1、[每题 0.2，共 0.6 分]

(1) 设 $A(x)$：x是有限个数因子的乘积；$B(y)$：y为0；$C(x)$：x的乘积为0；

$E(y)$：y是乘积中的一个因子。

$(\forall x)(A(x) \wedge C(x) \rightarrow (\exists y)(E(y) \wedge B(y)))$

(2) 设 $R(x)$：x是实数，$G(x,y)$：$y>x$。

$(\forall x)(R(x) \rightarrow (\exists y)(G(x,y) \wedge R(y)))$

(3) 设 $R(x)$：x是实数，$G(x,y)$：$x>y$

$\exists x \exists y \exists z(R(x) \wedge R(y) \wedge R(z) \wedge G(x+y, x\cdot z))$

2、[每题 0.4，共 1.6 分]

	约束变元	自由变元	全称量词	存在量词
(1)	x	y	$P(x)$	无
(2)	x	无	$P(x) \wedge Q(x)$	$S(x)$
(3)	x, y	无	$A(x), B(y)$	$A(x)$
(4)	x, y	z	无	$E(x,y)$

3、[每题 0.6，共 1.8 分]

(1) 左式 $\equiv \neg \exists x(\neg P(x) \wedge \neg Q(x))$

$\vDash \neg(\exists x(\neg P(x)) \wedge \exists x(\neg Q(x)))$

$\equiv \forall x P(x) \vee \forall x Q(x) \vDash \forall x P(x) \vee \exists x Q(x)$

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(2) 左式 $\equiv \forall x(\neg P(x) \vee (Q(x) \wedge R(x))) \wedge \exists x(P(x) \wedge Q(x))$

$\equiv \forall x(\neg P(x) \vee Q(x)) \wedge \forall x(\neg P(x) \vee R(x)) \wedge \exists x(P(x) \wedge Q(x))$

$\equiv \neg \exists x(P(x) \wedge \neg Q(x)) \wedge \neg \exists x(P(x) \wedge \neg R(x)) \wedge \exists x(P(x) \wedge Q(x))$

$\models \neg(\exists x P(x) \wedge \exists x \neg Q(x)) \wedge \neg(\exists x P(x) \wedge \exists x \neg R(x)) \wedge \cancel{\exists x(P(x) \wedge Q(x))}$ $\exists x P(x) \wedge \exists x Q(x)$

$\equiv (\neg \exists x P(x) \vee \neg \exists x \neg Q(x)) \wedge (\neg \exists x P(x) \vee \neg \exists x \neg R(x)) \wedge \exists x P(x) \wedge \exists x Q(x)$

$\equiv ((\neg \exists x P(x) \wedge \exists x P(x)) \vee (\neg \exists x \neg Q(x) \wedge \exists x P(x))) \wedge (\neg \exists x P(x) \vee \neg \exists x \neg R(x)) \wedge \exists x Q(x)$

$\equiv \forall x Q(x) \wedge \exists x Q(x) \wedge ((\exists x P(x) \wedge \neg \exists x P(x)) \vee (\exists x P(x) \wedge \forall x R(x)))$

$\equiv \forall x Q(x) \wedge \exists x Q(x) \wedge \exists x P(x) \wedge \forall x R(x)$

$\models \forall x Q(x) \wedge \forall x R(x) \wedge \exists x P(x)$

$\equiv \forall x(Q(x) \wedge R(x)) \wedge \exists x P(x)$

$\models \exists x(Q(x) \wedge R(x)) \wedge \exists x P(x) \models \exists x(Q(x) \wedge R(x))$

(3) 左式 $\equiv \exists x(\neg P(x) \vee Q(x)) \equiv \neg \forall x P(x) \vee \exists x Q(x) \equiv \forall x P(x) \to \exists Q(x)$

4. [(1) (2) 各0.3分, (3) 0.4分, 共1分]

(1) $\exists x A(x) \wedge \neg \exists x A(x)$

$\equiv \neg(\neg \exists x A(x) \vee \exists x A(x)) \equiv \neg(\exists x A(x) \to \exists x A(x)) \equiv F$. 永假式.

(2) $\neg \exists x P(x) \to \forall x P(x) \equiv \exists x P(x) \vee \forall x P(x)$

设域为 $\{1, 2\}$. 若 $P(1) = P(2) = T$, 则原式 $= (P(1) \vee P(2)) \vee (P(1) \wedge P(2)) = T$.

若 $P(1) = P(2) = F$, 则原式 $= (P(1) \vee P(2)) \vee (P(1) \wedge P(2)) = F$. 可满足式

(3) 设域为实数集 R.

若 $F(x,y)$ 表示 "$x=y$", 则 $\exists x \forall y(F(x,y) \to F(y,x)) \equiv \exists x \forall y((x=y) \to (y=x)) \equiv T$

若 $F(x,y)$ 表示 "$x>y$", 则 $\exists x \forall y(F(x,y) \to F(y,x)) \equiv \exists x \forall y((x>y) \to (y>x)) \equiv F$

$\therefore$ 可满足式.

「最后提供两个同学在作业中提交的关于3(2)比较好的解法」

解法1：左式 $\models \forall x(P(x)\to Q(x)\wedge R(x))\wedge\exists x P(x)$

$\equiv \forall x(\neg P(x)\vee(Q(x)\wedge R(x))\wedge\exists x P(x)$

由(1)中结论

$\models \cancel{\forall x}\,[\neg\exists x P(x)\vee\exists x(Q(x)\wedge R(x))]\wedge\exists x P(x)$

$\equiv \quad F\vee[\exists x(Q(x)\wedge R(x)) \quad \wedge\exists x P(x)]$

$\equiv \exists x(Q(x)\wedge R(x))\wedge\exists x P(x) \quad \models \exists x(Q(x)\wedge R(x))$

解法2：左边中有 $\exists x(P(x)\wedge Q(x))$. 令左边为真，并设c为成真赋值.

则 $\cancel{\exists x}\; P(c)\wedge Q(c)=T \Rightarrow P(c)=Q(c)=T$.

$\therefore$ 左边 $\models \forall(P(x)\to Q(x)\wedge R(x))\wedge T$

$\models (P(c)\to Q(c)\wedge R(c))$

$\equiv T\to Q(c)\wedge R(c)$

$\equiv Q(c)\wedge R(c)$

$\equiv \exists x(Q(x)\wedge R(x))$