Notes for AdS

Darsh Kodwani,[1,2,*] Ue-Li Pen,[1,3,4,5,†] and I-Sheng Yang[1,5,‡]

[1] *Canadian Institute of Theoretical Astrophysics, 60 St George St, Toronto, ON M5S 3H8, Canada.*
[2] *University of Toronto, Department of Physics, 60 St George St, Toronto, ON M5S 3H8, Canada.*
[3] *Canadian Institute for Advanced Research, CIFAR program in Gravitation and Cosmology.*
[4] *Dunlap Institute for Astronomy & Astrophysics, University of Toronto, AB 120-50 St. George Street, Toronto, ON M5S 3H4, Canada.*
[5] *Perimeter Institute of Theoretical Physics, 31 Caroline Street North, Waterloo, ON N2L 2Y5, Canada.*

* dkodwani@physics.utoronto.ca
† pen@cita.utoronto.ca
‡ isheng.yang@gmail.com

I. INTRODUCTION AND SUMMARY

Choose the metric

$$ds^2 = -\left(1 - b^2r^2\right) dt^2 + \left(1 - b^2r^2\right)^{-1} dr^2 + r^2 d\Omega_2^2 \tag{1}$$

where $b^2 = \frac{\Lambda}{3}$. For a black hole of mass M we have

$$ds^2 = -\left(1 - \frac{2M}{r} - b^2r^2\right) dt^2 + \left(1 - \frac{2M}{r} - b^2r^2\right)^{-1} dr^2 + r^2 d\Omega_2^2 \tag{2}$$

we know

$$\frac{\Delta T}{T} = \sqrt{\frac{g_{00}(b, r_2) g_{00}(b, \Delta M, r_0)}{g_{00}(b, r_1) g_{00}(b, \Delta M, r_1)}} - 1 \tag{3}$$

where

$$\begin{aligned} g_{00}(\Lambda, r) &= -\left(1 - b^2r^2\right) \\ g_{00}(\Lambda, \Delta M, r) &= -\left(1 - \frac{2\Delta M}{r} - b^2r^2\right) \end{aligned} \tag{4}$$

The induced metrics we will use are

$$ds_I^2 = g_{\alpha\beta}dx^\alpha dx^\beta = -(1 - b^2r^2)dt^2 + (1 - b^2r^2)dr^2 + r^2 d\Omega_2^2 \tag{5}$$

$$ds_{II}^2 = g_{\alpha\beta}dx^\alpha dx^\beta = -(1 - b^2r^2)dt^2 + (1 - b^2r^2)dr^2 + r^2 d\Omega_2^2 \tag{6}$$

The induced metrics are

$$ds_{3(I)}^2 = h_{ab}^{(I)} dy^a dy^b = h_{00}^{(I)} d\hat{t}^2 + h_{11}^{(I)} d\hat{\theta}^2 + h_{22}^{(I)} d\hat{\phi}^2 \tag{7}$$

$$ds_{3(II)}^2 = h_{ab}^{(II)} dy'^a dy'^b = h_{00}^{(II)} d\hat{t'}^2 + h_{11}^{(II)} d\hat{\theta'}^2 + h_{22}^{(II)} d\hat{\phi'}^2 \tag{8}$$

matching the induced metrics at r_1 gives

$$dt = \frac{g_{00}^{(II)}(r_1)}{g_{00}^{(I)}(r_1)} dt' \tag{9}$$

We will need the stress energy of the shell and use the IJC for that. Lets look at the extrinsic curvature components first

$$K_{ab}^{(I)} = \frac{1}{2} g_{\alpha\beta}^{(I)} e_a^\alpha e_b^\beta \tag{10}$$

where

$$e_a^\alpha = \frac{dx^\alpha}{dy^a} \tag{11}$$

which we can simplify

$$K_{tt}^{(I)} = \frac{1}{2}(g_{tt}^{(I)})^{-\frac{1}{2}}\partial_r g_{tt}^{(I)} \tag{12}$$

similarly the other components of the extrinsic curvature are

$$K_{\theta\theta}^{(I)} = \frac{1}{2}n^r\partial_r g_{\theta\theta}^{(I)} = \frac{1}{2}(g_{tt}^{(I)})^{-\frac{1}{2}}\partial_r g_{\theta\theta}^{(I)} = K_{\phi\phi}^{(I)} \tag{13}$$

$$K_{tt}^{(II)} = \frac{1}{2}(g_{tt}^{(II)})^{-\frac{1}{2}}\partial_r g_{tt}^{(II)} \tag{14}$$

$$K_{\theta\theta}^{(II)} = \frac{1}{2}n^r\partial_r g_{\theta\theta}^{(II)} = \frac{1}{2}(g_{tt}^{(II)})^{-\frac{1}{2}}\partial_r g_{\theta\theta}^{(II)} = K_{\phi\phi}^{(II)} \tag{15}$$

The surface stress energy is

$$S_{00} = \sigma = \frac{1}{4\pi}((K_\theta^\theta)^{(I)} - (K_\theta^\theta)^{(II)}) \tag{16}$$

$$\sigma = \frac{1}{4\pi r_1^2}\left(\left(1 - b^2 r_1^2 - \frac{2\Delta M}{r_1}\right)^{-\frac{1}{2}} - \left(1 - b^2 r_1^2\right)^{-\frac{1}{2}}\right) \tag{17}$$