This key should allow you to understand why you choose the option you did (beyond just getting a question right or wrong). More instructions on how to use this key can be found here.

If you have a suggestion to make the keys better, please fill out the short survey here.

Note: This key is auto-generated and may contain issues and/or errors. The keys are reviewed after each exam to ensure grading is done accurately. If there are issues (like duplicate options), they are noted in the offline gradebook. The keys are a work-in-progress to give students as many resources to improve as possible.

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1. Add the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = \sqrt{-3x - 16} \text{ and } g(x) = 3x^3 + 9x$$

The solution is The domain is all Real numbers less than or equal to$x = -5.33.$, which is option B.

A. The domain is all Real numbers except $x = a,$ where $a \in [4.2, 12.2]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-7.33, 0.67]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [5.33, 11.33]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [5.67, 8.67]$ and $b \in [4.6, 12.6]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.

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2. Find the inverse of the function below. Then, evaluate the inverse at $x = 7$ and choose the interval that $f^-1(7)$ belongs to.

$$f(x) = \ln(x - 5) - 2$$

The solution is $f^{-1}(7) = 8108.084$, which is option B.

A. $f^{-1}(7) \in [153.41, 157.41]$

This solution corresponds to distractor 1.

B. $f^{-1}(7) \in [8107.08, 8112.08]$

This is the solution.

C. $f^{-1}(7) \in [2.39, 7.39]$

This solution corresponds to distractor 4.

D. $f^{-1}(7) \in [8096.08, 8099.08]$

This solution corresponds to distractor 3.

E. $f^{-1}(7) \in [162748.79, 162755.79]$

This solution corresponds to distractor 2.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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3. Determine whether the function below is 1-1.

$$f(x) = 25x^2 + 220x + 484$$

The solution is no, which is option D.

A. Yes, the function is 1-1.

Corresponds to believing the function passes the Horizontal Line test.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

D. No, because there is a y-value that goes to 2 different x-values.

* This is the solution.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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4. Find the inverse of the function below. Then, evaluate the inverse at $x = 10$ and choose the interval that $f^-1(10)$ belongs to.

$$f(x) = e^{x+5} - 3$$

The solution is $f^{-1}(10) = -2.435$, which is option D.

A. $f^{-1}(10) \in [-1.1, -0.84]$

This solution corresponds to distractor 2.

B. $f^{-1}(10) \in [7.32, 8.32]$

This solution corresponds to distractor 1.

C. $f^{-1}(10) \in [-0.32, 0.71]$

This solution corresponds to distractor 4.

D. $f^{-1}(10) \in [-2.55, -1.81]$

This is the solution.

E. $f^{-1}(10) \in [-2.19, -1.3]$

This solution corresponds to distractor 3.

General Comment: Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

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5. Determine whether the function below is 1-1.

$$f(x) = (5x + 17)^3$$

The solution is yes, which is option B.

A. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

B. Yes, the function is 1-1.

* This is the solution.

C. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

D. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

E. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

General Comment: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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6. Choose the interval below that f composed with g at $x = -1$ is in.

$$f(x) = -4x^3 - 2x^2 + 4x \text{ and } g(x) = -3x^3 - 4x^2 - x - 3$$

The solution is 78.0, which is option C.

A. $(f \circ g)(-1) \in [10, 23]$

Distractor 3: Corresponds to being slightly off from the solution.

B. $(f \circ g)(-1) \in [83, 91]$

Distractor 2: Corresponds to being slightly off from the solution.

C. $(f \circ g)(-1) \in [75, 84]$

* This is the correct solution

D. $(f \circ g)(-1) \in [3, 15]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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7. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = 2x^3 + 2x^2 - 3x \text{ and } g(x) = -4x^3 + x^2 + 2x - 3$$

The solution is -84.0, which is option C.

A. $(f \circ g)(1) \in [-9, -3]$

Distractor 1: Corresponds to reversing the composition.

B. $(f \circ g)(1) \in [1, 4]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-84, -83]$

* This is the correct solution

D. $(f \circ g)(1) \in [-76, -73]$

Distractor 2: Corresponds to being slightly off from the solution.

E. It is not possible to compose the two functions.

General Comment: f composed with g at x means $f(g(x))$. The order matters!

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8. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -10$ and choose the interval that $f^-1(-10)$ belongs to.

$$f(x) = \sqrt[3]{4x - 3}$$

The solution is -249.25, which is option A.

A. $f^{-1}(-10) \in [-250, -247]$

* This is the correct solution.

B. $f^{-1}(-10) \in [246.4, 250.6]$

This solution corresponds to distractor 2.

C. $f^{-1}(-10) \in [249.9, 253]$

This solution corresponds to distractor 3.

D. $f^{-1}(-10) \in [-250.9, -249.9]$

Distractor 1: This corresponds to

E. The function is not invertible for all Real numbers.

This solution corresponds to distractor 4.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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9. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -15$ and choose the interval that $f^-1(-15)$ belongs to.

$$f(x) = 5x^2 - 2$$

The solution is The function is not invertible for all Real numbers. , which is option E.

A. $f^{-1}(-15) \in [5.33, 5.75]$

Distractor 4: This corresponds to both distractors 2 and 3.

B. $f^{-1}(-15) \in [1.81, 2.05]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

C. $f^{-1}(-15) \in [1.48, 1.65]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

D. $f^{-1}(-15) \in [4.33, 4.91]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comment: Be sure you check that the function is 1-1 before trying to find the inverse!

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10. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 9x^2 + 8x + 6 \text{ and } g(x) = 5x^3 + 8x^2 + 5x + 6$$

The solution is $(-\infty, \infty)$, which is option E.

A. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [0.5, 5.5]$

B. The domain is all Real numbers except $x = a,$ where $a \in [6.2, 7.2]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [2.67, 9.67]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-7.33, 1.67]$ and $b \in [-5.25, -2.25]$

E. The domain is all Real numbers.

General Comment: The new domain is the intersection of the previous domains.
