Objective 2 - Graph Rational functions

Convert between a rational function and its graph.

Link to section in online textbook.

First, watch this video to learn about what rational functions look like. We will work with two specific rational functions: $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x^2}$. By using what we know about shifting and leading coefficients from Quadratics, Radicals, and Polynomials, we have two basic equations for rational functions:

$$f(x) = \frac{a}{x-h} + k$$

Graph of $f(x) = a/(x-h) + k, a = 1, h = 0, k = 0, x = h, y = k$

$$g(x) = \frac{a}{(x-h)^2} + k$$

Graph of $f(x) = a/(x-h)^2 + k, a = 1, h = 0, k = 0, x = h, y = k$

Thinking back to the previous objective, our rational functions **are not defined at** $x = h$**.** So while (h, k) acts like our vertex for quadratics, it is not actually a point on the graph! Check out the Desmos graphs to see how a, h, and k affect the graphs of these two functions.

We will focus on working from graphs to the equation. *If you master this, you'll be able to work backwards and graph a radical function from the equation.*

Question   1   *Write an equation of the function graphed below. Assume $a = 1$ or $a = -1$.*
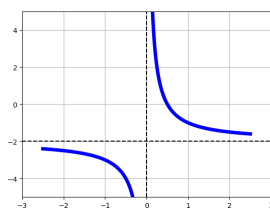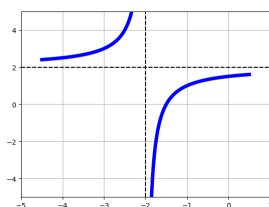

Learning outcomes:
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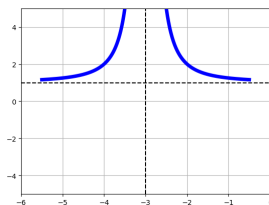$$f(x) = \frac{\boxed{1}}{(x - \boxed{0})^{\boxed{1}}} + \boxed{-2}$$

Hint: The leading coefficient is either $a = 1$ or $a = -1$. Try going back to the Desmos graphs and switch between 1 and -1. For the other parts, what acts like the "vertex" of the graph?

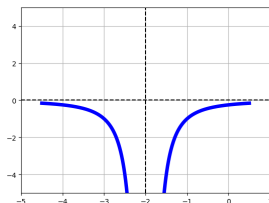**Question   2** *Write an equation of the function graphed below.*

$$f(x) = \frac{\boxed{-1}}{(x - \boxed{-2})^{\boxed{1}}} + \boxed{2}$$

Question   3 *Write an equation of the function graphed below.*

$$f(x) = \frac{\boxed{1}}{(x - \boxed{-3})^{\boxed{2}}} + \boxed{1}$$

Question   4 *Write an equation of the function graphed below.*

$$f(x) = \dfrac{\boxed{-1}}{(x - \boxed{-2})^{\boxed{2}}} + \boxed{0}$$

Question 5 *Write an equation of the function graphed below.*

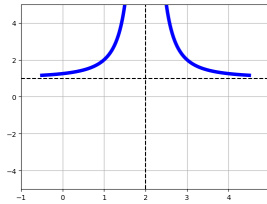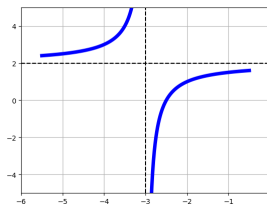$$f(x) = \dfrac{\boxed{1}}{(x - \boxed{2})^{\boxed{2}}} + \boxed{1}$$

Question 6 *Write an equation of the function graphed below.*

$$f(x) = \dfrac{\boxed{-1}}{(x - \boxed{-3})^{\boxed{1}}} + \boxed{2}$$

Question 7 *Main takeaway:* *Before looking, you should work through the previous problems. Have you finished working through the examples?* $\boxed{Yes}$

***Feedback*(`correct`):** *The important components of a basic rational function are:*

- *The vertical asymptote (vertical line where the function is not defined);*
- *Horizontal asymptote (horizontal line normally at $y = 0$, shifted by k); and*
- *The power of the denominator (1 has curves in opposite corners, 2 has curves side-by-side).*