1. Given: elevator ~~w~~ accl. downward with acc. a, $a<g$ ①

Lisa's speed w.r.t. $= u$

ball intial speed w.r.t. Lisa $= u'$ & $u' < u$

$$F_{drag} = -kv$$

a) From Lisa's frame

$$F = +mg + ma - ku'$$

b) From Amaan's frame

$$F = +mg - k(u' - u)$$

2.

a) For any general vector $\vec{B}$

$$\left(\frac{d\vec{B}}{dt}\right)_{inertial} = \left(\frac{d\vec{B}}{dt}\right)_{N.I.} + \omega \times \vec{B} \quad -①$$

Here Lisa - rotational frame.

So using above eqn ①

Raman - Inertial.

$$\left(\frac{d\omega}{dt}\right)_{Aman} = \left(\frac{d\omega}{dt}\right)_{Lisa} + \underbrace{\vec{\omega} \times \vec{\omega}}_{0}$$

So

$$\left(\frac{d\omega}{dt}\right)_{Amain} = \left(\frac{d\omega}{dt}\right)_{Lisa.}$$

(b)

$$\left(\frac{d\vec{r}}{dt}\right)_{iner} = \left(\frac{d\vec{r}}{dt}\right)_{Rotational} + \vec{\omega}\times\vec{r}$$

$$\vec{v}_{inertial} = \vec{v}_{Rot} + \vec{\omega}\times\vec{r}$$

Aman- Inertial

$$\vec{v}_{Aman} = \vec{v}_{Lisa} + \vec{\omega}\times\vec{r}$$

$$\vec{v}_A = \vec{v}_L + \vec{\omega}\times\vec{r}$$

Now

$$\left(\frac{d v_A}{dt}\right)_{Amain} = \frac{d}{dt}(\vec{v}_L + \vec{\omega}\times\vec{r}) + \vec{\omega}\times(\vec{v}_R + \vec{\omega}\times\vec{r})$$

$$\vec{a}_{Aman} = \frac{d(\vec{v}_L)}{dt} + \frac{d}{dt}(\vec{\omega}\times\vec{r}) + \vec{\omega}\times\vec{v}_R + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$= \vec{a}_{Lisa} + (\dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times\dot{\vec{r}})_{Lisa} + \vec{\omega}\times\vec{v}_{R=Lisa} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$= \vec{a}_{Lisa} + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times\vec{v}_L + \vec{\omega}\times\vec{v}_{Lisa} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{a}_A = \vec{a}_L + 2(\vec{\omega}\times\vec{v}_L) + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$$\vec{a}_L = \vec{a}_A - 2(\omega\times\vec{v}_L) - \dot{\vec{\omega}}\times\vec{r} - \underline{\vec{\omega}\times(\vec{\omega}\times\vec{r})}$$

3. Given:

Light spring of relaxed length $= l$, &

Stiffness $= k$.

& Angular velocity of spring $= \vec{\omega}$ (constt.)

(Rotating with constt. $\vec{\omega}$)

Note: Ignore gravity.

Distance (fixed) of particle having mass m from the rotation axis $= l + \Delta l$.

a) In non inertial frame.

$$m\omega^2 (l + \Delta l) - k\Delta l = 0$$

$$m\omega^2 (l + \Delta l) = k\Delta l$$

b) In inertial frame.

$$F = -k\Delta l.$$

4. Considering origin at center of large sphere.

$$R_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} \quad \text{(In general)}$$

given Mass and Radius of bigger sphere = M, B respec.

" " " small sphere removed from

large solid sphere ~~is~~ = m, b respectively.

$$R = \frac{M(\vec{r}_B) + (-m)\vec{r}_b}{M+(-m)}$$

$$= M(0) + (-m)(x_b\hat{x} + y_b\hat{y}) / M-m$$

$$\boxed{R = \frac{-m(x_b\hat{x} + y_b\hat{y})}{M-m}}$$

Alternate Method:

In terms of vol. of spheres

$$R = \frac{-b^3(x_b\hat{x} + y_b\hat{y})}{B^3 - b^3}$$

5. given: $|\vec{v}_2| = |\vec{v}_3|$

$$m_1 v_1 + m_2 v_2 + m_3 v_3 = 0 \qquad (\because m_1 = m_2 = m_3 = m)$$

$$v_1 + v_2 + v_3 = 0$$

$$|\vec{v}_1| = 2|\vec{v}_2|$$

For first fragment

$$h = v_1 t_1 + \frac{1}{2} g t_1^2$$

$$h = 2 v_2 t_1 + \frac{1}{2} g t_1^2 \quad —(1)$$

For other two fragments.

$$h = -v_2 t_2 + \frac{1}{2} g t_2^2 \quad —(2)$$

multiplying (1) by t_2 and (2) by $2t_1$ & adding, we get

$$ht_2 + 2ht_1 = \cancel{2v_2 t_2 t_1} + \frac{1}{2} g t_1^2 t_2 + (\cancel{-2 v_2 t_1 t_2}) + g t_1 t_2^2$$

$$ht_2 + 2ht_1 = \frac{1}{2} g t_1^2 t_2 + g t_1 t_2^2$$

$$h(t_2 + 2t_1) = \frac{1}{2} g t_1 t_2 (t_1 + 2t_2)$$

$$h = \frac{\frac{1}{2} g t_1 t_2 (t_1 + 2t_2)}{(2t_1 + t_2)} \quad \text{Ans}$$

6. given:

Mass of Bucket = M (when empty)

Initially $(t=0)$ total mass of water in bucket $= m$

Total mass of bucket (with water) at any time, m'

$$m' = M + m - \frac{mt}{T} \quad (I),$$

when it losses all water $t = T$.

Now

eq. of motion

$$m' \frac{dv}{dt} = F - m'g$$

$$\frac{dv}{dt} = \frac{F}{m'} - g \quad — (II)$$

Use $m' = M + m - \frac{mt}{T}$ in (II)

$$\frac{dv}{dt} = \frac{F}{M + m - \frac{mt}{T}} - g$$

Integrating on both sides.

$$\int_0^v dv = \int_0^T \left[\frac{F}{\left(M + m - \frac{mt}{T}\right)} - g \right] dt$$

$$v = -\frac{TF}{m} \left[\ln\left(m + M - \frac{mT}{T}\right) - \ln(M + m) \right] - gT$$

$$\boxed{v = -\frac{TF}{m} \ln\left(\frac{M}{m + M}\right) - gT}$$

7 kepler's 3rd law

$$T^2 = \frac{4\pi^2 a^3}{GM}$$

$$T_e = T_m = \frac{4\pi^2 R_e^3}{GM_e} = \frac{4\pi^2 R_m^3}{GM_m}$$

$$= \frac{R_e^3}{M_e} = \frac{R_m^3}{M_m}$$

$$\frac{1}{\rho_e} = \frac{1}{\rho_m}$$

$$\frac{\rho_e}{\rho_m} = 1$$