This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Multiply the following functions, then choose the domain of the resulting function from the list below.

   $$f(x) = 4x^2 + 4x + 4 \text{ and } g(x) = 5x^2 + 8x + 2$$

   The solution is $(-\infty, \infty)$, which is option E.

   A. The domain is all Real numbers except $x = a,$ where $a \in [-12.67, -3.67]$

   B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [0.25, 6.25]$

   C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-5.4, 0.6]$

   D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [3.67, 9.67]$ and $b \in [-4.33, 0.67]$

   E. The domain is all Real numbers.

   General Comment: The new domain is the intersection of the previous domains.

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2. Choose the interval below that f composed with g at $x = 1$ is in.

   $$f(x) = -x^3 - 2x^2 + x \text{ and } g(x) = -4x^3 - 4x^2 + 4x + 3$$

   The solution is -2.0, which is option A.

   A. $(f \circ g)(1) \in [-3.2, 0.5]$

      * This is the correct solution

   B. $(f \circ g)(1) \in [9.5, 11.2]$

      Distractor 1: Corresponds to reversing the composition.

   C. $(f \circ g)(1) \in [4.5, 8]$

      Distractor 3: Corresponds to being slightly off from the solution.

   D. $(f \circ g)(1) \in [2.6, 4.9]$

      Distractor 2: Corresponds to being slightly off from the solution.

   E. It is not possible to compose the two functions.

   General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Choose the interval below that f composed with g at $x = 1$ is in.

   $$f(x) = -3x^3 + 4x^2 + 4x \text{ and } g(x) = x^3 - 1x^2 - 3x + 1$$

   The solution is 32.0, which is option B.

   A. $(f \circ g)(1) \in [81, 89]$

      Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(1) \in [28, 36]$

* This is the correct solution

C. $(f \circ g)(1) \in [35, 42]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [88, 96]$

Distractor 3: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^-1(7)$ belongs to.

$$f(x) = e^{x-2} - 2$$

The solution is $f^{-1}(7) = 4.197$, which is option E.

A. $f^{-1}(7) \in [-1.06, 0.03]$

This solution corresponds to distractor 4.

B. $f^{-1}(7) \in [-0.08, 0.39]$

This solution corresponds to distractor 1.

C. $f^{-1}(7) \in [-1.06, 0.03]$

This solution corresponds to distractor 2.

D. $f^{-1}(7) \in [-0.08, 0.39]$

This solution corresponds to distractor 3.

E. $f^{-1}(7) \in [3.32, 4.75]$

This is the solution.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -12$ and choose the interval that $f^-1(-12)$ belongs to.

$$f(x) = \sqrt[3]{2x - 5}$$

The solution is -861.5, which is option A.

A. $f^{-1}(-12) \in [-863.5, -860.5]$

* This is the correct solution.

B. $f^{-1}(-12) \in [-866.5, -865.5]$

Distractor 1: This corresponds to

C. $f^{-1}(-12) \in [866.5, 870.5]$

This solution corresponds to distractor 3.

D. $f^{-1}(-12) \in [855.5, 862.5]$

This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^-1(9)$ belongs to.

$$f(x) = e^{x-5} + 2$$

The solution is $f^{-1}(9) = 6.946$, which is option A.

A. $f^{-1}(9) \in [6.88, 7.09]$

This is the solution.

B. $f^{-1}(9) \in [-3.15, -2.83]$

This solution corresponds to distractor 1.

C. $f^{-1}(9) \in [4.54, 4.85]$

This solution corresponds to distractor 3.

D. $f^{-1}(9) \in [4.28, 4.42]$

This solution corresponds to distractor 2.

E. $f^{-1}(9) \in [3.38, 3.42]$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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7. Determine whether the function below is 1-1.

$$f(x) = 25x^2 - 90x - 319$$

The solution is no, which is option C.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

C. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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8. Determine whether the function below is 1-1.

$$f(x) = 9x^2 - 60x + 100$$

The solution is no, which is option A.

A. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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9. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \frac{5}{3x - 20} \text{ and } g(x) = 7x + 3$$

The solution is The domain is all Real numbers except $x = 6.67$, which is option C.

A. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-7.4, 3.6]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [2, 4]$

C. The domain is all Real numbers except $x = a,$ where $a \in [5.67, 7.67]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-0.33, 8.67]$ and $b \in [2.83, 11.83]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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10. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 14$ and choose the interval that $f^-1(14)$ belongs to.

$$f(x) = 5x^2 - 4$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(14) \in [2.69, 3.04]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B. $f^{-1}(14) \in [5.7, 5.94]$

Distractor 4: This corresponds to both distractors 2 and 3.

C. $f^{-1}(14) \in [0.88, 1.83]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(14) \in [1.81, 2.14]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!
