Objective 2 - Vertical Asymptotes

Link to section in online textbook.

Introduction video describing holes/vertical asymptotes *without limits*.

When we learned about the domain of rational functions, we set the denominator equal to 0 and solved. This gave us all values that the function is not defined for. As we saw in Objective 1, some of these x-values are holes of the function. The rest are **vertical asymptotes** of the function, or vertical lines where the function approaches positive or negative infinity.

Graph of $f(x) = 1/((x-1)(x-3))$

Unlike holes, vertical asymptotes affect the function around where they are defined. With left- and right-sided limits, we can determine how the function is behaving near these vertical lines.

Theorem 1. ***Vertical Asymptotes of a Rational Function***

A rational function has a vertical asymptote $x = a$ (vertical line) if

$$\lim_{x\to a^-} f(x) = \pm\infty \text{ or } \lim_{x\to a^+} f(x) = \pm\infty$$

Thus, to determine if a rational function has any vertical asymptotes, we need to factor the denominator and evaluate the limit. If a one-sided limit is positive or negative infinity, it is a vertical asymptote.

Practice with the questions below.

Question 1 *Find all vertical asymptotes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{x+1}{6x+6}$$

Vertical Asymptote: the vertical line $x = \boxed{NA}$.

Question 2 *Find all vertical asymptotes of the rational function below. If they do not exist, answer "NA".*

Learning outcomes:
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$$f(x) = \frac{-4}{x+1}$$

Vertical Asymptote: the vertical line $x = \boxed{-1}$.

Question 3 *Find all vertical asymptotes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{4}{-3x^2 + x + 4}$$

Vertical Asymptote: the vertical line $x = \boxed{-1}$ *and* $x = \boxed{\frac{4}{3}}$.

Question 4 *Find all vertical asymptotes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{3x^2 - 14x + 15}{9x^3 - 24x^2 + 19x - 4}$$

Vertical Asymptote: the vertical line $x = \boxed{\frac{1}{3}}$, $x = \boxed{1}$, *and* $x = \boxed{\frac{4}{3}}$.

Question 5 *Find all vertical asymptotes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{-6x^2 + 11x - 3}{-3x^3 - 17x^2 - 18x + 8}$$

Vertical Asymptote: the vertical line $x = \boxed{-4}$ *and* $x = \boxed{-2}$.

Question 6 *Find all vertical asymptotes of the rational function below. If they do not exist, answer "NA".*

$$f(x) = \frac{x^2 - 6x + 5}{x^3 - 5x^2 + 26x - 130}$$

Vertical Asymptote: the vertical line $x = \boxed{NA}$.

Main takeaway: Values not in the domain of the function can be one of two things:

- Vertical Asymptotes: Values the function will come close to, but will not touch at. These are vertical lines $x = a$, where a makes the denominator zero. The limit of the function at these points are $\pm\infty$.
- Holes: Values that only affect the function exactly at that point (rather than nearby by vertical asymptotes). The limit of the function at these points are $\frac{0}{0}$.