This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{20x^3 + 105x^2 - 128}{x + 5}$$

The solution is $20x^2 + 5x - 25 + \frac{-3}{x+5}$, which is option B.

A. $a \in [19, 27], b \in [-15, -11], c \in [89, 92]$, and $r \in [-670, -662]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [19, 27], b \in [2, 11], c \in [-30, -24]$, and $r \in [-7, -2]$.

* This is the solution!

C. $a \in [-105, -94], b \in [-397, -394], c \in [-1976, -1973]$, and $r \in [-10008, -9998]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

D. $a \in [-105, -94], b \in [602, 607], c \in [-3027, -3024]$, and $r \in [14989, 15000]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [19, 27], b \in [203, 206], c \in [1023, 1026]$, and $r \in [4997, 5002]$.

You divided by the opposite of the factor.

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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2. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 10x^3 - 39x^2 - 61x + 30$$

The solution is $[-1.5, 0.4, 5]$, which is option A.

A. $z_1 \in [-2.4, -0.9]$, $z_2 \in [0.36, 0.97]$, and $z_3 \in [4.87, 5.67]$

* This is the solution!

B. $z_1 \in [-5.1, -4.1]$, $z_2 \in [-0.78, -0.09]$, and $z_3 \in [0.97, 1.69]$

Distractor 1: Corresponds to negatives of all zeros.

C. $z_1 \in [-1.4, 0.1]$, $z_2 \in [2.08, 3.12]$, and $z_3 \in [4.87, 5.67]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-5.1, -4.1]$, $z_2 \in [-3.19, -2.32]$, and $z_3 \in [0.45, 0.75]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-5.1, -4.1]$, $z_2 \in [-2.36, -1.87]$, and $z_3 \in [0.14, 0.36]$

Distractor 4: Corresponds to moving factors from one rational to another.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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3. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 15x^3 - 1x^2 - 52x + 20$$

The solution is $[-2, 0.4, 1.67]$, which is option E.

A. $z_1 \in [-1.85, -1.24]$, $z_2 \in [-0.43, -0.35]$, and $z_3 \in [1.81, 2.03]$

Distractor 1: Corresponds to negatives of all zeros.

B. $z_1 \in [-2.78, -2.35]$, $z_2 \in [-0.6, -0.46]$, and $z_3 \in [1.81, 2.03]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

C. $z_1 \in [-5.02, -4.61]$, $z_2 \in [-0.17, -0.02]$, and $z_3 \in [1.81, 2.03]$

Distractor 4: Corresponds to moving factors from one rational to another.

D. $z_1 \in [-2.33, -1.98]$, $z_2 \in [0.59, 0.64]$, and $z_3 \in [2.15, 2.76]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-2.33, -1.98]$, $z_2 \in [0.38, 0.48]$, and $z_3 \in [1.36, 1.85]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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4. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{8x^3 - 62x + 33}{x + 3}$$

The solution is $8x^2 - 24x + 10 + \frac{3}{x+3}$, which is option E.

A. $a \in [4, 9], b \in [-39, -31], c \in [62, 69]$, and $r \in [-232, -225]$.

You multipled by the synthetic number and subtracted rather than adding during synthetic division.

B. $a \in [-27, -21], b \in [-72, -67], c \in [-280, -277]$, and $r \in [-804, -800]$.

You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

C. $a \in [4, 9], b \in [20, 26], c \in [7, 15]$, and $r \in [58, 66]$.

You divided by the opposite of the factor.

D. $a \in [-27, -21], b \in [71, 77], c \in [-280, -277]$, and $r \in [867, 868]$.

You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [4, 9], b \in [-28, -21], c \in [7, 15]$, and $r \in [2, 5]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

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5. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{4x^3 - 22x^2 + 4x + 26}{x - 5}$$

The solution is $4x^2 - 2x - 6 + \frac{-4}{x-5}$, which is option A.

A. $a \in [2, 5]$, $b \in [-2, 2]$, $c \in [-6, -5]$, and $r \in [-7, -1]$.

* This is the solution!

B. $a \in [20, 23]$, $b \in [75, 79]$, $c \in [394, 399]$, and $r \in [1991, 1997]$.

You multiplied by the synthetic number rather than bringing the first factor down.

C. $a \in [2, 5]$, $b \in [-8, -5]$, $c \in [-24, -18]$, and $r \in [-58, -52]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

D. $a \in [2, 5]$, $b \in [-43, -39]$, $c \in [213, 221]$, and $r \in [-1044, -1043]$.

You divided by the opposite of the factor.

E. $a \in [20, 23]$, $b \in [-125, -115]$, $c \in [610, 618]$, and $r \in [-3050, -3036]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

General Comment: Be sure to synthetically divide by the zero of the denominator!

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6. Factor the polynomial below completely, knowing that $x + 3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 4x^4 + 4x^3 - 51x^2 - 36x + 135$$

The solution is $[-3, -2.5, 1.5, 3]$, which is option A.

A. $z_1 \in [-5, 1]$, $z_2 \in [-2.54, -2.45], z_3 \in [1.24, 1.53]$, and $z_4 \in [3, 4]$

* This is the solution!

B. $z_1 \in [-5, 1]$, $z_2 \in [-0.8, -0.68], z_3 \in [2.74, 3.16]$, and $z_4 \in [5, 7]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [-5, 1]$, $z_2 \in [-0.72, -0.56], z_3 \in [0.32, 0.59]$, and $z_4 \in [3, 4]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

D. $z_1 \in [-5, 1]$, $z_2 \in [-0.5, -0.35], z_3 \in [0.42, 0.9]$, and $z_4 \in [3, 4]$

Distractor 2: Corresponds to inversing rational roots.

E. $z_1 \in [-5, 1]$, $z_2 \in [-1.5, -1.46], z_3 \in [2.24, 2.69]$, and $z_4 \in [3, 4]$

Distractor 1: Corresponds to negatives of all zeros.

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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7. Factor the polynomial below completely, knowing that $x+4$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$. *To make the problem easier, all zeros are between -5 and 5.*

$$f(x) = 12x^4 + 101x^3 + 165x^2 - 248x - 240$$

The solution is $[-5, -4, -0.75, 1.333]$, which is option E.

A. $z_1 \in [-0.46, 0.02]$, $z_2 \in [2.74, 3.09], z_3 \in [3.87, 4.03]$, and $z_4 \in [3.99, 5.65]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-5.22, -4.73]$, $z_2 \in [-4.54, -3.29], z_3 \in [-2.25, -0.9]$, and $z_4 \in [-0.17, 1]$

Distractor 2: Corresponds to inversing rational roots.

C. $z_1 \in [-1.56, -0.95]$, $z_2 \in [0.63, 0.84], z_3 \in [3.87, 4.03]$, and $z_4 \in [3.99, 5.65]$

Distractor 1: Corresponds to negatives of all zeros.

D. $z_1 \in [-0.96, -0.61]$, $z_2 \in [1.26, 1.46], z_3 \in [3.87, 4.03]$, and $z_4 \in [3.99, 5.65]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

E. $z_1 \in [-5.22, -4.73]$, $z_2 \in [-4.54, -3.29], z_3 \in [-1, -0.5]$, and $z_4 \in [0.79, 1.62]$

* This is the solution!

General Comment: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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8. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{25x^3 - 85x^2 + 15x + 40}{x - 3}$$

The solution is $25x^2 - 10x - 15 + \frac{-5}{x-3}$, which is option E.

A. $a \in [73, 76]$, $b \in [-314, -306]$, $c \in [945, 951]$, and $r \in [-2795, -2791]$.

You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

B. $a \in [25, 26]$, $b \in [-163, -157]$, $c \in [492, 496]$, and $r \in [-1445, -1441]$.

You divided by the opposite of the factor.

C. $a \in [73, 76]$, $b \in [136, 145]$, $c \in [432, 438]$, and $r \in [1340, 1346]$.

You multiplied by the synthetic number rather than bringing the first factor down.

D. $a \in [25, 26]$, $b \in [-42, -31]$, $c \in [-60, -51]$, and $r \in [-71, -65]$.

You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

E. $a \in [25, 26]$, $b \in [-19, -9]$, $c \in [-17, -12]$, and $r \in [-5, -1]$.

* This is the solution!

General Comment: Be sure to synthetically divide by the zero of the denominator!

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9. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 3x^2 + 5x + 4$$

The solution is $\pm 1, \pm 2, \pm 4$, which is option A.

A. $\pm 1, \pm 2, \pm 4$

* This is the solution **since we asked for the possible Integer roots**!

B. All combinations of: $\dfrac{\pm 1, \pm 3}{\pm 1, \pm 2, \pm 4}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

C. All combinations of: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3}$

This would have been the solution **if asked for the possible Rational roots**!

D. $\pm 1, \pm 3$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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10. What are the *possible Integer* roots of the polynomial below?

$$f(x) = 5x^2 + 5x + 2$$

The solution is $\pm 1, \pm 2$, which is option B.

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 2}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. $\pm 1, \pm 2$

* This is the solution **since we asked for the possible Integer roots**!

C. $\pm 1, \pm 5$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

D. All combinations of: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 5}$

This would have been the solution **if asked for the possible Rational roots**!

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comment: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.