Anova Stat Inference

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An **ANOVA** test is a way to find out if survey or experiment results are significant. In other words, they help to figure out if one needs to reject the null hypothesis or accept the alternate hypothesis.

- **One-way** or **two-way** refers to the number of independent variables (IVs) in Analysis of Variance test.

I

a) Derive pdf of chi-square distribution

A direct relation exists between a chi-square-distributed random variable and a gaussian random variable. The chi-square random variable is in a certain form a transformation of the gaussian random variable. If we have X as a gaussian random variable and we take the relation $Y=X^2$ then Y has a chi-square distribution with one degree of freedom [21].

If we define the random variable Y as

χ^2_n-distribution. χ^2_n-distribution with n degrees of freedom as a distribution of the sum $X_1^2 + \ldots + X_n^2$, where X_is are i.i.d. standard normal. We will now show that which χ^2_n-distribution coincides with a gamma distribution $\Gamma(\frac{n}{2}, \frac{1}{2})$, i.e.

$$\chi^2_n = \Gamma\Big(\frac{n}{2}, \frac{1}{2}\Big).$$

Consider a standard normal random variable $X \sim N(0,1)$. Let us compute the distribution of X^2. The c.d.f. of X^2 is given by

$$\mathbb{P}(X^2 \le x) = \mathbb{P}(-\sqrt{x} \le X \le \sqrt{x}) = \int_{-\sqrt{x}}^{\sqrt{x}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt.$$

The p.d.f. can be computed by taking a derivative $\frac{d}{dx}\mathbb{P}(X \le x)$ and as a result the p.d.f. of X^2 is

$$\begin{aligned} f_{X^2}(x) &= \frac{d}{dx}\int_{-\sqrt{x}}^{\sqrt{x}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\sqrt{x})^2}{2}} (\sqrt{x})' - \frac{1}{\sqrt{2\pi}} e^{-\frac{(-\sqrt{x})^2}{2}} (-\sqrt{x})' \\ &= \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{x}} e^{-\frac{x}{2}} = \frac{1}{\sqrt{2\pi}} x^{\frac{1}{2}-1} e^{-\frac{x}{2}}. \end{aligned}$$

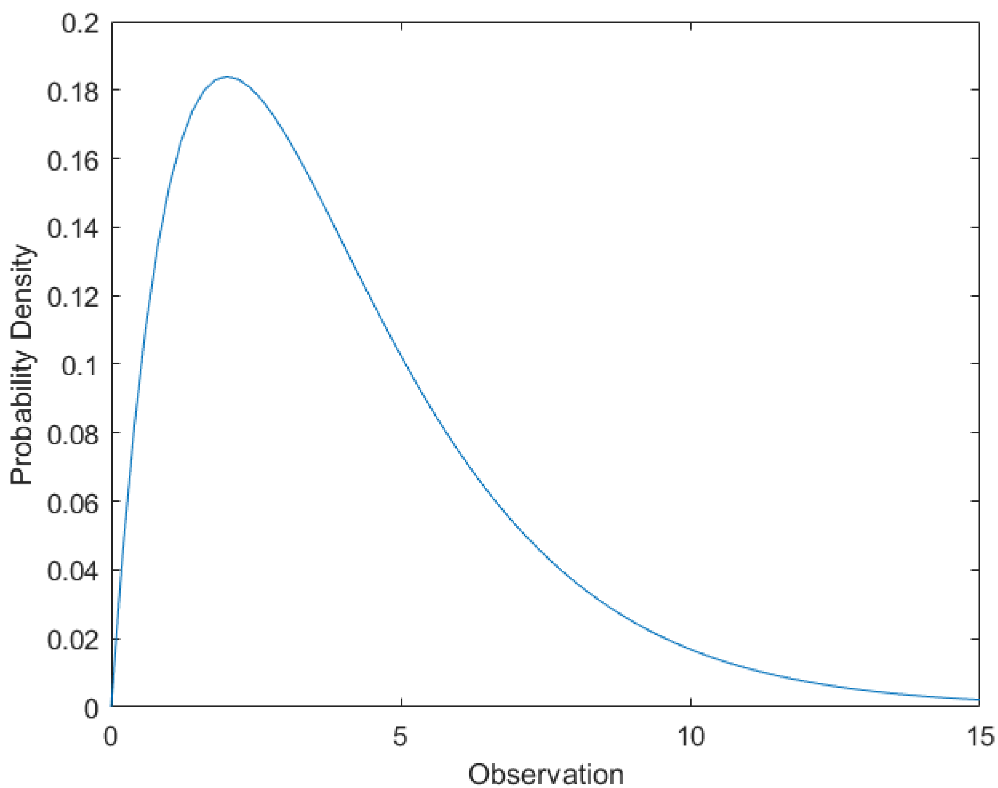
b) How do you plot the pdf in matlab

```
x = 0:0.2:15;
```

```
x
```

```
x = 1×76
         0    0.2000    0.4000    0.6000    0.8000    1.0000    1.2000    1.4000 ⋯
```

```
y = chi2pdf(x,4); %here 4 is the degree of freedom
figure;
plot(x,y)
xlabel('Observation')
ylabel('Probability Density')
```



c) How to get different percentile of F-distribution in matlab

The F-distribution function is approximated using the following formul

$$percentile = 1 - \frac{(1 - a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4)^{-4}}{2} + \varepsilon(y)$$

where,

$$a_1 = 0.196854$$
$$a_2 = 0.115194$$
$$a_3 = 0.000344$$
$$a_4 = 0.019527$$

$$y = \left(F^{1/3}\left(1 - \frac{2}{9d_2}\right) - \left(1 - \frac{2}{9d_1}\right) \right) \left(\frac{2}{9d_1} + F^{2/3}\frac{2}{9d_2} \right)^{-1/2}$$

$$d_1 = \text{degrees of freedom in numerator}$$
$$d_2 = \text{degrees of freedom in denominator}$$

$$|\varepsilon(y)| < 2.5 \times 10^{-4}$$

```
clc; clear

% Asks the user for the relevant input
f = 0.8457
```

```
f = 0.8457
```

```
d1 = 3
```

```
d1 = 3
```

```
d2 = 2
```

```
d2 = 2
```

```


x = 1;
```

```
% Computes using inverse for small F-values
if f < 1
    s = d2;
    t = d1;
    z = 1/f;
else
    s = d1;
    t = d2;
    z = f;
end
j = 2/(9*s);
k = 2/(9*t);

% Uses approximation formulas
y = abs((1 - k)*z^(1/3) - 1 + j)/sqrt(k*z^(2/3) + j);
if t < 4
    y = y*(1 + 0.08*y^4/t^3);
end

a1 = 0.196854;
a2 = 0.115194;
a3 = 0.000344;
a4 = 0.019527;
x = 0.5/(1 + y*(a1 + y*(a2 + y*(a3 + y*a4))))^4;
x = floor(x*10000 + 0.5)/10000;

% Adjusts if inverse was computed
if f < 1
    x = 1 - x;
end

% Displays results
str1 = ['Percentile = ' num2str(1-x)];
str2 = ['Tail end value = ' num2str(x)];
disp(str1)
```

```
Percentile = 0.4189
```

```
disp(str2)
```

```
Tail end value = 0.5811
```

d) Show an example of the use of

chi-square distribution in testing of

hypothesis.

Let A,B,C,D be the choices for each questions.Probability of choosing any one choice is 25%

H_0: Equal Distribution of correct choices.

H_1: Unequal Distribution.

Assuming Null Hypothesis is true,

```
%choice=[A B C D]
Expected=[25 25 25 25]
```

```
Experted = 1×4
    25    25    25    25
```

```
Actual=[20 20 25 35]
```

```
Actual = 1×4
    20    20    25    35
```

```
alpha=0.05
```

```
alpha = 0.0500
```

```
ChisSquare = sum((Actual-Expected).^2 ./ Expected)
```

```
ChisSquare = 6
```

```
p=1-chi2cdf(ChisSquare,3)
```

```
p = 0.1116
```

```
[h,p,stats] = chi2gof([1 2 3 4],'freq',Actual,'Expected',Expected,'ctrs',[1 2 3 4],'nparams',3)
```

```
h = 0
p = NaN
stats = struct with fields:
    chi2stat: 6
          df: 0
       edges: [0.5000 1.5000 2.5000 3.5000 4.5000]
           O: [20 20 25 35]
           E: [25 25 25 25]
```

```
%Here, probability of getting chi-squared value greater than or equal to
%six is going to be greater than 10%
%And so far our probability assuming the Null Hypothesis is greater than
% 10%.This is greater than significance level.
%P(Chi-square>=6)>10%
%Hence, we will fail to reject the H_0
```

e) Find and Discuss other uses of chi-square distribution .

- Chi-square test of goodness of fit of observed data to hypothetical distributions: The primary reason for which the chi-square distribution is extensively used in hypothesis testing is its relationship to the normal distribution. The simplest chi-square distribution is the square of a standard normal distribution. So wherever a normal distribution could be used for a hypothesis test, a chi-square distribution could be used.
- It is also a component of the definition of the t-distribution and the F-distribution used in t-tests, analysis of variance, and regression analysis.

- Likelihood-ratio test for nested models:In statistics, the **likelihood-ratio test** assesses the goodness of fit of two competing statistical models based on the ratio of their likelihoods.

2

a).Derive pdf of F distribution

Let X & Y both have independent chi-square distributions with degree of freedom v_1 & v_2 respectively.

Define the quotient $Z = \frac{Y}{X}$. Then the PDF of Z can be determined from the result of the joint distribution of a quotient.

$$f_Z(z) = \int_0^\infty \frac{(x z)^{\frac{v_1}{2}-1} e^{-x^{z}/2}}{\Gamma\left(\frac{v_1}{2}\right) 2^{v_1/2}} \frac{(xz)^{\frac{v_2}{2}-1} e^{-\frac{xz}{2}}}{\Gamma\left(\frac{v_2}{2}\right) 2^{v_2/2}} dx$$

$$= \frac{z^{\frac{v_1}{2}-1}}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)2^{\frac{(v_1+v_2)}{2}}} \int_0^\infty x^{(v_1+v_2)/2-1} e^{-x(z+1)/2} dx$$

Now the gamma f^n is actually defined by

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt.$$

After using the substitution, $t = x\left(\frac{z+1}{2}\right)$, we recognize the integral above as a value of the gamma f^n.

$$f_Z(z) = \frac{z^{v_1/2-1}}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)2^{\frac{(v_1+v_2)}{2}}\left(\frac{z+1}{2}\right)^{\frac{(v_1+v_2)}{2}}} \int_0^\infty t^{\frac{(v_1+v_2)}{2}-1} e^{-t} dt$$

$$= \frac{\Gamma\left(\frac{v_1+v_2}{2}\right) z^{v_1/2-1}}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)(z+1)^{\frac{(v_1+v_2)}{2}}}$$

The F- distribution is actually not the quotient, but the quotient $F_0 = \dfrac{Y/v_1}{X/v_2}$. Therefore, if X is a random variable with an F-distribution its DF is

$$f_x(x) = \frac{v_1}{v_2} f_Z\left(\frac{v_1}{v_2}\right)x$$

$$= \frac{\frac{v_1}{v_2}\Gamma\left(\frac{v_1+v_2}{2}\right)\left(\frac{v_1}{v_2}x\right)^{\frac{v_1}{2}-1}}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)\left(\frac{v_1}{v_2}x+1\right)^{\left(\frac{v_1+v_2}{2}\right)}}$$

$$= \frac{\Gamma\left(\frac{v_1+v_2}{2}\right)\left(\frac{v_1}{2}\right)^{\frac{v_1}{2}}x^{\frac{v_1}{2}-1}}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)\left(1+\frac{v_1}{v_2}x\right)^{\frac{v_1+v_2}{2}}}$$

where v_1, v_2 are shape parameters & Γ is the gamma f^n.

b) How do you plot the pdf of f-ditribution in matlab

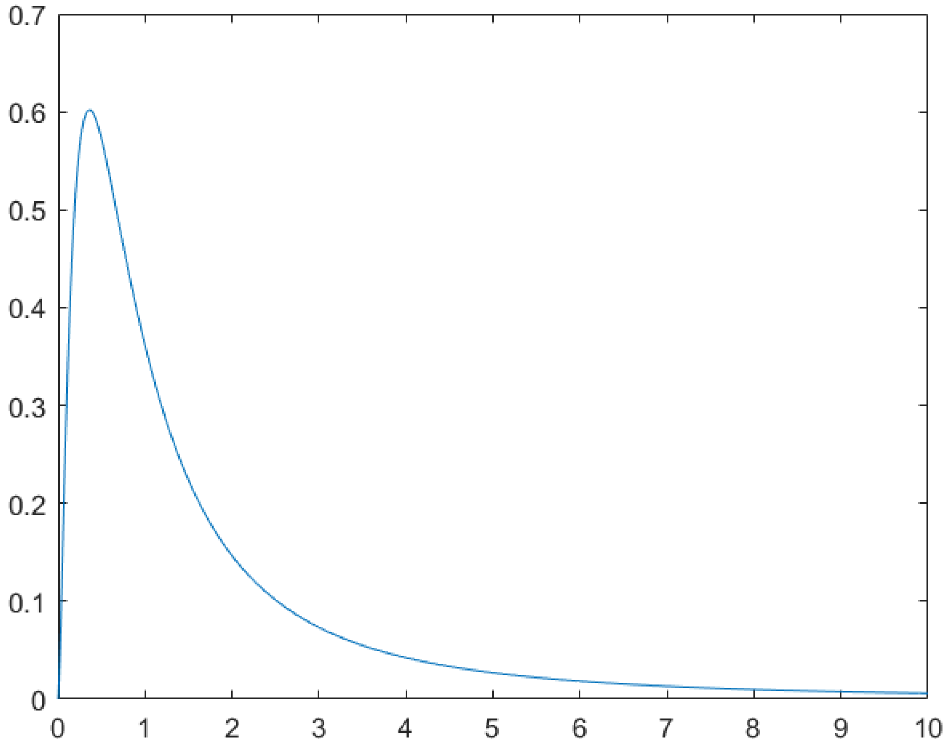
```
x = 0:0.01:10
```

```
x = 1×1001
         0    0.0100    0.0200    0.0300    0.0400    0.0500    0.0600    0.0700 ···
```

```
% F distribution with 5 numerator degrees of freedom and 3 denominator degrees of freedom.
y = fpdf(x,5,3)
```

```
y = 1×1001
         0    0.0171    0.0453    0.0781    0.1129    0.1483    0.1833    0.2175 ···
```

```
figure
plot(x,y)
```



d) Why F test in Anova is one tailed

An *F*-test is used to test if the variances of two populations are equal. This test can be a two-tailed test or a one-tailed test. The two-tailed version tests against the alternative that the variances are not equal. The one-tailed version only tests in one direction, that is the variance from the first population is either greater than or less than (but not both) the second population variance. The choice is determined by the problem.

For example, if we are testing a new process, we may only be interested in knowing if the new process is less variable than the old process.

IV. Give a method to generate 5 population data such that

4 means are statistically equal and one mean is different from the

Rest. In each population take 10 data.

Apply oneway ANOVA and show that most of the time null hypothesis

is rejected.

Sol: The **null hypothesis** for an **ANOVA is** that there **is no** significant difference among the groups. If the **null hypothesis is rejected**, one concludes that the means of all the groups are **not** equal.

ANOVA rejects the null hypothesis that all group means are equal.

Using the multiple comparisons to determine which group means are different from others. In this example, `anova1` rejects the null hypothesis that the mean of data from all 5 population groups are equal to each other, i.e., $H0: \mu 1=\mu 2=\mu 3=\mu 4=\mu 5.$

Graphical assurance that the means are different by looking at the box plots.

Write a Matlab code to do every thing

Step1:8

```
clear all;clc;
clf;
rng(12345);
sigma=3;
mu1=160;
mu2=150;
m=10;  % number of data in each column
n=4;  % number columns (no of population)
for i=1:10
A=normrnd(mu1,sigma,m,n);
[m,n]=size(A);
B=normrnd(mu2,sigma,m,1);
end
C=[A B]
```

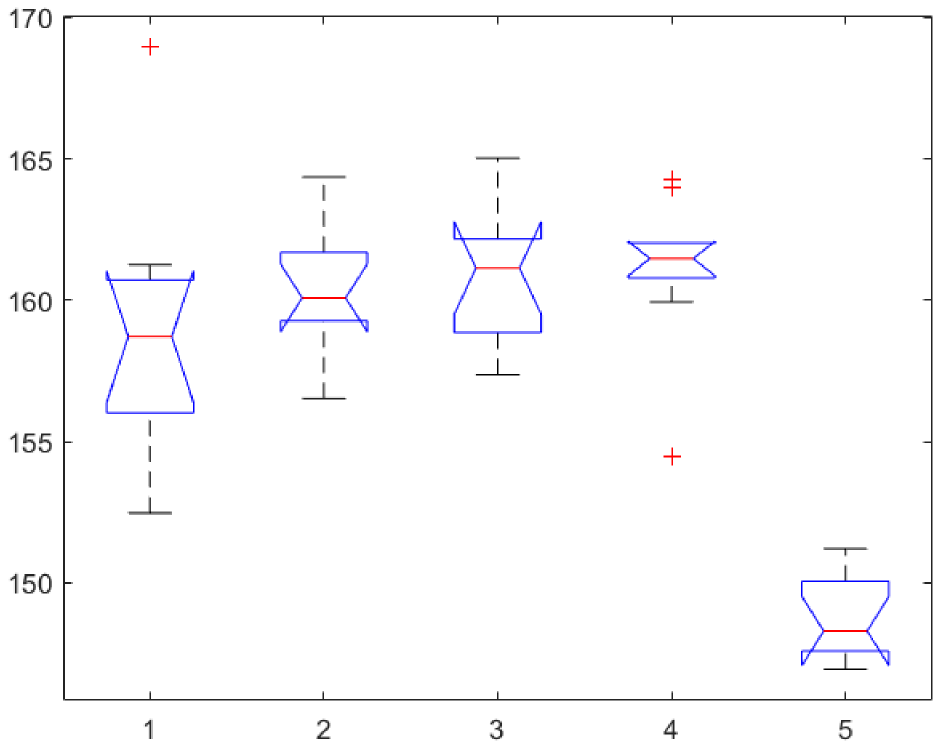
```
C = 10×5
  152.4643  160.0348  162.3452  159.9328  147.5889
  161.2525  161.6983  158.8613  163.9731  150.0472
  160.6205  164.3541  160.8715  160.7845  150.3422
  154.6573  156.5117  157.3612  161.0915  146.9326
  160.7071  159.2705  162.0211  161.9667  148.0689
  159.4307  161.3213  159.5143  164.2879  147.0921
  157.9835  163.7618  165.0271  161.8561  149.3168
  168.9721  157.1811  157.7458  162.0225  151.2025
  156.0101  160.1246  161.4048  154.4458  148.5179
  157.3899  160.0037  162.1639  160.9601  147.7095
```

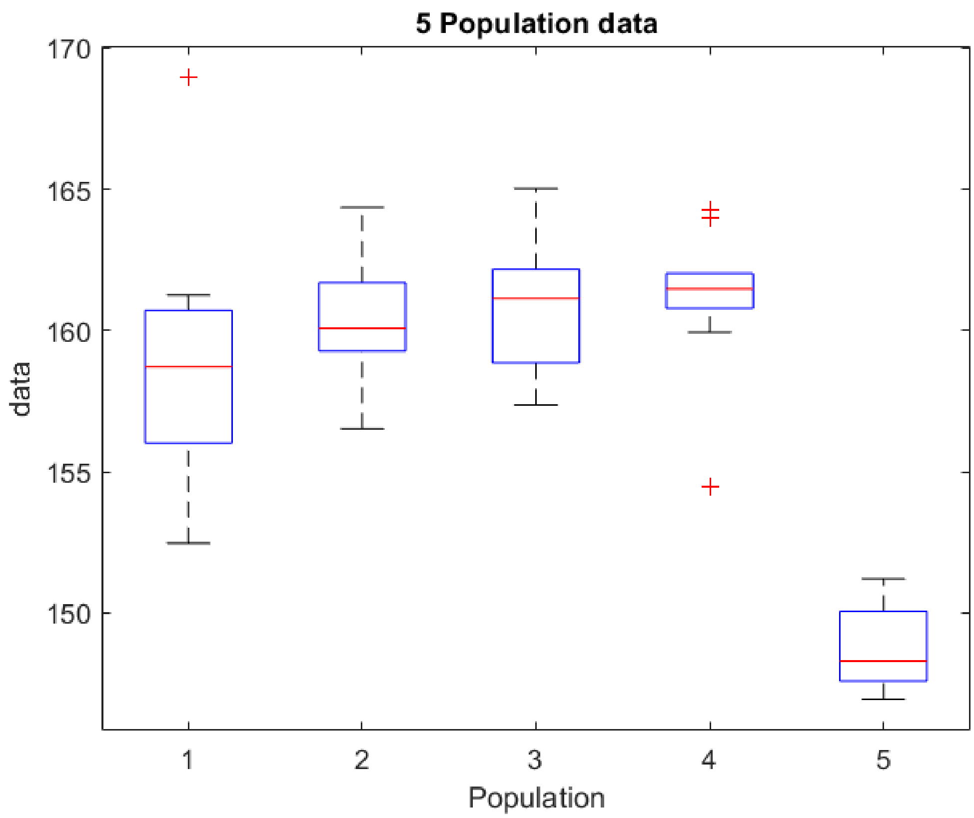
```
boxplot(C)
title('5 Population data')
xlabel('Population')
```

```
ylabel('data')
[p,tbl,stats] = anova1(C);
```

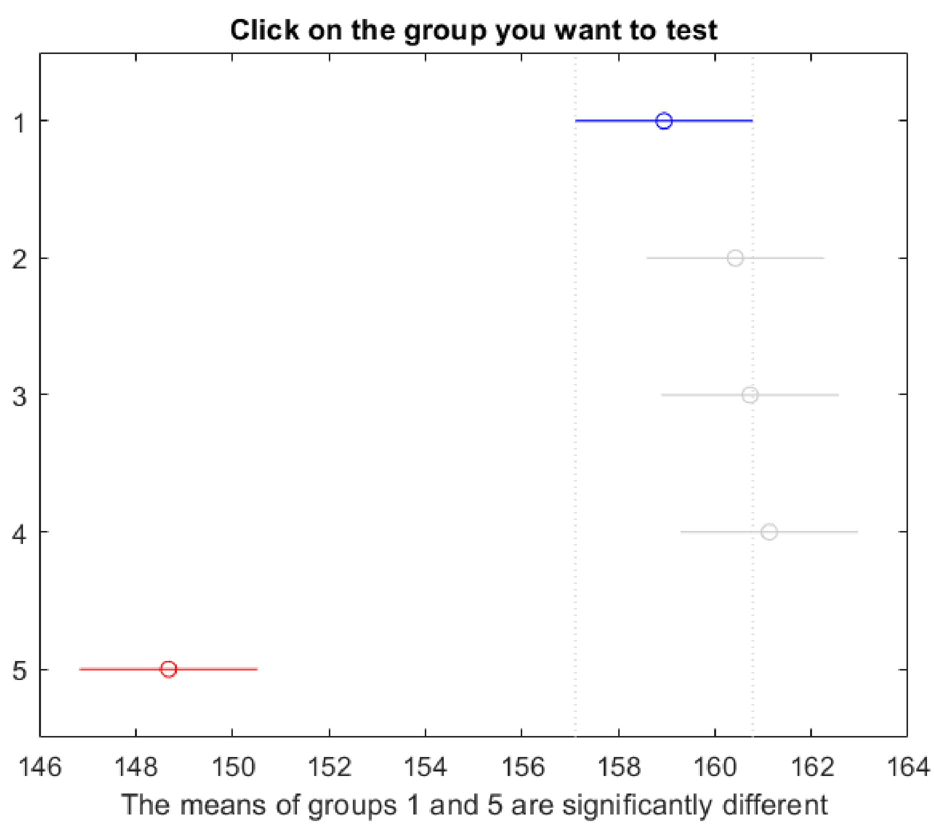
ANOVA Table

Source	SS	df	MS	F	Prob>F
Columns	1108.85	4	277.212	33.06	7.14324e-13
Error	377.29	45	8.384		
Total	1486.13	49			






```
multcompare(stats)
```




```
ans = 10×6
    1.0000    2.0000   -5.1568   -1.4774    2.2021    0.7840
```

```
    1.0000    3.0000   -5.4623   -1.7828    1.8966    0.6453
    1.0000    4.0000   -5.8628   -2.1833    1.4962    0.4526
    1.0000    5.0000    6.5875   10.2669   13.9464    0.0000
    2.0000    3.0000   -3.9849   -0.3054    3.3740    0.9993
    2.0000    4.0000   -4.3854   -0.7059    2.9735    0.9820
    2.0000    5.0000    8.0649   11.7443   15.4238    0.0000
    3.0000    4.0000   -4.0799   -0.4005    3.2790    0.9979
    3.0000    5.0000    8.3703   12.0498   15.7292    0.0000
    4.0000    5.0000    8.7708   12.4502   16.1297    0.0000
```

```
p<0.05
```

```
ans = logical
   1
```

```
%OR

[m,n]=size(C);  % n= number of population groups, m=#of data
ytbars=sum(C)/m; % n group averages
ybar=sum(ytbars)/n; % grand mean
ytbarsmatrix=repmat(ytbars,m,1);
ybarmatrix=ybar*ones(m,n);
SSTotal=sumsqr(C-ybarmatrix);
SSbetween=sumsqr(ytbarsmatrix-ybarmatrix);
SSError=sumsqr(C-ytbarsmatrix);
dfbetween=n-1;
dfError=m*n-n;
MSbetween=SSbetween/dfbetween;
MSError=SSError/dfError;
Fval_comp=MSbetween/MSError
```

```
Fval_comp = 33.0640
```

```
p = fcdf(Fval_comp,dfbetween,dfError) % cumulative f distribution
```

```
p = 1.0000
```

```
pbar=1-p;   % right tail area beyond Fval_computed
fdist_cutoff=finv(0.95,dfbetween,dfError)
```

```
fdist_cutoff = 2.5787
```

V. Give a method to generate 2 way ANOVA data with A at 3 levels and B at 4 levels. For each combination take 10 data.

Data should be such that 2 way ANOVA almost always accept the null hypothesis

(means of all levels in A are equal, means of all levels in B are equal and there is no interaction between A and B)

```
%Synthetic data Visualization
clear all;clc;
clf;
rng default;
A=3;
B=4;
n=10;
X=cell(A,B,1);
for j=1:A
    for k=1:B
        X(j,k)={normrnd(160,0.05,1,n)};
    end

end
X
```

X = 3×4 cell

	1	2	3	4
1	[160.026...	[159.932...	[160.033...	[160.044...
2	[159.994...	[159.956...	[159.945...	[159.969...
3	[160.071...	[159.942...	[160.042...	[159.893...

```
%Synthetic data Creation

clear all;clc;
clf;
rng default;
A=3;
B=4;
n=10;
for j=1:A
    for k=1:B
        X(:,j,k)=normrnd(160,1,1,n);
        cellavg(j,k)=mean(X(:,j,k));
        SSerr(j,k)=sum(sum(((X(:,j,k)-cellavg(j,k)).^2),2));
    end

end
Y=reshape(X(:),n*A,B)
```

```
Y = 30×4
  160.5377  158.6501  160.6715  160.8884
  161.8339  163.0349  158.7925  158.8529
  157.7412  160.7254  160.7172  158.9311
  160.8622  159.9369  161.6302  159.1905
  160.3188  160.7147  160.4889  157.0557
  158.6923  159.7950  161.0347  161.4384
  159.5664  159.8759  160.7269  160.3252
  160.3426  161.4897  159.6966  159.2451
  163.5784  161.4090  160.2939  161.3703
  162.7694  161.4172  159.2127  158.2885
      ⋮
```

```
cellavg
```

```
cellavg = 3×4
  160.6243  160.7049  160.3265  159.5586
  160.2059  159.8905  160.1282  159.7354
  160.3030  159.7536  160.1958  159.6460
```

```
%anova two way using inbuilt function
[p,~,stats]=anova2(Y,n)
```

ANOVA Table

```
Source          SS      df       MS        F      Prob>F
-----------------------------------------------------
Columns        8.883     3    2.96109    2.26    0.0855
Rows           2.757     2    1.37865    1.05    0.3526
Interaction    3.851     6    0.64184    0.49    0.8146
Error        141.447   108    1.3097
Total        156.939   119
```

```
p = 1×3
    0.0855    0.3526    0.8146
stats = struct with fields:
      source: 'anova2'
     sigmasq: 1.3097
    colmeans: [160.3777 160.1163 160.2168 159.6467]
        coln: 30
    rowmeans: [160.3036 159.9900 159.9746]
        rown: 40
       inter: 1
        pval: 0.8146
          df: 108
```

In a 3 way ANOVA with factors A , B and C

A is having 4 levels

B is having 3 levels

C is having 3 levels.

10 experiment is done for each factor level combination.

(4x3x3x10 experimental data).

Assume all interactions are absent.

What is

1)DF of A

2)DF of B

3)DF of C

4)DF of Error

5)Total DF

6)For 5% significant level , what are the 3 critical values of F distribution

In 3 way Anova with factors A,B,C:

1) A is having 4 levels => **DF of A is** 4-1=3

2) B is having 3 levels => **DF of B is** 3-1=2

3) C is having 3 levels => **DF of C is** 3-1=2

Given that : 10 experiment is done for each factor level combination

DF of AB = (DF of A)*(DF of B) = 3*2=6

DF of AC = (DF of A)*(DF of C) = 3*2=6

DF of BC = (DF of B)*(DF of C) = 2*2=4

DF of ABC = (DF of A)*(DF of B)*(DF of C) =3* 2*2=12

4) DF of Error = **Total DF - (DF of A+DF of B+DF of C +DF of AB +DF of BC +DF of AC)**

= 359-(3+2+2+6+6+4+12)

= 324

OR

4) DF of Error = 4*3*3*(10-1)

= 324

5) Total DF : (4*3*3*10) -- 1 = 360-1 = 359

6)For 5% significant level , what are the 3 critical values of F distribution

Using F-distribution table

F_A = F(3,324) = 2.6325

F_B =F(2,324) =3.0236

F_C =F(2,324) =3.0236

```
fdist_cutoff=finv(0.95,3,324)
```

```
fdist_cutoff = 2.6325
```

```
fdist_cutoff=finv(0.95,2,324)
```

```
fdist_cutoff = 3.0236
```