Objective 1 - Divide with Synthetic Division

Divide two polynomials using Synthetic Division.

Link to section in online textbook.

First, watch this video to learn how to divide polynomials using synthetic division. This process can be used to divide polynomials by *linear factors*. To divide by non-linear factor *(e.g.,* $x^2 + 1$*)*, you would need to perform long division. However, this is rarely necessary to solve an equation or graph a function.

Question 1 *First, we start with some questions to learn the terminology for this section.*

If we were to divide $\frac{9}{2}$ *using long division, we would get* 4 *with a remainder of* 1*. Write what this looks like as an equation, then practice the terminology for each part of the equation.*

$$\frac{9}{2} = \boxed{4} + \frac{\boxed{1}}{2}$$

$$\frac{\boxed{\textit{dividend}}}{\boxed{\textit{divisor}}} = \boxed{\textit{quotient}} + \frac{\boxed{\textit{remainder}}}{\boxed{\textit{divisor}}}$$

Feedback(*`attempt`*): *Terminology: "quotient", "divisor", "dividend", "remainder"*

Some terms will be used more than once.

Hint: *You likely know some of these words (remainder, quotient) but not others. You also know "divisor" though! Try to think of questions that normally ask for "divisors", like "What is the GCD (greatest common divisor) of 12 and 9?" In this question, is it asking for something that is being divided or something that is dividing other numbers?*

Let's see how we can apply this same idea to polynomials.

Learning outcomes:
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Question 2 *Complete the division below. Then, rewrite the equation to remove the fractions.*

$$\frac{6\,x^3 - 20\,x^2 - 14\,x + 57}{x - 3} = \boxed{6\,x^2 - 2\,x - 20} + \frac{\boxed{-3}}{x - 3}$$

$$6\,x^3 - 20\,x^2 - 14\,x + 57 = \boxed{6\,x^2 - 2\,x - 20}(x - 3) + \boxed{-3}$$

Feedback(`correct`): *Great! Based on the second equation, you can see why we care about Synthetic Division: it gives us a new way to factor polynomials! If only there were no remainder...*

Now watch this video to see how to deal with linear terms that are not in the form $x - a$. The process is *nearly* the same, except we need to divide our quotient by the a in the factor $ax - b$.

Question 3 *Complete the division below. Then, rewrite the equation to remove the fractions. This time after completing the synthetic division, you will need to factor our the a term.*

$$\frac{48\,x^3 - 88\,x^2 - 13\,x + 64}{3\,x - 4} = \boxed{16\,x^2 - 8\,x - 15} + \frac{\boxed{4}}{3\,x - 4}$$

$$48\,x^3 - 88\,x^2 - 13\,x + 64 = \boxed{16\,x^2 - 8\,x - 15}(3\,x - 4) + \boxed{4}$$

Hint: *You likely got the remainder correct but not the quotient. To see why, let's focus on* $\frac{48\,x^3}{3\,x}$. *This should be the first term of your quotient. Is there a relationship between what the first term should be and what your quotient is? Maybe check the instructions/video just before this question...*

Feedback(`correct`): *Right again! Synthetic division can be used for linear terms of the form $ax + b$. To do so, we solve the factor equal to zero and synthetically divide by this zero. Then we divide the quotient by a since synthetic division only focuses on the zero and not the coefficients of the divisor.*

Question 4 *Complete the division below. Then, rewrite the equation to remove the fractions.*

$$\frac{10\,x^3 - 70\,x - 55}{x-3} = \boxed{10\,x^2 + 30\,x + 20} + \frac{\boxed{5}}{x-3}$$

$$10\,x^3 - 70\,x - 55 = \boxed{10\,x^2 + 30\,x + 20}(x-3) + \boxed{5}$$

Feedback(*`attempt`*): *We need to make sure there is a coefficient for each term in the divisor. Did you put a 0 if we were missing a term?*

Feedback(*`correct`*): *Great! Synthetic division relies on positions to remove terms and strip down the division to numbers. Not including a 0 for a missing term would be the same as thinking 111 is 1101.*

Question 5 *Complete the division below. Then, rewrite the equation to remove the fractions.*

$$\frac{32\,x^3 - 96\,x - 59}{4\,x+4} = \boxed{8\,x^2 - 8\,x - 16} + \frac{\boxed{5}}{4\,x+4}$$

$$32\,x^3 - 96\,x - 59 = \boxed{8\,x^2 - 8\,x - 16}(4\,x+4) + \boxed{5}$$

Hint: *You likely got the remainder correct but not the quotient. To see why, let's focus on* $\frac{32\,x^3}{4\,x}$. *This should be the first term of your quotient. Is there a relationship between what the first term should be and what your quotient is? Maybe check the instructions/video just before question three...*

Feedback(*`correct`*): *Perfect! You seem to understand how to synthetically divide under any conditions. We'll use a lot of synthetic division in the next two objectives.*
