Objective 1 - Holes

Link to section in online textbook.

Introduction video describing holes/vertical asymptotes *without limits*.

A *hole* in a function occurs when the value of that function is $\frac{0}{0}$. For example, the function

$$f(x) = \frac{(x+2)(x-3)}{x-3}$$

has a hole at $x = 3$ because $f(3) = \frac{0}{0}$. If we want to describe this with limits, we would say $\lim_{x \to 3} f(x) = \frac{0}{0}$. Holes only affect the function *exactly* at that point. Notice for our example that

$$f(x) = x + 2 \text{ when } x \neq 3 \text{ and } f(x) \text{ is undefined at } x = 3.$$

That means the rational function actually looks like a line almost everywhere! Recognizing if a rational function has holes will be our first step in graphing these functions.

Theorem 1. ***Holes of a Rational Function:***

A rational function $f(x)$ has a hole at $x = a$ if

$$\lim_{x \to a} f(x) = \frac{0}{0}.$$

Thus, to determine if there are any holes in a rational function, we need to factor the denominator and check if that value is a zero of the numerator (using Synthetic Division, if necessary).

Practice with the questions below.

Question 1 *Find all holes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{\textbf{??}}{\textbf{??}}$$

Learning outcomes:
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Holes: at the x-value $x = \boxed{\textbf{??}}$

Question 2 *Find all holes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{\textbf{??}}{\textbf{??}}$$

Holes: at the x-value $x = \boxed{NA}$

Question 3 *Find all holes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{\textbf{??}}{\textbf{??}}$$

Holes: at the x-value $x = \boxed{NA}$

Question 4 *Find all holes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{\textbf{??}}{\textbf{??}}$$

Holes: at the x-value $x = \boxed{NA}$

Question 5 *Find all holes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{\textbf{??}}{\textbf{??}}$$

Holes: at the x-value $x = \boxed{\textbf{??}}$

Question 6 *Find all holes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{\mathbf{??}}{\mathbf{??}}$$

Holes: at the x-value $x = \boxed{\mathbf{??}}$