## MTH 102 - ANALYSIS IN ONE VARIABLE
## ASSIGNMENT 2

**1.** Show that for all $a, b \in \mathbb{R}$,

i. $|b| \leq a$ if and only if $-a \leq b \leq a$.
ii. $||a| - |b|| \leq |a - b|$.

**2.** Let $a, b \in \mathbb{R}$. Show if $a \leq b_1$ for every $b_1 > b$, then $a \leq b$.

**3.** Show :

i. Supremum and infimum of a set are uniquely determined when they exist.
ii. If $S$ is a finite set, $\sup S, \inf S \in S$.
iii. If $S$ is a nonempty subset of $\mathbb{R}$ and $b \in \mathbb{R}$ is such that $b < s$ for all $s \in S$, then $\inf S = -\sup(-S)$.

**4.** Determine supremum and infimum of the following sets if they exist.

i. $\{1/n - 1/m : n, m \in \mathbb{N}\}$, ii. $\{\cos(n\pi/3) : n \in \mathbb{N}\}$, iii. $\{1 - (-1)^n/n : n \in \mathbb{N}\}$.

**5.** Let $S$ be a non-empty bounded subset of $\mathbb{R}$. If $b < 0$ and $bS = \{bs : s \in S\}$ prove:

i. $\inf bS = b \sup S$, ii.$\sup bS = b \inf S$.

**6.** Let $A$ and $B$ be two set of positive numbers that are bounded above. Let $C = \{xy : x \in A, y \in B\}$. If $a = \sup A$, $b = \sup B$, show that $\sup C = ab$.

**7.** Prove :

i. If $a < 0$, then there exists $n \in \mathbb{N}$ such that $-n < a < -\frac{1}{n}$.
ii. The set of all negative integers is not bounded below.
iii. If $y > 0$, there exists $n \in \mathbb{N}$ such that $\frac{1}{2^n} < y$.
iv. If $a, b \in \mathbb{R}$ are such that $a \leq b + \frac{1}{n}$ for all $n \in \mathbb{N}$, then $a \leq b$.

**8.** Given a real number $x$, prove that there exists a unique integer $n$ such that $n - 1 \leq x < n$.

**9.** Let $S$ be a non-empty subset of $\mathbb{R}$. If a number $u \in \mathbb{R}$ has the properties that $u - 1/n$ is not an upper bound of $S$ for all $n \in \mathbb{N}$ and $u + 1/n$ is an upper bound of $S$ for all $n \in \mathbb{N}$, prove that $u = \sup S$. Conversely if $u = \sup S$, then show that $u - 1/n$ is not an upper bound of $S$ for all $n \in \mathbb{N}$ and $u + 1/n$ is an upper bound of $S$ for all $n \in \mathbb{N}$

**10.** Consider $a, b \in \mathbb{R}$ where $a < b$. Using Denseness of rationals in $\mathbb{R}$ show :

i. There are infinitely many rationals between $a$ and $b$.
ii. If $\mathbb{I}$ is the set of real numbers that are not rational, then there exists $x \in \mathbb{I}$ such that $a < x < b$.
iii. For any real number $u > 0$, there exists $r \in \mathbb{Q}$ such that $a < ru < b$.