Creating diagnostic assessments: Automated distractor generation with integrity

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OUTLINE

Motivation for Automated Assessment Generation

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- Generating good answer choices is challenging.
- Develop assignments that let students reflect and learn from their exams *(formative assessments)*.

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Why the large discrepancy? Are students just really good at guessing? Are the free response questions that much harder than the multiple choice?

Example Exam Item (Multiple Choice)

12. Solve the equation.

$$\frac{8}{x-2}+\frac{8}{x+2}=3$$

A. $\left\{-\frac{3}{2},-6\right\}$

B. $\left\{\frac{2}{3}\right\}$

C. $\left\{-\frac{2}{3},6\right\}$

D. $\emptyset$

14. Solve the equation.

$$-5[-3x-3-6(x+1)]=-5x+2$$

A. $\left\{-\frac{43}{50}\right\}$

B. $\left\{-\frac{13}{50}\right\}$

C. $\left\{\frac{43}{3}\right\}$

D. $\left\{\frac{13}{3}\right\}$

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Questionable integrity may be to blame!

EXAMPLE EXAM (FREE RESPONSE)

18. When asked to solve the equation $4x^2 - 2x = 5$, a student submitted the following:

Line 1: $4\left(x^2 - \frac{1}{2}x + \frac{1}{4}\right) = 5 + \frac{1}{4}$

Line 2: $4\left(x - \frac{1}{4}\right)^2 = \frac{21}{4}$

Line 3: $2\left(x - \frac{1}{4}\right) = \frac{\sqrt{21}}{2}$

Line 4: $x - \frac{1}{4} = \frac{\sqrt{21}}{4}$

Line 5: $x = \frac{1}{4} + \frac{\sqrt{21}}{4}$

A. The student made an error in 2 of the 5 lines. state what the error is and why it is an error.

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Can this be written as a multiple choice item that captures student understanding?

Automated Item Generation

[Stem]

Solve the linear equation below.

[Problem]

$$\frac{-3x-6}{3} - \frac{-8x-8}{5} = \frac{7x+6}{2}$$

[Options]

A. $x = -\frac{40}{29}$ **[Distractor]**

B. $x = -\frac{34}{29}$ **[Solution]**

C. $x = -\frac{66}{29}$ **[Distractor]**

D. $x = -\frac{17}{10}$ **[Distractor]**

Figure 1: Illustration of AIG terms.

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Theoretically, automated item generation should be able to generate distractors using any of the methods above.

EXAMPLE

Dubinsky and Wilson (2013)

1. Suppose f and g are two functions. Find the compositions $f \circ g$ and $g \circ f$.
2. Suppose $h = f \circ g$ is the composition of two functions f and g. Given h and g, find f.
3. Suppose $h = f \circ g$ is the composition of two functions f and g. Given h and f, find g. (Dubinsky & Wilson, 2013, p. 97)

Example - Continued

In both the written instrument and the interviews, we asked students questions, some of which we considered to be difficult, about composition of functions. Our intention was to investigate the depth of their understanding of the function. We also felt that success in solving these problems was an indication of a process conception of function and in some cases, an indication of a process conception that was strong enough so that it could be reversed in the mind of a participant in order to solve a difficult composition problem (pgs. 96-97).

AIG Example - Question Type 1

Specific: Suppose $f(x) = (x+1)^2$ and $g(x) = \frac{1}{3}x^2$ are two functions. Find the composition $(f \circ g)(x)$ at the point $x = 5$.

A. $\frac{784}{9}$

B. 300

C. $\frac{1296}{3}$

Generalized: Suppose $f(x) = (x+c)^2$ and $g(x) = \frac{b_1}{b_2}x^2$ are two functions. Find the composition $(f \circ g)(x)$ at the point $x = a$.

A. $f(g(a)) = \left(\frac{b_1}{b_2}a + c\right)^2$

B. $(f \cdot g)(a) = \frac{b_1}{b_2}a^2\,(a+c)^2$

C. $g(f(a)) = \frac{b_1}{b_2}\,(a+c)^4$

Figure 2: Dynamically generating a procedural composition of functions question.

AIG Example - Question Type 2

Specific: Given **only** the information in the following table, find $f(2)$ *(if possible)*.

x	$h(x)$	$g(x)$
2	1	-3
-3	4	1
-2	0	2

A. $f(2) = 4$ $[h(g(2))]$

B. $f(2) = 0$ $[g(x) = 2 \rightarrow h(x)]$

C. $f(2) = 1$ $[h(x) = 2 \rightarrow g(x)]$

D. It is not possible to find $f(2)$ based only on the information in the table.

General: Given **only** the information in the following table, find $f(a_1)$ *(if possible)*.

x	$h(x)$	$g(x)$
a_1	c_1	b_2
b_2	b_3	c_1
a_2	a_3	a_1

A. $f(a_1) = b_3$ $[h(g(a_1))]$

B. $f(a_1) = a_3$ $[g(x) = a_1 \rightarrow h(x)]$

C. $f(a_1) = c_1$ $[h(x) = a_1 \rightarrow g(x)]$

D. It is not possible to find $f(a_1)$ based only on the information in the table.

Conceptions in Example

General: Given **only** the information in the following table, find $f(a_1)$ *(if possible)*.

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a_2	a_3	a_1

A. $f(a_1) = b_3$ $[h(g(a_1))]$

B. $f(a_1) = a_3$ $[g(x) = a_1 \to h(x)]$

C. $f(a_1) = c_1$ $[h(x) = a_1 \to g(x)]$

D. It is not possible to find $f(a_1)$ based only on the information in the table.

A. Does not recognize need for reversal (Action).

B. Recognizes need for reversal and order of composition (Successful Process).

C. Recognizes need for reversal but incorrect order of composition (Unsuccessful Process).

D. Views a function as a single algebraic formula (PreAction).

A Typical Example: Solving Linear Equations

Solve the linear equation below.

$$\frac{a_1 x + b_1}{d_1} - \frac{a_2 x + b_2}{d_2} = \frac{a_3 x + b_3}{d_3}$$

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How do we manipulate the original problem to create *distractor solutions*?

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"Nearby Problem":

$$\frac{a_1 x + b_1}{d_1} - \frac{a_2 x - b_2}{d_2} = \frac{a_3 x + b_3}{d_3}.$$

We will call the solution to this problem x_1^d.

A Typical Example: Solving Linear Equations

Issue: Student neglects to divide b_i by d_i.

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We will call the solution to this problem x_3^d.

By solving these "nearby" problems, we are able to generate *realistic* distractor solutions from common student mistakes during problem solving.

A Typical Example: Solving Linear Equations

Returning to our original problem:

1. Solve the linear equation below.

$$\frac{6x-7}{2} - \frac{3x-4}{3} = \frac{4x+8}{5}$$

A. $x = \frac{113}{36} \to x^*$

B. $x = \frac{119}{36} \to x_1^d$

C. $x = \frac{55}{6} \to x_2^d$

D. $x = -\frac{113}{30} \to x_3^d$

E. There are no Real solutions.

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5. Use these errors to generate “nearby problems.”

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4. Determine common errors students make when solving this type of problem.
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6. Algorithmically solve these “nearby problems.”

QUESTIONABLE INTEGRITY

1. Solve the linear equation below.

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How do we protect the integrity of this question?

INTERVAL MASKS

We use intervals to mask the numeric values to a solution. These intervals should:

- Not give clues as to the specific values being disguised;
- Have minimal overlap with each other; and
- Be “easy enough" to read.

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The method I use is included in a public GitHub repository.

NEW-LOOK EXAMS

10. Solve the linear equation below. Then, choose the interval that con

$$\frac{-6x+7}{5} - \frac{-8x-3}{4} = \frac{5x-6}{2}$$

$x = \boxed{}$

A. $x \in [2.13, 2.65]$

B. $x \in [1.7, 1.92]$

C. $x \in [9.28, 9.63]$

D. $x \in [2.93, 3.37]$

E. There are no Real solutions.

Auto-generated Key

10. Solve the linear equation below. Then, choose the interval that contains the solution.

$$\frac{-6x+7}{5} - \frac{-8x-3}{4} = \frac{5x-6}{2}$$

The solution is 3.029

A. $x \in [2.13, 2.65]$

Corresponds to not distributing the negative correctly for the second fraction.

B. $x \in [1.7, 1.92]$

Corresponds to dividing only the second term for each fraction (rather than multiplying to remove the fractions).

2

C. $x \in [9.28, 9.63]$

Corresponds to dividing only the first term for each fraction (rather than multiplying to remove the fractions).

D. $x \in [2.93, 3.37]$

* Correct option.

E. There are no Real solutions.

Corresponds to students thinking a fraction means there is no solution to the equation.

General Comments: If you are having trouble with this problem, try to remove a fraction at a time by multiplying each term by the denominator.

On to the GitHub and questions from the audience!

https://github.com/archdoc/ufMathAssessmentAIG