Announcements

- HW1 is due **Tuesday, January 30**, 11:59 PM PT
- Project 1 is due **Friday, February 2**, 11:59 PM PT

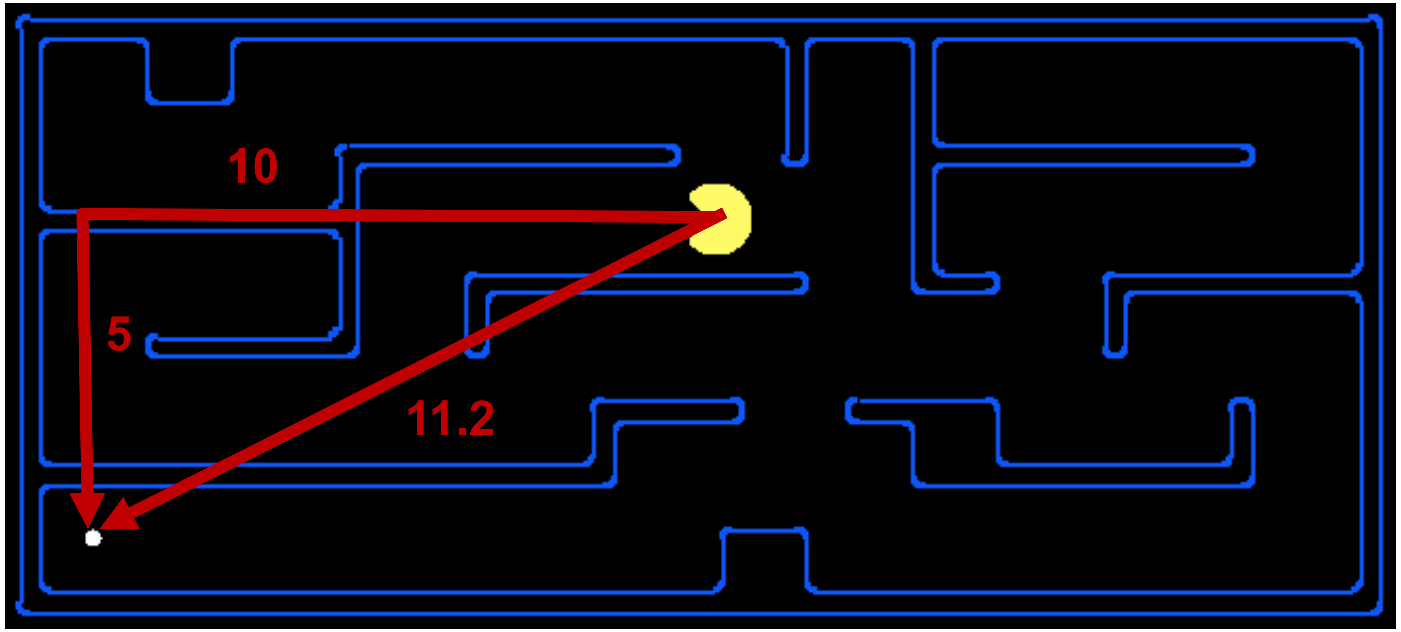
Pre-scan attendance QR code now!

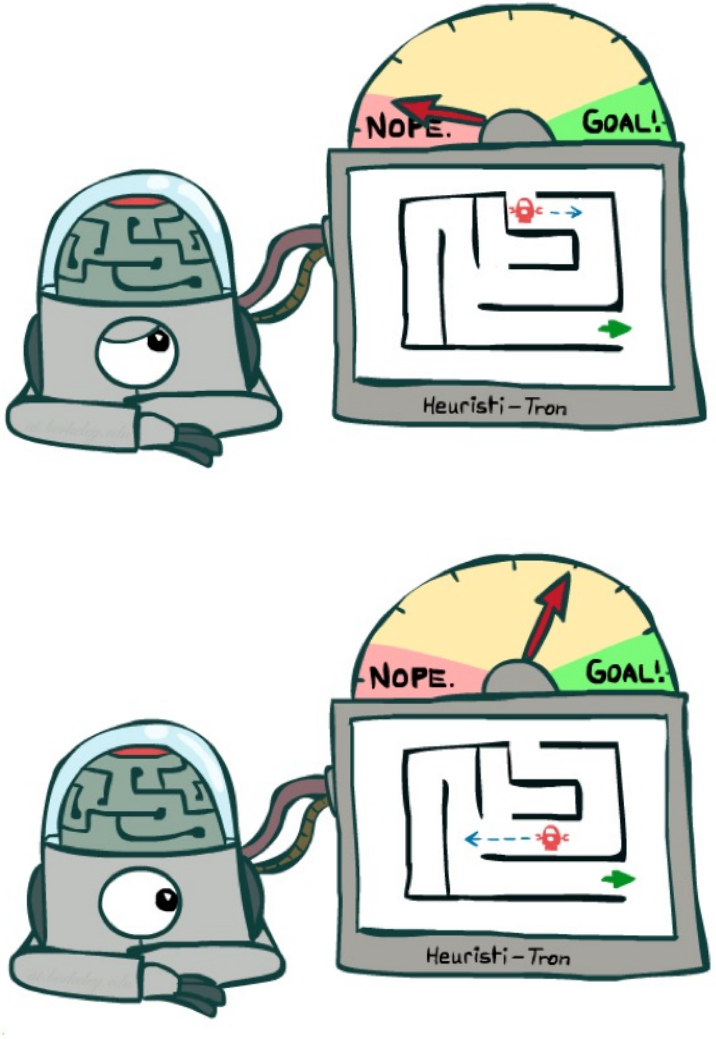
(Password appears later)

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at http://ai.berkeley.edu.]
[Updated slides from: Stuart Russell and Dawn Song]

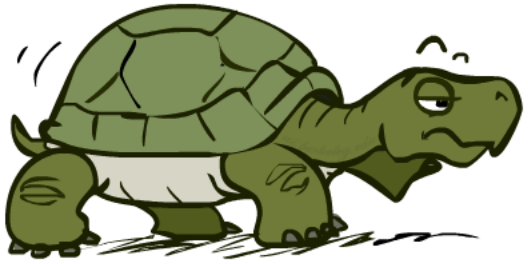
Recap: Search Heuristics

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing

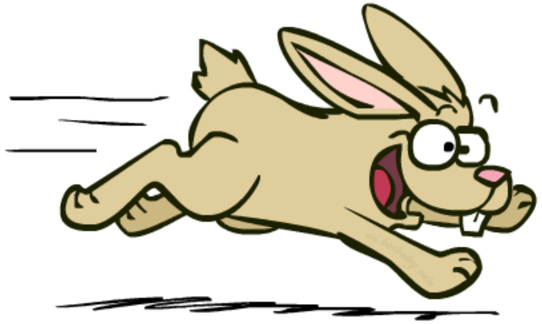




Recap: Cost- vs. Heuristic-Guided Search



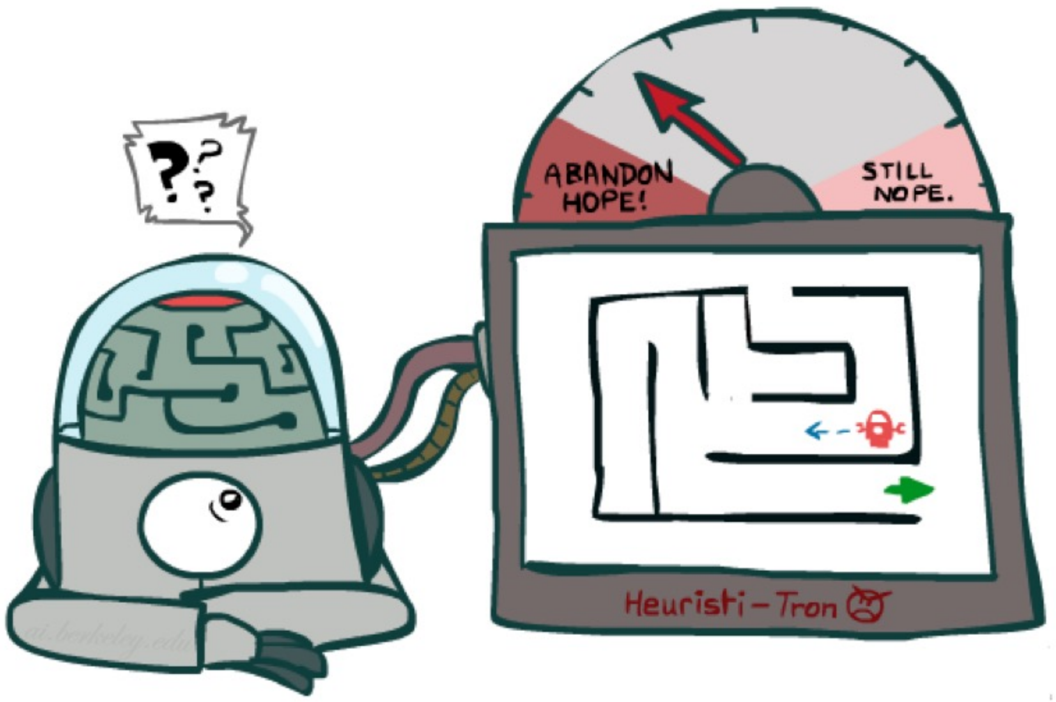
Uniform-Cost Search
(only costs, g)



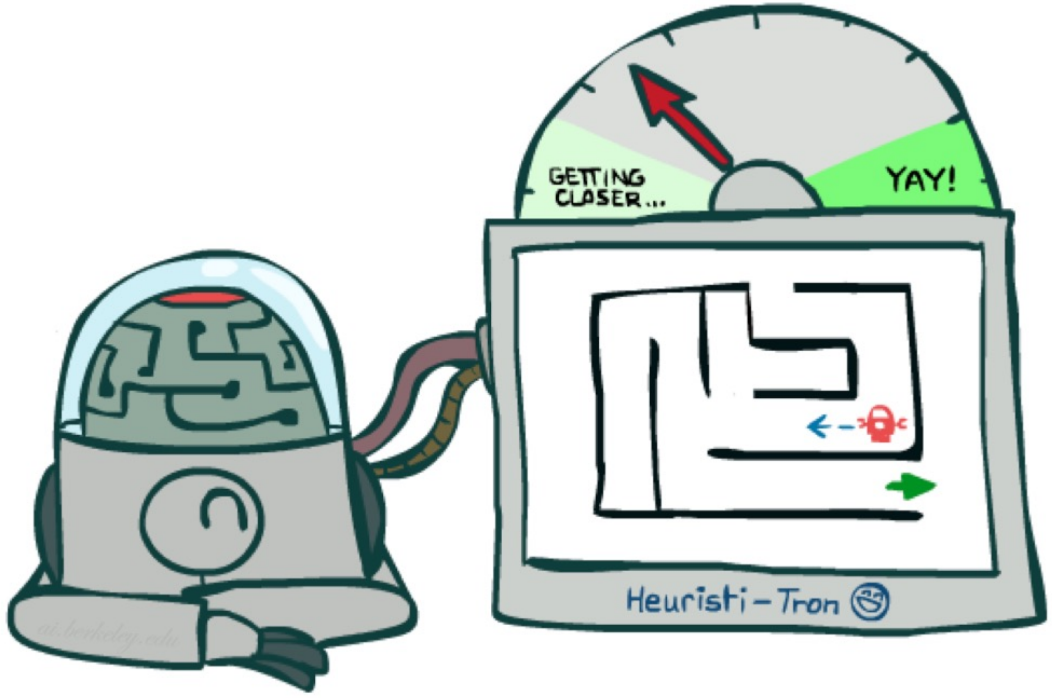
Greedy Best-First Search
(only heuristic, h)

A* Search
(both, f=g+h)

Recap: Admissibility

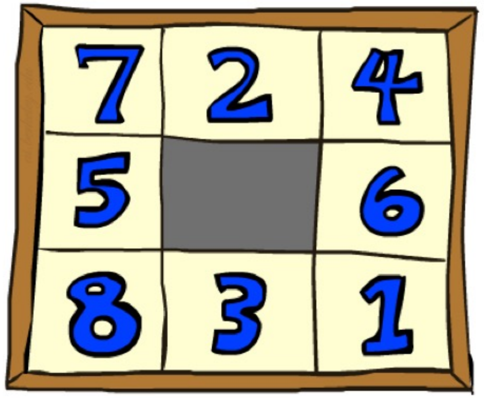


Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe

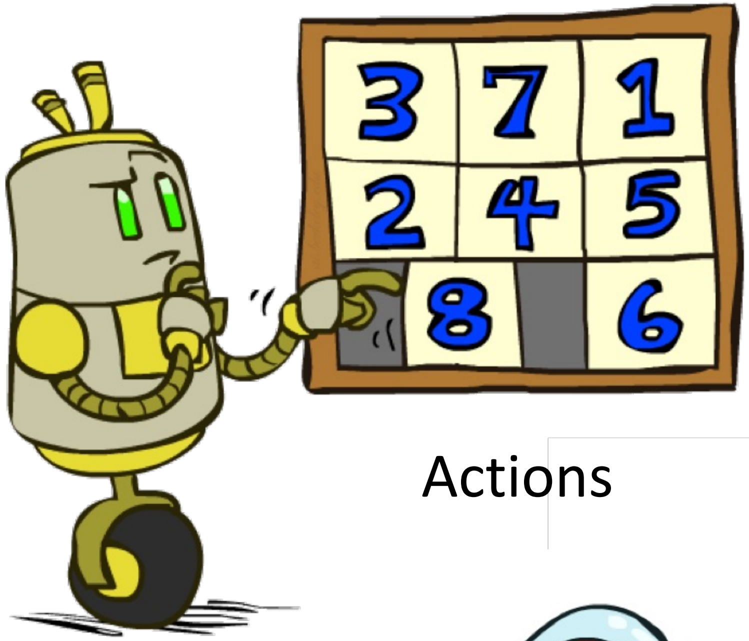


Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

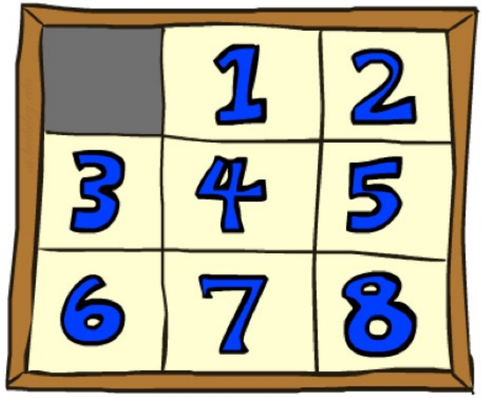
Recap: 8-Puzzle



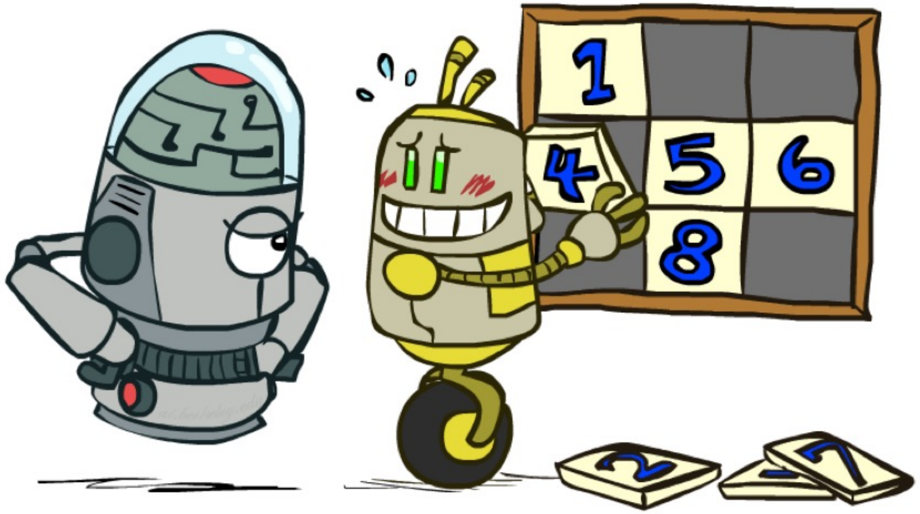
Start State



Actions



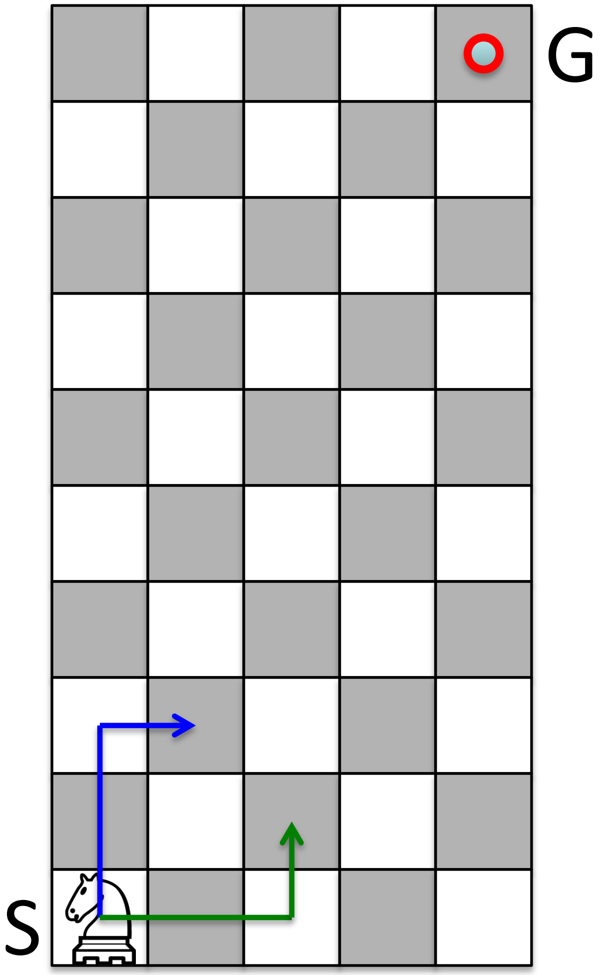
Goal State



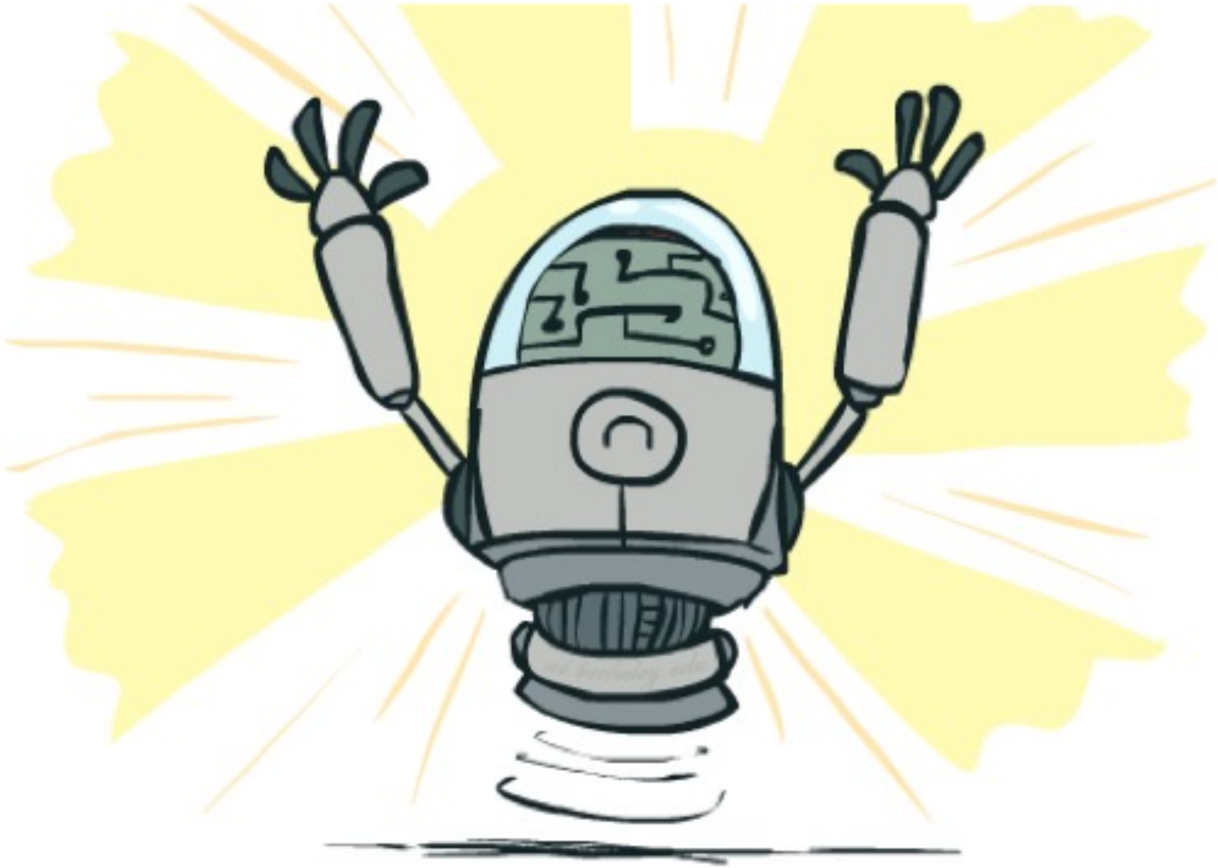
Designing a Heuristic: Knight’s moves

- Minimum number of knight’s moves to get from S to G?
  - h_1 = (Manhattan distance)/3 because each step of Knight can cover manhattan distance of 3
    - h_1' = h_1 rounded up to correct parity (even if S, G same color, odd otherwise)
  - h_2 = (Euclidean distance)/$\sqrt{5}$
    - h_2' = h_2 rounded up to correct parity
  - h_3 = (maximum horizontal or vertical distance)/2
    - h_3' = h_3 rounded up to correct parity
- $h(n) = \max(h_1'(n), h_2'(n), h_3'(n))$ is admissible!

because we definitely underestimate the steps using current heuristic methods
(even we use the max number of the three)
remember the concept of h(n) < h*(n)

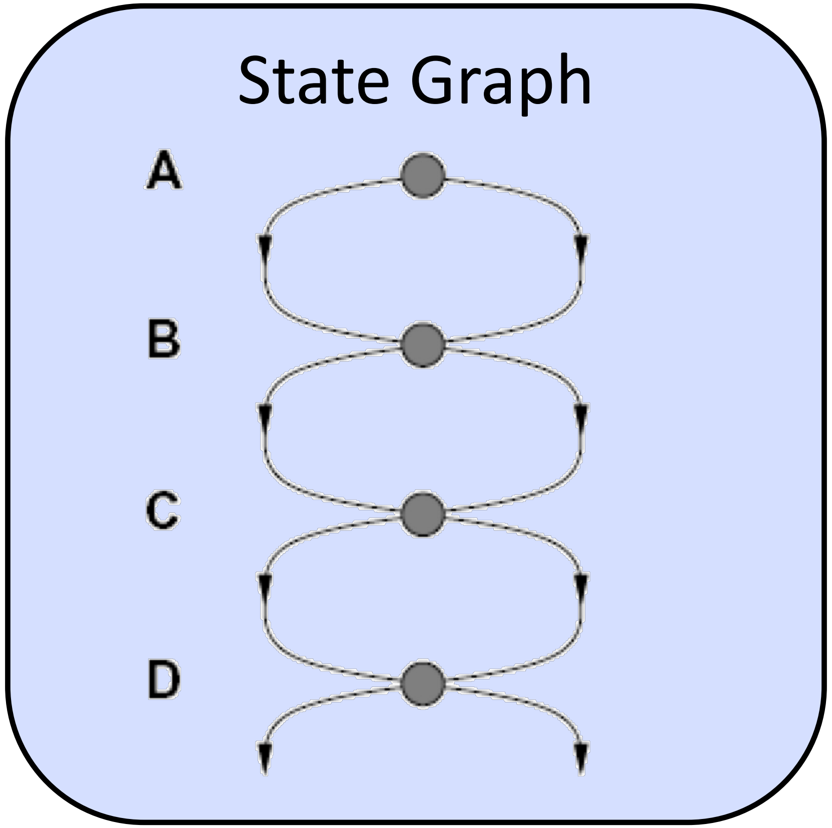


Recap: Optimality of A* Tree Search

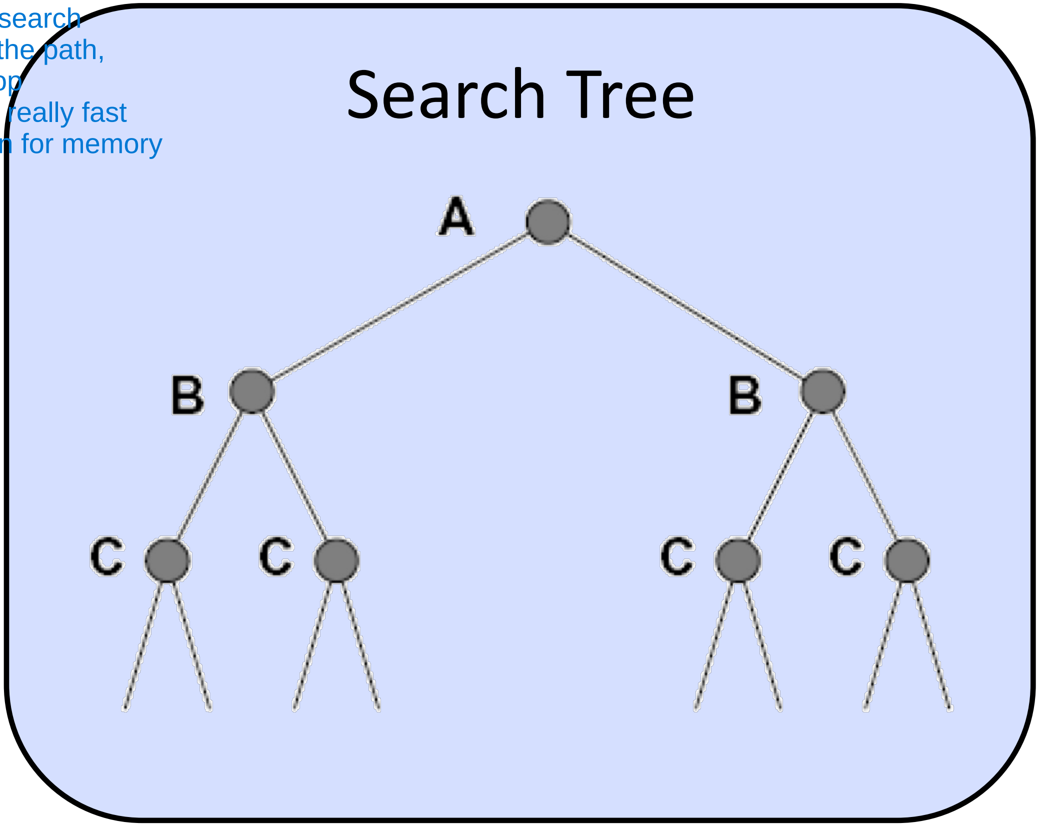


Tree Search: Extra Work!

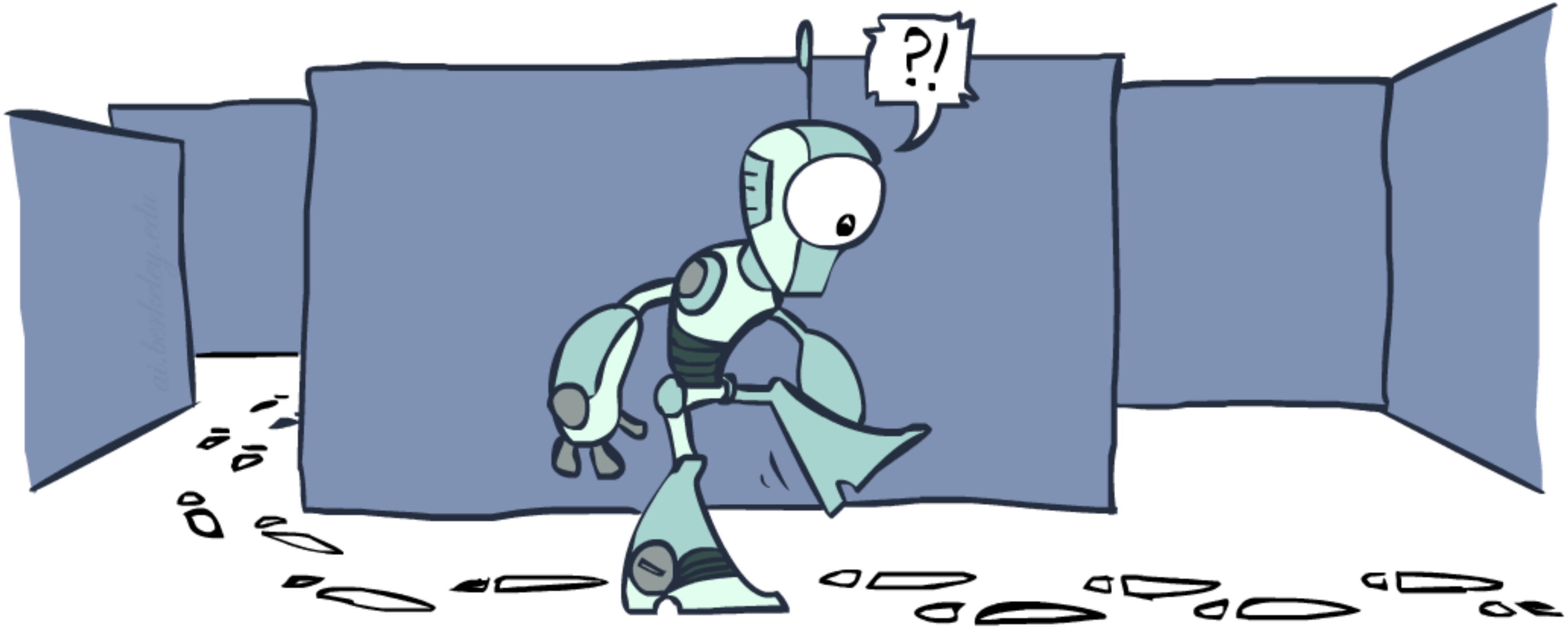
- Failure to detect repeated states can cause exponentially more work.



The setbacks of tree search
if we have a circle in the path,
we will fall in dead loop
and the tree expands really fast
which is heavy burden for memory

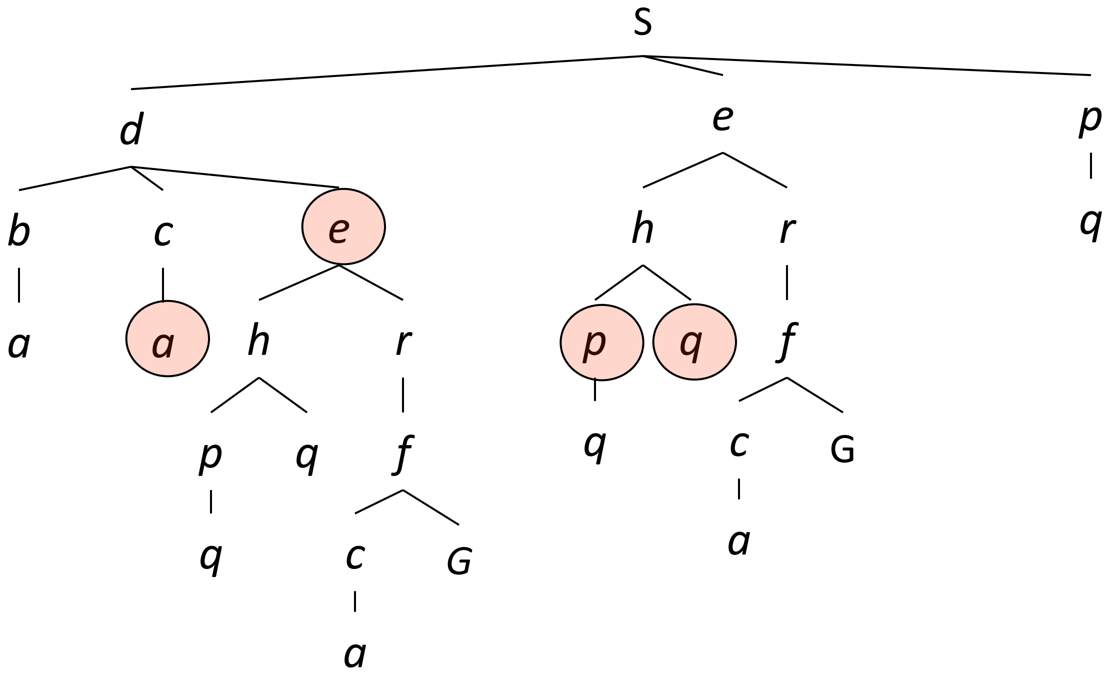


Graph Search



Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



Graph Search

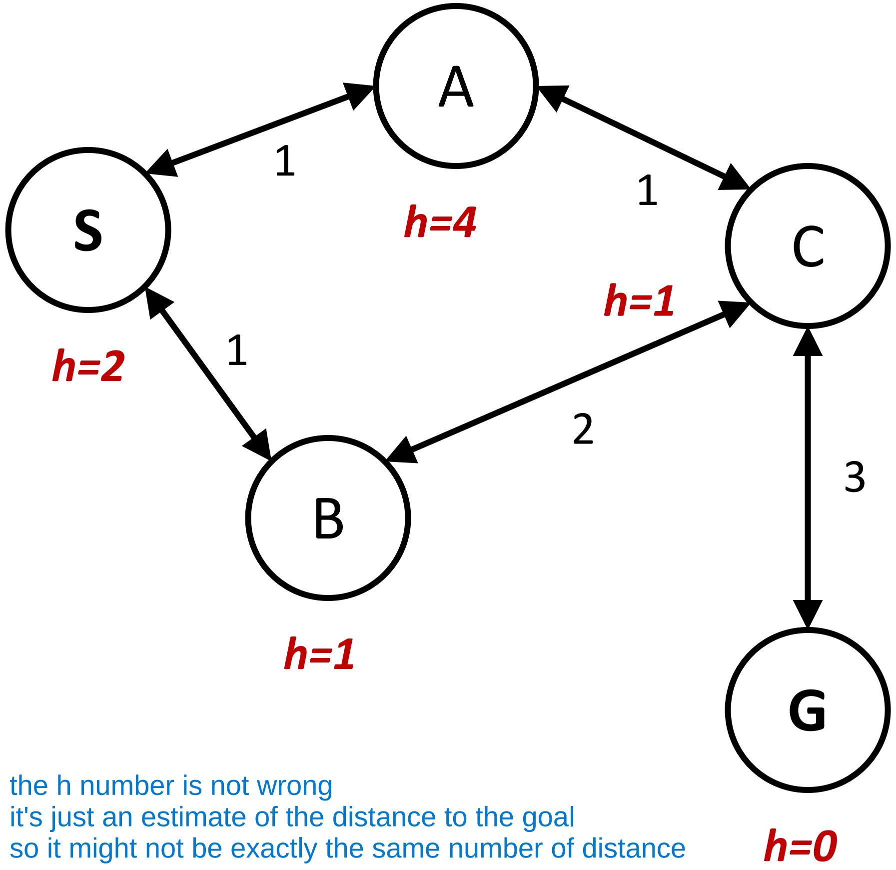
- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list

in search, we can meet a node times and times, so we must make sure this node is not duplicated in the storage structure

- Can graph search wreck completeness? Why/why not?
- How about optimality?

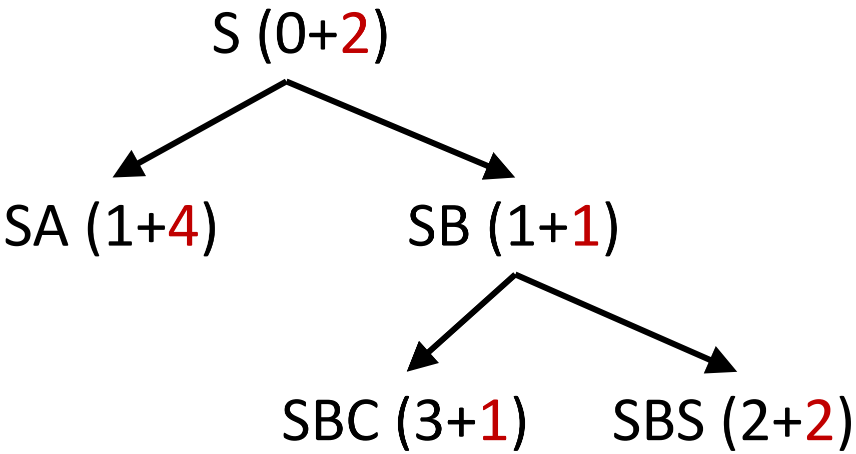
A* Graph Search Gone Wrong?

State space graph



the h number is not wrong
it's just an estimate of the distance to the goal
so it might not be exactly the same number of distance

Search tree



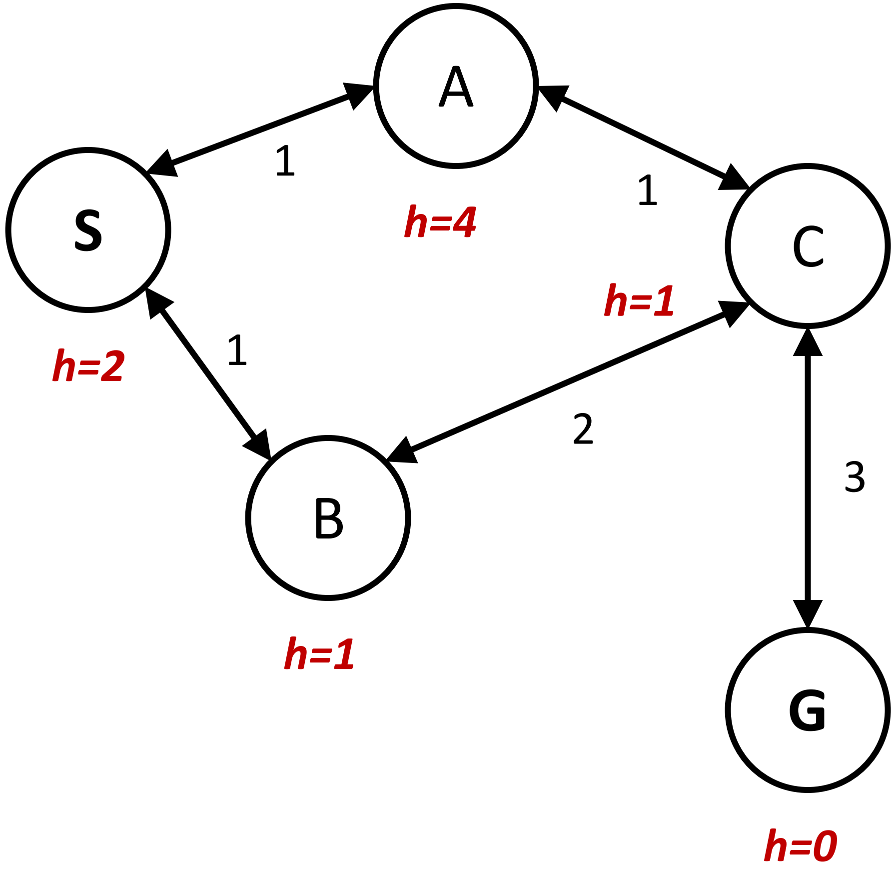
in graph search, we have to add the visited node to the closed set

Closed set

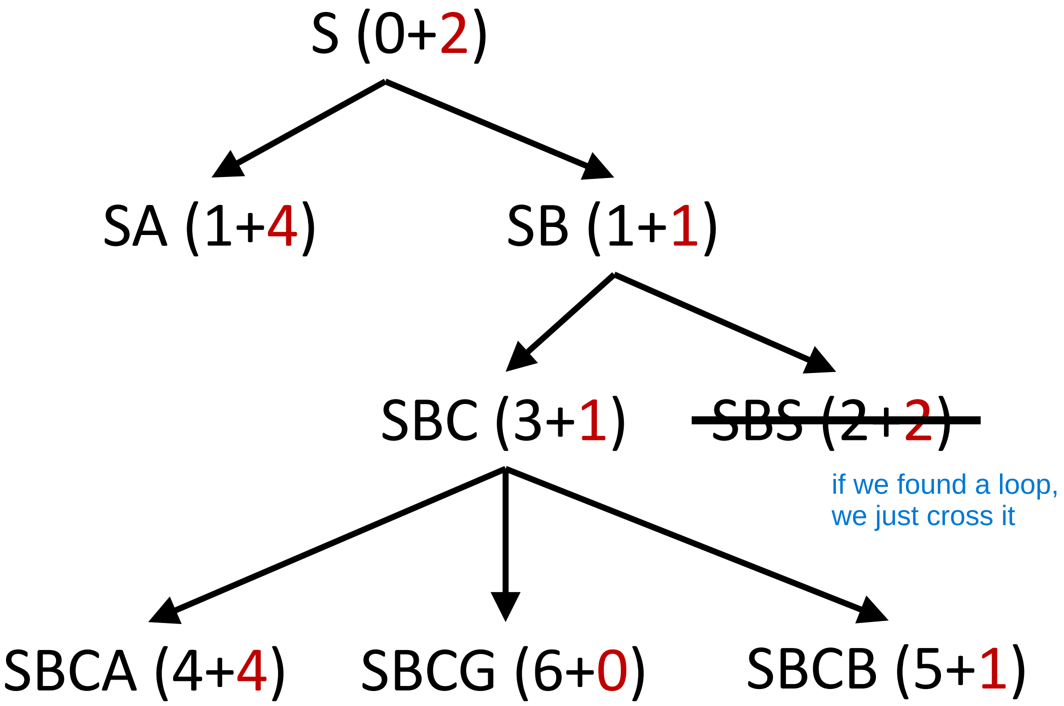
{ S B }

A* Graph Search Gone Wrong?

State space graph



Search tree

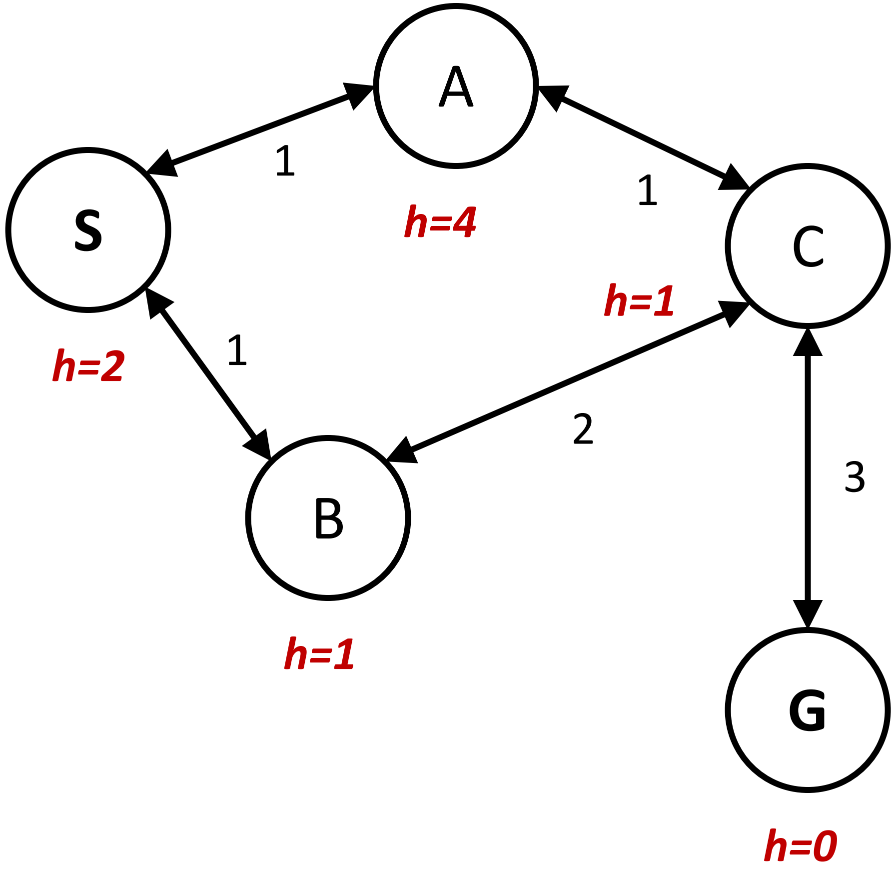


Closed set

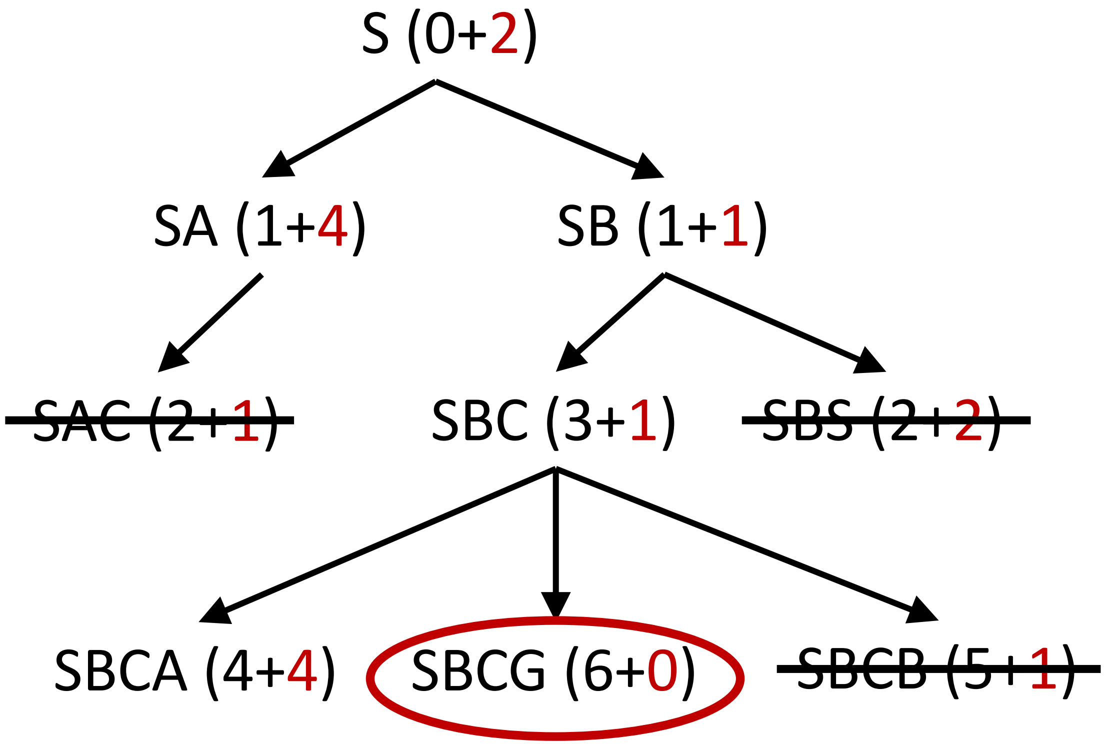
{ S B }

A* Graph Search Gone Wrong?

State space graph



Search tree

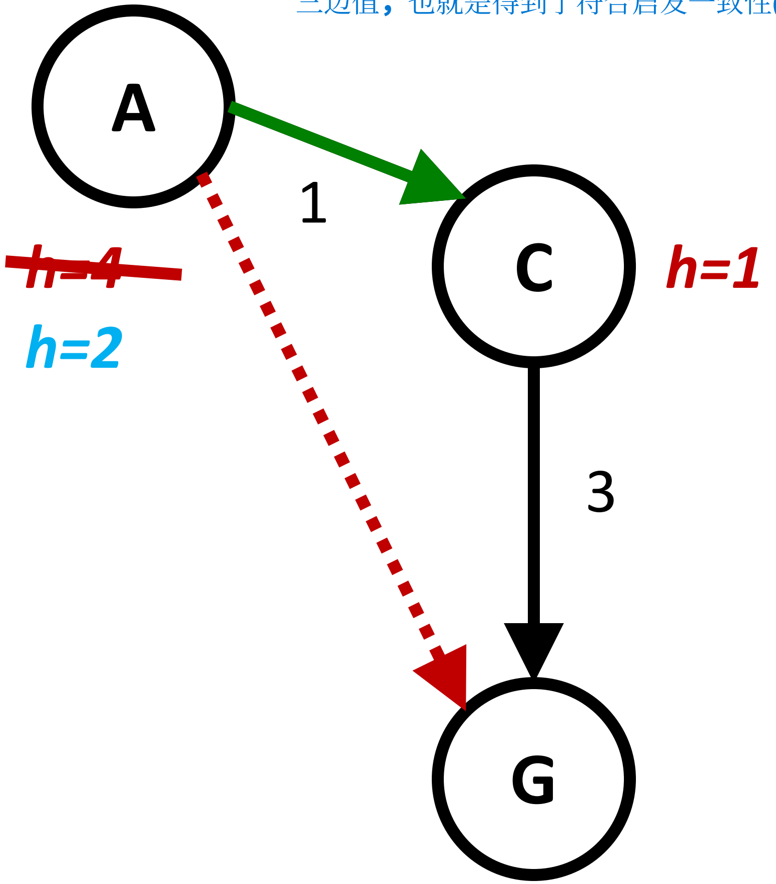


Closed set

{ S B C A }

wrong ans!!! why? sometimes we overestimate the h value

Consistency of Heuristics



用三角形法则进行优化，得到满足三角形关系的三边值，也就是得到了符合启发一致性(lower than actual cost)的h的值

- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost *h** from A to G

  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

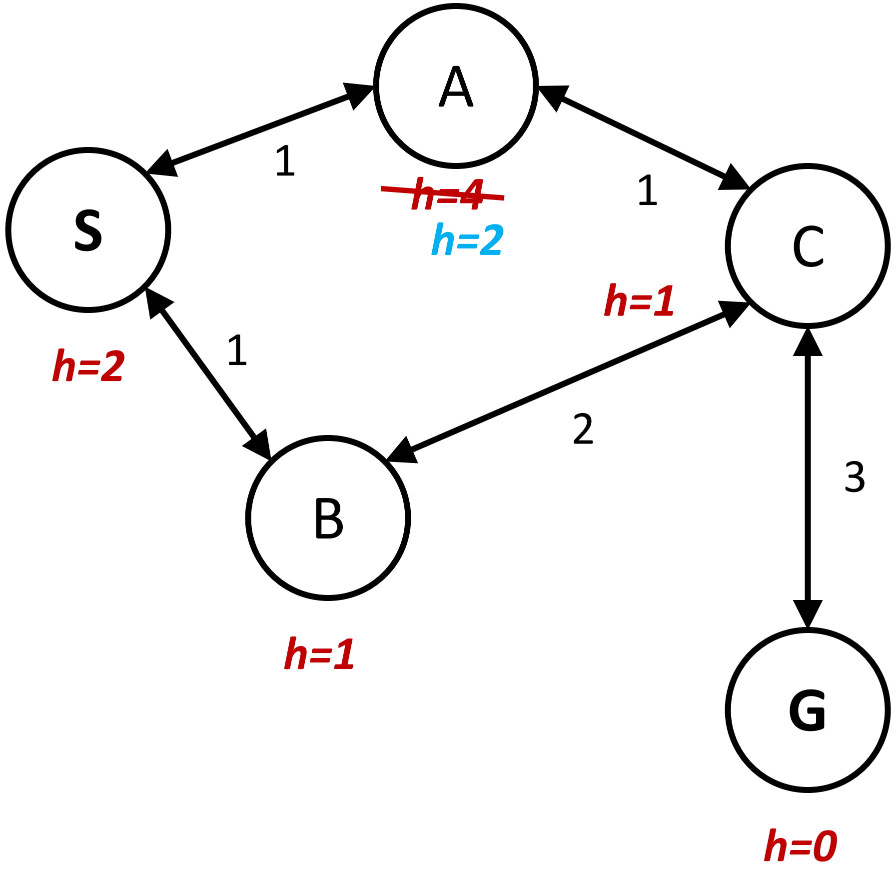
    - a.k.a. "triangle inequality": h(A) ≤ cost(A to C) + h(C)
    - Note: true cost *h** necessarily satisfies triangle inequality
- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)

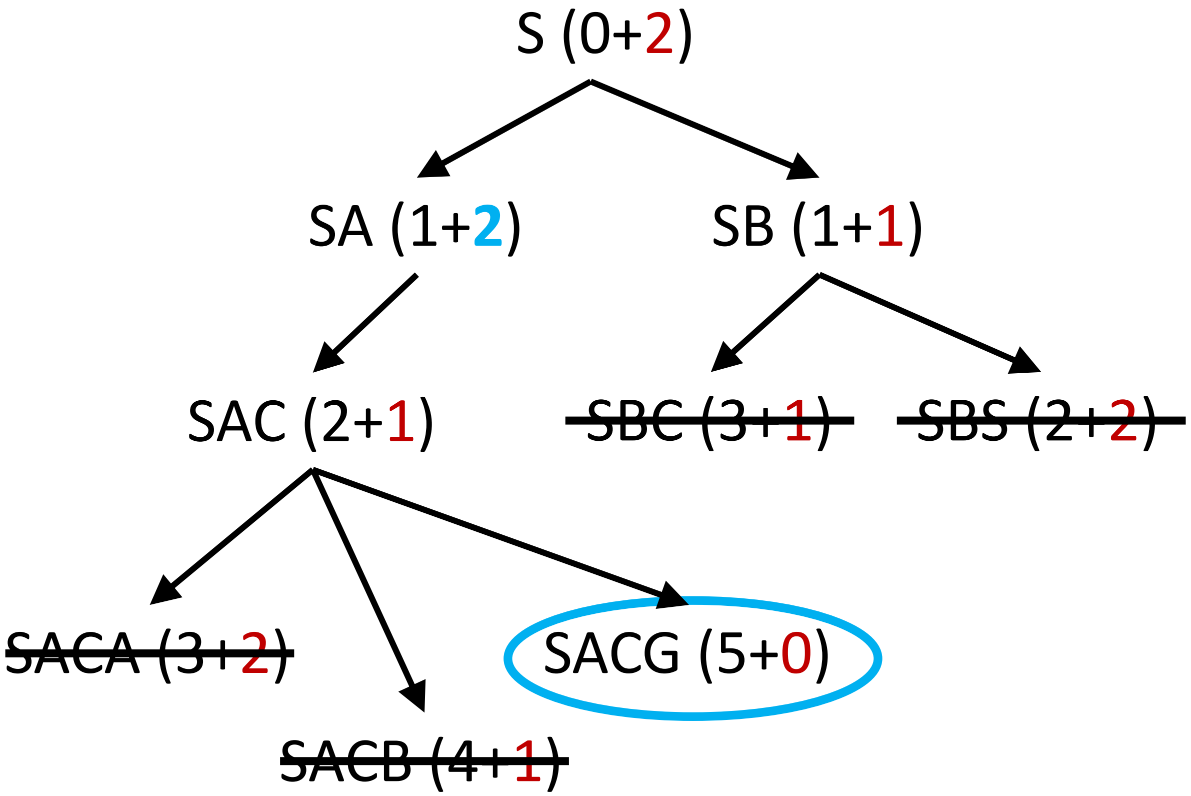
  - A* graph search is optimal

A* Graph Search with Consistent Heuristic

State space graph



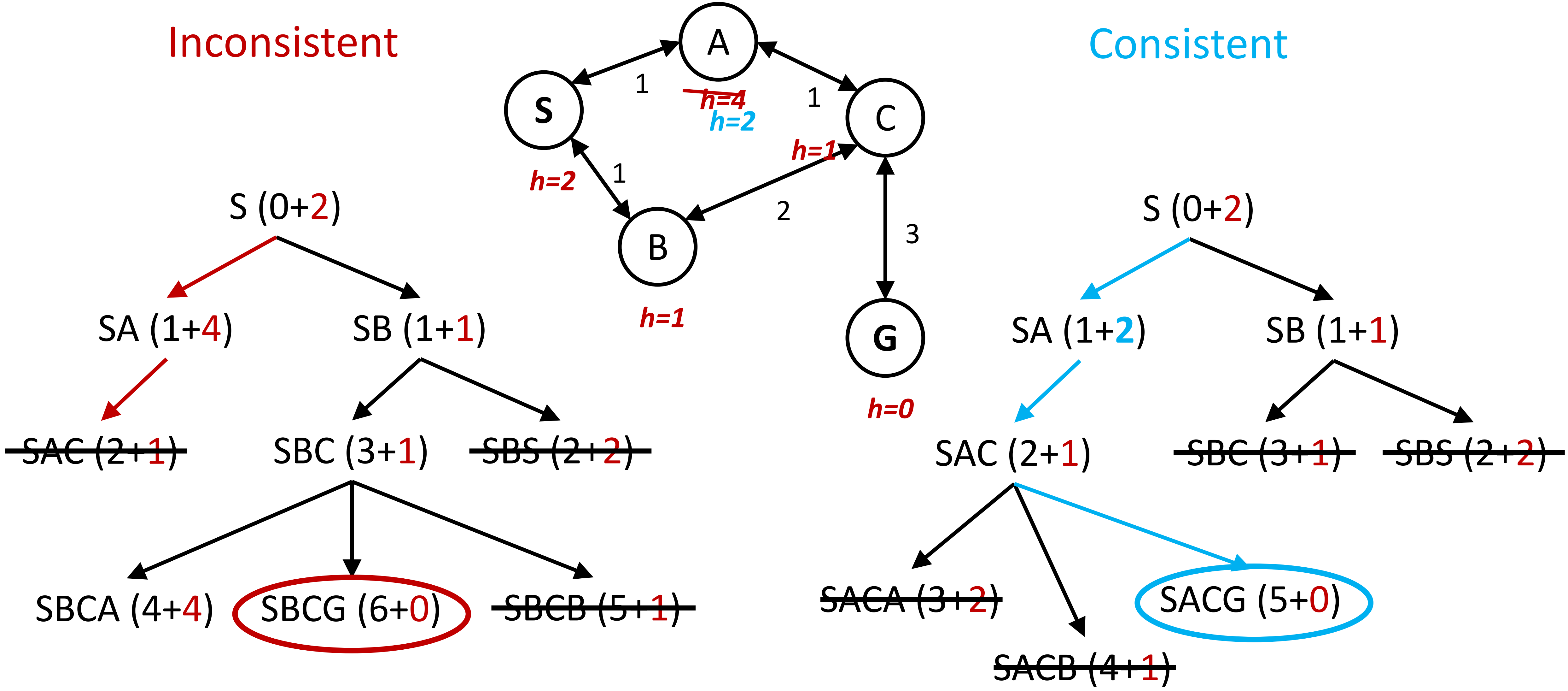
Search tree



Closed set

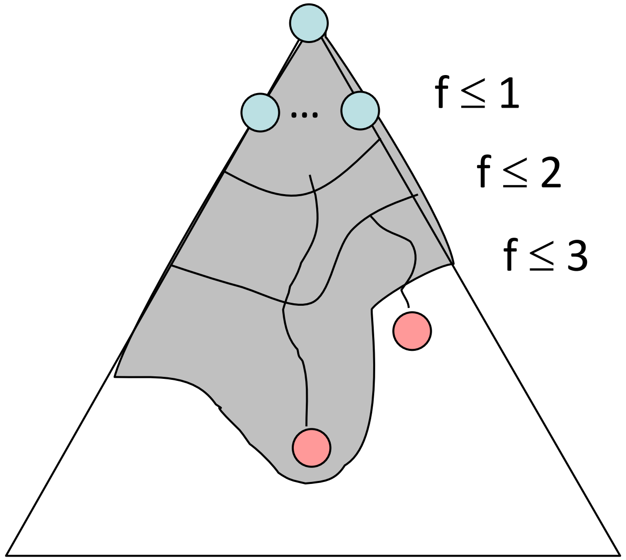
{ S B A C }

Consistency => non-decreasing f-score



Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal



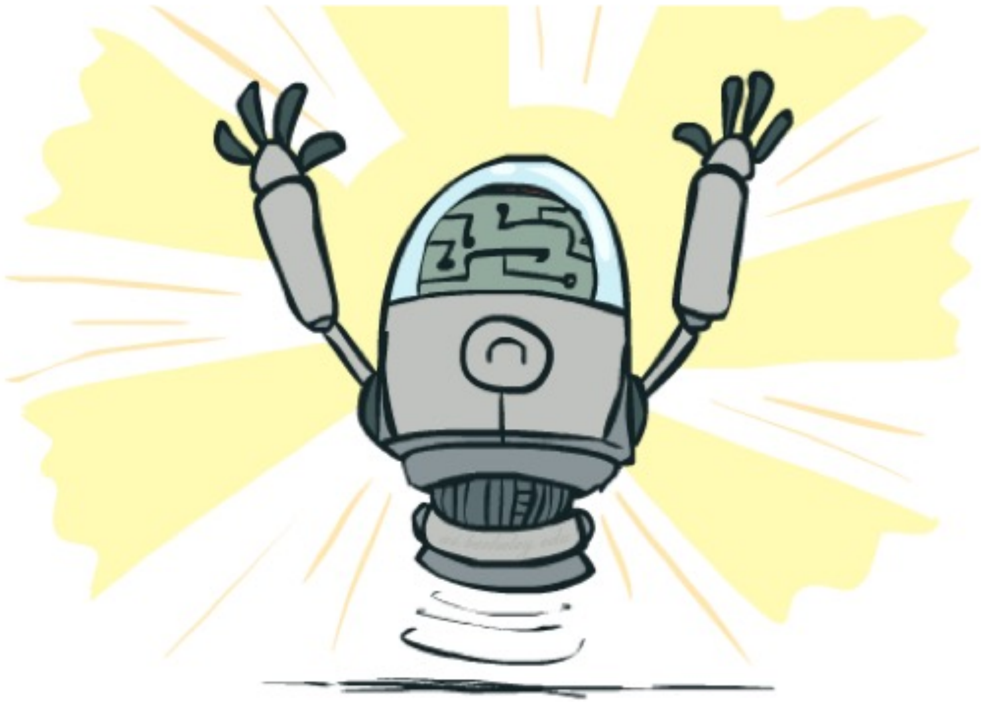
Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)

- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)

- Consistency implies admissibility

- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

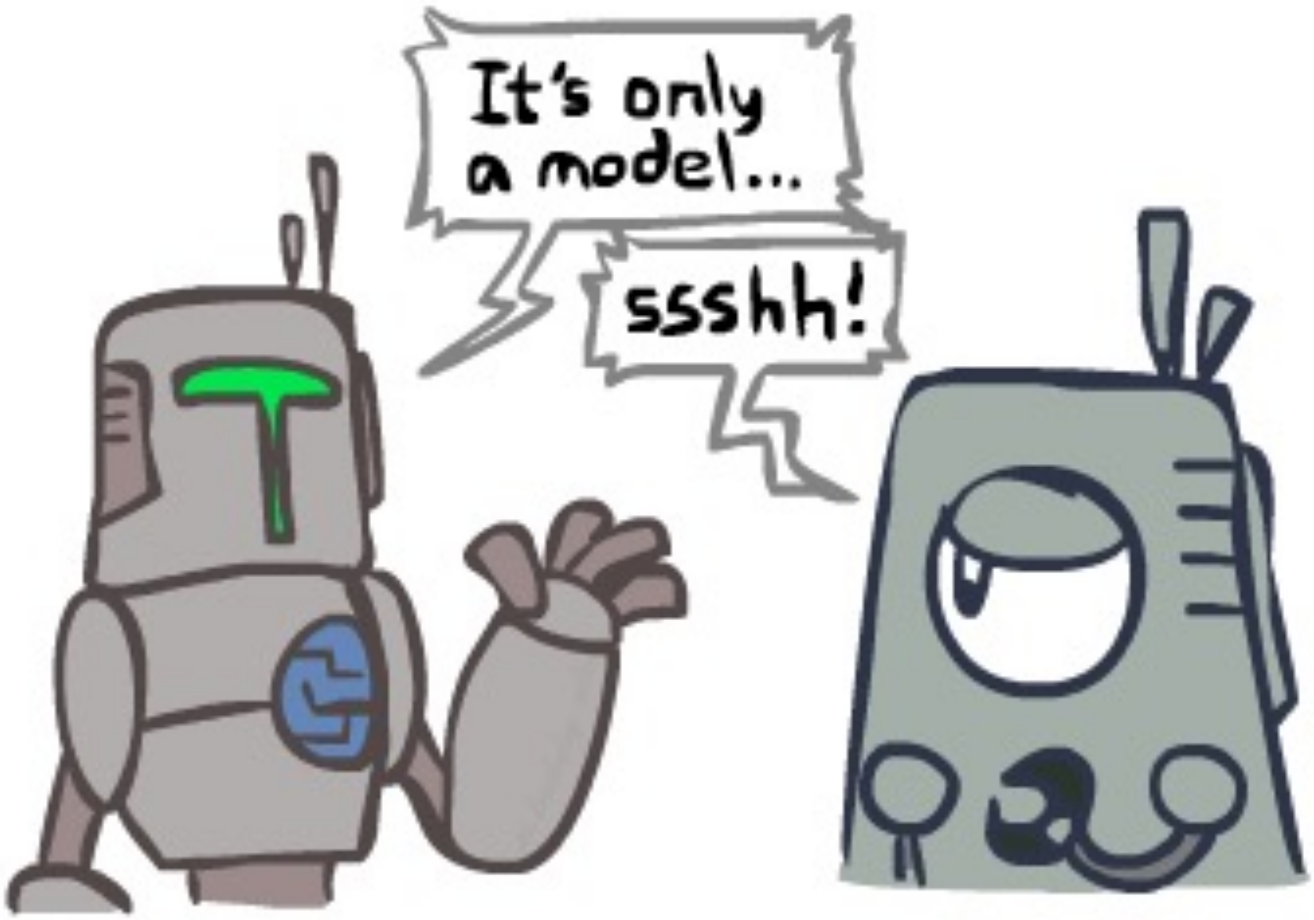


But…

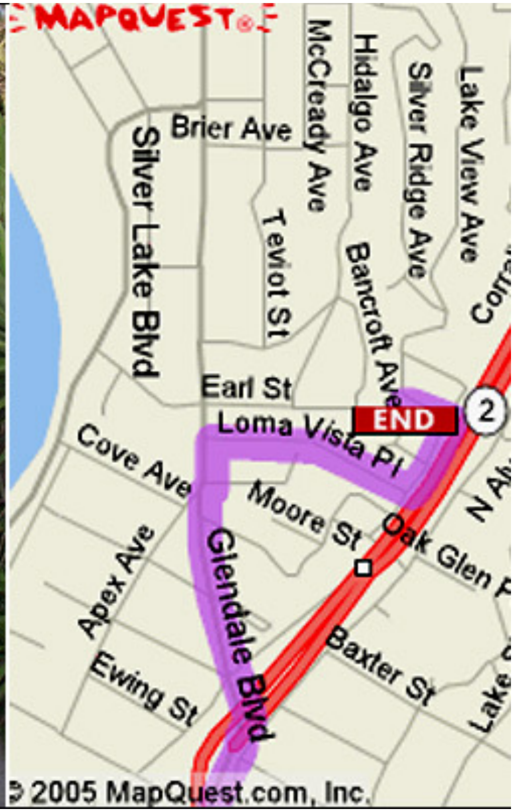
- A* keeps the entire explored region in memory
- => will run out of space before you get bored waiting for the answer ☹️
- There are variants that use less memory (Section 3.5.5):
  - IDA* works like iterative deepening, except it uses an f-limit instead of a depth limit
    - On each iteration, remember the smallest f-value that exceeds the current limit, use as new limit
    - Very inefficient when f is real-valued and each node has a unique value
  - RBFS is a recursive depth-first search that uses an f-limit = the f-value of the best alternative path available from any ancestor of the current node
    - When the limit is exceeded, the recursion unwinds but remembers the best reachable f-value on that branch
  - SMA* uses *all available memory* for the queue, minimizing thrashing
    - When full, drop worst node on the queue but remember its value in the parent

Search and Models

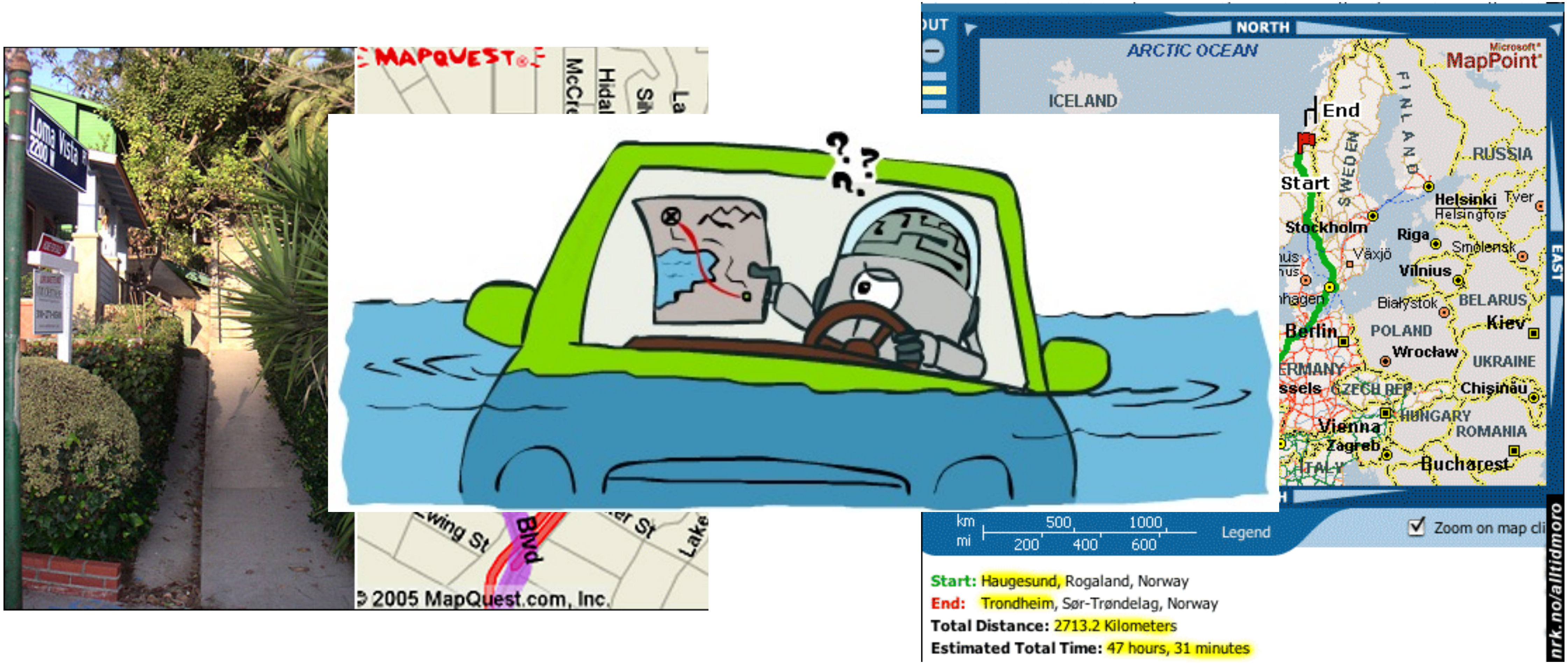
- Search operates over models of the world
  - The agent doesn't actually try all the plans out in the real world!
  - Planning is all "in simulation"
  - Your search is only as good as your models…



Search Gone Wrong?



Search Gone Wrong?



Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```

Local Search

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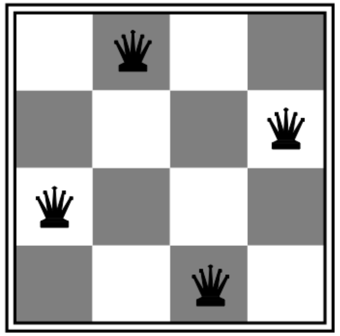
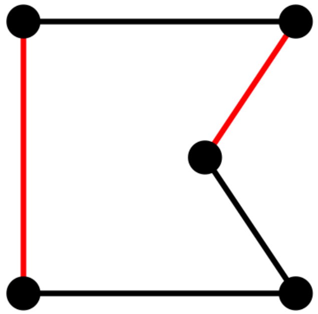
[Updated slides from: Stuart Russell and Dawn Song]

Local search algorithms

- In many optimization problems, ***path*** is irrelevant; the goal state ***is*** the solution
- Then state space = set of "complete" configurations;
  find ***configuration satisfying constraints***, e.g., n-queens problem; or, find ***optimal configuration***, e.g., travelling salesperson problem





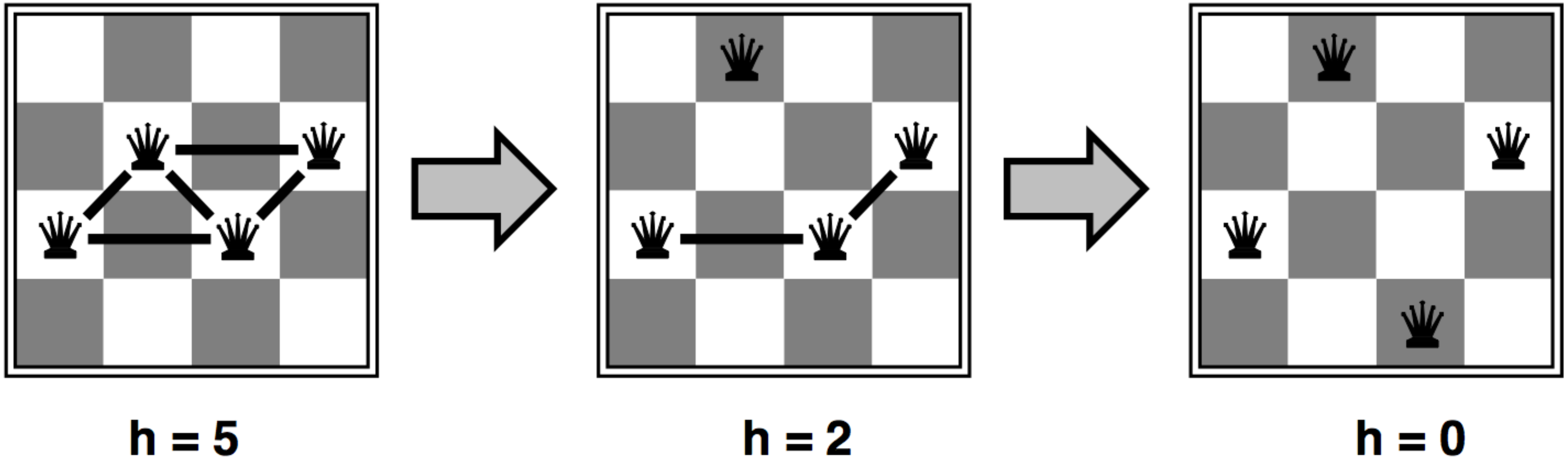
- In such cases, can use ***iterative improvement*** algorithms: keep a single "current" state, try to improve it
- Constant space, suitable for online as well as offline search
- More or less unavoidable if the "state" is yourself (i.e., learning)

Hill Climbing

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best neighboring state
  - If no neighbors better than current, quit

Heuristic for *n*-queens problem

- Goal: n queens on board with no ***conflicts***, i.e., no queen attacking another
- States: n queens on board, one per column
- Actions: move a queen in its column
- Heuristic value function: number of conflicts

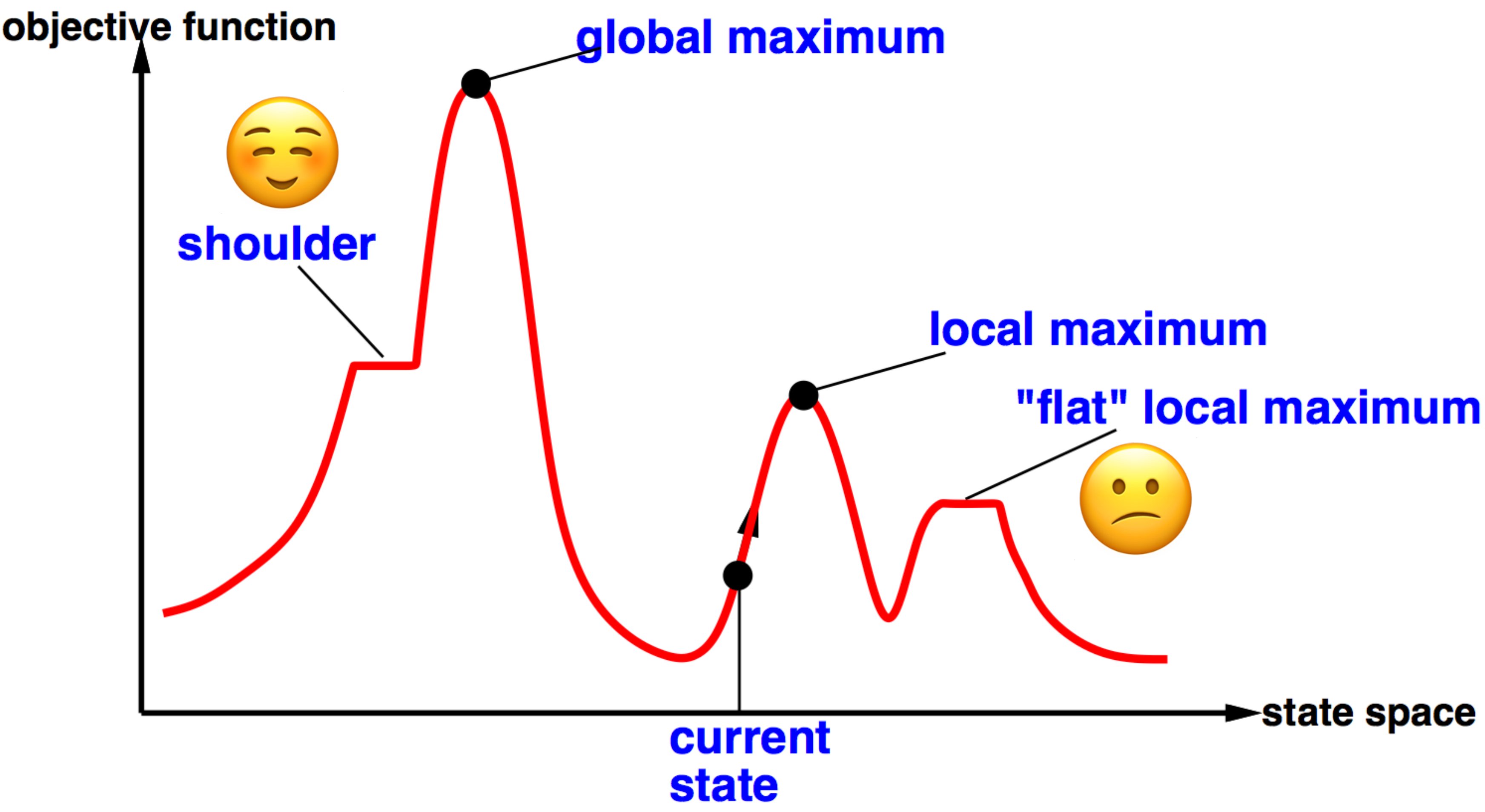


Hill-climbing algorithm

```
function HILL-CLIMBING(problem) returns a state
    current ← make-node(problem.initial-state)
    loop do
        neighbor ← a highest-valued successor of current
        if neighbor.value ≤ current.value then
            return current.state
        current ← neighbor
```

"Like climbing Everest in thick fog with amnesia"

Global and local maxima



Random restarts

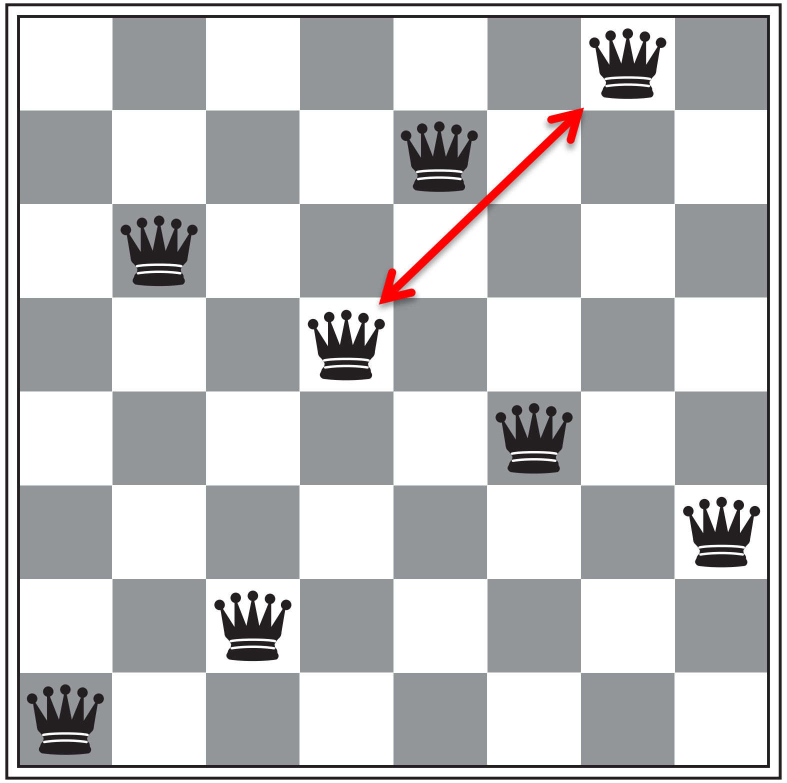
- find global optimum
- duh

Random sideways moves

- Escape from shoulders
- Loop forever on flat local maxima

Hill-climbing on the 8-queens problem

- No sideways moves:
  - Succeeds w/ prob. 0.14
  - Average number of moves per trial:
    - 4 when succeeding, 3 when getting stuck
  - Expected total number of moves needed:
    - 3(1-p)/p + 4 =~ 22 moves
- Allowing 100 sideways moves:
  - Succeeds w/ prob. 0.94
  - Average number of moves per trial:
    - 21 when succeeding, 65 when getting stuck
  - Expected total number of moves needed:
    - 65(1-p)/p + 21 =~ 25 moves



Moral: algorithms with knobs to twiddle are irritating

Simulated annealing

- Resembles the annealing process used to cool metals slowly to reach an ordered (low-energy) state
- Basic idea:
  - Allow “bad” moves occasionally, depending on “temperature”
  - High temperature => more bad moves allowed, shake the system out of its local minimum
  - Gradually reduce temperature according to some schedule
  - Sounds pretty flaky, doesn’t it?

Simulated annealing algorithm

function SIMULATED-ANNEALING(problem,schedule) **returns** a state

current ← problem.initial-state

for t = 1 **to** ∞ **do**

  T ←schedule(t)

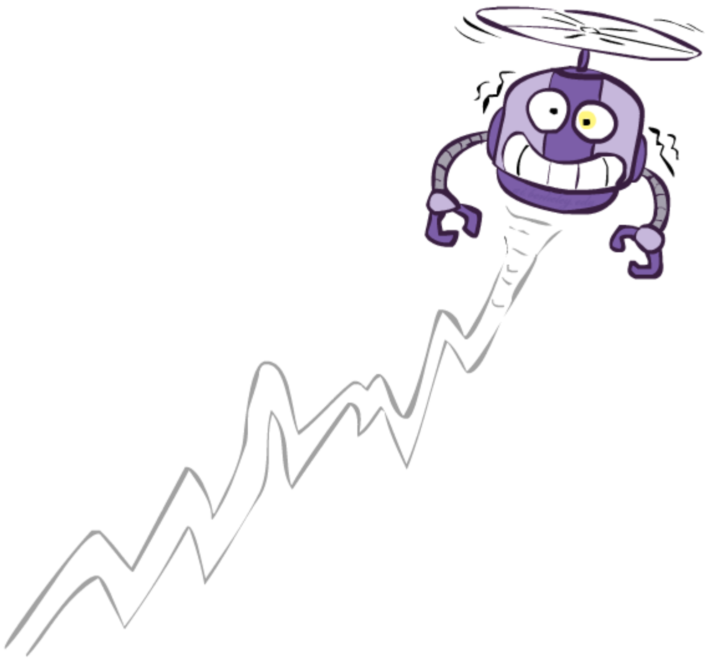
  if T = 0 **then return** current

  next ← a randomly selected successor of current

  ΔE ← next.value – current.value

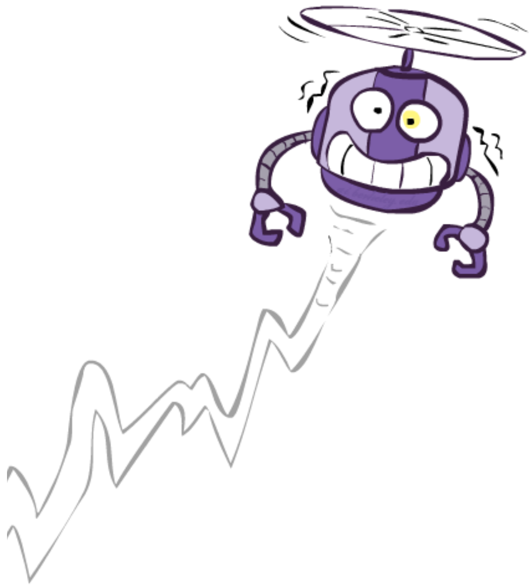
  if ΔE > 0 **then** current ← next

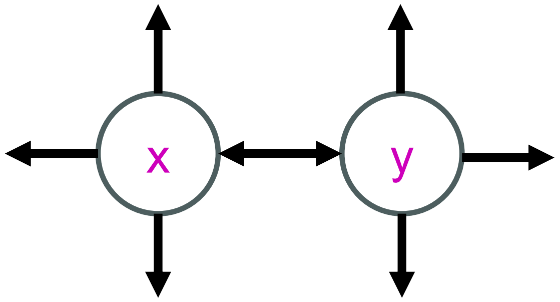
    else current ← next only with probability $e^{\Delta E/T}$



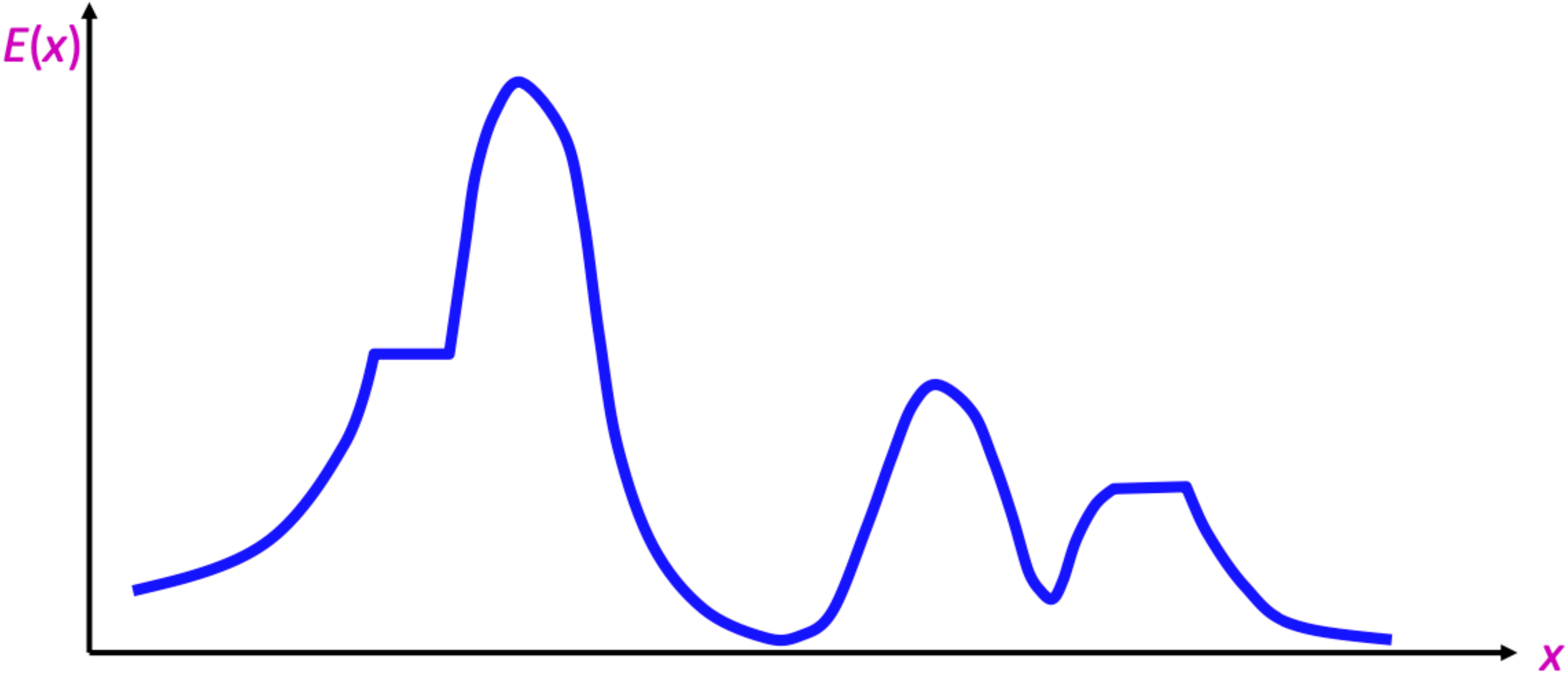
Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution (Boltzmann): $P(x) \propto e^{E(x)/T}$
  - If T decreased slowly enough, will converge to optimal state!
- Proof sketch
  - Consider two adjacent states x, y with $E(y) > E(x)$ [high is good]
  - Assume $x \rightarrow y$ and $y \rightarrow x$ and outdegrees $D(x) = D(y) = D$
  - Let $P(x)$, $P(y)$ be the equilibrium occupancy probabilities at T
  - Let $P(x \rightarrow y)$ be the probability that state x transitions to state y



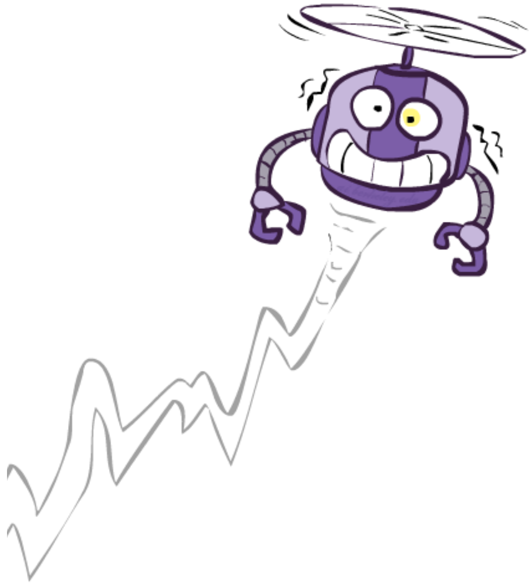


Occupation probability as a function of T



Simulated Annealing

- Is this convergence an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row
  - “Slowly enough” may mean exponentially slowly
  - Random restart hillclimbing also converges to optimal state…
- Simulated annealing and its relatives are a key workhorse in VLSI layout and other optimal configuration problems

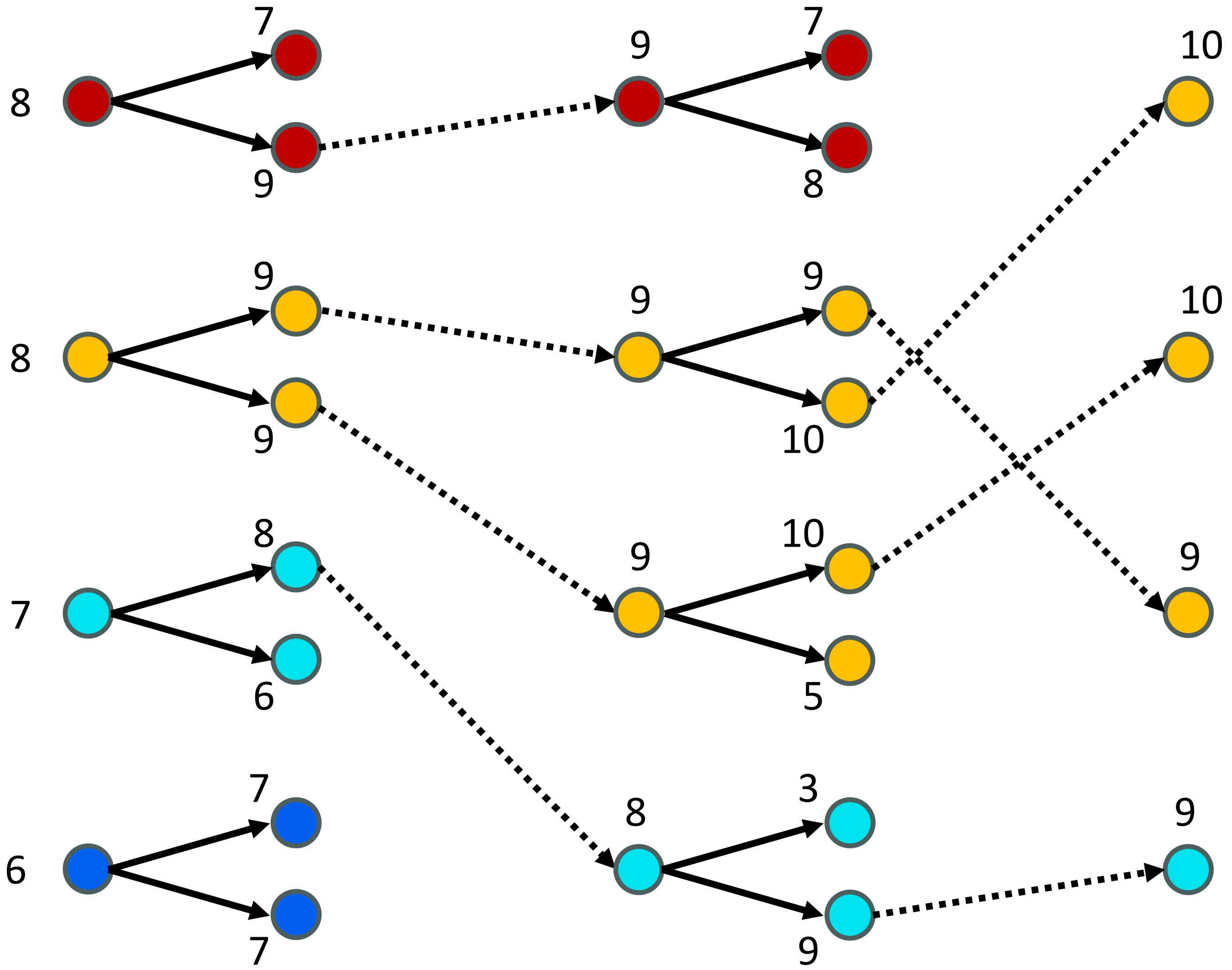


Local beam search

- Basic idea:
  - *K* copies of a local search algorithm, initialized randomly
  - For each iteration
    - Generate ALL successors from *K* current states
    - Choose best *K* of these to be the new current states

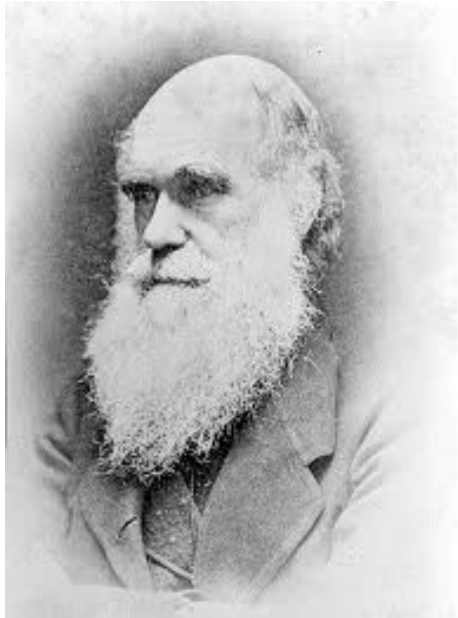
Or, K chosen randomly with
a bias towards good ones

Beam search example (*K*=4)

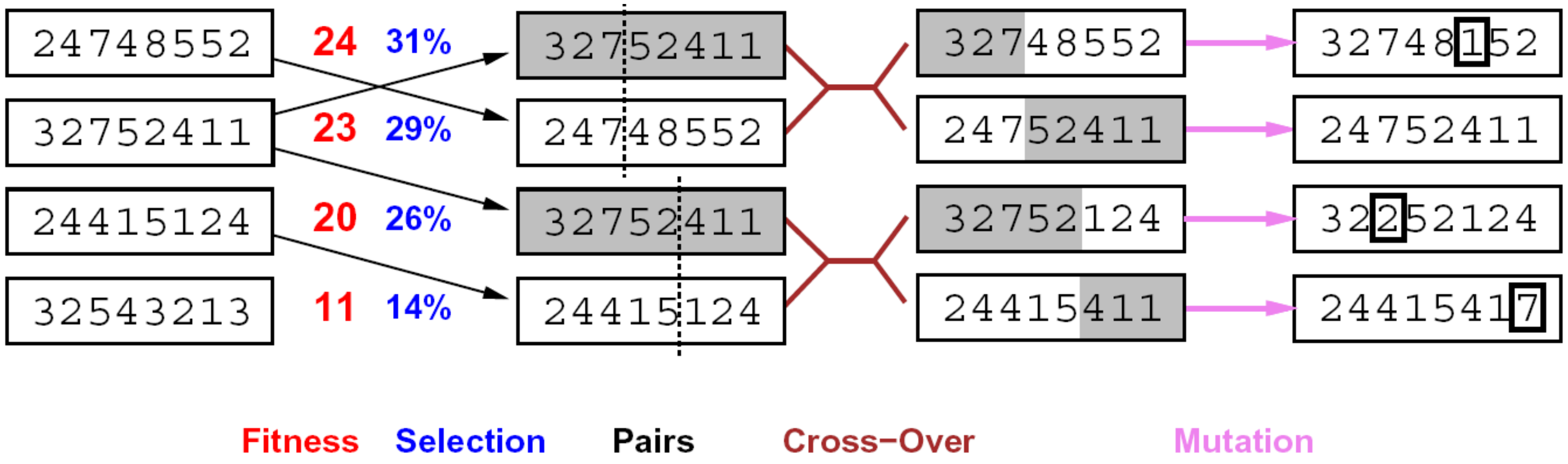


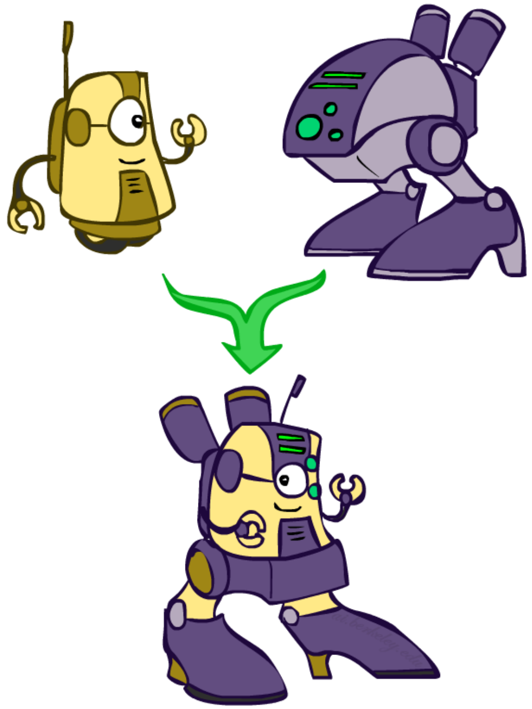
Local beam search

- Why is this different from *K* local searches in parallel?
  - The searches ***communicate***! “Come over here, the grass is greener!”
- What other well-known algorithm does this remind you of?
  - Evolution!



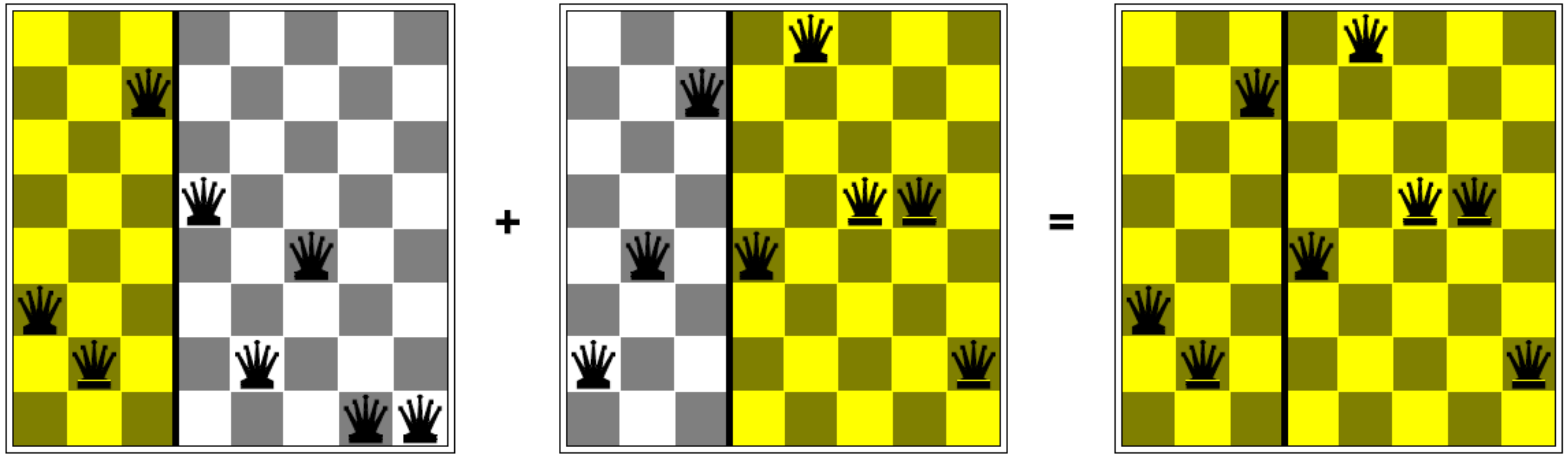
Genetic algorithms





- Genetic algorithms use a natural selection metaphor
  - Resample *K* individuals at each step (selection) weighted by fitness function
  - Combine by pairwise crossover operators, plus mutation to give variety

Example: N-Queens

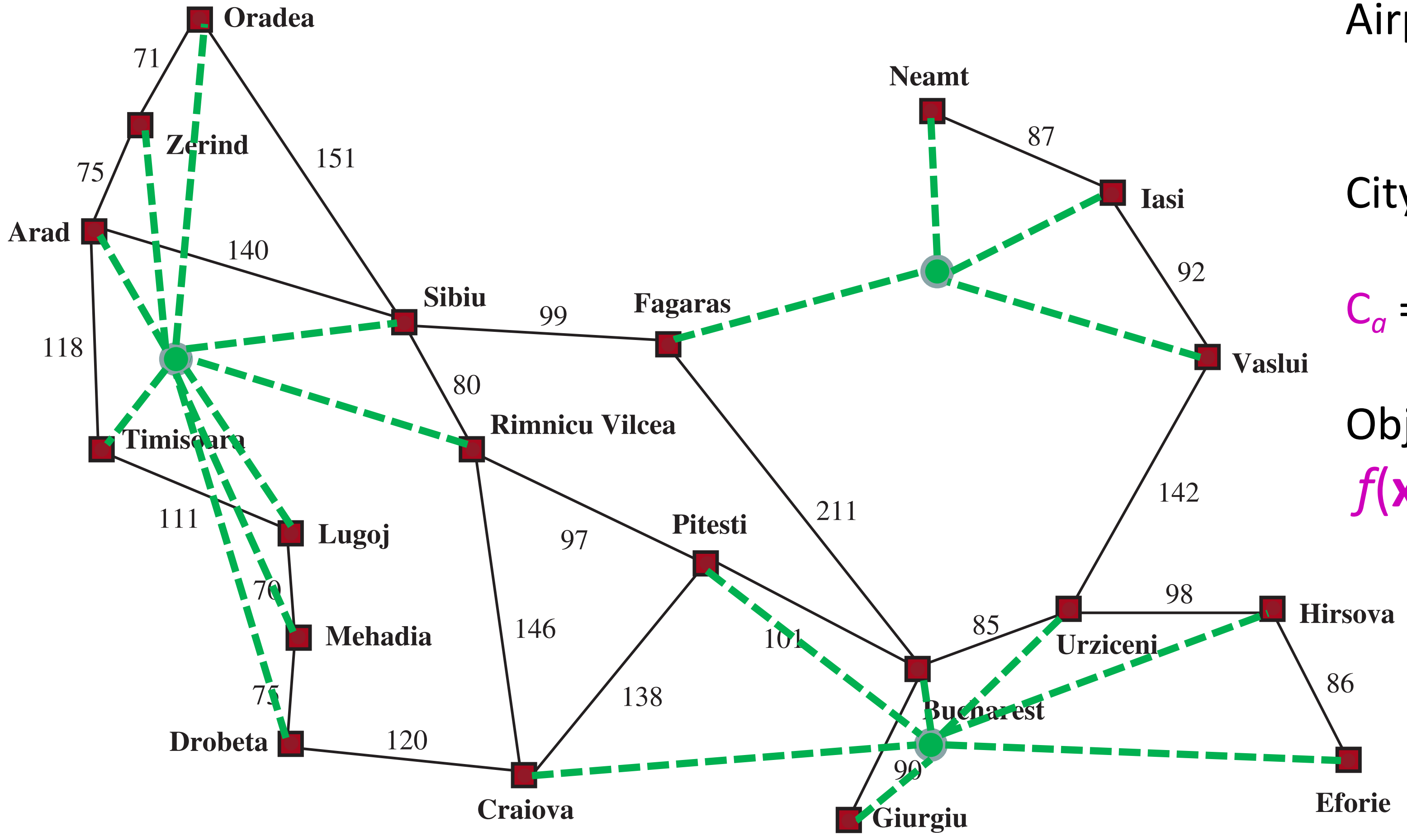


- Does crossover make sense here?
- What would mutation be?
- What would a good fitness function be?

Local search in continuous spaces

Example: Placing airports in Romania

Place 3 airports to minimize the sum of squared distances from each city to its nearest airport



Airport locations

$\mathbf{x} = (x_1,y_1), (x_2,y_2), (x_3,y_3)$

City locations (x_c,y_c)

C_a = cities closest to airport a

Objective: minimize

$$f(\mathbf{x}) = \Sigma_a \, \Sigma_{c \in C_a} \, (x_a - x_c)^2 + (y_a - y_c)^2$$

Handling a continuous state/action space

1. Discretize it!
   - Define a grid with increment δ, use any of the discrete algorithms
2. Choose random perturbations to the state
   a. First-choice hill-climbing: keep trying until something improves the state
   b. Simulated annealing
3. Compute gradient of $f(\mathbf{x})$ analytically

Finding extrema in continuous space

- Gradient vector $\nabla f(\mathbf{x}) = (\partial f/\partial x_1, \partial f/\partial y_1, \partial f/\partial x_2, \ldots)^T$
- For the airports, $f(\mathbf{x}) = \Sigma_a \Sigma_{c \in C_a} (x_a - x_c)^2 + (y_a - y_c)^2$
- $\partial f/\partial x_1 = \Sigma_{c \in C_1} 2(x_1 - x_c)$
- At an extremum, $\nabla f(\mathbf{x}) = 0$
- Can sometimes solve in closed form: $x_1 = (\Sigma_{c \in C_1} x_c)/|C_1|$
  - Is this a local or global minimum of f?
- If we can't solve $\nabla f(\mathbf{x}) = 0$ in closed form…
  - Gradient descent: $\mathbf{x} \leftarrow \mathbf{x} - \alpha \nabla f(\mathbf{x})$
- Huge range of algorithms for finding extrema using gradients

Summary

- Many configuration and optimization problems can be formulated as local search
- General families of algorithms:
  - Hill-climbing, continuous optimization
  - Simulated annealing (and other stochastic methods)
  - Local beam search: multiple interaction searches
  - Genetic algorithms: break and recombine states

Many machine learning algorithms are local searches