This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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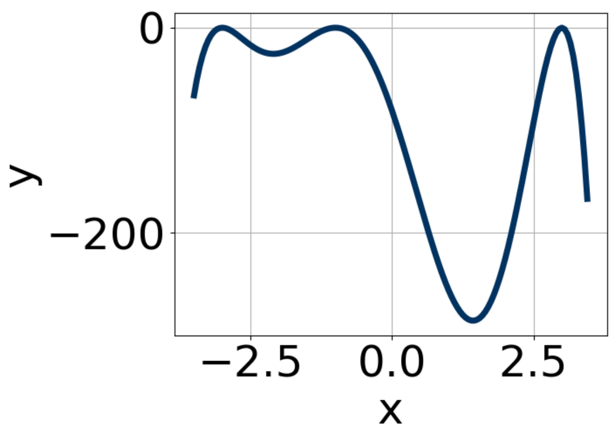
1. Which of the following equations *could* be of the graph presented below?



The solution is $-18(x+3)^6(x+1)^4(x-3)^8$, which is option A.

A. $-18(x+3)^6(x+1)^4(x-3)^8$

* This is the correct option.

B. $12(x+3)^8(x+1)^4(x-3)^7$

The factor $(x-3)$ should have an even power and the leading coefficient should be the opposite sign.

C. $-7(x+3)^6(x+1)^{10}(x-3)^7$

The factor $(x-3)$ should have an even power.

D. $8(x+3)^6(x+1)^6(x-3)^6$

This corresponds to the leading coefficient being the opposite value than it should be.

E. $-12(x+3)^{10}(x+1)^{11}(x-3)^5$

The factors $(x+1)$ and $(x-3)$ should both have even powers.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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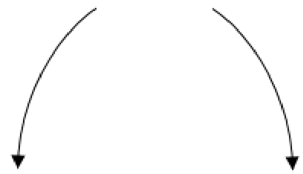
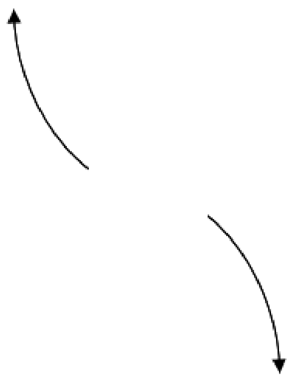
2. Describe the end behavior of the polynomial below.

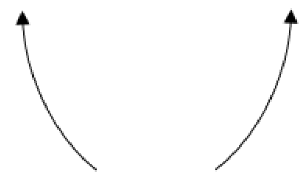
$$f(x) = -9(x+8)^2(x-8)^5(x-6)^2(x+6)^3$$

The solution is the graph below, which is option B.

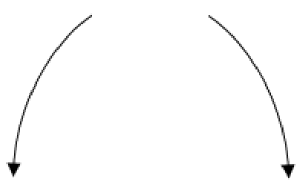




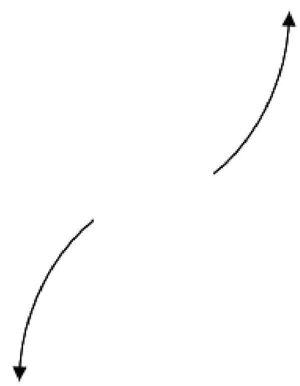
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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3. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 3i \text{ and } 1$$

The solution is $x^3 - 9x^2 + 33x - 25$, which is option C.

A. $b \in [1, 2], c \in [-6, -4.35],$ and $d \in [3.48, 4.06]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

B. $b \in [1, 2], c \in [-4.24, -2.63],$ and $d \in [2.96, 3.37]$

$x^3 + x^2 - 4x + 3$, which corresponds to multiplying out $(x - 3)(x - 1)$.

C. $b \in [-17, -7], c \in [31.51, 33.82],$ and $d \in [-25.2, -24.86]$

* $x^3 - 9x^2 + 33x - 25$, which is the correct option.

D. $b \in [8, 10], c \in [31.51, 33.82],$ and $d \in [24.44, 25.5]$

$x^3 + 9x^2 + 33x + 25$, which corresponds to multiplying out $(x-(4+3i))(x-(4-3i))(x+1)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a+bi$ is $a-bi$. Since these zeros always come in pairs, we need to multiply out $(x-(4+3i))(x-(4-3i))(x-(1))$.

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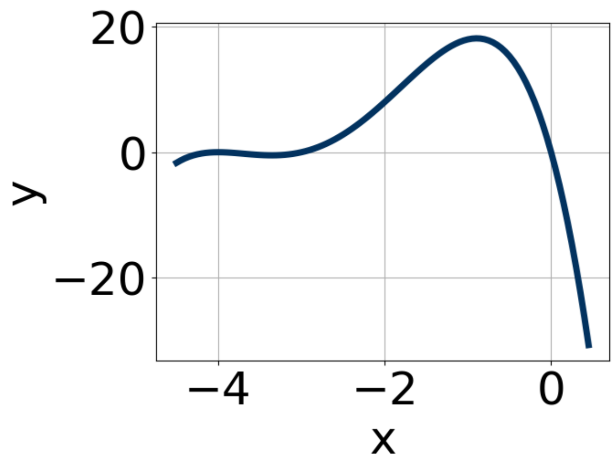
4. Which of the following equations *could* be of the graph presented below?



The solution is $-5x^5(x+4)^8(x+3)^7$, which is option A.

A. $-5x^5(x+4)^8(x+3)^7$

* This is the correct option.

B. $-19x^6(x+4)^6(x+3)^7$

The factor x should have an odd power.

C. $-14x^{10}(x+4)^9(x+3)^5$

The factor -4 should have an even power and the factor 0 should have an odd power.

D. $10x^{11}(x+4)^8(x+3)^{10}$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

E. $19x^{11}(x+4)^4(x+3)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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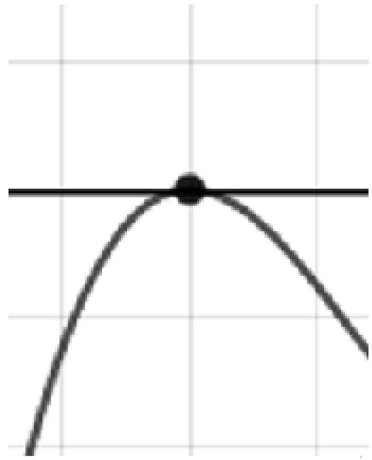
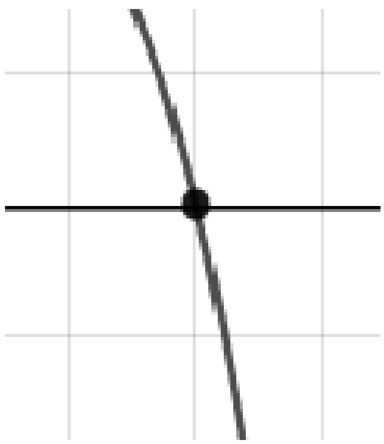
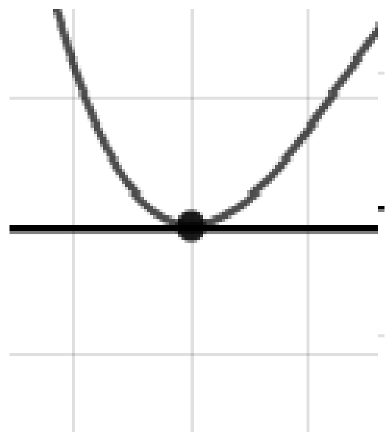
5. Describe the zero behavior of the zero $x = 8$ of the polynomial below.

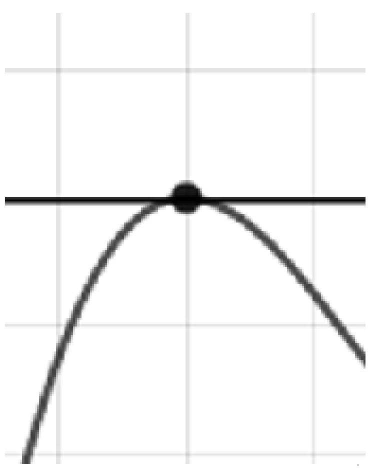
$$f(x) = -9(x-6)^9(x+6)^6(x-8)^{12}(x+8)^9$$

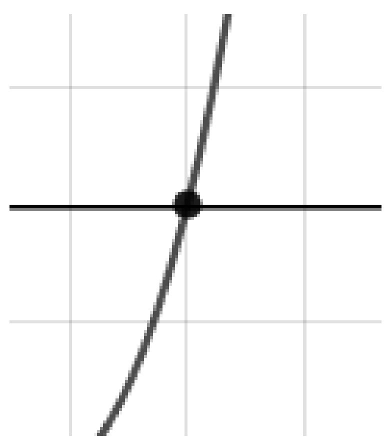
The solution is the graph below, which is option B.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

6. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain

the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{3}{2}, 5, \text{ and } \frac{-4}{3}$$

The solution is $6x^3 - 31x^2 - 7x + 60$, which is option B.

A. $a \in [0, 10], b \in [45, 52], c \in [97, 103], \text{ and } d \in [55, 64]$

$6x^3 + 47x^2 + 97x + 60$, which corresponds to multiplying out $(2x + 3)(x + 5)(3x + 4)$.

B. $a \in [0, 10], b \in [-34, -27], c \in [-9, -4], \text{ and } d \in [55, 64]$

* $6x^3 - 31x^2 - 7x + 60$, which is the correct option.

C. $a \in [0, 10], b \in [-34, -27], c \in [-9, -4], \text{ and } d \in [-66, -56]$

$6x^3 - 31x^2 - 7x - 60$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [0, 10], b \in [26, 37], c \in [-9, -4], \text{ and } d \in [-66, -56]$

$6x^3 + 31x^2 - 7x - 60$, which corresponds to multiplying out $(2x + 3)(x + 5)(3x - 4)$.

E. $a \in [0, 10], b \in [-13, -7], c \in [-76, -68], \text{ and } d \in [-66, -56]$

$6x^3 - 13x^2 - 73x - 60$, which corresponds to multiplying out $(2x + 3)(x - 5)(3x + 4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x - 3)(x - 5)(3x + 4)$

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7. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-3 + 2i \text{ and } 2$$

The solution is $x^3 + 4x^2 + x - 26$, which is option C.

A. $b \in [-5.5, -1.2], c \in [-3, 3], \text{ and } d \in [24, 27]$

$x^3 - 4x^2 + x + 26$, which corresponds to multiplying out $(x - (-3 + 2i))(x - (-3 - 2i))(x + 2)$.

B. $b \in [0.7, 1.5], c \in [-6, -3], \text{ and } d \in [0, 9]$

$x^3 + x^2 - 4x + 4$, which corresponds to multiplying out $(x - 2)(x - 2)$.

C. $b \in [3.8, 5.3], c \in [-3, 3], \text{ and } d \in [-32, -24]$

* $x^3 + 4x^2 + x - 26$, which is the correct option.

D. $b \in [0.7, 1.5], c \in [-3, 3], \text{ and } d \in [-10, -3]$

$x^3 + x^2 + x - 6$, which corresponds to multiplying out $(x + 3)(x - 2)$.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-3 + 2i))(x - (-3 - 2i))(x - (2))$.

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8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{5}, \frac{-7}{2}, \text{ and } \frac{-3}{2}$$

The solution is $20x^3 + 112x^2 + 165x + 63$, which is option C.

A. $a \in [15, 23], b \in [110, 119], c \in [165, 169], \text{ and } d \in [-64, -58]$

$20x^3 + 112x^2 + 165x - 63$, which corresponds to multiplying everything correctly except the constant term.

B. $a \in [15, 23], b \in [-117, -109], c \in [165, 169], \text{ and } d \in [-64, -58]$

$20x^3 - 112x^2 + 165x - 63$, which corresponds to multiplying out $(5x - 3)(2x - 7)(2x - 3)$.

C. $a \in [15, 23], b \in [110, 119], c \in [165, 169], \text{ and } d \in [54, 68]$

* $20x^3 + 112x^2 + 165x + 63$, which is the correct option.

D. $a \in [15, 23], b \in [-53, -49], c \in [-85, -80], \text{ and } d \in [54, 68]$

$20x^3 - 52x^2 - 81x + 63$, which corresponds to multiplying out $(5x - 3)(2x - 7)(2x + 3)$.

E. $a \in [15, 23], b \in [88, 93], c \in [36, 51], \text{ and } d \in [-64, -58]$

$20x^3 + 88x^2 + 45x - 63$, which corresponds to multiplying out $(5x - 3)(2x + 7)(2x + 3)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 3)(2x + 7)(2x + 3)$

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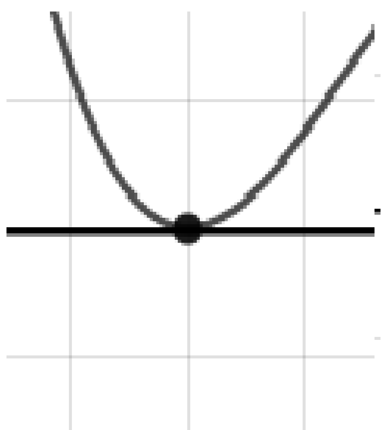
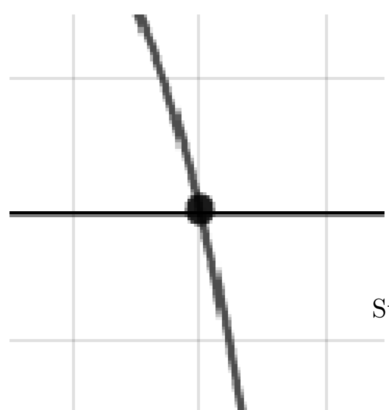
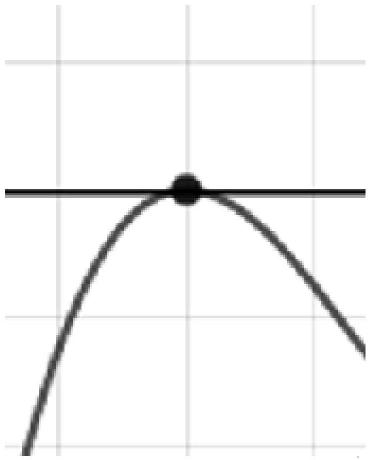
9. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

$$f(x) = 2(x + 7)^7 (x - 7)^{10} (x - 3)^4 (x + 3)^8$$

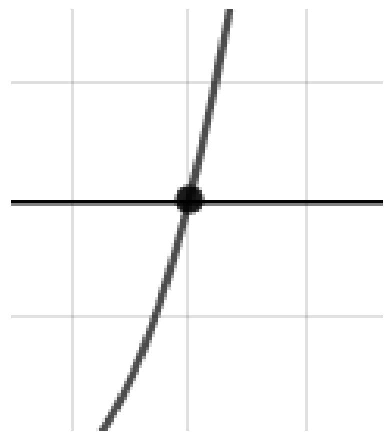
The solution is the graph below, which is option C.





B. 

C. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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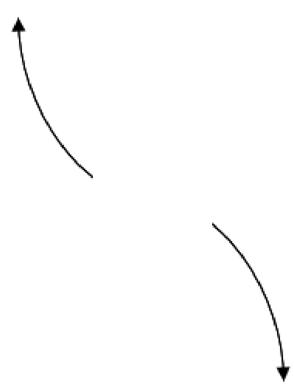
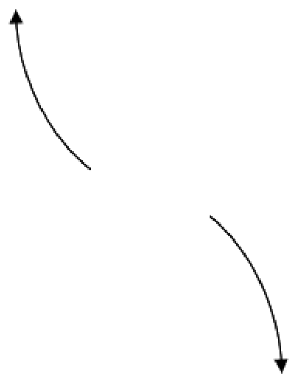
10. Describe the end behavior of the polynomial below.

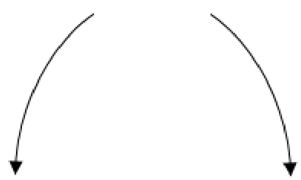
$$f(x) = -3(x+4)^3(x-4)^6(x-5)^5(x+5)^7$$

The solution is the graph below, which is option A.

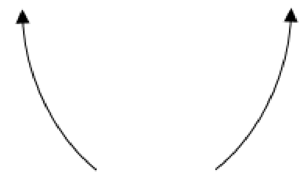




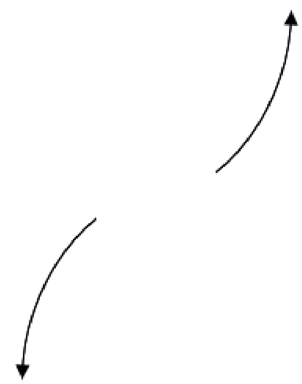
A.



B.



C.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.