MTH101: Symmetry
Problem Set 3

Problem 1. Determine whether the following vectors are linearly independent.

$$\begin{bmatrix} 2 \\ -1 \\ 4 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} 4 \\ 7 \\ 9 \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} 3 \\ 3 \\ 4 \\ 2 \end{bmatrix}$$

Problem 2. Determine whether the vector $\mathbf{v}_3$ is in $Span(\mathbf{v}_1, \mathbf{v}_2)$ where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} \qquad \text{and} \qquad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

Problem 3. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 3 & 2 & -1 \\ 12 & -10 & 7 \end{bmatrix}$. What is $\dim(Im(T))$? What is $Ker(\dim(T))$? Find a basis for $Im(T)$.

Problem 4. Find a basis for the space of solutions of the following system:

$$\begin{aligned} 2X_1 + 4X_2 - 4X_3 + 7X_4 &= 0 \\ -X_1 + 4X_2 + 2X_3 - 4X_4 &= 0 \end{aligned}$$