MATH112

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Contents

Chapter 1

Integration

Integration is the anti-derivative of a function.

$$\frac{d}{dx}(F(x)) = f(x) \leftrightarrow \int f(x) = F(x)$$

1.1 Basic integration formulas

$\int$	$\frac{d}{dx}$
$x^n \cdot dx$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{f(x)} \cdot dx$	$\ln(f'(x))$
$a^x \cdot dx$	$\frac{a^x}{\ln(a)}$

1.2 Basic trignometric identites

$\sin^2(x) + \cos^2(x)$	1
$\cosh^2(x) - \sinh^2(x)$	1
$\sec^2(x) - \tan^2(x)$	1
$\tanh^2(x) + \operatorname{sech}^2(x)$	1
$\csc^2(x) - \cot^2(x)$	1

Chapter 2

Integration by Substitution

Integrating by substitution is considered a ”backwards” chain rule.

$$\frac{d}{dx}\sin(x^2) = \cos(x^2) \cdot 2x$$
$$\int (\cos(x^2) \cdot 2x) \cdot dx = \sin(x^2) + C$$

To easily integrate, x^2 can be considered as u and $2x \cdot dx$ as du which can be easily integrated.

$$\int \cos(u) \cdot du = \sin(u) + C$$

After re-substituting:

$$\int \cos(u) \cdot du = \sin(u) + C \to \sin(x^2) + C$$

Therefore to integrate by substitution, the derivatice of the part of the function substituted needs to be present for the integration to be successful.

Examples

1. $\int \frac{1}{x \times \ln(x)} \cdot dx$

$$u = \ln(x), \qquad du = \frac{1}{x} \cdot dx$$
$$\int \frac{1}{u} \cdot du = \ln(u) + c = \ln(\ln(x)) + C$$

2. $\int \frac{e^{\tan(x)}}{\cos^2(x)} \cdot dx$

$$\int \frac{e^{\tan(x)}}{\cos^2(x)} \cdot dx = e^{\tan(x)} \times \sec^2(x) \cdot dx$$
$$u = \tan(x), \qquad du = \sec^2(x) \cdot dx$$

$$\int e^u \cdot du = e^u + C = e^{\tan(x)} + C$$

3. $\int \frac{e^{\frac{1}{x}}}{x^2} \cdot dx$

$$u = \frac{1}{x}, \qquad du = -\frac{1}{x^2} \cdot dx$$

$$-\int e^u \cdot du = -e^u + C = -e^{\frac{1}{x}} + C$$

4. $\int x^3 \sqrt{x^2 - 1} \cdot dx$

$$\int x \times x^2 \sqrt{x^2 - 1} \cdot dx$$

$$u = x^2 - 1 \rightarrow x^2 = u + 1, \qquad du = 2x \cdot dx$$

$$\frac{1}{2} \times \int (u+1)\sqrt{u} \cdot du = \ldots$$

Integration Shortcuts

$$\begin{aligned} \int \frac{z'}{z} &= \ln(z) + C \\ \int \frac{z'}{\sqrt{z}} &= 2\sqrt{z} + C \\ \int z^n \times z' &= \frac{z^{n+1}}{n+1} + C \end{aligned}$$

Chapter 3

Integration by Parts

Integration by parts is used to find the integral of the product of two functions.

$$d(u \times v) = u \cdot dv + v \cdot du$$
$$\therefore u \cdot dv = d(u \times v) - v \cdot du$$

By integrating both sides of the equation.

$$\int u \cdot dv = \int d(u \times v) - \int v \cdot du$$

Therefore:

$$\boxed{\int u \cdot dv = \int uv - \int v \cdot du}$$

Examples

- $x \times e^x \cdot dx$

$$\begin{aligned} u &= x, & du &= 1 \cdot dx \\ v &= e^x, & dv &= e^x \end{aligned}$$

$$\begin{aligned} \therefore \int x \times e^x &= x \times e^x - \int e^x \cdot dx \\ &= x \times e^x - e^x + C \end{aligned}$$

3.1 First Category

A polynomial function and a function easily integrated.

Examples

1. $\displaystyle\int x \times \cos(2x) \cdot dx$

$$
\begin{aligned}
u &= x & du &= 1 \cdot dx \\
v &= \frac{\sin(2x)}{2} & dv &= \cos(2x)
\end{aligned}
$$

$$
\begin{aligned}
\int x \times \cos(2x) &= x \times \frac{\sin(2x)}{2} - \int \frac{\sin(2x)}{2} \cdot dx \\
&= \frac{x\sin(2x)}{2} + \frac{\cos(2x)}{4}
\end{aligned}
$$

2. $\displaystyle\int x \times \sec^2(x) \cdot dx$

$$
\begin{aligned}
u &= x & du &= 1 \cdot dx \\
v &= \tan(x) & dv &= \sec^2(x)
\end{aligned}
$$

$$
\begin{aligned}
\int x \times \sec^2(x) &= x \times \tan(x) - \int \tan(x) \cdot dx \\
&= x\tan(x) - \int \frac{\sin(x)}{\cos(x)} \cdot dx \\
&= x\tan(x) + \ln(\sec(x))
\end{aligned}
$$

3.1.1 Tabular form