DHIRUBHAI AMBANI INSTITUTE OF INFORMATION AND COMMUNICATION TECHNOLOGY

SC-374

Computational and Numerical Methods

ASSIGNMENT

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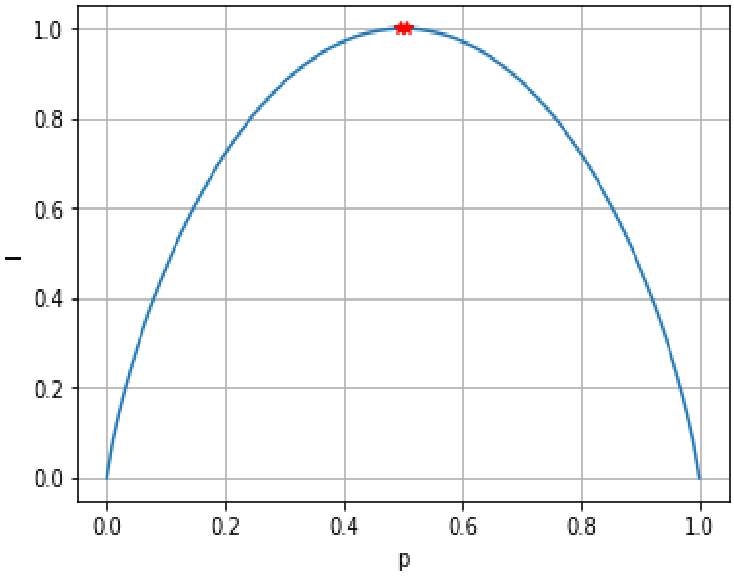
Contents

Assignment

1 The Binary Search and Information Entropy

1.1

Figure 1:



The information $\langle I \rangle$ in a two outcome experiment is maximum when $p = \frac{1}{2}$, and this maximum value is 1.

Analytically:

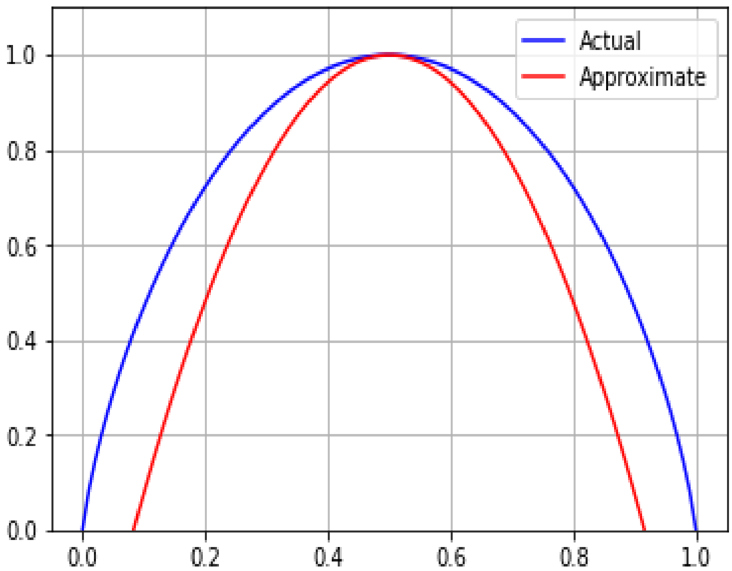
$$\langle I \rangle = -k[p \log_2 p + (1-p) \log_2 (1-p)]$$
$$\frac{d\langle I \rangle}{dp} = -k[\log_2 p + 1 + - \log_2 (1-p) - 1]/ln(2) = 0$$
$$=> -k[\log_2 p - \log_2 (1-p)]/ln(2) = 0$$
$$=> \log_2 p = \log_2 (1-p)$$
$$=> p = 1 - p$$
$$=> p = \frac{1}{2}$$

1.2

$$
\begin{aligned}
\langle I\rangle &= -k[p\log_2 p + (1-p)\log_2(1-p)] \\
&= -k[(\frac{1}{2}+\epsilon)\log_2(\frac{1}{2}+\epsilon) + (\frac{1}{2}-\epsilon)\log_2(\frac{1}{2}-\epsilon)] \\
&= -k[\frac{1}{2}(\log_2(\frac{1}{2}+\epsilon) + \log_2(\frac{1}{2}-\epsilon)) + \epsilon(\log_2(\frac{1}{2}+\epsilon) - \log_2(\frac{1}{2}-\epsilon))] \\
&= -\frac{k}{\ln 2}[\frac{1}{2}(\ln(\frac{1}{2}+\epsilon) + \ln(\frac{1}{2}-\epsilon)) + \epsilon(\ln(\frac{1}{2}+\epsilon) - \ln(\frac{1}{2}-\epsilon))] \\
&= -\frac{k}{\ln 2}[\frac{1}{2}(\ln(\frac{1+2\epsilon}{2}) + \ln(\frac{1-2\epsilon}{2})) + \epsilon(\ln(\frac{1+2\epsilon}{2}) - \ln(\frac{1-2\epsilon}{2}))] \\
&= -\frac{k}{\ln 2}[\frac{1}{2}(\ln(1+2\epsilon) + \ln(1-2\epsilon) - 2\ln 2) + \epsilon(\ln(1+2\epsilon) - \ln(1-2\epsilon) + \ln 2 - \ln 2)] \\
&\approx -\frac{k}{\ln 2}[\frac{1}{2}(2\epsilon - 2\epsilon - 2\ln 2) + \epsilon(2\epsilon - (-2\epsilon))] \\
&\approx -\frac{k}{\ln 2}[\frac{1}{2}(-2\ln 2) + \epsilon(4\epsilon)] \\
&\approx -\frac{k}{\ln 2}[-\ln 2 + 4\epsilon^2] \\
&\approx k - (\frac{4k}{\ln 2})\epsilon^2 \\
&\approx a - (b)\epsilon^2
\end{aligned}
$$

1.3

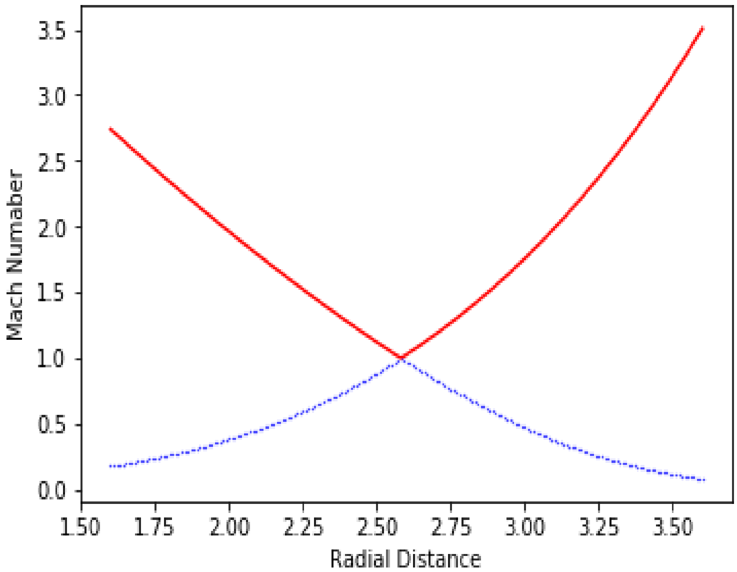
Figure 2:



=> At $p = \frac{1}{2}$ the value of ϵ is much smaller than $\frac{1}{2}$ and so the approximate function is almost equal to the actual function, which deviates on increasing value of ϵ on the ends of the plot.

2 An Astrophysical Inflow

Figure 3: Radial Distance on log scale vs. Mach Number $\frac{v(r)}{c_s(r)}$



$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

$$M = 2 \times 10^{30} \text{ kg}$$

$$c_s(\infty) = 10^4 \text{ ms}^{-1}$$

$$\rho_\infty = 10^{-21} \text{ kg m}^{-3}$$

$$n = 2.5$$

$$\gamma = \tfrac{1}{n} + 1 = 1.4$$

The rate of the fluid flow (matter flowing in unit time) on to it is given as

$$\dot{m} = \pi G^2 M^2 \frac{\rho_\infty}{c_s(\infty)^3} \left(\frac{2}{5 - 3\gamma} \right)^{\left(\frac{5-3\gamma}{2(\gamma-1)}\right)}$$

$$\dot{\mu} = \left(\frac{\dot{m}}{4\pi\rho_\infty} \right) c_s^{2n}(\infty)$$

The velocity of the fluid flow v, as a function of the radial distance from the centre r, is given by the equation :

$$f(v, r) = \frac{v^2}{2} + n \left(\frac{\dot{\mu}}{v r^2} \right) - \frac{GM}{r} - n c_s^2(\infty) = 0$$

$c_s(r)$ is given by

$$c_s(r) = c_s(\infty)\left(\frac{\rho(r)}{\rho(\infty)}\right)^{(\frac{\gamma-1}{2})}$$

where $\rho(r)$ is given by

$$\rho(r) = \frac{\dot{m}}{4\pi v r^2}$$

3 A Nuclear Outflow

3.1 a

$$=> R^4Y^4 - BR^4Y^2 - 4R^2Y + 3 = 0$$
$$=> \frac{dY}{dR} = \frac{2(3 - 2YR^2)}{R^3(2Y^3R^2 - BYR^2 - 2)}$$

Turning Point is obtained at :

$$=> \frac{dY}{dR} = 0$$
$$=> 2(3 - 2YR^2) = 0$$
$$=> R = \sqrt{\frac{3}{2Y}}$$

Singular Point is obtained at :

$$R^3(2Y^3R^2 - BYR^2 - 2) = 0$$

hence,

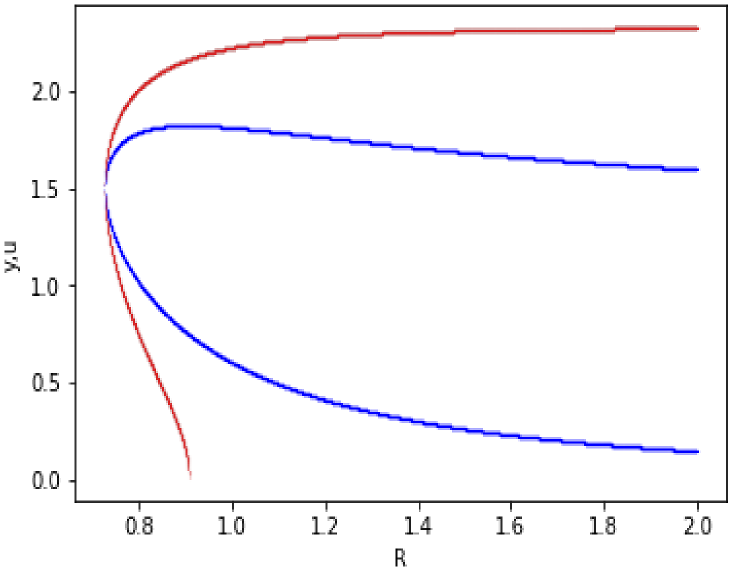
$$R = 0$$
$$OR$$
$$R = \sqrt{\frac{2}{2Y^3 - BY}} \; where \; Y \neq 0 \; and \; Y \neq \sqrt{\frac{B}{2}}$$

3.2 b

$$Y = -\frac{-R^4 + 4R^2 \pm \sqrt{(R^4 - 4R^2)^2 + 12R^4B}}{2R^4B}$$

3.3 c

Figure 4: y(R) is the inner continuous curve and u(R)is the outer dotted curve



4 The Hydraulic Jump

4.1 a

$$=> 4H - H^4 = 3(X - D)$$
$$=> 4\left(\frac{dH}{dX}\right) - 4H^3\left(\frac{dH}{dX}\right) = 3(1 - 0)$$
$$=> \frac{dH}{dX}(4 - 4H^3) = 3$$
$$=> \frac{dH}{dX} = 3/4(1 - H^3)$$

4.2 b

Figure 5: A plot of two solutions of H against X.