

This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found [here](#).

If you have a suggestion to make the keys better, please fill out the short survey [here](#).

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

1. Determine whether the function below is 1-1.

$$f(x) = (3x - 21)^3$$

The solution is yes, which is option B.

- A. No, because there is a y -value that goes to 2 different x -values.

Corresponds to the Horizontal Line test, which this function passes.

- B. Yes, the function is 1-1.

* This is the solution.

- C. No, because there is an x -value that goes to 2 different y -values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

- D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

- E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y -value that goes to 2 different x -values.

2. Find the inverse of the function below. Then, evaluate the inverse at $x = 8$ and choose the interval that $f^{-1}(8)$ belongs to.

$$f(x) = e^{x+3} - 4$$

The solution is $f^{-1}(8) = -0.515$, which is option D.

- A. $f^{-1}(8) \in [-2, -1.42]$

This solution corresponds to distractor 4.

- B. $f^{-1}(8) \in [-2.82, -2.48]$

This solution corresponds to distractor 2.

- C. $f^{-1}(8) \in [5.43, 5.73]$

This solution corresponds to distractor 1.

- D. $f^{-1}(8) \in [-0.6, -0.48]$

This is the solution.

- E. $f^{-1}(8) \in [-2.56, -2.07]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y , use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

3. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{4x - 25} \text{ and } g(x) = 3x^2 + 5$$

The solution is The domain is all Real numbers greater than or equal to $x = 6.25$., which is option A.

- A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [4.25, 9.25]$
- B. The domain is all Real numbers except $x = a$, where $a \in [0.25, 7.25]$
- C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-8.8, -0.8]$
- D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [-6.4, -1.4]$ and $b \in [3.33, 10.33]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

4. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^{-1}(-15)$ belongs to.

$$f(x) = 5x^2 - 3$$

The solution is The function is not invertible for all Real numbers. , which is option E.

- A. $f^{-1}(-15) \in [2.29, 3.07]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

- B. $f^{-1}(-15) \in [4.54, 5.58]$

Distractor 4: This corresponds to both distractors 2 and 3.

- C. $f^{-1}(-15) \in [0.83, 1.66]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

- D. $f^{-1}(-15) \in [1.78, 1.96]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

- E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

5. Find the inverse of the function below. Then, evaluate the inverse at $x = 6$ and choose the interval that $f^{-1}(6)$ belongs to.

$$f(x) = \ln(x - 4) + 3$$

The solution is $f^{-1}(6) = 24.086$, which is option D.

- A. $f^{-1}(6) \in [9.39, 11.39]$

This solution corresponds to distractor 4.

- B. $f^{-1}(6) \in [22028.47, 22031.47]$

This solution corresponds to distractor 2.

C. $f^{-1}(6) \in [11.09, 17.09]$

This solution corresponds to distractor 3.

D. $f^{-1}(6) \in [23.09, 30.09]$

This is the solution.

E. $f^{-1}(6) \in [8102.08, 8111.08]$

This solution corresponds to distractor 1.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y , use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

6. Determine whether the function below is 1-1.

$$f(x) = (5x - 31)^3$$

The solution is yes, which is option A.

- A. Yes, the function is 1-1.

* This is the solution.

- B. No, because there is an x -value that goes to 2 different y -values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

- C. No, because there is a y -value that goes to 2 different x -values.

Corresponds to the Horizontal Line test, which this function passes.

- D. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

- E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y -value that goes to 2 different x -values.

7. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = 10$ and choose the interval that $f^{-1}(10)$ belongs to.

$$f(x) = 4x^2 - 5$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(10) \in [0.75, 1.29]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

B. $f^{-1}(10) \in [1.22, 2.45]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

C. $f^{-1}(10) \in [2.14, 4.12]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

D. $f^{-1}(10) \in [6.06, 7.1]$

Distractor 4: This corresponds to both distractors 2 and 3.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

8. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{5x + 26} \text{ and } g(x) = 8x^2 + 6x$$

The solution is The domain is all Real numbers greater than or equal to $x = -5.2$, which is option A.

- A. The domain is all Real numbers greater than or equal to $x = a$, where $a \in [-6.2, -3.2]$
- B. The domain is all Real numbers except $x = a$, where $a \in [1.17, 9.17]$
- C. The domain is all Real numbers less than or equal to $x = a$, where $a \in [-1.8, 3.2]$
- D. The domain is all Real numbers except $x = a$ and $x = b$, where $a \in [5.33, 6.33]$ and $b \in [-8.6, -5.6]$
- E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

9. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -x^3 - 3x^2 - 2x - 4 \text{ and } g(x) = x^3 + 4x^2 + 3x - 3$$

The solution is 2.0, which is option A.

- A. $(f \circ g)(-1) \in [2, 3]$

* This is the correct solution

- B. $(f \circ g)(-1) \in [-7, -4]$

Distractor 2: Corresponds to being slightly off from the solution.

- C. $(f \circ g)(-1) \in [-20, -13]$

Distractor 1: Corresponds to reversing the composition.

- D. $(f \circ g)(-1) \in [-30, -17]$

Distractor 3: Corresponds to being slightly off from the solution.

- E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

10. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -x^3 + 3x^2 + 4x - 1 \text{ and } g(x) = -x^3 + 2x^2 - x + 2$$

The solution is 11.0, which is option C.

- A. $(f \circ g)(1) \in [-83, -72]$

Distractor 1: Corresponds to reversing the composition.

- B. $(f \circ g)(1) \in [-90, -82]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [10, 18]$

* This is the correct solution

D. $(f \circ g)(1) \in [-4, 6]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!
