

Objective 3 - Construct Linear Model

Construct a model equation for the real-life situation.

Scenarios we normally use linear functions to model are:

Finance Cost to produce, Utility Bill, Depreciation of Value;

Motion Relative distance of two objects, Using round-trip times to calculate distance,

Chemistry Mixing two different concentrations of solutions;

Statistics Line of best fit.

After you complete each of the questions below, see if you can put together a “general form” for the linear model you built. You can print out [these notes](#) to follow along with the video below and keep notes to organize your thoughts.

YouTube link: <https://www.youtube.com/watch?v=y1p6rx1N4P0>

Exercise 1 A company sells doughnuts. They incur a fixed cost of \$20000 for rent, insurance, and other expenses. It costs \$0.15 to produce each doughnut. The company sells each doughnut for \$0.3.

Part A. Construct a linear model that describes their total costs, C , as a function of the number of doughnuts, x , they produce.

$$C(x) = \boxed{0.15x + 20000}$$

Part B. Construct a linear model that describes their total profits, P , as a function of the number of doughnuts, x , they produce.

$$P(x) = \boxed{0.3x}$$

Part C. Construct a linear model that describes their total revenue, R , as a function of the number of doughnuts, x , they produce.

$$R(x) = \boxed{0.15x - 20000}$$

Exercise 2 Aubrey is a college student going into her first year at UF. She will receive Bright Futures, which covers her tuition plus a \$300 educational expense each Fall and Spring semester. Before college, Aubrey saved up \$7000.

Learning outcomes:

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She knows she will need to pay \$900 in rent a month, \$40 for food a week, and \$32 in other weekly expenses.

Part A. Construct a linear model that describes her total costs, C as a function of the number of months, x , she is at UF **during Fall semester**.

$$C(x) = \boxed{1188x}$$

Part B. Construct a linear model that describes her total income, I , as a function of the number of months, x , she is at UF **during Fall semester**.

$$I(x) = \boxed{7300}$$

Part C. Construct a linear model that describes their total budget, B , as a function of the number of months, x , she is at UF **during Fall semester**.

$$B(x) = \boxed{-1188x + 7300}$$

Try to write down notes on how to solve the first two questions *in general*.

Exercise 3 Two UFPD are patrolling the campus on foot. To cover more ground, they split up and begin walking in different directions. Office A is walking at 4 mph while Office B is walking at 4 mph.

Part A. Construct a linear model that describes Officer A's distance from their starting point, D , as a function of minutes, m , that have passed.

$$D(m) = \boxed{\frac{1}{15}m}$$

Hint: Speed = $\frac{\text{distance}}{\text{time}}$

Can you re-solve this for distance?

Part B. Construct a linear model that describes their total distance from each other, T_1 , as a function of minutes, m , that have passed if they were walking in exactly opposite directions (e.g., North/South).

$$T_1(m) = \boxed{\frac{2}{15}m}$$

Part C. Construct a linear model that describes their total distance from each other, T_2 , as a function of minutes, m , that have passed if they were walking in exactly 90 degrees from each other (e.g., North/East).

$$T_2(m) = \boxed{\frac{1}{15}\sqrt{2}m}$$

Exact value needed for the coefficient! DO NOT ROUND.

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Figure 1: Training path.

Hint: For Part C, draw a picture and think about how the Pythagorean Theorem could be used. Remember: these are all linear models! So if your model has a non-linear **variable**, is there a reason we would ignore part of the domain?

Exercise 4 A bicyclist is training for a race on a hilly path. Their bike keeps track of their speed at any time, but not the distance traveled. Their speed traveling up a hill is 2mph, 7mph when traveling down a hill, and 4mph when traveling along a flat portion.

Hint: Distance is equal to rate times time.

Part A. Construct linear models that describe their distance, D in miles, on a particular portion of the path in terms of the time, t in hours, spent on that part of the path.

$$D_{\text{up}}(t) = \boxed{2t}$$

$$D_{\text{down}}(t) = \boxed{7t}$$

$$D_{\text{flat}}(t) = \boxed{4t}$$

Part B. Construct linear models that describe their time, t in hours, on a particular portion of the path in terms of the length, D in miles, of that part of the path.

$$t_{\text{up}}(D) = \boxed{\frac{1}{2}D}$$

$$t_{\text{down}}(D) = \boxed{\frac{1}{7}D}$$

$$t_{\text{flat}}(D) = \boxed{\frac{1}{4}D}$$

Part C. Construct a linear model that describes the total distance of the path, D , in terms of the time spent on a particular path if we knew that the time spent on each path was equal.

$$D(t) = \boxed{13t}$$

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Part D. Construct a linear model that describes the total time T spent on the path in terms of the distance of a particular part of the path if we knew that all parts of the path are equal length.

$$T(D) = \boxed{\frac{25}{28} D}$$

Exercise 5 Kappa Delta is hosting an all-you-can-eat pancake fundraiser to support the prevention of child abuse. Adult (18+) tickets are \$12 and teen (10-17) tickets are \$6. Children under 10 are let in without a ticket. The ticket-sellers only kept track of the total number of tickets sold, 100, and total revenue, \$792.

Part A. Construct a linear model that describes the total number of adult tickets, y , sold in terms of the number of teen tickets, x , sold.

$$y = \boxed{-x + 100}$$

Part B. Construct a linear model that describes the revenue made from selling many adult tickets, R_a , in terms of the number of teen tickets, x , sold.

$$R_a = \boxed{-12x + 1200}$$

Part C. Construct a linear model that describes the total revenue made, R_T , in terms of the number of teen tickets, x , sold.

$$R_T = \boxed{-6x + 1200}$$

Hint: Part A: Is there way to build an equation that relates adult tickets, teen tickets, and total tickets? Solving this equation for adult tickets would give you the linear model.

Part B: It may be easier to first build this model in terms of y , then use your answer from part A.

Part C: Think about how to model total revenue, then use your answer in Part B to make part of the model.

Exercise 6 Chemists commonly create a solution by mixing two products of differing concentrations together. For example, a chemist could have large amounts of a 10% acid solution and a 40% acid solution, but need a 9 liter 22% solution.

Part A. Construct a linear model that describes the volume of the 40% acid solution, v_{40} , in terms of the volume of the 10% acid solution, v .

$$v_{40} = \boxed{-v + 9}$$

Part B. Construct a linear model that describes the amount of acid in a 10% acid solution, A_{10} , in terms of the volume of the 10% acid solution, v .

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$$A_{10} = \boxed{0.1000000000000000 v}$$

Part C. Construct a linear model that describes the amount of acid in a 40% acid solution, A_{40} , in terms of the volume of the 10% acid solution, v .

$$A_{40} = \boxed{-0.4000000000000000 v + 3.6000000000000000}$$

Part D. Construct a linear model that describes the amount of acid in a 22% acid solution, A_{22} , in terms of the volume of the 10% acid solution, v .

$$A_{22} = \boxed{-0.3000000000000000 v + 3.6000000000000000}$$

Hint: Parts A-D: Think about what you did in the last problem. How can we use that same structure in this new setting?