IE-415
Control Of Autonomous System

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<u>Assignment - 2</u>

Questions:

1) Describe ‘non-minimum phase’ system transfer functions. Why are they of particular concern in feedback control systems. Discuss whether you can use a PID controller to successfully control a non-minimum phase system.

→ Description of ‘non-minimum phase’:

- A non-minimum phase system refers to a system that has one or more zeros located in the right-half of the complex plane (i.e., with positive real parts in the Laplace domain). This characteristic can lead to problematic behavior when designing control systems, particularly when using feedback control.

  In control theory, the transfer function of a system is typically expressed as:

  $$G(s)=N(s)/D(s)$$

  Where **N(s)** is the numerator polynomial representing the system's zeros, and **D(s)** is the denominator polynomial representing the system's poles.

- If the zeros of N(s) have positive real parts, the system is classified as a non-minimum phase. The term **"non-minimum phase"** specifically indicates that the system’s phase response is greater than that of a system with all of its zeros in the left-half plane (minimum phase system), for the same magnitude respons.

Key Characteristics of Non-Minimum Phase Systems:

- **Right-Half Plane Zeros (RHP Zeros)**: The defining feature of a non-minimum phase system is that it has zeros in the right-half plane.

- **Phase Lag**: Non-minimum phase systems exhibit excessive phase lag, which makes the system more challenging to stabilize using feedback control. This lag often increases the risk of instability when designing a control system.

- **Initial Undershoot**: When attempting to follow a reference signal or step input, a non-minimum phase system typically exhibits an initial undershoot. This means that the system initially moves in the opposite direction of the desired output before settling on the correct trajectory.

→ Concerned about the feedback control system:

Stability Issues:

● Feedback control systems aim to achieve stable operation by manipulating the phase and magnitude of the system's response. In non-minimum phase systems, the presence of RHP zeros creates a significant phase lag, making it difficult to design controllers that can stabilize the system without pushing it toward instability.

Performance Limitations:

● Because of the initial undershoot and phase lag, it becomes difficult to achieve fast and accurate tracking of a reference input. The system may take longer to reach the desired state, and the control effort may increase significantly to overcome the effects of the RHP zeros.

Controller Design Challenges:

● The excessive phase lag restricts the controller's ability to provide high-gain feedback, which is crucial for improving performance metrics such as transient response, rise time, and settling time.

● Attempting to directly cancel the RHP zeros using traditional feedback may lead to instability or poor control performance.

→ Using PID Controllers for Non-Minimum Phase Systems:

A PID controller (Proportional-Integral-Derivative) is a common type of feedback controller used for stabilizing and controlling systems. The transfer function of a PID controller is typically expressed as:

$$C(s)=K_p+K_i/s+K_ds$$

Where:

● K_p is the proportional gain,

● K_i is the integral gain,

● K_d is the derivative gain.

PID controllers are widely used because they offer simplicity and effectiveness in controlling many types of systems. However, they may face challenges when applied to non-minimum phase systems.

Challenges of Using PID on Non-Minimum Phase Systems:

1**. Phase Lag Compensation:**
○ A PID controller can help compensate for phase lag to some extent by adjusting the gains (e.g., using derivative action to increase phase margin), but it cannot fully overcome the phase lag introduced by the RHP zeros in a non-minimum phase system.
2**. Initial Undershoot:**
○ A PID controller may struggle with the initial undershoot caused by non-minimum phase behavior. The system might initially move in the opposite direction of the desired output, which could lead to undesirable performance.

3. Instability Risk:
○ The RHP zeros introduce a risk of instability if the PID controller's gains are not carefully tuned. Since PID controllers often rely on feedback with high gains, attempting to apply this approach to a non-minimum phase system may destabilize the system.

Can a PID Controller Be Used for Non-Minimum Phase Systems?

While a PID controller can be used for non-minimum phase systems, it is not always the best solution. The key concerns are:

● **Limited Performance:** A PID controller might stabilize a non-minimum phase system, but it may not provide optimal performance (e.g., long settling time, large overshoot).
● **Careful Tuning:** The PID gains must be tuned very carefully to avoid instability, and in many cases, additional compensation (e.g., lead-lag compensators) might be required.

In some cases, advanced control strategies, such as LQR (Linear Quadratic Regulator) or model predictive control (MPC), which take into account the system’s dynamics more thoroughly, may provide better performance for non-minimum phase systems.

2) Discuss the kinematics of a differential drive mobile robot. Also discuss what is a Braitenberg vehicle and its peculiarity.

→ Differential Drive Mobile Robot:

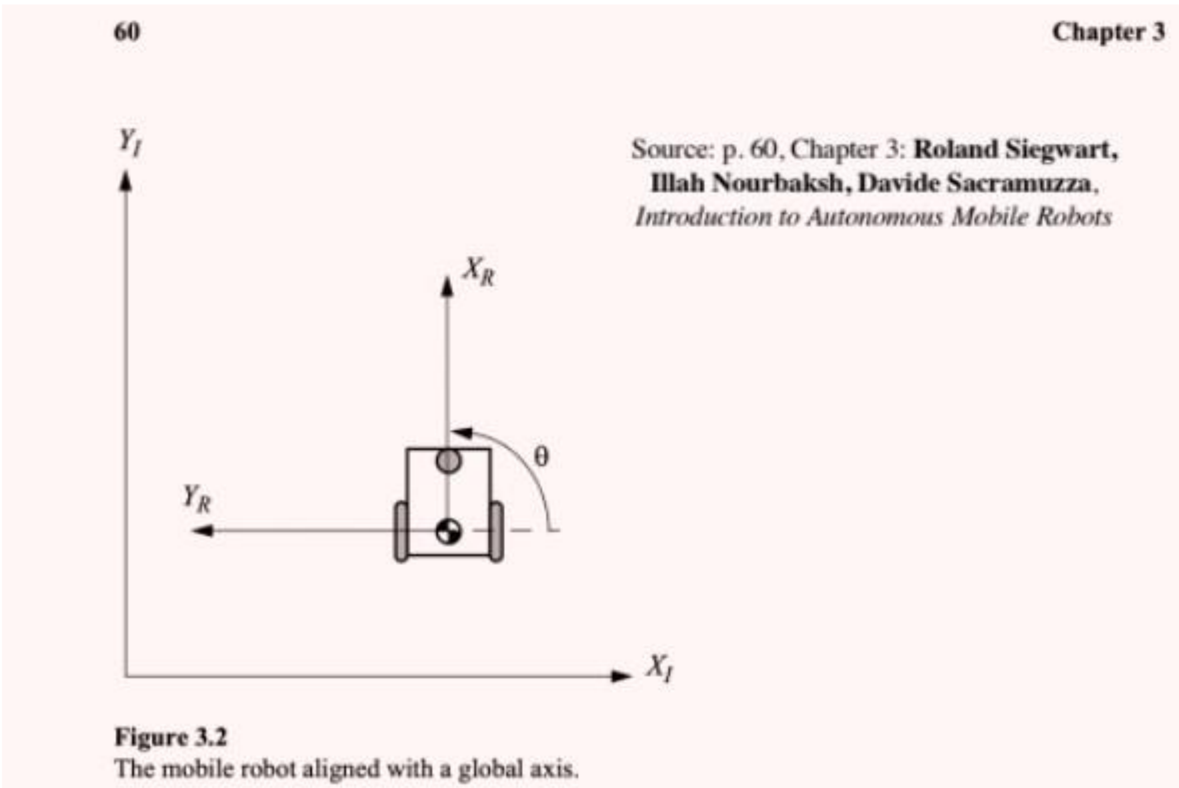


Figure 3.2
The mobile robot aligned with a global axis.

- A **differential drive mobile robot** is a robot that has two independently driven wheels, typically mounted on either side of the robot chassis. The robot's motion is determined by the relative velocities of these two wheels.

→ Key Components of Kinematics:

1. **Robot's Configuration:**
   - The robot's position and orientation in the 2D plane can be represented by the state vector x(t),y(t),θ(t)

   where:

- x(t) and y(t) are the coordinates of the robot's position.

  reference frame.

- Pose of the robot
  - $\xi = (x\ y\ \theta)\ (transpose)$

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos\theta & \cos\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- $\dot{\xi}_R = \mathbf{R}(\theta)\dot{\xi}_I$
- The forward kinematic model of a differential drive robot is given by
- $\dot{\xi}_I = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \mathbf{f}(\ell, r, \theta, \dot{\varphi}_{\text{left}}, \dot{\varphi}_{\text{right}})$ $\quad$ ℓ — wheel distance from point P $\quad$ r — wheel radius
- $\dot{\xi}_I = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{r(\dot{\varphi}_{\text{left}} + \dot{\varphi}_{\text{right}})}{2} \\ 0 \\ \frac{r(\dot{\varphi}_{\text{left}} - \dot{\varphi}_{\text{right}})}{2\ell} \end{pmatrix}$

θ(t) is the robot’s orientation (heading) with respect to a global

→ Braitenberg vehicle:

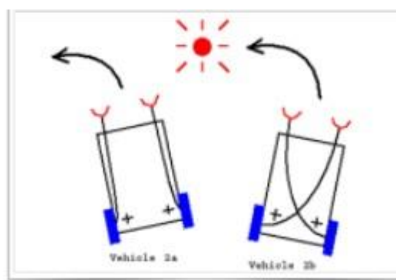
Braitenberg Vehicles:

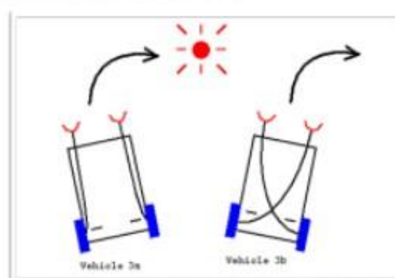
Braitenberg vehicles are simple, autonomous robots that exhibit complex behaviors based on straightforward sensory-motor connections. They are named after **Valentino Braitenberg**, who introduced them in his book *"Vehicles: Experiments in Synthetic Psychology."*

Basic Structure of Braitenberg Vehicles:

- A Braitenberg vehicle consists of sensors (for detecting environmental stimuli) and actuators (for driving motors attached to wheels).
- The key idea is to directly connect the sensor outputs to the motors in a simple, predefined way. For example, a sensor detecting light could be connected to the motor controlling the robot's wheel, and the strength of the sensor signal might control the speed of the motor.

- Autonomous agent based on sensor inputs
- Primitive sensors measure stimulus at a point
- Wheels function as actuators or effectors
- Sensors directly connected to effectors
- Vehicle's behaviours vary based on sensor and wheel connection
- Can exhibit complex goal-oriented behaviours





→ Peculiarities of Braitenberg Vehicles:

1. **Simple Mechanisms, Complex Behavior**:
    - Despite their simple design, Braitenberg vehicles can exhibit surprisingly complex behaviors that resemble biological organisms. For example, a vehicle might appear to "fear" light by moving away from it, or it might "love" light by moving toward it.
2. **Behavioral Modes**:
    - Braitenberg vehicles are often described based on the kind of behavior they exhibit, which emerges from the interaction between their sensors and actuators. Common behaviors include:
        - **Attraction**: The vehicle moves toward a stimulus (e.g., light, heat, or sound).
        - **Repulsion**: The vehicle moves away from a stimulus.
        - **Aggression**: The vehicle speeds up when moving toward a stimulus.
        - **Fear**: The vehicle slows down or turns away from a stimulus.
3. **Connections**:
    - The behavior of the vehicle depends on how its sensors are connected to its motors. For example:
        - **Excitatory connections**: A sensor increases the speed of the motor, making the vehicle move toward the stimulus.
        - **Inhibitory connections**: A sensor decreases the speed of the motor, making the vehicle move away from the stimulus.

Key Features of Braitenberg Vehicles:

- **Reactive Nature**: The vehicle's actions are purely reactive to the environment. There is no internal decision-making or computation, yet the vehicle behaves in ways that can seem intentional.
- **Biological Inspiration**: The vehicles are often seen as metaphors for animal behavior, showing how simple neural structures could give rise to complex and adaptive behaviors.

Braitenberg Vehicle Example:

One classic Braitenberg vehicle design is where two light sensors are connected to two motors in a crisscross fashion:

- The right sensor is connected to the left motor.
- The left sensor is connected to the right motor.

This design causes the vehicle to turn toward the light, giving the appearance that it is "attracted" to the light source.

3. Consider the system shown in Figure 8–76(a), where K is an adjustable gain and G(s) and H(s) are fixed components. The closed-loop transfer function for the disturbance is

$$\frac{C(s)}{D(s)} = \frac{1}{1+KG(s)H(s)}$$

To minimize the effect of disturbances, the adjustable gain K should be chosen as large as possible.

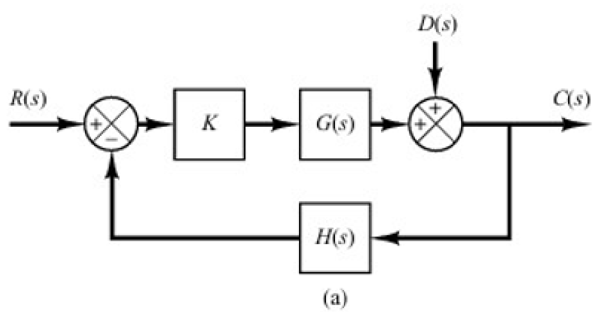


Figure 8_76 (a) Control system with disturbance entering in the feedforward

Is this true for the system in Figure 8–76(b), too?

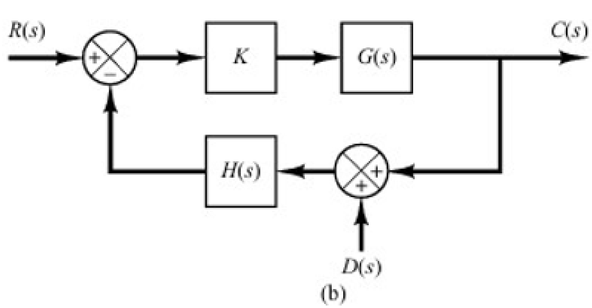


Figure 8–76 (b) control system with disturbance entering in the feedback path.

→ We know the transfer function gain for the first figure is:

$$\frac{C(s)}{D(s)} = \frac{1}{1+KG(s)H(s)}$$

Calculation for the gain for the second figure:

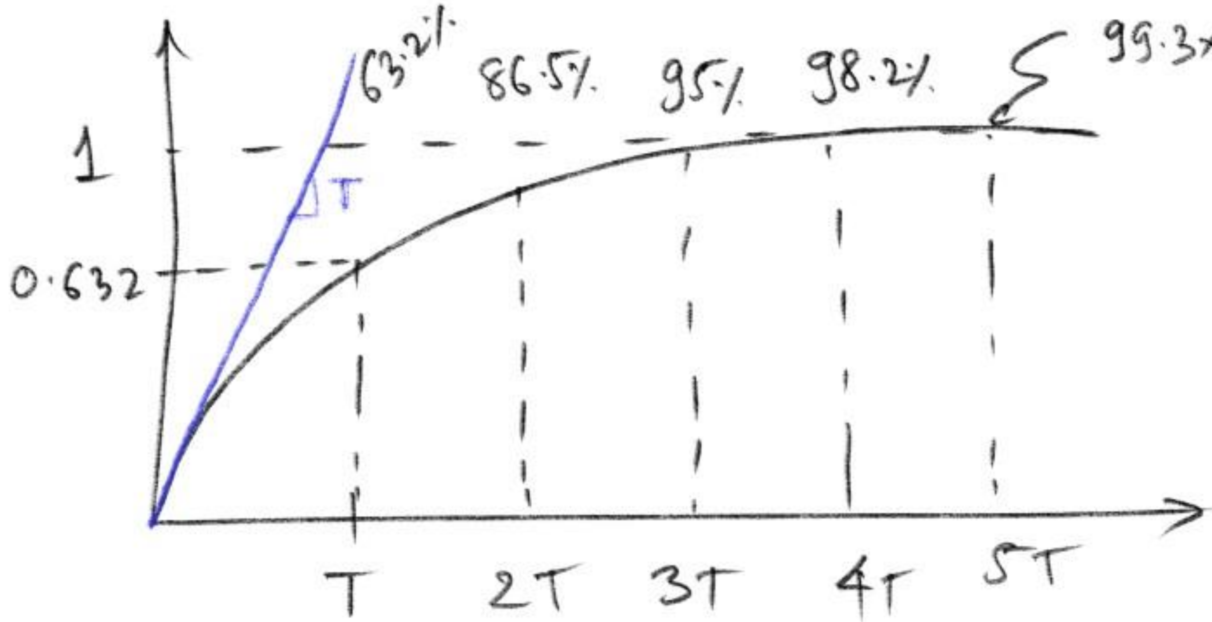
$$-\ \ \mathrm{KC}(s)\mathrm{G}(s) - KG(s)D(s)H(s) = C(s)$$

$$\frac{C(s)}{\mathrm{D}(s)} = \frac{-1}{\frac{1}{KG(s)H(s)}+1}$$

Transfer function of figure 8-76(b) we can say the following answer of

increase the K the overall answer of the disturbance would increase as increasing the K the denominator becomes smaller so the increase.

decreasing of the disturbance We would decrease the K.



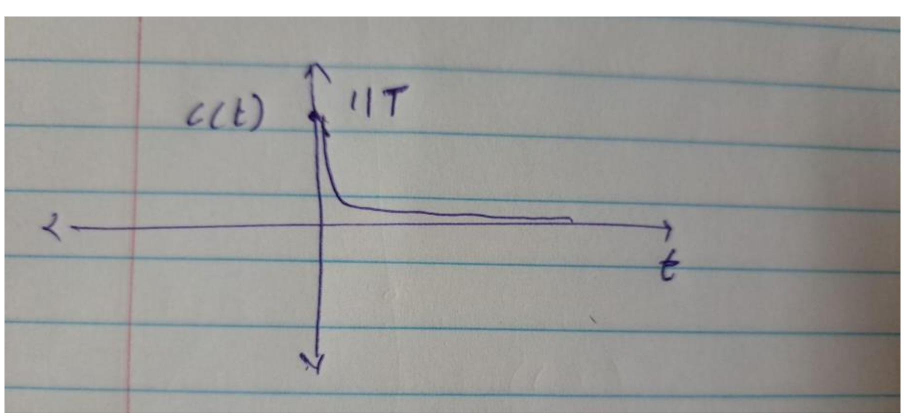
2) **Taking R(s) = 1 (Unit Response)**

$C(s) \ = \ \frac{1}{Ts+1}$

- **Taking Laplace Inverse both the side**

$C(t) \ = \ \frac{1}{T} e^{\frac{-t}{T}}$

- **The Graph for the above equation is like below:**



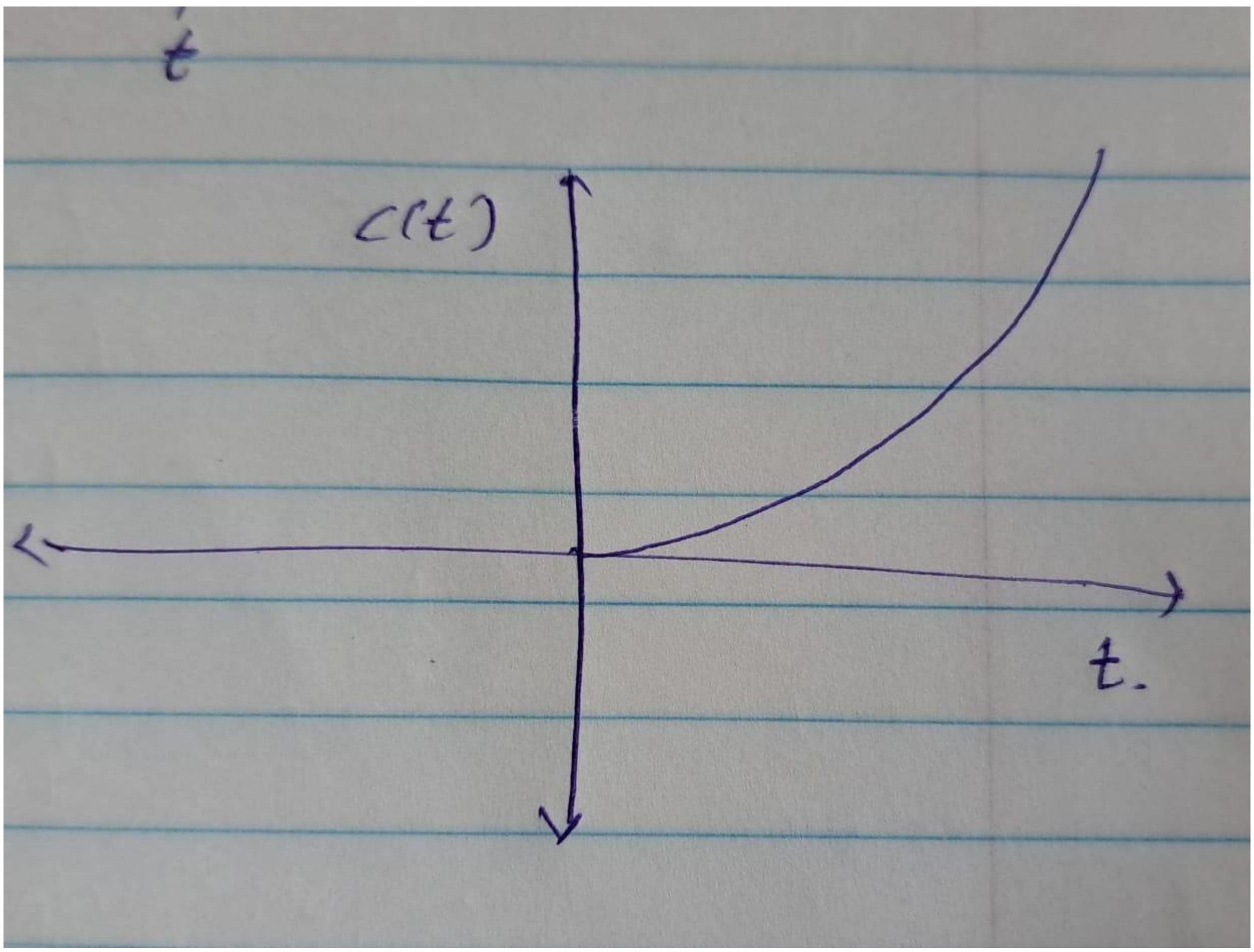
3) Likewise for R(s) = ramp signal (1/s^2)

$$C(s) = \frac{1}{s^2(Ts+1)}$$

$$C(t) = (L^{-1})(\frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{Ts+1})$$

$$C(t) = t - T + Te^{\frac{-t}{T}}$$

- **The graph is like below:**



→ **Second order System:**

$$C(s) \; = \; \frac{R(s)\,K}{Js^{2}+Bs+K}$$

Taking,

$$w_{n}^{\;2} \; = \; K/J,\; 2rw_{n} \; = \; B/J$$

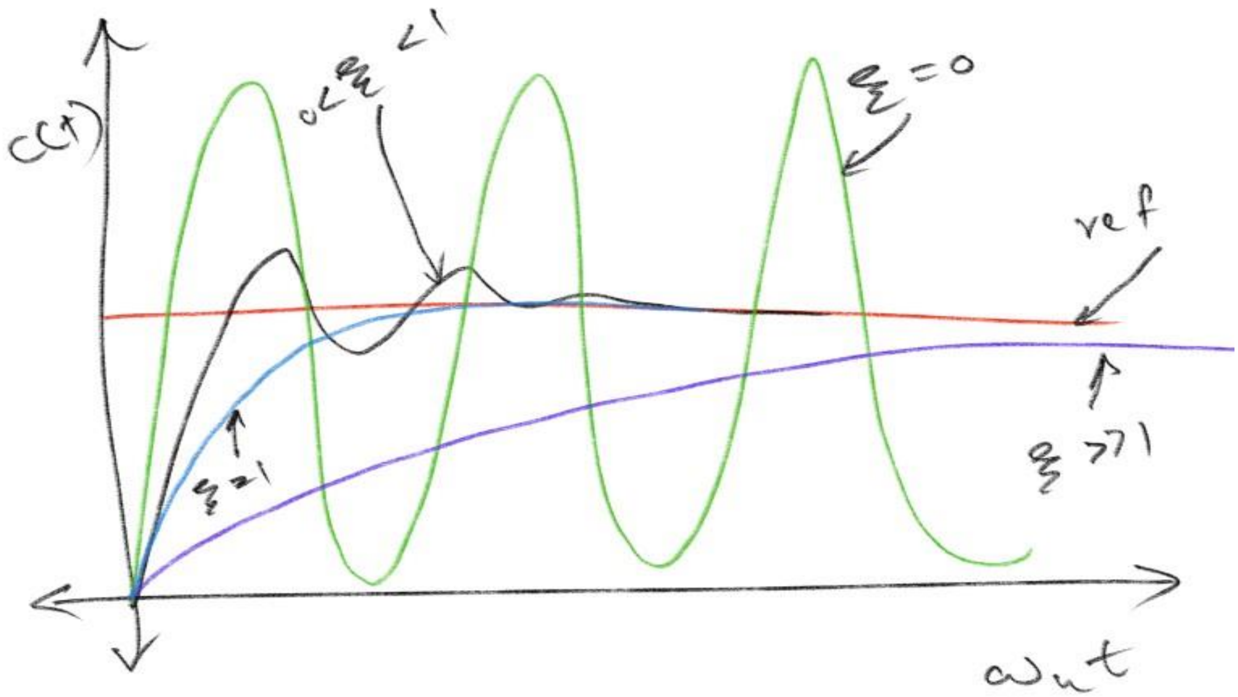
$$C(s) \; = \; \frac{R(s)\,{}^{*}w_{n}^{\;2}}{s^{2}+2rw_{n}+w_{n}^{\;2}}$$

Where ,

w_{n} is the natural Frequency

r is damping ratio

- The plot is like below:



5) What is ‘tracking control’ and ‘regulatory control’? Can a PID controller perform both? Illustrate with a complete numerical example and simulations in MATLAB.

1.
2.

→ **Tracking Control & Regulatory Control:**

the goal is for the output of a system to **follow** or
r(t)r(t)r(t). The system output is continuously
and the control system adjusts e input to

the desired output change such
trajectory, or cruise contr system aintai ng a

, the goal is intain the s m output at a
of disturbances. e pr ary focus is reject
the system back to t se henever deviations

Regulatory control is co ly used stems like temperature
pressure control in proce in ies, and water level control

—> **Can a PID Controller Perform oth cking and Regulatory Control?**

Yes, a **PID controller** can perform **trac co ol** and **regulatory control**, depending on how it is applied:

- **For Tracking Con** The PID nt er adjusts the system input to follow a time-varying refe nce nal.
- **For Regulatory ontr** The PI controller maintains the setpoint, nsating urbances or errors in the

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d \quad k_d \frac{d}{dt}$$

Where.

- u(t) is control input (e.g.,
- e(t)=r(t)−y(t) is the error,
  output.
- Kp, Ki, and Kd are the

3. **Tracking Control:**
   - **Objective**: In **tracking control**,
     track a desired reference signal
     compared to the reference signal,
     minimize the error.

- **Application**: Tracking is used when as in robotic arms following a specific speed.

4. **Regulatory Control:**
   - **Objective**: In **regulatory control** **fixed setpoint** in the face disturbances and bring occur.
   - **Application**: regulation,

system output at a constant process.

$\tau +$ $e(t)$

voltage to a motor, etc.). with r(t) being the

reference signal and y(t) the system proportional,

integral, and derivative gains, respectively.

→ **Example with MATLAB Simulation**:

$$G(s) = \frac{1}{ms^2+bs+k}$$

Where:

- m is the mass,
- b is the damping coefficient,
- k is the spring constant.

For this example, let's use:

- m = 1 kg,
- b = 1 Ns/m,
- k = 10 N/m

$$G(s) = \frac{1}{s^2 + s + 10}$$

Tracking Control:

1. The system needs to track a sinusoidal reference signal r(t)=sin(2t)
2. We will use a PID controller with gains Kp=100, Ki=1, and Kd=20.

Regulatory Control:

1. The system will regulate the output at a constant setpoint r(t)=1, compensating for an external disturbance that applies a sudden force.

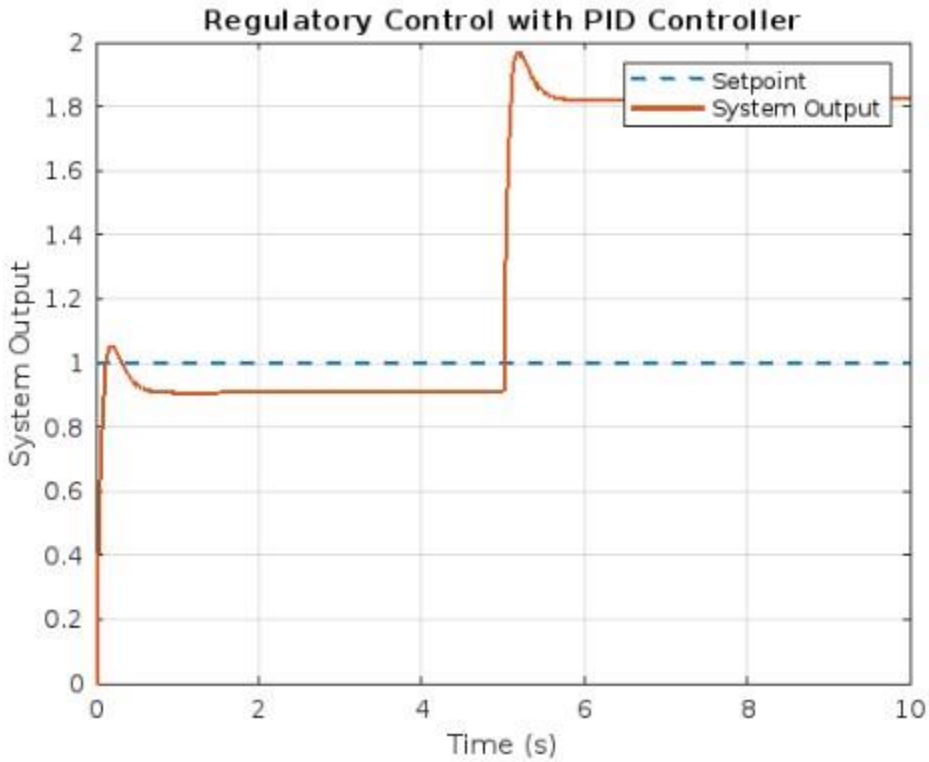
→ **Matlab Execution:**

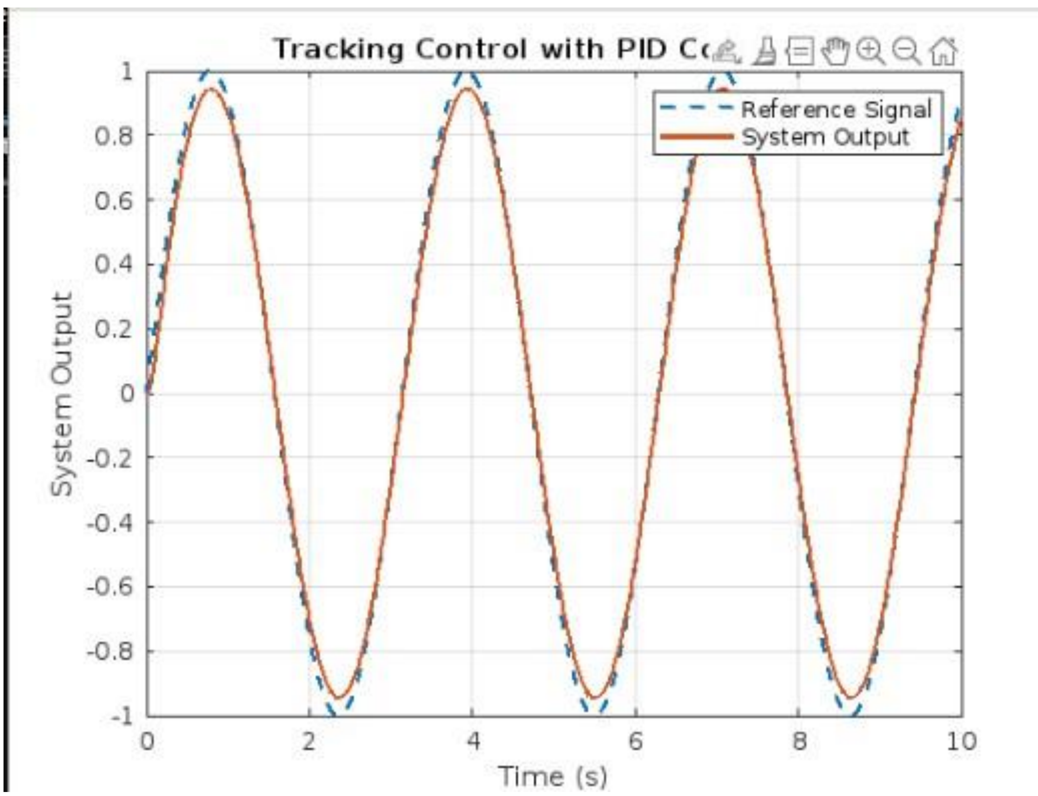
- **Matlab code:**

```
m = 1;
b = 1;
k = 10;

num = 1;
den = [m b k];
sys = tf(num, den);

% PID Controller Parameters
Kp = 100;
Ki = 1;
Kd = 20;

C = pid(Kp, Ki, Kd);

closed_loop_sys = feedback(C * sys, 1);

% --- Tracking Control Example ---

t = 0:0.01:10;
r_tracking = sin(2 * t);


[y_tracking, t_tracking] = lsim(closed_loop_sys,
r_tracking, t);

% Plot the tracking control result
```

```
figure; plot(t_tracking, r_tracking, '--',
'LineWidth', 1.5); hold on;

plot(t_tracking, y_tracking, 'LineWidth', 2);
title('Tracking Control with PID Controller');
xlabel('Time (s)');
ylabel('System Output');
legend('Reference Signal', 'System Output');
grid on;

% --- Regulatory Control Example ---


r_regulation = ones(size(t));
disturbance = [zeros(1, 500) 1*ones(1, 501)];


[y_regulation, t_regulation] = lsim(closed_loop_sys,
r_regulation + disturbance, t);


figure;
plot(t_regulation, r_regulation, '--', 'LineWidth',
1.5); hold on;
plot(t_regulation, y_regulation, 'LineWidth', 2);
title('Regulatory Control with PID Controller');
xlabel('Time (s)');
ylabel('System Output');
legend('Setpoint', 'System Output');
grid on;
```

- **Matlab Output:**


Regulatory Control with PID Controller
Setpoint
System Output
System Output
Time (s)


Tracking Control with PID Cc
Reference Signal
System Output
System Output
Time (s)