Adapting the Argumentative Knowledge Construction Framework to Asynchronous Mathematical Discussions

Darryl Chamberlain

Zack Reed

Karen Keene

EMBRY-RIDDLE
Aeronautical University

EMBRY-RIDDLE Aeronautical University

Context and Motivation

Despite overall downward trends in higher education enrollment, online mathematics education has seen nontrivial growth. (Trenholm et al., 2019; Blair et al., 2012; Blaire et al, 2018)

35% of math departments (2010) in 4-year institutions offer online education, with 72% of the courses being fully online. (Blair et al., 2012)

88% of math departments (2010) in 2-year institutions offer online education, with 73% of the courses being fully online. (Blair et al., 2012)

EMBRY-RIDDLE Aeronautical University

Context and Motivation

52% of math departments (2015) in 4-year institutions offer online education, with 69% of the courses being fully online. (Blair et al., 2018)

87% of math departments (2015) in 2-year institutions offer online education, with 69% of the courses being fully online. (Blair et al., 2018)

Online courses are societally relevant, increasing access to higher education.

Context and Motivation

Despite their prevalence, we still know little about how students develop mathematical meanings in a fully online format (Trenholm et al., 2019).

From what we do know, fully online mathematics courses are among the more difficult to teach (Engelbrecht & Harding, 2005).

Context and Motivation

“Clearly, from multiple perspectives, FO mathematics instruction has not been successful in comparison with traditional F2F mathematics instruction.”

”Further research should be directed at FO mathematics pedagogies that enable better learning expeireces and outcomes … including retention, transfer, and depth of understaning” (Trenholm et al., 2019).

Context and Motivation

While researchers have examined aspects of online or at-home learning (Lew & Zazkis, 2019; Dorko, 2020a; 2020b; 2021), we still lack research on fully online courses.

Discussion activities give rich opportunities for students to co-develop robust mathematical meanings.

Relatively few online math courses utilize discussion activities (Trenholm et al., 2019)

Emergent Perspective

Cobb and Yackel (1996) sought to "account for students' mathematical development *as it occurs in the context of the classroom*".

Social – "focus on the patterns and regularities in [students'] interactions and on the consensual meanings that emerge between them" (p. 184).

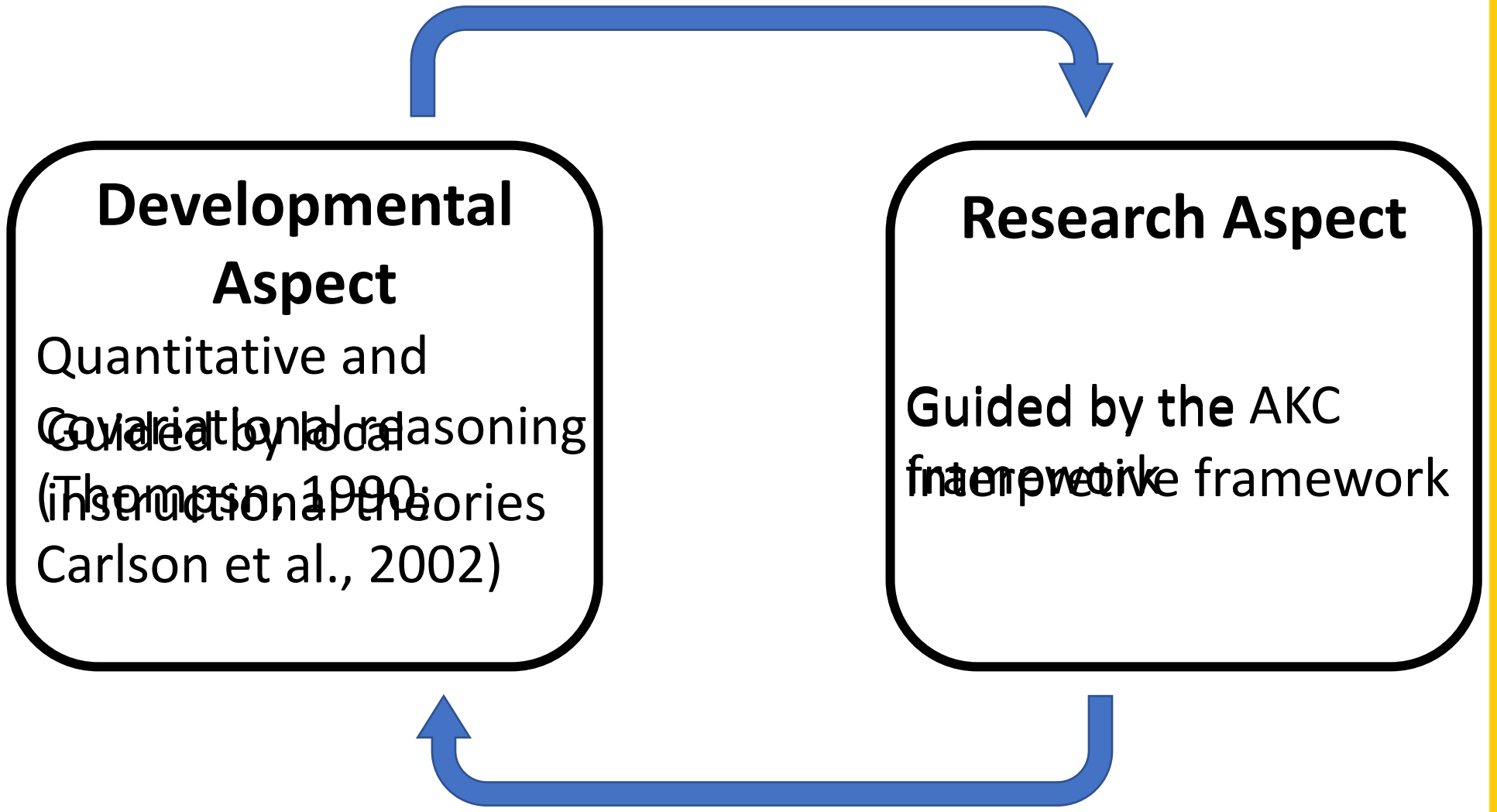
Psychological – Students necessarily draw from their extant mathematical meanings (in the sense of Thompson et al. 2014). These meanings might develop over the course of a discussion activity.

Emergent Perspective

This perspective *"coordinates* analyses of classroom processes that are conducted in psychological and social terms".

We seek to coordinate the psychological and social aspects of students' mathematical development in the specific context of an asynchronous discussion activity.

Emergent Perspective – Developmental Research



Theory Development in Research

1. Develop Initial Theory
2. Implement Framework(s) on Pilot Data
3. Refine Framework(s) based on Pilot Analysis
4. Implement Framework(s) on Broader Scale
5. Make claims based on broader analysis
5. Further Refine Theory

AKC (Weinberger and Fischer, 2006)

Argumentative Knowledge Construction

Four dimensions to analyzing “Knowledge Acquisition” in an asynchronous online discussion format.

1. Participation
2. Epistemic
3. Argumentative
4. Social

EMBRY-RIDDLE Aeronautical University

Research Questions

1. How do the social, epistemic, and argumentative dimensions of knowledge construction interact with each other in the context of a mathematics classroom?

2. How does the AKC Framework change when contextualized in a mathematics class?

3. What alterations to the AKC Framework increase our understanding of student learning in online calculus courses with discussions?

Data

3 Discussions Activities - Asynchronous Differential Calculus

Prior to “Research Phase”, discussion activities designed to:

1. Promote deep thinking about core calculus concepts
2. Promote Engagement in Quantitative and Covariational Reasoning
3. Promote peer-to-peer and teacher-to-student interaction
    a. Requiring that students justify their responses was a key design principle.

One group of 3 students – Highest participation

EMBRY-RIDDLE Aeronautical University

Discussion 1: Torty and Harry

Task Situation: Torty and Harry are competing in a 100m footrace. Torty's average speed on ***any possible*** 5-second interval is always less than Harry's average speed on any 5-second interval, but Torty wins the race!

(No head starts, no backward motion, no stopping)

Produce graphs of Torty's and Harry's distance vs time relationships that reflect these constraints.

Justify that your graphs satisfy the constraints.

EMBRY-RIDDLE
Aeronautical University

Discussion 4: Chain Rule

Your overall goal is to draw a random curve representing a function f defined on the interval $[2, 12]$, and then construct the graph of a second function g such that the following requirement is satisfied: For the composite function $h(x) = g(f(x))$ (meaning g composed with f), $h'(x) = 2$ at each x-value on $[0, 10]$.

https://www.geogebra.org/m/vrrj9hqr

EMBRY-RIDDLE Aeronautical University

Methods and Iterative Analysis

1. Analyze Discussion 1 pilot data using AKC
2. Suggest Epistemic Reconceptualization
3. Analyze Discussion 2 pilot data with revised AKC
4. Suggest Argumentative Reconceptualization
5. Analyze Discussion 3 pilot data with revised AKC
6. Suggest Social Reconceptualization
7. Reconceptualize AKC into new framework
8. Re-analyze Discussion 1 pilot data
9. Upscale Data Collection and Analysis

AKC (Weinberger and Fischer, 2006)

1. Participation
2. Epistemic
3. Argumentative
4. Social

Deficit perspective

Enabled primarily content-general findings

Limited cross-dimensional insights

AKC (Weinberger and Fischer, 2006)

1. Participation
2. Epistemic
3. Argumentative
4. Social

Too sharply delineated the discursive activity into disparate dimensions.

We adopt different *lenses* to extract insights from the same discursive activity and *coordinate* the results of multiple analyses.

EMBRY-RIDDLE Aeronautical University

Reconceptualized Epistemic Lens

Domain-Specific Codebooks, usable in multiple discussions.

Leverage constructs from the literature on students' meanings for core concepts.
(Quantitative Reasoning, Covariational Reasoning, Students' Meanings for Slope)

Code based on how students describe the task-specific concepts in the problem-solving process.

Macro codes based on the mathematical concepts described within students' justifications.

EMBRY-RIDDLE Aeronautical University

Reconceptualized Epistemic Lens

Constant Rate of Change Described as determining amount of increase or decrease	CROC-INC	i.e. reference to how high or low the graph goes
Constant Rate of Change Described as the "slope constant" m in $y = mx + b$.	CROC-NRS	Very formula driven, similar to F-NRS
Average Rate of Change Described as an arithmetic mean of instantaneous rates	AROC-MEAN	"Take an average of all of the rates of change over an interval"
Average Rate of Change Described as slope of secant line	AROC-SEC	
Average Rate of Change Described as linear substitute	AROC-LIN	AROC described as the slope of a line substituting the variation in the function over the set interval.

(Nagel et al., 2013)

(Harel, 2008)

(Musgrave & Carlson, 2017)

Reconceptualized Social Lens

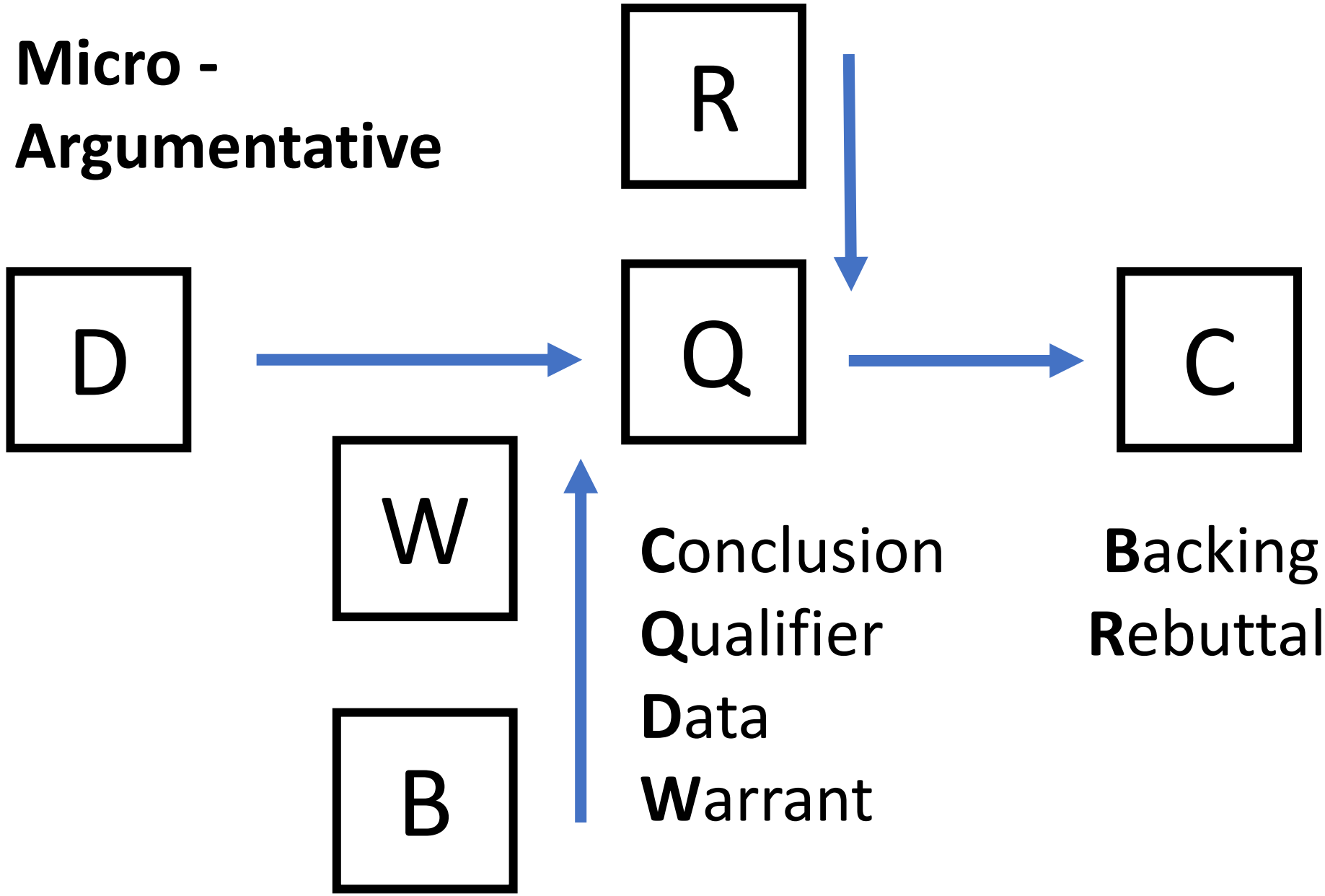
Mathematical discourse is largely argumentative

Social Lens –
Macro-argumentative codes
Toulmin's model for argumentation
Network analysis of interaction chains

This afforded explanatory power for math-specific discourse, and cross-dimensional insights

Reconceptualized Argumentative-Social

Micro - Argumentative



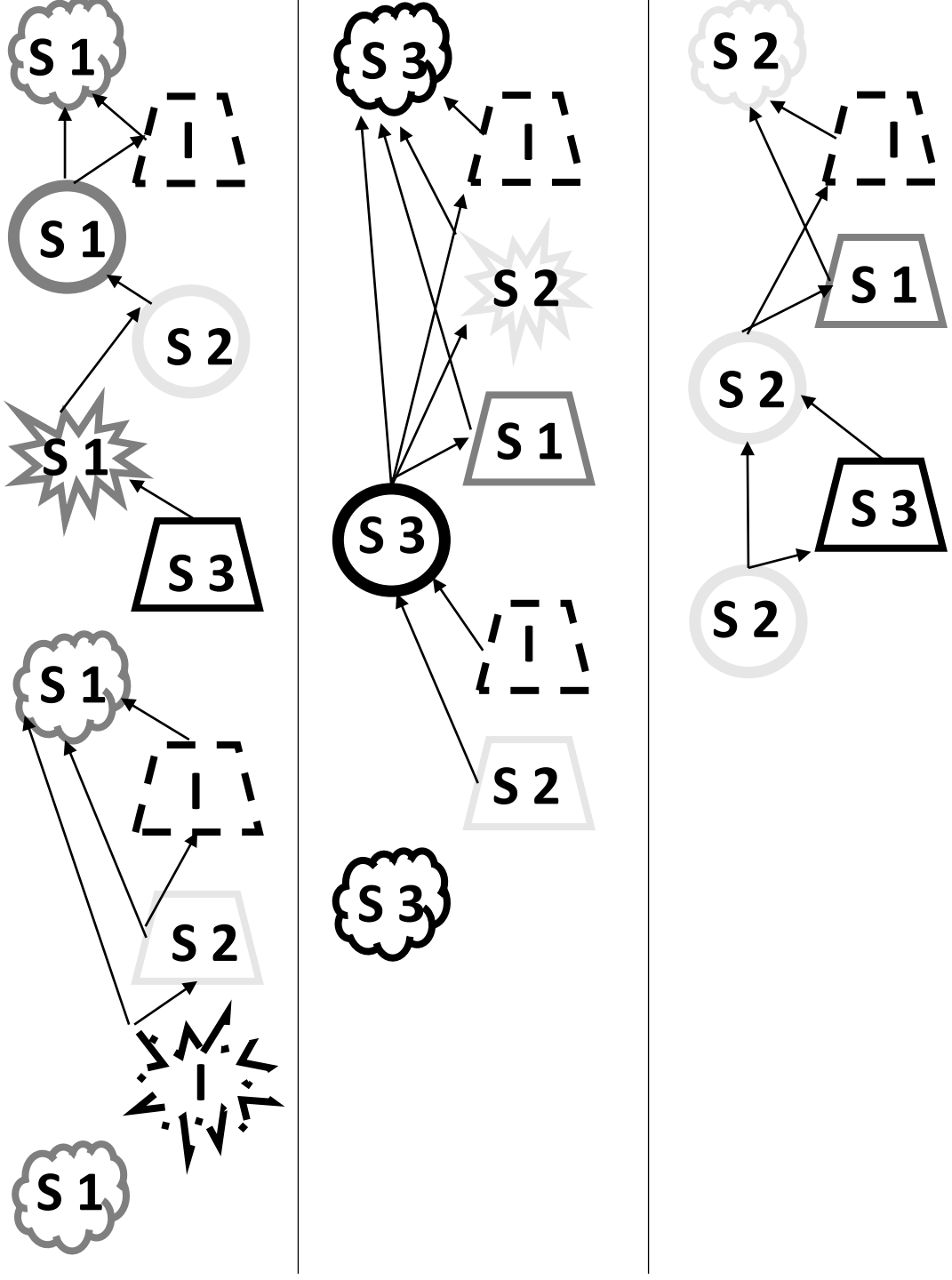
Conclusion
Qualifier
Data
Warrant

Backing
Rebuttal

EMBRY-RIDDLE Aeronautical University

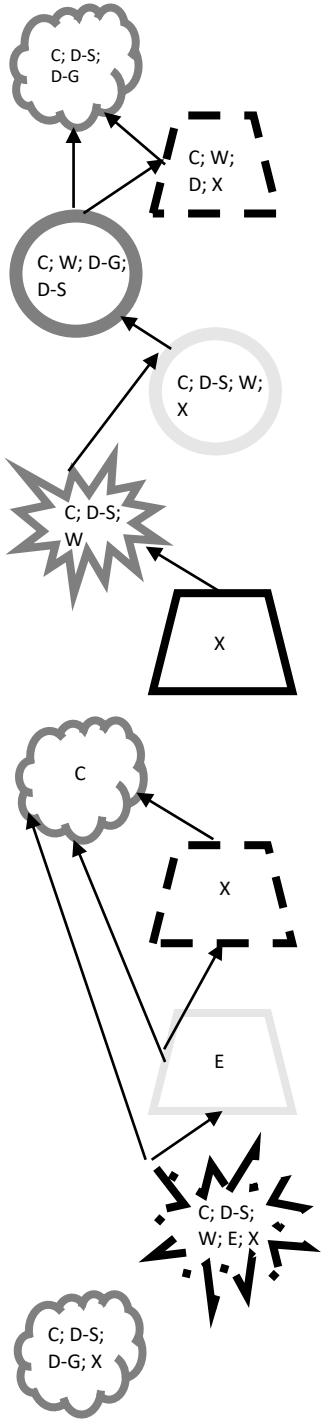
Data Analysis

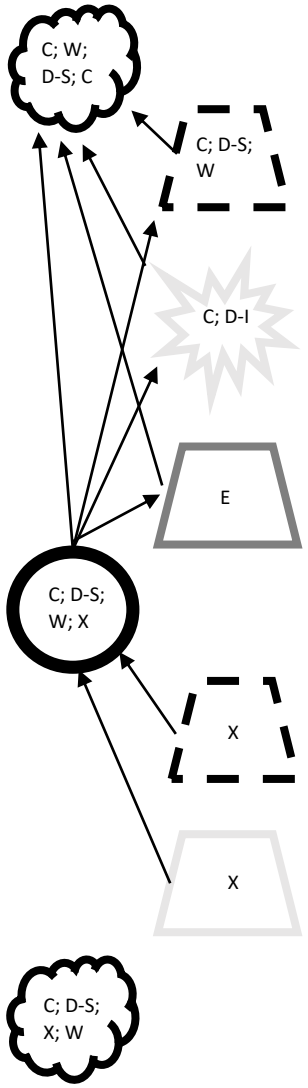
	A	B	C	D	E	F	
1	Speaker ID	C	Statement	C, D Claim, Data	W, B Warrant, Backing	Q, R Qualifier, Rebuttal	Ev
3	Black	1	Torty still managed to win the race, but how?				
4	Black	1	For this to happen, Harry's top speed in every interval had to be significantly higher than Torty's top speed to have these results.				
5	Black	1	Torty won this race due to going slower at a more steady, manageable pace.				
6	Black	1	Harry lost the race because he went all out and had to slow down repetitively in each race interval.				
7	Black	1	So to conclude, each five-second interval probably looked like Harry taking a big lead, and at the end of the interval Torty has caught up and passed Harry; this cycle continues the rest of the race.				
8	Instructor	1	[Name], great first conjecture!				
9	Instructor	1	You're definitely on the right track with how you're trying to envision the race unfolding.				
10	Instructor	1	I'll wait to give more substantive feedback until your group members post as well, but I think moving forward by being a little more specific (possible with some drawings) about the timing of the events you describe in relation to the start and finish of the race will be helpful.				
11	Instructor	1	Nice work!				
12	Dark Gray	1	[Name],				
13	Dark Gray	1	I agree with your initial assessment, but the issue I ran into when I started to draft a graph of the two racers following this initial idea was keeping the average rate of change higher for Harry with him losing.				



Reconceptualized Participation Analysis

Shape	Macro Argument
Star	Counterargument
Circle	Integration
Trapezoid	Evaluation
Cloud	Argument

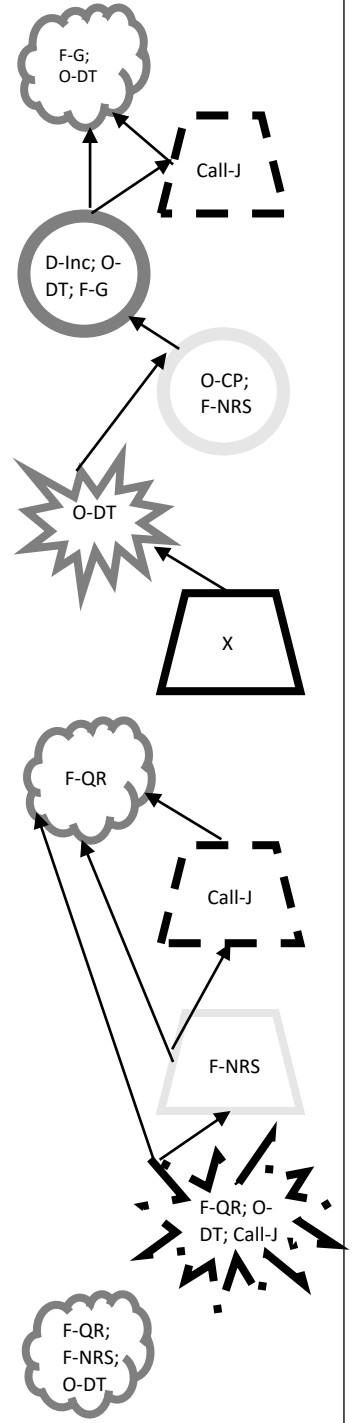


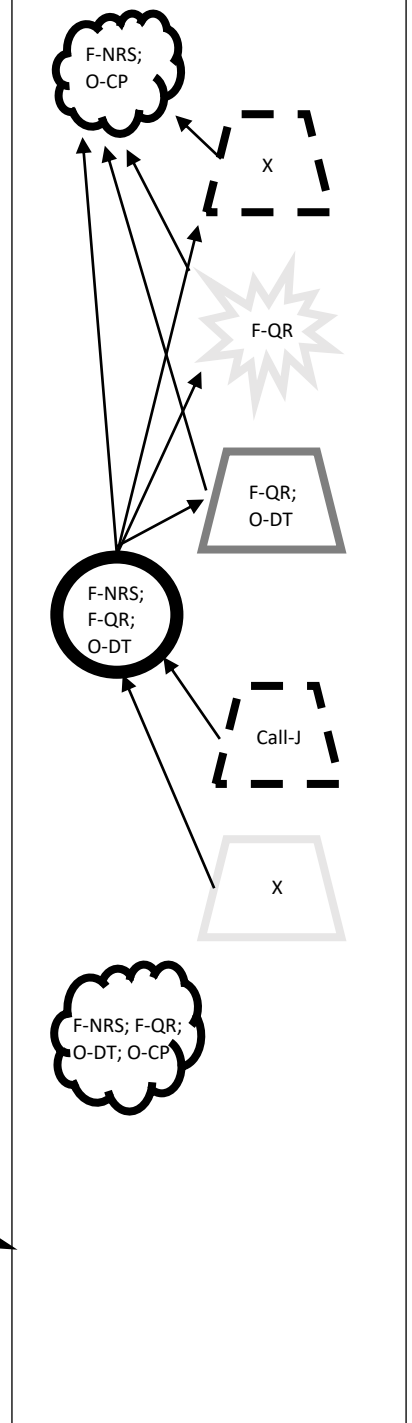


Social Analysis

Code	Micro Argument
C	Claim
D	Data (Symbolic, Graphical, Tabular, Contextual)
W	Warrant
B	Backing
Q	Qualifier
X	Nonargumentative

Shape	Macro Argument
Star	Counterargument
Circle	Integration
Trapezoid	Evaluation
Cloud	Argument





Code	Epistemic
F	Function (Geometric, Non-referential symbolic, Quantitative, Memorized Class)
D	Derivative (Increasing/Decreasing, Tangent Slope, Rate of Change)
O	Optimization (Derivative Tests, Rate of Change Analyses, Critical Points)
Code	**Justification - Macro**
J-Geo	Justify by appeal to geometry
J-Der	Justify. By Derivative
J-Ctxt	Justify by Context

Epistemic Analysis

Shape	Macro Argument
Star	Counterargument
Circle	Integration
Trapezoid	Evaluation
Cloud	Argument

Data Analysis

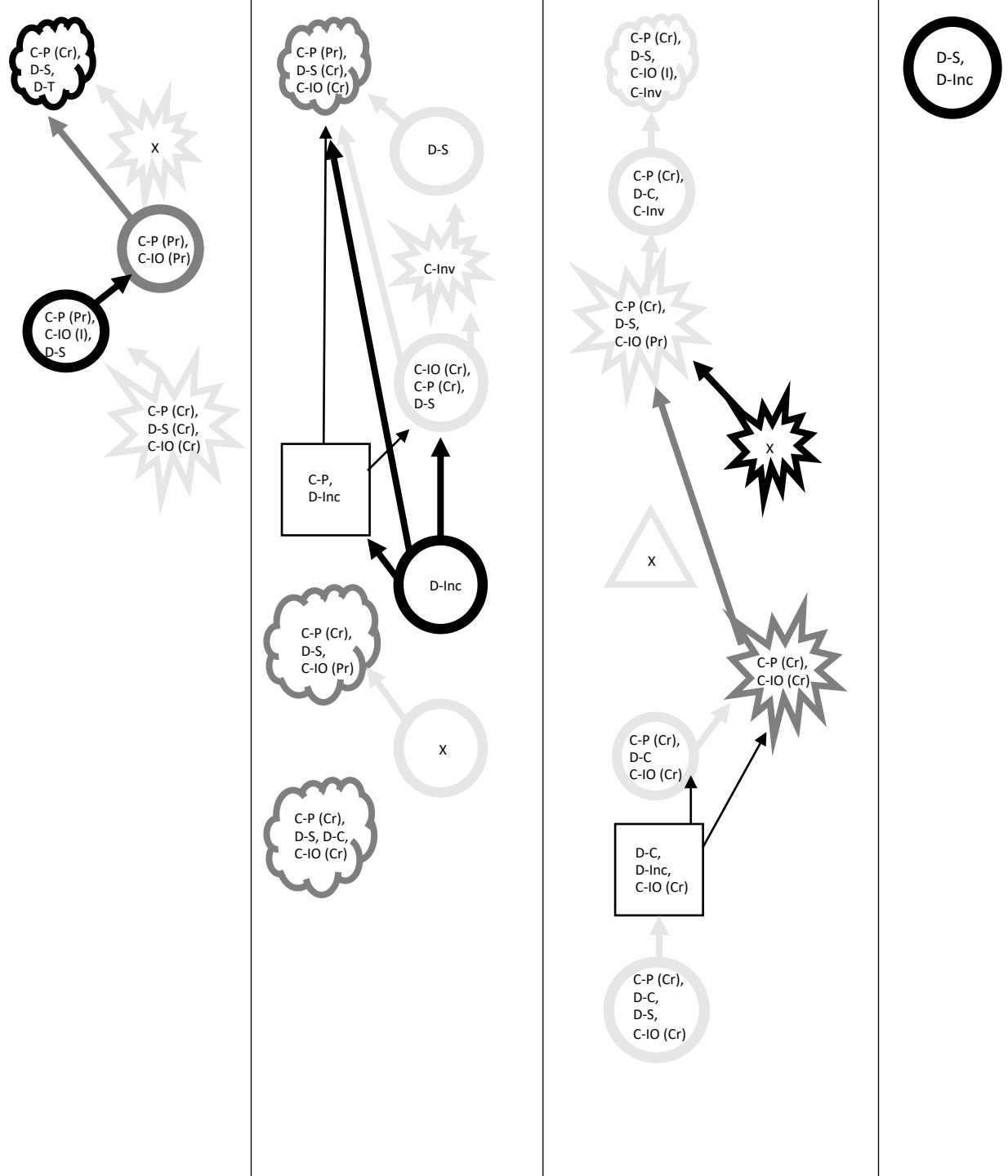
Multiple grain sizes – Micro and Macro for both Epistemic and Argumentative

1. Independent Argumentative coding
2. Compare codebooks and resolve conflicts
3. Independent Epistemic coding
4. Compare codebooks and resolve conflicts
5. Look for cross-dimensional Insights

Cross-Lens Insights

Clear links between epistemic progression and macro argumentative codes.

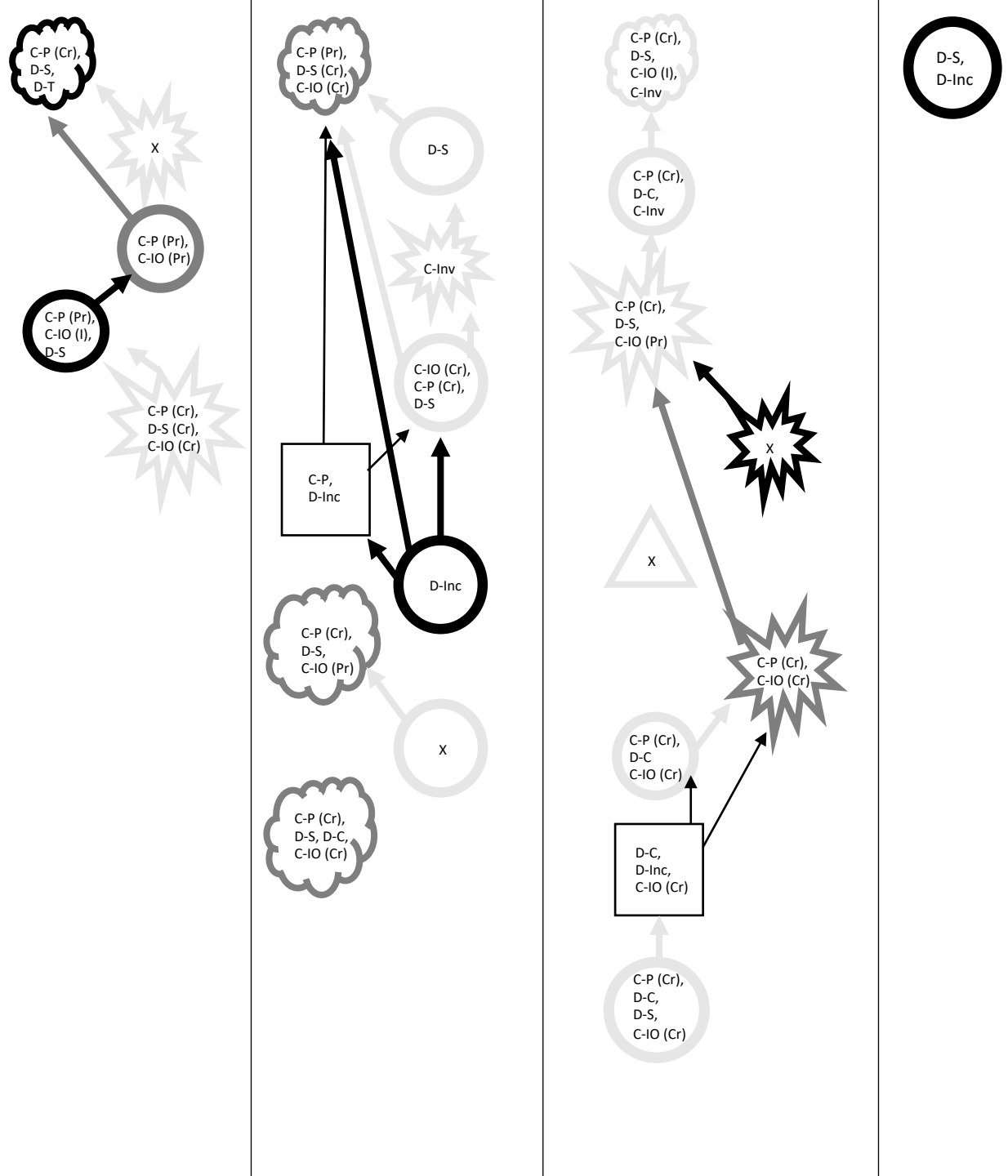
Introducing new epistemic codes makes room for integration or counterargument.

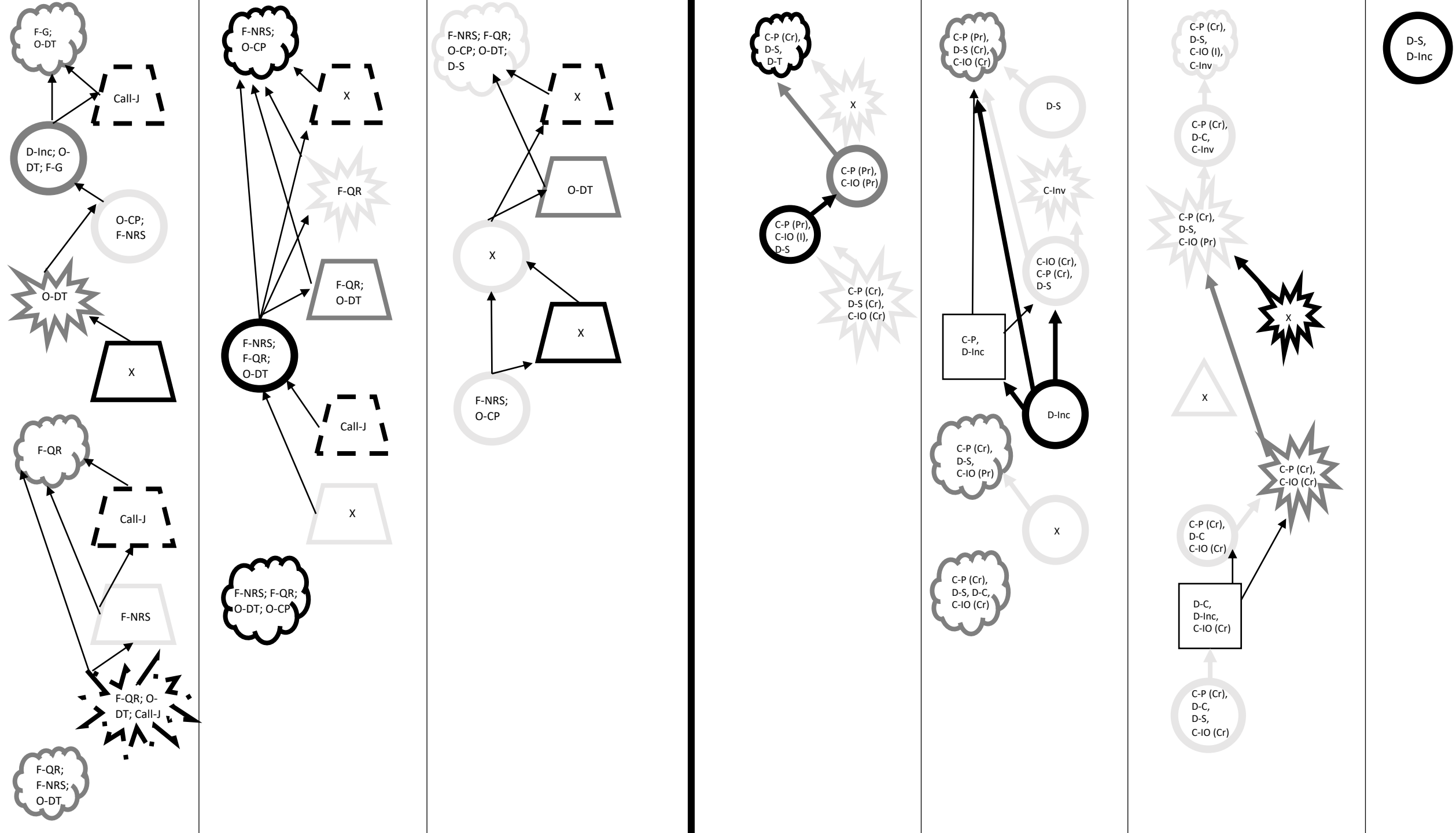


Cross-Lens Insights

Integration of epistemic codes point to otherwise invisible student interactions (reading others posts)

Under-diversified macro argumentative codes pairs with a dearth of epistemic codes.




F-G; O-DT
Call-J
D-Inc; O-DT; F-G
O-CP; F-NRS
O-DT
X
F-QR
Call-J
F-NRS
F-QR; O-DT; Call-J
F-QR; F-NRS; O-DT
F-NRS; O-CP
X
F-QR
F-QR; O-DT
F-NRS; F-QR; O-DT
Call-J
X
F-NRS; F-QR; O-DT; O-CP
F-NRS; F-QR; O-CP; O-DT; D-S
X
O-DT
X
X
F-NRS; O-CP
C-P (Cr), D-S, D-T
X
C-P (Pr), C-IO (Pr)
C-P (Pr), C-IO (I), D-S
C-P (Cr), D-S (Cr), C-IO (Cr)
C-P (Pr), D-S (Cr), C-IO (Cr)
D-S
C-Inv
C-IO (Cr), C-P (Cr), D-S
C-P, D-Inc
D-Inc
C-P (Cr), D-S, C-IO (Pr)
X
C-P (Cr), D-S, D-C, C-IO (Cr)
C-P (Cr), D-S, C-IO (I), C-Inv
C-P (Cr), D-C, C-Inv
C-P (Cr), D-S, C-IO (Pr)
X
X
C-P (Cr), C-IO (Cr)
C-P (Cr), D-C C-IO (Cr)
D-C, D-Inc, C-IO (Cr)
C-P (Cr), D-C, D-S, C-IO (Cr)
D-S, D-Inc

Future Directions

Attend to the role of the instructor, particularly when students can take similar social actions as the instructor.

Upscale the analysis to lend more descriptive and explanatory insights on a much larger sample

EMBRY-RIDDLE
Aeronautical University

Thank You!

chambd17@erau.edu

reedz@erau.edu