IDC410

A course on Image Processing and Machine Learning

(Lecture 05)

Shashikant Dugad,

IISER Mohali

Filters (contd..)

Properties of Convolution Filter

- **Notation: $b = c \star a$**
- **Convolution is a multiplication-like operation**
  - Commutative: $a \star b = b \star a$
  - Associative: $a \star (b \star c) = (a \star b) \star c$
  - Distributes over addition: $a \star (b + c) = (a \star b) + (a \star c)$
  - Scalars factor out: $\alpha a \star b = a \star \alpha b = \alpha(a \star b)$
  - Identity: unit impulse $e = [\ldots, 0, 0, 1, 0, 0, \ldots]$: $a \star e = a$
- **Usefulness of associativity**
  - Often apply several filters one after another: $(((a \star b1) \star b2) \star b3)$
  - This is equivalent to applying one filter: $a * (b1 \star b2 \star b3)$

Padding and Strides

- In Convolutional Neural Networks (CNNs), stride refers to the step size by which the filter/kernel moves across the input image during the convolution operation in horizontal and vertical direction
  - Stride defines how big of steps filters should take (i.e., how many pixels our filters should skip) while sliding over the image
  - Minimum value of stride = 1
  - Smaller strides (1 or 2) offer detailed feature extraction, while larger strides (3+) helps in down-sampling.

- CNN is like a collection of small, overlapping magnifying glasses called filters. These filters scan over different parts of a image to find interesting features, like edges, shapes, or colours. These filters slide or convolve over the entire image as defined by the stride.

- The kernel size, stride value can substantially reduce the output size and computational efficiency of the network, influencing feature extraction and spatial dimensions of image.

Movement of Filter

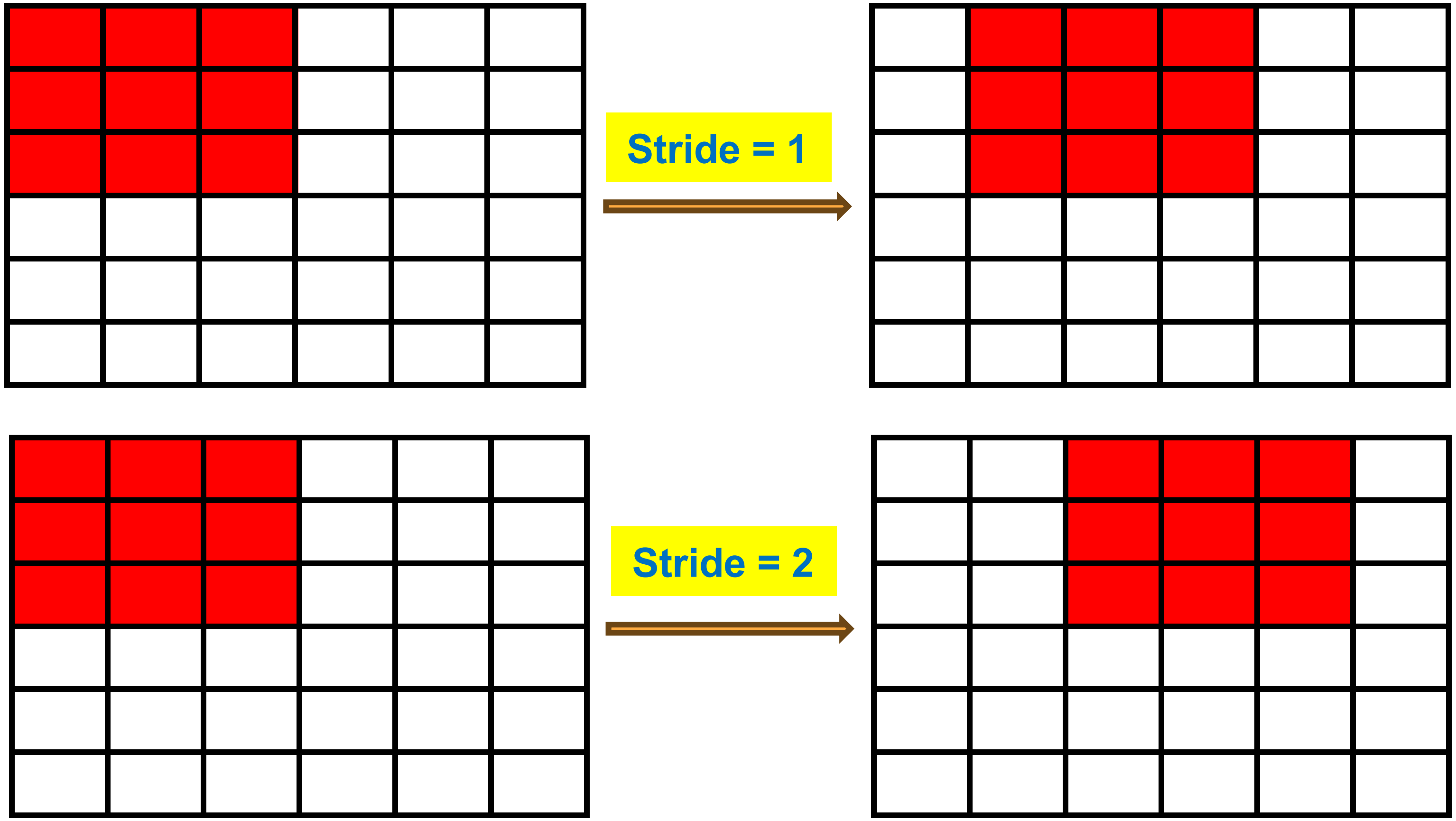


Image Padding

Input Image (5 x 5)

22	145	23	167	67
45	110	45	119	29
78	99	78	88	112
145	100	99	38	164
223	110	23	45	29

Pad = 1 →

Output Image (7 x 7) [Pad = 1]

0	0	0	0	0	0	0
0	22	145	23	167	67	0
0	45	110	45	119	29	0
0	78	99	78	88	112	0
0	145	100	99	38	164	0
0	223	110	23	45	29	0
0	0	0	0	0	0	0

Input Image (5 x 5)

22	145	23	167	67
45	110	45	119	29
78	99	78	88	112
145	100	99	38	164
223	110	23	45	29

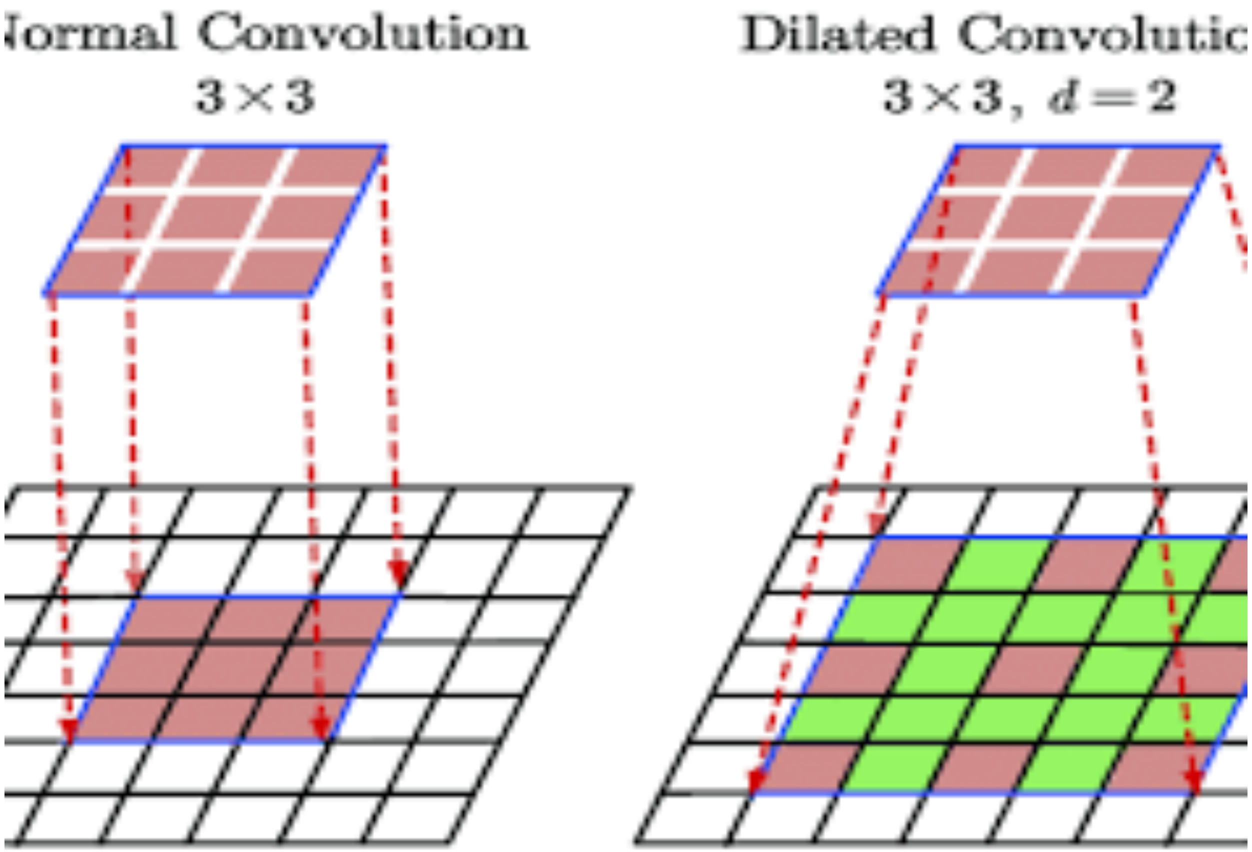
Pad = 2 →

Output Image (9 x 9) [Pad = 2]

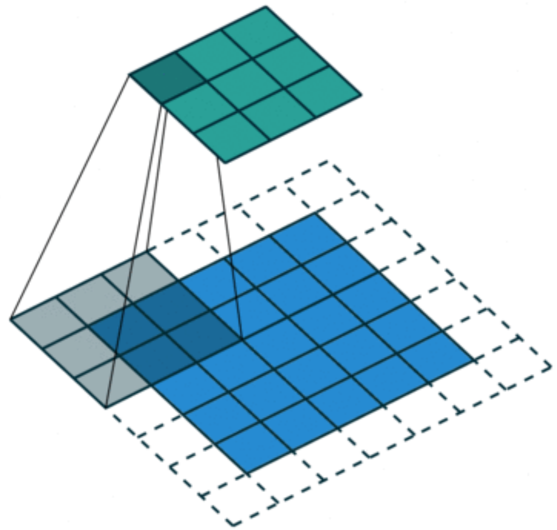
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	22	145	23	167	67	0	0
0	0	45	110	45	119	29	0	0
0	0	78	99	78	88	112	0	0
0	0	145	100	99	38	164	0	0
0	0	223	110	23	45	29	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

- **Padding means adding extra columns and rows with ZERO pixel intensity before doing any operations.**
- **Helps in keeping the spatial info intact, particularly prevents data loss at the edges of the image, hence stabilises training**
- **It also helps to keep the output size consistent with the input and makes training more stable.**

Dialted Image Convolution



Padding and Strides



- **Image Dimension: N_{row} X M_{col}**
- **Padding: P   Strides: S_{row}, $S_{col.}$ Dilation: D_{row}, D_{col}**
- **Kernal Size: K_{row}, K_{col}**

$$N_{row}^{out} = \frac{N_{row}^{in} + 2\,X\,P \quad - D_{row}\,X\,(K_{row} - 1) - 1}{S_{row}} + 1$$

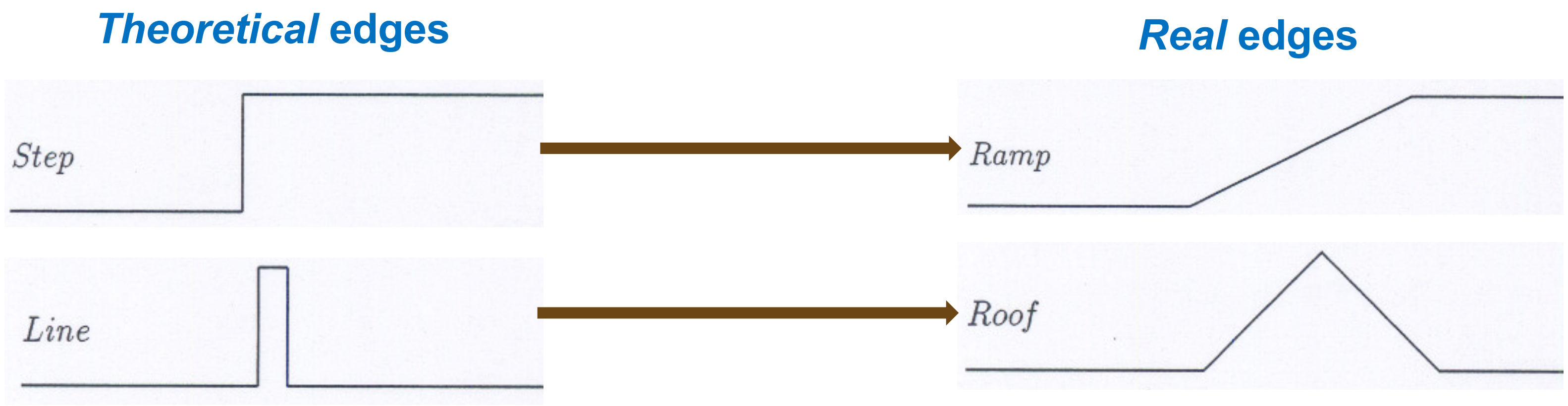
$$M_{col}^{out} = \frac{M_{col}^{in} + 2\,X\,P \quad - D_{col}\,X\,(K_{col} - 1) - 1}{S_{col}} + 1$$

Edges

Edges

- Feature extraction is one of the crucial steps in image processing and machine learning
- Edges in the image is one such key feature.
  - Edges typically occur on the boundary between two different regions in an image
  - An edge in an image is usually associated with a discontinuity in the image intensity resulting large difference or a large amplitude of the first derivative of the image intensity
- Discontinuities in the image intensity can be either
  - Step discontinuities: the intensity abruptly changes from one value on one side of the discontinuity to a different value on the opposite side
  - Line discontinuities: the intensity abruptly changes value but then returns to the starting value within some short distance.
- However, step and line edges are rare in real images due to the low-frequency components or the smoothing of the image.
- Step edges become ramp edges and line edges become roof edges

Edge Profile and Gradient



- An edge is associated with the maxima in the first derivative (gradient) of intensity in local region of an image
- The gradient is the two-dimensional equivalent of the first derivative and is defined as the vector

$$\vec{G}(f(x,y)) = \begin{pmatrix} G_x \\ G_y \end{pmatrix} = \begin{pmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{pmatrix}$$

Gradient

- **The vector G[f(x, y)] points in the direction of the maximum rate of increase of the function f(x, y)**

- **The magnitude of the gradient, given by**

$$|\boldsymbol{G}(\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})| = \sqrt{\boldsymbol{G}_{\boldsymbol{x}}^{\mathbf{2}} + \boldsymbol{G}_{\boldsymbol{y}}^{\mathbf{2}}}$$

- **The *direction* of the gradient is defined as,**

$$\boldsymbol{\alpha}(\boldsymbol{x},\boldsymbol{y}) = \tan^{-1}\left(\frac{\boldsymbol{G}_{\boldsymbol{y}}}{\boldsymbol{G}_{\boldsymbol{x}}}\right)$$

Gradient for digital image

- **Gradient for a digital image can be defined as,**

$$G_x(i,j) \cong f(i,j+1) - f(i,j). \qquad G_y(i,j) \cong f(i+1,j) - f(i,j)$$

- **These can be implemented with simple convolution masks as shown below:**

$G_x = \begin{array}{|c|c|} \hline -1 & +1 \\ \hline \end{array}$ $\qquad G_y = \begin{array}{|c|} \hline +1 \\ \hline -1 \\ \hline \end{array}$

- **Gradient has to be computed at exactly the same position in space.**
    - **However, gradients G_x and G_y are calculated at different points, [i, j+ 1/2] and [i+1/2, j] which are NOT the same**
    - **3x3 convolution mask is preferred to maintain this criteria**

Edge Detection Operators

- **Roberts Operator: Provides a convolution mask for following simple gradient operation: $G[f(i, j)] = G_x + G_y = |f(i, j) - f(i+1, j+1)| + |f(i+1, j) - f(i, j+1)|$**

- $G_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\qquad G_y = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

- **The Roberts operator is NOT located at the desired point [i,j].**

- **In order to have an edge operation at fixed point [i,j] in both the directions, we should have a mask of 3x3 size**

- **Sobel operator is the magnitude of the gradient computed by,**

$$M = \sqrt{S_x^2 + S_y^2}$$

Edge Detection Operators

- Partial derivative are defined as per following matrix:

a0	a1	a2
a7	[i, j]	a3
a6	a5	a4

$S_x = (a2-a0) + c(a3-a7) + (a4-a6)$

$S_y = (a0-a6) + c(a1-a5) + (a2-a6)$

Higher weightage can be given to closer pixels by appropriately choosing c

- *For Sobel operator c = 2;* S_x and S_y can be implemented using the following convolution masks:

S_x =

-1	0	1
-2	0	2
-1	0	1

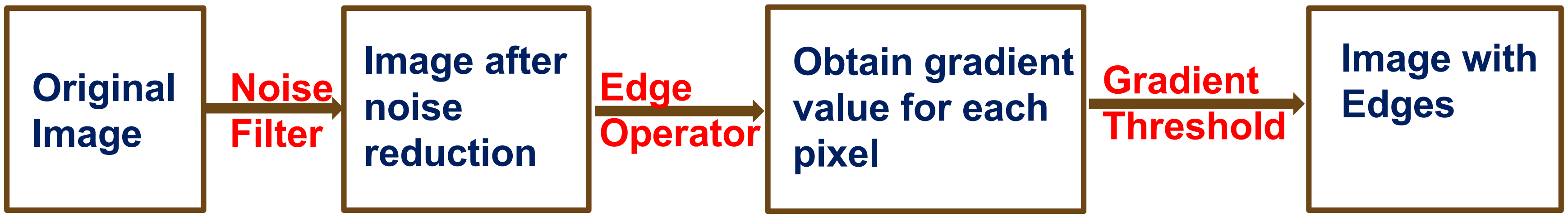
S_y =

1	2	1
0	0	0
-1	-2	-1

- For Prewitt Operator c = 1: No emphasis is given to the closer pixels! Edge operators obtained with c = 1

Application of Edge Detection Technique

- Direct application of edge operator on a image may result in fake edges due to the noise in image
- First remove noise in the image
- Choose one of the edge detection operator appropriately and apply it on the image
- Obtain gradient for each pixel and apply a threshold on the gradient value to identify the pixel as a pixel representing edge



Edge Detection with and without Noise Filter

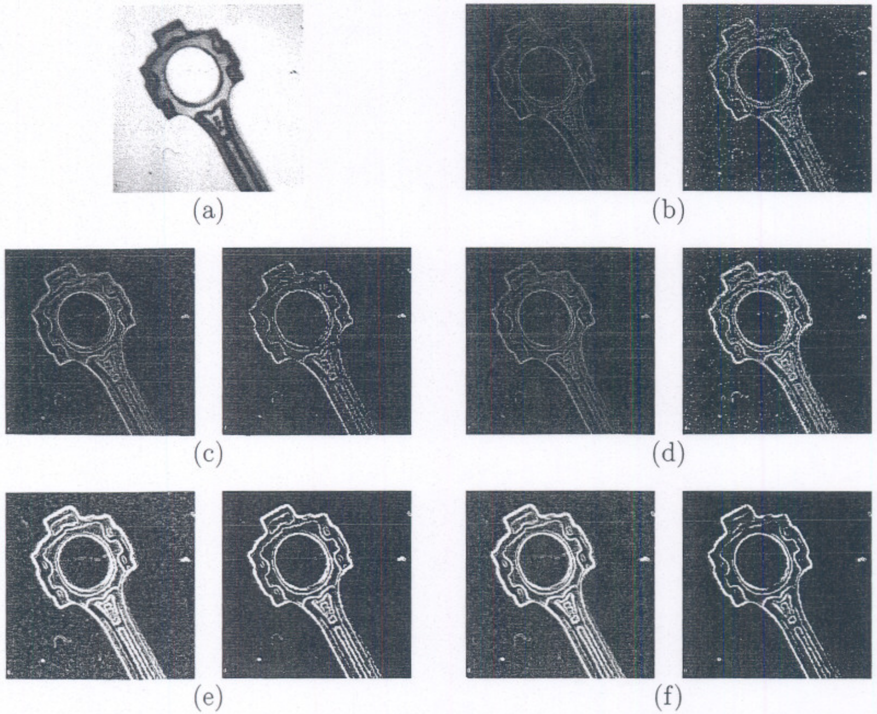


Figure 5.7: A comparison of various edge detectors on a noisy image without filtering. (a) Noisy image. (b) Simple gradient using 1×2 and 2×1 masks, $T = 64$. (c) Gradient using 2×2 masks, $T = 128$. (d) Roberts cross operator, $T = 64$. (e) Sobel operator, $T = 225$. (f) Prewitt operator, $T = 225$.

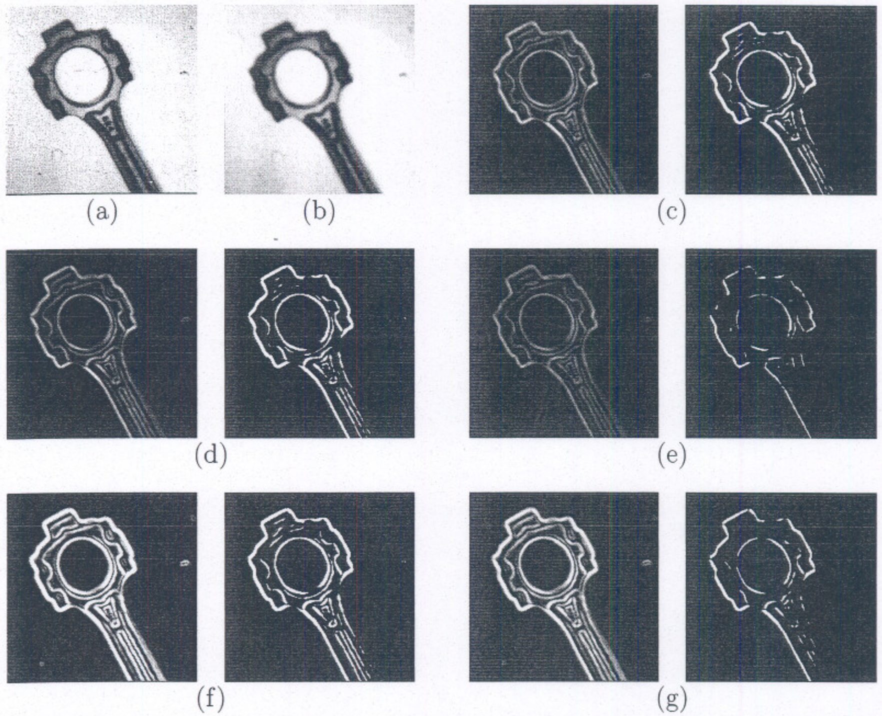
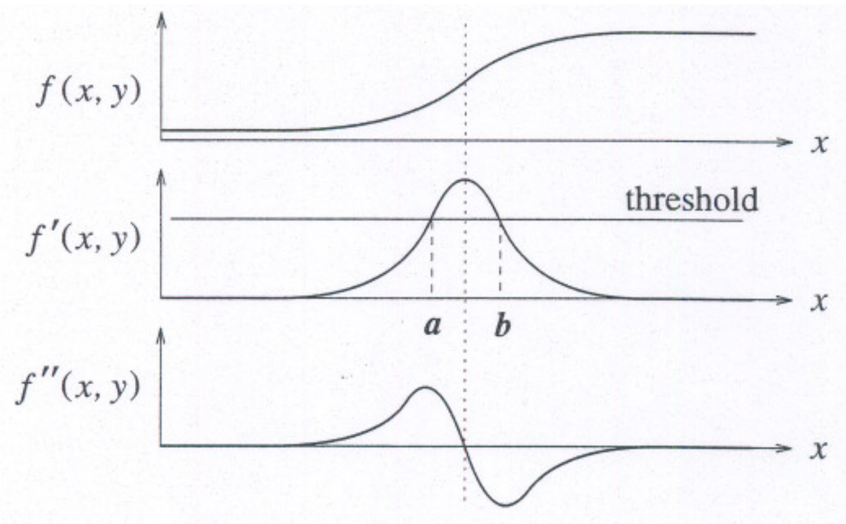


Figure 5.6: A comparison of various edge detectors on a noisy image. (a) Noisy image. (b) Filtered image. (c) Simple gradient using 1×2 and 2×1 masks, $T = 32$. (d) Gradient using 2×2 masks, $T = 64$. (e) Roberts cross operator, $T = 64$. (f) Sobel operator, $T = 225$. (g) Prewitt operator, $T = 225$.

https://cse.usf.edu/~r1k/MachineVisionBook/MachineVision.files/

Edge Detection with Second Derivative Operators

- A single derivate edge operators with a threshold provides too many edge points depending on noise in the image
- A better approach would be to find only the points that have local maxima in gradient values and consider them a edge points.
  - This means that at edge points, there will be a peak in the first derivative and, equivalently, there will be a zero crossing in the second derivative.



If a threshold is used for detection of edges, all points between *a* and *b* will be marked as edge pixels. However, by removing points that are *not* a local maximum in the first derivative, edges can be detected more accurately. Local maximum in the first derivative corresponds to a zero crossing in the second derivative.

Edge Detection with Second Derivative Operators

- There are two operators in two dimensions that correspond to the second derivative: the Laplacian and second directional derivative.

$$Gx = \frac{\partial f(x,y)}{\partial x} = f(i,j+1) - f(i,j)$$

$$\therefore \quad \frac{\partial^2 f(x,y)}{\partial x^2} = \frac{\partial G_x}{\partial x} = \frac{\partial f(i,j+1)}{\partial x} - \frac{\partial f(i,j)}{dx} = f(i,j+2) - f(i,j+1) - [f(i,j+1) + f(i,j)]$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} = f(i,j+2) - 2f(i,j+1) + f(i,j)$$

- However, this approximation is centered about the pixel *[i,j* + 1]. Therefore, by replacing j with j - 1,

$$\frac{\partial^2 f(x,y)}{\partial x^2} = f(i,j+1) - 2f(i,j) + f(i,j-1). \qquad \frac{\partial^2 f(x,y)}{\partial y^2} = f(i+1,j) - 2f(i,j) + f(i-1,j)$$

Second Derivative Laplacian Operators

- **By combining these two equations into a single operator, the following mask can be used to approximate the Laplacian:**

$$\nabla^2 = \nabla^2_x + \nabla^2_y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- **It is desired to give weight to the corner pixels as below:**

$$\nabla^2 = \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

Laplacian of Gaussian (LoG) Operators

- Very small local peaks in the first derivative will also result in zero crossings the double derivative operators are quite sensitive to the noise

- The Laplacian operators has to be used in conjunction with powerful filtering methods

- Laplacian operator combined Gaussian filter referred as LoG operator

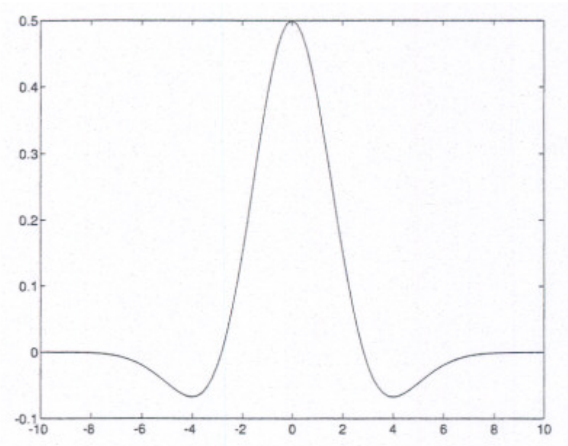
- The detection criterion: presence of a zero crossing in the second derivative with a corresponding large peak in the first derivative.

Laplacian of Gaussian (LoG) Operators

- **The output of the LoG operator, h(x, y), is obtained by the convolution operation**
- **h(x,y) = ∇^2[g(x,y)★f(x,y)] = [∇^2g(x,y)]★f(x,y)**

$$\nabla^2 g(x,y) = \left(\frac{x^2 + y^2 - 2\sigma^2}{\sigma^4}\right) e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}$$

- **∇^2g(x,y) is offerent referred as *Mexican Hat operator* as shown in Figure**



5x5 LoG Convolution Mask

0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

Laplacian of Gaussian (LoG) Operators

- **Zero crossings may happen due to the noisy region of image**
- **The slope of the zero crossing depends on the contrast or sharpness of the change in image intensity across the edge.**
- **To obtain real edges in an image, it may be necessary to combine information from operators with several filter sizes or look at the amplitude of variation (1^{st} derivative)**
- **A larger σ results in better noise filtering but may lose important edge information, which may affect the performance of an edge detector. If a small filter is used, there is likely to be more noise due to insufficient averaging.**

Laplacian of Gaussian (LoG) Operators

- **The LoG operator which is symmetric; can reduce noise by smoothing the image, but it also dilutes the real edges resulting in uncertainty to the accurate location of the edge**

- **The gradient may have greater sensitivity to the presence of edges, but it has higher sensitivity to the noise.**

- **There is a trade-off between noise suppression, edge determination and localization.**

- **The linear operator that provides the best compromise between noise immunity and localization, is the first derivative of a Gaussian.**