Mathematical Formulae

Sad Baguette

Part I

Trigonometry

1 Sum and Difference

When the argument is a sum or difference:

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

2 Double Angle

When the argument is twice of a desired angle:

$$\sin(2u) = 2 \sin u \cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u$$

$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$$

3 Half Angle

When the argument is half of a desired angle:

$$\sin\frac{u}{2} = \frac{1-\cos u}{2}$$

$$\cos\frac{u}{2} = \frac{1+\cos u}{2}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{1+\cos u}$$

4 Sum to Product

When the sum of two sine/cosine functions needs to be expressed as a product:

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\sin\left(\frac{u-v}{2}\right)\cos\left(\frac{u+v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

5 Product to Sum

When the product of two sine/cosine functions needs to be expressed as a sum:

$$\sin u \sin v = -\frac{1}{2}[\cos(u+v) - \cos(u-v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$