Problem 1: A rocket based observer (uniform speed)and her ground (earth) based twin set their respective clocks to zero at the instant they crossed each other. sometime later, the ground based twin observers the rocket's......

If the rocket based twin had sent a light signal the moment she crossed the earth, that signal would have reached the earth instantaneously - when both twins watches read zero. thus, we can say that where the rocket based twin sent out signal at an interval of 4 hours by her watch, the ground based twin received them at an interval of 5 hours according to her. this means that the K factor involved here is 5/4. Since

$$K = \sqrt{\frac{1+\beta}{1-\beta}} \tag{1}$$

This means that

$$\frac{1+\beta}{1-\beta} = \frac{25}{16} \tag{2}$$

and thus

$$\beta = \frac{9}{41} \tag{3}$$

So, the speed of the rocket with respect to the ground is (9/41) c

Problem 2: Alice has two clocks -one at her origin and another 8 light hours away . She sunchronized..

The first part is easy. According to Alice,Bob is covering a distance of 8 light hours at 0.8 C- which takes him 10 hours.So, Alice's second clock shows 10 o'clock when Bob reaches it. This is the time interval between bob's crossing the two clocks belonging to Alice, as compared by alice's clocks. Since Bob is present at both the events, he measures the proper time interval between them which is equation 10 $\sqrt{1-0.8^2} = 6 hours$ Since his watch was showing 0 when the first event (his crossing alice) occurred, his watch shows 6' o clock when he crosses the second of alice's clocks. The difficulty arises when one tries to analyse the events from bob's frame of reference. To Bob, the clocks that alice carries are running slow- so how does he reconcile with the fact that when his clock is only showing 6 o' clock, alice's clock is showing 10? The Secret is to realise that whenever we measure time, we are, actually,measuring a time interval. The time interval of 10 hours that alice's second clock shows is the interval between this clock being set to zero (event A) and bob crossing it (event B). Of course, alice reckons the interval between Bob crossing her (event C) and event B is also 10 hours- but that is because she is sure that event C (when both their clocks had been set to 0) and event A are simultaneous. This is where the relativity of simultaneity come is. To Bob, the event A does not occur at the same time event C, but actually occurs a time

$$\gamma l \frac{v}{c^2} = \frac{1}{\sqrt{1-0.8^2}} \times 8\mathrm{lt}-\mathrm{hrs} \times \frac{0.8c}{c^2} = \frac{32}{3} hrs$$

earlier! so, according to Bob, the interval between events A and B is not 6 hours, but actually equation

$$6 + \frac{32}{3} = \frac{50}{3}$$

hours

an interval that alice's clock shows to be only 10 hours! so, according to Bob alice's clock is running slow why a factor of

$$0.6 = \gamma^{-1}$$

exactly the same factor by which alice had observed bob's clock to slow down!

Problem 3: A train of proper length L_0 is approaching a station at ....

Let the coordinates of the front and back ends of the train be $x_f(t)$ and $x_b(t)$, respectively with respect to an observer standing on the station. The length of the train is given by

$$x_f(t) - x_b(t) = L = \frac{L_0}{\gamma}$$

let the times at which the two light signals got emitted be t_f and t_b, respectively. Then, since the signals reach the observer simultaneously, we have

$$t_f + \frac{x_f\left(t_f\right)}{c} = t_b + \frac{x_b\left(t_b\right)}{c}$$

Thus

$$t_f - t_b = \frac{1}{c}\left(x_f\left(t_f\right) - x_b\left(t_b\right)\right)$$

$$= \frac{1}{c}\left(\left[x_f\left(t_f\right) - x_f\left(t_b\right)\right] + \left[x_f\left(t_b\right) - x_b\left(t_b\right)\right]\right)$$

$$= \frac{1}{c}\left(\beta c\left(t_f - t_b\right) + L\right)$$

So,

$$x_f\left(t_f\right) - x_b\left(t_b\right) = c\left(t_f - t_b\right) = \frac{L}{1-\beta} = L_0\frac{\sqrt{1-\beta^2}}{1-\beta} = L_0\sqrt{\frac{1+\beta}{1-\beta}}$$

Note that in spite of length contraction, the moving train actually appears longer!

Problem 4 :- Find the moment of inertia and products of inertia of a uniform right angled triangle bounded by the x - axis - - - - .

Solution :-

$$I_{xx} = \sigma \int_0^a dx \int_0^{a-x} y^2 dy = \sigma \int_0^a \frac{(a-x)^3}{3} dx = \frac{\sigma a^4}{12} = \frac{Ma^2}{6}$$

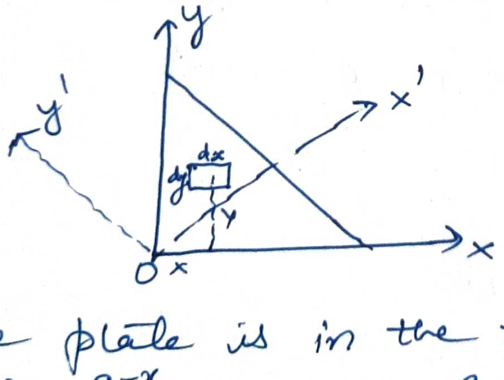
where σ is the mass per unit area and $M = \frac{1}{2}\sigma a^2$ is the mass of the triangular plate.

From symmetry $I_{xx} = I_{yy}$ (refer figure) & from the theorem of perpendicular axis

$$I_{zz} = I_{xx} + I_{yy} = \frac{Ma^2}{3}$$

Alternatively, one can calculate I_{zz} directly

$$I_{zz} = \sigma \int_0^a dx \int_0^{a-x} (x^2+y^2)\, dy = \frac{Ma^2}{3}$$



As the plate is in the $z=0$ plane, $I_{xz} = I_{yz} = 0$

$$I_{xy} = -\sigma \int_0^a dx \int_0^{a-x} xy\, dy = -\sigma \int_0^a x \frac{(a-x)^2}{2} dx = -\frac{\sigma a^4}{24} = -\frac{Ma^2}{12}$$

Thus the inertia tensor can be written as

$$I = \frac{Ma^2}{12} \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

The principal moments of inertia may be found from the secular equation

$$\det \begin{pmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{pmatrix} = 0$$

The characteristic eqn is thus given by

$$(4-\lambda)\left((2-\lambda)^2-1\right) = (1-\lambda)(3-\lambda)(4-\lambda) = 0$$

The roots of the above equation are $\lambda = 1, 3, 4$

So the principal moments of inertia are

$$I_1 = \frac{Ma^2}{12}, \quad I_2 = \frac{Ma^2}{4} \text{ and } I_3 = \frac{Ma^2}{3}$$

The principal axes can be found from the equation

$$\begin{pmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

For $\lambda = 1$ the equations are

$$x - y = 0, \quad x - y = 0, \quad 3z = 0$$

The nontrivial solution of which is $x = y$ and $z = 0$. Thus the normalized eigenvector can be written as $\left(\frac{\hat{i}}{\sqrt{2}}, \frac{\hat{j}}{\sqrt{2}}, 0\right)$

For $\lambda = 3$ the non trivial solution is $x = -y$ and $z = 0$. Thus the normalized eigen vector corresponding to $\lambda = 3$ can be written as $\left(-\frac{\hat{i}}{\sqrt{2}}, \frac{\hat{j}}{\sqrt{2}}, 0\right)$ and these are the x' & y' axes as shown in figure.

For $\lambda = 4$ the eigenvector is $(0, 0, \hat{k})$.

Problem 5 :- Observer A, standing on earth, sends out pulsed signals using a laser pointer every six minutes, observer B is on a space station that is stationary with respect to ...

Solution :- It is trivial to see that the time interval between the signals reaching B is the same as the interval at which the signals leave A, namely 6 minutes.

For the sake of simplicity, let us assume that a laser signal is sent out by A at the instant when C is just alongside him. That signal, of course reaches C immediately. By the time that A sends out the next signal, C is a distance of $0.6c \times 6$ minutes, i.e. 3.6 light-minutes away. If this signal travels for t minutes before it reaches C, then in this time C will have moved an additional distance of $0.6t$ light-minutes. Since the light pulse travels a distance of 't' light-minutes in this interval, we must have

$$t = 3.6 + 0.6t$$

$$t = 9$$

So that the next light pulse will reach C $9+6 = 15$ minutes after the first one reaches him. There is a subtle point here, though the time interval that we have calculated above is the correct time interval - but as observed by the observer A. This is not the time

interval that C observes between the two signals - Simply because of time dilation.
Now, Since C is present at both the events that we are discussing (after all, both events involve light signals reaching C), the time interval that C measures between them is the proper time interval which can be easily Calculated from the time interval that A has measured. It is

$$15 \text{ minutes} \times \sqrt{1-0.6^2} = 12 \text{ minutes}$$

which is the interval at which C receives the signals.

It is instructive to repeat the calculation from the point of view of the observer C. As far as C is concerned, the time interval between the sending of the two signals by A

$$\frac{6 \text{ min}}{\sqrt{1-0.6^2}} = 7.5 \text{ min}$$

(Remember that the 6 minute interval that A observes between the two events is the proper time b/w them)
This means that when the second signal is being emitted, A is at a distance

$$7.5 \text{ min} \times 0.6\,c = 4.5 \text{ light minutes from C.}$$

light takes 4.5 minutes to cover that distance and thus the time when light reaches C is

7.5 minute + 4.5 minutes = 12 minutes.

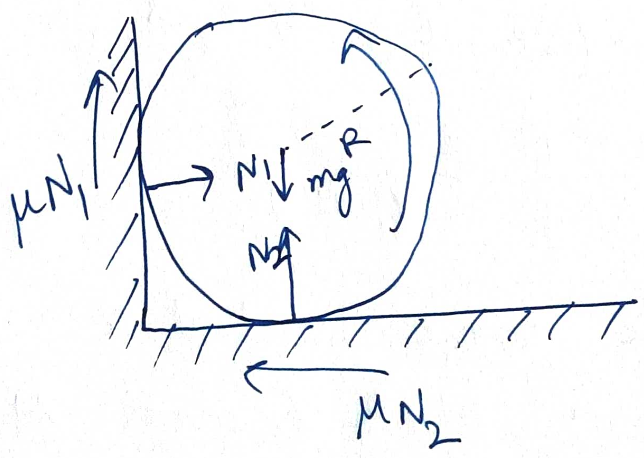
Problem [illegible] solution:-

Problem — 6½ Solution

For the final part of the problem, we can calculate the answer explicitly. However, it is easier to just note that the two situations are identical - except that the two observers A and C have been switched. Hence, according to the principle of relativity, the answer is the same in this case - observer A will receive the signals at 12 minute intervals (according to his watch)

Problem 4:- ~~Dee~~ ~~Continuation~~ A uniform cylinder of R is made to rotate with an angular velocity w_0 - - - -

Solution:- As the center of mass of the cylinder does not move, the resultant of all the forces acting on it is zero. ∴



i.e. $\mu N_1 + N_2 = mg$ & $N_1 = \mu N_2$

Solving for N_1 & N_2

$$N_1 = \frac{\mu}{\mu^2+1} mg \quad \& \quad N_2 = \frac{1}{\mu^2+1} mg$$

For the rotational motion we have

$$I \frac{d\omega}{dt} = -\mu (N_1 + N_2) = -\frac{\mu(\mu+1)}{\mu^2+1} mg R$$

where $I = \frac{1}{2} mR^2$ is the moment of inertia of the cylinder about its axis of rotation

Thus $$\frac{1}{2} m R^2 \omega \frac{d\omega}{d\theta} = -\frac{\mu(\mu+1)}{\mu^2+1} mg R$$

Integrating & using the condition $\omega = \omega_0$ at $\theta = 0$

$$\frac{1}{4} m R^2 \omega_0^2 = \frac{\mu(\mu+1)}{\mu^2+1} mg R \theta$$

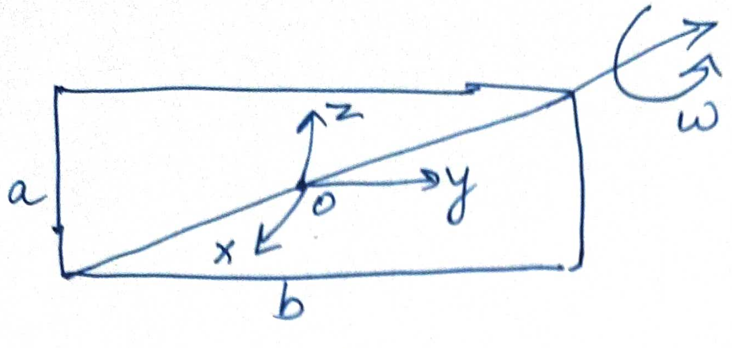
$$\theta = \frac{\omega_0^2 R}{4g} \frac{\mu^2+1}{\mu(\mu+1)}$$

So the no. of turns that the cylinder will complete before it stops is

$$n = \frac{\theta}{2\pi} = \frac{\omega_0^2 R}{8\pi g} \frac{\mu^2+1}{\mu(\mu+1)}$$

Problem 8 ;- Find the torque needed to rotate a rectangular plate of mass M - - -

Solution ;- The principal axes along the symmetry directions as shown in the figure.



The principal axes along the symmetry directions)

The principal inertial tensor is given by

$$I = M \begin{pmatrix} \frac{a^2}{12} & 0 & 0 \\ 0 & \frac{b^2}{12} & 0 \\ 0 & 0 & \frac{a^2+b^2}{12} \end{pmatrix}$$

The angular velocity is

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} = \frac{\omega b}{\sqrt{a^2+b^2}} \hat{i} + \frac{\omega a}{\sqrt{a^2+b^2}} \hat{j}$$

Substituting these in the Euler eqns.

$$I_1 \dot{\omega}_1 + (I_3 - I_2)\omega_2 \omega_3 = N_1$$

$$I_2 \dot{\omega}_2 + (I_1 - I_3)\omega_1 \omega_3 = N_2$$

$$I_3 \dot{\omega}_3 + (I_2 - I_1)\omega_1 \omega_2 = N_3$$

we get $N_1 = 0$, $N_2 = 0$, $N_3 = \frac{M(b^2-a^2)ab\omega^2}{12(a^2+b^2)}$

Thus the required torque about O is

$$\vec{N} = \frac{M(b^2-a^2)ab\omega^2}{12(a^2+b^2)} \hat{k}$$

It may be noted that if the plate is a

square plate i.e $a = b$, then $\vec{N} = 0$. Thus, in the case of a square set spinning ~~abo~~ along a diagonal, no further torque is needed to maintain the rotation.