Lecture 5

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Example

$$f : \mathbb{R}^3 \to \mathbb{R}$$
$$(x, y, z) \mapsto f(x, y, z) = \vec{0}$$

Critical Points

$$\nabla f(x, y, z) = \vec{0}$$
$$\iff (f_x, f_y, f_z) = (0, 0, 0)$$
$$\iff (yz, xz, xy) = (0, 0, 0)$$
$$xy = yz = zx = 0$$
$$(x, 0, 0) \forall x \in \mathbb{R}$$
$$(0, y, 0) \forall y \in \mathbb{R}$$
$$(0, 0, z) \forall z \in \mathbb{R}$$
$$(0, 0, 0)$$

$$H(x, y, z) = \begin{pmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{pmatrix}$$
$$= \begin{pmatrix} 0 & z & y \\ z & 0 & x \\ y & x & 0 \end{pmatrix}$$

At the origin

$$H(\vec{0}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

All eigenvalues are $0 \Rightarrow$ inconclusive.
But we can see that $\vec{0}$ is a saddle point as for
any $r > 0 \in B(\vec{0}, r)$, we take $(\frac{r}{2}, \frac{r}{2}, \frac{r}{2})$, $(-\frac{r}{2}, -\frac{r}{2}, -\frac{r}{2})$

At the point $(0, 0, 1)$

$$H(0,0,1) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$