EE208: CONTROL ENGINEERING LAB PROJECT REPORT

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Objectives:

The project requires design of a cascade feedback controller for a given analog transfer function,

according to desired specifications.

A sensitivity analysis for variation of key parameters is further required.

Given System:

$$G(s)H(s) = \frac{k(s + 1.5)}{s(1 + Ts)(1 + 2.5s)}$$

(to include the parameter variation let 1.5 be denoted by a and 2.5 by b).

$$G(s)H(s) = \frac{k(s + a)}{s(1 + Ts)(1 + bs)}$$

The closed loop transfer function:

$$G_{cl}(s) = \frac{k(s + a)}{Tbs^3 + (T + b)s^2 + (k + 1)s + ak}$$

Also, given T and k assume positive values.

Using Routh-Hurwitz to get Design Requirements:

For a cubic polynomial $P(x) = Ax^3 + Bx^2 + Cx + D$,

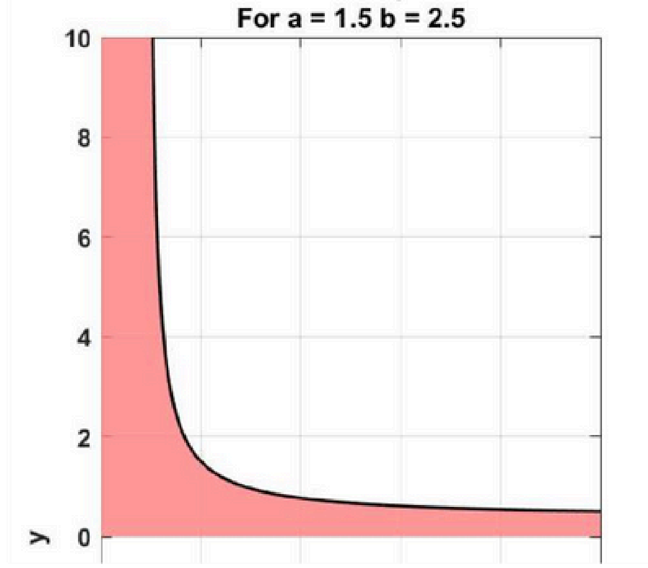
the Routh-Hurwitz conditions are:

- B > 0
- C > 0
- B*C > A*D

If these conditions are satisfied, all roots will have negative real parts, ensuring the real part of the complex roots is also negative.

(Note: In all the plots Vertical axis is k and horizontal axis is T)

For a=1.5 and b =2.5;



- B > 0

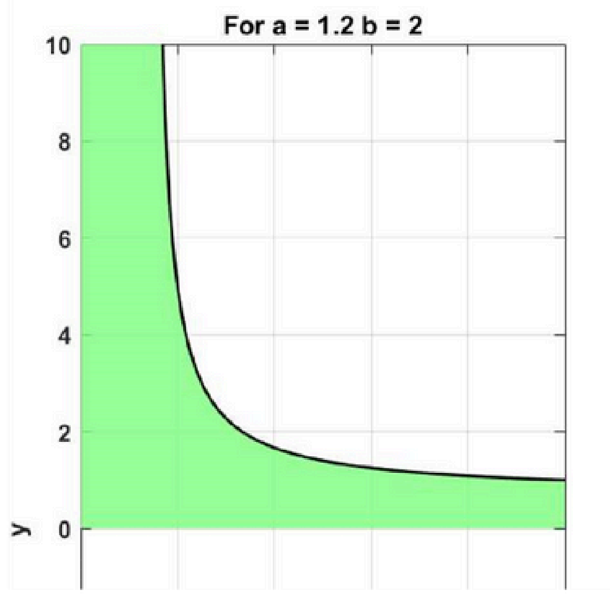
  We get, T > -2.5
- C > 0

  We get, k > -1
- B*C > A*D

  We get (T+2.5)(k+1)>T*2.5*(1.5k)

  2.5k + T +2.5 – 2.75Tk > 0

For a=1.2 and b =2;



- B > 0

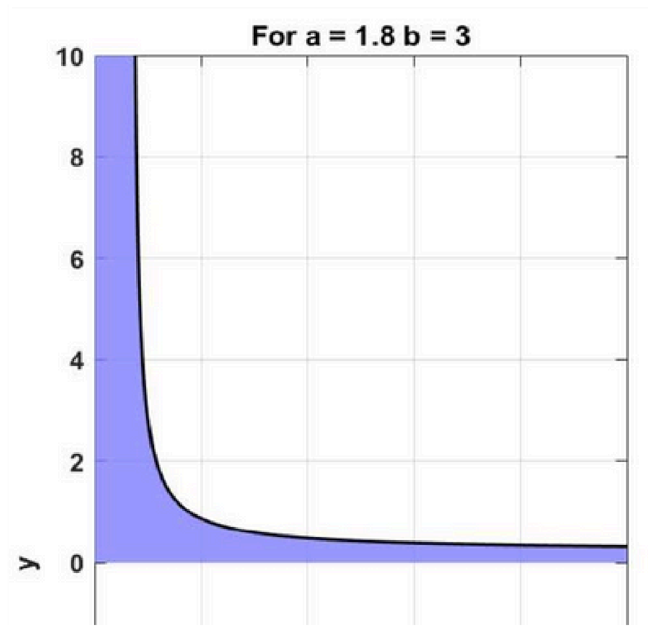
  We get, T > -2
- C > 0

  We get, k > -1
- B*C > A*D

  We get (T+2)(k+1)>T*2*(1.2k)

  -1.4Tk + 2k + T +2> 0

For a=1.8 and b =3;



- B > 0

  We get, T > -3
- C > 0

  We get, k > -1
- B*C > A*D

  We get (T+3)(k+1)>T*3*(1.8k)

  -4.4Tk + 3k + T + 3> 0

For a=1.8 and b =2;

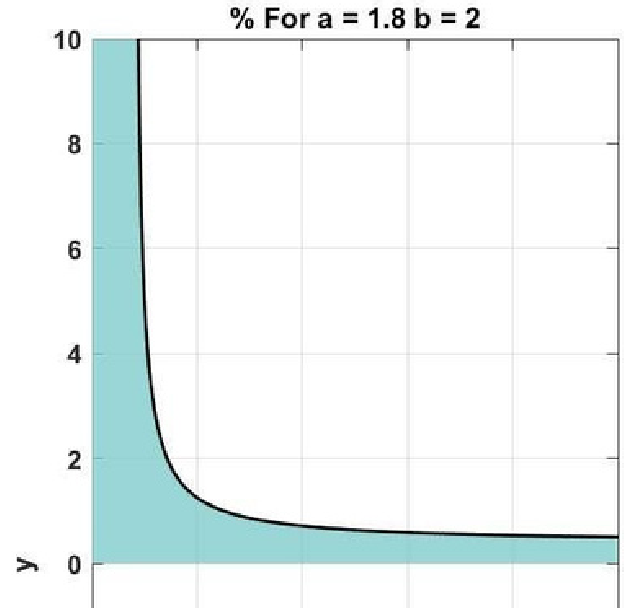
- B > 0

  We get, T > -2
- C > 0

  We get, k > -1
- B*C > A*D

  We get (T+2)(k+1)>T*2*(1.8k)

  -2.6Tk + 2k + T + 2> 0



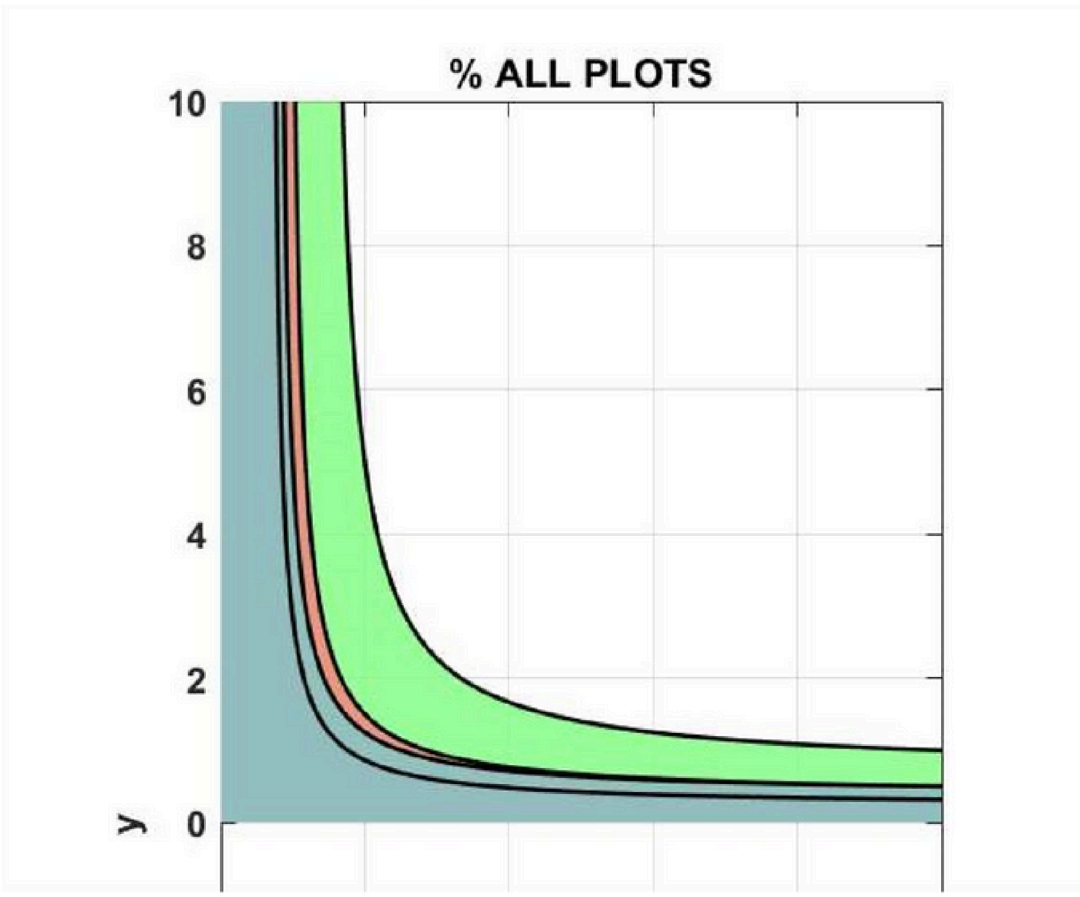
MATLAB Code for the Plots:

untitled.m

```
% Define range for x and y
xrange = 0:0.01:10;
yrange = 0:0.01:10;
[X, Y] = meshgrid(xrange, yrange);

% Evaluate the inequalities numerically
Z1 = 2*Y + X + 2 - 1.4*X.*Y;     % For a = 1.2 b = 2
Z2 = -4.4*X.*Y + 3*Y + X + 3;     % For a = 1.8 b = 3
Z3 = -2.75*X.*Y + 2.5*Y + X + 2.5; % For a = 1.5 b = 2.5
Z4 = -2.6*X.*Y + 2*Y + X + 2; % For a = 1.8 b = 2

% Create a figure
figure('Color', 'w'); % Set background color to white
hold on;

% Define masks for the feasible regions
mask1 = Z1 > 0;
mask2 = Z2 > 0;
mask3 = Z3 > 0;
mask4 = Z4 > 0;
% Plot the feasible regions with contour
contourf(X, Y, mask1, [1 1], 'LineColor', 'none', 'FaceColor', [0.5 1 0.5], 'FaceAlpha', 0.8); % Green
contourf(X, Y, mask2, [1 1], 'LineColor', 'none', 'FaceColor', [0.5 0.5 1], 'FaceAlpha', 0.8); % Blue
contourf(X, Y, mask3, [1 1], 'LineColor', 'none', 'FaceColor', [1 0.5 0.5], 'FaceAlpha', 0.8); % Red
contourf(X, Y, mask4, [1 1], 'LineColor', 'none', 'FaceColor', [0.5 0.8 0.8], 'FaceAlpha', 0.8); % LIGHT BLUE

% Draw the boundaries
contour(X, Y, Z1, [0 0], 'LineColor', 'k', 'LineWidth', 1.5); % Boundary for Z1=0
contour(X, Y, Z2, [0 0], 'LineColor', 'k', 'LineWidth', 1.5); % Boundary for Z2=0
contour(X, Y, Z3, [0 0], 'LineColor', 'k', 'LineWidth', 1.5); % Boundary for Z3=0
contour(X, Y, Z4, [0 0], 'LineColor', 'k', 'LineWidth', 1.5); % Boundary for Z4=0

% Set axis limits and labels
xlim([-10 10]);
ylim([-10 10]);
xlabel('x', 'FontSize', 12);
ylabel('y', 'FontSize', 12);
title('% ALL PLOTS', 'FontSize', 14);
grid on;
axis equal;
```

Combined Plot For All The Cases Of a And b:

Here, the outer refers to a=1.2 and b = 2 and the innermost refers to a=1.8 and b =3. Here we can see that the contour of the innermost has all the regions of the other so to always remain in the stable region one should choose k and T in the closed region



defined by

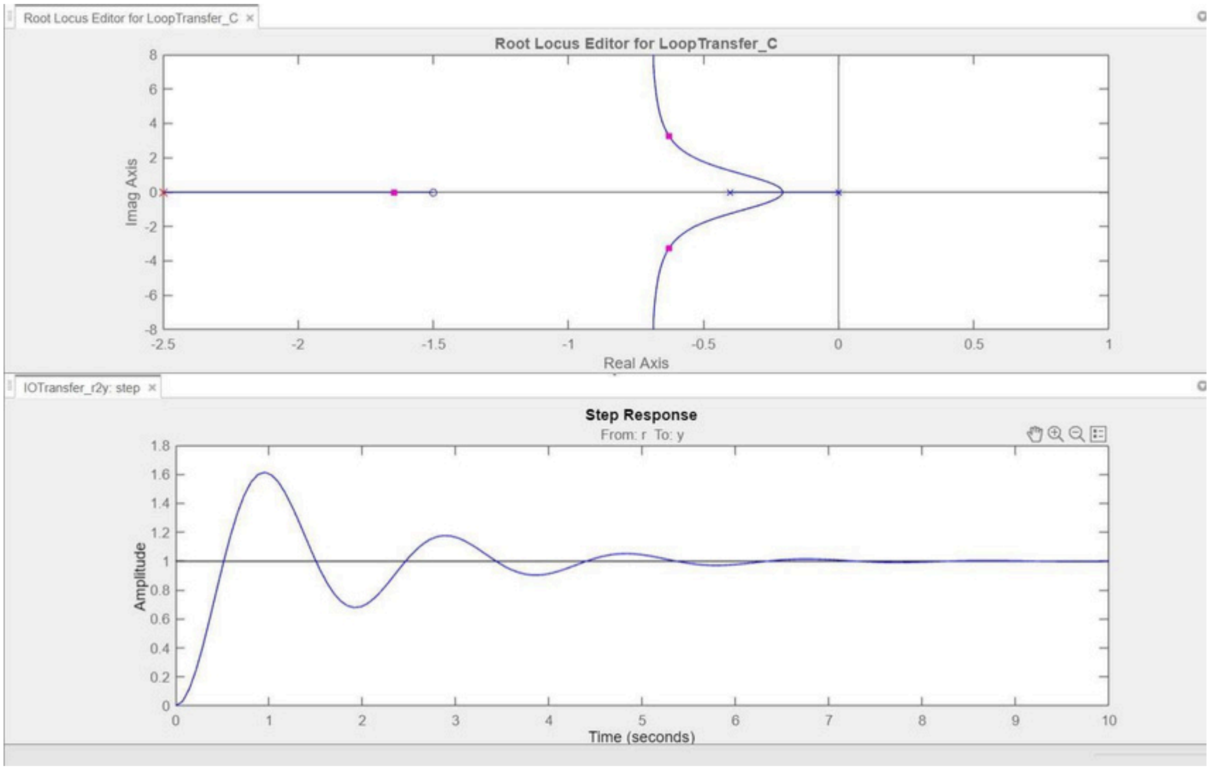
- -4.4Tk + 3k + T + 3> 0
- T>0
- k>0

It also provides stability with 20% variation in the parameters a and b.

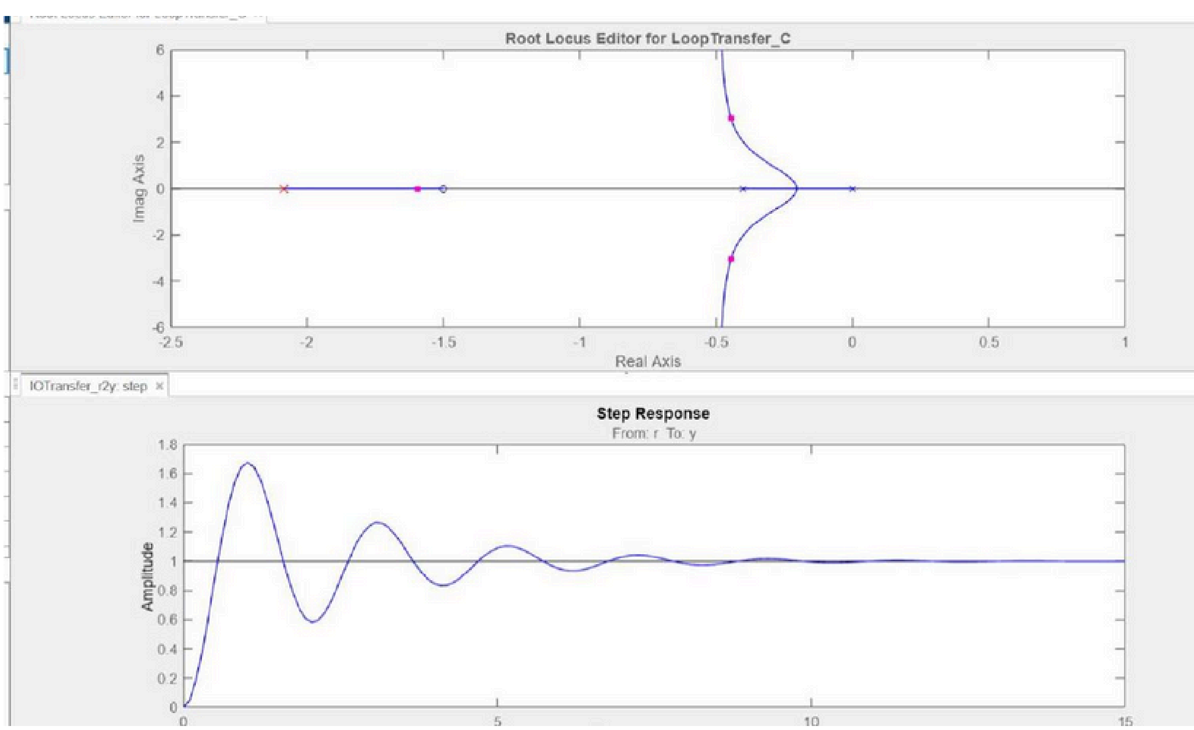
Root Locus Analysis:

Using the above plot now we can choose k and t for which there is 20% variation in T will not push the system into instability

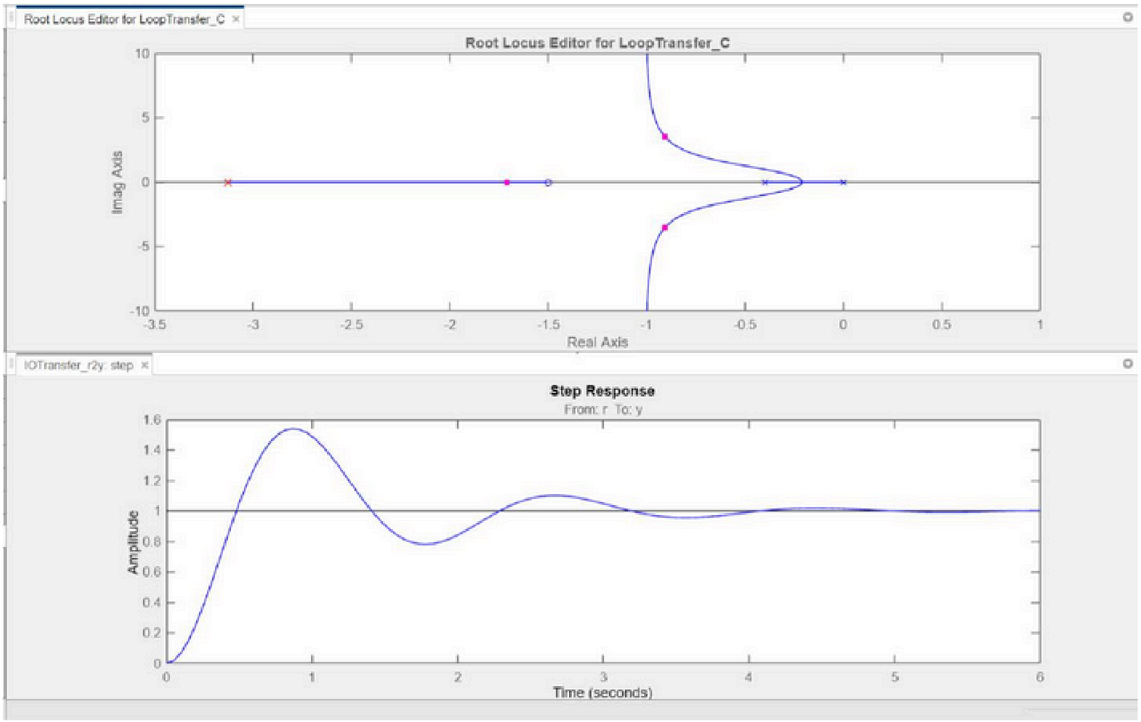
- **T = 0.4 and K = 12**



20% variation in T **T=0.48**

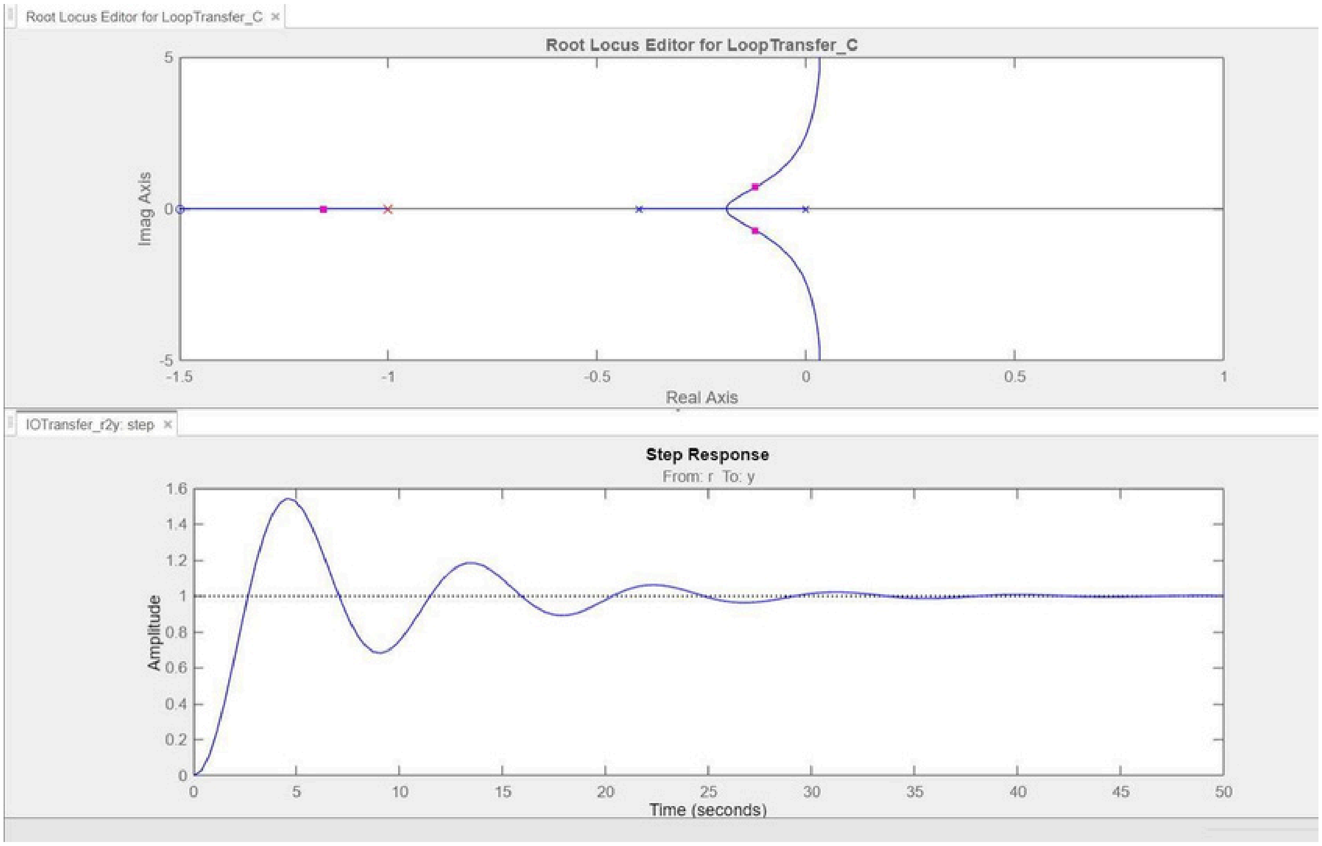


T = 0.32

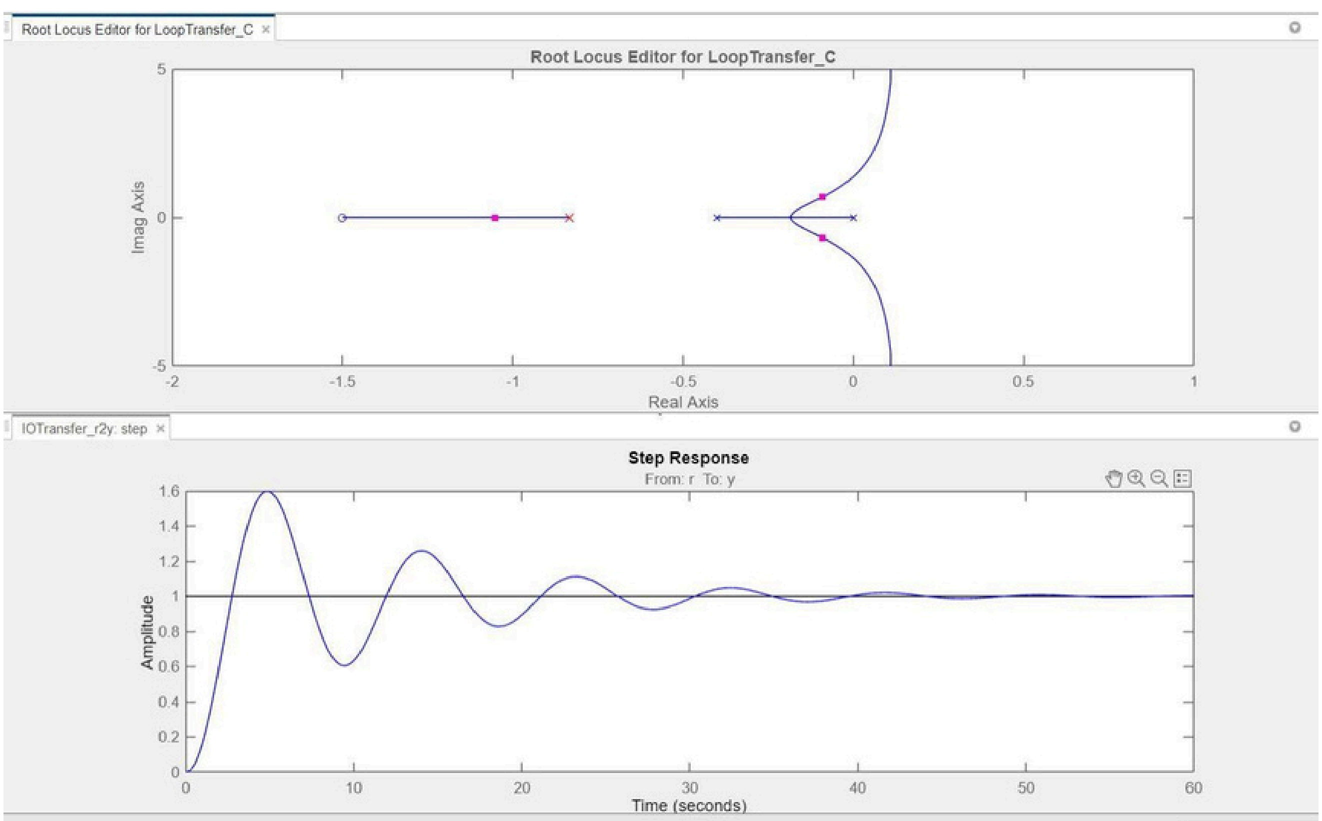


At T = 0.4, system remains stable with 20% variation at k=12. It is selected from the routh Hurwitz criteria in which, T=0.4 and k =12 and its variation lies in the stable region. Increment in T leads to increasing oscillatory and slow to settle response due to its integral control which is slow in nature. For the specific choice of using T =0.4 and K = 12 only symbolizes the maximum K value we should use, since above 12 stable region gets narrower and there is risk of T leaving the stable region.

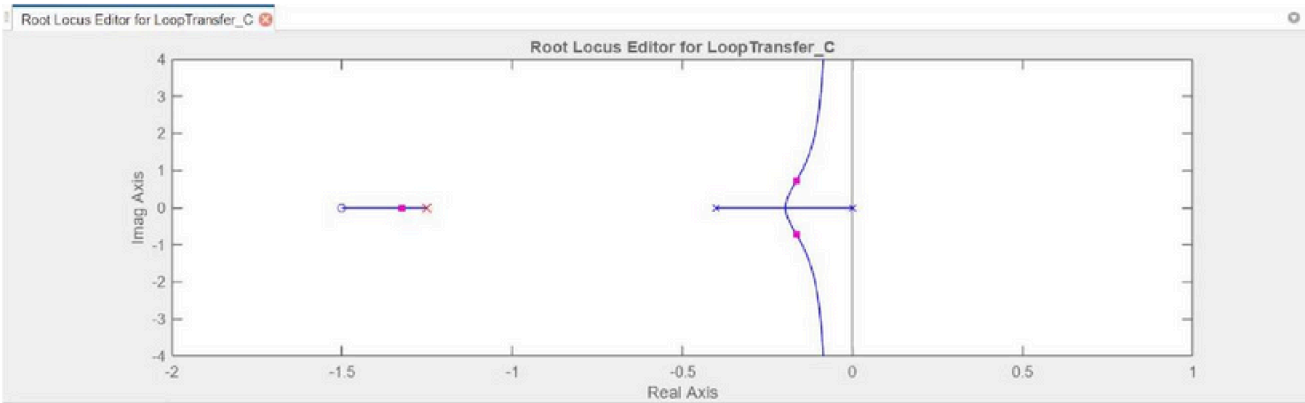
T = 1 and K = 1

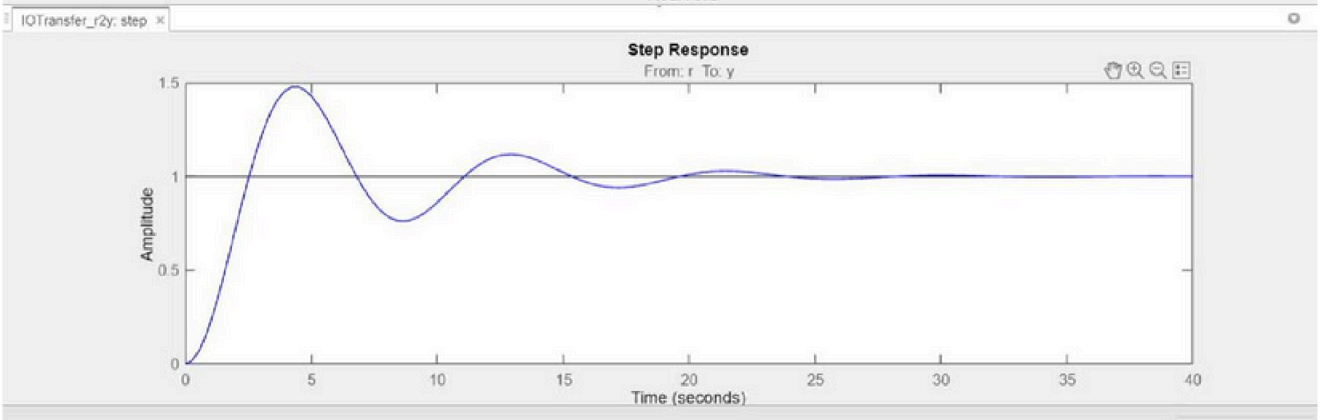


20% variation in T   T = 1.2



T = 0.8





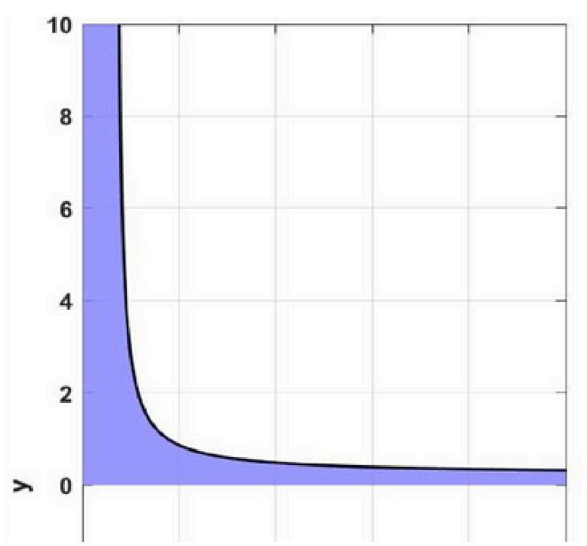
By changing the T= 1 and k=1, overall response becomes significantly oscillatory in nature. Furthermore the 20% variation doesn't lead to unstable or highly undesired CLTF behaviour. When T=1, k can take maximum value upto 4.5 and after that it will drive system to instability and choice of k as 1 represents that further increasing k is unreasonable for step response consideration. Simply meaning choice of k <1 is optimal for the system.

Further increasing the T leads to very high oscillatory nature which is irrelevant. Since a system is supposed to have low settling time and low peak overshoot.

T vs K Graph:

Here in the analysis, we can see for low value of T i.e. upto 2 can have wide range of k values to adjust the requirements as for in regioin T>2, we will have lesser choice of k for the system. So, limit analysis of T upto 2.

(Here vertical axis is k and horizontal is T and each block represents 2 units)



In the T vs K graph, there are two distinct regions highlighted: the **green region** and the **red region**.

- **Blue Region**: This area represents the **stable region** for various values of K (gain) and T (time constant). Observations show that the blue region predominates on the left side of the graph. It covers a wide range of T values, indicating stability for lower time constants. Furthermore, the stable region is predominant when the gain (K) values are less than approximately 15.
- White region: The area marked in white indicates the **unstable region**. As gain (K) increases beyond approximately 15, the system tends to become highly oscilatory for almost every value of T. This suggests that higher gain values significantly affect the system's stability, with the white region encompassing nearly all higher T values except for very low time constants. The significance of this white area is that it highlights the critical transition point where stability can no longer be assured as the gain rises.

This analysis emphasizes the importance of maintaining low gain values to ensure stability across a range of time constants. The transition from the green (stable) to the red (unstable) region reflects the delicate balance in control systems, where exceeding certain limits can lead to instability, impacting system performance.

Conclusion:

In this experiment, we worked on designing a control system using MATLAB's root locus tools, aiming to achieve both stability and good dynamic performance. The root locus method was very helpful for designing controllers, especially for analog systems. It allowed us to see how changing important system parameters, like gain (k) and time constant (T), affected the position of closed-loop poles, which directly impacts the system's stability and performance. By adjusting these parameters and observing how the system responded, we gained a better understanding of how sensitive the system is to changes in these values and how they affect the overall system behaviour.

We also performed a sensitivity analysis by looking at how a ±20% change in parameters influenced the system's stability. This showed us that even small changes can have a big impact on stability and performance, highlighting the need for a robust design that can handle variations. The root locus diagrams were especially useful in helping us visualize and understand the complex relationship between the system's parameters and its stability.

Additionally, the K vs. T stability region graph played an important role in helping us identify the ranges of parameters where the system stays stable and responsive. This graph allowed us to make better decisions about which parameter values to choose in order to balance stability with dynamic performance.

Overall, this experiment emphasized the importance of a careful and systematic approach to designing control systems. It showed how crucial it is to fine-tune parameters to ensure both stability and robustness, as even small variations in parameters can greatly affect system performance.