

Oral Examination Dec - 2021
Medicaps University

DATE

Subject : Discrete Mathematics

Unit - 1

① Cartesian Product of set A and B is defined as the set of all ordered pairs (x, y) such that $x \in A$ and $y \in B$.

② Example: $A = \{1, 2\}$ $B = \{a, b\}$
 $A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

③ Cartesian Product of a set with itself?

If $A = B$, we can denote the Cartesian product of A with itself as $A \times A = A^2$, also known as Cartesian Square.

④ Size of Cartesian product of two sets :

$$\underbrace{|A \times B|}_{\text{Size of } A \times B} = \underbrace{|A|}_{\text{Size of } A} \times \underbrace{|B|}_{\text{Size of } B}$$

⑤ Cartesian product of two sets A and B, denoted $A \times B$ is set of all ordered pairs where a is in A,

and b is in B .

Ans \rightarrow Yes

6. Relation: A collection of ordered pair containing one object from each set.

Example: $\{(2, -1), (3, 3), (7, 5), (8, 4)\}$

7. Empty Relation: A relation in which there is no relation between any element of a set.

In other words, a relation R on set A is called empty relation, if no element of A is related to any element of A .

Set $A = \{1, 2, 3, 4\}$

$R = \{(a, b) : a + b = 10\}$ defined on A .

8. Symmetric Relation: In a symmetric Relation, if $a = b$ is true then $b = a$ is also true.

In other words, a R Relation is symmetric only if $(b, a) \in R$ is true when $(a, b) \in R$.

Set $B = \{1, 2\}$

$R = \{(1, 2), (2, 1)\}$

9. how many symmetric relations on a set with n elements

Ans. The number of symmetric relations on a set with the 'n' number of elements is given by $N = 2^{n(n+1)/2}$, where N is the

number of symmetric relations and n is the number of elements in the set.

10 . What is antisymmetric relation example?

Ans. In set theory, the relation R is said to be antisymmetric on a set A , if xRy and yRx hold when $x = y$. Or it can be defined as, relation R is antisymmetric if either $(x,y) \notin R$ or $(y,x) \notin R$ whenever $x \neq y$.

11.How many reflexive relations are possible with n elements?

Ans. $2^{(n^2-n)}$

12. Write the smallest reflexive relation on set $\{1, 2, 3, 4\}$.

Ans. Smallest Reflexive Set= $\{(1,1),(2,2),(3,3),(4,4)\}$

13. what is the cardinality of smallest reflexive relation if no. of elements in set is n .

Ans. CARDINALITY will be n .

Example : set $A = \{1, 2, 3, 4\}$.

$$|A| = 4$$

Smallest Reflexive Set= $\{(1,1),(2,2),(3,3),(4,4)\}$

$$|\text{Smallest Reflexive Set}| = 4$$

14.what is the cardinality of largest reflexive relation if no. of elements in Set is n .

Ans. CARDINALITY will be 2^{n^2} (maybe)

15 | 15 How many reflexive relations are there on a set with n elements?

Ans. $2^{(n^2-n)}$

16 | 16 what is the cardinality of smallest symmetric relation if no. of elements in set is n.

Ans. CARDINALITY will be $2n$ (maybe)

17 | 17 what is the cardinality of largest symmetric relation if no. of elements in set is n.

Ans. CARDINALITY will be

18 | 18 How many symmetric relations are there on a set with n elements?

Ans. The number of symmetric relations on a set with the 'n' number of elements is given by **$N = 2^{n(n+1)/2}$** , where N is the number of symmetric relations and n is the number of elements in the set.

19 | 19 what is the difference between reflexive relation and symmetric.

Ans. The Reflexive Property states that for every real number x , $x=x$. The Symmetric Property states that for all real numbers x and y , if $x=y$, then $y=x$.

20 | 20 what is transitive relation?

Ans. A relation R on a set X is transitive if, for all elements a, b, c in X , whenever R relates a to b and b to c , then R also relates a to c .

21 | 21 Give an example of a relation which is reflexive and symmetric but not Transitive.

Ans. If $a, b, c \in A$ such that $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$ so this is called a transitive relation. So as we see that, $(4,6), (6,8) \in R$ for all $a, b, c \in A$ Hence R is a reflexive and symmetric but not transitive.

22 | 22 What is meant by transitive relation?

Ans. A relation R on a set X is transitive if, for all elements a, b, c in X , whenever R relates a to b and b to c , then R also relates a to c .

23 | 23 What is an example of the transitive property?

Ans. Example of a transitive law is “If a is equal to b and b is equal to c, then a is equal to c.”

24 | 24 What is Irreflexive relation with example?

Ans. A relation R on set A is said to be irreflexive if $(a, a) \notin R$ for every $a \in A$.

Example : A relation R on set A is said to be irreflexive if $(a, a) \notin R$ for every $a \in A$. Example: Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 2), (3, 1), (1, 3)\}$.

25 | 25 What is the difference between reflexive relation and irreflexive Relation?

Ans. Reflexive: every element is related to itself.

Irreflexive: no element is related to itself.

26 | 26 What is the cardinality smallest reflexive relation?

Ans. We have set $\{a, b, c, d\}$ then the smallest reflexive relation on this set is $\{(a, a), (b, b), (c, c), (d, d)\}$.

CARDINALITY will be n .

27 | 27 What is Irreflexive relation with example?

Ans. Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 2), (3, 1), (1, 3)\}$.

28 | 28 What is the cardinality largest reflexive relation?

Ans.

29 | 29 How many irreflexive relations are possible with n elements?

Ans. Irreflexive Relations on a set with n elements : $2^{n(n-1)}$.

30 | 30 How many antisymmetric relations are possible with n elements?

Ans. Anti-Symmetric Relations on a set with n elements: $2^n \cdot 3^{n(n-1)/2}$

31 | 31 What is the cardinality smallest antisymmetric relation?

Ans.

32 | 32 What is the cardinality largest antisymmetric relation?

Ans.

33 | 33 What is asymmetric relation with example?

Ans. A relation is asymmetric if and only if it is both antisymmetric and irreflexive.

Example: the relation R on a set A is asymmetric if and only if, $(x,y) \in R \implies (y,x) \notin R$ For example: If R is a relation on set $A = \{12,6\}$ then $\{12,6\} \in R$ implies $12 > 6$, but $\{6,12\} \notin R$, since 6 is not greater than 12.

34 | 34 What is asymmetric and antisymmetric relation?

Ans. Antisymmetric means that the only way for both aRb and bRa to hold is if $a = b$. It can be reflexive, but it can't be symmetric for two distinct elements.

Asymmetric is the same except it also can't be reflexive. An asymmetric relation never has both aRb and bRa , even if $a = b$.

35 | 35 How many asymmetric relations are there in a set with n elements?

Ans. Number of Asymmetric Relations on a set with n elements : $3n(n-1)/2$.

36 | 36 For a given set, relation is defined as (a,b) in R then (b,a) is also in R , Then R should be ?

Ans. R should be symmetric.

37 | 37 For every relation R on some set, if (t,t) is not in, then R cannot be

Ans. Reflexive

38 | 38 Product of first n natural numbers

Ans. Product = $n!$

39 | 39 What is the Cartesian product of $A = \{1, 2\}$ and $B = \{a, b\}$?

Ans. $A \times B = \{(1,a), (1,b), (2,a), (2,b)\}$

40 | 40 A partial ordered relation is transitive, reflexive and

Ans. Partial order relation is a homogeneous relation that is transitive and antisymmetric.

41 | 41 The set of positive integers is _____ .

Ans. Natural Number/ Never ending/ Infinte

42 | 42 Power set of empty set has exactly _____ subset.

Ans. The empty number of elements is 0. So the number of subset in the power set of an empty set is $2^0=1$.

43 | 43 The set O of odd positive integers less than 10 can be expressed by

Ans. $O=\{1,3,5,7,9\}$

44 | 44 The members of the set $S = \{x \mid x \text{ is the square of an integer and } x < 100\}$ is

Ans. $S=\{1,4,9,16,25,36,49,64,81\}$

45 | 45 If a set has n elements, how many relations are there from A to A.

Ans. A contains n^2 elements. A relation is just a subset of $A \times A$, and so there are 2^{n^2} relations on A. So a 3-element set has $2^9 = 512$ possible relations.

46 | 46 If A has m elements and B has n elements. How many relations are There from A to B and vice versa?

Ans. $m \times n$ elements

47 | 47 If a set $A = \{1, 2\}$. Determine all relations from A to A.

Ans. There are $2^2 = 4$ elements i.e., $\{(1, 2), (2, 1), (1, 1), (2, 2)\}$ in $A \times A$. So, there are $2^4 = 16$ relations from A to A .

They are : $\{ \{(1, 2), (2, 1), (1, 1), (2, 2)\}, \{(1, 2), (2, 1)\}, \{(1, 2), (1, 1)\}, \{(1, 2), (2, 2)\}, \{(2, 1), (1, 1)\}, \{(2, 1), (2, 2)\}, \{(1, 1), (2, 2)\}, \{(1, 2), (2, 1), (1, 1)\}, \{(1, 2), (1, 1), (2, 2)\}, \{(2, 1), (1, 1), (2, 2)\}, \{(1, 2), (2, 1), (2, 2)\}, \{(1, 2), (2, 1)\} \}$

48 | 48 Let $A = \{1, 2, 3, 4\}$ $B = \{a, b, c, d\}$

$R = \{(1, a), (1, b), (1, c), (2, b), (2, c), (2, d)\}$. Find domain and Range.?

Ans. Domain of $R = \{1, 2\}$

Range of $R = \{a, b, c, d\}$

49 | 49 Consider the relation R from X to Y

$X = \{1, 2, 3\}$

$Y = \{8, 9\}$

$R = \{(1, 8), (2, 8), (1, 9), (3, 9)\}$

Find the complement relation of R .

Ans. Compliment of a relation will contain all the pairs where pair do not belong to relation but belongs to Cartesian product.

$R' = P * Q - R$

$X * Y = \{(1, 8), (1, 9), (2, 8), (2, 9), (3, 8), (3, 9)\}$

Complement: $R' = \{(2,9), (3,8)\}$

50 | 50 Let $X = \{4, 5, 6\}$, $Y = \{a, b, c\}$ and $Z = \{l, m, n\}$. Consider the relation R_1 from X to Y and R_2 from Y to Z .

$$R_1 = \{(4, a), (4, b), (5, c), (6, a), (6, c)\}$$

$R_2 = \{(a, l), (a, n), (b, l), (b, m), (c, l), (c, m), (c, n)\}$ Find the Composition of relation $R_1 \circ R_2$.

Ans. $X = \{4, 5, 6\}$
 $Y = \{a, b, c\}$
 $Z = \{l, m, n\}$

$R_1 = \{(4, a), (4, b), (5, c), (6, a), (6, c)\} : X \rightarrow Y$
 $R_2 = \{(a, l), (a, n), (b, l), (b, m), (c, l), (c, m), (c, n)\}$

Composition of $R_1 \& R_2$:

$R_1 \circ R_2$

$R_1 \circ R_2 \Rightarrow \{(4, l), (4, n), (5, l), (5, m), (5, n), (6, l), (6, m), (6, n)\}$

51 | 51 Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 2), (3, 1), (1, 3)\}$. Is the relation R Reflexive or irreflexive?

Ans. Irreflexive.

52 | 52 Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2)\}$. Is the relation R Antisymmetric?

Ans. Yes , because $1=1$ and $2=2$.

53 | 53 Let $A = \{4, 5, 6\}$ and $R = \{(4, 4), (4, 5), (5, 4), (5, 6), (4, 6)\}$. Is the Relation R antisymmetric?

Ans. No, Asymmetric Because $(6,5)$ and $(6,4)$ is not present.

54 | 54 Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$. Is the relation Transitive?

Ans. Yes

55 | 55 Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 3), (2, 2), (2, 4), (3,1), (3, 3), (4, 2), (4, 4)\}$.

Show that R is an Equivalence Relation.

Ans. $A = \{1, 2, 3, 4\}$

$$R = \{(1, 1), (1, 3), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$$

for R to be Equivalence

1) It should be Reflexive

as R contain $(1, 1)$ $(2, 2)$ $(3, 3)$ $(4, 4)$ i.e. $(a, a) \in R$

$\therefore R$ is Reflexive.

2) It should be Transitive

as all $(a, b) \in R$ have $(b, a) \in R$ i.e. $(1, 3) \in R \Rightarrow$
 $(3, 1) \in R$ and $(2, 4) \in R \Rightarrow (4, 2) \in R$.

$\therefore R$ is transitive

3) It should be Symmetric

as all $(a, b) \in R$, $(b, c) \in R \Rightarrow (a, c) \in R$ i.e.

$$(1, 1) \in R \quad (1, 3) \in R \Rightarrow (1, 3) \in R$$

$$(2, 4) \in R \quad (4, 2) \in R \Rightarrow (2, 2) \in R$$

$$(1, 3) \in R \quad (3, 1) \in R \Rightarrow (1, 1) \in R$$

--- and so on.

$\therefore R$ is symmetric

Hence, R is Equivalence Relation.

56 | 56. $A = \{1, 2, 3\}$

$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$ find $R_1 \cap R_2$.

Ans. $R_1 \cap R_2 = \{(1, 1), (2, 2), (3, 3)\}$

57 | 57 $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (3, 2)\}$ find R^{-1} .

Ans. $R^{-1} = \{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (2, 3)\}$

58 | 58 Show that the relation 'Divides' defined on N is a partial order relation.

Ans. **Reflexive:** We have a divides a , $\forall a \in N$. Therefore, relation 'Divides' is reflexive.

Antisymmetric: Let $a, b, c \in N$, such that a divides b . It implies b divides a iff $a = b$. So, the relation is antisymmetric.

Transitive: Let $a, b, c \in N$, such that a divides b and b divides c .

Then a divides c . Hence the relation is transitive. Thus, the relation being reflexive, antisymmetric and transitive, the relation 'divides' is a **partial order relation**.

59 | 59 Find the Domain, Co-Domain, and Range of function.

Domain: Set of first element of the ordered pair which belongs to R.

Range : Set of all second elements of ordered pairs.

Codomain: set of all possible elements of ordered pairs.

Ex: Let $A = \{1, 2\}$ $B = \{a, b, c, d, e, f, g, h, i, j\}$

$R = \{(1, a), (1, b), (1, c), (2, b), (2, c), (2, d)\}$.

Domain of $R = \{1, 2\}$

Range of $R = \{a, b, c, d\}$

Co domain of $R = \{a, b, c, d, e, f, g, h, i, j\}$

60 | 60 A function is said to be _____, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f .

Ans. One-one

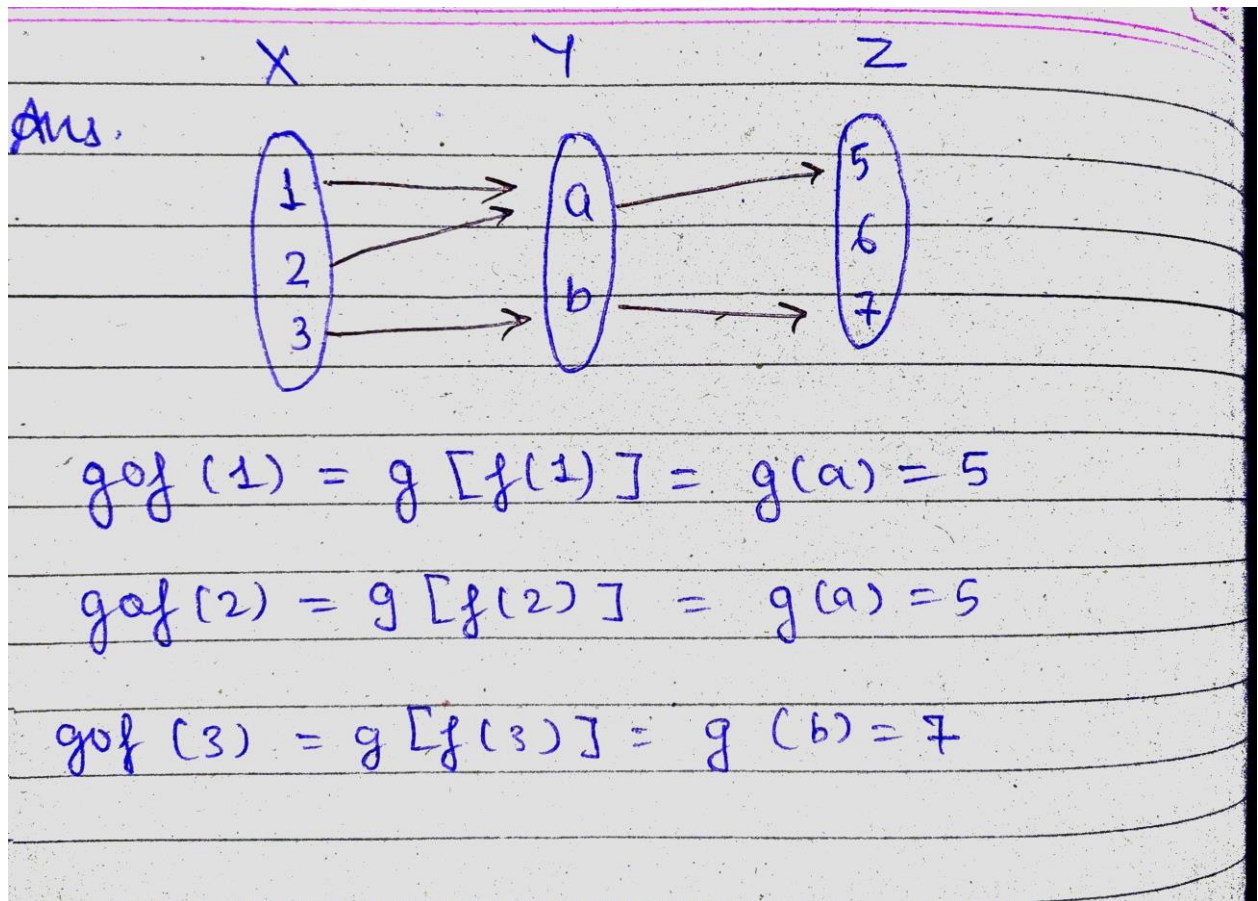
61 | 61 If a set A has n elements, how many functions are there from A to A .

Ans. The number of functions that can be defined from a set into another set can be found by:

(Number of elements in co-domain) $^{\text{Number of elements in domain}}$.

No. Of Function = n^n

62 | 62. Let $X = \{x, y, z, k\}$ and $Y = \{1, 2, 3, 4\}$. Give reasons if it is not. Find range if it is a function.



Ans. What is function 🧐.

63 | 63. Let $X = \{1, 2, 3\}$ $Y = \{a, b\}$ $Z = \{5, 6, 7\}$. Consider the function $f = \{(1, a), (2, a), (3, b)\}$ and $g = \{(a, 5), (b, 7)\}$ as in figure. Find the composition of $g \circ f$.

Ans.

64 | 64. Consider f, g and h , all functions on the integers, by $f(n) = n^2$, $g(n) = n + 1$ and $h(n) = n - 1$.

Determine (i) $h \circ f \circ g$

Ans.

Handwritten solution for part (i) of a composition problem. The steps are as follows:

$$\begin{aligned} \text{Ans (i) } h \circ f \circ g(n) & \\ \Rightarrow h \circ f[g(n)] & \\ \Rightarrow h \circ f(n+1) & \\ \Rightarrow h[f(n+1)] & \\ \Rightarrow h[(n+1)^2] & \\ \Rightarrow (n+1)^2 - 1 & \\ \Rightarrow n^2 + 1 + 2n - 1 \Rightarrow n^2 + 2n \Rightarrow n(n+2) & \end{aligned}$$

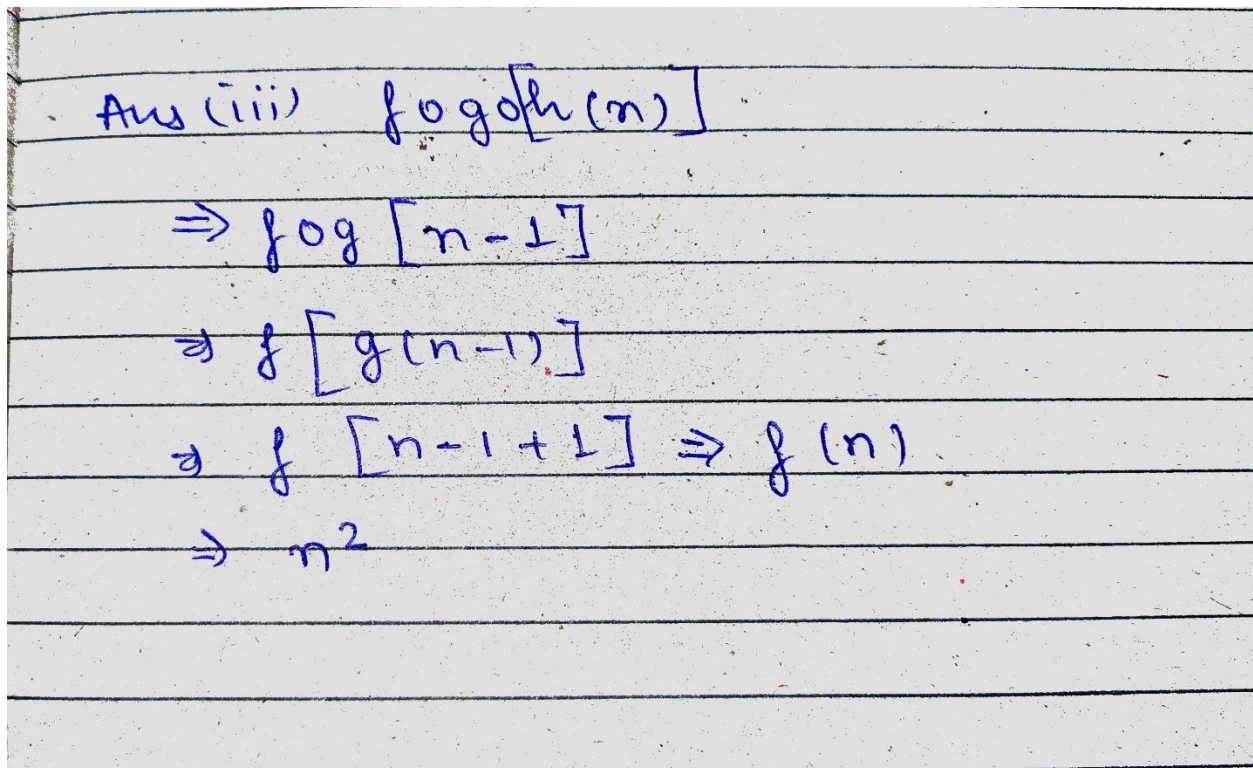
65 | 65. Consider f , g and h , all functions on the integers, by $f(n) = n^2$, $g(n) = n + 1$ and $h(n) = n - 1$. Determine (ii) $g \circ f \circ h$

Ans.

Handwritten solution for part (ii) of a composition problem. The steps are as follows:

$$\begin{aligned} \text{Ans (ii) } g \circ f \circ h(n) & \\ \Rightarrow g \circ f(n-1) & \\ \Rightarrow g[f(n-1)] & \\ \Rightarrow g[(n-1)^2] & \\ \Rightarrow (n-1)^2 + 1 & \\ \Rightarrow n^2 + 1 - 2n + 1 & \\ \Rightarrow n^2 + 2 - 2n. \text{ or } n^2 - 2n + 2. & \end{aligned}$$

66 | 66 Consider f , g and h , all functions on the integers, by $f(n) = n^2$, $g(n) = n + 1$ and $h(n) = n - 1$. Determine (iii) $f \circ g \circ h$.



Handwritten solution for problem 66(iii) on lined paper:

$$\begin{aligned} \text{Ans (iii)} \quad & f \circ g \circ h(n) \\ \Rightarrow & f \circ g [n-1] \\ \Rightarrow & f [g(n-1)] \\ \Rightarrow & f [n-1+1] \Rightarrow f(n) \\ \Rightarrow & n^2 \end{aligned}$$

67 | 67. Find the minimum number of students in a class to be sure that three of them are born in the same month.

Ans. The generalized Pigeonhole principle:

Assume N objects are placed into k boxes. $N=12, k+1=3; k=2$.

So, there is at least one box containing at least N/k objects.

To use the pigeonhole principle, first, find boxes and objects.

Suppose that for each month, we have a box that contains persons who were born in that month. The number of boxes is

12, by the generalized pigeonhole principle, to have at least 3 ($= N/12$) students at the same box, the total number of the students must be at least $N = 2*12 + 1 = 25.$ ($k.n+1$)

So, the minimum number of students required= **25**.

68 | 68. Show that at least two people must have their birthday in the same month if 13 people are assembled in a room.

Ans. Suppose that for each month, we have a box that contains persons who were born in that month. The number of boxes is 12, by the generalized pigeonhole principle, to have at least 2 ($= N/12$) students at the same box, the total number of the students must be at least $N = 12*1 + 1 = 13$.

So, the minimum number of students required= **13**.

69 | 69 The Cartesian Product $B \times A$ is equal to the Cartesian product $A \times B$. Is it True or False?

Ans. False.

70 | 70 What is Commutative, Associative and Distributive laws in maths?

Ans. **Commutative law**, in mathematics, either of two laws relating to number operations of addition and multiplication, stated symbolically: $a + b = b + a$ and $ab = ba$.

Associative Laws ,in mathematics, either of two laws relating to number operations of addition and multiplication, stated symbolically: $(a + b) + c = a + (b + c)$, $(a \times b) \times c = a \times (b \times c)$.

Distributive Law: $a \times (b + c) = a \times b + a \times c$.

71 | 71. In a class of 120 students numbered 1 to 120, all even numbered students opt for Physics, those whose numbers are divisible by 5 opt for Chemistry and those whose numbers are divisible by 7 opt for Math. How many opt for none of the three subjects?

Ans.

Ans.

$$P = \{2, 4, 6, \dots, 118, 120\} \quad n(P) = 60$$
$$C = \{5, 10, 15, \dots, 115, 120\} \quad n(C) = 24$$
$$M = \{7, 14, 21, \dots, 112, 119\} \quad n(M) = 17$$
$$n(P \cap C) = 12 \rightarrow \text{LCM } 2 \& 5 = 10 \rightarrow \text{multiple of } 10 (1-12)$$
$$n(C \cap M) = 3 \rightarrow \text{LCM } 5 \& 7 = 35 \rightarrow \text{multiples of } 35 (1-20)$$
$$n(P \cap M) = 8 \rightarrow (\text{similarly})$$
$$\text{and } n(P \cap C \cap M) = 1$$
$$\therefore n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(P \cap M) + n(P \cap C \cap M)$$
$$1 = 60 + 24 + 17 - (12 + 3 + 8) + x$$
$$[x = 79]$$

Who opt none = $120 - 79 = 41$ students.

Number of students who opt for none of the subjects
 $= 120 - 79 = 41$.

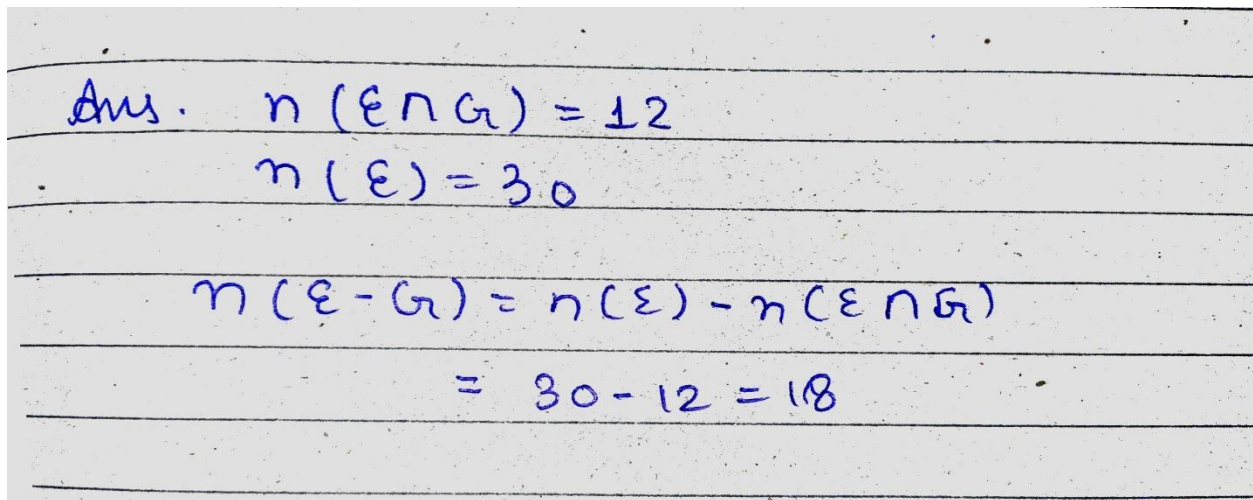
72 | 72 .In a class of 40 students, 12 enrolled for both English and German. 22

enrolled for German. If the students of the class enrolled for at least

one of the two subjects, then how many students enrolled for only

English and not German?

Ans.



Handwritten solution on lined paper:

$$\begin{aligned}\text{Ans. } n(E \cap G) &= 12 \\ n(E) &= 30 \\ n(E - G) &= n(E) - n(E \cap G) \\ &= 30 - 12 = 18\end{aligned}$$

73 | 73. In a class 40% of the students enrolled for Math and 70% enrolled for Economics. If 15% of the students enrolled for both Math and Economics, what % of the students of the class did not enroll for either of the two subjects?

Ans $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, where $(A \cup B)$ represents the set of people who have enrolled for at least one of the two subjects Math or Economics and $(A \cap B)$ represents the set of people who have enrolled for both the subjects Math and Economics.

$$N(A \cup B) = 40 + 70 - 15 = 95\%$$

That is 95% of the students have enrolled for at least one of the two subjects Math or Economics.

Therefore, the balance $(100 - 95)\% = 5\%$ of the students have not enrolled for either of the two subjects.

74 | 74 what is equivalence set.?

Ans. Two sets A and B are said to be equivalent if they have the same cardinality i.e. $n(A) = n(B)$.

75 | 75. what is equal set.?

Ans. Two sets A and B can be equal only on the condition that each element of set A is also the element of set B. Also, if two sets happen to be the subsets of each other, then they are stated to be equal sets.

76 | 76 what is the difference between subset and proper subset.?

Ans. **Subset of a set A can be equal to set A but a proper subset of a set A can never be equal to set A.** A proper subset of a set A is a subset of A that cannot be equal to A. In other words, if B is a proper subset of A, then all elements of B are in A but A contains at least one element that is not in B.

77 | 77 what is power subset.

Ans. A power set includes all the subsets of a given set including the empty set.

78 | 78 what is Idempotent laws on a set.?

Ans. Intersection and union of any set with itself revert the same set.

Idempotent Laws (a) $A \cup A = A$ (b) $A \cap A = A$

79 | 79 what is Distributive law .?

Ans. For all sets A,B and C, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

80 | 80. Let U be the set of positive integer not exceeding 1000. Then $|U| = 1000$ and find S where S is the set of such integer which is not divisible by 3, 5 or 7?

Ans. Let A be the subset of integer which is divisible by 3

Let B be the subset of integer which is divisible by 5

Let C be the subset of integer which is divisible by 7

Then $S = A^c \cap B^c \cap C^c$ since each element of S is not divisible by 3, 5, or 7.

By Integer division,

$$|A| = 1000/3 = 333$$

$$|B| = 1000/5 = 200$$

$$|C| = 1000/7 = 142$$

$$|A \cap B| = 1000/15 = 66$$

$$|B \cap C| = 1000/21 = 47$$

$$|C \cap A| = 1000/35 = 28$$

$$|A \cap B \cap C| = 1000/105 = 9$$

Thus by Inclusion-Exclusion Principle

$$|S| = 1000 - (333 + 200 + 142) + (66 + 47 + 28) - 9$$

$$|S| = 1000 - 675 + 141 - 9 = 457$$

81 | 81 What is the cardinality of the set of odd positive integers less than 10?

Ans. Set S of odd positive an odd integer less than 10 is {1, 3, 5, 7, 9}. Then, Cardinality of set S = |S| which is 5.

82 | 82. Which of the following two sets are equal?

- a) $A = \{1, 2\}$ and $B = \{1\}$
- b) $A = \{1, 2\}$ and $B = \{1, 2, 3\}$
- c) $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$
- d) $A = \{1, 2, 4\}$ and $B = \{1, 2, 3\}$

Ans. C) $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$

83 | 83 What is the Cardinality of the Power set of the set $\{0, 1, 2\}$.

Ans. 2^n ; $2^3=8$.

84 | 84 Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

Ans. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$$n(A \cap B) = 20 + 28 - 36 = 12.$$

85 | 85. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and Each person likes at least one of the two drinks. How many like both Coffee and tea?

Ans. Let A = Set of people who like cold drink

And, B = Set of people who like hot drink

Given,

$$N(A \cup B) = 60$$

$$N(A) = 27 \text{ and } n(B) = 42$$

$$N(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$N(A \cap B) = 27 + 42 - 60 = 9$$

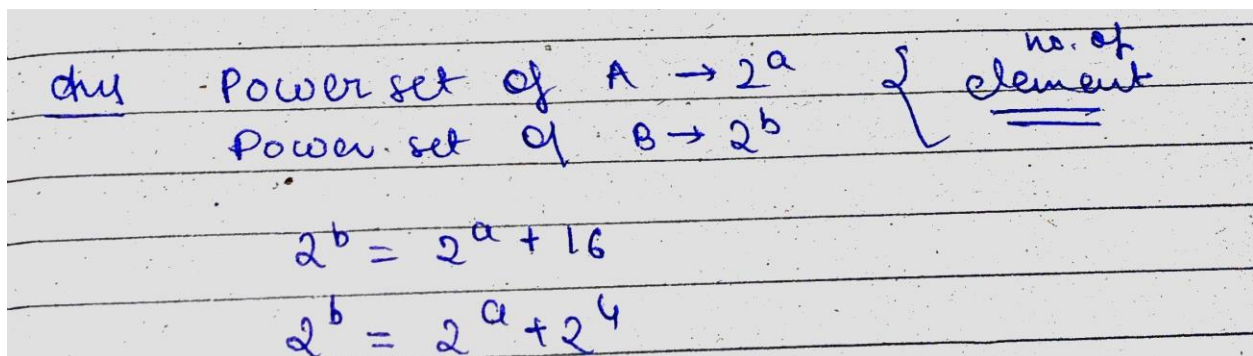
Therefore, 9 people like both cold drink and hot drink.

86 | 86. If A has 4 elements B has 8 elements then the minimum and maximum Number of elements in $A \cup B$ are _____

Ans. Minimum: 4 (Minimum would be when 4 elements are same as in 8, maximum would be when all are distinct.)

Maximum: 12.

87 | 87. Two sets A and B contains a and b elements respectively. If power set Of A contains 16 more elements than that of B, value of 'b' and 'a' are



Handwritten solution for problem 87:

thus Power set of $A \rightarrow 2^a$
Power set of $B \rightarrow 2^b$

no. of element

$$2^b = 2^a + 16$$
$$2^b = 2^a + 2^4$$

Therefore, $a=4$ and $b=5$.

88 | 88 If $n(A)=20$ and $n(B)=30$ and $n(A \cup B) = 40$ then $n(A \cap B)$ is?

Ans. $N(A \cap B) = n(A) + n(B) - n(A \cup B)$

$$N(A \cap B) = 20 + 30 - 40 = 20.$$

89 | 89. Let the students who like table tennis be 12, the ones who like lawn Tennis 10, those who like only table tennis are 6, then number of Students who like only lawn tennis are, assuming there are total of 16 Students.

90 | 90 Let the set A is $\{1, 2, 3\}$ and B is $\{2, 3, 4\}$. Then the set $A - B$ is?

$$A - B = \{1\}.$$

91 | 91 What Is Discrete Mathematics?

Ans. Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous.

92 | 92 how many Categories Of Mathematics?

Ans. Modern mathematics can be divided into three main branches: continuous mathematics, algebra, and discrete mathematics.

93 | 93 what are difference between continuous and discrete mathematics.

Ans. Continuous mathematics deals with real numbers.

Discrete math deals with sets of items, numbers.

94 | 94 . What Is Sets In Discrete Mathematics?

Ans. Set is an unordered collection of different elements.

95 | 95 Give Some Example of Sets

Ans. $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$.

96 | 96 How Many Ways Represent A Set?

Ans. There are three ways to represent a set.

- a) Tabular form / Roaster form
- b) Representative form
- c) Set-Builder form

97 | 97 Write Set of vowels in English alphabet, in roster form .

Ans. Vowel = $\{a, e, i, o, u\}$

98 | 98 Write Set of odd numbers less than 10, in roster form

Ans. Odd = $\{1, 3, 5, 7, 9\}$

99 | 99. Write Set of vowels in English alphabet, in set builder notation.

Ans .Vowel $= \{x: x \text{ is vowel}, x \text{ belongs to alphabet}\}$

100 | 100. Explain Some Important Sets?

Ans. Natural numbers $= \{1, 2, 3, \dots\}$

Integers $= \{1/2, 3/4, 4/5, \dots\}$, etc.

101 | 101 What Is Cardinality Of A Set?

Ans .Number of elements of set is cardinality.

102 | 102 What is cardinality of $|\{1, 4, 3, 5, \dots\}|$

Ans. Infinite.

103 | 103 What is cardinality of $|\{1, 4, 3, 5\}|$

Ans. 4

104 | 104 what is Finite Set

Ans. Finite set : Set which have finite no. Of elements in it.

105 | 105 what is Infinite set.

Ans. INFINITE SET : Set in which no. Of elements are infinite.

106 | 106 what is Subset

Ans. Set A is a subset of a set B if all elements of A are also elements of B; B is then a superset of A.

107 | 107 What is Proper Subset

Ans. A proper subset of a set A is a subset of A that is not equal to A. In other words, if B is a proper subset of A, then all elements of B are in A but A contains at least one element that is not in B.

108 | 108 What is Universal Set

Ans. The universal set is the set of all elements or members of all related sets.

109 | 109 What is Empty Set or Null Set

Ans. Any Set that does not contain any element is called the empty or null or void set. The symbol used to represent an empty set is $\{\}$ or ϕ . Examples: Let $A = \{x : 9 < x < 10, x \text{ is a natural number}\}$ will be a null set because there is NO natural number between numbers 9 and 10.

110 | 110 what is cardinality of null set

Ans.zero.

111 | 111 What is Singleton Set or Unit Set

Ans. Set that consist exactly one element.

112 | 112 What is Equal Set

Ans. Two set are equal if they are subset of each other and have exactly same elements.

113 | 113 What Is Set Operations

Ans. Union , intersection , complement , difference.

114 | 114 What Is Power Set

Ans.power set (or power set) of a Set A is defined as the set of all subsets of the Set A including the Set itself and the null or empty set.

115 | 115 What Is Discrete Mathematics Functions?

Ans.Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous.

116 | 116 What is the minimum number of students required in a discrete Mathematics class to be sure that at least six will receive the same Grade, if there are five possible grades, A, B, C, D, and F?

Ans. We have $k = 5$ and $r = 6$. We need to compute N such that $\lceil N/k \rceil = r$ or more precisely $\lceil N/5 \rceil = 6$. We can compute the smallest integer with this property as $N = K(r - 1) + 1$. Plugging k and r into this equation gives us $N = 5(6-1)+1 = 26$. Thus, 26 is the minimum number of Students needed to ensure that at least six students will Receive the same grade.

117 | 117 Find the minimum number of students in a class to be sure that three of Them are born in the same month.

Ans. The generalized Pigeonhole principle:

Assume N objects are placed into k boxes. $N=12, k+1=3; k=2$.

So, there is at least one box containing at least N/k objects.

To use the pigeonhole principle, first, find boxes and objects.

Suppose that for each month, we have a box that contains persons who were born in that month. The number of boxes is

12, by the generalized pigeonhole principle, to have at least 3 ($= N/12$) students at the same box, the total number of the students must be at least $N = 2 \cdot 12 + 1 = 25$. ($k \cdot n + 1$)

So, the minimum number of students required = 25.

Repeated 67

118 | 118 Show that at least two people must have their birthday in the same Month if 13 people are assembled in a room.

Repeated 68

119 | 119 Bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly From the bag to ensure that we get 4 marbles of same color?

Ans.

120 | 120

A box contains 6 red, 8 green, 10 blue, 12 yellow and 15 white balls.

What is the minimum no. of balls we have to choose randomly from

The box to ensure that we get 9 balls of same color?

Ans. On the assumption that you draw the balls at random without replacement, the minimum number of balls that gives you the certainty to get 9 balls of the same colour is:

$6+8+3*8+1=39$. In fact, in the most unfavorable case you draw, in some order, 6 red + 8 green + 8 blue + 8 yellow + 8 white balls = 38 and only the 39th allows you to get either 9 blue or 9 yellow or 9 white balls.