

## Objective 2 - Graph Rational functions

*Convert between a rational function and its graph.*

Link to section in online textbook.

First, watch [this video](#) to learn about what rational functions look like. We will work with two specific rational functions:  $f(x) = \frac{1}{x}$  and  $g(x) = \frac{1}{x^2}$ . By using what we know about shifting and leading coefficients from Quadratics, Radicals, and Polynomials, we have two basic equations for rational functions:

$$f(x) = \frac{a}{x-h} + k$$

Graph of  $f(x) = a/(x-h) + k$ ,  $a = 1, h = 0, k = 0, x = h, y = k$

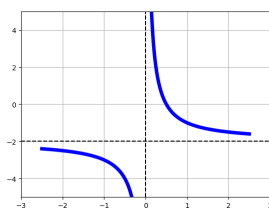
$$g(x) = \frac{a}{(x-h)^2} + k$$

Graph of  $f(x) = a/(x-h)^2 + k$ ,  $a = 1, h = 0, k = 0, x = h, y = k$

Thinking back to the previous objective, our rational functions **are not defined at**  $x = h$ . So while  $(h, k)$  acts like our vertex for quadratics, it is not actually a point on the graph! Check out the Desmos graphs to see how  $a$ ,  $h$ , and  $k$  affect the graphs of these two functions.

We will focus on working from graphs to the equation. *If you master this, you'll be able to work backwards and graph a radical function from the equation.*

**Question 1** Write an equation of the function graphed below. Assume  $a = 1$  or  $a = -1$ .




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Learning outcomes:

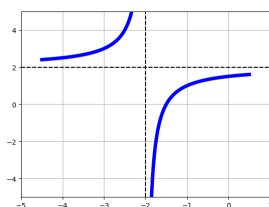
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$$f(x) = \frac{\boxed{1}}{(x - \boxed{0})^{\boxed{1}}} + \boxed{-2}$$

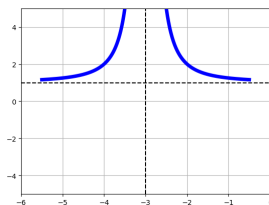
**Hint:** The leading coefficient is either  $a = 1$  or  $a = -1$ . Try going back to the Desmos graphs and switch between 1 and -1. For the other parts, what acts like the “vertex” of the graph?

**Question 2** Write an equation of the function graphed below.



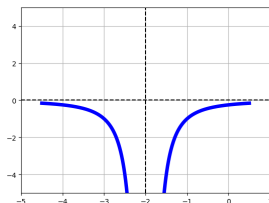
$$f(x) = \frac{\boxed{-1}}{(x - \boxed{-2})^{\boxed{1}}} + \boxed{2}$$

**Question 3** Write an equation of the function graphed below.



$$f(x) = \frac{\boxed{1}}{(x - \boxed{-3})^{\boxed{2}}} + \boxed{1}$$

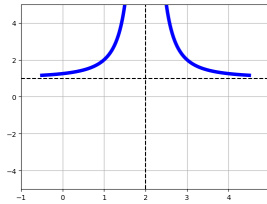
**Question 4** Write an equation of the function graphed below.



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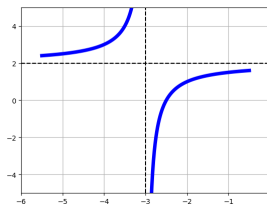
$$f(x) = \frac{\boxed{-1}}{(x - \boxed{-2})^{\boxed{2}}} + \boxed{0}$$

**Question 5** Write an equation of the function graphed below.



$$f(x) = \frac{\boxed{1}}{(x - \boxed{2})^{\boxed{2}}} + \boxed{1}$$

**Question 6** Write an equation of the function graphed below.



$$f(x) = \frac{\boxed{-1}}{(x - \boxed{-3})^{\boxed{1}}} + \boxed{2}$$

**Question 7 Main takeaway:** Before looking, you should work through the previous problems. Have you finished working through the examples?

**Feedback(correct):** The important components of a basic rational function are:

- The vertical asymptote (vertical line where the function is not defined);
- Horizontal asymptote (horizontal line normally at  $y = 0$ , shifted by  $k$ ); and
- The power of the denominator (1 has curves in opposite corners, 2 has curves side-by-side).