IT575 Computational Shape Modeling

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Abstract

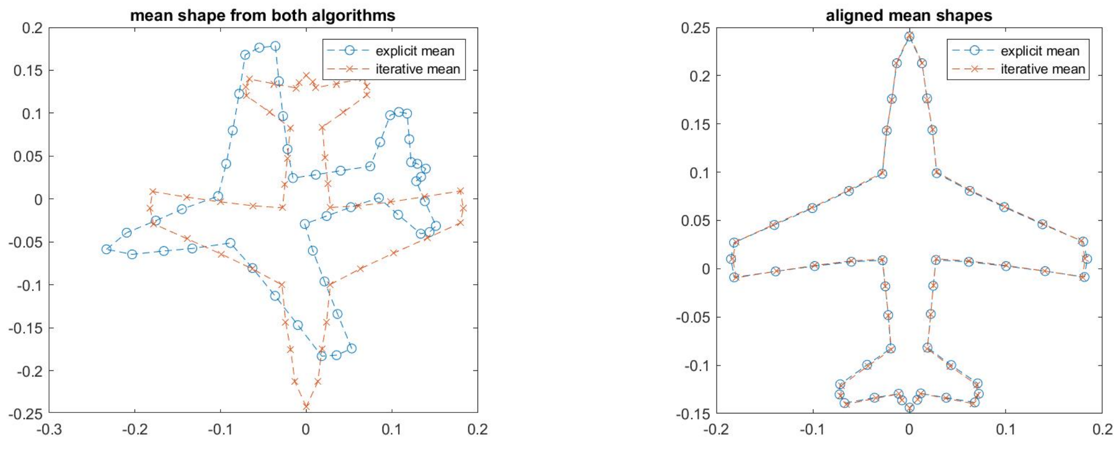
Performing shape analysis on different shapes of same type of object gives idea about general shape of that object. This assignment introduces mean shape and shape variation using landmark point sets.

Dataset

This assignment is implemented in MATLAB. For implementation purpose plane landmark points are used which is the collection of landmark point sets of 56 planes where each plane is represented using 51 landmarks points. Translation and scaling is performed on raw data to convert into centered unit vectors.

1 Mean Shape

For all plane landmark points sets, the mean shape is computed using two algorithms, one is explicit method in which the mean shape is the unit eigenvector of the covariance matrix and the other which is an iterative algorithm.



(a) (b)

Figure 1: explicit and iterative means of plane landmark points sets

As shown in figure (a) both mean are rotated version of each other which is obvious because in iterative method all landmark points are aligned to the mean shape in every iteration while in explicit method alignment is not needed so it gives rotated shape.

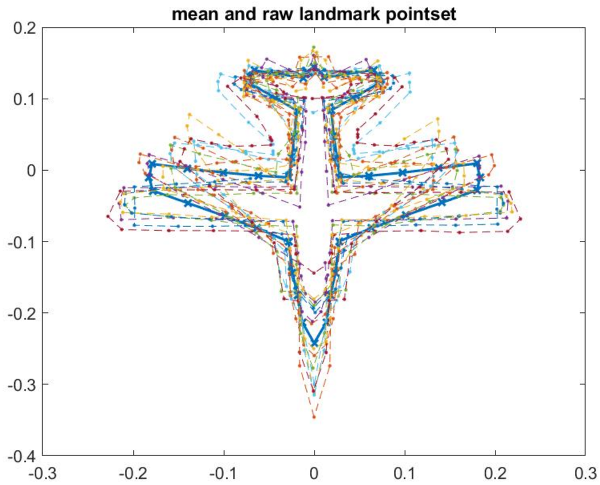


Figure 2: Mean shape with plane landmark points

2 Shape Variation

Shape variation means distribution of shape in the given collection. Shape variation contains a direction and amount of variation in that particular direction. Here first 3 direction $v_1, v_2, v_3 \in \mathbb{C}^{51}$ are selected which contains maximum variance.

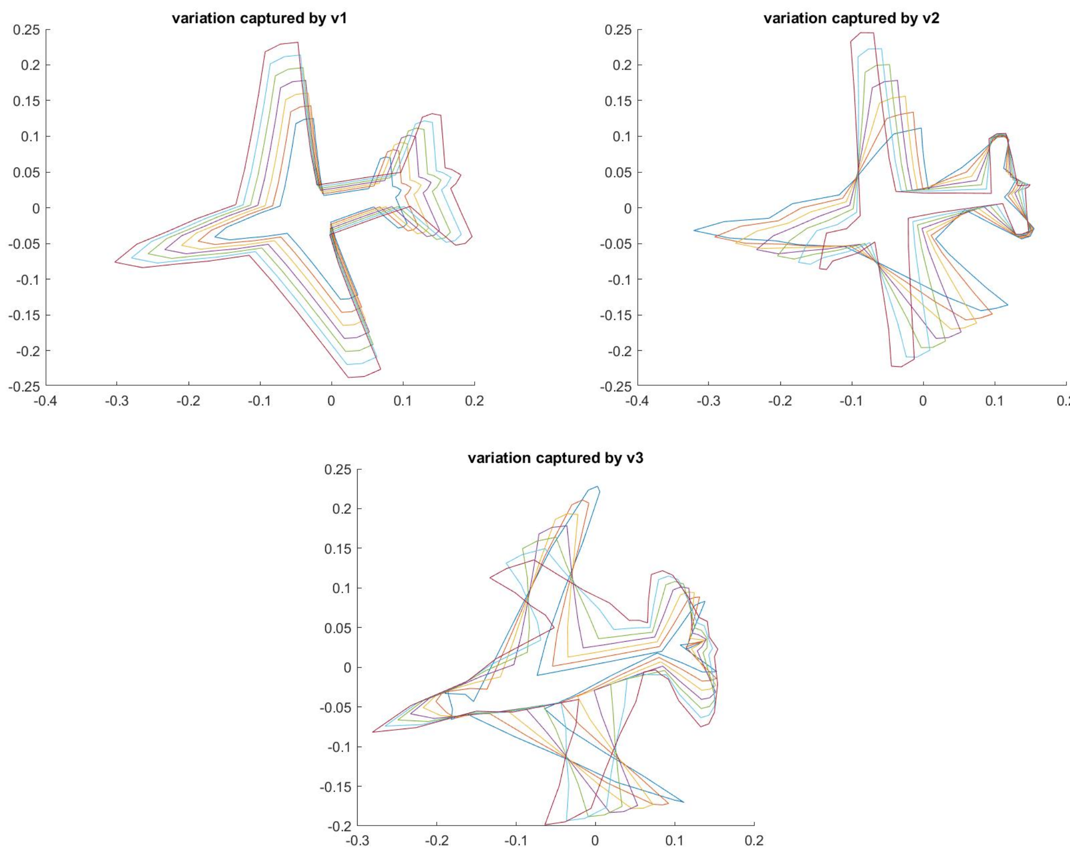


Figure 3: variation along with v_i

In 1^{st} figure, variation on v_1 is related to the mean shape. It means in the direction of the mean the variance is maximum.

In 2^{nd} figure, variation on v_2 represents deviation in the length of the wings and length of the head.

In 3^{rd} figure, variation on v_3 represents deviation of the angle between wings and body as well as the size of the tail of the plane.

there are total 51 principal directions $v_1, v_2, ..., v_{51}$ available which are the unit eigenvector of the shape covariance matrix. The eigenvalue λ_i related to the eigenvector v_i represent amount of the variation on v_i and sum of all eigenvalues gives total variance.

Here, total variance in all direction is 194.2086. v_1 contains major portion of variance which is 192.0787, v_2 contains 1.2822 variance and v_3 contains 0.2858.

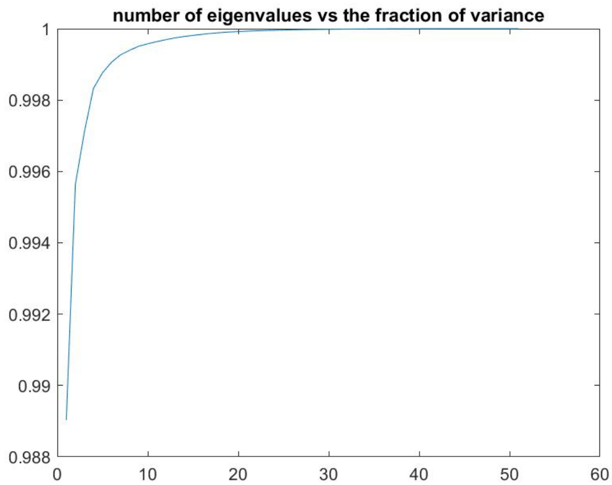


Figure 4: number of eigenvalues vs the fraction of variance

As shown in figure 4 1st principal direction v_1 captures 98.9% variance and first two and three principal directions captures 99.56% and 99.7% variance respectively.

So, v_1 is enough to capture 90% or 95% variance. that means we can store landmark point set using just single complex number instead of 51 complex numbers if we can tolerate loss of 1.1% of the variance.

There are 8 such eigenvectors which contains 0 eigenvalue that means in these direction variance is 0.

3 Tangent PCA

Let us assume that the collection of landmark point sets $\{x^p \in \mathbb{R}^{2n}, p = 1, ..., m\}$ is aligned with the mean shape μ. As a simplified pictorial representation, assume that these landmark point sets all lie in a circular arc around μ as shown in Figure 5. The amount of variation in the second direction will depend upon the spread of data around the mean. In order to eliminate the second variation, we can project the data so that it belongs to the tangent space of unit-sized shapes at the mean μ. Let $x \in \mathbb{R}^{2n}$ with $\|x\| = 1$ represent one of the landmark point set which is aligned to the mean $\mu \in \mathbb{R}^{2n}$ with $\|\mu\| = 1$. Let $x + s\mu$, $s \in \mathbb{R}$ represent the projected point $\tilde{x}$ that belongs to the tangent space at μ, T_μ.

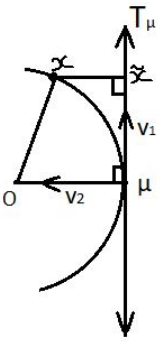


Figure 5: orthogonal projection on tangent space

Here,

$$\|\mu\|^2 = \langle \mu, \mu \rangle = 1$$

Where, $\langle \cdot, \cdot \rangle$ is standard inner product on $\mathbb{R}^{2n}$

As shown in figure,

$$\begin{aligned} \langle \tilde{x} - \mu, \mu \rangle &= 0 \\ \langle (x + s\mu) - \mu, \mu \rangle &= 0 \\ \langle x + (s-1)\,\mu, \mu \rangle &= 0 \\ \langle x, \mu \rangle + \langle (s-1)\,\mu, \mu \rangle &= 0 \\ \langle x, \mu \rangle + (s-1)\,\langle \mu, \mu \rangle &= 0 \\ s - 1 &= -\langle x, \mu \rangle \\ s &= 1 - \langle x, \mu \rangle \end{aligned}$$

Thus one should perform PCA on $\left\{ x^p + (1 - \langle x^p, \mu \rangle)\, \mu \in \mathbb{R}^{2n}, p = 1, ..., m \right\}$, in order to obtain incorrect variations due to curvature of the unit-sized shapes, especially when the shapes are well spread around the mean.