Objective 3 - Lowest-Degree Polynomial

Construct a lowest-degree polynomial given its zeros.

Link to section in online textbook

First, watch this video to learn how to construct a polynomial, given its zeros. Now practice constructing polynomials from zeros with the questions below.

Question 1 *Construct the lowest-degree polynomial given the zeros below.*

$$5, 4, -4$$

$f(x) = \boxed{1}x^3 + \boxed{-5}x^2 + \boxed{-16}x + \boxed{80}$

Question 2 *Construct the lowest-degree polynomial given the zeros below.*

$$-3, -4, 3$$

$f(x) = \boxed{1}x^3 + \boxed{4}x^2 + \boxed{-9}x + \boxed{-36}$

Question 3 *Construct the lowest-degree polynomial given the zeros below.*

$$\frac{4}{3}, -\frac{5}{2}, \frac{4}{3}$$

$f(x) = \boxed{18}x^3 + \boxed{-3}x^2 + \boxed{-88}x + \boxed{80}$

Hint: *Remember back to what it meant to be in Standard Form for linear functions: we did not have any fractions as coefficients. How would we rewrite a factor that has a fraction in it, like* $\left(x - \frac{3}{4}\right)$*?*

Question 4 *Construct the lowest-degree polynomial given the zeros below.*

Learning outcomes:
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$$-\frac{4}{3}, -5, -\frac{4}{3}$$

$f(x) = \boxed{9}x^3 + \boxed{69}x^2 + \boxed{136}x + \boxed{80}$

Hint: *Remember back to what it meant to be in Standard Form for linear functions: we did not have any fractions as coefficients. How would we rewrite a factor that has a fraction in it, like* $\left(x - \frac{3}{4}\right)$*?*

We focused on building polynomials with integer and rational zeros. What would we do if we had other types of zeros, like irrational or complex?

Theorem 1. *Complex and Irrational roots for polynomials come in "_________" pairs.*

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

tells us something about the types of zeros a quadratic function may have:

- *2 different, rational zeros*
  - *e.g.,* $\frac{1}{4}$ *and* -3 *for the polynomial* $4x^2 + 11x - 3$
- *2 copies of a rational zero*
  - *e.g.,* $\frac{1}{3}$ *and* $\frac{1}{3}$ *for the polynomial* $9x^2 - 2x + 1$
- *2 different, irrational zeros*
  - *e.g.,* $\frac{1}{2} - \sqrt{2}$ *and* $\frac{1}{2} + \sqrt{2}$ *for the polynomial* $4x^2 - 4x - 7$
- *2 different, complex zeros*
  - *e.g.,* $\frac{1}{4} - 3i$ *and* $\frac{1}{4} + 3i$ *for the polynomial* $16x^2 - 8x + 145$

Let's focus on the irrational and complex zeros. These occur when the number under the square root is either (1) not a perfect square or (2) negative. Let's look closer at the form these zeros take by looking at the subgroups the numbers belong to.

Case 1: $b^2 - 4ac$ *is positive and is* ***not*** *a perfect square.*

$$x = \frac{\textit{integer}}{\textit{integer}} \pm \frac{\textit{irrational}}{\textit{integer}}$$

$$x = \textit{rational} \pm \textit{irrational}$$

Case 2: $b^2 - 4ac$ *is negative.*

$$x = \frac{\textit{integer}}{\textit{integer}} \pm \frac{\textit{complex}}{\textit{integer}}$$

$$x = \textit{rational} \pm \textit{complex}$$

Question 5 *What word describes the relationship between the zeros* $x = \textit{rational} - \textit{complex}$ *and* $x = \textit{rational} + \textit{complex}$*?*

They are $\boxed{\textit{conjugate}}$ *pairs!*

Hint: *What are* $3 + 4i$ *and* $3 - 4i$ *to each other?*

We use this theorem to construct polynomials with irrational and/or complex roots.

Question 6 *Construct the lowest-degree polynomial given the zeros below.*

$$2, -\frac{5}{2}$$

$f(x) = \boxed{2}x^3 + \boxed{5}x^2 + \boxed{-8}x + \boxed{-20}$

Hint: *If 2 is a zero to the polynomial, then -2 is also! Multiply $(x-2)(x+2)$ first, then use the third zero to finish building the polynomial.*

Question 7 *Construct the lowest-degree polynomial given the zeros below.*

$$5 + \sqrt{5}, \frac{3}{2}$$

$f(x) = \boxed{2}x^3 + \boxed{-23}x^2 + \boxed{70}x + \boxed{-60}$

Hint: *Be careful with how you set up this problem. Again, multiply the conjugate factors together first. If you did this right, there should be no radicals left!*

Question 8 *Construct the lowest-degree polynomial given the zeros below.*

$$4i, \frac{3}{4}$$

$f(x) = \boxed{4}x^3 + \boxed{-3}x^2 + \boxed{64}x + \boxed{-48}$

Question 9 *Construct the lowest-degree polynomial given the zeros below.*

$$3 + 5i, -\frac{5}{7}$$

$f(x) = \boxed{7}x^3 + \boxed{-37}x^2 + \boxed{208}x + \boxed{170}$

Hint: *First, you want to set up the factors:*

$$(x - (3 + 5i))(x - (3 - 5i))(7x - -5)$$

Now, we want to be careful how we multiply out.

$$(x^2 - (3 + 5i)\,x - (3 - 5i)\,x + (3 + 5i)(3 - 5i))\,(7x - -5)$$

$$(x^2 - 6x + (3^2 + 5^2))(7x - -5)$$

Distribute this last part out carefully and you will have completed the problem.
