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Parity breaking correlation functions and fifth forces

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I. INTRODUCTION

Why modified gravity, why screening. How this translates into the phenomenology of fifth forces.

Various different regimes in which it manifests itself – cosmological, cluster scale, galaxy scale, stars, lab. Each scale has specific types of signal that are most useful for searching for it.

Within the context of specific models there are stronger constraints from some of these than others (eg galaxies and stars for f(R)). But still worth exploring consequences more generally, as there may be models with stronger/different effects in some regimes than others.

Tests between the lab and cosmological scales generally rely on the altered dynamics of matter either with respect to light or screened regions, which means either intra-galaxy or intra-cluster dynamics or changes to stellar lifecycles. Some cosmological tests (weak) use overall changes in halo clustering or abundance, which requires N-body sims.

Here we use the relative clustering of galaxies of different types within clusters. The asymmetric part of the CF picks out the relativistic contribution which is most sensitive to the effects of MG, which provides a fast-track to probing fifth forces without the need either for expensive simulations or detailed dynamical data within galaxies or clusters.

Layout of paper: 1) Overview of screening and asymmetric part of CF. 2) Derivation of general impact of F5 on this, and hence which signals (multipoles etc) provide the most sensitive probes. Results of simplest approximation (alpha(r) step function) and successively more complex ones. 3) Cluster F5 profiles by numerical integration -¿ best estimate for total effect. 4) Application to real data. 5) Summary and further work.

Astrophysical and cosmological tests of gravity have been done before. Here we focus on a test of screening using clusters of galaxies. Previously various probes have been used such as

- X-ray measurements using hot intra-clusters gas dynamics
- Strong and Weak lensing

Screening can be accomplished in the following ways

- Weak coupling between the field and matter in regions of high density, thus inducing a weak fifth force as in symmetron theories.
- The field can acquire a large mass in high density environments, being short-ranged and undetectable and be light and long ranged in lower density regions, as for chameleon fields and $f(R)$ models.
- Finally one may change the kinetic contribution of the field to the Lagrangian with first and second order derivatives being important in a certain range, as happens in Vainshtein screening.

II. SYMMETRON

A. Intro

Here we summarise the results from [1, 2]. The scalar equation of motion for a symmetron is

$$\nabla^2\phi - \partial_\phi V + A^3(\phi)\partial_\phi \tilde{T} = 0 \tag{1}$$

where $A^2(\phi)$ is the conformal factor that relates the Jordan frame metric $\tilde{g}_{\mu\nu}$ to the Einstein frame metric

$$\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} \tag{2}$$

and the stress energy tensor is defined as the trace of the Jordan frame metric,

$$\begin{aligned}\tilde{T}_{\mu\nu} &= -\frac{2}{\sqrt{-\tilde{g}}}\frac{\delta L_m}{\delta g^{\mu\nu}} \\ \tilde{T} &= \tilde{T}_{\mu\nu}\tilde{g}^{\mu\nu}\end{aligned} \tag{3}$$

If we ignore backreaction of the scalar field on gravity, and consider spherically symmetric pressureless sources, the density $\rho = A^3\tilde{\rho}$ is conserved in the Einstein frame. The Laplacian operator also simplifies in this case to give

$$\nabla^2\phi = \frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} \tag{4}$$

For cases of roughly homogenous ρ, the field evolves according to an effective potential

$$V_{eff}(\phi) = V(\phi) + \rho A(\rho) \tag{5}$$

For the model which has a spontaneously broken $\mathbb{Z}_2$.

B. Equations of motion

The general equation of motion

$$\partial_r^2\phi + \frac{2}{r}\partial_r\phi = \partial_\phi V(\phi) + \partial_\phi A(\phi)\rho \tag{6}$$

For homogenous ρ such as the NFW profile, the field evolves according to

$$\partial_r^2\phi + \frac{2}{r}\partial_r\phi = \partial_\phi V_{eff} \tag{7}$$

where

$$V_{eff} = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4 \tag{8}$$

We can separate the solution in terms of the solution inside and outside the object. We can start with separating the effective potential for top hat density profile

$$\rho = \begin{cases} \rho_0 & (r < R) \\ 0 & (r > R) \end{cases} \tag{9}$$

$$V_{eff} = \begin{cases} \frac{\rho\phi^2}{2M^2} & (r < R) \\ m_0^2(\phi^2 - \phi_0^2)^2/2 & (r > R) \end{cases} \tag{10}$$

The solution for the scalar field can also be split in this way

$$\phi(r) = \begin{cases} C\frac{R}{r}\sinh\left(\frac{\sqrt{\rho}r}{M}\right) & (r < R) \\ D\frac{R}{r}e^{-m_0(r-R)} + \phi_0 & (r > R) \end{cases} \tag{11}$$

Coefficients C and D are fixed by matching the field and its radial derivative at the interface $r = R$

$$\begin{aligned} C &= \phi_0\sqrt{\frac{\Delta R}{R}}sech\left(\sqrt{\frac{R}{\Delta R}}\right) \\ D &= -\phi_0\left(1 - \sqrt{\frac{\Delta R}{R}}\tanh\left(\sqrt{\frac{\Delta R}{R}}\right)\right) \end{aligned} \tag{12}$$

where $\frac{\Delta R}{R} \equiv \frac{M^2}{\rho R^2} = \frac{M^2}{6M_{\rm pl}^2\phi} = \frac{\phi_0}{6gM_{\rm pl}\Phi}$ is the thin shell radius in analogy with chameleon screening. $\Phi \equiv \frac{\rho R^2}{6M_{\rm pl}^2}$ is the gravitational potential of the source.

Note that so far we have not said anything about the screening environment. We are interested in the dark matter halos so we can use galaxy cluster measurements to test for screening. So we can assume $R = R_{vir}$ in this case and $M = M_{300}$ (or some definition of the halo mass).

C. Ratio of forces

The fifth force for the symmetron is

$$F_\phi = \frac{\phi\nabla\phi}{M_s^2} = -\frac{1}{M_s^2}\left(\frac{BR}{r}e^{-\sqrt{2}\mu r} + \phi_0\right)\left(\frac{BR}{r}e^{-\sqrt{2}\mu r}\left(\frac{1}{r} + \sqrt{2}\mu\right)\right) \tag{13}$$

D. Putting the action into Horndeski form

We begin with the canonical action of a single scalar field ϕ:

$$\tilde{S} = \int d^4x\sqrt{-\tilde{g}}\left\{\frac{M_{\rm pl}^2}{2}\tilde{\mathcal{R}} - \frac{1}{2}(\tilde{\partial}\phi)^2 - V(\phi)\right\} - \int d^4x\,\mathcal{L}_m(\psi_m, \exp(-2\beta\phi/M_{\rm pl})\tilde{g}_{\mu\nu}), \tag{14}$$

where β is the coupling coefficient, ψ_m is the matter field and $\mathcal{L}_m$ is the matter Lagrangian. This is written in the Einstein frame (denoted by tilde), where the gravitational part of the action takes its GR form and hence the left-hand side of Einstein's equation is unaffected by the scalar field, but ϕ couples to matter in $\mathcal{L}_m$ so that particles do not move on geodesics but rather feel a fifth force. The form of the potential $V(\phi)$ determines the field's screening properties. To transform to the Jordan frame, in which matter moves on geodesics but the gravitational action is altered, we perform a conformal transformation with factor $\Omega \equiv \exp(\beta\phi/M_{\rm pl})$. The components transform as follows:

$$\tilde{g}_{\mu\nu} = \exp\left(\frac{2\beta\phi}{M_{\rm pl}}\right) g_{\mu\nu}, \tag{15}$$

$$\tilde{g}^{\mu\nu} = \exp\left(-\frac{2\beta\phi}{M_{\rm pl}}\right) g^{\mu\nu}, \tag{16}$$

$$\sqrt{-\tilde{g}} = \exp\left(\frac{4\beta\phi}{M_{\rm pl}}\right)\sqrt{-g}, \tag{17}$$

$$\tilde{\mathcal{R}} = \exp\left(-\frac{2\beta\phi}{M_{\rm pl}}\right)\left[\mathcal{R} - 6g^{\alpha\beta}\nabla_\alpha\nabla_\beta\left(\frac{\beta\phi}{M_{\rm pl}}\right) - 6g^{\alpha\beta}\nabla_\alpha\left(\frac{\beta\phi}{M_{\rm pl}}\right)\nabla_\beta\left(\frac{\beta\phi}{M_{\rm pl}}\right)\right] \tag{18}$$

$$\Rightarrow \tilde{\mathcal{R}} = \exp\left(-\frac{2\beta\phi}{M_{\rm pl}}\right)\left[\mathcal{R} - 6\frac{\beta}{M_{\rm pl}}\Box\phi - 6\frac{\beta^2}{M_{\rm pl}^2}g^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi\right], \tag{19}$$

$$(\tilde{\partial}\phi)^2 \equiv \tilde{g}^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = \exp\left(-\frac{2\beta\phi}{M_{\rm pl}}\right)g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = \exp\left(-\frac{2\beta\phi}{M_{\rm pl}}\right)(\partial\phi)^2, \tag{20}$$

and ϕ itself is unaffected. Putting this all together, we find for the Jordan-frame action

$$S = \int d^4x\sqrt{-g}\exp\left(\frac{2\beta\phi}{M_{\rm pl}}\right)\left(\mathcal{R} - \frac{6\beta}{M_{\rm pl}}\Box\phi - \frac{6\beta^2}{M_{\rm pl}^2}\partial_c\phi\partial^c\phi - \frac{1}{2}\partial_c\phi\partial^c\phi - \exp\left(\frac{2\beta\phi}{M_{\rm pl}}\right)V(\phi)\right) - \int d^4x\,\mathcal{L}_m(\psi_m, g_{\mu\nu}). \tag{21}$$

Comparing this to the Horndeski form [3], the only non-zero coefficients are

$$G_2 = -\exp\left(\frac{4\beta\phi}{M_{\rm pl}}\right)V(\phi) + \exp\left(\frac{2\beta\phi}{M_{\rm pl}}\right)X\left(1 + 12\frac{\beta^2}{M_{\rm pl}^2}\right), \tag{22}$$

$$G_3 = \frac{6\beta}{M_{\rm pl}} \exp\left(\frac{2\beta\phi}{M_{\rm pl}}\right), \tag{23}$$

$$G_4 = \exp\left(\frac{2\beta\phi}{M_{\rm pl}}\right), \tag{24}$$

where X is the scalar field kinetic term $-\frac{1}{2}\partial_c\phi\partial^c\phi$.

III. PARITY BREAKING CORRELATION FUNCTIONS

The overdensity of bright galaxies can be written as

$$\Delta_B(z,\hat{n}) = \Delta_B^{st}(z,\hat{n}) + \Delta_B^{rel}(z,\hat{n}) + \Delta_B^{lens}(z,\hat{n}) + \Delta_B^{AP}(z,\hat{n}) \tag{25}$$

where

$$\Delta_B^{st}(z,\hat{n}) = b_B\delta(z,\hat{n}) - \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\hat{n}) \tag{26}$$

$$\Delta_B^{rel}(z,\hat{n}) = \frac{1}{\mathcal{H}}\left(\partial_r\psi + \dot{\mathbf{v}}\cdot\hat{n}\right) - \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} - 1 + 5s_B\left(1 - \frac{1}{r\mathcal{H}}\right)\right)\mathbf{v}\cdot\hat{n} \tag{27}$$

$$\Delta_B^{lens}(z,\hat{n}) = (5s_B - 2)\int_0^r dr'\frac{(r-r')r'}{2r}\nabla_\perp^2(\phi+\psi) \tag{28}$$

$$\Delta_B^{AP}(z,\hat{n}) = (\partial_r - \partial_\eta)(\Delta_B^{st} + \Delta_B^{rel} + \Delta_B^{lens})\frac{\partial r(z,\Omega)}{\partial\Omega}\delta\Omega \tag{29}$$

The quantities with a B index denotes the value of a variable that will change for bright and faint galaxies. b_B is the bias of the bright galaxies and s_B is their effective number count slope. These expressions are valid in any conformal theory of gravity (where null geodesics are unchanged). Thus are perfectly suite for our purposes as all screened theories are conformally invariant. When timelike curves are also geodesics, we can use the Euler equation

$$\dot{\mathbf{v}}\cdot\hat{n} + \mathcal{H}\mathbf{v}\cdot\hat{n} + \partial_r\psi = 0 \tag{30}$$

Using this equation we can rewrite Eq (27) as

$$\Delta_B^{rel}(z,\hat{n}) = -\mathbf{v}\hat{n}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} + 5s_B\left(1 - \frac{1}{r\mathcal{H}}\right)\right) \tag{31}$$

In presence of screened theories of gravity we include a new force which is sourced by the gravitational potential

$$\nabla\phi = -\alpha\frac{GM}{r} \tag{32}$$

Thus we can write a general paramterized form of the effect of screened fifth forces

$$\partial_r\psi \to (1+\alpha)\partial_r\psi \tag{33}$$

Thus we can write the modified Euler equation as

$$\dot{\mathbf{v}}\cdot\hat{n} + \mathcal{H}\mathbf{v}\cdot\hat{n} + (1+\alpha)\partial_r\psi = 0 \tag{34}$$

Under this modification to the Euler equation we can rewrite the Eq (27) as

$$\begin{aligned}
\Delta_B^{rel(MG)}(z,\hat{n}) &\equiv \Delta_B^{MG}(z,\hat{n}) + \Delta_B^{rel}(z,\hat{n}) \\
\Delta_B^{MG}(z,\hat{n}) &\equiv \left(1 - \frac{1}{1+\alpha^2}\right)\left(\frac{\dot{\mathbf{v}}n}{\mathcal{H}} + \mathbf{v}\hat{n}\right) \\
\Delta_B^{rel}(z,\hat{n}) &\equiv -\left(\frac{\mathcal{H}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} + 5s_B\left(1 - \frac{1}{r\mathcal{H}}\right)\right)\vec{v}\cdot\hat{n}
\end{aligned} \tag{35}$$

Using these we can write down correlation functions as normal. The only terms that will change will be terms that previously dependent on the relativistic part.

Lets start with writing down the correlation function in normal GR.

$$\begin{aligned}
\xi^{rel}(r,r',\theta) &= \langle \Delta_B^{st}(z,\hat{n})\Delta_F^{rel}(z',\hat{n}')\rangle \\
&= A\int \frac{d^3k}{(2\pi)^2} e^{i\mathbf{k}(\mathbf{x}'-\mathbf{x})}\frac{(k\eta_0)^{n_s-1}}{k^3}\left(\left[\frac{\dot{\mathcal{H}}(r')}{\mathcal{H}^2(r')} + \frac{2}{r'\mathcal{H}(r')} + 5s_B(r')\left(1 - \frac{1}{r'\mathcal{H}(r')}\right)\right] i(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}T_V(k,r')) \right. \\
&\left. \left(b_B T_D(k,r) - \frac{k}{\mathcal{H}(r)}(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})^2 T_V(k,r)\right) - (r\leftrightarrow r')\right)
\end{aligned} \tag{36}$$

Now we can look at the correlation function for modified gravity.

$$\begin{aligned}
\langle \Delta_B^{st}(z,\hat{n})\Delta_F^{rel(MG)}(z',\hat{n}')\rangle &= A\int \frac{d\Omega_k}{2\pi} e^{i\vec{k}(\vec{x}'-\vec{x})}\frac{(k\eta_0)^{n_s-1}}{k}\left[\left(1 - \frac{1}{1+\alpha^2}\right)\left((i\vec{k}\cdot\hat{n})\left(\frac{\dot{T}_v(k,r)}{\mathcal{H}} + T_v(k,r)\right)\right)\right] \\
&\times\left[\left(b_B T_D(k',r') - \frac{k'}{\mathcal{H}}\left((\vec{k}'\cdot\hat{n}')^2 T_v(k',r')\right)\right)\right] \\
&= \frac{A}{2\pi}\left(1 - \frac{1}{1+\alpha^2}\right)\int d\Omega_k\ e^{i\vec{k}(\vec{x}'-\vec{x})}(i\vec{k}\cdot\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}\frac{b_B}{\mathcal{H}}\dot{T}_v(k,r)T_D(k',r') \\
&- \frac{A}{2\pi}\left(1 - \frac{1}{1+\alpha^2}\right)\int d\Omega_k\ e^{i\vec{k}(\vec{x}'-\vec{x})}(i\vec{k}\cdot\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}(\vec{k}'\cdot\hat{n}')^2\frac{k'}{\mathcal{H}^2}\dot{T}_v(k,r)T_v(k',r') \\
&+ \frac{A}{2\pi}\left(1 - \frac{1}{1+\alpha^2}\right)\int d\Omega_k\ e^{i\vec{k}(\vec{x}'-\vec{x})}(i\vec{k}\cdot\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}b_B(r')T_v(k,r)T_D(k',r') \\
&- \frac{A}{2\pi}\left(1 - \frac{1}{1+\alpha^2}\right)\int d\Omega_k\ e^{i\vec{k}(\vec{x}'-\vec{x})}(i\vec{k}\cdot\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}(\vec{k}'\cdot\hat{n}')^2\frac{k'}{\mathcal{H}^2}T_v(k,r)T_v(k',r') \\
&= \frac{A}{2\pi}\left(1 - \frac{1}{1+\alpha^2}\right)\int d\Omega_k\ e^{i\vec{k}(\vec{x}'-\vec{x})}(i\vec{k}\cdot\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}\left[\underbrace{\frac{b_B}{\mathcal{H}}\dot{T}_v(k,r)T_D(k',r')}_{T1}\right. \\
&\left. - \underbrace{(\vec{k}'\cdot\hat{n}')^2\frac{k'}{\mathcal{H}^2}\dot{T}_v(k,r)T_v(k',r')}_{T2} + \underbrace{b_B(r')T_v(k,r)T_D(k',r')}_{T3} - \underbrace{(\vec{k}'\cdot\vec{n}')^2\frac{k'}{\mathcal{H}^2}T_v(k,r)T_v(k',r')}_{T4}\right]
\end{aligned} \tag{37}$$

Lets focus on one term first, $T1$

$$\begin{aligned}
T1 &= \int d\Omega_k\ e^{i\vec{k}(\vec{x}'-\vec{x})}(i\vec{k}\cdot\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}\frac{b_B}{\mathcal{H}}\dot{T}_v(k,r)T_D(k',r') \\
&= \frac{(4\pi)^2}{3}\int d\Omega_k \sum_{LM}\sum_{m=-1}^{1} i^{L+1}\ j_L(kd)Y^*_{LM}(\vec{k})Y_{lm}(\hat{N})Y^*_{1m}(\hat{k})Y_{1m}(\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}\frac{b_B}{\mathcal{H}}\dot{T}_v(k,r)T_D(k',r') \\
&= \frac{(4\pi)^2}{3}\sum_{M,m=-1}^{1}(-1)\delta_{Mm}Y_{1M}(\hat{N})Y_{1m}(\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}\frac{b_B}{\mathcal{H}}j_i(kd)\dot{T}_v(k,r)T_d(k',r')
\end{aligned} \tag{38}$$

Similarly $T3$ is

$$T3=\frac{(4\pi)^2}{3}\sum_{M,m=-1}^{1}(-1)\delta_{Mm}Y_{1M}(\hat{N})Y_{1m}(\hat{n})\frac{(k\eta_0)^{n_s-1}}{k}b_Bj_i(kd)T_v(k,r)T_d(k',r') \tag{39}$$

Next lets compute $T2$.

A. Fourier space transformations

Here we write down all the ingredients we need and their Fourier space transformation

$$\begin{aligned}
&\partial_r(\vec{v}\cdot\hat{n})=-k(\hat{k}\cdot\hat{n})^2V(k,\eta)\sim -k(\hat{k}\cdot\hat{n})^2T_V(k,r)\\
&\vec{v}\cdot\hat{n}=i(\hat{k}\cdot\hat{n})V(k,\eta)\sim i(\hat{k}\cdot\hat{n})T_V(k,r)\\
&\dot{\vec{v}}\cdot\hat{n}=i(\hat{k}\cdot\hat{n})\dot{V}(k,\eta)\sim i(\hat{k}\cdot\hat{n})\dot{T}_V(k,r)
\end{aligned} \tag{40}$$

IV. SHOOTING METHOD

Our differential equation is

$$\partial_r^2\phi+\frac{2}{r}\partial_r\phi=\left(\frac{\rho}{M_s^2}-\mu^2\right)\phi+\lambda\phi^3 \tag{41}$$

The BC's are

$$\begin{aligned}
\partial_r\phi(0)&=0\\
\phi(\infty)&=1
\end{aligned} \tag{42}$$

We can convert this second order DE into two first order DE's

$$\begin{aligned}
y_1&=\phi\\
y_2&=\partial_r\phi
\end{aligned} \tag{43}$$

Thus the DE is

$$y_2'+\frac{2y_2}{r}=\left(\frac{\rho}{M_s^2}-\mu^2\right)y_1+\lambda y_1^3 \tag{44}$$

V. CHECKS IN CLUSTERS

If all galaxies in a cluster are screened we have nothing to measure. So we must check that out to the screening radius we see galaxies around clusters.

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[3] M. Zumalacrregui, E. Bellini, I. Sawicki, J. Lesgourgues, and P. G. Ferreira, JCAP **1708**, 019 (2017), arXiv:1605.06102 [astro-ph.CO].

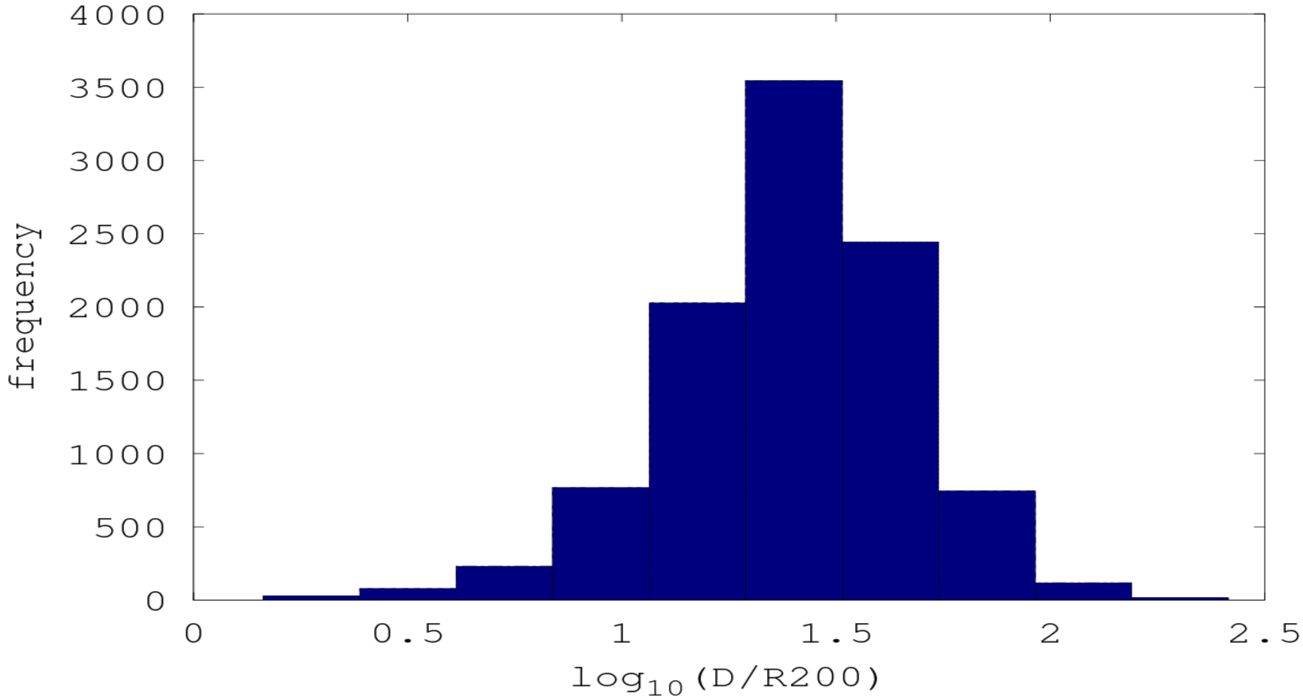

FIG. 1. SDSS

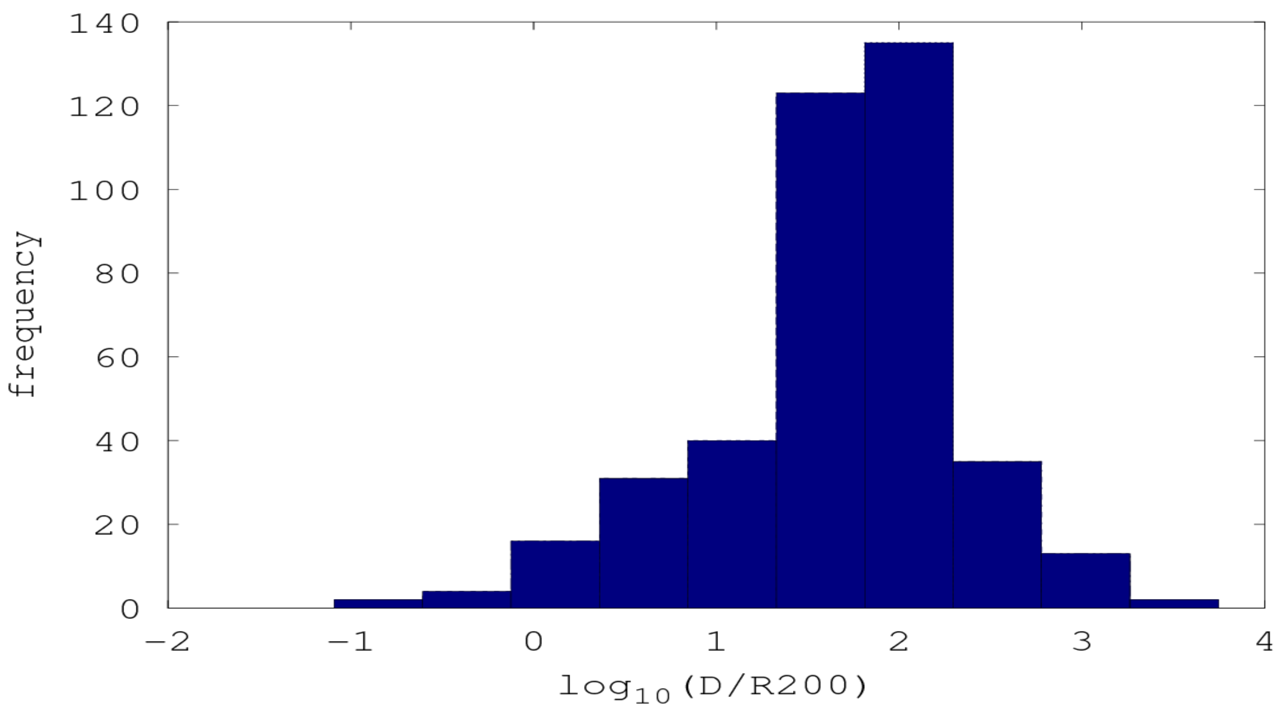

FIG. 2. XMM