This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 6x^2 + x + 6 \text{ and } g(x) = \sqrt{4x + 25}$$

The solution is The domain is all Real numbers greater than or equal to$x = -6.25$., which is option B.

A. The domain is all Real numbers except $x = a,$ where $a \in [2.4, 11.4]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [-9.25, -4.25]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-2.5, 10.5]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [5.67, 11.67]$ and $b \in [-7.83, -0.83]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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2. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 3x^3 + 2x^2 - x - 4 \text{ and } g(x) = 2x^3 - 4x^2 + 2x + 2$$

The solution is 26.0, which is option D.

A. $(f \circ g)(1) \in [34, 39]$

Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [2, 4]$

Distractor 1: Corresponds to reversing the composition.

C. $(f \circ g)(1) \in [8, 18]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [21, 31]$

* This is the correct solution

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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3. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 2x^3 + 3x^2 - 2x \text{ and } g(x) = -3x^3 - 3x^2 + 4x$$

The solution is 0.0, which is option C.

A. $(f \circ g)(1) \in [1, 11]$

Distractor 2: Corresponds to being slightly off from the solution.

B. $(f \circ g)(1) \in [-109, -103]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-1, 1]$

* This is the correct solution

D. $(f \circ g)(1) \in [-102, -90]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 6$ and choose the interval that $f^-1(6)$ belongs to.

$$f(x) = \ln(x + 4) + 4$$

The solution is $f^{-1}(6) = 3.389$, which is option A.

A. $f^{-1}(6) \in [1.39, 5.39]$

This is the solution.

B. $f^{-1}(6) \in [22019.47, 22024.47]$

This solution corresponds to distractor 1.

C. $f^{-1}(6) \in [9.39, 14.39]$

This solution corresponds to distractor 2.

D. $f^{-1}(6) \in [22029.47, 22035.47]$

This solution corresponds to distractor 4.

E. $f^{-1}(6) \in [9.39, 14.39]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 15$ and choose the interval that $f^-1(15)$ belongs to.

$$f(x) = 2x^2 - 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(15) \in [5.93, 6.5]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(15) \in [2.58, 3.53]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C. $f^{-1}(15) \in [4.86, 5.77]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(15) \in [2.06, 2.97]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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6. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^-1(8)$ belongs to.

$$f(x) = e^{x-5} - 2$$

The solution is $f^{-1}(8) = 7.303$, which is option D.

A. $f^{-1}(8) \in [-0.01, 1.1]$

This solution corresponds to distractor 3.

B. $f^{-1}(8) \in [-3.28, -2.68]$

This solution corresponds to distractor 1.

C. $f^{-1}(8) \in [-2.24, -0.83]$

This solution corresponds to distractor 4.

D. $f^{-1}(8) \in [7.11, 8.12]$

This is the solution.

E. $f^{-1}(8) \in [-0.33, 0.41]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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7. Determine whether the function below is 1-1.

$$f(x) = 36x^2 - 204x + 289$$

The solution is no, which is option C.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

B. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

C. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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8. Determine whether the function below is 1-1.

$$f(x) = 9x^2 - 21x - 228$$

The solution is no, which is option B.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

D. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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9. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-6x + 22} \text{ and } g(x) = 8x + 3$$

The solution is The domain is all Real numbers less than or equal to$x = 3.67$., which is option C.

A. The domain is all Real numbers except $x = a,$ where $a \in [-4.4, 2.6]$

B. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [4.25, 6.25]$

C. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [1.67, 4.67]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-5.83, -0.83]$ and $b \in [4.83, 12.83]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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10. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 15$ and choose the interval that $f^-1(15)$ belongs to.

$$f(x) = \sqrt[3]{5x - 2}$$

The solution is 675.4, which is option B.

A. $f^{-1}(15) \in [-675.69, -675.24]$

This solution corresponds to distractor 2.

B. $f^{-1}(15) \in [675.16, 675.65]$

* This is the correct solution.

C. $f^{-1}(15) \in [673.87, 674.67]$

Distractor 1: This corresponds to

D. $f^{-1}(15) \in [-675.01, -674.59]$

This solution corresponds to distractor 3.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!
