

1. Which property must be satisfied for a set to be algebraic structure?

Ans- A non empty set A is called algebraic structure with respect to a binary operation if $a * b \in A$ for all values of $a, b \in A$.

2. Which property must be satisfied for a set to be a semi group?

Ans- An algebraic structure $(A, *)$ is called semi group if it follows the associative property.

$(a * b) * c = a * (b * c)$ for all values of a, b, c belongs to A. Here $*$ is a binary operation.

3. Which properties must be satisfied for a set to be a monoid?

Ans- If an algebraic structure satisfies the associativity and if there exist an identity element (e). That is $a * e = a$.

For example $a * 1 = a$. and $a + 0 = a$

4. Which property must be satisfied for a set to be a group?

Ans- $(A, *)$ is a group if

- A is closed.
- A is associative.
- There exists an identity element (e).
- There must be inverse of each element that is $a * a^{-1} = e$.

5. Which property must be satisfied for a set to be an abelian group?

Ans- Set is an abelian group we must satisfy the following five properties that is Closure Property, Associative Property, Identity Property, Inverse Property, and Commutative Property.

6. If a set satisfies only Closure property it is called

Ans- Algebraic Structure.

7. If a set satisfies Closure and associative property it is called

Ans- Semi group.

8. If a set satisfies Closure, Associative and Identity property it is called

Ans- Monoid.

9. If a set satisfies Closure, Associative, Identity and inverse property it is called

Ans- Group.

10. Name the 5 properties for a set to be abelian group

Ans- Closure Property, Associative Property, Identity Property, Inverse Property, and Commutative Property.

11. What is identity property?

Ans- An identity property is a property that applies to a group of numbers in the form of a set. It is named identity property because when applied to a number, the number keeps its 'identity.'

12. What is inverse property?

Ans- Consider a non-empty set A, and a binary operation $*$ on A. Then the operation is the inverse property, if for each $a \in A$, there exists an element b in A such that $a * b$ (right inverse) = $b * a$ (left inverse) = e, where b is called an inverse of a and e is an identity element.

13. What is closure property?

Ans- In a set A if $a * b \in A$ for all values of $a, b \in A$. Then set A satisfies the closure property.

14. What is associative property?

Ans- If in a set A if $(a*b)*c=a*(b*c)$ for all values of a,b,c belongs to A. Then it satisfies the associativity property Here * is a binary operation.

15. A monoid is called a group if _____

Ans- A monoid $(B,*)$ is called Group if to each element there exists an element c such that $(a*c)=(c*a)=e$. Here e is called an identity element and c is defined as the inverse of the corresponding element.

16. A group $(M,*)$ is said to be abelian if _____

Ans- If it satisfies the commutative property that is $x*y=y*x$.

17. What is commutative property?

Ans- For a set A if $x*y=y*x$ then A satisfies the commutative property. Where x,y belongs to A.

18. A cyclic group is always _____

Ans- A cyclic group is always an abelian group.

19. $a*H=H*a$ relation holds if

Ans- H is **subgroup of an abelian group**.

20. $a*H$ is a set of which coset?

Ans- $(a * H)$ is the set of **a left coset of H in G**.

21. What is generator in cyclic group?

Ans- In a set A there is an **element g**, such that every other element of the group can be written as a power of g. This element g is the generator of the group.

22. What is cyclic group?

Ans- A group where there exist a generator is called a cyclic group.

23. What is ring?

Ans- An algebraic structure $(A,+,*)$ is said to be a ring if:

- $(A,+)$ should be an abelian group.
- $(A,*)$ should be monoid.
- $(*)$ should be distributive over first operation(+).

24. What is field?

Ans- An algebraic structure $(A,+,*)$ is said to be a field if:

$(A,+)$ should be abelian group.

$(A,*)-(e)$ is an abelian group.

25. What are the conditions for a group to be a ring?

Ans-

- $(A,+)$ should be an abelian group.
- $(A,*)$ should be monoid.
- $(*)$ should be distributive over first operation(+).

26. What is subgroup?

Ans- A subgroup is a subset of group elements of a group. that satisfies the four group requirements. It must therefore contain the identity element.

27. $eH=He=H$, True or false

Ans-

28. No of elements in coset are(order of subgroup)

Ans- Number of elements in coset is equal to number of elements in subgroup from which coset is generated.

29. What is order of element?

Ans- The order of an element in a group is the smallest positive power of the element which gives you the identity element.

30. What is the order of an identity element?

Ans- By definition, the order of the identity, **e**, is **one**, since $e^{-1} = e$.

31. What is Semi group?

Ans- An algebraic structure $(A, *)$ is called semi group if it follows the associative property.
 $(a*b)*c=a*(b*c)$ for all values of a, b, c belongs to A . Here $*$ is a binary operation.

32. What are the properties that are to be satisfied by a relation to be an abelian group?

Ans- Set is an abelian group we must satisfy the following five properties that is Closure Property, Associative Property, Identity Property, Inverse Property, and Commutative Property.

33. Is it compulsory for an abelian group to be reflexive or not ?

Ans- To satisfy the closure property abelian group must be reflexive.

34. Is a set of integer along with + operator i.e. $(\mathbb{Z}, +)$ satisfies all the properties required for an abelian group or not ?

Ans- Yes, it satisfies the all properties of abelian group.

35. When we call a ring as a ring with 0 divisor?

Ans- A ring R is said to be a ring with zero divisors if there exist a, b belongs to R such that $a \neq 0$ and $b \neq 0$ yet $a.b=0$

36. Define a Ring .

Ans- An algebraic structure $(A, +, *)$ is said to be a ring if:

- $(A, +)$ should be an abelian group.
- $(A, *)$ should be monoid.
- $(*)$ should be distributive over first operation $(+)$.

37. A non empty set is said to be an algebraic structure with respect to unary , binary , ternary which type of operation.

Ans- A non empty set A is called an algebraic structure w.r.t binary operation.

38. Which type of property does matrix multiplication holds?

Ans- Associative property of multiplication $\rightarrow A.(BC)=(AB).C$

Distributive property of multiplication $\rightarrow A(B+C)=AB+AC$ $(B+C)A=BA+CA$

Multiplicative identity property $\rightarrow IA=AI$

Multiplicative property of zero $\rightarrow OA=AO$

Dimension property: The product of an $m \times n$ matrix and an $n \times k$ matrix is an $m \times k$ matrix.

39. Is cyclic group always be an abelian group or not?

Ans- cyclic group is **an abelian group**.

40. Is every abelian group can be considered as a cyclic group?

Ans- **All cyclic groups are Abelian**, but an Abelian group is not necessarily cyclic.

41. Singular element is responsible to generate which type of group?

Ans- Singular element is responsible to generate cyclic group.

42. Which type of group is this :- $\{1, -i, i, 1\}$

Ans- Abelian group;

43. A function is define by $f(x)=2x$ and $f(x+y)=f(x)+f(y)$ is called .

Ans- Isomorphic.

44. How we can call a group i.e. onto itself as an isomorphic?

Ans- **An automorphism** is defined as an isomorphism of a group onto itself.

45. What we call a set which is representing all cosets?

Ans- A set of representatives of all the cosets is called **a transversal**.

46. If a group has order 8 then how many non isomorphic groups can be obtained through it .

Ans- Five.

47. What is the multiplicative identit of natural numbers?

Ans- 1

48. If X is an idempotent non singular matrix then X must be an :

Ans- Since X is idempotent, we have $X^2=X$. As X is nonsingular, it is **invertible**.

49. When the sum of the elements in each row of $N \times N$ matrix is 0 then the matrix is .

Ans- If the Sum of Entries in Each Row of a Matrix is Zero, then the Matrix is **Singular**.

50. A semi group S that has an identity is called as .

Ans- A semigroup with an identity is called **a monoid**.

51. A group is also a semigroup , but can a semigroup always be an abelian group?

Ans- No.

52. If an element a is said to be idempotent then it means.

Ans- An element ' a ' of a set S equipped with a binary operator \cdot is said to be idempotent under \cdot **if. $a \cdot a = a$** .

53. $a \cdot H = H \cdot a$ a relation holds if

Ans- H is subgroup of an abelian group.

54. A set of all non singular matrices form a group under multiplication . is it true or false.

Ans- The set of all matrices **doesn't form a group under multiplication**.

55. A relation $(34 \cdot 78) \cdot 57 = 57 \cdot (78 \cdot 34)$ is satisfying which property?

Ans- Associativity property.

56. If a group has total 65 elements and it has two sub groups with the order 14 and 30 then what is the order of the group formed from the intersection of both the subset ?

Ans- 5

57. $\text{GCD}(a,b) = \text{GCD}(b, a \bmod b)$, is it true or not .

Ans- True.

58. A Modular arithmetic $(a/b) = b (a^{-1})$ is true ?

Ans- False.

59. if $+abxab$, $aabaaa$, $bbabab$ are true the relation $(S, +, *)$ where $S = \{a, b\}$. then Can S be called as Ring.

Ans- Yes.

60. A set of all real number under the normal multiplication operator is not a group because

Ans- . A set of all real number under the normal multiplication operator is not a group because zero has no inverse.

61. The inverse of $-i$ in multiplicative group $(1, -1, i, -i)$ is :

Ans- The inverse of $-i$ is i .

62. if $(G, .)$ is a group such that $(ab)^2 = a^2b^2$ where a & b belongs to G , then G is an abelian group is it true or not .

Ans- True.

63. If the binary operation $*$ is defined on ordered pair of real numbers as $(a,b) * (c,d) = (ad + bc, bd)$ and is associative then $(1,2) * (3,5) * (3,4)$ is equals to :-

Ans-

$$\text{Sol: } (a,b) * (c,d) = (ad + bc, bd)$$

$$(1,2) * (3,5) = (1 \times 5 + 2 \times 3, 2 \times 5)$$

$$= (11, 10)$$

$$(11, 10) * (3,4) = (11 \times 4 + 10 \times 3, 10 \times 4)$$

$$= (44 + 30, 40)$$

$$= (74, 40)$$

1:57 / 1:58

64. An algebraic structure $(P, *)$ is a semi group or not .

Ans- An algebraic structure $(P, *)$ is called a **semigroup** if $a*(b*c) = (a*b)*c$ for all a, b, c belongs to S

65. When a monoid will become a group ?

Ans- A monoid will become a group when there must exist an inverse of each element.

66. if $(G, .)$ is a group such that $(ab)^{-1} = a^{-1}b^{-1}$ where a & b belongs to G , then G is an abelian group is it true or not .

Ans- True.

67. Matrix multiplication represents which type of property?

Ans- Associative property of multiplication $\rightarrow A.(BC)=(AB).C$

Distributive property of multiplication $\rightarrow A(B+C)=AB+AC$ $(B+C)A=BA+CA$

Multiplicative identity property $\rightarrow IA=AI$

Multiplicative property of zero $\rightarrow OA=AO$

Dimension property: The product of an $m \times n$ matrix and an $n \times k$ matrix is an $m \times k$ matrix.

68. Is $(1, i, -i, -1)$ set a cyclic group?

Ans- Yes.

69. If $A(1, 2, 3, 4)$, Let $\sim = \{(1, 2), (1, 3), (4, 2)\}$ then \sim is :-

Ans- Transitive

70. If $(G, +)$ is a group such that $(ab)^2 = a^2b^2$ where a & b belongs to G , then G is an abelian group is it true or not.

Ans- True

71. The inverse of $-i$ in additive group $(1, -1, i, -i)$ is i or not

Ans- True.

72. Is every group is a monoid statement if true or false ?

Ans- True

73. If $(G, +)$ is a group such that $(ab)^{-1} = a^{-1}b^{-1}$ where a & b belongs to G , then G is not an abelian group

Ans- False it is an abelian group.

74. If $(G, +)$ is a group such that $(ab)^5 = a^5b^5$ where a & b belongs to G , then G is an abelian group is it true or not.

Ans-

75. A set of all positive integers under the normal multiplication operator is a group.

Ans- The set of integers under multiplication **is not a group** it does not have the INVERSE PROPERTY.

76. A set of all positive integers under the normal multiplication operator for being a group what all properties it must satisfy.

Ans- $(A, *)$ is a group if

- A is closed.
- A is associative.
- There exists an identity element (e) .
- There must be inverse of each element that is $a^{-1}a = e$.

77. How many properties are to be satisfied by an abelian group? Name them.

Ans- Closure Property, Associative Property, Identity Property, Inverse Property, and Commutative Property.

78. If a group dissatisfies the distributive property then can it be considered as a ring?

Ans- No.

79. What all the properties a ring must satisfy to be a field?

Ans- An algebraic structure $(A, +, *)$ is said to be a field if:
 $(A, +)$ should be abelian group.
 $(A, *) - (e)$ is an abelian group.

80. If n is the smallest positive integer that satisfies $a^n = e$. then what is n here ?
 Ans- n is order.

81. Is set of all integer over the $+$ operator is considered as a monoid.
 Ans- True.

82. Given set $(2, 4, 8, 16)$ what will be the generator element?
 Ans-

83. $H * a$ is a set of which coset?
 Ans- Right coset.

84. What is H in an abelian group ?
 Ans- Subgroup.

85. Define Generator element with an example.

Ans- In a set A there is an **element g** , such that every other element of the group can be written as a power of g . This element g is the generator of the group.
 For example let $A = \{0, 1, 2, 3, 4, 5\}$

$$\left. \begin{aligned} 5^1 \bmod 6 &= 5 + 0 \bmod 6 = 5 \\ 5^2 \bmod 6 &= 5 + 5 \bmod 6 = 4 \\ 5^3 \bmod 6 &= 4 + 5 \bmod 6 = 3 \\ 5^4 \bmod 6 &= 3 + 5 \bmod 6 = 2 \\ 5^5 \bmod 6 &= 2 + 5 \bmod 6 = 1 \\ 5^6 \bmod 6 &= 1 + 5 \bmod 6 = 0 \end{aligned} \right\} \begin{array}{l} \text{Generator} \\ \text{All the elements} \\ \text{of set are} \\ \text{generated} \end{array}$$

The order of 5 is 6

$$\begin{aligned} 1^1 \bmod 6 &= 1 \\ 1^2 \bmod 6 &= 2 \\ 1^3 \bmod 6 &= 3 \\ 1^4 \bmod 6 &= 4 \\ 1^5 \bmod 6 &= 5 \\ 1^6 \bmod 6 &= 0 \end{aligned}$$

1 & 5
are the generators

86. Is a set of \mathbb{Z} and $\mathbb{Z} \bmod n$ is a cyclic group or not ?
 Ans-

87. If $a^2 = a \cdot a = a$ is true, then what a represents here?

Ans- ' a ' is an idempotent.

88. Is a set of natural numbers with $+$ operator can be an abelian group?

Ans- The set of natural numbers under **addition is not an abelian group**, because it does not satisfy all of the group PROPERTIES: it does not have the IDENTITY PROPERTY.

89. Is a set of natural numbers with $+$ operator can be an algebraic structure or not?

Ans- Yes it can be considered as an algebraic structure.

90. Is a set of real number satisfying the associative property ?

Ans- Yes

91. Set $S(2,4,6,8,\dots)$ set of even number along with the operator $+$ can be a group ?

Ans- Yes it can be considered as group.

92. Ring $(R, +, \cdot)$ must satisfy the associative property, state whether it is true or false

Ans- False.

93. To be a ring is it mandatory for the relation to be an abelian group?

Ans- True

94. Define idempotent element .

Ans- An element x of a group G is called idempotent if $x \cdot x = x$.

95. What do you understand by term closure?

Ans- if $a \cdot b \in A$ for all values of $a, b \in A$. Then A satisfies the closure property.

96. Define Inverse property

Ans- Consider a non-empty set A , and a binary operation $*$ on A . Then the operation is the inverse property, if for each $a \in A$, there exists an element b in A such that $a \cdot b$ (right inverse) $= b \cdot a$ (left inverse) $= e$, where b is called an inverse of a and e is an identity element.

97. Why a set of odd number along with $+$ operator cannot form an abelian group ?

Ans- Because set of odd number does not satisfy the property of closure that is why set of odd numbers along with $+$ operator cannot form an abelian group.

98. If I be the Identity element and A be any element of the given set and $AI=IA$, then what is AI ?

Ans-

99. When a set is considered as a monoid?

Ans- - If an algebraic structure satisfies the associativity and if there exist an identity element (e). That is $a \cdot e = a$.

For example $a \cdot 1 = a$. and $a + 0 = a$

100. If in a given group its element's product is also included then what does this statement represents?

Ans-

101. What do you understand by the commutative property?

Ans- For a set A if $x \cdot y = y \cdot x$ then A satisfies the commutative property. Where x, y belongs to A .

102. Why a Set of Natural numbers along with + operator cannot be a group ?

Ans- The set of natural numbers under addition is not a group, **because it does not satisfy all of the group PROPERTIES**: it does not have the IDENTITY PROPERTY

103. Why the set of integers over the operator * is not a group ?

Ans- The set of integers under multiplication is not a group, **because it does not satisfy all of the group PROPERTIES**: it does not have the INVERSE PROPERTY

104. Among the various number system who are eligible to be a valid Abelian group?

Ans- Set of real number with + operator.

105. A set of integer , real number and even number along with + operator are eligible to form a valid abelian group ?

Ans- Only the set of real number along with + operator are eligible to form a valid abelian group.

106. A set of Matrix along with * operator is fail to satisfy the inverse property . what does it means ?

Ans- It means that a set of matrix can not be a group.

107. There exist two matrices of order $N \times N$ i.e. A and B if we are taking $A+B$ then the resultant matrix is a valid group .

Ans-

108. When a set will considered as a valid group?

Ans- $(A, *)$ is a group if

- A is closed.
- A is associative.
- There exists an identity element(e).
- There must be inverse of each element that is a^{-1} .

109. if $AA^{-1} = A^{-1}A = I$ where A is the element of a set and A^{-1} is its inverse, then what does it represents?

Ans- It represents that set satisfies the inverse property.

110. If a set S (1,2,3,4,5,6) is satisfying the associative property along with * operator and 30 will be the product of a & b . What is the value of a & b?

Ans- If value of a is 5 then value of b is 6.

If value of a is 6 then value of b is 5.

111. If a set S (1,2,3,4,5,6) is satisfying the associative property along with * operator. What we will call (3,4)?

Ans-

112. A set S(2,4,6,8....) with + operator satisfying the condition $(2+4)+6 = 2+(4+6)$, then (2,4,6,8) is considered as :-

Ans-

113. A set S(2,4,6,8.....) with * operator satisfying the condition $(2*4)*6 = 2*(4*6)$, then (2,4,6,8) is not considered as semi group why?

Ans-

114. What is Commutative Property is also called?

Ans- This property is also called **the order property of multiplication**. Commutative property only applies to multiplication and addition.

115. What is grouping property of multiplication?

Ans- Associative property.

116. What is associative and distributive properties?

Ans- The **associative property** states that when adding or multiplying, the grouping symbols can be rearranged and it will not affect the result. This is stated as $(a+b)+c=a+(b+c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

The **distributive property** is a multiplication technique that involves multiplying a number by all of the separate addends of another number. This is stated as $a(b+c)=ab+ac$ and $a(b \cdot c)=ab \cdot ac$.

117. Properties of group under group theory.

Ans- $(A, *)$ is a group if

- A is closed.
- A is associative.
- There exists an identity element (e) .
- There must be inverse of each element that is $a \cdot a^{-1} = e$.

118. Why group theory is important?

Ans- group theory is **the study of symmetry**. When we are dealing with an object that appears symmetric, group theory can help with the analysis.

119. Where group theory is used in real life?

Ans- Perhaps a most prominent example of an application of group theory (a la symmetry study) in real life is **for the study of crystals**.

120. How Matrices are related to group theory?

Ans- In mathematics, a matrix group is a group G **consisting of invertible matrices over a specified field K , with the operation of matrix multiplication**.