PHY101, Mechanics
Mid-semester Exam
17/02/2022, IISER Mohali
Time Limit: 60 Minutes, Maximum points: 20

You will need to write down your answers clearly on your notebooks. After the exam, you will need to immediately make pdf files (as **YOURFULLNAME-ROLLNUMBER.pdf**) of your answer sheets and then send the files to your tutorial group email ID. The files must be sent before 10:06 am. Please also Cc your email to goutam@iisermohali.ac.in.

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1. Lisa is standing inside an elevator (with glass walls and no roof) which is accelerating downward with an acceleration a and $a < g$, the acc. due to gravity. Aman is standing on the ground. When Lisa is at a height h from the ground and her speed is just u (w.r.t. Aman), she throws a soft ball of mass m in the vertically upward direction with an initial speed u' (w.r.t. Lisa) and $u' < u$. In general, the air drag is given by $F_{drag} = -kv$, where k is a constant and v is the velocity. Assume g to be the acceleration due to gravity and does not depend on h.

   (a) (1.5 points) Write the force equation for the ball from Lisa's frame.

   (b) (1.5 points) Write the force equation for the ball from Aman's frame.

   Hint: *The drag force depends on the relative velocity between an object and the air (which, as you can imagine, is not moving in this case).*

2. A point P is fixed at $\vec{r}$ as observed by Aman who is standing at the origin of an inertial coordinate system. Imagine Lisa is also standing at the same origin, but is using a different coordinate system which is rotating about Aman's z-axis, with an angular velocity $\vec{\omega}(t)$. Note that the angular velocity is changing with time.

   (a) (1 point) Show that the time derivative of the vector $\vec{\omega}(t)$ is same in both Aman and Lisa's frames.

   (b) (2.5 points) Find the acceleration of the point P as seen from Lisa's frame.

   Hint: *For the sake of simplicity (and without loss of generality), you may assume that the position vector seen by both Aman and Lisa at the instant of interest is the same. This means, when the observation is made, the two coordinate systems are "instantaneously aligned".*

3. Consider a light (negligible mass) spring of relaxed length l and stiffness κ. The spring is rotating with a constant angular velocity ω on a smooth (frictionless) horizontal plane, with one end of the spring fixed on the rotation axis. A particle of mass m is attached at the other end of the spring which remains at a fixed distance $l + \Delta l$ from the rotation axis. Ignore gravity.

(a) (1.5 points) Write down the force equation in the rotating frame in which the spring and the mass are at rest.

(b) (2 points) Write down the force equation in the frame of an inertial observer located at the fixed end of the spring.

Hint: *I am sure you know what are the forces acting here.*

4. (2 points) Consider a solid sphere of initial mass M and radius B is at rest. You somehow (just imagine!) removed a smaller solid sphere of mass m and radius b from the bigger sphere without making it move. The position vector of the center of the hollow spherical part is given by $\vec{r} = x_b\hat{x} + y_b\hat{y}$ as measured from the center of the bigger sphere. Find out the position of the center of mass of the sphere with the hole.

   Hint: *There are multiple ways of doing it. In one of the simple methods, one may actually argue that removing some mass from an object is equivalent to adding a "negative" mass. Well, at least mathematically!* .

5. (3 points) A bomb (mass M) thrown vertically upward from the ground (initial velocity u) explodes into three equally massive fragments as soon as it reaches the maximum height h. The fragments fly in different directions. One of the fragments comes straight to the ground in a time t_1 after the explosion. The other two reach the ground at a time t_2 after the explosion. Find h in terms of g, t_1 and t_2.

   g is the acceleration due to gravity which remains almost constant within the height h.

   Hint: *Notice that the relationship between the vertical component of the velocity of the fragments after the explosion may help you get it quick.*

6. (3 points) Consider a bucket of Mass M (when empty) contains water in it. Initially the bucket is at rest on the ground. At time $t = 0$, someone starts lifting the bucket from the top of a tower by a rope exerting a constant force F on the bucket. At $t = 0$, the total mass of water in the bucket was m. The bucket loses water through a leak at a steady rate and it becomes completely empty after a time t_0. Find the velocity of the bucket at the instant it just becomes completely empty.

   Hint: *The problem is (relatively) easy because the loss happens at a steady rate and you just need to write down the equation of motion and solve it. The solution will involve a simple integration which all of you said that you know about.*

7. (2 points) An artificial satellite of mass m orbits around the Earth (mass M_e) following a circular trajectory near the surface of the Earth and takes a time T per complete revolution. The same satellite also takes the same time for one complete revolution if it orbits in a circular trajectory around the Moon (mass M_m) and near the surface of the Moon. Find out the ratio of the mass densities of the Earth and the Moon.

   Hint: *This needs no hints, I guess! You know what are the forces to be considered and the definition of mass density. Do not forget that both the Earth and the Moon can be safely assumed to be spherical and with uniform mass density!*