Program: Mathematics and Physics

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Assignment No.	**1**

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2) Let

x_T □ 0.3676□10^3 y_T □ 0.4562□10^1 Assume 3 decimal mantissa with rounding are used so that x_A □ 0.368□10^3 y_A □ 0.456□10^1

a) Calculate the propagated error after x+y

② $x_T = 0.3676\times10^3$

$y_T = 0.4562\times10^1$

⇒ Assume 3 decimal mantissa with rouding are used so that

$x_A = 0.368\times10^3$

$y_A = 0.456\times10^1$

(a) Propagated Error:-

$= Ex + Ey$

$Ex = x_T - x_A$

$= 0.3676\times10^3 - 0.368\times10^3$

$\boxed{Ex = -0.4\times10^{-1}}$

$Ey = y_T - y_A$

$= 0.4562\times10^1 - 0.456\times10^1$

$\boxed{Ey = 0.3\times10^{-3}}$

$\Rightarrow Ex + Ey = -0.4\times10^{-1} + 0.3\times10^{-3}$

$= 10^{-1}(0.4 + 0.003)$

$= -0.397\times10^{-1}$

$\boxed{Ex + Ey = -0.0397}$

b)

b) Calculate the round-off error after x+y

(b) Rounding Error :-

$$x_A + y_A = 0.368 \times 10^3 + 0.456 \times 10^1$$

$$= 0.37256 \times 10^3$$

⇒ Representation Form

$$f(x_A + y_A) = 0.373 \times 10^3$$

⇒ So, total rounding error

$$= (x_T + y_T) - f(x_A + y_A)$$

$$= (0.3721 \times 10^3) - (0.373 \times 10^3)$$

$\boxed{= -0.90}$ Rounding Error.

3) If we want to approximate $e^{5.04}$ with an error less than 10^{-10} using the Taylor series for $f(x)=e^x$ around 5, at least how many terms are needed?

③

$$R_n = \frac{e^x}{(n+1)!} \times x^{n+1}$$

$$\Rightarrow e^x = e^{5.04} = 154.4700$$

$$\Rightarrow \frac{e^{5.04}}{(31+1)!} \times 5.04^{31+1}$$

$$\Rightarrow \frac{154.4700}{32!} \times 5.04^{32}$$

$$\boxed{R_n = 1.76380110E^{-11} < 10^{-10}}$$

So, atleast 31 terms required to get Error less than 10^{-10}. using Taylor Series.

4) For the following linear equation system:

2x_1 □x_2 □ 4x_3 □ 1 □2 1 41 □ x_1 □ 2x_2 □3x_3 □1.5 | □□1 2 31.5□□

4x_1 □x_2 □ 2x_3 □ 2 □□4 □1 22 | □□

Use Scaling and if necessary pivoting strategy to solve based on

a) Jordan Gauss-Elimination method

b) Naïve Gauss Eliminiation method

(4) $x_2 + 3x_3 = 8$

$4x_1 + 6x_2 + 7x_3 = -3$

$2x_1 + x_2 + 6x_3 = 5$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 3 \\ 4 & 6 & 7 \\ 2 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ 5 \end{bmatrix}$$

$$\Rightarrow \text{swap } R_1 \Leftrightarrow R_2 \quad \begin{bmatrix} 4 & 6 & 7 \\ 0 & 1 & 3 \\ 2 & 1 & 6 \end{bmatrix} = \begin{bmatrix} -3 \\ 8 \\ 5 \end{bmatrix}$$

$$\Rightarrow R_1/4 \rightarrow R_1 \quad \begin{bmatrix} 1 & 3/2 & 7/4 \\ 0 & 1 & 3 \\ 2 & 1 & 6 \end{bmatrix} = \begin{bmatrix} -3/4 \\ 8 \\ 5 \end{bmatrix}$$

$$\Rightarrow 2R_1 \rightarrow R_1 \quad \begin{bmatrix} 2 & 3/2 & 7/2 \\ 0 & 1 & 3 \\ 2 & 1 & 6 \end{bmatrix} = \begin{bmatrix} -3/2 \\ 8 \\ 5 \end{bmatrix}$$

$$\Rightarrow -R_1 + R_3 \Rightarrow R_3 \quad \begin{bmatrix} 1 & 3/2 & 7/4 \\ 0 & 1 & 3 \\ 0 & -2 & 5/2 \end{bmatrix} = \begin{bmatrix} -3/4 \\ 8 \\ 13/2 \end{bmatrix}$$

$$\tfrac{3}{2}R_2 \to R_2 \begin{bmatrix} 1 & 3/2 & 7/4 \\ 0 & 3/2 & 9/2 \\ 0 & -2 & 5/2 \end{bmatrix} = \begin{bmatrix} -3/4 \\ 12 \\ 13/2 \end{bmatrix}$$

$$-R_2 + R_1 \to R_1 \begin{bmatrix} 1 & 0 & -11/4 \\ 0 & 1 & 3 \\ 0 & -2 & 5/2 \end{bmatrix} = \begin{bmatrix} -51/4 \\ 8 \\ 13/2 \end{bmatrix}$$

$$2R_2 \to R_2 \begin{bmatrix} 1 & 0 & -11/4 \\ 0 & -2 & -6 \\ 0 & -2 & 5/2 \end{bmatrix} = \begin{bmatrix} -51/4 \\ -16 \\ 13/2 \end{bmatrix}$$

$$-R_2 + R_3 \Rightarrow R_3 \begin{bmatrix} 1 & 0 & -11/4 \\ 0 & 1 & 3 \\ 0 & 0 & 17/2 \end{bmatrix} = \begin{bmatrix} -51/4 \\ 8 \\ 45/2 \end{bmatrix}$$

$$\tfrac{17}{2}R_3 \to R_3 \begin{bmatrix} 1 & 0 & -11/4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -51/4 \\ 8 \\ 45/17 \end{bmatrix}$$

$$-\tfrac{11}{4}R_3 \to R_3 \begin{bmatrix} 1 & 0 & -11/4 \\ 0 & 1 & 3 \\ 0 & 0 & -11/4 \end{bmatrix} = \begin{bmatrix} -51/4 \\ 8 \\ -495/68 \end{bmatrix}$$

$-R_3 + R_1 \Rightarrow R_1$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -93/17 \\ 8 \\ 45/17 \end{bmatrix}$

$-3R_3 + R_1 \Rightarrow R_1$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -93/17 \\ 1/17 \\ 45/17 \end{bmatrix}$

$\boxed{X_1 = -93/17}$, $\boxed{X_2 = 1/17}$, $\boxed{X_3 = 45/17}$

5) Solve the following system by using the Gauss-Jordan elimination method:

$$\begin{cases} A + B + 2C = 1 \\ 2A - B + D = -2 \\ A - B - C - 2D = 4 \\ 2A - B + 2C - D = 0 \end{cases}$$

(5) Gauss-Jordan Elemination

$A + B + 2C = 1$
$2A - B + D = -2$
$A - B - C - 2D = 4$
$2A - B + 2C - D = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & -1 & 0 & 1 \\ 1 & -1 & -1 & -2 \\ 2 & -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{array}{l} \cancel{2R_1 \Rightarrow R_1} \\ \cancel{-R_1 + R_2 \Rightarrow R_2} \\ 2R_1 \Rightarrow R_1 \end{array} \begin{bmatrix} 2 & 2 & 4 & 0 \\ 2 & -1 & 0 & 1 \\ 1 & -1 & -1 & -2 \\ 2 & -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 4 \\ 0 \end{bmatrix}$$

$$\to -R_1 + R_2 \Rightarrow R_2 \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -3 & -4 & 1 \\ 1 & -1 & -1 & -2 \\ 2 & -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 4 \\ 0 \end{bmatrix}$$

$$\to -R_1 + R_3 \Rightarrow R_3 \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & -2 & -3 & -2 \\ 2 & -1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 3 \\ 0 \end{bmatrix}$$

$$-2R_2 + R_4 \Rightarrow R_4 \quad \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & -2 & -3 & -2 \\ 0 & -3 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 3 \\ -2 \end{bmatrix}$$

$$-\frac{1}{3}R_2 + R_1 \Rightarrow R_1 \quad \begin{bmatrix} 1 & 0 & 2/3 & 1/3 \\ 0 & 1 & 4/3 & -1/3 \\ 0 & -2 & -3 & -2 \\ 0 & -3 & -2 & -1 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 4/3 \\ 3 \\ -2 \end{bmatrix}$$

$$-2R_2 + R_3 \Rightarrow R_3 \quad \begin{bmatrix} 1 & 0 & 2/3 & 1/3 \\ 0 & 1 & 4/3 & -1/3 \\ 0 & 0 & -1/3 & -8/3 \\ 0 & -3 & -2 & -1 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 4/3 \\ 17/3 \\ -2 \end{bmatrix}$$

$$-3R_2 + R_4 \Rightarrow R_4 \quad \begin{bmatrix} 1 & 0 & 2/3 & 1/3 \\ 0 & 1 & 4/3 & -1/3 \\ 0 & 0 & -1/3 & -8/3 \\ 0 & 0 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 4/3 \\ 17/3 \\ 2 \end{bmatrix}$$

$$-1/3 R_3 \Rightarrow R_3 \quad \begin{bmatrix} 1 & 0 & 2/3 & 1/3 \\ 0 & 1 & 4/3 & -1/3 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 2 & -2 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 4/3 \\ 17 \\ 2 \end{bmatrix}$$

$$+2/3 R_3 \Rightarrow R_3$$

$$-2/3 R_3 + R_1 \Rightarrow R_1 \quad \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 2/3 & -1/3 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 11 \\ 4/3 \\ -17 \\ 2 \end{bmatrix}$$

$$-\tfrac{4}{3}R_3 + R_2 \quad \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 11 \\ 24 \\ -17 \\ 2 \end{bmatrix}$$

$$-2R_3 + R_4 \quad \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & -18 \end{bmatrix} = \begin{bmatrix} 11 \\ 24 \\ -17 \\ 36 \end{bmatrix}$$

$$-\tfrac{1}{18}R_4 \Rightarrow R_4 \quad \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 11 \\ 24 \\ -17 \\ 10 \end{bmatrix}$$

$$-5R_4 + R_1 \Rightarrow R_1 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 24 \\ -17 \\ -2 \end{bmatrix}$$

$$-11R_4 + R_2 \Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -17 \\ -2 \end{bmatrix}$$

$$-8R_4 + R_3 \Rightarrow R_3 \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$$

$\boxed{A = 1}$, $\boxed{B = 2}$, $\boxed{C = -1}$, $\boxed{D = -2}$

6) Use Gaussian elimination to solve the system of linear equations:

$$\begin{aligned} x_1 - 2x_2 - 6x_3 &= 12 \\ 2x_1 + 4x_2 + 12x_3 &= -17 \\ x_1 - 4x_2 - 12x_3 &= 22. \end{aligned}$$

⑥ Gauss elimination of Linear System

$X_1 - 2X_2 - 6X_3 = 12$

$2X_1 + 4X_2 + 12X_3 = -17$

$X_1 - 4X_2 - 12X_3 = 22.$

$$\begin{bmatrix} 1 & -2 & -6 \\ 2 & 4 & 12 \\ 1 & -4 & -12 \end{bmatrix} = \begin{bmatrix} 12 \\ -17 \\ 22 \end{bmatrix}$$

$-2R_1 + R_2 \Rightarrow \begin{bmatrix} 1 & -2 & -6 \\ 0 & 8 & 24 \\ 0 & -2 & -12 \end{bmatrix} = \begin{bmatrix} 12 \\ -41 \\ 22 \end{bmatrix}$

$-R_1 + R_3 \Rightarrow R_3 \begin{bmatrix} 1 & -2 & -6 \\ 0 & 8 & 24 \\ 0 & -2 & -6 \end{bmatrix} = \begin{bmatrix} 12 \\ -41 \\ 10 \end{bmatrix}$

$\frac{1}{8}R_2 \Rightarrow R_2 \quad \begin{bmatrix} 1 & -2 & -6 \\ 0 & 1 & 3 \\ 0 & -2 & -6 \end{bmatrix} = \begin{bmatrix} 12 \\ -5.12 \\ 10 \end{bmatrix}$

$2R_2 + R_1 \Rightarrow R_1$

$2R_2 + R_3 \Rightarrow R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1.75 \\ -5.125 \\ -0.25 \end{bmatrix}$

$\Rightarrow$ system equation has not solved

becaused $\boxed{Z = 0 \neq -0.25}$