Objective 2 - Possible Rational Roots

Determine the possible rational roots of a polynomial.

Link to section in online textbook.

First, watch this video to learn about the rational root theorem.

Theorem 1. *Rational Root Theorem: The possible rational roots of the polynomial* $a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x + a_0$ *are of the form* $\pm \frac{p}{q}$*, where* p *is a divisor of* a_0 *and* q *is a divisor of* a_n*.*

Question 1 *This question will walk you through how to list the possible rational roots for a given polynomial.*

$f(x) = 6x^3 - 17x^2 + 6x + 8$

First, we identify a_0 *and* a_n*:*

$a_0 = \boxed{8}$

$a_n = \boxed{6}$

Next, we list the divisors of each number. We'll list them smallest to largest.

Divisors of a_0 : $\boxed{1}, \boxed{2}, \boxed{4}, \boxed{8}$

Feedback(`attempt`): *Don't forget 1 and the number itself!*

Divisors of a_n : $\boxed{1}, \boxed{2}, \boxed{3}, \boxed{6}$

Now, we list every possible combination. This example illustrates a large number of possible rational roots.

$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6},$

$\pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{2}{3}, \pm\frac{2}{6},$

$\pm\frac{4}{1}, \pm\frac{4}{2}, \pm\frac{4}{3}, \pm\frac{4}{6},$

$\pm\frac{8}{1}, \pm\frac{8}{2}, \pm\frac{8}{3}, \pm\frac{8}{6}$

This may look like a lot (and it is!) but we also have a lot of copies. Fill in the reduced forms for each combination above.

Learning outcomes:
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$\pm\boxed{1}, \pm\boxed{1/2}, \pm\boxed{1/3}, \pm\boxed{1/6},$

$\pm\boxed{2}, \pm\boxed{1}, \pm\boxed{2/3}, \pm\boxed{1/3},$

$\pm\boxed{4}, \pm\boxed{2}, \pm\boxed{4/3}, \pm\boxed{2/3},$

$\pm\boxed{8}, \pm\boxed{4}, \pm\boxed{8/3}, \pm\boxed{4/3}$

Our final answer is listing each number once:

$\pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, 2, \pm\frac{2}{3}, \pm 4, \pm\frac{4}{3}, \pm 8, \pm\frac{8}{3}$

It is important to keep in mind this is a list of all possible rational roots. It is usually far more than the actual number of roots of the polynomial. It could even miss some of our roots as roots can be irrational! We will illustrate that issue with a few examples below.

Question 2 *List the possible rational roots of the polynomial below. Then, find the actual rational roots by factoring or using the Quadratic Formula.*

$f(x) = x^2 - 21$

Possible rational roots: $\pm\boxed{1}, \pm\boxed{3}, \pm\boxed{7}, \pm\boxed{21}$

Feedback(*`attempt`*): *List possible roots in order from smallest to largest.*

Smaller root: $\boxed{-4.58257569495584}$

Larger root: $\boxed{4.58257569495584}$

Feedback(*`correct`*): *Here's an example where the rational root theorem doesn't help us. Both of the zeros of this polynomial are irrational!*

Question 3 *List the possible rational roots of the polynomial below. Then, find the actual rational roots by factoring or using the Quadratic Formula.*

$f(x) = x^2 + 49$

Possible rational roots: $\pm\boxed{1}, \pm\boxed{7}, \pm\boxed{49}$

Feedback(*`attempt`*): *List possible roots in order from smallest to largest.*

Smaller root: $\boxed{-7i}$

Larger root: $\boxed{7i}$

Feedback(*`correct`*): *Here's another example where the rational root theorem doesn't help us. Both of the zeros of this polynomial are complex!*

In the next section, you will get plenty of practice finding the possible rational roots. Let's get to it.