PHY101 (Mechanics): 2nd Mid-semester Examination

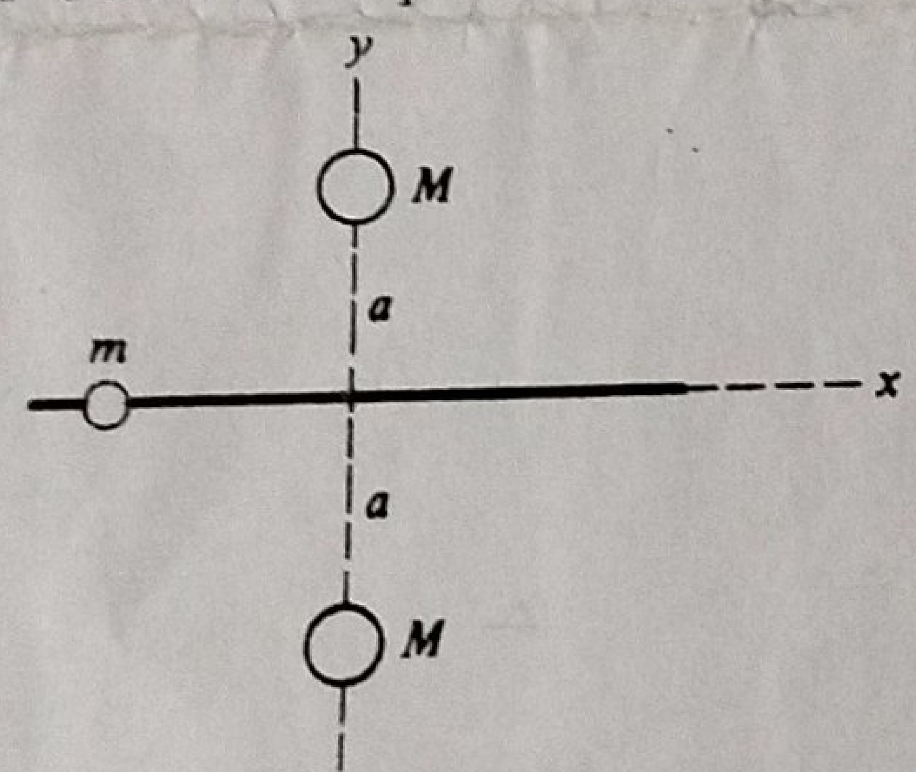
Duration: **60** minutes; Maximum marks: **20**

1. A rocket of initial mass M_0 ejects its burnt fuel at a constant rate $|dM/dt| = \mu$, and at a speed v_0 relative to the rocket.
   (a) Find the expression for the initial upward acceleration of the rocket.
   (b) If $v_0 = 200$ m/s and $M_0 = 1000$ kg, how many kg of fuel must be ejected per second to give an initial upward acceleration of $2g$ to the rocket?
   [Be careful with the signs while solving this problem.] **[3+2]**

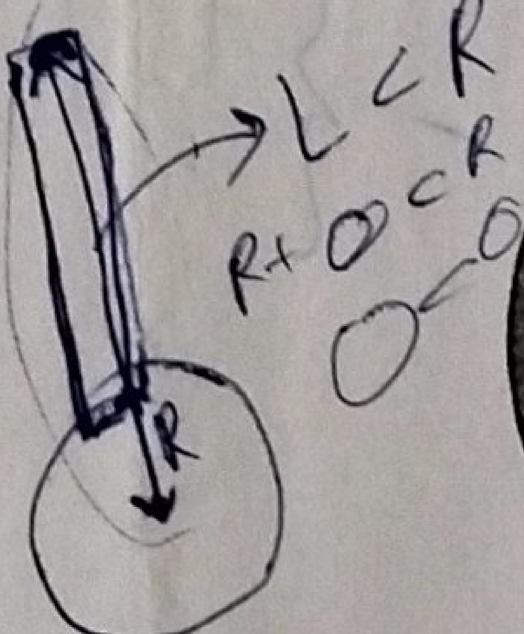
$\vec{v} = -\ \vec{v}_{EG}$ $\vec{V}_{RG} = \vec{V}_{RE} + \vec{V}_{EG}$ $\vec{V}_{EG} = \vec{V}_{RG} + \vec{V}_{ER}$

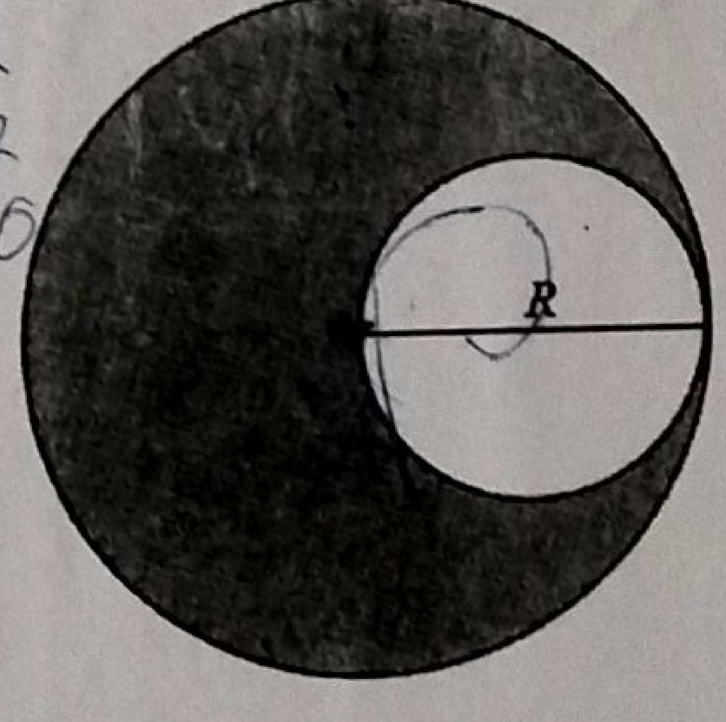
$\frac{M}{4}\left(\frac{R}{2}\right)^2 + \frac{1}{2} \times \frac{M}{4} \times \frac{R^2}{4}$ $\frac{2MR^2}{32} + \frac{MR^2}{32} = \frac{3MR^2}{32}$

2. A bead of mass m slides on a frictionless rod along x-axis. The rod is equidistant from two spheres, each of mass M, fixed at locations $(0, a)$ and $(0, -a)$ (as shown below), and attract the bead gravitationally.
   (a) Express the potential energy of the bead in terms of x.
   (b) Bead is released at $x = -3a$ with velocity v_0 towards the origin. Find the speed of the bead as it passes the point $(0, 0)$. **[3+3]**



(a) Figure for problem 2.





(b) Figure for problem 3.

$\frac{M}{4} \times \frac{R^2}{4} + \frac{MR^2}{4}$

$\frac{M}{4} \times \frac{R^2}{4} + \frac{M}{4} \times$

3. Consider a solid disk of mass M and radius R. A circular cavity of diameter R is created leading to the shape shown in the figure above. Find the moment of intertia about an axis perpendicular to the disk and passing through, (i) the center of the original disk and (ii) the centre of the cavity. **[2+2]**

$T = 2\pi \sqrt{\frac{2R\sqrt{2}}{2g}} = 2\pi\sqrt{\frac{\sqrt{2}R}{g}} = 2 \times 3.14 \sqrt{\frac{1.4 \times 1.4}{10}} = 2 \times 3.14 \times$

4. A physical pendulum is made of a uniform disk of radius $R = 14$ cm and mass $M = 0.4$ kg suspended from a rod of negligible mass. The distance from the pivot to the center of the disk is L. What value of L makes the period a minimum? What is the value of this minimum period in seconds? *[$L < R$ is also allowed. Use $g = 10$ m/s^2].* **[3+2]**

$2\pi\sqrt{\frac{\sqrt{2}R}{g}}$ 1.414 0.014 0.019796 $2\pi \frac{dT}{dL} = -\frac{4\pi^2 R^2}{2gL^2}$ $\sqrt{\frac{\frac{MR^2}{2} + ML^2}{MgL}}$ 0.28×3.14 $L^2 << R^2$ $2\pi\sqrt{\frac{R^2}{2gL}}$ $T^2 = \frac{4\pi^2 R^2}{2gL}$ 3.14 × 28 = 2512 + 628 = 0.8792

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PHY101 (Mechanics): End-semester Examination; Part B

Duration (**Part A + Part B**): **180** minutes; Maximum marks: **40**

1. Two blocks of masses 9 kg and 1 kg, hang from the ends of a string that passes around a pulley of mass 40 kg and radius 0.5 m ($I_p = \frac{1}{2}MR^2$). The system is released from rest and the pulley starts rotating without slipping against the string.
   (a) Write down the Newton's equations for the two masses.
   (b) What is the acceleration of the 9 kg mass?
   (c) What is the angular velocity of the pulley after the 9 kg mass has dropped by 2 m?
   (d) What are the tensions in the two parts of the string? **[2+3+3+2]**

2. Consider a potential energy function, $U(x) = \epsilon\left[\frac{1}{2}(x-x_0)^4 - 4(x-x_0)^2\right]$.
   (a) Find the values of x where the potential is minimum.
   (b) Find the minimum value of the potential.
   (c) Find the frequency of small oscillations about the minimum for a mass m. **[2+2+2]**

3. A child's hoop of radius R and mass M rolls in a straight line with velocity v. Its top is given a light tap with a stick at right angle to the direction of motion. The impulse of the tap is I. Assuming that the gyroscope approximation holds, and neglecting friction with the ground in comparison to the impulse force, find the angle by which the direction of the line of rolling changes after the tap. What is the condition on the peak force exerted during the impulse for the gyroscope approximation to hold? **[4+2]**

4. Consider a ladder leaning against a frictionless wall. If the coefficient of friction between the ground and the ladder is μ, find the smallest angle the ladder can make with the ground without slipping. **[4]**

5. Train A of proper length L_0 moves eastwards with speed $v = 0.8c$ and train B of proper length $2L_0$ moves westwards with speed $v = 0.8c$ as observed in the ground frame. How much time does it take for the trains to pass each other in, (a) A's frame, (b) B's frame? Passing interval is defined as the time between the front ends of the trains coinciding and the back ends coinciding. (c) In which frame is time interval for passing larger? **[3+3+1]**

6. An object of mass 0.2 kg is hung from a spring with $k = 80$ N/m. The object is subjected to a resistive force $-bv$, where v is the velocity and $b = 4$ Nm^{-1}sec.
   (a) Write down the differential equation for the oscillator.
   (b) Find the quality factor (Q value) of the oscillator.
   (c) Find the period of oscillations.
   (d) The object is now subjected to a driving force $F = F_0 \sin(\omega t)$. Derive the expression for the amplitude of forced oscillations in the steady state. what is the amplitude of forced oscillations if $F_0 = 2$ N and $\omega = 30$ rad/sec? **[1+1+2+3]**