Objective 3 - Construct Log/Exp Models

Link to textbook: Construct a model equation for the real-life situation.

Videos:

- pH
- Age using Half-Life
- Doubling time growth pt. 1
- Doubling time growth pt. 2
- Radioactive Decay and Law of Cooling

Question 1 *A population of bacteria* `quadruples` *every hours. If the culture started with* 400*, write the equation that models the bacteria population after* t *hours.*

$P(t) = \boxed{400}\,\boxed{4}^{\boxed{t}}$

Question 2 *There is initially* 976 *grams of element* X*. The half-life of element* X *is* 895520 *years. Describe the amount of element* X *remaining as a function of time,* t*, in years. Use* e *as the base of your exponential model and exact values.*

$X(t) = \boxed{976}\,e^{\boxed{-\frac{1}{895520}\log(2)\,t}}$

Hint: *The exponent's coefficient is not* $\frac{1}{895520}$ *nor is it* $\frac{-1}{895520}$*. To find the correct coefficient, take the base exponential equation current amount = initial amount* e^{ct}*, use what you know about the half-life time and amount, then solve the equation for* c*. If you are having trouble, refer back to the objective on "Solving exponential equations". Do not use a calculator - use exact values to get a correct answer.*

Learning outcomes:
Author(s): Darryl Chamberlain Jr.

Question 3 *The half-life of carbon-14 is 5,730 years.*

Part A. *Describe the amount of carbon-14 remaining after t years. The initial amount of carbon-14, C_0, is already included below.*

$$C = C_0 * \boxed{e}^{\boxed{-0.000120968094338559t}}$$

Part B. *Solve the equation above for t written in terms of the ratio of carbon-14 remaining, $r = \dfrac{C}{C_0}$.*

$$t = \boxed{-8266.64258429376}\ln(\boxed{r})$$

Part C. *The equation above is used to carbon-date objects. To solidify this idea, use the model in Part B. to solve the following problem.*

A bone fragment is found that contains 65% of its original carbon-14. To the nearest year, how old is the bone?

$\boxed{3561}$ *years old*

Hint: ***Part A.*** *The exponent's coefficient is not $\dfrac{1}{5730}$ nor is it $\dfrac{-1}{5730}$. To find the correct coefficient, take the base exponential equation current amount $=$ initial amount e^{ct}, use what you know about the half-life time and amount, then solve the equation for c. If you are having trouble, refer back to the objective on "Solving exponential equations". Do not use a calculator - use exact values to get a correct answer.*

Part B. *Now that you found the exponent's coefficient, you have modeled the general equation as $C = C_0e^{ct}$. Since it asks you to rewrite the equation in terms of $r = \dfrac{C}{C_0}$, start by converting the equation into one with r instead of C and C_0. Then, solve the equation for t.*

Part C. *What does $r = \dfrac{C}{C_0}$ represent? If we know this, then we could use the equation you built in Part B. to solve for the age of the bone.*