Project Report

Analysis of a Fourth-Order Butterworth Active Low-Pass Filter Using Tow–Thomas Topology

For

Curious Analog IC Design Course

By

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Abstract

This report presents the design and MATLAB-based pre-simulation of a fourth-order low-pass filter using the Tow–Thomas topology. The filter is analyzed at the system level to determine its transfer function, frequency and phase response, and pole–zero characteristics. Component values (resistors and capacitors) are calculated in MATLAB based on the desired specifications. These values are then used in Xschem with ideal and non-ideal op-amps to replicate the MATLAB simulation results. Additionally, the filter response is analyzed under variations in the op-amp's unity-gain bandwidth (UGB) and finite gain, demonstrating how these non-idealities affect the frequency response, stability, and overall performance. The work highlights the combined use of MATLAB and Xschem for validating filter designs and understanding the impact of op-amp limitations on practical circuit implementation.

1 Introduction

Filters are essential building blocks in electronics, used to remove noise, suppress unwanted frequencies, and allow only the desired portion of a signal to pass. Low-pass filters are particularly important because they preserve the signal of interest while attenuating higher-frequency components.

In this project, a fourth-order low-pass Butterworth filter is designed and pre-simulated in MATLAB. The MATLAB simulation is used to obtain the transfer function, frequency response, and pole–zero plot. Based on these results, the required resistor and capacitor values are calculated. These values are then applied in Xschem using ideal voltage-controlled voltage sources (VCVS) and op-amps to verify that the circuit-level implementation replicates the MATLAB results. This approach ensures the design meets specifications before proceeding to a full transistor-level realization.

2 Specification

The target specifications for the filter, which guided both MATLAB pre-simulation and Xschem implementation, are summarized in Table 1.

Parameter	**Target / Notes**
Filter Type	4th-order Butterworth (cascade of two biquads)
Passband Gain	0 to 0.5 dB
Passband Frequency	0 – 50 kHz
Stopband Gain	< -40 dB at 80 kHz

Table 1: Design specifications for the 4th-order Butterworth filter.

3 Methodology

The design and analysis of the fourth-order Butterworth low-pass filter are carried out using a MATLAB–based pre-simulation framework, followed by circuit-level verification in Xschem. The methodology combines classical filter theory with practical insights from biquadratic (biquad) realizations such as the Tow–Thomas topology. The design flow proceeds as follows.

3.1 Tow–Thomas Biquad Topology

The Tow–Thomas (TT) biquad topology is used to implement each second-order section of the filter. This topology is preferred because it provides:

- Independent control of natural frequency (ω_n) and quality factor (Q),
- Low component count and compact implementation,
- High linearity and stability for active filter realizations,
- Low input-referred noise and predictable bandwidth behavior.

The Tow–Thomas (TT) biquad can be interpreted as a negative-feedback loop that contains two integrators. A local feedback around one of the integrators introduces loss, which helps stabilize the overall circuit. Figure 1 illustrates the differential implementation of this biquad topology.

The transfer function of the TT biquad is expressed as:

$$\frac{V_{\text{out}}}{V_{\text{in}}}(s) = \frac{A_v\,\omega_n^2}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2},$$

where the gain factor, natural frequency, and quality factor are related to the component values by:

$$A_v = \frac{R_4}{R_1}, \qquad \omega_n = \frac{1}{\sqrt{R_3 R_4 C_1 C_2}}, \qquad Q = R_2\sqrt{\frac{C_1}{R_3 R_4 C_2}}.$$

In the TT topology, R_1 sets the passband gain, R_2 and R_3 determine Q and ω_n, while C_1 and C_2 set the frequency scaling. This makes it ideal for realizing a 4th-order Butterworth response by cascading two such biquads. The differential structure allows precise tuning of each stage and ensures that gain distribution does not degrade noise performance. Finite op-amp non-idealities such as limited gain, bandwidth, and output resistance are also considered to maintain stability and linearity.

3.2 Realization of Filter in MATLAB

The fourth-order Butterworth low-pass filter is realized and analyzed in MATLAB before circuit-level implementation. The target specifications and calculated component values are summarized below.

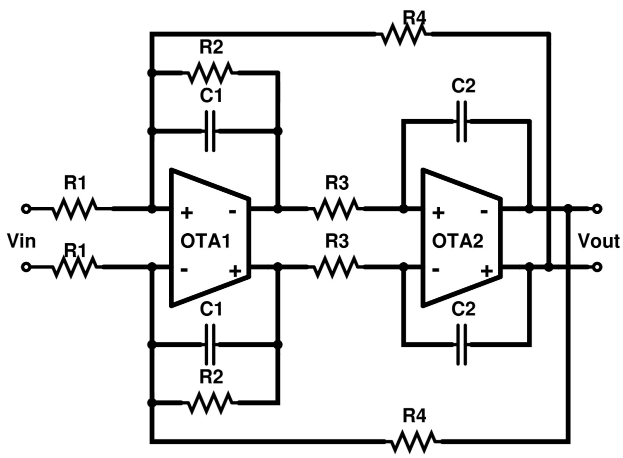

Figure 1: Tow–Thomas biquad circuit diagram.

3.2.1 Filter Specifications

- Cutoff frequency: $f_c = 50$ MHz
- Desired quality factors: $Q_1 = 0.9$, $Q_2 = 0.7$ (for a 4th-order Butterworth response)
- Capacitor values: $C_1 = C_2 = 0.5$ pF (initial assumption)

3.2.2 MATLAB-Based Component Values

The resistor and capacitor values obtained from MATLAB for the two biquad stages are:

Stage	R_1 (kΩ)	R_2 (kΩ)	R_3 (kΩ)	R_4 (kΩ)	C_1 (pF)	C_2 (pF)
1	5.84	5.26	5.84	5.84	0.50	0.50
2	5.84	4.09	5.84	5.84	0.50	0.50

3.2.3 Analytical Characterization of the Filter

The transfer functions of the individual biquad stages and the overall cascaded filter are expressed below. Each stage shows the general formula followed by the MATLAB-obtained transfer function.

$$H_1(s) = \frac{A_{v1}\,\omega_{n1}^2}{s^2 + \frac{\omega_{n1}}{Q_1}s + \omega_{n1}^2} \quad = \frac{9.87 \times 10^{16}}{s^2 + 3.491 \times 10^8 s + 9.87 \times 10^{16}},$$

$$H_2(s) = \frac{A_{v2}\,\omega_{n2}^2}{s^2 + \frac{\omega_{n2}}{Q_2}s + \omega_{n2}^2} \quad = \frac{9.87 \times 10^{16}}{s^2 + 4.488 \times 10^8 s + 9.87 \times 10^{16}},$$

$$H_{\text{total}}(s) = H_1(s) \cdot H_2(s) \quad = \frac{9.741 \times 10^{33}}{s^4 + 7.979 \times 10^8 s^3 + 3.541 \times 10^{17} s^2 + 7.875 \times 10^{25} s + 9.741 \times 10^{33}}.$$

Filter component	Zero	Pole
Cascaded Filter	$0 + 0i$	$-1.7453 \times 10^8 - 1.7453 \times 10^8 i$
	$0 + 0i$	$-2.2439 \times 10^8 - 2.2439 \times 10^8 i$
	$0 + 0i$	$2.6122 \times 10^8 - 2.6122 \times 10^8 i$
	$0 + 0i$	$2.1987 \times 10^8 - 2.1987 \times 10^8 i$

Table 2: Pole-zero locations of the cascaded 4th-order filter.

The poles and zeros of 4th-order filter, obtained from MATLAB, are summarized in Table 2.

The analytical behavior is further validated using MATLAB-generated frequency response and pole-zero plots. Figure 2 shows the Bode magnitude and phase plots.

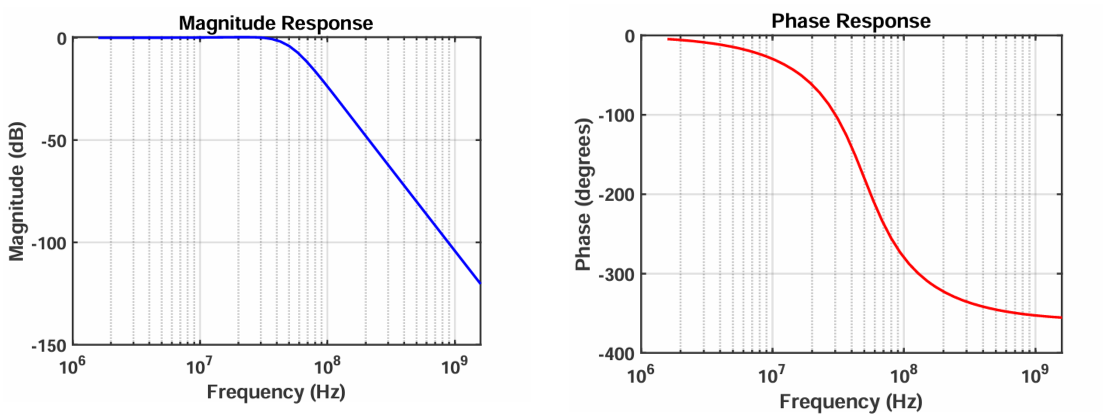

Figure 2: Bode magnitude and phase plots of the fourth-order Butterworth filter.

4 Design Procedure

4.1 Circuit Implementation (Xschem)

The filter is implemented using the Skywater open-source PDK in Xschem. The design procedure follows a hierarchical approach, starting from idealized blocks and gradually including non-idealities.

4.1.1 Op-Amp Design with Ideal VCVS

A differential-input, differential-output operational amplifier was designed for implementing the Tow–Thomas topology in differential form. The op-amp is constructed using ideal voltage-controlled voltage sources (VCVS) to simplify modeling while focusing only on the key parameters: gain and bandwidth.

The gain of the model op-amp is determined by element **E1**, a VCVS with a value set to 10,000, corresponding to a gain of 80 dB. The bandwidth is set by an *RC* network placed at the output, chosen such that the cutoff frequency is 200 MHz. The relationship is given by:

$$f_{\mathrm{BW}} = \frac{1}{2\pi RC}$$

For the target bandwidth of 200 MHz, choosing $C = 1\,\mathrm{pF}$ gives:

$$R \approx \frac{1}{2\pi \cdot 200 \times 10^6 \cdot 1 \times 10^{-12}} \approx 8.0\,\mathrm{k\Omega}$$

In addition, **E2** and **E3** are included to convert the single-ended input into a differential form, each having a gain of unity. This configuration ensures that the designed op-amp can be directly used in the differential Tow–Thomas biquad topology.

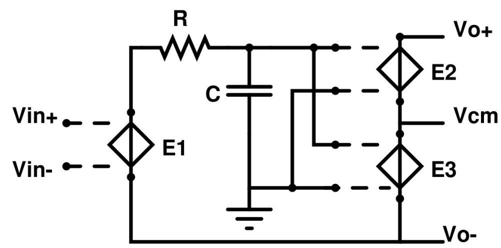

Figure 3: Op-amp implementation using ideal VCVS in Xschem.

4.1.2 Filter Design with Derived R and C Values

Using the designed op-amp model, the Tow–Thomas biquad filter was implemented in Xschem. The resistor and capacitor values chosen for the two second-order sections were derived based on the MATLAB design and verified for the target cutoff frequency and quality factor.

- **Biquad 1:** $R_1 = R_3 = R_4 = 5.84\,\mathrm{k\Omega}, \quad R_2 = 5.26\,\mathrm{k\Omega}, C_1 = C_2 = 0.50\,\mathrm{pF}$
- **Biquad 2:** $R_1 = R_3 = R_4 = 5.84\,\mathrm{k\Omega}, \quad R_2 = 4.09\,\mathrm{k\Omega}, C_1 = C_2 = 0.50\,\mathrm{pF}$

These values were mapped into the schematic to realize the complete 4th-order Butterworth filter. The cascaded configuration of the two biquads ensures the required frequency response as obtained in MATLAB simulations.

5 Results

The circuit shown in Figure 4 was simulated in Xschem, and the filter performance was evaluated. Two sets of simulations were carried out:

1. Varying the **op-amp gain** (20 dB, 40 dB, 60 dB, and 80 dB).
2. Varying the **op-amp unity-gain bandwidth (UGB)** (20 MHz, 50 MHz, 100 MHz, 150 MHz, 200 MHz, and 400 MHz).

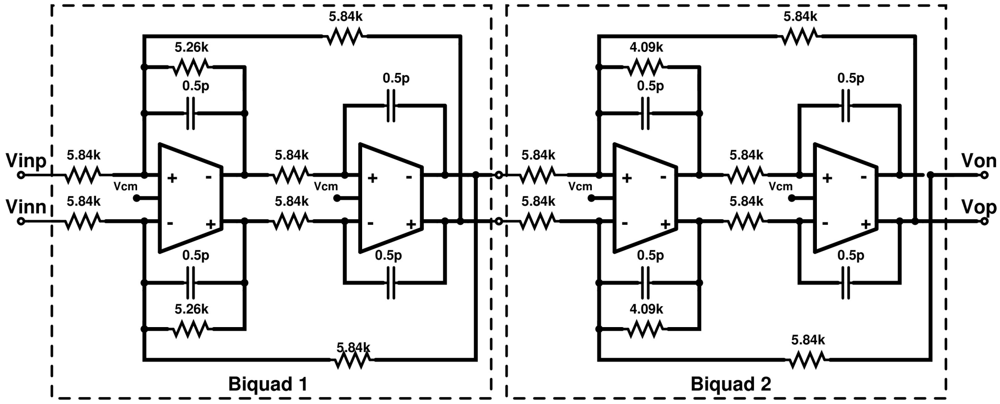

Figure 4: Tow–Thomas 4th-order Butterworth filter implementation in Xschem.

5.1 Effect of Op-Amp Gain

The Tow–Thomas filter was simulated for different op-amp gains. The table below summarizes the passband gain, −3 dB gain, and corresponding cutoff frequency.

Op-Amp Gain (dB)	Passband Gain (dB)	−3 dB Gain (dB)	Cutoff Frequency (MHz)
20	-2.61	-5.61	38.1
40	-0.23	-3.23	49
60	-0.02	-3.02	50
80	0	-3	50

Table 3: Filter performance for different op-amp gain values.

5.2 Effect of Op-Amp UGB

The effect of finite bandwidth on filter performance was studied by varying the op-amp unity-gain bandwidth (UGB) while keeping the op-amp gain constant at 50 dB. The results are summarized in the following table.

Op-Amp UGB (MHz)	Cutoff Frequency (MHz)
20	12.2
50	23.6
100	34.6
150	39.1
200	44.9
400	50.2

Table 4: Filter performance for different op-amp unity-gain bandwidths at fixed 50 dB gain.

Overall, the results show that higher op-amp gain improves passband flatness and ensures accurate cutoff frequency, with the 80 dB case closely matching the MATLAB ideal filter.

For finite UGB, the cutoff frequency increases with bandwidth, and a minimum UGB of about 100 MHz is required for the filter to meet its design specifications without distortion. At very low UGB values (20–50 MHz), the response degrades significantly, while at 200–400 MHz the behavior approaches the theoretical Butterworth response.

6 Conclusion

In this project, a fourth-order low-pass Butterworth filter was designed, analyzed in MATLAB, and implemented in Xschem using the Tow–Thomas biquad topology. The MATLAB simulations provided transfer functions, Bode plots, and pole-zero analyses, which were used to calculate resistor and capacitor values for the two filter stages. The Xschem implementation with ideal op-amps and VCVS accurately replicated the MATLAB results, demonstrating correct passband gain, cutoff frequency, and frequency response.

The effect of op-amp non-idealities was studied by varying both gain and unity-gain bandwidth (UGB). Results show that higher op-amp gain (80 dB) and sufficient UGB (100 MHz) are necessary to maintain passband flatness and achieve the desired cutoff frequency. Lower gain and bandwidth lead to degraded response and reduced accuracy. Overall, the design successfully meets the target specifications at the system level, providing a reliable foundation for further transistor-level implementation, PVT analysis, and layout-aware verification.

Acknowledgment

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References

1. B. Razavi, *Design of Analog CMOS Integrated Circuits*, McGraw-Hill, 2001.
2. Behzad Razavi, "The Biquadratic Filter: A Circuit for All Seasons," IEEE Solid-State Circuits Magazine, 2018.
3. L. Lei *et al.*, "A 5.4-mW 50-MHz 29.3-dBm-IIP3 Fourth-Order Low-Pass Filter," ESSCIRC 2022.
4. MATLAB documentation — Signal Processing Toolbox.

Appendix A: MATLAB code

MATLAB code for calculating component values, transfer functions, Bode and pole-zero plots of the cascaded Tow–Thomas filter.

```
clc; clear; close all;

% === Plot settings ===
fsize = 14;  % Font size
lw = 2;      % Line width
%% === Filter Specifications ===
fc = 50e6;         % Cutoff frequency in Hz
w0 = 2*pi*fc;      % Angular frequency
C1 = 0.5e-12;      % F
C2 = 0.5e-12;      % F
Q1 = 0.9;
Q2 = 0.7;

%% === Assume R1 = R3 = R4 and calculate R2 for each biquad ===
R1_1 = 5.84e3; R3_1 = R1_1; R4_1 = R1_1;
R2_1 = Q1 * R3_1 * C1 / sqrt(C1*C2);

R1_2 = 5.84e3; R3_2 = R1_2; R4_2 = R1_2;
R2_2 = Q2 * R3_2 * C1 / sqrt(C1*C2);

%% === Display Component Values ===
fprintf('--- Biquad 1 ---\n');
fprintf('R1 = R3 = R4 = %.2f kOhm, R2 = %.2f kOhm\n', R1_1/1e3, R2_1/1e3);
fprintf('C1 = C2 = %.2f pF\n', C1*1e12);

fprintf('--- Biquad 2 ---\n');
fprintf('R1 = R3 = R4 = %.2f kOhm, R2 = %.2f kOhm\n', R1_2/1e3, R2_2/1e3);
fprintf('C1 = C2 = %.2f pF\n', C1*1e12);

%% === Define Transfer Functions ===
num1 = [w0^2];
den1 = [1 w0/Q1 w0^2];
H1 = tf(num1, den1);

num2 = [w0^2];
den2 = [1 w0/Q2 w0^2];
H2 = tf(num2, den2);

%% Cascaded Filter
H_total = series(H1, H2);
```

```
%% === Display Transfer Functions ===
disp('H1(s) Transfer Function:'); H1
disp('H2(s) Transfer Function:'); H2
disp('H_total(s) Transfer Function:'); H_total

%% === Bode Plot (Separate Magnitude and Phase) ===
[mag, phase, w] = bode(H_total);  % Get data from transfer function
w = squeeze(w); mag = squeeze(mag); phase = squeeze(phase);
freq = w / (2*pi);  % Convert rad/s to Hz

%% --- Magnitude Plot ---
figure;
semilogx(freq, 20*log10(mag), 'b-', 'LineWidth', lw);
xlabel('Frequency (Hz)', 'FontSize', fsize, 'FontWeight', 'bold');
ylabel('Magnitude (dB)', 'FontSize', fsize, 'FontWeight', 'bold');
title('Magnitude Response', 'FontSize', fsize+2, 'FontWeight', 'bold');
grid on;

%% --- Phase Plot ---
figure;
semilogx(freq, phase, 'r-', 'LineWidth', lw);
xlabel('Frequency (Hz)', 'FontSize', fsize, 'FontWeight', 'bold');
ylabel('Phase (degrees)', 'FontSize', fsize, 'FontWeight', 'bold');
title('Phase Response', 'FontSize', fsize+2, 'FontWeight', 'bold');
grid on;

%% === Pole-Zero Plot ===
figure;
hPZ = pzplot(H_total); grid on;
title('Poles and Zeros of Cascaded Filter', 'FontSize', 16, 'FontWeight', 'bold');

pz_lines = findall(gcf, 'Type', 'line');
set(pz_lines, 'MarkerSize', 12, 'LineWidth', 2);
for hline = pz_lines.'
    m = get(hline, 'Marker');
    if strcmp(m, 'x') set(hline, 'Color', [0.85,0,0]);
    elseif strcmp(m, 'o') set(hline, 'Color', [0,0,0.85]); end
end

[z_total, p_total, k_total] = tf2zpk(H_total.num{1}, H_total.den{1});
fprintf('\n--- Cascaded Filter Pole-Zero Data ---\nGain: %.4e\n', k_total);
fprintf('Zeros:\n'); for i=1:length(z_total)
fprintf('   %.2e + %.2ei\n', real(z_total(i)), imag(z_total(i))); end
fprintf('Poles:\n'); for i=1:length(p_total)
fprintf('   %.2e + %.2ei\n', real(p_total(i)), imag(p_total(i)));
end
```