DHARMSINH DESAI UNIVERSITY, NADIAD
FACULTY OF TECHNOLOGY
B.TECH SEMSTER-I (CE, IT, EC)
SUBJECT: (BS101) Mathematics-I

Examination	**: First Internal**	**Seat No.**	**:** 112
Date	**: 18/09/2023**	**Day**	**:Monday**
Time	**: 1 Hr 15min.**	**Max. Marks**	**: 36**

INSTRUCTIONS:

1. Figures to the right indicate maximum marks for that question.
2. The symbols used carry their usual meanings.
3. Assume suitable data, if required & mention them clearly.
4. Draw neat sketches wherever necessary.

Q.1 **Do as directed.** **[12]**

CO4 A (a) Check whether the set $S = \{(1,1,2),(2,1,4),(1,0,3),(1,1,1)\}$ is linearly dependent or not? If so, find the relation between them. [4]

CO3 A (b) Expand $y = \log(1+\sin x)$ in powers of x up to the term containing x^4. [4]

CO3 A (c) Verify Roll's theorem for the function $f(x) = x^4 \; ; x \in [-1,1]$. [4]

Q.2 Attempt ***Any TWO*** from the following questions. [12]

CO4 A (a) Determine the value of λ for which the equations $x+2y+z=3,\ x+y+z=\lambda,\ 3x+y+3z=\lambda^2$ are consistent and Solve them for each λ. [6]

CO4 A (b) Using Gauss Jordan method, find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & 4 & -4 \end{bmatrix}$. [6]

CO4 A (c) Find the value of λ for which the equations $(\lambda-1)x+(3\lambda+1)y+2\lambda z=0,\ (\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0,\ 2x+(3\lambda+1)y+(3\lambda-3)z=0$ have a nontrivial solution and solve them completely in each cases. [6]

Q.3 [12]

CO3 A (a) Derive the series for $y=\tan^{-1}x$ and also expand $y=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}$ in powers of x up to the term containing x^4. [6]

CO3 A (b) Prove that, $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$ and hence deduce that $\frac{\pi}{6}+\frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6}+\frac{1}{8}$, where $0<a<b<1$. [6]

OR

CO3 A (a) Derive the series for $y=e^x$ and also expand $y=\frac{x}{2}\left[\frac{e^x+1}{e^x-1}\right]$ in powers of x up to the term containing x^4. [6]

CO3 A (b) Verify Cauchy mean value theorem for $f(x)=e^x$ *and* $g(x)=e^{-x}$ $; x\in[a,b],\ a,b>0$ [6]

DHARMSINH DESAI UNIVERSITY, NADIAD
FACULTY OF TECHNOLOGY
B.TECH. SEMESTER-I [CE, IT, EC]
SUBJECT: (23BS101) MATHEMATICS-I

Examination	**: Second sessional**	**Seat No.**	**:** 112
Date	**: 23/10/2023**	**Day**	**: Monday**
Time	**: 11:45 to 1:00 pm**	**Max. Marks**	**: 36**

INSTRUCTIONS:
1. Figures to the right indicate maximum marks for that question.
2. The symbols used carry their usual meanings.
3. Assume suitable data, if required & mention them clearly.
4. Draw neat sketches wherever necessary.

Q.1 **Do as directed.** **[12]**

CO1 A (a) (1) Prove that $\beta(m, n) = \beta(n, m)$ [6]

(2) The value of $\int_0^{\frac{\pi}{2}} \sqrt{cot\theta}\ d\theta$ *is* ____ *using* Beta *function.*

(3) Find the value of $m(> 0)$ given that $\beta(m, 2) = \frac{1}{6}$.

CO6 A (b) (1) If $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ then find the eigenvalues of A^{-1}, A^5. [6]

(2) Check whether $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is orthogonal matrix or not? If so, find A^{-1}.

(3) If two eigenvalues of $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ are 1 and 2 then the third eigenvalue is________ .

Q.2 Attempt any ***TWO*** from the following **[12]**

(a) Find the eigenvalue and eigenvector for the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. [6]

CO6 A (b) Diagonalize the matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ and hence find the matrix A^4 [6]

(c) Find the eigenvalue and eigenvector for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Hence find the eigenvalues of $A^2 - 2A$. [6]

Q.3

CO1 A (a) State and prove relation between Beta and Gamma function and hence evaluate $\int_0^{\infty} \frac{\sqrt{x}}{1 + 2x + x^2} dx$. [6]

(b) Evaluate $\int_0^{\infty} xe^{-x^8} dx \cdot \int_0^{\infty} x^2 e^{-x^4} dx$ in terms of Gamma function. [6]

OR

Q.3 (a) Prove that $\beta(n, n) = 2^{1-2n}\beta(n, \frac{1}{2})$ and hence find the value of $\Gamma\frac{1}{4}\Gamma\frac{3}{4}$. [6]

CO1 A (b) (1) Prove that $\beta(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$ [6]

(2) Evaluate $\int_5^9 \sqrt[4]{(x-5)(9-x)}\ dx$ using Beta and Gamma function.

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B.TECH. SEMESTER-I [CE, IT, EC]
SUBJECT: (23BS101) MATHEMATICS-I

Examination	**: Third sessional**	**Seat No.**	**:** 112
Date	**: 18/12/2023**	**Day**	**: Monday**
Time	**: 09:15 to 10:30 am**	**Max. Marks**	**: 36**

INSTRUCTIONS:
1. Figures to the right indicate maximum marks for that question.
2. The symbols used carry their usual meanings.
3. Assume suitable data, if required & mention them clearly.
4. Draw neat sketches wherever necessary.

Q.1 **Do as directed.** **[12]**

A (a) **[6]**

CO5

(1) Does $f(x,y)=\begin{cases}\dfrac{x^2-y^2}{y^2+x^2}, & \text{for } (x,y)\neq(0,0)\\ 0, & \text{for } (x,y)=(0,0)\end{cases}$ continuous at (0,0)?

(2) If $u=x^2e^yz^3$ and $x=\sin 2t, y=\cos t, z=t^3$ find $\dfrac{du}{dt}$.

(3) If $u=\tan^{-1}\left(\dfrac{x}{y}\right)$ then prove that $\dfrac{\partial^2 u}{\partial x\partial y}=\dfrac{\partial^2 u}{\partial y\partial x}$.

A (b) **[6]**

CO2

(1) Find constants a,b,c so that $\overline{V}=(x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k$ is irrotational.

(2) For $\overline{F}=(y^2-z^2+3yz-2x)i+(3xz+2xy)j+(3xy-2xz+2z)k$, show that $\overline{F}$ is solenodial vector.

(3) Find the unit normal vector to the surface $x^2+2y^2+z^2=7$ at $(1,-1,2)$.

Q.2 Attempt any ***TWO*** from the following **[12]**

CO5 A

(a) If $\theta=t^{\frac{-3}{2}}e^{\frac{-r^2}{4t}}$, prove that $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial\theta}{\partial r}\right)=\dfrac{\partial\theta}{\partial t}$ **[6]**

(b) Find the extreme values for $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$ **[6]**

(c) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2+y^2+z^2=1$. **[6]**

Q.3

CO2 A

(a) Find the angle between the normals to the surface $xy=z^2$ at the points $(1,9,-3)$ and $(-2,-2,2)$. **[6]**

(b) Find the directional derivative of $\phi=xy^2+yz^2$ at (2, -1, 2) along the normal to the surface $xy+yz+zx=3$ at the point $(1,1,1)$. **[6]**

OR

Q.3

CO2 A

(a) Show that $\overline{F}=(2xyz)i+(x^2z+2y)j+(x^2y)k$ is irroational and find a scalar function ϕ such that $\overline{F}=\nabla\phi$. **[6]**

(b) Find the directional derivative of $\phi=4xz^3-3x^2y^2z$ at (2, -1, 2) along tangent to the curve $x=e^t\cos t, y=e^t\sin t, z=e^t$ at t=0. **[6]**