Objective 3 - Construct Linear Model

Construct a model equation for the real-life situation.

Scenarios we normally use linear functions to model are:

Finance Cost to produce, Utility Bill, Depreciation of Value;

Motion Relative distance of two objects, Using round-trip times to calculate distance,

Chemistry Mixing two different concentrations of solutions;

Statistics Line of best fit.

After you complete each of the questions below, see if you can put together a "general form" for the linear model you built. You can print out these notes to follow along with the video below and keep notes to organize your thoughts.

YouTube link: `https://www.youtube.com/watch?v=ylp6rx1N4P0`

Exercise 1 *A company sells doughnuts. They incur a fixed cost of* $13000 *for rent, insurance, and other expenses. It costs* $0.1 *to produce each doughnut. The company sells each doughnut for* $0.2.

Part A. *Construct a linear model that describes their total costs, C, as a function of the number of doughnuts, x, they produce.*

$C(x) = \boxed{0.1\,x + 13000}$

Part B. *Construct a linear model that describes their total profits, P, as a function of the number of doughnuts, x, they produce.*

$P(x) = \boxed{0.2\,x}$

Part C. *Construct a linear model that describes their total revenue, R, as a function of the number of doughnuts, x, they produce.*

$R(x) = \boxed{0.1\,x - 13000}$

Exercise 2 *Aubrey is a college student going into her first year at UF. She will receive Bright Futures, which covers her tuition plus a $300 educational expense each Fall and Spring semester. Before college, Aubrey saved up* $8000.

Learning outcomes:
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She knows she will need to pay \$1000 in rent a month, \$50 for food a week, and \$40 in other weekly expenses.

Part A. *Construct a linear model that describes her total costs, C as a function of the number of months, x, she is at UF **during Fall semester**.*

$C(x) = \boxed{1360\,x}$

Part B. *Construct a linear model that describes her total income, I, as a function of the number of months, x, she is at UF **during Fall semester**.*

$I(x) = \boxed{8300}$

Part C. *Construct a linear model that describes their total budget, B, as a function of the number of months, x, she is at UF **during Fall semester**.*

$B(x) = \boxed{-1360\,x + 8300}$

Try to write down notes on how to solve the first two questions *in general.*

Exercise 3 *Two UFPD are patrolling the campus on foot. To cover more ground, they split up and begin walking in different directions. Office A is walking at 4 mph while Office B is walking at 4 mph.*

Part A. *Construct a linear model that describes Officer A's distance from their starting point, D, as a function of minutes, m, that have passed.*

$$D(m) = \boxed{\frac{1}{15}\,m}$$

Hint: $\textit{Speed} = \dfrac{\textit{distance}}{\textit{time}}$

Can you re-solve this for distance?

Part B. *Construct a linear model that describes their total distance from each other, T_1, as a function of minutes, m, that have passed if they were walking in exactly opposite directions (e.g., North/South).*

$$T_1(m) = \boxed{\frac{2}{15}\,m}$$

Part C. *Construct a linear model that describes their total distance from each other, T_2, as a function of minutes, m, that have passed if they were walking in exactly 90 degrees from each other (e.g., North/East).*

$$T_2(m) = \boxed{\frac{1}{15}\,\sqrt{2}m}$$

Exact value needed for the coefficient! DO NOT ROUND.

Figure 1: Training path.

Hint: *For Part C, draw a picture and think about how the Pythagorean Theorem could be used. Remember: these are all linear models! So if your model has a non-linear **variable**, is there a reason we would ignore part of the domain?*

Exercise 4 *A bicyclist is training for a race on a hilly path. Their bike keeps track of their speed at any time, but not the distance traveled. Their speed traveling up a hill is 4mph, 8mph when traveling down a hill, and 5mph when traveling along a flat portion.*

Hint: *Distance is equal to rate times time.*

Part A. *Construct linear models that describe their distance, D in miles, on a particular portion of the path in terms of the time, t in hours, spent on that part of the path.*

$D_{up}(t) = \boxed{4\,t}$

$D_{down}(t) = \boxed{8\,t}$

$D_{flat}(t) = \boxed{5\,t}$

Part B. *Construct linear models that describe their time, t in hours, on a particular portion of the path in terms of the length, D in miles, of that part of the path.*

$t_{up}(D) = \boxed{\frac{1}{4}\,D}$

$t_{down}(D) = \boxed{\frac{1}{8}\,D}$

$t_{flat}(D) = \boxed{\frac{1}{5}\,D}$

Part C. *Construct a linear model that describes the total distance of the path, D, in terms of the time spent on a particular path if we knew that the time spent on each path was equal.*

$D(t) = \boxed{17\,t}$

Part D. *Construct a linear model that describes the total time T spent on the path in terms of the distance of a particular part of the path if we knew that all parts of the path are equal length.*

$T(D) = \boxed{\frac{23}{40}D}$

Exercise 5 *Kappa Delta is hosting an all-you-can-eat pancake fundraiser to support the prevention of child abuse. Adult (18+) tickets are \$10 and teen (10-17) tickets are \$4. Children under 10 are let in without a ticket. The ticket-sellers only kept track of the total number of tickets sold, 97, and total revenue, \$640.*

Part A. *Construct a linear model that describes the total number of adult tickets, y, sold in terms of the number of teen tickets, x, sold.*

$y = \boxed{-x + 97}$

Part B. *Construct a linear model that describes the revenue made from selling many adult tickets, R_a, in terms of the number of teen tickets, x, sold.*

$R_a = \boxed{-10\,x + 970}$

Part C. *Construct a linear model that describes the total revenue made, R_T, in terms of the number of teen tickets, x, sold.*

$R_T = \boxed{-6\,x + 970}$

Hint: *Part A: Is there way to build an equation that relates adult tickets, teen tickets, and total tickets? Solving this equation for adult tickets would give you the linear model.*

Part B: It may be easier to first build this model in terms of y, then use your answer from part A.

Part C: Think about how to model total revenue, then use your answer in Part B to make part of the model.

Exercise 6 *Chemists commonly create a solution by mixing two products of differing concentrations together. For example, a chemist could have large amounts of a 15% acid solution and a 40% acid solution, but need a 5 liter 18% solution.*

Part A. *Construct a linear model that describes the volume of the 40% acid solution, v_{40}, in terms of the volume of the 15% acid solution, v.*

$v_{40} = \boxed{-v + 5}$

Part B. *Construct a linear model that describes the amount of acid in a 15% acid solution, A_{15}, in terms of the volume of the 15% acid solution, v.*

$A_{15} = \boxed{0.150000000000000\, v}$

Part C. *Construct a linear model that describes the amount of acid in a* 40% *acid solution,* A_{40}*, in terms of the volume of the* 15% *acid solution,* v*.*

$A_{40} = \boxed{-0.400000000000000\, v + 2.00000000000000}$

Part D. *Construct a linear model that describes the amount of acid in a* 18% *acid solution,* A_{18}*, in terms of the volume of the* 15% *acid solution,* v*.*

$A_{18} = \boxed{-0.250000000000000\, v + 2.00000000000000}$

Hint: *Parts A-D: Think about what you did in the last problem. How can we use that same structure in this new setting?*