

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } 4$$

The solution is $x^3 + 6x^2 + x - 164$, which is option A.

- A. $b \in [5, 14]$, $c \in [-1, 5]$, and $d \in [-165, -162]$

* $x^3 + 6x^2 + x - 164$, which is the correct option.

- B. $b \in [-2, 4]$, $c \in [-1, 5]$, and $d \in [-22, -18]$

$x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

- C. $b \in [-2, 4]$, $c \in [-10, -7]$, and $d \in [11, 20]$

$x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

- D. $b \in [-12, -3]$, $c \in [-1, 5]$, and $d \in [164, 169]$

$x^3 - 6x^2 + x + 164$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x + 4)$.

- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (4))$.

2. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{7}{4}, 5, \text{ and } \frac{7}{3}$$

The solution is $12x^3 - 109x^2 + 294x - 245$, which is option D.

- A. $a \in [12, 14]$, $b \in [-112, -107]$, $c \in [293, 297]$, and $d \in [245, 252]$

$12x^3 - 109x^2 + 294x + 245$, which corresponds to multiplying everything correctly except the constant term.

- B. $a \in [12, 14]$, $b \in [-76, -65]$, $c \in [-15, -8]$, and $d \in [245, 252]$

$12x^3 - 67x^2 - 14x + 245$, which corresponds to multiplying out $(4x + 7)(x - 5)(3x - 7)$.

- C. $a \in [12, 14]$, $b \in [108, 115]$, $c \in [293, 297]$, and $d \in [245, 252]$

$12x^3 + 109x^2 + 294x + 245$, which corresponds to multiplying out $(4x + 7)(x + 5)(3x + 7)$.

D. $a \in [12, 14]$, $b \in [-112, -107]$, $c \in [293, 297]$, and $d \in [-246, -240]$

* $12x^3 - 109x^2 + 294x - 245$, which is the correct option.

E. $a \in [12, 14]$, $b \in [49, 58]$, $c \in [-88, -83]$, and $d \in [-246, -240]$

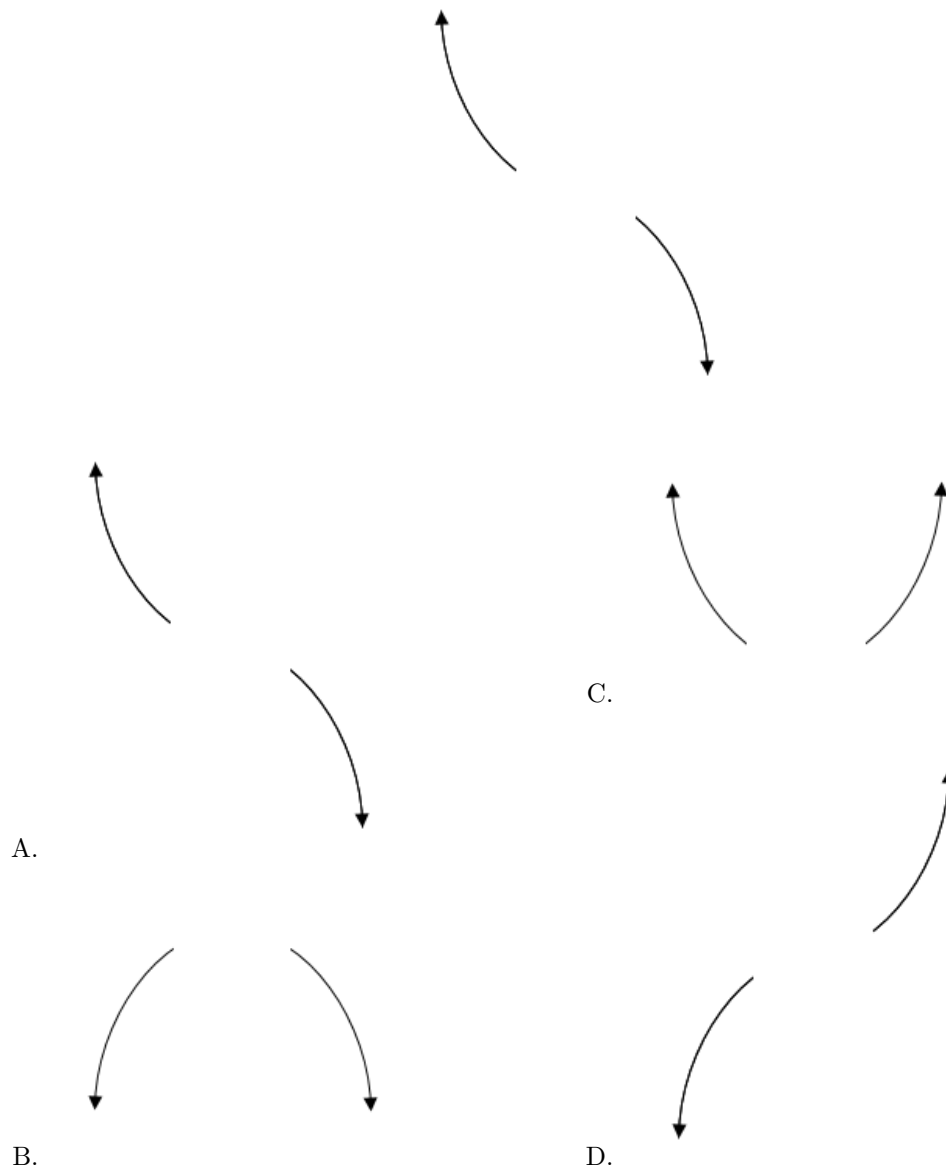
$12x^3 + 53x^2 - 84x - 245$, which corresponds to multiplying out $(4x + 7)(x + 5)(3x - 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 7)(x - 5)(3x - 7)$

3. Describe the end behavior of the polynomial below.

$$f(x) = -5(x + 3)^4(x - 3)^5(x + 2)^3(x - 2)^5$$

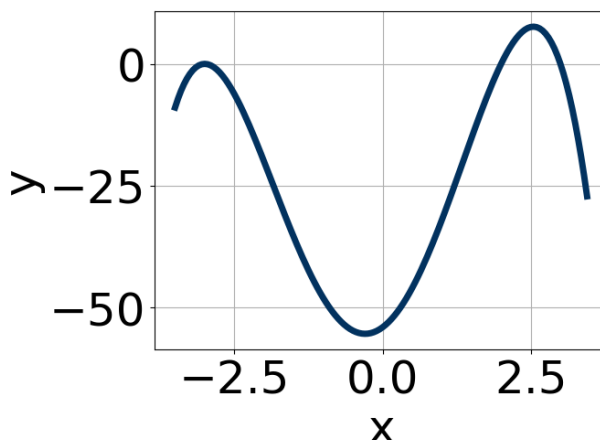
The solution is the graph below, which is option A.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

4. Which of the following equations *could* be the graph presented below?



The solution is $-5(x+3)^6(x-3)^5(x-2)^9$, which is option B.

A. $18(x+3)^{10}(x-3)^9(x-2)^7$

This corresponds to the leading coefficient being the opposite value than it should be.

B. $-5(x+3)^6(x-3)^5(x-2)^9$

* This is the correct option.

C. $4(x+3)^4(x-3)^9(x-2)^4$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

D. $-16(x+3)^5(x-3)^8(x-2)^7$

The factor -3 should have an even power and the factor 3 should have an odd power.

E. $-13(x+3)^{10}(x-3)^8(x-2)^{11}$

The factor $(x-3)$ should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

5. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{2}, \frac{5}{4}, \text{ and } \frac{7}{5}$$

The solution is $40x^3 - 86x^2 + 17x + 35$, which is option A.

A. $a \in [35, 45], b \in [-86, -81], c \in [16, 18], \text{ and } d \in [32, 42]$

* $40x^3 - 86x^2 + 17x + 35$, which is the correct option.

B. $a \in [35, 45], b \in [-128, -124], c \in [121, 126], \text{ and } d \in [-38, -33]$

$40x^3 - 126x^2 + 123x - 35$, which corresponds to multiplying out $(2x-1)(4x-5)(5x-7)$.

C. $a \in [35, 45], b \in [-33, -17], c \in [-75, -64]$, and $d \in [32, 42]$

$40x^3 - 26x^2 - 67x + 35$, which corresponds to multiplying out $(2x - 1)(4x + 5)(5x - 7)$.

D. $a \in [35, 45], b \in [-86, -81], c \in [16, 18]$, and $d \in [-38, -33]$

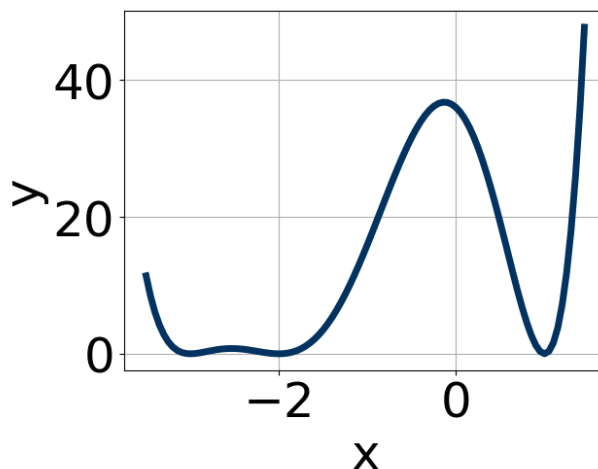
$40x^3 - 86x^2 + 17x - 35$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [35, 45], b \in [81, 94], c \in [16, 18]$, and $d \in [-38, -33]$

$40x^3 + 86x^2 + 17x - 35$, which corresponds to multiplying out $(2x - 1)(4x + 5)(5x + 7)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(2x + 1)(4x - 5)(5x - 7)$

6. Which of the following equations *could* be of the graph presented below?



The solution is $8(x + 2)^8(x + 3)^6(x - 1)^6$, which is option A.

A. $8(x + 2)^8(x + 3)^6(x - 1)^6$

* This is the correct option.

B. $-12(x + 2)^4(x + 3)^8(x - 1)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $10(x + 2)^6(x + 3)^6(x - 1)^5$

The factor $(x - 1)$ should have an even power.

D. $-13(x + 2)^{10}(x + 3)^8(x - 1)^9$

The factor $(x - 1)$ should have an even power and the leading coefficient should be the opposite sign.

E. $19(x + 2)^{10}(x + 3)^{11}(x - 1)^9$

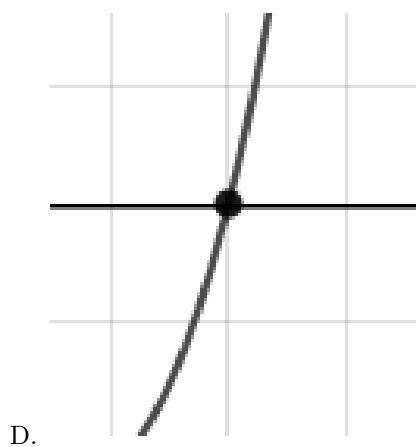
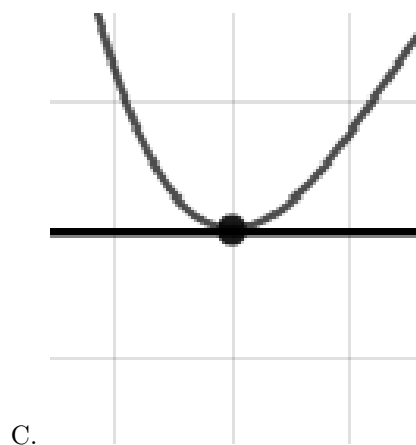
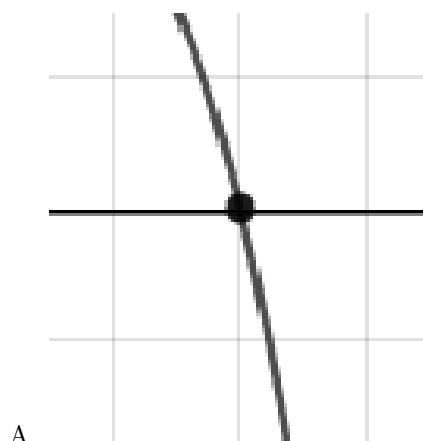
The factors $(x + 3)$ and $(x - 1)$ should both have even powers.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

7. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

$$f(x) = -2(x - 2)^6(x + 2)^3(x + 3)^{11}(x - 3)^6$$

The solution is the graph below, which is option B.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

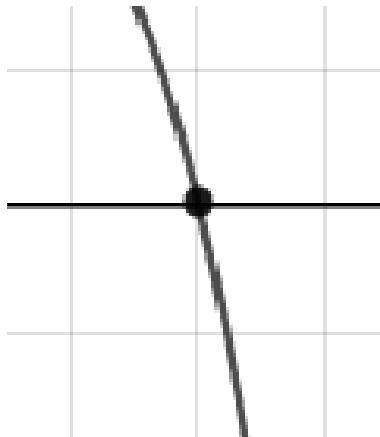
8. Describe the zero behavior of the zero $x = 2$ of the polynomial below.

$$f(x) = -2(x - 2)^2(x + 2)^7(x + 9)^4(x - 9)^8$$

The solution is the graph below, which is option B.



A.



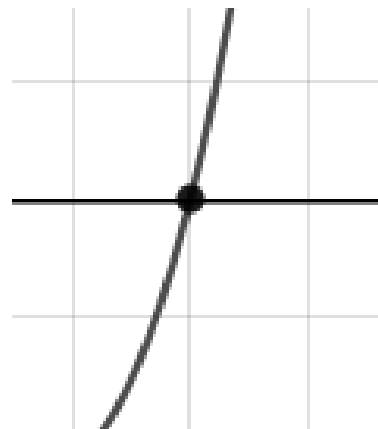
C.



B.



D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-2 + 2i \text{ and } 1$$

The solution is $x^3 + 3x^2 + 4x - 8$, which is option C.

- A. $b \in [-3.8, -1.5]$, $c \in [2.1, 5.2]$, and $d \in [6.4, 9.3]$

$$x^3 - 3x^2 + 4x + 8, \text{ which corresponds to multiplying out } (x - (-2 + 2i))(x - (-2 - 2i))(x + 1).$$

- B. $b \in [0.7, 2.7]$, $c \in [-3.6, -0.8]$, and $d \in [0.3, 4.3]$

$$x^3 + x^2 - 3x + 2, \text{ which corresponds to multiplying out } (x - 2)(x - 1).$$

- C. $b \in [2, 4.5]$, $c \in [2.1, 5.2]$, and $d \in [-8.4, -7.4]$

$$* x^3 + 3x^2 + 4x - 8, \text{ which is the correct option.}$$

- D. $b \in [0.7, 2.7]$, $c \in [-0.3, 2.5]$, and $d \in [-4.2, -1.1]$

$$x^3 + x^2 + x - 2, \text{ which corresponds to multiplying out } (x + 2)(x - 1).$$

- E. None of the above.

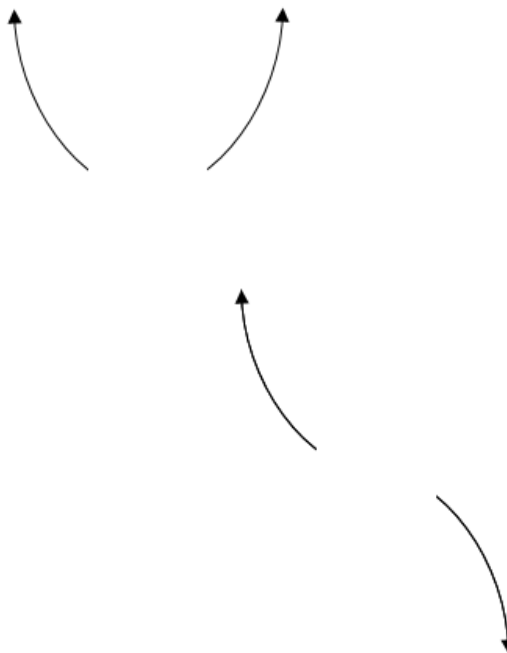
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

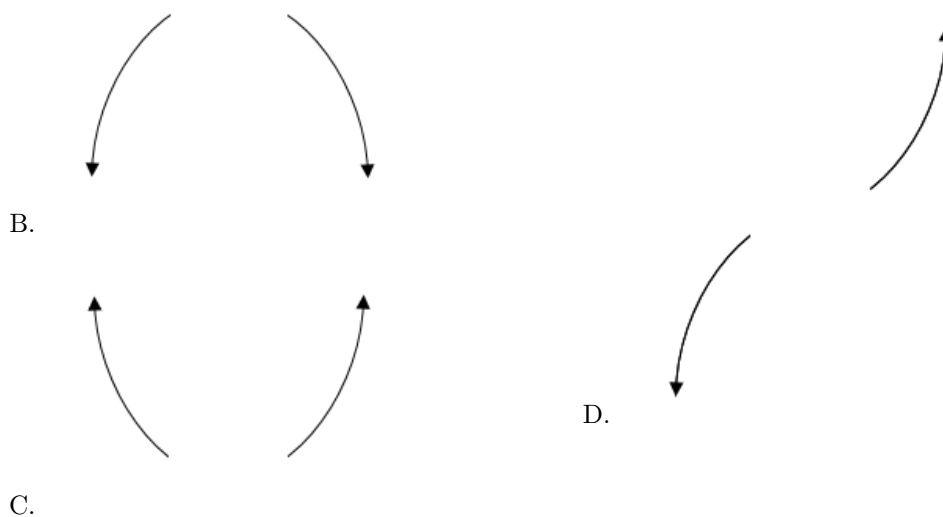
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-2 + 2i))(x - (-2 - 2i))(x - (1))$.

10. Describe the end behavior of the polynomial below.

$$f(x) = 6(x + 3)^3(x - 3)^6(x - 2)^5(x + 2)^6$$

The solution is the graph below, which is option C.





General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

11. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } 1$$

The solution is $x^3 + 9x^2 + 31x - 41$, which is option A.

- A. $b \in [7, 12]$, $c \in [31, 38]$, and $d \in [-48, -38]$

* $x^3 + 9x^2 + 31x - 41$, which is the correct option.

- B. $b \in [0, 5]$, $c \in [-2, 10]$, and $d \in [-10, -2]$

$x^3 + x^2 + 4x - 5$, which corresponds to multiplying out $(x + 5)(x - 1)$.

- C. $b \in [-10, -7]$, $c \in [31, 38]$, and $d \in [35, 42]$

$x^3 - 9x^2 + 31x + 41$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x + 1)$.

- D. $b \in [0, 5]$, $c \in [-7, 0]$, and $d \in [2, 5]$

$x^3 + x^2 - 5x + 4$, which corresponds to multiplying out $(x - 4)(x - 1)$.

- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (1))$.

12. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-7}{3}, \frac{-3}{2}, \text{ and } -1$$

The solution is $6x^3 + 29x^2 + 44x + 21$, which is option E.

A. $a \in [0, 8], b \in [-18, -13], c \in [-7, -1]$, and $d \in [20, 27]$

$6x^3 - 17x^2 - 2x + 21$, which corresponds to multiplying out $(3x - 7)(2x - 3)(x + 1)$.

B. $a \in [0, 8], b \in [-31, -26], c \in [43, 48]$, and $d \in [-21, -18]$

$6x^3 - 29x^2 + 44x - 21$, which corresponds to multiplying out $(3x - 7)(2x - 3)(x - 1)$.

C. $a \in [0, 8], b \in [-2, 12], c \in [-29, -22]$, and $d \in [-21, -18]$

$6x^3 + x^2 - 26x - 21$, which corresponds to multiplying out $(3x - 7)(2x + 3)(x + 1)$.

D. $a \in [0, 8], b \in [26, 34], c \in [43, 48]$, and $d \in [-21, -18]$

$6x^3 + 29x^2 + 44x - 21$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [0, 8], b \in [26, 34], c \in [43, 48]$, and $d \in [20, 27]$

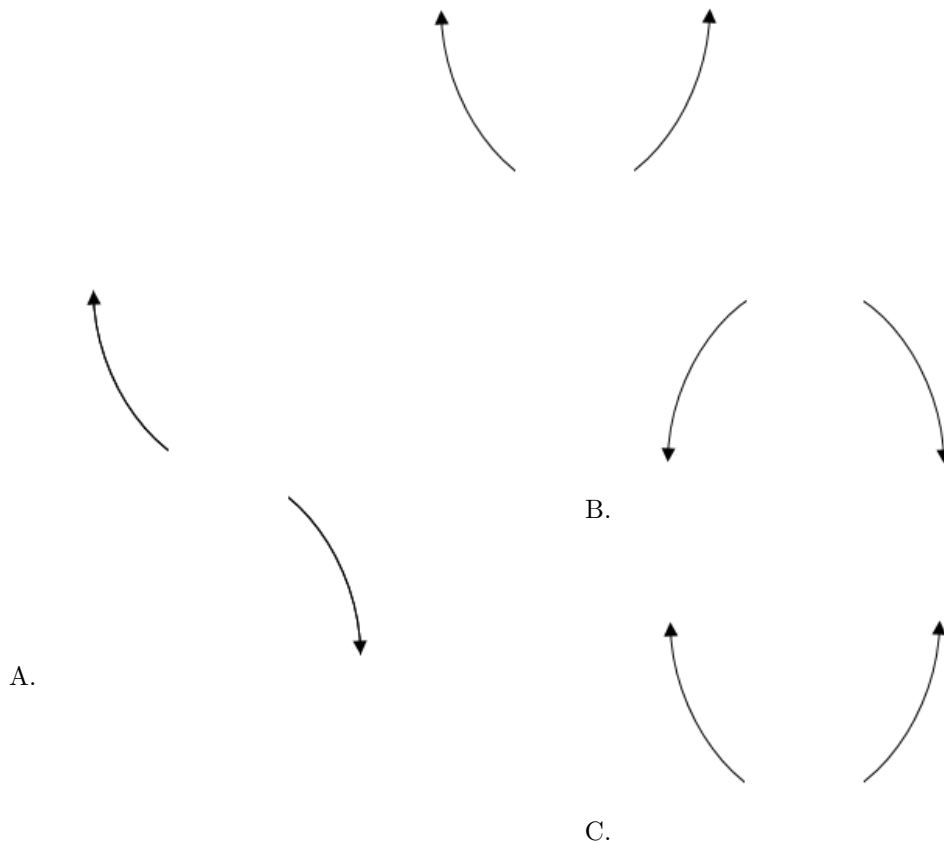
* $6x^3 + 29x^2 + 44x + 21$, which is the correct option.

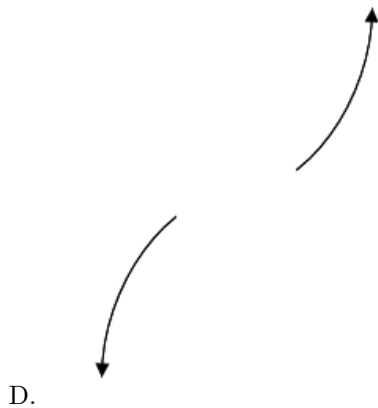
General Comment: To construct the lowest-degree polynomial, you want to multiply out $(3x + 7)(2x + 3)(x + 1)$

13. Describe the end behavior of the polynomial below.

$$f(x) = 8(x + 3)^5(x - 3)^{10}(x + 9)^2(x - 9)^3$$

The solution is the graph below, which is option C.



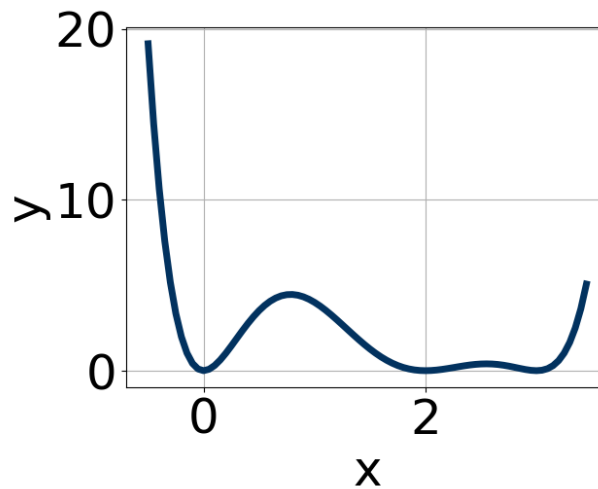


D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

14. Which of the following equations *could* be of the graph presented below?



The solution is $20x^8(x-3)^4(x-2)^6$, which is option C.

A. $-6x^{10}(x-3)^8(x-2)^{11}$

The factor $(x-2)$ should have an even power and the leading coefficient should be the opposite sign.

B. $-12x^8(x-3)^{10}(x-2)^{10}$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $20x^8(x-3)^4(x-2)^6$

* This is the correct option.

D. $19x^{10}(x-3)^{10}(x-2)^5$

The factor $(x-2)$ should have an even power.

E. $17x^7(x-3)^8(x-2)^7$

The factors x and $(x-2)$ should both have even powers.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

15. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-3}{4}, 4, \text{ and } \frac{4}{3}$$

The solution is $12x^3 - 55x^2 + 16x + 48$, which is option C.

A. $a \in [6, 19], b \in [54, 56], c \in [13, 20], \text{ and } d \in [-52, -44]$

$12x^3 + 55x^2 + 16x - 48$, which corresponds to multiplying out $(4x-3)(x+4)(3x+4)$.

B. $a \in [6, 19], b \in [-58, -53], c \in [13, 20], \text{ and } d \in [-52, -44]$

$12x^3 - 55x^2 + 16x - 48$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [6, 19], b \in [-58, -53], c \in [13, 20], \text{ and } d \in [47, 52]$

* $12x^3 - 55x^2 + 16x + 48$, which is the correct option.

D. $a \in [6, 19], b \in [22, 29], c \in [-90, -84], \text{ and } d \in [47, 52]$

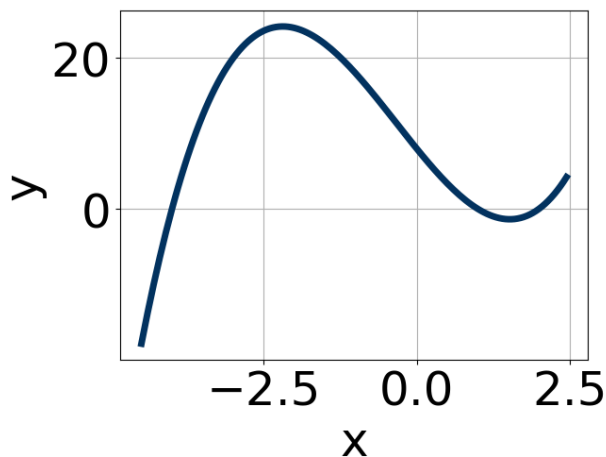
$12x^3 + 23x^2 - 88x + 48$, which corresponds to multiplying out $(4x-3)(x+4)(3x-4)$.

E. $a \in [6, 19], b \in [-77, -65], c \in [111, 121], \text{ and } d \in [-52, -44]$

$12x^3 - 73x^2 + 112x - 48$, which corresponds to multiplying out $(4x-3)(x-4)(3x-4)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x+3)(x-4)(3x-4)$

16. Which of the following equations *could* be of the graph presented below?



The solution is $11(x-1)^7(x-2)^9(x+4)^7$, which is option C.

A. $-7(x-1)^8(x-2)^7(x+4)^{11}$

The factor $(x-1)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $-17(x-1)^5(x-2)^9(x+4)^9$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $11(x-1)^7(x-2)^9(x+4)^7$

* This is the correct option.

D. $20(x-1)^{10}(x-2)^8(x+4)^{11}$

The factors 1 and 2 have been odd power.

E. $18(x-1)^6(x-2)^{11}(x+4)^5$

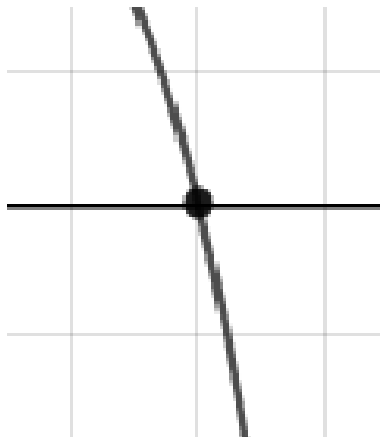
The factor 1 should have been an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

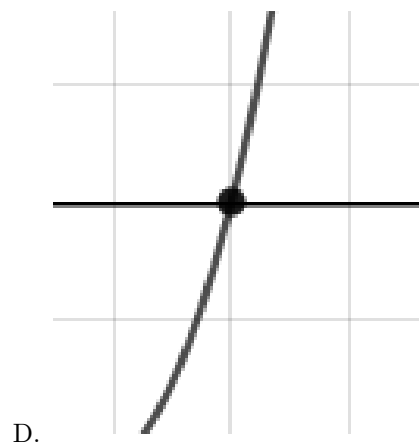
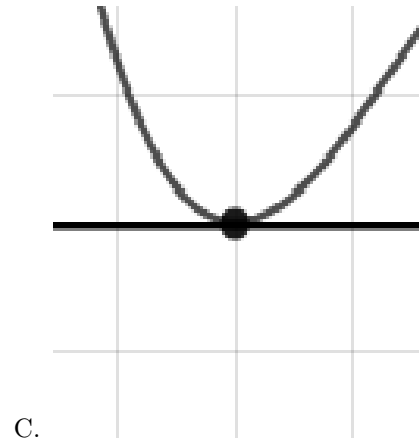
17. Describe the zero behavior of the zero $x = -4$ of the polynomial below.

$$f(x) = 4(x+4)^8(x-4)^{13}(x-8)^2(x+8)^6$$

The solution is the graph below, which is option B.



A.



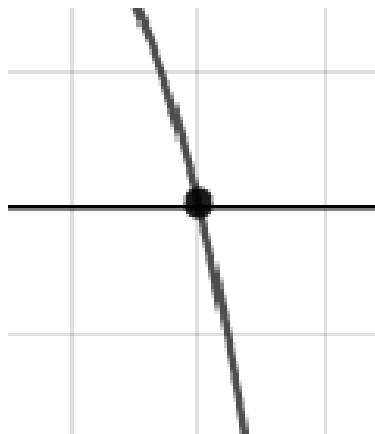
E. None of the above.

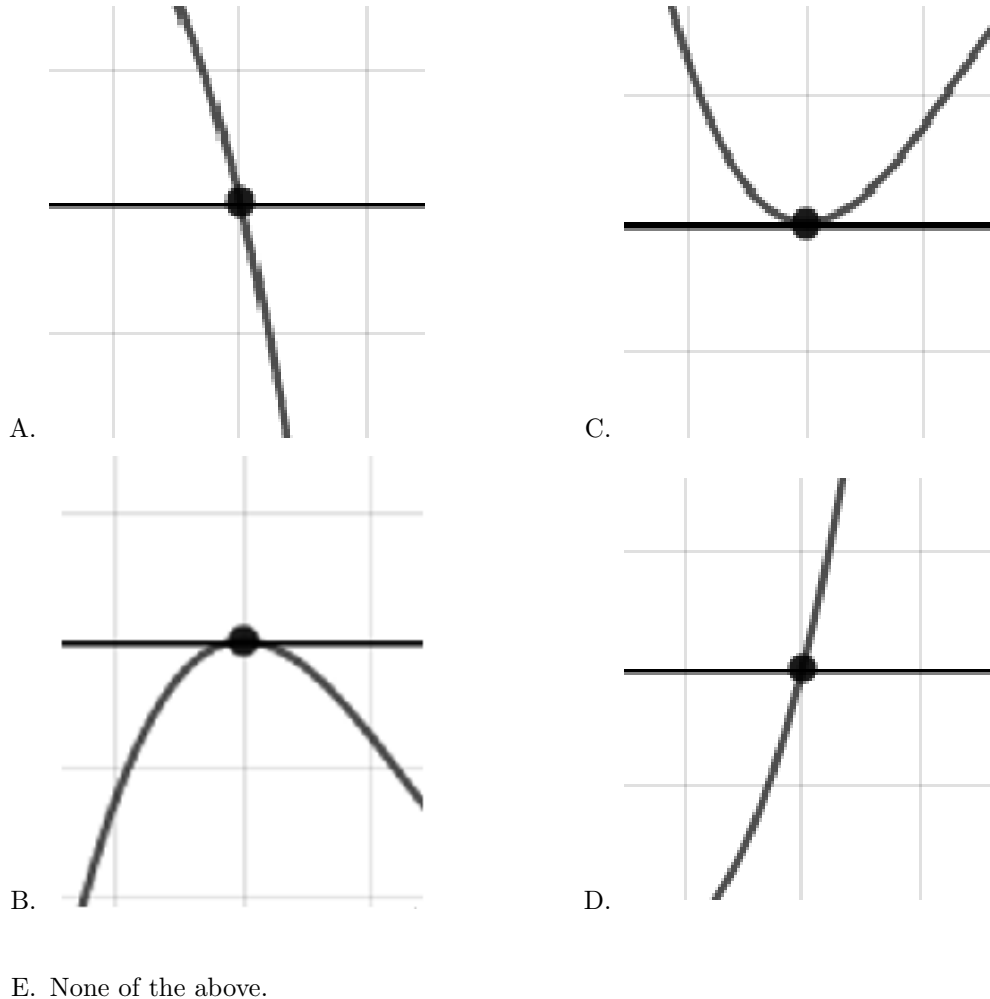
General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

18. Describe the zero behavior of the zero $x = -6$ of the polynomial below.

$$f(x) = -4(x - 8)^{12}(x + 8)^8(x + 6)^{11}(x - 6)^8$$

The solution is the graph below, which is option A.





General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

19. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } 4$$

The solution is $x^3 + 6x^2 + x - 164$, which is option D.

- A. $b \in [-7.2, -3.1]$, $c \in [-6, 9]$, and $d \in [157, 173]$

$x^3 - 6x^2 + x + 164$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x + 4)$.

- B. $b \in [-1.8, 4.3]$, $c \in [-6, 9]$, and $d \in [-26, -16]$

$x^3 + x^2 + x - 20$, which corresponds to multiplying out $(x + 5)(x - 4)$.

- C. $b \in [-1.8, 4.3]$, $c \in [-10, -5]$, and $d \in [16, 23]$

$x^3 + x^2 - 8x + 16$, which corresponds to multiplying out $(x - 4)(x - 4)$.

- D. $b \in [5.4, 7.5]$, $c \in [-6, 9]$, and $d \in [-165, -163]$

* $x^3 + 6x^2 + x - 164$, which is the correct option.

E. None of the above.

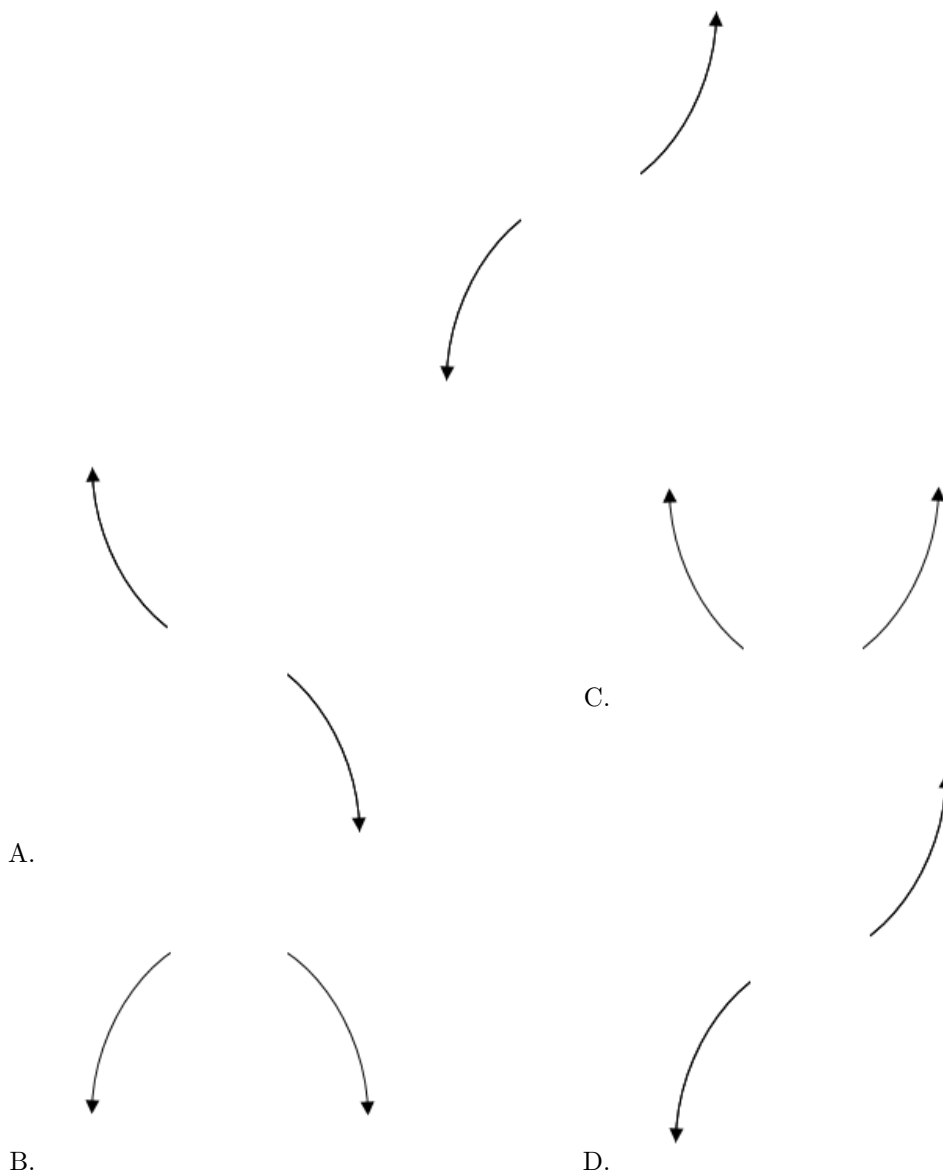
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (4))$.

20. Describe the end behavior of the polynomial below.

$$f(x) = 4(x - 9)^3(x + 9)^8(x - 8)^4(x + 8)^4$$

The solution is the graph below, which is option D.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

21. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 - 2i \text{ and } -4$$

The solution is $x^3 - 4x^2 - 12x + 80$, which is option B.

- A. $b \in [2.8, 5.1], c \in [-12, -11]$, and $d \in [-87, -78]$

$x^3 + 4x^2 - 12x - 80$, which corresponds to multiplying out $(x - (4 - 2i))(x - (4 + 2i))(x - 4)$.

- B. $b \in [-7.8, -3.5], c \in [-12, -11]$, and $d \in [78, 84]$

* $x^3 - 4x^2 - 12x + 80$, which is the correct option.

- C. $b \in [-0.6, 1.1], c \in [3, 7]$, and $d \in [3, 9]$

$x^3 + x^2 + 6x + 8$, which corresponds to multiplying out $(x + 2)(x + 4)$.

- D. $b \in [-0.6, 1.1], c \in [0, 5]$, and $d \in [-20, -13]$

$x^3 + x^2 - 16$, which corresponds to multiplying out $(x - 4)(x + 4)$.

- E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 - 2i))(x - (4 + 2i))(x - (-4))$.

22. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$-6, \frac{1}{3}, \text{ and } \frac{-3}{2}$$

The solution is $6x^3 + 43x^2 + 39x - 18$, which is option B.

- A. $a \in [6, 12], b \in [-25.3, -24.5], c \in [-64, -61]$, and $d \in [-24, -15]$

$6x^3 - 25x^2 - 63x - 18$, which corresponds to multiplying out $(x - 6)(3x + 1)(2x + 3)$.

- B. $a \in [6, 12], b \in [40.1, 45.7], c \in [33, 40]$, and $d \in [-24, -15]$

* $6x^3 + 43x^2 + 39x - 18$, which is the correct option.

- C. $a \in [6, 12], b \in [-30.7, -26], c \in [-53, -38]$, and $d \in [11, 26]$

$6x^3 - 29x^2 - 45x + 18$, which corresponds to multiplying out $(x - 6)(3x - 1)(2x + 3)$.

- D. $a \in [6, 12], b \in [-44.1, -41], c \in [33, 40]$, and $d \in [11, 26]$

$6x^3 - 43x^2 + 39x + 18$, which corresponds to multiplying out $(x - 6)(3x + 1)(2x - 3)$.

- E. $a \in [6, 12], b \in [40.1, 45.7], c \in [33, 40]$, and $d \in [11, 26]$

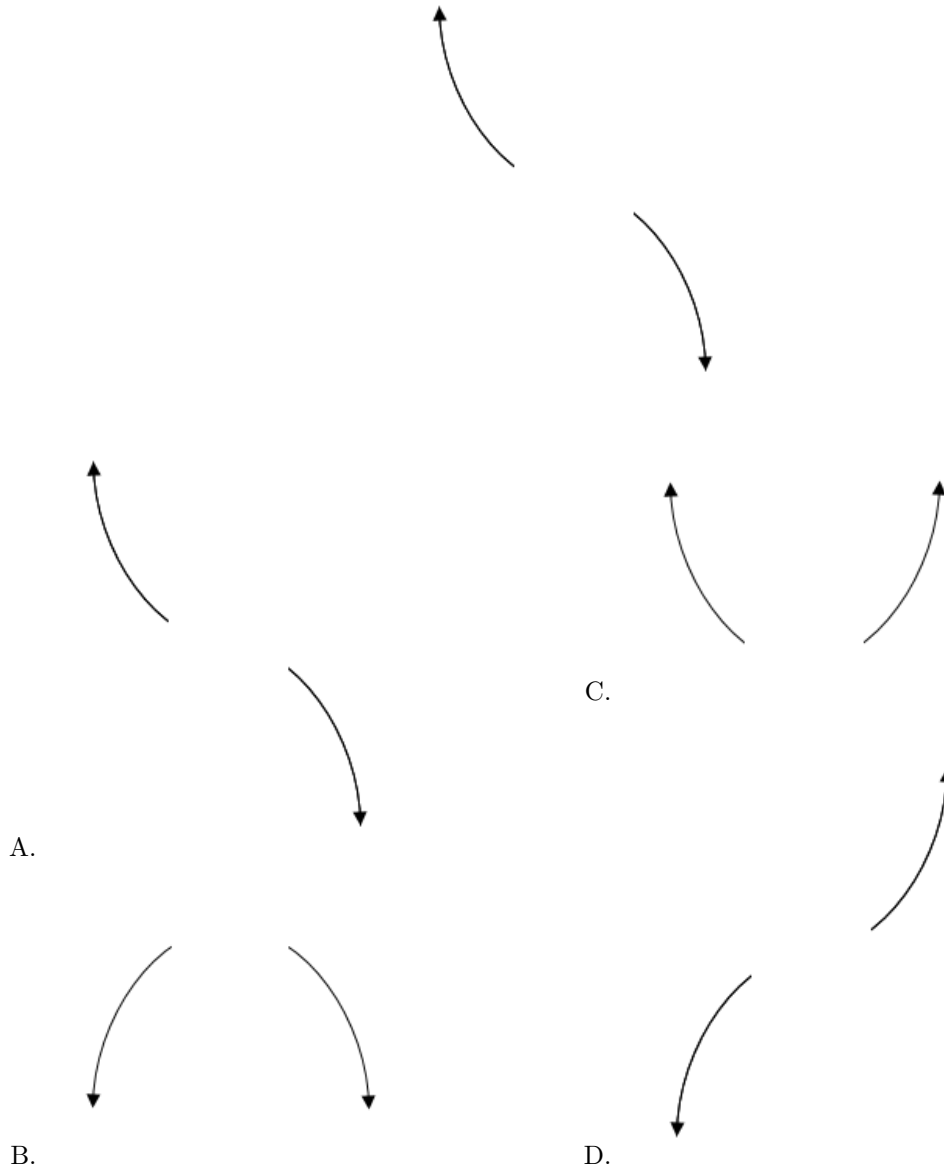
$6x^3 + 43x^2 + 39x + 18$, which corresponds to multiplying everything correctly except the constant term.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(x+6)(3x-1)(2x+3)$

23. Describe the end behavior of the polynomial below.

$$f(x) = -6(x+7)^5(x-7)^{10}(x-8)^3(x+8)^3$$

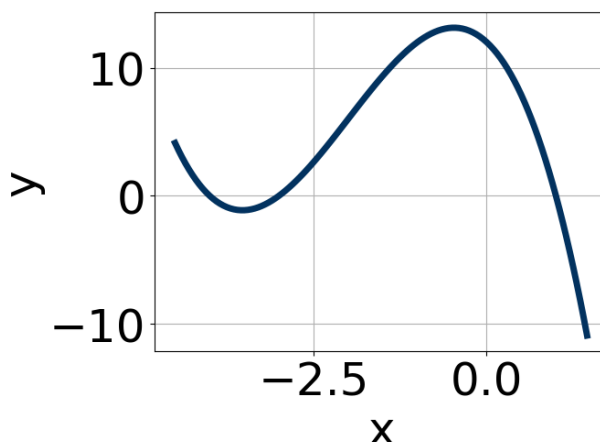
The solution is the graph below, which is option A.



E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

24. Which of the following equations *could* be of the graph presented below?



The solution is $-13(x+3)^9(x-1)^7(x+4)^5$, which is option E.

A. $-10(x+3)^{10}(x-1)^{11}(x+4)^9$

The factor -3 should have been an odd power.

B. $9(x+3)^{10}(x-1)^5(x+4)^5$

The factor $(x+3)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $16(x+3)^7(x-1)^7(x+4)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $-17(x+3)^{10}(x-1)^6(x+4)^{11}$

The factors -3 and 1 have been odd power.

E. $-13(x+3)^9(x-1)^7(x+4)^5$

* This is the correct option.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

25. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{1}{4}, \frac{7}{4}, \text{ and } \frac{-2}{3}$$

The solution is $48x^3 - 64x^2 - 43x + 14$, which is option C.

A. $a \in [45, 50], b \in [123, 130], c \in [83, 89], \text{ and } d \in [12, 19]$

$48x^3 + 128x^2 + 85x + 14$, which corresponds to multiplying out $(4x+1)(4x+7)(3x+2)$.

B. $a \in [45, 50], b \in [-41, -33], c \in [-70, -66], \text{ and } d \in [-20, -13]$

$48x^3 - 40x^2 - 69x - 14$, which corresponds to multiplying out $(4x+1)(4x-7)(3x+2)$.

C. $a \in [45, 50], b \in [-66, -60], c \in [-43, -33], \text{ and } d \in [12, 19]$

* $48x^3 - 64x^2 - 43x + 14$, which is the correct option.

D. $a \in [45, 50]$, $b \in [-66, -60]$, $c \in [-43, -33]$, and $d \in [-20, -13]$

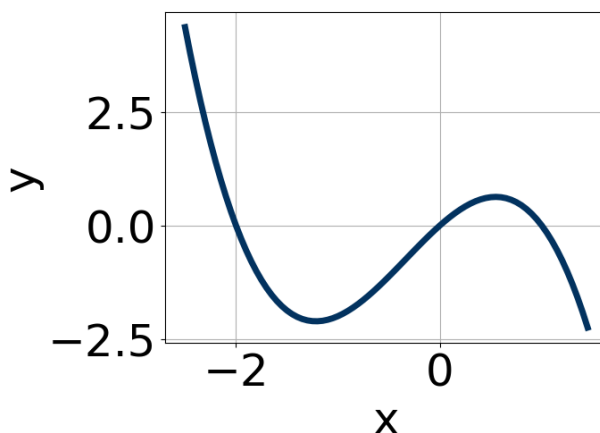
$48x^3 - 64x^2 - 43x - 14$, which corresponds to multiplying everything correctly except the constant term.

E. $a \in [45, 50]$, $b \in [64, 70]$, $c \in [-43, -33]$, and $d \in [-20, -13]$

$48x^3 + 64x^2 - 43x - 14$, which corresponds to multiplying out $(4x + 1)(4x + 7)(3x - 2)$.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x - 1)(4x - 7)(3x + 2)$

26. Which of the following equations *could* be of the graph presented below?



The solution is $-12x^5(x + 2)^5(x - 1)^9$, which is option C.

A. $11x^9(x + 2)^6(x - 1)^5$

The factor $(x + 2)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $11x^{11}(x + 2)^5(x - 1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

C. $-12x^5(x + 2)^5(x - 1)^9$

* This is the correct option.

D. $-6x^7(x + 2)^{10}(x - 1)^7$

The factor -2 should have been an odd power.

E. $-9x^6(x + 2)^4(x - 1)^7$

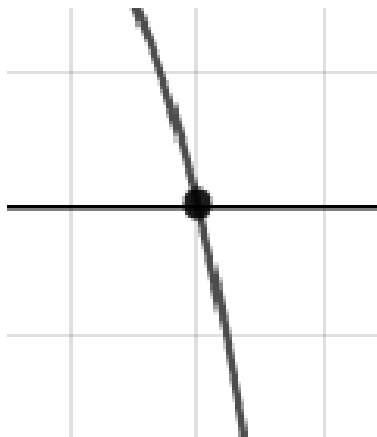
The factors -2 and 0 have have been odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

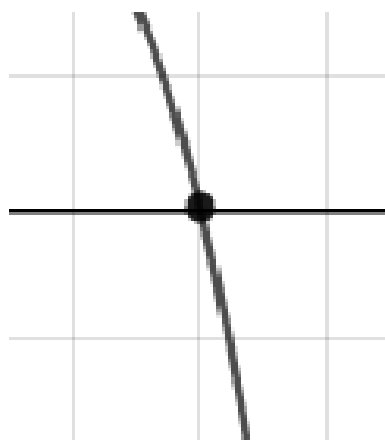
27. Describe the zero behavior of the zero $x = -8$ of the polynomial below.

$$f(x) = 3(x + 2)^5(x - 2)^2(x + 8)^7(x - 8)^2$$

The solution is the graph below, which is option A.



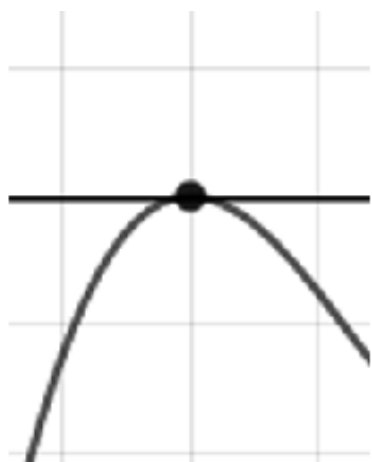
A.



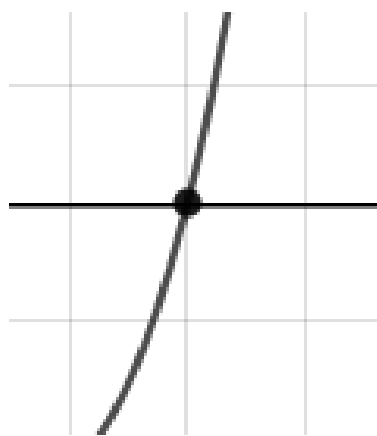
C.



B.



D.



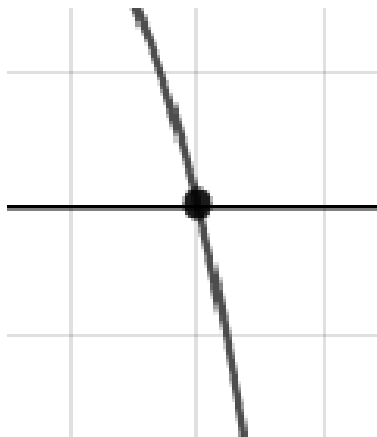
E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

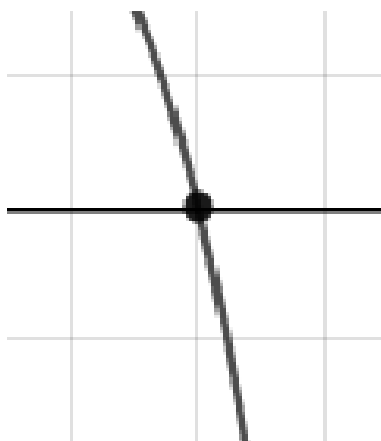
28. Describe the zero behavior of the zero $x = -5$ of the polynomial below.

$$f(x) = 7(x - 5)^2(x + 5)^5(x + 9)^8(x - 9)^{11}$$

The solution is the graph below, which is option A.



A.



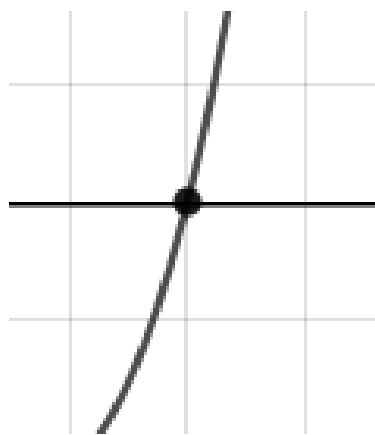
C.



B.



D.



E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

-
29. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-4 + 3i \text{ and } 3$$

The solution is $x^3 + 5x^2 + x - 75$, which is option B.

A. $b \in [-0.5, 2]$, $c \in [-15, -4]$, and $d \in [7, 12]$

$x^3 + x^2 - 6x + 9$, which corresponds to multiplying out $(x - 3)(x - 3)$.

B. $b \in [3.6, 8.1]$, $c \in [0, 5]$, and $d \in [-77, -74]$

* $x^3 + 5x^2 + x - 75$, which is the correct option.

C. $b \in [-5.2, 0.5]$, $c \in [0, 5]$, and $d \in [70, 77]$

$x^3 - 5x^2 + x + 75$, which corresponds to multiplying out $(x - (-4 + 3i))(x - (-4 - 3i))(x + 3)$.

D. $b \in [-0.5, 2]$, $c \in [0, 5]$, and $d \in [-14, -5]$

$x^3 + x^2 + x - 12$, which corresponds to multiplying out $(x + 4)(x - 3)$.

E. None of the above.

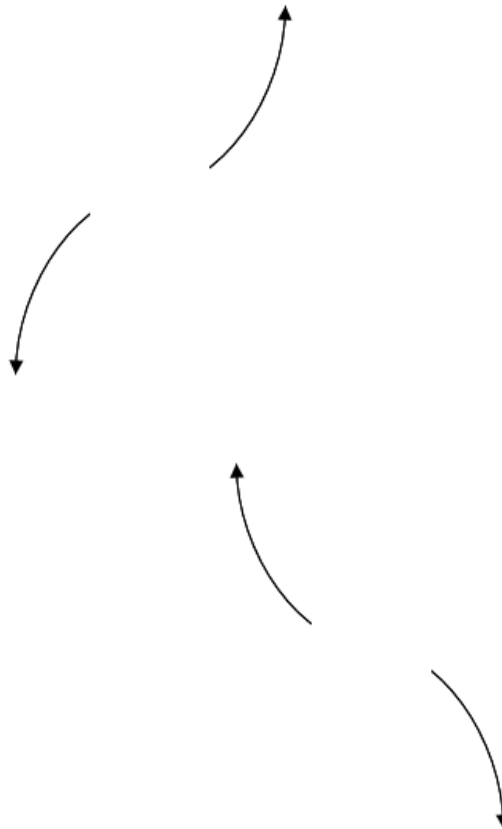
This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

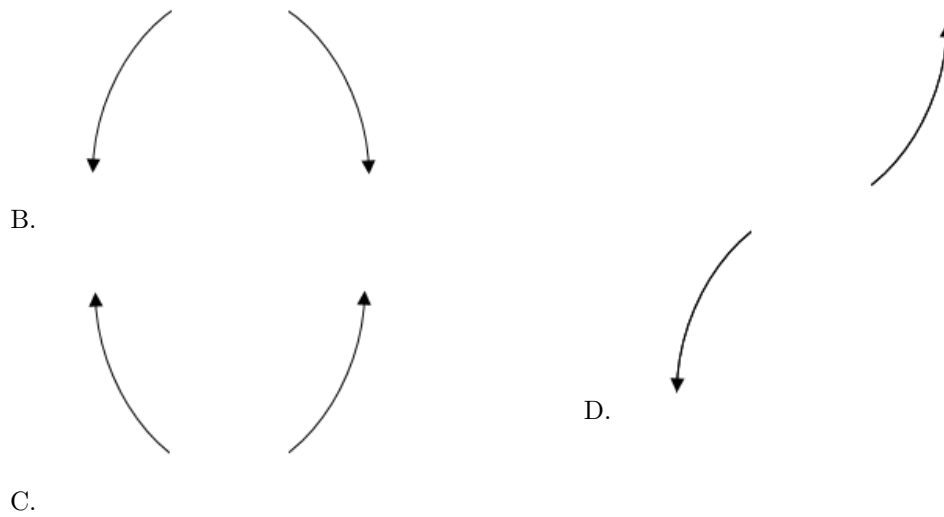
General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-4 + 3i))(x - (-4 - 3i))(x - (3))$.

30. Describe the end behavior of the polynomial below.

$$f(x) = 5(x - 6)^2(x + 6)^3(x + 3)^5(x - 3)^7$$

The solution is the graph below, which is option D.





E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.
