This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Choose the interval below that f composed with g at x = 1 is in.

$$f(x) = 2x^3 + x^2 - 3x$$
 and $g(x) = -x^3 + 2x^2 - 3x$

The solution is -6.0, which is option A.

- A. $(f \circ g)(1) \in [-8, -5]$
 - * This is the correct solution
- B. $(f \circ g)(1) \in [-8, -5]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ q)(1) \in [-3, 2]$

Distractor 1: Corresponds to reversing the composition.

D. $(f \circ g)(1) \in [-16, -10]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means f(g(x)). The order matters!

2. Determine whether the function below is 1-1.

$$f(x) = 16x^2 - 80x + 100$$

The solution is no, which is option D.

A. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

B. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

C. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

- D. No, because there is a y-value that goes to 2 different x-values.
 - * This is the solution.
- E. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

3. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-3x + 12}$$
 and $g(x) = 5x + 5$

The solution is The domain is all Real numbers less than or equal to x = 4.0, which is option C.

- A. The domain is all Real numbers greater than or equal to x = a, where $a \in [-6.25, -2.25]$
- B. The domain is all Real numbers except x = a, where $a \in [-6.4, 0.6]$
- C. The domain is all Real numbers less than or equal to x = a, where $a \in [4, 6]$
- D. The domain is all Real numbers except x = a and x = b, where $a \in [-7.8, -0.8]$ and $b \in [3.75, 7.75]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

4. Find the inverse of the function below. Then, evaluate the inverse at x = 8 and choose the interval that $f^{-}1(8)$ belongs to.

$$f(x) = e^{x+4} - 2$$

The solution is $f^{-1}(8) = -1.697$, which is option D.

A.
$$f^{-1}(8) \in [-0.66, -0.48]$$

This solution corresponds to distractor 3.

B.
$$f^{-1}(8) \in [6.02, 6.73]$$

This solution corresponds to distractor 1.

C.
$$f^{-1}(8) \in [-0.35, -0.08]$$

This solution corresponds to distractor 2.

D.
$$f^{-1}(8) \in [-1.75, -1.28]$$

This is the solution.

E.
$$f^{-1}(8) \in [0.38, 0.63]$$

This solution corresponds to distractor 4.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

5. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -12 and choose the interval that $f^{-1}(-12)$ belongs to.

$$f(x) = 4x^2 - 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A.
$$f^{-1}(-12) \in [3.32, 4.02]$$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

B.
$$f^{-1}(-12) \in [1.6, 2.66]$$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

C.
$$f^{-1}(-12) \in [5.18, 5.92]$$

Distractor 4: This corresponds to both distractors 2 and 3.

D. $f^{-1}(-12) \in [1.28, 1.69]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

- E. The function is not invertible for all Real numbers.
 - * This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

6. Choose the interval below that f composed with q at x = 1 is in.

$$f(x) = -2x^3 + 2x^2 + 3x - 1$$
 and $g(x) = -x^3 + 2x^2 - 2x + 4$

The solution is -28.0, which is option B.

A. $(f \circ g)(1) \in [-7, 5]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(1) \in [-28, -27]$

* This is the correct solution

C. $(f \circ g)(1) \in [-14, -6]$

Distractor 3: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [-26, -22]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means f(g(x)). The order matters!

7. Multiply the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 5x^2 + 3x + 3$$
 and $g(x) = 2x^4 + x^3 + 7x^2 + 8x + 9$

The solution is $(-\infty, \infty)$, which is option E.

- A. The domain is all Real numbers less than or equal to x = a, where $a \in [3.5, 8.5]$
- B. The domain is all Real numbers except x = a, where $a \in [4.8, 6.8]$
- C. The domain is all Real numbers greater than or equal to x = a, where $a \in [-12.67, -5.67]$
- D. The domain is all Real numbers except x = a and x = b, where $a \in [4.2, 8.2]$ and $b \in [-10.6, -2.6]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

8. Find the inverse of the function below. Then, evaluate the inverse at x = 5 and choose the interval that $f^{-1}(5)$ belongs to.

$$f(x) = e^{x+2} - 3$$

The solution is $f^{-1}(5) = 0.079$, which is option C.

A.
$$f^{-1}(5) \in [-2.77, -1.99]$$

This solution corresponds to distractor 2.

B.
$$f^{-1}(5) \in [-1.09, -0.58]$$

This solution corresponds to distractor 4.

C.
$$f^{-1}(5) \in [-0.74, 0.63]$$

This is the solution.

D.
$$f^{-1}(5) \in [-1.97, -1.86]$$

This solution corresponds to distractor 3.

E.
$$f^{-1}(5) \in [3.78, 4.92]$$

This solution corresponds to distractor 1.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

9. Determine whether the function below is 1-1.

$$f(x) = 12x^2 - 114x + 252$$

The solution is no, which is option B.

A. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

- B. No, because there is a y-value that goes to 2 different x-values.
 - * This is the solution.
- C. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

D. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

E. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

10. Find the inverse of the function below (if it exists). Then, evaluate the inverse at x = -13 and choose the interval that $f^{-1}(-13)$ belongs to.

$$f(x) = \sqrt[3]{5x+4}$$

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The solution is -440.2, which is option C.

A.
$$f^{-1}(-13) \in [438.59, 439.36]$$

This solution corresponds to distractor 3.

B.
$$f^{-1}(-13) \in [-439.72, -437.92]$$

Distractor 1: This corresponds to

C.
$$f^{-1}(-13) \in [-440.63, -439.67]$$

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^{*} This is the correct solution.

D. $f^{-1}(-13) \in [439.31, 440.26]$

This solution corresponds to distractor 2.

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!