Objective 3 - Lowest-Degree Polynomial

Construct a lowest-degree polynomial given its zeros.

Link to section in online textbook

First, watch this video to learn how to construct a polynomial, given its zeros. Now practice constructing polynomials from zeros with the questions below.

Question 1 *Construct the lowest-degree polynomial given the zeros below.*

$$??, ??, ??$$

$f(x) = \boxed{??}x^3 + \boxed{??}x^2 + \boxed{??}x + \boxed{??}$

Question 2 *Construct the lowest-degree polynomial given the zeros below.*

$$??, ??, ??$$

$f(x) = \boxed{??}x^3 + \boxed{??}x^2 + \boxed{??}x + \boxed{??}$

Question 3 *Construct the lowest-degree polynomial given the zeros below.*

$$??, ??, ??$$

$f(x) = \boxed{??}x^3 + \boxed{??}x^2 + \boxed{??}x + \boxed{??}$

Hint: *Remember back to what it meant to be in Standard Form for linear functions: we did not have any fractions as coefficients. How would we rewrite a factor that has a fraction in it, like* $\left(x - \frac{3}{4}\right)$*?*

Question 4 *Construct the lowest-degree polynomial given the zeros below.*

$$??, ??, ??$$

$f(x) = \boxed{??}x^3 + \boxed{??}x^2 + \boxed{??}x + \boxed{??}$

Learning outcomes:

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Hint: *Remember back to what it meant to be in Standard Form for linear functions: we did not have any fractions as coefficients. How would we rewrite a factor that has a fraction in it, like* $\left(x - \frac{3}{4}\right)$*?*

We focused on building polynomials with integer and rational zeros. What would we do if we had other types of zeros, like irrational or complex?

Theorem 1. *Complex and Irrational roots for polynomials come in "________" pairs.*

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

tells us something about the types of zeros a quadratic function may have:

- *2 different, rational zeros*
  - *e.g.,* $\frac{1}{4}$ *and* -3 *for the polynomial* $4x^2 + 11x - 3$
- *2 copies of a rational zero*
  - *e.g.,* $\frac{1}{3}$ *and* $\frac{1}{3}$ *for the polynomial* $9x^2 - 2x + 1$
- *2 different, irrational zeros*
  - *e.g.,* $\frac{1}{2} - \sqrt{2}$ *and* $\frac{1}{2} + \sqrt{2}$ *for the polynomial* $4x^2 - 4x - 7$
- *2 different, complex zeros*
  - *e.g.,* $\frac{1}{4} - 3i$ *and* $\frac{1}{4} + 3i$ *for the polynomial* $16x^2 - 8x + 145$

Let's focus on the irrational and complex zeros. These occur when the number under the square root is either (1) not a perfect square or (2) negative. Let's look closer at the form these zeros take by looking at the subgroups the numbers belong to.

Case 1: $b^2 - 4ac$ *is positive and is* ***not*** *a perfect square.*

$$x = \frac{\text{integer}}{\text{integer}} \pm \frac{\text{irrational}}{\text{integer}}$$

$$x = \text{rational} \pm \text{irrational}$$

Case 2: $b^2 - 4ac$ *is negative.*

$$x = \frac{\textit{integer}}{\textit{integer}} \pm \frac{\textit{complex}}{\textit{integer}}$$

$$x = \textit{rational} \pm \textit{complex}$$

Question 5 *What word describes the relationship between the zeros* $x = \textit{rational} - \textit{complex}$ *and* $x = \textit{rational} + \textit{complex}$*?*

They are $\boxed{\textit{conjugate}}$ *pairs!*

Hint: *What are* $3 + 4i$ *and* $3 - 4i$ *to each other?*

We use this theorem to construct polynomials with irrational and/or complex roots.

Question 6 *Construct the lowest-degree polynomial given the zeros below.*

$$\textbf{??}, \textbf{??}$$

$f(x) = \boxed{\textbf{??}}x^3 + \boxed{\textbf{??}}x^2 + \boxed{\textbf{??}}x + \boxed{\textbf{??}}$

Hint: *If* **??** *is a zero to the polynomial, then* $-$**??** *is also! Multiply* $(x - \textbf{??})(x + \textbf{??})$ *first, then use the third zero to finish building the polynomial.*

Question 7 *Construct the lowest-degree polynomial given the zeros below.*

$$\textbf{??} + \textbf{??}, \textbf{??}$$

$f(x) = \boxed{\textbf{??}}x^3 + \boxed{\textbf{??}}x^2 + \boxed{\textbf{??}}x + \boxed{\textbf{??}}$

Hint: *Be careful with how you set up this problem. Again, multiply the conjugate factors together first. If you did this right, there should be no radicals left!*

Question 8 *Construct the lowest-degree polynomial given the zeros below.*

$$??i, ??$$

$f(x) = \boxed{??}x^3 + \boxed{??}x^2 + \boxed{??}x + \boxed{??}$

Question 9 *Construct the lowest-degree polynomial given the zeros below.*

$$?? + ??i, ??$$

$f(x) = \boxed{??}x^3 + \boxed{??}x^2 + \boxed{??}x + \boxed{??}$

Hint: *First, you want to set up the factors:*

$$(x - (?? + ??i))(x - (?? - ??i))(??x - ??)$$

Now, we want to be careful how we multiply out.

$$(x^2 - (?? + ??i)\,x - (?? - ??i)\,x + (?? + ??i)(?? - ??i))\,(??x - ??)$$

$$(x^2 - ??x + (??^2 + ??^2))(??x - ??)$$

Distribute this last part out carefully and you will have completed the problem.