Undergraduate's Covariational Reasoning Across Function Representations

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Defining Covariational Reasoning

Covariational reasoning is the mental actions, processes, and constructions used to coordinate two or more varying quantities and interpret the relation between them [1]

Two types of covariational Reasoning

- **Simultaneous-Independent Reasoning:** Coordination of how two independent quantities change with respect to each other[2]

  Example: A ferris wheel and relating the vertical change and horizontal change.

- **Change-Dependent Reasoning:** Coordination of how a quantity changes with respect to change in another quantity[2]

  Example: A ferris wheel and relating the change between time and the vertical change.

Defining Parametric Functions

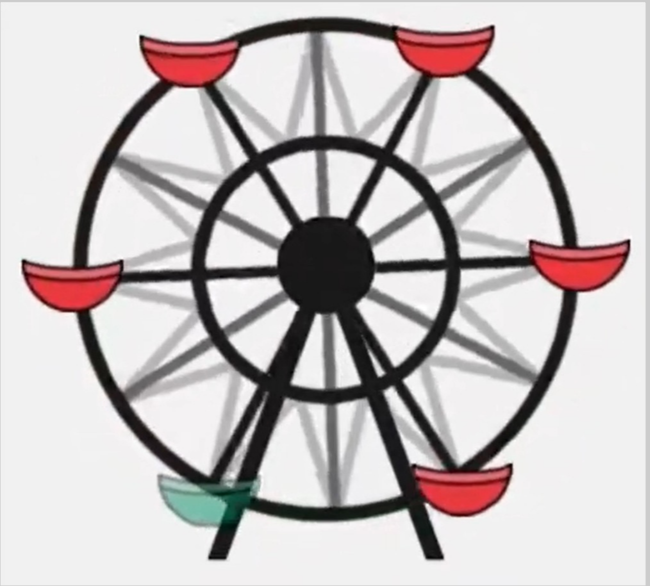
Parametric functions are a relation from **R** to $\mathbf{R}^n$ where for each unique input, there is a unique n-tuple output[2]

Example from Taking a ride[3]

- $x(t) = \cos(t-\pi/2)$
- $y(t) = \sin(t-\pi/2)$
- $f(t) = (x(t),y(t))$

Change-dependent reasoning can be used to understand both x(t) and y(t) since changes in x(t) and y(t) can be coordinated with time (t).

Simultaneous-Independent reasoning can be applied to f(t) because x(t) and y(t) are *independent* quantities which can be coordinated by their changes with respect to one another.



Taking a ride[3]

Research Question:

How are students able to use covariational reasoning to create a parametric representation to a real-life 3-dimensional problem?

Theoretical Framework: APOS Theory

Parametric Function

	Action	Process	Object
Definition	Individuals can take a mathematical object and perform an explicit transformation based on external cues.	Individuals can implicitly carry out the transformation and even deviate from the external cues they previously relied on.	Individuals can act on the dynamic, internal procedure as a static object.
Example	Participants can construct a specific, point-to-point representation based on a finite collection of key points.	Participants are able to act independently from the explicit point process and construct a continuous representation.	Participants are able to treat existing representations as static set of functions and transform this set

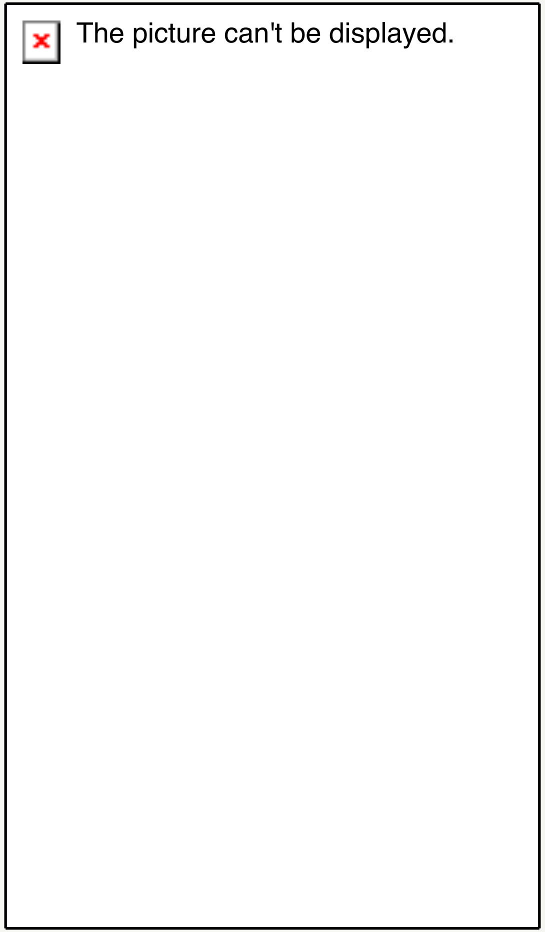
Study Structure

- Instrument
  - Construct a graph representing each component of the birds location relative to time
  - Using previous representations, graphically construct a polar parametric representation
  - Use previous representations to construct a cylindrical representation
- 60 minute interview

The picture can't be displayed.

Task 1

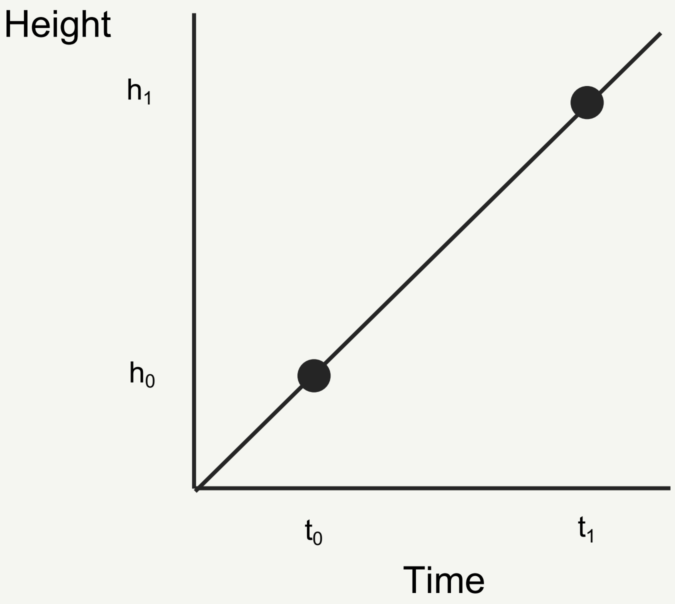
Construct a representation between height and time relative to the following point of view



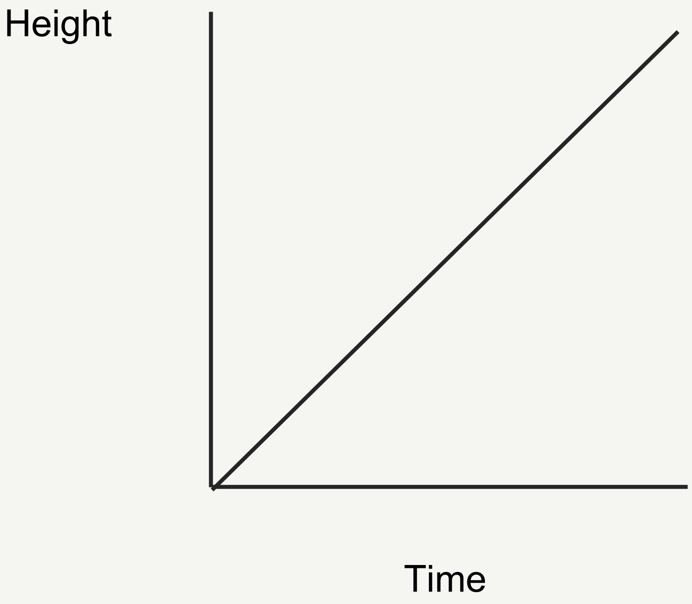
Expected Results: Task 1

Graphing the birds height with respect to time

Action Level Response



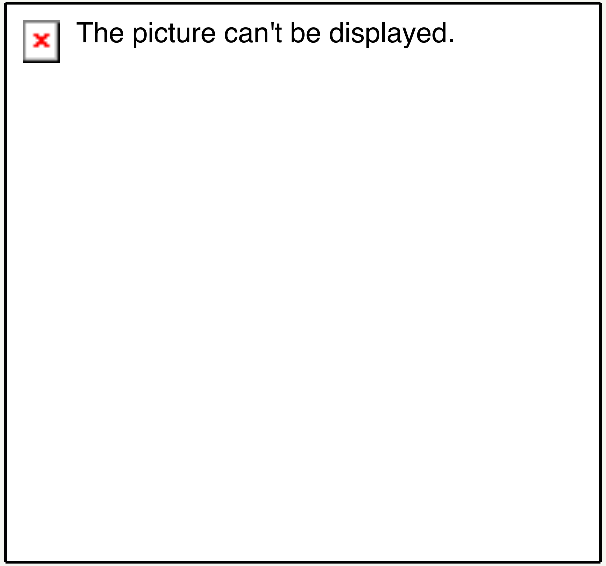
Process Level Response



Task 4

Intermediate tasks asked participants to construct a representation between time and horizontal and vertical position, relative to this point of view

This task challenged participants to combine these representation without referencing time

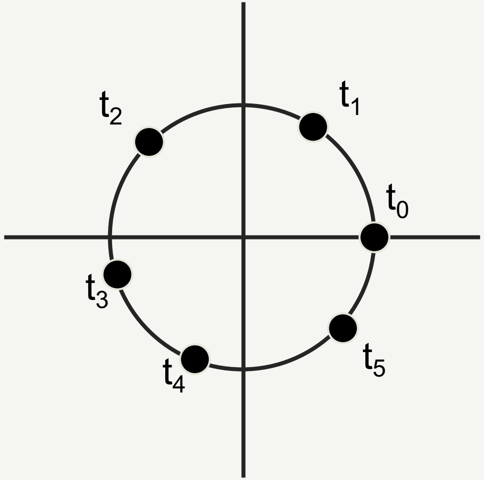


Expected Results: Task 4

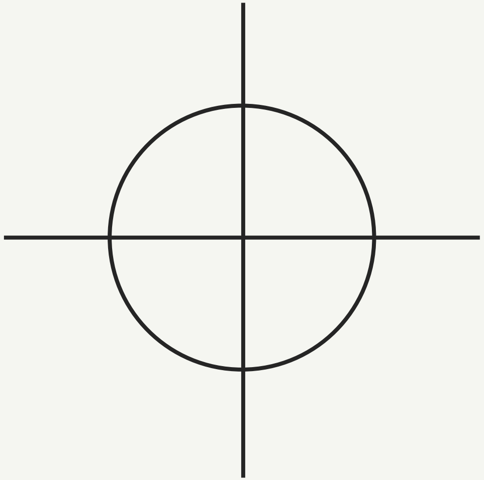
Constructing a graph showing the birds position around the tower (Constructing a polar representation)

Action Level Response

Time	X	Y
t_0	x_0	y_0
t_1	x_1	y_1
t_2	x_2	y_2
t_3	x_3	y_3
t_4	x_4	y_4
t_5	x_5	y_5



Process Level Response



Expected Results

The tasks can be classified based on whether we would expect students to apply change-dependent reasoning or simultaneous-independent reasoning order to be successful

- Task 1
  - Focusing on analyzing students abilities with change-dependent reasoning
  - Graphing positional components with respect to time

- Task 4
  - Prompting students to transition to simultaneously independent reasoning
  - Graphing positional components with time being implicitly represented

Results Overview

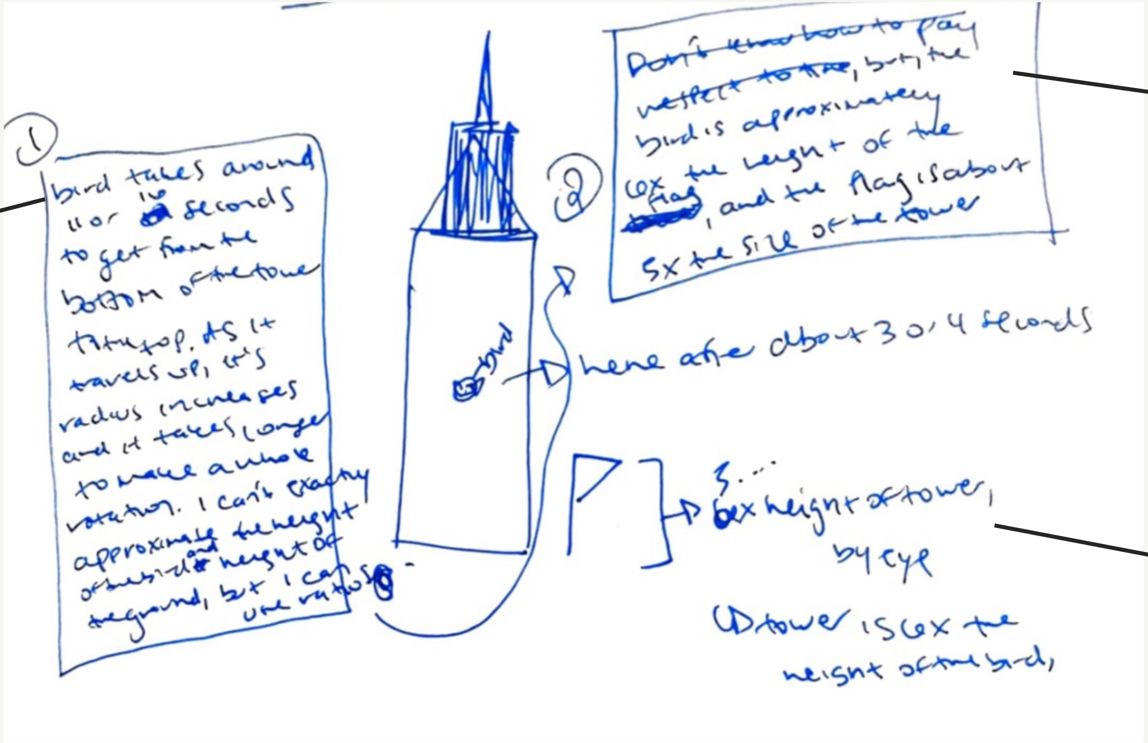
The student, pseudonym Jane, possessed an action level understanding of covariational reasoning which was identified through:

- Task 1
    - Constructing a measurement system to mark determine points
    - Utilizing these points to create a “chunky” representation
- Task 4
    - Took points from prior representations and used time as a key to explicitly correlate with other representations
    - Took “snapshots” at these key times to develop a “chunky” representation
- Their personal definition of a function and derivative during the interview

Task 1 Results

Graph the birds height with respect to time

Bird takes 11 or 16 seconds to get from bottom of the tower to the top



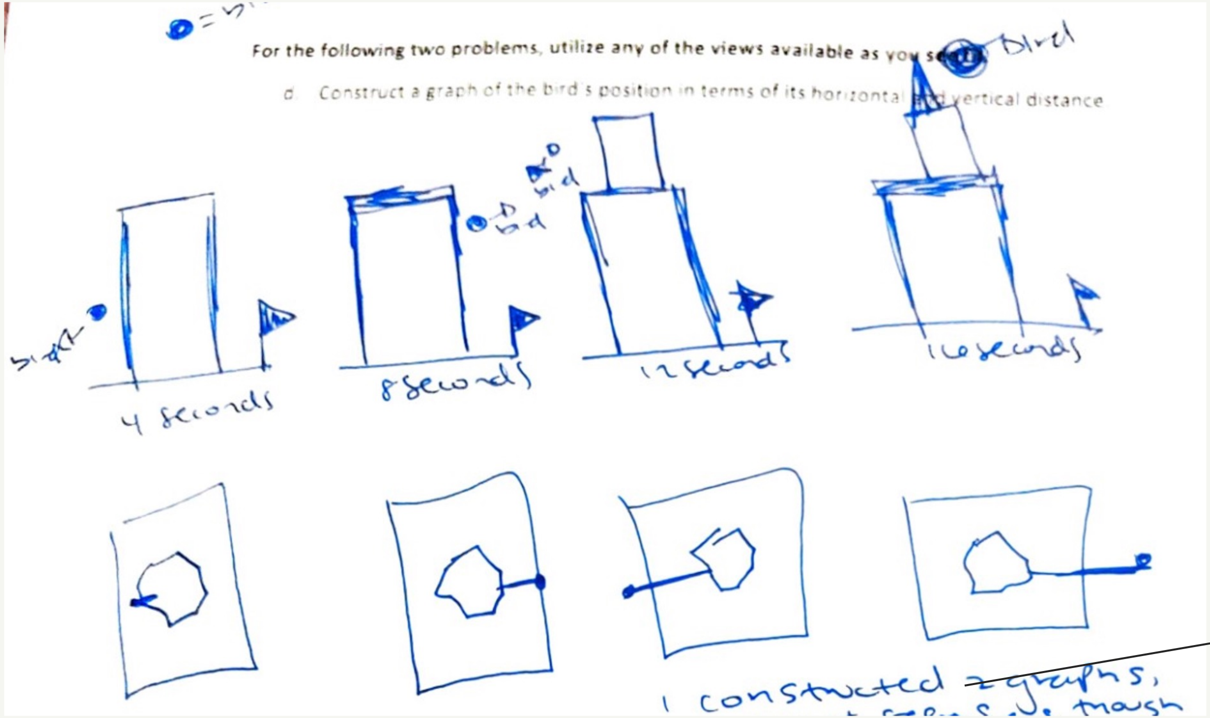
Bird is approximately 6x the height of the flag... flag is about 5x the size of the tower

5x height of tower...

Tower 156x the height of the bird

Task 4 Results

Construct a graph showing the birds position around the tower (Constructing a polar representation)



Constructed 2 graphs... feels as though height and horizontal distance grow at a rate of 2

Exploring Student's Schema of Parametric Functions

Function

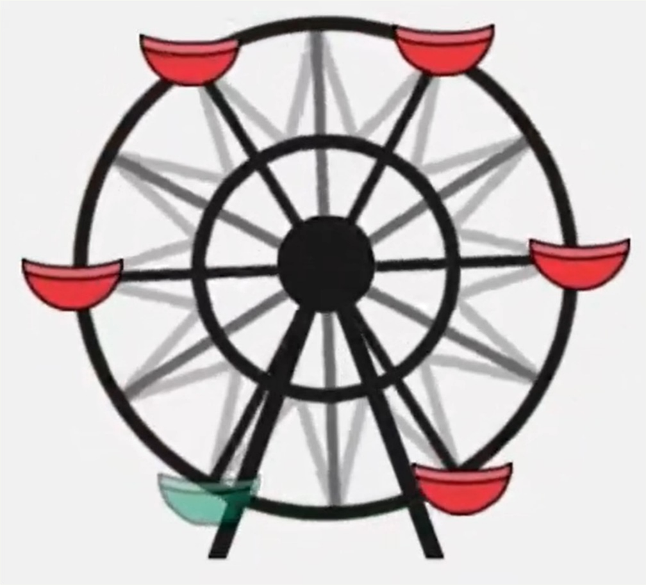
"A function is a formula with variables and numbers and coefficients that you plug in independent data you are observing"

Derivative

"Derivatives are rates of change that can sometimes be a function that pay respect to time"

Conclusions

- Student was unable to coordinate the change in two quantities with respect to each other.
  - Student could not internalize and control the point construction process and was reliant on finding external key points to construct their representation.
- Student was unable to coordinate the the change in three quantities.
  - Their action conception of function hindered them when trying to coordinate multiple quantities.
  - When asked to apply simultaneous independent reasoning, they relied on time to interpret the relationship between quantities.



Taking a ride[3]

Implications

- Covariational reasoning may need to be explicitly taught to better prepare students for higher level courses
    - Awareness of two ways to reason covariationally (Simultaneous Independent vs Change Dependent)
    - Encouraging students to step away from chunky reasoning
        - Stepping away from specific values
- Students may be unable to separate their definition of the derivative from being with respect to time
    - Deemphasizing time as a variable in derivatives
        - Time cannot be removed but we can choose to not pay attention
        - Ex. how volume of a cone changes with respect to the radius

Future Work

- Expansion of the number of participants
  - As a pilot study, this work focused on a single student. Our results were consistent with prior studies and showed the instrument was able to yield results
  - Future work could not only include more participants, but also include students with different conceptions for function and derivative
- Expansion of functions used
  - This study was constructed to be used with first semester calculus students and as such the functions that were used were linear and trigonometric
  - Future work may focus on other function representations and how students identify those functions and reason with them

References

1. Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying Covariational Reasoning While Model Dynamic Events: A Framework and a Study. *Journal For Research in Mathematics Education*, 352-378.

2. Stalvey, H. E., & Vidakovic, D. (2015). Students' Reasoning about relationships between variables in a real-world problem. *The Journal of Mathematical Behaviour*, 192-210.

3. Stevens, I. E., & Moore, K. C. (2016). The Ferris wheel and justifications of curvature. In M. B. Wood, E. E. Turner, M. Civil, & J. A. Eli (Eds.), Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 644-651). Tucson, AZ: The University of Arizona.

Extended Instrument Demo