第五周作业答案

1.[每道题0.3分，共1.2分]

(1) $\{\varnothing, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$

(2) $\{\varnothing, \{a\}, \{\varnothing\}, \{\{a\}\}, \{\varnothing, a\}, \{a, \{a\}\}, \{\varnothing, \{a\}\}, \{\varnothing, a, \{a\}\}\}$

(3) $\{\varnothing, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}$

2.[每道题0.4分，共1.2分]

(1) $A \times B = \{< 0, 1 >, < 0, 2 >, < 1, 1 >, < 1, 2 >\}$

(2) $A^2 = \{< 0, 0 >, < 0, 1 >, < 1, 0 >, < 1, 1 >\}$

所以 $A^2 \times B = \{< 0, 0, 1 >, < 0, 0, 2 >, < 0, 1, 1 >, < 0, 1, 2 >, < 1, 0, 1 >, < 1, 0, 2 >, < 1, 1, 1 >, < 1, 1, 2 >\}$

(3) $B \times A = \{< 1, 0 >, < 1, 1 >, < 2, 0 >, < 2, 1 >\}$

3.[每道题0.4分，共1.6分]

(1)正确。因为 $B \subseteq C$, 表示 $\forall x \in B$，有$x \in C$，又因为$A \in B$，所以$A \in C$

(2)错误。反例: $A = \{1\}, B = \{\{1\}\}, C = \{\{1\}\}$

(3)错误。反例: $A = \{1\}, B = \{1, 2\}, C = \{\{1, 2\}\}$

(4)错误。反例: $A = \{1\}, B = \{1, 2\}, C = \{\{1, 2\}\}$

4.[每道题0.5分，共1分]

(1)证明: $(A \cap B) - (A \cap C)$

$= \{x \mid (x \in A \wedge x \in B) \wedge \neg(x \in A \wedge x \in C)\}$

$= \{x \mid x \in A \wedge x \in B \wedge (\neg x \in A \vee \neg x \in C)\}$

$= \{x \mid (x \in A \wedge x \in B \wedge \neg x \in A) \vee (x \in A \wedge x \in B \wedge \neg x \in C)\}$

$= \{x \mid F \vee (x \in A \wedge x \in B \wedge \neg x \in C)\}$

$= \{x \mid x \in A \wedge x \in B \wedge \neg x \in C\}$

$= \{x \mid x \in A\} \cap \{x \mid x \in B \wedge \neg x \in C\} = A \cap (B - C)$

(2)证明: $(A-B)-C$

$= \{x \mid x \in A \wedge \neg x \in B\} - C$

$= \{x \mid x \in A \wedge \neg x \in B \wedge \neg x \in C\}$

$= \{x \mid x \in A \wedge \neg(x \in B \vee x \in C)\}$

$= \{x \mid x \in A\} - \{x \mid x \in B \vee x \in C\}$

$= A - (B \cup C)$