MTH 102 - ANALYSIS IN ONE VARIABLE
ASSIGNMENT 1

1. Prove $3 + 11 + \; + (8n5) = 4n^2 - n$ for all positive integers n.

2. Prove $1 + \frac{1}{2} + \frac{1}{4} + \; + \frac{1}{2^n} = 2 - \frac{1}{2^n}$ for all positive integers n.

3. The principle of mathematical induction can be extended as follows. A list $P_m, P_{m+1}, ...$ of propositions is true provided

(1) P_m is true,
(2) P_{n+1} is true whenever P_n is true and $n \geq m$.

a. Prove $n^2 > n + 1$ for all integers $n \geq 2$.
b. Prove $n! > n^2$ for all integers $n \geq 4$. [Recall $n! = n(n-1)\cdots 21$;]

4. Determine for which integers the inequality $2^n > n^2$ is true. Prove your claim by mathematical induction.

5. For each $n \in \mathbb{N}$, let P_n denote the assertion $n^2 + 5n + 1$ is an even integer.

(1) Prove P_{n+1} is true whenever P_n is true.
(2) For which n is P_n actually true? What is the moral of this exercise ?

6. Prove $7^n - 6n - 1$ is divisible by 36 for all positive integers n.

7. Show $\sqrt[3]{5 - \sqrt{3}}$ is not a rational number.

8. Show $[3 + \sqrt{2}]^{\frac{2}{3}}$ is not a rational number.

9. Show the following irrational-looking expressions are actually rational numbers:

(1) $\sqrt{4 + 2\sqrt{3}} - \sqrt{3}$,
(2) $\sqrt{6 + 4\sqrt{2}} - \sqrt{2}$.

10. . Given $e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots$, and $s_k = \sum_{k=0}^{n} \frac{(-1)^k}{k!}$, prove

$$e^{-1} - s_{2n+1} > 0, \qquad \text{for all } n \in \mathbb{N}.$$