FE 630 Project Report
A Factor Model Based Allocation

Viswanathan Raman, Brahmani Pingle, Darsh Kachhara

A. Introduction

In this project, we have come up with an investment strategy that maximizes the rate of return given by the portfolio subject to a constraint of target beta, which is the usual single factor market risk measure. We also reallocate the portfolio on the first trading day of each week from August 2007 to November 2021. Finally, we evaluate the performance of each strategy on mean return, volatility, and Sharpe ratio to showcase the optimal strategy.

B. Purpose

This project aims to understand, analyze, and compare the behavior of the Long/Short Macro Strategy, using the French Fama 3-Factor model, with factors momentum, value, and size. It is aimed to calculate the portfolio's sensitivity to different lengths of look-back periods and target beta during several historical periods: before the subprime (2008) crisis, during that crisis, and after the crisis.

C. Portfolio Components and Specification

1. Financial Crisis Periods

Time periods	Dates	
Before crisis	08/2007-06/2008	
During Crisis	07/2008-06/2009	
After Crisis	07/2009-11/2013	

2. Term Structure

Long Term (LT)	120 Days
Mid Term (MT)	60 Days

Short Term(ST)	40 Days

3. Portfolio Environment

For practical considerations, the project assumes that the universe of investment is a set of ETFs large enough to represent the World global economy. The benchmark in this project is S&P 500 Index, which SPY represents. The ETFs are listed as below:

1. CurrencyShares Euro Trust (FXE)
2. iShares MSCI Japan Index (EWJ)
3. SPDR GOLD Trust (GLD)
4. Powershares NASDAQ-100 Trust (QQQ)
5. SPDR S&P 500 (SPY)
6. iShares Lehman Short Treasury Bond (SHV)
7. PowerShares DB Agriculture Fund (DBA)
8. United States Oil Fund LP (USO)
9. SPDR S&P Biotech (XBI)
10. iShares S&P Latin America 40 Index (ILF)
11. iShares MSCI Pacific ex-Japan Index Fund (EPP)
12. SPDR DJ Euro Stoxx 50 (FEZ)

4. French Fama 3-Factor Model

The Fama French 3-factor model is an asset pricing model that expands on the capital asset pricing model by adding size risk and value risk factors to the market risk factors.

The Fama and French model has three factors: the size of firms, book-to-market values, and excess return on the market.

The model is essentially the result of an econometric regression of historical stock prices.

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1(R_{Mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{it}$$

where:

$R_{it} = \text{total return of a stock or portfolio } i \text{ at time } t$

$R_{ft} = \text{risk free rate of return at time } t$

$R_{Mt} = \text{total market portfolio return at time } t$

$R_{it} - R_{ft} = \text{expected excess return}$

$R_{Mt} - R_{ft} = \text{excess return on the market portfolio (index)}$

$SMB_t = \text{size premium (small minus big)}$

$HML_t = \text{value premium (high minus low)}$

$\beta_{1,2,3} = \text{factor coefficients}$

D. Strategy

We present our strategies for each period with three look-back periods: before the crisis, during the crisis and after the crisis. We build a factor model to find the optimized weight for the portfolio and reoptimize it each week. There are three important parameters in our strategy: the time estimating variable for expected return, the time estimating variable for covariance and the target beta of the portfolio.

For example, a strategy $S^{90}_{120}(1.5)$ means at each week, we will look back for 90 days to do the regression of returns and get the expected return ρ for the portfolio. Afterward, we will look back for 120 days to do the regression of returns and get the covariance of Σ, and 1.5 is the target beta of our portfolio. Finally, we record the daily return and analyze our strategies by comparing different financial stats for each period.

E. Functions

i) Regression

The first step in the model is to conduct regression. The regression does the following three important tasks: Firstly, it predicts the return of a security based on the predefined term-structure estimator.

According to the French-Fama data, the independent variables consist of market factor, SMB factor and HML factor. The dependent variable is real security return minus risk-free rate, which show as the function below:

$$(r_i - r_f) = \beta_i^3 * (r_M - r_f) + \beta_i^s * \rho_{SMB} + \beta_i^v * \rho_{HML} + \varepsilon_i$$

Based on this function, we can forecast the return of security i. And then compute the expected return of this security. Secondly, regression predicts the sigma of a security based on another time estimator in our strategy. Again, this difference is the look-back period we used from the above function. In the same process, we can produce the expected return of this security. Thirdly, we use regression to compute the beta between market and security i.

In the French-Fama table, we obtain columns of market return and the risk-free rate. Thus, we can produce daily market volatility. After that, we will use the following formula to compute the beta. The length of data is the same as the covariance time estimator.

$$\beta_i^m = \frac{cov(r_i, r_M)}{\sigma^2(r_M)}$$

We will do this regression for each security each week. Then we can get two lists of expected returns, the first is rho. In addition, the second list of expected returns is used

to compute the covariance. In the end, we can get the inputs required by quadratic solvers.

ii) Portfolio optimization model

The portfolio optimization problem is to find the optimal weights of the stocks to maximize the utility function given as follows. ϱ is expected returns and Σ is covariance matrix from the previous French- Fama factor model.

$$\max_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda(\omega - \omega_p)^T \Sigma(\omega - \omega_p)$$
$$\sum_{i=1}^{n} \beta_i^m \omega_i = \beta_T^m$$
$$\sum_{i=1}^{n} \omega_i = 1, \ -2 \leq \omega_i \leq 2,$$

iii) Rebalance

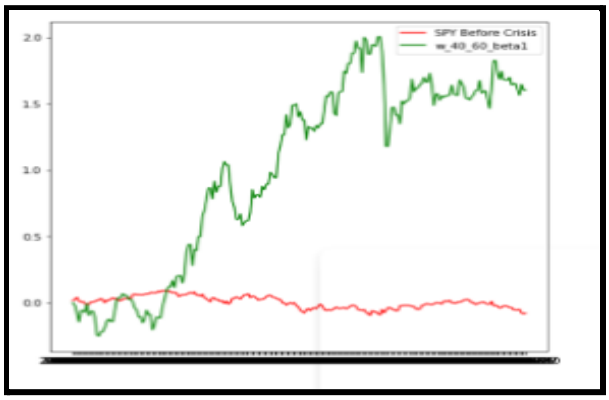
We rebalance our portfolio on the first trading day every week and get optimal weights for each strategy. On the first day of each trading week, we used the optimal weight to buy instruments and hold the same shares for a week. Then we multiply shares by prices to obtain the portfolio's capital for each strategy and compute daily return.

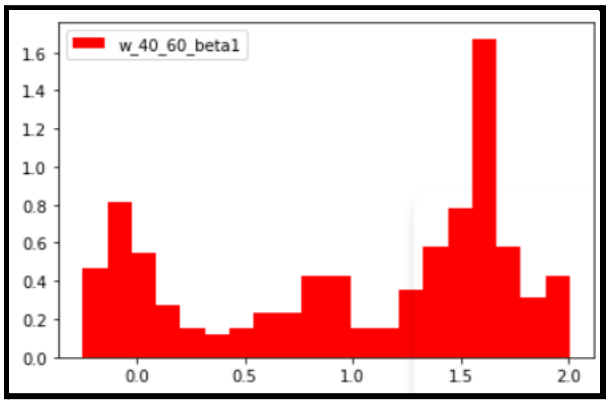
F. Result and Analysis

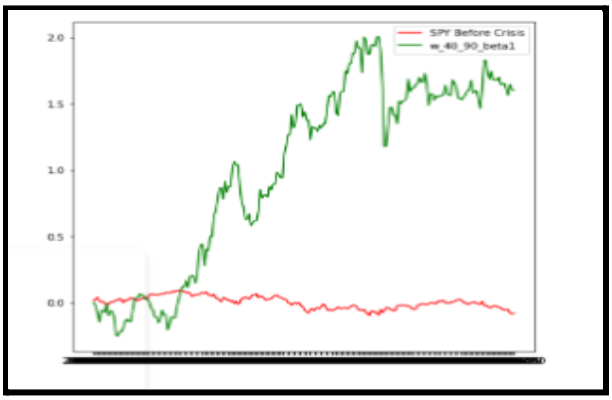
1. **BEFORE CRISIS**

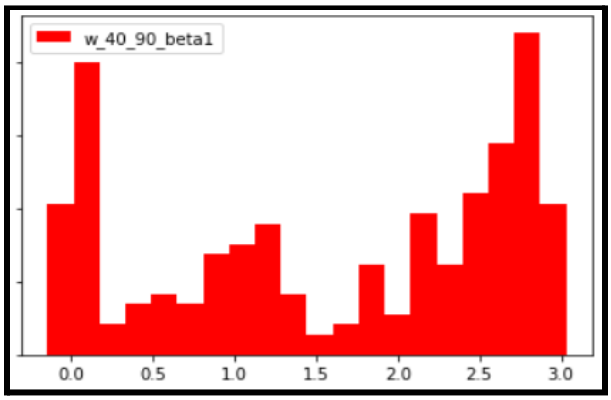
	w_40_60_beta1	w_40_90_beta1	w_90_120_beta1	40_60_beta2	w_40_90_beta2	w_90_120_beta2	w_40_60_beta3	40_90_beta3	w_90_120_beta3
Mean	1.010830	1.010830	1.624167	1.034408	1.133113	1.783389	1.092185	1.218907	1.847111
Volatility	11.245767	11.245767	16.957934	10.647220	12.204342	17.411571	10.461783	12.193853	16.955067
Skewness	-0.483131	-0.483131	-0.296339	-0.599545	-0.477810	-0.372169	-0.678442	-0.596975	-0.487512
Kurtosis	-1.275854	-1.275854	-1.469835	-1.130821	-1.280462	-1.397420	-0.966871	-1.176533	-1.272806
Sharp Ratio	0.089885	0.089885	0.095776	0.097153	0.092845	0.102425	0.104398	0.099961	0.108942
Variance	0.501854	0.501854	1.141157	0.449854	0.591055	1.203027	0.434321	0.590040	1.140771
Minimum Return	-0.250975	-0.250975	-0.143528	-0.251629	-0.255461	-0.143528	-0.231321	-0.233765	-0.143528
Maximum Return	2.004632	2.004632	3.031698	1.954916	2.135401	3.164496	1.990088	2.142825	3.132060

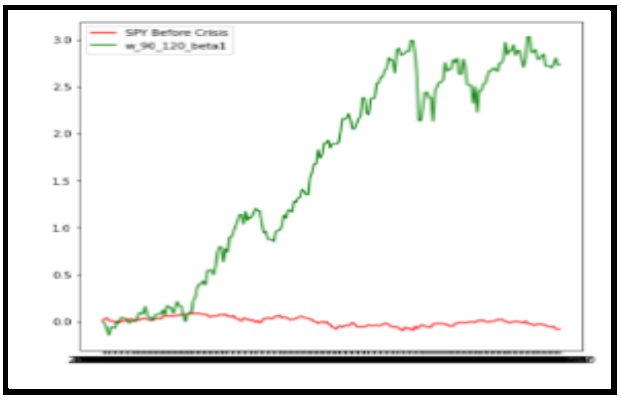
TABLE 1.1 BETA -0.5

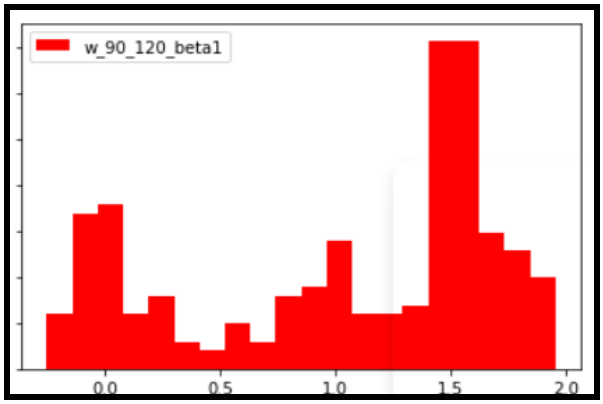












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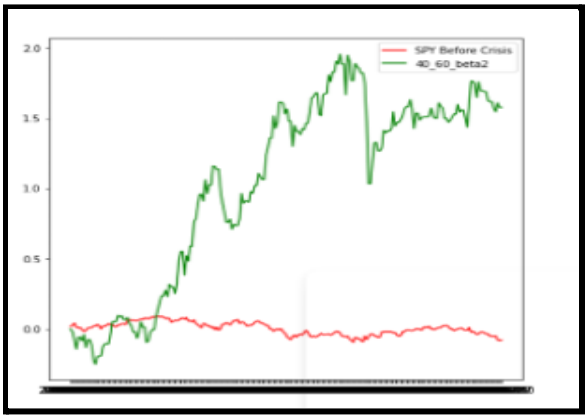
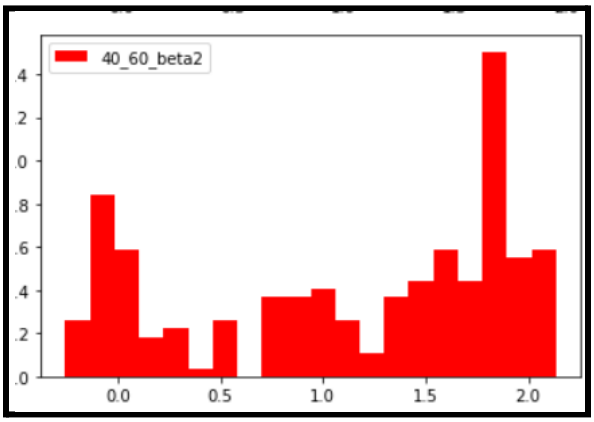
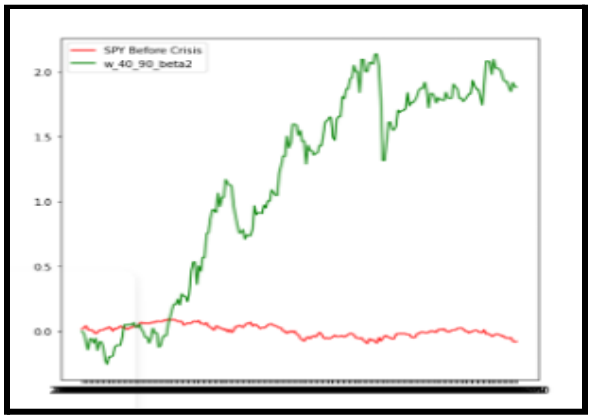
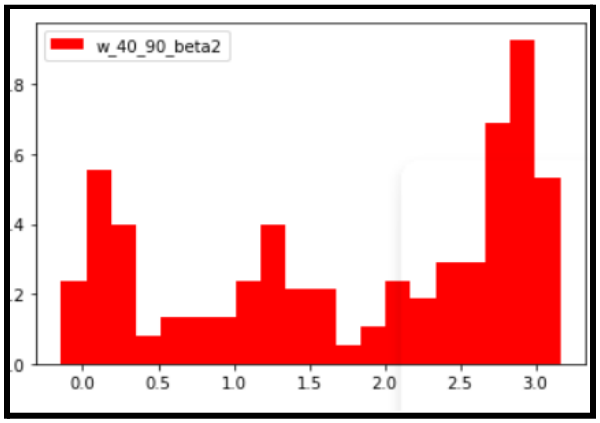
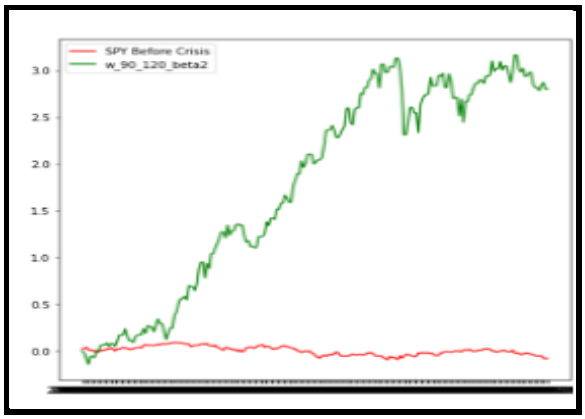










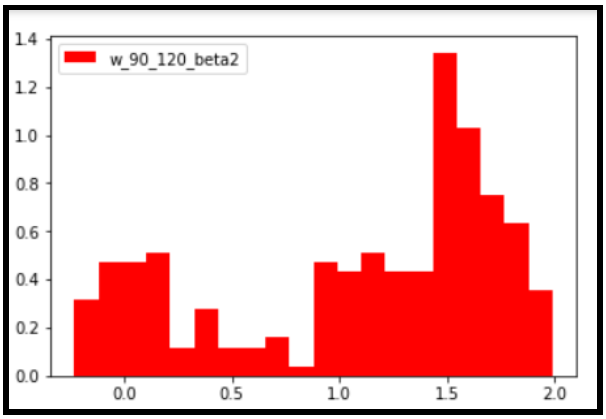
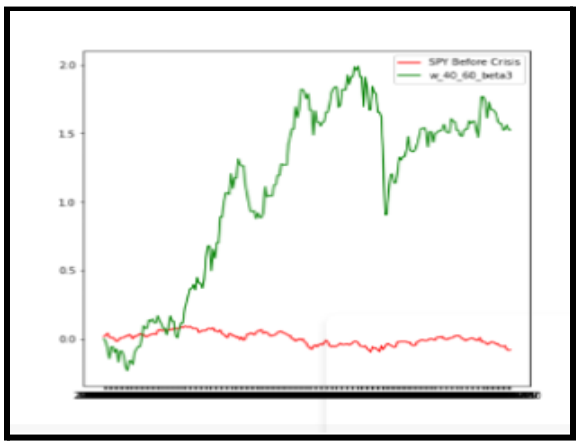
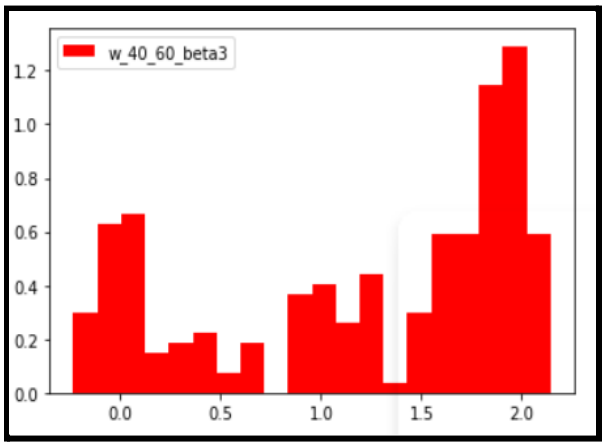
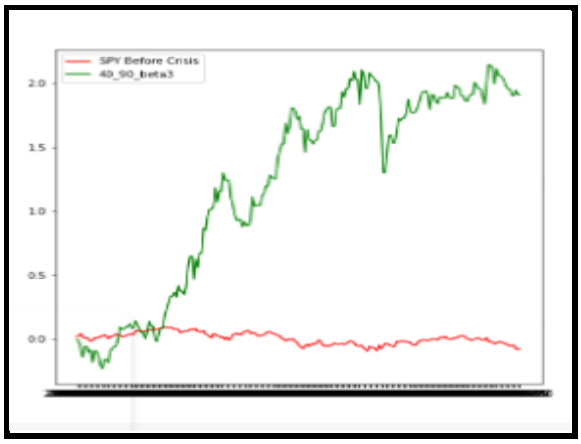
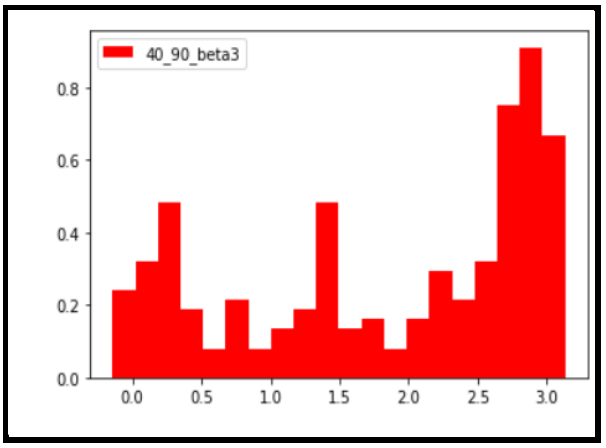


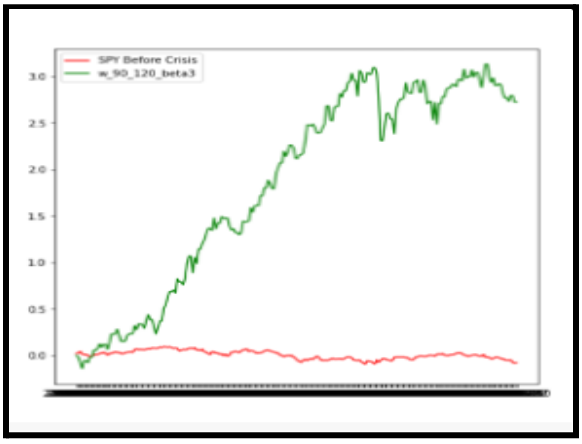
TABLE 1.3 BETA 1.5

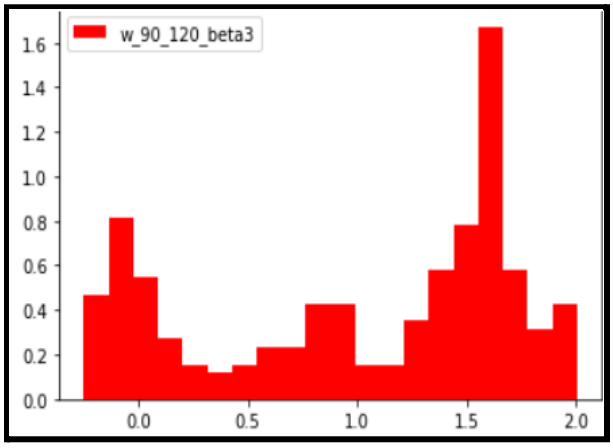








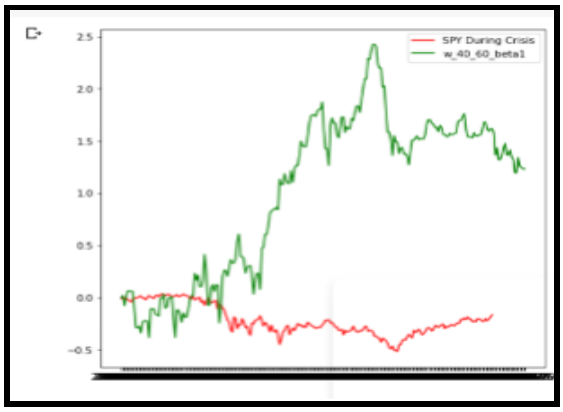


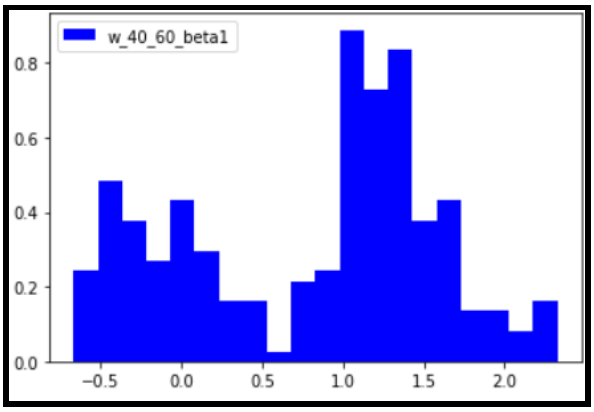


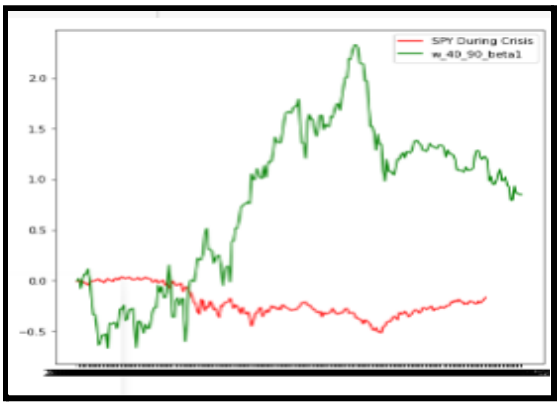
2. DURING CRISIS

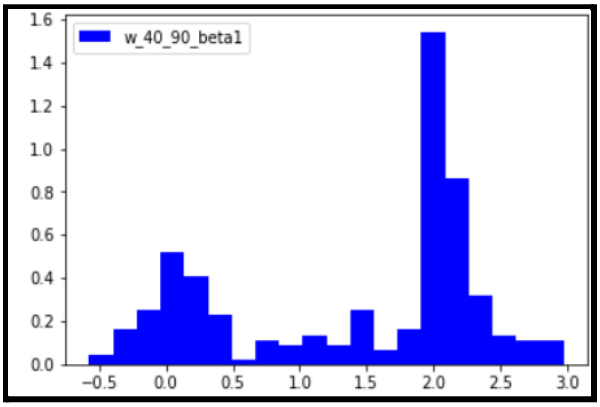
	w_40_60_beta1	w_40_90_beta1	w_90_120_beta1	w_40_60_beta2	w_40_90_beta2	w_90_120_beta2	w_40_60_beta3	w_40_90_beta3	w_90_120_beta3
Mean	1.010345	0.798342	1.424932	0.898795	0.702769	1.506396	0.745935	0.479743	1.563053
Volatility	12.252731	12.469924	15.108290	11.565496	11.629008	15.338514	10.312617	9.603648	16.165267
Skewness	-0.354804	-0.291967	-0.575608	-0.298518	-0.281503	-0.542585	-0.199201	-0.033309	-0.433289
Kurtosis	-1.309586	-1.074718	-1.206985	-1.458697	-1.182450	-1.297341	-1.432481	-1.054933	-1.408507
Sharp Ratio	0.779000	0.516477	1.155956	0.830736	0.588477	1.431487	0.733670	0.423664	1.791463
Variance	0.595752	0.617060	0.905795	0.530796	0.536642	0.933611	0.422024	0.365992	1.036960
Minimum Return	-0.381983	-0.666302	-0.575993	-0.407586	-0.701132	-0.369383	-0.424291	-0.677601	-0.196963
Maximum Return	2.424644	2.327587	2.978061	2.171030	2.110939	3.020387	1.911160	1.780425	2.963048

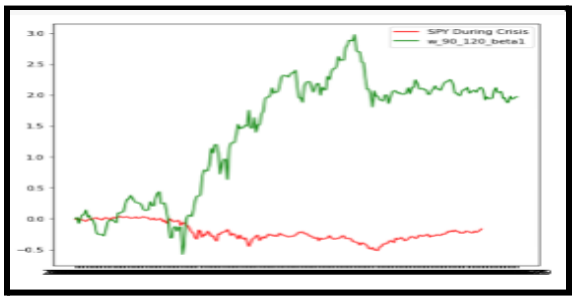
TABLE 2.1 BETA -0.5











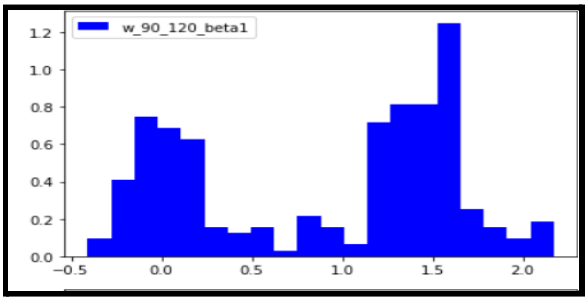
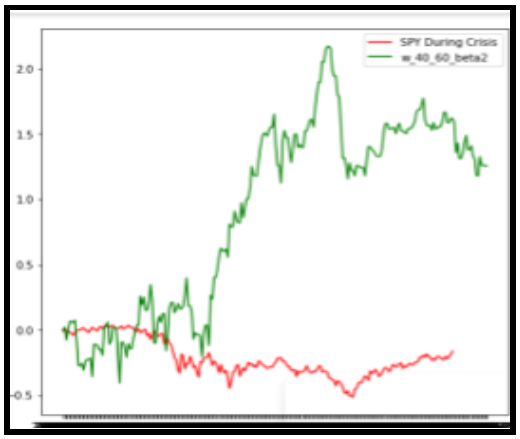
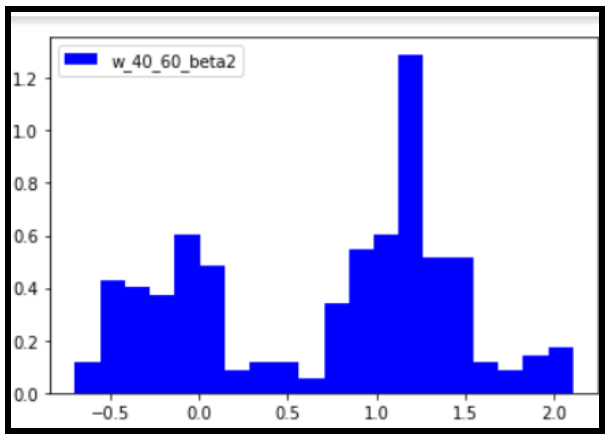
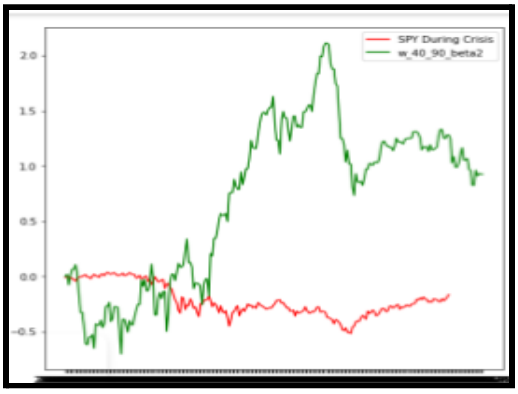
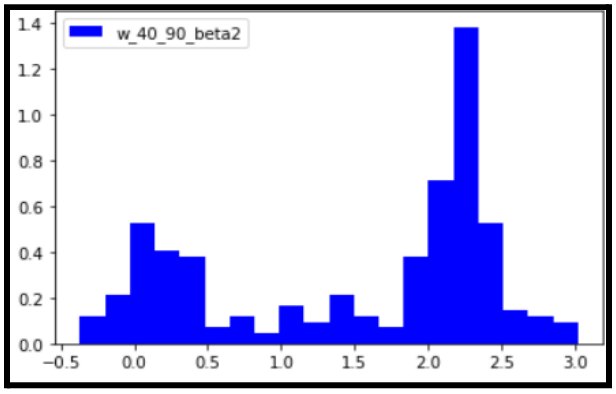


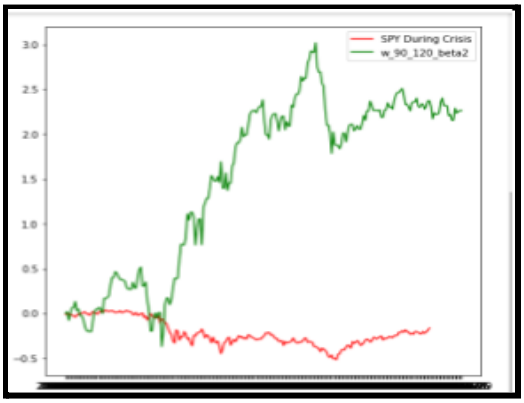
TABLE 2.2 BETA 0.5











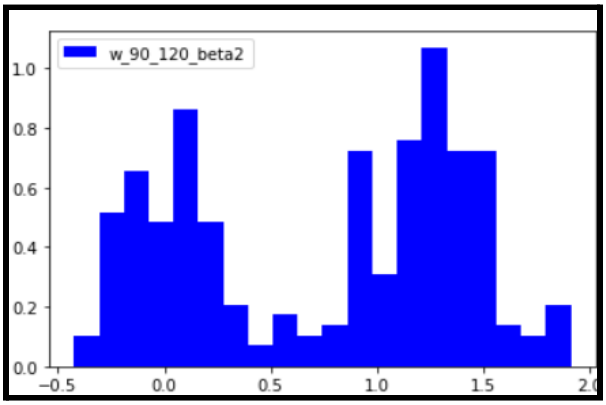
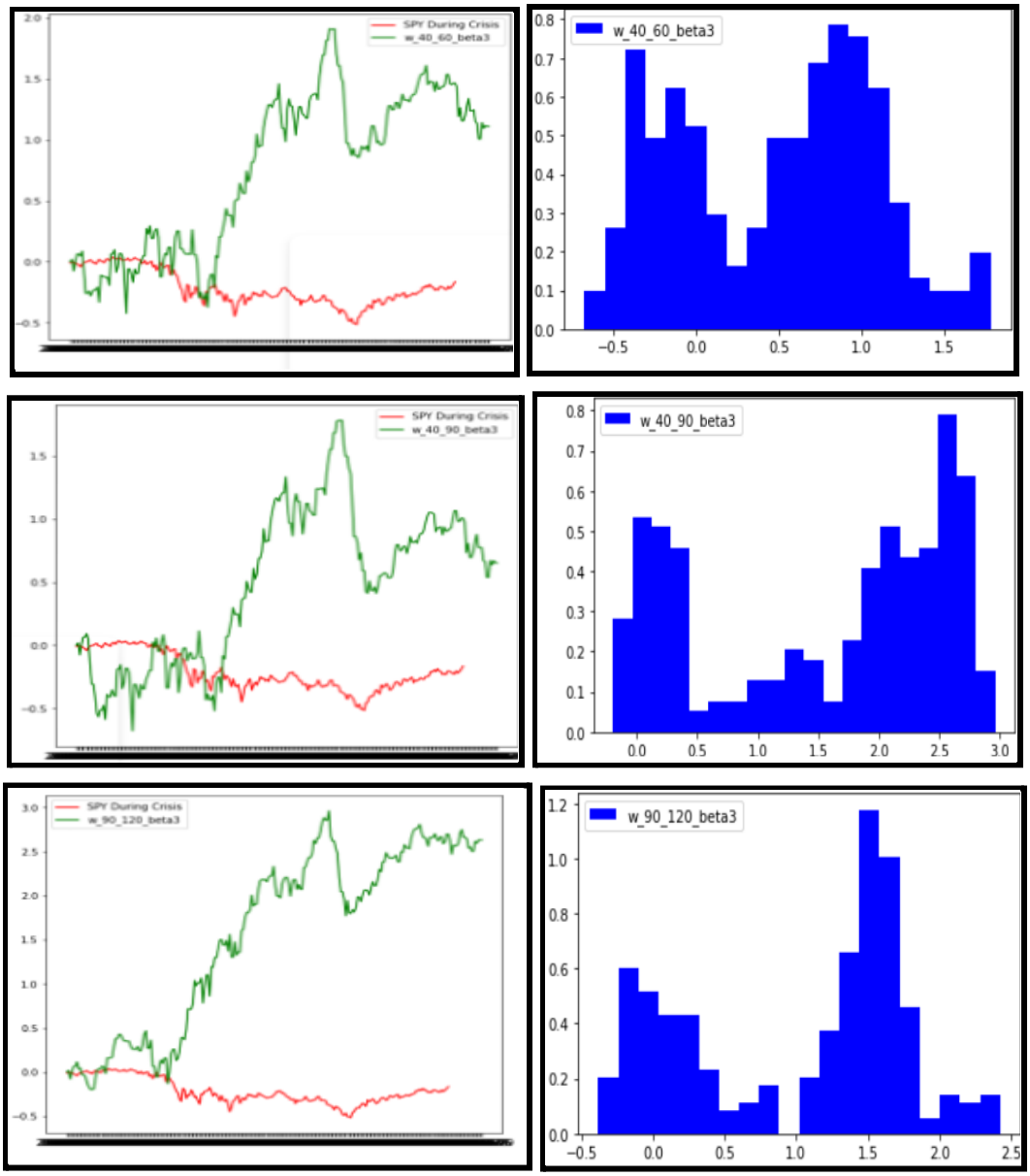


TABLE 2.3 BETA 1.5



As clearly visible in the plots, all our strategies outperform the market before and during the crisis. The returns follow a normal distribution and have a heavy tail risk. Certain specifications that showcase our strategy broadly are, mean return, Sharpe ratio and volatility. The mean return for SPY was negative for both periods, but our portfolio provided not only a positive mean return but a much better mean return. Under S_{90}^{120}, the Sharpe ratio performs better than other strategies, and with a beta of 1.5, it performs the best result. The volatility of our strategy is much bigger than those

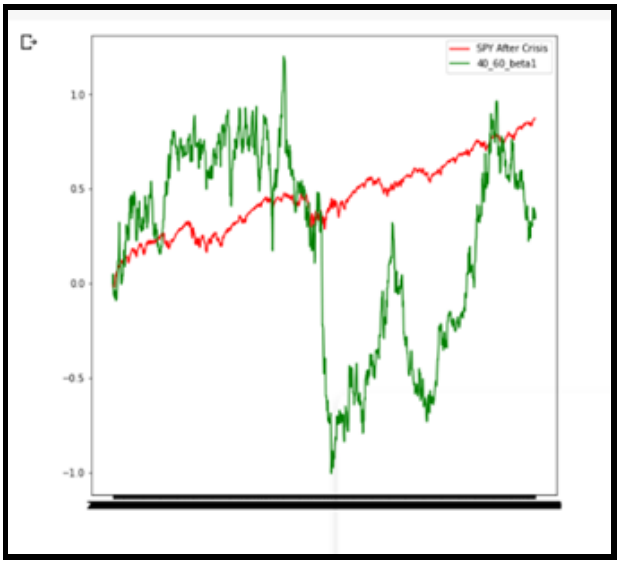
of the market because it is riskier. And with short-term return prediction, we have smaller volatilities. Therefore, the nearest period is more valuable when considering the historical return. The distributions of daily return under each time structure are similar.

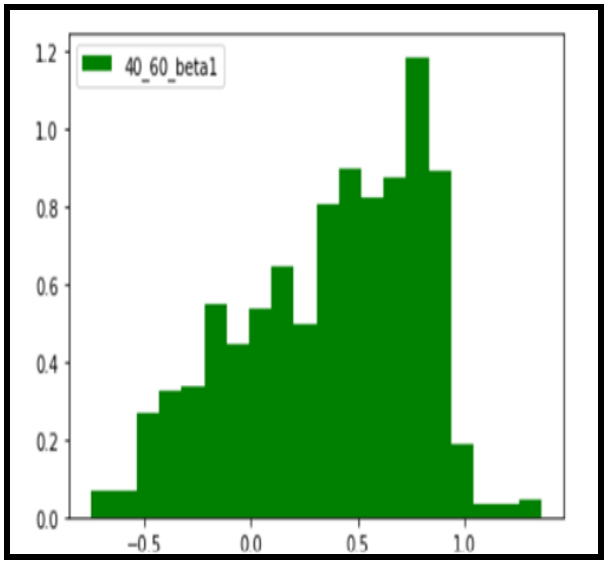
3. AFTER CRISIS

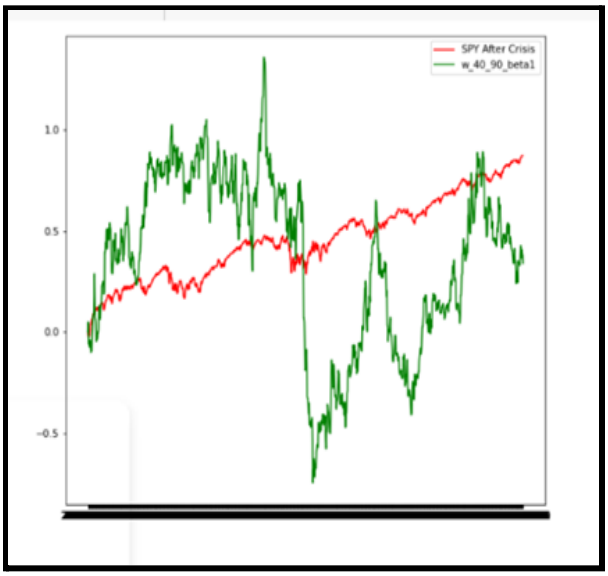
[95] dfAfter

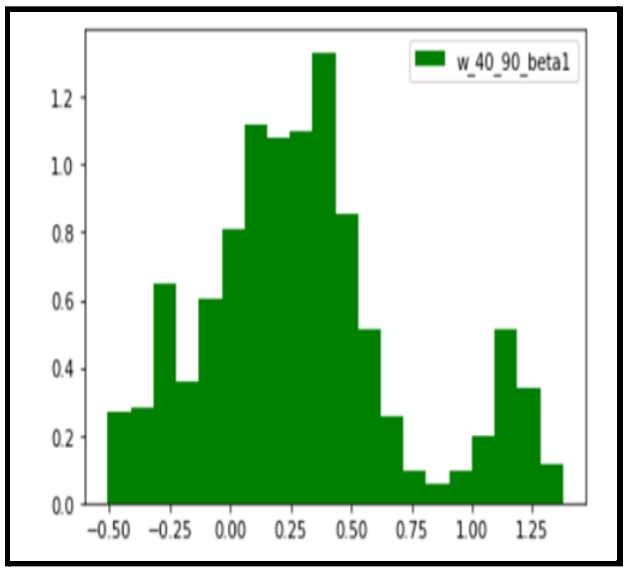
	40_60_beta1	w_40_90_beta1	w_90_120_beta1	40_60_beta2	90_120_beta2	40_60_beta3	40_90_beta3	aop_90_120_beta3
Mean	0.215958	0.364789	0.293485	0.151135	0.219547	0.446641	0.228419	0.252937
Volatility	8.184688	6.688845	6.648092	8.221811	7.597128	8.442565	8.259765	8.611132
Skewness	-0.509573	-0.439083	0.631446	-0.626350	0.869224	0.155149	-0.568627	0.798263
Kurtosis	-1.002613	-0.702397	0.137841	-0.904669	0.329303	-0.359552	-0.747733	0.094617
Sharp Ratio	0.026386	0.054537	0.044146	0.018382	0.028899	0.052904	0.027654	0.029373
Variance	0.265830	0.177542	0.175385	0.268247	0.229033	0.282845	0.270729	0.294252
Minimum Return	-1.006017	-0.746955	-0.506231	-1.090702	-0.631507	-0.786732	-1.066623	-0.738246
Maximum Return	1.200766	1.358755	1.376270	1.053054	1.475763	1.704313	1.111191	1.592512

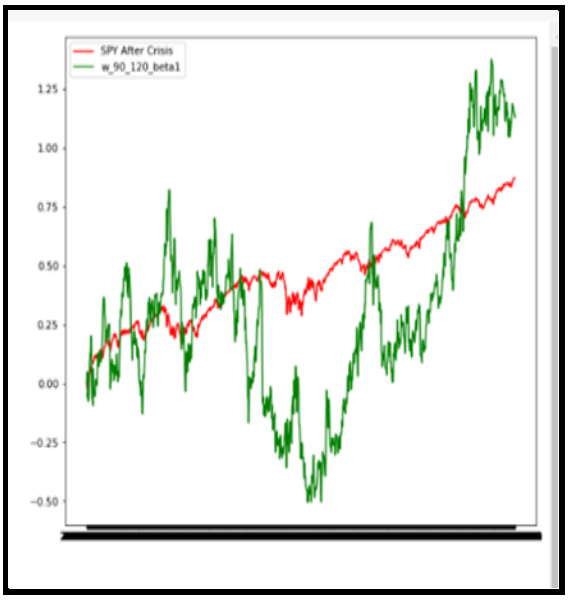
TABLE 3.1 BETA -0.5


SPY After Crisis
40_60_beta1


40_60_beta1


SPY After Crisis
w_40_90_beta1


w_40_90_beta1



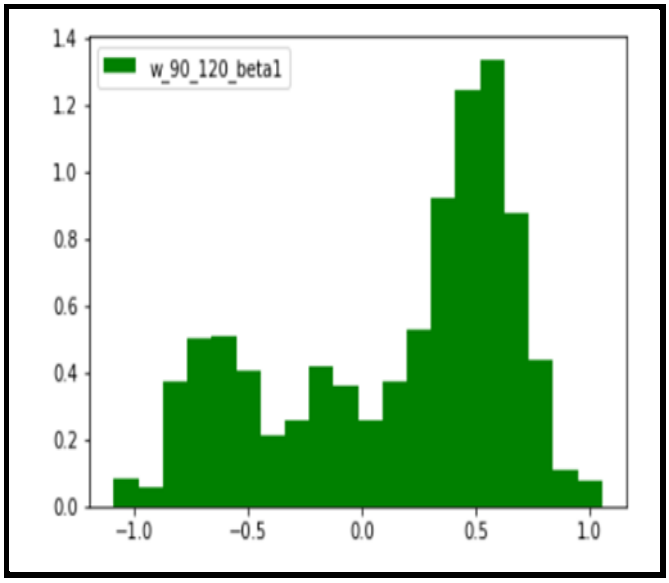
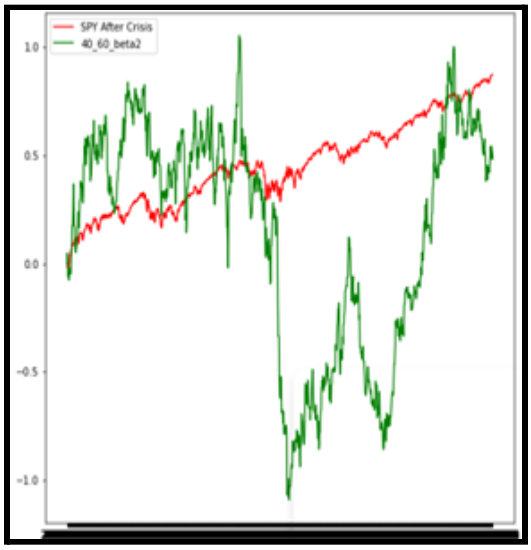
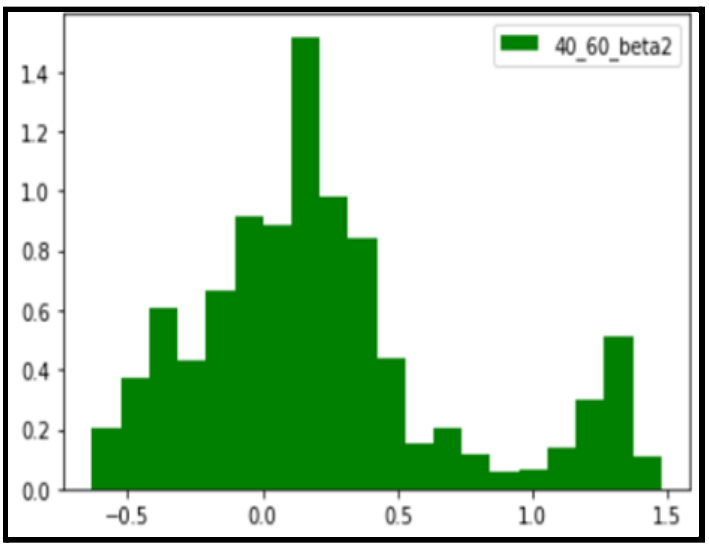
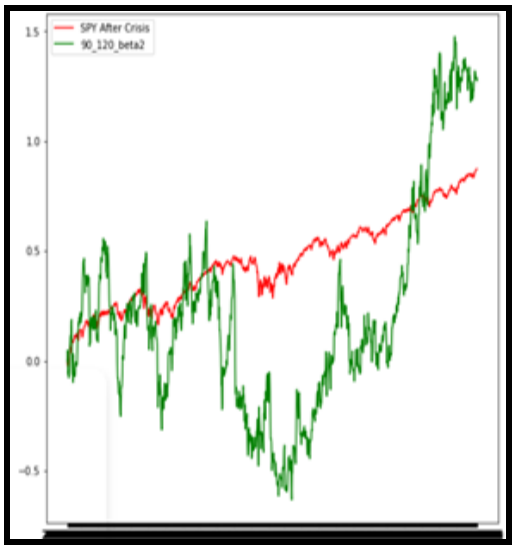


TABLE 3.2 BETA 0.5







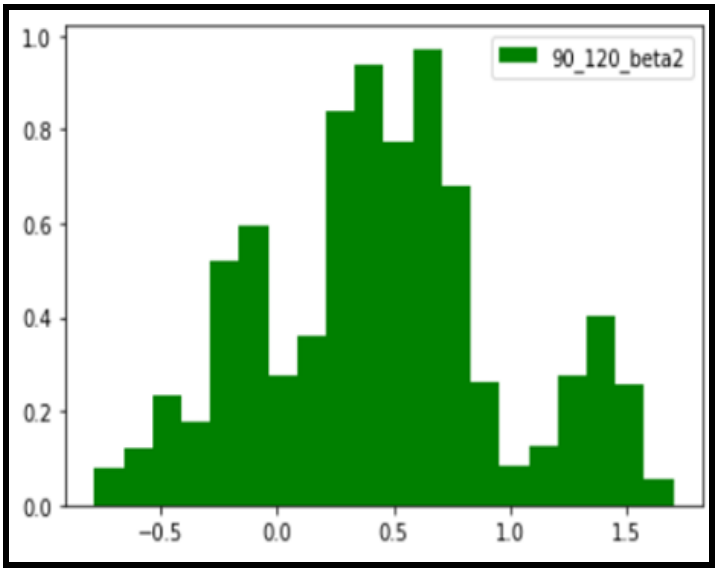
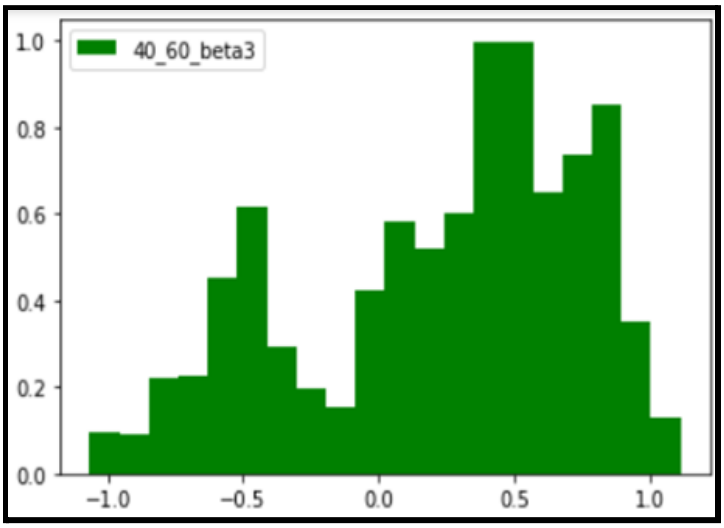
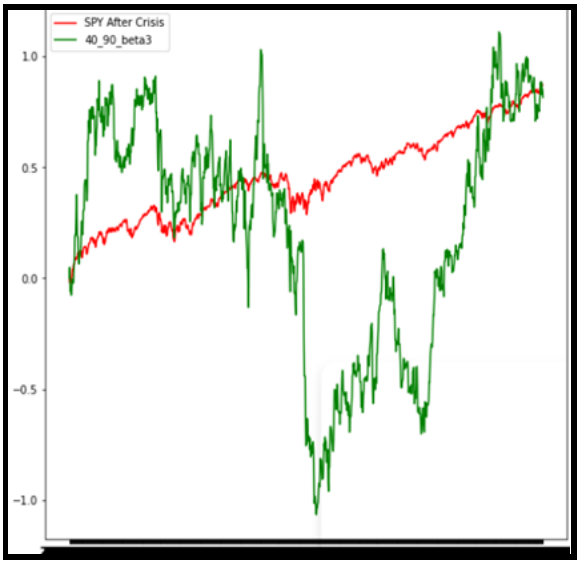
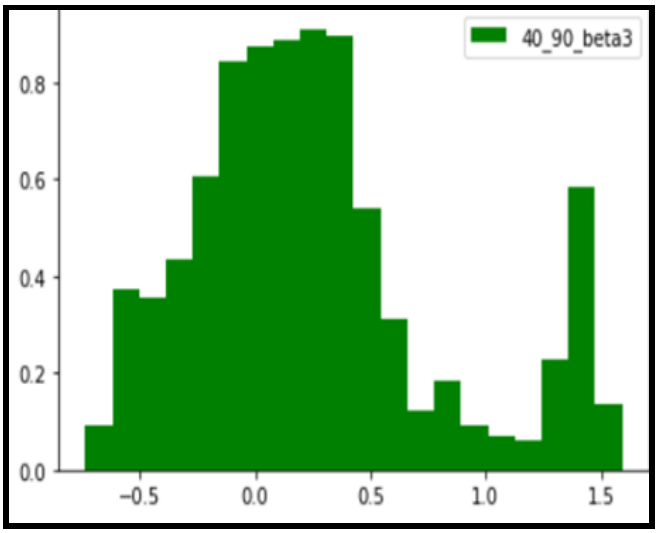
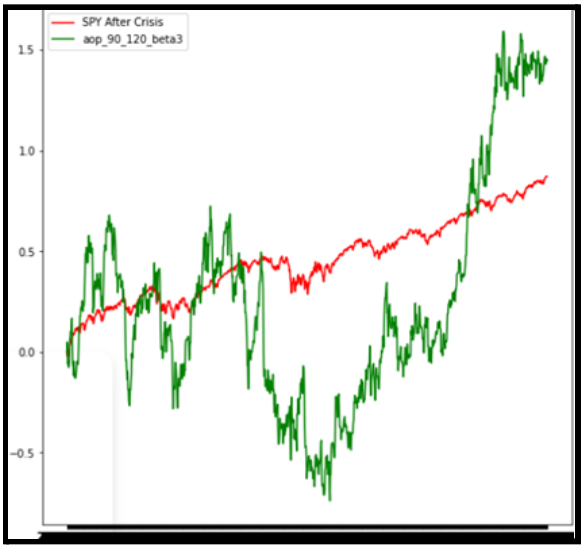


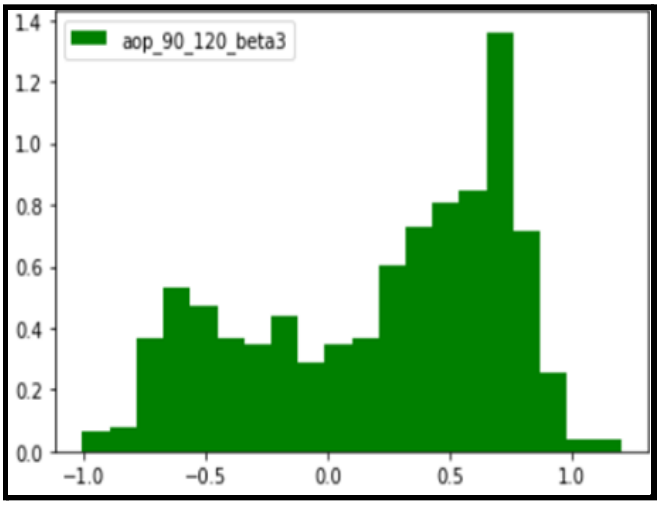
TABLE 3.3 BETA 1.5









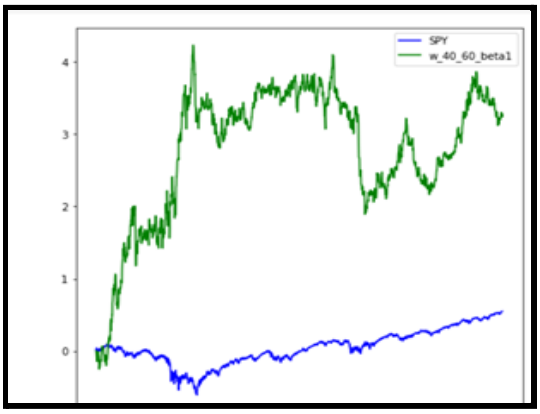


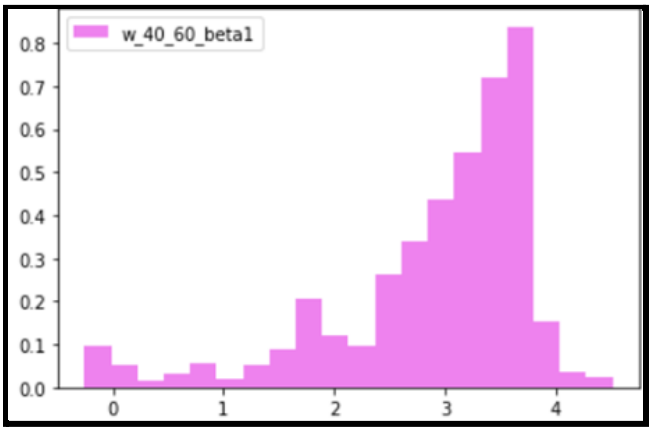
After the crisis, we observed that our strategy was not able to beat the market. The returns of SPY500 are all positive whereas the portfolio returns have considerable dips. The volatility of our strategies is much bigger than those of the market which reflects that the risk of our strategies is larger than the market. The distribution of daily returns appears to be similar.

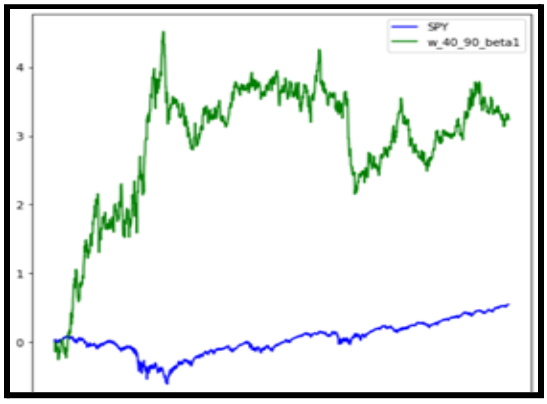
4. TOTAL PERIOD

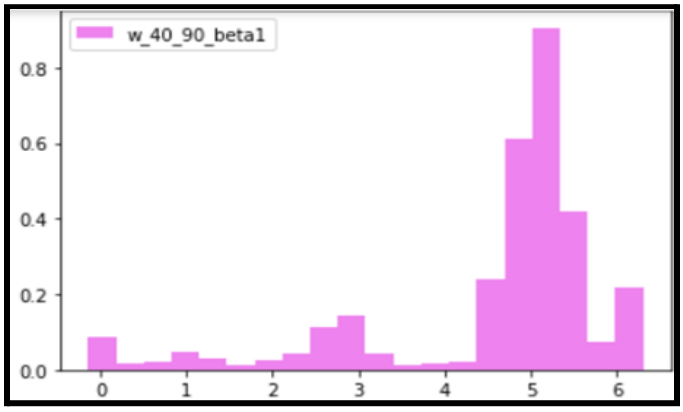
We now observe what is the output of the entire period under consideration.

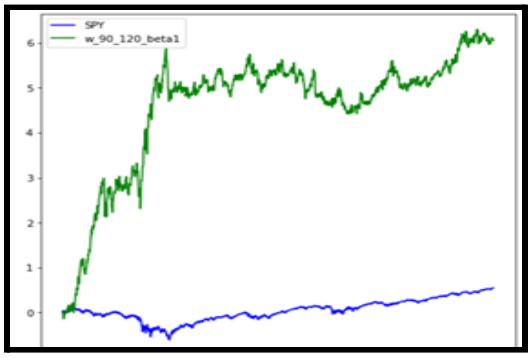
TABLE 4.1 BETA -0.5











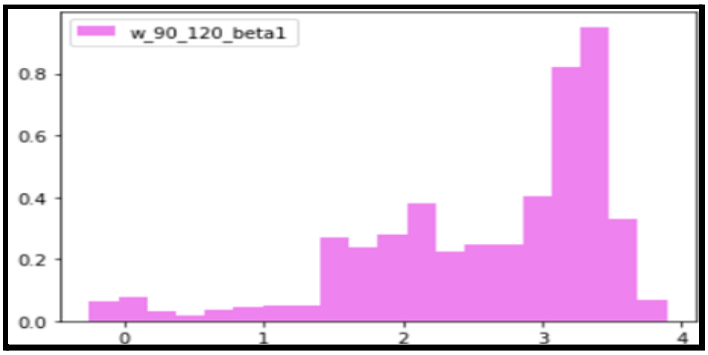
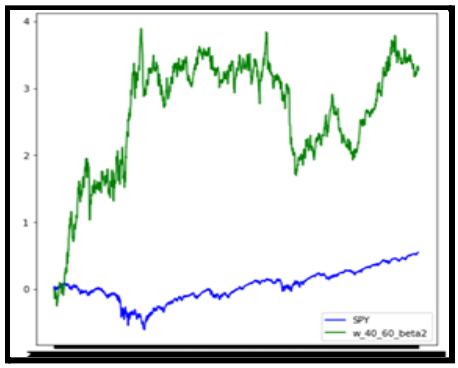
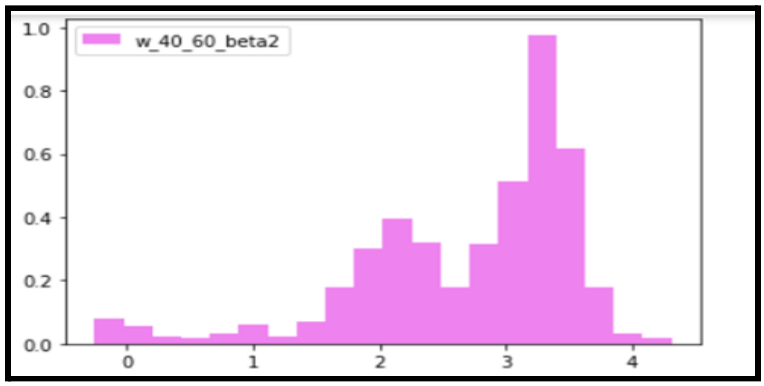
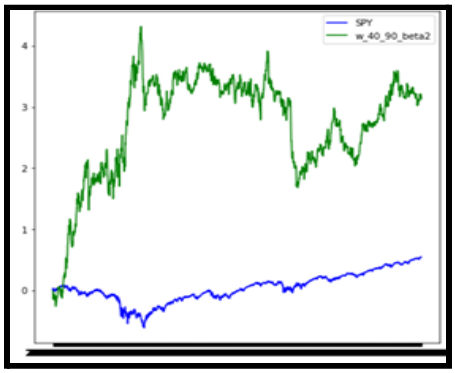
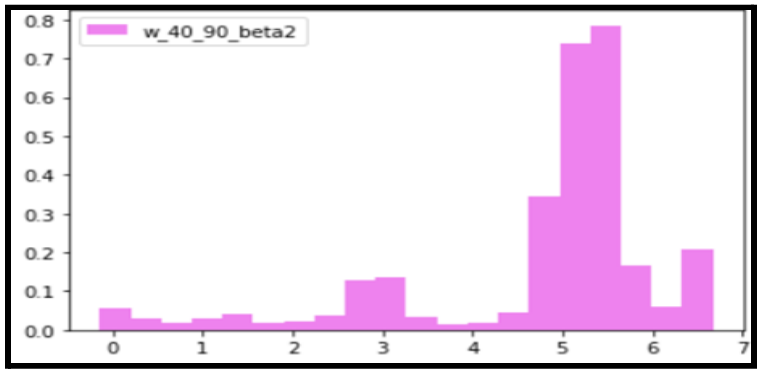


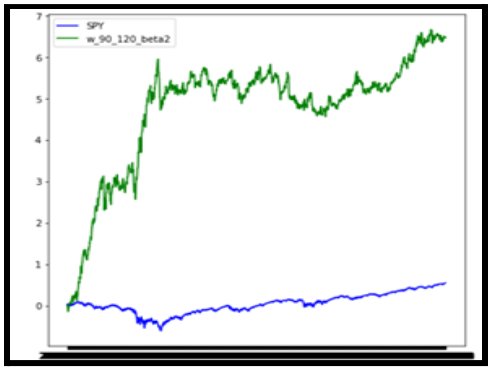
TABLE 4.2 BETA 0.5











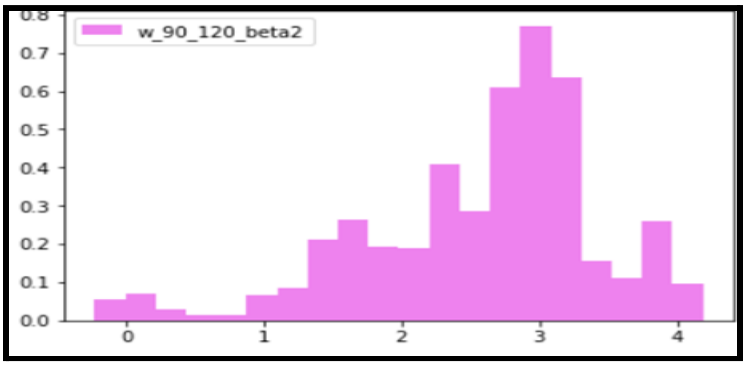
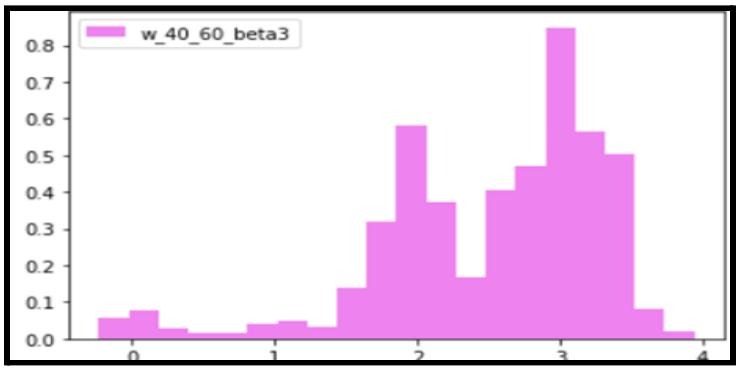
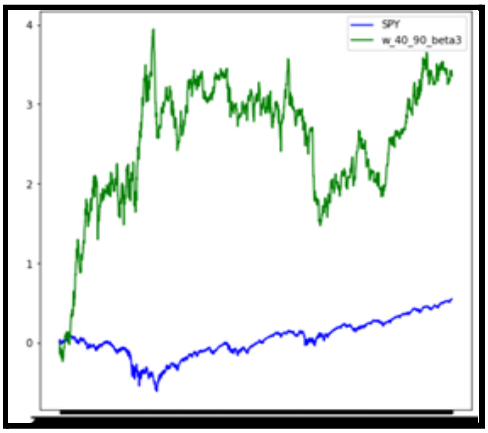
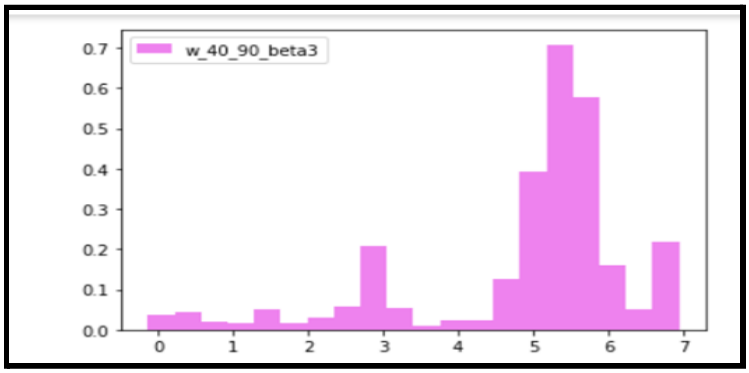
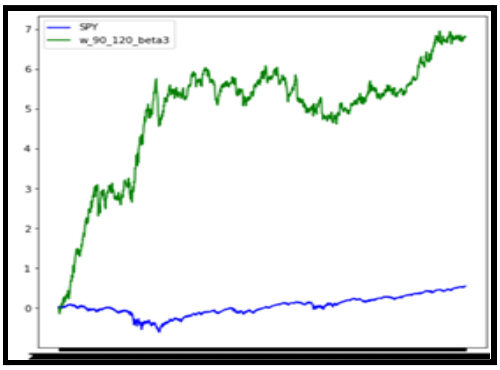


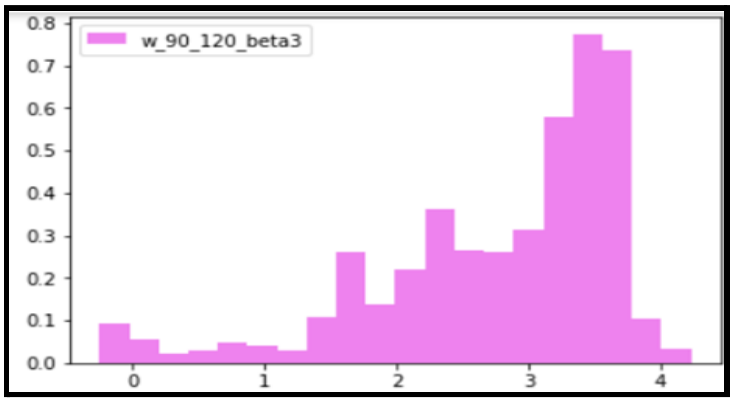
TABLE 4.3 BETA 1.5











In conclusion, the analysis of how our portfolio functions with respect to the market gives us a fair understanding of the strength of our strategy under varied economic

conditions. Our strategy produces much better results than the market during the financial crisis however post crisis, it is a different scenario. We observe similar trends whenever an economy goes through any kind of a crisis.