1-A) Prior and Posterior Distributions:

1. <u>Likelihood Function</u>:

It is a fundamental concept in statistical inference. It indicates how likely a particular population is to produce an observed sample.

Bayesian Probability Distribution:

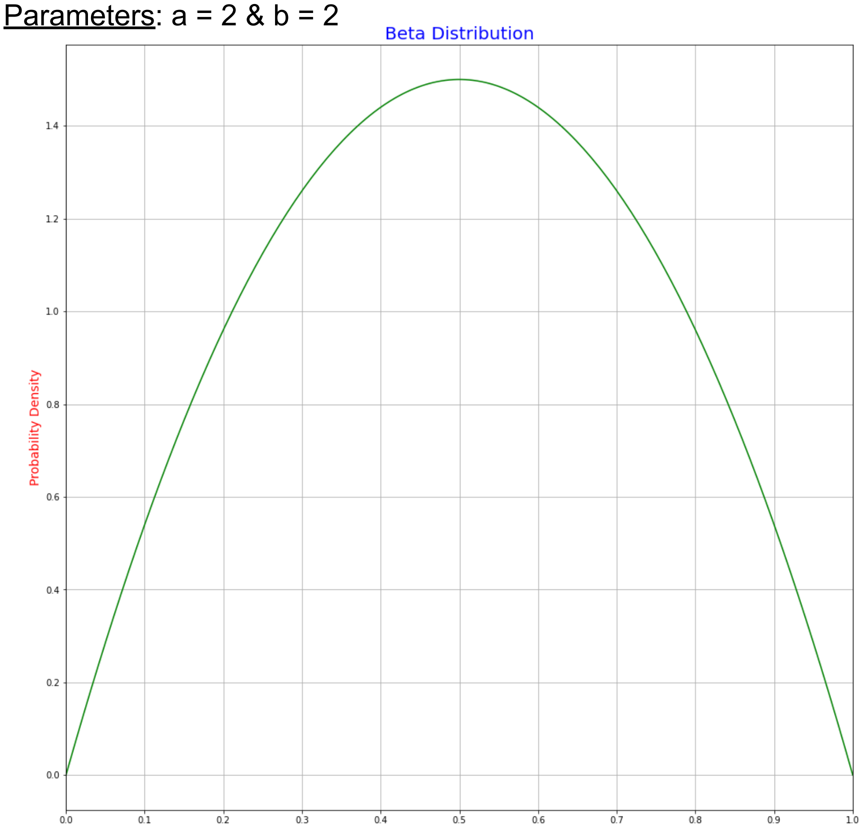
$$\underbrace{\boxed{P(A|B)}}_{\text{posterior}} = \underbrace{\boxed{P(A)}}_{\text{prior}} \times \frac{\boxed{P(B|A)}\ \text{likelihood}}{\boxed{P(B)}\ \text{marginal}}$$

$$E[X] = \frac{a}{a+b}$$

$$Var[X] = \frac{ab}{(a+b)^2(a+b+1)}$$

Initial Prior Distribution (Beta Distribution):

<u>Parameters</u>: a = 2 & b = 2

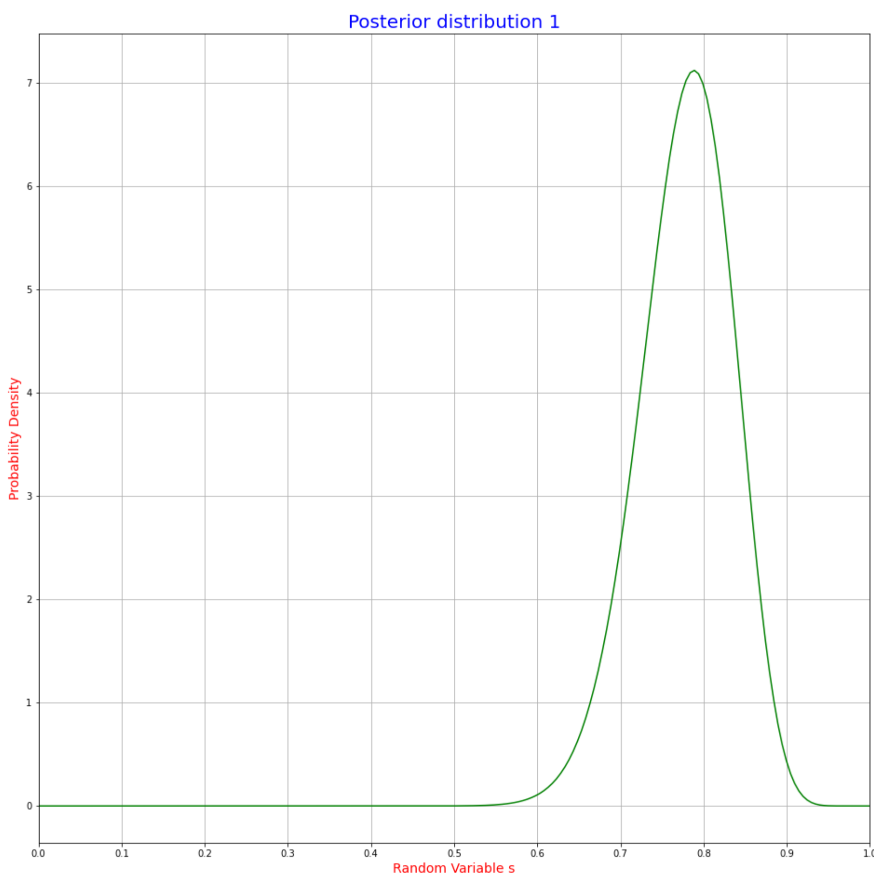


E[x] = a / (a + b)
= 2 / (2 + 2)
= 0.5

Maximum Probability Density is achieved at value of random variable s = 0.5.

Likelihood of s:
It indicates the probability of people liking the new update in the survey.

2. Posterior Distribution of s:
1st Survey:
It is said that in the initial survey, 40 out of 50 people liked the new update.
Let p be the probability that a person who is randomly picked in the sample likes the new update. Then (1 - p) is the probability that he does not like the update.
Of the given data, we can be sure that if we conduct this experiment 50 times, we will get a positive outcome 40 times i.e. 40 of 50 people will like the new update.
We calculate the likelihood of the particular population liking the new update. The likelihood of random variable S becomes 40/50 i.e. 0.8.
Thus we add this new likelihood to our prior distribution (The previous beta distribution). So, our total number of trial becomes (2 + 50) = 52, in which we can expect a positive outcome (2 + 40) = 42 times.
Hence, the parameters for the beta distribution change as follows: a = 2 + 40, b = 2 + 10.

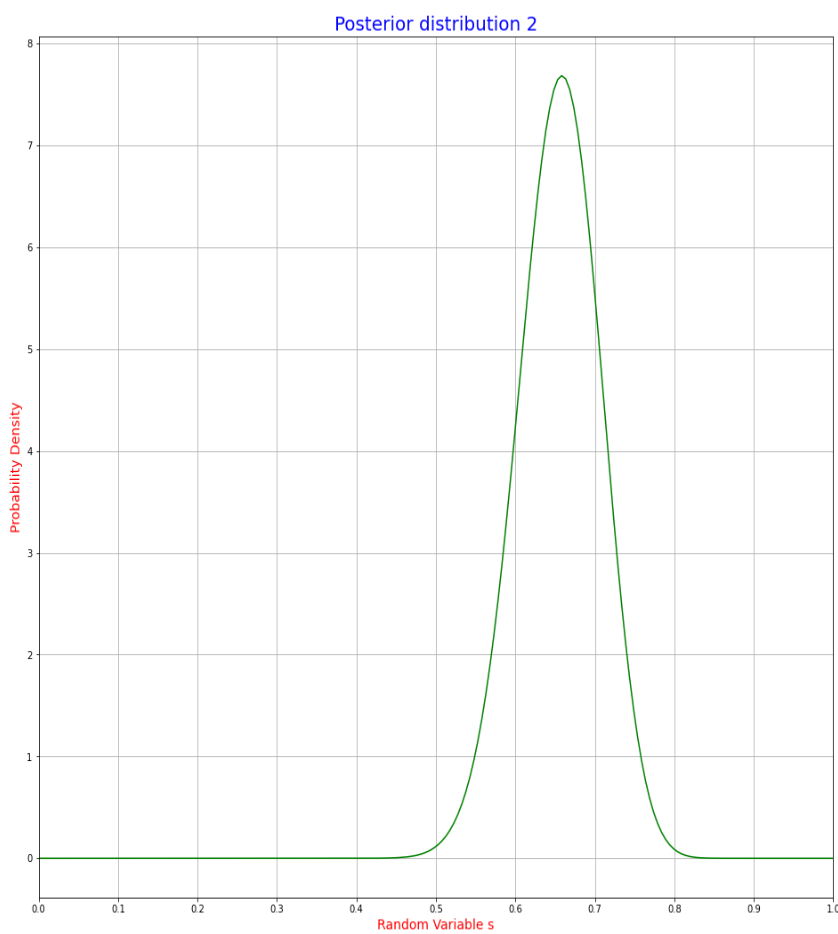


E[x] = a / (a + b)
= 42 / (42 + 12)
= 0.77

Maximum Probability Density is achieved at value of random variable s = 0.77.

2nd Survey:

It is said that in the second survey 13 out of 30 people liked the new update.
Let p be the probability that a person who is randomly picked in the sample likes the new update. Then (1 - p) is the probability that he does not like the update.
Of the given data, we can be sure that if we conduct this experiment 30 times, we will get a positive outcome 13 times i.e. 13 of 30 people will like the new update.
We calculate the likelihood of the particular population liking the new update. The likelihood of random variable S becomes 13/30 i.e. 0.43.
Thus we add this new likelihood to our prior distribution (The last posterior distribution). So, our total number of trial becomes (50 + 30) = 80, of which we can expect a positive outcome (40 + 13) = 53 times.
Hence, the parameters for the beta distribution change as follows: a = 2 + 40 + 13, b = 2 + 10 + 17.
Posterior distribution obtained in previous figure becomes the prior distribution for this plot.

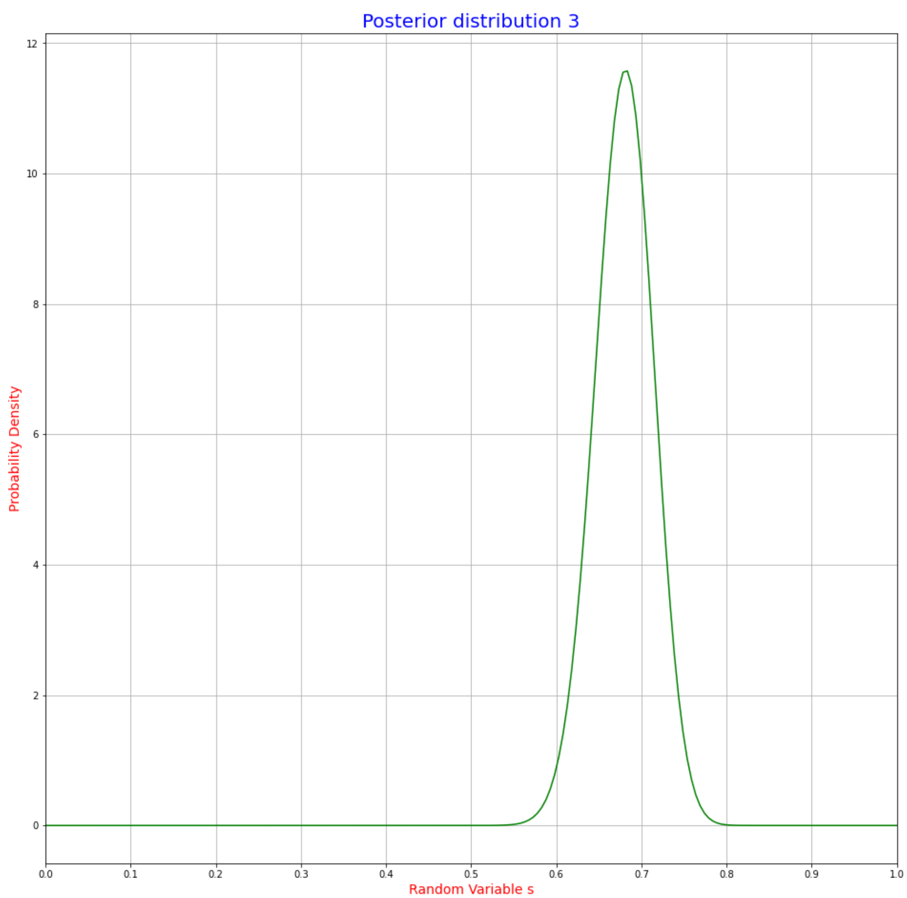


E[x] = a / (a + b)
= 55 / (55 + 29)
= 0.65

Maximum Probability Density is achieved at value of random variable s = 0.65.

3rd Survey:

It is said that in the third survey 70 out of 100 people liked the new update.
Let p be the probability that a person who is randomly picked in the sample likes the new update. Then (1 - p) is the probability that he does not like the update.
Of the given data, we can be sure that if we conduct this experiment 100 times, we will get a positive outcome 70 times i.e. 70 of 100 people will like the new update.
We calculate the likelihood of the particular population liking the new update. The likelihood of random variable S becomes 70/100 i.e. 0.7.
Thus we add this new likelihood to our prior distribution (The last posterior distribution). So, our total number of trial becomes (50 + 30 + 100) = 180, of which we can expect a positive outcome (40 + 13 + 70) = 123 times.
Hence, the parameters for the beta distribution change as follows: a = 2 + 40 + 13 + 70, b = 2 + 10 + 17 + 30.



E[x] = a / (a + b)
= 125 / (125 + 59)
= 0.68

Maximum Probability Density is achieved at value of random variable s = 0.68.