Objective 3 - Evaluating Limits

Interpret the notation for limits.

Link to section in online textbook.

Video for evaluating a limit.

Now that we have learned about left- and right-hand limits, we can evaluate the limit of a function at a point.

Theorem 1. *Evaluating the limit of a function at a point $x = a$:*

$$\lim_{x\to a}(f(x)) = L$$
$$\textit{if and only if}$$
$$\lim_{x\to a^-}(f(x)) = L = \lim_{x\to a^+}(f(x))$$

*Note: The limit **exists** if L is a Real number. The limit can be equal to ∞ or $-\infty$, but we would not say that it exists. If the left- and right-hand limits do not agree, we say **the limit does not exist** (or DNE for short).*

This objective will allow you to practice evaluating the left- and right-hand limits to determine if the limit at a point exists. **This would be where you want to practice before the exam.**

Answers are either a Real number, ∞, $-\infty$, or DNE.

Question 1 $\lim_{x\to-\infty} f(x) = \boxed{-\infty}$

$\lim_{x\to-1} g(x) = \boxed{3}$

$\lim_{x\to\infty} f(x) = \boxed{+\infty}$

Question 2 $\lim_{x\to-\infty} f(x) = \boxed{-\infty}$

$\lim_{x\to-2} f(x) = \boxed{3}$

$\lim_{x\to1} f(x) = \boxed{DNE}$

$\lim_{x\to3} f(x) = \boxed{-\infty}$

Learning outcomes:
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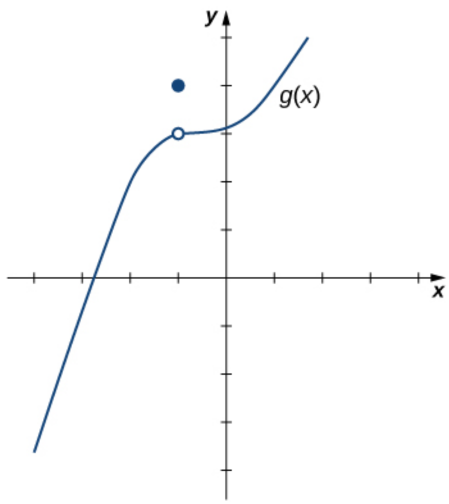

Figure 1: Function with a hole at $x = -1$.

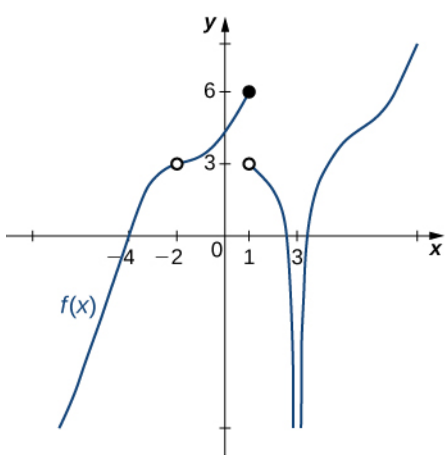

Figure 2: Piecewise function to evaluate.

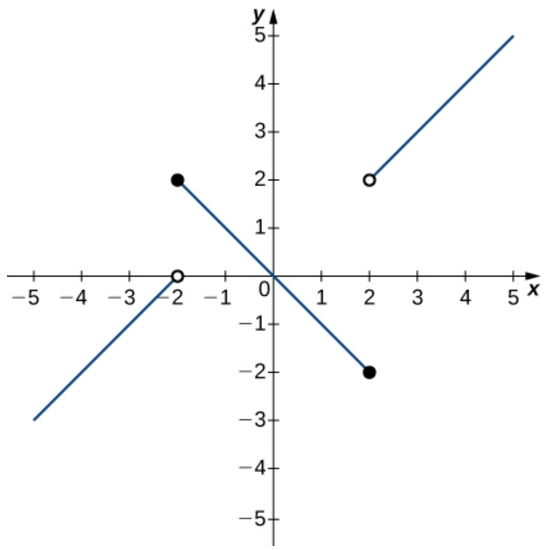

Figure 3: Piecewise function to evaluate.

$\lim_{x\to\infty} f(x) = \boxed{+\infty}$

Question 3 $\lim_{x\to-\infty} f(x) = \boxed{-\infty}$

$\lim_{x\to-2} f(x) = \boxed{DNE}$

$\lim_{x\to0} f(x) = \boxed{0}$

$\lim_{x\to2} f(x) = \boxed{DNE}$

$\lim_{x\to\infty} f(x) = \boxed{+\infty}$

Question 4 $\lim_{x\to-\infty} f(x) = \boxed{+\infty}$

$\lim_{x\to-8} f(x) = \boxed{-6}$

$\lim_{x\to-2} f(x) = \boxed{DNE}$

$\lim_{x\to6} f(x) = \boxed{DNE}$

$\lim_{x\to10} f(x) = \boxed{0}$

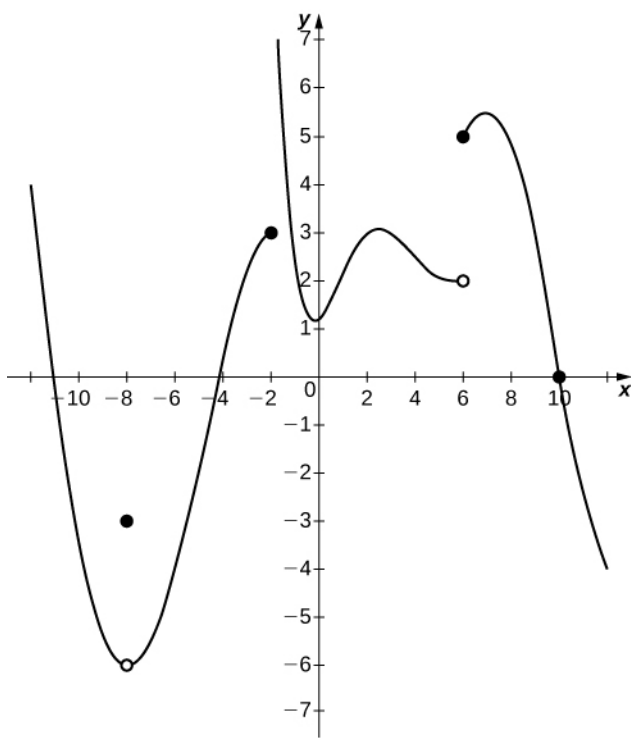

Figure 4: Piecewise function to evaluate.

$\lim_{x \to \infty} f(x) = \boxed{-\infty}$

Question 5 $f(x) = \dfrac{8\,(2\,x+1)(x+1)}{16\,x+8}$

$\lim_{x \to -\infty} f(x) = \boxed{-\infty}$

$\lim_{x \to -\frac{1}{2}} f(x) = \boxed{\dfrac{1}{2}}$

$\lim_{x \to -1} f(x) = \boxed{0}$

$\lim_{x \to \infty} f(x) = \boxed{+\infty}$

Question 6 $f(x) = \dfrac{4\,x-2}{-6\,(3\,x+2)(2\,x-1)}$

$\lim_{x \to -\infty} f(x) = \boxed{0}$

$\lim_{x \to -\frac{2}{3}} f(x) = \boxed{DNE}$

$\lim_{x \to \frac{1}{2}} f(x) = \boxed{-\dfrac{2}{21}}$

$\lim_{x \to \infty} f(x) = \boxed{0}$

Question 7 $\lim_{x \to 0} (x+1)^{\left(\frac{1}{x}\right)} = \boxed{e}$

Hint: *We can't plug in the exact value, so we will need to plug in values very near* 0.

Question 8 $\lim_{x \to 10} \dfrac{\sqrt{x-1}-3}{x-10} = \boxed{\dfrac{3}{2}}$

Hint: *We can't plug in the exact value, so we will need to plug in values very near* 10.
