CCC '20 S5 Let b_i be the burger i^{th} person ordered.
Let $dp[i]$ be the probability that in $[i, \dots, N-1]$, coach starts (replacing b_i w/ b_0) and Josh gets to eat his burger.

BC: $dp[N-1] = 0$.

Recurrence: Fix i, compute $dp[i]$.
If coach eats his own burger $\Rightarrow$ 1, Josh wins.
If coach eats burger b, the last person who ordered burger b will end up w/ coach's situation.

Let a_b be last orderer of burger b.
We simply get $\frac{c_b}{N-i} \cdot dp[a_b]$, and sum. $O(M^2)$.

From $dp[i]$ to $dp[i-1]$: There is an extra b_i.

Then $$dp[i] = \sum_{\substack{a_b > i \\ b \neq b_i}} \frac{c_b}{N-i} \cdot dp[a_b] + \frac{c_{b_0}+1}{N-i}.$$

Notice $$dp[i-1] = \sum_{a_b > i-1} \frac{c_b}{N-i+1} \cdot dp[a_b] + \frac{c_{b_0}+1}{N-i+1}.$$

Drop the $\frac{c_\cdot}{i}$ terms: we have

$$\sum_{a_b > i-1} \frac{c_b}{N-i+1} \cdot dp[a_b] = \frac{N-i}{N-i+1} \cdot \sum_{a_b > i} \frac{c_b}{N-i} dp[a_b]$$

$$+ \frac{dp[a_{b_i}]}{N-i+1},$$ as c_x increments iff $x = b_i$,

and if $x \neq b_i$, c_x remains same.

This doesn't work for the case $b_i = b_0$. In this case we just evaluate $dp[i-1] = \frac{(N-i)\,dp[i] + 1}{N-i+1}$ $(b_i = b_0)$

o/w $$dp[i-1] = \frac{(N-i)\,dp[i] + dp[a_{b_i}]}{N-i+1} \quad (\text{o/w})$$

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