- $\int \frac{1}{1+e^{2x}} \cdot dx$

$$a = 1 \qquad\qquad\qquad\qquad x^2 = e^{2x} \to x = e^x$$

$$= \frac{1}{a}\arctan(\frac{x}{a}) = \arctan(e^x)$$

- $\int \sqrt{\frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)}} \cdot dx$

$$\cosh(x) + \sinh(x) = e^x$$
$$\cosh(x) - \sinh(x) = e^{-x}$$

$$\int \sqrt{\frac{e^{-x}}{e^x}} = \int \sqrt{e^{-2x}} = \int e^{-x}$$
$$= -e^{-x} + C$$

- $\int \frac{1}{\sqrt{2-x^2}} \cdot dx$

$$a^2 = 2 \rightarrow a = \sqrt{2} \qquad x^2 = x^2 \rightarrow x = x$$

$$= \arcsin(\frac{x}{a}) = \arcsin(\frac{x}{\sqrt{2}}) + C$$

- $\int \sqrt{e^x} \cdot dx$

$$
\begin{aligned}
&= \int e^{\frac{1}{2}x} \\
&= \frac{e^{\frac{1}{2}x}}{\frac{1}{2}} \\
&= 2\sqrt{e^x} + C
\end{aligned}
$$

- $\int \frac{1}{x\sqrt{x^2-2}} \cdot dx$

$$a^2 = 2 \to a = \sqrt{2} \qquad\qquad x^2 = x^2 \to x = x$$

$$\begin{aligned} &= \frac{1}{a}\operatorname{arcsec}(\frac{x}{a}) \\ &= \frac{1}{\sqrt{2}}\operatorname{arcsec}(\frac{x}{\sqrt{2}}) \end{aligned}$$

- $\int \frac{1}{x^2+6x+13} \cdot dx$

$$= \int \frac{1}{(x+\frac{6}{2})^2 - (\frac{6}{2})^2 + 13}$$

$$= \int \frac{1}{(x+3)^2 + 4}$$

$$= \frac{1}{2} \arctan(\frac{x+3}{2}) + C$$

- $\int (e^x + 3^x)^2 \cdot dx$

$$= \int e^{2x} + 9^x + 2 \times e^x \times 3^x$$

$$= \int e^{2x} + 9^x + 2 \times 3^x e^x$$

$$= \frac{e^{2x}}{2} + \frac{9^x}{\ln(9)} + 2 \times \frac{3^x e^x}{1 + \ln(3)} + C$$

- $\displaystyle\int \frac{1}{\sqrt{x^2+4x+5}} \cdot dx$

$$= \int \frac{1}{\sqrt{(x+2)^2+1}} \cdot dx$$

$$= \operatorname{arcsinh}((x+2)) + C$$

- $\int \frac{1}{4+16x^2} \cdot dx$

$$= \int \frac{1}{16(x^2 + \frac{4}{16})}$$

$$= \frac{1}{16} \times 2 \arctan(2x) + C$$

$$= \frac{1}{8} \times \arctan(2x) + C$$

- $\int \frac{1}{\sqrt{x}(\sqrt{x}-1)} \cdot dx$

$$u = \sqrt{x} \qquad du = \frac{1}{2\sqrt{x}} \cdot dx$$

$$= 2\int \frac{1}{2\sqrt{x}(\sqrt{x}-1)} \cdot dx$$

$$= 2\int \frac{1}{(u-1)} \cdot du$$

- $\int e^x \coth(e^x) \cdot dx$

$$u = e^x \qquad\qquad du = e^x \cdot dx$$

$$= \int \coth(u) \cdot du$$

$$= \ln(\sinh(u))$$

$$= \ln(\sinh(e^x)) + C$$

- $\int \frac{e^x}{\sqrt{e^{2x} - 2e^x}} \cdot dx$

$$u = e^x \qquad du = e^x \cdot dx$$

$$= \int \frac{1}{\sqrt{u^2 - 2u}} \cdot du$$

$$= \int \frac{1}{\sqrt{(u-1)^2 - 1}}$$

$$= \operatorname{arccosh}(u - 1)$$

$$= \operatorname{arccosh}(e^x - 1) + C$$

- $\frac{e^x}{4+e^2x} \cdot dx$

$$u = e^x \qquad du = e^x \cdot dx$$

$$= \int \frac{1}{u^2+4} \cdot du$$

$$= \int \frac{1}{u^2+4}$$

$$= \frac{1}{2} \arctan(\frac{u}{2})$$

$$= \frac{1}{2} \arctan(\frac{e^x}{2}) + C$$

- $\int \frac{\operatorname{sech}(x)\tanh(x)}{1+\operatorname{sech}(x)} \cdot dx$

$$u = \operatorname{sech}(x) \qquad\qquad du = -\operatorname{sech}(x)\tanh(x) \cdot dx$$

$$= -\int \frac{1}{1+u} \cdot du$$

$$= -\ln(u+1)$$

$$= \ln\left(\frac{1}{\operatorname{sech}(x)+1}\right) + C$$

- $\displaystyle\int \frac{\cosh(x)}{\sqrt{\sinh^2(x) - 2\sinh(x)}} \cdot dx$

$$u = \sinh(x) \qquad\qquad du = \cosh(x) \cdot dx$$

$$= \int \frac{1}{\sqrt{u^2 - 2u}} \cdot du$$

$$= \int \frac{1}{\sqrt{(u-1)^2 - 1}} \cdot du$$

$$= \operatorname{arccosh}(u - 1)$$

$$= \operatorname{arccosh}(\sinh(x) - 1) + C$$

- $\frac{1}{\sqrt{x}(x+1)} \cdot dx$

$$u = \sqrt{x} \qquad du = \frac{1}{2\sqrt{x}} \cdot dx \qquad x = u^2$$

$$= 2\int \frac{1}{u^2+1} \cdot du$$

$$= \arctan(u)$$

$$= \arctan(\sqrt{x}) + C$$

- $\int \frac{\operatorname{sech}^2(x)}{3+\operatorname{sech}^2(x)} \cdot dx$

$$u = \tanh(x) \qquad du = \operatorname{sech}^2(x) \cdot dx$$

$$= \frac{\operatorname{sech}^2(x)}{3+1-\tanh^2(x)} \qquad = \frac{1}{4-u^2} \cdot du$$

$$= \frac{1}{2}\operatorname{arctanh}(\frac{u}{2}) \qquad = \frac{1}{2}\operatorname{arctanh}(\frac{\tanh(x)}{2}) + C$$

- $\int (x\ln(x))^2 \cdot dx$

$$u = \ln(x)^2 \qquad du = \frac{2\ln(x)}{x}$$

$$v = \frac{x^3}{3} \qquad dv = x^2$$

$$= \int x^2 \ln(x)^2 \cdot dx$$

$$= \frac{x^3}{3} \times \ln(x)^2 - \int \frac{2x^2\ln(x)}{3}$$

$\frac{2}{3} \int x^2 \ln(x) \cdot dx$

$$u = \ln(x) \qquad du = \frac{1}{x}$$

$$v = \frac{x^3}{3} \qquad =dv = x^2$$

$$= \frac{2x^3\ln(x)}{9} - \int \frac{2x^2}{9}$$

$$= \frac{2x^3\ln(x)}{9} - \frac{2x^3}{27}$$

$$= \frac{x^3}{3}\ln(x)^2 - \frac{2x^3\ln(x)}{9} - \frac{2x^3}{27}$$