Objective 3 - Evaluating Limits

Interpret the notation for limits.

Link to section in online textbook.

Video for evaluating a limit.

Now that we have learned about left- and right-hand limits, we can evaluate the limit of a function at a point.

Theorem 1. Evaluating the limit of a function at a point x = a:

$$\lim_{x\to a}(f(x))=L$$
 if and only if
$$\lim_{x\to a^-}(f(x))=L=\lim_{x\to a^+}(f(x))$$

Note: The limit exists if L is a Real number. The limit can be equal to ∞ or $-\infty$, but we would not say that it exists. If the left- and right-hand limits do not agree, we say the limit does not exist (or DNE for short).

This objective will allow you to practice evaluating the left- and right-hand limits to determine if the limit at a point exists. This would be where you want to practice before the exam.

Answers are either a Real number, ∞ , $-\infty$, or DNE.

Question 1
$$\lim_{x \to -\infty} f(x) = \boxed{-\infty}$$
 $\lim_{x \to -1} g(x) = \boxed{3}$ $\lim_{x \to \infty} f(x) = \boxed{+\infty}$

Question 2
$$\lim_{x \to -\infty} f(x) = \boxed{-\infty}$$

$$\lim_{x \to -2} f(x) = \boxed{3}$$

$$\lim_{x \to 1} f(x) = \boxed{DNE}$$

$$\lim_{x \to 3} f(x) = \boxed{-\infty}$$

Learning outcomes:

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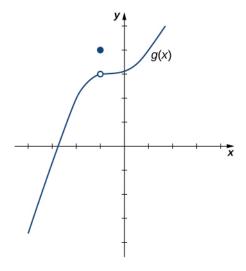


Figure 1: Function with a hole at x = -1.

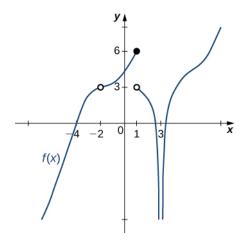


Figure 2: Piecewise function to evaluate.

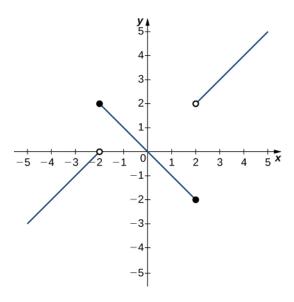


Figure 3: Piecewise function to evaluate.

$$\lim_{x \to \infty} f(x) = \boxed{+\infty}$$

Question 3
$$\lim_{x \to -\infty} f(x) = \boxed{-\infty}$$

$$\lim_{x \to -2} f(x) = \boxed{DNE}$$

$$\lim_{x \to 0} f(x) = \boxed{0}$$

$$\lim_{x \to 2} f(x) = \boxed{DNE}$$

$$\lim_{x \to \infty} f(x) = \boxed{+\infty}$$

Question 4 $\lim_{x \to -\infty} f(x) = \boxed{+\infty}$

$$\lim_{x \to -8} f(x) = \boxed{-6}$$

$$\lim_{x \to -2} f(x) = \boxed{DNE}$$

$$\lim_{x \to 6} f(x) = \boxed{DNE}$$

$$\lim_{x \to 10} f(x) = \boxed{0}$$

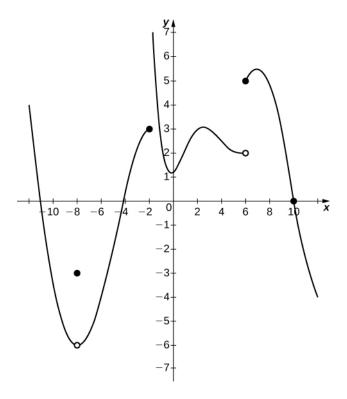


Figure 4: Piecewise function to evaluate.

$$\lim_{x \to \infty} f(x) = \boxed{-\infty}$$

Question 5
$$f(x) = \frac{6(2x-1)(x+1)}{-6x-6}$$

$$\lim_{x \to -\infty} f(x) = \boxed{+\infty}$$

$$\lim_{x \to -1} f(x) = \boxed{3}$$

$$\lim_{x \to \frac{1}{2}} f(x) = \boxed{0}$$

$$\lim_{x \to \infty} f(x) = \boxed{-\infty}$$

Question 6 $f(x) = \frac{3x+4}{4(4x-3)(3x+4)}$

$$\lim_{x \to -\infty} f(x) = \boxed{0}$$

$$\lim_{x \to \frac{3}{4}} f(x) = \boxed{DNE}$$

$$\lim_{x \to -\frac{4}{3}} f(x) = \boxed{-\frac{3}{100}}$$

$$\lim_{x \to \infty} f(x) = \boxed{0}$$

Question 7 $\lim_{x\to 0} (x+1)^{\left(\frac{1}{x}\right)} = e$

Hint: We can't plug in the exact value, so we will need to plug in values very near 0.

Question 8 $\lim_{x \to 7} \frac{\sqrt{x-3}-2}{x-7} = \boxed{\frac{1}{4}}$

Hint: We can't plug in the exact value, so we will need to plug in values very near 7.