MTH101: Symmetry

Tutorial 03

Problem 1. In the group of symmetries of the regular n-gon, denoted by D_n, let ρ denote the rotation through $2\pi/n$ radians and let τ denote one of the reflections. Then, we know that $\rho^n = 1$, $\tau^2 = 1$ and $\rho\tau = \tau\rho^{-1}$. Let H denote the group $\{1, \rho^2\tau\}$. (Check that this really is a subgroup of D_n.) Describe all the left and right cosets of H.

Problem 2. Let G be a group such that $ord(x) \leqslant 2$ for any $x \in G$. Prove that $xy = yx$ for any $x, y \in G$. (In other words, G is an *abelian* group.)

Problem 3. Let S be a set and let $G = Perm(S)$. Let us fix an element x of S. Let $H = \{\sigma \in Perm(S) | \sigma(x) = x\}$. Prove that $\sigma, \tau \in G$ are in the same left coset of H if and only if $\sigma(x) = \tau(x)$. Can you formulate and prove a similar statement for right cosets?

Problem 4. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Let $G = Perm(S)$. Find an element $\sigma \in G$ such that $ord(\sigma) = 12$.