ECON 102: Mathematical Operations and Index Numbers Formulas

In this document, we present the most important formulas used in Module 2.

Date Convention

Flow Variables or Variables Representing Averages

The Date

The date corresponds to the center of the period:

- Annual: July 1^{st}.
- Monthly: The 15^{th} of each month.
- Quarterly: The dates of each quarter are respectively February 15^{th}, May 15^{th}, August 15^{th} and November 15^{th}.

The Decimal Date

- Annual: Year + 0.5. For example, 1980.5 for the year 1980.
- Monthly: Year + month/12 -1/24. For example, the decimal date for March 2020 is $2020 + 3/12 - 1/24 = 2020.208$.
- Quarterly: Year + Quarter/4 - 1/8. For example, the decimal date for the third quarter of 2019 is $2019 + 3/4 - 1/8 = 2019.625$

Stock Variables

In this course, we rarely work with stock variables. Most macro-economic variables that we analyze are either flow variables (GDP, consumption, etc.) or averages (interest rate, price index, population etc.).

The Date

The date corresponds to the last day of the period:

- Annual: December 31^{st}.
- Monthly: The last day of the month (28, 29, 30 or 31).
- Quarterly: The dates of each quarter are respectively March 31^{st}, June 30^{th}, September 30^{th} and December 31^{th}.

The Decimal Date

- Annual: Year + 1 For example, 1981 for the year 1980.
- Monthly: Year + month/12. For example, the decimal date for March 2020 is $2020 + 3/12 = 2020.25$.
- Quarterly: Year + Quarter/4. For example, the decimal date for the third quarter of 2019 is $2019 + 3/4 = 2019.75$

Growth Rate

Notation: X_t is the value of X at time t, X_{t-s} is the value of X s periods before t and X_{t+s} is the value of X s periods after t.

Growth Rate Between Two Periods (g_t)

Not in percentage:

$$g_t = \frac{X_t - X_{t-1}}{X_{t-1}}$$

In percentage

$$g_t = \frac{X_t - X_{t-1}}{X_{t-1}} \times 100$$

We can also write:

$$1 + g_t = \frac{X_t}{X_{t-1}}$$

Growth Rate of a Product

The growth rate of $(X_t Y_t)$ between $t-1$ and t is:

$$(1 + g_x) \times (1 + g_y)\,,$$

where g_x and g_y are the growth rates of X_t and Y_t between $t-1$ and t not expressed in percentage. You can then multiply by 100 to express the growth rate in percentage.

Growth Rate of a Product: An Approximation

The growth rate of $(X_t Y_t)$ between $t-1$ and t is approximately equal to:

$$g_x + g_y\,,$$

where g_x and g_y are the growth rates of X_t and Y_t between $t-1$ and t expressed in percentage or not. The approximation is better when g_x and g_y are small.

Growth Rate of a Ratio

The growth rate of (X_t/Y_t) between $t-1$ and t is:

$$\frac{(1 + g_x)}{(1 + g_y)} - 1\,,$$

where g_x and g_y are the growth rates of X_t and Y_t between $t-1$ and t not expressed in percentage. You can then multiply by 100 to express the growth rate in percentage.

Growth Rate of a Ratio: An Approximation

The growth rate of (X_t/Y_t) between $t-1$ and t is approximately equal to:

$$g_x - g_y\,,$$

where g_x and g_y are the growth rates of X_t and Y_t between $t-1$ and t expressed in percentage or not. The approximation is better when g_x and g_y are small.

Annualized Growth Rate: Monthly Series

Let X_t be a monthly series. The annualized growth rate is:

$$(1+g_t)^{12} - 1\,,$$

where g_t is the growth rate of X_t between $t-1$ and t not expressed in percentage. You can then multiply by 100 to express the growth rate in percentage.

Annualized Growth Rate: Quarterly Series

Let X_t be a quarterly series. The annualized growth rate is:

$$(1+g_t)^{4} - 1\,,$$

where g_t is the growth rate of X_t between $t-1$ and t not expressed in percentage. You can then multiply by 100 to express the growth rate in percentage.

Annualized Growth Rate: An Approximation

For monthly series, the annualized growth rate is approximately equal

$$g_t \times 12$$

and for quarterly series, if it approximately equal to

$$g_t \times 4\,,$$

where g_t is the growth rate of X_t between $t-1$ and t expressed in percentage or not. The approximations are better when the g_t is small.

Average Annual Growth Rate

Let X_t and X_s, $s > t$, be two observations distant by $(s-t)$ years, then the average annual growth rate over the period is

$$(1+g)^{1/(s-t)} - 1\,,$$

where

$$1 + g = \frac{X_s}{X_t}\,.$$

It is also approximately equal to

$$\frac{g}{s-t}$$

when g is small.

Log Approximation

The following only applies to strictly positive variables.

Log Properties

$$\log(ab) = \log(a) + \log(b)$$
$$\log(a/b) = \log(a) - \log(b)$$
$$\log(a^b) = b \times \log(a)$$

Growth rates

The growth rate between X_{t-1} and X_{t-1} is approximately equal to

$$\log(X_t) - \log(X_{t-1}) = \log(X_t/X_{t-1}) = \log(1 + g_t)\,,$$

where $(1 + g_t) = X_t/X_{t-1}$. The approximation is better when g_t is small.

Index Numbers

An index number I_{t/t_0} is a ratio of a variable X_t over the same variable at a base period X_{t_0} multiplied by 100:

$$I_{t/t_0} = \frac{X_t}{X_{t_0}} \times 100\,.$$

The index is called base $100 = t_0$.