Definite Integral

The definite integral

The definite integral, unlike the indefinite integral, returns a numerical value.

$$\int_a^b f(x).\ \mathrm{dx} = F(x)\ |_a^b = F(b) - F(a)$$

Examples

1. $\int_2^5 8.\ \mathrm{dx}$

sol:

$$= \ 8x\ |_2^5 = 8(5) - 8(2) = 24$$

2. $\int_1^4 (5x - 4).\ \mathrm{dx}$

sol:

$$= \ \left(\frac{5}{2}\right)x^2 - 4x\ |_1^4 = \left(\frac{5(4)^2}{2} - 4(4)\right) - \left(\frac{5(1)^2}{2} - 4(2)\right)$$
$$= 25.5$$

3. $\int_{-3}^4 \frac{8}{x^3}.\ \mathrm{dx}$

sol:

$$= \ \left(-\frac{8}{2}\right)x^{-2}\ |_{-3}^4 = \left(-\frac{8}{2(4)^2}\right) - \left(\frac{8}{(2(-3))^2}\right)$$
$$= -\frac{1}{4} + \frac{4}{9} = \frac{7}{36}$$

4. $\int_1^e \frac{5}{x}.\ \mathrm{dx}$

sol:

$$= \ 5\int_1^e \frac{1}{x} = 5\ln(x)\ |_1^e = 5\ln(e) - 5\ln(1)$$
$$= 5(1) - 5(0) = 5$$

5. $\int_4^9 \frac{1}{\sqrt{x}}.\ \mathrm{dx}$

sol:

$$= \ 2\sqrt{x}\ |_4^9 = 2\sqrt{9} - 2\sqrt{4}$$
$$= 2$$

6. $\displaystyle\int_0^{\frac{\pi}{2}} \cot^4(x) \times \sin^7(x).\ \mathrm{dx}$

sol:

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^4(x)}{\sin^4(x)} \times \sin^7(x)$$

$$= \int_0^{\frac{\pi}{2}} \cos^4(x) \times \sin^3(x)$$

$$= \int \cos^4(x) \times (1 - \cos^2(x)) \times \sin(x)$$

$$\rightarrow u = \cos(x) \rightarrow \mathrm{du} = -\sin(x)$$

$$\rightarrow \int u^4 \times (u^2 - 1).\ \mathrm{du}$$

$$= \int u^6 - u^4.\ \mathrm{du} = \frac{u^7}{7} - \frac{u^5}{5}$$

$$\rightarrow \left[\frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5}\right]_0^{\frac{\pi}{2}} = \frac{\cos^7(\frac{\pi}{2})}{7} - \frac{\cos^5(0)}{5}$$

7. $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cot^n(x)}{\sin^2(x)}.\ \mathrm{dx} = \frac{1}{k-1}$

Find the value of k.

sol:

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{\cos^n(x)}{\sin^{2+n}(x)}\right)$$

$$\rightarrow u = \sin(x) \rightarrow \mathrm{du} = \cos(x)$$

$$\int \left(\frac{\cos^{n-1}(x)}{u^{2+n}}\right).\ \mathrm{du} =$$