Predicting Students' Thoughts to Provide Elaborative Feedback

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Types of Feedback: Perspectives

	'feed-up'	'feed-back'	'feed-up'
What	Comparison of actual status with target status	Comparison of actual status with previous status	Explanation of target status based on actual status
Why	To provide information about learning goals	To provide information about accomplishments	To provide information for deeper understanding

Table 1: Different perspectives of feedback (Hattie & Timperley, 2007).

Types of Feedback: Cognitive Complexity

Task	Process	Self-Regulation	Self
Content, facts, surface information	Strategies of student performance	Individual's regulation of strategies	Personal characteristics of recipient
Correct/Incorrect	What needs to be done	Meta-cognitive evaluation	Affective praise

Table 2: Cognitive complexity of feedback (Hattie & Timperley, 2007).

Meta-Analysis on Effect Size of Feedback

$$d = \frac{\bar{X}_1 - \bar{X}_2}{\sigma}$$

	Reinforcement or Punishment	Corrective	High-Information
Cohen's d effect size	Small (0.24)	Medium (0.46)	High (0.99)

Table 3: Effect sizes by type of feedback (Wisniewski et al., 2020).

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Upshot: High-information feedback has an oversized effect on student learning.

ChatGPT to the rescue?

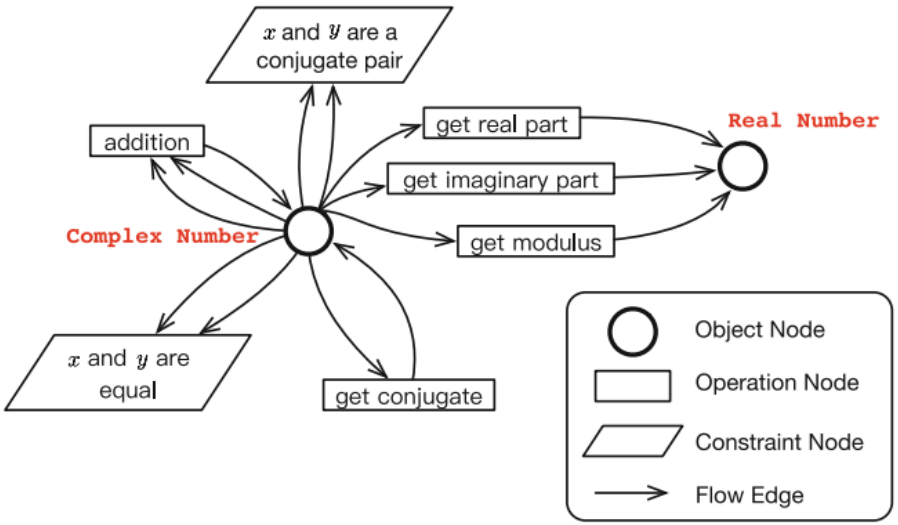

Figure 1: Knowledge graph from Zhao et al. (2019).

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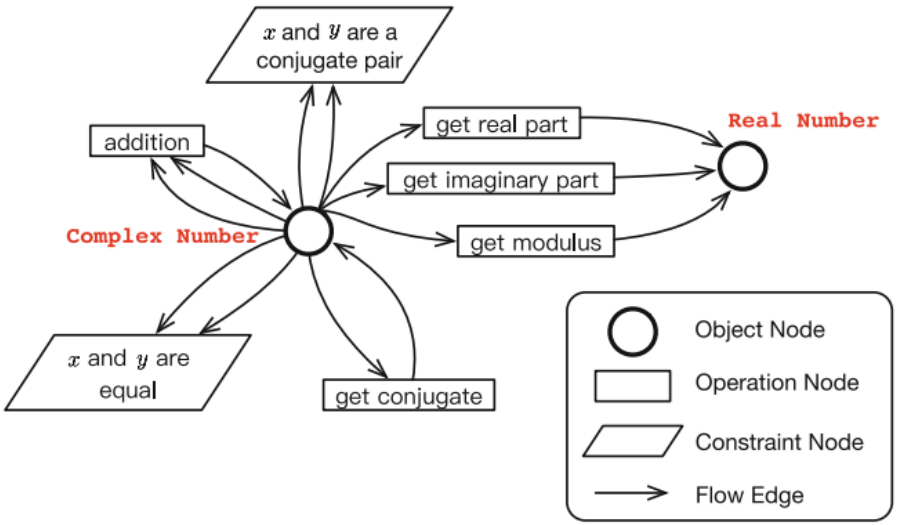

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2 Methods

Background The BERT model architecture (Devlin et al., 2019) is based on a multilayer bidirectional Transformer (Vaswani et al., 2017). Instead of the traditional left-to-right language modeling objective, BERT is trained on two tasks: predicting randomly masked tokens and predicting whether two sentences follow each other. SCIBERT follows the same architecture as BERT but is instead pretrained on scientific text.

Figure 2: Background for natural language processing (NLP) language model SciBert (Beltagy et al., 2019).

Reflect

Take one minute to reflect on the challenges of providing 'high-information' feedback (as an instructor or as a computer).

Type what you think is the biggest challenge and, after one minute has passed, hit enter.

Reflect

Take one minute to reflect on the challenges of providing 'high-information' feedback (as an instructor or as a computer).

For this presentation, I'll focus on assessing knowledge. If we want to provide 'high-information' feedback based on *how* a student responds to a prompt, we first need to talk about types of assessments we use to deliver prompts.

Types of Singular Responses

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How do we tell the difference?

Types of Singular Responses

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How do we tell the difference? Tracking across multiple prompts!

Types of Assessment

	Of Learning (Summative)	**For Learning** (Formative)	**As Learning**
Identifies	Proficiency on objective(s)	Understanding	Procedures
Feedback	R&P or Corrective	Elaborative	Corrective & Elaborative
Goal	Classify level of knowledge	Modify future learning	Learn skills and concepts

Table 4: Different types of assessment (Trenholm et al., 2015; Dann, 2014).

ALEKS: Knowledge Space

Figure 1.1. Graph of the miniature learning space on the items $a, b, \ldots, j$ described in Table 1.1. Each circle stands for a knowledge state. There are 34 states in this example. The top circle is the empty state, and learning proceeds from top to bottom, the items being mastered successively. The number of dots in each circle represents the number of items contained in the corresponding state. For instance, the state represented by the red circle contains the items c, g, h, and i, which can be verified by moving down from the empty circle at the top and following the arrows. According to this graph, there are 16 ways to learn these four items and reach the red state: either item i or item g has to precede item h. (Ignore the blue circle and the yellow circles for the moment.)

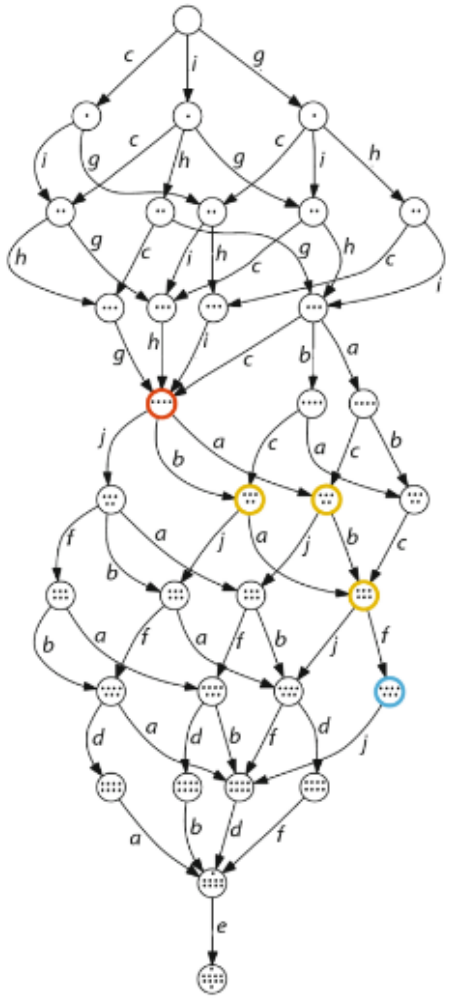

Figure 3: Example knowledge space from Falmagne et al. (2013).

ALEKS: Placement Test (Summative)

Algorithm 1 Assessment stopping algorithm.

Inputs:

α, stopping threshold probability

$\mathbf{x}_n$, the input features of the classification model after question n

$P(k \mid \mathbf{x}_n)$, predicted probability of class k, $k = 1, \ldots, 6$, after question n

$K_n = \arg\max_{k=1,\ldots,6} P(k \mid \mathbf{x}_n)$; i.e., the most likely class after question n

C_{29}, the recommended course placement after question 29 (based on computing the student's percentage score and applying the cut scores in Table 1)

Iterations:

for $n = 5$ to 29 **do**
  if $n == 29$ **then**
    Return C_{29}
  else if $P(K_n \mid \mathbf{x}_n) > \alpha$ **then**
    Stop the assessment and return K_n
  end if
end for

Output:

The predicted course placement recommendation

Figure 4: Stopping algorithm for ALEKS placement test (Matayoshi et al., 2021)

Concept Inventory: Elements of Understanding

1. the ability to distinguish between functions and equations;
2. the ability to recognise and relate different representations of functions and use them interchangeably;
3. the ability to classify relationships as functions or not functions;
4. the ability to have a working familiarity with properties of functions such as one-one/many-one, increasing/decreasing, linearity, composition, inverses;
5. the ability to use functions in context, modelling and interpreting;
6. the ability to engage with co-variational reasoning.

Figure 5: Elements of understanding functions from O'Shea et al. (2016).

Concept Inventory: Example Question

Q1. Let $f(x) = 3x + 5$. Given the equation $3a + 5 = 2$, which of the following are true? (There may be more than one true statement.)

- $f(a) = 3a + 5$,
- $f(a) = 2$,
- $f(x) = 2$ for all x,
- $f(x) = 2$ for some value of x.

Recreated example question targeting E1 and relates to E4 (O'Shea et al., 2016).

Choose-Your-Own-Adventure: Prompt

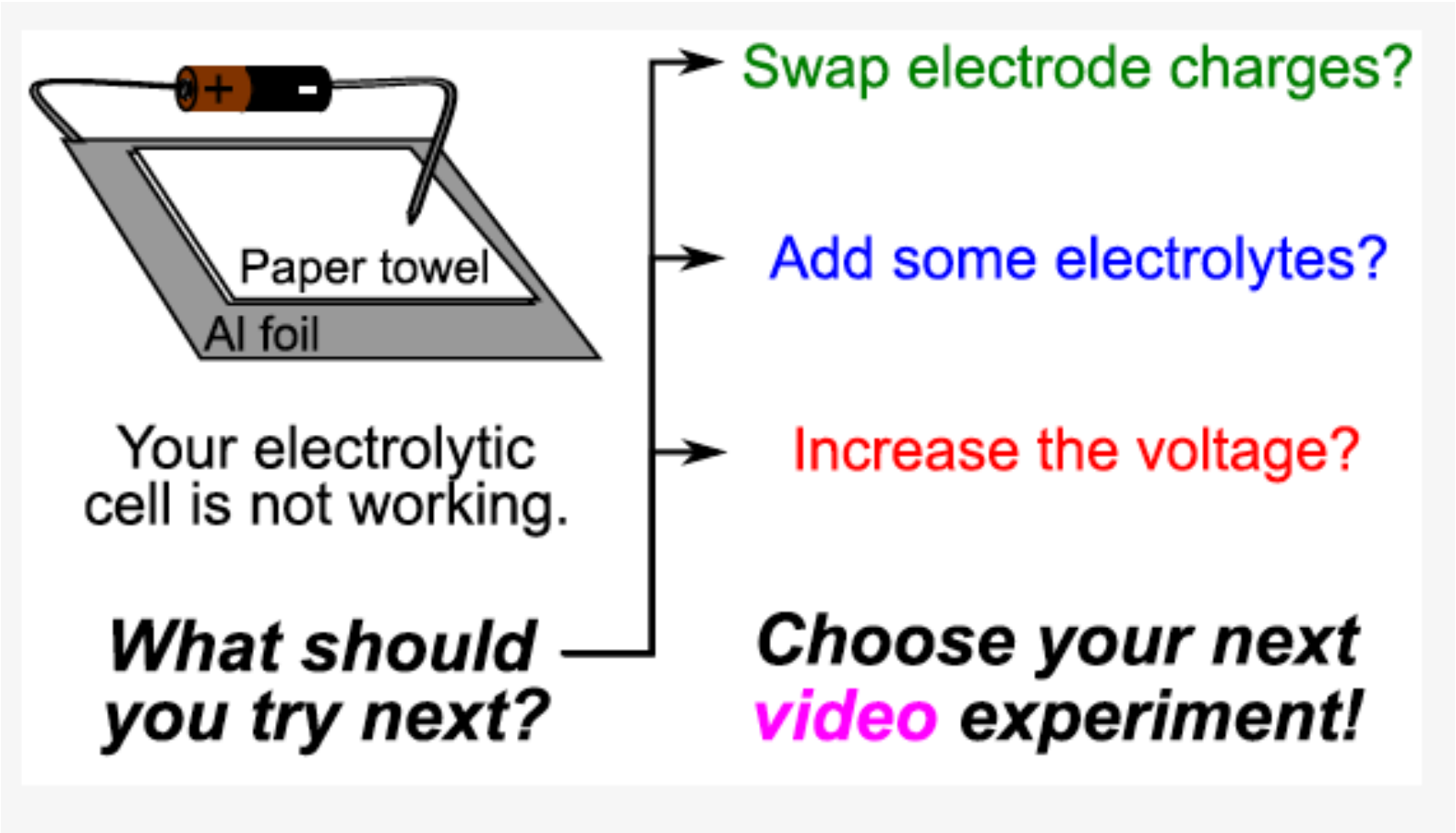

Figure 6: Initial prompt for electrochemistry simulated lab activity Warning & Kobylianskii (2021).

Choose-Your-Own-Adventure: Choice Flowchart

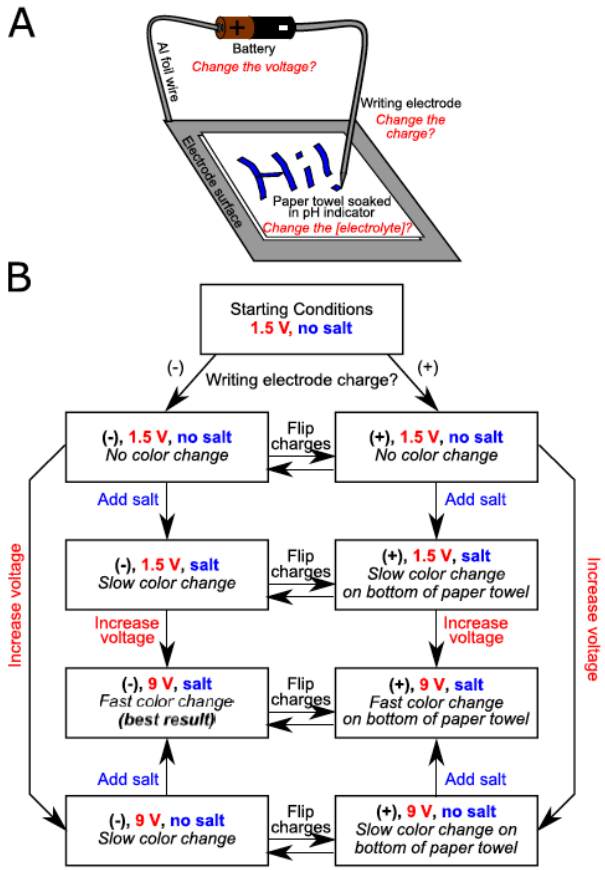

Figure 7: Flowchart for different states based on student choices Warning & Kobylianskii (2021).

Automated Homework Feedback

Solve the equation below. Do not round.

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

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Student: $x = \frac{54}{13}$

Automated Homework Feedback

Solve the equation below. Do not round.

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

Student: $x = \frac{54}{13}$

System: Close! Check your first step – did you distribute the 'minus' sign to both terms in the second fraction?

Automated Homework Feedback

Solve the equation below. Do not round.

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

Student: $x = \frac{54}{13}$

Student: $x = 9.69$

Automated Homework Feedback

Solve the equation below. Do not round.

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

Student: $x = \frac{54}{13}$

Student: $x = 9.69$

System: Almost! You were expected to solve for the exact value. If you only rounded at the end, submit your final fraction. Otherwise, you'll need to redo the problem without rounding.

Automated Homework Feedback

Solve the equation below. Do not round.

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

Student: $x = \frac{54}{13}$

Student: $x = 9.69$

Student: $x = \frac{126}{13}$

Automated Homework Feedback

Solve the equation below. Do not round.

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Student: $x = \frac{54}{13}$

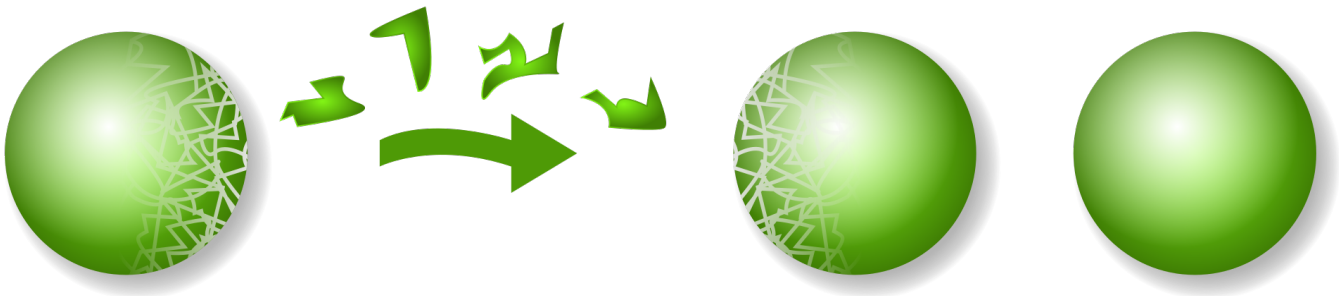
Student: $x = 9.69$

Student: $x = \frac{126}{13}$

System: Awesome!

Banach-Tarski Assessment Paradox

By tracking all responses for a question through an entire assessment, we can identify more student conceptions than the individual questions provide.



Reflect

Take one minute to reflect on the types of assignments we discussed: 'of learning', 'for learning', and 'as learning'.

Choose one example assessment (ALEKS, Function Concept Inventory, Choose-Your-Own-Adventure virtual lab, Automated Homework) and type one way predicting students' thoughts was incorporated into the design. After one minute has passed, hit enter.

Reflect

Take one minute to reflect on the types of assignments we discussed: 'of learning', 'for learning', and 'as learning'.

Now we will shift to considering how we can incorporate elaborative feedback based on the predictions of student thought already incorporated into activities.

Solve Linear Equation: Spring 2020

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

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$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

- 59 - Correct answer.

Solve Linear Equation: Spring 2020

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

- 59 - Correct answer.
- 22 - Not distributing the minus in front of the second fraction.

Solve Linear Equation: Spring 2020

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

- 59 - Correct answer.
- 22 - Not distributing the minus in front of the second fraction.
- 13 - Dividing only the second coefficient in each numerator by the denominator.

Solve Linear Equation: Spring 2020

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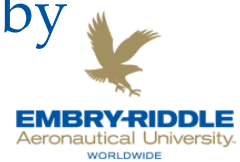
- 59 - Correct answer.
- 22 - Not distributing the minus in front of the second fraction.
- 13 - Dividing only the second coefficient in each numerator by the denominator.
- 6 - No solution

Solve Linear Equation: Spring 2020

$$\frac{4x-3}{3} - \frac{-3x+6}{5} = \frac{3x+4}{2}$$

- 59 - Correct answer.
- 22 - Not distributing the minus in front of the second fraction.
- 13 - Dividing only the second coefficient in each numerator by the denominator.
- 6 - No solution
- 0 - Dividing only the first coefficient in each numerator by the denominator.



Fall 2018 and Fall 2019

Fall 2018

$$\frac{-4x-6}{2} - \frac{-4x+6}{5} = \frac{3x+7}{4}$$

- 68 - Correct answer.
- 20 - Not distributing the minus in front of the second fraction.
- 4 - No solution
- 4 - Dividing only the first coefficient in each numerator by the denominator.
- 3 - Dividing only the second coefficient in each numerator by the denominator.

Fall 2019

$$\frac{-8x+7}{4} - \frac{-3x+4}{5} = \frac{-3x+6}{2}$$

- 66 - Correct answer.
- 17 - Not distributing the minus in front of the second fraction.
- 10 - Dividing only the first coefficient in each numerator by the denominator.
- 4 - No solution
- 3 - Dividing only the second coefficient in each numerator by the denominator.

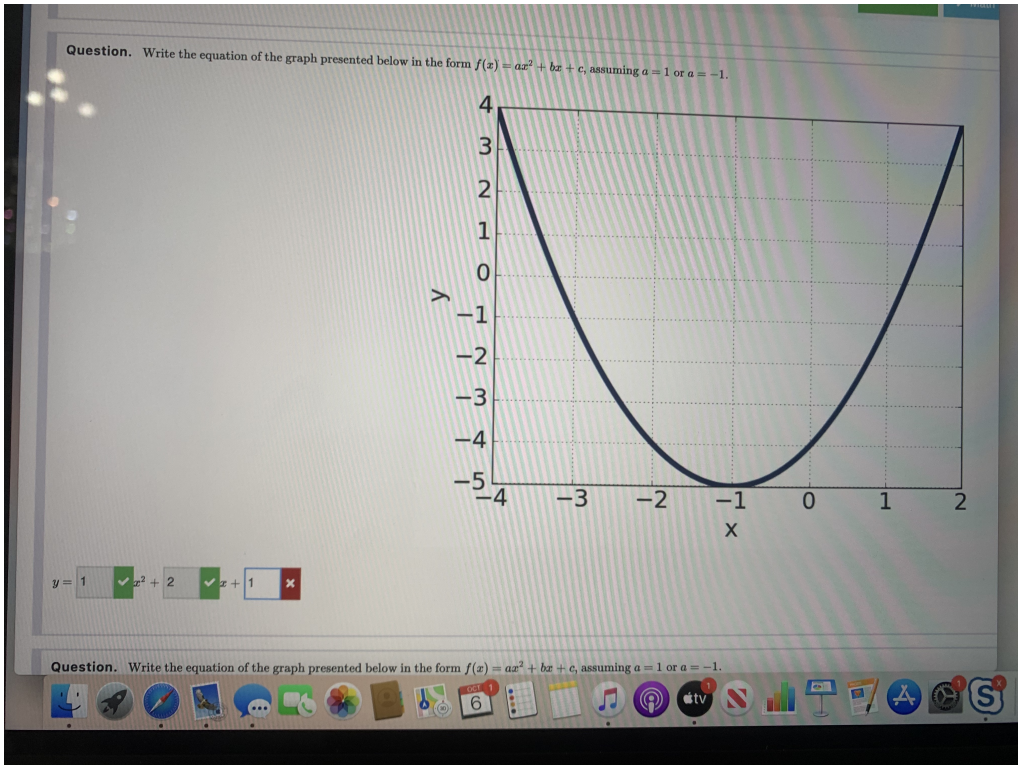
Choice Associated to Student Thinking

Fall 2018 Q9	Fall 2019 Q5
$\frac{-4x-6}{2} - \frac{-4x+6}{5} = \frac{3x+7}{4}$	$\frac{-8x+7}{4} - \frac{-3x+4}{5} = \frac{-3x+6}{2}$
A. $x \in [-1.9, -0.7]$	A. $x \in [2, 5]$
*B. $x \in [-5.1, -2.1]$	B. $x \in [27, 31]$
C. $x \in [-11.6, -8.8]$	C. $x \in [-5, 0]$
D. $x \in [-3, -1.9]$	*D. $x \in [18, 25]$
E. There are no Real solutions.	E. There are no Real solutions.
20%, **68%**, 3%, 4%, 4%	10%, 3%, 17%, **66%**, 4%

Figure 8: Multiple-choice presentation of questions.

Construct Quadratic Equation

Write the equation of the graph presented below in the form $f(x) = ax^2 + bx + c$, assuming $a = 1$ or $a = -1$.



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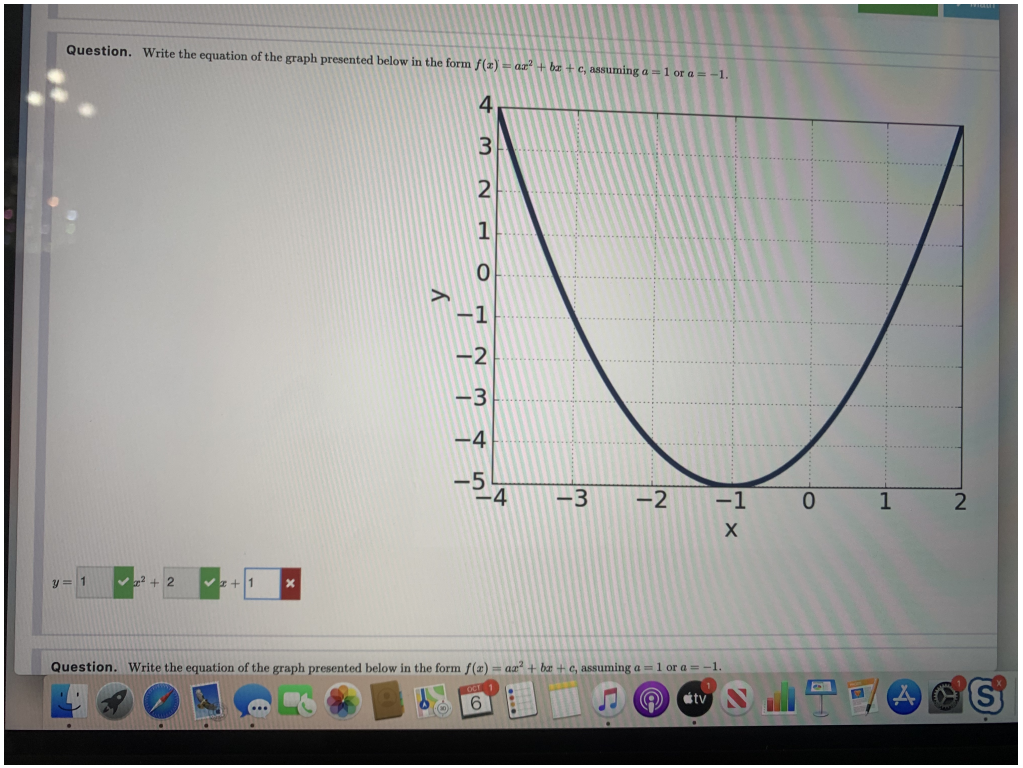

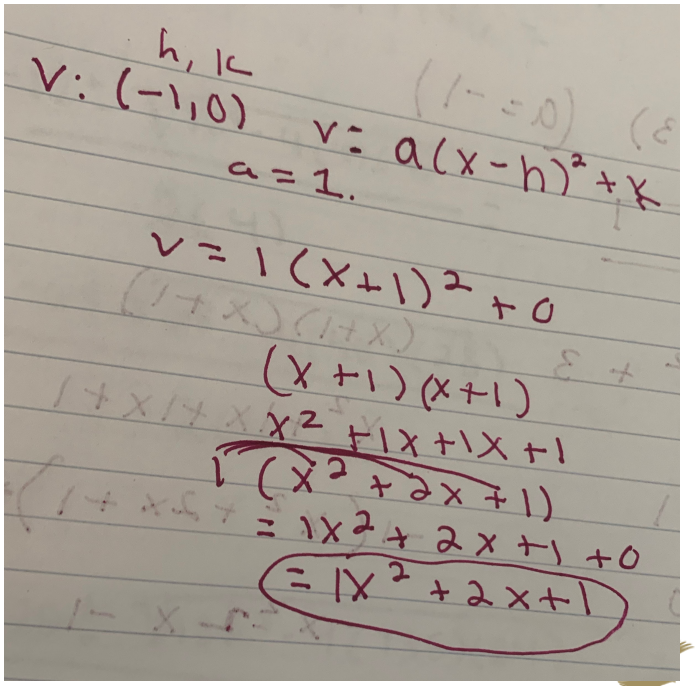

Assessment Report

Objective	Sub-Objective	Score	Possible Issue (if applicable)
Construct linear functions	...using a slope and point.	1/1	
	...using two points.	1/1	
	...using a parallel/perpendicular line and point.	1/1	
Convert between linear forms.	Standard to Slope-Intercept.	1/1	
	Slope-Intercept to Standard.	0/1	Standard form requires integer coefficients.
	Point-Slope to Standard.	0/1	Standard form requires integer coefficients.
Convert between linear representations.	Graph to Equation.	1/1	
	Equation to Graph.	1/1	
Solve linear equations	...with integer coefficients.	1/1	
	...with rational coefficients.	0/1	Did not distribute the negative in front of the second fraction.

Progress in Project

- Constructed numerous (80+) question structures with common student responses and associate elaborative feedback

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- Using these structures, designed and implemented a Mastery-Based College Algebra course with an open-source online homework system
- Using new data, labeled and cataloged student responses in a new course (Precalculus for Aviation)
- Designing exploratory 'Choose-Your-Own-Adventure' lessons with integrated skills practice that provide elaborative feedback

Calculus Discussion: Torty and Harry

Situation: Torty and Harry are competing in a 100m sprint race. Torty's average speed on any 5-second interval is always less than Harry's average speed on any 5-second interval, but Torty wins the race! *(Note: For consistency, let's say they keep running after 100 meters so that their speeds can always be calculated by looking forward in time, but we stop the race at 100 meters)*

Prompt: Discuss as a group (using the definitions of constant and average rate of change) how it is possible that Torty wins the race.

Concepts and Interpretations I

- **Constant Rate of Change described as...**
  - Quantitative Change
  - Geometric Mean
  - Algebraic Ratio
  - Contextual Ratio
  - Perceivable Geometric Property
  - Amount of Increase/Decrease
  - Slope Constant in $y = mx + b$
- **Average Rate of Change described as...**
  - Arithmetic Mean of Instantaneous Rates
  - Slope of Secant Line
  - Linear Substitution

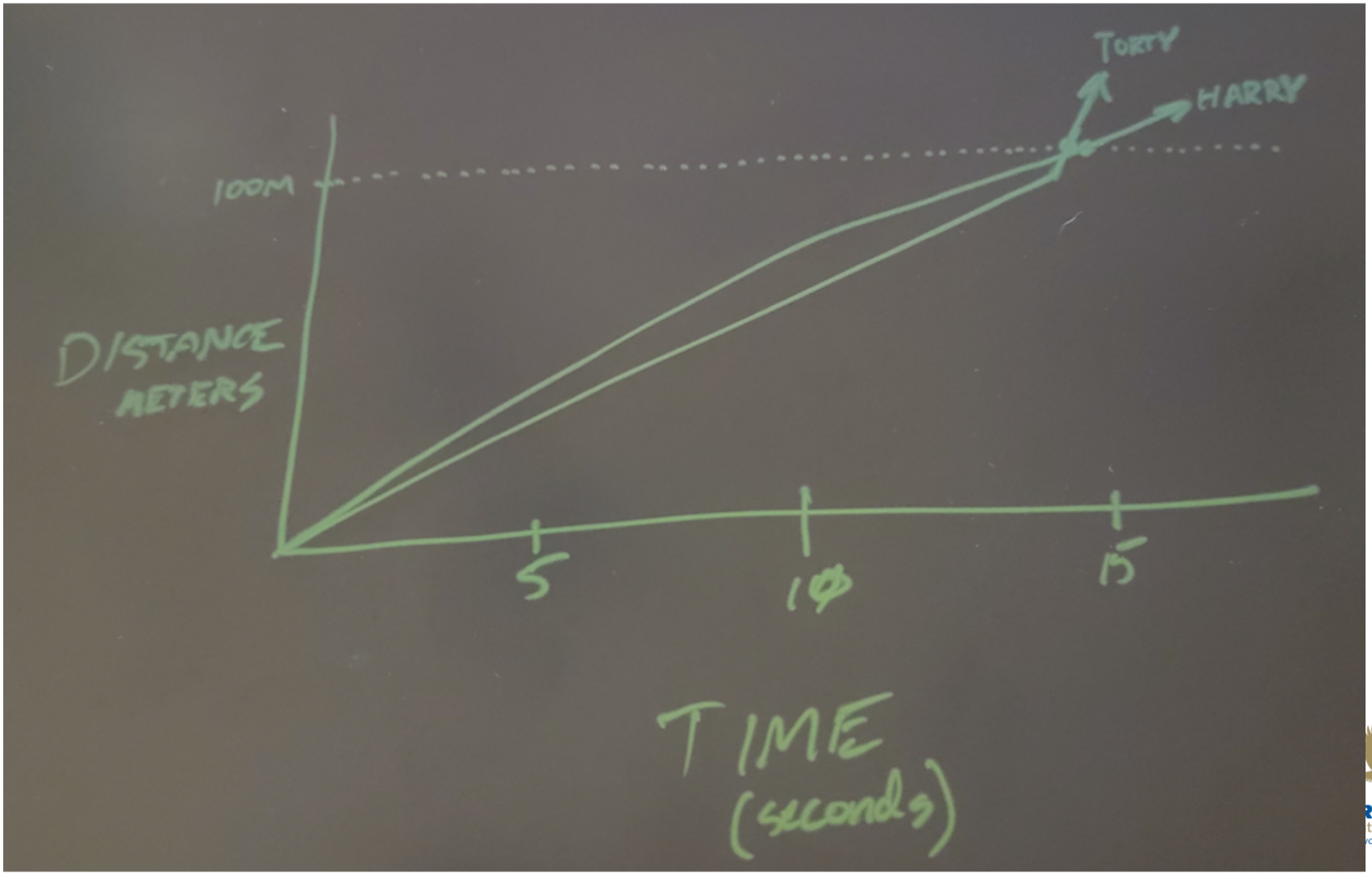
Concepts and Interpretations II

- **Derivative described as...**
  - Giving Increase/Decrease
  - Giving Slope
  - Giving Rate of Change
- **Function described as...**
  - Class of Functions
  - Nonreferential Symbolic
  - Geometrically
  - Input-Output
  - Contextually

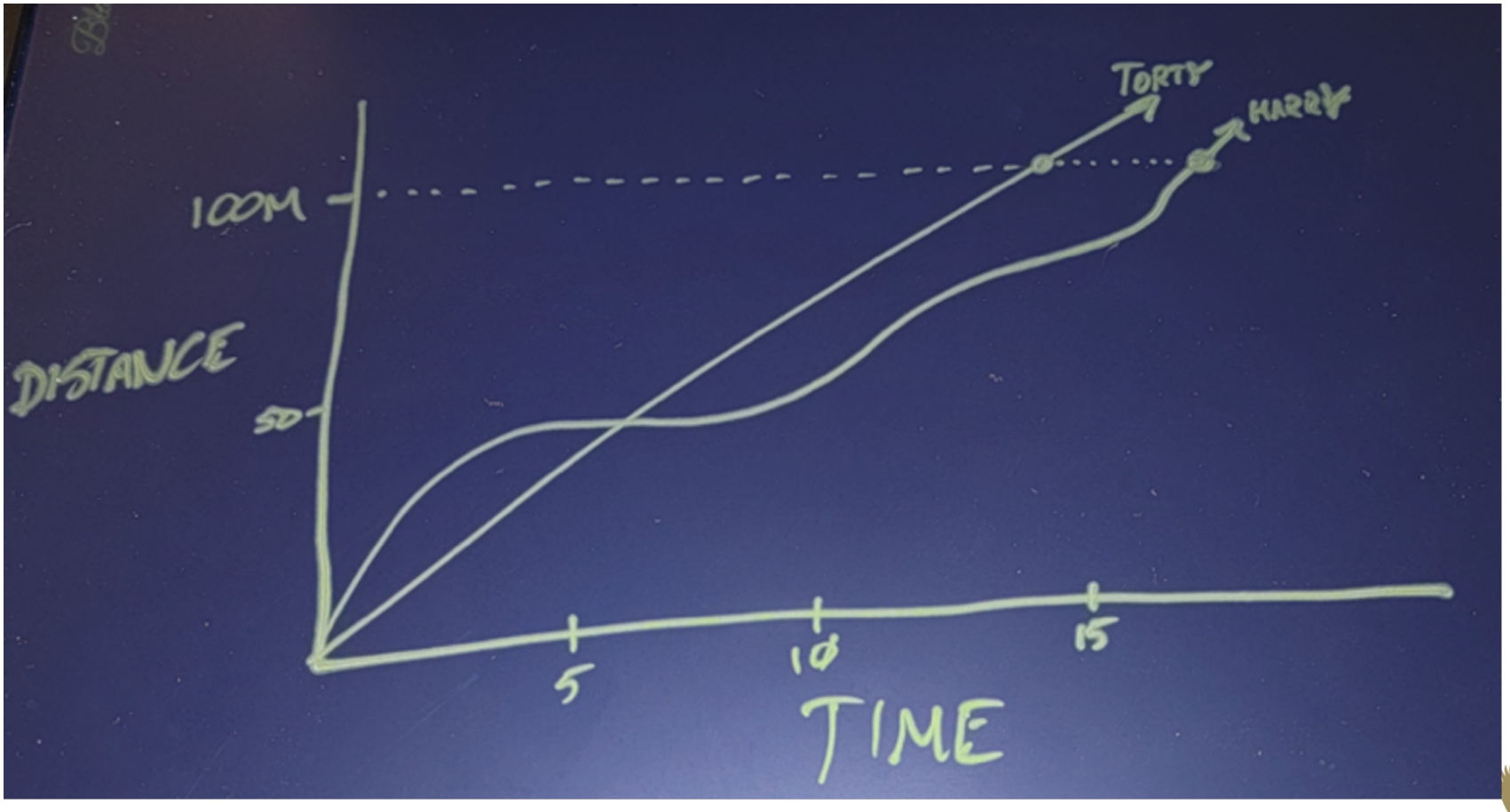
Patterns of Thought - Justification

- **Justify by Derivative** - Student might calculate derivatives to say things about the graphs' rates of change.
- **Justify by Geometry (Chunky/Scaling/Continuous)** - Student focuses on the comparative ROCs of the two graphs over contiguous/different scales/variety of intervals
- **Justify by Context -** Student appeals to possible physical aspects of the race (e.g. Torty moving in "spurts" or "going back and forth" or "winding around")

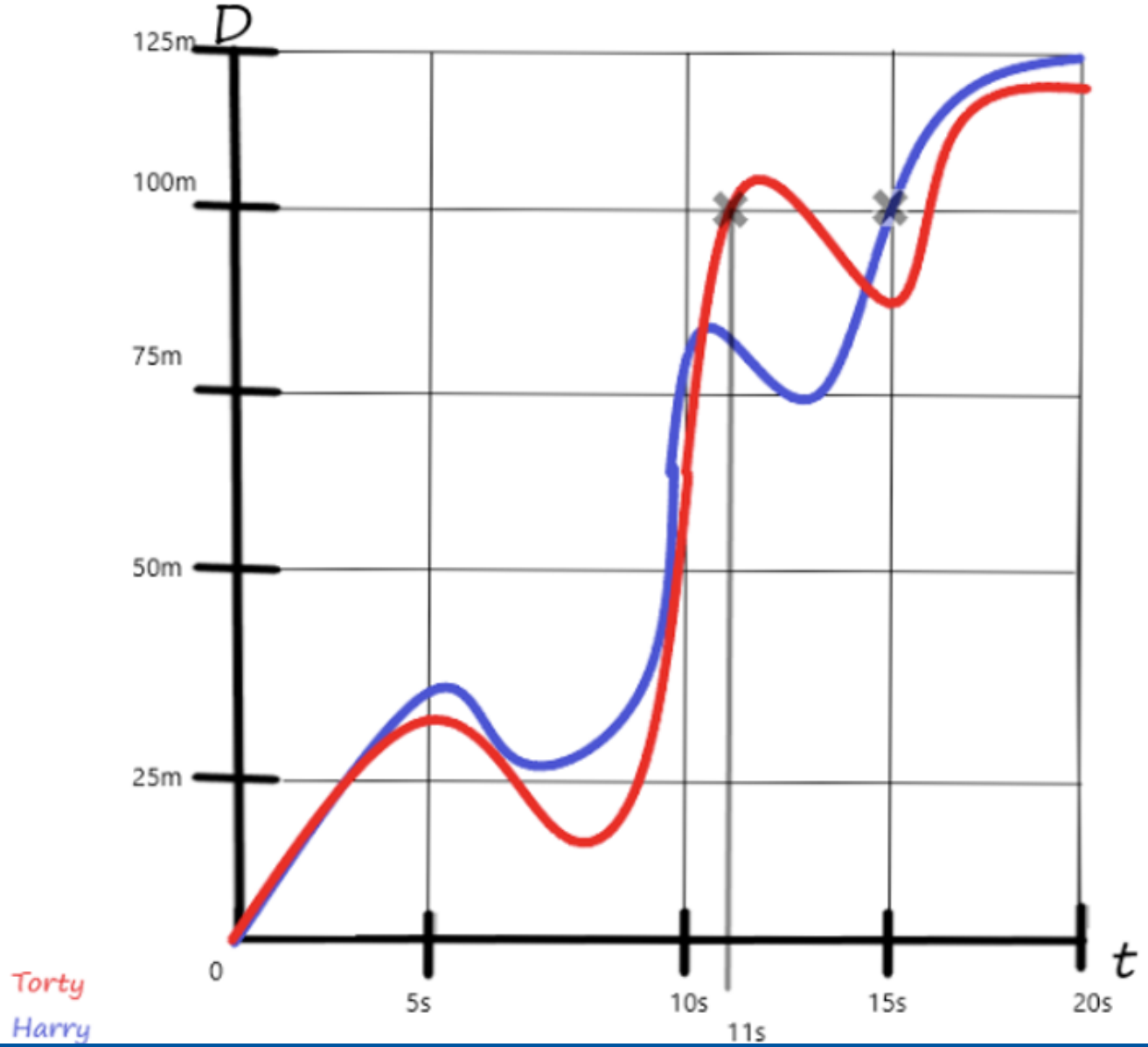
Torty and Harry Group Constructions I



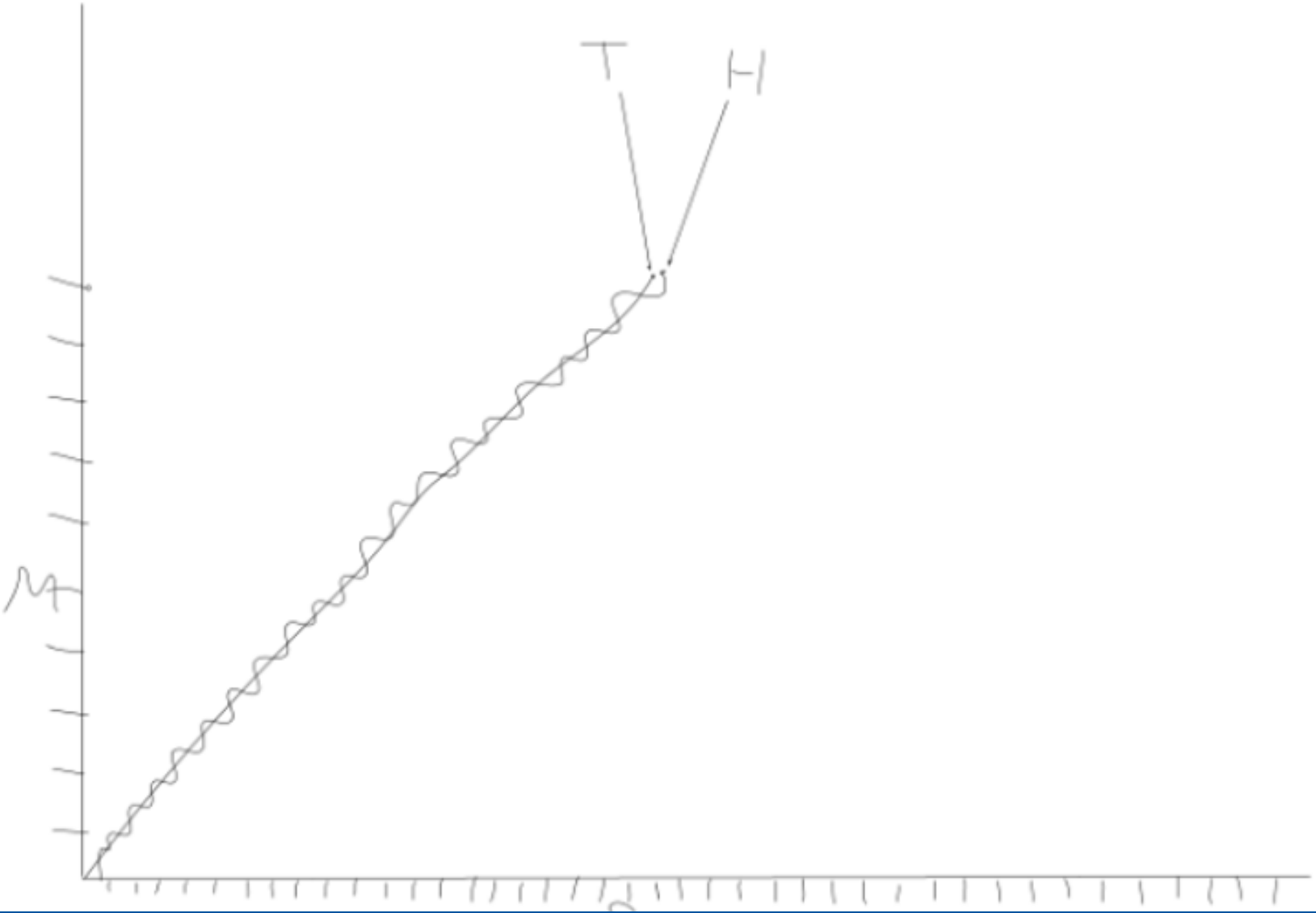
TORTY AND HARRY GROUP CONSTRUCTIONS II



Torty and Harry Group Constructions III



Torty and Harry Group Final Response



Rails for Instructor Elaborative Feedback

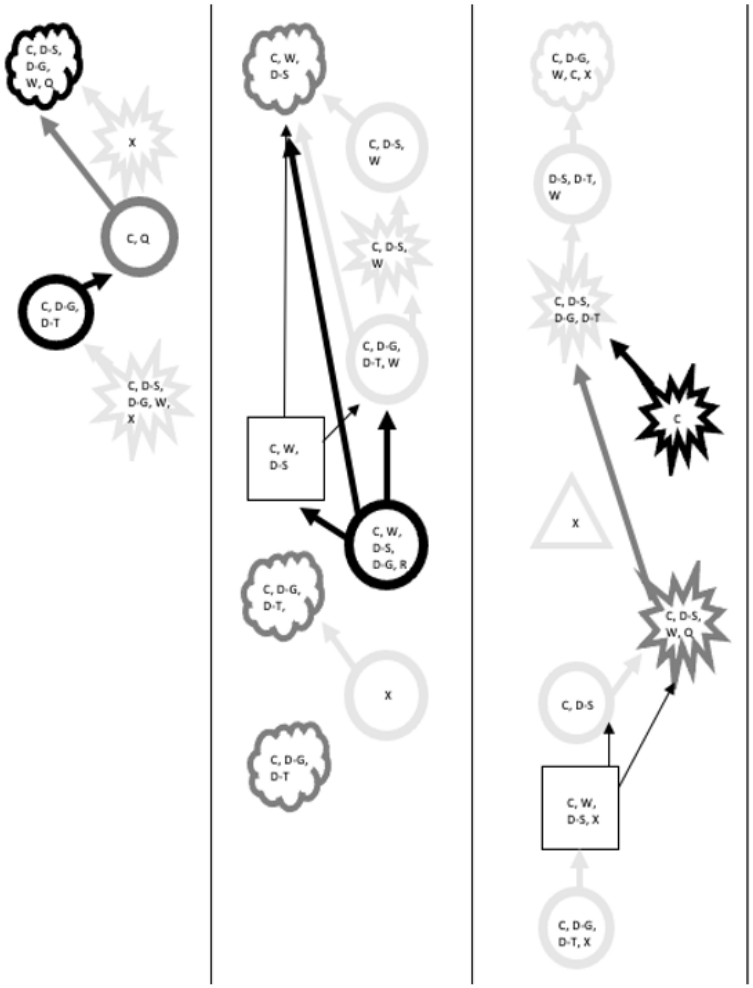
This page provides opportunities for Math 241 instructors to share resources and common hints/responses given throughout the course of **Module 1-2 Discussion: Torty and Harry**

The Table of Contents link to specific hints given throughout the course of the discussion, or link to common questions that arise from students. The linked pages then give the hints/responses, and you can leave comments to offer further suggestions/hints, or to ask questions.

I (Zack) will be adding to the pages based on your comments as time progresses, the idea is to build up a collection of shared responses that have been seen to help students over the course of the two-week discussion.

Table of Contents

Feedback from Students I



Legend

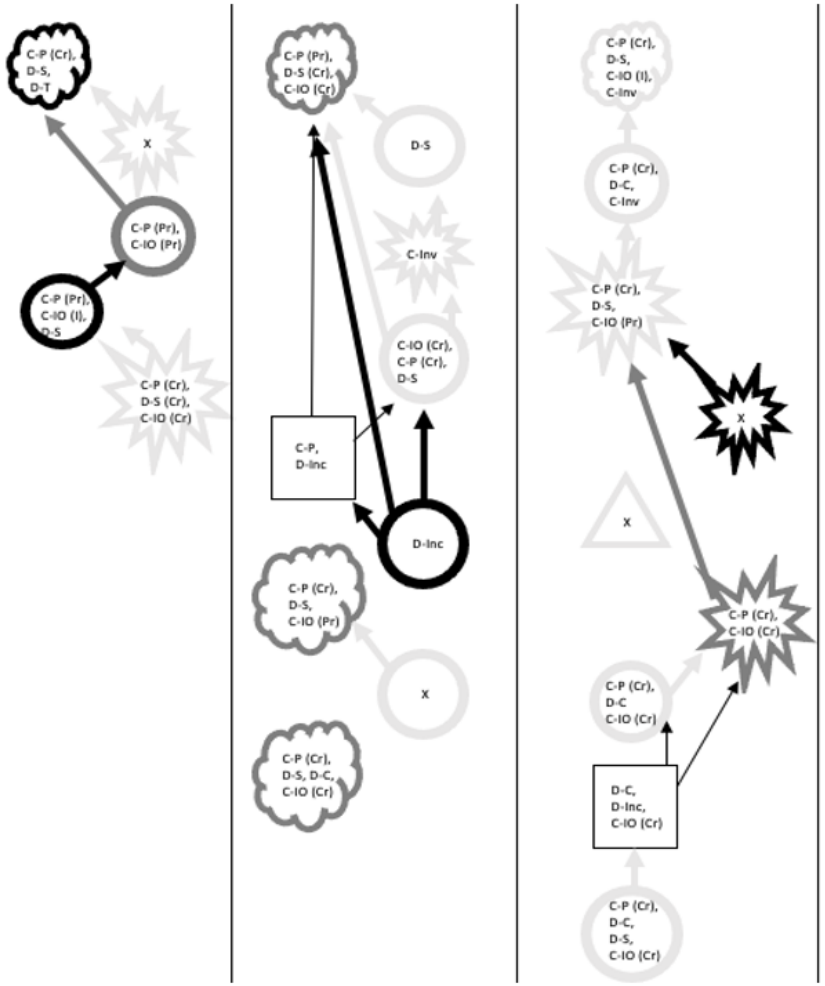
Macro-Argumentative

Cloud	Argument
Star	Counterargument
Circle	Integration
Square	Instructor
Triangle	Non-Argumentative Move

Micro-Argumentative

C	Claim
D-S D-G D-T	Data represented as Symbols, Graph, or Table
W	Warrant
Q	Modal Qualifier
R	Rebuttal
X	Non-Argumentative Move

Feedback from Students II



D-S, D-Inc

Legend

Macro-Argumentative	
Cloud	Argument
Star	Counterargument
Circle	Integration
Square	Instructor
Triangle	Non-Argumentative Move
Epistemic	
C-P C-IO C-INV	Composition as Product, Input/Output, or Inverses
D-INC D-S D-RoC D-C D-T	Derivative as Increasing, Slope of Tangent, Rate of Change, Computation, or Tangent Line
I Pr Cr	Inattention, Progress, or Coordination

ChatGPT to the rescue! (eventually)

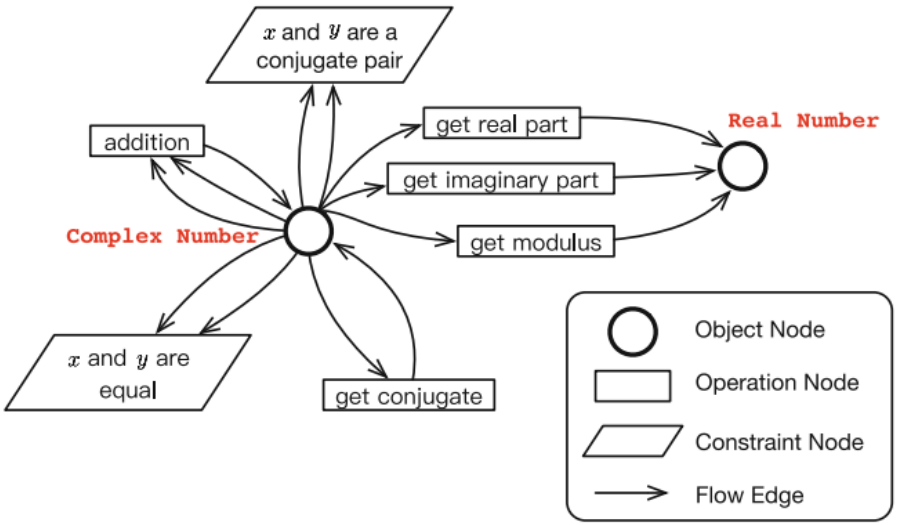


Figure 9: Knowledge graph from Zhao et al. (2019).

2 Methods

Background The BERT model architecture (Devlin et al., 2019) is based on a multilayer bidirectional Transformer (Vaswani et al., 2017). Instead of the traditional left-to-right language modeling objective, BERT is trained on two tasks: predicting randomly masked tokens and predicting whether two sentences follow each other. SCIBERT follows the same architecture as BERT but is instead pretrained on scientific text.

Figure 10: Background for natural language processing (NLP) language model SciBert (Beltagy et al., 2019).

References I

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