4.79],\ z_2 \in [-2.35, -1.49], z_3 \in [-1.1, 1.6],$ and $z_4 \in [-0.93, 0.31]$

* This is the solution!

E. $z_1 \in [0.5, 0.72],\ z_2 \in [-0.1, 0.82], z_3 \in [1.3, 2.2],$ and $z_4 \in [4.71, 5.09]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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25. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 + 5x^2 - 22x - 24$$

The solution is $[-1.5, -1.33, 2]$, which is option A.

A. $z_1 \in [-1.67, -1.39], \; z_2 \in [-1.42, -1.18], \text{ and } z_3 \in [1.7, 2.6]$

* This is the solution!

B. $z_1 \in [-2.13, -1.96], \; z_2 \in [0.46, 0.55], \text{ and } z_3 \in [3.7, 4.4]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-2.13, -1.96], \; z_2 \in [0.62, 0.81], \text{ and } z_3 \in [-0.5, 1.2]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-2.13, -1.96], \; z_2 \in [1.22, 1.4], \text{ and } z_3 \in [1, 1.9]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-1.04, -0.67], \; z_2 \in [-0.83, -0.6], \text{ and } z_3 \in [1.7, 2.6]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

26. Factor the polynomial below completely, knowing that $x - 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 - 53x^3 - 23x^2 + 202x - 120$$

The solution is $[-2, 0.75, 1.667, 4]$, which is option A.

A. $z_1 \in [-3.4, -1.4], \; z_2 \in [0.68, 0.95], z_3 \in [1.54, 1.71], \text{ and } z_4 \in [4, 6]$

* This is the solution!

B. $z_1 \in [-3.4, -1.4], \; z_2 \in [0.52, 0.7], z_3 \in [1.2, 1.38], \text{ and } z_4 \in [4, 6]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-5.6, -4.6], \; z_2 \in [-4.05, -3.87], z_3 \in [-0.43, -0.2], \text{ and } z_4 \in [0, 3]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-4.7, -3.1], \; z_2 \in [-1.44, -1.16], z_3 \in [-0.71, -0.32], \text{ and } z_4 \in [0, 3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-4.7, -3.1], \; z_2 \in [-1.75, -1.65], z_3 \in [-0.85, -0.62], \text{ and } z_4 \in [0, 3]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

27. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 + 28x^2 - 33}{x + 3}$$

The solution is $8x^2 + 4x - 12 + \frac{3}{x+3}$, which is option A.

A. $a \in [5, 12], b \in [4, 6], c \in [-13, -3],$ and $r \in [0, 8]$.

   * This is the solution!

B. $a \in [5, 12], b \in [52, 57], c \in [156, 158],$ and $r \in [435, 437]$.

   You divided by the opposite of the factor.

C. $a \in [5, 12], b \in [-6, 1], c \in [13, 19],$ and $r \in [-104, -92]$.

   You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [-24, -23], b \in [97, 102], c \in [-300, -290],$ and $r \in [864, 875]$.

   You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [-24, -23], b \in [-48, -40], c \in [-135, -124],$ and $r \in [-432, -427]$.

   You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

28. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 41x^2 - 54x + 45$$

The solution is $[-1.5, 0.6, 5]$, which is option E.

A. $z_1 \in [-6, -4.8]$, $z_2 \in [-0.8, -0.3]$, and $z_3 \in [1, 1.6]$

   Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-6, -4.8]$, $z_2 \in [-3.3, -2.7]$, and $z_3 \in [-0.7, 0.6]$

   Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-6, -4.8]$, $z_2 \in [-2.9, -1.5]$, and $z_3 \in [0.6, 0.9]$

   Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-1, -0.1]$, $z_2 \in [0.8, 2.1]$, and $z_3 \in [4.4, 5.9]$

   Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-1.9, -1.1]$, $z_2 \in [-0.1, 1.2]$, and $z_3 \in [4.4, 5.9]$

   * This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

---

29. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{12x^3 + 45x^2 - 21x - 39}{x+4}$$

The solution is $12x^2 - 3x - 9 + \frac{-3}{x+4}$, which is option E.

A. $a \in [8, 13]$, $b \in [90, 102]$, $c \in [342, 359]$, and $r \in [1363, 1367]$.

You divided by the opposite of the factor.

B. $a \in [-49, -44]$, $b \in [-152, -144]$, $c \in [-611, -607]$, and $r \in [-2475, -2471]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [-49, -44]$, $b \in [232, 242]$, $c \in [-970, -961]$, and $r \in [3834, 3839]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [8, 13]$, $b \in [-16, -10]$, $c \in [53, 55]$, and $r \in [-310, -305]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [8, 13]$, $b \in [-3, 4]$, $c \in [-19, -8]$, and $r \in [-5, 4]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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30. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 3x^2 + 6x + 5$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 3}$, which is option B.

A. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 3}$

* This is the solution **since we asked for the possible Rational roots**!

C. $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots**!

D. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.
