Objective 3 - Lowest-Degree Polynomial

Construct a lowest-degree polynomial given its zeros.

Link to section in online textbook

You can print out these notes to follow along with the video below and keep notes to organize your thoughts.

YouTube link: `https://www.youtube.com/watch?v=g3cpbm3fQjw`

Now practice constructing polynomials from zeros with the questions below.

Question 1 *Construct the lowest-degree polynomial given the zeros below.*

$$4, -4, -3$$

$f(x) = \boxed{1}x^3 + \boxed{3}x^2 + \boxed{-16}x + \boxed{-48}$

Question 2 *Construct the lowest-degree polynomial given the zeros below.*

$$-4, 5, -5$$

$f(x) = \boxed{1}x^3 + \boxed{4}x^2 + \boxed{-25}x + \boxed{-100}$

Question 3 *Construct the lowest-degree polynomial given the zeros below.*

$$\frac{4}{3}, \frac{5}{2}, 5$$

$f(x) = \boxed{6}x^3 + \boxed{-53}x^2 + \boxed{135}x + \boxed{-100}$

Hint: *Remember back to what it meant to be in Standard Form for linear functions: we did not have any fractions as coefficients. How would we rewrite a factor that has a fraction in it, like* $\left(x - \frac{3}{4}\right)$*?*

Learning outcomes:
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Question 4 *Construct the lowest-degree polynomial given the zeros below.*

$$-5, 4, -\frac{4}{3}$$

$f(x) = \boxed{3}x^3 + \boxed{7}x^2 + \boxed{-56}x + \boxed{-80}$

Hint: *Remember back to what it meant to be in Standard Form for linear functions: we did not have any fractions as coefficients. How would we rewrite a factor that has a fraction in it, like $\left(x - \frac{3}{4}\right)$?*

We focused on building polynomials with integer and rational zeros. What would we do if we had other types of zeros, like irrational or complex?

Theorem 1. *Complex and Irrational roots for polynomials come in "_________" pairs.*

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

tells us something about the types of zeros a quadratic function may have:

- *2 different, rational zeros*
  - *e.g.,* $\frac{1}{4}$ *and* -3 *for the polynomial* $4x^2 + 11x - 3$
- *2 copies of a rational zero*
  - *e.g.,* $\frac{1}{3}$ *and* $\frac{1}{3}$ *for the polynomial* $9x^2 - 2x + 1$
- *2 different, irrational zeros*
  - *e.g.,* $\frac{1}{2} - \sqrt{2}$ *and* $\frac{1}{2} + \sqrt{2}$ *for the polynomial* $4x^2 - 4x - 7$
- *2 different, complex zeros*
  - *e.g.,* $\frac{1}{4} - 3i$ *and* $\frac{1}{4} + 3i$ *for the polynomial* $16x^2 - 8x + 145$

Let's focus on the irrational and complex zeros. These occur when the number under the square root is either (1) not a perfect square or (2) negative. Let's *look closer at the form these zeros take by looking at the subgroups the numbers belong to.*

Case 1: $b^2 - 4ac$ *is positive and is* ***not*** *a perfect square.*

$$x = \frac{\textit{integer}}{\textit{integer}} \pm \frac{\textit{irrational}}{\textit{integer}}$$

$$x = \textit{rational} \pm \textit{irrational}$$

Case 2: $b^2 - 4ac$ *is negative.*

$$x = \frac{\textit{integer}}{\textit{integer}} \pm \frac{\textit{complex}}{\textit{integer}}$$

$$x = \textit{rational} \pm \textit{complex}$$

Question 5 *What word describes the relationship between the zeros* $x = \textit{rational} - \textit{complex}$ *and* $x = \textit{rational} + \textit{complex}$*?*

They are $\boxed{\textit{conjugate}}$ *pairs!*

Hint: *What are* $3 + 4i$ *and* $3 - 4i$ *to each other?*

We use this theorem to construct polynomials with irrational and/or complex roots.

Question 6 *Construct the lowest-degree polynomial given the zeros below.*

$$\sqrt{5}, -\frac{4}{3}$$

$f(x) = \boxed{3}x^3 + \boxed{4}x^2 + \boxed{-15}x + \boxed{-20}$

Hint: *If* $\sqrt{5}$ *is a zero to the polynomial, then* $-\sqrt{5}$ *is also! Multiply* $(x - \sqrt{5})(x + \sqrt{5})$ *first, then use the third zero to finish building the polynomial.*

Question 7 *Construct the lowest-degree polynomial given the zeros below.*

$$2 + \sqrt{7}, \frac{3}{2}$$

$f(x) = \boxed{2}x^3 + \boxed{-11}x^2 + \boxed{6}x + \boxed{9}$

Hint: *Be careful with how you set up this problem. Again, multiply the conjugate factors together first. If you did this right, there should be no radicals left!*

Question 8 *Construct the lowest-degree polynomial given the zeros below.*

$$4i, -\frac{1}{7}$$

$f(x) = \boxed{7}x^3 + \boxed{1}x^2 + \boxed{112}x + \boxed{16}$

Question 9 *Construct the lowest-degree polynomial given the zeros below.*

$$2+6i, \frac{4}{3}$$

$f(x) = \boxed{3}x^3 + \boxed{-16}x^2 + \boxed{136}x + \boxed{-160}$

Hint: *First, you want to set up the factors:*

$$(x - (2+6i))(x - (2-6i))(3x-4)$$

Now, we want to be careful how we multiply out.

$$(x^2 - (2+6i)\,x - (2-6i)\,x + (2+6i)(2-6i))\,(3x-4)$$

$$(x^2 - 4x + (2^2+6^2))(3x-4)$$

Distribute this last part out carefully and you will have completed the problem.
