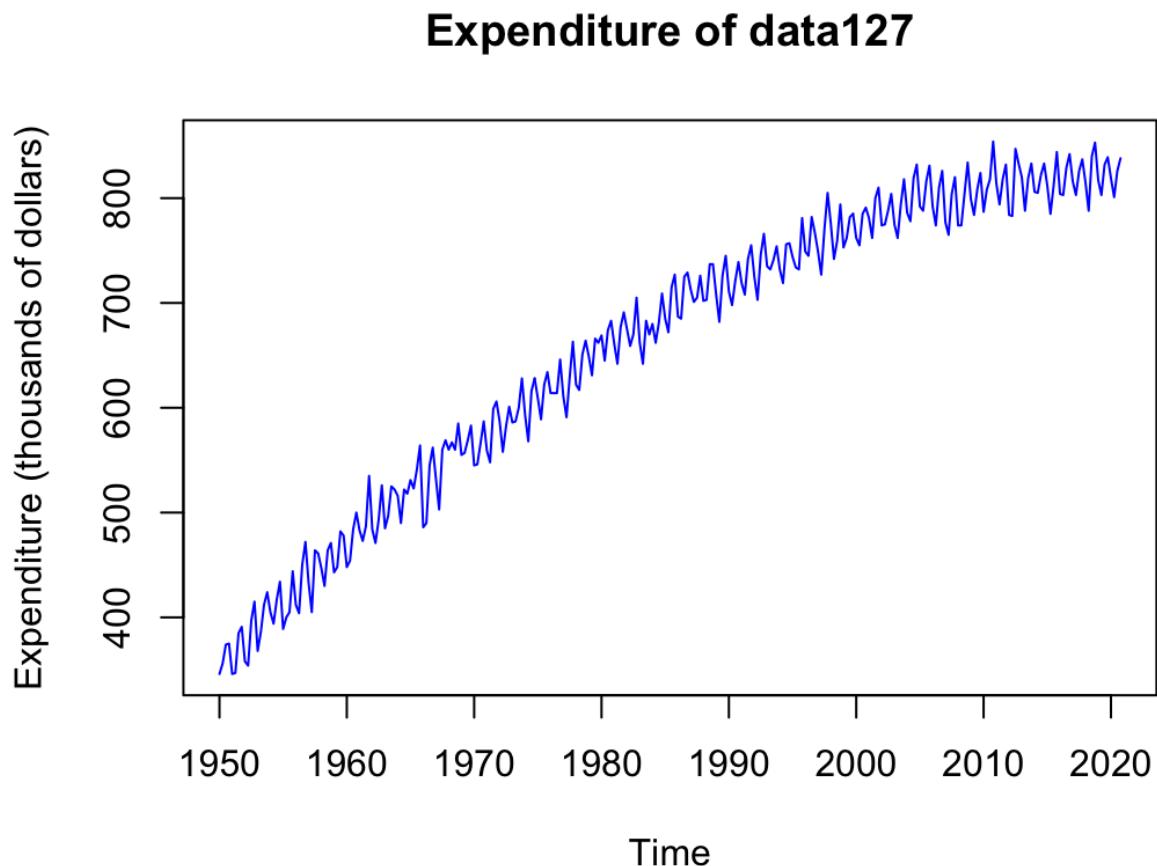


Part A Visualization

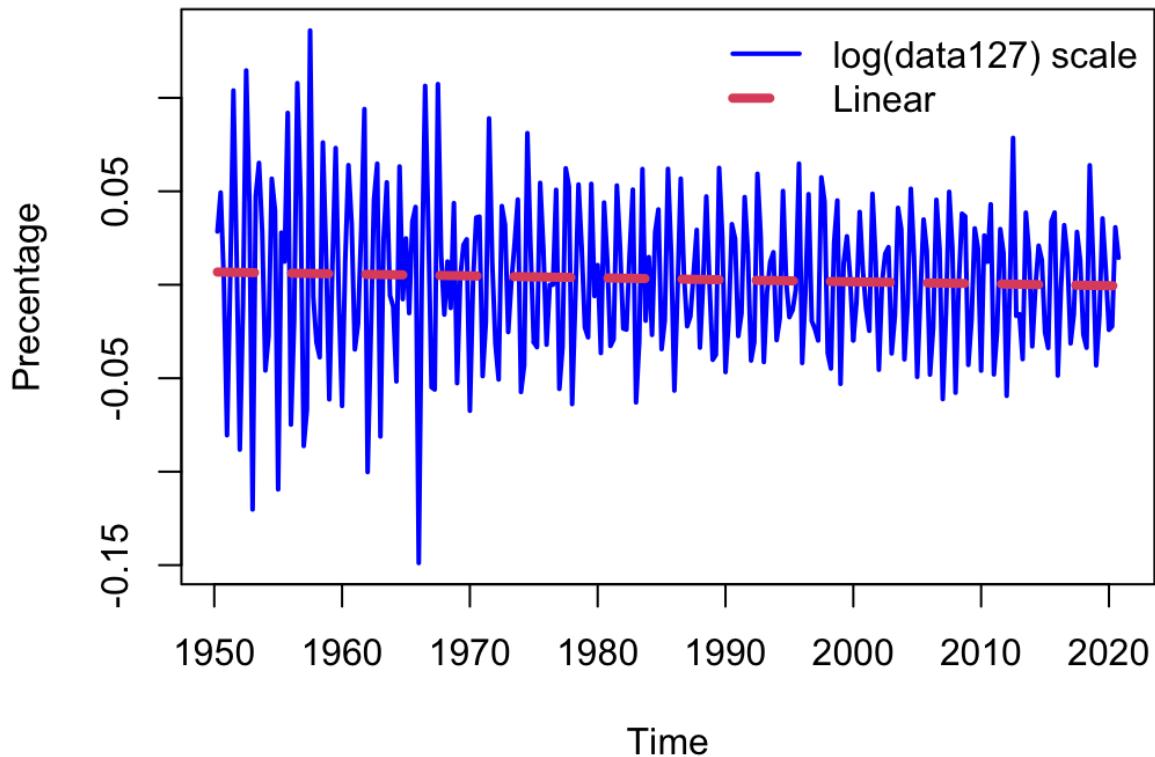
a)



There is a clear positive trend in the above chart, as the data starts steep and eventually flattens after the year 2000. There exist no short-term fluctuations as the graph steadily increased from 1950 to 2000 and finally flattened from 2000 to 2020.

b)

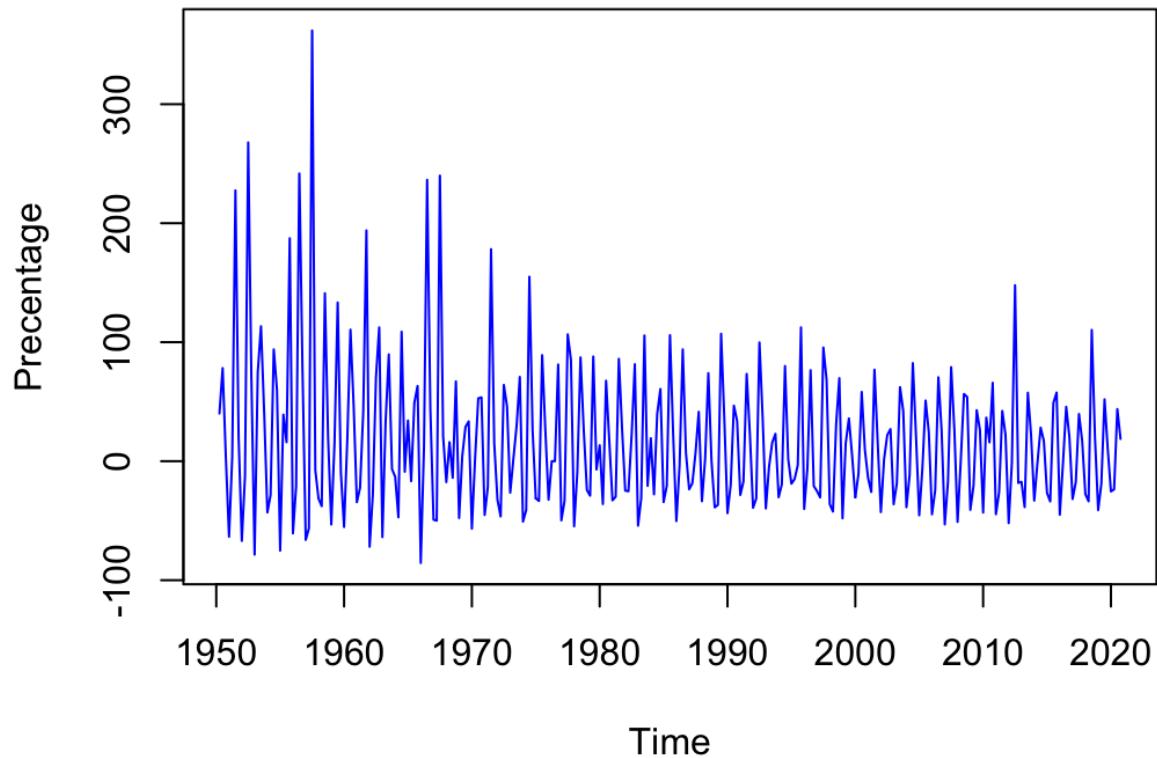
Growth Rate of data127



The growth rate decreased steadily from 1950 to 2020, being relatively constant around the mean as expected. The growth rate did approach 0 by the start of 2020. The growth rate decreased from 1950 to 2000 since the trend is mostly concave down during that period.

c)

Growth Rate of data127 Annualized

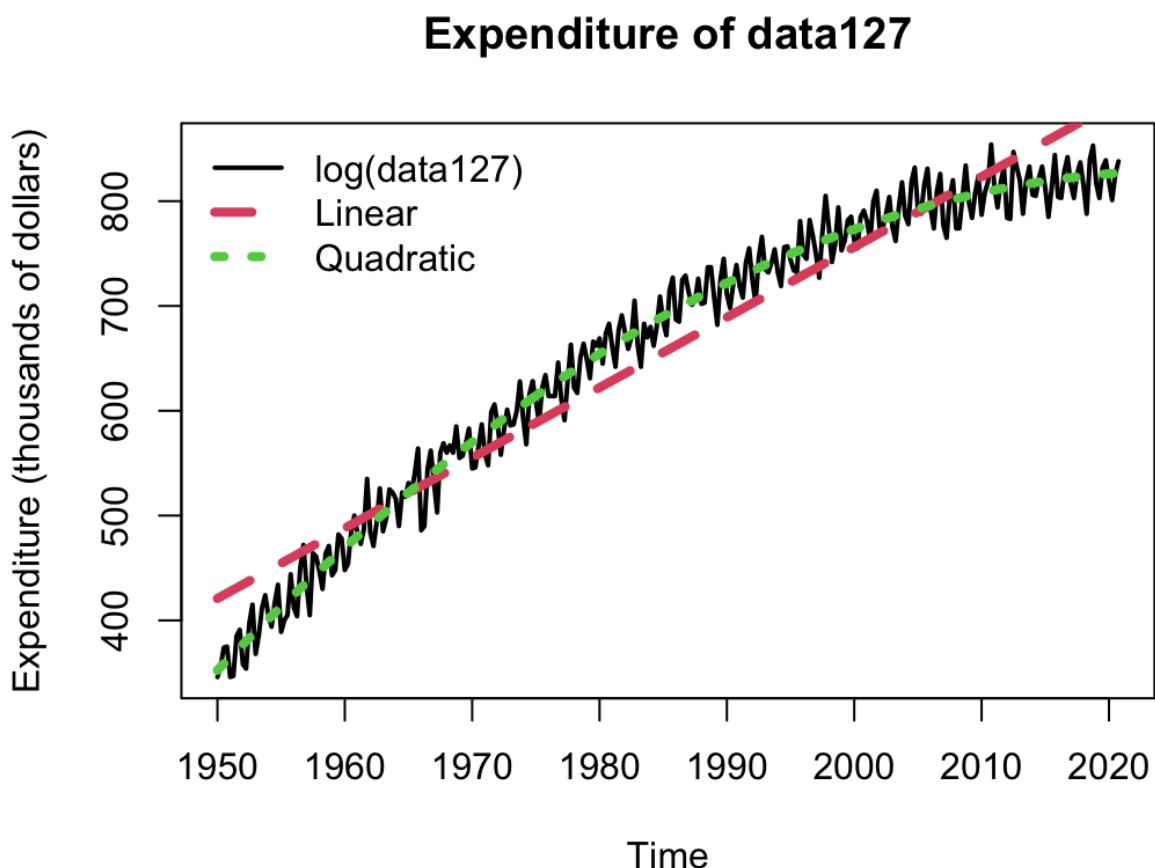


There exists a decreasing growth rate. From the year 1950 to 1970, the growth rate was higher and decreased faster, and after 1970 the growth rate remained steadily decreasing. Overall the growth rate remains constant around 0.

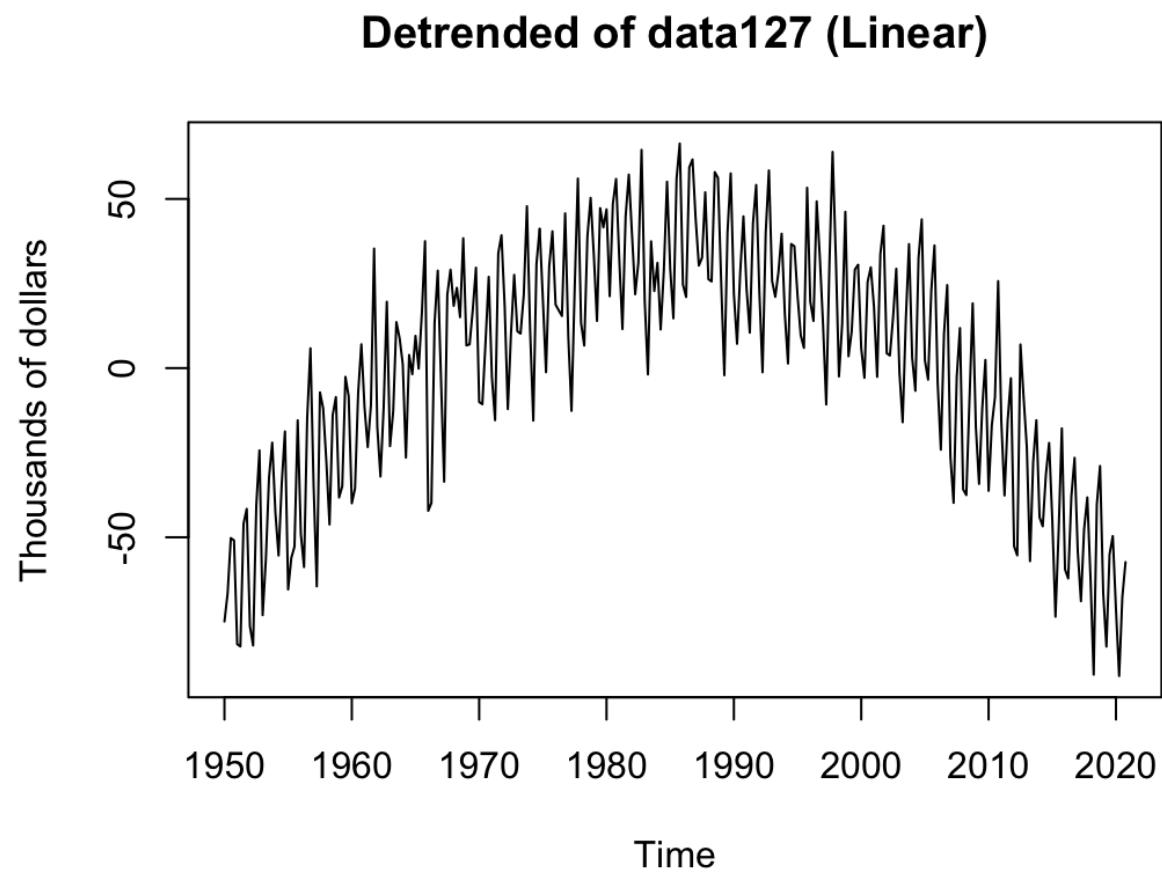
Part B

I have produced both a linear trend and a quadratic trend on data 127, as shown in the graphs below.

a)

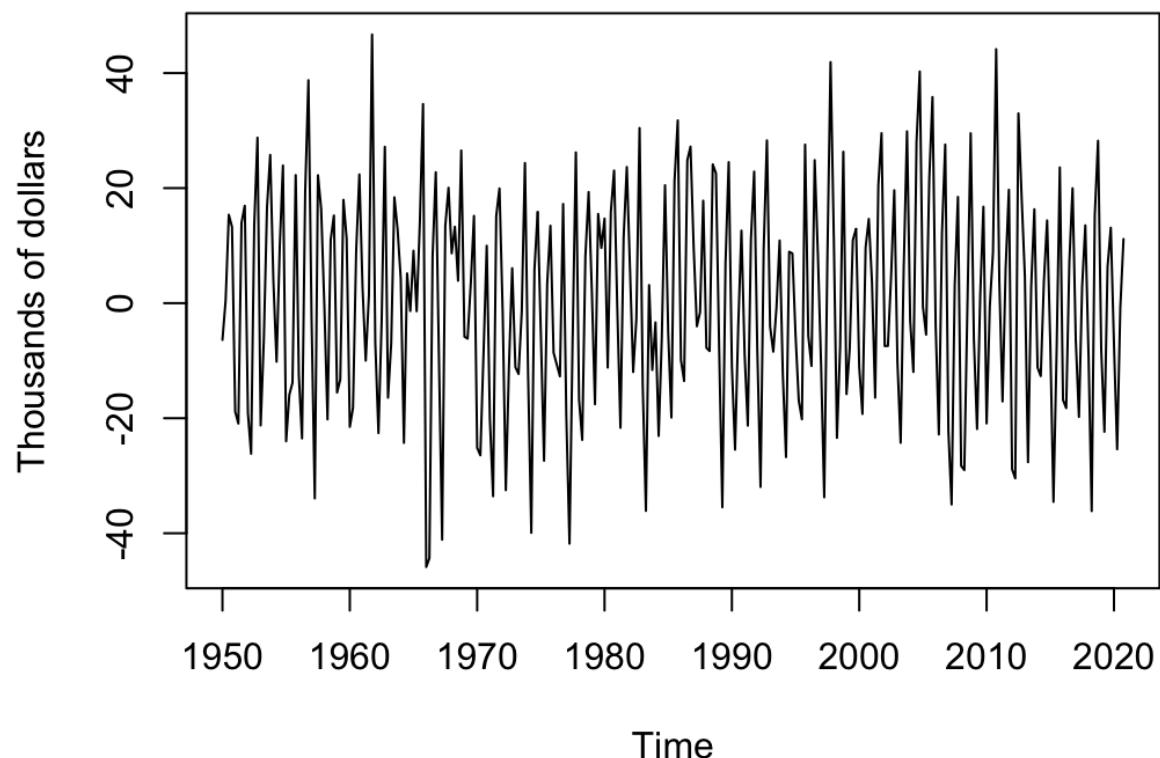


b) The following two graphs illustrates the detrended series



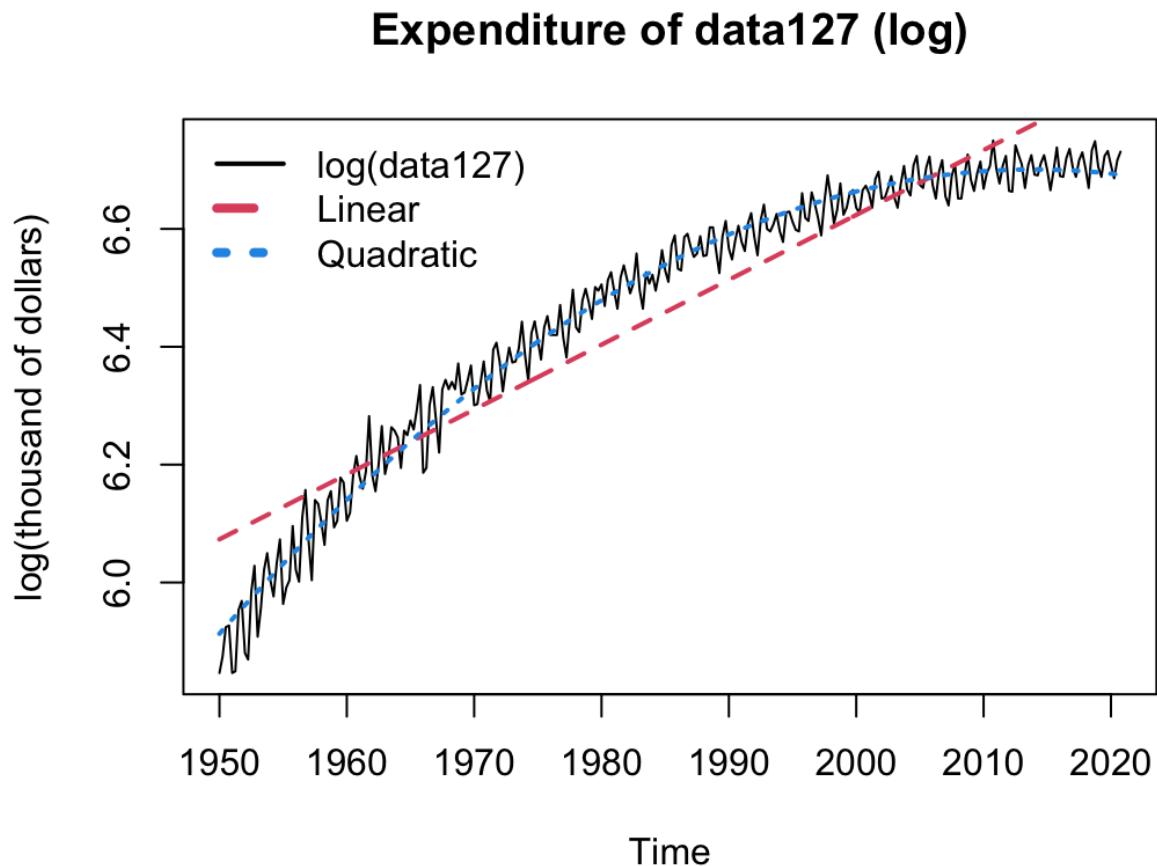
As seen from the graph, there is a low-frequency component with half a cycle per 70 years. Therefore the linear trend is not a good fit for the trend.

Detrended of data127 (Quadratic)



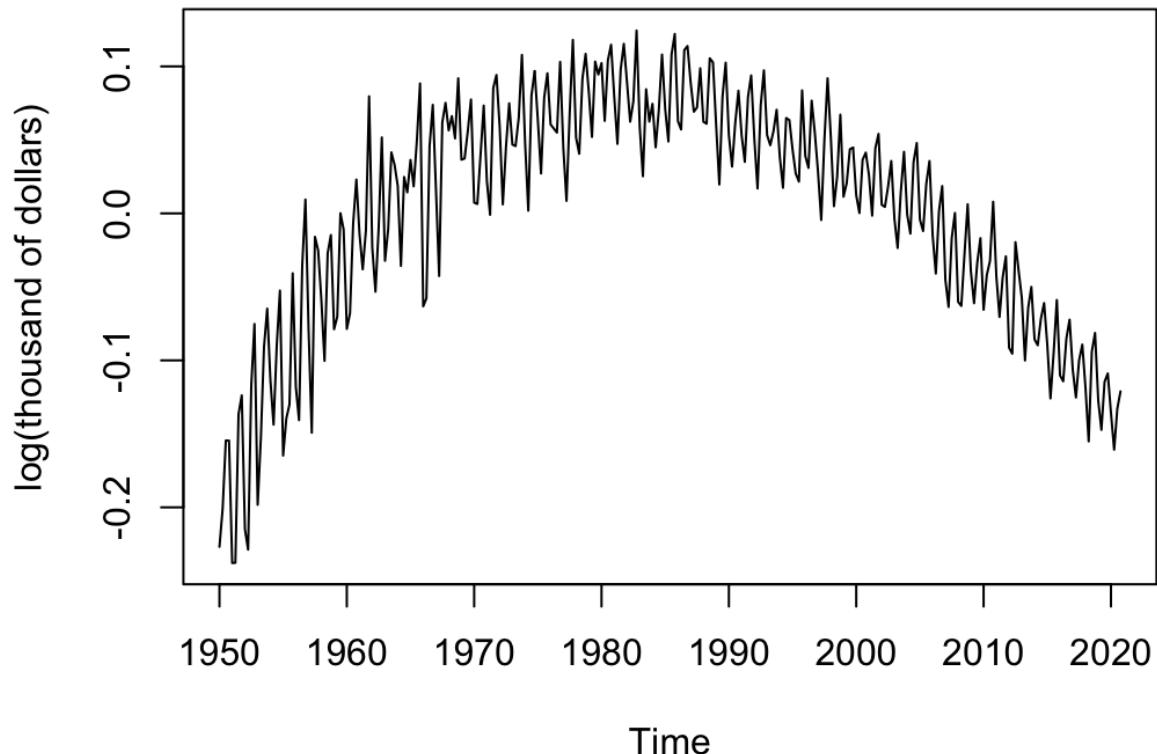
As with the quadratic trend, there exists no low-frequency component in the graph shown. Suggesting a quadratic trendline is a better fit for the series.

c) Plots done with log scale



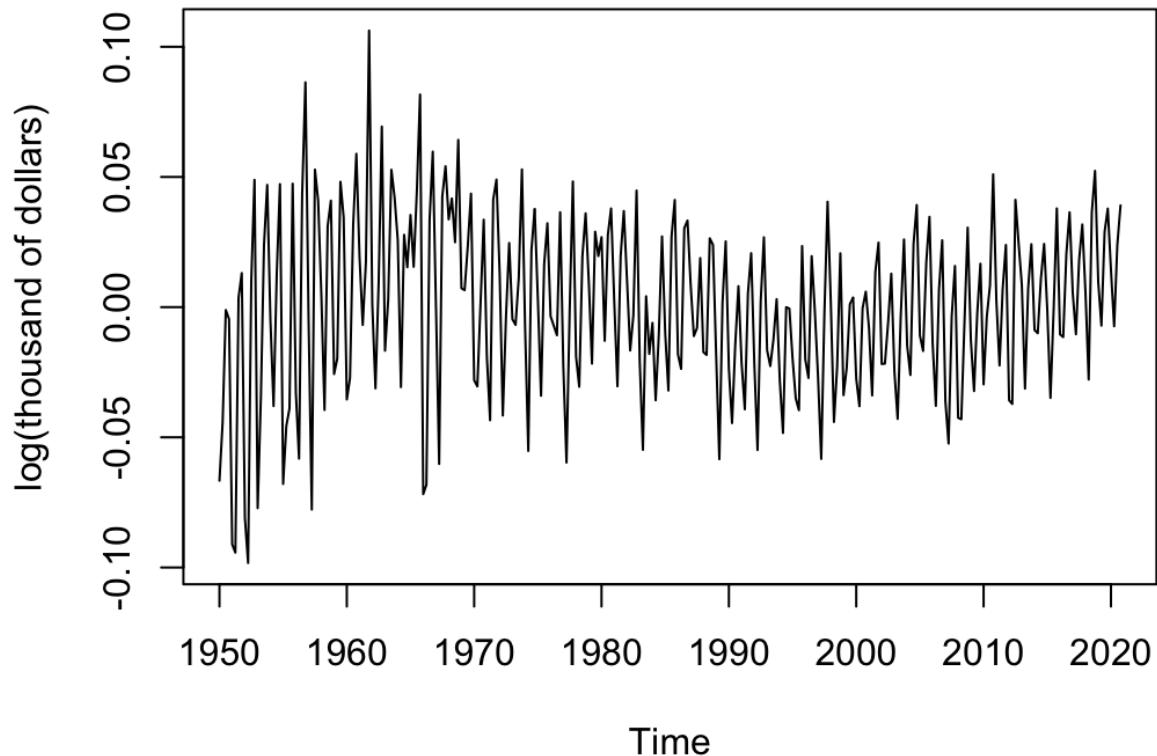
In the log series, the quadratic trend fits the series better.

Detrended of data127 (log)



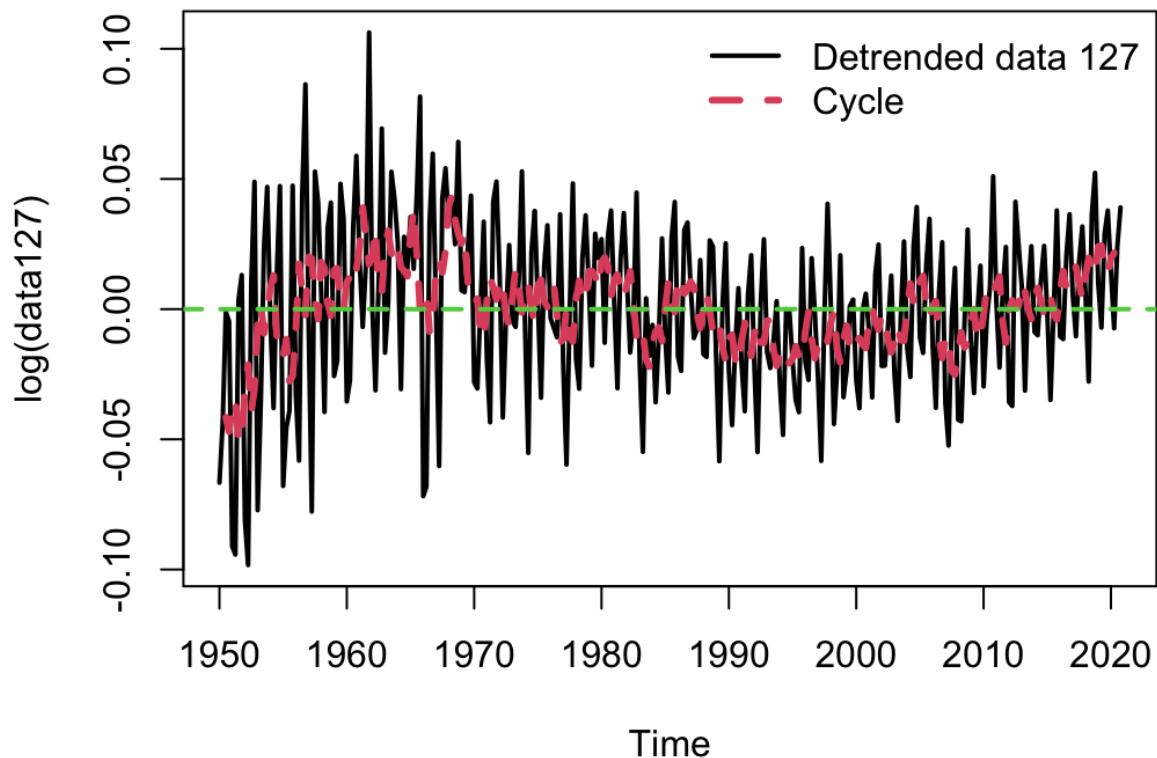
The linear trend suffers from the same elements as it did in part B a). It tells that the growth rate is not constant.

Detrended of data127 (log)



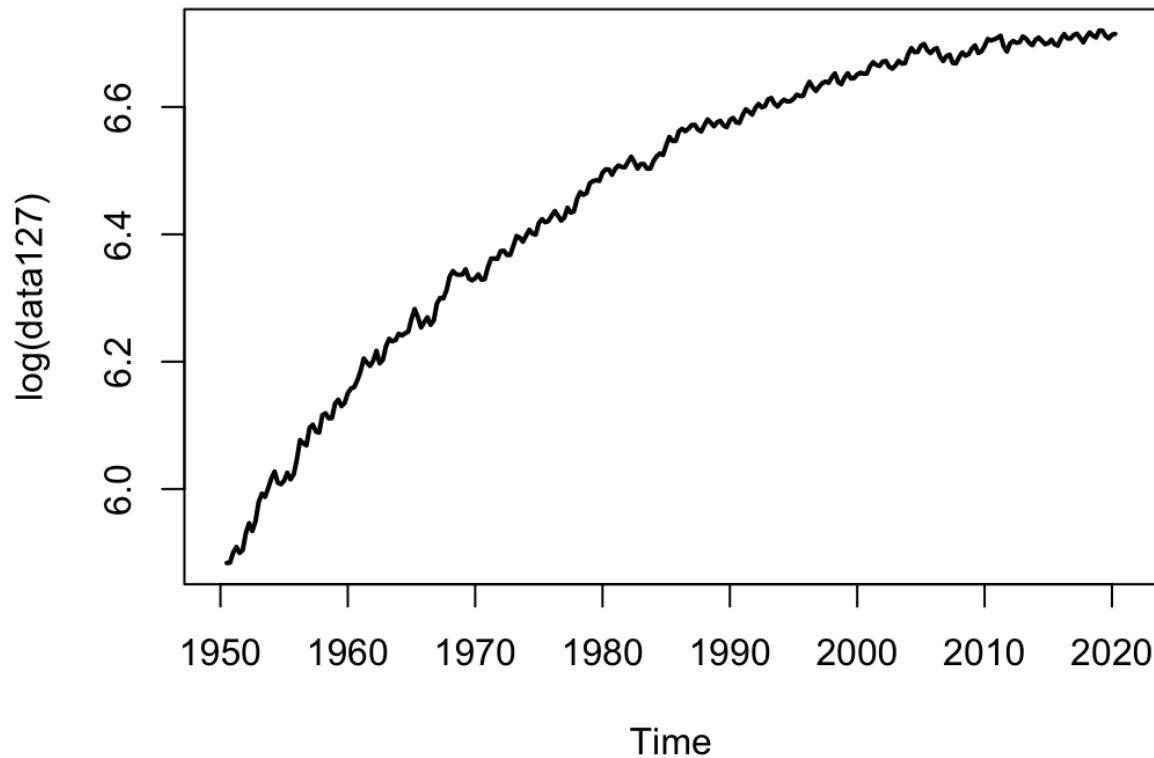
The series detrended with the quadratic trend exists a cycle that spans 70 years. The series increased from 1950 to 1965 and started rising again from 2000 to 2020.

log detrended of data127 (log-scale)



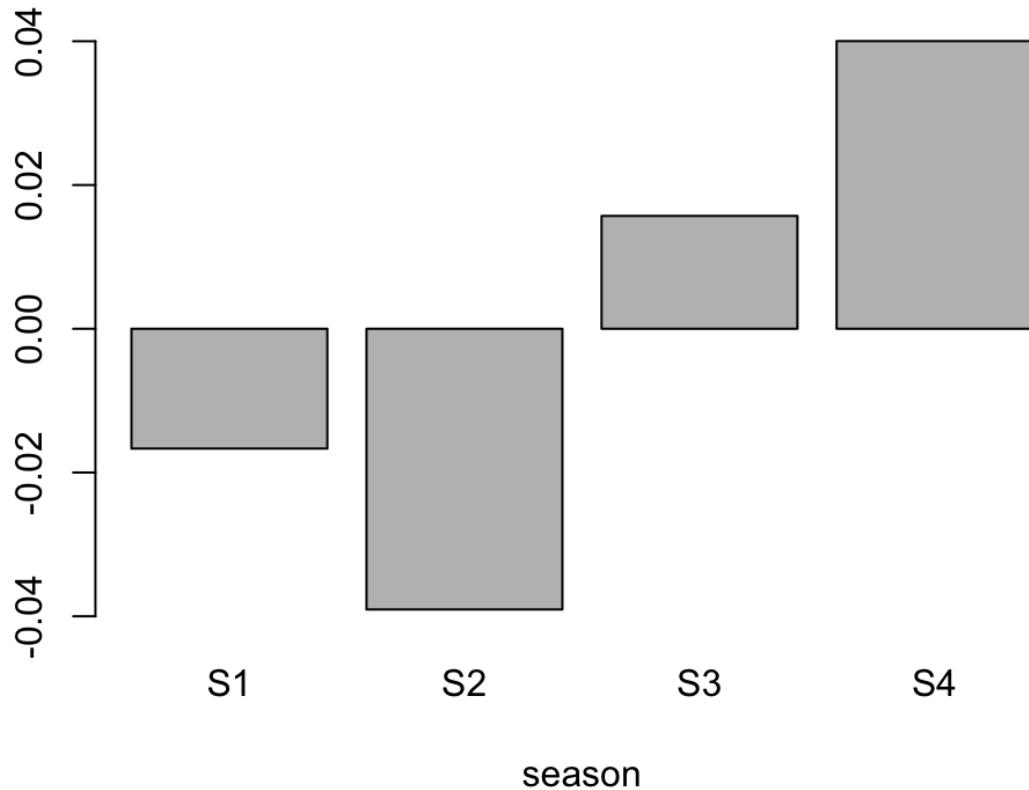
- The above chart shows that the Detrended series with the Cyclical component is computed with a moving average of size 5.
- It appears to have a local max near the year 1970 and a local minimum near the year 2000.
- There is higher volatility between 1950 and 1970 with a fluctuation of -10% to 10% of the trend.
- There is lower volatility after 1970 with a fluctuation between -5% to 5% of the trend.

Low Freq Component of data127 (log-scale)



There exist lower fluctuations. The curve is flatter. However, there are some fluctuations with a higher frequency. That could be the high-frequency component.

Co2 Seasonal Component

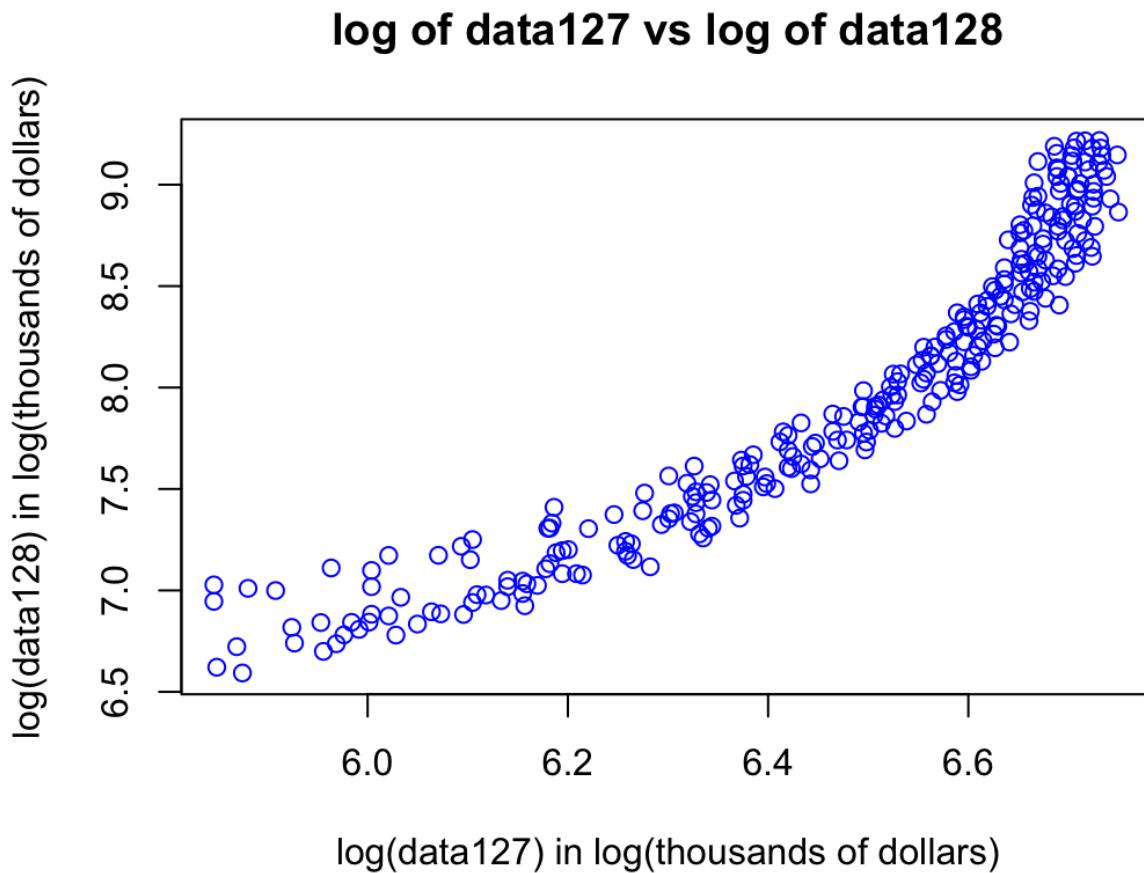


During the second season, the expenditure is around 4% less than the trend.

In the fourth season, it is around 4% higher than the trend.

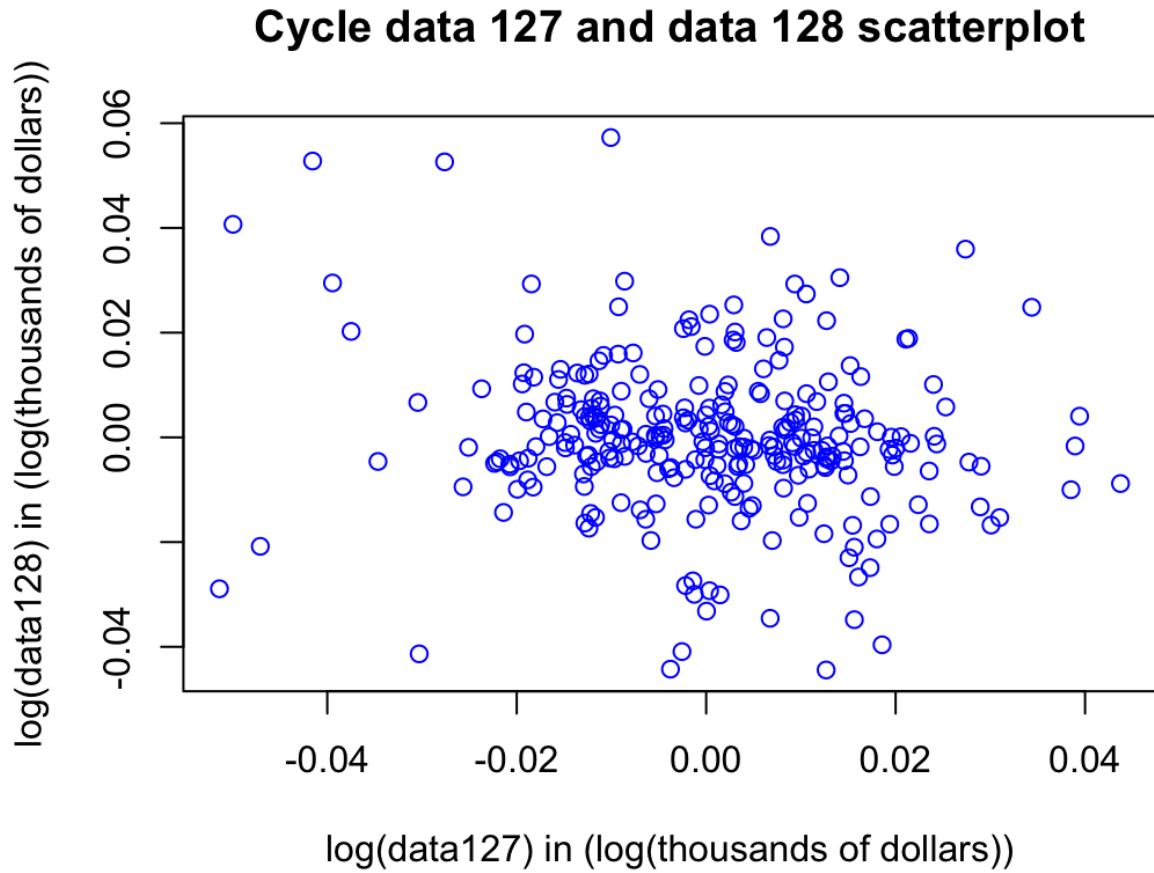
Part C:

a)



There exists a positive nonlinear co-movement between the log of data128 and the log of data 127. The relationship resembles the original data127 graph plotted in part A (a). However, while the log of data127 approaches 5.8, log data 128 approaches both 6.5 and 7.0. The branching behaviour could be explained by higher volatility as the log of data 127 is highly volatile while it's values are low, as seen in Part B - 3.

b)



There also exists weak linear negative comovement between the cyclic component of data 127 and data 128.