Fundamentals of Mathematical Applications - Summary

Unit 1: Set Theory

- **Set:** Collection of distinct objects.
- **Representation:** Roster {1,2}, Set-builder {x | x is even}, Venn diagram.
- **Types of Sets:** Finite, Infinite, Subset, Superset, Universal, Empty, Equal, Disjoint, Proper/Improper, Power set.
- **Set Operations:** Union ($A \cup B$), Intersection ($A \cap B$), Complement (A'), Symmetric difference ($A \Delta B$).
- **Properties:** Commutative, Associative, Distributive, De Morgan's laws.
- **Cartesian Product:** $A \times B$ = all ordered pairs (a,b).

Unit 2: Function

- **Function:** Each element of domain maps to exactly one codomain element.
- **Types (Mapping):** One-one, Many-one, Onto, Into, Constant.
- **Types (Equation):** Linear, Quadratic, Cubic, Polynomial, Exponential, Power.
- **Graphs:** Plot points (x, f(x)) and join smoothly.

Unit 3: Relations

- **Relation:** Shows connection between elements of two sets.
- **Universal & Empty Relation:** All pairs / No pairs.
- **Inverse:** Swap each pair.
- **Total Relations:** 2^(m*n) for sets A(m) and B(n).
- **Types:** Reflexive, Symmetric, Transitive, Antisymmetric, Asymmetric, Irreflexive.
- **Equivalence Relation:** Reflexive + Symmetric + Transitive.
- **Partial Order:** Reflexive + Antisymmetric + Transitive; Hasse Diagram.
- **Lattice:** Each pair has Join ($\vee$) and Meet ($\wedge$).

Unit 4: Lattice & Boolean Algebra

- **Lattice Types:** Bounded, Complete, Distributive, Complemented.
- **Boolean Algebra:** Binary variables (0,1) with AND ($\wedge$), OR ($\vee$), NOT (').
- **Properties:** Commutative, Associative, Distributive, Identity, Complement, Idempotent, Involution, De Morgan's.
- **Minterms & Maxterms:** Minterms give 1 for one combination, Maxterms give 0.
- **SOP & POS:** Sum of Product / Product of Sum forms.

Unit 5: Matrix

- **Matrix:** Rectangular array of numbers.
- **Types:** Row, Column, Rectangular, Square, Diagonal, Scalar, Identity, Zero, Upper/Lower Triangular, Symmetric, Skew-Symmetric, Unit.
- **Operations:** Addition, Subtraction, Multiplication.
- **Rank:** Number of independent rows/columns.
- **Inverse:** A^{-1} exists if determinant $\neq 0$.
- **Solving Equations:** $AX=B \rightarrow X = A^{-1} * B$

Unit 6: Logic and Propositional Calculus

- **Proposition:** Statement either True or False.
- **Compound Proposition:** Combine with AND ($\wedge$), OR ($\vee$), NOT ($\neg$).
- **Truth Table:** List all possible values.
- **Tautology & Contradiction:** Always True / Always False.
- **Laws:** Idempotent, Associative, Commutative, Distributive, Identity, Complement, Involution, De Morgan's.

Unit 7: Permutation & Combination

- **Factorial:** $n! = n*(n-1)*(n-2)...1$, $0! = 1$.
- **Permutation:** Arrangement in order.
  - n things all at a time: $n!$
  - n things r at a time: $P(n,r)=n!/(n-r)!$
- **Combination:** Selection without order.
  - $C(n,r)=n!/(r!(n-r)!)$
- **Difference:** Permutation order matters, Combination order does not.

Unit 8: Probability

- **Probability:** $P(E)$ = favorable / total outcomes ($0 \leq P \leq 1$).
- **Basic Terms:** Experiment, Sample Space, Event, Favorable Outcome, Mutually Exclusive.
- **Rules:**
  - Addition: $P(A \cup B)=P(A)+P(B)$ or $P(A)+P(B)-P(A \cap B)$
  - Multiplication: $P(A \cap B)=P(A)*P(B)$
  - Conditional: $P(A|B)=P(A \cap B)/P(B)$
- **Total Probability:** Sum of probabilities using partitions.
- **Bayes' Theorem:** $P(Bi|A)=P(A|Bi)P(Bi)/\Sigma P(A|Bj)P(Bj)$
- **Combinations in Probability:** With/Without replacement.

References: As per syllabus (Sangheti, Lipschutz, Rosen, etc.)

UNIT 1: SET THEORY

1. Introduction to Sets

A **set** is a collection of distinct objects, considered as a single entity. These objects are called **elements** or **members** of the set.

Example:

- Set of vowels in English: A={a,e,i,o,u}A = \{a, e, i, o, u\}A={a,e,i,o,u}
- Set of natural numbers less than 5: B={1,2,3,4}B = \{1, 2, 3, 4\}B={1,2,3,4}

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2. Methods of Representation

Sets can be represented in three ways:

1. **Roster or Tabular form:** List all elements inside curly braces.
   - Example: A={1,2,3,4,5}A = \{1, 2, 3, 4, 5\}A={1,2,3,4,5}
2. **Set-builder form:** Describe the elements using a property.
   - Example: B={x|x is an even number less than 10}={2,4,6,8}B = \{x \mid x \text{ is an even number less than 10}\} = \{2, 4, 6, 8\}B={x|x is an even number less than 10}={2,4,6,8}
3. **Venn diagram representation:** Visual representation using circles to show elements and relationships.

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3. Types of Sets

1. **Finite Set:** Contains a limited number of elements.
   - Example: {1,2,3}\{1, 2, 3\}{1,2,3}
2. **Infinite Set:** Contains unlimited elements.
   - Example: Set of all natural numbers {1,2,3,… }\{1, 2, 3, \dots\}{1,2,3,…}
3. **Subset (⊆):** Every element of set A is in set B.
   - Example: A={1,2},B={1,2,3}A = \{1, 2\}, B = \{1, 2, 3\}A={1,2},B={1,2,3}, A⊆BA \subseteq BA⊆B
4. **Superset (⊇):** Set B contains all elements of set A.
   - Example: B⊇AB \supseteq AB⊇A
5. **Universal Set (U):** Set containing all possible elements under consideration.
6. **Empty Set (∅):** A set with no elements.
7. **Equal Set:** Sets with exactly the same elements.
8. **Disjoint Set:** Sets with no common elements.

9. **Proper & Improper Subset:**
    - Proper subset: A ⊂ B, A has fewer elements than B
    - Improper subset: A = B
10. **Power Set (P(A)):** Set of all subsets of a set A.
    - Example: A={1,2},P(A)={∅,{1},{2},{1,2}}A = \{1, 2\}, P(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}A={1,2},P(A)={∅,{1},{2},{1,2}}

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4. Set Operations

1. **Union (A ∪ B):** Elements in A or B or both.
    - Example: A={1,2},B={2,3}A = \{1, 2\}, B = \{2, 3\}A={1,2},B={2,3}, A∪B={1,2,3}A \cup B = \{1, 2, 3\}A∪B={1,2,3}
2. **Intersection (A ∩ B):** Elements common to both A and B.
    - Example: A∩B={2}A \cap B = \{2\}A∩B={2}
3. **Complement (A'):** Elements not in set A, but in the universal set.
4. **Symmetric Difference (A Δ B):** Elements in A or B but not in both.

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5. Properties of Set Operations

1. **Commutative Law:** A ∪ B = B ∪ A, A ∩ B = B ∩ A
2. **Associative Law:** (A ∪ B) ∪ C = A ∪ (B ∪ C)
3. **Distributive Law:** A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
4. **De Morgan's Laws:**
    - (A ∪ B)' = A' ∩ B'
    - (A ∩ B)' = A' ∪ B'

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6. Cartesian Product

The **Cartesian product** of two sets A and B, denoted by A×BA \times BA×B, is the set of all ordered pairs (a, b) where a ∈ A and b ∈ B.

Example:

- A = {1, 2}, B = {x, y}
- A×B={(1,x),(1,y),(2,x),(2,y)}A \times B = \{(1,x), (1,y), (2,x), (2,y)\}A×B={(1,x),(1,y),(2,x),(2,y)}

UNIT 2: FUNCTION

1. Introduction

A **function** is a relation between two sets where **each element of the first set (domain) is related to exactly one element of the second set (codomain)**.

Notation:

- f:A→Bf: A \to Bf:A→B means function f maps elements from set A (domain) to set B (codomain).
- If f(a)=bf(a) = bf(a)=b, then "a in A is mapped to b in B".

Example:

- A = {1, 2, 3}, B = {2, 4, 6}
- Function f: A → B defined by f(x) = 2x
- Then f(1)=2, f(2)=4, f(3)=6

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2. Types of Functions

A. Based on Mapping

1. **One-One (Injective):**
   - Every element of domain maps to a **unique element** in codomain.
   - Example: f(x) = 2x (x ∈ integers)
2. **Many-One:**
   - Two or more elements of domain map to the **same element** in codomain.
   - Example: f(x) = x^2 (for x ∈ integers, f(2)=f(-2)=4)
3. **Onto (Surjective):**
   - Every element of codomain is mapped by **at least one element** of domain.
   - Example: f(x) = x^3, domain and codomain are all real numbers
4. **Into:**
   - Some elements of codomain **are not mapped** by domain elements.
   - Example: f(x) = x^2, codomain = {0, 1, 4, 9, 16}, domain = {0, 1, 2}
5. **Constant Function:**
   - Maps all elements of domain to the **same element** in codomain.
   - Example: f(x) = 5

B. Based on Equation

1. **Linear Function:** f(x) = mx + c (graph is a straight line)
2. **Quadratic Function:** f(x) = $ax^2 + bx + c$ (graph is a parabola)
3. **Cubic Function:** f(x) = $ax^3 + bx^2 + cx + d$
4. **Polynomial Function:** f(x) = $a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$

5. **Exponential Function:** f(x) = a^x (a > 0)
6. **Power Function:** $f(x) = x^n$

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3. Graphs of Functions

- A **graph** is a visual representation of a function.
- **Steps to plot a graph:**
    1. Take values of x (domain).
    2. Compute corresponding y = f(x).
    3. Plot points (x, y) on Cartesian plane.
    4. Join points smoothly (line/curve depending on function).

Examples:

- Linear: straight line
- Quadratic: parabola opening up/down
- Cubic: S-shaped curve
- Exponential: curve increasing rapidly

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4. Important Notes

- Function must map **each domain element to only one codomain element**.
- A relation that maps one domain element to more than one codomain element is **not a function**.

UNIT 3: RELATIONS

1. Introduction

A **relation** is a connection between elements of two sets.

- If A and B are sets, a relation R from A to B is a subset of the Cartesian product A×BA \times BA×B.
- In simple words, it shows which elements of A are related to which elements of B.

Example:

- A = {1, 2}, B = {x, y}
- Relation R = { (1, x), (2, y) }
- Here, 1 is related to x, and 2 is related to y.

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2. Universal and Empty Relations

1. **Universal Relation:**
   - Every element of A is related to **every element** of B.
   - Example: A = {1, 2}, B = {x, y}
     - Universal relation = { (1, x), (1, y), (2, x), (2, y) }
2. **Empty Relation:**
   - No element of A is related to any element of B.
   - Example: A = {1, 2}, B = {x, y}
     - Empty relation = { }

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3. Inverse of a Relation

- The **inverse** of relation R, denoted R−1R^{-1}R−1, swaps each pair in R.
- If (a, b) ∈ R, then (b, a) ∈ R−1R^{-1}R−1

Example:

- R = { (1, x), (2, y) }
- R−1R^{-1}R−1 = { (x, 1), (y, 2) }

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4. Total Number of Relations

- If set A has m elements and set B has n elements, then the **total number of relations** from A to B is:

2m·n2^{m \cdot n}2m·n

- Example: A = {1, 2} (2 elements), B = {x, y} (2 elements)
  - Total relations = 22·2=162^{2 \cdot 2} = 1622·2=16

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5. Types of Relations

1. **Reflexive Relation:**
   - Every element is related to itself.
   - Example: R on A = {1, 2}, R = { (1,1), (2,2) }
2. **Symmetric Relation:**
   - If (a, b) ∈ R, then (b, a) ∈ R.

    - Example: R = { (1,2), (2,1) }
3. **Transitive Relation:**
    - If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R.
4. **Antisymmetric Relation:**
    - If (a, b) ∈ R and (b, a) ∈ R, then a = b
5. **Asymmetric Relation:**
    - If (a, b) ∈ R, then (b, a) ∉ R
6. **Irreflexive Relation:**
    - No element is related to itself.

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6. Equivalence Relation

- A relation that is **reflexive, symmetric, and transitive**.
- Equivalence relations divide the set into **equivalence classes** (partitions).

Example:

- A = {1, 2, 3, 4}, relation R = "has same remainder when divided by 2"
- Equivalence classes: {1, 3}, {2, 4}

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7. Partial Order Relation

- A relation that is **reflexive, antisymmetric, and transitive**.
- Used to define order without requiring all elements to be comparable.

Hasse Diagram:

- A **graphical representation** of a partially ordered set (poset) showing elements and their order without drawing all connections.

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8. Lattice

- A **lattice** is a partially ordered set in which **every pair of elements has a least upper bound (join) and a greatest lower bound (meet)**.

Types of Lattices:

1. **Bounded Lattice:** Has a greatest (1) and least (0) element.
2. **Complete Lattice:** Every subset has a join and meet.
3. **Distributive Lattice:** Join and meet operations distribute over each other.

4. **Complemented Lattice:** Every element has a complement.

UNIT 4: LATTICE & BOOLEAN ALGEBRA

1. Lattice

A **lattice** is a partially ordered set (poset) in which **every two elements have both a least upper bound (join) and a greatest lower bound (meet)**.

- **Join (∨):** The smallest element greater than or equal to both elements.
- **Meet (∧):** The largest element smaller than or equal to both elements.

Example:

- Set A = {0, 1, 2, 3} with order ≤
- Meet (∧) of 2 and 3 = 2 (largest smaller)
- Join (∨) of 2 and 3 = 3 (smallest greater)

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2. Types of Lattices

1. **Bounded Lattice:**
   - Has a **least element (0)** and a **greatest element (1)**.
   - Example: Lattice of subsets of a set with ∅ as 0 and the set itself as 1
2. **Complete Lattice:**
   - Every subset has a **join** and **meet**.
3. **Distributive Lattice:**
   - Join and meet distribute over each other:
     - $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
     - $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
4. **Complemented Lattice:**
   - Every element has a **complement** such that:
     - $a \vee a' = 1$, $a \wedge a' = 0$

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3. Boolean Algebra

Boolean Algebra is a **mathematical structure** used to work with **binary variables (0 and 1)** and **logical operations**.

- Elements: 0 (false) and 1 (true)
- Operations: AND (∧), OR (∨), NOT (’ or ¬)

Example:

- $a = 1, b = 0$
- $a \land b = 0, a \lor b = 1, a' = 0$

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4. Properties of Boolean Algebra

1. **Commutative Laws:**
   - $a \lor b = b \lor a$
   - $a \land b = b \land a$
2. **Associative Laws:**
   - $(a \lor b) \lor c = a \lor (b \lor c)$
   - $(a \land b) \land c = a \land (b \land c)$
3. **Distributive Laws:**
   - $a \lor (b \land c) = (a \lor b) \land (a \lor c)$
   - $a \land (b \lor c) = (a \land b) \lor (a \land c)$
4. **Identity Laws:**
   - $a \lor 0 = a, a \land 1 = a$
5. **Complement Laws:**
   - $a \lor a' = 1, a \land a' = 0$
6. **Idempotent Laws:**
   - $a \lor a = a, a \land a = a$
7. **Involution Law:**
   - $(a')' = a$
8. **De Morgan's Laws:**
   - $(a \lor b)' = a' \land b'$
   - $(a \land b)' = a' \lor b'$

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5. Minterms and Maxterms

- **Minterm:** A product (AND) of all variables in true/false form giving 1 for exactly one combination.
- **Maxterm:** A sum (OR) of all variables in true/false form giving 0 for exactly one combination.

Example: For two variables x and y:

- Minterms: $x'y'$, $x'y$, xy', xy
- Maxterms: $(x + y)$, $(x + y')$, $(x' + y)$, $(x' + y')$

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6. Sum of Product (SOP) & Product of Sum (POS)

1. **SOP Form:** OR of all minterms giving 1.
   - Example: f(x,y) = x’y + xy’
2. **POS Form:** AND of all maxterms giving 0.
   - Example: f(x,y) = (x + y) ∧ (x’ + y’)

UNIT 5: MATRIX

1. Introduction

A **matrix** is a rectangular array of numbers arranged in **rows and columns**.

- Matrices are used to represent data, solve linear equations, and in many areas of mathematics and computer science.

Notation:

- A matrix is denoted by a capital letter, e.g., AAA
- A=[aij]A = [a_{ij}]A=[aij] where aija_{ij}aij is the element in the i-th row and j-th column

Example:

A=[123456]A = \begin{bmatrix} 1 & 2 & 3\\ 4 & 5 & 6 \end{bmatrix}A=[142536]

- 2 rows, 3 columns → 2×3 matrix

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2. Types of Matrix

1. **Row Matrix:** Only one row
   - Example: [1,2,3][1, 2, 3][1,2,3]
2. **Column Matrix:** Only one column
   - Example: [123]\begin{bmatrix} 1\\2\\3 \end{bmatrix}123
3. **Rectangular Matrix:** Rows ≠ Columns
   - Example: 2×3 matrix
4. **Square Matrix:** Rows = Columns
   - Example: 3×3 matrix
5. **Diagonal Matrix:** Non-diagonal elements are 0
   - Example: [2003]\begin{bmatrix} 2 & 0\\0 & 3 \end{bmatrix}[2003]
6. **Scalar Matrix:** Diagonal matrix with equal diagonal elements

- Example: [2002]\begin{bmatrix} 2 & 0\\0 & 2 \end{bmatrix}[2002]

7. **Identity Matrix (I):** Diagonal elements = 1, others = 0
    - Example: [1001]\begin{bmatrix} 1 & 0\\0 & 1 \end{bmatrix}[1001]
8. **Zero Matrix:** All elements = 0
    - Example: [0000]\begin{bmatrix} 0 & 0\\0 & 0 \end{bmatrix}[0000]
9. **Upper Triangular Matrix:** All elements below diagonal = 0
10. **Lower Triangular Matrix:** All elements above diagonal = 0
11. **Symmetric Matrix:** A=ATA = A^TA=AT (transpose = original)
12. **Skew-Symmetric Matrix:** A=−ATA = -A^TA=−AT
13. **Unit Matrix:** Same as identity matrix

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3. Algebra of Matrices

Matrices can be added, subtracted, and multiplied.

1. **Addition (A + B):** Only if dimensions are same
    - Add corresponding elements
    - Example:

[1234]+[5678]=[681012]\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} + \begin{bmatrix}5 & 6\\7 & 8\end{bmatrix} = \begin{bmatrix}6 & 8\\10 & 12\end{bmatrix}[1324]+[5768]=[610812]

2. **Subtraction (A - B):** Same as addition, subtract element-wise
3. **Multiplication (A × B):**
    - Number of columns in A = number of rows in B
    - Multiply rows of A with columns of B and sum products

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4. Rank of Matrix

- Rank = Number of **linearly independent rows or columns**
- Helps determine if a system of equations has **unique, infinite, or no solution**

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5. Inverse of a Matrix

- For square matrix A, inverse A−1A^{-1}A−1 satisfies:

A·A−1=IA \cdot A^{-1} = IA·A−1=I

- Only **non-singular** matrices (determinant ≠ 0) have an inverse

Example (2×2):

A=[abcd],A−1=1ad−bc[d−b−ca]A = \begin{bmatrix}a & b\\c & d\end{bmatrix},\quad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix}d & -b\\-c & a\end{bmatrix}A=[acbd],A−1=ad−bc1[d−c−ba]

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6. Solving Equations using Matrices

- System of linear equations:

AX=BAX = BAX=B

- Solution:

X=A−1B(if A−1 exists)X = A^{-1} B \quad \text{(if } A^{-1} \text{ exists)}X=A−1B(if A−1 exists)

Example:

- Equations:

x+y=3,2x+y=5x + y = 3, \quad 2x + y = 5x+y=3,2x+y=5

- Matrix form:

A=[1121], X=[xy], B=[35]A = \begin{bmatrix}1 & 1\\2 & 1\end{bmatrix},\ X = \begin{bmatrix}x\\y\end{bmatrix},\ B = \begin{bmatrix}3\\5\end{bmatrix}A=[1211], X=[xy], B=[35]

- Solution: X=A−1BX = A^{-1} BX=A−1B

UNIT 6: LOGIC AND PROPOSITIONAL CALCULUS

1. Introduction

Logic studies reasoning and the principles of valid inference.

- In mathematics and computer science, **propositional logic** is used to represent statements that are either **true (T)** or **false (F)**.

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2. Proposition

- A **proposition** is a statement that is either **true or false**, but **not both**.

Examples:

- "5 is greater than 3" → True
- "The Earth is flat" → False
- "x + 2 = 5" → Not a proposition if x is unknown

Compound Proposition:

- Formed by combining simple propositions using **logical connectives**.

Logical Connectives:

1. **AND (∧) / Conjunction:** True if **both** propositions are true
   - Example: $p \wedge q$
2. **OR (∨) / Disjunction:** True if **at least one** proposition is true
   - Example: $p \vee q$
3. **NOT (' or ¬) / Negation:** Opposite of a proposition
   - Example: $\neg p$

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3. Truth Table

- A **truth table** lists all possible truth values of propositions and compound propositions.

Example (p ∧ q):

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

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4. Tautologies and Contradictions

1. **Tautology:** Always true regardless of truth values of components

- Example: $p \lor \neg p$

2. **Contradiction:** Always false
    - Example: $p \land \neg p$

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5. Laws of Propositional Algebra

1. **Idempotent Law:**
    - $p \lor p = p$, $p \land p = p$
2. **Associative Law:**
    - $(p \lor q) \lor r = p \lor (q \lor r)$
    - $(p \land q) \land r = p \land (q \land r)$
3. **Commutative Law:**
    - $p \lor q = q \lor p$, $p \land q = q \land p$
4. **Distributive Law:**
    - $p \lor (q \land r) = (p \lor q) \land (p \lor r)$
    - $p \land (q \lor r) = (p \land q) \lor (p \land r)$
5. **Identity Law:**
    - $p \lor F = p$, $p \land T = p$
6. **Complement Law:**
    - $p \lor \neg p = T$, $p \land \neg p = F$
7. **Involution Law:**
    - $\neg(\neg p) = p$
8. **De Morgan's Laws:**
    - $\neg(p \lor q) = \neg p \land \neg q$
    - $\neg(p \land q) = \neg p \lor \neg q$

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6. Applications

- Used in **computer programming**, **digital circuits**, **decision-making**, and **problem-solving**.
- Logical operations can be represented with **truth tables, Boolean expressions, and logic gates**.

UNIT 7: PERMUTATION & COMBINATION

1. Introduction

- **Permutation** and **Combination** are methods in combinatorics used to **count arrangements and selections**.

2. Factorial Notation

- **n! (n factorial)** = product of all positive integers up to n.

n!=n×(n−1)×(n−2)⋯×2×1n! = n \times (n-1) \times (n-2) \dots \times 2 \times 1n!=n×(n−1)×(n−2)⋯×2×1

- Example: 5! = 5×4×3×2×1 = 120
- Special case: 0! = 1

3. Permutations

- **Permutation:** Arrangement of objects in a **specific order**.

Formulas:

1. **Permutation of n different things all at a time:**

Pn=n!P_n = n!Pn=n!

2. **Permutation of n things taken r at a time:**

P(n,r)=n!(n−r)!P(n,r) = \frac{n!}{(n-r)!}P(n,r)=(n−r)!n!

3. **With repetition:**

- If some elements are repeated, divide by factorial of repetitions:

n!p!⋅q!⋅r!…\frac{n!}{p! \cdot q! \cdot r! \dots}p!⋅q!⋅r!…n!

4. Combinations

- **Combination:** Selection of objects **without considering order**.

Formulas:

1. **n items taken r at a time:**

C(n,r)=n!r!(n−r)!C(n,r) = \frac{n!}{r!(n-r)!}C(n,r)=r!(n−r)!n!

2. **Restricted combinations:**

- Situations where certain conditions are applied (like certain items must/must not be included).

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5. Difference Between Permutation and Combination

Feature	Permutation	Combination
Order	Important	Not important
Formula	P(n,r)=n!/(n−r)!P(n,r) = n!/(n-r)!P(n,r)=n!/(n−r)!	C(n,r)=n!/r!(n−r)!C(n,r) = n!/r!(n-r)!C(n,r)=n!/r!(n−r)!
Example	ABC ≠ BAC	ABC = BAC = CAB

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6. Examples

1. **Permutation Example:**

- How many ways to arrange 3 letters A, B, C?

3!=6 ways (ABC, ACB, BAC, BCA, CAB, CBA)3! = 6 \text{ ways (ABC, ACB, BAC, BCA, CAB, CBA)}3!=6 ways (ABC, ACB, BAC, BCA, CAB, CBA)

2. **Combination Example:**

- How many ways to choose 2 letters from A, B, C?

C(3,2)=3!2!1!=3 ways (AB, AC, BC)C(3,2) = \frac{3!}{2!1!} = 3 \text{ ways (AB, AC, BC)}C(3,2)=2!1!3! =3 ways (AB, AC, BC)

UNIT 8: PROBABILITY

1. Introduction

- **Probability** measures the **likelihood of an event occurring**.
- Probability values always lie between **0 and 1**:
  - 0 → impossible event
  - 1 → certain event

Notation:

P(E)=Number of favorable outcomesTotal number of possible outcomesP(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}P(E)=Total number of possible outcomesNumber of favorable outcomes

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2. Basic Terms

1. **Experiment:** Any process with uncertain outcome (e.g., rolling a die)
2. **Sample Space (S):** Set of all possible outcomes
    - Example: Rolling a die, S={1,2,3,4,5,6}S = \{1,2,3,4,5,6\}S={1,2,3,4,5,6}
3. **Event (E):** Subset of sample space
    - Example: Getting an even number, E={2,4,6}E = \{2,4,6\}E={2,4,6}
4. **Favorable Outcome:** Outcome for which event occurs
5. **Mutually Exclusive Events:** Events that **cannot occur together**

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3. Probability Rules

1. **Addition Rule:**
    - For mutually exclusive events:

    P(A∪B)=P(A)+P(B)P(A \cup B) = P(A) + P(B)P(A∪B)=P(A)+P(B)

    - For non-mutually exclusive events:

    P(A∪B)=P(A)+P(B)−P(A∩B)P(A \cup B) = P(A) + P(B) - P(A \cap B)P(A∪B)=P(A)+P(B)−P(A∩B)

2. **Multiplication Rule:**
    - For independent events:

    P(A∩B)=P(A)·P(B)P(A \cap B) = P(A) \cdot P(B)P(A∩B)=P(A)·P(B)

3. **Conditional Probability:**
    - Probability of A given B has occurred:

    P(A|B)=P(A∩B)P(B), P(B)≠0P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0P(A|B)=P(B)P(A∩B), P(B)⍰=0

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4. Theorem of Total Probability

- If events B1,B2,…,BnB_1, B_2, \dots, B_nB1,B2,…,Bn form a **partition of sample space** S:

P(A)=P(A∩B1)+P(A∩B2)+⋯+P(A∩Bn)P(A) = P(A \cap B_1) + P(A \cap B_2) + \dots + P(A \cap B_n)P(A)=P(A∩B1)+P(A∩B2)+⋯+P(A∩Bn)

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5. Bayes' Theorem

- Used to find probability of an event based on **prior knowledge**:

P(Bi|A)=P(A|Bi)⋅P(Bi)∑j=1nP(A|Bj)⋅P(Bj)P(B_i|A) = \frac{P(A|B_i) \cdot P(B_i)}{\sum_{j=1}^{n} P(A|B_j) \cdot P(B_j)}P(Bi|A)=∑j=1nP(A|Bj)⋅P(Bj)P(A|Bi)⋅P(Bi)

Example:

- Disease testing: Probability that a person has disease given a positive test result

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6. Combinations in Probability

- Often we use **combination formulas** for probability problems:
- Example: Choosing 2 cards from 52-card deck:

P(2 Aces)=C(4,2)C(52,2)P(\text{2 Aces}) = \frac{C(4,2)}{C(52,2)}P(2 Aces)=C(52,2)C(4,2)

- **With replacement:** Items are returned before the next draw
- **Without replacement:** Items are **not returned**, changes total outcomes

7. Key Points

- Probability is always **between 0 and 1**
- Sum of probabilities of all events in sample space = 1
- Events can be independent, mutually exclusive, or dependent

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