This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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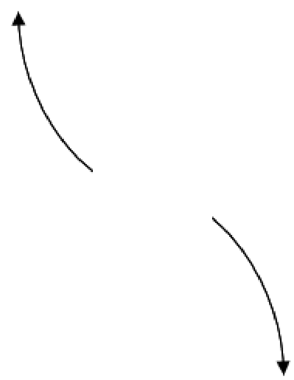
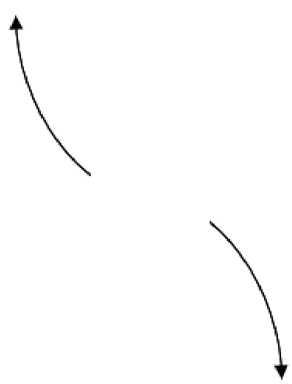
1. Describe the end behavior of the polynomial below.

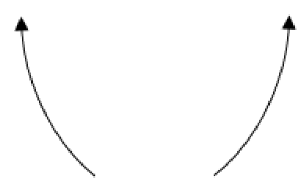
$$f(x) = -3(x+5)^4(x-5)^7(x-7)^4(x+7)^6$$

The solution is the graph below, which is option A.

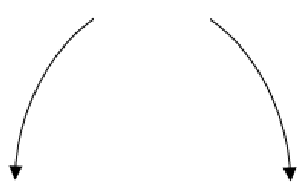




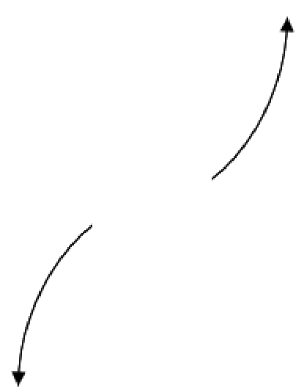
A.



C.



B.



D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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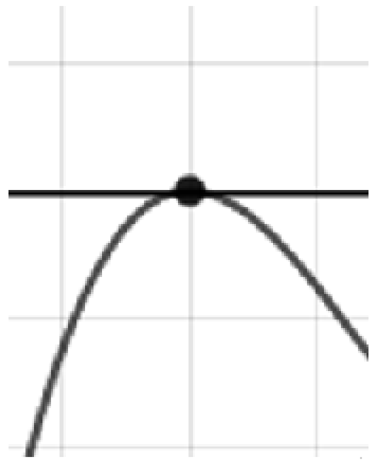
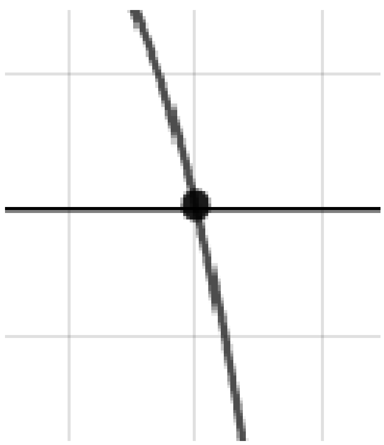
2. Describe the zero behavior of the zero $x = 3$ of the polynomial below.

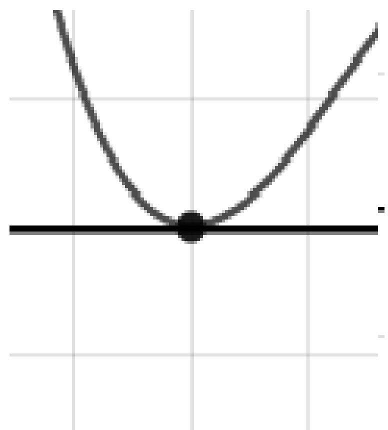
$$f(x) = 6(x-3)^8(x+3)^{13}(x-4)^9(x+4)^{12}$$

The solution is the graph below, which is option B.

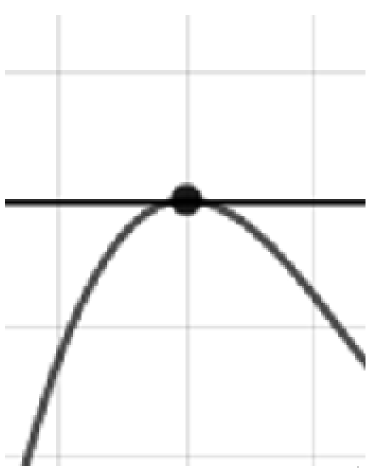




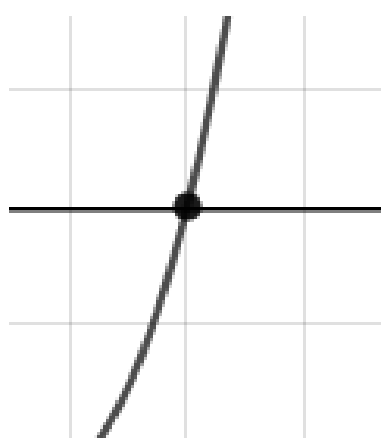
A.



C.



B.



D.

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

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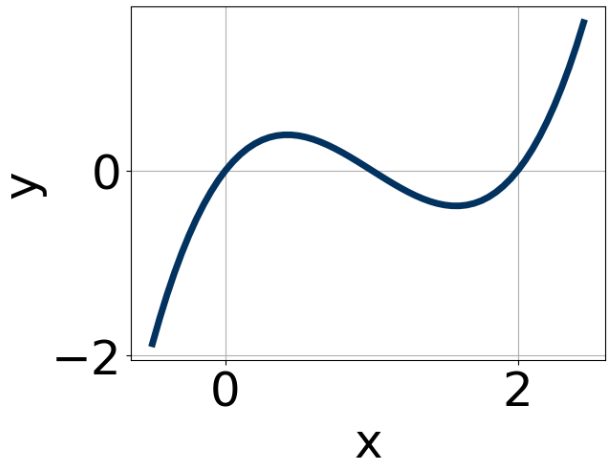
3. Which of the following equations *could* be of the graph presented below?



The solution is $18x^7(x-2)^7(x-1)^5$, which is option C.

A. $4x^7(x-2)^{10}(x-1)^5$

The factor 2 should have been an odd power.

B. $-8x^9(x-2)^6(x-1)^5$

The factor $(x-2)$ should have an odd power and the leading coefficient should be the opposite sign.

C. $18x^7(x-2)^7(x-1)^5$

* This is the correct option.

D. $7x^{10}(x-2)^8(x-1)^{11}$

The factors 2 and 0 have have been odd power.

E. $-12x^9(x-2)^5(x-1)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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4. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$4 + 5i \text{ and } 2$$

The solution is $x^3 - 10x^2 + 57x - 82$, which is option D.

A. $b \in [7, 16], c \in [56, 57.3], \text{ and } d \in [81, 86.3]$

$x^3 + 10x^2 + 57x + 82$, which corresponds to multiplying out $(x - (4 + 5i))(x - (4 - 5i))(x + 2)$.

B. $b \in [0, 2], c \in [-6.8, -1.8],$ and $d \in [3.8, 8.5]$

$x^3 + x^2 - 6x + 8$, which corresponds to multiplying out $(x - 4)(x - 2)$.

C. $b \in [0, 2], c \in [-8.8, -6.5],$ and $d \in [8.4, 13.1]$

$x^3 + x^2 - 7x + 10$, which corresponds to multiplying out $(x - 5)(x - 2)$.

D. $b \in [-13, -4], c \in [56, 57.3],$ and $d \in [-84.9, -79.5]$

* $x^3 - 10x^2 + 57x - 82$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (4 + 5i))(x - (4 - 5i))(x - (2))$.

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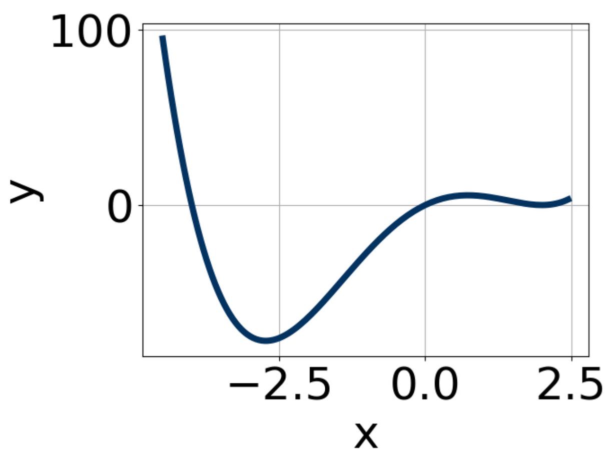
5. Which of the following equations *could* be of the graph presented below?



The solution is $11x^9(x - 2)^8(x + 4)^7$, which is option D.

A. $-18x^9(x - 2)^{10}(x + 4)^8$

The factor $(x + 4)$ should have an odd power and the leading coefficient should be the opposite sign.

B. $19x^6(x - 2)^9(x + 4)^9$

The factor 2 should have an even power and the factor 0 should have an odd power.

C. $-14x^9(x - 2)^6(x + 4)^5$

This corresponds to the leading coefficient being the opposite value than it should be.

D. $11x^9(x - 2)^8(x + 4)^7$

* This is the correct option.

E. $19x^6(x - 2)^{10}(x + 4)^9$

The factor x should have an odd power.

General Comment: General Comments: Draw the x-axis to determine which zeros are touching (and so have even multiplicity) or cross (and have odd multiplicity).

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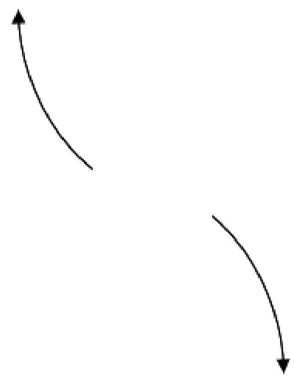
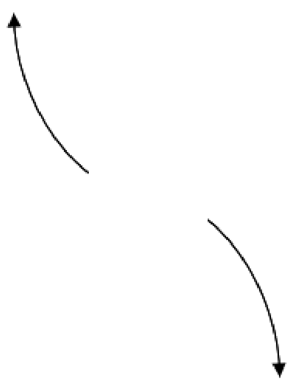
6. Describe the end behavior of the polynomial below.

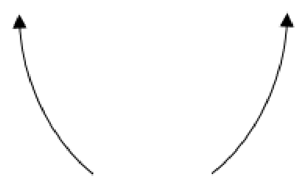
$$f(x) = -3(x+2)^3(x-2)^8(x+6)^5(x-6)^5$$

The solution is the graph below, which is option A.

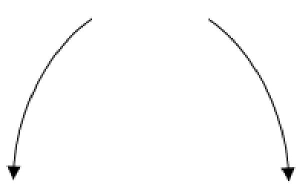




A.



C.



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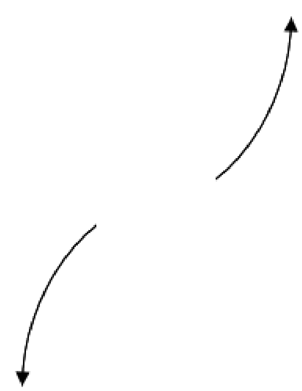

D.

E. None of the above.

General Comment: Remember that end behavior is determined by the leading coefficient AND whether the **sum** of the multiplicities is positive or negative.

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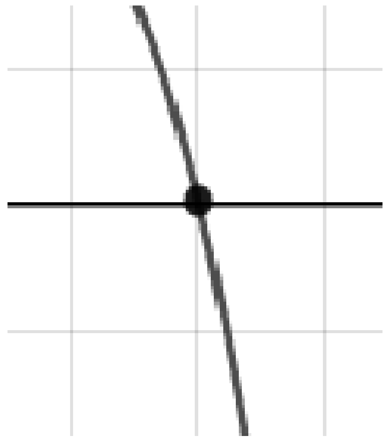
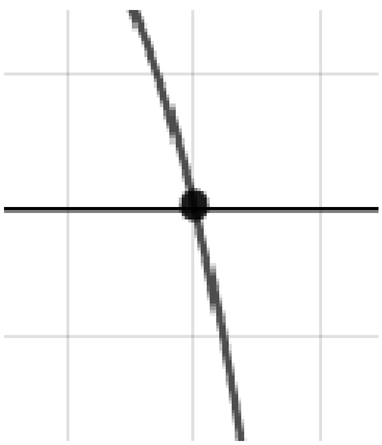
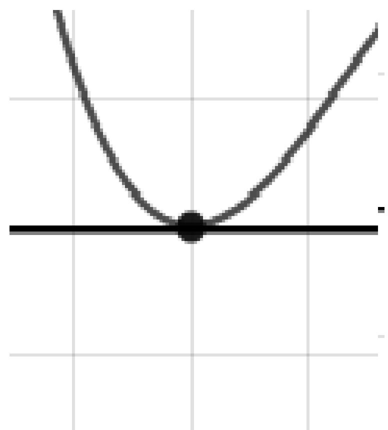
7. Describe the zero behavior of the zero $x = 7$ of the polynomial below.

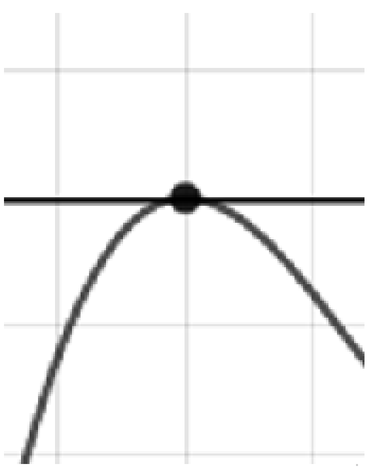
$$f(x) = 9(x+8)^9(x-8)^7(x-7)^7(x+7)^2$$

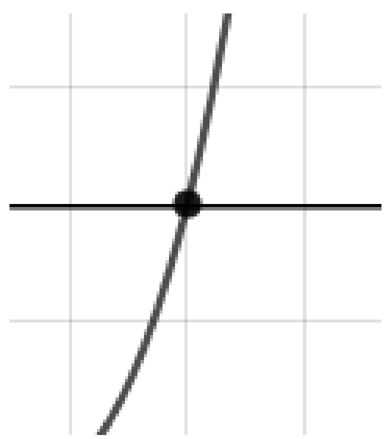
The solution is the graph below, which is option A.



A. 

C. 

B. 

D. 

E. None of the above.

General Comment: You will need to sketch the entire graph, then zoom in on the zero the question asks about.

8. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-1}{5}, \frac{-1}{4}, \text{ and } 6$$

The solution is $20x^3 - 111x^2 - 53x - 6$, which is option E.

A. $a \in [16, 21], b \in [107, 116], c \in [-55, -47], \text{ and } d \in [-2, 13]$

$20x^3 + 111x^2 - 53x + 6$, which corresponds to multiplying out $(5x - 1)(4x - 1)(x + 6)$.

B. $a \in [16, 21], b \in [-137, -123], c \in [51, 61], \text{ and } d \in [-9, -5]$

$20x^3 - 129x^2 + 55x - 6$, which corresponds to multiplying out $(5x - 1)(4x - 1)(x - 6)$.

C. $a \in [16, 21], b \in [-113, -106], c \in [-55, -47], \text{ and } d \in [-2, 13]$

$20x^3 - 111x^2 - 53x + 6$, which corresponds to multiplying everything correctly except the constant term.

D. $a \in [16, 21], b \in [-120, -116], c \in [-19, -4], \text{ and } d \in [-2, 13]$

$20x^3 - 119x^2 - 7x + 6$, which corresponds to multiplying out $(5x - 1)(4x + 1)(x - 6)$.

E. $a \in [16, 21], b \in [-113, -106], c \in [-55, -47], \text{ and } d \in [-9, -5]$

* $20x^3 - 111x^2 - 53x - 6$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(5x + 1)(4x + 1)(x - 6)$

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9. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $x^3 + bx^2 + cx + d$.

$$-5 + 4i \text{ and } -2$$

The solution is $x^3 + 12x^2 + 61x + 82$, which is option D.

A. $b \in [-14, -10], c \in [53, 67], \text{ and } d \in [-88, -77]$

$x^3 - 12x^2 + 61x - 82$, which corresponds to multiplying out $(x - (-5 + 4i))(x - (-5 - 4i))(x - 2)$.

B. $b \in [1, 6], c \in [-5, -1], \text{ and } d \in [-8, 0]$

$x^3 + x^2 - 2x - 8$, which corresponds to multiplying out $(x - 4)(x + 2)$.

C. $b \in [1, 6], c \in [7, 8], \text{ and } d \in [9, 17]$

$x^3 + x^2 + 7x + 10$, which corresponds to multiplying out $(x + 5)(x + 2)$.

D. $b \in [9, 25], c \in [53, 67], \text{ and } d \in [82, 90]$

* $x^3 + 12x^2 + 61x + 82$, which is the correct option.

E. None of the above.

This corresponds to making an unanticipated error or not understanding how to use nonreal complex numbers to create the lowest-degree polynomial. If you chose this and are not sure what you did wrong, please contact the coordinator for help.

General Comment: Remember that the conjugate of $a + bi$ is $a - bi$. Since these zeros always come in pairs, we need to multiply out $(x - (-5 + 4i))(x - (-5 - 4i))(x - (-2))$.

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10. Construct the lowest-degree polynomial given the zeros below. Then, choose the intervals that contain the coefficients of the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$\frac{-5}{4}, \frac{-3}{4}, \text{ and } -5$$

The solution is $16x^3 + 112x^2 + 175x + 75$, which is option E.

A. $a \in [12, 18], b \in [44, 51], c \in [-145, -143], \text{ and } d \in [73, 83]$

$16x^3 + 48x^2 - 145x + 75$, which corresponds to multiplying out $(4x - 5)(4x - 3)(x + 5)$.

B. $a \in [12, 18], b \in [104, 114], c \in [171, 181], \text{ and } d \in [-76, -74]$

$16x^3 + 112x^2 + 175x - 75$, which corresponds to multiplying everything correctly except the constant term.

C. $a \in [12, 18], b \in [72, 73], c \in [-60, -50], \text{ and } d \in [-76, -74]$

$16x^3 + 72x^2 - 55x - 75$, which corresponds to multiplying out $(4x - 5)(4x + 3)(x + 5)$.

D. $a \in [12, 18], b \in [-114, -109], c \in [171, 181], \text{ and } d \in [-76, -74]$

$16x^3 - 112x^2 + 175x - 75$, which corresponds to multiplying out $(4x - 5)(4x - 3)(x - 5)$.

E. $a \in [12, 18], b \in [104, 114], c \in [171, 181], \text{ and } d \in [73, 83]$

* $16x^3 + 112x^2 + 175x + 75$, which is the correct option.

General Comment: To construct the lowest-degree polynomial, you want to multiply out $(4x + 5)(4x + 3)(x + 5)$