This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 - 65x^2 + 82}{x - 4}$$

The solution is $15x^2 - 5x - 20 + \dfrac{2}{x-4}$, which is option B.

A. $a \in [13, 16], b \in [-24, -15], c \in [-60, -55], \text{ and } r \in [-99, -97]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [13, 16], b \in [-11, -1], c \in [-25, -13], \text{ and } r \in [-5, 4]$.

* This is the solution!

C. $a \in [60, 61], b \in [175, 181], c \in [697, 708], \text{ and } r \in [2882, 2889]$.

You multipled by the synthetic number rather than bringing the first factor down.

D. $a \in [13, 16], b \in [-125, -123], c \in [495, 504], \text{ and } r \in [-1919, -1912]$.

You divided by the opposite of the factor.

E. $a \in [60, 61], b \in [-309, -304], c \in [1220, 1223], \text{ and } r \in [-4803, -4794]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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2. What are the *possible Rational* roots of the polynomial below?

$$f(x) = 6x^2 + 5x + 2$$

The solution is All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$, which is option B.

A. $\pm 1, \pm 2$

This would have been the solution **if asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$

* This is the solution **since we asked for the possible Rational roots**!

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

D. $\pm 1, \pm 2, \pm 3, \pm 6$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Rational roots.

Distractor 4: Corresponds to not recalling the theorem for rational roots of a polynomial.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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3. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 38x^2 - 16x + 34}{x - 4}$$

The solution is $10x^2 + 2x - 8 + \frac{2}{x-4}$, which is option C.

A. $a \in [37, 41], \; b \in [119, 126], \; c \in [468, 475], \text{ and } r \in [1922, 1924]$.

You multiplied by the synthetic number rather than bringing the first factor down.

B. $a \in [5, 14], \; b \in [-78, -74], \; c \in [296, 303], \text{ and } r \in [-1152, -1147]$.

You divided by the opposite of the factor.

C. $a \in [5, 14], \; b \in [-3, 4], \; c \in [-11, -3], \text{ and } r \in [-1, 3]$.

* This is the solution!

D. $a \in [37, 41], \; b \in [-201, -193], \; c \in [776, 778], \text{ and } r \in [-3074, -3063]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

E. $a \in [5, 14], \; b \in [-10, -2], \; c \in [-42, -39], \text{ and } r \in [-86, -82]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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4. Factor the polynomial below completely, knowing that $x + 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^4 + 13x^3 - 253x^2 + 78x + 72$$

The solution is $[-4, -0.4, 0.75, 3]$, which is option E.

A. $z_1 \in [-3.3, -2.6], \; z_2 \in [-1.16, -0.5], z_3 \in [0.23, 0.44], \text{ and } z_4 \in [3.1, 4.6]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-3.3, -2.6], \; z_2 \in [-1.59, -1.31], z_3 \in [2.3, 2.69], \text{ and } z_4 \in [3.1, 4.6]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-4.7, -3.5], \; z_2 \in [-2.67, -2.31], z_3 \in [1.2, 1.91], \text{ and } z_4 \in [1.5, 3.2]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.3, -2.6], \; z_2 \in [-3.23, -2.61], z_3 \in [-0.05, 0.12], \text{ and } z_4 \in [3.1, 4.6]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-4.7, -3.5]$, $z_2 \in [-0.5, 0.04]$, $z_3 \in [0.72, 0.88]$, and $z_4 \in [1.5, 3.2]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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5. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 6x^3 - 1x^2 - 39x - 36$$

The solution is $[-1.5, -1.33, 3]$, which is option E.

A. $z_1 \in [-0.79, -0.48]$, $z_2 \in [-0.68, -0.58]$, and $z_3 \in [2.6, 3.4]$

Distractor 2: Corresponds to inversing rational roots.

B. $z_1 \in [-3.4, -2.82]$, $z_2 \in [1.28, 1.47]$, and $z_3 \in [1, 1.6]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-3.4, -2.82]$, $z_2 \in [0.56, 0.82]$, and $z_3 \in [-0.2, 1.1]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-3.4, -2.82]$, $z_2 \in [0.36, 0.66]$, and $z_3 \in [3.4, 5.4]$

Distractor 4: Corresponds to moving factors from one rational to another.

E. $z_1 \in [-2.03, -1.3]$, $z_2 \in [-1.4, -1.18]$, and $z_3 \in [2.6, 3.4]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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6. Factor the polynomial below completely, knowing that $x - 4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 8x^4 - 6x^3 - 189x^2 + 265x + 300$$

The solution is $[-5, -0.75, 2.5, 4]$, which is option A.

A. $z_1 \in [-5.9, -4.4]$, $z_2 \in [-0.82, -0.46]$, $z_3 \in [2.49, 2.51]$, and $z_4 \in [2.7, 4.9]$

* This is the solution!

B. $z_1 \in [-4.7, -2.8]$, $z_2 \in [-0.5, -0.38]$, $z_3 \in [1.33, 1.35]$, and $z_4 \in [4.7, 5.3]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.9, -4.4]$, $z_2 \in [-4.11, -3.8]$, $z_3 \in [0.35, 0.38]$, and $z_4 \in [4.7, 5.3]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-4.7, -2.8]$, $z_2 \in [-2.96, -2.39]$, $z_3 \in [0.74, 0.76]$, and $z_4 \in [4.7, 5.3]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-5.9, -4.4]$, $z_2 \in [-1.42, -1.05]$, $z_3 \in [0.39, 0.41]$, and $z_4 \in [2.7, 4.9]$

Distractor 2: Corresponds to inversing rational roots.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{10x^3 - 70x + 65}{x + 3}$$

The solution is $10x^2 - 30x + 20 + \frac{5}{x+3}$, which is option E.

A. $a \in [7, 12], b \in [30, 33], c \in [20, 26],$ and $r \in [124, 130]$.

You divided by the opposite of the factor.

B. $a \in [-38, -25], b \in [90, 93], c \in [-344, -335],$ and $r \in [1078, 1091]$.

You multipled by the synthetic number rather than bringing the first factor down.

C. $a \in [-38, -25], b \in [-91, -85], c \in [-344, -335],$ and $r \in [-958, -953]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [7, 12], b \in [-40, -39], c \in [89, 91],$ and $r \in [-298, -294]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [7, 12], b \in [-35, -29], c \in [20, 26],$ and $r \in [2, 13]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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8. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 20x^3 - 33x^2 - 20x + 12$$

The solution is $[-0.75, 0.4, 2]$, which is option B.

A. $z_1 \in [-2.02, -1.65],\ z_2 \in [-2.77, -1.28],$ and $z_3 \in [0.09, 0.38]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-1.2, -0.31],\ z_2 \in [0.22, 0.44],$ and $z_3 \in [1.92, 2.22]$

* This is the solution!

C. $z_1 \in [-1.63, -1.11],\ z_2 \in [1.83, 2.91],$ and $z_3 \in [2.28, 2.58]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-2.55, -2.31],\ z_2 \in [-2.77, -1.28],$ and $z_3 \in [1.1, 1.38]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-2.02, -1.65],\ z_2 \in [-0.52, -0.21],$ and $z_3 \in [0.69, 0.98]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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9. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{15x^3 + 67x^2 + 94x + 35}{x + 2}$$

The solution is $15x^2 + 37x + 20 + \dfrac{-5}{x+2}$, which is option A.

A. $a \in [13, 18]$, $b \in [37, 39]$, $c \in [16, 24]$, and $r \in [-11, -3]$.

   * This is the solution!

B. $a \in [-31, -28]$, $b \in [6, 13]$, $c \in [104, 113]$, and $r \in [251, 257]$.

   You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [-31, -28]$, $b \in [125, 129]$, $c \in [-161, -159]$, and $r \in [354, 357]$.

   You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [13, 18]$, $b \in [92, 101]$, $c \in [284, 289]$, and $r \in [606, 615]$.

   You divided by the opposite of the factor.

E. $a \in [13, 18]$, $b \in [20, 23]$, $c \in [24, 34]$, and $r \in [-50, -46]$.

   You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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10. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 6x^4 + 4x^3 + 7x^2 + 4x + 7$$

The solution is $\pm 1, \pm 7$, which is option B.

A. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 7}$

   Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 7$

   * This is the solution **since we asked for the possible Integer roots**!

C. $\pm 1, \pm 2, \pm 3, \pm 6$

   Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 2, \pm 3, \pm 6}$

   This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

   Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.