This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 7x^3 + 7x^2 + 3x + 5$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 7}$, which is option A.

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 7}$

* This is the solution **since we asked for the possible Rational roots**!

B. $\pm 1, \pm 7$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

C. $\pm 1, \pm 5$

This would have been the solution **if asked for the possible Integer roots**!

D. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 5}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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2. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 4x^4 + 3x^3 + 3x^2 + 3x + 6$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$, which is option A.

A. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

* This is the solution **since we asked for the possible Rational roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2, \pm 3, \pm 6}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. $\pm 1, \pm 2, \pm 4$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. $\pm 1, \pm 2, \pm 3, \pm 6$

This would have been the solution **if asked for the possible Integer roots**!

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Factor the polynomial below completely, knowing that $x+5$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 9x^4 + 27x^3 - 127x^2 - 155x + 150$$

The solution is $[-5, -1.667, 0.667, 3]$, which is option A.

A. $z_1 \in [-5.2, -4.9]$, $z_2 \in [-1.81, -1.57]$, $z_3 \in [0.65, 0.78]$, and $z_4 \in [1.9, 4.5]$

* This is the solution!

B. $z_1 \in [-5.2, -4.9]$, $z_2 \in [-0.62, -0.5]$, $z_3 \in [1.46, 1.56]$, and $z_4 \in [1.9, 4.5]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-4.5, -1.9]$, $z_2 \in [-0.8, -0.62]$, $z_3 \in [1.62, 1.74]$, and $z_4 \in [3.6, 5.7]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-4.5, -1.9]$, $z_2 \in [-0.25, -0.17]$, $z_3 \in [4.88, 5.04]$, and $z_4 \in [3.6, 5.7]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-4.5, -1.9]$, $z_2 \in [-1.51, -1.43]$, $z_3 \in [0.59, 0.62]$, and $z_4 \in [3.6, 5.7]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 62x^2 - 16}{x+3}$$

The solution is $20x^2 + 2x - 6 + \dfrac{2}{x+3}$, which is option C.

A. $a \in [-60, -57], b \in [242, 243], c \in [-730, -721]$, and $r \in [2161, 2168]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [-60, -57], b \in [-119, -110], c \in [-354, -349]$, and $r \in [-1080, -1074]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [18, 23], b \in [-2, 7], c \in [-13, -1]$, and $r \in [-2, 3]$.

* This is the solution!

D. $a \in [18, 23], b \in [-18, -15], c \in [70, 76]$, and $r \in [-311, -303]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [18, 23], b \in [120, 128], c \in [364, 373]$, and $r \in [1076, 1088]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 30x^2 + 43}{x - 2}$$

The solution is $10x^2 - 10x - 20 + \frac{3}{x-2}$, which is option D.

A. $a \in [16, 23], b \in [9, 11], c \in [12, 29]$, and $r \in [81, 88]$.

You multipled by the synthetic number rather than bringing the first factor down.

B. $a \in [16, 23], b \in [-70, -67], c \in [139, 141]$, and $r \in [-239, -231]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [6, 13], b \in [-20, -15], c \in [-21, -18]$, and $r \in [21, 24]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [6, 13], b \in [-19, -7], c \in [-21, -18]$, and $r \in [-1, 4]$.

* This is the solution!

E. $a \in [6, 13], b \in [-50, -48], c \in [95, 104]$, and $r \in [-158, -154]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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6. Factor the polynomial below completely, knowing that $x + 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 10x^3 - 101x^2 + 238x - 120$$

The solution is $[-4, 0.75, 2, 2.5]$, which is option B.

A. $z_1 \in [-2.15, -1.66]$, $z_2 \in [-1.58, -1.33], z_3 \in [-0.52, -0.16]$, and $z_4 \in [3.33, 4.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-4.34, -3.6]$, $z_2 \in [0.46, 0.95], z_3 \in [1.97, 2.28]$, and $z_4 \in [2.01, 2.65]$

* This is the solution!

C. $z_1 \in [-2.68, -2.27]$, $z_2 \in [-2.16, -1.71], z_3 \in [-0.92, -0.7]$, and $z_4 \in [3.33, 4.3]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-4.34, -3.6]$, $z_2 \in [0.37, 0.42]$, $z_3 \in [1.07, 1.43]$, and $z_4 \in [0.8, 2.23]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-3.35, -2.63]$, $z_2 \in [-2.16, -1.71]$, $z_3 \in [-0.64, -0.62]$, and $z_4 \in [3.33, 4.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 - 43x^2 - 3x + 18$$

The solution is $[-0.6, 0.75, 2]$, which is option D.

A. $z_1 \in [-1.74, -1.55]$, $z_2 \in [1.33, 1.4]$, and $z_3 \in [1.74, 2.19]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-2.07, -1.88]$, $z_2 \in [-0.9, -0.68]$, and $z_3 \in [0.53, 0.61]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-2.07, -1.88]$, $z_2 \in [-0.47, 0.06]$, and $z_3 \in [2.75, 3.01]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-0.8, -0.48]$, $z_2 \in [0.43, 0.86]$, and $z_3 \in [1.74, 2.19]$

* This is the solution!

E. $z_1 \in [-2.07, -1.88]$, $z_2 \in [-1.41, -1.21]$, and $z_3 \in [1.07, 1.95]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 25x^3 + 50x^2 - 9x - 18$$

The solution is $[-2, -0.6, 0.6]$, which is option E.

A. $z_1 \in [-3.1, -2.6]$, $z_2 \in [0.06, 0.51]$, and $z_3 \in [1.92, 2.63]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-1.9, -0.7]$, $z_2 \in [1.54, 1.86]$, and $z_3 \in [1.92, 2.63]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-1.5, 0.1]$, $z_2 \in [0.29, 1.22]$, and $z_3 \in [1.92, 2.63]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-2.1, -1.8]$, $z_2 \in [-1.91, -1.44]$, and $z_3 \in [1.3, 1.89]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-2.1, -1.8]$, $z_2 \in [-1.34, -0.43]$, and $z_3 \in [0.54, 0.97]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 29x^2 - 50x + 26}{x - 4}$$

The solution is $10x^2 + 11x - 6 + \frac{2}{x-4}$, which is option E.

A. $a \in [38, 42]$, $b \in [130, 134]$, $c \in [471, 482]$, and $r \in [1920, 1925]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [5, 15]$, $b \in [-73, -61]$, $c \in [220, 231]$, and $r \in [-883, -870]$.

You divided by the opposite of the factor.

C. $a \in [38, 42]$, $b \in [-190, -187]$, $c \in [704, 707]$, and $r \in [-2800, -2793]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

D. $a \in [5, 15]$, $b \in [0, 4]$, $c \in [-47, -45]$, and $r \in [-117, -112]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [5, 15]$, $b \in [9, 14]$, $c \in [-13, -3]$, and $r \in [0, 7]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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10. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 85x^2 + 200x - 129}{x - 5}$$

The solution is $10x^2 - 35x + 25 + \frac{-4}{x-5}$, which is option C.

A. $a \in [49, 53]$, $b \in [-335, -331]$, $c \in [1872, 1879]$, and $r \in [-9510, -9502]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [3, 11]$, $b \in [-136, -134]$, $c \in [875, 882]$, and $r \in [-4504, -4494]$.

You divided by the opposite of the factor.

C. $a \in [3, 11]$, $b \in [-41, -33]$, $c \in [25, 28]$, and $r \in [-4, 1]$.

* This is the solution!

D. $a \in [49, 53]$, $b \in [164, 171]$, $c \in [1020, 1033]$, and $r \in [4988, 5000]$.

You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [3, 11]$, $b \in [-45, -44]$, $c \in [18, 23]$, and $r \in [-51, -48]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!