PHY101, Mechanics
Final Exam
08/04/2022, IISER Mohali
Time Limit: 3 hours, Maximum points: 60

You will need to write down your answers clearly on your notebooks. After the exam, you will need to immediately make pdf files (as **YOURFULLNAME-ROLLNUMBER.pdf**) of your answer sheets and then send the files to your tutorial group email ID. The files must be sent before 12:06 pm. Please also Cc your email to iisergoutam@gmail.com

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1. $\vec{A} = (1/2)\hat{r} + (3/4)\hat{\theta} + (1/5)\hat{\phi}$, $\vec{B} = (2)\hat{r} + (4/3)\hat{\theta}$ and $\vec{C} = \vec{A} + \vec{B}$, where $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are three unit vectors and are orthogonal to each other. Find the following:
   (a) (1 point) $\vec{A} \times \vec{B}$
   (b) (1 point) Magnitude of $\vec{C}$
   (c) (1 point) $\vec{A}.\vec{C}$

2. (2 points) An elevator starts ascending vertically upward from the ground at time $t = 0$ with a uniform speed u. At time $t = T_1$ a child standing inside the elevator drops a marble of mass m through the floor. The marble falls with uniform acceleration due to gravity (g), and hits the ground at time $t = T_2$. Find the height of the elevator at time T_1.

3. Are the following forces conservative?
   (a) (1 point) $\vec{F} = \frac{y^2}{x}\ \hat{x} + 2y\ln(\frac{x}{b})\ \hat{y}$, b is a constant number.
   (b) (1 point) $\vec{F} = x^2\hat{x} + 2xy\hat{y} + y^2\hat{z}$

4. A bead of mass m slides on a frictionless rod along x-axis(see Figure 1). The rod is equidistant from two spheres, each of mass M, fixed at locations $(0, a)$ and $(0, -a)$ and attract the bead gravitationally.
   (a) (3 points) Express the potential energy of the bead in terms of x.
   (b) (4 points) The bead is released at $x = -3a$ with velocity v_0 towards the origin. Find the speed of the bead as it passes the point $(0, 0)$.

5. (5 points) See Figure 2. A ball of mass M is initially at rest at a distance L from a hard wall (located at $x = 0$). Another ball of mass m ($m < M$) starts moving from $x = l$ with uniform velocity $u\hat{x}$, given $l < L$. After some time, the mass m collides with the mass M and makes it move along $\hat{x}$. Following the collision, the mass m moves along $-\hat{x}$, collides with the hard wall after a while and again moves along $\hat{x}$. All collisions are elastic and there is no friction. For what ratio of m and M a second collision between the two balls will be possible?

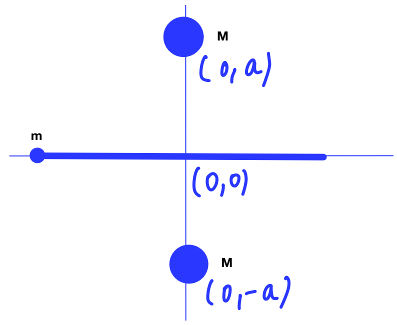

Figure 1: For problem 4

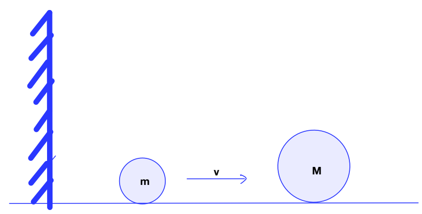

Figure 2: For problem 5

6. (4 points) A simple pendulum with a bob of mass m and string length l is hanging from a point A on the roof of a bus (mass M). The bus is accelerating with an acceleration $\vec{a} = -a\hat{x}$. Find the angle (θ_c) that the string will make with a vertical line (running through A) at which the pendulum will remain at rest relative to an observer sitting inside the bus (and accelerating with the bus).

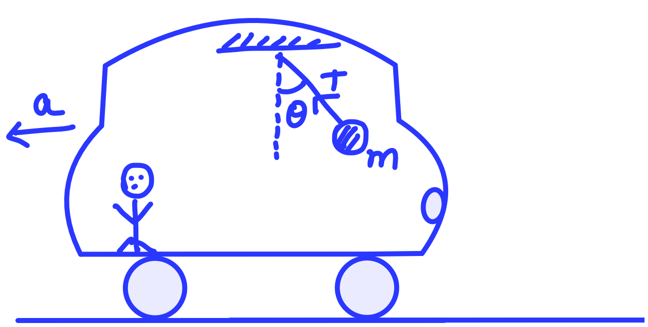

Figure 3: For problem 6

7. (4 points) As shown in Figure 4, consider a thin triangular (equilateral, side a) plate of

mass m and uniform mass density. A triangular (equilateral, side b) portion has been removed from the bigger plate. The centroids of both the triangles are at the same point. Calculate the moment of inertia of the system about the centroid.

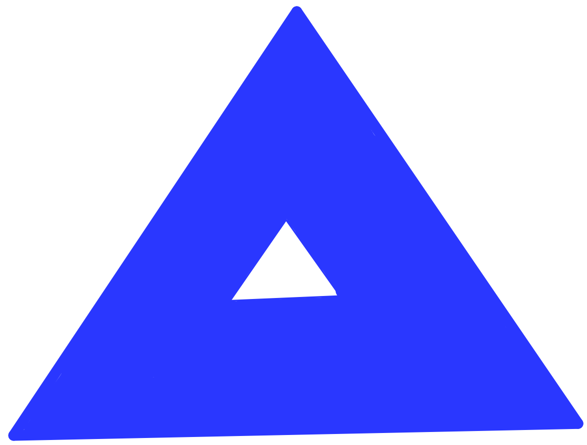

Figure 4: For problem 7

8. (4 points) Two masses m_1 and m_2 are connected by a massless spring of spring constant k and natural length l. The masses are free to slide on a frictionless plane. Initially m_2 is pressed (towards the wall) such that m_1 gets pressed against the wall, and the distance between m_1 and m_2 is $l/2$. The mass m_2 is released at $t = 0$. Find the position of the center of mass as a function of time.

9. A raindrop of mass M_0 starts falling from rest under gravity. Assume that the drop gains mass from the cloud at a rate proportional to its instantaneous mass and its instantaneous velocity $dM/dt = KMV$, where K is constant.
   (a) (3 points) Show that the speed of the drop eventually becomes constant.
   (b) (3 points) Find the expression for the terminal speed.

10. Consider a spherical planet (non-rotating) with mass M and radius R. A projectile of mass $m << M$ is fired from a point on the surface of the planet with a velocity v at an angle $\theta = 45^o$ with the radial direction ($\hat{r}$). In its subsequent trajectory, the projectile reaches its maximum altitude (from the surface) $h = (3/2)R$.
    (a) (4 points) Find out if there exists a point about which the angular momentum of the projectile is constant.
    (b) (4 points) Use the law of conservation of mechanical energy to find out v in terms of G (the universal gravitational constant), M and R.

11. (3 points) Find the expression for the average recoil force on a machine gun firing 500 shots per minute. The mass of each bullet is m and velocity is v.

12. A bomb of mass m is thrown vertically upward with a speed u from the surface of the Earth. The bomb explodes into two parts of masses m_1 and m_2 after the time t, much before the mass could reach the maximum possible height h. The mass m_1 keeps moving vertically upward with a speed u_1.
    (a) (2 points) What is the magnitude of the speed of the mass m_2?
    (b) (2 points) What is the speed of the center of mass of the system after explosion?

13. Lisa is sitting on the edge of a circular platform (radius $= r$) rotating about its axis at a constant angular velocity ω. Lisa throws an object of mass m parallel to the platform, and radially inward, at an initial velocity V_R with respect to her frame. Lisa was at point A (see Figure 5) when she did that.
    (a) (4 points) Aman is an inertial observer who is at rest with respect to the center of the circular platform. If the object had moved from A to C in the time Δt, how would Aman determine the tangential and the radial components of the velocity (at C) of the object?
    (b) (4 points) Find out how Lisa will measure a net radial acceleration (centrifugal acceleration) and a net tangential acceleration (Coriolis acceleration).

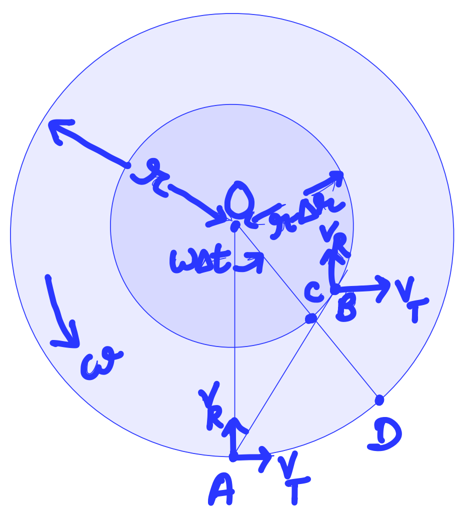

Figure 5: For problem 13